THE ABSORPTION OF HIGH FREQUENCY ELECTROMAGNETIC RADIATION.

Thesis by John Read

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SUMMARY

The different mechanisms of absorption of high frequency electromagnetic radiation are described. Theories of scattering by electrons are outlined, and a survey is given of data on photoelectric absorption, absorption by the production of positive and negative electron pairs, and nuclear absorption.

An experiment for the measurement of μ in carbon, aluminum, iron, copper, and lead, is described.

The results for carbon and aluminum indicate it is unlikely that the Klein-Nishina formula is in error by as much as one percent in the wave-length region 50 to 20 x-u.

The results for lead give values for the photoelectric absorption coefficient in agreement with Gray's empirical law between 100 and 38 x-u. Between 25 and 20 x-u. however, the experimental results are higher than Gray's values. An empirical relation is fitted to the experimental points, the origin, and Allen's value at 100 x-u. by the method of least squares. The predictions of this law are compared with available information on the photoelectric absorption coefficient. It is suggested that the true law should lie midway between Gray's law and the new law, and it is pointed out that such a law would leave a balance of absorption at $\lambda = 4.7$ x-u. close to that required for the production of the observed positive and negative electron pairs. Some results of Ellis and Aston obtained from the "magnetic spectrum" of photoelectrons ejected from platinum by the γ-rays of radium B+C are discussed, and it is shown that they cannot be explained without assuming photoelectric absorption coefficients lower than Gray's values, and at variance with the experimental results of this thesis. If the data is correct, the whole can scarcely be explained

without assuming some absorption in addition to Klein-Nishina and photoelectric absorption, exists at wave-lengths as long as 24 x-u.

The results for τ for iron, copper, and lead are in agreement with a law of variation $\tau \propto Z^{2.9}$ at both 24 and 49 x-u.

A discussion of nuclear absorption and the most likely conclusion to reconcile all the data are given

FOREWORD

The subject of this thesis is restricted to the absorption of electromagnetic radiation of wave-length shorter than 100 x-u., ie., the phenomenon of the absorption edges will not be discussed.

THE THEORY OF ABSORPTION OF ELECTROMAGNETIC RADIATION

Consider a beam of photons passing normally through a thin plane lamina of matter of thickness dx. The probability that a photon will be removed from the beam is proportional (a) to the intensity I of the beam when in the lamina; (b) to the thickness of the lamina dx. This thickness must be so small that I may be regarded as constant throughout the lamina.

Then $$dI = -\mu I\, dx \qquad (1)$$

Provided μ is not a function of x this may be integrated, giving:-

$$I = I_0 e^{-\mu x} \qquad (2)$$

The condition that μ is not a function of x requires that the beam be monochromatic throughout the thickness x of the absorbing matter. This necessitates that radiation, whose wave-length is changed by an absorption process, be excluded from the beam when measuring I. μ is called the absorption coefficient per cm and is written $\mu\ \mathrm{cm}^{-1}$. We may write the law of absorption

$$I = I_0 e^{-\left(\frac{\mu}{N_a}\right) N_a x} = I_0 e^{-{}_a\mu N} \qquad (3)$$

N_a is the number of atoms in unit volume. $\left(\frac{\mu}{N_a}\right)$ is written ${}_a\mu$ and called the absorption coefficient per atom. $(N_a x)$ is the number of atoms behind one square centimeter of surface of the absorber. In a similar manner we can define an absorption coefficient per electron, denoted by ${}_e\mu$.

If the loss of intensity dI can be split into parts dI_1, dI_2, --dI_n, each of which can be attributed to a definite mechanism of absorption, then:-

$$dI = dI_1 + dI_2 \text{ --- } + dI_n$$

$$= -(\mu_1 + \mu_2 \text{ --- } + \mu_n) I\, dx \qquad (4)$$

Then $$I = I_0 e^{-(\mu_1 + \mu_2 \text{ -- } \mu_n)x} \qquad (5)$$

Therefore the absorption coefficients μ, $_a\mu$, or $_e\mu$ can be split into parts, each of which can be associated with a definite mechanism of absorption. Such mechanisms are:- (a) A photon may be deflected from the beam by a Compton collision with an electron. (b) An atom may absorb a whole quantum and eject a photoelectron. (c) Recent evidence suggests that if the quantum energy is greater than twice the rest-mass energy of an electron, ie., 1,020,000 e.v., it can be converted into a positive and negative electron pair.[1] (d) It is possible that the nuclei might absorb or scatter the radiation[2]. Any loss of radiation by unknown mechanism will be included in this type of absorption. The coefficients for these processes will be denoted by σ, τ, π and κ respectively, with a subscript a or e to indicate whether the coefficient refers to the atom or to the electron.

Then $$\mu = \sigma + \tau + \pi + \kappa \qquad (6)$$

DISCUSSION OF THE DIFFERENT TYPES OF ABSORPTION

(a) Absorption by Compton scattering:

Theory:-

The Compton scattering may be expressed in terms of the scattering of each electron, since it is reasonable to assume all electrons scatter equally and independently. This is not strictly true unless the

quantum energy is large compared with the binding energy of the electrons. The following expression for ${}_e\sigma$ can be obtained from classical electromagnetic theory:-

$$ {}_e\sigma = \frac{8}{3}\frac{\pi e^4}{m^2c^4} \qquad (7) $$

It was early shown by Barkla that this was approximately true for soft radiation and the electrons of light elements. Since it was based on the conception of electrons executing vibrations under the influence of the electric vector of the incident radiation it could not be expected to hold for shorter wave-lengths, where the Compton recoil process becomes increasingly important. In 1923 Compton[3] deduced the formula

$$ {}_e\sigma = \frac{8}{3}\frac{\pi e^4}{m^2c^4} \cdot \frac{1}{1+2\alpha} \qquad \text{where} \quad \alpha = \frac{h\nu}{mc^2} \qquad (8) $$

To do this he made use of the correspondence principle and his formula for the Compton wave-length change. In 1927 Dirac[4] used the wave mechanics in arriving at the formula

$$ {}_e\sigma = 2 \cdot \frac{\pi e^4}{m^2c^4} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\log_e(1+2\alpha) \right] \right\} \qquad (9) $$

In 1928 Klein and Nishina[5] modified this by considering Dirac's relativistic interpretation of the quantum mechanics. They obtained

$$ {}_e\sigma = 2 \cdot \frac{\pi e^4}{m^2c^4} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\log_e(1+2\alpha) \right] + \frac{1}{2\alpha}\log_e(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \qquad (10) $$

Each of these formulae converges for small values of α (long wave-lengths) to the classical value.

The Compton formula, the Dirac formula and the Klein-Nishina formula are plotted in diagram **8**.

Experimental evidence:-

When seeking experimental evidence regarding ${}_e\sigma$ the difficulty arises that absorption measurements give a value for ${}_e\mu$ which equals ${}_e\sigma + {}_e\tau + {}_e\pi + {}_e\kappa$. Evidence will be given that for elements of atomic number less than 14 and for wave-lengths less than 50 x-u. ${}_e\tau$ is a negligible

part of $_e\mu$. Also $_e\pi$ should be zero for wave-lengths greater than about 12 x.u., while we would expect $_eK$ to be negligible for these wave-lengths also. Measurements of $_e\mu$ between 50 and 12 x.u. should therefore be measurements of $_e\sigma$. For wave-lengths less than 12 x.u. an excess is to be expected. The only suitable gamma-rays for such measurements are those emitted by thorium C" as by suitable filtration the longer wave-lengths can be reduced to negligible intensity without too strongly diminishing the energy of the ray of wave-length 4.7 x.u. The absorption coefficient of this ray has been measured in a number of elements by several experimenters. Their results are given in Table 6. The values for elements of low atomic number may be compared with the Klein-Nishina value for $_e\sigma$ for a wave-length of 4.7 x.u., which is 1.233×10^{-25}.

Chao[7] has also scattered the 4.7 x-u. radiation from aluminum at suitable angles to produce monochromatic beams of wave-length 7.0, 9.6, 15.5, 29.4, and 47 x-u., and measured the absorption coefficient per electron $_e\mu$, in aluminum. The values he obtained are given in Table I and plotted on diagram 8. In general the values for $_e\mu$ are about 4 percent greater than the value for $_e\sigma$ to be expected from the Klein-Nishina formula, and the excess appears to exist for wave-lengths from 4.7 to 29.4 x-u. However Chao considered his values to check the ~~K.N.~~ Klein-Nishina formula within his experimental errors. The question arises whether this excess, if real, is due to the other types of absorption, or to an error in the Klein-Nishina formula. The first object of this research will be to measure $_e\mu$ in carbon and aluminum for wave-lengths between 50 and 20 x-u. and to compare the results with $_e\sigma$ calculated from the Klein-Nishina formula.

TABLE I

Chao's values for $\epsilon\mu$ for aluminum.

Wave-length in x-u.	Experimental value.	Klein-Nishina value.
	$\times 10^{25}$	$\times 10^{25}$
4.7	1.301	1.233
7.0	1.63	1.55
9.6	1.94	1.82
15.5	2.45	2.32
29.4	3.2	3.07
47	3.7	3.66

(b) Photoelectric absorption:

Theory:-

Photoelectric absorption has been treated theoretically by Sauter[8] and by McDougall and Hulme[9]. Sauter has calculated the following values for the photoelectric absorption by the K electrons of 4.7 x-u. radiation:-

Aluminum	2.7×10^{-26}
Tin	2.4×10^{-25}
Lead	2.5×10^{-24}

McDougall and Hulme find the photoelectric absorption of the K shell for atomic number 84 and a wave-length of 9.15 x-u. should equal 4.0×10^{-24}.

Experimental:-

For wave-lengths greater than 100 x-u. and elements of high atomic number ${}_e\sigma$ calculated on any theory is small compared with ${}_e\mu$. ${}_e\pi$ and ${}_eK$ are zero. Therefore if from measured values of ${}_e\mu$ we subtract say the classical value of ${}_e\sigma$, the balance will practically equal ${}_e\tau$. In this way Allen[10] has found that in the region around 100 x-u. ${}_e\tau$ is given by:-

$$ {}_e\tau = 2.19 \times 10^{-26} \lambda^{2.92} Z^3 $$

Ahmad[11] measured the absorption coefficient of radium (B+C) γ-rays at five different degrees of filtering, in aluminum, copper, tin, and lead. The effective wave-lengths were of the order of 10 x-u. He concluded that an exponent for Z of 3.0 ± 0.5 held for all of his effective wave-lengths. Kohlrausch[12] arrived at the same conclusion from a similar experiment.

Gray[13] has pointed out that between 100 and 10 x-u. the dependence of ${}_e\tau$ on λ cannot be represented by the simple power law ${}_e\tau \propto \lambda^{2.92}$. He has suggested the following empirical relation for the photoelectric

absorption of lead:-

$$\log_{10} \tau = \bar{3}.6505 + 1.0 \log_{10} \lambda + 0.480 (\log_{10} \lambda)^2$$

λ is in x-u. and τ is per cm.

