The Effect of Small Amounts of Impurities on the Magnetic Susceptibility of Single Crystals of Bismuth

Thesis by

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One of the most fundamental problems in the study of the structure of matter is the origin of diamagnetism. The phenomenon was first observed by Brugmans1) who noticed that metallic bismuth was repelled from the pole pieces of a magnet. It was Faraday2), however, who first made a systematic investigation of a large number of materials, and found that bismuth was not a singular case, but that the diamagnetic behavior was common to many other substances in a greater or less degree.

In 1852 Weber gave a theoretical explanation of this effect, based on the supposition that the molecules of a diamagnetic substance contain resistanceless circuits, in which an electric current may be set up when the material is placed in a magnetic field. Since this is an induced current, by Lenz's law its direction would be such that its magnetic field would oppose the impressed field. In this way, the magnetic intensity within the substance would be reduced, and the diamagnetic property would result.

If the area of such a molecular circuit is A, and its coefficient of self induction is L, then the current produced by a field of strength H, which is at an angle θ with the normal to the plane of the circuit, is given by

$$i = -\frac{HA}{L}\cos\theta$$

The magnetic moment of such a current is

$$\mu = iA = -\frac{HA^2}{L}\cos\theta$$

and the component of this parallel to H is

$$\mu \cos \theta = -\frac{HA^2}{L} \cos \theta$$

Now if there are N such molecular circuits per unit volume, with their axes distributed at random, the number of axes lying between θ and $\theta + d\theta$ will be

$$\frac{N}{2} \sin \theta \, d\theta$$

Hence the resultant magnetization per unit volume will be given by

$$I = \int_0^{\pi} -\frac{NHA^2}{L} \cos^2\theta \sin \theta \, d\theta$$

whence the diamagnetic susceptibility per unit volume is

$$\frac{I}{H} = k = -\frac{1}{3} \frac{NA^2}{2}$$

Weber further postulated that if, instead of resistanceless circuits, the molecules contained currents which were already flowing, the material would be paramagnetic instead of diamagnetic. Maxwell combined the two ideas, assuming that there might be both; primitive currents, and resistanceless circuits in which no current flowed initially. In this case, the molecular current in a magnetic field would be

$$i = i_o - \frac{HA}{L} \cos \theta$$

where i_o represents the initial molecular current. The magnetic moment would then be

$$\mu = iA = i_oA - \frac{HA^2}{L} \cos \theta$$

and the component parallel to H

$$\mu \cos \theta = i_0 A \cos \theta (1 - \frac{HA^2}{Li_0} \cos \theta)$$

Thus this one theory would yield both paramagnetism and diamagnetism, the only determining factor being the i_0. If i_0 were small, the second term would predominate, and the substance would be diamagnetic, while if i_0 were large, only the first term would give any appreciable contribution, and the material would be paramagnetic.

At the time this theory was developed, the idea of such molecular currents was considered extremely artificial and difficult to believe. The discovery of the electron, however, provided a possible mechanism for such a conception, and in 1905, Langevin3) worked out his famous theory of magnetism. His primary assumption was that every atom consists of positive and negative electrical charges, which rotate in fixed orbits. Each of these circulating charges would thus have a magnetic field associated with it, and so could be treated as a magnetic dipole or a "magneton". The magnetic properties of the atom would then depend upon the relative orientations and directions of the axes of these magnetons. Working from this standpoint, Langevin found that if an atom possessed a resultant magnetic moment, it was paramagnetic, and its magnetic susceptibility followed the law $k = I_0 \frac{\mu}{3KT}$ where k is the volume susceptibility, I the maximum value of the intensity of magnetization, μ the magnetic moment of the atoms or molecules, K, Boltzmann's constant, and T, the absolute temperature. This law agrees with the empirical law that was established by Curie3) that the paramagnetic susceptibility should be inversely proportional to the absolute temperature.

If, however, the magnetic moment of the atom or molecule is zero then Langevin showed that the material would be diamagnetic, and its susceptibility would be given by $k = \frac{-Ne^2}{4mc^2} r^2$, where N is the number of electrons per unit volume, e, the charge of the electron, m its mass, c the velocity of light, and r the radius of gyration of the mean configuration of the electrons in the atom or molecule, with respect to an axis through their centroid. In this case, k is found to be independent of the absolute temperature, which is again in agreement with Curie's empirical law for diamagnetism.

The development of the Bohr theory of the atom gave further grounds for the existence of electrical charges in orbital motion, limiting these motions, however, to the extra-nuclear negative charges only. This at once gave a good explanation for the existence of the magnetons, but also showed that it should be expected that a monovalent atom of the alkali type should be paramagnetic, due to the resultant magnetic moment of the valence electron. This discrepancy was overcome by Pauli$^{4)}$, by considering the Lamor precession effect that would be produced, in the case of an atom possessing both a large mechanical moment of momentum and a magnetic moment, when this atom was placed in a magnetic field.

Ehrenfest$^{5)}$ attempted to explain the large value of the diamagnetic susceptibility of bismuth by postulating that it was due to electrons moving in orbits about two atoms. That is, in the crystalline phase, the valence electrons were no longer considered to be bound to one atom, but should be thought to be shared by two neighboring atoms. In order to understand the relative positions of the atoms, between which the interchange was assumed to take place, a photograph, Figure (1), of a model of the bismuth lattice, was taken in the plane normal to the principle axis. This shows that in this direction, the crystal consists of a system, in which a pair of planes are close together, and are separated from the next pair by a much larger distance.

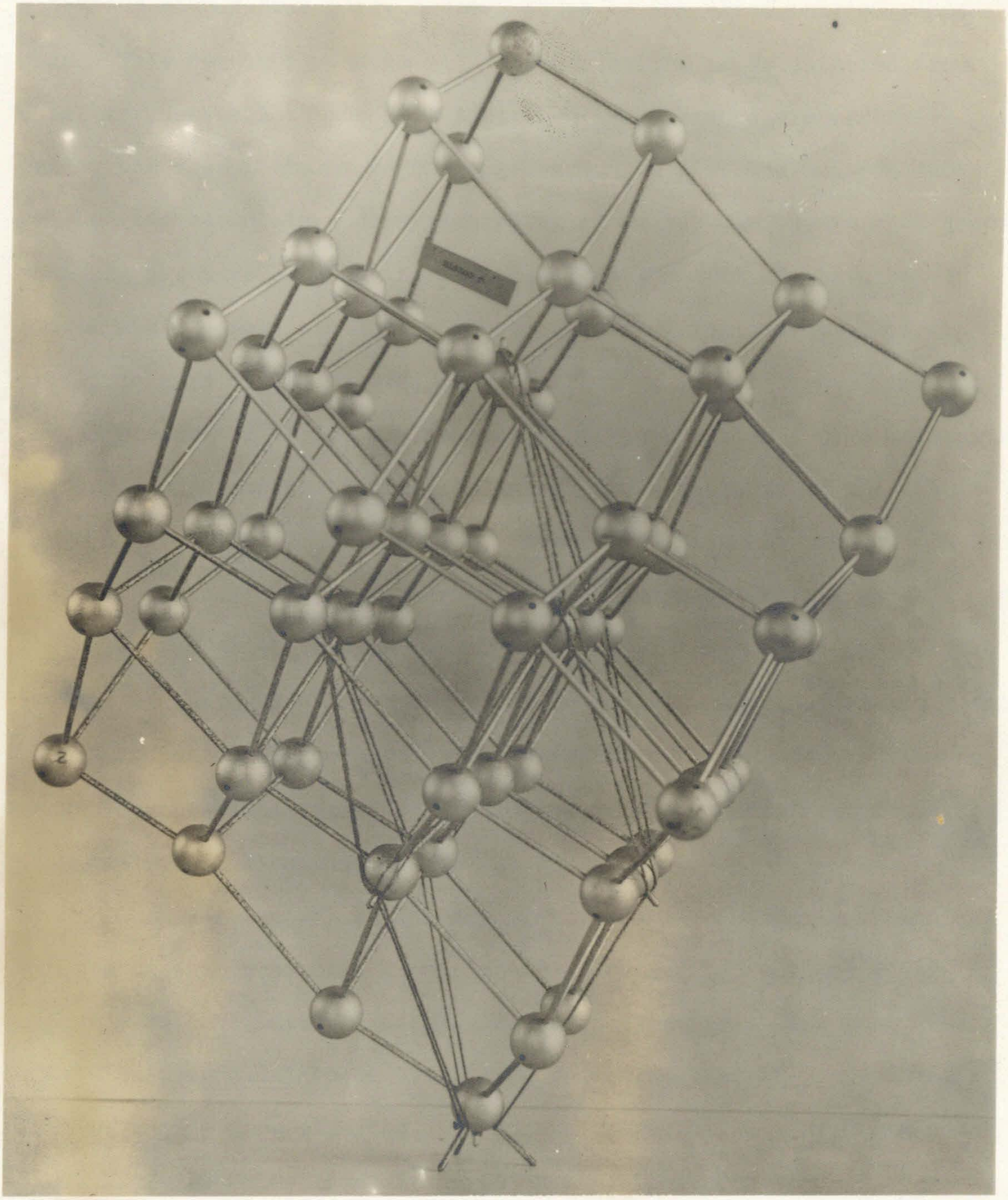
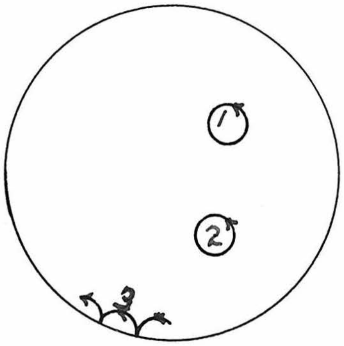


Figure 1. Model of the bismuth lattice.

The assumption, made by Ehrenfest, was that the interchange of electrons should take place between atoms in the close pair of planes only. This was in agreement with the prevailing ideas gathered from chemistry, as well as the fact that such interchanges are much more probable, the closer the atoms are together. The distance between two neighboring atoms, in a direction parallel to the planes, is much greater than the distance between the planes. Thus the area of the electron orbits must possess a larger projection in a direction normal to the principle axis of the crystal than parallel to it. This would result in the production of the maximum diamagnetism parallel to the main axis. Through some misunderstanding, Ehrenfest states this to be the result found experimentally by Tyndall, thus strengthening his theory. In fact however, Tyndall6) reported the ratio of the susceptibility parallel to the principle axis, to that normal to it, as 70:100, which is just the inverse of that which Ehrenfest's theory would give. Thus although this theory indicates the possibility of electronic orbits of more than one atomic diameter, it cannot be considered as the explanation of the large diamagnetism of bismuth.

Drude's theory, explaining electrical conduction in a metal as due to the existence of a free electron gas, led Schrodinger7) and Wilson8) to calculate the diamagnetic properties of such a gas. Their investigations led them to the same conclusion, that if λ is the mean free path of a free electron, the volume susceptibility of a gas consisting of such electrons, is given by the expression $k = -\frac{1}{3}\frac{Ne^2\lambda^2}{mc^2}$. All the other letters have the same meaning as in the Langevin formula.

At first, this theory seemed successful, as it was the only one by which a large enough value could be obtained to satisfy the experimental results in the particular case of bismuth. However, in this theory, one very important condition was neglected. This was the effect of the boundary upon the motion of the electrons. In his dissertation, Bohr[3)] has shown that if the electrons obey Maxwell's Law of distribution of velocities, the behavior of the electrons at the boundaries produces an equal but opposite effect to that of the electrons in the interior. This may be most simply illustrated by the following diagram.



If the electrons in the interior of a body are moving under the influence of a magnetic field, as indicated by the paths 1 and 2, an electron that happens to collide with the boundary will have some such path as 3. It will be seen at once that a path such as 3 results in an apparent current flowing around the boundary, in a direction just opposite to the direction of rotation of the electrons in the interior.

A further argument against a resultant diamagnetic behavior of free electrons has arisen with the need to postulate that an electron spins. The result of such an assumption would be to make the electron itself a magneton which, if it were free to move, would be definitely paramagnetic. In a paper immediately following his article on the diamagnetism of a monatomic gas, Pauli[9)] calculated this paramagnetic property of a free electron, and found it to be

$$k = \frac{1}{4\pi}\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\frac{N\ e^2}{mc^2}$$

More recently, Bitter10) attacked the problem of the diamagnetism of a free electron by using the method of quantum mechanics, and although he found an apparent diamagnetism, it possessed a smaller value than the paramagnetism found by Pauli. Thus, it seems that the free electron cannot be considered as contributing any diamagnetism, and of the foregoing theories, the only ones which yield a resultant diamagnetism, are those which treat the phenomenon as intrinsically atomic in nature. There are, however, several rather strong objections to explaining all the observed diamagnetic facts in this way.

The first of these is the discontinuity of magnetic properties in a number of metals when they pass through a change of phase. It is found in general that the phase which is stable at lower temperatures, is more diamagnetic than the higher temperature phase. This effect is shown very well in the work of Honda11) and Endo12). Figure 2 shows the results of the latter. In some instances, notably the cases of antimony and bismuth, the solid may be as much as fifteen to twenty times as diamagnetic as the liquid. The exact correspondence of this change in the susceptibility, with the change of phase, has been shown quite definitely by Webster13), who investigated the effect in the case of bismuth, and found that the magnetic change and the phase change took place at the same temperature, within a limit of one-hundredth of one degree.

Thus it appears that the diamagnetic susceptibility of an element is not alone a function of the atom itself, but also the particular configuration in which the atoms are arranged. A further striking example of this is found in the case of the element carbon, which can be obtained in three forms: amorphous, diamond and graphite. Here it is found that in the first two forms, carbon has a specific susceptibility

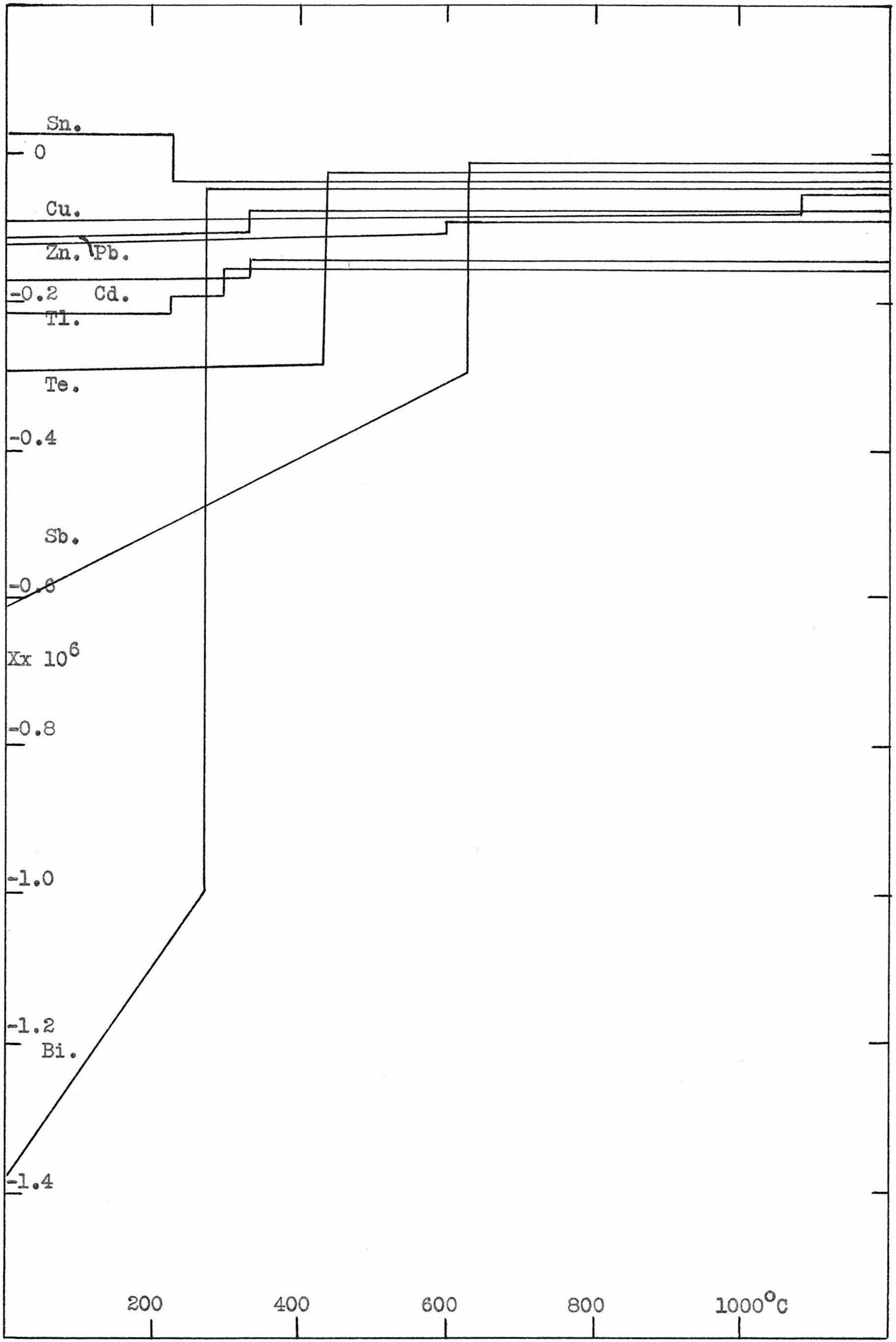


Figure 2. Change of susceptibility at the melting point.

of-0.49 x 10^{-6} while in the form of graphite, its value is more than twelve times as large, or about-6.0 x 10^{-6}.

A second serious objection to the atomic explanation is to be found in the recent work of Lowance and Constant14), who investigated the effect of cold working on the susceptibility of various metals. Figure 3 gives some of their results, showing that the metals become considerably less diamagnetic when subjected to any form of cold working, even if the metal was paramagnetic in the beginning. They found further that annealing restored the metals to their original and normal states.

A third, and perhaps the most striking published set of experiments that rather definitely point toward the necessity of a "crystal diamagnetism", that is, one which depends on the configuration of the atoms rather than on the atoms alone, is the work done in India during the past two years by Vaidyanathan15), Rao16), Mathur and Varma17), and Paramasivan18). In these papers, a study has been made of the dependence of susceptibility upon the size of grains of the substance under observation. The elements used were antimony, bismuth and graphite. As would be expected, the susceptibility was found to be independent of the particle size, as long as the particles were larger than two microns. When they became smaller than this, however, it was found that the diamagnetism decreased rapidly to approximately the value for the liquid or amorphous material, as shown in Figure 4.

The correspondence of this particle size with the dimensions of the secondary structures in bismuth as measured by Goetz19) leads to the further possible assumption that the large crystal diamagnetism of these elements may be very closely connected with the secondary structure in the crystals.

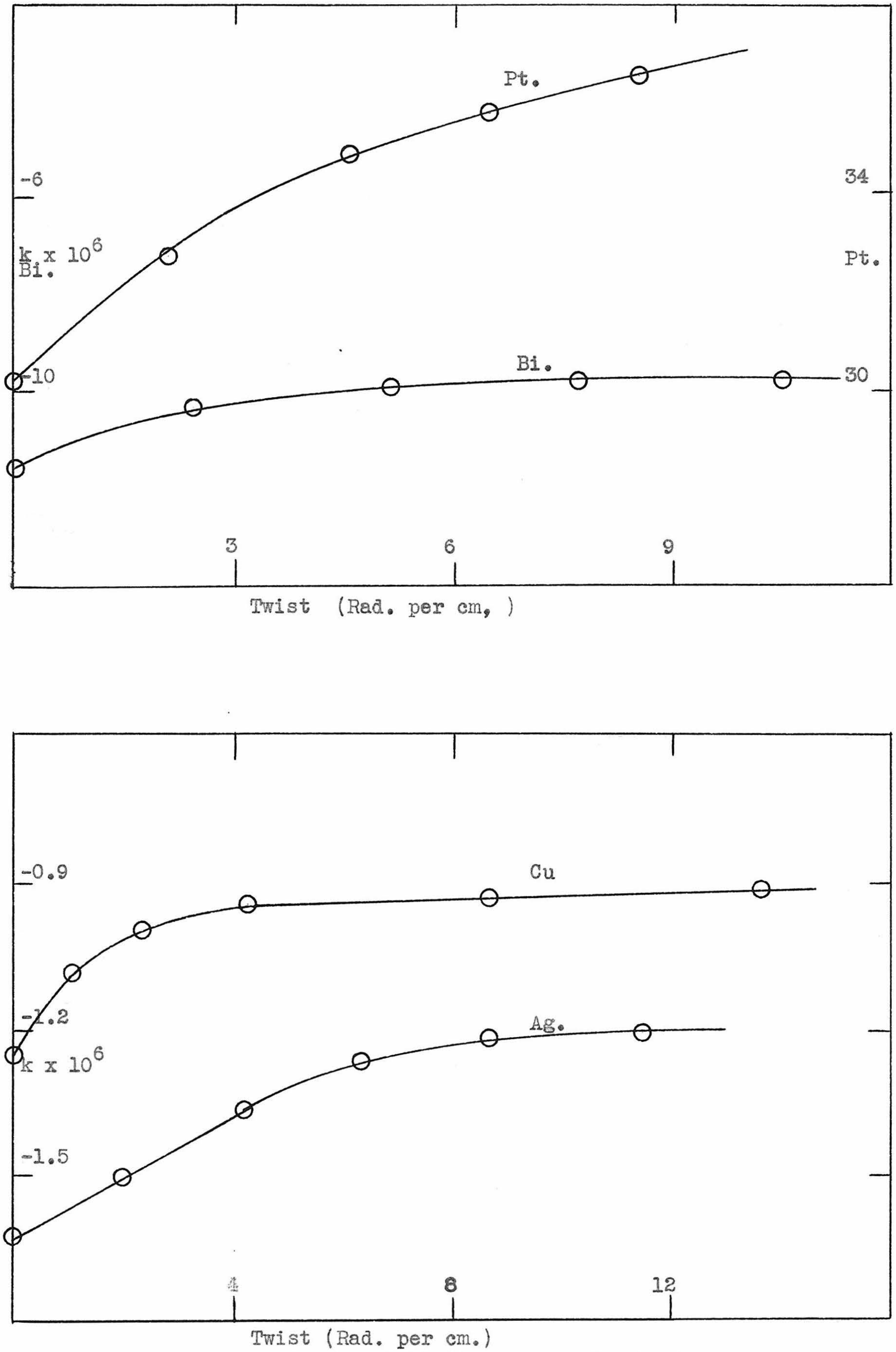


Figure 3. Dependence of susceptibility on amount of cold-working.

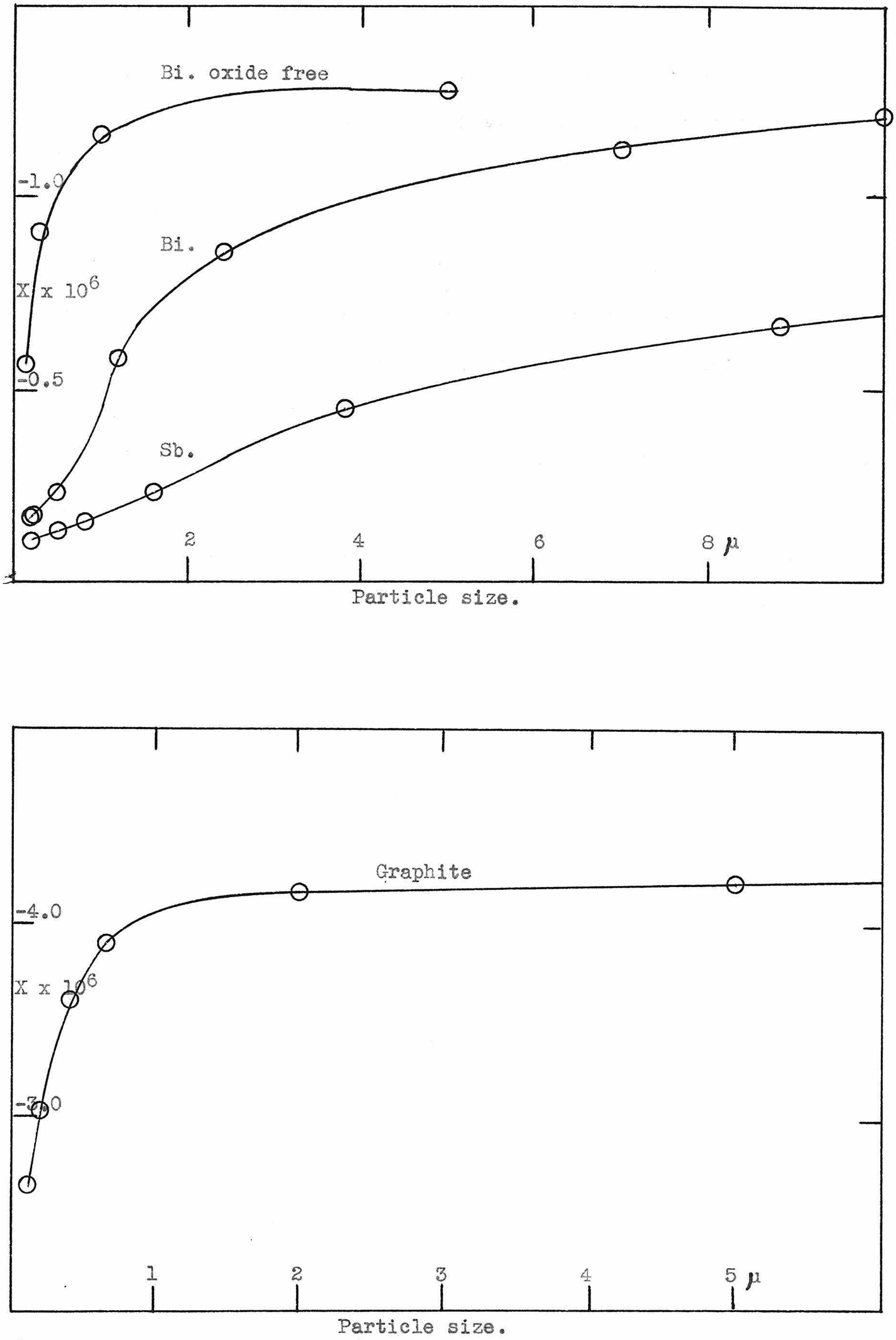


Figure 4. Dependence of susceptibility on particle size.

