THESIS

The Design of a Reinforced Concrete Bridge over Eaton's Canon on the Mount Wilson Toll Road

By

Earle A. Burt

Class of Nineteen Hundred and Fifteen

Department of Civil Engineering

THROOP COLLEGE OF TECHNOLOGY

Pasadena, California

1915

The Design of a Reinforced Concrete Arch Bridge, over Eaton's Canon on the Mt. Wilson Toll Road

The Mt. Wilson Toll Road was originally a two-foot burro-trail. Later on the grade was revised and it was made into a narrow wagon road. The ascent began in the upper part of the Eaton's Canon Wash. For several years the approach to the foot of the trail was up the bed of the canon, over a very rough road. About three years ago, that is in 1912, it was decided to eliminate this bad approach by building a wooden trestle bridge across the canon at the point where the Toll Road starts up the side of the mountain. This bridge is connected to the Santa Anita Boulevard by a short, well built approach.

In view of the fact that the Solar Observatory is hauling all of its materials of construction and supplies over this bridge, and probably will be for many years, and that Mt. Wilson offers also a most pleasing pleasure resort and is being used for that purpose, it seems reasonable and proper to assume that the traffic over the road will justify the replacement of the old wooden bridge by a more substantial and at the same time a more beautiful structure. This thesis offers a suggestion for such an improvement.

A single-center reinforced concrete Arch will be investigated. Although other types of bridge might be built which are much cheaper, it is believed that the beauty, safety against high water, and lasting qualities of the reinforced arch are factors which strongly recommend its use.

The site of the present bridge is at a point in the canon where the steep walls suddenly widen out into a rather wide wash. This point seems to be the best location for an arch bridge. By going farther up the canon, a shorter span could be used but the approaches would have to be blasted from the solid rock cliff on either side of the canon, at the present site. On the east side of the canon, the solid rock is visible at the surface, and on the west side of the canon a water tunnel shows solid rock about ten feet from the surface. It would therefore be possible to locate the abutments on solid rock, at the present bridge site. Plate III shows the main features and topography of the site selected.

The method used in designing the arch, is that outlined in "Principles of Reinforced Concrete Construction" by Tauneaure and Maurer, pages 333 - 369. The method in brief is as follows. A preliminary design is selected by the aid of approximate or empirical formulas or by reference to existing arches. The selected arch is then exactly analyzed and if necessary the results are used to correct the preliminary design.

The analysis consists of several steps. First, the half arch is divided into ten divisions along its axis, such that the length (ds) of each division divided by the moment of inertia (I) of each division is a constant. Second, the points at which the loads from the floor system are applied, are then determined and the values of the thrust at crown (Ho), the shear at crown (Vo), and the bending moment at crown (Mo), are found for unit loads at the load points. Third, the influence lines for stresses at several differentsections are constructed. Fourth, the stresses at the several sections are computed from the influence lines and the graphical layout of the arch. A Fifth step is made in this case, and consists of a graphical solution to show that the line of thrust stays within the middle third of the arch.

Selections of the Arch: Howe's "Symmetrical Masonry Arches" contains a table which gives the name, loading and principal dimensions of five hundred arch bridges. From this table an arch of 75 ft. span 17 ft. rise is selected. (With working stresses of 16000 lbs. per square inch, for steel (in tension) and 650 (lbs. per square inch) for concrete in compression) the amount of tension steel to be used should be about .66% of the area of the section. The section selected requires, from this, about 2 square inches per foot width in extrados and intrados. Reinforcement to be placed 3 inches from surface.

Calculation of Values of I and ds.

The half length of arch is found by scaling, to be 43.3 ft. Depth at crown 2.3 ft. and at springing line 3.0 ft. The moment of inertia $= I = I_c + 15I_s$, where I_c and I_s are the moment of inertia of the concrete and the steel sections respectively. In making $\frac{ds}{I}$ = constant it is first necessary to divide the half arch into a number of equal divisions. In this case the half arch is divided into ten equal divisions, (see Plate I) and the value of I determined at the center of each division. The reciprocal of I is found and is called (i). The results of these calculations are given in Table I. The preliminary divisions are each 4.33 ft. long. As shown in Table I the value of $\frac{ds}{I} = 2.46$. By reference to Plate I a new value for the moment of inertia for the first new division may be assumed and by substitution in the equation $\frac{ds}{I} = 2.46$, a value of ds may be found. Each value of ds as found is laid off on the arch axis. The first trial does not generally come out even at the end of the half arch but by paying close attention to the value of I it is quite possible to find ten values of ds whose sum is exactly equal to the length of the half arch. These results are also shown in Table I.

TABLE I

Divisions of Arch Ring.