This formula was based on the following knowledge:-

(1) Allen's value for e^{τ} and the slope of the $e^{\tau} - \lambda$ curve at 100 x-u.

(2) From the relative intensities of the "magnetic spectrum" lines produced from lead by radium (B+C) γ-rays quantities $\rho\tau$ can be determined, where ρ is the relative frequency of emission of a γ-ray of a certain wave-length and τ is the corresponding photoelectric coefficient. The empirical formula for τ was adjusted until values for ρ calculated by its use were in agreement with Skobelzyn's intensity measurements.

(3) The ratio of the γ-ray energy of radium B to that of radium C is 0.13 ± 0.02. Gray's law leads to a value of 0.17.

(4) Kohlrausch found that radium (B+C) γ-rays filtered through 0.35 cms of lead had an absorption coefficient in lead of 1.0 cm^{-1}. Gray's formula leads to a value of 0.87 cm^{-1}. The second aim of this research is to find whether Gray's formula is true in the region 50 to 20 x-u., and if possible to improve on it. A third aim is to test the accuracy of the Z^3 law of variation.

Values of μ for lead for wave-lengths from 80 to 100 x-u. obtained by Allen[14], using a spectrograph, are given in Table 7. Chao[15] and Gentner[16] using the method of Chao previously described measured μ in lead for several wave-lengths around 10 x-u. Their results are given in Table 8.

(c) Absorption with the production of positive and negative electron pairs:

In September 1932 Anderson[17] reported the discovery of free positive electrons, made while making cosmic ray experiments with a Wilson

Cloud Chamber. In March 1933 this was confirmed by Blackett and Occhialini[18] while in April 1933, Chadwick, Blackett and Occhialini,[19] Curie and Joliot,[20] and Meitner and Philipp,[21] showed that the γ-rays produced by the bombardment of beryllium by α particles could eject positive electrons from matter. Blackett and Occhialini[22] suggested on the basis of the Dirac theory of the positive electron that the absorption of a photon of energy greater than 1,020,000 e.v. might produce a positive and negative electron pair. Anderson and Neddermeyer[23] in April 1933 showed that when the 4.7 x-u. γ-ray of Thorium C" was passed through lead and aluminum positive and negative electron pairs, single positive electrons, and single negative electrons were ejected. It was suggested by Blackett and Occhialini[24] that that part of the absorption of the Thorium C" 4.7 x-u. radiation by heavy elements, which cannot be attributed to Compton or photoelectric absorption, should be attributed to the production of positive and negative electron pairs.

Oppenheimer and Plesset[25] have regarded the phenomenon as a photoelectric absorption by Dirac negative energy state electrons. Such an electron is raised to a state of positive energy and the resulting "hole" acts like a positive electron. The electron pair then possesses kinetic energy equal to $h\nu - 2mc^2$. The absorption by this process should be proportional to the square of the atomic number, and should rise very rapidly with increasing photon energy. For the 4.7 x-u. Thorium C" radiation in lead[18] should be 25 percent of the Klein-Nishina absorption, while in tin it should be 15 percent. Anderson[26] has found that for radiothorium radiation with 2.5 cms of lead filtration the absorption in lead with production of positive electrons is 20 percent of the remaining absorption. In aluminum it is 50 percent, while Skobelzyn[27] has found that in air it is 1 percent.

Gentner[28] has scattered the 4.7 x-u. ray of thorium C" from aluminum at suitable angles to give beams of wave-length 4.7, 5.9, 6.6, 7.9, and 9.3 x-u. He measured μ for these rays in lead. On subtracting σ calculated from the Klein-Nishina formula he found that the balance had a minimum at about 7 x-u. He explained this as being due to the counteraction of the decrease in τ by the increase in π. He added the values of τ calculated from Gray's empirical law to σ calculated from the Klein-Nishina formula. The sum he subtracted from μ and called π. On plotting π against wave-length a straight line was obtained giving $\pi = 0$ at 12 x-u. This he regarded as evidence that positive electron absorption commences at about 1,000,000 e.v.

Oppenheimer and Plesset[25] suggested that a positive electron would unite with a negative electron; their annihilation producing two quanta of about 510,000 e.v. Heiting[29] irradiated aluminum, iron, copper, and lead, with the thorium C" 4.7 x-u. ray and measured the scattered radiation at an angle of 130° with an ionization chamber. An absorption curve of this scattered radiation in lead was obtained. In addition to the Compton scattered radiation he found a radiation of wave-length 23.8 ± 1.0 x-u. from all elements. This he identified with the half-million volt quanta to be expected from the annihilation of positive electrons. In the case of lead there was also a radiation of wave-length 6-7 x-u. The intensity of the 24 x-u. radiation increased as the square of the atomic number of the radiator.

(d) Nuclear absorption:

(i) Nuclear Rayleigh scattering.

Meitner and Hupfeld[30] irradiated lead and iron scatterers with the γ-rays of radium and its products with 3 cm of lead filtration. They examined the radiation scattered at 90° by means of Geiger-Müller tube counters. An absorption curve in lead was obtained for this scattered radiation. In the case of iron only Compton scattered radiation could be detected, but for lead, in addition Meitner and Hupfeld claimed to have found a component of the same hardness as the primary radiation. They computed that this hard component had an intensity of 4 percent of the Compton scattered radiation.

$$_a\mu = 16.4 \times 10^{-24}$$

$$_a\sigma = 12.6 \times 10^{-24}$$ from the Klein Nishina formula.

Nuclear absorption $_aK$ 4 percent of $_a\sigma = 0.5 \times 10^{-24}$. This leaves a balance of 3.3×10^{-24} for $_a\tau$ compared with Sauter's value of 3.9×10^{-24}.

Later Meitner and Kösters[31] repeated the experiment but used the γ-rays of mesothorium in equilibrium with its products filtered through 3 cm of lead. The hard component of wave-length equal to that of the primary radiation, ie. practically 4.7 x-u., was found for both iron and lead. They calculated the nuclear absorption coefficient $_aK$ of lead to be 1.01×10^{-24} and for iron 3.37×10^{-25}. This gave a value for τ for lead of 3.05×10^{-24}. It must be noted that the Compton scattered radiation in both these experiments had a wave-length around 24 x-u. and consequently any radiation due to the annihilation of positive electrons would be obscured.

Gray and Tarrant[32] have irradiated several elements both with

the γ-rays of thorium C'' and those of radium (B+C). The radiation scattered at angles of 125°, 140° and 175° was examined with an ionization chamber and absorption curves in lead were obtained. These curves were analyzed and it was concluded that in addition to Compton scattered radiation bands of discrete wave-lengths were also emitted. These are set out in Table 10. They concluded that all of these bands had an excitation threshold of about 1.9×10^6 e.v. The intensity of the scattered radiation was calculated and the resulting nuclear absorption coefficients are given in Table 11. The nuclear absorption coefficient for the soft radiation was proportional to the atomic number squared, that of the hard component to the cube. They concluded that all of the nuclear absorption $a\mu - (a\sigma + a\tau)$ could be accounted for by these bands. The possibility of the softer bands being due to the annihilation of positive electrons was examined and rejected for the following reasons:-

(a) When a 2.6×10^6 e.v. quantum is absorbed with the production of a pair of electrons 1.6×10^6 e.v. goes into kinetic energy. Therefore only 40 percent of the absorbed energy should reappear as radiant energy, whereas Gray and Tarrant found 80 percent.

(b) The mean energy of quanta produced by the annihilation of positive electrons should not be less than 510,000 e.v. However Gray and Tarrant found that 98 percent of the quanta re-emitted when iron was irradiated with radium (B+C) γ-rays were of the order of 380,000 e.v.

Stahel and Ketelaar[33] have performed an experiment somewhat similar to that of Gray and Tarrant's. Lead, tin, and iron were irradiated by γ-rays from radium (B+C) and the radiation scattered at angles of 80°, 90°, 130°, and 135° was examined. Their results were somewhat similar to those of Gray and Tarrant and are given in Tables 10 and 11.

APPARATUS

The arrangement of the apparatus is shown in Figure 1. The source of radiation was an x-ray tube A, which could be excited by voltages up to 1,000 k. v. A spectrograph was used to select radiation of a desired wave-length from the continuous spectrum. Two lead blocks B and C had slits, each 0.02 by 1 by 3 inches, which defined a plane horizontal sheet of radiation. This passed through a rocksalt crystal D, incident on its internal atom planes at such an angle that the desired wave-length was reflected in a vertical plane. The lead block E, having a slit 0.04 inches wide, allowed the monochromatic beam to pass through, but absorbed the unreflected radiation. The beam then passed into the next room through an aperture 12 inches square in a concrete wall 10 inches thick. Stray radiation was excluded from this room by lead plates G 2 inches thick, and a lead block H 6 inches long pierced by a 0.25 inch slit to pass the beam. A lead filter F was chosen of a suitable thickness to cut down the background radiation, without too seriously weakening the monochromatic beam. The latter could be shut off by a sliding lead block J, or allowed to pass through two ionization chambers K and L. Each chamber was an aluminum box 8 by 8 by 4 inches, shielded from stray radiation by lead 3 inches thick at the front and back, 5/8 inch at the sides, and 3/8 inch at the top and bottom. The lead blocks had slits to pass the beam, and all slits except B and C were made just so large that the beam did not graze their sides.

The construction of the ionization chambers is shown in Figure 2. A low power microscope A projected into the aluminum box. The brass block B carried an amber rod C 1 cm long and 1 mm in diameter, which insulated the wire frame D. To this frame was attached a gold-coated L-shaped quartz fibre E, about 2μ in diameter. It was 7 mm long and had a "foot" 0.5 mm long. This "foot" could be brought into focus in the mic-

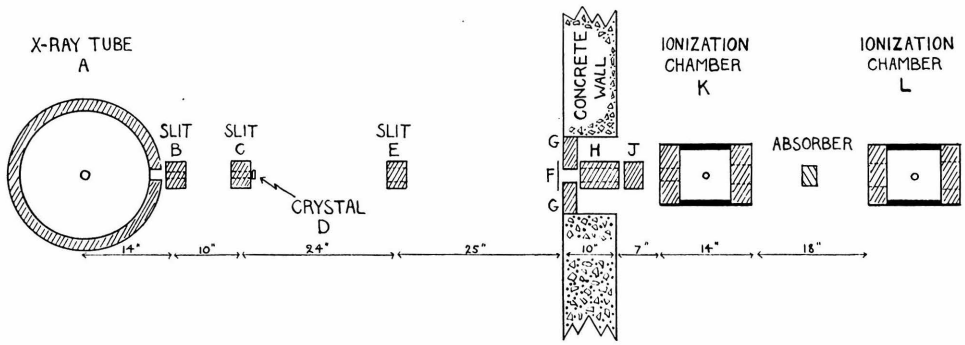


FIG. 1. Arrangement of the apparatus.

