Lorentz symmetry and non-unitary quantum field theories

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ABSTRACT

We know quantum field theory is unitary. However, since the early 1980s, there have been numerous attempts to construct quantum field theories where time evolution is non-unitary. Some of these endeavors aimed to address the issue of black hole information loss, as non-unitary evolution does not necessarily require information preservation. Some wanted to use it as a modification of quantum mechanics to allow objective collapse. Some wanted to construct classical-quantum gravity which could serve as an alternative to quantum gravity.

I embarked on a similar path, attempting to construct a Lorentz covariant non-unitary quantum field theory. At a certain point, I believed we were making significant progress. However, I gradually realized that our construction faced serious problems. We took a lot of assumptions and results from unitary quantum field theory for granted, and used them without justification. After struggling with it for a long time, I decide to make a complete reversal and prove that non-unitary quantum field theories fundamentally conflict with Lorentz covariance.

There are three approaches to constructing a Lorentz covariant non-unitary QFT. The first approach involves constructing a theory based on unitary quantum field theory, where a system is coupled to an environment. If we only consider the system and trace out the environment, the resulting equation of motion appears non-unitary. In this case, unitarity emerges as an emergent property. The second approach is to propose a theory from scratch where the time evolution is fundamentally non-unitary, described by the Lindblad master equation. Both in the emergent and fundamentally non-unitary theories, the dynamics are intended to be Lorentz covariant. The third approach employs the Schwinger-Keldysh formalism to construct a path integral, and examines the symmetry of the Keldysh action within the path integral. It is assumed that, similar to quantum field theory, the non-unitary theory will possess the same symmetry as the Keldysh action.

Regrettably, none of these three classes of theories prove successful. This thesis thoroughly analyzes the issues associated with these three constructions. The most significant problems include:

1, The fundamental assumption that the quantum fields (and their excitations) form a unitary representation of the Lorentz group is invalid, and they cannot form a non-unitary representation either.

2, The system Hamiltonian in the Lindblad equation is ill-defined and does not transform as the first component of a Lorentz four-vector.

3, Even if we overlook the aforementioned inconsistencies, the dynamics fail to produce expected results when applied to phenomena such as particle decay, as they exhibit a preferred reference frame.

4, The symmetry of the Keldysh action does not guarantee the corresponding symmetry in the dynamics. Invariant Keldysh actions can correspond to non-covariant equations of motion.

In conclusion, the Lorentz symmetry is incompatible with non-unitary quantum field theories.

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TABLE OF CONTENTS

Chapter 1

INTRODUCTION

1.1 Background and motivations

In 1974, Stephen Hawking made the groundbreaking discovery of Hawking radiation (Hawking, 1974). In his efforts to address the resulting black hole information paradox, Hawking later proposed an intriguing possibility: the evolution of a pure state into a mixed state (Hawking, 1982). This idea has sparked numerous discussions among researchers.

The most famous contribution came from Banks, Peskin, and Susskind (BPS) (Banks, Peskin, and Susskind, 1984), who highlighted the conflict between locality and energy-momentum conservation in such theories. Srednicki (M. Srednicki, 1993) briefly examined the constraints imposed by Lorentz covariance and claimed that the BPS toy model satisfied Lorentz covariance (although this was later proven incorrect). Unfortunately his conditions for Lorentz covariance are not accurate (as discussed in Chapter 5).

In 2007, John Preskill and David Poulin initiated a project (Poulin, 2017) aimed at constructing a theory that evades the constraints highlighted by BPS while maintaining Lorentz symmetry. The objective was to develop a theory where the fundamental time evolution is non-unitary but potentially suppressed at low energy scales, thereby ensuring the predictions of the theory remains consistent with our daily experiences.

Furthermore, a Lorentz covariant non-unitary theory has the potential to provide a mathematical foundation for objective collapse theories, which have long struggled to reconcile with Lorentz covariance. Additionally, such a theory is necessary in the context of post-quantum classical gravity, where gravity is assumed to be fundamentally classical and the dynamics of quantum systems coupled to gravity become non-unitary. Jonathan Oppenheim et al. (Oppenheim and Weller-Davies, 2022) are actively engaged in constructing such a theory, envisioning it as an alternative to the full quantum gravity.

The non-unitarity of quantum field theory can also arise when a system is coupled to the environment, and we trace out the environment and only look at the system. This scenario corresponds to the infinite-dimensional extension of open quantum

systems in the field of quantum information science, where non-unitarity emerges naturally. Alicki et al. (Alicki, Fannes, and Verbeure, 1986) laid the mathematical groundwork for such theories and applied them to describe particle decay.

The dynamics of all the aforementioned theories are described by the Lindblad master equation (Lindblad, 1976). They were constructed such that the Lorentz transformation on the Lindblad equations are Lorentz covariant.

Another class of theories, based on Lindblad equations, incorporates Lorentz symmetry through the analysis of the Keldysh action. John Preskill initially proposed the use of the Schwinger-Keldysh path integral formalism to construct fundamentally non-unitary quantum field theories (Preskill, 2007). Building upon his leaked notes, Avinash et al. developed the renormalization of certain "super-Lagrangian" models within the framework of open system quantum field theory (Avinash, Jana, Loganayagam, et al., 2017; Avinash, Jana, and Rudra, 2019). Jonathan Oppenheim et al. are also working towards constructing a classical-quantum gravity theory using this formalism, aiming to provide an alternative to full quantum gravity (Oppenheim and Weller-Davies, 2023).

1.2 To work or not to work, that is the question

When I started in the summer of 2016, I was also trying to construct a non-unitary quantum field theory that is Lorentz covariant. But what I am showing in this thesis is the opposite, that such theories cannot work.

I started with building a more solid conceptual and mathematical foundation by going back to the asymptotic past Hilbert space, and tried to add interaction to the original Preskill-Poulin model. At some point, I thought we were close to our goals. However, over the years, I noticed something was not right. First I noticed that, in order to keep Lorentz covariance, spatial translation also needs to be non-unitary. This is contrary to what other researchers believe. Then I discovered that our fundamental assumption that the field forms a unitary representation of the Lorentz group, or more precisely, $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$, which all previous researchers assumed, is not consistent with non-unitary time evolution.

I realized that what we have been doing might be wrong from the beginning. I felt very frustrated and did not know what to do. One option was to just make some hand-waving arguments to explain these uncomfortable facts, try to apply the theory to do some computations that can potentially be verified by experiments, and claim we were successful. The other was to turn 180 degrees around, and to prove

that Lorentz covariant quantum field theories cannot be constructed. Obviously, if we successfully constructed such a theory, it would be much more exciting than to prove it cannot be constructed. After struggling about it for some time, I decided to be completely honest. If I can clearly and decisively prove that non-unitarity QFT is fundamentally inconsistent with the Lorentz symmetry, it would help clarify a lot of misunderstandings among the community, and give a conclusive answer to this forty-year-old problem. Even if the result is not as exciting as the other way around, this would still be real contribution to our understanding of foundational questions in physics. My original motivation when I chose to study physics, was exactly to better understand the foundational questions in physics, NOT to get some great results and become a famous physicist.

After working in the opposite direction for about one year, I discovered more and more evidence supporting my decision. We have taken too many things from unitary QFT for granted, without carefully checking whether those assumptions and results are still valid in non-unitary settings. And in most cases, they are not. I organized these results in the following way.

In order to construct a non-unitary QFT that is Lorentz covariant, there are three approaches. The first is to start from the framework of unitary QFT. The mathematical and conceptual constructions are the same. The system field we are interested in is considered as a part of the global system, which is fundamentally unitary. However, since we only have access to the system field, and have no access to the environment it couples to, the reduced dynamics of the system field will look non-unitary. This is a direct extension of the theory of open quantum systems in quantum information science, with the additional constraint that it has to be Lorentz covariant. In this framework, the non-unitarity is due to our lack of knowledge of the environment. In another word, it is emergent.

The second approach is to start from scratch, to construct a theory that is fundamentally non-unitary. The dynamics would be the same as the first approach, but the conceptual and the mathematical framework would be different. Ideally, there should be a parameter γ that controls the non-unitarity. In the limit γ goes to zero, the theory will approach unitary QFT. In my opinion, this approach is closer to what Preskill and Oppenheim wanted, a fundamentally non-unitary theory that can potentially serve as the solution to the black information paradox, or an alternative to the full quantum gravity.

In the above two approaches, the Lorentz symmetry is seen at the level of the

dynamics. The equation of motion–the Lindblad equation–is Lorentz covariant. In the third approach, it is slightly different. As Preskill suggested, we look at the Keldysh action. In the path integral formulation of the normal quantum field theory, the invariance of the action always corresponds to the covariance of the equation of motion. It was believed that a similar relationship holds in non-unitary quantum field theory, that a Lorentz invariant Keldysh action corresponds to a Lorentz covariant dynamics. Therefore, we can focus on constructing a Lorentz invariant Keldysh action, that can be solved, and at the same time, preserves the vacuum state as a stable point.

Unfortunately, as I discovered, none of the above three approaches work. The major inconsistencies are:

1. the inconsistencies with the representation of the sLorentz group, both unitary and non-unitary.

2. the problem with non-unitary spatial translation.

3. the invalid assumption that the system Hamiltonian transforms as the first component of a Lorentz four-vector.

4. even if we disregard the above difficulties, these theories would not work as expected, since there will be a preferred time frame.

5. the invalid assumption that an invariant Keldysh action corresponds to covariant dynamics.

Because of these issues, we can draw the conclusion that Lorentz covariant non-unitary quantum field theories cannot be constructed.

1.3 The organization of this thesis

This thesis is organized as follows:

In the second chapter, I discussed the problem with the fundamentally non-unitary construction. I mainly focused on the conflict between the Lorentz symmetry and the representation of the Lorentz group. It turns out that the quantum field cannot form a representation of the Lorentz group, either unitary or non-unitary. I also discussed the issue with non-unitary spatial translation, and the case with non-Markovian dynamics.

In the third chapter, I discussed the problem with the emergent non-unitary theories. I started with the problem of the representation of the Lorentz group, which is

slightly different from the unitary case in the second chapter. I also discussed the assumption that the system Hamiltonian transforms as the first component of a Lorentz four-vector. Finally, I discussed the problem with the preferred time frame, which is also shared with the fundamentally non-unitary construction, since the dynamics of these two are the same.

In the fourth chapter, I discussed the construction based on the Schwinger-Keldysh formalism. It has been believed that a symmetric Keldysh action will lead to a symmetric dynamics, just like the case of the path integral in quantum field theory. However, this is not the case for Lindblad dynamics. The equation of motion, the Lindblad equation, is completely different from the Euler-Lagrange equation obtained from extremizing the Keldysh action, so they will not share its symmetry. I further showed that in the case of the Lorentz symmetry, invariant Keldysh actions actually comes from non-covariant equations of motion.

In the fifth chapter, I discussed the Lorentz transformation properties of operators and quantum channels that are defined as an integration on a time slice. It is widely believed that in quantum field theory, the momentum operator P^ν, defined as an integration of stress tensor on a time slice $P^\nu := \int dx^3 T^{0\nu}(x)$, transforms as a Lorentz vector. Some prominent physicists believe that a Lindblad quantum channel acting on an operator defined as an integration on a time slice could transform as the time derivative of that operator. These two views are incorrect. Operators and quantum channels defined as integrations on a time slice in general transform covariantly under the Lorentz transformation only if two conditions are met: 1, the integrand should contain an operator which is the time-like component of a Lorentz four-vector; 2, some conservation conditions are met.

Chapter six is a by-product of this thesis research, in which I analyzed the physical meaning of the momentum eigenstates. We were trying to construct an operational experiment in which we can measure some quantities to test if the theory is Lorentz covariant. And we were using momentum eigenstates and wavepackets. However, after careful analysis, I found out that, contrary to popular belief, momentum eigenstates cannot be interpreted as plane-waves. In fact, the physical plane-waves should be coherent states, and as a result, wavepackets are superpositions of coherent states.

Chapter seven summarizes the thesis with concluding remarks.

Chapter 2

NON-UNITARY LORENTZ COVARIANT THEORY CANNOT BE CONSTRUCTED

In this chapter, we analyze the theories where time evolution is fundamentally non-unitary. We will analyze the theories where the non-unitarity is emergent (i.e., open system quantum field theories) in the next chapter. These two classes of theories share similar dynamics, but are constructed on different conceptual and mathematical foundations. This chapter mainly focuses on the issues with the construction, especially the issues of the representation of the Lorentz group. We will leave the issues of self-Hamiltonian and dynamics to the next chapter.

As introduced in the first chapter, the closest to success in this genre is the Preskill-Poulin Theory. I have spent years trying to make it well-defined, and to extend it to interacting field theory. In this chapter, I first introduce the basic constructions, and the issue with non-unitary spatial translations. Then we focus on the fundamental assumption that the field in the theory forms a unitary representation of the Lorentz group, which is inconsistent with the non-unitary time evolution. I also analyze the possibility of the non-unitary representation of the Lorentz group, as suggested by Oppenheim (Oppenheim and Weller-Davies, 2022) and Diosi (Diosi, 2022).

2.1 Basic constructions

We first introduce the framework of the Preskill-Poulin theory, which is the closest to success in that genre. This construction starts from the level of space-time foliation and Hilbert spaces. The time evolution is fundamentally non-unitary, but can approach unitarity when the parameter γ controlling non-unitarity is taken to zero, and the theory converges to normal unitary quantum field theory.

The fundamental time evolution is non-unitary, as described by the Lindblad equation

$$\partial_t \rho = -i[H, \rho] + \gamma \int \widetilde{dp} \omega_p \left(a(p) \rho a(p)^\dagger - \frac{1}{2} \{ a^\dagger(p) a(p), \rho \} \right) \tag{2.1}$$

where $\widetilde{dp} := \frac{d^3 p}{(2\pi)^3 2\omega_p}$ is the Lorentz invariant integration measure. The term in the integrand is the non-unitary dressing to the normal unitary time evolution, which corresponds to environmentally induced decoherence in open system quantum field

theories. The operator $a(p)$ in the integrand is called the jump operator. In the original Preskill-Poulin theory, since they want the vacuum to be a stable point, they only used annihilation jump operators, but in principle there can be more complicated operators as well.

To see the Lorentz covariance property, it is better to transform it into the Heisenberg picture. (Part of the reason is that $U(\Lambda)^\dagger \rho(t) U(\Lambda)$ is not well defined. We address this issue later.)

It is assumed that the quantum field in this theory forms a unitary representation of the Lorentz group. The Lorentz transformations are generated by unitary generators $U(\Lambda)$

$$U(\Lambda) a(p) U(\Lambda)^\dagger = a(\Lambda p) \tag{2.2}$$

We can rewrite the Lindblad equation in the Heisenberg picture and further generalize it to include other jump operators $Q_a(p)$

$$\begin{aligned} \partial^0 A &= i[P^0, A] \\ &+ \sum_{a,b} \gamma_{ab} \int \widetilde{dp} p^0 \left(Q_a^\dagger(\vec{p}) A Q_b(\vec{p}) + \frac{1}{2} \{ Q_a^\dagger(\vec{p}) Q_b(\vec{p}), A \} \right). \end{aligned} \tag{2.3}$$

where $U(\Lambda) Q_a(p) U(\Lambda)^\dagger = Q_a(\Lambda p)$

It is not hard to show that $U(\Lambda)$ acting on Eqn (2.3) by conjugation will yield a Lorentz covariant form by using the invariance of the measure $\widetilde{dp}$.

(In Ref.(M. Srednicki, 1993), the author claimed that one general Lorentz-covariant framework is

$$\partial^0 \rho = -i[P^0, \rho] - g \int d\Sigma^0 \left[\varphi(\vec{x}\,), \left[\varphi(\vec{x}\,), \rho \right] \right].$$

Here $\vec{x}$ specifies a point on the spacelike hypersurface Σ whose unit normal defines the time direction, and $d\Sigma^\mu = \frac{1}{6} \varepsilon^{\mu ijk} dx_i dx_j dx_k$. However, this turns out to be incorrect, as discussed in detail in Chapter 5.)

It can also be shown that the time evolution of the operators commutes with the Lorentz transformation

$$\exp[\mathcal{L}_{\Lambda b}[U(\Lambda)^\dagger A U(\Lambda)]] = U(\Lambda)^\dagger \exp[\mathcal{L}_b[A]] U(\Lambda) \tag{2.4}$$

where $\mathcal{L}_b[A]$ generates the time evolution along a time-like vector b. These are basically the same properties with the non-fundamental quantum open system version, as described in (Alicki, Fannes, and Verbeure, 1986).

It should be noted that, in his seminar at KITP (Poulin, 2017), Poulin claimed that although time evolution is non-unitary, spatial translations are still generated by the momentum operator P. Poulin did not give detailed constructions or derivations to show why this is the case, nor did he explained whether it conflicts with Lorentz covariance. We found out that this could not be the case. With a non-unitary time evolution, Lorentz covariance would require non-unitary spatial translation, as we show as follows. We construct the framework of non-unitary quantum field theory from the most basic level. This is applicable to the Preskill-Poulin theory, and can also reduce to unitary QFT when the coupling γ goes to 0 in Eqn (2.1).