PURPOSE OF THIS INVESTIGATION

The purpose of the present investigation is to extend the knowledge of the factors governing crystal diamagnetism to the influence of small amounts of impurities on the susceptibility of single crystals of bismuth.

The element bismuth was chosen as the basis of this work for several reasons. In the first place, it has an unusually large crystal diamagnetism. In the second place, there had been considerable work done with bismuth at this Institute by Goetz19) 20), Goetz and Hasler21) 22), Hergenrother23) and Goetz and Focke24). The methods for producing and handling bismuth single crystals had already been well developed. In the third place, the results of Endo12) 25) in investigating the magnetic characteristics of several series of binary alloys, showed that the alloys of bismuth with small amounts of antimony, tellurium, lead, and tin, should be of considerable interest. His results for these four alloy series are shown in Figure 5 and Figure 6. These diagrams give the phase equilibrium diagram, the magnetic susceptibility of the solid alloys at room temperature, and the magnetic susceptibility of the liquid alloys at temperatures considerably above their melting points.

The phase diagram for the bismuth-antimony series, Figure 5, shows that these two metals form a simple solid solution. In agreement with existing theories of diamagnetism, the susceptibility is a linear function of the amount of antimony present in the bismuth. Thus a simple atomic theory would explain this case, if it could give a sufficiently high value for bismuth. Such a theory would also explain all cases of susceptibilities of the liquid alloys. The reason for the deviation from a straight line in the case of bismuth-tellurium liquid

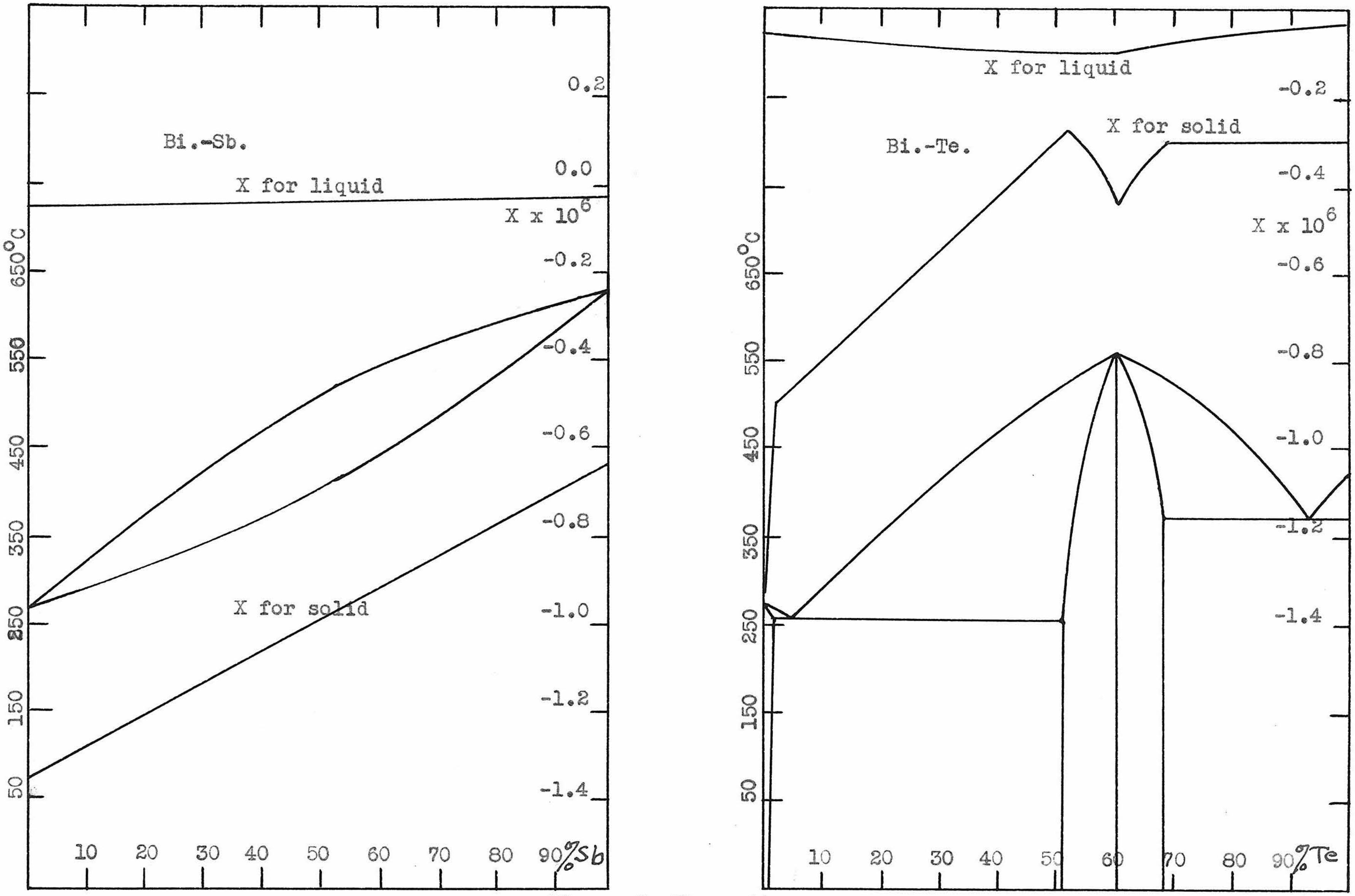


Figure 5. Phase diagrams.

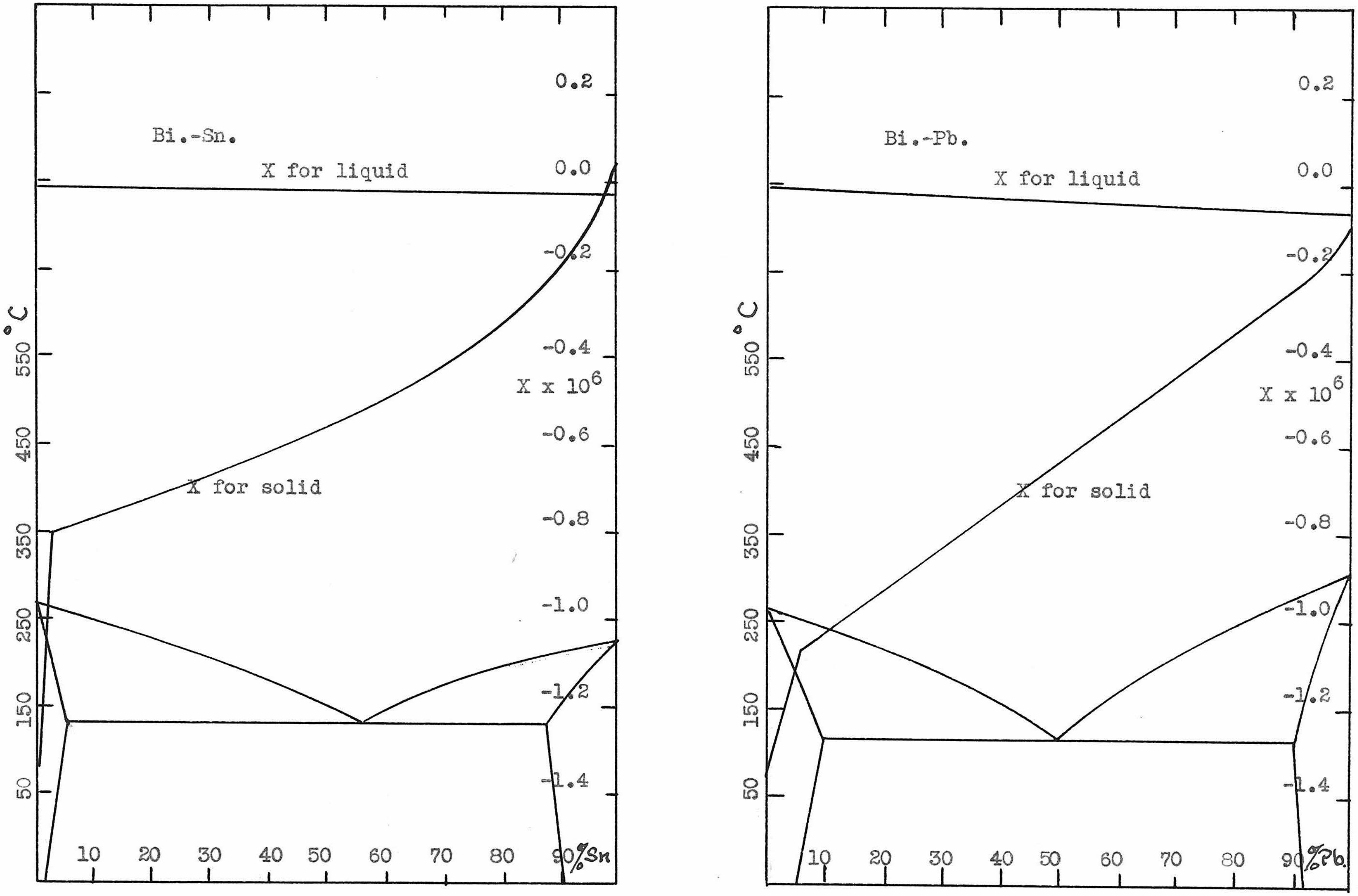


Figure 6. Phase diagrams.

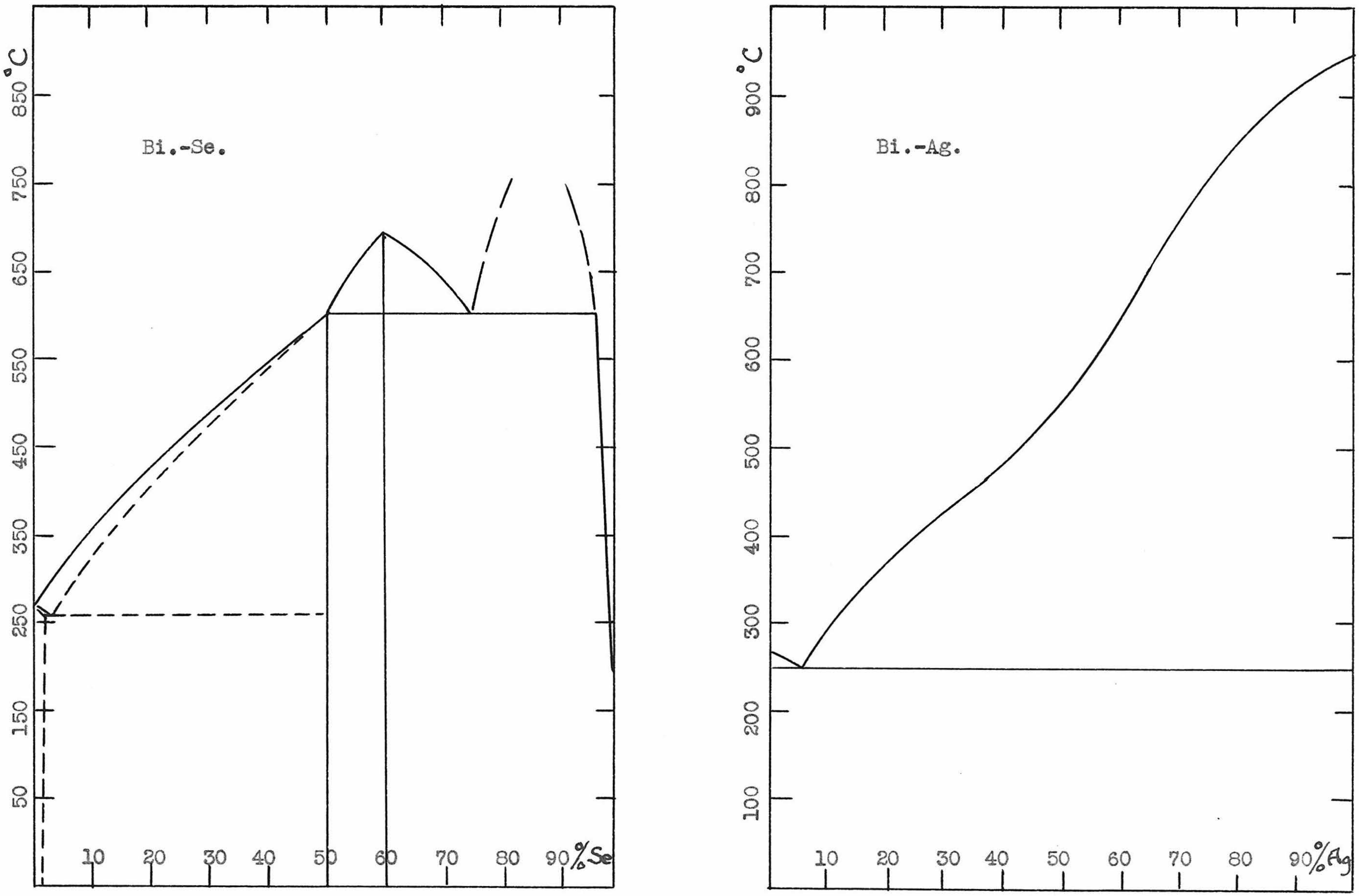


Figure 7. Phase diagrams.

alloys, is that the compound Bi_2Te_3 decomposes to a certain extent at the temperature at which the measurements were made.

This is as far as a simple atomic theory could go in explaining these curves, since the susceptibility of the alloys falls off very rapidly with increasing percentages of the impurity, in the range of limited solid solutions of tellurium, lead or tin in bismuth.

In view of the fact that the crystal structure of a pure metal is maintained through a region of solid solution, it appears that it would be possible to investigate this most interesting region of these diagrams by the study of single crystals of these alloys. Through this study, more accurate information could be sought concerning the mechanism by which the magnetic susceptibility is so greatly changed by comparatively small amounts of foreign atoms.

EXPERIMENTAL PROCEDURE IN THIS INVESTIGATION

The bismuth used in this investigation was manufactured by the Merk Chemical Company, and was of commercial grade. This particular kind was used because a spectroscopic analysis, performed by Mr. Hasler, proved that this grade of bismuth was freer from impurities than several so-called chemically pure grades from various sources. The following table gives the percentages of impurities for all the samples tested. The capital letters A, B, C, etc. refer to the samples of bismuth. Each letter denotes a separate consignment of the metal.

Impurity	At.Wt.	A	B	C	D	E	F
Copper	63.57	0.002	0.011	0.006	0.003	0.007	0.001
Silver	107.88	0.017	0.028	0.012	0.046	0.013	0.006
Tellurium	127.5	0.000	0.001	0.000	0.004	0.000	0.000
Thalium	204.4	0.000	0.002	0.000	0.005	0.000	0.000
Lead	207.2	0.025	0.061	0.019	0.020	0.014	0.002
TOTAL		0.044	0.103	0.037	0.078	0.034	0.009

A	Baker Corporation	Chemically pure
B	Hartmann Braun	Chemically pure
C	Hartmann Braun	Electrolytic
D	Karlbaum	Chemically pure
E	Merk	Commercial
F	Merk	Commercial

Table 1.

Subsequent to the spectroscopic determination, two more lots of Merk bismuth were obtained, for which no analysės were made. These are designated by the letters G and H. Their magnetic properties agree perfectly with bismuth E, and so may be considered as containing approximately equivalent amounts of impurity.

The other metals used in making up the alloy samples were of chemical reagent quality. No special precautions were taken to insure their great purity, as all the alloys were limited to small percentages of the added metal.

Two methods were used in determining the amounts of impurities added to the bismuth. The first, used in making up the first series of silver and lead alloys, was the preparation of a base solution containing ten percent of the impurity, which was subsequently diluted to the desired strength. The object in using this method, was the avoidance of relatively large inaccuracies, in weighing the very small amounts of the impurity metals used in the most dilute solutions. However, the uniformity of the ten percent alloys was uncertain, and as there was an apparent loss of lead each time an alloy containing it was melted this method was abandoned in favor of direct weighing.

It was by means of the latter procedure that all the later alloys were made. In all cases, the amounts of impurities were calculated in atomic percentages, in order to facilitate the comparison of the changes produced by different foreign atoms.

PRODUCTION OF THE CRYSTALS

In a general way, the procedure used in growing the single crystals followed that developed by Goetz[20)], but as there were several changes, it would be well to outline the method used. The first step, after the desired amounts of metals were weighed, was to melt the bismuth in a pyrex crucible, under an atmosphere of purified hydrogen. After being heated to about 500° C., to reduce any oxide present, the bismuth was cooled to about 350° C., and the impurity was added. This procedure was used in an effort to reduce the amount of chemical combination of the impurity with either the hydrogen or the glass crucible. It seemed to be satisfactory for all the alloys except lead and tin. In these cases, it was found that there was always some reaction with the glass, resulting in the loss of a part of the impurity. As this would cause considerable error in the estimated amount of impurity present in the finished crystal, crucibles made of quartz were used for these two metals and no further difficulty was encountered.

After the impurity metal was melted into the bismuth, the mixture was thoroughly stirred and then drawn into pyrex tubing, which had an inside diameter of two to three milimeters. The furnace described by Goetz20) in his method was not used because it was desired to cool the bismuth alloy as rapidly as possible, to further prevent an excessive loss of the impurity by reaction with the glass. Quartz was not used here, as it was necessary to break the tubing to get the metal rod out, and it was felt that if the alloy were cooled quickly, the loss of known impurity would not be great enough to warrant the expense involved in destroying the necessary amount of quartz tubing.

Considerable trouble was found, due to the tendency of the alloys to stick to the glass tubing, and various methods were used to prevent this. The method suggested by Bridgman26) was tried. It consisted of coating the inner surface of the tubing with oil, and then heating it until only a very light deposit of carbon remained. Although it was found to be successful in general, it did not work at all for the tellurium and selenium alloys. For these cases, a very simple procedure solved the problem. The tube used was cleaned before the alloy was drawn up, and after solidification of the alloy, the tube was immersed in water, and the glass cracked off while still wet. The capillary action of the water in the cracks formed in breaking the glass, seemed to admit a film of water between glass and metal, so that the glass came off with ease. After removal from the glass, the metal rods were straightened, and cut to lengths of about twelve centimeters.

The "second generation" of the crystals, as described by Goetz20), was omitted in all the samples used in this investigation. This was done because it was found that if the density of the impurity metal was considerably different from that of bismuth, there was a definite tendency toward separation. There was also a tendency to form gaseous hydrides, in the cases of tellurium, selenium and antimony. Due to these facts, it was thought best to make the single crystals in as few operations as possible.

The "third generation" was carried out in exactly the manner described by Goetz. It was found necessary to grow the crystals at the greatest possible speed, to assure a uniform distribution of the impurity metal along the whole length of the crystal. To test the necessity of this, several crystals were grown at one-sixth the normal speed, or about one millimeter a minute. It was found that in the case of very pure bismuth, there was no difference between the magnetic properties of the two ends of the crystal so grown. A rod originally containing 2 percent lead, when grown at one-sixth the normal speed, was found to have the magnetic properties of a normal 1% crystal at the end which crystallized first, and of a normal 3% crystal at the other. It was found, on the other hand, that a crystal grown at normal speed had the same characteristic magnetic properties throughout its entire length.

METHOD OF MEASUREMENT

There are two principle methods of measuring the magnetic susceptibility of a solid body: the Faraday or Curie method, and the Gouy method.[1)] In the Faraday method, the sample to be tested must be placed in a region of a magnetic field, where both the field strength and the field gradient are known. The product of the two must be constant throughout the whole sample. Because of this last requirement it is necessary that the sample be small, as it is difficult to obtain a region of any considerable size in which the product $H \frac{dH}{dx}$ is a constant. This restriction of the size of the sample would be a serious objection if the method were to be used for the measurement of single crystals of bismuth, because it is impossible to cut or cleave a bismuth crystal without causing the formation of some twinning lammellae. These twinning lammellae are in reality regions in which the orientation of the crystal has changed by an angle of sixty degrees. Thus, if small samples were cut from a bismuth crystal for magnetic measurement by the Faraday method, the resultant sample would no longer be a single crystal, but a composite of several different crystals. This is without doubt, one reason for Nusbaum's[27)] failure to obtain the correct values for the principle susceptibilities of bismuth by this method.

Therefore the Gouy method was chosen for making all the determinations in the present investigation. The principle requirement of this method is that the samples have a uniform cross-section over the entire part in which the field gradient is appreciable. The condition of the extreme ends is not important, as they are both located in uniform fields. Both of these facts were advantageous, because the crystals grown by the method described above may be of almost any desired length, and are of quite uniform area, while the unavoidable twinned areas due to cleaving would be in such a position as to have no effect upon the results.

One disadvantage of the Gouy method is that no quantitative measurement can be made of the dependence of susceptibility on field strength. Furthermore, if the susceptibility of the sample should depend on the strength of field, the result obtained would be the average value for all fields, from zero to the strongest used. Due to these facts, several tests were made at field strengths ranging from

2500 gauss to 22,500 gauss. In this region, no change of susceptibility with field strength was found, so it is unlikely that any errors have been introduced from this cause.

The principle factors limiting the accuracy of the measurements were: 1) the determination of the area of the cross-section of the crystals, 2) the sensitivity of the balance, and 3) the measurement of the strength of the magnetic field. Due to the irregular shape of the crystals, the cross-sectional area could not be measured directly, but had to be calculated from the weight, length and density of a portion of each crystal. That part in which the most of the field gradient occurred was used for this measurement. All forces were measured with an accuracy of one-tenth of a milligram, which was in general less than one-half of one percent of the total force. The field strength was measured with a Grassot fluxmeter, with an accuracy of one percent.

The arrangement of the apparatus is shown in Figure 8.

An aluminum suspension, S, was designed to take the place of one of the regular pans of a sensitive analytical balance, B. It was fitted with a torsion head T, so that the crystal could be rotated about a vertical axis. The lower end of the suspension was seven centimeters above the pole pieces P, of the magnet. This distance was great enough so that the suspension was but little affected by the magnetic field, the greater part of the gradient being quite well confined to the first centimeter from the edge of the pole pieces. These were flat faced, and ten centimeters in diameter, giving a very uniform field between them, with comparatively little stray field. The crystals, C, ranged from nine to twelve centimeters in length, thus placing the lower end well within the uniform region of the field, while the upper end was in a field of negligible intensity.

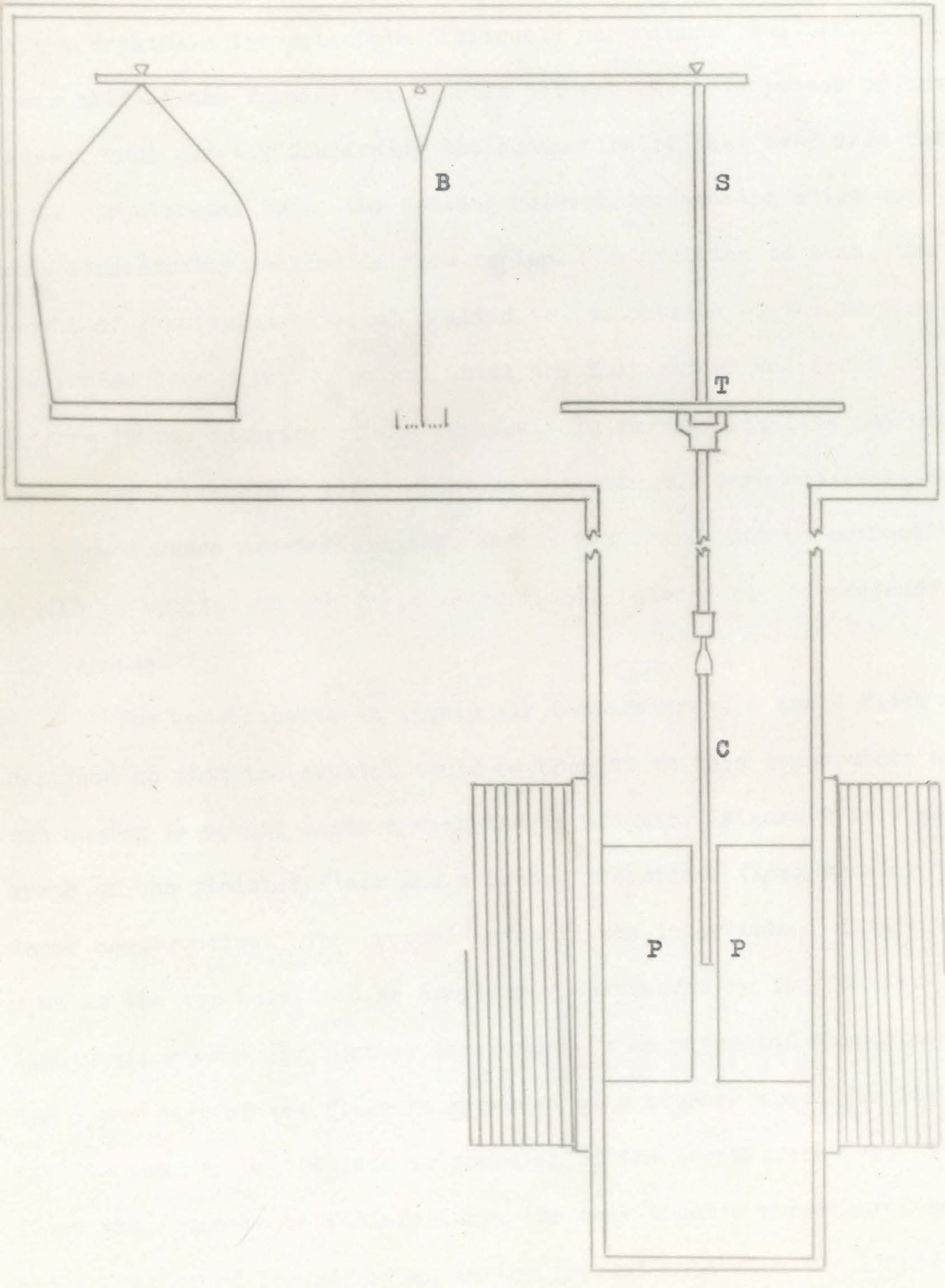


Figure 8. Arrangement of apparatus.