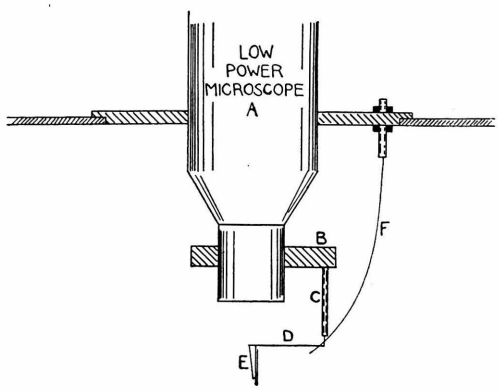


FIG. 2. Construction of the ionization chamber.

roscope by adjusting the block B, and when charged it moved across an eye-piece scale. It was charged by rotating the wire F to touch the wire frame D. When not in use the wire F was grounded in a fixed position. Maximum deflection of the fibre on the scale was obtained with a potential difference of about 400 volts between it and the box.

EXPERIMENTAL PROCEDURE

Characteristics of the ionization chambers:

For each chamber it was necessary to know the relation between a discharge through any range on the eye-piece scale, and the quantity of radiation producing it. Since the experiment required comparison of the intensities of beams of the same wave-length, ionization could be used as a measure of intensity. Some radium was placed in line with the slit system of the chambers, and photographs of their discharge were taken at equal time intervals. Graphs, with scale reading as ordinate and time as abscissa, were then constructed. Since the loss of charge due to insulation leakage was negligible compared with that due to the gamma-rays and cosmic rays, equal times of discharge were equivalent to equal quantities of ionization. Therefore a unit on the time axis could be taken as an arbitrary unit of ionization, and the ionization causing a discharge through any range on the eye-piece scale could be expressed in terms of this unit.

To determine whether the response of the chamber was affected by the size of the ionization current the radium was placed at different distances from the chambers so that discharge rates of 5, 10, and 15 divisions per minute were obtained. In each case scale reading was plotted against time, but the time units for the 10 and 15 rates were adjusted so that all three curves coincided between the first two points. The curves for the 5 and 10 rates then coincided throughout, but the curve for the

discharge of 15 divisions per minute lay higher. This indicated a relatively slower discharge, probably due to recombination of ions having greater effect. The discharge rates were therefore kept below 10 divisions per minute. It was also found that the ionization produced by two radioactive sources acting together, equalled the sum of the ionizations produced by the sources acting separately.

Measurement of absorption:

When the chambers had been charged, shutter J was opened for a time chosen so that the more rapidly moving fibre covered almost the whole eye-piece scale. This was done with no absorber between the chambers. The ionization causing each discharge could be determined in arbitrary units from the radium discharge curves, so that the ionization in chamber L could be expressed in terms of that in chamber K. The discharge was repeated with an absorbing screen midway between the chambers. From the ionization in chamber K, and the known response of L in terms of K when no absorber was present, the ionization which would have occurred in L had no absorber been present, was calculated. The actual ionization in L was known from the observed discharge, so that the absorption coefficient could be found. The ionizations were always corrected for ionization due to general and cosmic radiation. The correction was found by altering the angle of the crystal so that it no longer reflected the monochromatic beam down the slit system. The rate of discharge of each chamber was then measured under the same conditions as existed during the measurements to be corrected. To minimize the effect of any change in the ratio of the response of chamber L to that of K, this ratio was determined before and after measurements with an absorber, and the mean was used.

The absorption coefficient was measured for several thicknesses of absorber, to determine whether there was any hardening of the beam with increasing thickness. This did not occur.

The x-ray tube was operated at a peak voltage less than twice that corresponding to the wave-length reflected by the crystal. Therefore radiation of half the desired wave-length, reflected in the second order, was not present.

A sample set of observations is given in Table 2. K_1 and K_2 are constants of proportionality between the arbitrary unit of ionization and a true unit. When the absorber was present the ionization in chamber K was 1,653 k_1. Hence had no absorption occurred the ionization in chamber L would have been 1,653 k_1 x 1.2459k_2/k_1 = 2059 k_2. Actually it was 1,422 k_2 so that $I_o/I = 1.4480$ and $\mu t = \log_e 1.4480$.

Choice of absorber thickness:

We have:-

$$\mu = -\frac{1}{t}\log\frac{I}{I_o} = -\frac{1}{t}\left[\log I - \log I_o\right]$$

$$d\mu = \frac{\partial\mu}{\partial t}\delta t + \frac{\partial\mu}{\partial I}\delta I + \frac{\partial\mu}{\partial I_o}\delta I_o$$

$$= -\left[-\frac{1}{t^2}\log\frac{I}{I_o}\delta t + \frac{1}{t}\cdot\frac{1}{I}\delta I - \frac{1}{t}\frac{1}{I_o}\delta I_o\right]$$

$$\frac{d\mu}{\mu} = -\frac{1}{t}\delta t + \frac{1}{\log I/I_o}\left[\frac{\delta I}{I} - \frac{\delta I_o}{I_o}\right]$$

I and I_o are measured in the same way and may be taken equal. So δI and δI_o In Figure 3 are plotted curves showing how

i $\dfrac{1}{\log I/I_o}\left[\dfrac{1}{I} + \dfrac{1}{I_o}\right]$

ii $\dfrac{1}{\log I/I_o}\cdot\dfrac{1}{I}$

iii $\dfrac{1}{\log I/I_o}\left[\dfrac{1}{I} - \dfrac{1}{I_o}\right]$ Vary with I.

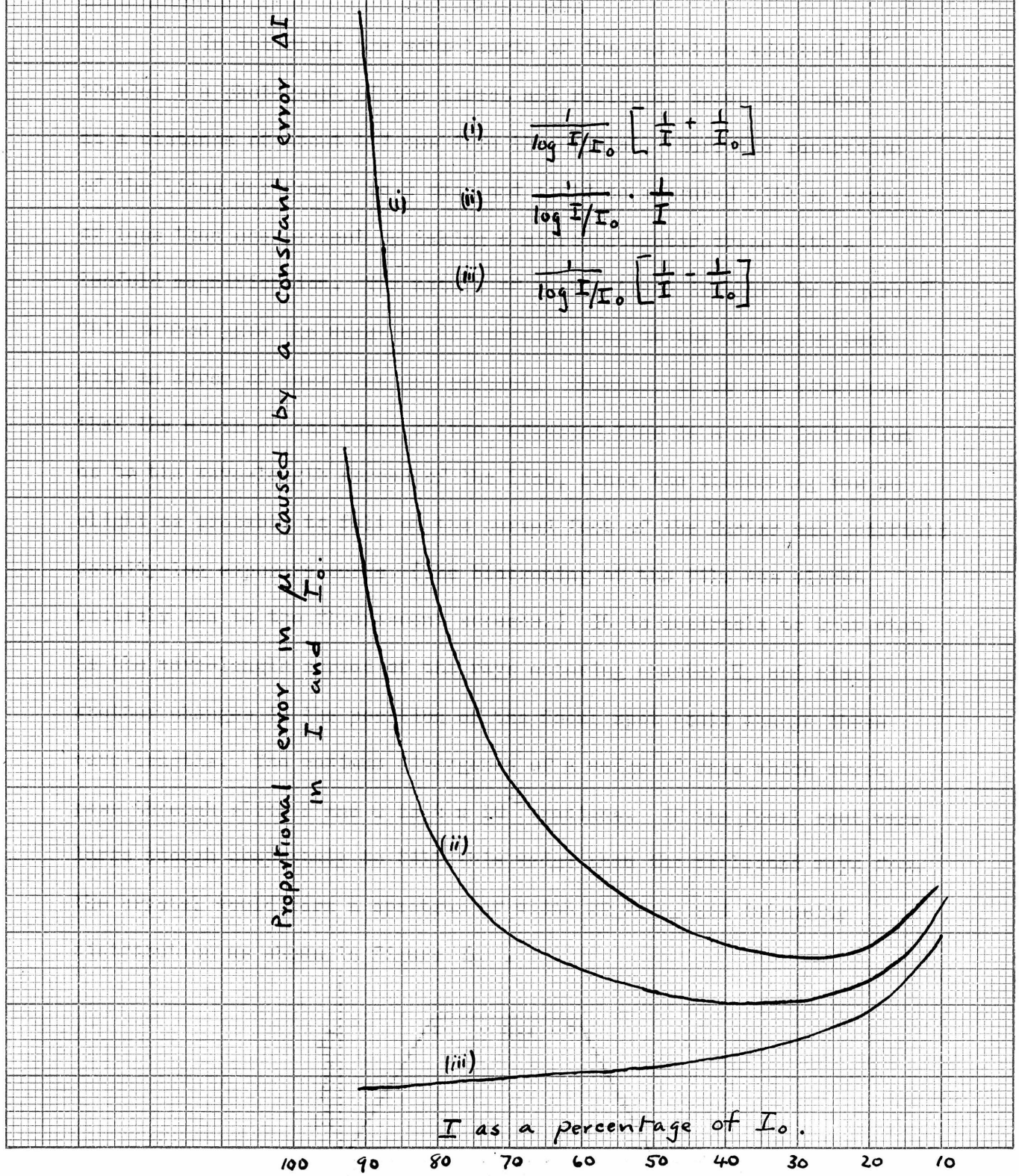
TABLE 2.

Specimen set of observations.

Observation.	Time of discharge.	Chamber.	Initial scale reading.	Final Scale reading.	Initial reading Radium curve.	Final reading Radium curve.	Ionization of monochomatic beam plus background.	Ionization per min. without monochromatic beam.	Background ionization.	Ionization due to monochromatic beam alone.	Ratio of ionization in L to K.	Mean ratio.
No absorber present.	10	K	90.3	27.15	303	1755	1452k	11.4k	114k	1338k	1.2399	
		L	94.3	8.15	195	2032	1837k	17.8k	178k	1659k		
Aluminum ½ inch thick.	12	K	89.9	13.0	312	2102	1790k	11.4k	137k	1653k		1.2459
		L	98.3	20.75	113	1737	1624k	16.8k	202k	1422k		
No absorber present.	10½	K	90.8	26.7	292	1766	1474k	11.4k	120k	1354k	1.2518	
		L	93.0	5.15	222	2104	1882k	17.8k	187k	1695k		

Figure 3.

EFFECT OF ABSORBER THICKNESS ON THE LIKELY ERROR.



From these it is apparent that the best thickness of absorber is one which reduces the intensity of the incident beam to about 30 percent. In this experiment the absorber thickness was chosen so that I was from 50 to 20 percent of I_o.

Measurement of wave-length and resolution.