We choose a foliation of space-time labeled by a reference system (x, t). At each time slice t, we associate a Hilbert space $\mathscr{H}(t)$. For now let us focus on the free theory, which furnishes a Fock space structure. There is a natural isomorphism between the Hilbert spaces defined on t and $t+\tau$. If we use the Fock space structure, $|p\rangle \in \mathscr{H}(t)$ is identified with $|p\rangle \in \mathscr{H}(t+\tau)$ (up to a global phase). This natural isomorphism is not the same as the time evolution map as in the unitary QFT, since time evolution is not unitary here. (Strictly speaking, $|p\rangle$ is outside of the physical Hilbert since it cannot be normalized, but in this paper, we would not go to that level of mathematical rigor.) Def: An operator O that acts on the Hilbert space $\mathscr{H}(t)$ maps a vector $|\phi\rangle \in \mathscr{H}(t)$ to another vector $|\psi\rangle = O|\phi\rangle \in \mathscr{H}(t)$. We notate it as $O^{(t)}$. (There are also operators that map an operator from the Hilbert space on one time slice to another, which we will see later.)

Some operators may have a spatial dependence $O^{(t)}(\vec{x})$. To be more precise, this notation means there is a family of operators $O^{(t)}(\vec{x})$ parameterized by spatial location $\vec{x}$ on the time slice. Here I deliberately make t and x on different footings, since their meanings are different. I am tempted to call them field operators.

We can define a super-operator that maps one $O^{(t)}(\vec{x})$ to another operator at the same time slice but a different spatial location. We call it a displacement super-operator.

Def: $\mathcal{D}_a[O^{(t)}(\vec{x})] := O^{(t)}(\vec{x}+\vec{a})$

At this stage this definition of displacement is abstract, purely dependent on the relationship between operators. We want such super-operators to satisfy the additive property: $\mathcal{D}_b[\mathcal{D}_a[O^{(t)}(\vec{x})]] = O^{(t)}(\vec{x}+\vec{a}+\vec{b}) = \mathcal{D}_{a+b}[O^{(t)}(\vec{x})]$.

Aside from this, we have not imposed any other restrictions on the relationship between operators on the same time slice but at different locations, so this relationship can be arbitrary. Later we will see that Lorentz covariance is one restriction, based on which we derive the form of the displacement operator.

Operators like $a^{(t)}(p)$ have no spatial label, but a momentum label. Such labels tell us how they map one vector in the Fock space to another one.

The time translation/evolution super-operator $\mathcal{E}_\tau$ maps an operator that acts on $\mathscr{H}(t)$ to an operator acting on $\mathscr{H}(t+\tau)$.

Def: $\mathcal{E}_\tau[O^{(t)}] : |\phi\rangle \in \mathscr{H}(t+\tau) \to |\psi\rangle \in \mathscr{H}(t+\tau)$. Here τ is purely in the temporal direction.

(All labels are implicitly assumed to be real numbers, unless declared otherwise.)

We can now notate $O^{(t)}$ acting on Hilbert space $\mathscr{H}(t)$ as $O(t)$. And for field operators, we can write it as $O(t,\vec{x})$. Hence $\mathcal{E}_\tau[O(t,\vec{x})] := O(t+\tau,\vec{x})$.

Let us think about the relationship between time and spatial translation. We want it to commute, which means if we first translate $O(t,\vec{x})$ to $O(t,\vec{x}+\vec{a})$, and then evolve it for time duration τ and get $O(t+\tau,\vec{x}+\vec{a})$, we should get the same result if we first time translate and then spatially translate:

$$\mathcal{E}_\tau[\mathcal{D}_a[O(t,\vec{x})]] = O(t+\tau,\vec{x}+\vec{a}) = \mathcal{D}_a[\mathcal{E}_\tau[[O(t,\vec{x})]] \tag{2.5}$$

Here we drop the superscript of the spatial translation super-operator since it is clear which Hilbert space it acts on.

Now let us look at how the Lorentz transformations act on operators. The textbook definition is, if we want to change our coordinate from x to $x' = \Lambda^{-1}x$, the corresponding (scalar) operator should change according to

$$U(\Lambda)^{-1}\phi(x)U(\Lambda) = \phi(\Lambda^{-1}x) \tag{2.6}$$

To simplify notation, in the following I will replace Λ^{-1} with Λ, so the new frame is $x' = \Lambda x$, and the corresponding transformation is $U(\Lambda)\phi(x)U(\Lambda)^{-1} = \phi(\Lambda x)$. In our case, we also assume the Lorentz group is represented unitarily. The super-operator $\mathcal{U}(\Lambda)$ maps an operator defined in one time slice t to an operator defined in another time slice in the boosted frame t' with a different foliation.

$$\mathcal{U}(\Lambda) : O^{(t)} \to O^{(t')} = U(\Lambda)O^{(t)}U^\dagger(\Lambda)$$

$$= e^{i\omega_{\mu\nu}M^{\mu\nu}} O^{(t)} e^{-i\omega_{\mu\nu}M^{\mu\nu}}.$$

This satisfies the composition rule $U(\Lambda)U(\Lambda') = U(\Lambda\Lambda')$. For infinitesimal transformations, we can write

$$U(\Lambda) = I + i\omega_{\mu\nu}M^{\mu\nu}$$

where $M^{\mu\nu}$ are the generators of the Lorentz transformations. From the composition rules we can get the Lie algebra satisfied by these generators.

From above we can see that $U^\dagger(\Lambda)$ maps a vector from $\mathscr{H}(t')$ to $\mathscr{H}(t)$.

Now let us think about what Lorentz invariance/covariance mean. After we do a active boost $x' = \Lambda^{-1}x$, we want a (family of) scalar operators to transform in the following way

$$U(\Lambda)O(t,\vec{x})U(\Lambda)^{-1} := O(\Lambda x) = O(\Lambda^0_0 t + \Lambda^0_i x^i, \Lambda^j_0 t + \Lambda^j_i x^i)$$

This means that under a the Lorentz transformation, an operator $O(t,\vec{x})$ acting on Hilbert space $\mathscr{H}(t)$ associated with location $\vec{x}$, will be transformed into an operator acting on Hilbert space $\mathscr{H}(t')$ associated with the time slice in the transformed frame $t' = \Lambda^0_0 t + \Lambda^0_i x^i$, with the location $\vec{x'} = \Lambda^j_0 t + \Lambda^j_i x^i$. Note that two operators at the same time slice but different locations will be transformed into Hilbert spaces of DIFFERENT time slices in the new frame. This is because after a boost, the points in the original time slice $\{(t,\vec{x})|x^i \in \{-\infty,\infty\}\}$ will be in different time slices in the new frame.

It seems it is not well defined to do a the Lorentz transformation for an operator that does NOT have a spatial label, because we do not know after the transformation which Hilbert space it acts on. This is actually not that surprising, since the Lorentz transformation is fundamentally transforming space and time by mixing them. If an operator has no spatial Label, what are we going to mix time with?

Another way to understand this is, as in the case of unitary QFT, $M^{\mu\nu}$ acting on scalar field operator as $x^\mu\partial^\nu - x^\nu\partial^\mu$. Without x^μ in the argument, the derivative is just 0. (Actually, even in unitary QFT, the derivation of how $M^{\mu\nu}$ acts on a scalar operator already assumes that the scalar operator is a function of x^μ.)

The only exception seems to be for operators with a label of four-momentum, such as $a(p)$, as p is in some sense the Fourier transformed x. We will discuss this later.

We can also derive how derivatives of scalar operators transform, which we skip here.

2.2 Problem with non-unitary spatial translation

In the above discussion, $\mathcal{D}_d$ is supposed to translate operators in the same Hilbert space, i.e. in purely spatial direction. Similarly, $\mathcal{E}_b$ is supposed to evolve the operators from one time slice to another, i.e, in purely time direction. But when we do a the Lorentz transformation, we will get $\mathcal{E}_{\Lambda b}$ and $\mathcal{D}_{\Lambda d}$. We have to extend the definition for $\mathcal{E}_{\Lambda b}$, since previously $\mathcal{E}$ is only defined in the time direction of the chosen frame. $\mathcal{E}_{\Lambda b}$ is defined as follow. We choose another frame (call it the primed frame) in which Λb is the coordinate of time direction, where b is in time direction. In the primed frame, each time slice is associated with a Hilbert space. And $\mathcal{E}_{\Lambda b}$ generates the time evolution for operators defined in those Hilbert spaces. An operator $O(t, \vec{x})$ in the old frame is labeled as $U(\Lambda)O(t, \vec{x})U(\Lambda)^\dagger$ in the new frame. Similarly, we can define $\mathcal{D}_{\Lambda d}$, where d is in pure space direction. We say that time and space translation super-operators are consistent or covariant with Lorentz covariance if they satisfy the following conditions:

$$U(\Lambda)\mathcal{E}_b[O(t,\vec{x})]U(\Lambda)^\dagger = \mathcal{E}_{\Lambda b}[U(\Lambda)O(t,\vec{x})U(\Lambda)^\dagger] \tag{2.7}$$

$$U(\Lambda)\mathcal{D}_d[O(t,\vec{x})]U(\Lambda)^\dagger = \mathcal{D}_{\Lambda d}[U(\Lambda)O(t,\vec{x})U(\Lambda)^\dagger] \tag{2.8}$$

We can generalize b to be a portion of a time-like curve, and d a portion of a space-like curve. In short, $\mathcal{E}$ and $\mathcal{D}$ commute with $\mathcal{U}$, which clearly must hold, otherwise there is no way to claim the theory is Lorentz covariant.

LHS means first move $b(d)$ in the old frame, then do the boost to the new frame. Let's do the calculation for Eqn 2.8 to see how LHS=RHS. In the old frame

$$\begin{aligned}
LHS &= U(\Lambda)[O(x+d)]U(\Lambda)^\dagger \\
&= O(\Lambda^\mu_\nu(x^\nu + d^\nu))\text{extract spatial component of } \Lambda d \text{ to form } \mathcal{D}_{\Lambda^i_\nu d^\nu} \\
&= \mathcal{D}_{\Lambda^i_\nu d^\nu}[O(\Lambda^\mu_\nu x^\nu + \Lambda^0_\nu d^\nu))]\text{extract temporal component of } \Lambda d \text{to form } \mathcal{E}_{\Lambda^0_\nu d^\nu} \\
&= \mathcal{D}_{\Lambda^i_\nu d^\nu}[\mathcal{E}_{\Lambda^0_\nu d^\nu}[O(\Lambda^\mu_\nu x^\nu)] \\
&= \mathcal{D}_{\Lambda^i_\nu d^\nu}[\mathcal{E}_{\Lambda^0_\nu d^\nu}[U(\Lambda)O(t,\vec{x})U(\Lambda)^\dagger] \\
&= RHS = \mathcal{D}_{\Lambda d}[U(\Lambda)O(t,\vec{x})U(\Lambda)^\dagger]
\end{aligned}$$

This means

$$\mathcal{D}_{\Lambda d} = \mathcal{D}_{\Lambda^i_\nu d^\nu} \circ \mathcal{E}_{\Lambda^0_\nu d^\nu}$$

Namely, the displacement super-operator in the new frame, can be decomposed as the displacement in the old frame plus the time evolution of the old frame. Since the

time evolution channel $\mathcal{E}_{\Lambda^0_\nu d^\nu}$ is non-unitary, if $\mathcal{D}_{\Lambda^i_\nu d^\nu}$ is unitary, their composition $\mathcal{D}_{\Lambda d}$ must always be non-unitary. And if $\mathcal{D}_{\Lambda^i_\nu d^\nu}$ is non-unitary, their composition $\mathcal{D}_{\Lambda d}$ must always be non-unitary, unless in the very special case when they happen to cancel out, which cannot be the general case since Λ and d are completely arbitrary. Hence, we can conclude that the displacement super-operator is also non-unitary.

To prove this I only used the definition of space and time translation, and Lorentz transformation must commute with time translation. Nothing more.

It is thus clear that in our case of non-unitary field theory, time evolution is non-unitary, but spatial translation should also be non-unitary. We will discuss the physical implications about this later.

2.3 Conflict with unitary representation of the Lorentz group

Almost all previous discussions and constructions, including the Preskill-Poulin theory as discussed above, assumed that the quantum field forms a unitary representation of the Lorentz group (Eqn (2.2)). Or equivalently

$$U(\Lambda)|p\rangle = |\Lambda p\rangle \tag{2.9}$$

In fact, this leads to contradictions both physically and mathematically.

Let us first look at $a(\Lambda p)$. Assume $a(p)$ is an operator in the Hilbert space associated with the time slice t_0, $a(p) \in \mathscr{H}(t_0)$. $a(\Lambda p)$ may mean two things. The first is an operator living in the same Hilbert space, but with a different momentum label. Namely, $a(\Lambda p)$ will annihilate a state in the same time-slice Hilbert space, but with different four-momentum $|\Lambda p\rangle$. In other words, we actively change the physical state, the configuration of the time-slice t_0 from $|p\rangle$ to $|\Lambda p\rangle$.

The second is that the operator $a(\Lambda p)$ is living on a time-slice in the boosted frame. For example, it may describe the situation of the time slice of a boosted observer. This corresponds to the passive transformation: we did not change anything physically, but just have a different observer. However, since the Lorentz boost mixes space and time, the time-slice in the boosted frame will contain part of the information not only of the original time-slice t_0, but all the time-slices in the old frame.

While this is fine in unitary QFT, since in any time slice, $|p\rangle$ will stay the same, it is problematic in non-unitary theories. Due to non-unitary time evolution, the states of the time-slices after t_0 will not stay $|p\rangle$ in general. Unless in the very special case that $|p\rangle$ is the stable state of the Lindblad equation, they will in general evolve into mixed states.

More explicitly, the physical configuration of the time slice at time t_0 in lab frame is $|\Lambda p\rangle\langle\Lambda p|$. According to an observer in the lab frame, at time t_1, the physical state would be

$$\rho(t_1) = e^{-i\mathcal{L}(t_1-t_0)}|\Lambda p\rangle\langle\Lambda p|$$

This would be a mixed state in general. The boosted time-slice containing $|\Lambda p\rangle$ will have intersections with those future time-slices, where the state is mixed. Say it intersects with time slice t_1 at position $\vec{x_1}$. Now, two observers both do a measurement around the neighborhood of $(t_1, \vec{x_1})$. The observer in the boosted frame will get a result corresponding to a pure state, since according to him, the physical situation in his time slice is described by $|\Lambda p\rangle$. However, for the observer in the lab frame, he will get a result corresponding to a mixed state, since for him, the time slice is described by the mixed state $e^{-i\mathcal{L}(t_1-t_0)}|\Lambda p\rangle\langle\Lambda p|$ that evolved from the original pure state $|\Lambda p\rangle\langle\Lambda p|$. While according to the observer in the boosted frame, the physical state is still the pure state $|\Lambda p\rangle\langle\Lambda p|$ The fact that two observers can tell the difference between inertial frames directly violates Lorentz covariance.

This contradiction is more obvious mathematically. In unitary QFT, $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$ can either be treated as an assumption, or derived from other assumptions or definitions. In non-unitary theories, we already see the assumption $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$ contradicts with Lorentz invariance, as discussed above. This cannot be derived as well, since in order to derive it, we need the equation of motion to be the Klein-Gordon equation, well in the non-unitary cases, the equation of motion is the Lindblad equation.

In the unitary QFT, $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$ can be derived as follow. From field quantization of free scalar fields

$$\phi(x) := \int \tilde{dp}\left(a_p e^{ipx} + a_p^\dagger e^{-ipx}\right)$$

One can solve for

$$a(p)^\dagger = -i\int d^3x e^{ipx}\overleftrightarrow{\partial}^0\phi(x) \tag{2.10}$$

In fact, Eqn 2.10 can be treated as the definition of creation operators, especially in the cases of interacting theories. Now let us look at how the Lorentz transformation works on both sides of Eqn 2.10.

$$
\begin{aligned}
RHS &= -i \int d^3x U(\Lambda) e^{ipx} \overleftrightarrow{\partial}^0 \phi(x) U(\Lambda)^\dagger \\
&= -i \int d^3x [U(\Lambda) e^{ipx} \partial^0 \phi(x) U(\Lambda)^\dagger - ip^0 e^{ipx} U(\Lambda) \phi(x) U(\Lambda)^\dagger] \\
&= -i \int d^3x e^{ipx} [(\Lambda^{-1})^0_\mu \partial^\mu \phi(\Lambda x) - ip^0 \phi(\Lambda x)]
\end{aligned}
$$

Let us mode expend LHS

$$
LHS = a(\Lambda p)^\dagger = -i \int d^3x [e^{i\Lambda px} \partial^0 \phi(x) - i(\Lambda p)^0 e^{i\Lambda px} \phi(x)]
$$

Now, how do we make sense of LHS=RHS?