The problem of determining the dependence on temperature of the magnetic properties of the crystals, required the construction of a furnace which would give a uniform temperature over the entire length of the crystal. The principle difficulty encountered was that the lower half of the furnace was clamped between the pole pieces of the magnet, thus greatly increasing the conduction of heat away from this part. To overcome this, the spacing between the heating coils was made considerably smaller in this region. In addition to this, the amount of insulation material applied to the outside of the furnace was varied from point to point, until the temperature was found to be uniform in the interior of the furnace. To further equalize the temperature, the furnace was lined with sheet aluminum. Care was taken to wind the furnace non-inductively, and it was found that no correction had to be applied to the field measurements because of the presence of the furnace.

For measurements at liquid air temperatures, a small flask was designed so that the crystal could be brought to this temperature without coming in actual contact with the liquid air. Figure 9 is a photograph of the finished flask and a second incomplete flask showing the inner construction. The crystal hangs in the inner tube. This tube is open at the top only, and is completely surrounded by liquid air. The liquid air chamber is further surrounded by an evacuated compartment. The upper part of the flask is provided as a storage space for liquid air, to insure the complete surrounding of the crystal chamber. This flask was found to be satisfactory, the only trouble encountered being the formation of liquid oxygen in the crystal tube when the liquid air was fresh.

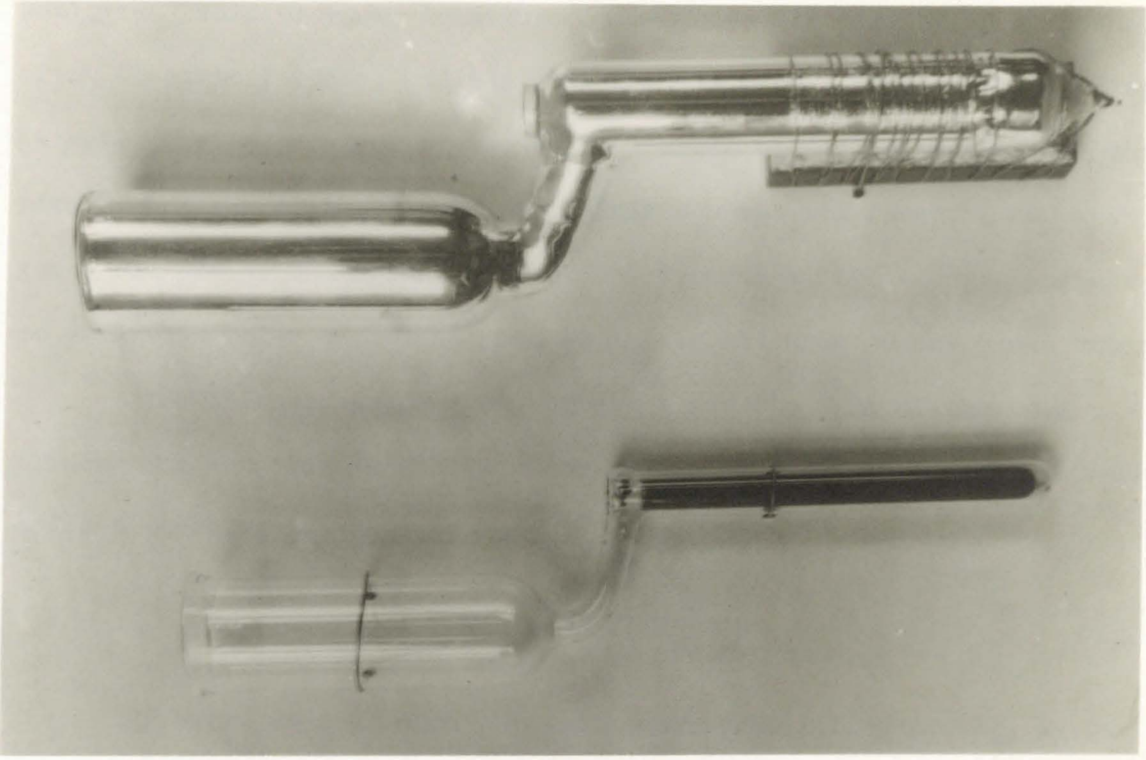


Figure 9. Liquid air flask.

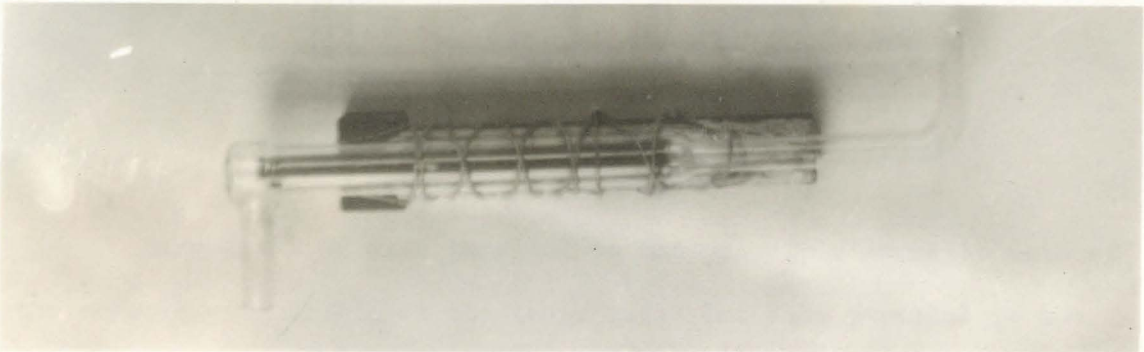


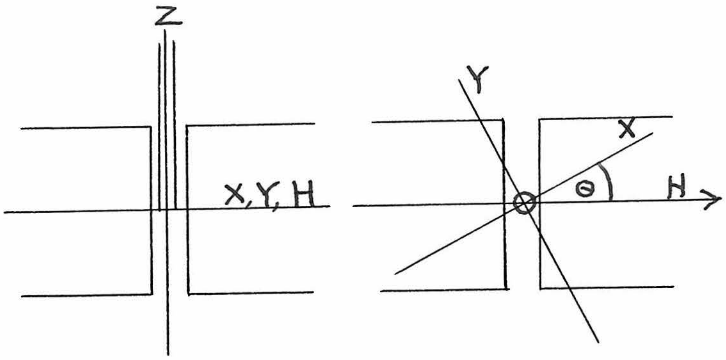
Figure 10. cooling jacket.

For a few measurements taken at temperatures between -185°C and 22°C, a cooling jacket similar to that used by Kapitza28) was employed. This is shown in Figure 10. Liquid air was boiled by means of an electric furnace, and the cold gas was passed through the cooling jacket. The temperature of the crystal could be varied by changing the rate of boiling of the liquid air. This control was found to be very good, but due to the arrangement of the apparatus, it was not possible to make the cooler efficient, and so it was not used to any great extent.

The balance, suspension, crystal, and the ends of the pole pieces were enclosed in a cabinet to prevent air currents from the warm coils disturbing the balance.

THEORY OF THE METHOD

Let a crystal with principle susceptibilities k_1, k_2, k_3 be placed in a magnetic field as in Figure 11.



Take a set of axes parallel to the principle magnetic axis of the crystal with origin at its lower end. Let Z be parallel to the length of the crystal, and let X make an angle θ with the magnetic lines of force. The boundary conditions will then be

$$H_x = H\cos\theta, \quad H_y = H\sin\theta, \quad H_z = 0$$

$$\frac{\partial H_x}{\partial x} = \frac{\partial H_y}{\partial y} = \frac{\partial H_x}{\partial y} = \frac{\partial H_y}{\partial x} = \frac{\partial H_z}{\partial x} = \frac{\partial H_z}{\partial y} = \frac{\partial H_z}{\partial z} = 0$$

if the diameter of the crystal is small in comparison to that of the pole pieces.

The equation for the energy of a magnetic field in an anisotropic medium is given as

$$E_c = \frac{1}{8\pi}(\mu_x H_x^2 + \mu_y H_y^2 + \mu_z H_z^2)$$

where $\mu_x = 1 + 4\pi k_x, \quad \mu_y = 1 + 4\pi k_y, \quad \mu_z = 1 + 4\pi k_z$

Thus

$$E_c = \frac{1}{8\pi}(H_x^2 + H_y^2 + H_z^2) + \frac{1}{2}(k_x H_x^2 + k_y H_y^2 + k_z H_z^2)$$

Now if we consider the case of a crystal hanging in air where $\mu = 1$, the force experienced by the crystal in the direction parallel to the axes, due to the magnetic field, will be

$$F_x = \frac{\partial E_c}{\partial x} - \frac{\partial E_a}{\partial x} = \frac{k_x}{2}\frac{\partial H_x^2}{\partial x} + \frac{k_y}{2}\frac{\partial H_y^2}{\partial x} + \frac{k_z}{2}\frac{\partial H_z^2}{\partial x}$$

$$F_y = \frac{\partial E_c}{\partial y} - \frac{\partial E_a}{\partial y} = \frac{k_x}{2}\frac{\partial H_x}{\partial y} + \frac{k_y}{2}\frac{\partial H_y}{\partial y} + \frac{k_z}{2}\frac{\partial H_z}{\partial y}$$

$$F_z = \frac{\partial E_c}{\partial z} - \frac{\partial E_a}{\partial z} = \frac{k_x}{2}\frac{\partial H_x^2}{\partial z} + \frac{k_y}{2}\frac{\partial H_y^2}{\partial z} + \frac{k_z}{2}\frac{\partial H_z^2}{\partial z}$$

or

$$F_x = k_x H_x \frac{\partial H_x}{\partial x} + k_y H_y \frac{\partial H_y}{\partial x} + k_z H_z \frac{\partial H_z}{\partial x}$$

$$F_y = k_x H_x \frac{\partial H_x}{\partial y} + k_y H_y \frac{\partial H_y}{\partial y} + k_z H_z \frac{\partial H_z}{\partial y}$$

$$F_z = k_x H_x \frac{\partial H_x}{\partial z} + k_y H_y \frac{\partial H_y}{\partial z} + k_z H_z \frac{\partial H_z}{\partial z}$$

Substituting the boundary conditions, these equations reduce to

$$F_x = 0$$

$$F_y = 0$$

$$F_z = k_x H_x \frac{\partial H_x}{\partial z} + k_y H_y \frac{\partial H_y}{\partial z}$$

or

$$F_z = k_x \cos^2\theta \, H \frac{\partial H}{\partial x} + k \sin^2\theta \, H \frac{\partial H}{\partial z}$$

$$F_z = \left[k_y + (k_x - k_y) \cos^2\theta \right] H \frac{\partial H}{\partial z}$$

Now if $H = H$ when $Z = 0$, and $H = H_o$ when $Z = Z$, and the expression for F_z is integrated over the volume of the crystal, the total force acting is given by

$$F = \int_V F_z \, dv = \int\int_V \left[k_y + (k_x - k_y) \cos^2\theta \right] dx \, dy \, dz \, H \frac{dH}{dz}$$

$$F = \left[k_y + (k_x - k_y) \cos^2\theta \right] \frac{H^2 - H_o^2}{2} A$$

so

$$k_y + \left[(k_x - k_y) \cos^2\theta \right] = \frac{2F}{(H^2 - H_o^2) A}$$

if F is given in grams, and if $H_o \ll H$, this becomes

$$k_y + (k_x - k_y) \cos^2\theta = \frac{2 g F}{A H^2}$$

which, if expressed in terms of the specific susceptibility instead of the volume susceptibility, becomes

$$X_y + (X_x - X_y) \cos^2\theta = \frac{2 g F}{dA H^2}$$

Now if we choose Y to be parallel to the principle crystallographic axis, the final expression will be

$$X_{\parallel} + (X_{\perp} - X_{\parallel}) \cos^2\theta = \frac{2 g F}{dA H^2}$$

Thus it is shown that if the magnetic field is parallel to the principle axis of the crystal, that is, $\theta = 90^\circ$, we get

$$X_{\parallel} = \frac{2 g F_{\parallel}}{dA H^2}$$

and if the field is normal to the principle axis, $\theta = 0^\circ$, the expression becomes

$$X_{\perp} = \frac{2 g F_{\perp}}{dA H^2}$$

Also the susceptibility of the crystal should follow a $\cos^2$ law between the maximum and minimum values.

Since the bismuth crystallizes in a hemihedral form of the hexagonal system, it should be isotropic with respect to all directions normal to the principle axis. If this were found to be correct, the problem of investigating the magnetic properties would be greatly simplified, as all the necessary data could be obtained with one crystal. By using a crystal in which the principle axis is normal to the length of the rod, the entire variation of susceptibility with direction, can be obtained by turning the crystal about the axis of the rod. This degree of freedom was provided for, as stated in the description of the apparatus.

TABLE OF ABBREVIATIONS USED IN DATA

A,B,C,D,E,F,G,H	indicate the source of the bismuth used in the crystal.
G 0.025%Ag	shows that the crystal is made of G bismuth, to which 0.025 atomic percent of silver has been added.
W_n	as all measurements are made by the counter balance method, this symbol indicates the weight added to the crystal pan to bring about a balance with no field acting.
$W_\perp$	gives the weight added with the magnetic field normal to the principle axis of the crystal.
$W_\parallel$	gives the weight added when the field is parallel to the principle axis.
W_θ	indicates the added weight when the field makes an angle θ with the principle axis.
F	gives the difference between W_n and the W with a field. The subscript of F corresponds to that of W.
R	is the ratio $X_\perp/X_\parallel$
H	gives the strength of the magnetic field.
T	gives the temperature in degrees Centigrade.
P_1P_2	indicates that the principle axis of the crystal is normal to the length of the rod; unless otherwise indicated, the crystals are of the P_1 type.
P_3	indicates that the principle axis of the crystal is parallel to the length of the rod.

θ ------------------- gives the angular position of the crystal as shown on divided torsion head.

θ' ------------------ gives the angle between the principle axis of the crystal and the direction of the magnetic field.

$X_{\parallel}$ ---------------- indicates specific susceptibility parallel to the principle axis of the crystal.

$X_{\perp}$ ----------------- indicates specific susceptibility perpendicular to the principle axis of the crystal.

C ------------------- is the constant by which F is multiplied in the calculation of the susceptibility, and is determined by the equation

$$C = \frac{2g}{dAH^2} = \frac{2g\cancel{d}l}{\cancel{d}WH^2}$$

where g is the acceleration of gravity, d, l, and W, the density, length and weight of that section of the crystal in which the greater part of the field gradient occurred in the magnetic measurements.

Data for Fig. 12

Variation of the maximum susceptibility with change of field strength.

Crystal 116/11, E P_1, Length: 2.078 centimeters, Weight: 1.8516 grams.

Area: 8.76 mm^2, W_n = 1.9498.

W	Fluxmeter	H	F	$-X \times 10^6$
2.2152	30.5	20,150	0.2654	1.489
2.1803	28.5	18,820	0.2305	1.480
2.1040	23.2	15,350	0.1542	1.488
2.0273	16.5	10,980	0.0775	1.489
1.9793	10.2	6,760	0.0295	1.467
1.9549	4.2	2,805	0.0051	1.473

Data for Fig. 13

Variation of the susceptibility in the plane normal to the principle axis.

Crystal 120/41. E-distilled P_3, Length: 3.64 cm., Weight: 1.1846 grams.

Area: 3.32 mm^2., $W_n = 3.4695$, $H = 13,200$.

θ	W	F	$-X \times 10^6$	θ	W	F	$-X \times 10^6$
0	3.5123	0.0428	1.483	180	3.5123	0.0428	1.483
15	3.5123	0.0428	1.483	195	3.5123	0.0428	1.483
30	3.5123	0.0428	1.483	210	3.5123	0.0428	1.483
45	3.5123	0.0428	1.483	225	3.5123	0.0428	1.483
60	3.5123	0.0428	1.483	240	3.5123	0.0428	1.483
75	3.5123	0.0428	1.483	255	3.5123	0.0428	1.483
90	3.5123	0.0428	1.483	270	3.5123	0.0428	1.483
105	3.5123	0.0428	1.483	285	3.5123	0.0428	1.483
120	3.5123	0.0428	1.483	300	3.5123	0.0428	1.483
135	3.5123	0.0428	1.483	315	3.5123	0.0428	1.483
150	3.5123	0.0428	1.483	330	3.5123	0.0428	1.483
165	3.5123	0.0428	1.483	345	3.5123	0.0428	1.483
180	3.5123	0.0428	1.483	360	3.5123	0.0428	1.483

Data for Fig. 14

Variation of the susceptibility in the plane containing the principle axis.

Crystal 120/11, E-distilled P_1, Length: 2.80 cm., Weight: 0.7149. Area: 2.60 mm^2,

$W_n = 4.5698$, $H = 13,200$.

θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} x 10^6$	θ	θ'	W_θ	$F_{\theta'}$	$-X_{\theta'} x 10^6$
0	103	4.6029	0.0331	1.463	180	283	4.6029	0.0331	1.463
15	118	4.6013	0.0315	1.392	195	298	4.6013	0.0315	1.392
30	133	4.5989	0.0291	1.287	210	313	4.5989	0.0291	1.287
45	148	4.5961	0.0263	1.162	225	328	4.5961	0.0263	1.162
60	163	4.5943	0.0245	1.083	240	343	4.5943	0.0245	1.083
75	178	4.5934	0.0236	1.042	255	358	4.5934	0.0236	1.042
90	193	4.5939	0.0241	1.065	270	13	4.5939	0.0241	1.065
105	208	4.5953	0.0255	1.128	285	28	4.5953	0.0255	1.128
120	223	4.5974	0.0276	1.220	300	43	4.5974	0.0276	1.220
135	238	4.6001	0.0303	1.340	315	58	4.6001	0.0303	1.340
150	253	4.6021	0.0323	1.429	330	73	4.6021	0.0323	1.429
165	268	4.6032	0.0335	1.481	345	88	4.6032	0.0335	1.481
180	283	4.6029	0.0331	1.463	360	103	4.6029	0.0331	1.463

Data for Fig. 14 (cont.)

Crystal 120/21, E-distilled P_1, Length: 2.97 cm., Weight: 0.9045 grams. Area: 3.105 mm^2,

$W_n = 3.8040$, $H = 13,200$.

θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} x 10^6$	θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} x 10^6$
0	152	3.8347	0.0307	1.138	180	332	3.8347	0.0307	1.138
15	167	3.8328	0.0287	1.068	195	347	3.8328	0.0287	1.068
30	182	3.8323	0.0283	1.048	210	2	3.8323	0.0283	1.048
45	197	3.8335	0.0295	1.093	225	17	3.8335	0.0295	1.093
60	212	3.8357	0.0317	1.175	240	32	3.8357	0.0317	1.175
75	227	3.8380	0.0340	1.260	255	47	3.8380	0.0340	1.260
90	242	3.8411	0.0371	1.375	270	62	3.8411	0.0371	1.375
105	257	3.8432	0.0392	1.453	285	77	3.8432	0.0392	1.453
120	272	3.8440	0.0400	1.482	300	92	3.8440	0.0400	1.482
135	287	3.8427	0.0387	1.433	315	107	3.8427	0.0387	1.433
150	302	3.8402	0.0362	1.341	330	122	3.8402	0.0362	1.341
165	317	3.8371	0.0331	1.227	345	137	3.8371	0.0331	1.227
180	332	3.8347	0.0307	1.138	360	152	3.8347	0.0307	1.138

Data for Fig. 14 (cont.)

Crystal 120/51, E-distilled P_1, Length: 1.98 cm., Weight: 0.5932 grams. Area: 3.06 mm^2,

$W_n = 3.4850$, $H = 13{,}200$.

θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} x\ 10^6$	θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} x\ 10^6$
0	218	3.5172	0.0322	1.217	180	38	3.5172	0.0322	1.217
15	233	3.5201	0.0351	1.328	195	53	3.5201	0.0351	1.328
30	248	3.5228	0.0378	1.429	210	68	3.5228	0.0378	1.429
45	263	3.5245	0.0395	1.492	225	83	3.5245	0.0395	1.492
60	278	3.5245	0.0395	1.492	240	98	3.5245	0.0395	1.492
75	293	3.5232	0.0382	1.442	255	113	3.5232	0.0382	1.442
90	308	3.5204	0.0354	1.339	270	128	3.5204	0.0354	1.339
105	323	3.5175	0.0325	1.228	285	143	3.5175	0.0325	1.228
120	338	3.5146	0.0296	1.119	300	158	3.5146	0.0296	1.119
135	353	3.5129	0.0279	1.054	315	173	3.5129	0.0279	1.054
150	8	3.5129	0.0279	1.054	330	188	3.5129	0.0279	1.054
165	23	3.5141	0.0291	1.100	345	203	3.5141	0.0291	1.100
180	38	3.5172	0.0322	1.217	360	218	3.5172	0.0322	1.217

Data for Fig. 14 (cont.)

Crystal 120/61, E-distilled P_1, Length: 3.39 cm., Weight: 0.7272 grams. Area: 2.19 mm.^{2},

W_n = 0.2241, H = 13,200.

θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} \times 10^6$	θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} \times 10^6$
0	215	0.2465	0.0224	1.174	180	35	0.2465	0.0224	1.174
15	230	0.2487	0.0246	1.290	195	50	0.2487	0.0246	1.290
30	245	0.2511	0.0270	1.416	210	65	0.2511	0.0270	1.416
45	260	0.2522	0.0281	1.473	225	80	0.2522	0.0281	1.473
60	275	0.2524	0.0283	1.483	240	95	0.2524	0.0283	1.483
75	290	0.2513	0.0272	1.427	255	110	0.2513	0.0272	1.427
90	305	0.2496	0.0255	1.338	270	125	0.2496	0.0255	1.338
105	320	0.2471	0.0230	1.207	285	140	0.2471	0.0230	1.207
120	335	0.2454	0.0213	1.117	300	155	0.2454	0.0213	1.117
135	350	0.2443	0.0202	1.059	315	170	0.2443	0.0202	1.059
150	5	0.2441	0.0200	1.048	330	185	0.2441	0.0200	1.048
165	20	0.2452	0.0211	1.107	345	200	0.2452	0.0211	1.107
180	35	0.2465	0.0224	1.174	360	215	0.2465	0.0224	1.174

Data for Fig. 14 (cont.)

Crystal 120/71, E-distilled P_1, Length: 2.99 cm., Weight: 0.6022 grams.

Area: 2.055 mm^2, W_n = 0.3114, H = 13,200.

θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} \times 10^6$	θ	θ'	$W_{\theta'}$	$F_{\theta'}$	$-X_{\theta'} \times 10^6$
0	126	0.3358	0.0244	1.364	180	306	0.3358	0.0244	1.364
15	141	0.3334	0.0220	1.230	195	321	0.3334	0.0220	1.230
30	156	0.3317	0.0203	1.134	210	336	0.3317	0.0203	1.134
45	171	0.3305	0.0191	1.068	225	351	0.3305	0.0191	1.068
60	186	0.3304	0.0190	1.061	240	6	0.3304	0.0190	1.061
75	201	0.3312	0.0198	1.108	255	21	0.3312	0.0198	1.108
90	216	0.3325	0.0211	1.180	270	36	0.3325	0.0211	1.180
105	231	0.3347	0.0233	1.303	285	51	0.3347	0.0233	1.303
120	246	0.3364	0.0250	1.398	300	66	0.3364	0.0250	1.398
135	261	0.3375	0.0261	1.460	315	81	0.3375	0.0261	1.460
150	276	0.3377	0.0263	1.470	330	96	0.3377	0.0263	1.470
165	291	0.3372	0.0258	1.442	345	111	0.3372	0.0258	1.442
180	306	0.3358	0.0244	1.364	360	126	0.3358	0.0244	1.364

DETERMINATION OF THE MAGNETIC RATIO FOR ALL THE TYPES OF "PURE" BISMUTH

Data for Table 2

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
109/11	A	5.0174	5.0492	5.0386	318	212	1.499
122/11	B	4.0012	4.0632	4.0398	620	386	1.606
124/11	C	3.3847	3.4148	3.4049	301	202	1.440
111/11	D	4.5715	4.6100	4.5979	385	264	1.460
119/11	E	3.4331	3.4799	3.4659	468	328	1.429
120/11	E-Dist.	4.5698	4.6034	4.5933	336	236	1.423
123/41	F	4.3021	4.3740	4.3520	719	509	1.411
234/11	G	4.4386	4.4902	4.4747	516	361	1.428
246/21	H	1.6255	1.7464	1.7100	1209	845	1.430

DEPENDENCE OF MAGNETIC RATIO ON THE SPEED OF GROWTH

Data for Table 3

n = Normal speed   s = 1/6 normal speed

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
188/11n	G 3.00%Pb	1.9947	2.0439	2.0135	492	188	2.619
		1.9947	2.0427	2.0130	480	183	2.625
188/21s	G 3.00%Pb	2.3675	2.4257	2.3948	582	273	2.130
		2.3673	2.4379	2.3928	706	255	2.764
166/12n	G 0.15%Pb	4.3316	4.3575	4.3475	259	159	1.630
		4.3316	4.3571	4.3473	255	157	1.627
166/22s	G 0.15%Pb	0.7715	0.8018	0.7882	303	167	1.605
		0.7715	0.7982	0.7867	267	152	1.751
192/11n	G	1.4795	1.5264	1.5123	469	328	1.430
		1.4537	1.4989	1.4852	452	315	1.428
192/21s	G	4.8896	4.9280	4.9163	384	267	1.438
		4.8886	4.9250	4.9141	364	255	1.429

FIRST SILVER SERIES

Data for Fig. 15

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
135/11	Bi G	3.1924	3.3467	3.3008	1543	1084	1.423
	" "	4.1339	4.1831	4.1687	492	348	1.414
136/11	G 0.097%Ag	4.9656	4.9946	4.9864	290	208	1.393
137/11	G 0.194%Ag	4.5273	4.5538	4.5460	265	187	1.418
138/11	G 0.291%Ag	4.4676	4.4943	4.4862	267	186	1.435
139/11	G 0.288%Ag	4.0805	4.1209	4.1088	404	283	1.429
140/11	G 0.049%Ag	3.9521	3.9798	3.9714	277	193	1.434
141/11	G 0.146%Ag	3.8035	3.8410	3.8300	375	265	1.415
142/11	G 0.242%Ag	3.7101	3.7444	3.7347	343	246	1.396
143/11	G 0.339%Ag	4.6209	4.6506	4.6413	297	204	1.453
144/11	G 0.436%Ag	3.9862	4.0186	4.0090	324	228	1.421
145/11	G 0.485%Ag	3.9109	3.9413	3.9326	304	217	1.400
146/11	G 0.533%Ag	3.5180	3.5472	3.5386	292	206	1.416
147/11	G 0.582%Ag	3.5435	3.5739	3.5651	304	216	1.409

FIRST SILVER SERIES (cont)

Data for Fig. 15

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
148/11	G 0.630%Ag	3.5712	3.6093	3.5980	381	268	1.420
149/11	Bi G	3.6107	3.6436	3.6338	329	231	1.422
149/21	Bi G	3.3653	3.4014	3.3906	361	253	1.428
150/11	Bi G	4.9243	4.9588	4.9486	345	243	1.421
151/11	G 0.024%Ag	4.4849	4.5188	4.5089	339	240	1.414
152/11	G 0.073%Ag	3.7965	3.8262	3.8174	297	209	1.432
153/11	G 0.121%Ag	4.6676	4.7061	4.6950	385	274	1.407
154/11	G 0.234%Ag	4.3155	4.3430	4.3347	275	192	1.434
155/11	G 0.332%Ag	4.4218	4.4397	4.4344	179	126	1.421
156/11	G 0.679%Ag	4.1701	4.1904	4.1843	203	142	1.427
157/11	G 0.727%Ag	4.1066	4.1250	4.1195	184	129	1.425
158/11	G 0.776%Ag	4.5510	4.5710	4.5651	200	141	1.420
162/11	G 0.970%Ag	4.0225	4.0488	4.0410	263	185	1.420
196/11	G 1.940%Ag	3.7384	3.7747	3.7635	363	251	1.440

SECOND SILVER SERIES

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
136/21	G 0.097%Ag	2.3893	2.4263	2.4151	370	258	1.434
137/21	G 0.194%Ag	1.7945	1.8488	1.8330	543	385	1.410
140/21	G 0.049%Ag	1.4884	1.5425	1.5260	541	376	1.439
141/21	G 0.146%Ag	1.0989	1.1477	1.1327	488	338	1.445
142/21	G 0.242%Ag	1.2770	1.3287	1.3132	517	362	1.428
151/21	G 0.024%Ag	1.9203	1.9676	1.9534	473	331	1.431
152/21	G 0.073%Ag	1.4554	1.5049	1.4903	495	349	1.418
153/21	G 0.121%Ag	1.0608	1.0929	1.0833	321	225	1.427
154/21	G 0.234%Ag	1.9267	1.9694	1.9562	427	295	1.447
155/21	G 0.332%Ag	1.8117	1.8646	1.8498	529	381	1.390

COPPER SERIES

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R
201/22	G 0.025%Cu	3.4812	3.5188	3.5077	376	365	1.420
202/11	G 0.050%Cu	2.3418	2.3889	2.3748	471	330	1.427
203/11	G 0.075%Cu	2.4514	2.4956	2.4825	442	311	1.420
204/11	G 0.100%Cu	2.7372	2.7817	2.7687	445	315	1.414
205/11	G 0.125%Cu	2.6516	2.6933	2.6808	417	292	1.426
206/11	G 0.150%Cu	2.8114	2.8557	2.8424	443	310	1.430
207/11	G 0.175%Cu	2.8215	2.8635	2.8510	420	295	1.423
208/11	G 0.200%Cu	2.8156	2.8584	2.8455	428	299	1.428
209/11	G 0.225%Cu	2.9151	2.9547	2.9430	396	279	1.421
210/11	G 0.250%Cu	2.5901	2.6350	2.6216	449	315	1.424
211/11	G 0.300%Cu	2.6427	2.6854	2.6726	427	299	1.428
212/11	G 0.350%Cu	2.9745	3.0194	3.0064	440	308	1.429
213/11	G 0.400%Cu	2.4863	2.5317	2.5183	454	320	1.419
214/11	G 0.450%Cu	2.2011	2.2550	2.2380	539	369	1.422
215/11	G 0.500%Cu	2.1982	2.2470	2.2325	488	343	1.423
216/11	G 0.600%Cu	2.0421	2.0913	2.0767	492	346	1.421
217/11	G 0.700%Cu	1.8240	1.8776	1.8618	536	378	1.418
218/11	G 0.800%Cu	2.5966	2.6391	2.6264	425	298	1.425
219/11	G 0.900%Cu	2.7201	2.7662	2.7525	461	324	1.422
220/11	G 1.000%Cu	2.6108	2.6646	2.6484	534	376	1.420

FIRST LEAD SERIES

Data for Figs. 16 and 18

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$
163/11	G 0.026%Pb	3.5766	3.6163	3.6026	397	260	1.526	0.659
		3.5766	3.6170	3.6035	406	269	1.510	
164/11	G 0.076%Pb	4.3775	4.4099	4.3991	324	216	1.501	0.665
		4.3775	4.4095	4.3987	320	212	1.510	
165/11	G 0.102%Pb	4.6387	4.6692	4.6579	305	192	1.590	0.630
		4.6387	4.6697	4.6583	310	196	1.584	
166/11	G 0.152%Pb	4.3316	4.3575	4.3475	259	159	1.630	0.614
		4.3316	4.3571	4.3473	255	157	1.627	
167/11	G 0.17%Pb	4.2280	4.2551	4.2444	271	164	1.655	0.604
		4.2280	4.2547	4.2441	267	161	1.659	
168/11	G 0.202%Pb	4.5348	4.5640	4.5522	292	174	1.680	0.594
		4.5348	4.5628	4.5514	280	166	1.688	
169/11	G 0.227%Pb	4.2062	4.2398	4.2264	336	202	1.666	0.602
		4.2062	4.2392	4.2261	330	199	1.661	
170/11	G 0.252%Pb	3.8633	3.8999	3.8848	366	215	1.704	0.587
		3.8633	3.8993	3.8845	360	210	1.700	

FIRST LEAD SERIES

Data for Figs. 16 and 18 (cont.)

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	F	$\frac{1}{R}$
171/11	G 0.303%Pb	3.8787	3.9148	3.8996	361	209	1.730	0.578
		3.8787	3.9127	3.8984	340	197	1.728	
172/11	G 0.353%Pb	3.7033	3.7374	3.7225	341	192	1.780	0.563
		3.7033	3.7366	3.7221	333	188	1.774	
173/11	G 0.404%Pb	3.7188	3.7556	3.7390	368	202	1.823	0.549
		3.7188	3.7548	3.7386	360	198	1.820	
174/11	G 0.454%Pb	4.0551	4.0865	4.0724	314	173	1.818	0.551
		4.0551	4.0872	4.0728	321	177	1.816	
175/11	G 0.505%Pb	4.0665	4.1015	4.0852	350	187	1.863	0.536
		4.0665	4.1011	4.0850	346	185	1.870	
177/11	G 0.605%Pb	4.0528	4.0864	4.0708	336	180	1.870	0.533
		4.0528	4.0857	4.0703	329	175	1.880	
179/11	G 0.710%Pb	3.4954	3.5394	3.5179	440	225	1.956	0.510
		3.4954	3.5392	3.5179	438	225	1.951	
180/11	G 0.760%Pb	3.7395	3.7832	3.7613	437	218	2.004	0.499
		3.7395	3.7836	3.7615	441	220	2.006	

FIRST LEAD SERIES

Data for Figs. 16 and 18 (cont.)

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
181/11	G 0.810%Pb	3.6128	3.6594	3.6361	470	237	1.985	0.504
		3.6128	3.6590	3.6359	466	235	1.983	
182/11	G 0.860%Pb	4.2057	4.2523	4.2283	466	226	2.060	0.487
		4.2057	4.2532	4.2289	475	232	2.050	
183/11	G 0.910%Pb	4.1980	4.2372	4.2154	432	214	2.020	0.494
		4.1980	4.2364	4.2149	424	209	2.027	
184/11	G 0.960%Pb	3.3168	3.3498	3.3325	328	157	2.093	0.477
		3.3491	3.3322	3.3168	323	154	2.098	
185/11	G 1.010%Pb	3.4117	3.4498	3.4298	381	181	2.108	0.474
		3.4117	3.4499	3.4297	382	180	2.112	
186/11	G 5.050%Pb	3.7863	3.8187	3.7972	324	109	2.975	0.337
		3.7863	3.8177	3.7969	314	106	2.965	
188/11	G 3.030%Pb	1.9947	2.0439	2.0135	492	188	2.619	0.381
		1.9947	2.0427	2.0130	480	183	2.625	
189/11	G 4.040%Pb	2.3092	2.3587	2.3272	495	180	2.753	0.364
		2.3092	2.3573	2.3268	481	176	2.740	
190/11	G 2.020%Pb	1.9791	2.1314	2.1007	523	216	2.420	0.416
		1.9791	2.1328	2.1014	537	223	2.405	

DEPENDENCE OF R ON TEMPERATURE

Data for Fig. 17

Temp.° C.	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
Crystal 224/21 G 5.00%Sn							
22	0.4820	0.5029	0.4828	208	8	26.125	0.038
-185	0.4821	0.4940	0.4745	119	-76	-1.565	-0.639
-135	1.3446	1.3608	1.3395	162	-51	-3.180	-0.314
-115	1.3576	1.3720	1.3540	144	-36	-4.000	-0.250
- 56	1.3584	1.3766	1.3566	182	-18	-10.110	-0.099
32	0.4828	0.5041	0.4842	213	14	15.20	0.066
62	0.4845	0.5069	0.4872	224	27	8.30	0.121
72	0.4844	0.5069	0.4877	225	33	6.82	0.147
126	0.4856	0.5075	0.4906	219	50	4.38	2.228
185	0.4867	0.5062	0.4933	195	66	2.85	0.351
232	0.4873	0.5028	0.4935	155	62	2.50	0.400
Crystal 123/41 F							
22	1.2517	1.3192	1.2999	675	482	1.406	0.711
-185	1.2648	1.3214	1.3010	566	362	1.565	0.639
-84	1.2521	1.3291	1.3023	770	502	1.534	0.652
126	1.2584	1.3145	1.3003	561	419	1.340	0.746
286	1.2592	1.3011	1.2938	419	346	1.210	0.826

FIRST TIN SERIES

Data for Fig. 18

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
224/21	G 5.00%Sn	0.4820	0.5029	0.4828	208	8	26.125	0.038
225/11	G 4.00%Sn	0.3202	0.3518	0.3232	316	30	10.510	0.095
226/11	G 3.00%Sn	1.9886	2.0303	1.9954	417	68	6.130	0.163
228/11	G 1.00%Sn	0.5586	0.6031	0.5734	445	148	3.005	0.324
229/11	G 0.01%Sn	0.8657	0.9006	0.8894	349	237	1.472	0.392
230/11	G 0.05%Sn	1.2918	1.3231	1.3106	313	188	1.662	0.534
231/11	G 0.10%Sn	1.1502	1.1981	1.1758	479	256	1.873	0.601
323/11	G 0.05%Sn	3.0633	3.0942	3.0752	309	119	2.595	0.679

DEPENDENCE OF SUSCEPTIBILITY ON TEMPERATURE

Data for Fig. 25

Temp. °C	H	W_n	$W_\perp$	$F_\perp$	R	$-X_\perp \times 10^6$	$-X_\parallel \times 10^6$
Crystal 123/41	F	Length: 2.64 cm.		Weight: 2.4073		Area: 9.27 mm^2	
22	9,900	1.2517	1.3192	675	1.406	1.482	1.063
-185	7,920	1.2648	1.3214	566	1.565	1.944	1.226
- 84	9,900	1.2521	1.3291	770	1.534	1.691	1.104
126	9,900	1.2584	1.3145	561	1.340	1.237	0.921
266	9,900	1.2592	1.3011	419	1.210	0.922	0.760

DEPENDENCE OF SUSCEPTIBILITY ON TEMPERATURE

Data for Fig. 25

Crystal 224/21 G 5.00%Sn Length: 1.94 cm Weight: 1.7605 grams Area: 9.23

Temp.° C.	H	W_n	$W_\perp$	$F_\perp$	R	$-X_\perp$ x 10^6	$-X_\parallel$ x 10^6
22	9,900	4.6925	4.7404	479	26.12	1.058	0.044
-185	7,650	4.6930	4.7049	119	-1.56	0.433	-0.283
-135	9,900	4.6890	4.7206	316	-3.18	0.698	-0.219
- 84	9,900	4.6885	4.7270	385	-4.48	0.850	-0.190
75	10,560	4.6967	4.7534	567	6.78	1.102	0.161
126	10,560	4.6995	4.7575	580	4.38	1.129	0.257
185	10,560	4.7007	4.7545	542	2.85	1.054	0.369
232	10,560	4.7010	4.7470	460	2.50	0.895	0.358

FIRST TELLURIUM SERIES

Data for Fig. 27

Temperature: 22° C

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
241/11	G 0.01%Te	0.8944	1.0964	1.0439	2020	1495	1.350	0.740
242/11	G 0.03%Te	0.5558	0.7542	0.7228	1984	1670	1.189	0.842
243/11	G 0.09%Te	1.0703	1.2418	1.2494	1715	1791	0.957	1.044
244/11	G 0.27%Te	1.2705	1.4092	1.4668	1387	1963	0.706	1.418
245/11	G 0.81%Te	1.1056	1.1985	1.2788	929	1732	0.536	1.867
Temperature: -185° C								
242/11	G 0.03%Te	0.2118	0.2433	0.2448	315	330	0.955	1.048
243/11	G 0.09%Te	0.9375	0.9633	0.9690	258	315	0.820	1.220
244/11	G 0.27%Te	1.2716	1.2811	1.3030	195	314	0.621	1.610
245/11	G 0.81%Te	1.0988	1.1117	1.1274	129	286	0.451	2.219

FIRST TELLURIUM SERIES

Data for Fig. 27, (cont.)

Temperature: 52° C

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
242/11	G 0.03%Te	0.2141	0.2729	0.2641	588	500	1.178	0.849
243/11	G 0.09%Te	0.9409	0.9932	0.9938	528	534	0.990	1.010
244/11	G 0.27%Te	not measured at this temperature						
245/11	G 0.81%Te	0.7790	0.8063	0.8275	273	485	0.562	1.778*
245/11	G 0.81%Te	0.7788	0.8060	0.8283	272	495	0.549	1.820

* This measurement was made before equilibrium was established.

Temperature: 76° C.

242/11	G 0.03%Te	0.2195	0.2768	0.2683	573	488	1.173	0.852
243/11	G 0.09%Te	0.9435	0.9971	0.9946	536	511	1.047	0.955
244/11	G 0.27%Te	0.9403	0.9802	0.9943	399	540	0.738	1.354
245/11	G 0.81%Te	0.7826	0.8105	0.8296	279	470	0.594	1.685

FIRST TELLURIUM SERIES

Data for Fig. 27, (cont.)

Temperature; 129° C

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$
242/11	G 0.03%Te	0.2206	0.2755	0.2675	549	469	1.170	0.855
243/11	G 0.09%Te	0.9463	0.9970	0.9939	507	476	1.062	0.941
244/11	G 0.27%Te	0.9418	0.9828	0.9922	410	504	0.813	1.230
245/11	G 0.81%Te	0.7856	0.8147	0.8310	291	454	0.641	1.560
Temperature: 179° C.								
242/11	G 0.03%Te	0.2230	0.2735	0.2664	505	434	1.165	0.859
243/11	G 0.09%Te	0.9485	0.9972	0.9923	487	438	1.100	0.900
244/11	G 0.27%Te	0.9437	0.9850	0.9906	413	469	0.891	1.134
245/11	G 0.81%Te	0.7867	0.8164	0.8291	297	424	0.701	1.430
Temperature: 232° C								
242/11	G 0.03%Te	0.2216	0.2707	0.2634	491	418	1.174	0.852
243/11	G 0.09%Te	0.9463	0.9885	0.9838	422	375	1.125	0.889
244/11	G 0.27%Te	0.9449	0.9843	0.9854	394	405	0.972	1.028
245/11	G 0.81%Te	0.7871	0.8172	0.8262	301	391	0.770	1.300

EVALUATION OF C

Crystal	Kind	W	l	$C \times 10^{10}$	$C \times 10^{10}$	$C \times 10^{10}$
			Field →	20,800	8,500	6,850
249/11	H 0.01%Ag	1.6834	2.024	5.45	32.7	50.2
251/11	H 0.09%Ag	1.983	2.028	5.35	32.1	49.3
253/11	H 0.81%Ag	1.5439	1.944	5.72	34.3	52.6
254/11	H 2.43%Ag	1.4652	2.054	6.71	40.2	
256/11	H 0.01%Sn	1.5090	2.003	6.03	36.1	65.5
257/11	H 0.03%Sn	1.5337	1.990	5.87	35.2	54.1
258/11	H 0.09%Sn	1.4192	1.894	6.06	36.3	55.7
260/11	H 0.09%Sn	1.6290	2.023	5.64	33.8	51.8
261/11	H 2.43%Sn	1.6537	1.979	5.42	32.5	50.0
263/11	H 0.01%Pb	1.6185	2.026	5.69	34.1	52.3
264/11	H 0.03%Pb	1.5194	1.998	5.96	35.7	54.9
265/11	H 0.09%Pb	1.3778	1.989	6.65	39.8	61.2
266/11	H 0.27%Pb	1.5622	2.018	5.85	35.1	53.9
267/11	H 0.81%Pb	1.7208	1.980	5.27	31.6	48.5
268/11	H 2.43%Pb	1.5588	2.046	5.95	35.8	54.7

EVALUATION OF C

Crystal	Kind	W	1	$C\times10^{10}$	$C\times10^{10}$	$C\times10^{10}$
			Field →	29,800	8,500	6,850
270/11	H 0.01%Te	1.6957	2.101	5.63	33.8	51.8
271/11	H 0.03%Te	1.3181	2.056	7.08	42.4	65.1
272/11	H 0.09%Te	1.5827	1.898	5.45	32.7	50.2
273/11	H 0.27%Te	1.4139	2.076	6.65	39.9	61.3
275/11	H 0.01%Se	1.7510	2.068	5.36	32.1	49.3
276/11	H 0.03%Se	1.6163	1.941	5.46	32.7	50.2
277/11	H 0.09%Se	1.6651	1.960	5.34	32.0	49.2
278/11	H 0.27%Se	1.5356	2.074	6.13	36.7	56.4
279/11	H 0.81%Se	1.4454	2.108	6.62	39.6	60.9
280/11	H 0.01%Sb	1.6364	2.028	5.62	33.7	51.7
281/11	H 0.03%Sb	1.3520	1.905	6.38	38.2	58.7
282/11	H 0.09%Sb	1.5514	2.032	5.94	35.6	54.7
283/11	H 0.27%Sb	1.7653	1.950	5.01	30.0	46.1
254/11	H 2.43%Ag	1.2665	1.879			58.5

FINAL SERIES OF MEASUREMENTS

Temperature: 22° C. $H = 20,800$ Suspension correction = 0.0008

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$Cx10^{10}$	$-X_{\perp}x10^{6}$	$-X_{\parallel}x10^{6}$
249/11	H 0.01%Ag	3.8626	4.1341	4.0522	2723	1904	1.430	0.699	5.45	1.489	1.041
251/11	H 0.09%Ag	3.4939	3.7710	3.6877	2779	1946	1.428	0.700	5.35	1.488	1.043
253/11	H 0.81%Ag	4.0015	4.2617	4.1834	2610	1827	1.430	0.699	5.72	1.492	1.043
254/11	H 2.43%Ag	4.3543	4.5754	4.5098	2219	1563	1.420	0.705	6.71	1.490	1.051
256/11	H 0.01%Sn	3.9663	4.2208	4.1390	2253	1735	1.470	0.680	6.03	1.539	1.048
257/11	H 0.03%Sn	4.1667	4.4317	4.3352	2658	1693	1.570	0.637	5.87	1.562	0.994
258/11	H 0.09%Sn	3.9286	4.2010	4.0789	2732	1511	1.803	0.555	6.06	1.658	0.916
260/11	H 0.81%Sn	3.6625	3.9405	3.7557	2788	940	2.965	0.337	5.64	1.573	0.531
261/11	H 2.43%Sn	4.1731	4.4093	4.2231	2370	509	4.655	0.215	5.42	1.289	0.265
263/11	H 0.01%Pb	4.0412	4.3001	4.2202	2597	1798	1.443	0.692	5.69	1.478	1.023
264/11	H 0.03%Pb	4.2465	4.5031	4.4200	2574	1743	1.478	0.677	5.96	1.533	1.040
265/11	H 0.09%Pb	3.9845	4.2120	4.1316	2283	1479	1.542	0.648	6.65	1.520	0.990
266/11	H 0.27%Pb	4.3302	4.6091	4.4903	2797	1609	1.737	0.576	5.85	1.636	0.941
267/11	H 0.81%Pb	3.5762	3.8921	3.7321	3167	1567	2.021	0.494	5.27	1.655	0.816
268/11	H 2.43%Pb	4.0439	4.3124	4.1526	2693	1095	2.459	0.407	5.95	1.604	0.652

FINAL SERIES OF MEASUREMENTS

Temperature: 22° C. $H = 20,800$ Suspension correction = 0.0008

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-X_\perp \times 10^6$	$-X_\parallel \times 10^6$
270/11	H 0.01%Te	3.9010	4.1532	4.0908	2530	1906	1.329	0.753	5.63	1.442	1.073
271/11	H 0.03%Te	4.7965	4.9827	4.9519	1870	1562	1.197	0.836	7.08	1.325	1.108
272/11	H 0.09%Te	3.8378	4.0430	4.0651	2060	2281	0.903	1.108	5.45	1.122	1.244
273/11	H 0.27%Te	4.8258	4.9598	5.0182	1348	1932	0.697	1.435	6.65	0.897	1.288
275/11	H 0.01%Se	3.6325	3.9080	3.8317	2763	2000	1.381	0.724	5.36	1.483	1.073
276/11	H 0.03%Se	3.7628	4.0227	3.9658	2607	2036	1.280	0.781	5.46	1.424	1.111
277/11	H 0.09%Se	3.3829	3.6436	3.5993	2615	2172	1.203	0.830	5.34	1.395	1.161
278/11	H 0.27%Se	4.4879	4.6898	4.6860	2027	1989	1.020	0.980	6.13	1.244	1.220
279/11	H 0.81%Se	5.1147	5.2916	5.2978	1777	1839	0.966	1.034	6.62	1.178	1.218
280/11	H 0.01%Sb	3.3142	3.5788	3.4997	2654	1863	1.423	0.702	5.62	1.490	1.046
281/11	H 0.03%Sb	4.3301	4.5610	4.4922	2317	1629	1.421	0.703	6.38	1.480	1.040
282/11	H 0.09%Sb	4.4434	4.6888	4.6160	2462	1734	1.420	0.704	5.94	1.468	1.032
283/11	H 0.27%Sb	3.4806	3.7723	3.6850	2925	2052	1.427	0.700	5.01	1.468	1.028

