THE EFFECT OF A STEEL CORE UPON THE ELECTRICAL CHARACTERISTICS OF LARGE STRANDED ALUMINUM CONDUCTORS.

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T H E S I S

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INFLUENCE OF STEEL CORE UPON THE ELECTRICAL CHARACTERISTICS OF LARGE ALUMINUM STRANDED CONDUCTORS.

The determination of the electrical characteristics of a large aluminum stranded power cable with a steel core has been divided into the following sections in this paper:

1. A general description of the cable in question.

2. Tests for the permeability of the steel core.

3. An outline of the method followed in the tests, with the data sheets.

4. A consideration of the resistances of core and cable for direct and alternating E. M. Fs., including the skin effect.

5. A comparison of test results for inductances with those calculated by formulae.

6. An appendix containing matter relevant to this subject.

The cable under test was a section of the stranded aluminum cable used by the Pacific Light & Power Corporation in transmitting power from their hydro-electric stations on Big Creek to Los Angeles two hundred and forty one miles distant. The system is three phase, fifty cycles, and is designed to transmit 60,000 kilowatts at 150,000 volts.

Fig. 1 is a photograph of an end of the cable showing the three layers of aluminum strands and the steel core in the center.

Conductor	Strands	Cir. Mils	Outside Diam.
Aluminum	54	605,000	.906 inches
Steel	7	78,500	.3125 inches

Weight per mile	Specific Gravity (assumed)	Ratio of Conductor area to total area	Ratio of wts. to total wt.
2940 lbs.	7.85	88.50 %	72.5 %
1118 lbs.	2.68	11.50 %	27.5 %

It is of interest to compare various qualities of this cable with those possessed by solid copper or solid aluminum cables.

The ratio of actual weight to that of a solid copper cable of the same size is 0.367, and to one of solid aluminum 1.22.

If specific conductances be taken for copper

10.37, for steel 74.0, and for aluminum 16.72, the actual conductance compared with that of a solid copper cable is 56.48%; of a solid aluminum cable 91.11%.

The ratio of a mile ohm of this cable to that of a solid copper cable is 0.651, and to one of solid aluminum 1.34.

The wire is wound in three layers outside the core. In the outer layer there are 24 strands with one complete twist each 18 inches, the next layer has 18 strands and completes a twist in the opposite direction every 10 inches, the layer next the core is made up of 12 wires with a still greater twist, the direction being again reversed.

PERMEABILITY OF THE STEEL CORE.

To determine the permeability of the steel core the method outlined in Pender's American Handbook for Electrical Engineers, page 913, was followed, using a Leeds and Northrup Astatic Galvanometer.

For the standard solenoid mentioned in this method a portion of a permeameter, the dimensions of which were accurately known, was used.

The length of the sample in comparison to its cross section made end corrections unnecessary.

The set up is shown in Fig. 2 and the results are plotted in Fig. 3. Calculations are all given below.

$$B = \frac{.4\pi n_1 N_1 A_1 I_1}{n A} - \frac{a - A}{A} H$$

N_1 = turns in primary of solenoid per cm. length (axial)

n_1 = total turns in secondary of solenoid.

n = " " " " " test sample.

A_1 = mean cross sectional area in cm. sq. of primary of standard if secondary be on outside, or mean cross section of secondary if secondary be on inside.

A = Cross section in cm. sq. of test sample.

a = mean cross section of magnetizing coil of test sample.

I_1 = current thru primary of test sample.

D = deflection when current reversed.

B = fluxdensity in test sample corresponding to deflection

D when current is magnetized coil on sample is reversed.

H = magnetizing force in sample corresponding to flux-density B.

Av. diam of core .312 inches, $n_1 = 200$; $N_1 = 31.5$;

$A_1 = 1.67$ sq. cm.; n = 36; A = .0764;

$$K = \frac{.4\pi n_1 N_1 A_1}{n A} = \frac{.4 \times \pi \times 200 \times 31.5 \times 1.67}{36 \times .0764} = 4820 \text{ const.}$$

$$\frac{a - A}{A} H = \frac{.109 - .0764}{.0764} H = .0326 H = C H$$ for values used this term is negligible