Inspired by Sidney Coleman (Coleman, 2011), we need to rewrite the integration $\int d^3x = \int d^4x n_0 \delta(x \cdot n)$, where $n = (1, 0, 0, 0)$ is a unit vector pointing in the time direction of the original frame.

$$
RHS = -i \int d^4x n_\rho \delta(x \cdot n) e^{ipx} [(\Lambda^{-1})^\rho_\mu \partial^\mu \phi(\Lambda x) - ip^\rho \phi(\Lambda x)]
$$

Change integration variable $y = \Lambda x$ and define $n' := \Lambda n$, $n_\rho = (\Lambda^{-1})^\nu_\rho n'_\nu = \Lambda_{\nu\rho} n'^\nu$

$$
\begin{aligned}
RHS &= -i \int d^4y \Lambda_{\nu\rho} n'^\nu \delta(y \cdot n') e^{ip \cdot \Lambda^{-1} y} [(\Lambda^{-1})^\rho_\mu \partial^\mu \phi(y) - ip^\rho \phi(y)] \\
&= -i \int d^4y \Lambda_{\nu\rho} (\Lambda^{-1})^\rho_\mu n'^\nu \delta(y \cdot n') e^{i\Lambda p \cdot y} [\partial^\mu \phi(y) - i(\Lambda p)^\mu \phi(y)] \\
&= -i \int d^4y n'_\rho \delta(y \cdot n') e^{i\Lambda p \cdot y} [\partial^\rho \phi(y) - i(\Lambda p)^\rho \phi(y)] \\
&= -i \int d^4x n'_\rho \delta(x \cdot n') e^{i\Lambda p \cdot x} [\partial^\rho \phi(x) - i(\Lambda p)^\rho \phi(x)]
\end{aligned}
$$

On the other hand

$$
LHS = -i \int d^4x n_\rho \delta(x \cdot n) [e^{i\Lambda px} \partial^\rho \phi(x) - i(\Lambda p)^\rho e^{i\Lambda px} \phi(x)]
$$

We can see that RHS almost equals LHS, except in RHS, n' replaced n. This means that RHS was defined on a different time slice n' from LHS, and the surface now is $t' = 0$. So how can these two be equal?

Let us take the differences of the two,

$$
LHS - RHS = -i \int d^4x [n_\rho \delta(x \cdot n) - n'_\rho \delta(x \cdot n')] e^{i\Lambda p \cdot x} [\partial^\rho \phi(x) - i(\Lambda p)^\rho \phi(x)]
$$

Note $n_\rho \delta(x \cdot n) = \partial_\rho \theta(x \cdot n)$,

$$LHS - RHS = -i \int d^4x \partial_\rho [\theta(x \cdot n) - \theta(x \cdot n')] e^{i\Lambda p \cdot x} [\partial^\rho \phi(x) - i(\Lambda p)^\rho \phi(x)]$$

Now integrate by parts and drop the surface term

$$\begin{aligned}
LHS - RHS &= -i \int d^4x [\theta(x \cdot n) - \theta(x \cdot n')] \partial_\rho [e^{i\Lambda p \cdot x} [\partial^\rho \phi(x) - i(\Lambda p)^\rho \phi(x)]] \\
&= -i \int d^4x [\theta(x \cdot n) - \theta(x \cdot n')] [(\partial_\rho e^{i\Lambda p \cdot x}) [\partial^\rho \phi(x) - i(\Lambda p)^\rho \phi(x)] \\
&+ e^{i\Lambda p \cdot x} [\partial_\rho \partial^\rho \phi(x) - i(\Lambda p)^\rho \partial_\rho \phi(x)]]] \\
&= -i \int d^4x [\theta(x \cdot n) - \theta(x \cdot n')] [e^{i\Lambda p \cdot x} [i(\Lambda p)_\rho \partial^\rho \phi(x) + m^2 \phi(x)] \\
&+ e^{i\Lambda p \cdot x} [\partial^2 \phi(x) - i(\Lambda p)^\rho \partial_\rho \phi(x)]]] \\
&= -i \int d^4x [\theta(x \cdot n) - \theta(x \cdot n')] e^{i\Lambda p \cdot x} \{\partial^2 \phi(x) + m^2 \phi(x)\}
\end{aligned}$$

Now we can see that the last term above will be equal to zero if the Klein-Gordon equation is satisfied!

This is exactly the case in unitary QFT for the free scalar field, where Klein-Gordon equation is the EOM. This is how we derive $U(\Lambda) a(p) U(\Lambda)^\dagger = a(\Lambda p)$.

However, in non-unitary theories, the EOM is the Lindblad equation, and in general $\partial^2 \phi(x) + m^2 \phi(x) \neq 0$. This means mathematically, $U(\Lambda) a(p) U(\Lambda)^\dagger \neq a(\Lambda p)$. Since the Lorentz covariance properties in all the previous research including the Preskill-Poulin theory, crucially rely on the unitary representation of the Lorentz boost Eqn 2.2, they are thus not self consistent.

(One might argue that based on the above derivation, for interacting QFT, Klein-Gordon equation is not the EOM, hence $U(\Lambda) a(p) U(\Lambda)^\dagger \neq a(\Lambda p)$ as well. However, for interacting QFT, we do not understand the structure of the Hilbert space in the scattering region. We only understand the asymptotic Hilbert space, which furnishes a Fock space structure as well. Hence, in the above derivation, we should replace $a(p)^\dagger$ with $a_{in}(p)^\dagger$, and $\phi(x)$ with $\phi_{in}(x)$, where the operators with subscripts live in the asymptotic Hilbert spaces. And in the asymptotic past, $\partial^2 \phi_{in}(x) + m^2 \phi_{in}(x) = 0$. The same applies for the asymptotic future case.

This argument cannot be applied for non-unitary theories. This is because the free unitary QFT corresponds to the non-unitary QFT where the Hamiltonian is free (or quadratic). But the theory is still fundamentally non-unitary, regardless of whether we go into the asymptotic past.)

2.4 Inconsistencies with non-unitary representation of the Lorentz group

One potential method to resolve the above contradiction is to use non-unitary generators for the Lorentz boosts, as suggested by Diosi (Diosi, 2022) and Oppenheim (Oppenheim and Weller-Davies, 2022). However, after close inspections, we found that this approach is unacceptable on conceptual, mathematical and physical grounds, as we show as follows.

The central idea to find out the non-unitary boost generators, as suggested by Oppenheim, is to dress the unitary generators with dissipation. Since a the Lorentz boost can be understood as a rotation in space-time, one can construct such generators by updating the time evolution and/or spatial translation generators with dissipative super-operators. For example, Oppenheim suggests that, since in unitary QFT, the generators for the Lorentz boost are

$$Q_U^{0i} = \int d^3x \left(x^0 T^{0i} - x^i T^{00}\right) \tag{2.11}$$

one can generalize these to non-unitary super-operators $\mathcal{Q}^{0i}[\cdot]$

$$\begin{aligned}\mathcal{Q}^{0i}[O(x)] = &- i[\int d^3y \left(y^0 T^{0i} - y^i T^{00}\right), O(x)] \\ &+ i \int dy^3 y^i \mathcal{D}[O(x)]\end{aligned}$$

where $\mathcal{D}[O(x)]$ is the non-unitary contribution. (They have different kinds of $\mathcal{D}[O(x)]$ corresponding to different physical interpretations.) Intuitively, $i \int dy^3 y^i \mathcal{D}[O(x)]$ is dressing the term $i[\int d^3y y^i T^{00}, O(x)]$, which schematically corresponds to a small movement in the time direction. (Remember $H = \int d^3x T^{00}$, and H and $\mathcal{D}[O(x)]$ together generate time evolution in the Lindblad equation.)

Such theories have the following difficulties. Firstly, it is taking assumptions and conclusions from unitary QFT without justification. We need to remember how we derived Eqn 2.11 in Unitary QFT. It comes from variation of the Lagrangian with respect to an infinitesimal tensor

$$D\mathcal{L} = \epsilon_{\lambda\sigma} x^\lambda \partial^\sigma \mathcal{L} = \partial_\mu [\epsilon_{\lambda\sigma} x^\lambda g^{\mu\sigma} \mathcal{L}]$$

From the Noether theorem, we find the conserved current

$$\begin{aligned}J^\mu &= \pi^\mu \epsilon_{\lambda\sigma} x^\lambda \partial^\sigma \phi - \epsilon_{\lambda\sigma} x^\lambda g^{\mu\sigma} \mathcal{L} \\ &= \epsilon_{\lambda\sigma} \left(\pi^\mu x^\lambda \partial^\sigma \phi - x^\lambda g^{\mu\sigma} \mathcal{L}\right).\end{aligned}$$

Since this current must be conserved for all six independent anti-symmetric matrices $\epsilon_{\lambda\sigma}$, the quantity inside the parentheses that is anti-symmetric in λ and σ must be conserved, i.e. $\partial_\mu M^{\mu\lambda\sigma} = 0$, where

$$\begin{aligned} M^{\mu\lambda\sigma} &= \left(\pi^\mu x^\lambda \partial^\sigma \phi - x^\lambda g^{\mu\sigma}\mathcal{L}\right) - (\lambda \leftrightarrow \sigma) \\ &= x^\lambda \left(\pi^\mu \partial^\sigma \phi - g^{\mu\sigma}\mathcal{L}\right) - (\lambda \leftrightarrow \sigma) \\ &= x^\lambda T^{\mu\sigma} - x^\sigma T^{\mu\lambda} \end{aligned}$$

Correspondingly, the conserved charges that generate space-time rotations/boosts are

$$Q^{\lambda\sigma} = \int d^3x M^{0\lambda\sigma} = \int d^3x \left(x^\lambda T^{0\sigma} - x^\sigma T^{0\lambda}\right). \tag{2.12}$$

By imposing canonical commutation relationship $[\phi(x), \Pi(x)] = i$ where $\Pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}$, one can show that

$$[Q^{\lambda\sigma}, \phi(x)] = ix^\lambda \partial^\sigma \phi(x) - ix^\sigma \partial^\lambda \phi(x). \tag{2.13}$$

We can see that in order to derive Eqn 2.12, we need to extremize the Lagrangian and use the Noether theorem. However, for the non-unitary theory, there is no Lagrangian description to start with. Although it may be formulated in Schwinger-Keldysh form, and some related literature uses the terms like "Keldysh-action" or "super-Lagrangian", these are not equivalent to the action and Lagrangian in unitary QFT, since extremizing them would not yield the equation of motion, and there is no Noether theorem to give us the conserved charges that generate the symmetries. (See Chapter 5 for a detailed analysis) Hence, there is no way to justify that directly dressing Eqn 2.12 with dissipation would serve as the non-unitary boost generators, since the foundations to derive $Q^{\lambda\sigma}$ do not exist in non-unitary theories. Moreover, since time evolution is non-unitary, energy conservation is explicitly broken, and we will not have a conserved stress energy tensor $T^{\mu\nu}$ at all, dressing $\int d^3x x^i T^{00}$ with $\int dx^3 x^i \mathcal{D}[O(x)]$ with the hope that it generates the non-unitary boost, makes even less sense.

Mathematically, such a non-unitary boost acting on the Lindblad equation will hardly be Lorentz covariant. In unitary QFT, an infinitesimal the Lorentz boost $\Lambda = 1 + \delta\epsilon$ is generated by $U(1 + \delta\epsilon) = I + i\epsilon_{\lambda\sigma} Q^{\lambda\sigma}$, and we can show that

$$U(\Lambda)\phi(x)U(\Lambda)^\dagger = \phi(\Lambda x) \tag{2.14}$$

However, this ceases to be true under the non-unitary boost Q^{0i}, even if we temporarily ignore the conceptual difficulties discussed above. Assuming the Lorentz

boost on $\phi(x)$ is generated by $\mathcal{V}_\Lambda[\phi(x)] = \exp[i\Lambda_{0i}Q^{0i}[\phi(x)]]$, since

$$Q^{0i}[\phi(x)] = ix^0\partial^i\phi(x) - ix^i\partial^0\phi(x) + i\int dy^3 y^i \mathcal{D}[\phi(x)]$$

the third term will mess things up when we try to reorganize the terms into closed forms like $\phi(\Lambda x)$. This actually makes sense, since a dissipative channel will generally introduce noise to the operator, just like it will map a pure state into a mixed state.

And without $\mathcal{V}_\Lambda[\phi(x)] = \phi(\Lambda x)$, it is hard to justify that the Lorentz transformation acting on the equation of motion, the Lindblad equation, will be in a covariant form, like the equation with the unitary generator.

Moreover, whether Q^{0i} satisfies the Poincare algebra is not clear as well. Unlike the unitary case, now the commutation relationship depends on the structure of the non-unitary term $\mathcal{D}[\cdot]$. Without knowing the explict $\mathcal{D}[\cdot]$ there is no way to check whether the Poincare algebra is satisfied.

Even if some special $\mathcal{D}[\cdot]$ could satisfy Poincare algebra, there are no guarantees that they generate the Lorentz boosts for all operators. This is a very serious problem that would make this theory not viable and computation impossible.

In unitary QFT, Eqn 2.14 guarantees $U(\Lambda)O[\phi(x)]U(\Lambda)^\dagger = O[\phi(\Lambda x)]$, where $O[\phi(x)]$ is any polynomial in $\phi(x)$.

For example, $U(\Lambda)\phi(x)^2U(\Lambda)^\dagger = U(\Lambda)\phi(x)U(\Lambda)^\dagger U(\Lambda)\phi(x)U(\Lambda)^\dagger = \phi(\Lambda x)^2$.

It also gives us the crucial property of the correlation functions. This is because

$$\langle 0|\phi(x)\phi(y)|0\rangle = \langle 0|U(\Lambda)\phi(x)U(\Lambda)^\dagger U(\Lambda)\phi(y)U(\Lambda)^\dagger|0\rangle = \langle 0|\phi(\Lambda x)\phi(\Lambda y)|0\rangle \tag{2.15}$$

Where we used the fact that the vacuum state is invariant under the Lorentz transformation.

However, when dissipative channels are involved, even if $\mathcal{V}_\Lambda[\phi(x)] = \phi(\Lambda x)$, it does not necessarily mean $\mathcal{V}_\Lambda[\phi(x)^2] = \phi(\Lambda x)^2$. The structure of a generic dissipative channel is

$$\mathcal{V}[O] = \sum^i K_i O K_i^\dagger \tag{2.16}$$

This means in general $\mathcal{V}[\phi(x)^2] \neq \mathcal{V}[\phi(x)]^2$.

As a result, while the non-unitary boost generators $\mathcal{V}_\Lambda$ may generate the Lorentz boost for field operators $\phi(x)$, it will not generate the Lorentz boost for other

generators $O(x)$. This means $\mathcal{V}_\Lambda$ do not really serve the role as the Lorentz boost generators.

Let us see what it means for correlation functions. Because of the structure of the non-unitary generators Eqn 2.16, in general

$$\langle 0|\phi(x)\phi(y)|0\rangle \neq \langle 0| \sum^{i} K_i \phi(x) K_i^\dagger \sum^{j} K_j \phi(y) K_j^\dagger |0\rangle = \langle 0|\phi(\Lambda x)\phi(\Lambda y)|0\rangle \tag{2.17}$$

Namely, correlation functions are no longer Lorentz invariant.

Finally, the physical results of non-unitary boost generators are not acceptable. Such non-unitary channels will inevitably boost pure states into mixed states, except in the very special cases where the input pure state is a stable point of the channel. For example, Q^{0i} will in most cases boost a single particle pure state like $|p\rangle\langle p|$ into a mixture of multi-particle states. This is fundamentally different than that in the unitary case, where a single particle pure state $|p\rangle\langle p|$ will be boosted into another single particle state with a different momentum $|\Lambda p\rangle\langle \Lambda p|$, which is what we originally mean by boosting.

One way to interpret this phenomenon is that when we are boosted in the frame of the observer with a constant speed, that observer will not see a pure state for the reason discussed above. However, if we take the active view of the boost, which simply means changing the speed/momentum of the state, obviously, a single particle pure state should be boosted into another single particle pure state, just like in the unitary case. This means physically, active transformations are generated by unitary generators, while passive transformations (that change coordinates) will be generated by non-unitary generators. This is fundamentally different from any other physical theory, where passive and active transformations are represented in the same way.

In fact, that a Lorentz boost will boost a pure state into a mixed state seems to contradict the notion of Lorentz covariance directly. Since it suggests that some observers are special (the ones corresponding to the pure state). In a Lorentz covariant theory, observers traveling at different constant speeds are equivalent, since they cannot tell any differences by local measurements. While in this case, they can, as long as they can measure the purity of some localized states.

2.5 Non-Markovian dynamics are even worse

Finally, we would like to point out that although the above discussion is based on Markovian time evolution, all the difficulties in either the unitary or non-unitary Lorentz boost still exist when the dynamics is non-Markovian. In fact, moving to non-Markovian dynamics would not save the theory, but will make the problems even worse.