No. of Div.	Depth(d)	I_c	$15I_s$	$I=I_c+15I_s$	$i=\frac{1}{I}$	ds	I
1	2.300'	1.012	.335	1.347	.743	3.30'	1.340
2	2.325'	1.041	.344	1.385	.722	3.39'	1.378
3	2.375'	1.110	.363	1.473	.678	3.50'	1.423
4	2.400'	1.150	.373	1.523	.657	3.68'	1.496
5	2.475'	1.253	.402	1.655	.605	3.83'	1.558
6	2.550'	1.375	.435	1.810	.553	4.09'	1.664
7	2.625'	1.500	.468	1.968	.508	4.48'	1.820
8	2.725'	1.670	.512	2.182	.458	4.89'	1.990
9	2.850'	1.920	.572	2.492	.402	5.51'	2.240
10	2.950'	2.130	.622	2.752	.364	6.56'	2.670
				sum	5.690	43.23'	

$$i_{avg.} = \frac{5.69}{10} = .569 \qquad \frac{ds}{I} = \frac{43.3 \times .569}{10} = 2.46$$

Calculations of Ho, Vo, and Mo for a load of unity at each load point.

Before values of Ho, Vo, and Mo can be found, it is necessary to know the points at which the loads are to be applied to the arch. For the present we may consider that the superstructure has been designed and that the load points are A, B, C & D, as shown in Plate I. Values of Ho, Vo and Mo are found with a load of unity at each of the load points.

Referring to Table II, X and y are the coordinates of the central points of the various divisions in which $\frac{ds}{I}$ is constant. Values of M_r, M_rX and M_ry are determined when the unit load is at A, B, C and D respectively. X_1 as shown in plate I is the distance of the respective loads from the crown. M_r is equal to $(X - X_1 \cdot 1)$

From Tauneaure and Maurer we have the following formulas for values of Ho, Vo and Mo, determined from a consideration of the arch as a curved beam.

$$H_o = \frac{N \sum my - \sum m \sum y}{2 \left\{ (\sum y)^2 - N \sum y^2 \right\}}$$

$$Vo = \frac{\sum \left\{ M_r \right\} X}{2 \sum X^2}$$

$$M_o = - \frac{\sum M + 2 \, Ho \sum y}{2 \, N}$$

Values from Table II are substituted in these formulas. The results follow.

TABLE II

Calculations for Ho, Vo & Mo

Pt.	X	Y	X^2	Y^2	M_r	$M_r\ y$ (Load at A. $X_1 = 0$)	$M_r\ X$
1	1.6	.02	2.56	.0004	1.6	.04	2.6
2	5.00	.200	25.	.400	5.00	1.00	25.0
3	8.45	.675	71.5	.456	8.45	5.70	71.5
4	12.00	1.400	144.0	1.96	12.00	16.80	144.0
5	15.75	2.400	248.0	5.77	15.75	37.80	248.0
6	19.5	3.650	380.0	13.30	19.50	71.20	380.0
7	23.35	5.400	545.0	29.2	23.35	127.00	545.0
8	27.50	7.65	757.0	58.5	27.50	210.00	757.0
9	31.75	10.55	1004.0	117.0	31.75	335.00	1004.0
10	36.35	14.35	1320.0	207.0	36.35	524.00	1320.0
Σ		46.29	4497.0	433.59	- 181.00	- 1328.54	- 4497.1

	Load at B. $X_1 = 11$			Load at C $X_1 = 21$		
Pt.	M_r	$M_r\ y$	$M_r\ X$	M_r	$M_r\ y$	$M_r\ X$
4	1.00	1.4	12			
5	4.75	11.4	75			
6	8.25	30.1	168			
7	12.35	66.7	289	2.35	12.7	55
8	16.50	126.2	455	6.50	49.6	179
9	20.75	219.0	660	10.75	113.5	342
10	25.35	364.0	920	15.35	220.0	558
Σ	- 88.95	- 818.8	- 2579	- 34.95	- 395.8	- 1134

Load at D $X_1 = 31$

Pt.	M_r	$M_r\ y$	$M_r\ X$
9	.75	7.92	23.8
10	5.35	76.80	194.0
Σ	- 6.10	- 84.72	- 217.8

$$2\left[(\Sigma y)^2 - N\Sigma y^2\right] = 2\ 46.29^2 - 10 \cdot 432.52 = -4391.8$$

$$2\Sigma X^2 = 2 \cdot 4469 = 8994$$

$$2N = 20$$

$$Ho = \frac{n\Sigma my - \Sigma m \Sigma y}{-4391.8} = \frac{10(-1328.5) - (-181.53 \cdot 46.29)}{-4391.8} = +1.11$$