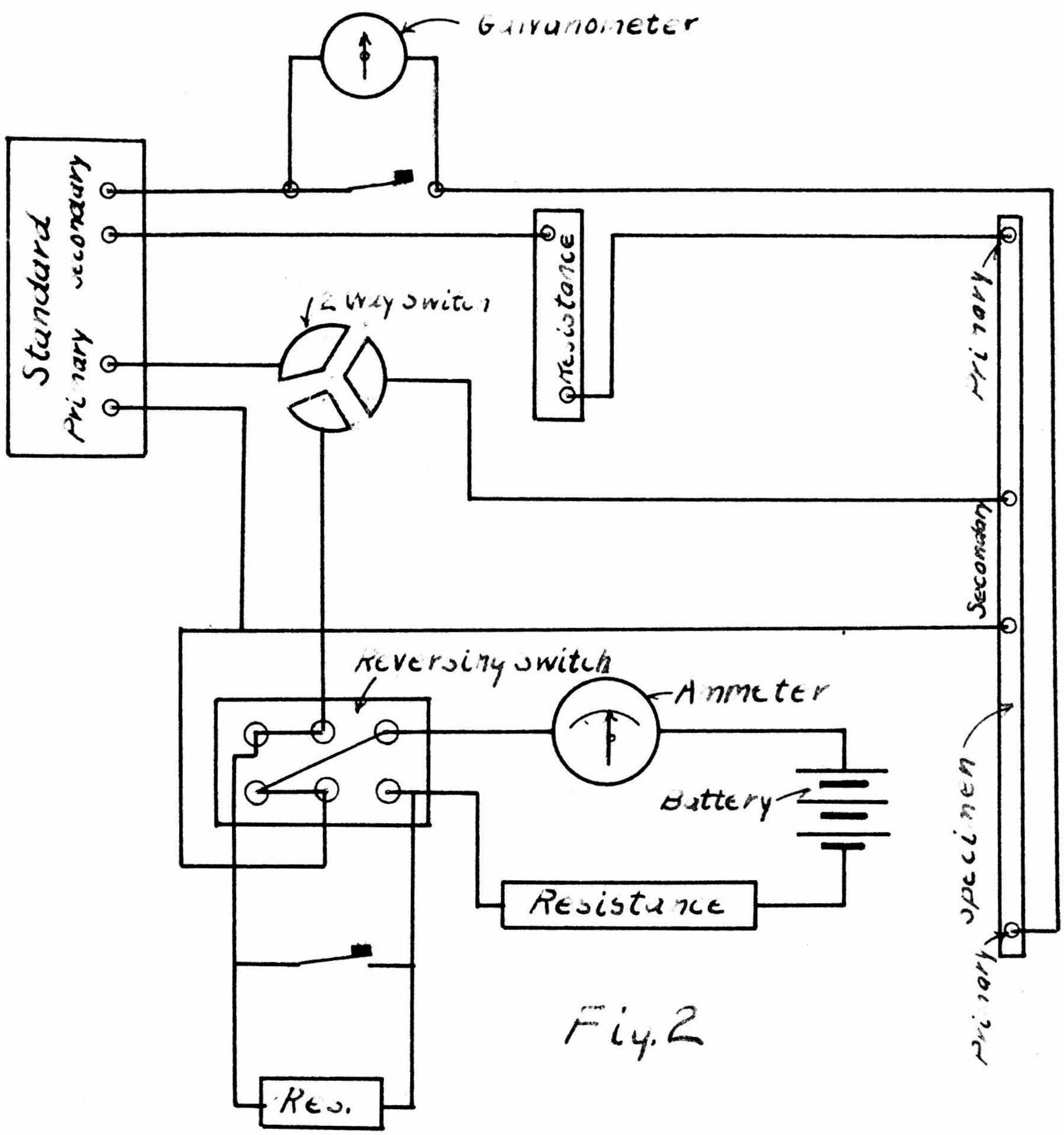
$B = K I_1$	B	D
4820 x .49	2360	.21
4820 x .59	2850	.28
4820 x .79	3810	.35
4820 x 1.00	4820	.46
4820 x 1.29	6220	.58
4820 x 1.62	7810	.70
4820 x 2.10	10100	.89
4820 x 2.28	11000	.99
4820 x 2.59	12500	1.11
4820 x 3.48	16800	1.52

These values were used in plotting flux density curve.

TEST PIECE

$H = .4\pi N I$; N = turns per cm. $N = \frac{360}{71} = 5.07$

Deflection	Current	B	H	$\mu = \frac{B}{H}$
.12	.9	1350	5.74	235
.16	1.05	1800	6.7	269
.21	1.28	2300	8.16	282
.23	1.4	2600	8.94	291
.30	1.63	3350	10.4	322
.42	1.91	4700	12.2	385
.71	2.3	7900	14.7	537
1.19	2.72	13300	17.5	760
1.62	2.97	18500	18.95	976
2.7	3.51	30000	22.4	1340
6.2	10.1			



Galvanometer
Standard
Primary
Secondary
Resistance
2 Way Switch
Primary
Secondary
Specimen
Primary
Reversing Switch
Ammeter
Battery
Resistance
Res.