Let us think about the corresponding quantum open system theory which has the same dynamics. The Markovian process means the reservoir (the environment that interacts with the system) is either infintely large, so that the change to the reservoir due to system-reservoir interaction is negligible, or the reservoir is refreshed constantly back to the initial state. And as a result, the back reaction to the system does not depend on the memory of the reservoir. On the contrary, in a non-Markvoian process, the memory of the reservoir does matter, and as a result the Lindblad equation describing the evolution of the system can depend explicitly on time. Ref. (Zhang, 2018) derived the non-Markovian Lindblad equation for a finite-dimensional system in an open quantum system setting,

$$\begin{aligned}\frac{d\rho(t)}{dt} =&\frac{1}{i}[\tilde{H}_S(t,t_0),\rho(t)] + \sum_{ij}\tilde{\boldsymbol{\gamma}}_{ij}(t,t_0)L_{a_i^\dagger,a_j}[\rho(t)]\\ &+\sum_{ij}[2\boldsymbol{\gamma}_{ij}(t,t_0) \pm \widetilde{\boldsymbol{\gamma}}_{ij}(t,t_0)]L_{a_j,a_i^\dagger}[\rho(t)]\,,\end{aligned} \tag{2.18}$$

where the super-operator $L_{a_i,a_j^\dagger}[\rho(t)]$ is defined as the standard Lindblad operator

$$L_{a_i,a_j^\dagger}[\rho(t)] \equiv a_i\rho(t)a_j^\dagger - \frac{1}{2}a_j^\dagger a_i\rho(t) - \frac{1}{2}\rho(t)a_j^\dagger a_i\,. \tag{2.19}$$

and t_0 notates the initial condition. The explicit time dependence of the system Hamiltonian and effective coupling coefficient matrix will break the Lorentz covariance. The non-Markovian Lindblad equation, even though generalized to infinite dimension, and written in the Lorentz covariant Heisenberg picture form like in Eqn (2.3), with some simplifying assumptions regarding the effective coupling coefficient matrix,

$$\begin{aligned}\partial^0 A =&i[P^0(t,t_0),A] + \gamma_{ab}(t,t_0)\int \widetilde{dp}p^0\\ &\left(Q_a^\dagger(\vec{p})AQ_b(\vec{p}) + \frac{1}{2}\{Q_a^\dagger(\vec{p})Q_b(\vec{p}),A\}\right)\end{aligned}$$

will not transform covariantly under conjugation of unitary boost generators $U(\Lambda)$, let alone the more complicated non-unitary boost generators.

In conclusion, neither unitary nor non-unitary boost generators could be consistent with non-unitary time evolution, as we have shown, from conceptual, mathematical and physical grounds. There are other difficulties of the non-unitary QFT as well, but we think this is the most serious problem. This seems to suggest that the objective collapse theories of the quantum measurement problems, which have a similar mathematical formulation and dynamics, cannot be consistent with Lorentz covariance as well.

Chapter 3

OPEN QUANTUM FIELD THEORY CANNOT BE LORENTZ COVARIANT

3.1 Introduction

Consider a global system consisting of the system and the environment. The dynamics of the global system is unitary and Lorentz covariant, just like in the normal quantum field theory. But if we only have access to the system and have no control of the environment, the system is described by the theory of quantum open system. Is it possible that the reduced dynamics, which is non-unitary, is still Lorentz covariant?

This question has been discussed since the 1980s. One side of the discussion is motivated by fundamental interests. The reduced non-unitary dynamics phenomenology has the same description as a theory in which the time evolution is fundamentally non-unitary, as pointed out in (Banks, Peskin, and Susskind, 1984), and that fundamental non-unitarity could potentially serve as the solution to the Black Hole information paradox, as suggested by Hawking in (Hawking, 1982). On the other side, the reduced dynamics is interesting by itself, which can be applied to describe the decay of unstable particles (Alicki, Fannes, and Verbeure, 1986) . Alicki et al. in Ref. (Alicki, Fannes, and Verbeure, 1986) laid out the mathematical foundation of such reduced non-unitary dynamics, which according to them is Lorentz covariant.

In Chapter 2, we pointed out that if a theory is fundamentally non-unitary, it cannot be Lorentz covariant. One of the must serious problems is that the representation of the Lorentz group, be it unitary or not, would not make sense in non-unitary dynamics. There are some differences for the case here, where the dynamics of the underlying global system is unitary, and the non-unitarity arises because we only have access to the system. However, this emergent non-unitarity still conflicts with the Lorentz covariance. One major issue is still the representation of the Lorentz group, although it is different from the fundamental non-unitary theory.

Moreover, for the equation of motion to be Lorentz covariant, one crucial assumption is that the self-Hamiltonian of the system would transform as the first component of the four-momentum operator. We find out that this assumption is invalid.

Even if we ignore these difficulties, the non-unitary quantum open system field

theory does not work as desired, since it does not apply to multiple particles with different momenta.

This paper is organized as follows. In section II, we introduce the main features of the non-unitary quantum open system field theory, and why Alicki et al. thought it is Lorentz covariant. In section III, we discuss the conflict with the representation of the Lorentz boost. In section IV, we discuss the unjustified assumption that the system Hamiltonian could transform in a Lorentz covariant fashion. In section V, we discuss the problems when applied to a multi-particle system. In section VI, we summarize the results, and clarify the potential confusion related to experiments with atomic clocks that confirm time dilation.

3.2 Non-unitary quantum open system field theory

We start with the Heisenberg picture. Consider a system with algebra of observables $\mathscr{B}(\mathscr{H})$, all bounded operators on a Hilbert space $\mathscr{H}$; as states of the system we consider the expectation functional defined by means of density matrices ρ on $\mathscr{H}$. The irreversible time evolution from time $t = 0$ up to time t is given by the dynamical map Γ_t, which is linear and complete positive and trace preserving (CPTP).

$$\Gamma_t : O \in \mathscr{B}(\mathscr{H}) \rightarrow \Gamma_t[O] \in \mathscr{B}(\mathscr{H})$$

Restricting to Markovian cases, Γ_t satisfies the following

$$\Gamma_0 = 1, \Gamma_{t_1}\Gamma_{t_2} = \Gamma_{t_1+t_2},$$

where $t_1, t_2 \in R^+$. It is clear that $\{\Gamma_t | t \in R^+\}$ is a representation of the semigroup R^+ into a one-parameter semigroup of unity preserving CPTP maps of $\mathscr{B}(\mathscr{H})$.

Under the uniform continuity condition of the map

$\Gamma : t \rightarrow \Gamma_t$,⁄

there exists a generator Lindbladian L such that

$$\Gamma_t = \exp(tL)$$

In relativistic settings, R^+ is replaced by the future cone

$$\mathscr{F} = \{b \in R^4 | b^2 = (b, b) \geq 0, b_0 \geq 0\}$$

$\mathscr{F}$ is an additive semi-subgroup of R^4.

For $a, b \in \mathscr{F}$, we have the semigroup property

$$\Gamma_a \Gamma_b = \Gamma_{a+b}$$

and Poincare covariance property

$$U(a,\Lambda)\Gamma_b U(a,\Lambda)^\dagger = \Gamma_{\Lambda b}. \tag{3.1}$$

In Eqn(3.1), we directly used unitary representation $U(a,\Lambda)$. This is different in the fundamentally non-unitary theory, where the representation can in principle be non-unitary, as discussed in chapter 2. This is because, for the theory here, the non-unitarity arises due to the lack of information about the environment. The underlying theory of the global system is still unitary, and has a unitary representation of Poincare group $U(a,\Lambda)$. Clearly $U(a,\Lambda)$ works on all observables of the system and the system plus environment. It is thus natural to use $U(a,\Lambda)$ here. In fact, all previous literature regarding non-unitary time evolution assumed $U(a,\Lambda)$ by default, no matter if the non-unitarity is fundamental or emergent.

As argued by Alicki et al., we can apply Γ_a on bosonic fields ϕ to describe its decay process. With $a(p)$ the usual annihilation operators, we have $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$. We can find that $\Gamma_b = \exp(b^\mu L_\mu)$, where

$$L_\mu[O] = \int \widetilde{dp} p_\mu \left(i[a(p)^\dagger a(p), O] + \frac{\gamma}{2}\{a(p)^\dagger a(p), O\}\right) \tag{3.2}$$

where $\widetilde{dp} := \frac{d^3p}{(2\pi)^3}$ is the Lorentz invariant measure, and O is an operator on ϕ.

We can see that the time component of Eqn (3.2) gives us the normal Lindblad equation

$$\partial_0[O] = i[P_0, O] + \frac{\gamma}{2}\int \widetilde{dp} p_0 \left(\{a(p)^\dagger a(p), O\}\right) \tag{3.3}$$

where $H_s := P_0 = \int \widetilde{dp} p_0 a(p)^\dagger a(p)$ is the Hamiltonian for the system. Eqn (3.3) is manifestly Lorentz covariant, as applying $U(\Lambda)$ by conjugation on it just transforms to different component in Eqn (3.2). Here it is assumed that H_s can indeed be considered as the time component of a four-vector operator P^μ. This assumption is widely used in the community, however, as we discuss in Section IV, it cannot be justified.

We can also show that the time evolution of the operators commute with the Lorentz transformation:

$$U(\Lambda)\Gamma_b[O]U(\Lambda)^\dagger = \Gamma_{\Lambda b}[U(\Lambda)OU(\Lambda)^\dagger].$$

Alicki et al. further discussed an example of how to apply it to describe the decay of moving particles. We will discuss this later.

3.3 Contradictions with unitary representation of the Lorentz group

In Chapter 2, we discussed the difficulties with the Lorentz representation of a fundamentally non-unitary theory. In this case, when the non-unitarity is emergent, the problems are slightly different, but the result is the same, namely, the non-unitarity conflicts with the unitary representation of the Poincare group.

First of all, in the fundamentally non-unitary case, the representation of the Poincare group may be either unitary or non-unitary, at least in theory. Here, the representation has to be unitary. This is because in the underlying unitary quantum field theory, the representation is unitary $U(\Lambda)$. This representation will transform both the system and the environment field in the global theory. When we trace out the environment, $U(\Lambda)$ obviously still works on the system field operators.

Could it be the case that a different representation of Poincare group $V(\Lambda)$ only works on the system field but not the environment field? The answer is no, simply because if that is the case, it will lead to non-localities. Consider the system field $\phi(x)$ coupled to the environment field $\psi(x)$ by the term $\phi(x)\psi(x)^2$. If $V(\Lambda)$ only acts on $\phi(x)$, it will directly lead to non-locality

$$V(\Lambda)\phi(x)\psi(x)^2V(\Lambda)^\dagger = \phi(\Lambda x)\psi(x)^2 \tag{3.4}$$

Hence, the only Poincare representation in this case, is the same one as in the global theory, namely,

$$U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p) \tag{3.5}$$

Here $a(p)$ is the annihilation operator for the free mode of the system field. As we discussed in Chapter 2, Eqn (3.5) is invalid when the theory is fundamentally non-unitary. This is because for Eqn (3.5) to hold, the Klein-Gordon equation must be satisfied. While in that theory, it does not, since the equation of motion is the Lindblad equation. The situation is different here, and there is a way to avoid the contradiction as in Chapter 2. This is because, in the global theory, the fundamental time evolution is unitary, and we can go back to the asymptotic past Hilbert space, where the theory is asymptotically free. On the other hand, in the fundamentally non-unitary theory, there is no asymptotic past Hilbert space where the theory is asymptotically unitary, so that the Klein-Gordon equation is satisfied.

Consider a system field $\phi(x)$ coupled to the environment field $\psi(x)$ through the term $\mathcal{L}_{int}$. The total Hamiltonian density is

$$\mathcal{H} = \frac{1}{2}(\partial^2 + m^2)\phi(x) + \frac{1}{2}(\partial^2 + M^2)\psi(x) - \mathcal{L}_{int}$$

In the asymptotic past Hilbert space, $\mathcal{H}$ equals the free Hamiltonian density for both fields,

$$\lim_{t\to-\infty} \mathcal{H} = \frac{1}{2}(\partial^2 + m_{in}^2)\phi_{in}(x) + \frac{1}{2}(\partial^2 + M_{in}^2)\psi_{in}(x)$$

As the result, the Klein-Gordon equation is satisfied for $\phi_{in}(x)$. Hence, at least in the asymptotic past(future) Hilbert space, Eqn (3.5) is valid.

However, this does not solve the full problem, since Eqn (3.5) is valid ONLY in the asymptotic past(future) Hilbert space. In the scattering regime, where the interaction is turned on, it no longer works, since the Klein-Gordon equation is no longer satisfied. More importantly, it is the scattering regime that we are mainly interested in. Because only in this regime, can we trace out the environment and get emergent non-unitary dynamics for the system. While in the asymptotic past (future) Hilbert space, since the system field $\phi_{in}(x)$ and the environment field $\psi_{in}(x)$ decouple, after tracing out the environment the dynamics of the system field is still unitary.

There is one interesting example that merits a deeper investigation. Consider the case for linear coupling, where $\mathcal{L}_{int} = g\phi(x)\psi(x)$. Let $a(p)^\dagger$ and $b(p)^\dagger$ be the creation operators that create one excitation of $\phi(x)$ and $\psi(x)$ in the mode p respectively. Precisely speaking, when the interaction is turned on, the notion of particle no longer makes sense since $a(p)^\dagger$ is time dependent and no longer creates a momentum eigenstate $|p\rangle$. However, in the case where g is small enough compared to the self Hamiltonian, we can treat $\mathcal{L}_{int}$ as a perturbation around the free theory, and continue using $a(p)^\dagger$ as if it creates a (slightly modified) system particle. This is also what we assumed when we traced out the environment and derived the Lindblad equation. Still, $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$ is invalid, for the reasons discussed above, but the special structure of $\mathcal{L}_{int}$ might save us.

Due to $\mathcal{L}_{int}$ being linear, it is possible to construct two new fields, $\phi_1(x)$ and $\phi_2(x)$, as the linear combination of $\phi(x)$ and $\psi(x)$, such that the Lagrangian density expressed in the new fields is free.

$$\mathcal{L} = \frac{1}{2}[(\partial\phi_1(x)) \cdot (\partial\phi_1(x)) - m_1^2\phi_1^2(x) + (\partial\phi_2(x)) \cdot (\partial\phi_2(x)) - m_2^2\psi_2(x)^2]$$

Let $\phi_1 = a_{11}\phi + a_{12}\psi$, $\phi_2 = a_{21}\phi + a_{22}\psi$,

$$
\begin{aligned}
2\mathcal{L} &= [(\partial\phi_1(x)) \cdot (\partial\phi_1(x)) - m_1^2\phi_1^2(x) + (\partial\phi_2(x)) \cdot (\partial\phi_2(x)) - m_2^2\psi_2(x)^2] \\
&= (a_{11}\partial\phi + a_{12}\partial\psi) \cdot (a_{11}\partial\phi + a_{12}\partial\psi) + (a_{21}\partial\phi + a_{22}\partial\psi) \cdot (a_{21}\partial\phi + a_{22}\partial\psi) \\
&- m_1^2(a_{11}\phi + a_{12}\psi)^2 - m_2^2(a_{21}\phi + a_{22}\psi)^2 \\
&= (a_{11}^2 + a_{21}^2)(\partial\phi)^2 + (a_{11}^2 + a_{22}^2)(\partial\psi)^2 - (m_1^2a_{11}^2 + m_2^2a_{21}^2)\phi^2 - (m_1^2a_{12}^2 + m_2^2a_{22}^2)\psi^2 \\
&+ (2a_{11}a_{12} + 2a_{21}a_{22})\partial\phi \cdot \partial\psi + (2m_1^2a_{11}a_{12} + 2m_2^2a_{21}a_{22})\phi\psi
\end{aligned}
$$

Comparing terms, we get

$$
\begin{aligned}
a_{11}^2 + a_{21}^2 = a_{11}^2 + a_{22}^2 &= 1 \\
(m_1^2a_{11}^2 + m_2^2a_{21}^2) &= m^2 \\
(m_1^2a_{12}^2 + m_2^2a_{22}^2) &= M^2 \\
a_{11}a_{12} + a_{21}a_{22} &= 0 \\
(2m_1^2a_{11}a_{12} + 2m_2^2a_{21}a_{22}) &= g
\end{aligned}
$$

Solving the above six equations for six unknown variables, we can get a_{ij} and m_1, m_2, although the results are too cumbersome to be included here.

Now the new field $\phi_1(x)$ and $\phi_2(x)$ are completely free, so we can mode expand them

$$
\begin{aligned}
\phi_1(x) &= \int \widetilde{dp}c(p)e^{ipx} + c(p)^\dagger e^{-ipx} \\
\phi_2(x) &= \int \widetilde{dp}d(p)e^{ipx} + d(p)^\dagger e^{-ipx}
\end{aligned}
$$

And $U(\Lambda)c(p)U(\Lambda)^\dagger = c(\Lambda p)$ is valid in the whole space-time.

