ABSORPTION MEASUREMENTS ON VERY HARD X-RAYS

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I. INTRODUCTION

Many - perhaps most - measurements of x-ray absorption coefficients have been made with inhomogeneous radiation, using the "end radiation" method. This method possesses the great advantage of involving but little apparatus and a simple experimental technique, but the measurements are difficult to interpret and it is usually impossible to determine the relation between the apparent absorption coefficient as determined by this method and the monochromatic absorption coefficient. Moreover, there are many pitfalls for the experimenter concealed in this apparently simple method, and it is the purpose of this paper to point out some of these pitfalls.

If the published values of absorption coefficients obtained by this method are compared, it is found that there is in general little agreement among the values obtained by various experimenters under supposedly identical conditions, and that most of these values are much higher than the values computed from the formulae of Klein and Nishina, Richtmyer, and others. This lack of agreement, together with the fact that the present active development of tubes for very high voltages will involve the taking of many new measurements by this method, has made it desirable to study the method in some detail, and this paper will discuss chiefly the process of "monochromatizing" radiation by filtration.

All of the experimental work reported herein was carried out at 550 kv. and the conclusions drawn from the work are to be regarded as applying strictly only to this type of radiation, namely, the continuous spectrum emitted by x-ray tubes working at 300 kv. and higher, although with suitable modifications they will also apply to softer radiation. They do not apply to measurements of absorption made with spectrographs, nor to methods making use of fluorescent or characteristic radiation, or of absorption edges, such as the balanced filter method of P. A. Ross. Finally, they do not apply to spectra consisting largely or wholly of well separated sharp lines or groups of lines, such as gamma-ray line spectra. Within these limits, however, it is believed that the conclusions are at least qualitatively correct.

II. THE ABSORPTION OF INHOMOGENEOUS X-RAYS: FILTRATION

The end-radiation method consists of interposing between a source of radiation and an intensity-measuring device a sample of the material to be studied, reading the radiation intensity with and without the sample in place, and computing from these readings and the thickness of the sample the absorption coefficient of the material. This procedure is carried out for successively greater and greater thicknesses, preferably using the same increment of thickness between each two readings, and if the absorption coefficient as determined in this way is plotted against thickness it will usually be found that a curve is obtained which descends more or less steeply for small thicknesses and then flattens out and approaches a more or less constant value for great thicknesses. A typical curve of this sort is shown in Fig. 1.

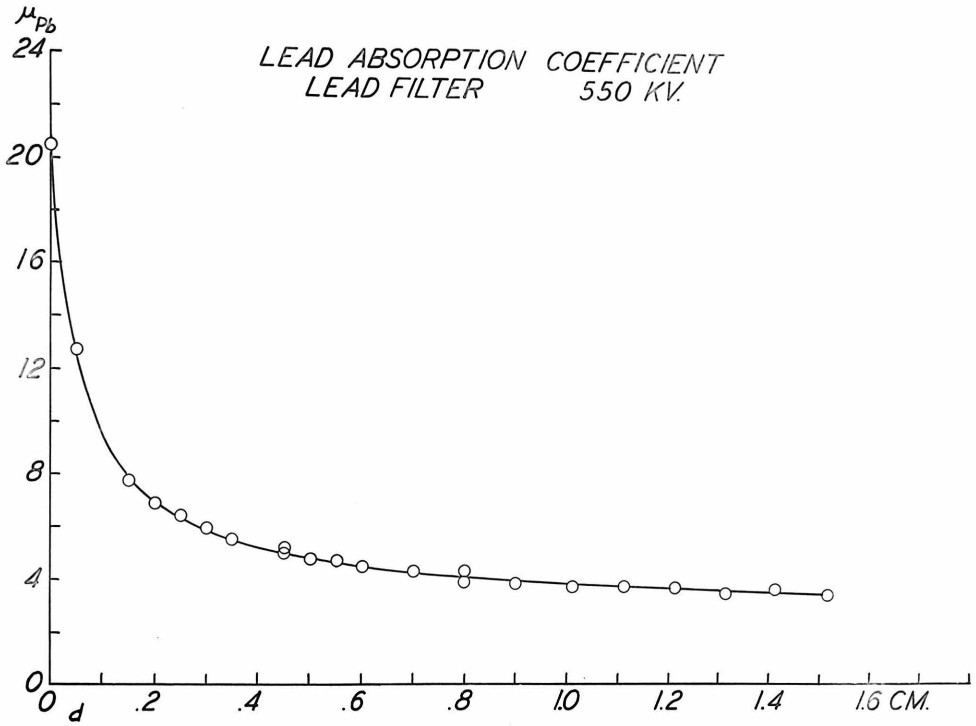

Figure 1. Typical Absorption Coefficient Curve.

This constant value is then taken as the absorption coefficient of the material for a wavelength identified with the short-wavelength limit of the original spectrum.

Such apparent absorption coefficients, obtained by the end-radiation method, have very little physical significance for two reasons. In the first place, small variations in the geometry of the set-up, in the experimental procedure, or in the method of computation, may produce considerable variations in the actual value of the absorption coefficient obtained. Second, and more important, is the question as to the value of the "effective wavelength" with which this coefficient is to be identified, and this point will be considered at some length in the following sections.

II-1. THE CONTINUOUS X-RAY SPECTRUM AT HIGH VOLTAGES

Since in using the end-radiation method we are working with the whole spectrum at once, and since the attenuation produced by an absorber is a function of wavelength, it is obviously desirable to know the relation between wavelength and intensity in our original spectrum. This relation has been established with considerable accuracy in the case of tubes working at low voltages; probably the best work on the subject is that of Kulenkampff (1). No comparable work has been done at voltages of the order of those used in the present investigation, and indeed the only published spectrum of a tube working above 200 kv. is that contained in Lauritsen's paper (2). This spectrum proves that there are no discontinuities in the region above 200 kv., but it is practically useless for the determination of the relation between actual intensity and wavelength in the absence of any knowledge of the relation between photographic density and x-ray intensity at these wavelengths. A number of attempts were made to utilize the microphotometer curve of this spectrum, as well as a set of similar curves just taken by Mr. C. E. Buffum at 550 kv. with an improved spectrograph, but they were finally abandoned since all of the spectrograms were taken with intensifying screens, which radically alter the photographic blackening law.

In the absence of any usable experimental data it was necessary to employ the formula of Kulenkampff as modified by Nicholas (1). This application to 550 k. v. radiation of an experimental formula established in the region below 10 kv. and confirmed at voltages only up to 50 kv. represents a rather extreme degree of extrapolation, but it is justified to a certain extent by the theoretical work of Kramers and of Wentzel (1).

It should be noted also that the formula applies only to the case of a beam of homogeneous cathode rays, whereas our tube was worked self-rectifying and we had all electron velocities present from zero to our maximum of 550 kv. This fact is not as serious as it seems, however, since a reference to the oscillogram (Fig. 6) will show that most of the current passed through the tube while the applied voltage was within a few percent of its maximum value, and since the X-ray excitation efficiency of cathode rays decreases linearly with decreasing voltage. The effect of using an alternating voltage on the tube is to decrease the intensity at the shorter wavelengths with respect to that at the longer, but the amount of this decrease is not very great, and is difficult to compute from the oscillograms.

The Kulenkampff formula actually gives an intensity curve which requires modification in two respects. In the first place the sharp corner at the short-wavelength limit must be rounded off somewhat in order to correspond to the experimental curves. In the second place, the intensity does not actually increase linearly with decreasing voltage below about 0.6 times the peak voltage, but falls off, giving a maximum at about one-half the peak voltage. These two modifications were made in the intensity curve which was finally chosen as a basis for computing the effect of filtration. Figure 2 shows the curve as given by the formula and as modified for the computations.