To determine the wave-length of the radiation admitted to chamber K a photograph of the direct and reflected beams was taken at a distance of about 167 cms from the crystal. Here the separation of the lines was from 1 to 2 cms, and could be measured with an error of less than one percent by taking a microphotometer trace with a tenfold magnification. The nature of such a trace is shown by figure 5. This method gave the wave-length corresponding to the intensity peak with an error of less than one percent. However, it is not considered that the width of the trace of the reflected line is a good indication as to the wave-length spread of the line, since the blackening was so slight as probably to fall on the curved foot of the Hurter and Driffield density-log exposure curve. (The blackening of the film was almost entirely due to the fluorescent screens.) It is desirable to know what is the wave-length spread of the rays entering chamber K, and also whether changes in the position of the focal spot could change these wave-lengths. The maximum divergence of a wedge of rays which could pass through slits B and C is 4×10^{-3} radians. If the focal spot were large enough completely to fill this wedge then the reflected rays from the crystal would form a spectrum of width 22 x-u., and intensity distribution as shown in figure 4.

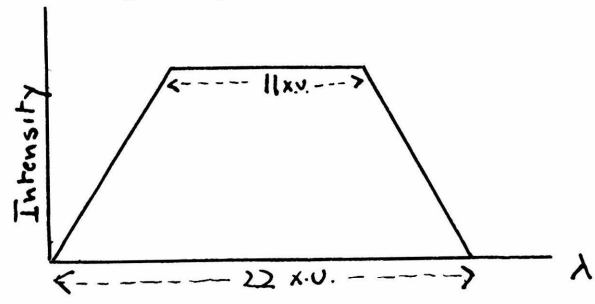
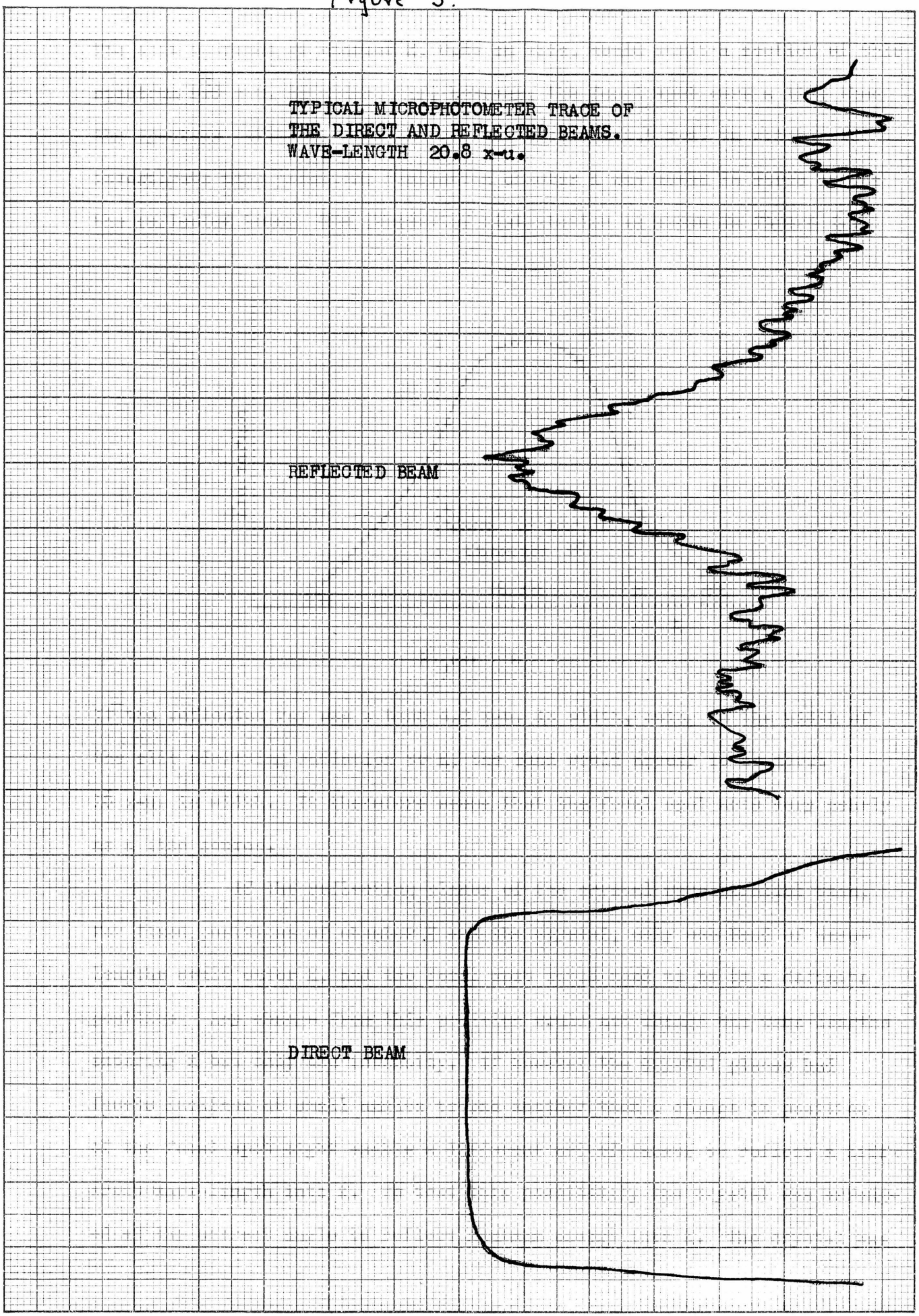


Figure 4.

Figure 5.


TYPICAL MICROPHOTOMETER TRACE OF
THE DIRECT AND REFLECTED BEAMS.
WAVE-LENGTH 20.8 x-u.
REFLECTED BEAM
DIRECT BEAM

The slit in front of chamber K, 0.35 cm wide, would admit a section of this spectrum ~~5.6~~ 11 x-u. in width. If, however, the focal spot were a line then the spectrum reflected from the crystal would be 4.5 x-u. in width. To investigate this point the ionization chamber K was moved in steps across the reflected beam. The ionization distribution is shown in Figure 6.

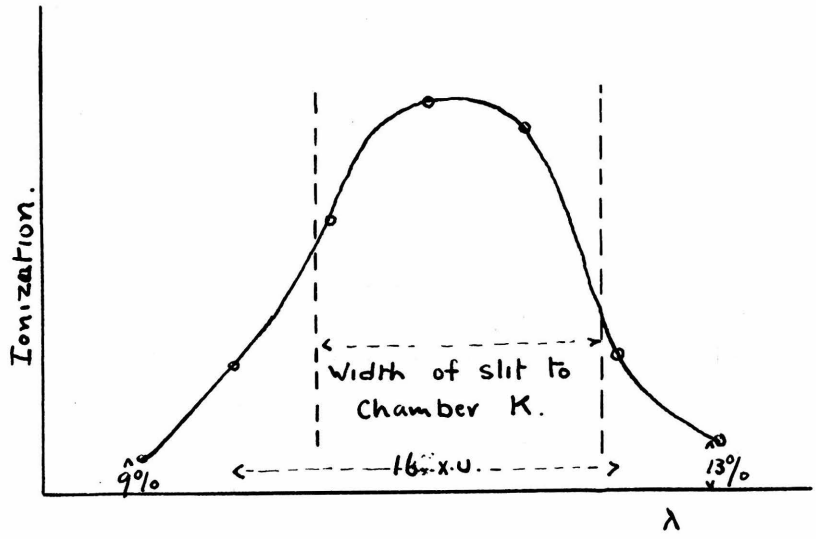


Figure 6.

If the reflected ray was a band 4.5 x-u. in width, owing to the width of the slit admitting radiation to K, the band would appear to be about 16 x-u. in width. It therefore seems that the focal spot acts very nearly as a line source.

If the reflecting planes of the crystal were parallel then, for fixed positions of chamber K and the crystal, only one band of wave-lengths could enter K, and the focal spot would need to be in a definite position. Any change from this position would not change the wave-lengths entering K but only the intensity. If however the crystal planes had facets inclined at small angles to one another then a change in position of the focal spot might enable a different set of facets to reflect a different wave-length into K. To test this possibility the crystal was arranged at the correct angle to reflect maximum energy into K. The crystal was

then rotated by small steps of angle, the ionization in K being noted at each step. The distribution of ionization is shown by Figure 7 .

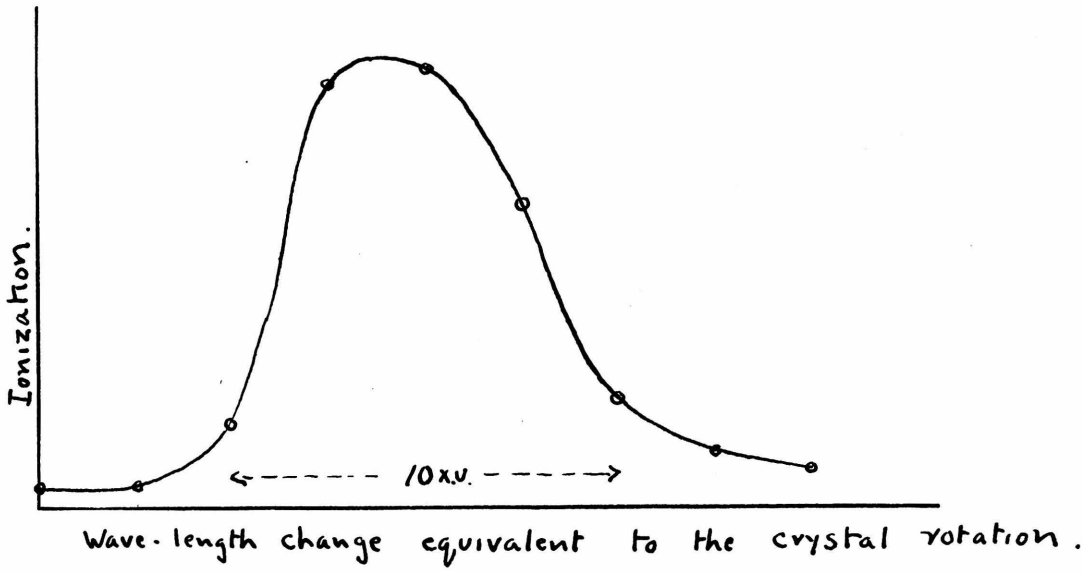


Figure 7.

Again there is a width of about 10 x-u., which is to be expected with a line source, and parallel reflecting planes. It therefore seems that for fixed positions of chamber K and the crystal a definite band of wave-lengths about 5 x-u. wide can enter chamber K, and this band is not changed in wave-length, but only in intensity by movement of the focal spot.

The wave-length spread of each beam used for an absorption measurement was investigated by moving ionization chamber K across the beam. Some typical curves are shown in Figure 8 .

