ANALYSIS OF CIRCULAR AND NON-CIRCULAR RINGS WITH A VARYING DISTRIBUTED LOAD

By

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INTRODUCTION

A rather extensive treatment of the stresses in fuselage main bulkheads is given in NACA Technical Report No. 509. The Least work equations developed in the report are for the most general case. The examples given are for concentrated loads, which are in balance, and applied at various points on a circular bulkhead.

Now the shear loads which are transmitted by the skin and stiffeners to a main ring exist in the form of a distributed load which varies around the bulkhead.

The purpose of this paper is to present an analysis of a main bulkhead in which the wing reactions are balanced by a varying distributed load around the bulkhead. The shear distribution although not exact because of the interaction of skin and stiffeners, seems however, closer to the actual existing conditions than would be a case in which the shear loads are treated as concentrated loads.

To somewhat simplify the energy equations the bending and torsional shears were treated separately, the work due to the shear and axial load deformations were also neglected. The error introduced is relatively small.

In the particular airplane to which this solution was applied 80% of the fuselage cross section area consisted of skin and the remaining 20% consisted of stiffeners. Hence the shear transferred by the skin is of major importance.

The case chosen as an illustrative example is an unsymmetrical condition since this is the most general case.

INTRODUCTION

A rather extensive treatment of the stresses in fuselage main bulkheads is given in several Technical Notes and Reports. In general all vertical shear loads are treated as concentrated external loads, which are in balance, and applied at various points on the bulkhead. The equations obtained are valid for this particular type of assumed loading. However they do not consider the fact that the skin plus stiffeners provide the external reaction. This reaction is obviously not concentrated but exists in the form of a distributed load which varies around the bulkhead.

The purpose of this paper is to present a method of analysis in which the wing reactions are balanced by a varying distributed load around the bulkhead. The shear distribution although not exact because of the interaction of skin and stiffener, seems however, closer to the actual existing conditions than the method of concentrated loads.

In the particular airplane to which this solution was applied 80% of the fuselage cross section area consisted of skin and the remaining 20% consisted of stiffeners. Hence the method of concentrated loads would give a rather poor approximation of the externally applied loads.

The case chosen as an illustrative example is an unsymmetrical condition since this is the most general case.

GENERAL DISCUSSION

For the unsymmetrical condition 70% of the load is applied on one wing and 100% on the other.

As specified in D.C. Bulletin 7A and 26, the resulting unbalanced moment will be balanced by the inertia of the wings and such masses, located in or on the wing, which may be assumed concentrated at the various points of location. The moment absorbed by each element being proportional to its distance from the lateral C.G. position of the airplane. The result is, there will exist a torsional moment in the fuselage due to the inertia of the fuselage and such masses as the tail surfaces which are attached to the fuselage.

The external applied loads in the bulkhead will then be, the wing reactions at the points of attachment to the main wing beam (assuming for simplicity a monospar construction) and a torsional moment in the fuselage.

Each loading will be treated separately and the resulting forces in the bulkhead added algebraically.

A cross section view of the fuselage for which the solution is worked out in detail is shown ~~on the next page~~ below - FIG I. The circular and two vertical portions are a close approximation of the actual shape of the fuselage.

With a slight modification the solution can be applied to circular bulkheads as will be shown.

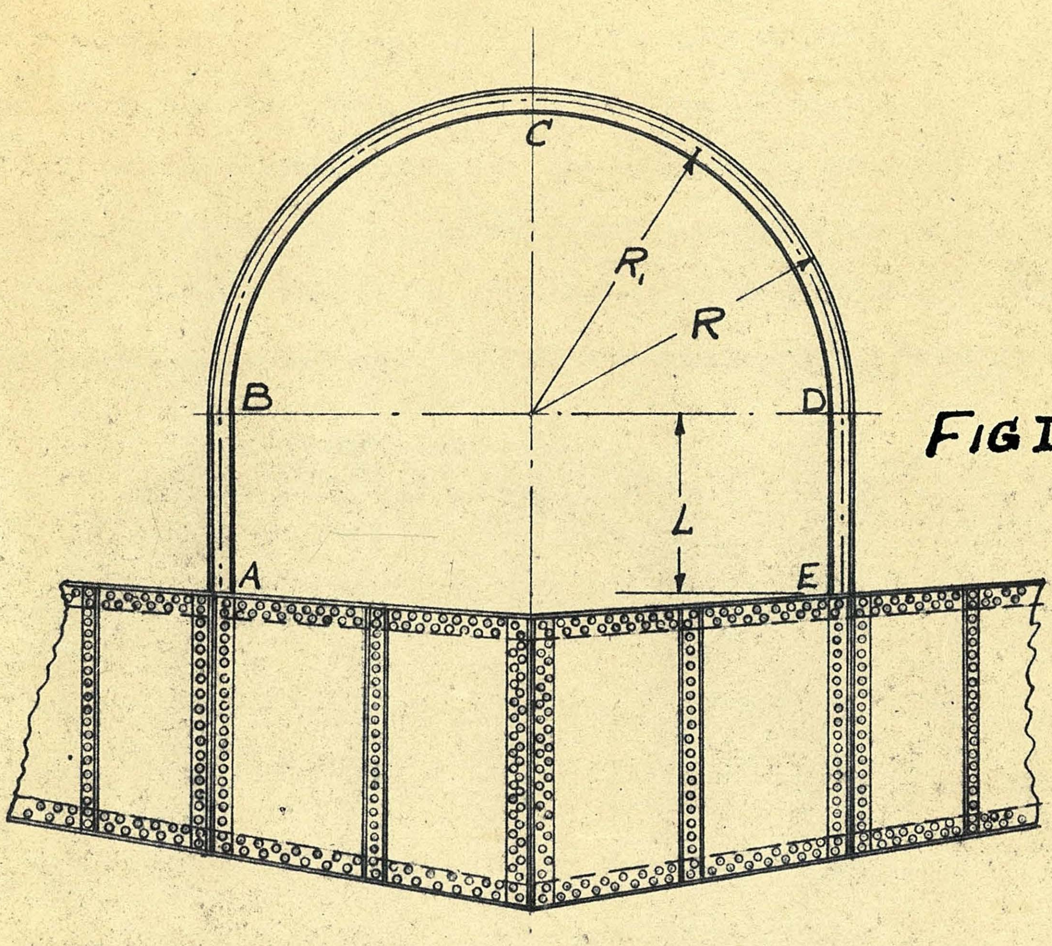


FIG I

CROSS SECTION VIEW OF THE FUSELAGE AT THE MAIN BULKHEAD

In order that a reasonable solution for the moments, shears and axial loads in the bulkhead may be arrived at the following assumptions will be made.

1. Since the moment of Inertia of the main beam is of the order of 1000 times that of the bulkhead it may readily be assumed that the bulkhead is attached to a member of infinite rigidity. Hence in the energy equations only the bulkhead will be considered.

2. The torsional moment in the fuselage will be assumed transmitted through the skin to the bulkhead by a uniform shear as shown in the sketch ~~on page 5~~ FIG. II . The magnitude of the unit shear being

$w = M_t/2A$ (lbs/in.) (1)

Where M_t = Torsional moment
A = Total cross sectional area of the fuselage

3. The reactions $P_{1,2}$ (see sketch below) are caused by the vertical shear in the fuselage and are transmitted to the fuselage bulkhead by the skin and stiffeners. The distribution being as follows.

a) Over the circular portion of the bulkhead the shear will be a function of the angle ϕ, where ϕ is measured from the vertical centre line of the bulkhead. The magnitude of the unit tangential shear being $q = \frac{P_0 \sin\phi}{\pi R}$ (#/")
~~The derivation for this value of "q" is shown on page 6~~. (see derivation)

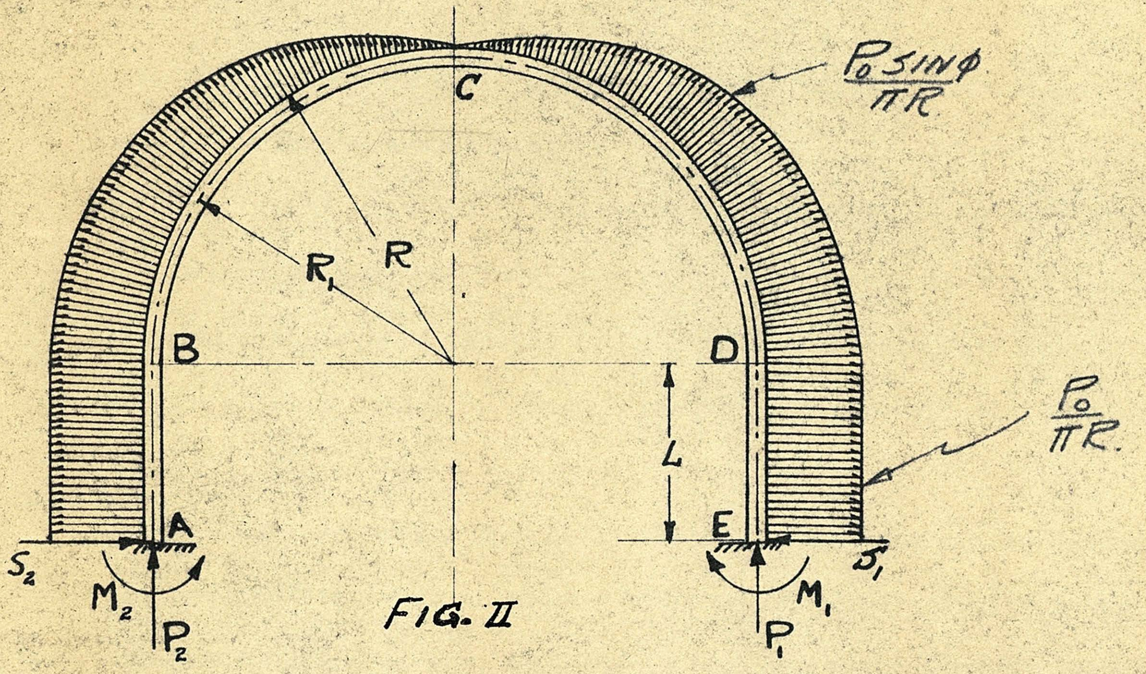


FIG. II

TANGENTIAL SHEAR, PLOTTED AT RIGHT ANGLES TO THE NEUTRAL AXIS OF THE BULKHEAD

For this type of loading the maximum shear occurs at the sides and is zero at the top. A result which is substantiated by tests on thin walled cylinders. Also a static test of the Lockheed Model 12 fuselage, in which a similar bulkhead is used, indicated essentially the same results.

b) At point B or D the unit tangential shear over the circular portion will be vertical and of the magnitude $q = P_0/\pi R$ lbs/in. (from $q = \frac{P_0 \sin\phi}{\pi R}$, and $\phi = \frac{\pi}{2}$)

(1) R. Bredt, V.D.I. Vol. 40, page 815 1896.

It is then assumed that the unit vertical shear along the straight portion is uniform and of the magnitude $q = \frac{P_o}{\pi R}$

C) The value of "q" is derived for a circular bulkhead, however, it is reasonable to assume that over the circular portion the equation for "q" will still be applicable. It will, however, be necessary to calculate an equivelent value of P_o in order that the total vertical shear calculated on the basis of $q = \frac{P_o \sin \phi}{\pi R}$ will equal the sum of the vertical reactions at A and E.

4) The derivation for the vertical shear distribution is based largely on the following assumptions:

a) That the skin which lies between two longitudinal stiffeners will behave as a shear web in a deep thin webbed beam, in which the stiffeners act as flanges, and that an element of skin adjacent to the ring will carry a shear load proportional to its static moment about the neutral axis.[2]

b) That even after buckling, the skin will continue to transmit shear as effectively as before buckling, a proportion, corresponding to the buckling stress, in direct shear and the proportion extending beyond the buckling load, in tension.[3]

From the above assumptions it will be possible to calculate the proportion of the total shear to be applied to the vertical and circular section.

(2) Herbert Wagner, " Flat Sheet Metal Girders with very Thin Metal Webs." Part I and II, N.A.C.A. Technical Memorandum No 604 and 605

(3) Herbert Wagner and W. Ballerstedt, " Tension Fields in Originally Curved, Thin Sheets during Shearing Stresses." Technical Memorandum No 774

SHEAR DISTRIBUTION OVER A UNIFORM CIRCULAR RING

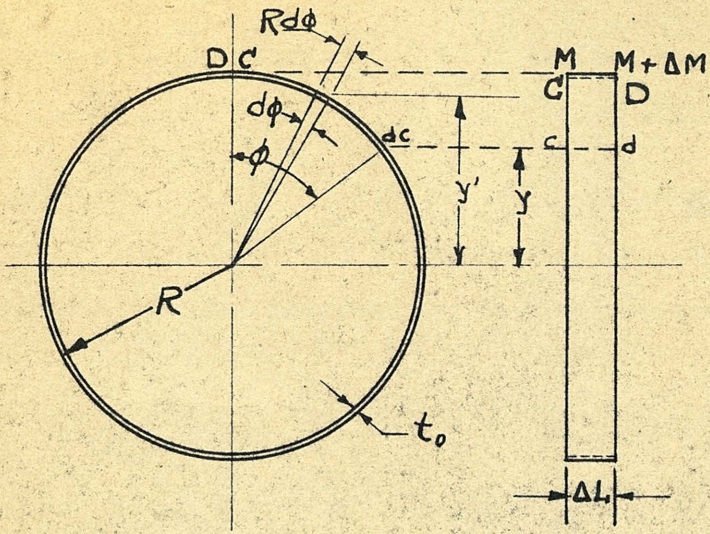


FIG III

To derive a mathematical expression for the shear distribution over the ring, it will be necessary to make the assumption that, the skin and stiffeners are replaced by an equivelent skin of a thickness t_0.