Temperature: -185° C.   H = 6,850   Suspension correction = 0.0007

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C\times10^{10}$	$-\chi_\perp\times10^6$	$-\chi_\parallel\times10^6$
249/11	H 0.01%Ag	3.8614	3.8989	3.8851	382	244	1.564	0.639	50.2	1.920	1.228
251/11	H 0.09%Ag	3.4927	3.5309	3.5163	389	243	1.595	0.626	49.3	1.920	1.200
253/11	H 0.81%Ag	4.0016	4.0329	4.0237	370	228	1.618	0.618	52.6	1.950	1.203
254/11	H 2.43%Ag	4.3510	4.3827	4.3708	324	305	1.575	0.634	58.5	1.900	1.203
256/11	H 0.01%Sn	3.9844	4.0239	4.0054	402	217	1.850	0.541	60.3	2.232	1.207
257/12	H 0.03%Sn	4.1830	4.2298	4.2027	475	204	2.325	0.429	54.1	2.568	1.104
258/11	H 0.09%Sn	3.9422	3.9823	3.9581	408	166	2.460	0.406	55.7	2.275	0.925
260/11	H 0.81%Sn	3.6752	3.6951	3.6800	206	55	3.740	0.267	51.8	1.070	0.286
261/11	H 2.43%Sn	4.1880	4.2014	4.1860	141	-13	-10.850	-0.092	50.0	0.704	-0.065
263/12	H 0.01%Pb	4.0571	4.0949	4.0791	385	227	1.698	0.589	52.3	2.017	1.189
264/11	H 0.03%Pb	4.2621	4.3014	4.2827	400	213	1.878	0.533	54.9	2.197	1.169
265/11	H 0.09%Pb	4.0990	4.0397	4.0164	414	181	2.283	0.438	61.2	2.530	1.108
266/11	H 0.27%Pb	4.2358	4.3902	4.3623	451	172	2.620	0.382	53.9	2.431	0.927
267/11	H 0.81%Pb	3.5896	3.6265	3.6044	376	155	2.425	0.412	48.5	1.827	0.752
268/11	H 2.43%Pb	4.0574	4.0816	4.0658	249	91	2.740	0.365	54.7	1.365	0.498

Temperature: -185° C.   H = 6,850   Suspension correction = 0.0007

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-X_\perp \times 10^6$	$-X_\parallel \times 10^6$
270/11	H 0.01%Te	3.9185	3.9486	3.9463	308	285	1.080	0.925	51.8	1.682	1.478
271/12	H 0.03%Te	4.8118	4.8330	4.8335	219	225	0.974	1.028	65.1	1.427	1.466
272/11	H 0.09%Te	3.8526	3.8763	3.8813	244	294	0.830	1.205	50.2	1.224	1.474
273/11	H 0.27%Te	4.8375	4.8518	4.8611	150	243	0.617	1.620	61.3	0.919	1.490
275/11	H 0.01%Se	3.6466	3.6824	3.6738	365	272	1.308	0.765	49.3	1.800	1.376
276/11	H 0.03%Se	3.7755	3.8079	3.8041	331	303	1.091	0.915	50.2	1.663	1.511
277/11	H 0.09%Se	3.3956	3.4260	3.4261	211	212	0.995	1.005	49.2	1.268	1.482
278/11	H 0.27%Se	4.5004	4.5222	4.5260	225	263	0.856	1.168	56.4	1.530	1.535
279/11	H 0.81%Se	5.1273	5.1470	5.1530	204	264	0.772	1.295	60.9	1.004	1.333
280/11	H 0.01%Sb	3.3305	3.3689	3.3550	391	252	1.552	0.644	51.7	2.025	1.303
281/11	H 0.03%Sb	4.3437	4.3757	4.3646	327	216	1.514	0.660	58.7	1.920	1.270
282/11	H 0.09%Sb	4.4556	4.4891	4.4773	342	234	1.462	0.684	54.7	1.875	1.330*
283/11	H 0.27%Sb	3.4936	3.5356	3.5205	427	276	1.545	0.647	46.1	1.978	1.279

* Large stranger may cause this deviation.

Temperature: 65° C.   H = 8,500 gauss   Suspension correction = 0.0005

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-\chi_\perp \times 10^6$	$-\chi_\parallel \times 10^6$
249/11	H 0.01%Ag	3.0972	3.1390	3.1275	423	305	1.387	0.729	32.7	1,387	0.998
251/11	H 0.09%Ag	2.7190	2.7621	2.7505	435	320	1.360	0.733	32.1	1.394	1.027
253/11	H 0.81%Ag	3.2276	3.2670	3.2566	399	295	1.352	0.735	34.3	1.370	1.012
254/11	H 2.43%Ag	3.5796	3.6138	3.6043	343	252	1.360	0.730	40.2	1.381	1.016
256/11	H 0.01%Sn	3.1914	3.2306	3.2196	397	287	1.385	0.730	36.1	1.436	1.038
257/12	H 0.03%Sn	3.3922	3.4333	3.4207	416	290	1.436	0.695	35.2	1.465	1.022
258/11	H 0.09%Sn	3.1536	3.1972	3.1793	441	262	1.683	0.590	36.3	1.563	0.915
260/11	H 0.81%Sn	2.8883	2.9334	2.9050	456	172	2.650	0.370	33.8	1.540	0.581
261/11	H 0.43%Sn	3.3982	3.4409	3.4093	432	116	3.722	0.260	32.5	1.406	0.378
263/11	H 0.01%Pb	3.2668	3.3067	3.2958	404	295	1.370	0.727	34.1	1.380	1.007
264/11	H 0.03%Pb	3.4721	3.5120	3.5001	405	285	1.419	0.702	35.7	1.444	1.018
266/11	H 0.27%Pb	3.5569	3.5992	3.5831	428	267	1.603	0.590	35.1	1.502	0.936
267/11	H 0.81%Pb	2.8026	2.8502	2.8288	481	267	1.800	0.549	48.5	1.508	0.835
268/11	H 2.43%Pb	3.2695	3.3131	3.2890	441	200	2.216	0.446	35.8	1.573	0.713

Temperature: 65° C. $H = 8,500$ gauss Suspension correction = 0.0005

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C\times10^{10}$	$-X_\perp\times10^6$	$-X_\parallel\times10^6$
270/11	H 0.01%Te	3.1274	3.1669	3.1580	400	311	1.287	0.775	33.8	1.350	1.050
271/12	H 0.03%Te	4.0229	4.0536	4.0480	312	256	1.220	0.820	42.4	1.332	1.084
272/11	H 0.09%Te	3.0655	3.0998	3.1012	347	362	0.958	1.041	32.7	1.132	1.182
273/11	H 0.27%Te	4.0529	4.0754	4.0838	230	314	0.733	1.372	39.9	0.917	1.252
275/11	H 0.01%Se	2.8586	2.9022	2.8910	441	329	1.340	0.743	32.1	1.418	1.058
276/11	H 0.03%Se	2.9884	3.0306	3.0205	429	326	1.315	0.760	32.7	1.405	1.067
277/11	H 0.09%Se	2.6088	2.6506	2.6437	423	354	1.197	0.833	32.0	1.358	1.133
278/11	H 0.27%Se	3.7138	3.7470	3.7457	337	324	1.040	0.961	36.7	1.236	1.189
279/11	H 0.81%Se	4.3425	4.3721	4.3714	301	294	1.022	0.979	39.6	1.194	1.167
280/11	H 0.01%Sb	2.5410	2.5816	2.5709	411	304	1.351	0.736	33.7	1.384	1.023
281/11	H 0.03%Sb	3.5572	3.5914	3.5826	347	259	1.340	0.742	38.2	1.328	0.990
282/11	H 0.09%Sb	3.6703	3.7102	3.6997	404	299	1.351	0.736	35.6	1.440	1.064
283/11	H 0.27%Sb	2.7081	2.7540	2.7419	464	343	1.351	0.736	30.0	1.400	1.040

Temperature: 75° C. $H = 8{,}500$ gauss Suspension correction = 0.0005

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-\chi_\perp \times 10^6$	$-\chi_\parallel \times 10^6$
249/11	H 0.01%Ag	3.1024	3.1441	3.1325	422	306	1.379	0.725	32.7	1.381	1.001
251/11	H 0.09%Ag	3.7356	3.7782	3.7672	431	321	1.342	0.745	32.1	1.381	1.030
253/11	H 0.81%Ag	3.2426	3.2822	3.2712	401	291	1.378	0.726	34.3	1.375	0.998
254/11	H 2.43%Ag	3.5958	3.6290	3.6200	337	247	1.364	0.733	40.2	1.358	0.995
256/11	H 0.01%Sn	2.2076	2.2470	2.2357	399	286	1.394	0.717	36.1	1.441	1.034
257/12	H 0.03%Sn	3.4070	3.4473	3.4357	408	292	1.398	0.715	35.2	1.438	1.029
258/11	H 0.09%Sn	3.1710	3.2114	3.1950	409	245	1.670	0.598	36.3	1.488	0.890
260/11	H 0.81%Sn	2.9020	2.9440	2.9204	425	189	2.245	0.445	33.8	1.437	0.639
261/11	H 2.43%Sn	3.4147	3.4573	3.4264	431	122	3.534	0.283	32.5	1.402	0.397
263/11	H 0.01%Pb	3.2795	3.3195	3.3085	405	295	1.372	0.728	34.1	1.383	1.007
264/11	H 0.03%Pb	3.4861	3.5251	3.5151	395	295	1.339	0.747	35.7	1.412	1.053
266/11	H 0.27%Pb	3.5700	3.6118	3.5963	423	268	1.581	0.629	35.1	1.484	0.940
267/11	H 0.81%Pb	2.8164	2.8638	2.8423	479	264	1.813	0.551	48.5	1.500	0.826
268/11	H 2.43%Pb	3.2926	3.3347	3.3122	426	201	2.120	0.466	35.8	1.520	0.717

Temperature: 75° C. $H = 8{,}500$ gauss. Suspension correction = 0.0005

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C\times10^{10}$	$-X_\perp\times10^6$	$-X_\parallel\times10^6$
270/11	H 0.01%Te	3.1387	1.1776	3.1709	394	327	1.205	0.829	33.8	1.330	1.103
271/12	H 0.03%Te	4.0361	4.0655	4.0616	299	260	1.149	0.870	42.4	1.268	1.102
272/11	H 0.09%Te	3.0770	3.1128	3.1143	363	378	0.974	1.026	32.7	1.184	1.233
273/11	H 0.27%Te	4.0648	4.0882	4.0950	239	307	0.778	1.290	39.9	0.954	1.225
275/11	H 0.01%Se	2.8714	2.9136	2.9052	427	343	1.242	0.801	32.1	1.372	1.102
276/11	H 0.03%Se	3.0003	3.0417	3.0338	419	340	1.230	0.810	32.7	1.371	1.112
277/11	H 0.09%Se	2.6213	2.6613	2.6556	405	348	1.163	0.858	32.0	1.298	1.114
278/11	H 0.27%Se	3.7257	3.7583	3.7568	331	316	1.048	0.953	36.7	1.214	1.160
279/11	H 0.81%Se	4.3506	4.3808	4.3796	307	295	1.040	0.960	39.6	1.219	1.170
280/11	H 0.01%Sb	2.5490	2.5906	2.5804	413	301	1.372	0.725	33.7	1.392	1.013
281/11	H 0.03$Sb	3.5657	3.6000	3.5916	348	264	1.319	0.755	38.2	1.332	1.011
282/11	H 0.09%Sb	3.6783	3.7172	3.7073	394	295	1.336	0.746	35.6	1.404	1.050
283/11	H 0.27%Sb	2.7133	2.7578	2.7466	450	338	1.332	0.748	30.0	1.358	1.020

Temperature: 101° C. $H = 8{,}500$ gauss Suspension correction = 0.0005

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$Cx10^{10}$	$-X_{\perp}x10^{6}$	$-X_{\parallel}x10^{6}$
249/11	H 0.01%Ag	3.1038	3.1430	3.1338	397	305	1.301	0.790	32.7	1.300	0.997
251/11	H 0.09%Ag	3.7375	3.7775	3.7685	405	315	1.285	0.778	32.1	1.299	1.010
253/11	H 0.81%Ag	3.2447	3.2817	3.2726	375	284	1.320	0.757	34.3	1.288	0.974
254/11	H 2.43%Ag	3.5970	3.6289	3.6206	324	241	1.343	0.744	40.2	1.305	0.970
256/11	H 0.01%Sn	2.2094	2.2463	2.2370	374	281	1.332	0.750	36.1	1.351	1.015
257/12	H 0.03%Sn	3.4094	3.4470	3.4370	381	281	1.354	0.738	35.2	1.341	0.990
258/11	H 0.09%Sn	3.1717	3.2095	3.1961	383	249	1.536	0.651	36.3	1.392	0.905
260/11	H 0.81%Sn	2.9040	2.9434	2.9247	399	212	1.890	0.532	33.8	1.348	0.716
261/11	H 2.43%Sn	3.4176	3.4590	3.4302	491	131	3.195	0.313	32.5	1.364	0.426
263/11	H 0.01%Pb	3.2826	3.3200	3.3110	379	289	1.310	0.764	34.1	1.293	0.986
264/11	H 0.03%Pb	3.4872	3.5258	3.5169	391	302	1.294	0.773	35.7	1.397	1.079
266/11	H 0.27%Pb	3.5717	3.6121	3.5996	409	284	1.440	0.690	35.1	1.435	0.996
267/11	H 0.81%Pb	2.8185	2.8627	2.8450	447	270	1.656	0.604	48.5	1.400	0.845
268/11	H 2.43%Pb	3.2936	3.3336	3.3142	405	211	1.920	0.515	35.8	1.445	0.753

Temperature: 101° C. $H = 8{,}500$ gauss Suspension correction $= 0.0005$

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$C\times10^{10}$	$-\chi_{\perp}\times10^{6}$	$-\chi_{\parallel}\times10^{6}$
270/11	H 0.01%Te	3.1417	3.1785	3.1724	373	312	1.196	0.835	33.8	1.259	1.052
271/12	H 0.03%Te	4.0376	4.0659	4.0631	288	260	1.109	0.902	42.4	1.221	1.102
272/11	H 0.09%Te	3.0792	3.1145	3.1145	358	358	1.000	1.000	32.7	1.169	1.169
273/11	H 0.27%Te	4.0668	4.0903	4.0970	240	307	0.776	1.285	39.9	0.958	1.225
275/11	H 0.01%Se	2.8735	2.9136	2.9061	406	331	1.228	0.813	32.1	1.304	1.063
276/11	H 0.03%Se	3.0026	3.0410	3.0346	389	320	1.197	0.833	32.7	1.273	1.063
277/11	H 0.09%Se	2.6236	2.6626	2.6568	395	337	1.171	0.852	32.0	1.264	1.080
278/11	H 0.27%Se	3.7281	3.7597	3.7582	321	306	1.050	0.952	36.7	1.178	1.123
279/11	H 0.81%Se	4.3537	4.3837	4.3820	305	288	1.060	0.941	39.6	1.210	1.142
280/11	H 0.01%Sb	2.5524	2.5906	2.5814	387	295	1.311	0.760	33.7	1.304	0.993
281/11	H 0.03%Sb	3.5673	3.6000	3.5926	332	258	1.287	0.774	38.2	1.269	0.986
282/11	H 0.09%Sb	3.6803	3.7172	3.7090	374	292	1.280	0.777	35.6	1.332	1.040
283/11	H 0.27%Sb	2.7159	2.7578	2.7478	424	324	1.309	0.760	30.0	1.279	0.978

Temperature: 159° C. $H = 8{,}500$ gauss. Suspension correction = 0.0005

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C\text{x}10^{10}$	$-\chi_\perp\text{x}10^6$	$-\chi_\parallel\text{x}10^6$
249/11	H 0.01%Ag	3.1053	3.1423	3.1344	375	296	1.267	0.790	32.7	1.228	0.968
251/11	H 0.09%Ag	2.7390	2.7763	2.7697	378	312	1.212	0.825	32.1	1.211	1.000
253/11	H 0.81%Ag	3.2460	3.2804	3.2733	349	278	1.256	0.796	34.3	1.198	0.953
254/11	H 2.43%Ag	3.5992	3.6274	3.6218	287	231	1.241	0.805	40.2	1.156	0.930
256/11	H 0.01%Sn	2.2112	2.2458	2.2377	351	270	1.300	0.769	36.1	1.268	0.975
257/11	H 0.03%Sn	3.4116	3.4460	3.4383	349	272	1.281	0.780	35.2	1.230	0.958
258/11	H 0.09%Sn	3.1732	3.2095	3.1997	368	260	1.416	0.706	36.3	1.338	0.944
260/11	H 0.81%Sn	2.9064	2.9429	2.9283	370	224	1.650	0.606	33.8	1.251	0.757
261/11	H 2.43%Sn	3.4192	3.4577	3.4336	390	149	2.619	0.382	32.5	1.270	0.484
263/11	H 0.01%Pb	3.2843	3.3198	3.3116	360	278	1.296	1.296	34.1	1.229	0.947
264/11	H 0.03%Pb	3.4904	3.5250	3.5166	351	277	1.268	0.789	35.7	1.254	0.989
266/11	H 0.27%Pb	3.5745	3.6096	3.6017	356	277	1.287	0.774	35.1	1.248	0.972
267/11	H 0.81%Pb	2.8216	2.8599	2.8480	388	269	1.442	0.693	48.5	1.217	0.842
268/11	H 2.43%Pb	3.2962	3.3304	3.3170	347	213	1.629	0.608	35.8	1.239	0.760

Temperature: 159° C. $H = 8,500$ gauss   Suspension correction = 0.0005

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-\chi_{\perp} \times 10^6$	$-\chi_{\parallel} \times 10^6$
270/11	H 0.01%Te	3.1440	3.1779	3.1734	344	299	1.151	0.868	33.8	1.160	1.008
271/12	H 0.03%Te	4.0394	4.0663	4.0636	274	247	1.133	0.882	42.4	1.161	1.048
272/11	H 0.09%Te	3.0812	3.1156	3.1149	349	342	1.020	0.980	32.7	1.139	1.116
273/11	H 0.27%Te	4.0694	4.0930	4.0975	241	286	0.843	1.190	39.9	0.961	1.141
275/11	H 0.01%Se	2.8754	2.9133	2.9066	384	317	1.211	0.823	32.1	1.234	1.019
276/11	H 0.03%Se	3.0044	3.0402	3.0349	363	310	1.170	0.852	32.7	1.188	1.014
277/11	H 0.09%Se	2.6263	2.6633	2.6575	375-	317	1.182	0.843	32.0	1.201	1.015
278/11	H 0.27%Se	3.7296	3.7608	3.7581	317	290	1.093	0.913	36.7	1.163	1.064
279/11	H 0.81%Se	4.3546	4.3849	4.3827	308	286	1.078	0.928	39.6	1.222	1.134
280/11	H 0.01%Sb	2.5549	2.5894	2.5829	350	285	1.228	0.810	33.7	1.179	0.958
281/11	H 0.03%Sb	3.5689	3.5997	3.5936	313	252	1.242	0.801	38.2	1.197	0.964
282/11	H 0.09%Sb	3.6813	3.7156	3.7086	348	278	1.252	0.796	35.6	1.240	0.990
283/11	H 0.27%Sb	2.7169	2.7562	2.7484	398	320	1.242	0.801	30.0	1.201	0.965

Temperature: 201° C.   H = 8,500 gauss   Suspension correction = 0.0005

Crystal	Kind	W_n	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$Cx10^{10}$	$-X_{\perp}x10^{6}$	$-X_{\parallel}x10^{6}$
249/11	H 0.01%Ag	3.1068	3.1404	3.1331	331	268	1.238	0.808	32.7	1.084	0.876
251/11	H 0.09%Ag	2.7403	2.7755	2.7688	357	290	1.251	0.798	32.1	1.145	0.930
253/11	H 0.81%Ag	3.2467	3.2788	3.2726	326	264	1.235	0.810	34.3	1.119	0.905
254/11	H 2.43%Ag	3.6006	3.6262	3.6216	261	215	1.214	0.823	40.2	1.050	0.865
256/11	H 0.01%Sn	2.2132	2.2449	2.2391	322	264	1.220	0.820	36.1	1.164	0.954
257/11	H 0.03%Sn	3.4128	3.4440	3.4392	317	269	1.178	0.848	35.2	1.116	0.930
258/11	H 0.09%Sn	3.1753	3.2081	3.1992	333	244	1.364	0.733	36.3	1.210	0.887
260/11	H 0.81%Sn	2.9078	2.9416	2.9302	343	219	1.566	0.638	33.8	1.160	0.740
261/11	H 2.43%Sn	3.4188	3.4552	3.4344	369	161	2.290	0.436	32.5	1.201	0.523
263/11	H 0.01%Pb	3.2868	3.3198	3.3126	335	263	1.273	0.785	34.1	1.143	0.896
264/11	H 0.03%Pb	3.4918	3.5245	3.5184	332	271	1.225	0.816	35.7	1.186	0.968
266/11	H 0.27%Pb	3.5769	3.6096	3.6022	332	258	1.288	0.774	35.1	1.163	0.905
267/11	H 0.81%Pb	2.8231	2.8596	2.8491	360	265	1.360	0.735	48.5	1.128	0.829
268/11	H 2.43%Pb	3.2982	3.3286	3.3182	309	205	1.508	0.658	35.8	1.103	0.732

Temperature: 201° C. $H = 8{,}500$ gauss. Suspension correction $= 0.0005$

Crystal	Kind	W_n	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C\text{x}10^{10}$	$-X_\perp\text{x}10^6$	$-X_\parallel\text{x}10^6$
270/11	H 0.01%Te	3.1456	3.1779	3.1731	328	280	1.172	0.853	33.8	1.107	0.945
271/12	H 0.03%Te	4.0412	4.0666	4.0636	259	229	1.131	0.884	42.4	1.098	0.970
272/11	H 0.09%Te	3.0828	3.1159	3.1132	336	309	1.088	0.919	32.7	1.098	1.008
273/11	H 0.27%Te	4.0711	4.0946	4.0974	240	268	0.895	1.120	39.9	0.957	1.070
275/11	H 0.01%Se	2.8763	2.0124	2.9063	366	300	1.200	0.830	32.1	1.177	0.979
276/11	H 0.03%Se	3.0057	3.0390	3.0345	338	293	1.153	0.865	32.7	1.107	0.958
277/11	H 0.09%Se	2.6272	2.6622	2.6570	355	303	1.171	0.852	32.0	1.137	0.970
278/11	H 0.27%Se	3.7304	3.7598	3.7577	299	278	1.075	0.930	36.7	1.097	1.020
279/11	H 0.81%Se	4.3563	4.3839	4.3817	281	259	1.084	0.919	39.6	1.115	1.028
280/11	H 0.01%Sb	2.5560	2.5878	2.5819	323	264	1.223	0.815	33.7	1.089	0.889
281/11	H 0.03%Sb	3.5701	3.5986	3.5933	290	237	1.223	0.814	38.2	1.109	0.906
282/11	H 0.09%Sb	3.6832	3.7149	3.7087	322	260	1.240	0.805	35.6	1.148	0.925
283/11	H 0.27%Sb	2.7180	2.7527	2.7471	352	296	1.189	0.839	30.0	1.062	0.894

Temperature: 245° C. $H = 8,500$ gauss Suspension correction $= 0.0005$

Crystal	Kind	W_{n}	$W_{\perp}$	$W_{\parallel}$	$F_{\perp}$	$F_{\parallel}$	R	$\frac{1}{R}$	$C\times10^{10}$	$-X_{\perp}\times10^{6}$	$-X_{\parallel}\times10^{6}$
249/11	H 0.01%Ag	3.1070	3.1358	3.1316	293	251	1.169	0.855	32.7	0.958	0.820
251/11	H 0.09%Ag	2.7403	2.7728	2.7670	330	272	1.213	0.824	32.1	1.058	0.872
253/11	H 0.81%Ag	3.2469	3.2750	3.2712	286	248	1.153	0.866	34.3	0.981	0.850
254/11	H 2.43%Ag	3.6006	3.6241	3.6211	240	210	1.142	0.875	40.2	0.968	0.845
256/11	H 0.01%Sn	2.2130	2.2414	2.2372	289	247	1.170	0.855	36.1	1.044	0.892
257/12	H 0.03%Sn	3.4138	3.4411	3.4378	278	245	1.135	0.880	35.2	0.979	0.863
258/11	H 0.09%Sn	3.1763	3.2062	3.1991	304	233	1.304	o.766	36.3	1.105	0.847
260/11	H 0.81%Sn	2.9086	2.9386	2.9288	305	207	1.473	0.679	33.8	1.030	0.700
261/11	H 2.43%Sn	3.4201	3.4499	3.4356	303	160	1.893	0.528	32.5	0.985	0.520
263/11	H 0.01%Pb	3.2875	3.3176	3.3122	306	252	1.215	0.823	34.1	1.044	0.859
264/11	H 0.03%Pb	3.4934	3.5215	3.5170	286	241	1.188	0.841	35.7	1.022	0.860
266/11	H 0.27%Pb	3.5769	3.6065	3.6010	301	246	1.223	0.813	35.1	1.057	0.863
267/11	H 0.81%Pb	2.8236	2.8552	2.8476	321	245	1.310	0.763	48.5	1.005	0.766
268/11	H 2.43%Pb	3.2895	3.3144	3.3072	254	182	1.397	0.710	35.8	0.906	0.650

Temperature: 245° C.   H = 8,500 gauss   Suspension correction = 0.0005

Crystal	Kind	W_H	$W_\perp$	$W_\parallel$	$F_\perp$	$F_\parallel$	R	$\frac{1}{R}$	$C \times 10^{10}$	$-\chi_\perp \times 10^6$	$-\chi_\parallel \times 10^6$
270/11	H 0.01%Te	3.1456	3.1752	3.1714	301	263	1.145	0.873	33.8	1.014	0.886
271/12	H 0.03%Te	4.0427	4.0656	4.0614	234	192	1.220	0.819	42.4	0.992	0.814
272/11	H 0.09%Te	3.0829	3.1142	3.1112	318	288	1.104	0.905	32.7	1.038	0.940
273/11	H 0.27%Te	4.0722	4.0950	4.0949	233	232	1.003	0.995	39.9	0.929	0.925
275/11	H 0.01%Se	2.8770	2.9103	2.9058	338	293	1.153	0.865	32.1	1.087	0.940
276/11	H 0.03%Se	3.0064	3.0370	3.0330	311	271	1.148	0.870	32.7	1.018	0.886
277/11	H 0.09%Se	2.6282	2.6587	2.6560	310	283	1.096	0.910	32.0	0.992	0.905
278/11	H 0.27%Se	3.7301	3.7552	3.7542	256	246	1.041	0.961	36.7	0.940	0.902
279/11	H 0.81%Se	4.3569	4.3813	4.3789	249	225	1.106	0.900	39.6	0.987	0.892
280/11	H 0.01%Sb	2.5569	2.5861	2.5810	297	246	1.208	0.825	33.7	0.998	0.828
281/11	H 0.03%Sb	3.5711	3.5960	3.5905	254	199	1.277	0.779	38.2	0.972	0.761
282/11	H 0.09%Sb	3.6825	3.7122	3.7076	302	256	1.180	0.845	35.6	1.076	0.911
283/11	H 0.27%Sb	2.7174	2.7484	2.7441	315	272	1.159	0.861	30.0	0.951	0.820

PRELIMINARY EXPERIMENTS

I. DEPENDENCE OF SUSCEPTIBILITY ON FIELD STRENGTH

Before the investigation could be carried out by using the above method, it was first necessary to test the variation of the susceptibility of the bismuth crystals, as a function of the magnetic field strength. This was done, and the result is given in Figure 12. In this case the field strength was varied from 20,151 gauss to 2,805 gauss. It is seen at once that the susceptibility was independent of the field strength in this region, and so no errors have been introduced from this source.