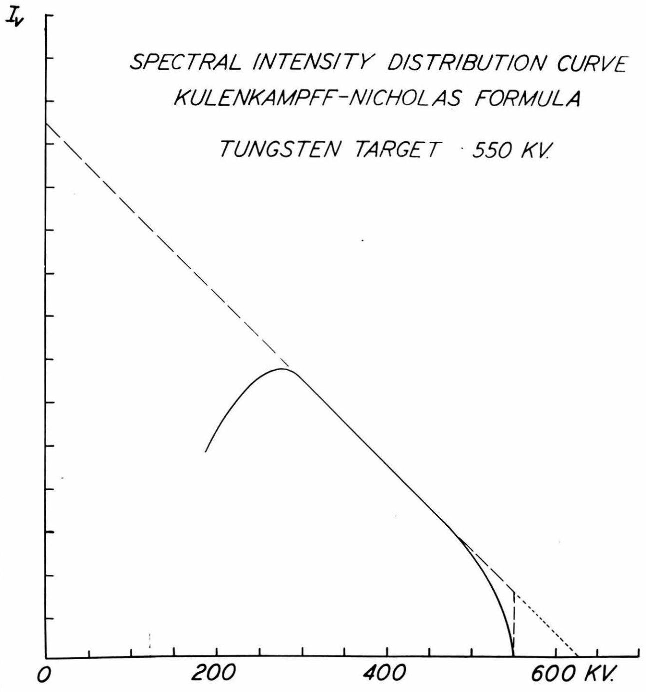

Figure 2

II-2. DEGRADED RADIATION: "SOFTENING" BY FILTRATION

In general, when a beam of heterogeneous x-rays passes through a filter two types of phenomena occur. In the first place, all of the wavelengths of the original spectrum are reduced in intensity by an amount depending upon the wavelength. In the second place, while much of the energy abstracted from the incident beam disappears as radiation, some of it is changed into radiation of greater wave-length. This is accomplished in two ways; some of the quanta are absorbed by atoms of the filter and excite them, and in returning to their normal state these excited atoms emit softer fluorescent radiation; and other quanta are scattered and softened by Compton encounters. Both fluorescent and Compton radiation are emitted in all directions, and some of this softened radiation will appear in the emergent beam and will increase the reading of the device used to measure its intensity.

The computation of the intensity of this softened radiation in the emergent beam is difficult, since the phenomenon is complicated by absorption in the filter and by the directional variation of the intensity of the secondary radiation, and in order to minimize its effect it is very desirable to make the measurements of intensity of the emergent beam at as great a distance from the filter and absorber as practicable. A very good discussion of the errors introduced into absorption measurements by this factor is given by Kohlrausch (3). –

If the emergent beam contains any considerable fraction of secondary radiation the "center of gravity" of its spectrum, and therefore its effective wavelength, will be shifted toward the longer waves; the beam will be partially softened. This softening effect may be small, or it may be great enough to more than counteract the hardening effect of the filter. One way to test for the presence of any considerable amount of such softened radiation, when the filter is made of a different material from that of the absorber, is to interchange the positions of the two; if the exponential absorption law is approximately followed such a change in the geometry of the beam will make no difference in the apparent absorption coefficient. Several such tests were made in the course of this work, and it was found that in each case the coefficients agreed within the errors of measurement.

II-3. THE EFFECT OF FILTRATION ON THE BEAM

If we assume that the exponential absorption law is followed and that there is no appreciable amount of softened secondary radiation present in the emergent beam, we can make an approximate calculation of the spectral composition of the emergent beam after any thickness of filter.

In order to do this it is necessary to know both the spectrum of the original radiation and the way in which the monochromatic absorption coefficient varies with wavelength. I have already discussed the spectrum of our radiation; for the absorption coefficient, in the absence of experimental values I have computed the coefficients from Richtmyer's formula for the photoelectric absorption coefficient and the Klein-Nishina formula for the scattering coefficient (4). The latter is quite accurate in this region, as far as known, but the validity of the Richtmyer formula is open to considerable doubt. It probably gives values which are too low throughout the region under consideration, but since in nearly all cases the photoelectric term is small compared with the scattering term the error is small. Oppenheimer (5) gives a formula which should be more accurate for extremely short wavelengths, but the constants involved are not known with sufficient accuracy to permit its use for numerical computation.

The method of calculating the spectral distribution of a filtered beam of radiation is illustrated in Figure 3.

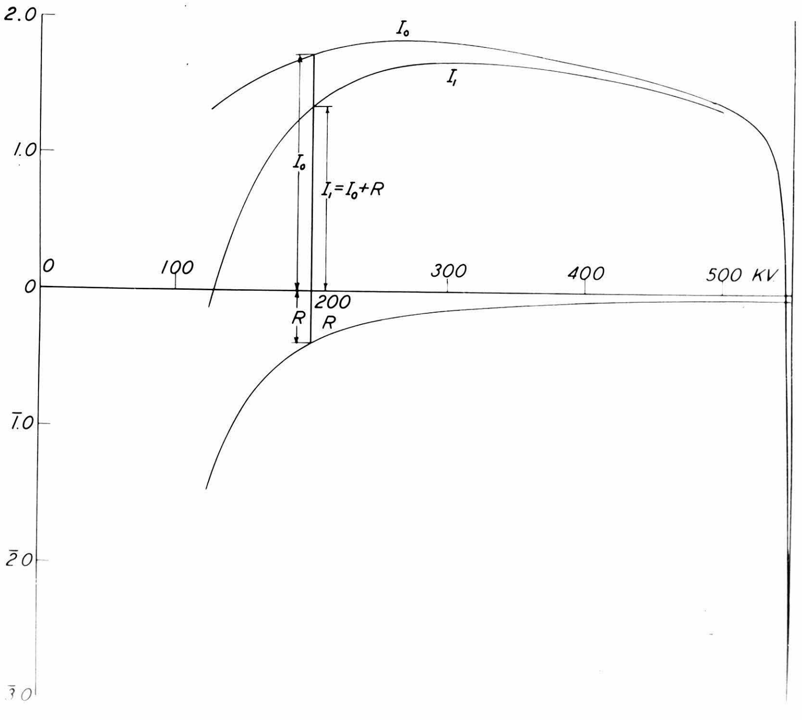

Figure 3. Graphical Construction of Filtration Curves.

Since the intensity at any wavelength is reduced in the ratio $e^{-\mu x}$, where x is the total thickness of the filter, it follows that if we plot a spectral intensity curve for our original radiation and then reduce all of the ordinates in the ratio $e^{-\mu x}$, remembering that μ is a function of wavelength, we will get a new curve which will represent the spectrum of the radiation after passage through x cm. of filter. Since we wish to work over a large intensity range it will be convenient to use a logarithmic intensity scale, and this has the further advantage that the amount of downward displacement of the curve at any wavelength is proportional to the filter thickness. In Figure 3 the curve I_0 represents the original spectrum; its rather odd appearance is due partly to the logarithmic scale of ordinates and partly to the fact that it represents the intensity in a unit frequency interval; in order to convert it into wavelength units it is necessary to divide all the intensities by the squares of the corresponding wavelengths. The abscissae represent frequencies - they are actually plotted as voltages in kv. The curve R represents the logarithm of the attenuation produced by the filter as a function of voltage. Now if we choose some thickness of filter that will give a convenient value of attenuation, plot the R-curve for this thickness x, and then add the two curves (algebraically), we will obtain a new curve I_x which will represent the spectrum of the radiation after passing through x cm. of filter. If R is then added to this new curve we will obtain a third spectral curve representing the radiation after filtration through 2x cm. of material, and the process can be kept up indefinitely.

Figures 4 and 5 have been prepared in this way, and represent the effect on the original spectrum of filtration through aluminum and lead, respectively, radiation softened by Compton scattering and fluorescent radiation being assumed to be absent in each case. It will be seen that as the thickness of filter is increased the longer-wave (lower-voltage) radiation rapidly decreases in intensity, while the decrease is much slower in the case of the harder components. Comparison of the two figures will show that this effect is much more pronounced for lead than for aluminum filtration.