From the well known equation:

$$\text{Bending stress} = Mc/I$$

The load on the differential length $Rd\phi$ is:

$$F_{d\phi} = \frac{M}{I} y' t_0 R d\phi$$

The force on the circular arc from $\phi = 0$ to $\phi = \phi$ is:

$$F_\phi = \frac{M t_0}{I} \int_0^\phi y R d\phi \quad , \quad y = R\cos\phi$$

$$= \frac{M t_0}{I} R^2 \int_0^\phi \cos\phi \, d\phi = \frac{M t_0}{I} R^2 \sin\phi$$

This is the total load on C - c

On D - d

$$F_\phi = \frac{(M + \Delta M)}{I} t_0 R^2 \sin\phi$$

Difference

$$= \frac{(M + \Delta M)}{I} t_0 R^2 \sin\phi - \frac{M}{I} t_0 R^2 \sin\phi$$

$$= \frac{\Delta M}{I} t_0 R^2 \sin\phi$$

SHEAR $= P_0 = \dfrac{\Delta M}{\Delta L} \quad \therefore \Delta M = \Delta L \, P_0$

Hence $\dfrac{\Delta M}{I} t_0 R^2 \sin\phi = \dfrac{P_0 R^2 t_0}{I} \sin\phi \cdot \Delta L$

If "q_h" is the horizontal shear in lbs per inch

Then $$\Delta L\, q_h = \frac{P_0 R^2 t_0}{I} \sin\phi . \Delta L$$

$$q_h = \frac{P_0 R^2 t_0 \sin\phi}{I} \qquad ①$$

For a uniform circular ring as shown - FIG III ~~on the preceding page we have~~:

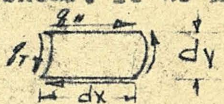
$$I = 2\int_0^{\pi} y^2 t_0 R\, d\phi = 2\int_0^{\pi} R^2 \cos^2\phi\, t_0 R\, d\phi$$

$$= 2 t_0 R^3 \left(\frac{\theta}{2} + \frac{1}{4}\sin\theta\right)\Big|_0^{\pi} = \pi t_0 R^3$$

Substituting in equation ①

$$q_h = \frac{P_0 R^2 t_0 \sin\phi}{\pi t_0 R^3} = \frac{P_0 \sin\phi}{\pi R}$$

This is the horizontal shear. If we now consider a small element of surface dy dx



From the stability of the sheet it follows that the horizontal shear is equal to the tangetial shear.

$\therefore$ Unit Tangential Shear = Unit Horizontal Shear = q = $\dfrac{P_0 \sin\phi}{\pi R}$

Where P_0 = total Shear

The above equation for "q" can also be directly derived from the standard beam equation.

Horizontal shear = VQ/It (lbs/sq.in.)

or q = VQ/I (lbs/in.)

Where Q = Static moment — V = Total vertical shear in lbs = P_0

t = Thickness — q = Shear in lbs/in.

$$Q = y t_0 R\, d\phi \qquad y = R\cos\phi$$

$$\therefore q = \frac{P_0 t_0 R^2}{I}\int_0^{\phi} \cos\phi\, d\phi$$

$$= \frac{P_0 t_0 R^2 \sin\phi}{I}, \quad I = \pi t_0 R^3$$

$$\therefore q = \frac{P_0 t_0 R^2 \sin\phi}{\pi t_0 R^3} = \frac{P_0 \sin\phi}{\pi R} \quad ″/″$$

SHEAR DISTRIBUTION

For the determination of the shear carried by the curved portion of the bulkhead and the vertical portion the assumption was made that:

At the points B and D the value of the unit vertical shear $= q_v = \frac{P_o}{\pi R}$ (lbs/in.)

It is then assumed that the value of the unit vertical shear along the straight portion will be uniform and of the magnitude $q_v = \frac{P_o}{\pi R}$ (lbs/in.)

Below is shown a curve for the vertical shear over half the bulkhead. The unit vertical shear over the curved portion being of the magnitude $q_v = \frac{P_o \sin^2\phi}{\pi R}$

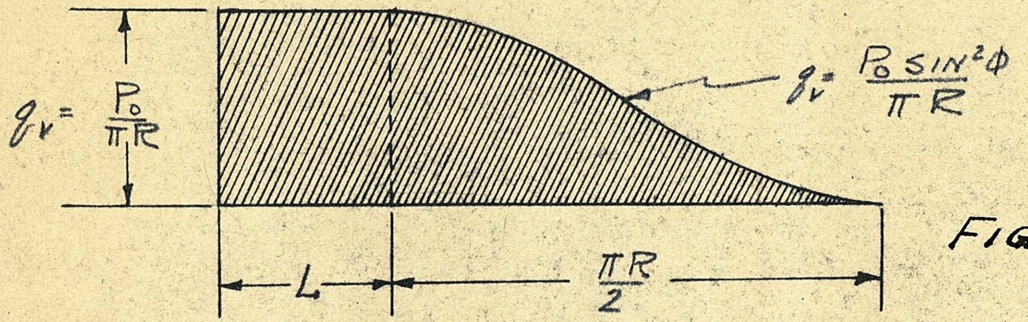


FIG IV

The area under the shear curve for the curved portion of the bulkhead is:

$$V_{sc} = \int_0^{\pi/2} \frac{P_o}{\pi R} \sin^2\phi \, R \, d\phi = \overset{\text{omit}}{\left\{ \int_0^{\pi/2} \frac{P_o}{\pi} \sin^2\phi \, d\phi = \frac{P_o}{\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\phi\right]_0^{\pi/2} \right\}} = \frac{P_o}{4} = P'$$

Hence the mean value of the unit shear along the curved portion is:

$$q_m = \frac{P_o}{4} \Big/ \frac{\pi R}{2} = \overset{\text{omit}}{\left\{ \frac{2 P_o}{4 \pi R} \right\}} = \frac{P_o}{2 \pi R}.$$

Which is half the value of the unit shear over the straight portion.

The total area under the shear curve is, if we assume w = unit shear $= \frac{P_o}{\pi R}$.

$$V_s = \omega L + \frac{\pi}{2} R \frac{\omega}{2} = \omega\left(L + \frac{\pi}{4} R\right)$$

Now V_s is the total vertical shear over one half the fuselage and equals the reaction P_1 ~~(Page 4)~~ FIG II

$$\text{Hence, } w = \frac{P_1}{L + \frac{\pi}{4} R}$$

Now the total vertical shear over the curved portion will be:

$$P' = \frac{\omega \pi R}{4}$$

Substituting the above value of "w"

$$P' = \frac{P_1}{\left(L + \frac{\pi}{4} R\right)} \frac{\pi R}{4} = \frac{\pi R P_1}{4\left(L + \frac{\pi}{4} R\right)}$$

It was shown on the preceding page that $P' = P_o/4$

Hence the equivalent P_o is:

$$P_o = 4P' = \frac{P_1 \pi R}{L + \frac{\pi}{4} R}$$

Check

The vertical shear over half the bulkhead is, from the curve on the preceding page

$$V_s = \frac{P_o L}{\pi R} + \frac{P_o}{2\pi R} \times \frac{\pi R}{2} = \frac{P_o L}{\pi R} + \frac{P_o}{4}$$

Substituting the equivalent value of P_o

$$V_s = \frac{P_1 \pi R L}{(L + \frac{\pi}{4} R)\pi R} + \frac{P_1 \pi R}{(L + \frac{\pi}{4} R)4}$$

$$= P_1 \left[\frac{L + \frac{\pi R}{4}}{L + \frac{\pi R}{4}} \right] = P_1$$

omit

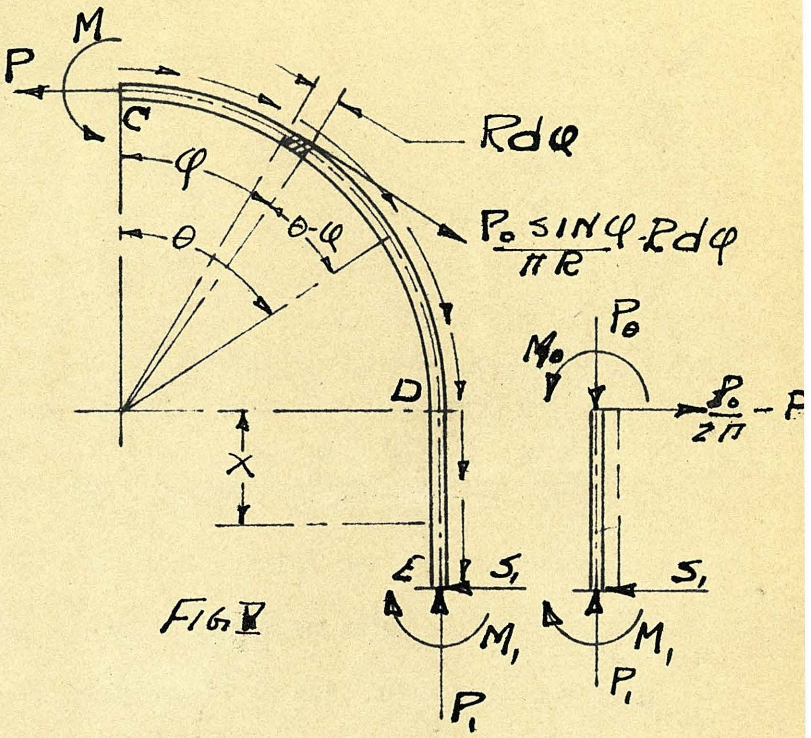


FIG V

VERTICAL SHEAR

Since the bulkhead is symmetrical about the vertical centre line, the bulkhead will be cut at the point C. The loads on each half will be identical.

From the symmetry of the bulkhead and the load it is obvious that the vertical shear at the point C will be zero.

In the above case the unit tangential shear load over the circular portion is $q = P_o \sin\varphi/\pi R$. Over the straight portion the unit vertical shear is $q = P_o/\pi R$

The moment at any point on the circular portion due to $q = P_o \sin\varphi/\pi R$ is:-

$$M_{\theta(q)} = \int_0^\theta P_o \frac{\sin\varphi}{\pi R} R\left[1-\cos(\theta-\varphi)\right] Rd\varphi = \frac{P_o R}{\pi}\left[1-\cos\theta - \frac{\theta}{2}\sin\theta\right]$$

Now if we designate the outer radius by R and the distance to the neutral axis by R_1, then the total moment over the circular portion is

$$M_{\theta(T)} = M + PR_1(1-\cos\theta) - \frac{P_o R}{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right) \quad (2)$$

When $\theta = \pi/2$, $M_{\theta(T)} = M + PR_1 - \frac{P_o R}{\pi}\left(1 - \frac{\pi}{4}\right)$

For one quarter of the circular portion the horizontal shear is

$$S_h = \int_0^{\pi/2} \frac{P_o}{\pi R} \sin\varphi \cos\varphi \, Rd\varphi = \frac{P_o}{2\pi}$$

From the equilibrium equation $\Sigma H = 0$

The horizontal shear at $\theta = \pi/2$ is $H_s = P_o/2\pi - P$

The moment at any point X along the vertical portion is then:

$$M_x = M + PR_1 - \frac{P_o R}{\pi}\left(1-\frac{\pi}{4}\right) - \frac{P_o x}{2\pi} + Px - \epsilon \frac{P_o x}{\pi R} \quad (3)$$

Where $\epsilon \frac{P_o x}{\pi R}$ is the eccentric moment for an eccentricity ϵ. ($\epsilon = R - R_1$)

Applying Castigliano's theorem: The Principle of Least Work.

The energy from C to E is then
$$U = \int_0^{\pi/2} \frac{M_{\theta(T)}^2}{2EI} R_1 d\theta + \int_0^L \frac{M_x^2 dx}{2EI}$$

Substituting the values of $M_{\theta(T)}$ and M_x the energy equation becomes

$$U = \frac{1}{2EI}\int_0^{\pi/2}\left[M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)\right]^2 R_1 d\theta$$
$$+ \frac{1}{2EI}\int_0^L\left[M + PR_1 - \frac{P_0 R}{\pi}\left(1-\frac{\pi}{4}\right) - \frac{P_0 x}{2\pi} + Px - \frac{\varepsilon P_0 x}{\pi R}\right]^2 dx$$

To determine the unknown quantities take the partial derivative with respect to each unknown and set the ~~reulting~~ resulting equation equal to zero.