II. VARIATION OF SUSCEPTIBILITY IN THE PLANE NORMAL TO THE PRINCIPLE AXIS.

It was necessary to investigate the variations of the susceptibility in the plane normal to the principle axis in order to show that all the magnetic properties of a single crystal of bismuth can be obtained by measuring only those that exist in any plane containing the principle axis. The result of this test is shown in Figure 13, and it is seen that the crystal was isotropic with respect to all directions normal to the principle axis, thus verifying the assumption that such should be the case.

III. DETERMINATION OF THE PRINCIPLE SUSCEPTIBILITIES OF BISMUTH

The next step necessary before an investigation could be made of the effects of impurities was the measurement of the principle susceptibilities of the pure bismuth itself. The bismuth used in this set

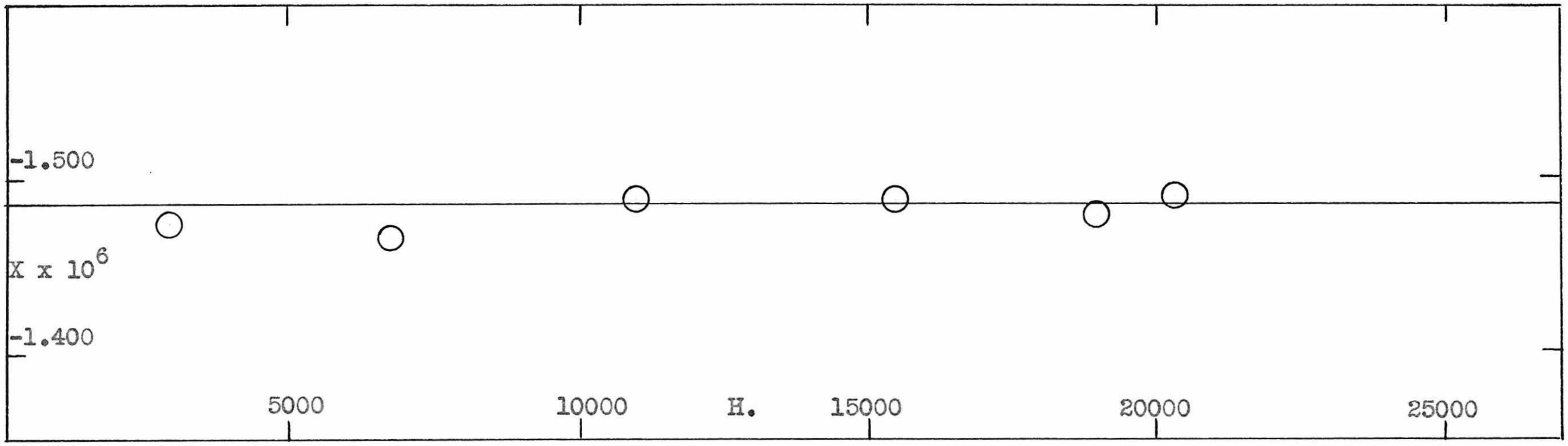


Figure 12. Dependence of susceptibility on field strength.

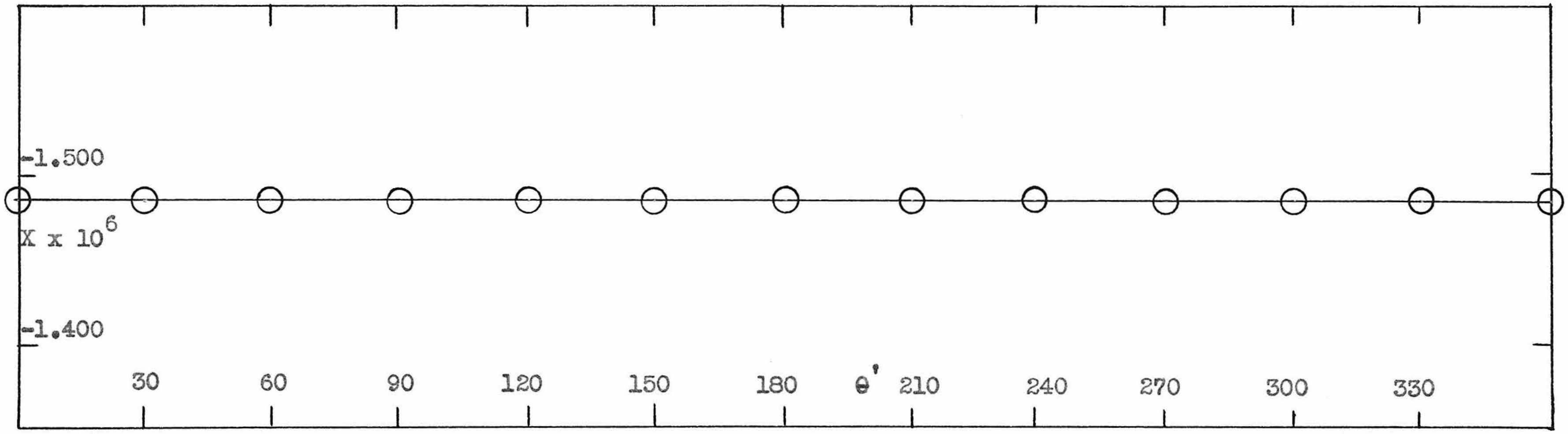


Figure 13. Variation of susceptibility in the plane perpendicular to the main axis.

of measurements was originally of the E type, and was subsequently distilled in an effort to remove some of the impurities. Spectroscopic tests showed, however, that very little improvement had been effected and the composition was still substantially the same as before. Measurements were made on five different crystals, every fifteen degrees, throughout a complete revolution about the axis of the rod. In addition to making an accurate determination of the principle susceptibilities, this allowed a further test of the theory of the method in regard to the $\cos^2$ relationship that should hold for intermediate positions. Figure 14 is a composite diagram on which all the data for the five crystals, as well as points calculated by the $\cos^2 \theta$ Law, have been plotted. The curve drawn is the theoretical one, and the agreement of experiment with theory is seen from the excellent correspondence of this curve with the experimental points.

The values of the principle susceptibilities as given by these measurements are

$$X_{\perp} = -1.487 \times 10^{-6}, \quad X_{\parallel} = -1.046 \times 10^{-6}, \quad R = 1.425$$

From these the value of the susceptibility to be expected of a polycrystalline aggregate is given by

$$\frac{2\,X_{\perp} + X_{\parallel}}{3} = -1.340 \times 10^{-6}$$

which is in very good agreement with the values given in the International Critical Tables.

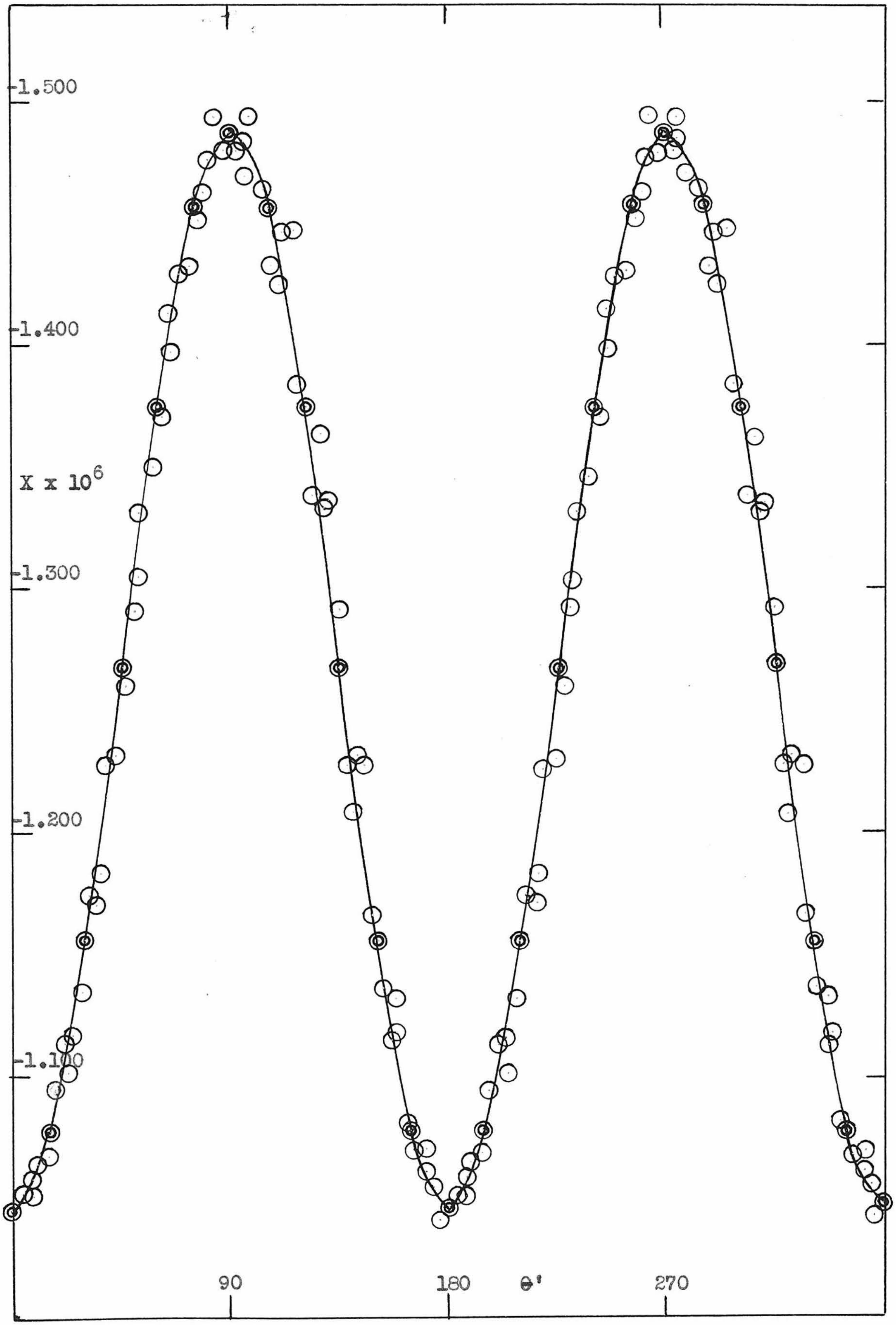


Figure 14. Variation of susceptibility in the plane of the main axis.

IV. DETERMINATION OF THE MAGNETIC RATIO $\chi_{\perp}/\chi_{\parallel}$ FOR ALL THE DIFFERENT TYPES OF "PURE" BISMUTH.

It is interesting to notice the wide variations in the magnetic properties of the various kinds of "pure" bismuth used in the earlier work done in this Laboratory. The ratio of the principle susceptibilities was the only quantity measured since this permits a direct comparison of the different samples without the introduction of errors due to the measurement of the cross-section and of the field strength. The results found for all the "pure" bismuths are given in Table 2.

A	1.499	D	1.460	G	1.430
B	1.597	E	1.430	H	1.435
C	1.440	F	1.410	E-Distilled	1.423

Table 2

These variations may well be one of the contributing causes of the great differences observed by Goetz and Hasler[22)] in their work on crystals grown in a magnetic field. From the above table, it would be expected that bismuth B would be most affected by growth in a magnetic field, due to the larger torque acting on it. This is exactly the result found.

V. DEPENDENCE OF THE DISTRIBUTION OF IMPURITY ON THE SPEED OF GROWTH OF THE CRYSTALS.

It is a well known fact that it is generally possible to purify a substance by crystallization. This effect is always amplified by causing the crystallization to proceed slowly, and this process was the method used by Schubnikow and de Haas[29] for producing their purest bismuth. It was therefore considered necessary to determine the extent to which this separation of pure bismuth from the alloys would affect the magnetic measurements. To make this test, several samples of bismuth containing different amounts of lead were grown into single crystals. Two rods of each alloy were made as nearly alike as possible. One of these was then grown at a speed of six millimeters per minute, while the other was grown at a speed of one millimeter per minute. The magnetic anisotropy was then measured at each end of each crystal. These measurements showed that the crystal grown at the more rapid speed was of uniform magnetic properties, while the other crystal's properties varied to a great extent. Table 3 gives a series of such crystals. The crystals grown at normal, or fast, speed are indicated by the letter n.

			R_1	R_2
188/11n	G	3.00% Pb	2.619	2.625
188/21s	G	3.00% Pb	2.130	2.764
166/12n	G	0.15% Pb	1.630	1.627
166/22s	G	0.15% Pb	1.605	1.751
192/11n	G		1.430	1.428
192/21s	G		1.438	1.429

Table 3.

R_1 is the magnetic anisotropy of the beginning of the crystal.

R_2 " " " " " " end " " "

From this series of measurements it was seen that if the crystals were grown rapidly there would be no resultant separation of metals to cause irregularity in the magnetic properties in one and the same crystal.

DISCUSSION OF RESULTS

The results of the first silver series are given in Figure 15. These show a definite variation of the magnetic anisotropy which is too large to be explained by experimental error. These variations are, however, irregular, and although reproducible in the same crystal, different crystals of the same composition do not agree with each other. A possible explanation of the variations will be given after the discussion of the tin and lead alloys.

The results for the series of copper alloys showed no changes in the magnetic anisotropy, and consequently are of little interest. This was to be expected from a consideration of the phase equilibrium diagram which shows there is no eutectic and no solid solution. Therefore regardless of the amount of copper in the molten alloy the bismuth will not crystallize out of the melt until all the copper is solid. Consequently the copper is unable to disturb the bismuth lattice in any way. This metal was therefore not considered throughout the rest of the investigation.

Figure 16 shows the result of the first series of lead alloys. These are in striking contrast with those obtained from silver and copper. Here, instead of finding little or no effect, we find a surprisingly large one. The most remarkable aspect of the curve is the tremendous effect of small amounts of lead, and the rapid falling off of the magnitude of the effectiveness of each additional atom.

This striking action of lead on the magnetic properties of bismuth led to the investigation of the behavior of tin as an impurity. This was a consequence of the similarity in action of tin and lead in

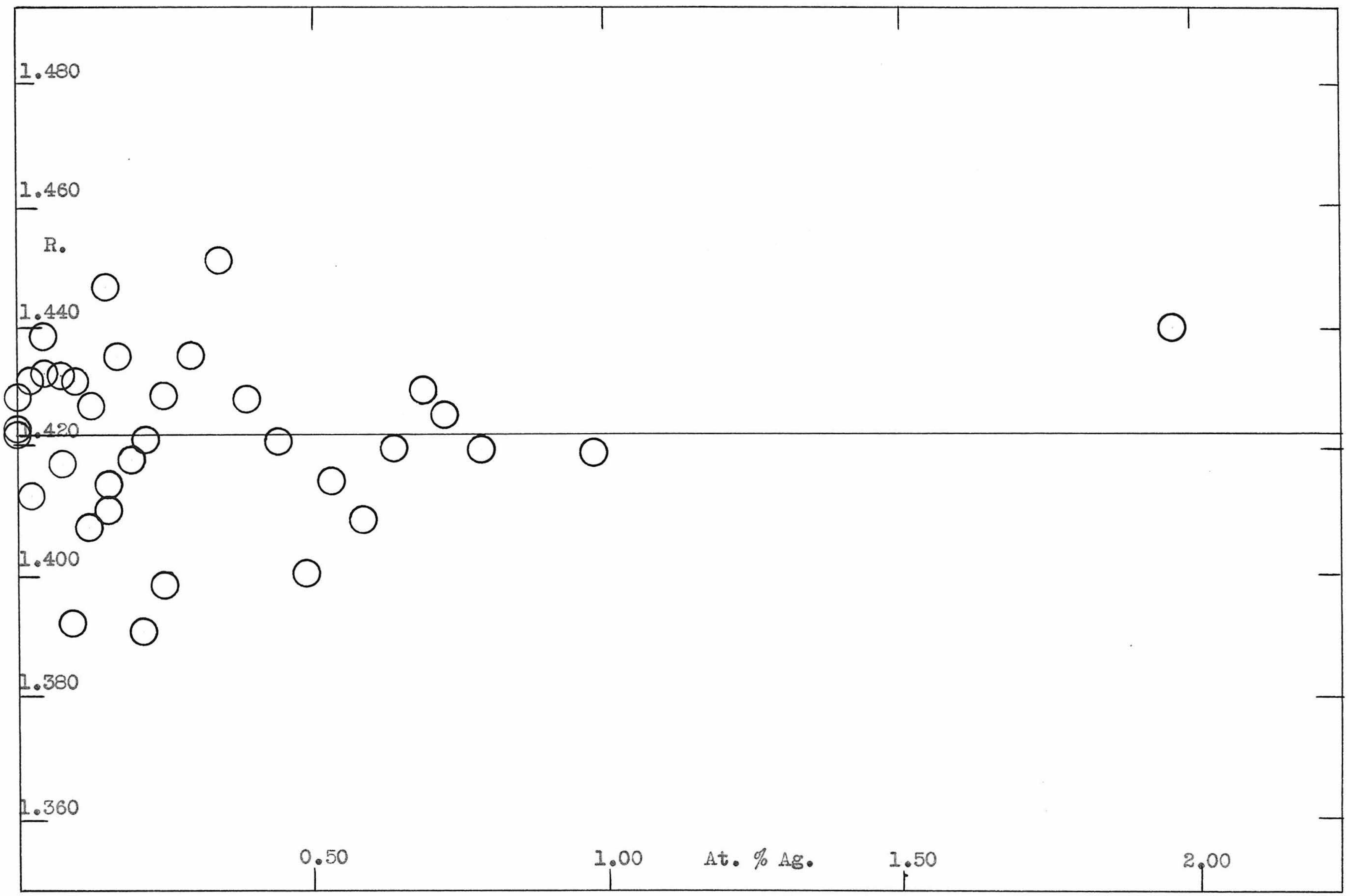


Figure 15. Silver series. Dependence of anisotropy on amount of impurity.

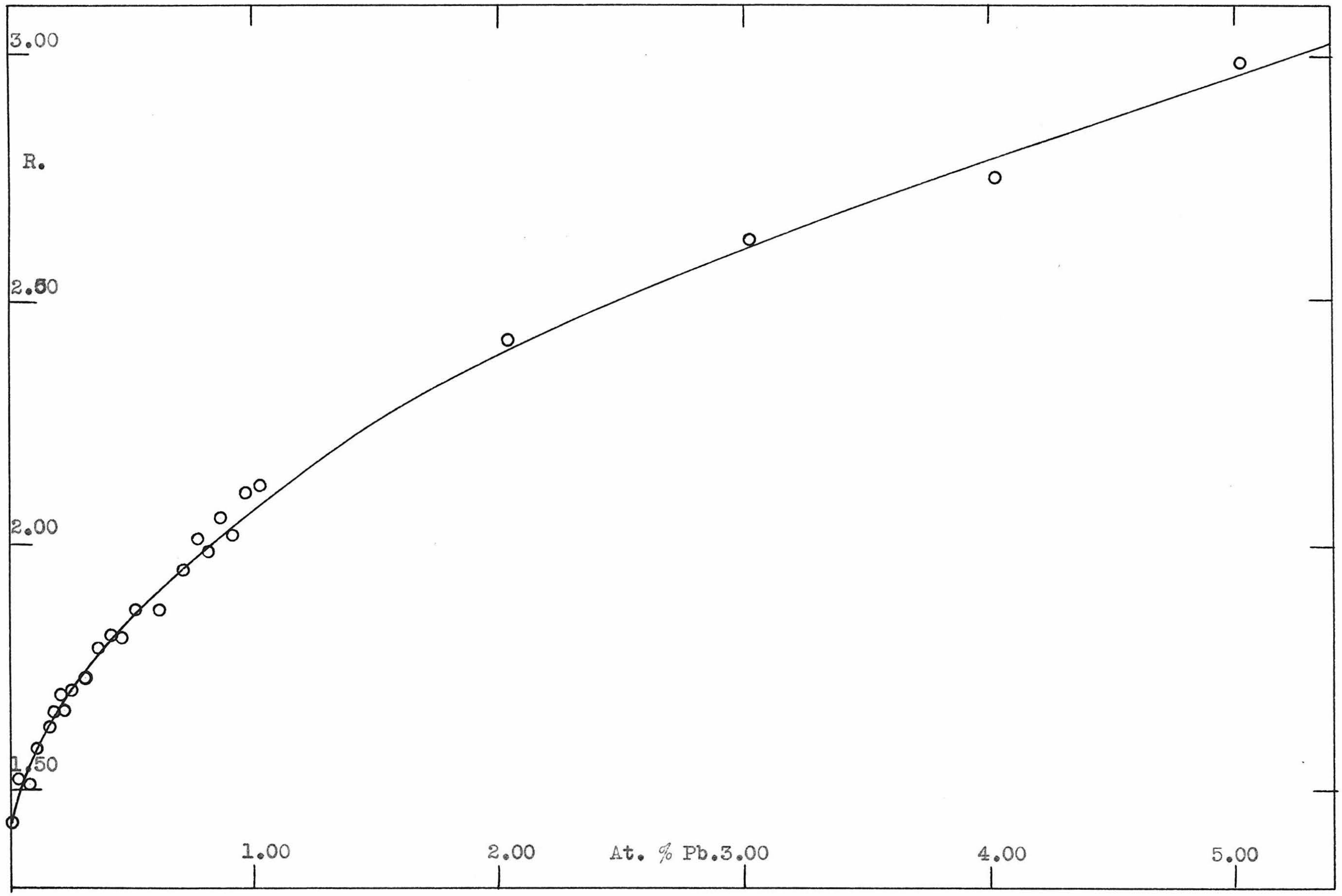


Figure 16. Lead series. Dependence of anisotropy on amount of impurity.

the alloys investigated magnetically by Endo.25) The first crystal investigated contained 5% tin. The anisotropy of this was so large at room temperature that it was decided to investigate its variation as a function of temperature. The solid line in Figure 17 shows the result of this investigation. This curve indicates that the anisotropy becomes infinite at some temperature in the neighborhood of 0°C. The shape of the curve suggests that it would be interesting to plot the inverse ratio $X_{\parallel}/X_{\perp}$ as a function of temperature instead of the anisotropy itself. The result of such a plot is shown in Figure 17, by the broken line. As is seen at once, this proves to be a continuous curve indicating the desirability of using the inverse ratio for plotting purposes. This has been used in all subsequent diagrams.

The results of the first tin series are plotted in Figure 18, with the revised curve for lead. This diagram shows that tin and lead behave in a similar fashion. The only difference is that each tin atom is about five times as effective as each lead atom. Since lead and tin are in the same column in the Periodic Table of Elements, the question arose as to the action of germanium. Through the courtesy of Professor Dennis of Cornell in supplying the necessary germanium, two crystals were grown with germanium as an impurity. These showed that the action of this metal was approximately that of lead, thus leaving tin as the most effective metal of the three.