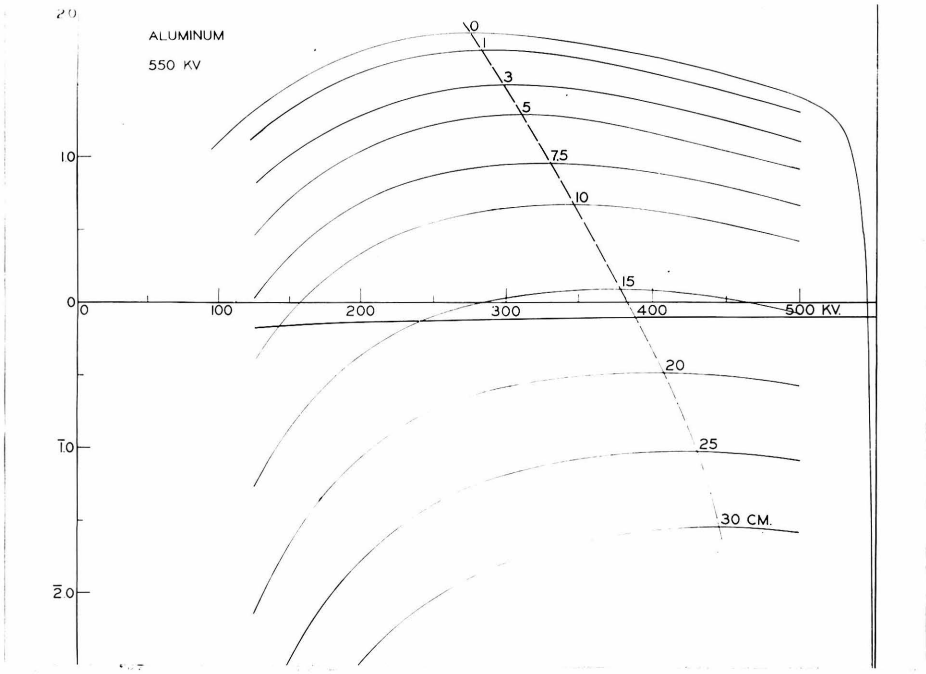

Figure 4. The Effect on the Spectrum of Filtration through Aluminum

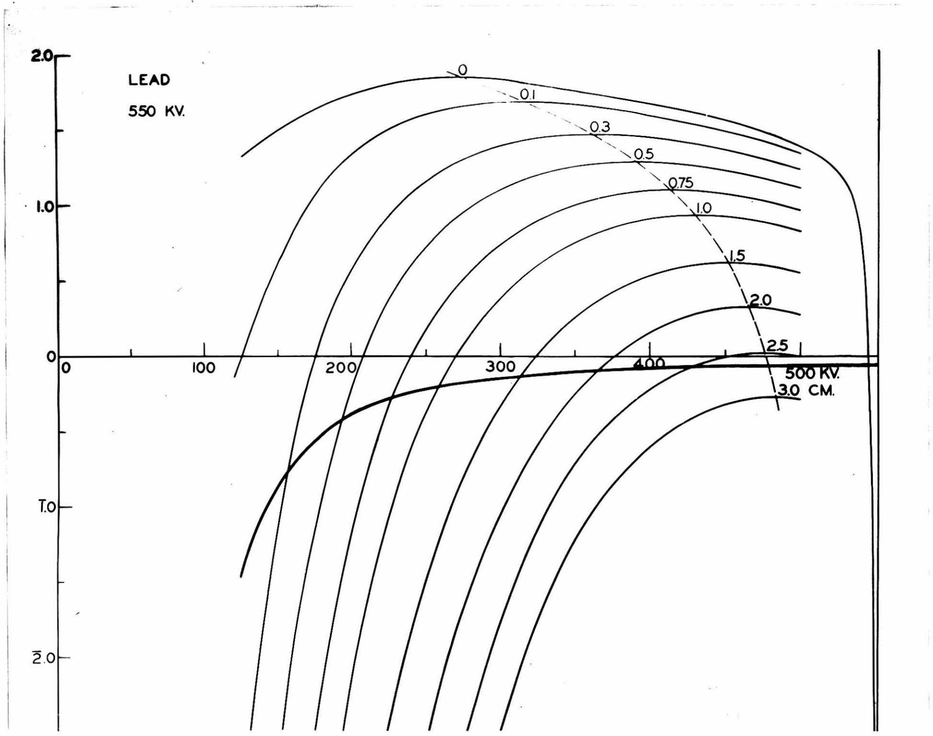

Figure 5. The Effect on the Spectrum of Filtration through Lead

II-4. THE "EFFECTIVE WAVELENGTH" OF A FILTERED BEAM

The two figures just described show that as the filter thickness is increased the intensity maximum moves downward and to the right, or that with increasingly heavy filtration the center of gravity of the spectrum moves toward the higher voltages. The "effective voltage" of the radiation - that voltage which corresponds to a monochromatic absorption coefficient equal to the apparent absorption coefficient as measured - cannot readily be obtained from the figures, but it can at least be said that it must vary with degree of filtration in much the same way as does the voltage of the intensity maximum. Since what we wish to accomplish in using a filter is to obtain as narrow a spectral region as possible without a too great decrease of intensity, it follows that the value of a filter is measured by the slope of a curve drawn through the successive maxima of the intensity curves. That filter is of the greatest value which gives the greatest attenuation of the softer radiation for a given decrease of the harder components. It can easily be seen that aluminum is practically worthless in comparison with lead; from the figures, using 550 kv. radiation, 15 cm of aluminum displaces the maximum only from 275 to 375 kv., which can be accomplished by 3.5 mm. of lead with a much smaller sacrifice of intensity. The actual (measured) total intensity ratio in the two cases is about 5. Since the curves through the maxima become steeper and steeper with increasing filtration, it follows that the gain in homogeneity of radiation becomes less and less as the filtration is increased, and in practice even with lead little gain in homogeneity would be gained by going to extremely thick filters in this region. This will be more and more true as the tube voltage is increased, and at very much higher voltages only comparatively thin filters could be used with much advantage. This conclusion applies, of course, only to continuous spectra; with gamma-ray line spectra it is not necessarily the case.

III. THE EXPERIMENTAL PROCEDURE

III-1. THE TUBE AND ITS ASSOCIATED APPARATUS

Because of the large number of factors which may affect the value of absorption coefficients determined by the "end-radiation" method it seems advisable to describe the experimental conditions in considerable detail, even if some of this detail may appear irrelevant. As a source of radiation I used the 550 kv. tube (No. 3) which was originally constructed in the spring of 1931 and which has been in constant use since that time for the medical work now being carried on under the auspices of the Institute. It differs from the tube described by Lauritsen and Cassen (6) principally in that it lacks the upper filament and the Lenard window of the earlier tube, and that it is mounted in a concrete cubicle separate from the one enclosing the control position, and with much more room inside so that as many as four patients may be treated at the same time. As before, the power supply was the million-volt testing transformer installation in the high-potential laboratory of the Institute. The filament heating current is normally supplied by a small step-down transformer, but it was found that when this source of current was replaced by a 12-volt storage battery the very troublesome output fluctuations caused by voltage fluctuations in the 110-volt power were materially reduced, and most of the runs were made with the battery. The filament temperature was regulated by means of a series rheostat in the control room, which was adjusted by the control operator so as to hold the emission current constant. It was found, however, that for long runs this scheme was impracticable, so a duplicate emission current milliameter and primary voltmeter were installed in the measuring position outside the tube housing, and a small carbon-pile radio rheostat, shunted with about 8 inches of #16 Chromel wire, was installed by the emission meter. This gave very smooth control of current over a small range, and when the current was held to within 1/3 of 1% of its normal value, which could easily be done under most conditions, the variations in the tube output became very much smaller.

The voltage applied to the tube was measured by an A.-C. voltmeter connected across a tertiary winding on the first transformer. Thanks largely to the fact that the high-voltage transformers are fed by a special 15 kv. line from the Pasadena municipal power station, which is usually connected directly to the main Southern California Edison Company system at the Eagle Rock substation, the transformer voltage fluctuations are usually small and more of the nature of slow drifts of a magnitude of $\frac{1}{4}$ to $\frac{1}{2}$% and a period of several minutes, than of sudden changes. Occasionally, however, especially on dark days, the voltage fluctuations were much larger, and when they exceeded 1% no consistent readings could be obtained. The primary voltmeter was originally calibrated against a large sphere gap, and this calibration was checked by the short-wavelength limit of the spectrum of the radiation, as obtained from a microphotometer curve (cf. Lauritsen, (2)). The voltage and current-waves could be watched at any time on a small two-element oscillograph. This instrument proved to be very useful in checking up on the condition of the tube, since the variations in the shapes of the curves were very sensitive indications of the presence of gas current or cold emission within the tube. A typical oscillogram is shown in Figure 6; it will be seen that the potential applied to the tube is quite accurately a sine-wave.