Load at A

$$Vo = \frac{\Sigma (MX)}{8994} = \frac{-4499.1}{8994} = -0.50 \qquad c = +3.9$$

$$Mo = -\frac{\Sigma m + \Sigma Ho \Sigma y}{20} = -\frac{-181.5 + 2 \cdot 1.11 \cdot 4629}{20} = +4.4$$

$$Ho = \frac{10(-818.8 - (-88.95 \cdot 46.29))}{-4391.8} = +.924$$

Load at B

$$Vo = -\frac{-2579.0}{8994} = -.287 \qquad C = +.187$$

$$Mo = -\frac{-88.95 + (2 \cdot 924 \cdot 46.29)}{20} = +.173$$

Load at C

$$Ho = \frac{10(-395.8) - (-34.95 \cdot 46.29)}{-4391.8} = +.532$$

$$Vo = \frac{-1134.0}{8994} = -.1265 \qquad C = \frac{Mo}{Ho} = -1.07$$

$$Ho = \frac{10(-84.72) - (-6.1 \cdot 46.29)}{-4391.8} = + .129$$

Load at D

$$Vo = \frac{-217.8}{8994} = - .0242 \qquad c = -2.27$$

$$Mo = - \frac{-6.1 + (2 \cdot .129 \cdot 46.29)}{20} = -.292$$

Influence Lines for Fiber Stress at any Section

Influence Lines are diagrams which show the character and amount of stress, at a particular section of the arch, caused by a unit load passing across the arch.

From a consideration of fundamental principles we know that

$F_c = \pm \frac{Mc}{I} + \frac{P}{A}$, in which Fc is the stress on the section, c the distance from the extreme fiber to the neutral axis, I the moment of inertia of the section, P the normal thrust, M the product of P and the excentricity of the resultant thrust.

If r equals the radius of gyration we have $A r^2 = I$,

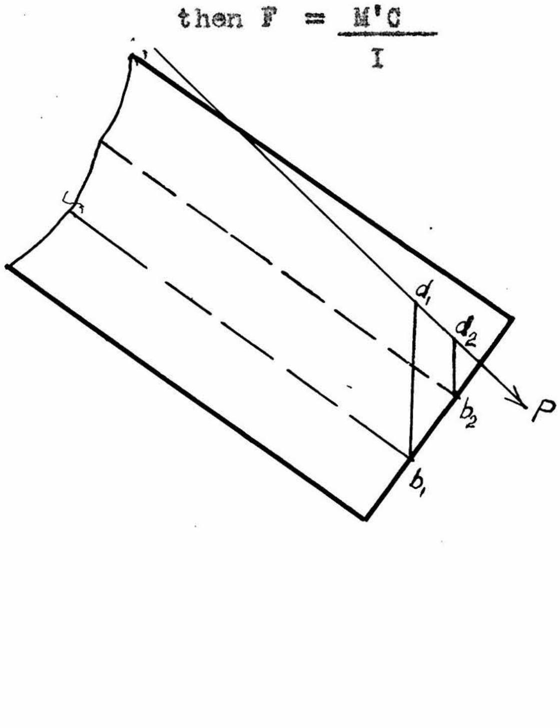
then:

$$F = \pm \frac{Pec}{I} + \frac{P\frac{r^2}{c} \cdot C}{I}$$

$$= \frac{P\left\{ e + \frac{r^2}{c} \right\} c}{I}$$

let $P\left\{ e + \frac{r^2}{c} \right\} = M'$

then $F = \frac{M'C}{I}$



In a rectangular section M' is equal to b, d, X Ho and B_2 d_2 X Ho for upper and lower fiber stress respectively. b, d, is vertical distance from lower third point to thrust line. $b_3 d_3$ is distance from upper third point to thrust line. Ho is taken from calculations following Table II. Values of M' are shown in Table III and are plotted in Plate lV. These Influence Lines show clearly the action of the arch under various loads. There is some difficulty in determining whether m' should be plotted as tension or compression. In general this can be determined by referring to Plate I and noting the position of the thrust line. When this line is well above the central axis we will have compression in the upper half of the section and tension in the lower half. When the thrust line is below the central axis there will be tension in the upper half and compression in the lower half of the section. When the line is doubtful, it is necessary to use the formula $F c = \pm \frac{M c}{I} + \frac{P}{A}$. When $\frac{P}{A}$ is larger than $\frac{M c}{I}$ we have compression over the entire section.

TABLE III

Solution by use of Influence Lines.

Unit Load at →	A				B			
			M				M	
	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob\ d_2$
Sec.AA	4.2'	3.20	4.67	3.52	0.4	0.53	0.37	0.49
" B	0.4	0.4	0.44	0.44	5.0	4.15	4.62	3.83
" C	0.90	1.80	1.00	2.00	0.50	0.50	0.46	0.46
" D	0.35	0.70	0.39	0.77	1.35	2.5	1.25	2.31
" E	3.6	2.55	4.00	2.83	0.45	1.70	.42	1.57

	C				D			
			M				M	
" A	0.70	1.65	.372	0.88	1.95	3.80	0.25	.49
" B	3.05	2.25	1.62	1.19	1.25	0.65	.16	.08
" C	8.5	7.5	4.52	4.0	16.4	15.7	1.82	1.74
" D	1.7	3.85	.92	1.78	14.0	11.0	1.81	1.42
" E	7.25	9.1	3.86	4.83	33.25	39.25	4.28	5.06