The investigation was extended further to the metals antimony, tellurium, and selenium. Antimony was chosen because of its similarity to bismuth. Tellurium was used since Endo's25) results pointed out that it should have an effect comparable to that of lead and tin. Selenium was added due to its location in the Periodic Table of Elements. In

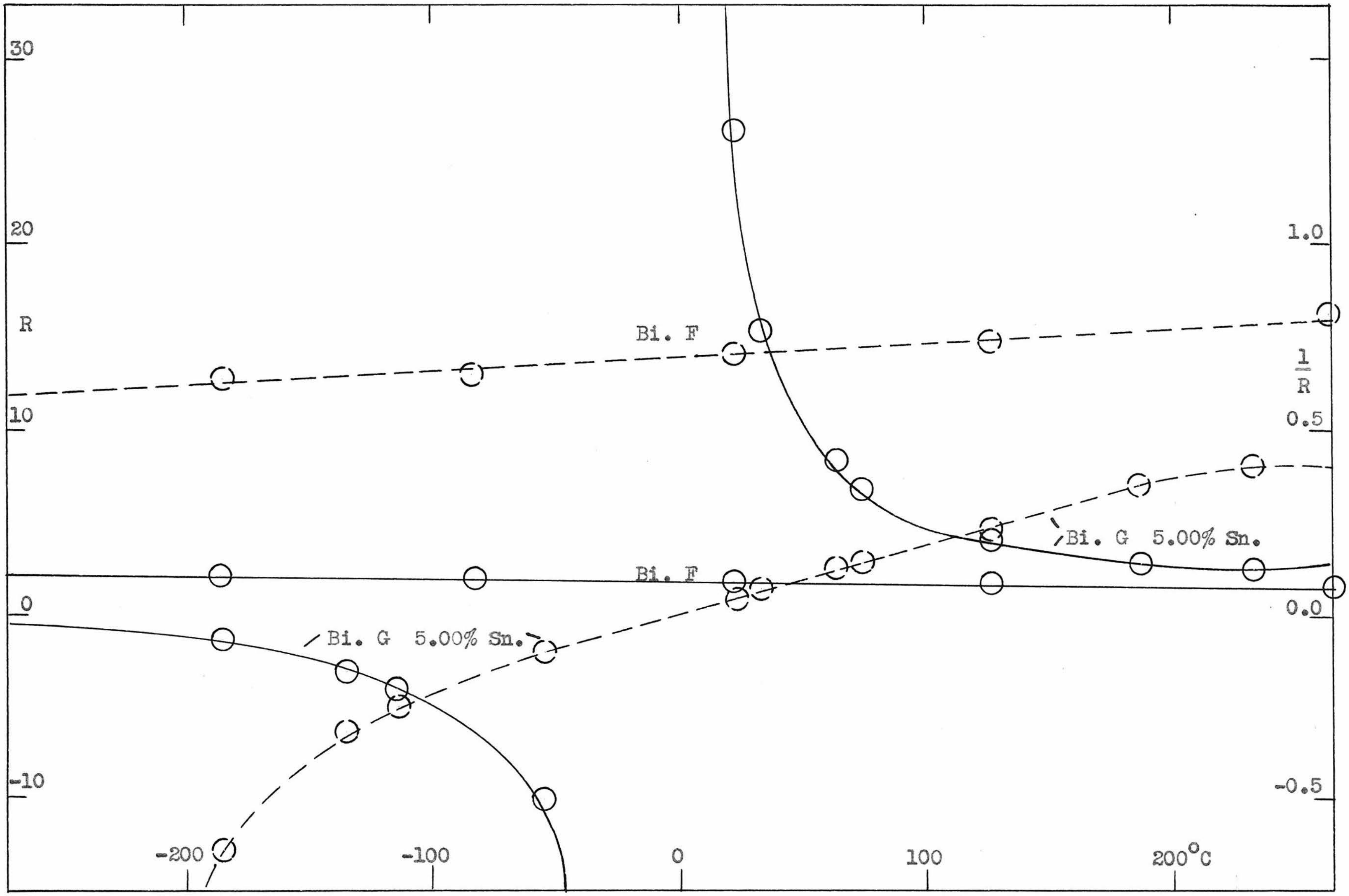


Figure 17. Dependence of anisotropy on temperature.

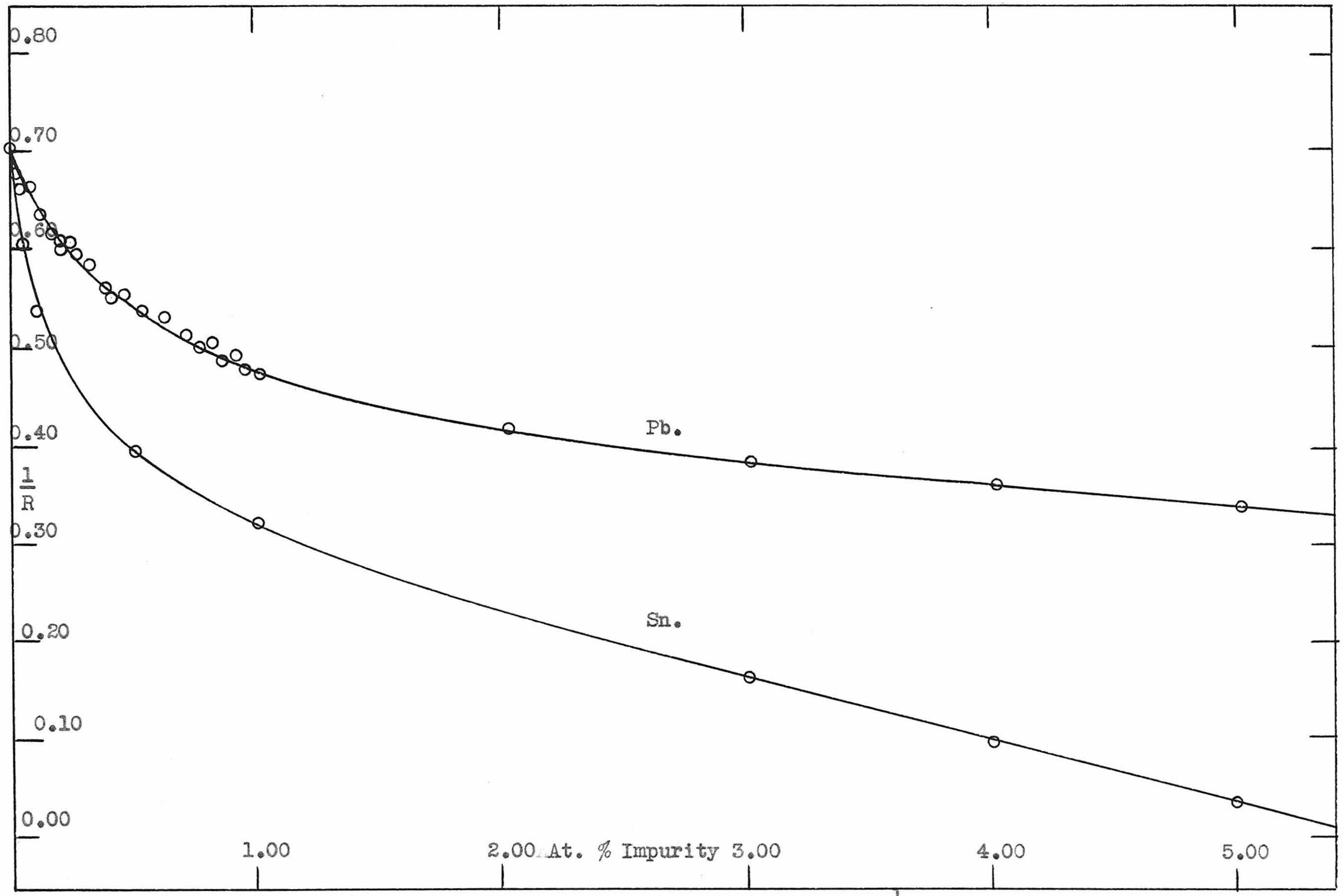


Figure 18. Tin and Lead series. Variation of $\frac{1}{R}$ with amount of impurity.

view of the success in obtaining crystals containing selenium it becomes necessary to change the phase equilibrium diagram for bismuth-selenium alloys from that given by the full lines in Figure 6 to one of the type shown by the broken lines.

A complete set of measurements were then made on all these series of alloys for the determination of the absolute values of the susceptibilities as a function of the amount of impurity. These measurements included the dependence on temperature for temperatures ranging from -185° C to 245° C.

Figure 19 gives the inverse ratio of susceptibilities as a function of the amount of impurity for all the alloys at -185° C and 22° C. The results of the antimony series show that in the case of a true replacement solid solution the properties of bismuth are but little changed by the addition of small amounts of the solute metal. The curves for tin and lead at liquid air temperature, -185° C, show a striking effect. Due to the similarity between the two curves, a discussion of the case of tin will serve for both.

The curve is seen to consist of two straight lines, the intersection occurring at 0.025% tin. This concentration corresponds to one tin atom in 4,000 bismuth atoms. The shape of the curve shows that if there is less than one atom of tin present for 4,000 bismuth atoms, in a crystal, each additional tin atom produces the same effect on the susceptibility ratio as each atom already present. However, if there is already present one tin atom per 4,000 bismuth atoms, and one more tin atom is added, this last atom cannot produce the same effect as the first group. Such a situation is difficult to explain if the distribution of the tin atoms is considered as statistical. For this to be the case, one of two mechanisms for destroying the susceptibility of

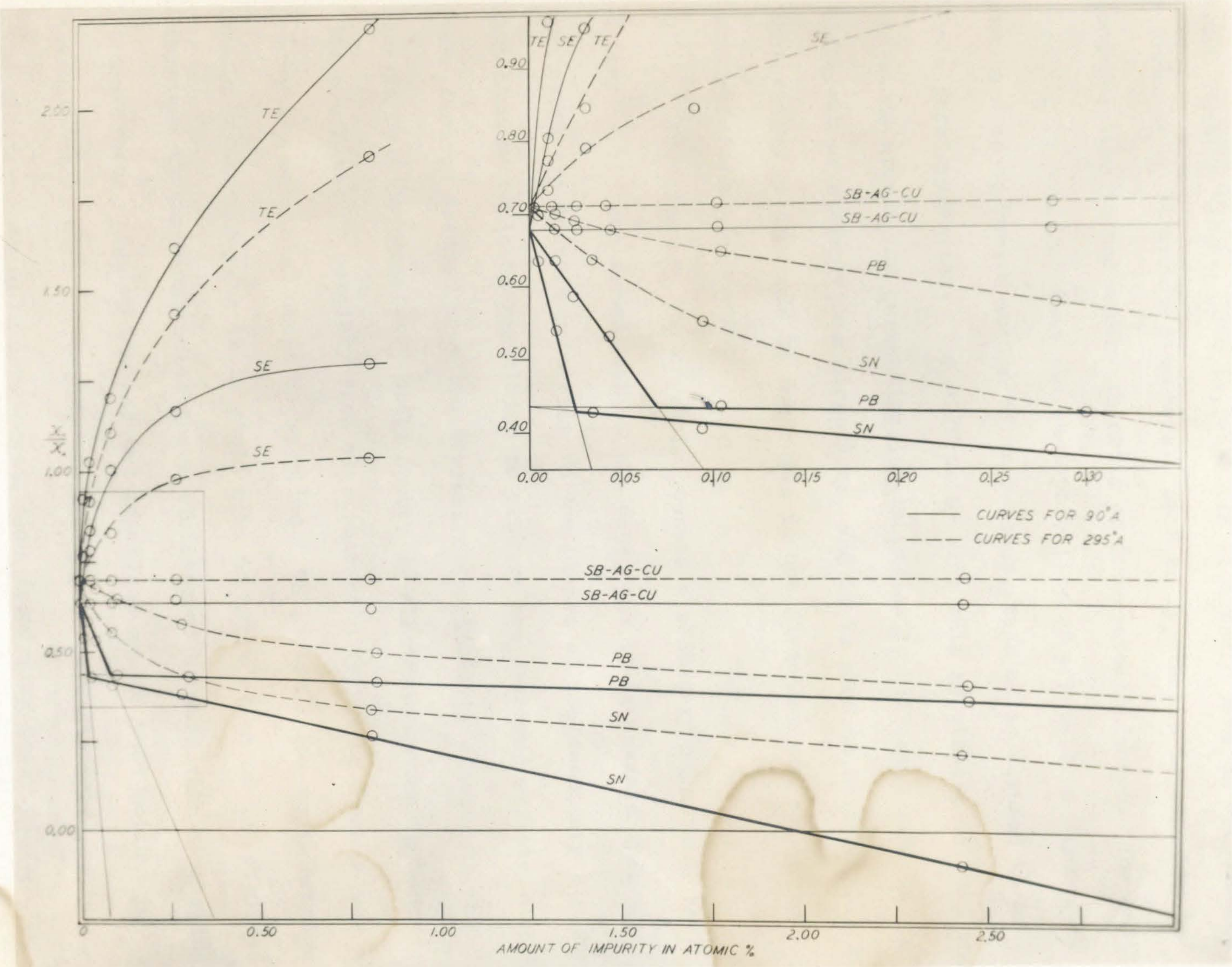


Figure 19. All impurities. Variation of $\frac{1}{R}$ with amount of impurity.

the crystal must be assumed. The first of these is that the sphere of influence of the tin atom has such a large volume that it contains 4,000 bismuth atoms. That is, each tin atom would have to act through a distance of 16 atomic diameters. Such an hypothesis is directly in contradiction to the modern theories of inter-atomic forces. The second possibility would be that the susceptibility of crystalline bismuth is due to a co-operative action of many atoms, such as is assumed to explain ferro-megnetism. The difficulty with this hypothesis is that the tin atoms affect the susceptibility in radically different ways in the different crystalographic directions, and it is hard to believe that a cooperative group could be destroyed in one direction and at the same time reenforced in another.

If, however, the crystals possess a secondary structure, as Zwicky30) 31) has shown must be the case, and it is assumed that the tin atoms are located primarily in the planes of the secondary structure a mechamism could be postulated to explain the observed effects. If it were further assumed that the crystal susceptibility of bismuth was caused by a comparatively small number of electrons moving in very large orbits, or possibly free electrons with very long free paths, the necessary mechamism would be supplied. This particular consideration would be advantageous as it would give an explanation of the observed increase in the diamagnetic susceptibility of bismuth with decreasing temperatures. Due to the reduced magnitude of the heat motions of the bismuth atoms with decreasing temperatures, the undisturbed paths of the electrons would be lengthened, thus increasing their diamagnetic effect.

A rough determination of the number of electrons which must be assumed to possess such large paths may be computed from Schrodinger's7) formula

$$k = -\frac{1}{3}\frac{N e^2 \lambda^2}{m c^2}$$

Substituting the value of the susceptibility of bismuth for k, and giving λ the value of 10^{-4} cm., it is possible to solve for N.

$$N = -\frac{3\ k\ m\ c^2}{e^2 \lambda^2}$$

$$N = \frac{3 \times 1.32 \times 10^{-5} \times 9 \times 10^{-28} \times 9 \times 10^{20}}{2.28 \times 10^{-19} \times 10^{-8}}$$

$N = 1.41 \times 10^{16}$ electrons per cubic centimeter

But there are 2.85×10^{22} bismuth atoms per cubic centimeter, so this N represents one electron for each 2×10^6 bismuth atoms.

The assumption of such a cause for crystal diamagnetism would fully explain the results obtained in regard to the variation of susceptibility with particle size, for as the particle size approximated the dimensions of 1μ, the diamagnetism would of necessity be reduced. The effect of cold working would likewise be explained due to the altered conditions of the secondary structure planes which might easily produce potential barriers, thus breaking up the long electron paths.

If the spacing of the secondary structure planes is that measured by Goetz19), it is seen that the postulated paths may also be destroyed by the introduction of foreign atoms into these planes and in this way causing them to become potential barriers.

The final assumption that is needed in dealing with the movements of the electrons is that the mechanism of collision with the potential walls is that of adsorption and reemission. The effect of this is best seen if the motions of the electrons are separated into components parallel and perpendicular to the wall. In this way it is seen that the component perpendicular to the wall is the only one that is affected by the mechanism of collision. The result of the assumed wall effect is that the number of electrons moving parallel to the wall is increased at the expense of those moving perpendicular to it.

Thus if the foreign atoms are supposed to be located in one set of the secondary planes only, it would be expected that the susceptibility in a direction parallel to the given planes should decrease, while that perpendicular to them would increase, as the amount of impurity was increased. Figure 20) shows that in the region from zero to 0.025% tin, the susceptibility parallel to the principle axis decreases and the susceptibility in a direction perpendicular to this axis increases. The experiment will be in agreement with the assumptions if the tin is considered as located in planes which are much more nearly parallel to the principle axis than perpendicular to it. One such possible set of planes is the $(II\bar{I})$ type. A stereographic diagram of bismuth, Figure 21, shows that the angle between the $(11\bar{1})$ planes and the (111) plane is 72°. The (111) plane is perpendicular to the principle axis.

In order to explain the falling off of the susceptibility in all directions at percentages greater than 0.025%, it is to be assumed that after this concentration has been reached, it becomes possible for the tin to enter the (111) planes also, thus breaking up the electron paths in all directions.

A very similar idea will explain the effect of lead, the only change being the shifting of the critical amount of impurity from 0,025% to 0.125%, as is seen from Figure 22.

In the cases of tellurium and selenium, see Figures 23 and 24,

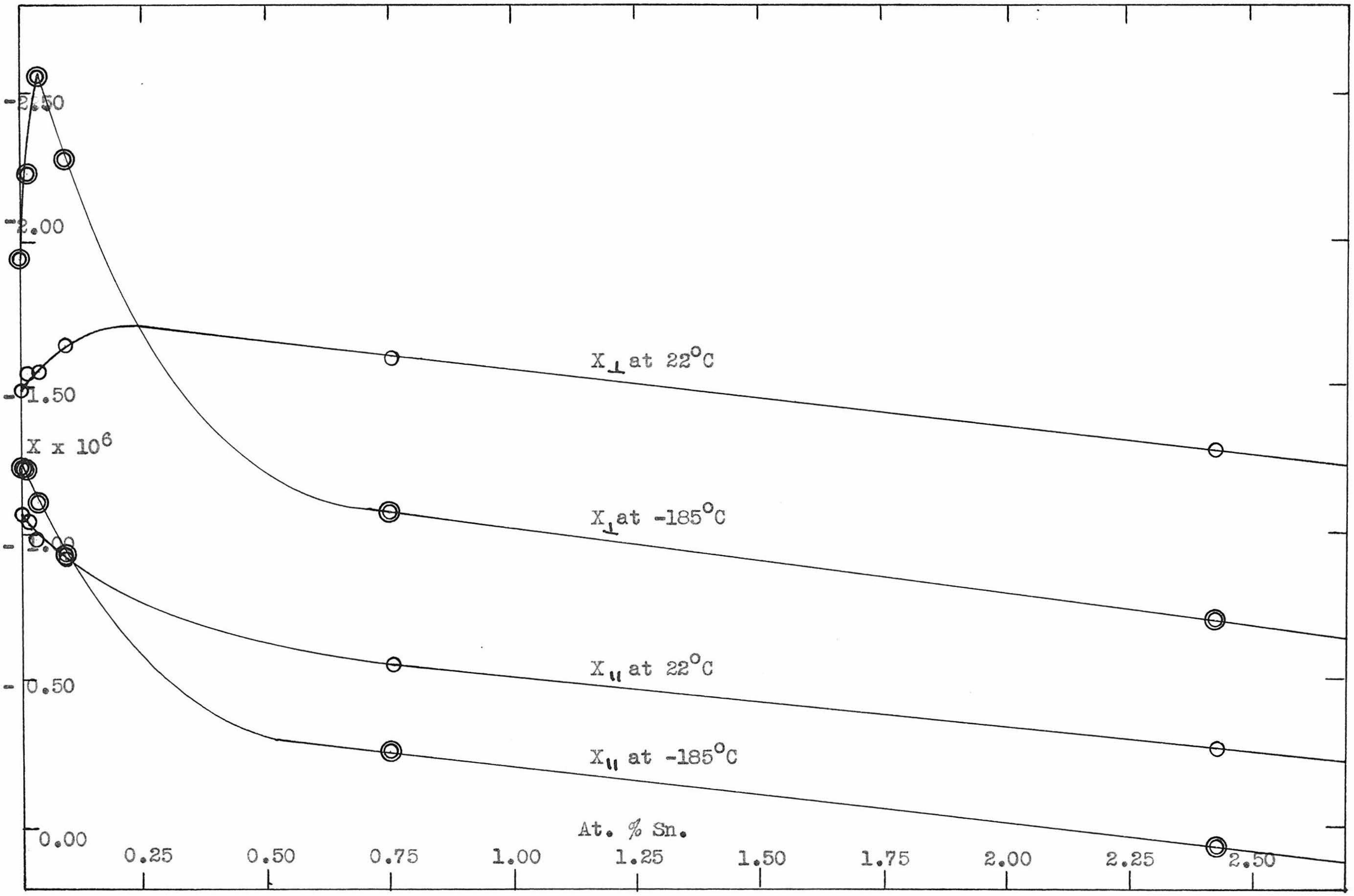
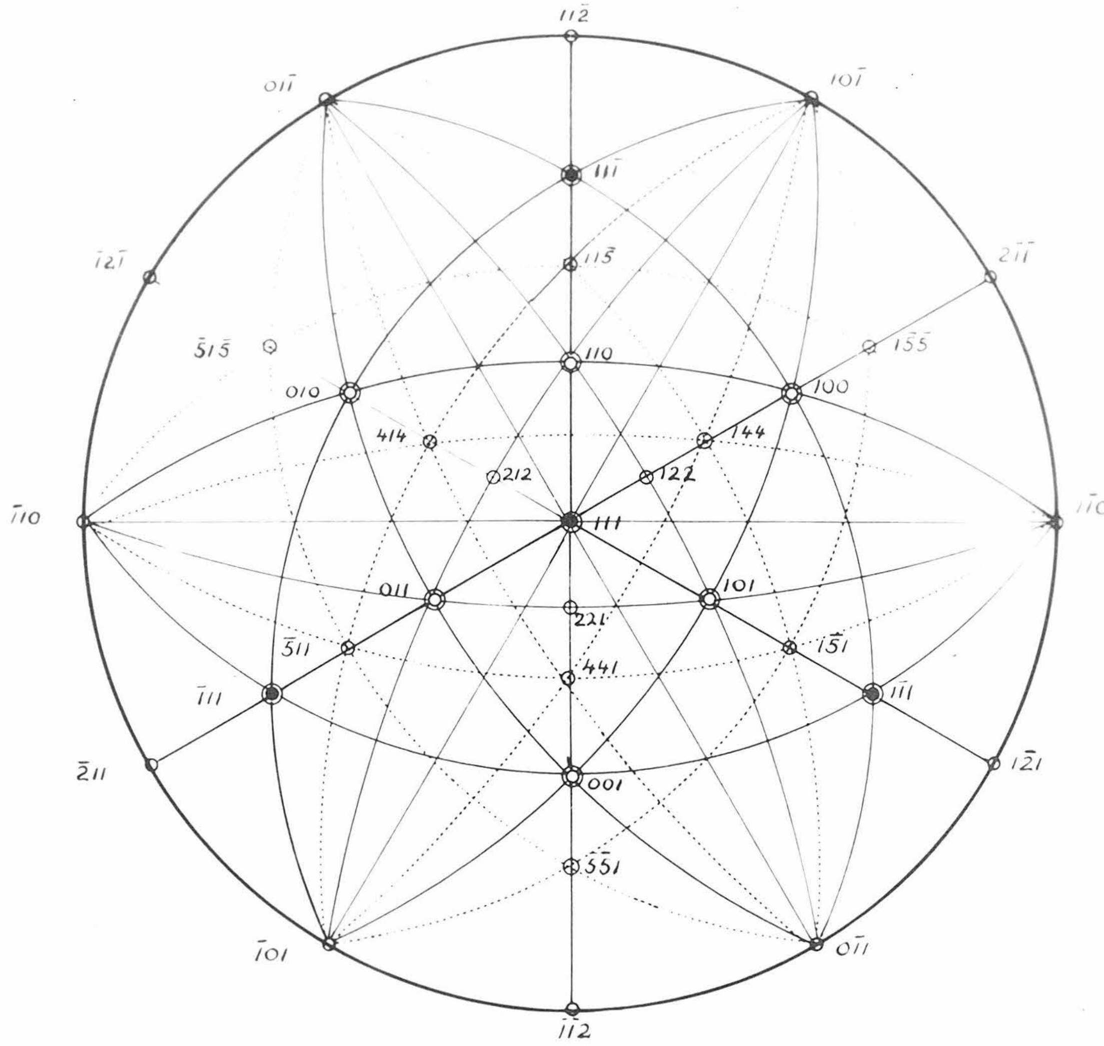


Figure 20. Tin. Variation of susceptibility with amount of impurity.



Bismuth. P_3.

Figure 21. Stereographic diagram of bismuth.