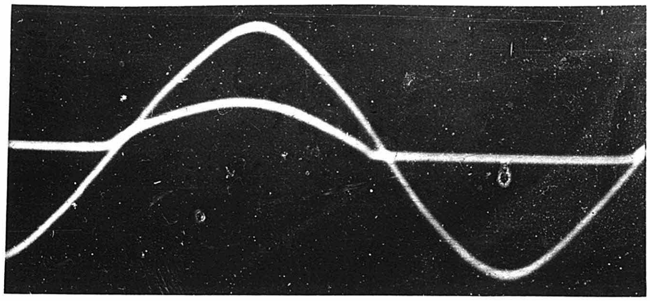

Figure 6. Oscillogram of Applied Voltage and Emission Current.

The slight phase-shift in the figure should be disregarded. It is due to incorrect alignment of the optical system of the oscillograph; actually the current and voltage were accurately in phase. The peak voltage was kept at 550 kilovolts and the average value of the emission current at 4.55 milliamperes throughout the whole series of measurements.

A number of early runs were made with the radiation filtered by the 6 mm. steel wall of the tube, but for all the work reported in this paper the measurements were made on the north beam, which comes through a hole cut in the steel wall of the tube and covered with an aluminum window .081 cm. thick. Figure 7 shows a section of the lower part of the tube and its concrete housing, with the track which supported the filters and absorbers, the lead diaphragms, shields, and shutters, and the Wulf electroscope which was used for the intensity measurements.

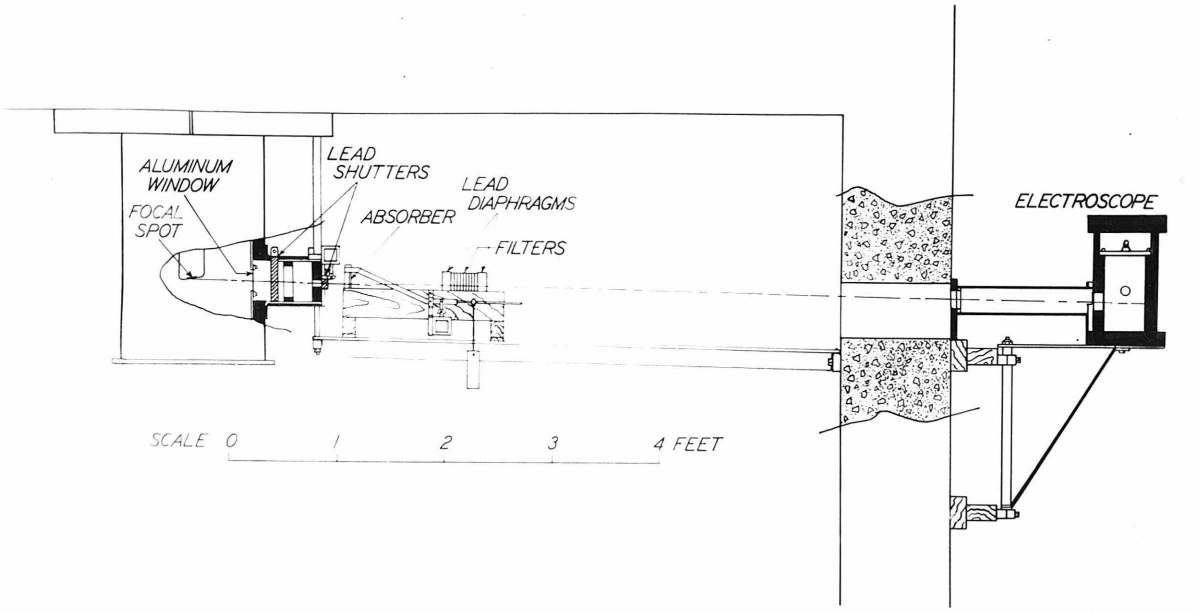

Figure 7. Elevation of Tube, Housing and Apparatus.

Figure 8 is a schematic diagram showing the geometry of the beam in a somewhat more easily understandable fashion.

The beam of radiation which was used left the focal spot nearly horizontally, the axis of the beam making an angle of about 11 degrees with the plane of the target. Directly in front of the aluminum window there was a 1" cast lead shutter which could be raised or lowered from outside the concrete housing, and which served to cut off the radiation when it was necessary to go inside to change filters or to make other adjustments. This shutter was built into the base of a lead-covered brass "spout", in the out~~h~~er end of which was placed a 1" lead block pierced with a 7/8" hole, which hole was normally covered by a 3/4" lead shutter. This second shutter could be raised by a small solenoid which was controlled by a switch outside the house. It opened in about 1/4 second and closed in about 1/10, both intervals being quite constant and small compared with the total time the shutter was kept open. This time was measured by a fifth-second stopwatch which was operated with one hand while the switch was thrown with the other. The shortest time interval used was 30 seconds, so that the maximum percentage error in timing was small compared with the errors from other sources.

III-2. THE FILTERS AND ABSORBERS

The metal slabs used as filters and absorbers were cut from ordinary flat rolled stock in the case of the harder metals, and the lead samples were rolled from commercial sheet lead in the case of the thinner pieces, while the 5 and 10 millimeter slabs were cast and faced off in the lathe, and the trued face waxed to a face-plate and the piece turned down to the proper thickness. All samples were measured with a micrometer at twelve points evenly distributed over the face, and any that showed variations in thickness of over 1% were discarded. Since the central portion of each sample was the only part used, the four thickness readings taken nearest the center were averaged for each piece and this average was recorded as the thickness of the piece. All of the samples were square and approximately 3" on a side. The length of each side was measured and the piece was weighed and the "equivalent thickness" computed from the area and the mass, using standard densities taken from the International Critical Tables. This equivalent thickness was found not to differ appreciably from the average measured thickness for any of the samples, although as might be expected the rolled lead samples showed a somewhat higher density than the cast ones. No analyses were made of the materials, but it is improbable that impurities were present in any of them sufficient in amount and of such a nature as seriously to affect their absorption coefficients in this region.

For each material one or two of the best samples were chosen and used for absorbers, the others being used as filters. The latter were held in a short wooden trough between two heavy lead blocks pierced with holes of sufficient size to permit the passage of the largest beam with which it was desired to work. These blocks were screwed to flat metal feet and held the filters together perpendicular to the beam much in the manner of book-ends on a shelf. The alignment of the filters was accurate to within 1 degree, so that the length of the path of the beam through the sample differed from its measured thickness by a negligible amount. The standard specimen~~s~~ whose absorption was to be measured was mounted in a holder which could be lifted out of the beam by a solenoid similar to the one used for lifting the second shutter, and all measurements were made by taking readings with the sample alternately in and out of the beam, in order to minimize errors due to gradually changing tube output.

III-3. THE IONIZATION CHAMBER

After passing through the absorber and the filters the beam continued outward through the 6" square window in the wall of the concrete house, through the lead tunnel, and into the ionization chamber, being finally stopped by its heavy lead armor. This instrument was the one described by Millikan and Cameron (7) and used by them in most of their cosmic-ray work. The only modifications made in preparing it for this work consisted of boring a 2" hole through the lead armor to permit the passage of the x-ray beam, and of replacing the old eyepiece and scale with a filar micrometer eyepiece, which permitted the determination of the position of the fiber to be made with much greater accuracy and ease. An extension handle was also fitted to the charging switch, which permitted its operation without removing the lead armor. A set of lead diaphragms was made which fitted into the 2" hole in the armor and allowed the sensitivity of the instrument to be varied over a wide range by varying the size of the opening. This was a great convenience, since it allowed measurements to be made over a large range of intensities without necessitating time intervals short enough to introduce timing errors, or long enough to slow up the readings and to demand corrections for leakage and for variation in the tube output.