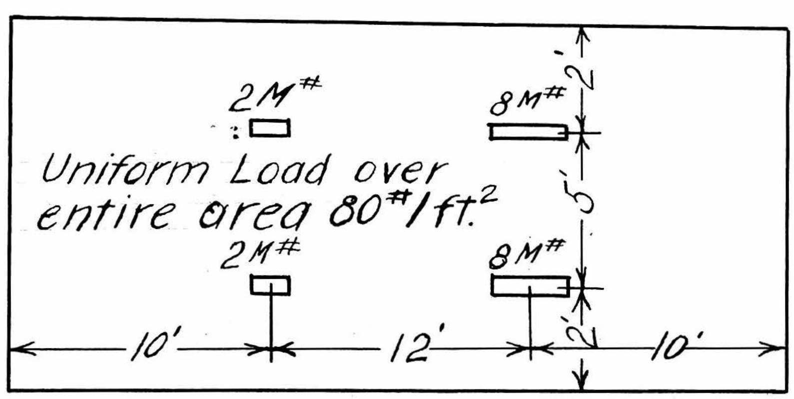
Unit Load at →	A				B			
			M				M	
Sec. B'	0.4	0.4	.44	.44	1.8	2.65	1.66	2.45
" C'	0.9	1.8	1.00	2.00	1.75	2.65	1.62	2.45
" D'	0.35	0.7	.39	.77	0.8	0.2	.74	.18
" E'	3.6	2.55	4.00	2.83	5.0	4.0	4.62	3.70

TABLE III Continued.

Unit Load at →	C				D			
			M				M	
	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$
Sec. B'	2.05	2.90	1.09	1.54	2.90	3.70	.37	.48
" c'	1.25	2.1	.66	1.11	1.60	2.45	.21	.37
" D'	2.15	1.25	1.14	.66	0.8	0.2	.10	.03
" E'	6.8	5.9	3.62	3.13	7.25	6.4	.93	.82

FLOOR SYSTEM

The present traffic over the Mt. Wilson Toll Road consists of a few pleasure automobiles, a small motor stage and two freight trucks, the heaviest being a three ton truck. As nearly as can be foretold, the heaviest load ever to be sent over the road will not exceed ten tons. In the Engineering News, Sept. 1914, pp 492, Vol. 72, there appears an article entitled "Motor Truck Loading on Highway Bridges", in which several diagrams are given; a ten ton diagram is selected and is shown here.



As the width of the road is at present eight feet and as there is no probability of its being made wider, it is reasonable to assume the roadway on the bridge to be nine feet.

The spandrel columns are assumed to spread the load equally over the entire width of arch. By comparing the diagram with plate I it will be seen that the wheel concentrations may be made to come almost exactly on the top of the spandrel columns. This is of advantage in designing the columns.

The floor slat is designed according to the method given by Turneaure and Maurer in "Principles of Reinforced Concrete Construction", pages 58- 65. For a beam with fixed ends and a concentrated load

$$M = \frac{P\,L}{8}$$

P = load L = length in inches

M = Bending moment.

Therefore

$$M = \frac{8000 \times 48}{8} = 48,000 \text{ in lbs.}$$

For a beam with fixed ends and uniform load

$$M' = \frac{W\,L^2}{12}$$

W = weight of load per foot.

Therefore:

$$M' = \frac{80 \times 48^2}{12} = 15,300 \text{ in lbs.}$$

Total moment = 48,000 + 15,300 = 63,300

$$R = \frac{63,300}{12 \times d^2} = \frac{M}{bd^2} = 105$$

D = depth to steel. With the working stresses in concrete and steel already noted and a resulting percentage of steel of about .78, the value of R should be about 105. Solving we have d = 7 inches. An eight inch slab will be used. The area of steel to be used should be .0078 X 12 X 8 = .75 square inches for each foot width.

$\frac{.75}{.25} = 3.$ Use 3 - 1/2 inch, square bars spaced 4 inches apart.

the shear on the floor slab will be:

Concentrated load	=	8000 lbs.
Uniform live load	=	320 lbs.
" dead "	=	400 lbs.
	Total	8720 lbs.

$\frac{8720}{2} = 4360 =$ maximum

<u>Reaction</u> <u>for</u> <u>one</u> <u>foot</u> <u>width</u>. Area of section = .67 square feet. $\frac{4360}{.67} =$ 6500 lbs. per square foot, = 45 lbs. per square inch. 45 lbs. per square inch is just a few pounds too high for plain concrete. The smallest size stirrup which has been found satisfactory is three eighth inch square rods. There will be used and will be spaced twelve inches apart.

Instead of using a floor system of beams and stringers, a system of stringers alone is used. Each stringer is supported at the load points by a column. The method of designing the stringer is the same as that used for the floor slab.

Uniform load

weight of slab = 4 X 10 X .67 X 150 =	4000	lbs.
" " live load = 80 X 40 =	3200	"
Total	7200	"

$\frac{7200}{10}$ = 720 lbs. per foot

Maximum concentrated load = 8000 lbs.

$M = \frac{8000 X}{8} = 120,000$ in lbs.

$M' = \frac{720 X 120^2}{12} = 863,000$ in lbs.

Total = 983,000 in lbs.