Note: When integrating the above expression $R_1 d\theta$ will become $R d\theta$ when operating on the term $\frac{P_0}{\pi}R\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)$, this will take into account the eccentricity. In taking the partial with respect to P the resulting R_1 will multiply through as such. The moment of Inertia will be considered constant.

$$\frac{\partial U}{\partial M} = \int_0^{\pi/2}\left[M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)\right] R_1 d\theta$$
$$+ \int_0^L\left[M + PR_1 - \frac{P_0 R}{\pi}\left(1-\frac{\pi}{4}\right) - \frac{P_0 x}{2\pi} + Px - \frac{\varepsilon P_0 x}{\pi R}\right] dx = 0$$

Integrating, collecting terms and simplifying

$$\frac{\partial U}{\partial M} = M\left(\frac{\pi}{2}R_1 + L\right) + P\left(0.5708 R_1^2 + R_1 L + L^2/2\right) - \frac{P_0}{\pi}\left(0.708 R^2\right.$$
$$\left. + 0.2146 RL + L^2/4 + \frac{\varepsilon L^2}{2R}\right) = 0 \qquad (4)$$

$$\frac{\partial U}{\partial P} = \int_0^{\pi/2}\left[M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)\right](1-\cos\theta) R_1 R_1 d\theta$$
$$+ \int_0^L\left[M + PR_1 - \frac{P_0 R}{\pi}\left(1-\frac{\pi}{4}\right) - \frac{P_0 x}{2\pi} + Px - \frac{\varepsilon P_0 x}{\pi R}\right](R + x) dx = 0$$

Integrating, collectong terms and simplifying

$$\frac{\partial U}{\partial P} = M\left[0.5708 R_1^2 + L(R_1 + L/2)\right] + P\left[0.3562 R_1^3 + R_1 L(R_1 + L)\right.$$
$$\left. + L^3/3\right] - \frac{P_0}{\pi}\left[0.05255 R^2 R_1 + 0.2146 R R_1 L + R_1 L^2\left(\frac{1}{4} + \frac{\varepsilon}{2R}\right)\right.$$
$$\left. + 0.1073 RL^2 + L^3\left(1/6 + \varepsilon/3R\right)\right] = 0 \qquad (5)$$

Substituting the values for the constants in equations ④ and ⑤

$L = 18''$, $R = 33.5''$, $R_1 = 32.5''$, $\varepsilon = 1''$ the equations reduce to the following.

$$69.1\ M + 1350\ P - 94.0\ P_o = 0 \qquad (4)$$

$$1350.0\ M + 43{,}844\ P - 3580\ P_o = 0 \qquad (5)$$

Solving similtaneously gives

$$M = -\ 0.593\ P_o \quad , \quad P = 0.0999\ P_o$$

Since P_o is a function of the weight, which is subject to many changes, it was felt that it would be advantageous from a practical point of view to give all moments, shears and axial loads in terms of P_o, the total vertical shear, as it will facilitate the checking of loads in the bulkhead for changes in weight.

Using the values obtained for M and P and evaluating the terms in the moment equation

θ	M	$PR_1(1-\cos\theta)$	$\frac{P_o R}{\pi}(1-\cos\theta-\frac{\theta}{2}\sin\theta)$	M_θ
$\pi/12$	$-\ 0.593\ P_o$	$0.1104\ P_o$	$-\ 0.0011\ P_o$	$-\ 0.483\ P_o$
$\pi/6$	$-\ 0.593\ P_o$	$0.4351\ P_o$	$-\ 0.032\ P_o$	$-\ 0.190\ P_o$
$\pi/4$	$-\ 0.593\ P_o$	$0.951\ P_o$	$-\ 0.139\ P_o$	$0.219\ P_o$
$\pi/3$	$-\ 0.593\ P_o$	$1.623\ P_o$	$-\ 0.5012\ P_o$	$0.529\ P_o$
$5\pi/12$	$-\ 0.593\ P_o$	$2.406\ P_o$	$-\ 1.158\ P_o$	$0.655\ P_o$
$\pi/2$	$-\ 0.593\ P_o$	$3.246\ P_o$	$-\ 2.285\ P_o$	$0.370\ P_o$

Substituting in eq. ③ the values obtained for M and P gives

$$M_x = 0.370\ P_o - 0.169\ P_oX + 0.0999\ P_oX - 0.0095\ P_oX$$

<u>When X = 6</u>

$$M_x = 0.370\ P_o - 0.4116\ P_o = -\ \underline{0.0416\ P_o}$$

<u>When X = 12</u>

$$M_x = 0.370\ P_o - 0.8232\ P_o = -\ \underline{0.4533\ P_o}$$

<u>When X = 18</u>

$$M_x = 0.370 P_o - 1.2348\ P_o = -\ \underline{0.8648\ P_o}$$

FIG. VII

SEE DIAGRAM ~~ON ...~~

SHEAR AT RIGHT ANGLES TO THE NEUTRAL AXIS

From the following shear moment equation

$$S = \frac{dM}{dx} = \frac{dM}{d\theta} \cdot \frac{d\theta}{dx}$$ (Given in any Strength of Materials) Text

$$dx = R\,d\theta$$

$$S = \frac{dM}{d\theta} \cdot \frac{d\theta}{R\,d\theta} = \frac{1}{R}\frac{dM}{d\theta}$$

$$SR = \frac{dM}{d\theta}$$

From equation @ page 10:

$$SR = \frac{dM}{d\theta} = \frac{d}{d\theta}\left[M + PR_1(1-\cos\theta) - \frac{P_oR}{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right)\right]$$

omit $$\left\{= PR_1 \sin\theta - \frac{P_oR}{\pi}\left(\sin\theta - \frac{\theta}{2}\cos\theta - \frac{1}{2}\sin\theta\right)\right\}$$

$$= PR_1 \sin\theta - \frac{P_oR}{2\pi}(\sin\theta - \theta\cos\theta)$$

$$P = .0999\,P_o$$

$$\therefore\ S = .0999\,P_o \sin\theta - \frac{P_o}{2\pi}(\sin\theta - \theta\cos\theta)$$

Note: The radii have been assumed equal in the above derivation. Since the the ratio $R/R_1 = 1.03$, the above result is close enough for practical purposes.

EVALUATING THE TERMS IN THE ABOVE EQUATION FOR VALUES OF θ FROM $\theta = 0$ TO $\theta = \frac{\pi}{2}$

θ	$.0999 P_o \sin\theta$	$\frac{P_o}{2\pi}(\sin\theta - \theta\cos\theta)$	S
$\frac{\pi}{12}$	$.02588\ P_o$	$-.0009\ P_o$	$.0249\ P_o$
$\frac{\pi}{6}$	$.0499\ P_o$	$-.0073\ P_o$	$.0426\ P_o$
$\frac{\pi}{4}$	$.0706\ P_o$	$-.0242\ P_o$	$.0464\ P_o$
$\frac{\pi}{3}$	$.0865\ P_o$	$-.0546\ P_o$	$.0319\ P_o$
$\frac{5\pi}{12}$	$.0965\ P_o$	$-.1003\ P_o$	$-.0038\ P_o$
$\frac{\pi}{2}$	$.0999\ P_o$	$-.1590\ P_o$	$-.0591\ P_o$

SEE DIAGRAM ~~ON PAGE 14~~ FIG VIII

For the straight portion from equation ~~(b) page 11~~ (3):-

$$\frac{dM}{dx} = \frac{d}{dx}\left[M + PR_1 - \frac{P_oR}{\pi}\left(1-\frac{\pi}{4}\right) - \frac{P_ox}{2\pi} + Px - \frac{\varepsilon P_ox}{\pi R}\right]$$

$$= P - \frac{P_o}{2\pi} - \frac{\varepsilon P_o}{\pi R}$$

$\frac{\varepsilon P_o}{\pi R}$ is small and may be neglected hence the shear will be constant and equal to the shear at $\theta = \pi/2$.

AXIAL LOAD at any point on the bulkhead.

Due to the symmetry of the bulkhead and the loading, the axial loads in the left half of the bulkhead will be equal to the axial loads in the right half.

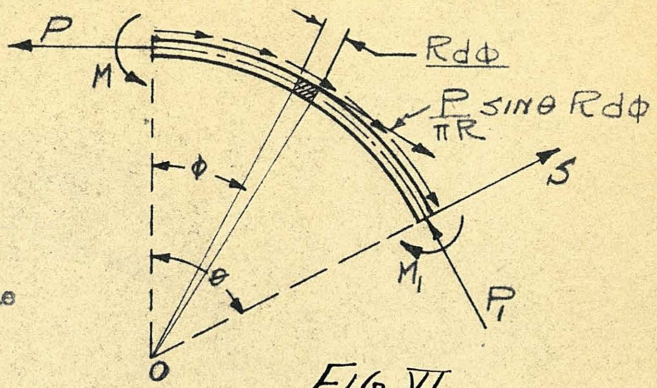


FIG VI

For equilibrium

$$\Sigma M_0 = M + PR + P_1R - M_1 - R\int_0^{\theta} \frac{P_0 \sin\phi}{\pi R} R\,d\phi = 0$$

$$M_1 = M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1 - \cos\theta - \frac{\theta}{2}\sin\theta\right) \qquad (P. \quad)$$

$$\text{HENCE } \Sigma M_0 = M + PR_1 + P_1R_1 - M - PR_1(1-\cos\theta) + \frac{P_0 R}{\pi}\left(1 - \cos\theta - \frac{\theta}{2}\sin\theta\right) - \frac{P_0 R}{\pi}(-\cos\theta)\Big|_0^{\theta}$$

omit

$$= M + PR_1 + P_1R_1 - M - PR_1 + PR_1\cos\theta + \frac{P_0 R}{\pi} - \frac{P_0 R\cos\theta}{\pi} - \frac{P_0 R}{\pi}\frac{\theta}{2}\sin\theta - \frac{P_0 R}{\pi} + \frac{P_0 R}{\pi}\cos\theta = 0$$

$$= P_1R_1 + PR_1\cos\theta - \frac{P_0 R}{2\pi}\theta\sin\theta = 0$$

$$P_1 = \frac{P_0}{2\pi}\theta\sin\theta - P\cos\theta \qquad (P = .0999\,P_0)$$

Evaluating P_1 for various values of θ

θ	$-.0999 P_o \cos$	$\frac{P_o}{2\pi}\sin\theta$	P_1
$\frac{\pi}{12}$	$-.0965\ P_o$	$.01078\ P_o$	$-.08572\ P_o$
$\frac{\pi}{6}$	$-.0865\ P_o$	$.0416\ P_o$	$-.0449\ P_o$
$\frac{\pi}{4}$	$-.0706\ P_o$	$.0885\ P_o$	$+.0179\ P_o$
$\frac{\pi}{3}$	$-.04995\ P_o$	$.1444\ P_o$	$+.0945\ P_o$
$\frac{5\pi}{12}$	$-.02558\ P_o$	$.2010\ P_o$	$+.1754\ P_o$
$\frac{\pi}{2}$	0	$.2500\ P_o$	$+.2500\ P_o$

For the straight portion

$$P_1 = .250\,P_o + \frac{P_o\,x}{R}$$

When $x = 18$

$$P_1 = .420\,P_o$$

SEE DIAGRAM ~~ON PAGE 18~~ FIG IX

MOMENT AND SHEAR DIAGRAMS

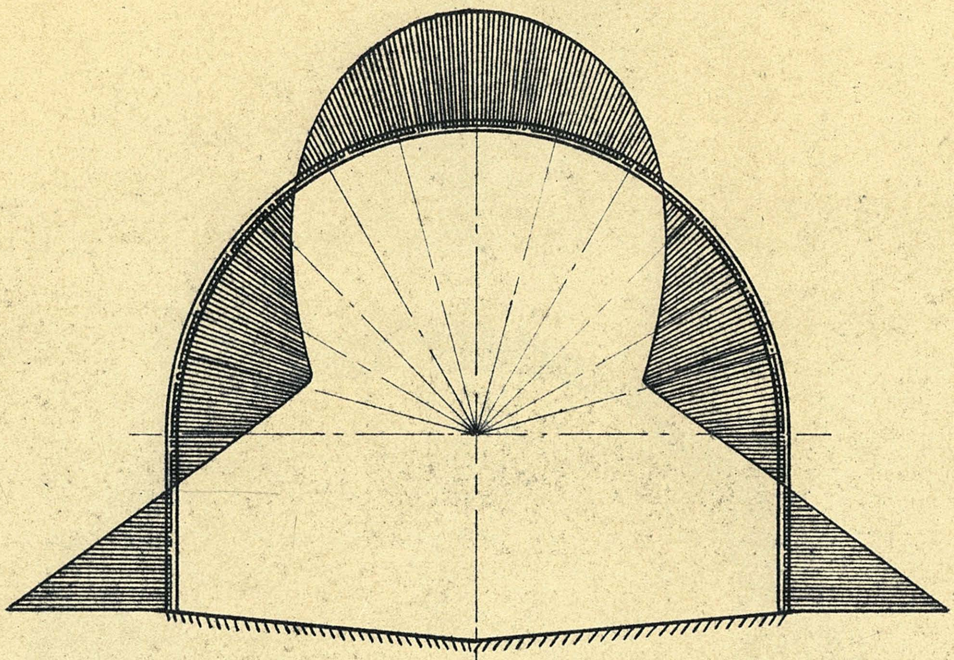


FIG VII

Moment diagram for vertical shear. Moments are plotted on Compression side. Scale 1" = P_o