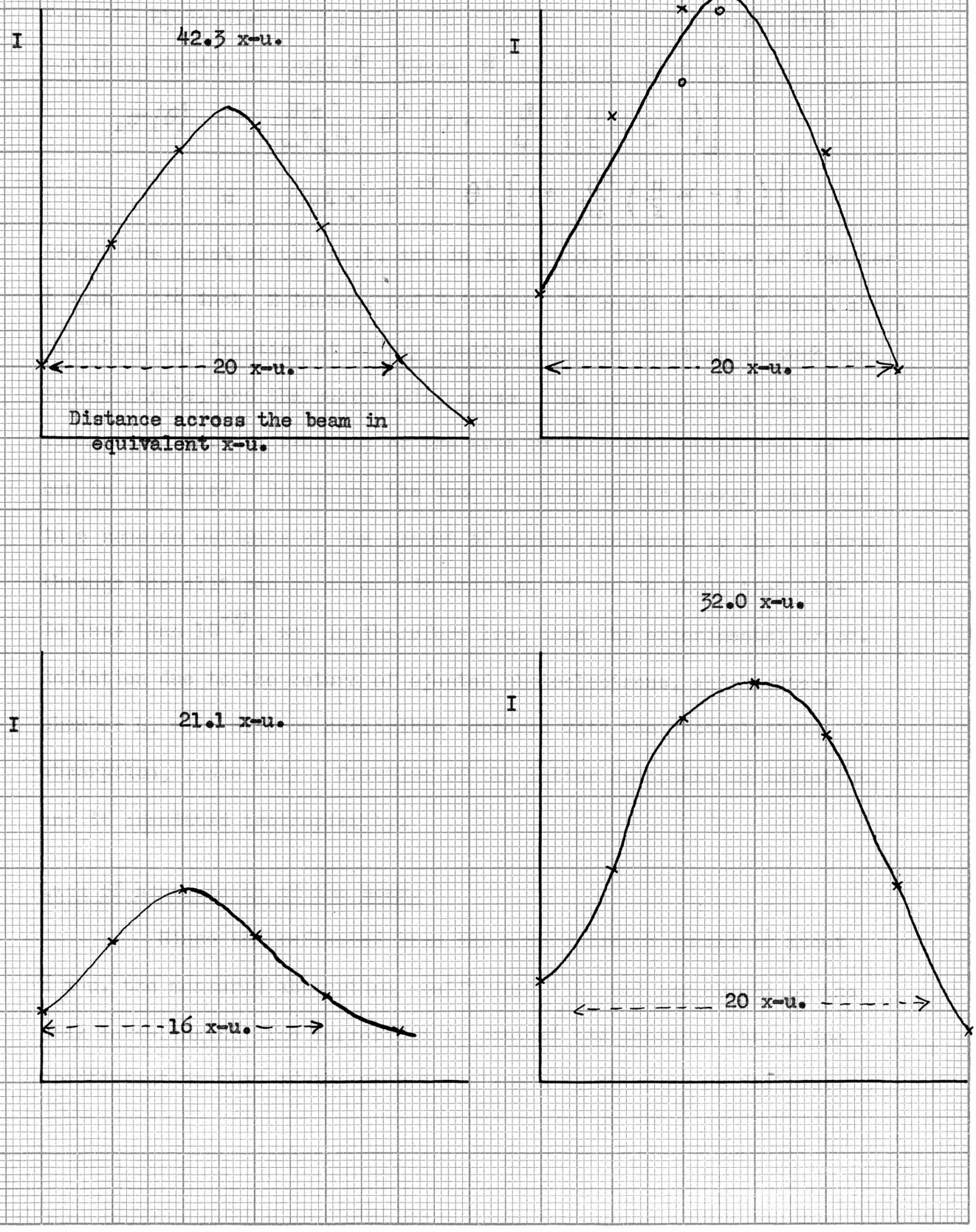
SOURCES OF ERROR

Scattering of x-rays into chamber L by the absorber:

X-rays scattered out of the beam, yet still able to enter chamber L, will cause the measured value of the absorption coefficient to be too small. This effect, per electron, will be less than that due to

Figure 8.

TYPICAL SHAPES OF THE X-RAY LINES,
AS EXPLORED WITH CHAMBER K.
WIDTH OF SLIT 11 x-u.



the electron in the absorber which is most efficient in scattering into chamber L. This electron is on the axis of the slit admitting radiation to L, and nearest to the slit. The slit was 4 inches by $\frac{1}{4}$ inch, and its more distant face was $10\frac{1}{2}$ inches from the nearest part of an absorber. From the Klein-Nishina formula, the fraction of energy per electron scattered inside an angle θ, where θ is small, is given by:-

$$\epsilon\sigma(\theta) = \frac{\pi e^4}{m^2c^4}\,\theta^2\left[1+\frac{\theta^2}{4}\left(\frac{11}{2}\alpha-1\right)\right]$$

$$= 2{\cdot}46\times10^{-25}\,\theta^2\left[1+\frac{\theta^2}{4}\left(\frac{11}{2}\alpha-1\right)\right]$$

Around the axis of the slit, and in its plane, draw circles of diameter $\frac{1}{4}$ inch, 3/4 inch, 2 inches, and 4 inches. By means of the formula above calculate the fraction of energy scattered by the electron through the centre circle, and the surrounding zones. The centre circle lies entirely within the slit;the fraction of the area of each zone which does so can be estimated. By taking the same fraction of the energy scattered in a zone an estimate of the amount of energy scattered by the electron through the slit can be made. In this way it can be shown that this is less than 10^{-27}, and is therefore less than the experimental error. Radiation due to the return of ejected photoelectrons, which enters chamber L, will also be negligible compared with the photoelectric absorption, since only a fraction of the latter reappears as radiation, and it is emitted isotropically.

Lack of homogeneity of the beam.

Since the reflected beam has a wave-length spread of about 6 x-u. the measured absorption coefficient will not correspond to the peak wave-length. The correction to convert the measured absorption coefficient to that of the peak wave-length is obtained as follows:-

Let λ represent the peak wave-length.

Suppose the wave-length limits of the line are $\lambda - \delta$ and $\lambda + \delta$.

Let the intensity incident on the screen with wave-length between $\lambda + x$ and $\lambda + x + dx$ be $I_{\lambda+x}\, dx$.

Suppose the absorption coefficient for this wave-length is $\mu_{\lambda+x}$.

Then if $I'_{\lambda+x}dx$ is the transmitted intensity of wave-length between $\lambda + x$ and $\lambda + x + dx$ we have:-

$$I'_{\lambda+x}\, dx = I_{\lambda+x}\, dx\, e^{-\mu_{\lambda+x} d}$$

For small values of δ we may assume $\frac{d\mu}{d\lambda}$ constant, so that

$\mu_{\lambda+x} = \mu_\lambda + sx$ where $s = \left(\frac{d\mu}{d\lambda}\right)_\lambda$.

Then $I'_{\lambda+x}\, dx = I_{\lambda+x}\, dx\, e^{-(\mu_\lambda + sx)d} = I_{\lambda+x}\, dx\, e^{-\mu_\lambda d} e^{-sxd}$

Now sxd is small compared with 1, so e^{-sxd} may be expanded.

$$I'_{\lambda+x}\, dx = I_{\lambda+x}\, dx\, e^{-\mu_\lambda d}\left[1 - sxd + \frac{s^2x^2d^2}{2} \cdots\right]$$

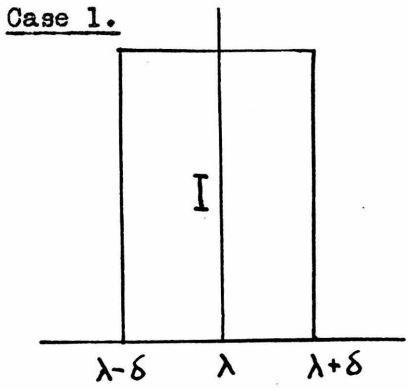
Similarly $I'_{\lambda-x}\, dx = I_{\lambda-x}\, dx\, e^{-\mu_\lambda d}\left[1 + sxd + \frac{s^2x^2d^2}{2} \cdots\right]$

Assume that the line is symmetrical.

Then $\left(I'_{\lambda+x} + I'_{\lambda-x}\right)dx = 2I_{\lambda+x}\, e^{-\mu_\lambda d}\left[1 + \frac{s^2x^2d^2}{2}\right]dx$

We must now make some assumption regarding $I_{\lambda+x}$ as a function of x.
Take two simple cases:-

Case 1.



Here $I_{\lambda+x} = I$ (a constant).

$$\therefore E' = 2\int_0^\delta I\, e^{-\mu_\lambda d}\left[1 + \frac{s^2x^2d^2}{2}\right]dx$$

$$= 2I\, e^{-\mu_\lambda d}\,\delta\left[1 + \frac{s^2\delta^2d^2}{6}\right]$$

Now $2I\delta$ = energy in the beam = E_0.

$$\therefore \quad E' = E_0 e^{-\mu_\lambda d}\left[1 + \frac{s^2\delta^2 d^2}{6}\right]$$

The measured value of $\mu_\lambda = \mu'_\lambda$ is defined by :-

$$E' = E_0 e^{-\mu'_\lambda d}$$

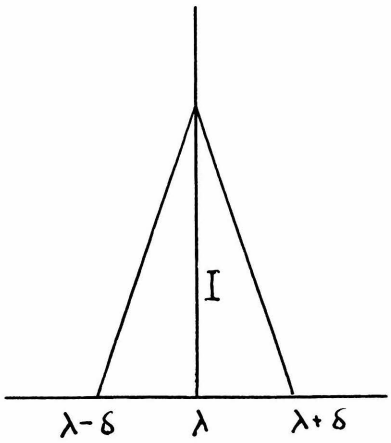
$$\therefore \quad e^{-\mu'_\lambda d} = e^{-\mu_\lambda d}\left[1 + \frac{s^2\delta^2 d^2}{6}\right]$$

$$-\mu'_\lambda d = -\mu_\lambda d + \log_e\left[1 + \frac{s^2\delta^2 d^2}{6}\right]$$

$$= -\mu_\lambda d + \frac{s^2\delta^2 d^2}{6}$$

$$\therefore \quad \mu_\lambda - \mu'_\lambda = \frac{s^2\delta^2 d}{6}$$

Case 2:-



Here $I_{\lambda+x} = I\left[1 - \frac{x}{\delta}\right]$

Proceeding as before we find :-

$$\mu_\lambda - \mu'_\lambda = \frac{s^2\delta^2 d}{12}$$

A real line will be midway between these two cases.

Therefore $\mu_\lambda - \mu'_\lambda = \frac{s^2\delta^2 d}{8}$

This correction is negligible except for lead, in which case it becomes of the same order of magnitude as the likely experimental error.