As used in these experiments, the electroscope had a voltage sensitivity (for all voltages above 100) of .0968 volts per division of the micrometer eyepiece, or 10.33 divisions per volt. The voltage calibration curve was accurately linear throughout this region, so that it was not necessary to translate discharge readings from divisions per second into volts per second. The capacity of the instrument was 1.41 centimeters, so that the above voltage sensitivity corresponded to a charge of 4.55×10^{-4} e.s.u., or 9.53×10^{5} ions, per division. Saturation was attained with this instrument well below 100 volts, and no readings were taken of sufficient length to permit unsaturation to occur. The discharge rate was linear with intensity of radiation within the errors of measurement for all intensities used and over the entire portion of the scale employed in the measurements. The natural leak of the instrument was very constant at about .41 divisions per minute; with the tube running the leak varied somewhat with diaphragm size and degree of filtration of the beam, but was never more than 2.5 divisions per minute, and as the slowest discharge rate used was of the order of 100 divisions per minute a correction for leakage was unnecessary in all but a few cases. The number of divisions traversed by the fiber in a single measurement varied from 200 to 1000, and as the cross-hair could be set on the edge of the fiber with an accuracy of plus or minus 1 division in all cases, the setting error was also negligible. It should be noted, however, that the precision of this setting was the factor which usually limited the length of the runs; after 2½ to 3 hours of measurement, making one or two settings per minute, my eyes became so fatigued that it was useless to try to continue, as I could no longer make settings with sufficient accuracy. For any work such as the present experiments in which a large number of readings must be made at short intervals, it is very desirable to use a projection-type instrument, if possible, in order to decrease eye-fatigue and its attendant errors.

III-4. THE GEOMETRY OF THE BEAM RADIATION

A horizontal section of the beam of radiation is shown in Figure 8.

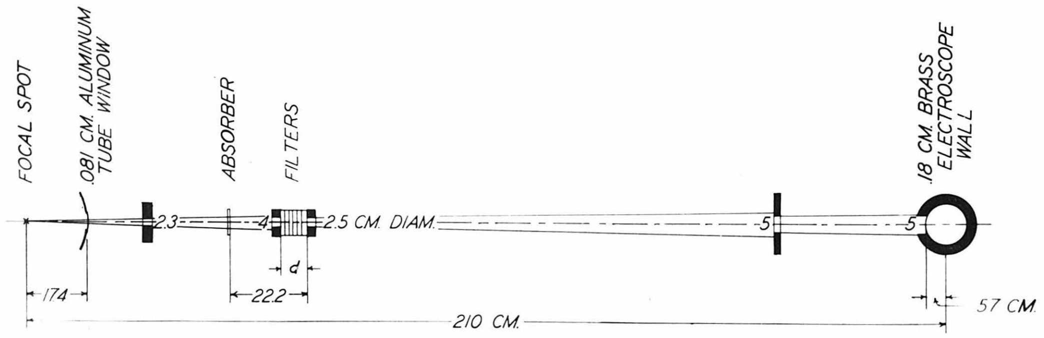

Figure 8. Geometry of the X-Ray Beam

The original beam was 3.7 cm. in diameter at the absorber and 4 cm. at the filters. It was stopped down to 2.5 cm. by the second filter diaphragm, and again to 5 cm. by the lead shield outside the window in the concrete wall. The diameter of the beam entering the electroscope was defined by the lead diaphragms previously described, and varied from

.11 to 5 cm. The absorber, filters, and filter diaphragms were all mounted on a wooden stand which could be placed at any desired position on the steel track, but throughout these experiments the stand was placed as close to the tube as possible, so that the distance from the shutter opening to the rear of the absorber was 22.2 cm.

The fraction of secondary radiation from the filter which passed through the electroscope diaphragm was very small; the total angular diameter of the largest opening used being only 1.7°. Kohlrausch (3) found for the case of gamma-rays that no serious error was introduced unless this angle exceeded 10°.

The radiation scattered from the last lead shield and from the edges of the electroscope diaphragm may be of considerably greater intensity, and lack of time made it impossible to investigate this point, but the effect, if present, is considerably reduced by the filtering action of the electroscope wall. This is of brass, .18 cm. thick, and should pass but very little fluorescent lead K-radiation. In any event the error cannot be very great in view of the agreement between absorption coefficients obtained with diaphragms of different sizes.

III-5. THE METHOD OF MAKING AND REDUCING THE OBSERVATIONS.

The following procedure was employed in taking the measurements. The tube was started and run until it reached a condition of fairly stable operation, with no gas current and but very few surges, preferably using a voltage somewhat higher than normal for this running-in process. After the warming-up period, which might require anything from a few minutes to two hours or more, the controls were transferred from the control room to the measuring position, the apparatus put in place both within and without the tube house, and the liquid air shoved up as far as possible on the trap. The tube was then restarted and the current and voltage accurately adjusted to their normal operating values. A few check measurements of tube output were made before and after each run, but were not usually recorded. If the output differed more than a few percent from its normal value, or if it fluctuated, no consistent readings could be obtained. This was frequently the case, especially if the liquid air had been allowed to get low and a little mercury vapor had diffused from the trap into the tube.

As soon as satisfactory operation was obtained the actual measurements were started. The form used in recording is illustrated in Table I:

TABLE I. SAMPLE SET OF MEASUREMENTS

5-16-32 Lead Absorption, Lead Filter 550 kv., 4.55 ma.

Filter	Abs.	d	t	0	Δ	δ	δ/t	av.	log	∂log	μ
Diaphragm #3											
--	N in	0	50	147	642	495	9.90				
	out		30	156	963	807	26.90	27.05	1.432		
	in		79.8	171	966	795	9.96	9.93	.997	.435	20.45
	out		30	176	992	816	27.20				
K	N in	.05	50	182	451	269	5.38				
	out		30	178	477	299	9.97	9.95	.998		
	in		50.2	182	448	266	5.32	5.35	.728	.270	12.70
	out		30	180	475	295	9.83				
I	N in	.10	79.8	180	447	267	3.35				
	out		50	179	445	266	5.32	5.32	.726		
	in		50	181	347	166	3.32	3.34	.523	.203	9.54
I,K	N in	.15	100	178	405	227	2.27				
	out		70	178	412	234	3.34	3.32	.521		
	in		100	179	405	226	2.26	2.27	.356	.165	7.75
	out		70	179	410	231	3.30				

The heading for each run contains such data as apply to the whole run, and any supplementary measurements on output, electroscope leak, etc., are inserted either just below the heading or at the end of the run. The absorption measurements are recorded in twelve columns, of which numbers 1, 2, 4, 5 and 6 represent experimental data while the others are derived from these. Column 1 contains the numbers or letters of the filters employed, in the order of their position counting toward the tube. In column 3, d signifies the total thickness of filter in centimeters. Column 2 gives the number of the absorber and its position, "in" signifying that it was in place, "out" that its solenoid was energized and the absorber lifted out of the beam. The individual intensity measurements were made as follows. The electroscope was charged, the charging switch grounded, and the crosshair set on the left edge of the fiber image; the micrometer reading was recorded in column 5. The filament rheostat was then adjusted, if necessary, and the shutter solenoid switch closed with the right hand while the stopwatch was started with the left. While the shutter was open, both current and voltage were carefully watched, and adjusted if necessary. When the desired time had elapsed, the shutter switch was opened and the stopwatch button simultaneously pressed with the other hand. The stop watch reading was recorded as "t" in column 4, and the crosshair was again set on the left edge of the fiber and the micrometer reading recorded as Δ in column 6. This completed one intensity measurement and the procedure was several times repeated with the absorber alternately in and out of the beam.

At first only the above quantities were recorded during the run and the computations were carried out later, but it was found that much greater consistency was obtained by carrying out the computations during the run and repeating measurements immediately if they were inconsistent.

The computations were made with a 10" slide rule.

In column 7, δ represents the difference between "0" in column 5 and Δ in column 6. It is the number of divisions traversed by the fiber in the time t. δ/t in column 8 is the rate of travel, and is proportional to the intensity of radiation. The several values of δ/t obtained with the absorber "in" were averaged, discarding values which did not agree with the others within 2%, and recorded as "av." in column 9, and the same thing was done for the absorber "out". The logarithms of these two averages were recorded in column 10, and their difference in column 11 was $\partial \log$. Finally, $\partial \log$ was multiplied by the proper factor to obtain μ, which was recorded in the last column. This factor is equal to 2.303 divided by the absorber thickness in centimeters.