$R = \frac{983,000}{bd^2} = 105$ for 78% steel

assume beam 18 X 30	R = 60
" " 12 X 25	R = 130
" " 14 X 26	R = 104 O.K.

26 inches is depth to steel; we will therefore use a beam 14 X 28

.0078 X 14 X 26 = 2.84 square inches steel required.

$\frac{2.84}{.39}$ = 7 .39 = area of 5/8 inch square Bar; use 7-5/8 inch, square Bars.

At each load point the load is to be carried to the arch ring, by four spandrel columns, each to take the load from one stringer.

The maximum weight to be carried by any four columns will be:

Weight of slab	12,000 lbs.
" " four beams	140,000 lbs.
Maximum concentrated load	16,000 lbs.
Uniform live load	9,600 lbs.
Wt. of road surface	12,000 lbs.
" " railing	10,800 lbs.
Total	200,400 lbs.

$\frac{200,400}{500}$ = 400 square inches area required four 14" X 14" columns have and area of 800 square inches. 14 inches is a minimum dimension since a dimension less than 14 inches will look weak in connection with the heavier members. In the case of the largest columns at load point , "C" an 18 inch column is used, mainly to give a better appearance.

It has been found in practice that it is desirable to analyze an arch with the entire length; two-thirds the length; one-third the length and with the middle two-thirds loaded. By referring to the diagram of a ten ton truck and to Plate I, it will be seen that the nearest approach to these conditions will be : First, when the front wheels are at point B' and the rear wheels two feet to the right of A. This loads the middle two-thirds. Second: Front wheels at A, rear wheels two feet to right of B. It is not possible to have two trucks on the arch at the same time if going in opposite directions. If two trucks were to follow each other as closely as possible, it would not put a greater stress on the arch because as shown by the diagram, the truck occupies thirty two feet, while the half length of arch is thirty seven and one half.

The loads which the two assumed loadings bring upon the arch, will now be given.

Load conditions Number I.

Front wheels at B' rear wheels at point two feet to right of A.

	Dead		Live		Total
Load at D' =	19,500	+	0	=	19,500 lbs.
" " C' =	19,000	+	300	=	19,300 lbs.

Load	at	B'	=	18,600	+ 930	=	19,530 lbs.
"	"	A	=	18,250	+ 1,900	=	20,150 lbs.
"	"	B	=	18,600	+ 300	=	18,900 lbs.
"	"	C	=	19,000	+ 0	=	19,000 lbs.
"	"	D	=	19,500	+ 0	=	19,500 lbs.

Load Condition Number 2:

Front wheels at A, rear wheels two feet to right of B.

				Dead		Live		Total
Load	at	D'	=	19,500	+	0	=	19,500
"	"	C'	=	19,000	+	0	=	19,000
"	"	B'	=	18,600	+	300	=	18,900
"	"	A	=	18,250	+	930	=	19,180
"	"	B	=	18,600	+	1,930	=	19,530
"	"	C	=	19,000	+	300	=	19,300
"	"	D	=	19,500	+	0	=	19,500

The actual stresses at any section A, B, C or D, may now be computed from the formula

$$f = \frac{M'C}{I}$$

M' is taken from Table III.

Relation between Table III and Influence Lines.

Values of M' taken from Table III are used as ordinates in the construction of the Influence Lines (Plate IV), as mentioned before. M' can be taken from (Plate IV) or Table III. The results taken from the Table are more accurate (than the scaled values from Plate IV), and are therefore used. The Influence Lines serve chiefly in (giving a quick method of) determining the character of stress.

$\frac{C}{I}$ from Table IV

TABLE IV.

Properties of Sections A, B, C, D, E.

Section	Depth	$C = \frac{I}{2} d$	I	$A = A_c + 15\,A_s$	$\frac{C}{I}$
A	2.30	1.15	1.34	2.27 + .42 = 2.69	.86
B	2.37	1.19	1.52	2.34 + .42 = 2.76	.78
C	2.53	1.26	1.75	2.50 + .42 = 2.92	.72
D	2.80	1.40	2.24	2.77 + .42 = 3.19	.63
E	3.00	1.50	2.75	2.97 + .42 = 3.39	.55

One example of the calculation of stress at a section (of one foot width) will now be given.

Upper half section A

Load Condition No. I

$$M' = (19{,}500 \times .25) + (19{,}300 \times .372) - (19{,}530 \times .37)$$
$$- (20{,}150 \times 4.67) - (18{,}900 \times .37) + (19{,}000 \times .372)$$

$+ (19,500 \times .25) = -84,320$

$$F = \frac{M'c}{\frac{I}{144}} = \frac{-84,320 \times .86}{144} = 500 \text{ lbs. per square inch compression.}$$

There follows the complete calculations for upper and lower fiber stresses at each section, for two conditions of loading. The highest compressive stress found out is 500 lbs. per square inch, which is allowable. The only cases of tension are in the lower half section A under Loading No. 1 and in the lower half of sections A and B under loading No. 2. The maximum tension found is 30 lbs. per square inch; On account of this low value and of the amount of heavy reinforcement it is believed safe to allow these values. This completes the analytical solution of the arch. A brief statement of the method used in the graphical solution will now be given.