Errors of measurement:

The eye-piece scales of the microscopes in the ionization chambers had 100 divisions. The position of the fibre could be read with an accuracy of 1/10 th of a division. Eight observations of fibre positions were necessary to obtain one value of the absorption coefficient. In addition an error of 1% might possibly be made in determining the mass per square cm of the absorbing screens. If we assume all nine errors cumulative they would lead to an error of 3% in the final value. The target of the x-ray tube was ~~covered~~ carried by a steel column 15 feet long and was cooled by water. When the observations on aluminum and carbon were made the water was circulated by thermo-syphon action which was not dependable. A small change of temperature would cause the target to move with respect to the slit system, altering the ratio of response of the chambers and also the amount of back ground radiation. This was the most troublesome source of error as it could be detected only by a diminution of x-ray intensity, and by the lack of agreement between values of the absorption coefficient obtained under otherwise identical conditions. It is probable that the values in Table 4 , marked by asterisks, were affected by such an occurrence. Before the results for the heavier elements were obtained a water-pump was installed in the cooling system. This produced a great improvement, but the trouble was not completely eliminated. Some of the results for lead showed a progressive hardening of the radiation with increasing absorber thickness. Such results were rejected and the trouble was reduced by splaying the slits so that less of their surfaces was illuminated, so that less softer scattered radiation was produced.

RESULTS OF THE ABSORPTION MEASUREMENTS

(1) Aluminum and Carbon:

In Table 3 all the values of e/μ obtained for aluminum and carbon are included. They were all calculated in an identical manner and no observations were rejected. In Table 4 the means are tabulated with the values to be expected from the Klein-Nishina formula. Mention has been made of a possible cause of the larger probable errors of the four values marked by asterisks. If a smooth curve were drawn among all the points these four would deviate most from it. In each case the value obtained with the same wave-length, but with the other element has a much smaller probable error and would fall much closer to the curve. It would therefore seem that little weight should be attached to the four values marked by asterisks, as compared with the corresponding values for the other element. The weighted means in Table 4 are obtained by compounding the values for aluminum with those for carbon, giving each a weight inversely proportional to the square of its probable error. If these weighted means were plotted, and a smooth curve were drawn among them, it would not deviate from the Klein-Nishina curve by more than one percent at any point between 50 and 20 x-u. Evidence will be given later that e/τ for aluminum and carbon in this wave-length range, should be less than the probable error in the measurement of e/μ, so that values of the latter may be regarded as values of e/σ.

It will be noticed from the values in Table 3, that in three cases, viz:- carbon 24.8 x-u., aluminum 36.5 x-u., and carbon 50 x-u., the deviation from the Klein-Nishina value is largely due to a single bad value.

It is considered that these observations check the Klein-Nishina formula to within one percent.

TABLE 3

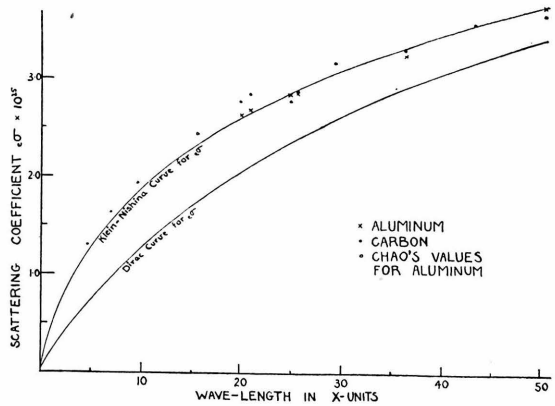
Values of $\rho\mu$ for Aluminum and Carbon

Wave-length in x-u.	Experimental Values. x 10^{25}	Mean$_{25}$ x 10^{25}	Most probable$_{25}$ error x 10^{25}	Experimental Values x 10^{25}	Mean$_{25}$ x 10^{25}	Most probable$_{25}$ error x 10^{25}
19.9	2.62 2.68 2.65 2.57	2.63	0.01	2.67 2.78 2.75 2.85	2.76	0.03
20.8	2.58 2.71 2.75	2.68	0.03	2.80 2.74 3.00	2.84	0.10
24.8	2.74 2.81 2.88 2.95	2.85	0.02	2.85 2.82 2.63	2.77	0.05
25.6	2.87 2.85	2.86	0.01	2.88 2.89	2.88	0.01
36.5	3.43 2.95 3.18 3.53 3.12	3.24	0.07	3.28 3.35 3.30	3.31	0.01
50	3.74 3.70 3.82 3.74	3.75	0.02	3.69 3.72 3.51	3.64	0.04

TABLE 4.

~~TABLE I~~. *Measured values of x-ray scattering coefficient and comparison with Klein-Nishina values.*

Wave-length λ (x.u.)	Δλ (x.u.)	Element	Experimental value ($\times 10^{25}$)	Probable error ($\times 10^{25}$)	No. of obs.	Weighted mean ($\times 10^{25}$)	Deviation from Klein-N. value	Klein-Nishina value ($\times 10^{25}$)
50.4	4.1	Aluminum	3.75	0.02	4	3.73	−0.5%	3.755
		Carbon	3.64	0.04	3			
36.3		Aluminum	3.24*	0.07	5	3.31	−1.2	3.350
		Carbon	3.31	0.01	3			
25.6	5.0	Aluminum	2.86	0.01	2	2.87	−1.0	2.902
		Carbon	2.88	0.01	2			
24.8	3.7	Aluminum	2.85	0.02	4	2.84	−0.7	2.864
		Carbon	2.77*	0.05	3			
20.8	3.3	Aluminum	2.68	0.03	3	2.69	+1.1	2.657
		Carbon	2.85*	0.10	3			
19.9	3.7	Aluminum	2.63	0.01	4	2.64	+1.1	2.606
		Carbon	2.76*	0.03	4			



~~FIG. 3~~.

Figure 8

(2) Lead.

The experimental results for lead are given in table 5 . In figure 10 is plotted the sum of τ calculated from Gray's formula and σ calculated from the Klein-Nishina formula. The spread of experimental points is indicated by a vertical bar, against which is written the number of values which lie within the length of the bar. It is seen that there is no systematic variation from the curve between 53.8 and 32 x-u., but between 26 and 20 x-u. the values all lie high. It is just possible that this may be due to some systematic error effective with short wave-lengths; further experiment must decide this point. However, it is considered that the 24 separate values, made on four occasions, with two different screens, all lying above the curve, afford strong evidence that Gray's empirical photoelectric curve is too low in this region.

The experimental values have been fitted with a curve by the method of least squares. Gray made his curve pass through Allen's value τ cm^{-1} = 37.2 at 100 x-u. When this value and the value $\tau = 0$ when $\lambda = 0$ are included along with the values in table 5 the following law is obtained:-

$$\tau \text{ cm}^{-1} = 0.02540\,\lambda - 0.0001570\,\lambda^2 + 0.00003624\,\lambda^3 \quad \text{(Formula A.)}$$

where λ is in x-u.

This curve is plotted in figure 11 .

Between 100 x-u. and 38 x-u. it lies too close to Gray's curve to be distinguished from it on the scale of the diagram.

In table 7 are some values of τ cm^{-1} obtained by Allen for wave-lengths between 80 and 102 x-u. It will be noticed that these by no means agree with Gray's values. This casts doubt on the usefulness of Allen's value at 100 x-u. as a basis for the formula. Therefore the

Figure 10.

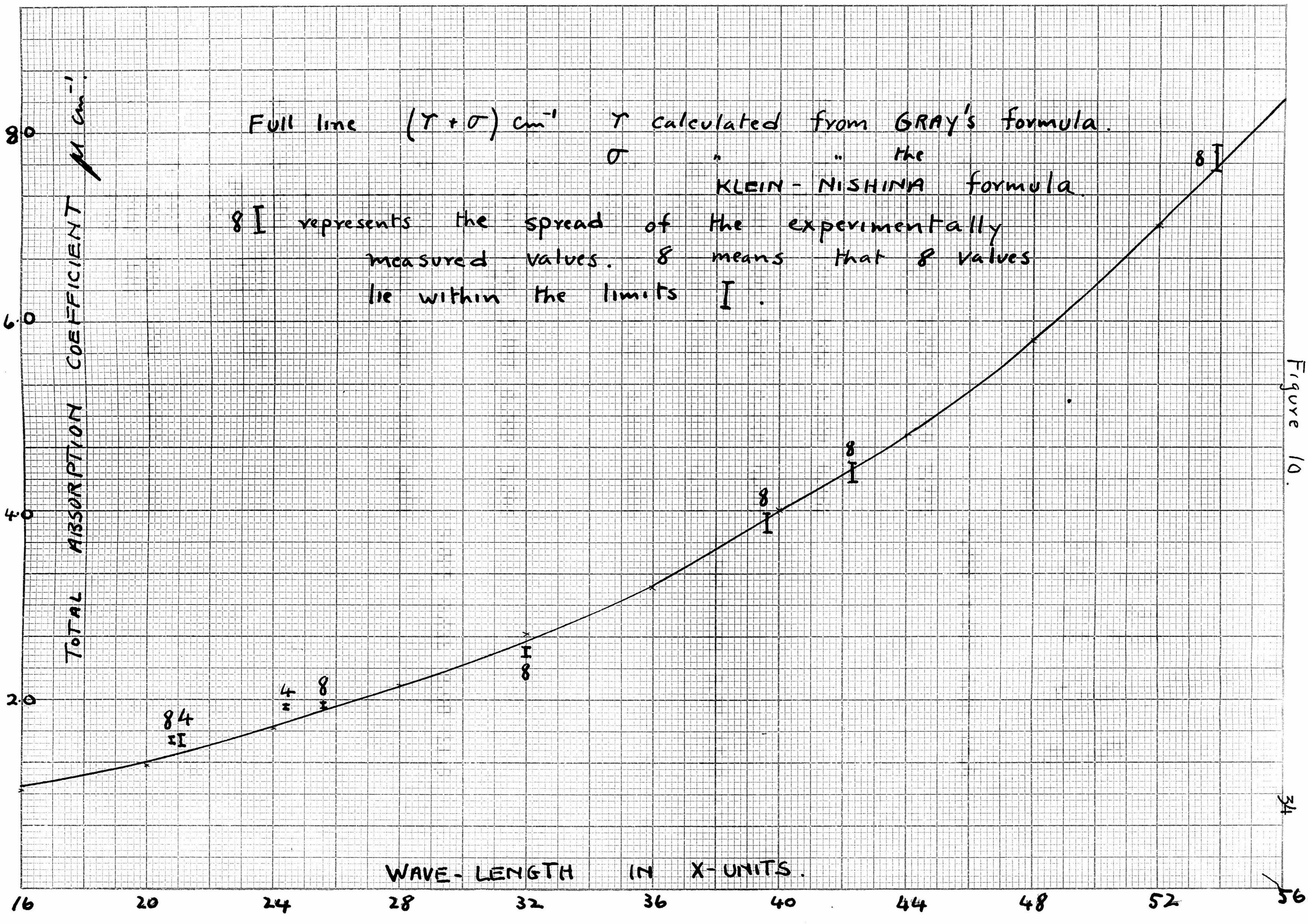

Full line (τ + σ) cm⁻¹
τ calculated from GRAY's formula.
σ " " the KLEIN - NISHINA formula.
8 I represents the spread of the experimentally measured values. 8 means that 8 values lie within the limits I.
WAVE-LENGTH IN X-UNITS.
TOTAL ABSORPTION COEFFICIENT μ cm⁻¹
16
20
24
28
32
36
40
44
48
52
56
2.0
4.0
6.0
8.0

TABLE 5

Experimental results for Lead.