Values of μ were determined for a number of thicknesses of filter varying from zero (actually the .081 cm. aluminum window) to a maximum determined by the minimum easily measurable intensity. The increments in filter thickness were made small for small thicknesses in order to obtain more points in the neighborhood of the bend in the absorption coefficient curve. The ratio of maximum to minimum intensities measured was about 5000 to 1. As the intensity decreased the electroscope diaphragms were increased in size in order to keep the discharge times approximately the same, from three to five points being obtained with each diaphragm. In order to see if diaphragm size had any effect on the apparent absorption coefficients, whenever a diaphragm was changed the last point was redetermined with the new diaphragm. Although the coefficients with the two diaphragms did not always check, the differences were small and showed no inconsistency in sign, so that it was concluded that any error due to this cause was within the experimental error.

IV. PRECISION OF THE MEASUREMENTS: SOURCES OF ERROR

IV-1. INCONSTANCY OF TUBE OUTPUT.

The principal source of error in most x-ray intensity measurements in which a balance method is not used is the constant variation of both the quantity and quality of the radiation emitted by the tube, this variability of output being by far the greatest practical difficulty encountered in the present work. The precautions taken to minimize variations in voltage and current have already been described, but it may be worth while to consider the sources of these variations and their effects on the values obtained.

The principal source of variation in filament current in the earlier measurements was voltage fluctuation on the 110-volt line feeding the filament transformer, but when this was replaced with a storage battery the filament current became quite steady. The fluctuations in emission current, though much smaller, were still present, and were sometimes large enough to prevent consistent readings from being obtained. These fluctuations arose partly from the microphonic contacts in the filament rheostat, and partly from sudden changes in the emissivity of the filament, due perhaps to the periodic formation and destruction of surface layers on the tungsten. There was also a slow drift of battery voltage, but as this could easily be followed with the auxiliary rheostat it caused little trouble. The primary effect of variation in emission current is to cause the output to vary in the same proportion as the current, but any change in current causes a change in the IR drop in the water column and therefore a change in the potential actually applied to the tube. This effect is small with light filtration, but with heavy filtration this change in potential causes a considerable variation in the intensity of the filtered beam.

Voltage variations also caused much trouble. The voltage of the 15 kv. power line was fairly steady, but the best results were obtained between 1:30 and 4:00 AM, when the power load is light and constant. Much more serious, however, were the variations in the water-column IR drop. These were due to several causes. Variations in the current through the column, due to varying emission current or to gas current, caused the applied potential to vary by an amount estimated at about plus or minus 5%. Furthermore, variations in the rate of flow of water through the column produced variations of its average temperature and resistivity, which again caused variations in the applied potential. These resistance variations were rather large in the early part of the work, but the old water column finally became stopped up and burned out, and a new one was installed. This column permitted a much greater flow of water, and the water remained quite cool.

For more accurate work it would be necessary to measure the applied voltage directly with some sort of constant-resistance multiplier, rather than to depend upon a voltmeter connected to the transformer. The current measurements are quite accurate, although it is desirable to have an oscillograph for checking the wave-shape, and not to work if the wave departs seriously from its normal form.

IV-2. LEAKAGE AND STRAY RADIATION

The errors produced by electroscope leakage and stray radiation have been discussed elsewhere, and were probably small. The natural leak of the electroscope, due to conductivity in the quartz supports and to ionization by cosmic rays and local radioactivity, was small and very constant.

With the tube running the rate of leakage was slightly greater, but not enough so to require a correction to be applied to the readings. The tube itself was very well shielded with lead, and although the stray radiation inside the concrete house was intense enough to discharge an unshielded electroscope in a few minutes, very little of this radiation entered the electroscope in the measuring position. The radiation scattered from the filters was largely screened off by the rear filter diaphragm, and such scattered radiation as did come through the hole was of much lower intensity than the direct beam. The radiation scattered from the electroscope diaphragm may have been of sufficient intensity to affect the readings somewhat, but the agreement between coefficients determined with various sizes of diaphragm is at least a good indication that this source of error is not serious. The effect of diaphragming close to the electroscope should be further investigated; it may be serious with thin-walled instruments.

IV-3. MISCELLANEOUS ERRORS

There are several sources of error that do not fall under either of the above headings. One is introduced by trying to use too thin an absorber. Since the measurement of absorption coefficients essentially involves a differential method, it follows that if the difference between the two intensity measurements is too small the percentage error in the difference will be large with respect to the percentage errors in the quantities actually measured. On the other hand, if a very thick absorber is used in order to minimize this error, there arises the question of what the measured coefficient means. The absorption coefficient is defined on the basis of an infinitesimal absorber thickness, and since for inhomogeneous radiation the absorption coefficient is a function of thickness a determination with a finite absorber leads to some sort of mean value of absorption coefficient to which it is difficult to ascribe a meaning. This ambiguity is naturally more serious with light filtration, where the absorption coefficient curve is descending steeply. With the more homogeneous heavily filtered radiation it is of little significance.

V. EXPERIMENTAL RESULTS

Figures 9 to 12 show the apparent absorption coefficients of aluminum and of lead as obtained from these experiments, plotted against filter thickness for aluminum and lead filters. The most important conclusion to be drawn from the curves is that even after the heaviest filtration used the radiation was still far from homogeneous, as the curve is still descending rapidly. Obviously also lead is a much more effective "hardener" than aluminum, since the final absorption coefficients of both metals are much smaller with lead than with aluminum filtration.

Since the maximum thickness of filter that can be used is that which will just pass a beam of the minimum intensity with which it is possible to work, it is useful to plot the absorption coefficients against the intensity of the emergent beam. This has been done in figures 13 and 14. Here the ordinates represent the absorption coefficients of aluminum and of lead, respectively, and the abscissae represent the logarithm of the attenuation, i.e. the logarithm of the ratio of incident to emergent intensity.

The equivalent voltage after any given thickness of filter may be determined by an absorption measurement. Table II shows the equivalent voltages after filtration through various thicknesses of lead and of aluminum, as determined by absorption measurements on both metals.

TABLE II

Filter Thickness	Effective Voltage	
	Al Absorber	Pb Absorber
0	122 kv.	148 kv.
Aluminum Filter:		
1 cm.	136	151
3	145	156
5	150	160
7.5	160	166
10	166	177
15	185	183
Lead Filter:		
0.1	166	202
0.3	245	236
0.5	275	256
0.75	295	271
1.0	315	282
1.5	385	295

In the case of the aluminum filter the effective voltages agree fairly well with each other, but for lead filters the measured lead absorption coefficient gives much lower effective voltages for the heavily filtered radiation than the aluminum absorption coefficient does. This is probably to be ascribed to the inapplicability of the Richtmyer formula in this region. It leads to values of the photoelectric absorption coefficient which are too low, and in the case of lead this is not negligible in comparison with the scattering coefficient. The agreement is better with aluninum filtration because with the lower effective voltage the formula is in better agreement with the facts. At considerably higher voltages the two results should again come into agreement, since even for lead the photoelectric absorption coefficient for 1000 kv. is small compared with the scattering coefficient. This work should be repeated at 1000 kv. as soon as the new Kellogg Laboratory tube is available. The disagreement between the effective voltages as measured with aluminum and with lead for small filter thicknesses should be disregarded; it is due to the averaging effect of the finite absorber thickness.

Beside the data upon which the conclusions of the previous paragraph were based, some miscellaneous tests were carried out to check up on the presence of certain experimental errors. The most important of these was a set of tests for softened radiation in the filtered beam. If the emergent beam contains any radiation manufactured by the filter, the amount and quality of this radiation should be dependent upon the thickness and material of the filter. If the intensity of this soft scattered radiation forms any appreciable fraction of the intensity of the beam as it enters the electroscope, the measured absorption coefficient should depend upon the position of the absorber, whether in front of or behind the filters, while if true exponential absorption occurs at all wavelengths the position of the absorber should make no difference. All of the absorption measurements used in the curves were taken with the absorber in front of the filters, i.e. between the filters and the tube. In order to test for softened radiation, however, a number of check measurements were made with the absorber behind the filter. These measurements were made with both heavy and light filtration and both with aluminum filter and lead absorber and with lead filter and aluminum absorber. The two coefficients agreed in all cases within the experimental error, so it was concluded that the softened radiation, if present, contained only a small fraction of the energy of the emergent beam.