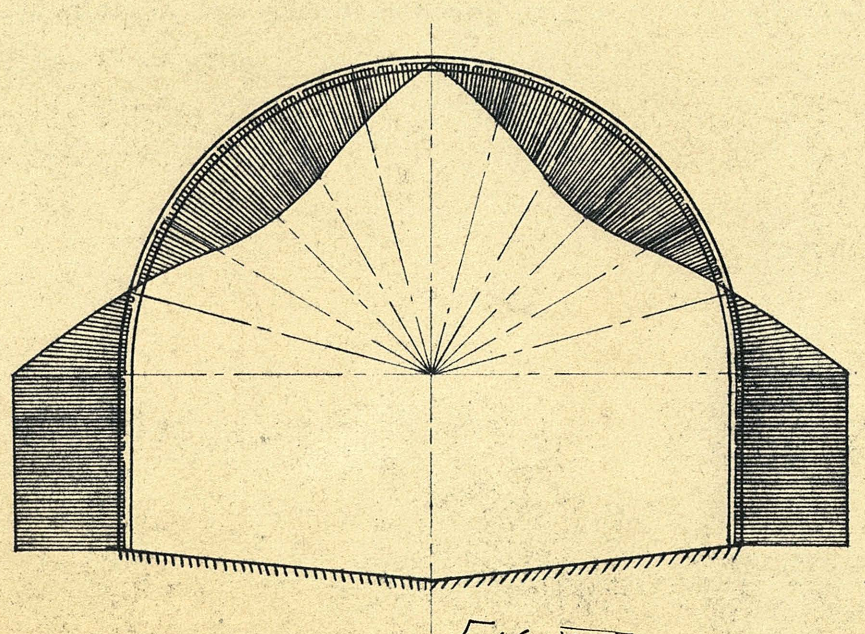


FIG VIII

Shear diagram for vertical shear. Shear perpendicular to neutral axis. Scale 1" = 0.1 P_o

AXIAL LOAD DIAGRAM

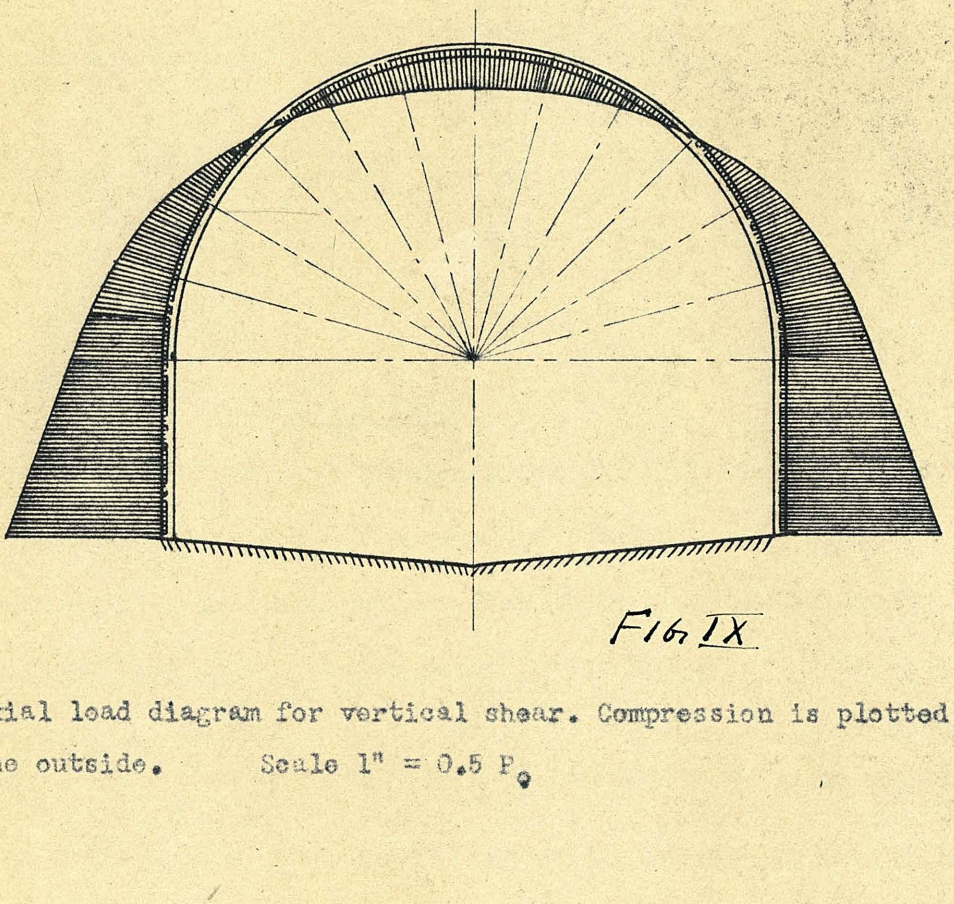


FIG IX

Axial load diagram for vertical shear. Compression is plotted on the outside. Scale 1" = 0.5 P_o

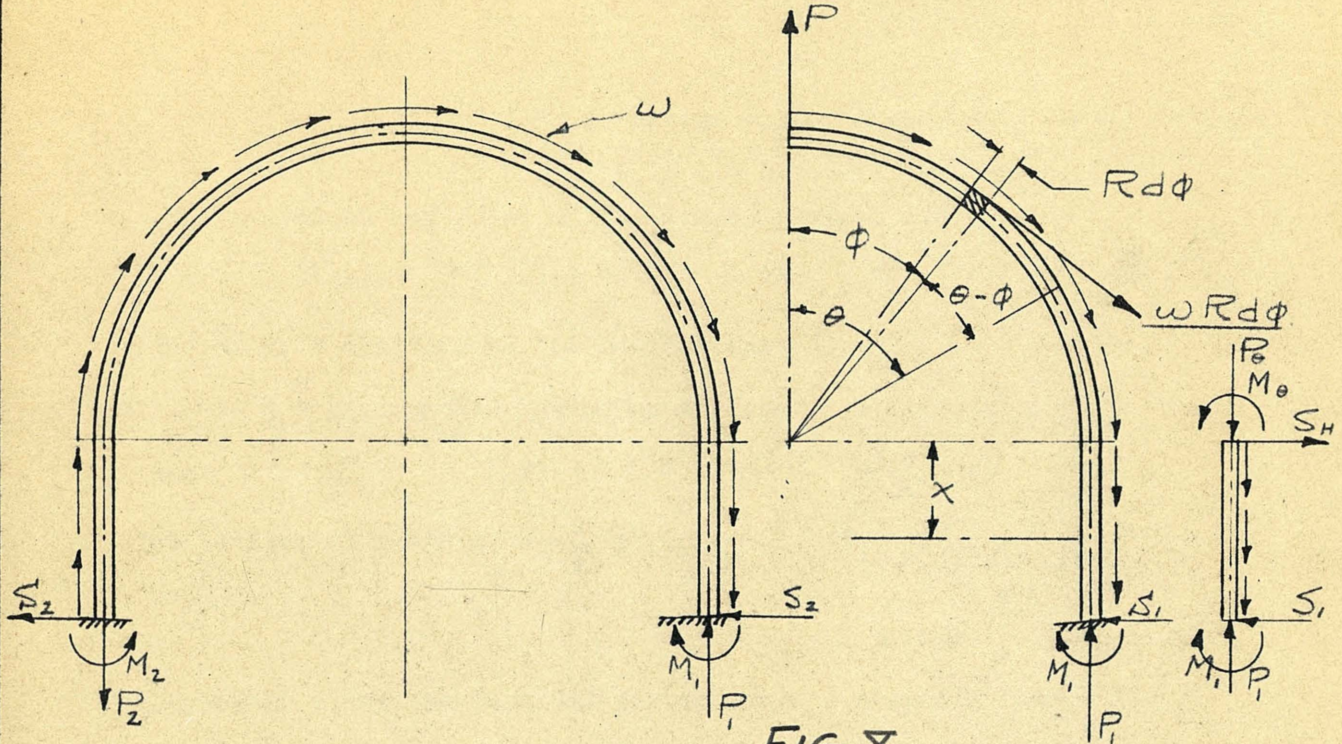


FIG X

TORSIONAL SHEAR

In the above case the torsional moment in the fuselage introduce a uniform torsional shear "w" around the fuselage ($w = M_t/2A$).

Since the bulkhead is symmetrical about the vertical centre line, the bulkhead will be cut at the point C. The loads in the left half will be equal in magnitude to those in the right half but of opposite sign.

From the symmetry of the bulkhead it can be seen that at C there will be a point of inflection hence the moment at C will be zero. It can also be shown that the axial load at C is zero ~~Also the axial load will be zero at C, a proof for this latter statement is given in Appendix I~~

The moment due to the unit shear "w" at any point on the circular portion is:

$$M_{\theta(w)} = \int_0^\theta wR^2[1-\cos(\theta-\phi)]\,d\phi$$

omit

$$= \int_0^\theta wR^2(1-\cos\theta\cos\phi+\sin\theta\sin\phi)\,d\phi$$

$$= wR^2(\phi-\cos\theta\sin\phi-\sin\theta\cos\phi)\Big|_0^\theta$$

$$\left\{M_{\theta(w)} = wR^2(\theta \overset{\text{omit}}{- \cos\theta\sin\theta + \sin\theta\cos\theta} - \sin\theta)\right\} = wR^2(\theta - \sin\theta)$$

The moment at any point on the curved surface is:

$$M_{\theta(T)} = -PR_1\sin\theta - wR^2(\theta - \sin\theta) \quad (6)$$

Where R_1 = distance to the neutral axis

R = distance to the outer surface

When $\theta = \frac{\pi}{2}$, $M_{(\frac{\pi}{2})} = -PR_1 - wR^2(\frac{\pi}{2} - 1)$

The horizontal shear at $\theta = \frac{\pi}{2}$ is: $S_H = \int_0^{\frac{\pi}{2}} wR\cos\theta\, d\theta$

$$= wR\sin\theta \Big|_0^{\frac{\pi}{2}} = wR$$

The moment along the vertical portion at a distance "X" from "D" is:

$$M_x = M_{\frac{\pi}{2}} + S_H X + \varepsilon w x$$

$$= -PR_1 - wR^2(\tfrac{\pi}{2} - 1) - wRx - \varepsilon w x \quad (7)$$

Where $\varepsilon w x$ is the eccentric moment due to an eccentricity ε $(\varepsilon = R - R_1)$

Writing the energy equations and following the procedure as given for the vertical shear condition, we have:

The total energy from C to E is:

$$U = \frac{1}{2EI}\int_0^{\frac{\pi}{2}}\left[PR_1\sin\theta + wR^2(\theta - \sin\theta)\right]^2 R_1 d\theta - \frac{1}{2EI}\int_0^L\left[PR_1 + wR^2(\tfrac{\pi}{2} - 1) + wRx + \varepsilon w x\right]^2 dx$$

Taking the partial derivative with respect to P and noting that $R_1 d\theta$ becomes $R d\theta$ when operating on the term $wR^2(\theta - \sin\theta)$ in order that the eccentricity may be taken into account, we have:

$$\frac{\partial U}{\partial P} = \int_0^{\frac{\pi}{2}}\left[PR_1\sin\theta + wR^2(\theta - \sin\theta)\right]R_1\sin\theta\, R_1 d\theta + \int_0^L\left[PR_1 + wR^2(\tfrac{\pi}{2} - 1) + wRx + \varepsilon w x\right]R_1 dx = 0$$

omit

$$\left\{ = \int_0^{\frac{\pi}{2}}\left[PR_1\sin^2\theta + wR^2(\theta\sin\theta - \sin^2\theta)\right]R_1 R_1 d\theta + \int_0^L\left[PR_1^2 + wR^2R_1(\tfrac{\pi}{2} - 1) + wRR_1x + \varepsilon w R_1 x\right]dx = 0\right.$$

omit
$$\frac{\partial u}{\partial P} = \Big\{ \Big[PR_1^3\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right) + wR^3R_1\Big(\sin\theta - \theta\cos\theta - \frac{\theta}{2} + \frac{1}{4}\sin 2\theta\Big)\Big]_0^{\pi/2} + \Big[PR_1x + wR^2R_1x\left(\frac{\pi}{2}-1\right) + wRR_1\frac{x^2}{2} + \varepsilon w R\frac{x^2}{2}\Big]_0^{L}$$

$$= \frac{\pi}{4}PR_1^3 + wR^3R_1\left(1-\frac{\pi}{4}\right) + PR_1^2L + wR^2R_1L\left(\frac{\pi}{2}-1\right) + \frac{w}{2}RR_1L + \frac{\varepsilon}{2}wL^2R_1 = 0$$

Collecting terms:

$$PR_1^2\left[\frac{\pi}{4}R_1 + L\right] + wR_1R\left[R^2 - \frac{\pi}{4}R^2 + \frac{\pi}{2}RL - RL + \frac{L^2}{2}\right] + \varepsilon\frac{wL^2}{2}R_1 = 0 \qquad (8)$$

Simplifying and solving for P:

$$P = \frac{-1}{R_1^2\left(\frac{\pi}{4}R_1 + L\right)}\left[wR_1R\left(.2146R^2 + .5708RL + .5L^2\right] - .5\varepsilon wL^2R_1\right.$$