Fig. 2

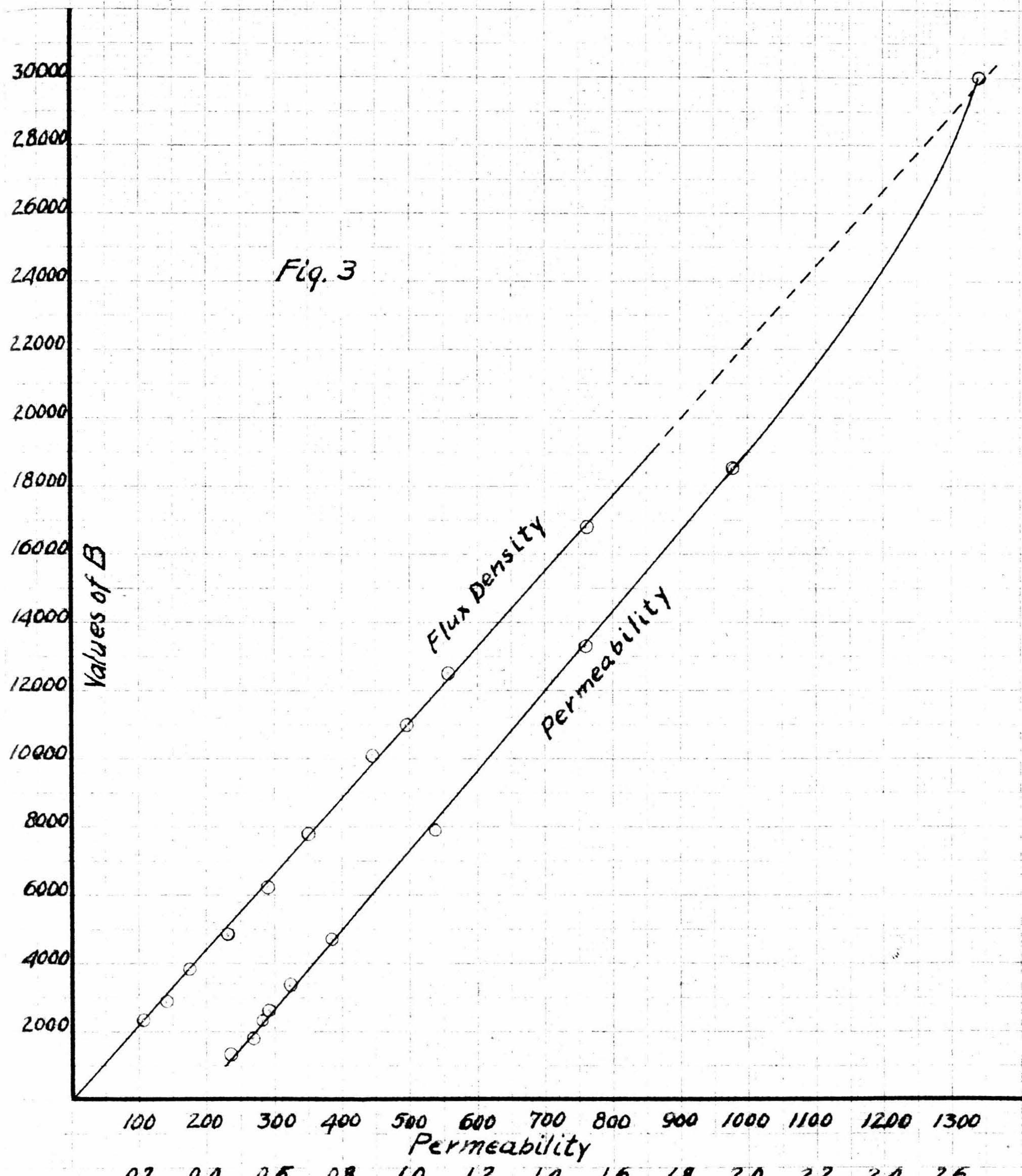


Fig. 3
Values of B
30000
28000
26000
24000
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
Flux Density
Permeability
100 200 300 400 500 600 700 800 900 1000 1100 1200 1300
Permeability
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6
Deflections, Galvanometer Scale

The permeability thus found is that lengthwise of the wire, whereas in the actual tests for inductance the flux is perpendicular to the length of the wire. No method for investigating the permeability crosswise appeared feasible, the high cost per foot of the cable precluded stripping the aluminum from any considerable length of the core.

OUTLINE OF METHOD USED IN RESISTANCE AND INDUCTANCE TESTS.

The following method was used in determining the alternating current resistance and inductance of both core and cable. A "fence" was built somewhat more that a foot high and in the shape of a rectangle 366.3 ft. long and of a variable width. The wire was stretched upon the boards forming the sides of the rectangle. Strong clamps of large cross section made good contact between cable and instrument connections. The photographs, Fig. 4 and Fig. 5, give a good idea of the whole set up.

All instruments used were carefully calibrated and curves were used to correct the readings obtained.

Readings were taken of volts, amperes, watts, frequency and temperature. In some runs frequency and temperature remained constant, in other cases simultaneous readings were taken of all five values.

The a. c. resistance was determined for each reading from the relation $W = I^2R$. These values were then corrected for temperature based on the resistance at 20° C. Temperature corrections were based on a solid aluminum conductor, the core carrying a very small proportion of the total current as appears later. The resistance per thousand feet of cable was next found and the arithmetical average

Fig 4

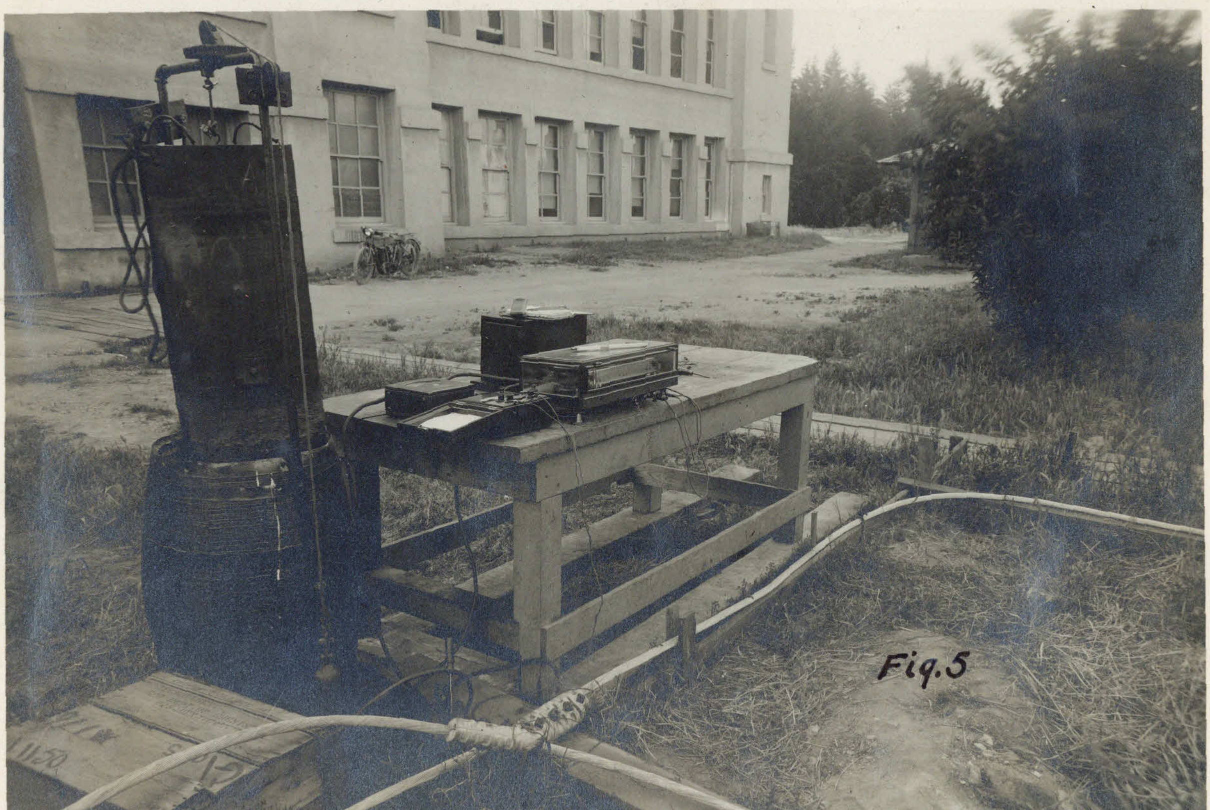


Fig.5

of the separate runs taken as the true resistance per thousand feet. Finally, these values were reduced to resistances for the actual lengths tested for use in calculating the inductance.