Another test was made to determine the effect of electroscope diaphragm size on the measured absorption coefficient. As soon as the discharge rate of the electroscope had decreased with increasing filter thickness to about 3 divisions per second, the diaphragm was removed and a larger one substituted and the last set of measurements repeated. The two coefficients frequently agreed exactly (i.e. within $\frac{1}{4}$% - the minimum perceptible difference); in no case were they greater than the experimental error while the tube was operating properly. This test does, however, seem to be particularly sensitive to fluctuations in output, and the effect of diaphragm location and size should be further investigated.

The measurements herein reported naturally cannot serve as a direct check upon the absorption formulae, since the radiation is exceedingly heterogeneous even after the maximum filtration employed, but the values of effective voltage of filtered radiation given in table II could be explained by the hypothesis there advanced that the Richtmyer formula gives too small values in this region. If we knew accurately the intensity distribution in our original spectrum the graphical method described in the second section would permit us to compute our apparent absorption coefficients by assuming some law for the variation of monochromatic absorption coefficient. In the absence of such information, however, any such computation is impossible.

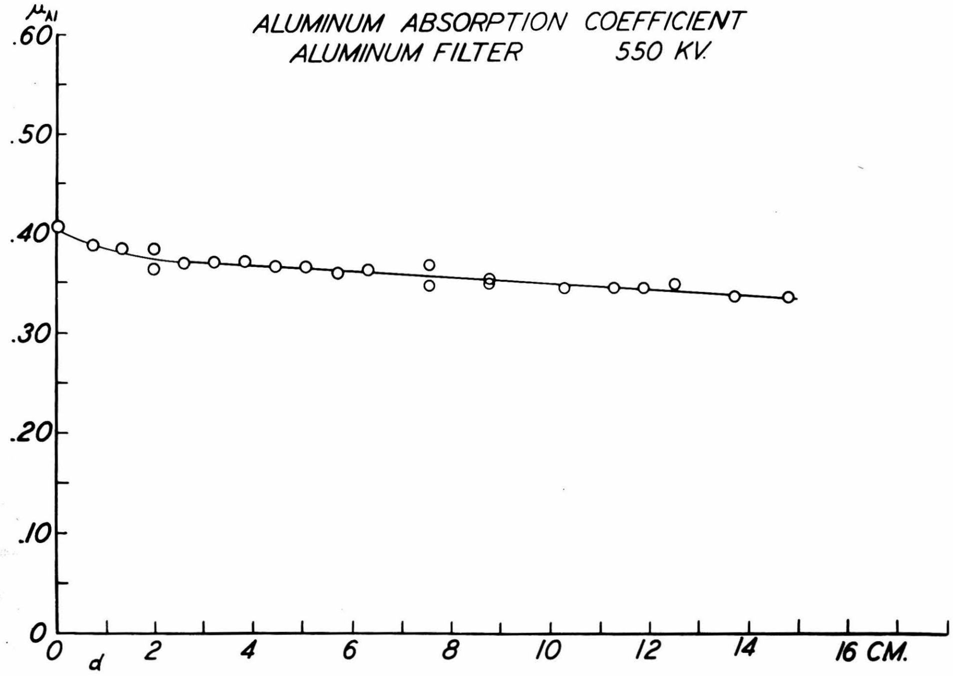

Figure 9.

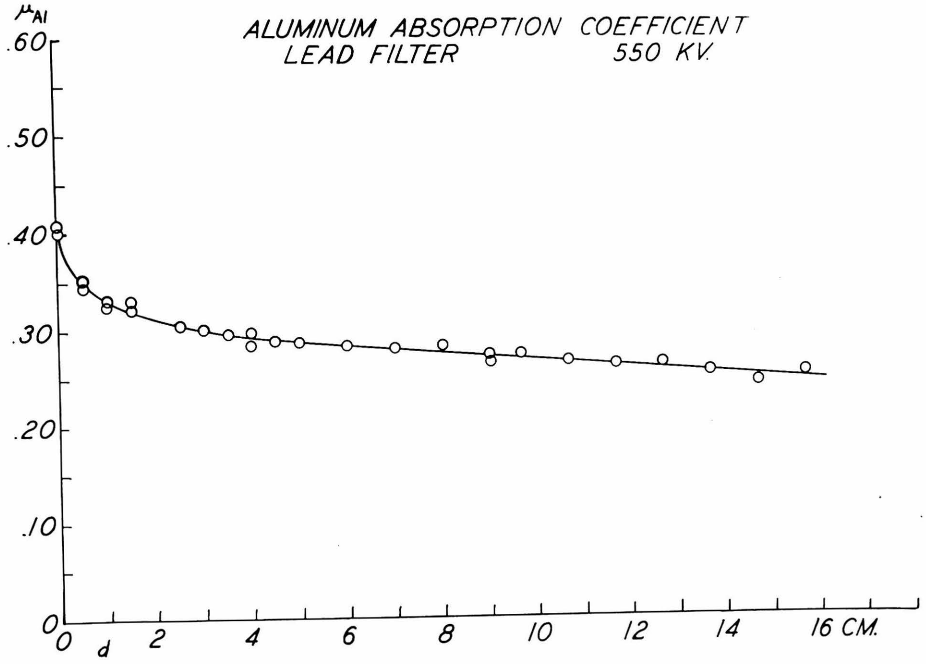

Figure 10.

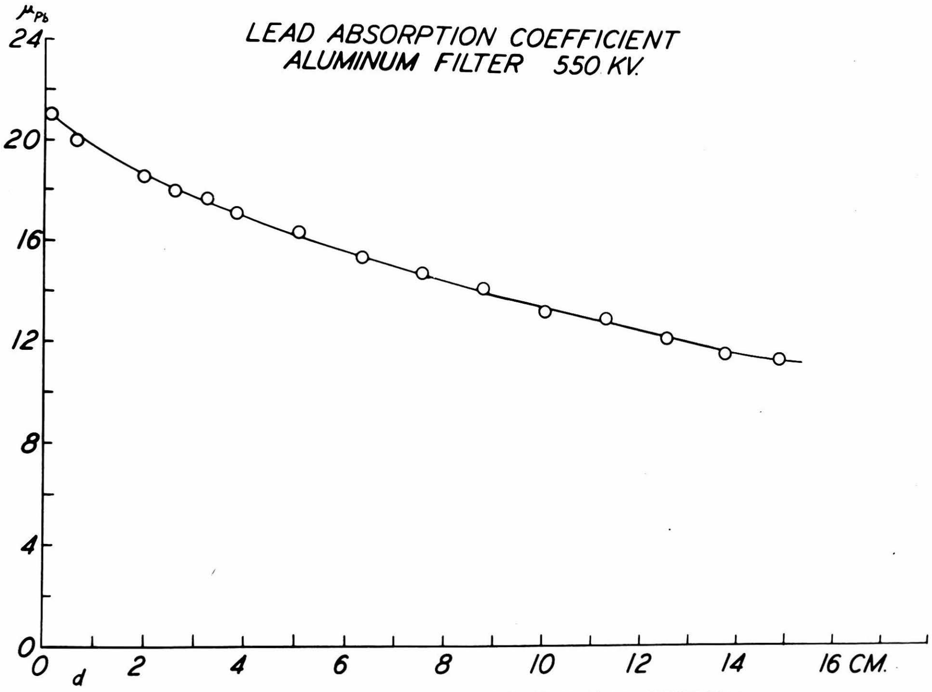

Figure 11.