Substituting the values for the constants in the above equation

$R = 33.5"$, $R_1 = 32.5"$, $L = 18"$, $\varepsilon = 1"$

$$P = -\frac{32.5\,wR\left[241 + 344.5 + 162\right] - 5575w}{46000}$$

$$= -\frac{819,600}{46000} = -17.85\,w$$

Evaluating the terms in equation (6) ~~page 20~~

θ	$PR_1\sin\theta$	$wR^2(\theta - \sin\theta)$	M_θ
$\frac{\pi}{12}$	+ 150 w	− 3.37 w	+ 146.7 w
$\frac{\pi}{6}$	+ 290 w	− 26.9 w	+ 263.1 w
$\frac{\pi}{4}$	+ 410 w	− 88.0 w	+ 322.0 w
$\frac{\pi}{3}$	+ 502 w	− 203.2 w	+ 304.8 w
$\frac{5\pi}{12}$	+ 560 w	− 384.9 w	+ 175.1 w
$\frac{\pi}{2}$	+ 580 w	− 640.0 w	− 60.0 w

SEE DIAGRAM - FIG XII

MOMENT ALONG THE VERTICAL PORTION

$$M_x = -PR_1 - wR^2\left(\frac{\pi}{2}-1\right) - wRx - \mathcal{E}wx$$

When X = 6

$$M_x = {}^{-}61.4\ w - 201\ w - 6\ w = {}^{-}\underline{268\ w}$$

When X = 12

$$M_x = {}^{-}61.4\ w - 402\ w - 12\ w = {}^{-}\underline{475\ w}$$

When X = 18

$$M_x = {}^{-}61.4\ w - 603\ w - 18\ w = {}^{-}\underline{682\ w}$$

SEE DIAGRAM ON PAGE 24

SHEAR AT RIGHT ANGLES TO THE NEUTRAL AXIS

From the shear moment equation ~~given on page 15 & 20~~

$$SR = \frac{dM}{d\theta} = \frac{d}{d\theta}\left[-PR_1 \sin\theta - wR^2(\theta - \sin\theta)\right]$$

$$= -PR_1 \cos\theta - wR^2(1-\cos\theta)$$

$$\therefore\ S = -P\cos\theta - wR\ (1-\cos\theta)$$

Evaluating Terms:

θ	$P\cos\theta$	$wR(1-\cos\theta)$	S
$\frac{\pi}{12}$	+ 17.2 w	− 1.14 w	+ 16.06 w
$\frac{\pi}{6}$	+ 15.43 w	− 4.5 w	+ 10.93 w
$\frac{\pi}{4}$	+ 12.60 w	− 9.8 w	+ 2.8 w
$\frac{\pi}{3}$	+ 8.92 w	− 16.75 w	− 7.83 w
$\frac{5\pi}{12}$	+ 4.56 w	− 24.9 w	− 20.34 w
$\frac{\pi}{2}$	+ 0	− 33.5 w	− 33.5 w

FOR HORIZONTAL SHEAR ALONG THE VERTICAL PORTION

$$S = \frac{dM}{dx} = -\frac{d}{dx}\left[PR_1 + wR^2\left(\frac{\pi}{2}-1\right) + wRx + \mathcal{E}wx\right] \quad (\text{Eq. } \cancel{3}\ 7\ \cancel{\text{Page } 20})$$

$$S = -[wR + \mathcal{E}w] = -w(R+\mathcal{E}) = \underline{-34.5w}.$$

SEE DIAGRAM ~~ON PAGE 24~~ FIG XIII

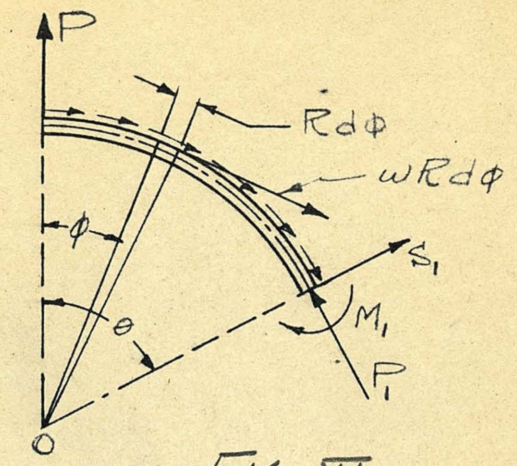


FIG. XI

Axial load at any point on the bulkhead.

The axial loads in the left half of the bulkhead will be equal to the axial loads in the right half but opposite in sign.

For equilibrium

$$\Sigma M_0 = 0 = P_1R_1 - M_1 - R\int_0^\theta wR\,d\phi$$

$$M_1 = -PR_1\sin\theta - wR^2(\theta - \sin\theta)$$

$$\text{HENCE}\quad \Sigma M_0 = P_1R + PR_1\sin\theta + wR^2(\theta - \sin\theta) - wR^2\theta$$

$$= P_1R_1 + PR_1\sin\theta - wR^2\sin\theta = 0$$

$$\therefore\quad P_1 = -P\sin\theta + wR\sin\theta \qquad P = -17.85w,\quad R = 33.5''$$

$$P_1 = +17.85\,w\sin\theta + 33.5\,w\sin\theta = +51.35w\sin\theta$$

For the straight portion:

$$P_1 = +51.35w + wx = +w(51.35 + x)$$

Evaluating P_1 for various values of θ

θ	$+51.35w\sin\theta$	P
$\frac{\pi}{12}$	+ 13.3 w	+ 13.3 w
$\frac{\pi}{6}$	+ 25.7 w	+ 25.7 w
$\frac{\pi}{4}$	+ 36.3 w	+ 36.3 w
$\frac{\pi}{3}$	+ 44.5 w	+ 44.5 w
$\frac{5\pi}{12}$	+ 49.6 w	+ 49.6 w
$\frac{\pi}{2}$	+ 51.35 w	+ 51.35 w

For the straight portion:

$P_1 = +51.5\text{ w} + \text{wx}$

For x = 6

$P_1 = +57.35\text{ w}$

For x = 12

$P_1 = +63.35\text{ w}$

For x = 18

$P_1 = +69.35\text{ w}$

SEE DIAGRAM ~~[illegible]~~ FIG. XIV

MOMENT AND SHEAR DIAGRAMS

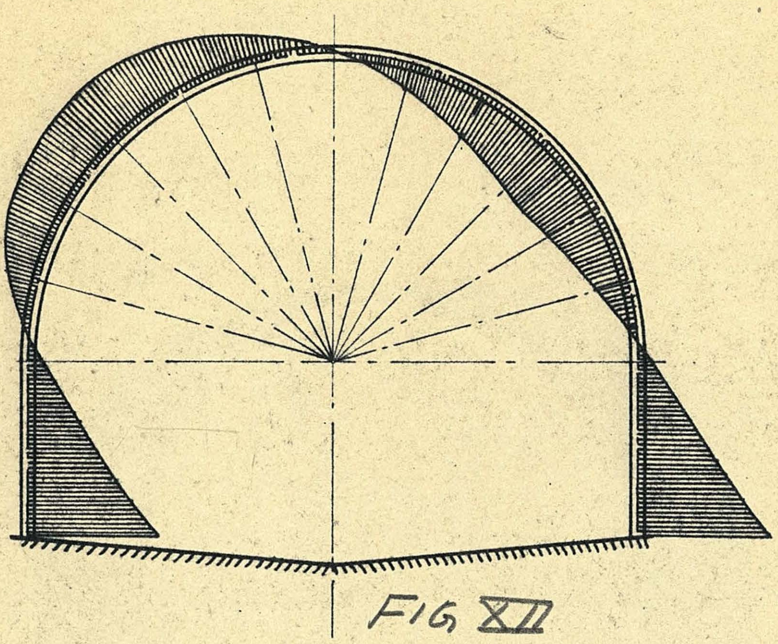


FIG XII

Moment diagram for torsional shear. Moments are plotted on compression side. Scale 1" = 1000 w

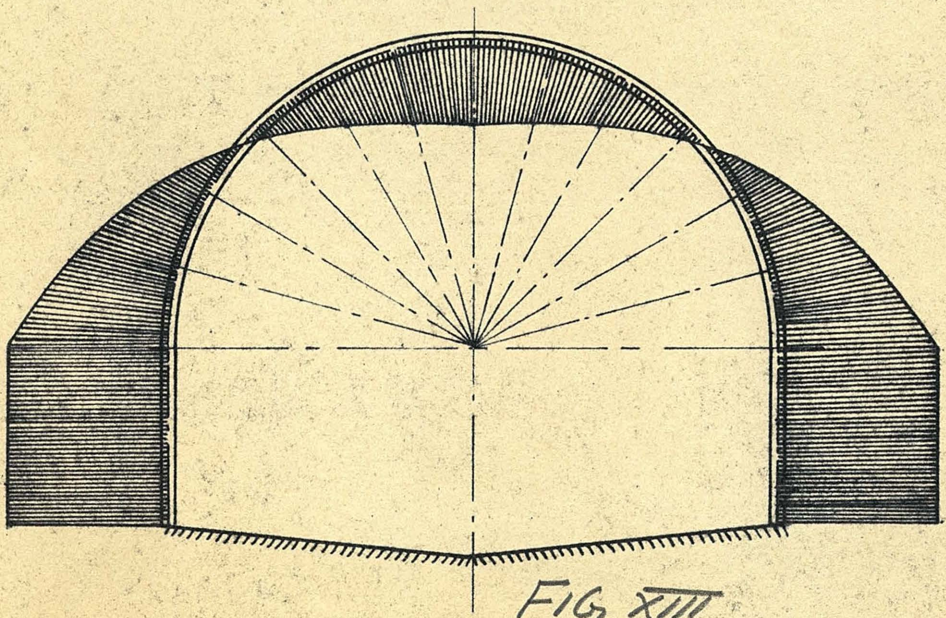


FIG XIII

Shear diagram for torsional shear. Shear perpendicular to neutral axis. Scale 1" = 40 w

AXIAL LOAD DIAGRAM

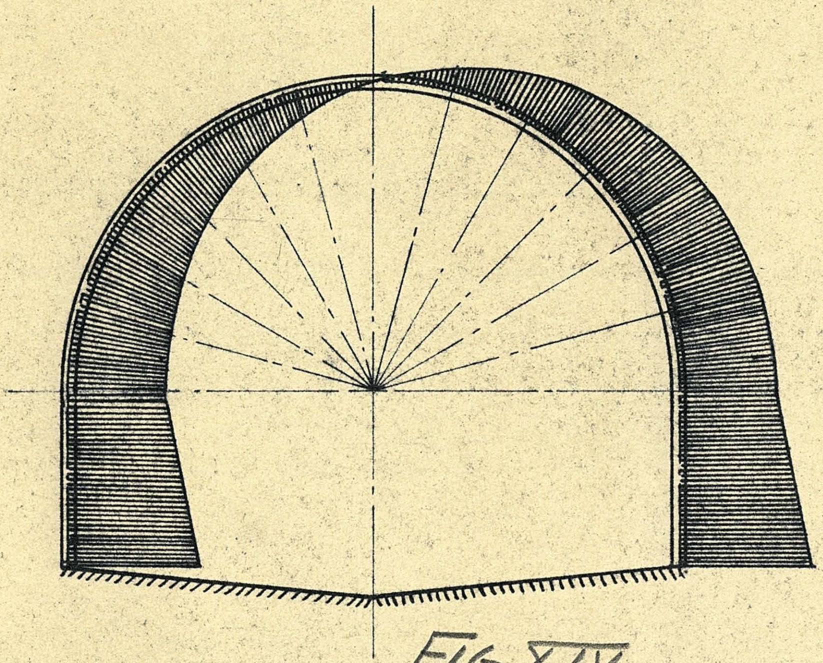


FIG XIV

Axial load diagram for torsional shear. Compression is plotted on the outside. Scale 1" = 100 w

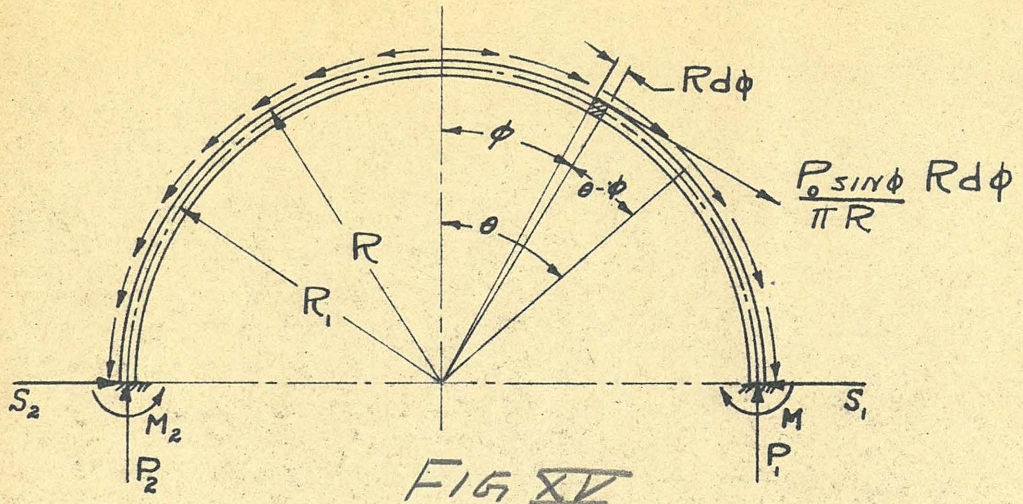


FIG XV

VERTICAL SHEAR SOLUTION FOR A CIRCULAR BULKHEAD.