However, this is not really what we want. What we want is a system that is (weakly) coupled to the environment. And when we trace out the environment field, its dynamics will be described by the Lindblad equation. The ϕ_1 particle as created by $c\dagger(p)$ is the combination of our system and environment particles. Since we cannot measure the environment, we will not be able to measure ϕ_1 as well. Besides, the ϕ_1 is just a normal free Klein-Gordon field, which we are not interested in. What we are interested in is original system field, because its non-unitary dynamics could potentially be used to describe interesting phenomena such as particle decay.

As we can see in the above examples, only in free theory, would $U(\Lambda)a(p)U(\Lambda)^\dagger = a(\Lambda p)$ hold. While for our non-unitary field, it does not. As a result, our non-unitary field cannot form a unitary representation of the Poincare group as expected.

3.4 System Hamiltonian and Momentum operators do not form a four-momentum

In general, to make the Lindblad equation Eqn (3.3) fully covariant, a key assumption is that the system Hamiltonian H_s will transform as the first component of the Lorentz four-momentum operator P^μ under the Lorentz transformation. Although this looks plausible, as we find out, this assumption is invalid.

In quantum open system theory, we define system Hamiltonian H_s as the part of the total Hamiltonian that only includes the operator acting on the system. In the limit of the system-environment coupling going to 0, H_s drives the time evolution of the system. As the coupling turns on, the time evolution will be affected by the coupling to the environment, and the equation of motion becomes the Lindblad equation as a result. With such a definition of H_s, it will not form a four-momentum operator with some "system momentum operator P_s^i" . The first reason is, that such "system momentum operator P_s^i" cannot be well-defined.

Let us remember how we define Hamiltonian and momentum operators, and why they form a four-momentum operator in quantum field theory. There are two ways to define them. The first one comes from Noether theorem and the stress-energy tensor.

$$P^\mu := \int d^3x T^{0\mu}$$

P^μ as defined here can be proven to transform as a four-vector operator under the Lorentz transformation in the same way as described in the above section. The key step of the proof is by using the conservation of the stress-energy tensor $\partial_\mu T^{\mu\nu} = 0$.

Such a definition of the global four-momentum operator cannot be generalized to define the "system momentum operator". This is because the system is not a closed system due to the interaction with the environment. There is no spatial and temporal translation invariance for the system alone, and we cannot use the Noether theorem to get the conserved Noether charge correspondingly. As a result, we cannot define the system momentum operator as $P_s^\mu := \int d^3x T_s^{0\mu}$, simply because we don't know what this $T_s^{0\mu}$ is, what it means physically, how we compute or define it, etc.

One might really want to push the limit, and define the system field operator by manually making all terms in $T^{0\mu}$ that involve the environment field operator to be 0. This is the same as dropping all the terms in the full Lagrangian that involve the environment field operator, and applying the Noether theorem on the remaining terms. Yet this will not do the job, because the resulting $T_s^{0\mu}$ is not conserved. And without the conservation of $T_s^{0\mu}$, we cannot prove that $P_s^\mu := \int d^3x T_s^{0\mu}$ transforms

as a four-vector operator, for the exact same reason as in the previous section and Chapter 2.

Another way to define the Hamiltonian and momentum operators in QFT is to define them as the generators of temporal and spatial translation. This method cannot be applied to define P_s^μ either. The time evolution is generated by the global Hamiltonian, or the Lindbladian for the system alone by approximation. The system Hamiltonian H_s as defined above does not generate the time evolution for the system at all. For spatial translation, it is also generated by the global momentum operator. There are no ways to define some P_s^i that generates the spatial translation for the system alone, and that forms a four-momentum with H_s.

Some readers may point out the special case in Eqn (3.2), where if the system Hamiltonian is defined as $H_s := \int \widetilde{dp}\omega a(p)^\dagger a(p)$, it seems one can define a four-momentum operator

$$P_s^\mu := \int \widetilde{dp} p^\mu a(p)^\dagger a(p)$$

where H_s can be considered as the first component P_s^0 that transforms properly under the Lorentz transformations.

There are a few caveats. Most importantly, if we define $a(p)^\dagger$ as the operator that creates a free mode of the system field, it will not transform as desired $U(\Lambda)a(p)^\dagger U(\Lambda)^\dagger = a(\Lambda p)^\dagger$, due to the coupling with the environment, as discussed in Section II. Even if we pretend this problem does not exist, there are still other difficulties. The first is that this only applies to the free theory, where $H_s := \int \widetilde{dp}\omega a(p)^\dagger a(p) = \frac{1}{2}(\partial^0\phi)^2 + \frac{1}{2}(\partial^i\phi)^2 + \frac{1}{2}m^2\phi^2$. This is because only in free theory, can we expand the field operators $\phi(x)$ into modes as created by $a(p)^\dagger$

$$\phi(x) = \int \widetilde{dp} a(p)^\dagger e^{-ipx} + h.c.$$

This breaks down whenever we introduce any interactions. For example, if we add a simplest ϕ^3 interaction term, we cannot write the system Hamiltonian $H_s := \int \widetilde{dp}\omega a(p)^\dagger a(p) = \frac{1}{2}(\partial^0\phi)^2 + \frac{1}{2}(\partial^i\phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^3$ into $a(p)$ and $a(p)^\dagger$since the field is no longer free.

Even if someone may want to do this as an "approximation" (which is completely unjustified), this will not serve the purpose. The additional self interaction term will explicitly break the Lorentz covariance. This is because, the reason that the free Hamiltonian $H_s := \int \widetilde{dp}\omega a(p)^\dagger a(p) = \frac{1}{2}(\partial^0\phi)^2 + \frac{1}{2}(\partial^i\phi)^2 + \frac{1}{2}m^2\phi^2$ transforms

nicely is that after the mode expansion and some computation, we end up with $\int \widetilde{dp} p_0 a(p)^\dagger a(p)$. The key here is p_0 provides the desired the Lorentz label. When adding the interacting terms, be it ϕ^3 or ϕ^4, the mode expansion of them will only be $O[a(p)]^3$or $O[a(p)]^4$, and we will not get that p_0 provides the desired the Lorentz label.

When we try to Lorentz transform the Lindblad equation, the covariance breaks down explicitly. This is because to show the Lorentz covaraince of the Lindblad equations Eqn (3.3) and Eqn (3.2), we must either define the system Hamiltonian part as a component of a four-momentum operator, or write it into modes that transform properly. In the interacting theory, we can do neither, for the reason discussed above. As a result, we would be left with

$$\partial_0[O] = i[\frac{1}{2}(\partial^0\phi)^2+\frac{1}{2}(\partial^i\phi)^2+\frac{1}{2}m^2\phi^2, O]+\frac{\gamma}{2}\int \widetilde{dp} p_0 \left(a(p)^\dagger O a(p) - \{a(p)^\dagger a(p), O\}\right)$$

If we do a Lorentz transformation, the self-Hamiltonian term would explicitly breaks the Lorentz covariance, since it is the only term that doesn't have a Lorentz index. (As a side note, in normal free QFT, the momentum operator obtained through Noether theorem is $P = -\int d^3x \Pi(x)\nabla\phi(x)$. When an interacting term like $\lambda\phi^3$ is added to the Lagrangian it is also added to the Hamiltonian, but P stays the same.)

3.5 Problem of preferred time frame

Even if we ignore the above difficulties in constructing this theory, it still will not work as expected. The reason is, it can only be applied to a single point-like particle. It cannot be applied when there are multiple particles with different momentum.

In Ref. Alicki, Fannes, and Verbeure, 1986, Alicki et al. applied this theory to describe particle decay. Basically, they evolve the particle number operator $n_p = a_p^\dagger a_p$ as follows:

$$n_p(b) = \mathcal{E}^*{}_b[n_p(0)] = n_p(0)e^{-\gamma p b}.$$

Here $b = (t, \vec{v}t)$ is aligned with the direction of the momentum p, and $\vec{v} = \vec{p}/p_0$, and $\mathcal{E}^*{}_b = \exp(b^\mu L_\mu)$ is the time evolution super operator as introduced in Section II. This object is manifestly Lorentz invariant, as it should be.

However, there is a very strong restriction, that we have to evolve operators along one particular direction $b = (t, \vec{v}t)$. This basically selects one preferred time direction. Let us think about the case where b_{lab} is our lab frame time vector $b_{lab} = (t_{lab}, 0)$. For a particle with non-zero momentum p, at time t_{lab} it will arrive at location $\vec{x}$. If

we evolve with the lab frame time vector, instead of the inertial time of the particle, we end up with

$$n_p(b_{lab}) = \mathcal{E}^*{}_b[n_p(0)] = n_p(0)e^{-\gamma E t_{lab}}$$

Since Et_{lab} is not Lorentz invariant, this result is manifestly frame dependent, so that different observers will read different numbers of remaining particles. This clearly is not acceptable. (This is the "incorrect" method in the Preskill paradox.)

Let us consider another example. Say we have some other particles with momentum q. How do we use this theory to describe these two groups of particles together? If we chose to evolve alone b_p ,which is aligned with the direction of p, the total particle number operator $n = n_p + n_q$,

$$\begin{aligned} n(b) = \mathcal{E}^*{}_b[n(0)] &= n_p(0)e^{-\gamma p b} + n_q(0)e^{-\gamma q b} \\ &= n_p(0)\exp[-\gamma(p_0 t - \vec{p}^2 t/p_0)] + n_q(0)\exp[-\gamma(q_0 t - \vec{q}\cdot\vec{p}t/p_0)] \\ &= n_p(0)\exp[-\gamma m^2/p_0)] + n_q(0)\exp[-\gamma(q_0 t - \vec{q}\cdot\vec{p}t/p_0)]. \end{aligned}$$

Here in the third line, we used the relativistic energy-momentum relation $p_0^2 - \vec{p}^2 = m^2$. We can see that, in the first term, we got the correct result for particles with momentum p, but the second term does not work, since q is not aligned in the direction of p.

Now, let us look at the difference in the result for these two methods, namely evolving with the internal time b_{int} or lab time b.

$$n_p(b_{int})/n_p(b) = e^{-\gamma\vec{p}\cdot\vec{b}}$$

We can see the difference is the spatial damping term. Namely, if at time t_{lab}, the lab is at spatial location $\vec{x} = 0$, and the particle is at spatial location $\vec{x} = \vec{b}$, the difference is exactly as if there is a spatial damping factor between 0 and $\vec{b}$.

Let us look at this from a different angle. For the field decomposition in unitary QFT

$$\phi(x) = \int \tilde{dp} a_p e^{-ipx} + a_p^\dagger e^{ipx},$$

if we want to evolve this field from x to $x + b$, we should use

$$\mathcal{E}^*{}_b[\phi(x)] = \int \tilde{dp} a_p e^{-ip(x+b)} e^{-\frac{\gamma}{2}pb} + a_p^\dagger e^{ip(x+b)} e^{-\frac{\gamma}{2}pb}.$$

If instead, we choose to evolve the field only with our lab frame time $b_{lab} = (t_{lab}, 0)$, we have

$$\mathcal{E}^*{}_{b_{lab}}[\phi(x)] = \int \tilde{dp} a_p e^{-ipx} e^{-iEt_{lab}} e^{-\frac{\gamma}{2}Et_{lab}} + a_p^\dagger e^{ipx} e^{-iEt_{lab}} e^{-\frac{\gamma}{2}Et_{lab}}.$$

These two fields differ by a factor of $e^{-i\vec{p}\cdot\vec{b}(1-\frac{i\gamma}{2})}$ and $e^{i\vec{p}\cdot\vec{b}(1+\frac{i\gamma}{2})}$. However, if we somehow believe that fields at spatial location $\vec{x}$ and the spatial location $\vec{x}+\vec{b}$ differ by a factor generated by the displacing super operator $\mathcal{D}_c^*$, where c is a space-like vector, then

$$\mathcal{D}^*{}_c[\phi(x)] = \int \tilde{dp} a_p e^{-ip(x+c)} e^{-\frac{\gamma}{2}pc} + a_p^\dagger e^{ip(x+c)} e^{-\frac{\gamma}{2}pc}$$

We can show

$$\mathcal{E}^*{}_b[\phi(x)] = \mathcal{D}^*_{b-b_{lab}}[\mathcal{E}^*{}_{b_{lab}}[\phi(x)]]$$

This is to say, there is an intrinsic spatial damping factor between field operators at different spatial locations. In this way, we can avoid the problem of the preferred time, and regain Lorentz invariance/covariance. For example, if we want to compute the correlation

$$\langle\phi(x+b)\phi(x)\rangle$$

(here the notation $\phi(x+b)$is a little sloppy) , we can either evolve the left field operator to the point that we want to make the measurement $x+b$ by $\mathcal{E}^*{}_b[\phi(x)]$. Or we evolve the field with our lab frame time $\mathcal{E}^*{}_{b_{lab}}[\phi(x)]$, but since the measurement is at $x+b$, not $x+b_{lab}$, we need to displace the field operator to the desired point using $\mathcal{D}^*_{b-b_{lab}}$.

Let us go back to the case of particle decay. If we can agree with the following,

1) If we evolve an operator to some space-time point, operationally, we are supposed to "measure" it at that point.

For example, when we say we want to measure the two-point correlation function $\langle\phi(x+b)\phi(x)\rangle$, we are supposed to measure the field strength at x and at $x+b$. In QFT classes, some professors say we create a particle at x using $\phi(x)$, and compute the probably we can detect it at $x+b$, using $\langle\phi(x+b)\phi(x)\rangle$. This implicitly assumed that the particle is evolved from x to $x+b$. This is actually very imprecise, if not simply wrong, since firstly, the two-point correlation function cannot be interpreted in that way, and secondly, the two-point correlation function cannot be directly measured in reality. What happens is we use an Unruh-DeWitt detector with a trajectory $x(\tau) = (t(\tau), \vec{x}(\tau))$, and try to deduce

$$W(\tau) = \langle\phi(x(\tau))\phi(x(0))\rangle$$

by the transition probability P of the two-level system with some proper settings.

In our non-unitary case, the corresponding physical picture is, the Unruh-DeWitt detector would start at location $x(0)$ and end at location $x(\tau)$. We should either run the Lindblad equation along the trajectory of the Unruh-DeWitt detector and end up at $x(\tau)$, just as for the "correct" method in John's paradox. Or we evolve with the lab frame time, but then displace the field to the ending point of the Unruh-DeWitt detector.

2) We are going to assume the existence of point-like particle, and ignore the uncertainty principle for now.

Say there is a source in our lab, at spatial location 0. At time $t = 0$, we have a number of particles $\hat{n}(0) = \int \tilde{dp} a_p^\dagger a_p$. We evolve this operator with the lab frame time t_{lab}

$$\hat{n}(t_{lab}, 0) = \mathcal{E}^*{}_{t_{lab},0}[\hat{n}(0)]$$

This is the number of the particles at the source at $(t_{lab}, 0)$. Basically, the observer is sitting with this source in the lab, and measures the number at a later time.

However, when the particles have nonzero momentum, they are not going to stay at the source with the observer. For particle with momentum p, after lab frame time t_{lab}, it will arrive at location $\vec{x}$. In order to compute the number of particles at location $(t_{lab}, \vec{x})$, there are two ways. The first is to evolve the number operator with the world line of these particles $b = (t_{lab}, \vec{x})$.

$$\hat{n}(b) = \mathcal{E}^*{}_b[\hat{n}(0)] = e^{-\gamma p b}\hat{n}(0)$$

This is the "correct method" in John's paradox.

The second way is to first evolve the number operator with the lab frame time t_{lab}, and then displace it to the desired location, where we have the assistant who does the measurement.

$$\hat{n}(b) = \mathcal{D}^*{}_{\vec{x}}[\mathcal{E}^*{}_b[\hat{n}(0)]] = e^{-\gamma p b}\hat{n}(0)$$

Using my non-unitary displacement operator, we can see that these two methods yield the same result.

Now, what if we want to know the total number of particles emitted from this source at lab time t_{lab}? Then

$$n_C = \int_{\vec{x}\in C} d\vec{x}\mathcal{D}^*{}_{\vec{x}}[\mathcal{E}^*{}_b[\hat{n}(0)]]$$

where C is the time slice at t_{lab}.

One may want to see how n_C transforms under a Lorentz transformation. The result is invariant, if one does it correctly. After the Lorentz transformation, the points on C are no longer on the same time slice of the boosted observer. Hence, the physical meaning of N_C is, the observer hired an army of assistants distributed on the time slice C (in reality only those in the light cone are needed), and after they all did the measurement at t_{lab}, the observer adds them up. If for some reason the observer is no longer sitting with the source but moving, he will see those assistants doing measurements at different times. After they all finished, the observer adds the numbers together. And the result will be the same.

3.6 Summary and remarks

In summary, due to the difficulties discussed above, quantum open system field theory cannot be considered as Lorentz covariant. Some readers may wonder why, if particle decay is a Lorentz covariant process, the theory aiming to describe it is not Lorentz covariant? Firstly, it is not clear what we mean by saying particle decay is a Lorentz covariant process. The global dynamics including the particle and the environment is indeed fully Lorentz covariant, but it does not mean that after tracing out the environment, the reduced dynamics for the system is still Lorentz covariant. In fact, as we have shown here, the claim that the reduced dynamics is Lorentz covariant cannot be justified.