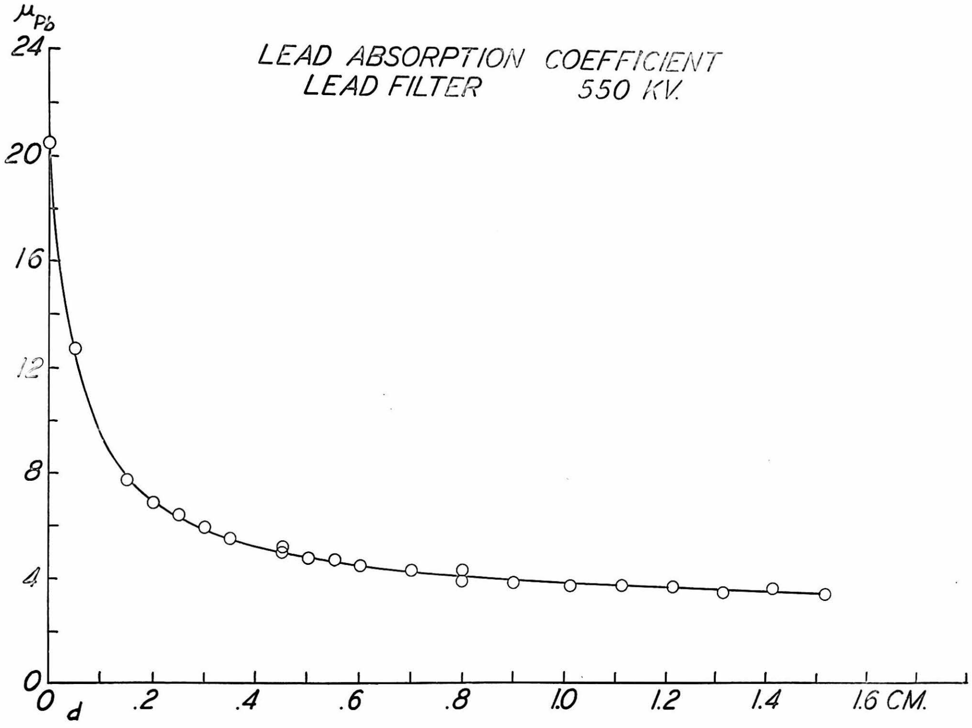

Figure 12.

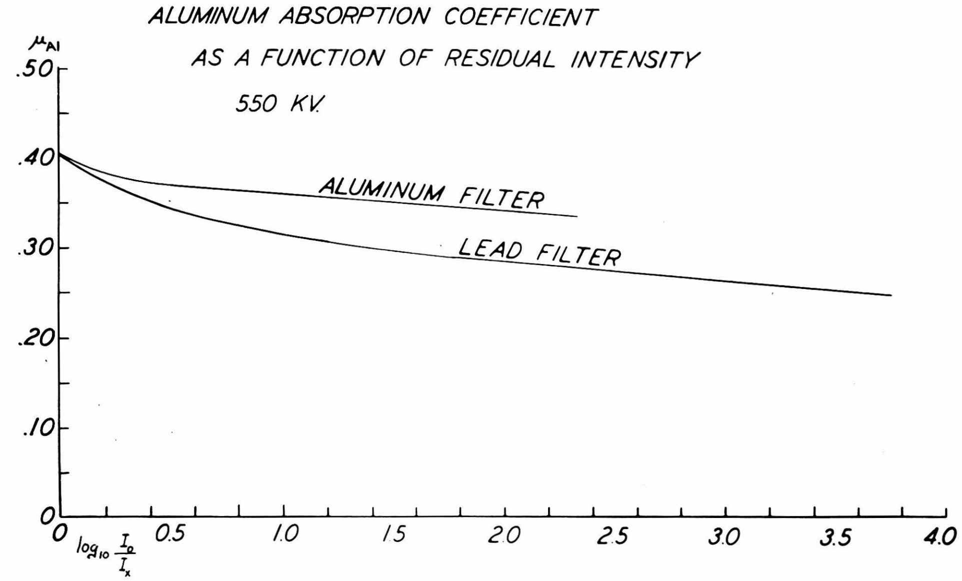

Figure 13.

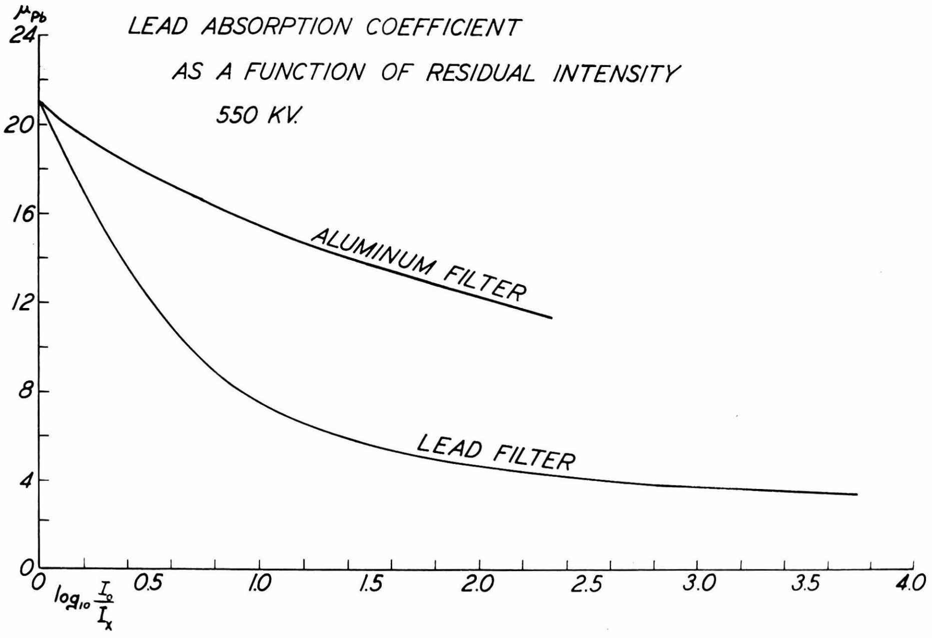

Figure 14.

VI. SUMMARY AND CONCLUSIONS

The principal facts of the foregoing paper may be summarized as follows:

A series of measurements of the apparent absorption coefficients of lead and aluminum was made, using the continuous radiation emitted by an x-ray tube run at 550 kv. peak voltage, filtered through various thicknesses of aluminum and of lead. A sensitive Wulf electroscope was used for measuring the intensity of the radiation. Tests were made for the presence of softened secondary radiation from the filters, and for the effect upon the measured absorption coefficient of varying the diameter of the beam of radiation admitted to the electroscope, and it is believed that the errors introduced by these factors were less than the error due to fluctuating tube output. The relative accuracy of the measurements was of the order of 3%.

The apparatus used and the experimental procedure are discussed in considerable detail. Various sources of experimental error are discussed, the precautions taken to minimize them are described, and, where possible, estimates of the magnitude of the errors are given.

A graphical method is developed by means of which the spectral composition of a beam of radiation after passing through a filter may be computed if the spectrum of the original radiation and the relation between wavelength and monochromatic absorption coefficient are known. The method is applied to the case of 550 kv. radiation filtered through aluminum and through lead, but the results are only qualitatively comparable with the experimental results because of the absence of quantitative information concerning the original spectrum.

The experimental date are presented in the curbes of Figures 9 to 14, and are summarized in the following table:

TABLE III.

Absorber	Al	Al	Pb	Pb
Thickness of Sample	1.245 cm.		.0490 cm.	
Initial absorption coefficient (no filter)	.405		21	
Filter	Al	Pb	Al	Pb
Maximum thickness	14.8 cm.	1.56 cm.	14.8 cm.	1.52 cm.
Absorption coefficient with heaviest filter	.335	.248	11.5	3.42
Effective voltage for this coefficient (computed)	182 kv.	392 kv.	182 kv.	295 kv.
Computed absorption coefficient for 550 kv.		.218	1.119	

Range of intensities covered: Al Filter, 1:200
Pb " 1:5000

In none of the curves was there any indication of an approach to a constant value of absorption coefficient, which indicates that even with the thickest filters employed the radiation was far from homogeneous.

From both the experimental and the theoretical work it was concluded that filtration is a very ineffective means of "monochromatizing" continuous radiation in the region above 300 kv, and that its effectiveness decreases with increasing voltage. It was also concluded that the absorption coefficients as measured by the "end-radiation" method, as well as the effective voltages calculated from these measured coefficients, are of little physical significance, and that in order to obtain quantities capable of exact physical interpretation it is necessary to employ some type of spectroscopic apparatus.

Finally, certain suggestions may be made as to future work on this method. From an experimental standpoint it is very desirable to study the effect on the apparent absorption coefficient of variations in a number of the experimental conditions, particularly voltage, angular aper~~a~~ture of electroscope opening, position of electroscope diaphragm, electroscope material and wall thickness, etc. From the theoretical point of view the most important thing is to obtain an accurate spectral intensity curve, together with some sufficiently accurate empirical formulae for this curve and for the curve expressing attenuation as a function of wavelength.

This will permit an analytical computation of apparent absorption coefficients, and a comparison of these with the measured values will permit a check on the Richtmyer formula and an estimate of the amount of softening of the original radiation by the process of filtration.

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