The vertical shear solution given ~~on pages 10 to 16~~ for the non-circular bulkhead can be used for a circular bulkhead, as shown in the figure, by letting L = 0.

The moment equation will then be:

$$M_\theta = M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right)$$

And the energy equation from C to D is:

$$U = \frac{1}{2EI}\int_0^{\frac{\pi}{2}}\left[M + PR_1(1-\cos\theta) - \frac{P_0 R}{\pi}\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)\right]^2 R\,d\theta$$

From equation 4 ~~(1) page 12~~, when L = 0

$$\frac{\partial U}{\partial M} = \frac{\pi}{2}R_1 M + \left(\frac{\pi}{2}-1\right)R_1^2 P - \frac{P_0 R^2}{\pi}\left(\frac{\pi}{2}-1.5\right) = 0 \qquad (4')$$

From equation 5 ~~(2) page 13~~, when L = 0

$$\frac{\partial U}{\partial P} = MR_1^2\left(\frac{\pi}{2}-1\right) + PR_1^3\left(\frac{\pi}{2}-2+\frac{\pi}{4}\right) - \frac{P_0 R^2}{\pi}R_1\left(\frac{\pi}{2}-2.5+\frac{\pi}{4}+\frac{\pi}{16}\right) = 0$$

$$= MR_1^2\left(\frac{\pi}{2}-1\right) + PR_1^3\left(\frac{3\pi}{4}-2\right) - \frac{P_0 R^2}{\pi}R'\left(\frac{13\pi}{16}-2.5\right) = 0 \qquad (5')$$

Simplifying:

$$1.5708\,M + .5708\,PR_1 - .0708\,\frac{P_0 R^2}{\pi R_1} = 0 \qquad (4')$$

$$.5708\,M + .3562\,PR_1 - .0520\,\frac{P_0 R^2}{\pi R_1} = 0 \qquad (5')$$

Solving similtaneously:

$$1.5708\,M + .5708\,PR_1 - .0708\,\frac{P_0 R^2}{\pi R_1} = 0$$

$$\underline{.9140\,M + .5708\,PR_1 - .0833\,\frac{P_0 R^2}{\pi R_1}} = 0$$

$$.6568\,M + .0125\,\frac{P_0 R^2}{\pi R_1} = 0$$

$$M = -\frac{.0125}{.6568}\frac{P_o R^2}{\pi R_1} = -.019\frac{P_o R^2}{\pi R_1}$$

Since $\dfrac{R}{R_1} = 1.03$ $\qquad M = -.00624\, P_o R$

$$= -.209\, P_o$$

Substituting this value of M in eq. (4')

$$-.02995\frac{P_o R^2}{\pi R_1} + .5708\, P R_1 - .0708\frac{P_o R^2}{\pi R_1} = 0$$

$$P = \frac{.10075}{.5708}\frac{P_o R^2}{\pi R_1^2}$$

$$= .0595\, P_o$$

Substituting the values of M and P in the moment equation the moment at any point on the bulkhead may be obtained in terms of P_o .

$$M_\theta = -.209 P_o + .0595 P_o R(1-\cos\theta) - \frac{P_o R}{\pi}\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)$$

Substituing the values for R and R_1

$$M_\theta = -.209 P_o + 1.95 P_o(1-\cos\theta) - 10.65 P_o\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right)$$

Evaluating terms in the moment equation:

θ	M	$1.935 P_o(1-\cos\theta)$	$10.65 P_o(1-\cos\theta-\frac{\theta}{2}\sin\theta)$	M_θ
$\frac{\pi}{12}$	$-.209\ P_o$	$.066\ P_o$	$-.0011\ P_o$	$-.1441\ P_o$
$\frac{\pi}{6}$	$-.209\ P_o$	$.2595\ P_o$	$-.0320\ P_o$	$-.0185\ P_o$
$\frac{\pi}{4}$	$-.209\ P_o$	$.5670\ P_o$	$-.1390\ P_o$	$.2190\ P_o$
$\frac{\pi}{3}$	$-.209\ P_o$	$.9680\ P_o$	$-.5012\ P_o$	$.2578\ P_o$
$\frac{5\pi}{12}$	$-.209\ P_o$	$1.4350\ P_o$	$-1.1580\ P_o$	$.0680\ P_o$
$\frac{\pi}{2}$	$-.209\ P_o$	$1.9350\ P_o$	$-2.2850\ P_o$	$-.559\ P_o$

For cases in which the spar attachment is such that $\theta > \frac{\pi}{2}$, the upper limit of integration in the general solution may be changed to the required value of θ. The solution can then be obtained by the proper differentiation and integration.

Due to symmetry the moments in the left side will be equal to those in the right side.

The shear and axial load solution is similar to that of the non-circular bulkhead.

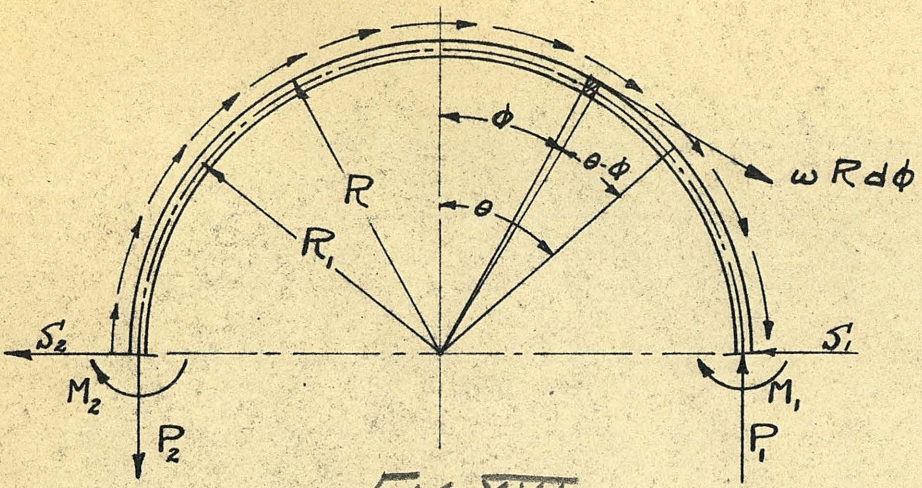


FIG XVI

TORSIONAL SHEAR SOLUTION FOR A CIRCULAR BULKHEAD

The torsional shear solution ~~given on pages 19 to 23,~~ may be used for the circular bulkhead, as shown in the figure, by letting L = 0

The moment equation will thenbe:-

$$M_\theta = -PR_1 \sin\theta - wR^2(\theta - \sin\theta)$$

And the energy from C to D is:

$$U = \frac{-1}{2EI}\int_0^{\frac{\pi}{2}} \left[PR_1 \sin\theta + wR^2(\theta - \sin\theta)\right]^2 R\,d\theta$$

From equation ~~(1)~~ (8) ~~page 21, when L = 0~~, when L = 0

$$\frac{\partial U}{\partial P} = \frac{\pi}{4} PR_1^3 + wR^3R_1\left(1 - \frac{\pi}{4}\right) = 0$$

$$P = -\frac{4}{\pi}\ \frac{wR^3R_1}{R_1^3}\left(1 - \frac{\pi}{4}\right)$$

$$= -\frac{4}{\pi} \times 1.03^2\ wR\left(1 - \frac{\pi}{4}\right)$$

$$= -.29\ wR. \qquad = -9.72\ wR.$$

Substituting the value of P in the moment equation, the moment at any point on the bulkhead may be obtained in terms of "w"

Evaluating the terms in the moment equation.

θ	$PR_1 \sin\theta$	$wR^2(\theta - \sin\theta)$	M_θ
0	0	0	0
$\frac{\pi}{12}$	+ 81.7 w	− 3.4 w	+ 78.3 w
$\frac{\pi}{6}$	+ 158.0 w	− 26.9 w	+ 131.1 w
$\frac{\pi}{4}$	+ 223.0 w	− 88.0 w	+ 135.0 w
$\frac{\pi}{3}$	+ 273.5 w	− 203.2 w	+ 70.3 w
$\frac{5\pi}{12}$	+ 305.0 w	− 384.9 w	− 79.9 w
$\frac{\pi}{2}$	+ 316.0 w	− 604.5 w	− 324.5 w

Again for cases where $\theta > \frac{\pi}{2}$ the solution may be obtained by changing the upper limit of integration.

The moments in the left half will be equal in magnitude to those in the right half but of opposite sign.

The shear and axial load solution is similar to that of the non-circular bulkhead.

CONCLUSIONS

The results obtained from this type of load distribution seem reasonable. A bulkhead design based on this analysis compares favorably with bulkheads which, through static tests of the completed airplane, have proved to be satisfactory from a weight as well as strength standpoint.

It may be of interest to note that when a straight section is added to the circular section the loads increase rather rapidly.

The analysis and procedure may seem somewhat too specific, however, it was felt that there exist a definite need in the industry for a solution of this type, dealing with a specific design problem and yet general enough so that by a slight modification it may be applied to problems of similar nature. The representation of a number of abstract formulae and equations lead too often to misinterpretation and confusion.