Peak wave-length in x-u.	Thickness in mm.	Experimental Values of $\mu\rho$ x 10^{27}.	Mean $\mu\rho$ x 10^{27}.	Mean μ cm^{-1} uncorrected.	Correction for inhomogeneity.	Corrected μ cm^{-1}.	Mean for one wave-length.	σ cm^{-1} from K-N. formula.	τ cm^{-1}.	$e\tau$ x 10^{25}
53.8	1	2897 2969 2875 2904	2911 ±14	7.688	0.009	7.697				
	2	2963 2939 2935 2914	2937 ± 7	7.757	0.018	7.775	7.736	1.015	6.72	25.4
42.3	1½	1637 1663	1650	4.358	0.009	4.367				
	2	1685 1702	1693	4.471	0.012	4.483				
	2½	1642 1631	1636	4.321	0.015	4.336	4.398	0.932	3.47	13.2
	3	1682 1640	1661	4.387	0.018	4.405				
39.6	2	1427 1445 1466 1473	1453 ± 8	3.837	0.009	3.846				
	3	1492 1502 1456 1497	1487 ± 8	3.927	0.014	3.941	3.893	0.911	2.98	11.3
		x 10^{26}								
32.0	3	9681 9310 9683 9619	9573 ±61	2.528	0.007	2.535				
	5	9417 9594 9457 9529	9499 ±27	2509	0.011	2.520	2.523	0.839	1.68	6.36

Peak wave-length in X-U.	Thickness in mm.	Experimental values of $e^{\mu} \times 10^{26}$.	Mean $e^{\mu} \times 10^{26}$.	Mean μ cm^{-1} Uncorrected.	Correction for inhomogeneity.	Corrected μ cm^{-1}.	Mean for one wave-length.	σ cm^{-1} from K-N. formula	τ cm^{-1}	$e^{\tau} \times 10^{25}$
25.6	5	7129 7163 7153 7491	7234 ±60	1.910	0.004	1.914				
							1.945	0.767	1.18	4.47
	7	7466 7535 7498 7302	7450 ±36	1.968	0.009	1.977				
24.4	5	7274 7343 7348 7267	7307 ±15	1.930	0.005	1.935	1.935	0.750	1.18	4.47
21.1	5	5852 5779 6009 5932	5893 ±36	1.556	0.004	1.560	1.560	0.705	0.855	3.24
20.8	5	5839 5879 6005 5961	5921 ±26	1.562	0.004	1.566				
							1.562	0.700	0.862	3.26
	7	5891 5972 5838 5818	5880 ±23	1.553	0.005	1.558				

The conversion factor used to convert values of e^{τ} to τ cm^{-1} was 2.641×10^{24}, based on a density of lead of 11.005.

TABLE 6

Absorption coefficients $\epsilon\mu$ for $\lambda = 4.7$ x-u. x 10^{27}

Element	Atomic number	Chao	Meitner	Jacobsen	Tarrant
Carbon	6		111.6		128.8
Aluminum	13	129.2	131.8		128.9
Iron	26		136.2		133.2
Copper	29	134.8	137.3		135.6
Tin	50	147.5	158.4		146.8
Lead	82	170.2	173.3	174.7	171.5

TABLE 7

Allen's values for τcm^{-1} for lead.

Wave-length in x-u.	81	85	90	95	102
Allen's measured value τ cm^{-1}	26.7	29.4	31.8	36.8	41.7
Value calculated from Gray's formula	20.3	23.3	27.4	32.0	39.4
Value calculated from formula A	20.5	23.3	27.4	32.1	39.3
Value calculated from formula B	23.3	26.41	31.5	37.2	46.4

TABLE 8

Gentner & Chao's values for τcm^{-1} for lead.

Wave-length in x-u.	7.0	9.6	15.5	29.4	47	Chao.
τ cm^{-1}	0.2	0.26	0.41	1.19	3.86	
Wave-length in x-u.	4.7	5.9	6.6	7.9	9.3	Gentner.
τ cm^{-1}	0.14	0.13	0.14	0.14	0.17	

best curve passing through the origin and the points up to 53.8 x-u. has been found. This is:-

$$\tau \text{ cm}^{-1} = 0.04476\lambda - 0.001313\lambda^2 + 0.00005226\lambda^3 \quad \text{(Formula B.)}$$

Values calculated from this formula for τ between 80 and 102 x-u. are given in table 7 for comparison with the values of Gray and Allen. The agreement is better, except for $\lambda = 102$ x-u. This, however, is probably the most accurate of Allen's values.

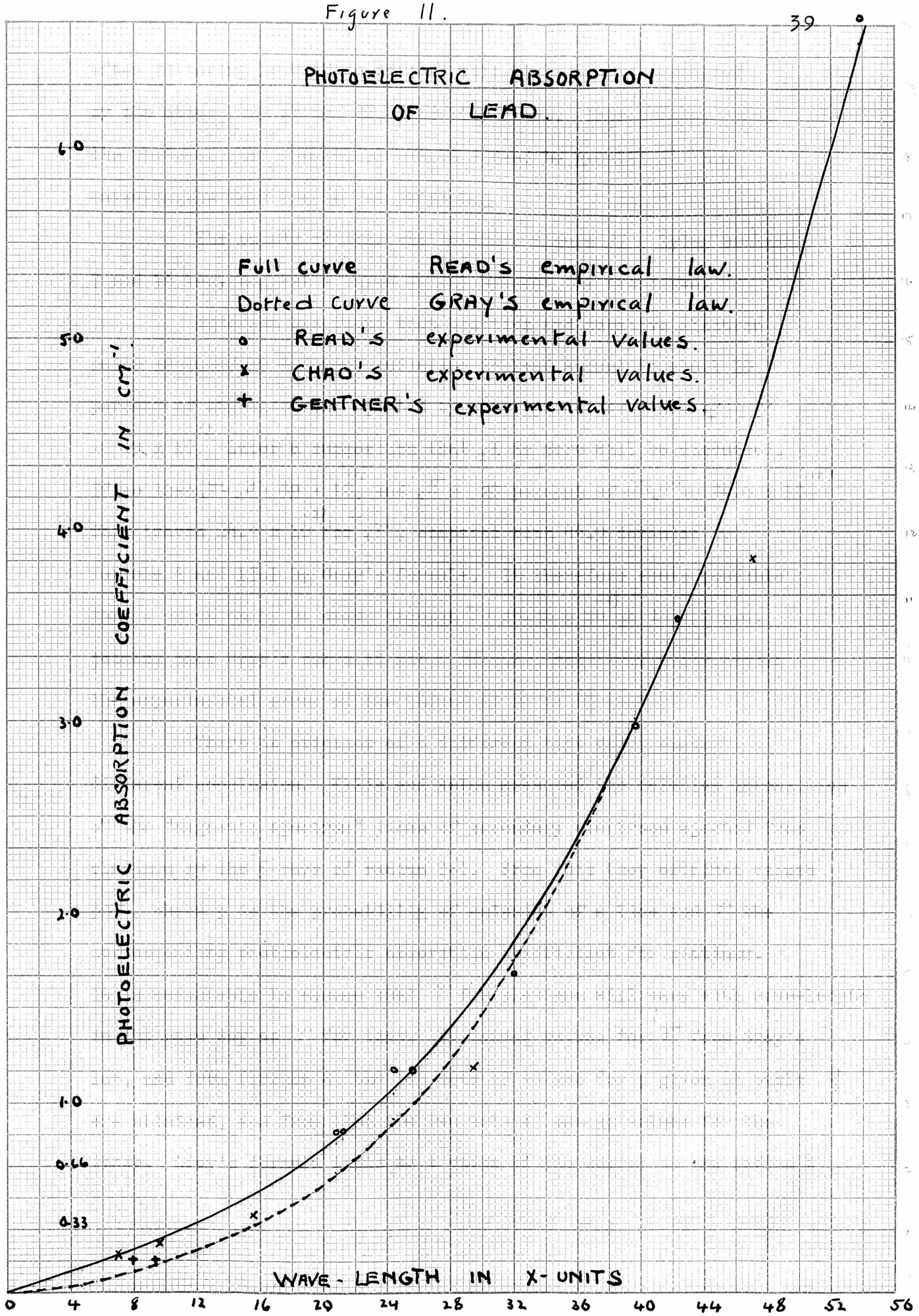
At 4.7 x-u. formula	A gives	4.49×10^{-26}
" " " "	B "	7.08×10^{-26}
" " " "	of Gray gives	1.31×10^{-26}
At 4.7 x-u. $e\mu$ =		17.24×10^{-26}
$e\sigma$ =		12.33×10^{-26}

Therefore even if no allowance is made for π and K, τ could scarcely be greater than 5.0×10^{-26}. Therefore formula A is preferred.

Let us compare the predictions of formula A with all known information regarding the photoelectric absorption coefficient. Sauter calculated τ for the K electrons of lead to be 2.5×10^{-24}. If we assume this to be 5/6 of the total for the atom we get $_e\tau = 3.68 \times 10^{-26}$, compared with 4.49×10^{-26} from formula A and 1.31×10^{-26} from Gray's formula. McDougall and Hulme have calculated τ for the K shell, $Z = 84$, $\lambda = 9.15$ x-u. to be 4.0×10^{-24}. This gives 3.6×10^{-24} for the K electrons of lead, which should be about 4/5 that for the whole atom. Therefore $_e\tau = 5.5 \times 10^{-26}$. Formula A gives 7.55×10^{-26} while Gray's formula gives 4.30×10^{-26}.

It has been mentioned that Kohlrausch found the absorption coefficient for radium B+C γ-rays, with an aluminum lined ionization chamber and 0.35 cm filtration to be 1.0 cm^{-1}. Gray's law applied to the γ-ray energy distribution as determined by Skobelzyn leads to a value of 0.87 cm^{-1}. The first three will have been absorbed strongly in the 0.35 cm filter; the bulk of the remaining energy will lie in the rays of wave-length 20.2, 10.9, and 6.95 x-u. The rate of absorption

Figure 11.