One may further argue that there have been many experiments using atomic clocks that verified that unstable particles travelling at higher speed have a longer lifetime that matches relativistic predictions. While this is true, it does not mean the reduced dynamics is the Lorentz covariant. There are two ways to understand it.

Firstly, when we say some process is Lorentz covariant, what we mean is that the same process viewed by a different observer in a different reference frame will appear in a covariant fashion. The Lorentz transformation is a transformation into a different reference frame, as we discussed in the above sections. However, the atomic clock experiments don't involve observers in different reference frames. One just measures the initial and final particle numbers, and compares with the results of another process in which the atomic clock is stationary. It is not one process observed by observers in different frames, but two different process.

Secondly, the atomic clock experiment just verifies that the special relativity is correct. One can argue that the key fact is that the global dynamics is Lorentz

covariant. This alone will guarantee that time dilation factor. Whether the reduced dynamics is Lorentz covariant is irrelevant. Or even further, this just verifies that relativity is correct. Think about the experiment of a ruler being shorter when moving faster. This simply means that special relativity is correct, not that the theory describing the ruler is Lorentz covariant. In fact, even if some complicated unstable particles cannot be described by the seemingly Lorentz covariant dynamics, they should still have a longer lifetime when moving at a faster speed.

Chapter 4

SYMMETRIES IN KELDYSH ACTIONS DO NOT TRANSLATE INTO CORRESPONDING DYNAMICAL SYMMETRIES

4.1 Introduction

The Schwinger-Keldysh path integral is a powerful tool in condensed matter physics, which enables us to compute various correlation functions for non-equilibrium systems. It has been extended to study driven-dissipative open quantum systems (Sieberer, Buchhold, and Diehl, 2016). It has also been introduced to study more fundamental physics. John Preskill first proposed to use Schwinger-Keldysh path integral formalism to construct a fundamentally non-unitary quantum field theory, in order to solve the black hole information paradox. Inspired by his notes, Avinash et al. worked out the renormalization of some "super-Lagrantians" of the open system quantum field theory (Avinash, Jana, Loganayagam, et al., 2017; Avinash, Jana, and Rudra, 2019). Jonathan Oppenheim et al. are trying to build a classical-quantum gravity based on such formalism, which they hope could serve as a alternative to the full quantum gravity (Oppenheim and Weller-Davies, 2023).

One of the main reasons the above authors used Schwinger-Keldysh path integral to construct a fundamentally non-unitary open quantum field theory, is that they think that a Lorentz invariant Keldysh action would guarantee a Lorentz covariant dynamics, just as in the path integral formulation of the quantum field theory, a Lorentz invariant action guarantees a Lorentz covariant dynamics. And for a theory to be as fundamental as to serve as a potential solution to the black hole information paradox, or a potential candidate for quantum gravity, Lorentz covariance is clearly a necessity.

However, unfortunately, this is not the case. Lorentz-invariant Keldysh actions do not guarantee Lorentz covariant dynamics. In fact, this is not limited to the Lorentz symmetry. Any on-shell symmetry that a Keldysh action processes will not automatically translates into the corresponding dynamical symmetry. In this chapter, we use the Lorentz symmetry to illustrate this point.

Indeed, the Schwinger-Keldysh formalism is very similar to the path integral. There is a Keldysh action that corresponds to the action. Both are tools to compute various correlation functions. However, one key is missing – the action principle. The

way the Keldysh action is derived means that such formulation does not have the structure of the action principle. Extremizing the Keldysh action does not yield the desired equation of motion, which provides the full information of the physical system (with canonical commutation relationship).

In fact, if we assume there is a Noether theorem for the Keldysh action, and try to work out the symmetry generators from the Keldysh in the same way as the unitary quantum field theory, we find out something very interesting. In order to make sure that the Lindblad equation has same symmetry with the Keldysh action, we have to change one fundamental assumption of the quantum field theory. One underlying assumption of quantum field theory (Whiteman axiom) is that field operators form a (unitary) representation of the Poincare group. But for the Keldysh theory, we need to upgrade this assumption to that the super-operators form a representation of the Lorentz group. This means that the open quantum field theory constructed from Keldysh actions will be categorically different from quantum field theory. There are no guarantee that it could be self-consistent (in fact, I believe it is not). Those theories would be out of the scope of this thesis (which focus on quantum field theory).

In this chapter, we point out the following facts and results:

1, in classical mechanics and quantum field theory, if the action is invariant up to a boundary term (or is quasi-symmetric), the Euler-Lagrange equation, obtained by extremizing the action, has the same symmetry, or is covariant (form-invariant). The dynamical equation of motion, which is the same as the Euler-Lagrange equation, is thus covariant. (However, the same covariant equation of motion may also be derived from an action that does not have the same symmetry. This means even in classical mechanics and quantum field theory, there is not a one to one correspondence between an invariant action and a covariant dynamics.(Castillo and Moreno-Ruiz, 2017))

2, in the Schwinger-Keldysh path integral, the dynamical equation of motion, which is the Lindblad equation, is NOT obtained by directly extremizing the Keldysh action. And because of the way the Keldysh action is derived, extremizing the Keldysh action will not yield the desired equation of motion. As the result, the symmetry of the Keldysh action would not be automatically shared by the equation of motion.

3, Some Lorentz-invariant Keldysh actions can be derived from non-covariant equa-

tions of motion. This means Lorentz invariance of the Keldysh action cannot be a criterion for the covariance of the dynamics.

4.2 Keldysh action from Lindblad equations

We first derive the Keldysh action from the Lindblad master equation. While the Keldysh action can be derived by other methods, in this context we focus on the ones that correspond to Lindblad equations. The Lindblad master equation is dynamical equation of open quantum systems. Given a physical system that couples with the environment which we have no control of, after tracing out the environment and making the Born-Markov approximation, one can derive the standard Lindblad form of master equation:

$$\partial_t \rho(t) := \mathcal{L}[\rho(t)] = -i[H, \rho(t)] + \sum_n (Q_n \rho(t) Q_n^\dagger - \frac{1}{2}\{Q_n^\dagger Q_n, \rho(t)\}) \tag{4.1}$$

.Here H is the system Hamiltonian, Q_n are a set of jump operators, which correspond to the effect on the system due to the interaction with the environment. The super-operator $\mathcal{L}[\cdot]$ is called the Liouvillian, which generates the time evolution of the density operator.

We can write the formal solution of Eqn (4.1)

$$\rho(t) = e^{t\mathcal{L}} \rho(0)$$

Just as we did in the path integral of quantum field theory, we can split the time evolution into infinitesimal time steps $\delta t = \frac{t}{N}$, and write

$$e^{t\mathcal{L}} = \lim_{N\to\infty} (1 + \delta t \mathcal{L})^N$$

Applying this expression to the time dependent density matrix, we find

$$\rho(t) = [\lim_{N\to\infty} \overset{[}{a}= 1]N \prod \prod (1 + \delta t^{(a)} \mathcal{L})^N] \rho(0)$$

where the time step $\delta t^{(a)} = t^{(a)} - t^{(a-1)} = \frac{t}{N}$. The partition function is

$$Z(t) = Tr[\rho(t)] = Tr([\lim_{N\to\infty} \overset{[}{a}= 1]N \prod \prod (1 + \delta t^{(a)} \mathcal{L})^N] \rho(0)) \tag{4.2}$$

To derive the path integral of $Z(t)$, we need to insert sets of basis. When the Hamiltonian and jump operators can be expressed by creation and annihilation operators, as is the case in condensed matter physics, the typical choice is the coherent state basis (assuming we are working with bosonic fields).

$$|\phi\rangle = e^{\Sigma_i \phi_i b_i^\dagger}|0\rangle$$

where $b_i^\dagger$ is the creation operator of the bosonic field, and the variable (i) labels a complete set of single particle quantum states in the relevant system. In the continuous case, i can be replaced by p, which labels the momentum of the mode. $|\phi\rangle$ is the eigenstate of the annihilation operator $b_i|\phi\rangle = \phi_i|\phi\rangle$.

We can normalize these coherent states to form a complete basis

$$\mathbf{1} = \int D[\phi^*, \phi]|\phi\rangle\langle\phi|$$

where $\int D[\phi^*, \phi] = \prod_i \int \frac{d\phi_i d\phi_i^*}{\pi} e^{-\phi_i^*\phi_i}]$.

One difference with path integral quantum field theory is that we need to insert complete basis to both the left and right of the density operator ρ. As the result, we will have two time series, labeled with + and - respectively. Basically, at time step a, we insert coherent basis $\mathbf{1}_{a+}$ and $\mathbf{1}_{a-}$ from the left and right of the density operator $\rho(t)^{(a)}$ respectively, and then apply the infinitesimal evolution $1+\delta t^{(a+1)}\mathcal{L}$, and repeat the process.

$$\begin{aligned} Z(t) &= Tr\left(... \left[(1+\delta t^{(a+1)}\mathcal{L})\left(\mathbf{1}_{a+}\rho(t)^{(a)}\mathbf{1}_{a-}\right)\right] ...\right) \\ &= \int D[\phi_{N+}^*, \phi_{N+}]D[\phi_{N-}^*, \phi_{N-}]...D[\phi_{a+}^*, \phi_{a+}]D[\phi_{a-}^*, \phi_{a-}]... \\ &\quad Tr\left(|\phi_{N+}\rangle\langle\phi_{N+}|...[(1+\delta_t^{(a+1)}\mathcal{L})|\phi_{a+}\rangle\langle\phi_{a+}|\rho(t)^{(a)}|\phi_{a-}\rangle\langle\phi_{a-}|]...|\phi_{N-}\rangle\langle\phi_{N-}|\right) \end{aligned}$$

At time step a, we would need to evaluate terms like $|\phi_{a+}\rangle\langle\phi_{a+}|H|\phi_{a-1+}\rangle\langle\phi_{a-1+}|...$ $|\phi_{a-1-}\rangle\langle\phi_{a-1-}|\phi_{a-}\rangle\langle\phi_{a-}|$ and $|\phi_{a+}\rangle\langle\phi_{a+}|Q^\dagger Q|\phi_{a-1+}\rangle\langle\phi_{a-1+}|...|\phi_{a-1-}\rangle\langle\phi_{a-1-}|\phi_{a-}\rangle\langle\phi_{a-}|$. When the Hamilton and jump operators can be expressed as functions of creation and annihilation operators $H = H(b^\dagger, b)$, $Q = Q((b^\dagger, b))$, and when the jump operators Q are normal ordered, we will have relationships like

$$|\phi_{a+}\rangle\langle\phi_{a+}|H|\phi_{a-1+}\rangle\langle\phi_{a-1+}| = H(\phi_{a+}^*, \phi_{a-1+})|\phi_{a+}\rangle\langle\phi_{a+}|\phi_{a-1+}\rangle\langle\phi_{a-1+}|$$

After careful computation, (detailed procedures can be seen from Ref. Michael Buchhold, 2015), we will be able to get

$$Z(t) = \int D[\phi^*, \phi]e^{i\mathcal{K}\mathcal{S}}\mathcal{K}\mathcal{S} = \int_{t_i}^{t_f} dt[\phi_+^* i\partial_0\phi_+ - \phi_-^* i\partial_0\phi_- - i\mathcal{L}(\phi_+^*, \phi_+, \phi_-^*, \phi_-)]$$

where

$$\mathcal{L}(\phi_+^*, \phi_+, \phi_-^*, \phi_-) = -i(H(\phi_+^*, \phi_+) - H(\phi_-^*, \phi_-)) - \gamma \sum_n \left(Q_n^\dagger Q_n(\phi_+^*, \phi_+) + Q_n^\dagger Q_n(\phi_-^*, \phi_-) - 2Q_n(\phi_+^*, \phi_+) Q_n^\dagger(\phi_-^*, \phi_-) \right)$$

If the Hamiltonian and jump operators are easily expressed in other operators, we usually choose the corresponding eigenstate basis for insertion, and the derived form of the Keldysh action will be slight different. For example, when the Hamiltonian and jump operators are easily expressed in field operators $\phi(x)$ and its conjugate momentum operators $\pi(x)$, we insert their eigenstate basis instead. When the Lindblad equation is of the form

$$\dot{\rho}(t) = \int dx^3 i[-\mathcal{H}, \rho(t)] + \gamma_{ij}[Q_i \rho(t) Q_j^\dagger - \frac{1}{2}\{Q_j^\dagger Q_i, \rho(t)\}])$$

where $Q_i = Q_i(\phi(x), \pi(x), \nabla\phi(x))$, a Keldysh action can be derived

$$\mathcal{KS} = \int_{t_i}^{t_f} dt \int dx^3 [\pi_+ \dot{\phi}_+ - \pi_- \dot{\phi}_- - i\mathcal{L}(\phi_+^*, \phi_+, \phi_-^*, \phi_-)] \tag{4.3}$$

where

$$\mathcal{L}(\phi_+^*, \phi_+, \phi_-^*, \phi_-) = -i(\mathcal{H}(\phi_+^*, \phi_+) - \mathcal{H}(\phi_-^*, \phi_-)) - \gamma_{ij} \left(Q_i^\dagger Q_j(\phi_+^*, \phi_+) + Q_i^\dagger Q_j(\phi_-^*, \phi_-) - 2Q_j(\phi_+^*, \phi_+) Q_i^\dagger(\phi_-^*, \phi_-) \right)$$

4.3 Equation of motion is not the Euler-Lagrange equations from Keldysh actions

From the above construction, we can see that the Schwinger-Keldysh path integral is very similar to the path integral in QFT, except for one key difference: there are two time series, one for the left field and one for the right field. Because of this key difference, there is no action principle that by extremizing the Keldysh action we can get the equation of motion. And the lacking of action principle means that unlike the path integral in QFT, even if the Keldysh action looks Lorentz invariant, the corresponding dynamics is not guaranteed Lorentz covariant.

Let us remember ourselves that, in the path integral in QFT, why the action and equation of motion have the same symmetry. The invariant action leads to a covariant Euler-Lagrange equation, which is obtained by extremzing the action. And if the equation of motion is the same with the Euler-Lagrange equation, as is the case here, it will share the same symmetry (Castillo and Moreno-Ruiz, 2017).

This is not the case for the Keldysh action in the Schwinger-Keldysh path integral. In the Keldysh action, the left and right field operators, which we introduce because $\mathcal{L}$ acts on both the right and left of the density operator, are treated as independent field variables. If we extremize the Keldysh action in the same way when we extremze the action, we will end up with a set of coupled differential equations of the left and right fields. This is completely different from the desired equation of motion, i.e., the Lindblad equation. extremizing the Keldysh action will give us some equations that are Lorentz covariant, but they do not contain full information of the physical system. From them, we cannot reconstruct the Lindblad equation. While we can derive the Lindblad equation starting from the Keldysh action, it is not by extremzing the Keldysh action. As the result, there are no grantee that a invariant Keldysh action gives a covariant dynamics. In fact, the way we derive the Lindblad equation from the Keldysh action require us to choose a time frame, and that breaks Lorentz covariance.

Let us see a simply example. Assuming the Lindblad equation is

$$\mathcal{KS} = \int d^4x \frac{i}{2}(\partial^\mu \phi_+ \partial_\mu \phi_+ - m^2\phi_+^2) - \frac{i}{2}(\partial^\mu \phi_- \partial_\mu \phi_- - m^2\phi_-^2) - \frac{\gamma}{2}(\phi_+ - \phi_-)^2$$

We can reorganize the fields, namely, do the Keldysh rotation

$$\phi_c := \frac{1}{\sqrt{2}}(\phi_+ + \phi_-)$$

$$\phi_q := \frac{1}{\sqrt{2}}(\phi_+ - \phi_-)$$

so

$$\phi_+ = \frac{\phi_c + \phi_q}{\sqrt{2}}$$

$$\phi_- = \frac{\phi_c - \phi_q}{\sqrt{2}}$$

And the Keldysh Lagrangian is

$$\begin{aligned}\mathcal{KL} &= \frac{i}{4}[(\partial\phi_c + \partial\phi_q)^2 - m^2(\phi_c + \phi_q)^2] - \frac{i}{4}[(\partial\phi_c - \partial\phi_q)^2 - m^2(\phi_c - \phi_q)^2] - \lambda\phi_q^2 \\ &= i[\partial\phi_c \cdot \partial\phi_q - m^2\phi_c\phi_q] - \lambda\phi_q^2 \end{aligned} \tag{4.4}$$

Assume the Keldysh Lagrangian (Eqn 4.4) has an action principle, and directly apply Euler-Lagrange equations

$$\partial_\mu \frac{\partial \mathcal{KL}}{\partial(\partial_\mu \phi_c)} = \frac{\partial \mathcal{KL}}{\partial \phi_c}$$

$$\frac{\partial \mathcal{KL}}{\partial \phi_c} = im^2\phi_q$$
$$\frac{\partial \mathcal{KL}}{\partial(\partial_\mu \phi_c)} = i\partial^\mu \phi_q$$
$$\partial_\mu \frac{\partial \mathcal{KL}}{\partial(\partial_\mu \phi_c)} = i\partial^2 \phi_q$$

so the E-L equation for ϕ_c is

$$\partial^2 \phi_q = m^2 \phi_q \tag{4.5}$$

Eqn 4.5 has the standard free KG solution

$$\phi_q = \int \widetilde{dp} a(p) e^{ipx} + h.c.$$

The E-L equation for ϕ_q is

$$\partial_\mu \frac{\partial \mathcal{KL}}{\partial(\partial_\mu \phi_q)} = \frac{\partial \mathcal{KL}}{\partial \phi_q}$$
$$\frac{\partial \mathcal{KL}}{\partial \phi_q} = im^2\phi_c - 2\lambda\phi_q$$
$$\partial_\mu \frac{\partial \mathcal{KL}}{\partial(\partial_\mu \phi_q)} = i\partial^2 \phi_c$$

hence

$$i\partial^2 \phi_c = im^2\phi_c - 2\lambda\phi_q$$
$$(\partial^2 - m^2)\phi_c = 2i\lambda\phi_q \tag{4.6}$$

Eqn (4.5 and 4.6) are clearly Lorentz covariant, since they are obtained by directly extremzing a Lorentz invariant Keldysh action. The physical meaning of them is interesting. If we treat ϕ_c as the classical average of the field in the saddle point approximation, and set $\phi_q = 0$ as in Section 3.2 of Alex Kamenev's note (Kamenev, 2004), we will have the classical field satisfies the free Klein-Gordon equation like a free theory. However, from the underlying physics, we know that this model corresponds to the physical situation that infinite particles will be created (Banks, Peskin, and Susskind, 1984; M. Srednicki, 1993), it is not likely that the mean field looks like free.