The impedance was found from the equation $E = Z I$ and the average impedance based on the arithmetical average taken for the run.

The value of the reactance came from the equation $X = \sqrt{Z^2 - R^2}$ and from these values knowing the frequency the inductance was easily calculated from the relation $X = 2\pi f L$.

For watt readings the Kelvin Balance was used, a previous run with a small wattmeter and current transformer proving unsatisfactory. The Balance was checked with direct current and found accurate.

The system of averaging resistances and impedances was used only after complete calculations from the separate readings were worked out and this method shown to be accurate.

The direct current resistances were taken with laboratory standard instruments and a storage battery.

The data sheets follow.

STEEL CORE, 15 Foot Spacing

Length of Side 22.75', Total Length 86.5' of Cable

D. C. Measurements

M. V.	M. A.	R.
112.6	.977	.11525
78.8	.682	.1155
59.8	.518	.1154
47.9	.413	.1158
40.5	.348	.1164
113.1	.978	.1155
112.9	.9775	.1155
48.0	.4125	.1163
40.5	.348	.1164

Average d. c. resistance .1158 ohms for the length tested, or 1.339 ohms per 1000 ft.

A. C. Measurements

Volts	Volts Corrected	Amperes	Current Squared	Watts	Watts Corrected
4.61	4.64	37.2	1384	13.2	165.0
5.27	5.40	42.9	1840	18.3	228.7
5.76	5.98	46.8	2190	21.5	268.75
6.12	6.40	49.6	2460	24.7	308.75
4.19	4.12	33.0	1089	11.1	138.75
4.69	4.72	37.8	1429	13.7	171.25
5.00	5.09	40.5	1640	16.25	203.1
5.29	5.41	43.2	1866	18.3	228.7
5.65	5.85	46.1	2125	20.8	260.0
4.92	5.00	40.6	1648	15.8	197.5
5.45	5.62	45.4	2061	20.3	253.7
5.82	6.06	48.6	2362	23.4	292.5
5.05	5.15	41.5	1722	16.9	211.25

Temperature	Cycles	Resistance	Temp. Corrections	Resist. Corr.	Impedance
92.5	50.25	.1192	.0068	.1124	.1247
92.5	"	.1243	.0071	.1172	.1259
92.0	"	.1227	.0069	.1158	.1278
92.0	"	.1255	.0071	.1184	.1290
91.0	"	.1274	.0069	.1205	.1249
91.0	"	.1198	.0065	.1133	.1249
90.0	"	.1238	.0064	.1174	.1257
90.0	"	.1226	.0066	.1160	.1252
90.0	"	.1233	.0064	.1169	.1269
82.0	50.3	.1198	.0038	.1161	.1232
81.5	"	.1231	.0038	.1193	.1238
81.0	50.4	.1238	.0037	.1201	.1247
80.0	50.6	..1227	.0035	.1192	.1241

Average impedance .1254 ohms.
Average a. c. resistance .1175 ohms, at 20° Cent.
" " " " per 1000 ft. 1.360 ohms.

$Z^2 - R^2 = .00192$
$X = .0438$
$L = 138.6$ millihenrys

RESISTANCE OF CABLE TO DIRECT CURRENT

Milli Volts	Milli Amperes	Resistance
13.7	637	.02150
22.0	1020	.02158
26.4	1230	.02145
40.5	1890	.02143
27.7	1285	.02158
20.8	962	.02160
17.0	786	.02163
14.25	660	.02160

Temperature 28.5° C.
Average resistance at 20° C. of 750.5 ft. of cable is .02077 ohms.
Resistance per 1000 ft. is .02767 ohms.
Resistance per mile, 0.1461 ohms at 20° C.
Weight is 592.9 pounds per mile-ohm.
If aluminum only, d. c. resistance would be .0281 ohms per 1000 ft.

CABLE 9 ft. SPACING

Volts	Volts Corrected	Amperes	Amperes Corrected	Current Squared	Watts
9.40	10.39	130.0	127.6	16280	31.7
5.15	5.67	77.0	75.3	5660	11.2
6.05	6.65	89.5	88.0	7740	15.5
6.65	7.31	96.0	94.25	8880	17.6
7.80	8.57	109.0	107.7	11600	22.7
8.40	9.21	118.7	116.5	13580	26.0
8.70	9.53	121.3	118.6	14060	27.8
9.50	10.40	131.5	129.1	16700	32.3
9.00	9.85	123.5	120.9	14600	28.9
8.70	9.53	120.0	117.6	13820	27.3
8.15	8.94	113.0	111.3	12400	24.3
6.70	7.37	85.5	84.0	7050	13.7

Watts Corrected	Resistance	Temperature	Temp. Corr.	Rest. Corr.	Impedance
396.0	.0243	92.0	.00133	.0230	.0815
140.0	.0247	91.0	.00136	.0233	.0754
194.0	.0251	86.0	.00106	.0237	.0756
220.0	.0248	85.0	.00105	.0237	.0776
284.0	.0245	83.0	.00083	.0237	.0796
325.0	.0240	82.5	.00081	.0232	.0791
347.7	.0237	82.0	.00080	.0229	.0784
406.0	.0244	80.5	.00072	.0237	.0806
361.5	.0241	80.0	.00068	.0234	.0798
341.0	.0240	79.0	.00062	.0234	.0794
304.0	.0245	78.5	.00060	.0239	.0790
171.2	.0243	78.0	.00058	.0237	.0879

Average a. c. resistance at 20° C. .0235
Corrected for length tested .0237 ohms.
Resistance per 1000 ft. at 20° C. .03131
Frequency 50.25 cycles constant.