FIBER STRESSES.

Upper Fiber — Section A — Loading No. I.

Tension	Compression
19,500 X .25 = 4,880	19,530 X .37 = 7,220
19,300 X .372 =7,180	20,150 X4.67 = 94,000
19,000 X .372 =7,180	18,900 X .37 = 7,100
+ 24,120	- 108,320
	24,120
	84,200

$$\frac{84,200 \times .86}{144} = 500 \text{ (lbs. per square inch) compression}$$

Lower Fiber | Section A | Loading No. I.

Tension	Compression
2o,150 X 3.52 = + 71,500	19,500 X .45 = 8,800
- 71,400	19,300 X .88 = 17,300
+ 100	19,530 X .49 = 9,600
	35,700
	2
	- 11,400

$\frac{100 \times .86}{144}$ = .6 lbs. per square inch Tension

---

Upper Fiber | Section B | Loading No. I.

Tension	Compression
19,500 X .37 = 7,230	20,150 X .44 = 8,940
19,300 X 1.09 = 21,000	18,900 X 4.62 = 87,200
19,530 X 1.66 = 32,400	19,000 X 1.62 = 30,800
60,630	19,500 X .16 = 3,120
	- 130,060
	60,630
	- 69,430

$\frac{69,430 \times .78}{144}$ = 375 lbs. per square inch Compression

---

Lower Fiber | Section B | Load No. I.

Tension	Compression
18,900 X 3.83 = 72,300	19,500 X .48 = 9,500
19,000 X 1.19 = 22,500	19,300 X 1.54 = 29,900
19,500 X .08 = 1,550	19,530 X 2.46 = 48,100
96,350	20,150 X .44 = 8,950
	96,450
	96,350
	100 comp.

$\frac{100 \times .78}{144}$ = .55 lbs. per square inch compression

Upper Fiber — Section C — Loading No. I.

Tension

19,300 X .21 = 4,050

19,530 X .66 = 12,900

20,150 X1.00 = 20,150

37,100

Compression

19,500 X 1.00 = 19,500

18,900 X .46 = 8,700

19,000 X 4.52 = 86,000

19,500 X .82 = 16,000

130,200

37,100

93,100

$\frac{93{,}100 \times .72}{144}$ = 465 lbs. per square inch compression.

Lower Fiber — Section C — Loading No. I.

Tension

19,500 X .37 = 7,200

19,000 X 4.00 = 76,000

19,500 X .74 = 14,000

97,200

Compression

19,300 X 1.11 = 21,500

19,530 X 2.45 = 48,000

20,150 X 2.00 = 42,600

18,900 X .46 = 8,700

120,800

97,200

23,600

$\frac{23{,}600 \times .72}{144}$ = 120 lbs. per square inch compression

---

Upper Fiber — Section D — Loading No. I.

Tension

B 18.900 X 1.25 = 23,600

C 19,000 X .92 = 17,100

40,700

Compression

D' 19,500 X .10 = 1,950

C' 19,530 X 1.14 = 22,000

B' 19,530 X .74 = 14,500

A 20,150 X .39 = 8,300

D 19,500 X 1.81 = 35,300

82,050

40,700

42,750

$\frac{42{,}750 \times .63}{144}$ = 187 lbs. in square inch compression.

Lower Fiber | Section D | Loading No. I.

Tension					Compression				
D'	19,500	X .03	=	585	A	20,150	X .77	=	15,600
C'	19,300	X .66	=	12,750	B	18,900	X 2.31	=	43,700
B'	19,530	X .18	=	3,500	C	19,000	X 1.78	=	33,800
				44,335					93,100
									44,335
									48,765

$$\frac{48,765 \times .63}{144} = 213 \text{ lbs. per square inch compression}$$

Upper Fiber | Section E | Loading No. I

Tension					Compression				
B	18,900	X .42	=	7,950					
C	19,000	X 3.86	=	73,400	D'	19,500	X .93	=	18,100
D	19,500	X 4.28	=	83,500	C'	19,300	X 3.63	=	70,200
				164,850	B'	19,530	X 4.63	=	90,000
					A	20, 150	X 4.00	=	81,200
									259,500
									164,850
									94,750

94,750 X .55 = 360 lbs. per square inch compression

Lower Fiber | Section E | Loading No. I

Tension					Compression				
D'	19,500	X .82	=	16,000	B	18,900	X 1.57	=	29,700
C'	19,300	X 3.13	=	60,500	C	19,000	X 4.83	=	92,000
B'	19,530	X 3.70	=	72,300	D	19,500	X 5.06	=	98,500
				206,300					220,200
									206,300
									14,900

$$\frac{14,900 \times .55}{144} = 57 \text{ lbs. per square inch compression.}$$

Upper Fiber   Section A   Loading No. 2.