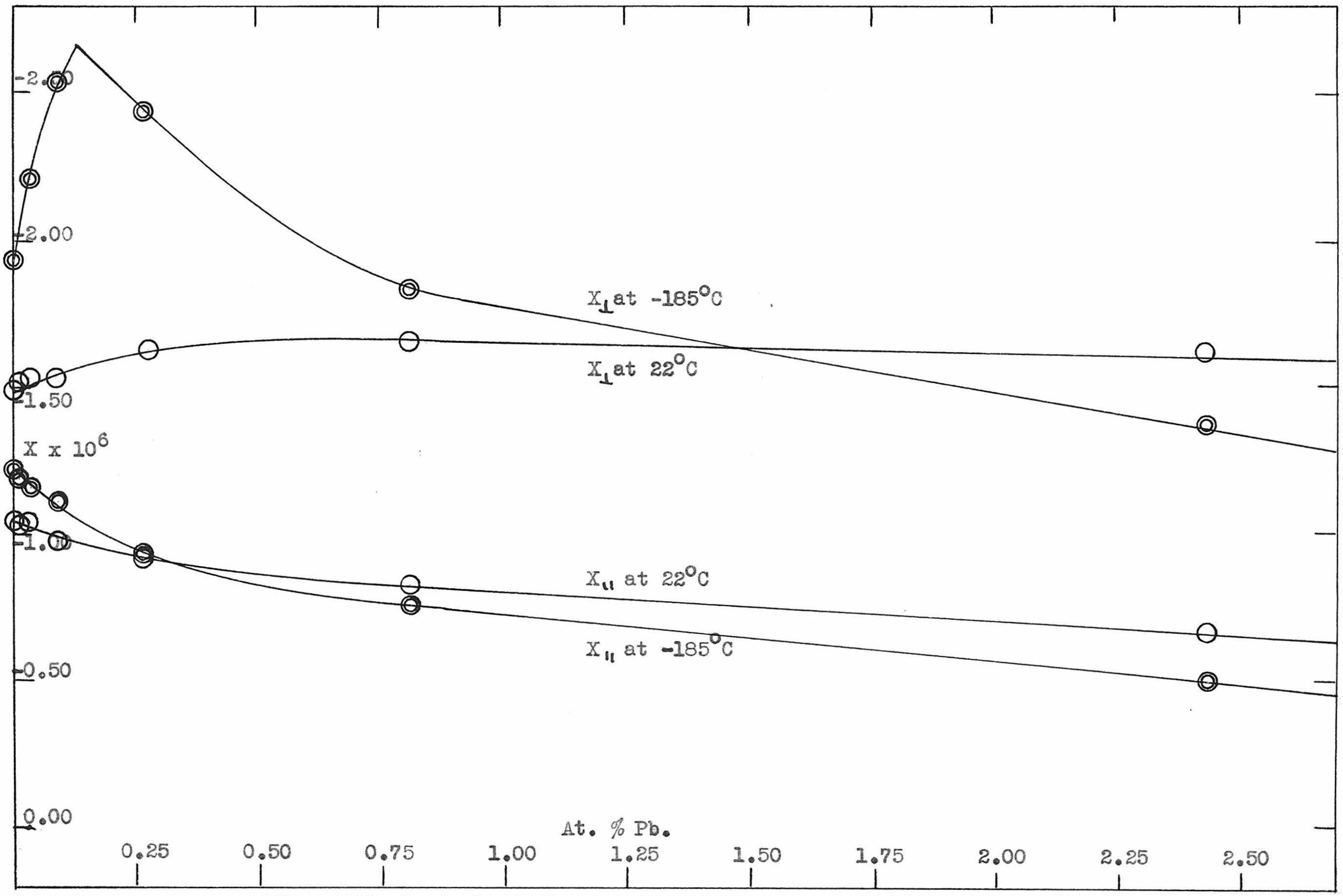


Figure 22. Lead. Variation of susceptibility with amount of impurity.

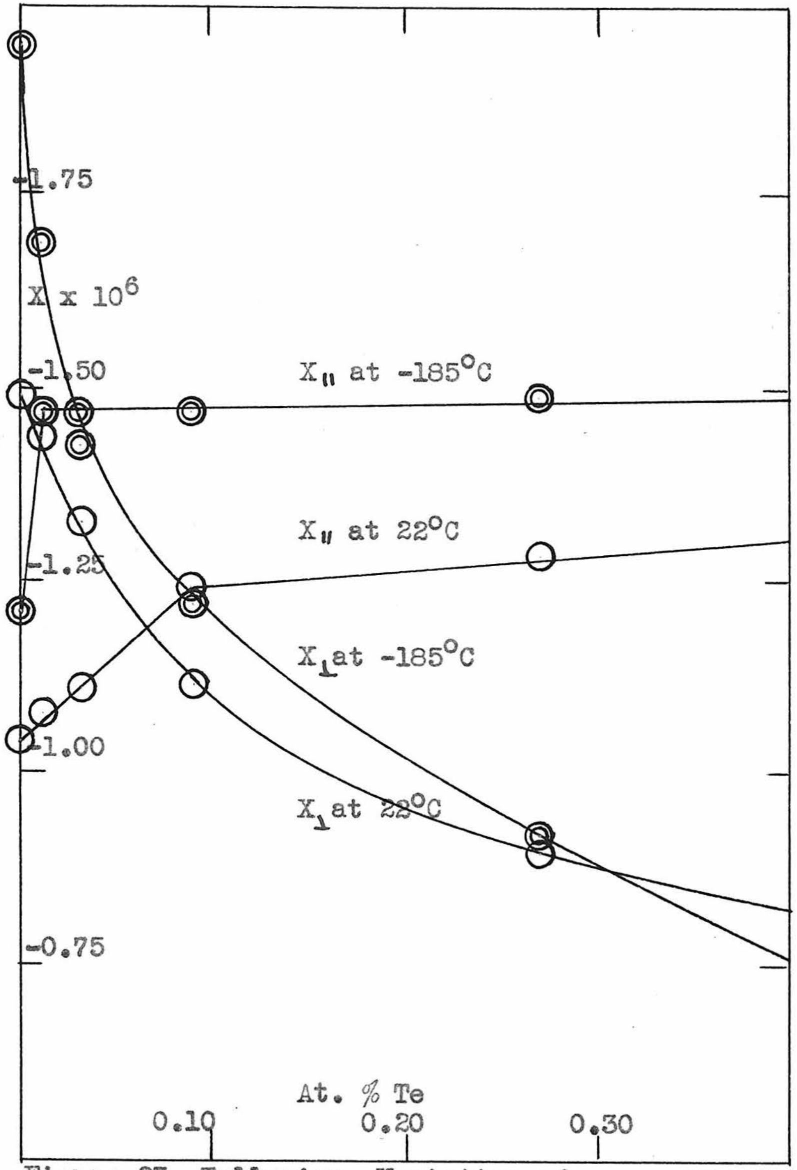


Figure 23. Tellurium. Variation of susceptibility with amount of impurity.

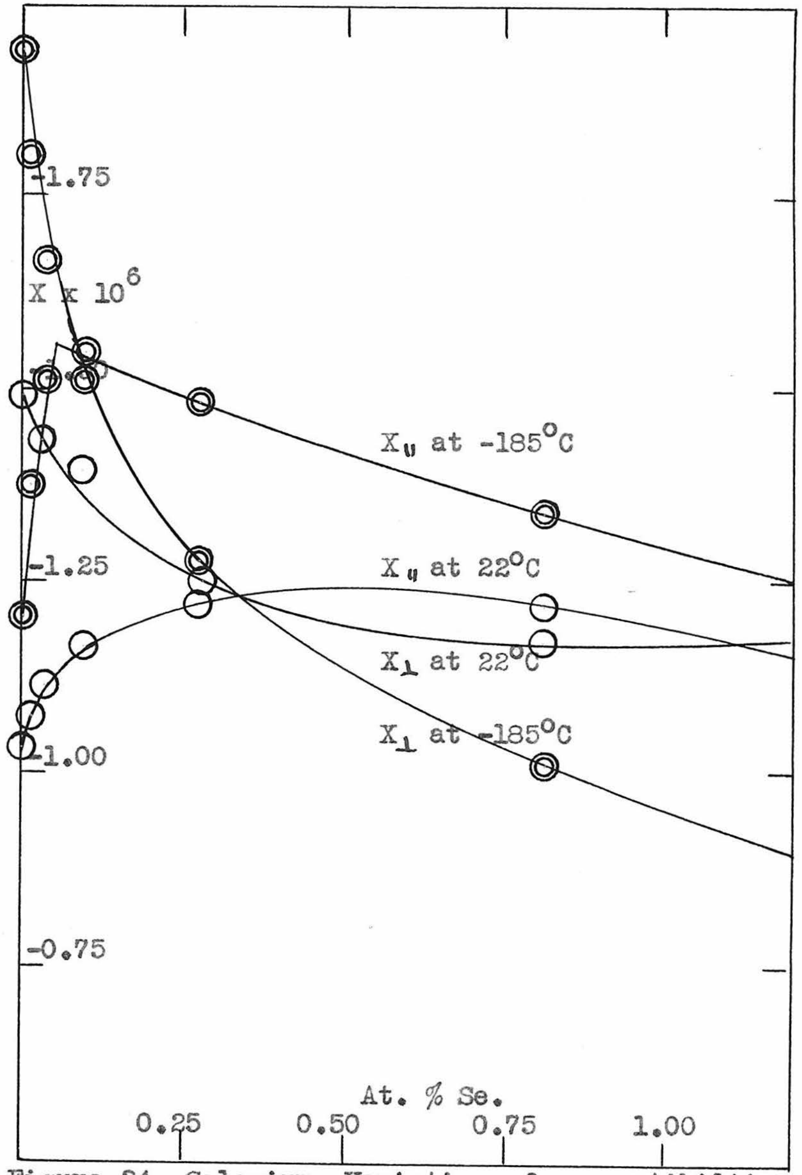


Figure 24. Selenium. Variation of susceptibility with amount of impurity.

it must be assumed that these metals are first filled into the (111) planes and later into the $(11\bar{1})$ planes. The critical amount of tellurium must be less than 0.01%, while that of selenium lies between 0.02% and 0.03%. From the shape of the curves it must further be assumed that for these elements the probability of being located in the $(11\bar{1})$ planes must always be considerably less than the probability of being placed in the (111) planes. In the cases of tin and lead, however, the probabilities of entering the different types of planes must be very nearly the same, after the critical value has been passed.

The hypothesis of potential barriers may be extended to explain the temperature dependence of the susceptibility of the impure crystals. If the deviation from the normal susceptibility is caused by the disturbing action of potential walls it would be expected that, due to the heat motions of the atoms, these deviations should decrease with increasing temperature. Super-imposed upon this would be the changes due to the normal crystal lattice as observed in pure bismuth. Figure 25 shows the relation between temperature and susceptibility for the two particularly interesting crystals 0.03%Sn and 5.00%Sn as compared to a pure crystal. Figure 26 gives similar curves for a tellurium crystal (0.27%Te), and a selenium crystal (0.81%Se). These all show that the variations are of the type to be expected.

The irregularities observed in the silver alloys may now be explained by assuming that there is no preferred position of the silver in the bismuth crystal, and that due to conditions of growth, some silver may be forced into the $(11\bar{1})$ type planes. In this case the anisotropy of the crystal would be increased. It would be just as probable, however, that the silver was forced into the (111) type planes, and the result would be a decrease in the magnetic anisotropy. Thus, the

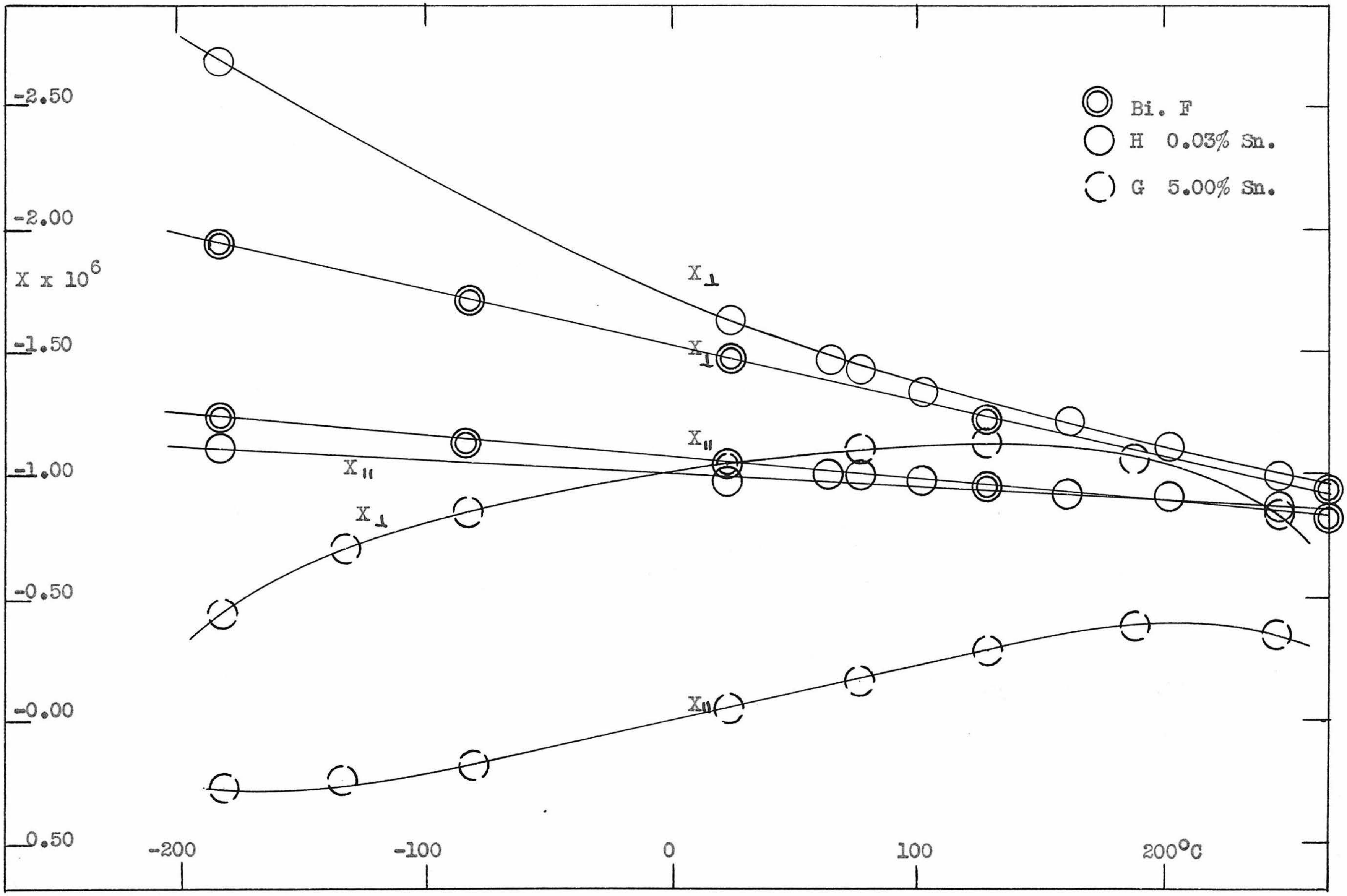


Figure 25. Variation of susceptibility with temperature.

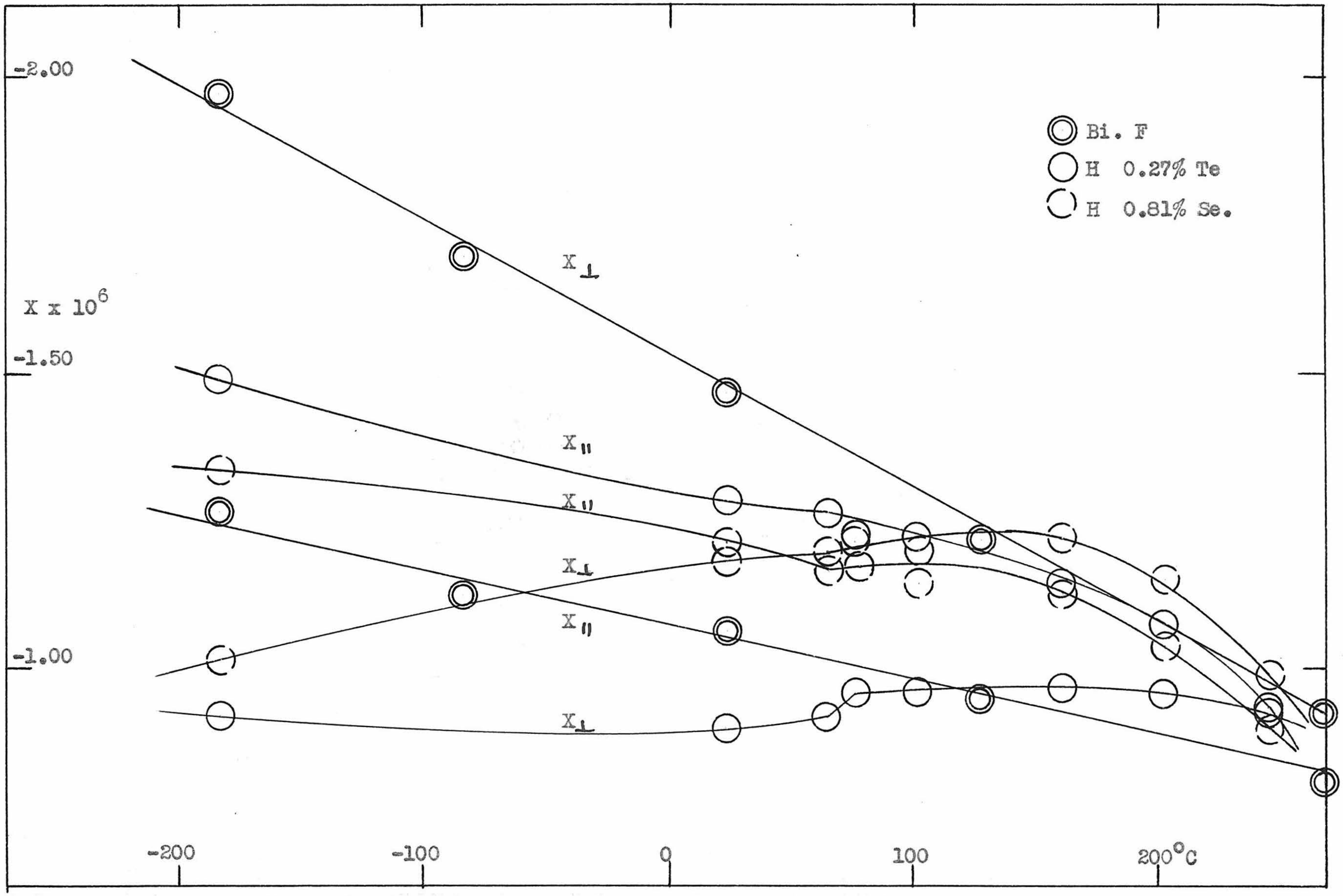


Figure 26. Variation of susceptibility with temperature.

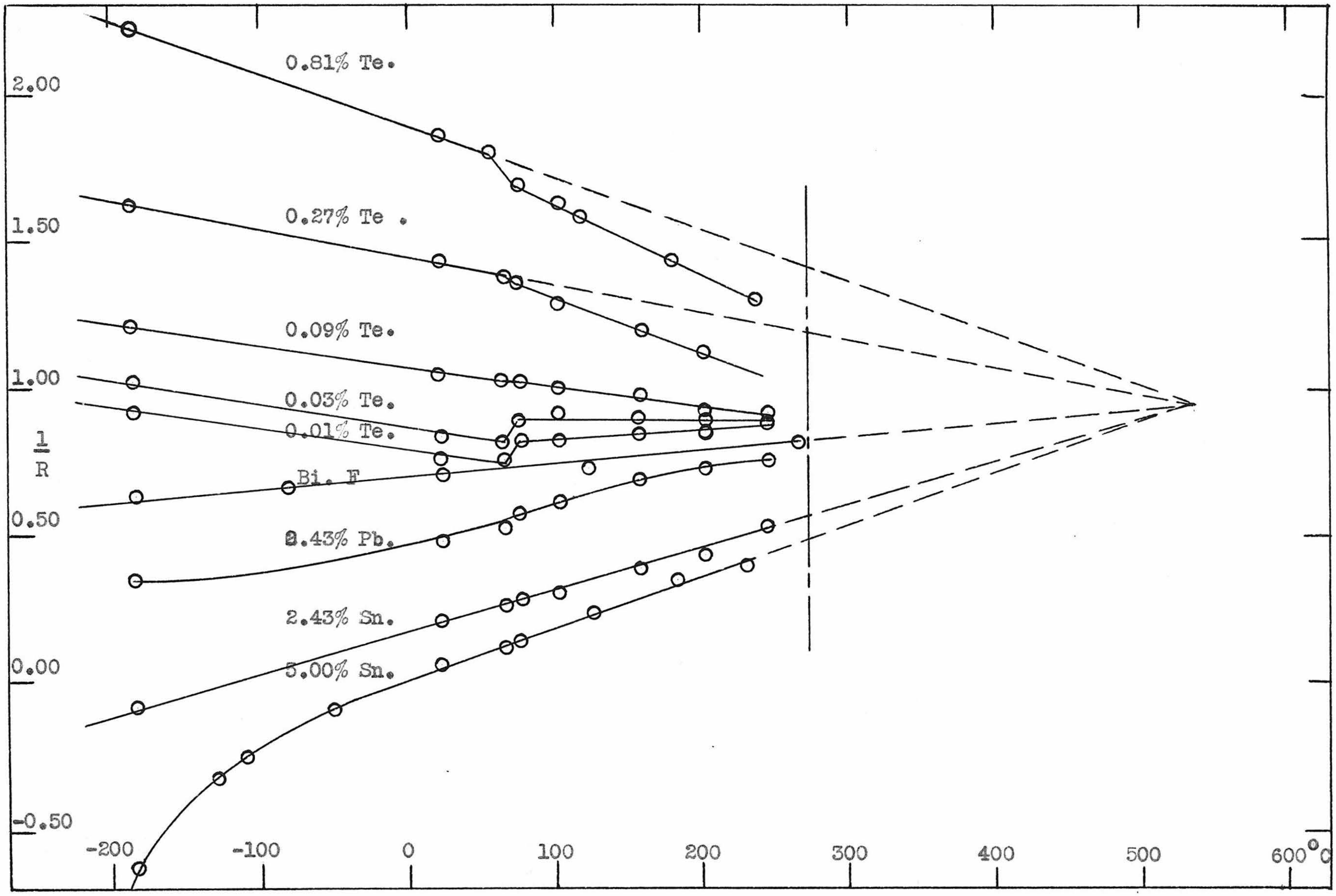


Figure 27. Variation of $\frac{1}{R}$ with temperature.

effect to be expected would depend on the conditions governing the formation of the crystal, and if no special precautions were taken, the behavior of the silver alloys would be erratic.

The discontinuity in the temperature-ratio curves for tellurium at 70° C, Figure 27, indicates a transformation in the bismuth at this temperature. There has been considerable discussion as to the reality of a transformation at this point, as evidence of it has been found in thermo-electric and other measurements, but no change in the x-ray lattice structure has ever been reported. In view of the hypothesis used to explain the magnetic phenomena under discussion this apparent transformation may be explained as a change in the secondary structure planes alone.

A further important result of the investigation, which may best be indicated by Figure 25, is the co-existence of paramagnetism and diamagnetism. A number of theories have postulated this fact in order to explain the varying degrees of magnetic properties observed in nature. The case of $X_{\parallel}$ shows that this must be true, and that the mechanism that destroyed the diamagnetism continues to destroy it further, even though the crystal has become paramagnetic.

SUMMARY OF RESULTS

Important conclusions that may be drawn from this investigation are: 1. In the region of partial solid solutions of lead, tin, tellurium, and selenium in bismuth, these impurities are not distributed at random through the crystal, but are located in definite planes. The planes in which these metals are located are probably the secondary structure planes of the (111) and $(11\bar{1})$ types. It is found that when the foreign metal is present in extremely small amounts it tends to be located in one of these sets of planes only. These preferred planes are, in the case of metals in the next column to the left of bismuth in the Periodic Table of the Elements, the $(11\bar{1})$ type, while the (111) plane is the preferred one for metals in the next column to the right. For each metal there exists a critical amount that may be taken into the preferred planes before any is forced into the other type of plane, the amount being a function of the impurity metal rather than that of the bismuth. 2. All the observed deviations of the magnetic properties may be explained if the crystal diamagnetism of bismuth is assumed to be due to a comparatively small number of electrons moving in orbits with diameters of the order of 10^{-4} centimeters, or free electrons whose free paths are of the same dimensions. The mechansim of collision of an electron with the potential barriers created by the impurity atoms in the secondary structure planes, must be that of adsorbtion and re-emission. 3. It is shown that diamagnetism and paramagnetism must co-exist in the same material. 4. If the added metal does not exist in the planes of the secondary structure, as in the cases of antimony, silver, and copper, it has little to no effect upon the magnetic properties of the crystals. 5. The principle magnetic susceptibilities of bismuth are found to be:

$$X_{\perp} = -1.487 \times 10^{-6}$$

$$X_{\parallel} = -1.046 \times 10^{-6}$$

I wish to express my deepest appreciation to Dr. Goetz for his kind suggestions throughout the investigation. I wish also to thank Mr. Hasler for his spectroscopic determination of the purity of the bismuth, and Dr. Hergenrother for his distillation of the bismuth used in making the determination of the principle susceptibilities.

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