Average impedance = .0787

$Z^2 - X^2$ .005632
X = .0750
L = 237.6 millihenrys inductance

CABLE 11.8 ft. SPACING

Volts	Volts Corr.	Amperes	Amperes Corr.	Current Squared	Watts
8.12	8.82	103.5	102.	10400	20.0
8.45	9.26	109.0	107.8	11620	22.3
8.78	9.69	116.0	114.0	13000	24.7
9.20	10.375	124.0	121.5	14760	28.2
9.50	10.80	130.0	127.6	16280	31.1
9.55	10.88	132.0	129.6	16810	32.6
9.90	11.40	136.5	133.8	17900	34.0

Watts Corr.	Resist-ance	Temper-ature	Temp. Corr.	Rest. Corr.	Impedance
250.0	.02430	78.5	.00062	.02368	.08647
278.7	.02398	"	.00061	.02337	.08590
308.7	.02375	"	.00060	.02315	.08500
352.5	.02388	"	.00061	.02327	.08539
388.7	.02388	"	.00061	.02327	.08464
407.5	.02427	"	.00062	.02365	.08394
425.0	.02374	79.0	000060	.02314	.08521

Average a. c. resistance = .0234 ohms at 20° C.
Average a. c. resistance per 1000 ft. at 20° C.= .03093
Corrected value for length tested = .02389.
Frequency 51 cycles constant.

Average impedance = .08522 ohms
$Z^2 - R^2 = .006691$
$X = .08179$
L = 255.3 millihenrys inductance

CABLE 15 foot SPACING

Volts	Volts Corr.	Amperes	Amps. Corr.	Curr. Squared	Watts
7.95	8.58	94.0	92.3	8520	16.5
8.60	9.50	103.0	101.5	10300	20.05
8.85	9.835	107.0	105.7	11180	21.3
9.35	10.57	115.0	113.1	12790	25.0
10.25	10.52	114.0	112.2	12590	24.6
11.97	12.15	133.0	130.05	16915	33.3
12.30	12.52	137.0	134.1	17970	34.8
11.05	11.14	121.0	118.3	14000	27.9
7.80	8.40	91.0	89.3	7974	15.9
8.20	8.91	98.0	96.4	9292	17.9

Watts Corr.	Resist-ance	Temp. Corr.	Rest. Corr.	Impedance
206.23	.02421	.00020	.02421	.09286
250.60	.02433	.00021	.02412	.09359
266.16	.02381	.00020	.02361	.09304
312.50	.02443	.00021	.02422	.09346
307.50	.02442	.00021	.02421	.09366
416.20	.02460	.00021	.02439	.09343
435.00	.02421	.00020	.02401	.09337
348.74	.02491	.00021	.02470	.09416
198.74	.02492	.00020	.02471	.09407
223.73	.02408	.00020	.02388	.09244

Average a. c. resistance at 20° C. = .02421 ohms
Average a. c. resistance per 1000 ft. = .03173 ohms
Corrected value for length tested = .02409 ohms
Average impedance = .09341
Frequency 50.9 cycles, Temperature 71.5° constants.

$Z^2 - R^2 = .008146$
$X = .09022$
$L = 282.1$ millihenrys inductance

CABLE, 18 foot SPACING

Volts	Volts Corr.	Amperes	Amps. Corr.	Current Squared	Watts
7.67	8.23	85.0	83.4	6956	13.4
8.20	8.91	92.5	90.9	8262	16.7
9.15	10.27	106.0	104.6	10940	21.9
9.28	10.46	108.0	106.7	11370	22.8
9.80	11.26	116.0	114.0	13000	26.6
11.25	11.33	119.0	116.7	13620	27.6
11.85	11.925	126.0	123.6	15270	31.0
12.60	12.96	135.0	132.3	17500	35.3
11.25	11.33	119.5	116.1	13480	27.6
13.10	13.45	141.0	138.	19040	38.4

Watts Corr.	Resist-ance	Temp. Corr.	Rest. Corr.	Impedance
167.5	.02408	.00085	.02323	.09868
208.7	.02526	.00086	.02440	.09802
273.8	.02503	.00085	.02418	.09690
285.0	.02506	.00085	.02421	.09804
332.5	.02557	.00087	.02470	.09868
345.0	.02533	.00086	.02447	.09708
387.5	.02538	.00086	.02452	.09648
441.3	.02522	.00086	.02456	.09797
345.0	.02559	.00086	.02473	.09844
480.0	.02522	.00086	.02436	.09746

Average a. c. resistance at 20° C. = .02439 ohms.
" " " " " " " per 1000 ft.= .03171
Corrected value for length tested = .02428 ohms.
Frequency 50.8 cycles, Temp. 82° Constants.
Average impedance = .09777 ohms.

$$Z^2 - R^2 = .008969$$
$$X = .0947$$
L = 296.7 millihenrys inductance.

CABLE, 20 foot SPACING

Volts	Volts Corr.	A Amperes	Amps. Corr.	Current Squared	Watts
8.25	8.99	91.5	89.9	8082	16.8
9.70	11.10	111.5	110.0	12100	25.2
12.30	12.51	129.0	126.7	16050	33.9
13.45	13.85	142.5	139.5	19460	40.9
12.00	12.177	126.0	123.5	15250	31.8
12.85	13.15	136.5	133.8	17900	37.3
8.00	8.63	88.3	86.8	7534	16.15
8.51	9.35	95.2	93.5	8743	18.90
8.94	9.94	101.4	99.9	9980	21.50
9.10	10.20	104.0	102.4	10480	28.25

Watts Corr.	Resist-ance	Temp. Corr.	Resist. Corr.	Impedance
210.0	.02598	.00143	.02455	.10000
315.0	.02603	.00143	.02460	.10090
423.8	.02640	.00145	.02495	.09874
511.2	.02627	.00144	.02483	.09930
397.5	.02606	.00143	.02463	.09860
466.5	.02606	.00143	.02463	.09840
201.9	.02680	.00147	.02533	.09942
236.2	.02710	.00149	.02561	.10000
268.8	.02693	.00148	.02545	.09950
282.5	.02696	.00148	.02548	.09962

Average a. c. resistance = .02499 ohms.
" " " " per 1000 ft. = .0322 ohms.
Corrected value for length tested = .02441 ohms.
Frequency 50.8 cycles, Temp. 91° constants.
Average impedance = .09945 ohms.