Directly from Eqn (4.5 and 4.6), it does not seem we can reconstruct the original Lindblad dynamics. Hence we can see that the Euler-Lagrange equation obtained by extremizing the Keldysh action is different from the equation of motion. As a result, there is no basis to believe that the symmetry of the Keldysh action is share by the equation of motion.

4.4 Lorentz invariant Keldysh actions from not Lorentz covariant EOMs

We can also see that some Lorentz invariant Keldysh action can be derived from a Lindblad equation that is NOT Lorentz covariant. This should not surprise us since the invariance of the Keldysh action is causally independent with the covariant equation of motion.

For example, consider the following Lindblad equation

$$\partial_t \rho(t) = -i\frac{1}{2}[(\partial^0\phi)^2+(\partial^i\phi)^2+m^2\phi^2, \rho(t)]+\gamma\int dx^3\phi(x)\rho(t)\phi(x)-\frac{1}{2}\{\phi(x)^2, \rho(t)\} \tag{4.7}$$

This describes the physics of a scalar field coupled to a classical Gaussian random source J, as first introduced by Banks, Peskin and Susskind (Banks, Peskin, and Susskind, 1984; M. Srednicki, 1993). It can be described by a Lagrangian density $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - J\phi$ (M. Srednicki, 1993). These authors think that the theory is the Lorentz covariant because it has an invariant Lagrangian $\mathcal{L}$. This is actually not correct. Remember J is a classical Gaussian random source, not a quantum operator. When we do a the Lorentz transformation by conjugation of $U(\Lambda)$, $U(\Lambda)$ only acts on quantum operators, not on classical functions. Hence, the correct transformation is

$$U(\Lambda)\mathcal{L}U(\Lambda)^\dagger = \frac{1}{2}(\bar{\partial}\phi(\Lambda x))^2 - \frac{1}{2}(m\phi(\Lambda x))^2 - J(x)\phi(\Lambda x)$$

As we can see, the Lorentz transformation generates non-locality, because $\phi(\Lambda x)$ is now coupled to the source at a different location. This is clearly not Lorentz invariant.

Interestingly, the corresponding Keldysh action $\mathcal{KS}$ obtained from Eqn (4.7) by the method in Section II is Lorentz invariant (Oppenheim and Weller-Davies, 2022).

$$\mathcal{KS} = \int d^4x \frac{i}{2}(\partial^\mu\phi_+\partial_\mu\phi_+ - m^2\phi_+^2) - ig\phi_+^4 - \frac{i}{2}(\partial^\mu\phi_-\partial_\mu\phi_- - m^2\phi_-^2) + ig\phi_-^4 - \frac{\gamma}{2}(\phi_+ - \phi_-)^2$$

However, the equation of motion – Eqn (4.7) is not Lorentz covariant, contrary to the belief of (Oppenheim and Weller-Davies, 2022; Banks, Peskin, and Susskind, 1984; M. Srednicki, 1993). This can be seen as follow.

Let us write Eqn (4.7) in a more covaraint Heisenberg picture form

$$\partial^0 O = i[P^0, O(x)] + \gamma\int dx^3\phi(x)O\phi(x) + \frac{1}{2}\{\phi(x)^2, O\})$$

In the unitary term, $P^0 = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 = \int \widetilde{dp} p^0 a(p)^\dagger a(p)$, where $\widetilde{dp} := \frac{d^3p}{(2\pi)^3 2\omega_p}$ is the Lorentz invariant measure. We can see that the unitary term transforms in the same way as left hand side.

$$LHS = U(\Lambda)\partial^0 O' U(\Lambda)^\dagger = (\Lambda^{-1})^0_\mu \partial^\mu O'$$

Here O will also be the Lorentz transformed. But since we don't know exactly what it is, we will keep the general form and write it as O' .

$$\begin{aligned} U(\Lambda) i[P^0, O] U(\Lambda)^\dagger &= [\int \widetilde{dp} p^0 a(\Lambda p)^\dagger a(\Lambda p), O'] \text{Let } q = \Lambda p \\ &= [\int \widetilde{dq} (\Lambda^{-1} q)^0 a(q)^\dagger a(q), O'] \\ &= (\Lambda^{-1})^0_\mu [\int \widetilde{dq} q^\mu a(q)^\dagger a(q), O'] \end{aligned}$$

Here the crucial part is because $\widetilde{dp}$ is invariant, the term p^0 will provide us the $(\Lambda^{-1})^0_\mu$, which is the same the Lorentz matrix that we get by transforming ∂^0.

However, the non-unitary terms will not transform in this way. Let us look at the $\int dx^3 \phi(x) O \phi(x)$ first. We can convert it into momentum basis by using the mode expansion $\phi(x) = \int \widetilde{dp} a(p)^\dagger e^{ipx} + a(p) e^{-ipx}$,

$$\begin{aligned} & U(\Lambda) \int dx^3 \phi(x) O \phi(x) U(\Lambda)^\dagger \\ &= U(\Lambda) [\int dx^3 \widetilde{dp}\widetilde{dq} (a(p)^\dagger e^{ipx} + a(p) e^{-ipx}) O (a(q)^\dagger e^{iqx} + a(q) e^{-iqx})] U(\Lambda)^\dagger \\ &= \int dx^3 \widetilde{dp}\widetilde{dq} (a(\Lambda p)^\dagger e^{ipx} + a(\Lambda p) e^{-ipx}) O' (a(\Lambda q)^\dagger e^{iqx} + a(\Lambda q) e^{-iqx}) \\ &= \int dx^3 \widetilde{dp'}\widetilde{dq'} (a(p')^\dagger e^{i\Lambda^{-1}p'x} + a(p') e^{-i\Lambda^{-1}p'}) O' (a(q')^\dagger e^{i\Lambda^{-1}q'} + a(q') e^{-i\Lambda^{-1}q'}) \end{aligned}$$

We can see that there are no terms like p^0 which provides us $(\Lambda^{-1})^0_\mu$, so this term will nottransform in the same fashion as $\partial^0 O$ and $i[P^0, O]$. We can see it more clearly by looking at the case where the argument of O is not x.

$$\begin{aligned} \int dx^3 \phi(x) O \phi(x) &= \int dx^3 \widetilde{dp}\widetilde{dq} (a(p)^\dagger e^{ipx} + a(p) e^{-ipx}) O (a(q)^\dagger e^{iqx} + a(q) e^{-iqx}) \\ &= \int dx^3 \widetilde{dp}\widetilde{dq} [a(p)^\dagger O a(q)^\dagger e^{i(p+q)x} + h.c. + a(p) O a(q)^\dagger e^{i(q-p)x} + h.c. \\ &= \int \widetilde{dp} [a(p)^\dagger O a(-p)^\dagger + h.c. + a(p) O a(p)^\dagger + h.c. \end{aligned}$$

Doing a the Lorentz transformation on the above term, we can see

$$U(\Lambda)\int dx^3\phi(x)O\phi(x)U(\Lambda)^\dagger = \int \widetilde{dq}[a(q)^\dagger O' a(-q)^\dagger + h.c. + a(q)O'a(q)^\dagger + h.c.$$

This differs with the unitary term by $(\Lambda^{-1})^0_\mu$, as pointed out before. The $\{\phi(x)^2, O\}$ term will have exactly the same problem.

As the result, even though Eqn (4.7) has a invariant Keldysh action, it does not transform covariantly under the Lorentz transformation. Hence, there is no one to one correspondence between the invariance of the super action and the covariance of the dynamics. (See Chapter 5 for a more detailed discuss about the Lorentz transformation properties.)

4.5 Symmetry, invariance and covariance

From the above sections, we can see that Lorentz invariance in Keldysh actions do not correspond to covariant Lindblad equations. Hence, having a invariant Keldysh action does not mean the theory is Lorentz covariant. Some readers may argue that, maybe the true the Lorentz symmetry means the invariance of the Keldysh action, instead of the covariance of the Lindblad equation. This is incorrect.

Physicists, especially field theorist, may be more familiar with invariance, and tend to think the Lagrangian/action plays a more fundamental role. But this is not the case here. What we care most, is how a physical system changes with time, namely, the equation of motion. When the physical system has some symmetry, under that symmetry transformation, the equation of motion will be form invariant, or covariant. At the same time, some quantitative objects, such as a functional that maps a field operator in to a number, could be invariant under that symmetry group. Usually, it is not easy to see whether a equation of motion is covariant, while it would be much easier to see whether that quantitative object is invariant. This is why we like to use the invariance of that object (the action) as the criteria to judge whether the physical system has such symmetry.

In classical mechanics and quantum field theory, the invariance of the action guarantees the covariance of the equation of motion. This is because, extremizing the action gives us the Euler-Lagrange equation, which serves as the equation of motion of the system. (Castillo and Moreno-Ruiz, 2017)

In the case for non-unitary Lindblad dynamics, the things are different. Our starting point is the Lindblad equation, which is the equation of motion. The Keldysh action is a object derived from the Lindblad equation, as we have shown. More importantly,

extremizing the Keldysh action would not give us the equation of motion. This is why we believe that in the non-unitary cases, when we say a physical system has some symmetry, we really mean the equation of motion (the Lindblad equation) is covariant. And the invariance of Keldysh action cannot serve as the criteria to judge whether the physical system has such symmetry.

4.6 the Lorentz Transformation on density matrix

We have mentioned that if we want to the Lorentz transform the Lindblad equation, we have to do it in the Heisenberg picture form. The main reason is, we cannot the Lorentz transform a density operator $\rho(t)$, both conceptually, mathematically and physically.

When we do a the Lorentz transformation on a field operator,

$$U(\Lambda)\phi(x)U(\Lambda)^{\dagger} = \phi(\Lambda x)$$

the argument of $\phi(x)$ has four components, so we can multiply it with a the Lorentz matrix Λ^{μ}_{ν}. However, a density operator $\rho(t)$ is defined on a Hilbert space associated with a time slice. It specifies the initial conditions of the time slice (or a local patch of the time slice). Hence, it only have a time index, but no spatial index. As the result, it makes no sense to do something like $U(\Lambda)\rho(t)U(\Lambda)^{\dagger}$. We don't know how to do it at all.

One may argue that in the case of point-like particle, we would be able to do it, since it will have a definite location. However, a point-like particle with precise location will have infinite energy, so it is outside of the physical Hilbert space.

One may also argue that we can the Lorentz transform every point on the time slice. This will notwork as well. If we do it, those points will be transformed into different time slices in the new frame. So after this imaginary the Lorentz transformation, the density matrix will describe different locations at different time slices. This makes no sense.

Finally, as discussed in Chapter 2, doing a the Lorentz transformation on the density matrix, which means transforming initial conditions on a time slice, we would face the problem non-unitary boost. A the Lorentz transformation is fundamentally a mixing of space and time. When the time evolution is non-unitary, the effect of a the Lorentz transformation would involve non-unitary process as well, and this will lead to many difficulties, as discussed in details in Chapter 2.

In fact, in the non-unitary cases, when the initial density operator is given, the initial condition on the initial time slice is set. This also select a preferred time frame. This directly contradict with the Lorentz symmetry, which says every inertial frames should be equivalent. And because the starting point of the Schwinger-Keldysh path integral is the density operator, it wound not be serve a criteria for the Lorentz covariance from this angle, even if we ignore the issue of the missing of action principle as discussed above.

In summary, the symmetric of a open quantum system is independent of the symmetry of the Keldysh action, especially in the case of the Lorentz symmetry.

Chapter 5

THE LORENTZ TRANSFORMATION PROPERTIES OF OPERATORS AND QUANTUM CHANNELS THAT ARE DEFINED AS AN INTEGRATION ON A TIME SLICE

5.1 Introduction

Many physicists many believe that in quantum field theory, the Momentum operator P^ν transforms as a the Lorentz vector. For example, in the second section of Quantum field theory by Mark Srednicki, one of the most used textbook of quantum field theory nowadays,

"For example, consider the energy-momentum four-vector P^ν, where P^0 is the Hamiltonian H and P^i are the components of the total three-momentum operator. We expect $U(\Lambda)^{-1}P^\nu U(\Lambda) = \Lambda^\nu_\mu P^\mu$."(Mark Srednicki, 2007)

Here the momentum operator is defined as an integration of the stress tensor over a time slice

$$P^\nu := \int dx^3 T^{0\nu}(x) \tag{5.1}$$

Some physicists even believe that an object defined as integrations of operators on a time slice could transform in the same way as the first time derivative ∂_t. For example , in the context of open quantum field theory, the dynamics of the system is described by the Lindblad master equations, such as

$$\partial_t \rho(t) = -i[H, \rho(t)] + \gamma \int dx^3 (\phi(x)\rho(t)\phi(x) - \frac{1}{2}\{\phi(x)^2, \rho(t)\}) \tag{5.2}$$

Eqn (5.2)describes the physics of a scalar field is coupled to a Gaussian random source, as in the famous paper by Bank, Peksin and Susskind (PBS) in 1983 (Banks, Peskin, and Susskind, 1984).

In fact, Eqn (5.2) can be treated as an extension of the Lindblad equations from finite dimension to infinite dimension. Exponentiating it will give us a quantum channel that maps the density matrix from time t to $t + dt$.

$$\rho(t + dt) = e^{dt\mathcal{L}}\rho(t)$$

where

$$\mathcal{L}[\rho(t)] = \partial_t \rho(t) = -i[H, \rho(t)] + \gamma \int dx^3 (\phi(x)\rho(t)\phi(x) - \frac{1}{2}\{\phi(x)^2, \rho(t)\})$$

The operators such $\phi(x)$ is called jump operators, which describes the effect of the system due to interaction with the environment. There can be other types of jump operators as well.

Prominent physicists such as BPS (Banks, Peskin, and Susskind, 1984), Srednicki (M. Srednicki, 1993), Oppenheim (Oppenheim and Weller-Davies, 2022), believe that the PBS theory is the Lorentz covariant, and that the term $\gamma \int dx^3 \phi(x)\rho(t)\phi(x) - \frac{1}{2}\{\phi(x)^2, \rho(t)\})$ transform in the same fashion as $\partial_t \rho(t)$. Srednicki (M. Srednicki, 1993) further modified the integration into

$$\partial^0 \rho(t) = -i[H, \rho(t)] + \gamma \int d\Sigma^0 (\phi(x)\rho(t)\phi(x) - \frac{1}{2}\{\phi(x)^2, \rho(t)\}) \tag{5.3}$$

where $d\Sigma^\mu := \epsilon^{\mu ijk} dx_i dx_j dx_k$ carries a vector index. He argued that Eqn (5.3) can the "weakest possible form" of Lorentz covariance which can be demanded of a Lindblad equation. The rationale could be that $H = P^0$ already carries a the Lorentz index, by writing the integration measure as $d\Sigma^0$, it provides a vector index, so every terms in the equation carries the same the Lorentz index, and the equation is thus Lorentz covariant.

Unfortunately, as we find out, these are not correct. Operators and objects defined as integrations on a time slice in general do not transform covariantly under the Lorentz transformation. Only when two conditions are met, they can be treated as transforming as the Lorentz scalar, vector, etc.

The first condition is that the integrand should contain an operator which is the time-like component of a the Lorentz four-vector. One common example is $P^\nu := \int dx^3 T^{0\nu}$.

The second condition is that some conservation conditions should be met. For the P^μ case, the condition is that the stress energy tensor is conserved $\partial_\mu T^{\mu\nu} = 0$.