Tension						Compression				
D'	19,500	x .25	=	4,880		B	19,530	x .37	=	7,230
C'	19,000	x .372	=	7,070		A	19,180	x 4.67	=	89,500
C	19,300	x .372	=	7,200		B'	18,900	x .37	=	6,990
				24,030						103,720
										24,030
										79,700

$$\frac{79,700 \times .86}{144} =$$ 477 lbs. per square inch compression.

Lower Fiber   Section A   Loading No. 2.

Tension						Compression				
A	19,180	x 3.52	=	67,500		D'	19,500	x .49	=	9,550
						C'	19,000	x .88	=	16,700
						B'	18,900	x .49	=	9,250
						B	19,530	x .49	=	9,580
						C	19,300	x .88	=	17,000
						D	19,500	x .49	=	9,570
										72,150
										67,500
										4,650

$$\frac{4,650 \times .86}{144} =$$ 28 lbs. per square inch compression.

Upper Fiber   Section B   Loading No. 2.

Tension						Compression				
D'	19,500	x .37	=	7,200		A	19,180	x .44	=	8,400
C'	19,000	x1.09	=	20,700		B	19,530	x 4.62	=	90,000
B'	18,900	x1.66	=	31,400		C	19,300	x 1.62	=	31,300
				59,300		D	19,500	x .16	=	3,100
										132,800
										59,500

$$\frac{73,500 \times .78}{144}$$ = 380 lbs. per square inch compression

Lower Fiber — Section B — Loading No. 2.

	Tension						Compression			
B	19,530	x 3.82	=	74,600		D'	19,500	x .48	=	9,300
C	19,300	x 1.19	=	23,000		C'	19,000	x 1.54	=	29,200
D	19,500	x .08	=	1,500		B'	18,900	x 2.45	=	46,300
				99,100		A	19,180	x .44	=	8,500
				93,300						93,300
				5,800						

$$\frac{5{,}800 \times .78}{144} =$$ 30 lbs. per square inch compression

---

Upper Fiber — Section C — Loading No. 2.

	Tension						Compression			
C'	19,000	x .66	=	12,500		D'	19,500	x .21	=	4,100
B'	18,900	x 1.62	=	30,600		B	19,530	x .46	=	9,000
A	19,180	x 1.00	=	19,200		C	19,300	x 4.52	=	87,500
						D	19,500	x .82	=	16,000
										116,600
										19,200
										97,400

$$\frac{97{,}400 \times .72}{144} =$$ 480 lbs. per square inch compression

---

Lower Fiber — Section C — Loading No. 2

	Tension						Compression			
D'	19,500	x !37	=	7,200		C'	19,000	x 1.11	=	21,100
C	19,300	x 4.0	=	77,000		B'	18,900	x 2.45	=	46,400
D	19,500	x .74	=	14,400		A	19,180	x 2.00	=	38,400
				98,600		B'	19,530	x .46	=	9,000
										114.900
										98,600
										16,300

$$\frac{16{,}300 \times .72}{144} =$$ 82 lbs. per square inch compression

Upper Fiber | Section D | Loading No. 2.

Tension					Compression				
B	19,530	x 1.25	=	24,400	D'	19,500	x .1	=	1,950
C	19,300	x .92	=	17,800	C'	19,000	x 1.14	=	21,700
				42,200	B'	18,900	x .74	=	14,000
					A	19,180	x .39	=	7,500
					D	19,500	x 1.81	=	35,300
									80,450

$\frac{38,250 \text{ X } .63}{144} =$ 168 lbs. per square inch compression.

Lower Fiber | Section D | Loading No. 2

Tension					Compression				
D'	19,500	x .03	=	590	A	19,180	x .77	=	44,300
C'	19,000	x .66	=	12,500	B	19,530	x 2.31	=	45,200
B'	18,900	x .18	=	3,400	C	19,300	x 1.78	=	34,400
D	19,500	x 1.42	=	27,700					123,900
				44,190					44,190
									79,710

$\frac{79,700 \text{ X } .63}{144} =$ 350 lbs. per square inch compression.

Upper Fiber | Section E | Loading No. 2

Tension					Compression				
B	19,530	x .42	=	8,200	D'	19,500	x .93	=	18,100
C	19,300	x 3.86	=	74,500	C'	19,000	x 3.62	=	68,700
D	19,500	x 4.28	=	83,500	B'	18,900	x 4.62	=	87,500
				166,200	A	19,180	x 4.00	=	77,000
									251,300
									166,200
									95,100

$\frac{95,100 \text{ x } .55}{144} =$ 360 lbs. per square inch compression

Lower Fiber   Section E   Loading No. 2.