$Z^2 - R^2 = .009294$
$X = .09638$
L = 302 millihenrys inductance

RESISTANCES

The resistances to alternating currents for both core and cable are plotted in Fig. 6 and Fig. 7 and the averages indicated.

The increase in cable resistance to alternating currents over that to direct currents is 14.9 %.

Kennelly in a paper entitled "Experimental Researches on Skin Effect in Conductors" published in the Proceedings of the American Institute, August 1915, makes the following remarks.

Current distortion is due to three classes of effects:

1. Skin effect proper.
2. "Spirality" effect in stranded conductors.
3. "Proximity" effect due to the spacing of the return conductor.

The "spirality" effect adds somewhat to the skin effect in the case of stranded conductors. The "proximity" effect is imperceptible for frequencies under 100 and spacings above 20 cms.

Pender's American Handbook makes the following statement: "Change in resistance due to skin effect is always relatively much higher than the change in inductance.

The following theoretical formula for skin

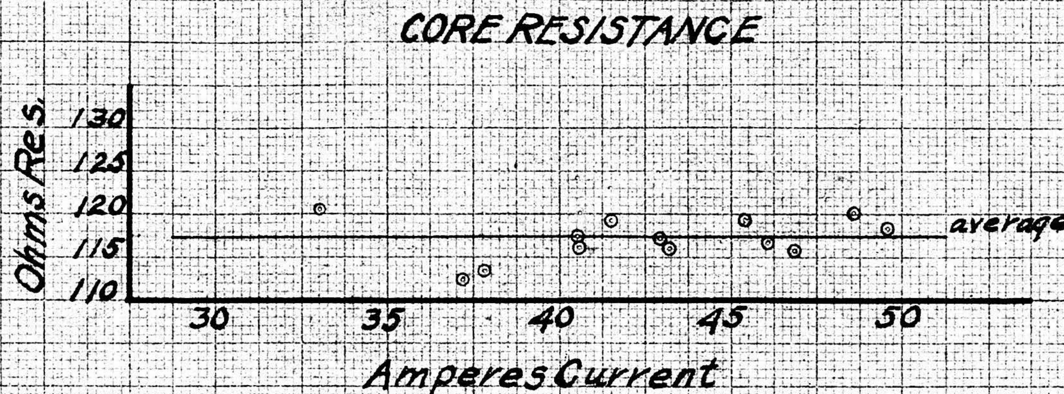

CORE RESISTANCE
Ohms Res.
130
125
120
115
110
average
30
35
40
45
50
Amperes Current

FIG. 6

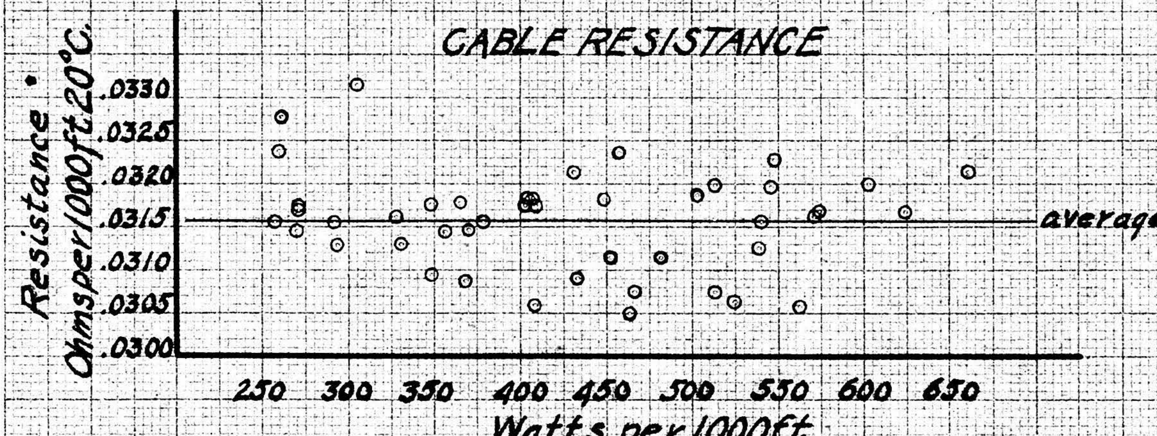

CABLE RESISTANCE
Resistance • Ohms per 1000 ft. 20°C.
.0330
.0325
.0320
.0315
.0310
.0305
.0300
average
250
300
350
400
450
500
550
600
650
Watts per 1000 ft.

FIG. 7

effect was taken from the Standard Handbook.

$$r = R\left\{1+\frac{1}{12}\left(\frac{2\pi f\, l\, \mu}{R\times 10^9}\right) - \frac{1}{180}\left(\frac{2\pi f\, l\, \mu}{R\times 10^9}\right) + - - - \text{etc}\right.$$

where, r = a. c. resistance

R = d. c. resistance

l = length in cms.

f = frequency in cycles per. sec.

μ = permeability

using this equation to solve for the permeability of the core, $\mu = 58$ for a current value of around 43 amperes flowing in the core.

Solving the same equation for the cable gives a value of 3.8 as an equivalent permeability for the whole cable.

Penders American Handbook gives the following equations for calculating skin effect.

$$x = .02768\sqrt{\frac{\mu f}{R}}\ ;$$

where μ, f and R have the same significance as above and x is a number which determines values for K_1 and K_2, constants for skin effect conditions. K_1 and K_2 are listed in a table.

Solving for skin effect in core, the alternating current resistance to be expected would be 1.0144 × 1.339 or 1.360 ohms per 1000 ft., a value which is identical with the test result.

Using the same equation for the cable the solution gives .0312 ohms a. c. resistance per 1000 ft.

of cable, thus checking closely the value .0316 the test average.