FIG 1

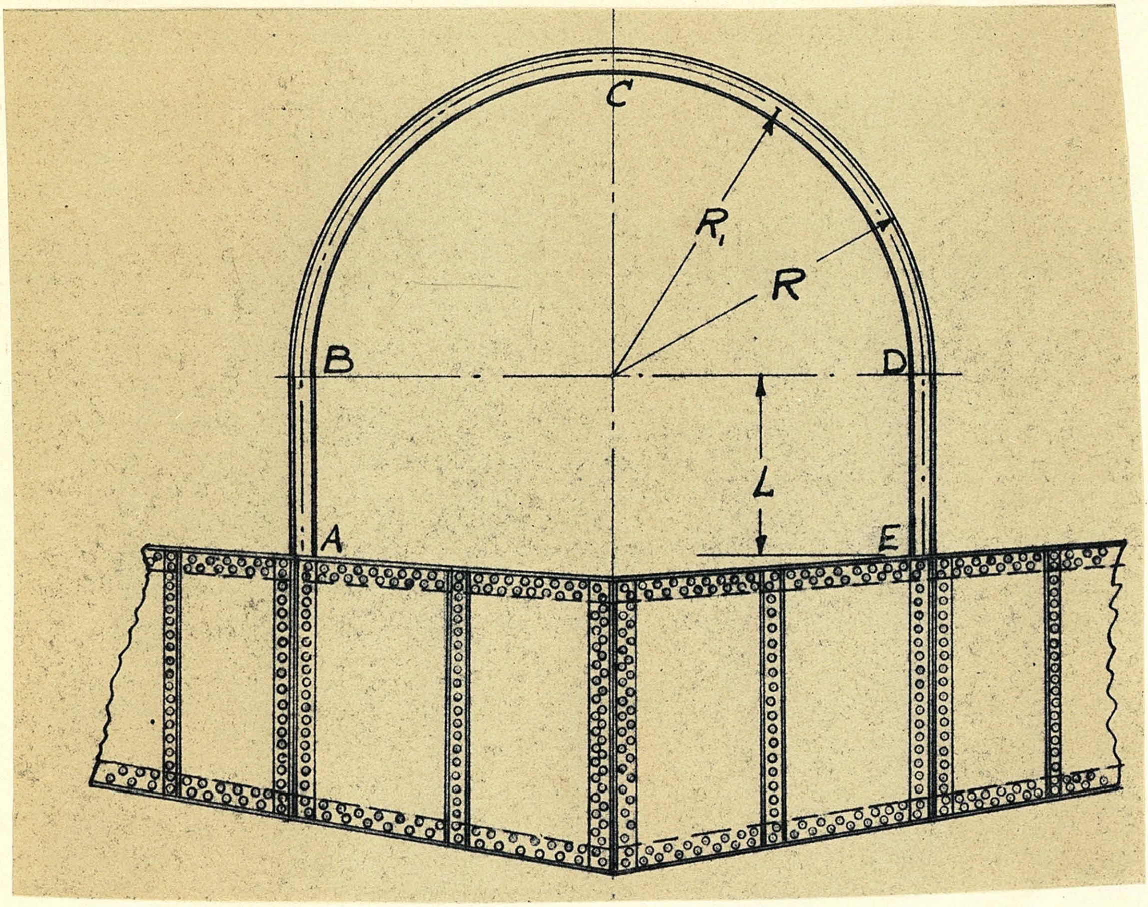


FIG 2

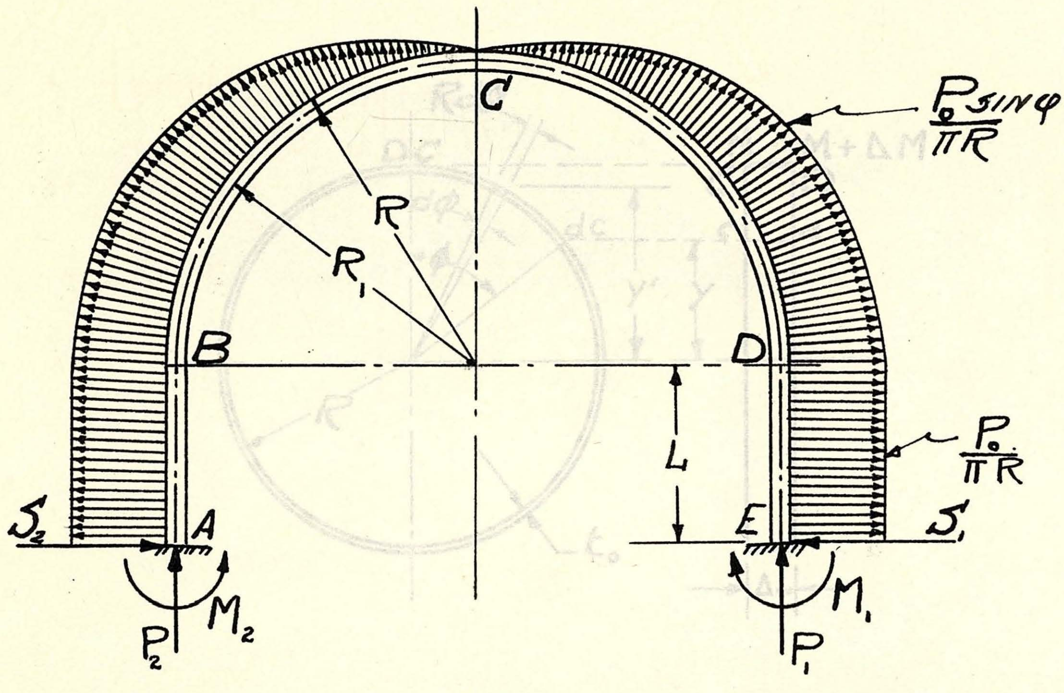


FIG. 3

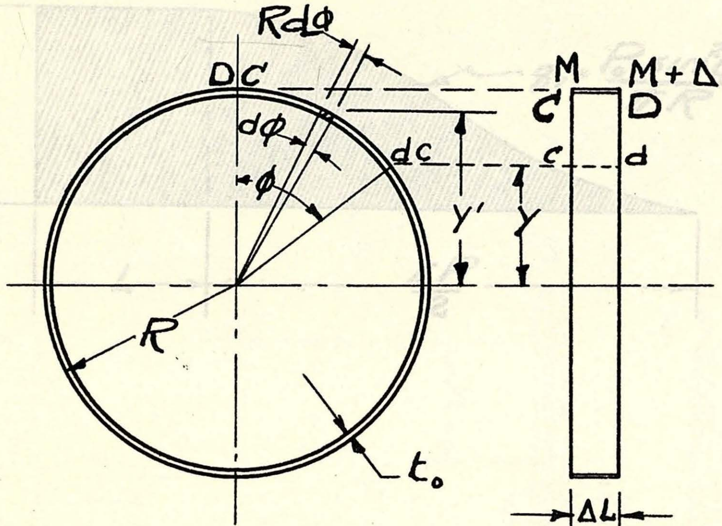


FIG. 4

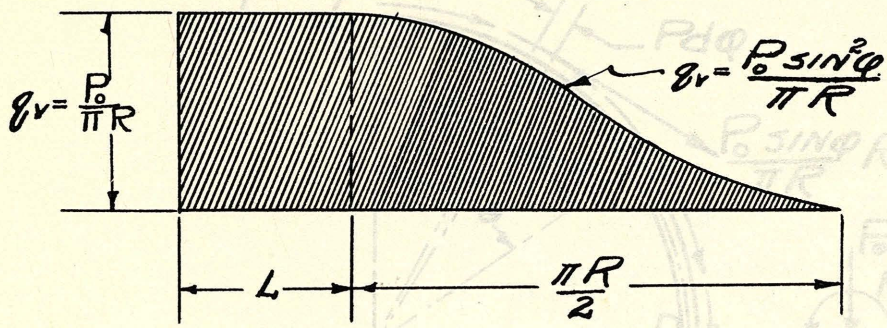
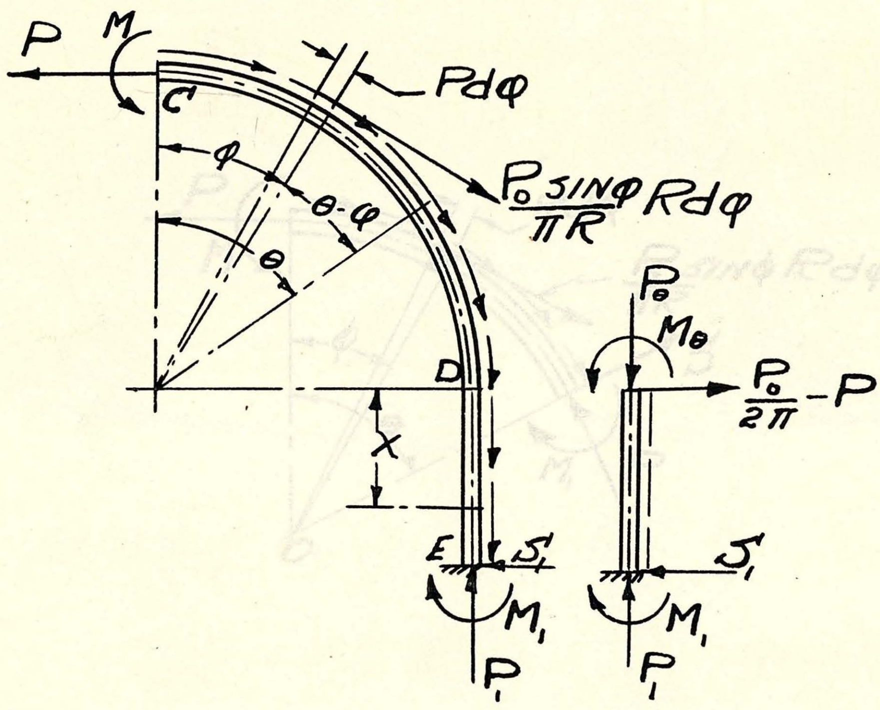


FIG 5



P
M
C
$Pd\varphi$
φ
$\theta-\varphi$
θ
$\frac{P_0 \sin\varphi}{\pi R} Rd\varphi$
D
X
E
S_1
M_1
P_1
P_0
M_θ
$\frac{P_0}{2\pi} - P$
S_1
M_1
P_1