PHOTOELECTRIC ABSORPTION OF LEAD.
Full curve READ'S empirical law.
Dotted curve GRAY'S empirical law.
o READ'S experimental values.
x CHAO'S experimental values.
+ GENTNER'S experimental values.
PHOTOELECTRIC ABSORPTION COEFFICIENT IN CM⁻¹.
6·0
5·0
4·0
3·0
2·0
1·0
0·66
0·33
WAVE-LENGTH IN X-UNITS
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56

after filtering by 0.35 cm of lead will therefore be dominated by the absorption of the 20.2 and 10.9 lines. This suggests that the photoelectric absorption should indeed be greater at these wave-lengths than Gray's law predicts.

The points of Chao and Gentner, figure 11 , lying between 7 and 16 x-u. suggest a curve midway between the two drawn.

Anderson has found that positive and negative electron pairs produced by 4.7 x-u. radiation in lead amount to about 20 percent of the electrons ejected by other processes. Now $_e\mu - _e\sigma = 49 \times 10^{-27}$. Later a suggestion that $_e K$ is zero will be discussed. If so then $_e\tau + _e\pi = 49 \times 10^{-27}$, and $_e\pi$ = 20 percent of $_e\tau + _e\sigma = 24.6 \times 10^{-27}$. This leaves 24.4×10^{-27} for $_e\tau$, compared with 44.9 demanded by formula A and 13.1 by Gray's formula. It therefore seems possible that a curve could be drawn through $\tau = 25\text{-}30 \times 10^{-27}$ at $\lambda = 4.7$ x-u. joining onto Gray's curve at about 38 x.u., which would lie within the experimental error of the points at 20 to 25 x-u.

There is evidence of a different type which suggests a different conclusion. Ellis and Aston measured the relative intensities of the "magnetic spectrum" lines of secondary electrons ejected from platinum by the γ-rays of radium B+C. From this they obtained values $p\tau$, where p is the probability of emission of a line and τ the corresponding photoelectric absorption coefficient for platinum. It is reasonable to assume that τ for platinum will vary with wave-length in the same way as τ for lead. By taking values for τ from Gray's law, and from formula A, the two sets of values for p given in table 9 are obtained, and from them the two sets of energy values for the lines. Gray's formula leads to a ratio of the energy in the radium B

TABLE 9

Energy distribution in the radium B C spectrum.

Line photon energy in e.v. x 10^6	Wave-length.	p	p from Gray's law.	Energy from Gray's law.	p from formula A.	Energy from formula A.
Radium B.						
2.43	50.8	67	11.5	27.44	11.88	28.87
2.97	41.6	89	25.8	76.63	26.22	77.87
3.54	34.9	100	45.0	159.3	44.72	158.31
				264		265
Radium C.						
6.12	20.2	40	65.8	402.7	53.58	327.3
7.73	16.0	2.4	6.5	50.2	4.67	36.1
9.41	13.1	1.6	6.7	63.1	4.12	38.8
11.30	10.9	3.4	20.6	232.8	11.15	126
12.48	9.89	0.85	6.3	78.6	3.14	39.1
13.90	8.88	0.70	6.4	89.0	2.93	40.7
14.26	8.65	-				
17.78	6.95	1.8	25.8	440.9	9.94	176.82
22.19	5.56	0.35	7.4	164.2	2.46	54.7
				1521		839

spectrum to that in the radium C spectrum of 0.17. Formula A leads to a value of 0.32. A reliable experimental value is 0.13± 0.02. This indicates that even Gray's law gives too high values for τ. There is little doubt that the values for τ for the lines of the radium B spectrum are correct, so that the energy of the radium B spectrum should be 265 arbitrary units (table 9). Therefore that of the radium C spectrum should be 2040 arbitrary units, instead of 1521, given by Gray's law. It would be difficult to obtain the extra 520 units without lowering the value of τ at 20 x-u., thus making the discrepancy between it and the experimental value even greater. Possible explanations are:-

(1) The experimental error for points between 20 and 25 x-u. is greater tham estimated.

(2) Ellis and Aston's values for $p\tau$ are incorrect.

(3) Absorption other than photoelectric and Klein-Nishina absorption commences around 25 x-u. and so influences the shape of the curve that for shorter wave-lengths it more nearly represents the total rather than the photoelectric absorption, which remains when the Klein-Nishina absorption has been subtracted.

(3) Iron and Copper.

The following values for τa^{-1} for iron and copper have been obtained:-

	24.4 x-u.	49 x-u.
Iron	0.042	0.18
Copper	0.057	0.31

In figure 12 log τ is plotted against log Z.

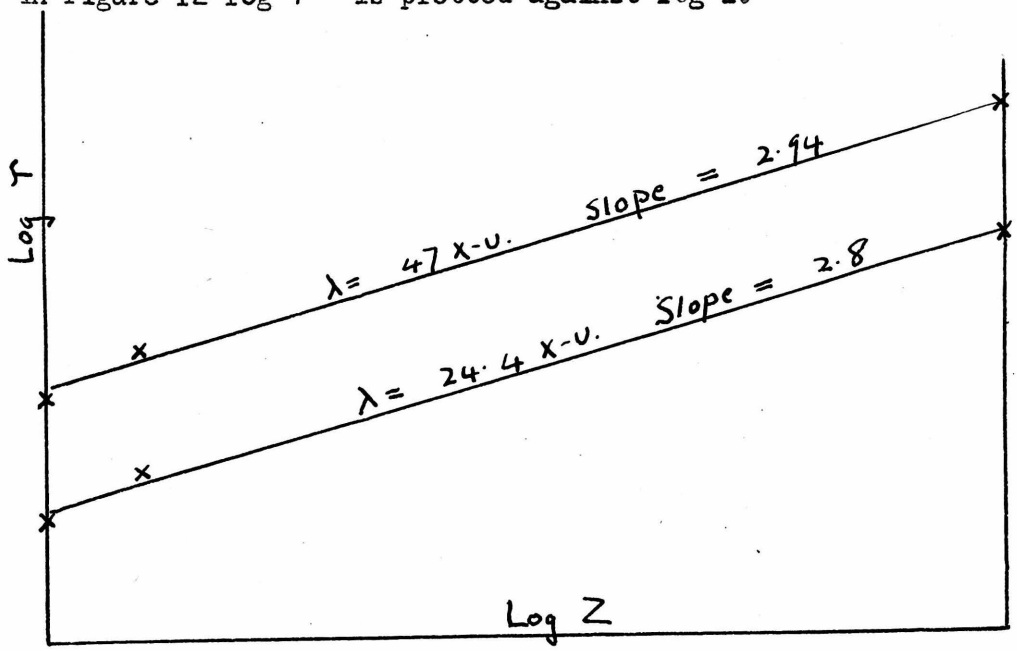


fig. 12

The slopes of the lines are 2.94 and 2.80, lying within Ahmad's limits 3.0 ± 0.5. It therefore seems that between 24 and 47 x-u. the variation of τ with Z is closely $\tau \propto Z^{2.9}$.

If we calculate the values of τ for aluminum and carbon from this law and the values of τ for lead, it is found that the assumption made earlier that this is less than the experimental error in the measurement of μ is confirmed.

DISCUSSION OF NUCLEAR ABSORPTION

From table 10 it can be seen that all of the radiation ascribed to nuclear scattering or reradiation falls into two bands:- (i) of wave-length 7-10 x-u., (ii) of wave-length 20 - 30 x-u. The total absorption coefficient for $\lambda = 4.7$ x-u. in lead is 172.4×10^{-27}. The Klein-Nishina value of $_e\sigma$ is 123.3×10^{-27}. If we assume the energy of the positive electrons comes from the γ-ray beam the number observed would require an absorption of 25×10^{-27}. If as a minimum, we assume a value for τ equal to half that predicted by Gray's formula, this would require 6.5×10^{-27}. Any nuclear absorption could therefore not exceed 17×10^{-27} per electron, or 1400×10^{-27} per atom. From table 11 it will be seen that Gray and Tarrant observed a nuclear absorption per atom of 3150×10^{-27} as reradiated energy. This suggests that at least the major part of it must consist of the return of radiation considered as absorbed. This could occur in two ways:-

(1) From the annihilation of positive electrons radiation of wave-length 24 x-u. will be obtained.

(2) The 4.7 x-u. γ-ray passing through lead will produce Compton and photoelectrons of energy up to about 2.5×10^6 e.v. These will be capable of producing "bremstrahlung".* It is possible that this radiation is analysed by the absorption curve method into bands of wave-length around 10 x-u. and 30 x-u. This would account for the hard band, while the softer "bremstrahlung" might account for the low value which Gray and Tarrant found for the average energy of the soft quanta.

It is therefore suggested that the most coherent picture of all the nuclear absorption phenomena can be obtained by assuming that the only types of absorption are (i) Klein-Nishina (ii) photoelectric

and (iii) absorption with the production of positive and negative electron pairs. The reradiated energy consists of half-million volt quanta due to the annihilation of positive electrons, and a "bremstrahlung" spectrum, due to the fast Compton and photoelectrons.

* I am indebted to Dr. R. Oppenheimer for this suggestion.

TABLE /o

Summary of results on nuclear absorption.

Experimenter	Source of primary rays	Element	Soft Component μ cm^{-1} in Pb	λ	Hard Component μ cm^{-1} in Pb	λ	Ratio of intensities. Hard:soft.
Meitner & Hupfeld	Radium B C 3cm Pb filtration	Pb			Same as primary beam.		
Meitner & Kosters	Thorium C" 3cm Pb filtration	Pb Fe			Same as primary beam.		
Heiting	Thorium C"	Al	1.55	23.8			
		Fe	1.50	23.0			
		Cu	1.41	21.8			
		Pb	1.75	26.5	0.59	6.6	Not stated.
Gray & Tarrant	Thorium C"	C) K)	2.0	27.4			
		Fe)					
		Sn	2.0	27.4	0.75	11.3	0.14
		Pb	1.9	26.8	0.75	11.3	0.32
	Radium B C	Fe	2.6	32.6	0.75	11.3	0.04
		Sn	2.4	30.8	0.75	11.3	0.06
		Pb	2.1	28.0	0.75	11.3	0.12
Stahel & Ketelaar	Radium B C	Fe		22.0			
		Sn		23.0			
		Pb		31.0		17	0.07

TABLE

Nuclear absorption coefficients.

Worker	Source	Coefficient	Pb	Sn	Cu	Fe	C
M. & H.	Ra B+C	$_{a}K$ hard $\times 10^{27}$	500				
M. & K.	Th C"	$_{a}K$ hard $\times 10^{27}$	1,010			337	
G. & T.	Ra B+C	$_{a}K$ hard $\times 10^{27}$	132				
		$_{a}K$ soft $\times 10^{27}$	1,880				
	Th C"	$_{a}K$ hard $\times 10^{27}$	485	99	15	10	0
		$_{a}K$ soft $\times 10^{27}$	2,670	950	315	282	19
S. & K.	Ra B+C	$_{a}K$ hard $\times 10^{27}$	311				
		$_{a}K$ soft $\times 10^{27}$	1,380				

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