Pender's American Handbook also gives the following equation for calculating the added effect of inductance caused by skin effect.

$L' = L + .0152 \{ \mu k_2 - 1 \}$, L' and L being total inductance and inductance minus skin effect respectively, and μ and K_2 having the same significance as stated above.

The second term on the right then is the added inductance due to skin effect. This term is so small as to be nearly negligible in these tests having a value of about .04 millihenrys per 1000 feet.

The percentages of current carried by core and aluminum may be approximated by considering them as two circuits in parallel and solving by means of comparing admittances. Since the resistances and reactances are known the conductances and susceptances are easily found, and hence the admittances. This method gives for the admittance of the core 0.688 ohms and for the aluminum 7.84 ohms. The core should then carry 8.78% of the current and the aluminum 91.22%

The ratio of core a. c. resistance to cable a. c. resistance is 43.1 : 1.

If copper took the place of aluminum, based on

61% conductivity of aluminum the cable resistance would be..0193 ohms per 1000 feet and the ratio of core resistance to cable resistance in that case would be 70.54 : 1.

INDUCTANCE.

The test results of inductances are shown in Fig. 8. For comparative purposes the inductance was also calculated by formulae.

The first formula used was that of E. B. Rosa, of the Bureau of Standards, and is for solid conductors. The formula is given below.

$$L = 4 \left\{ (a+b) \log \frac{2\,ab}{\rho} - a \log (a+d) - b \log (b+d) - \frac{7}{4} (a+b) + 2(d+\rho) \right\}.$$

L = inductance in centimeters.

a = length in centimeters.

b=width or spacing in centimeters

ρ = radius of wire in centimeters = 1.205.

$d = \sqrt{a^2 + b^2}$

Spacing	a	b	d	L
9'	11,175	274.5	11,200	263,000
12'	11,175	366.0	11,200	273,000
15'	11,175	456.5	11,200	284,500
18'	11,175	548.0	11,200	296,000
20'	11,175	610.0	11,200	302,000