FIG 6

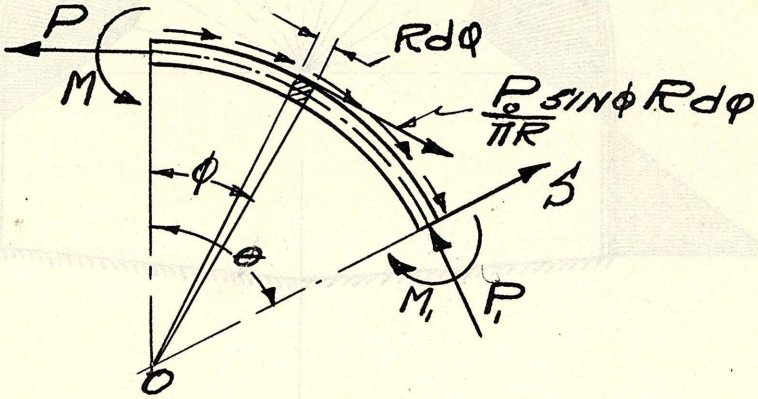


FIG 7

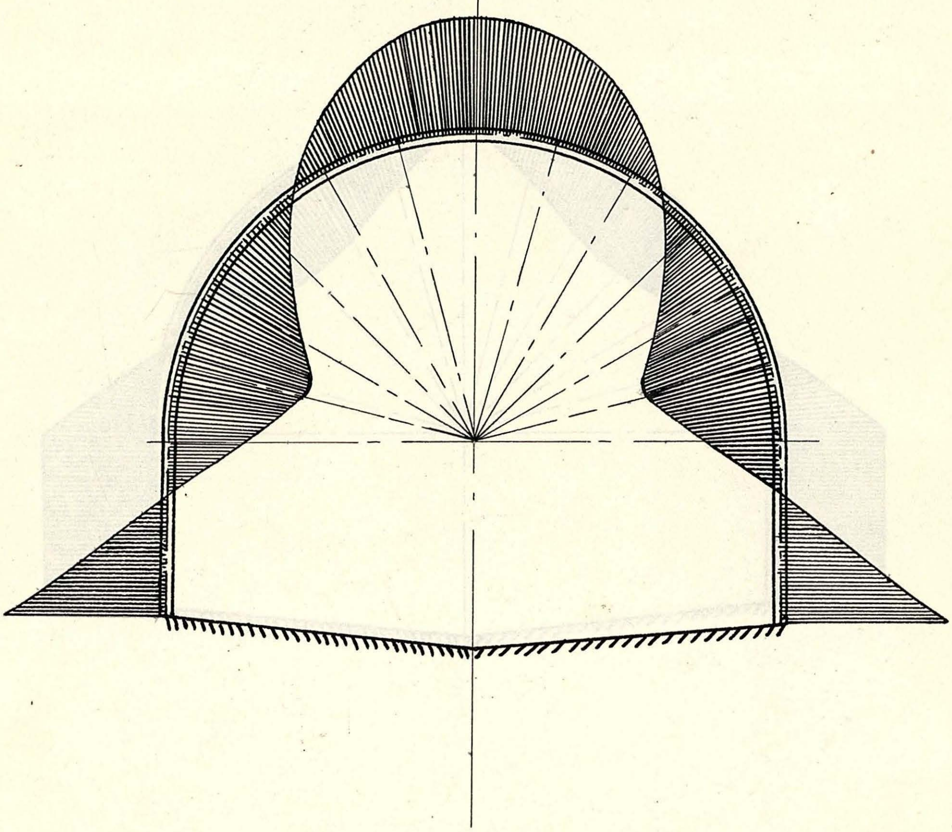


FIG 8

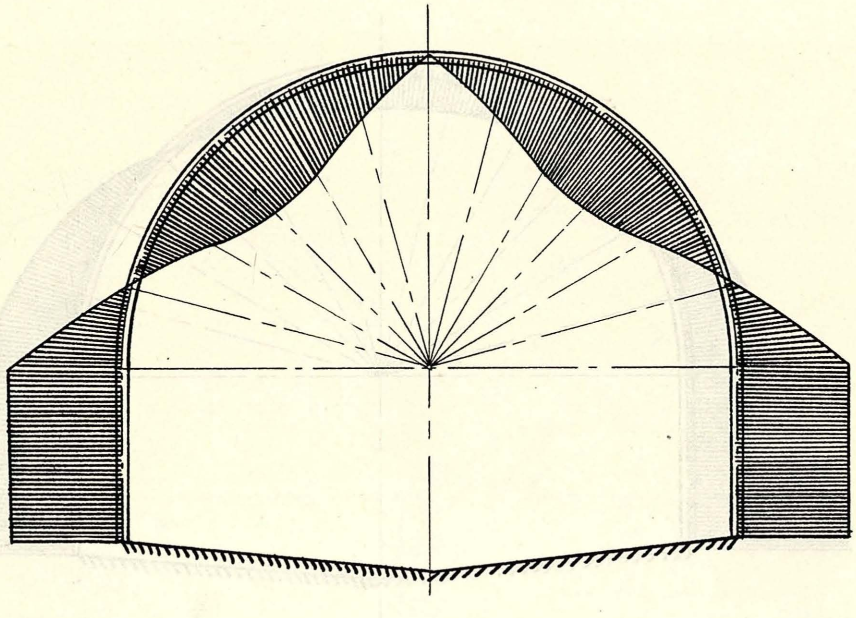


Fig 9

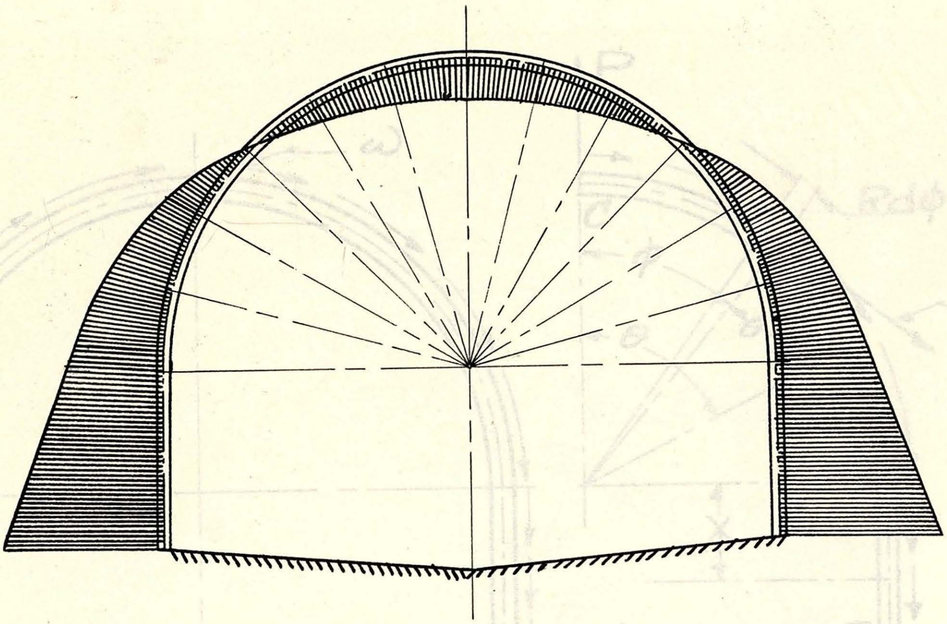


FIG 10

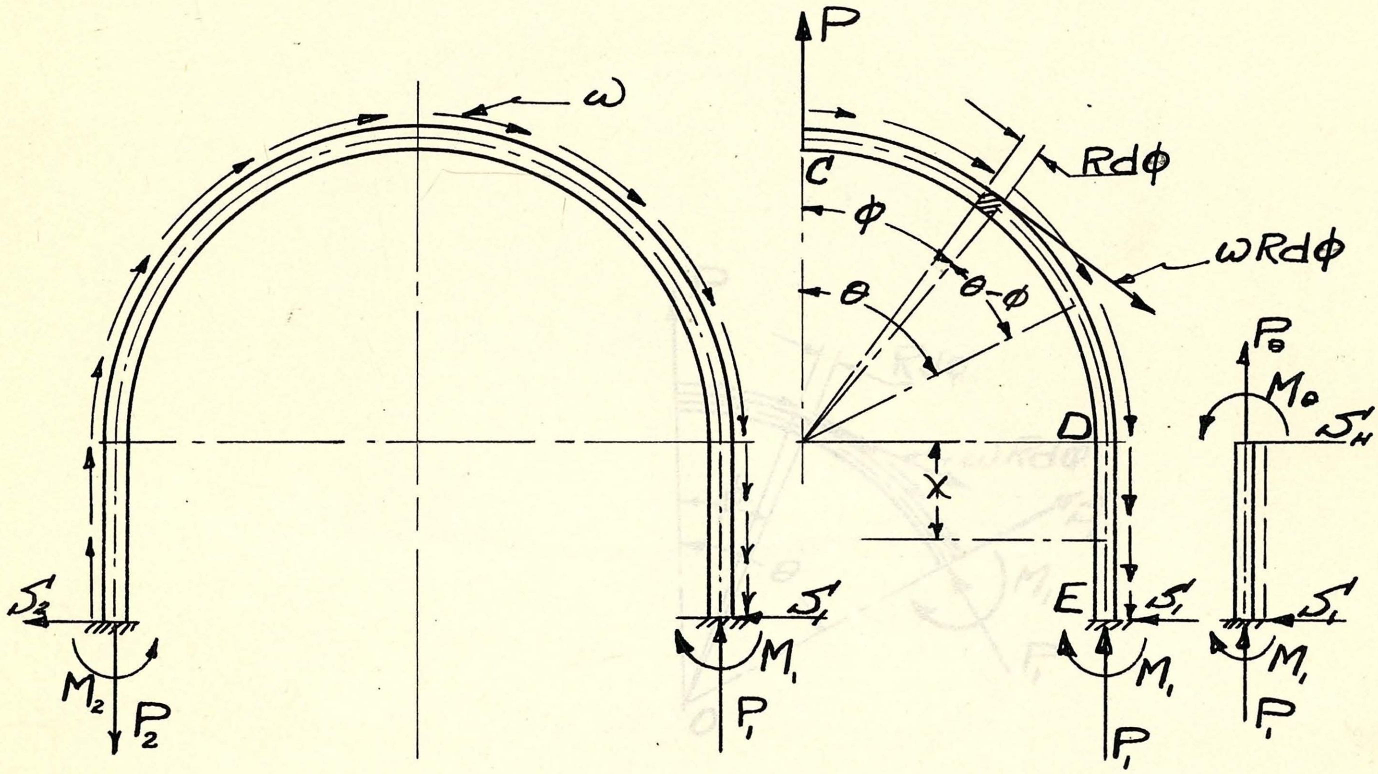


Fig 11

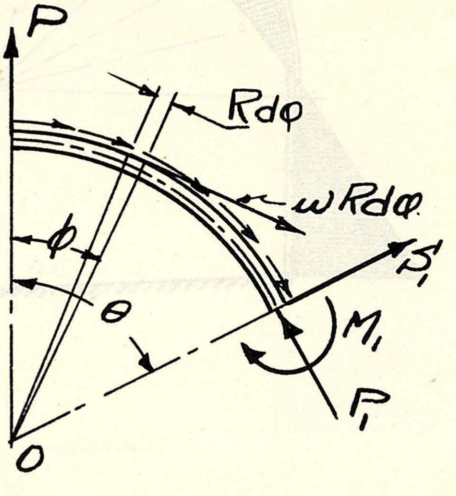


FIG 12

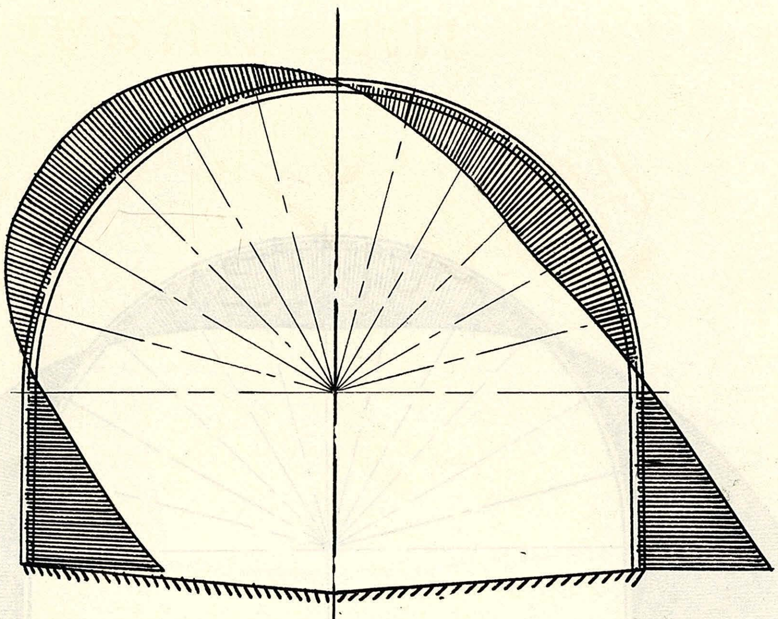


FIG 13

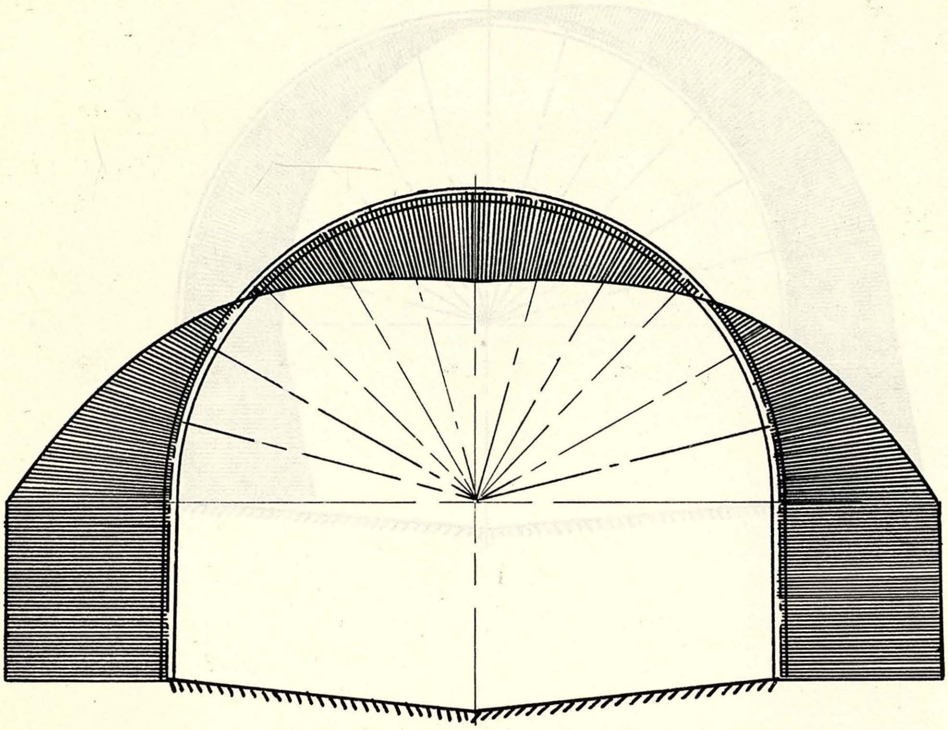
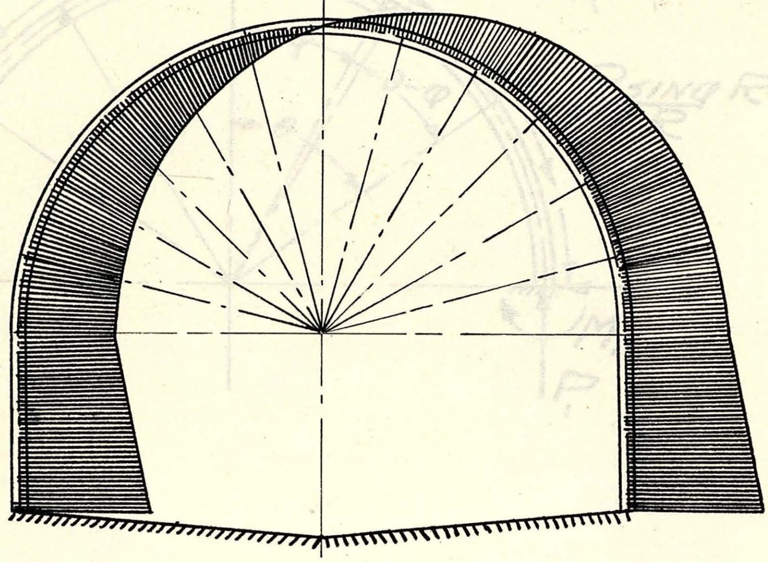
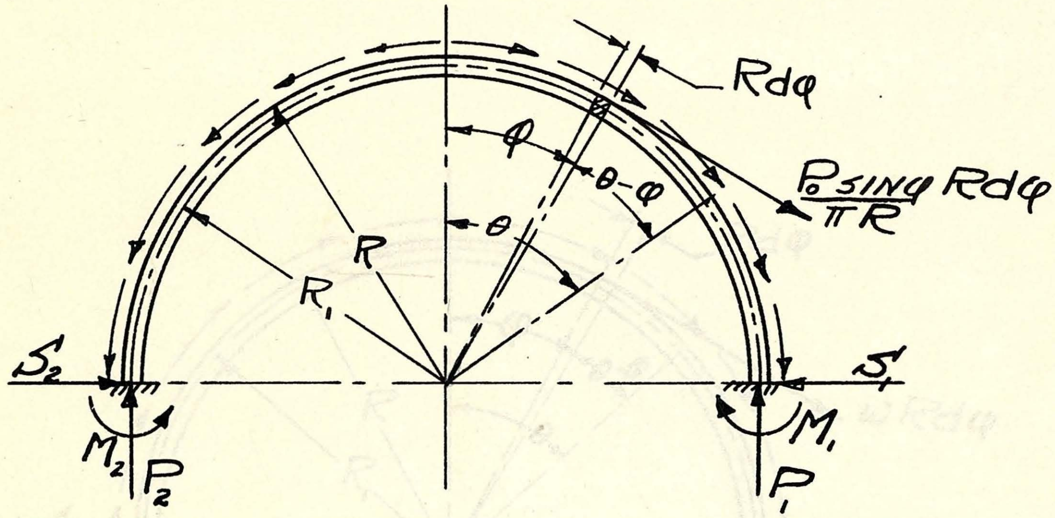


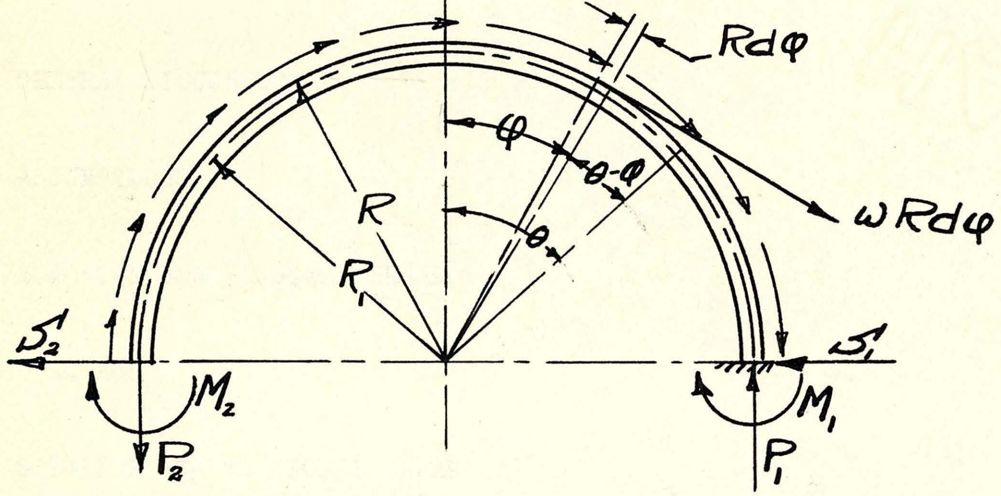
Fig 14





Rdφ
φ
θ-φ
θ
$\frac{P_0 \sin\varphi}{\pi R} R d\varphi$
R
R_1
S_2
S_1
M_2
P_2
M_1
P_1

FIG. 16.



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