Tension					Compression				
D'	19,500	x .82	=	16,000	B	19,530	x 1.57	=	30,700
C'	19,000	x 3.13	=	59,500	C	19,300	x 4.83	=	93,500
B'	18,900	x 3.70	=	70,000	D	19,500	x 5.06	=	99,000
A	19,180	x 2.82	=	54,000					223,200
				199,500					199,500
									24,700

$\frac{24,700 \times .55}{144} = 95$ lbs. per square inch compression

Plate II is a graphical calculation of the position of the thrust line under loading No. 2. The arch is laid out very carefully and the position of the middle third shown. The loads are then laid off on the vertical load line and some point "0" selected as a ray center. The first equilibrium polygon drawn in (not shown on plate). The dotted line H is drawn from the point 0 to the load line. H is parallel to the closing line of the equilibrium polygon. From the point where line H strikes the load line, a horizontal line Ho is drawn. The length of Ho is equal to the thrust at the crown and is determined in the same way that Ho for unity loads was determined. Table V gives all calculations. With the new ray center a new equilibrium polygon is drawn. This polygon must pass the crown at a distance e from the axis. $e = \frac{Mo}{Ho}$. Note on Plate II that the polygon stays within the middle third. This means that there is no tension at any section of the arch, and may seem to disagree with the analytical solution. The explanation is that the graphical solution is not refined enough to detect such small amounts of tension as are shown to exist by the analytical solution. It should also be noted that at the sections which show tension in the analytical method, the equilibrium polygon approaches very close to the edges of the hem. Taking these things into consideration, we may say that the graphical solution shows the arch to be safe and therefore checks the analytical method. This completes the analysis of the arch. All stresses have been found allowable. The original arch can therefore be used.

Plate V gives the general plans, elevations, and sections of the completed design. To add to the appearance of the structure, false spandrel walls are put in to cover the beam and column construction. Also a large fillet has been put in at the abutment to give the appearance of a three center arch. Several sketches of the different spandrel arrangements, railing, etc, show these particular architectural details to be the best sutied to the conditions of the problem (as shown in Plate V).

In order to make the problem complete, an estimate of cost is given below.

COST DATA

Top

Railing

(81 X .67 X 1.5)2 + (67 X .67 X 1.5)2 = 298 cubic feet

Web

(81 X 1.67)2 + (67 X 1.67)2 = 331 cubic feet

Base

(81 X 2 X .67)2 + (67 X 2 X .67)2 = 397 cubic feet

Floor slat

(.67 X 12 X 86.5) = 695 cubic feet

4 Floor beams

(1.67 X 1.17 X 86.5) 4 = 675 cubic feet

Column at center

(.5 X 12 X 1.17) + for base = 8 cubic feet

Column at 11 feet from center

(1.17 X 1.17 X 1.5) 8 + for base = 20 cubic feet

Column at 21 feet from center

(1.17 X 1.17 X 5.5)8 + = 65 cubic feet

Column at 31 feet

(1.5 X 1.5 X 9.75) 8 + = 185 cubic feet

Wall at end

(2 X 12 X 18.5)2 = 890 cubic feet

4 Retaining walls approximate

$\left(\frac{38}{2} \times 18 \times 1\right)4$ = 850 cubic feet

Arch ring

(2.8 X 12 X 86.5) = 2900 cubic feet

2 Abutments

(17 X 12 X 10)2 = 4000 cubic feet

11314

27) 11,314 Cubic feet
420 Cubic yards

420 X 15.0 = $6,300 cost of Concrete

150 yards at $1.00 = $150.00 cost of excavation

Excavation	=	$ 150.00
Concrete at $15.0	=	6,300.00
10% for engineering	=	645.00
10% for contingencies	=	645.00
Estimated total cost	=	$7,740.00

TABLE V.

Investigation of line of thrust within Arch Ring.

Point	X	Y	X^2	Y^2	ML	MR	(ML + MR)Y
1	1.60	.05	2.56	0.00	14,800	14,800	1,500
2	5.0	.30	25.00	0.90	46,300	46,300	27,700
3	8.5	.75	72.00	.56	78,700	78,700	118,000
4	12.0	1.4	144.00	1.96	129,000	131,000	364,000
5	15.75	2.3	248.00	5.30	224,000	243,000	1,100,000
6	19.5	3.75	382.00	14.10	339,000	355,000	2,600,000
7	23.4	5.5	550.00	30.20	494,000	572,000	5,530,000
8	27.5	7.6	758,000	57.90	685,000	722,000	10,700,000
9	31.75	10.6	1010.00	113.00	699,000	948,000	17,500,000
10	36.25	14.45	1320.00	208.00	1,198,000	1,364,000	37,000,000
		46.66	4509.56	431.92	3,917,800	4,417,800	- 74,941,200

-8,332,600

TABLE V Continued.

$$Ho = \frac{N \times \sum my - \sum m \sum y}{2 \; (\sum y)^2 - N \sum y^2}$$

$$Ho = \frac{(10 \times -74{,}941{,}200) - (-8{,}332{,}600 \times 46.65)}{2 \; (46.65)^2 - (10 \times 431.9)}$$

$$Ho = 84{,}500 \text{ lbs.}$$

$$Mo = \frac{\sum m + 2 \; Ho \; Ey}{2 \; N} = \frac{-8{,}333{,}000 + 2 \times 84{,}500 \times 46.65}{2 \times 10}$$

$$= + 23{,}600$$

$$C = \frac{Mo}{Ho} = \frac{23{,}600}{84{,}500} \cong .28$$