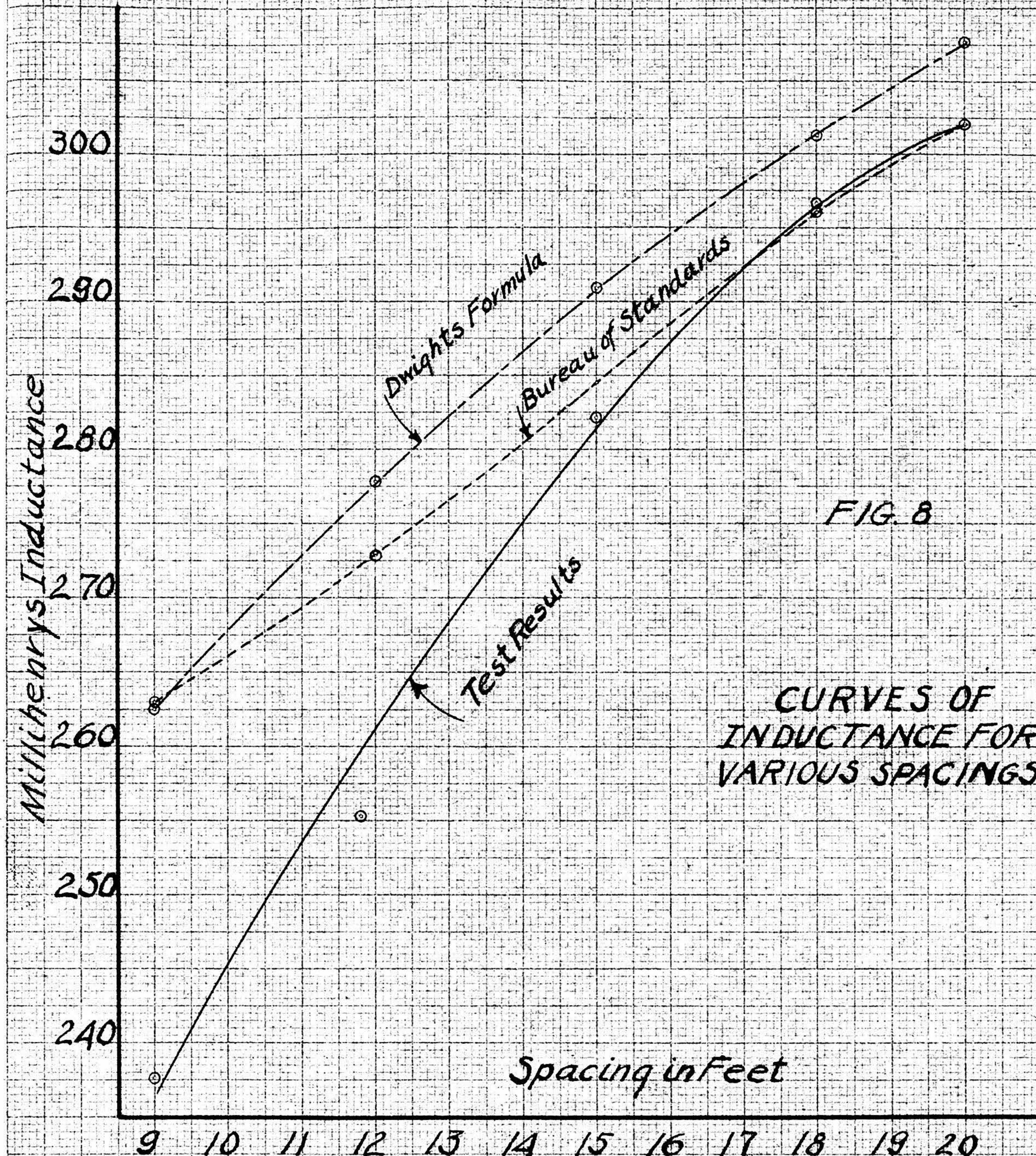


FIG. 8

CURVES OF INDUCTANCE FOR VARIOUS SPACINGS

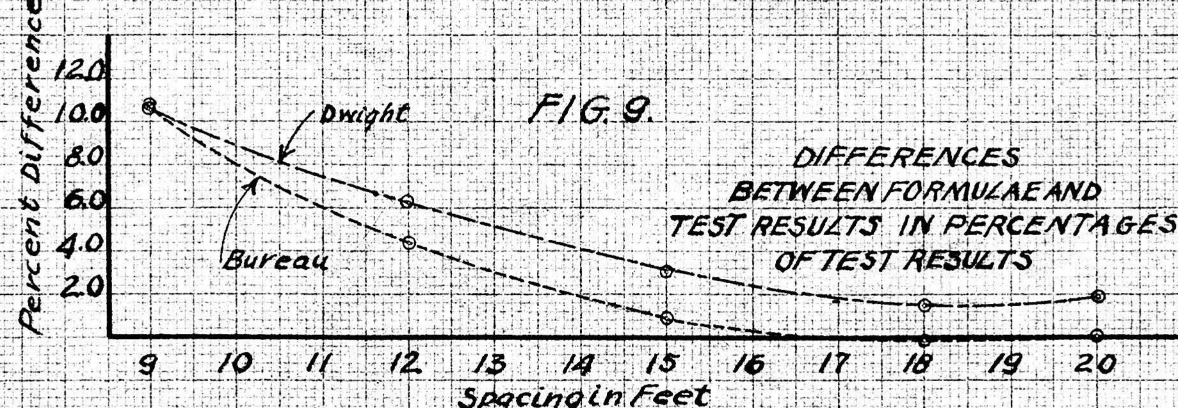


FIG. 9.

DIFFERENCES BETWEEN FORMULAE AND TEST RESULTS IN PERCENTAGES OF TEST RESULTS

H. B. Dwight on page 115 of the appendix to his book entitled "Constant Voltage Transmission" gives the following formula calculated for a 61 strand one metal conductor.

$$L = 741.13 \log_{10} \frac{2.590 \times S}{d} \times 10^{-6}$$

S = spacing in inches

d = actual diameter of cable

L = inductance in henrys per mile

Spacing in feet	Inches	Inductance in millihenrys
9	108	262.5
12	144	277.8
15	182	290.9
18	216	301.25
20	240	307.7

The inductance is that for the number of feet of cable in the rectangle, the formula being developed for a line of infinite length.

It will be seen in Fig. 8 that the test result coincides exactly with the inductance calculated by the Bureau of Standards formula at the 20 foot spacing only.

Fig. 9 shows the percent differences between formulae and test results at different spacings.

As a result of these tests it is recommended that the value of the a. c. resistance used in calculations of the Big Creek Cable be that of the d. c. resistance increased 14.9 %, and that the reactance values be those found by the standard methods.

APPENDIX

When preliminary work was under way, and before any results were obtained it was believed the inductance would plainly show an upward slope as the current increased due to the flux in the iron core. At this time the discussion led to the development of a formula whereby the permeability could be calculated, knowing the inductance, currents in core and aluminum and the physical dimensions. Credit is chiefly due to Mr. J. W. DuMond of the Class of 1916 for developing this formula which is hereby given as possibly being of value in the future.

Let ϕ_1 = flux due to current in iron
ϕ_2 = " " " " " aluminum

$$\phi_1 = \frac{1}{2} i_1 \mu + 2 i_1 \times \log \frac{d - r_1}{r_1} ;$$

d is the spacing of cable
i_1 is the current in the core
i_2 is the current in the aluminum
r_1 is the radius of the core
r_2 is the radius of the cable
μ is the permeability
l is the length

ϕ_2 is the same as flux if the tube were solid, minus the flux due to aluminum core.

$$\phi_2 = \frac{1}{2} i_2 \frac{r_2^2}{r_2^2 - r_1^2} - \frac{1}{2} i_2 \frac{r_1^2}{r_2^2 - r_1^2} + 2 i_2 \log \frac{d - r_2}{r_2}$$

$$\phi = \frac{1}{2} i_2 + 2 i_2 \log \frac{d - r_2}{r_2} + \frac{1}{2} i_1 \mu + 2 i_1 \log \frac{d - r_1}{r_1}$$

$$= i_2 \left(\frac{1}{2} + 2 \log \frac{d - r_2}{r_2}\right) + 2 i_1 \log \frac{d - r_1}{r_1} + \frac{1}{2} i_1 \mu$$

$\mu = a \phi_i + b$; but $\frac{1}{2} i_1 \mu = \phi_i = \frac{1}{2} i_1 (a \phi_i + b)$

solving for $\phi_i = \frac{1}{2} \frac{i_1 b}{1 - \frac{1}{2} i_1 a}$

Let $K_1 = 2 \log \left(\frac{d - r_1}{r_1}\right)$; $K_2 = \left(\frac{1}{2} + 2 \log \frac{d - r_2}{r_2}\right)$

$K_1' = 2 \log \frac{1 - r_1}{r_1}$; $K_2' = \left(\frac{1}{2} + 2 \log \frac{1 - r_2}{r_2}\right)$

$\phi = K_2 i_2 + K_1 i_1 + \frac{i_1 b}{2 - i_1 a}$; Let $i_2 = n i_1$ and the total current $I = i_1 + i_2$ then $i_1 = \frac{I}{n+1}$ and $i_2 = \frac{n I}{n+1}$

$$\phi = \frac{n}{n+1} K_2 I \quad \frac{1}{n+1} K_1 I + \frac{I b}{2n+2 - Ia}$$; *I is total current.*

$$L_{total} = 2 l \left(\frac{n}{n+1} K_2 + \frac{1}{n+1} K_1 + \frac{b}{2n + 2 - I a}\right) + 2 d \left(\frac{n}{n+1} K_2' + \frac{1}{n+1} K_1' + \frac{b}{2n+2 - Ia}\right).$$