We well analyze these two conditions in detail as follow.

5.2 conservation requirements

We consider the second condition first.

Let us try to do a the Lorentz transformation on both sides of Eqn (5.1).

$$LHS = U(\Lambda) P^\nu U(\Lambda)^\dagger = (\Lambda^{-1})^\nu_\mu P^\mu = (\Lambda^{-1})^\nu_\mu \int dx^3 T^{0\mu}(\Lambda x)$$

$$RHS = U(\Lambda) \int dx^3 T^{0\nu}(x) U(\Lambda)^\dagger$$
$$= (\Lambda^{-1})^\nu_\mu (\Lambda^{-1})^0_\sigma \int dx^3 T^{\sigma\mu}(\Lambda x)$$

Why could LHS equal RHS?

Let us use the trick to rewrite the integration to

$$P^\nu = \int dx^4 \delta(x \cdot n) n_\mu T^{\mu\nu}(x)$$

where $n = (1, 0, 0, 0)$ is the unit vector pointing to the time direction.

$$RHS = U(\Lambda) \int dx^4 \delta(x \cdot n) n_\mu T^{\mu\nu}(x) U(\Lambda)^\dagger$$
$$= (\Lambda^{-1})^\mu_\sigma (\Lambda^{-1})^\nu_\tau \int dx^4 \delta(x \cdot n) n_\mu T^{\sigma\tau}(\Lambda x)$$

Change integration variable $y = \Lambda x$ and define $n' := \Lambda n$, so $n_\mu = (\Lambda^{-1})^\rho_\mu n'_\rho$

$$RHS = (\Lambda^{-1})^\mu_\sigma (\Lambda^{-1})^\nu_\tau \int dy^4 \delta(y \cdot n') (\Lambda^{-1})^\rho_\mu n'_\rho T^{\sigma\tau}(y)$$
$$= (\Lambda^{-1})^\nu_\tau \int dy^4 \delta(y \cdot n') n'_\sigma T^{\sigma\tau}(y) \tag{5.4}$$

$$LHS = (\Lambda^{-1})^\nu_\tau P^\tau = (\Lambda^{-1})^\nu_\tau \int dx^4 \delta(x \cdot n) n_\rho T^{\rho\tau}(x) \tag{5.5}$$

Comparing Eqn (5.4) to Eqn (5.5), we can see the only difference is the integration surface is changed from $x_0 = 0$ to $y_0 = 0$. So how can these two be equal?

Let us take the difference of these two. Note that $n_\mu \delta(x \cdot n) = \partial_\mu \theta(x \cdot n)$,

$$LHS - RHS = (\Lambda^{-1})^\nu_\tau \int dx^4 n_\rho T^{\rho\tau}(x) [\delta(x \cdot n) - \delta(x \cdot n')]$$
$$= (\Lambda^{-1})^\nu_\tau \int dx^4 T^{\rho\tau}(x) \partial_\rho [\theta(x \cdot n) - \theta(x \cdot n')]$$

Perform integration by part, and not that in the far future or past $\theta(x \cdot n) = \theta(x \cdot n')$, so the surface term vanishes. And we are left with

$$LHS - RHS = (\Lambda^{-1})^{\nu}_{\tau} \int dx^4 \partial_\rho T^{\rho\tau}(x)[\theta(x \cdot n) - \theta(x \cdot n')]$$

For this to vanish, we need to have $\partial_\rho T^{\rho\tau}(x) = 0$ satisfied.

The above calculation shows that, for $U(\Lambda)P^\nu U(\Lambda)^\dagger = (\Lambda^{-1})^{\nu}_{\mu} P^\mu$ to hold, we need the conservation of the stress tensor. The reason behind this is that, when we do a the Lorentz transformation on a operator defined as the spatial integration over a time slice, that time slice is transform to a new one. And when we take the difference between the original operator and the transformed operator, for that to vanish, we need the integrand to vanish.

Another example is the creation operator $a(p)^\dagger$. This has been discussed in Section 2.4 in detail.

5.3 first component requirement

Now let us take a look at how the quantum channel defined by Eqn (5.2) transforms under the Lorentz transformation. Directly transforming Eqn (5.2) by conjugating $U(\Lambda)$ is controversial, because it would involve $U(\Lambda)\rho(t)U(\Lambda)^\dagger$, which is not well defined, since $\rho(t)$ is defined on the whole time slice, and only carries one time like variable t. However, we can look at the Heisenberg picture version of it, which acts on operators.

$$\partial^0 O = i[H, O] + \gamma \int dx^3 \phi(x) O(x) \phi(x) + \frac{1}{2}\{\phi(x)^2, O(x)\})$$

Let us try to see explicitly if the RHS does transform like the LHS. We can just focus the $\phi(x)O(x)\phi(x)$ since the others have the same transformation property.

$$\begin{aligned}
RHS = U(\Lambda) \int dx^3 \phi(x) O(x) \phi(x) U(\Lambda)^\dagger &= \int dx^4 \delta(n \cdot x) n^0 \phi(\Lambda x) O(\Lambda x) \\
\text{(define } y = \Lambda x, n' = \Lambda n\text{)} & \\
&= \int dy^4 \delta(n \cdot \Lambda^{-1} y) n^0 \phi(y) O(y) \phi(y) \\
&= \int dy^4 \delta(\Lambda n \cdot y) n^0 \phi(y) O(y) \phi(y) \\
&= \int dy^4 \delta(n' \cdot y) (\Lambda^{-1} n')^0 \phi(y) O(y) \phi(y) \\
&= (\Lambda^{-1})^0_\mu \int dy^4 \delta(n' \cdot y) n'^\mu \phi(y) O(y) \phi(y)
\end{aligned}$$

On the other hand, since

$$LHS = U(\Lambda)\partial^0 O U(\Lambda)^\dagger = (\Lambda^{-1})^0_\mu \partial^{\bar{\mu}} O'$$

It seems that one can indeed $claim$ that $\int dx^3 \phi(x) O(x) \phi(x)$ do transform as the same way as $\partial^0 O$.

However, if one write $\int dx^3 \phi(x) O = \int dx^4 \delta(n \cdot x) n_0$, the result of the calculation is

$$\begin{aligned}
RHS &= \int dx^4 \delta(n \cdot x) n_0 \phi(\Lambda x) O(\Lambda x) \\
&\quad (\text{ define } y = \Lambda x, n' = \Lambda n) \\
&= \int dy^4 \delta(n \cdot \Lambda^{-1} y) n_0 \phi(y) O(y) \phi(y) \\
&= \int dy^4 \delta(\Lambda n \cdot y) n_0 \phi(y) O(y) \phi(y) \\
&= \int dy^4 \delta(n' \cdot y) (\Lambda^{-1} n')_0 \phi(y) O(y) \phi(y) (\Lambda^{-1} n')_0 = (\Lambda^{-1})^\mu_0 n'_\mu \\
&= (\Lambda^{-1})^\mu_0 \int dy^4 \delta(n' \cdot y) n'_\mu \phi(y) O(y) \phi(y)
\end{aligned}$$

one can $claim$ it transforms as $\partial_0 O$. The computation are almost exactly the same.

And writing $\int dx^3 = \int dx^4 \delta(n \cdot x) n_0$ or $\int dx^3 = \int dx^4 \delta(n \cdot x) n^0$ seems arbitrary, since they at most differ by a sign depending on the convention one uses.

So, how can one term transform as both covariant and covariant vector?

Let us look back at the claim by Srednicki. He thinks $d\Sigma^\mu := \epsilon^{\mu i j k} dx_i dx_j dx_k$ can give the vector index to get the term transform like ∂^μ. But I can also write the integration measure as $d\Sigma_\mu = \frac{1}{6} \varepsilon_{\mu i j k} dx^i dx^j dx^k$. These two terms are essentially the same, at most differ by a sign.

Our answer is, because $n = (1, 0, 0, 0)$, there is simply just one component in the four-vector that exists, which is the original term. All other three are zero by definition. We *cannot* combine something with three zeros to form a four-vector, and claim it transforms covariantly as a the Lorentz four-vector.

In fact, because $n = (1, 0, 0, 0)$, the results of the above two calculations, $(\Lambda^{-1})^0_\mu \int dy^4 \delta(n' \cdot y) n'^\mu \phi(y) O(y) \phi(y)$ and $(\Lambda^{-1})^\mu_0 \int dy^4 \delta(n' \cdot y) n'_\mu \phi(y) O(y) \phi(y)$ are exactly the same, as they should be.

From the above analysis we can conclude that, yes, the integration over the time slice $\int dx^3$ can serve to contract with the time-like component of the first vector

in the integrand. But if there is no time-like component of the first vector in the integrand to contract with, it will notserve to provide a the Lorentz index, such that the operator or quantum channel can transform as a contravariant or covariant vector.

In summary, we can see that an operator or quantum channel can indeed transform covariantly under the Lorentz transformation, if the above two conditions are met.

Chapter 6

UNDERSTANDING MOMENTUM EIGENSTATES

One of the key assumptions of the non-unitary Lorentz covariant QFT is

$$U(\Lambda)|p\rangle = |\Lambda p\rangle \tag{6.1}$$

However, it is not clear if this assumption is valid in the non-unitary setting. In this chapter, we are going to look deep into the issues. We start from clarifying the physical meaning of momentum eigenstates $|p\rangle$. We analyze why Eqn (6.1) makes sense in unitary QFT, and why it does not make sense in non-unitary QFT.

6.1 Momentum eigenstates are not plane-waves

It is widely believed among physicists that momentum eigenstates $|p\rangle$ are "plane-waves", and superposition of them are wave packets. For example, in Peskin and Schroeder's "An Introduction to Quantum Field Theory", one of the standard textbook for QFT, around Eqation 4.65,

"A wavepacket representing some desired state $|\phi(x)\rangle$ can be expressed as

$$|\phi(x)\rangle = \int \frac{d^3k}{(2\pi)^3\sqrt{2E_k}}\phi(k)|k\rangle \tag{6.2}$$

where $\phi(k)$ is the Fourier transform of the spatial wavefunction, and $|k\rangle$ is a one-particle state of momentum k in the interacting theory. "(Michael E. Peskin, 1995)

In Srednicki's Quantum field theory, another standard textbook of QFT, one can also find similar statements (Mark Srednicki, 2007).

But why is this true? If we are in such a state, can we do any measurements to show that we are indeed in a plane-wave state?

In this chapter, We will analysis this problem in detail. We show that, country to the popular belief, momentum eigenstates are not plane-wave states, and superposition of them do not make a wave packet. In fact, the physical states that has the property of plane-waves are (second quantized) coherent states.

Let us first look at what a plane-wave means in classical physics. We say a field E is a plane-wave in space-time if it is has the following form

$$E(\vec{x}, t) = E_0 \cos[\vec{p}\cdot\vec{x} - \omega t + \phi_0] \tag{6.3}$$

This basically says the field amplitude is oscillating in time and space, with wave length $\lambda = \frac{2\pi}{p}$.

Moreover, the field is propagating in the space-time. This means the wave front will start from the source and propagate further as time goes by. At one point in time, there are portions in space where the amplitudes of the field is 0, since the wave front has not arrived yet.

Do the momentum eigenstates $|p\rangle$ have such properties? The answer is no.

Firstly, $|p\rangle$ is stationary and not dynamical, as it is an eigenstate of the (Free) Hamiltonian. If we are in $|p\rangle$ state, which specifies the situation on the whole time slice, we will not see the propagating behavior, that the wave has not arrived at some point in the time-slice yet but will arrive later. The excitation is spread out on the whole time-slice already.

Operationally, if a team of experimentalists try to detect the excitation at different locations on a time-slice, they should always find the same result on average, as required by the position-momentum uncertainty principle.

Secondly, we cannot measure the oscillating behavior or confirm that the wavelength is inversely proportional to p. The only information encoded in $|p\rangle$ is that it is an eigenstate of P and H operators. If we can measure the energy density, we will find it is uniform across the time slice. We can measure the field amplitude with a detector such as an Unruh-DeWitt detector, which couples our field to some two level system

$$H_D = \lambda\chi(\tau)[\sigma^+(\tau) + \sigma^-(\tau)]\phi[x_D(\tau)] \tag{6.4}$$

where $\chi(\tau)$ is the switching function, τ is the proper time of the detector, $\sigma^+(\tau)$, $\sigma^-(\tau)$ are ladder operators of the two level system, $x_D(\tau)$ is the trajectory of the detector parameterized by proper time, and ϕ is our field to detect.

When measuring the field amplitude with this detector, since the trace of the state with this operator is 0 everywhere, the detector will never click.

$$Tr[|p\rangle\langle p|\phi(x)] = \langle p| \int \tilde{d}p a_p e^{ipx} + a_p^\dagger e^{-ipx}|p\rangle = 0 \tag{6.5}$$

This means $|p\rangle$ will not display any property of a classical plane-wave. It is thus incorrect to interpret $|p\rangle$ as plane wave states.

In fact, the quantum state closest to the classical plane-wave is the (second-quantized) coherent state (for bosonic systems).

$$|\alpha_p\rangle = e^{-\frac{|\alpha_p|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha_p^n}{\sqrt{n!}} |n_p\rangle \tag{6.6}$$

Here the subscript p is the wave vector, and $|n_p\rangle$ means n excitations in momentum mode p. This state is different from the first-quantized coherent states in undergraduate quantum mechanics textbooks, for the reasons as follow.

Some readers may wonder that quantum mechanical coherent states are localized in phase space, so how can they be plane-wave, which are have a fixed wave vector (momentum), but are spread out in position space? The reasons are as follow.

When we say coherent states, we mean two classes of states. The first is coherent states in quantum optics. Such states are localized in quadrature space, as spanned by quadrature operators

$$X = \frac{1}{2}(a^\dagger + a), P = \frac{i}{2}(a^\dagger - a) \tag{6.7}$$

They are not localized in the physical phase spaces. For example, a quantum optical coherent state in a single momentum mode k has one fixed wave vector k, and are spread out in space.

The second class of states are first quantized coherent states, which is usually describing the motion of (massive) physical systems. Such states are indeed localized in the phase space. They usually describe the state of some localized physical systems such as a pendulum or some other forms of harmonic oscillators. The particle numbers (of the physical systems) are fixed. For example the amount of atoms in the harmonic oscillators are fixed and not fluctuating. What is quantized is the motion or excitation of the physical system.

But the coherent states correspond to plane-waves are second-quantized states, in which particle numbers are fluctuating.

We will find the field amplitude when we measure the field operator $\phi(x)$ in a coherent state $|\alpha_p\rangle$

$$\bar{\phi(x)} = Tr[|\alpha_p\rangle\langle\alpha_p|\phi(x)] = \alpha_p e^{ipx} + \alpha_p^* e^{-ipx} \tag{6.8}$$

$$= 2|\alpha_p| cos[\vec{p}\cdot\vec{x} - \omega t] \tag{6.9}$$

This is exactly what a plane-wave should behave, as in Eqn (6.3).

In reality, we are never able to prepare momentum eigenstates. Our experimental setup is local, so we cannot prepare a state that suddenly fills out the whole space. In quantum optics experiments, what we actually create are by Gaussian operations, such as displacement, squeezing, passing through beam splitters, and detections. We usually use attenuated coherent states to approximate single photon states. When we need to know when the single photo has been created, we use spontaneous parametric down-conversion to create entangled photon pairs and detect one of them. There are other constructions of single photon source, such as Nitrogen vacancy center, but those are further away from either momentum eigenstates or plane-waves.

6.2 Superpositions of $|p\rangle$ are not wave packets

Another common misunderstanding is superpersition of momentum eigenstates forms a wave packet (Mark Srednicki, 2007; Michael E. Peskin, 1995). A classical wave packet has an field amplitude that is localized around some space-time region. However, the field amplitude of states which are claimed to be wave packets

$$|f(p)\rangle = \int dp f(p)|p\rangle$$

are not localized. If we measure the amplitude using the Unruh-DeWitt detector, we will get 0 everywhere again.

The quantum states that actually describe wave packets are superpositions of coherent states.

$$|f_\alpha(p)\rangle = \int dp f(p)|\alpha_p\rangle \tag{6.10}$$

Let us compute the result of the the amplitude measurement of these states.

$$\begin{aligned}
\bar{\phi(x)} &= Tr[|f_\alpha(p)\rangle\langle|f_\alpha(p)|\phi(x)] = \alpha_p e^{ipx} + \alpha_p^* e^{-ipx} = 2|\alpha_p| cos[\vec{p}\cdot\vec{x} - \omega t] \\
&= \int dk dq dp f(k)^* f(q) \langle\alpha_k|a_p e^{ipx}|\alpha_q\rangle + \langle\alpha_k|a_p^\dagger e^{-ipx}|\alpha_q\rangle \\
&= \int dk dp f(k)^* f(p) e^{ipx}\langle\alpha_k|\alpha_p\rangle + \int dp dq f(p)^* f(q) e^{-ipx}\langle\alpha_p|\alpha_q\rangle \\
&= \int dk dp f(k)^* f(p) e^{ipx} e^{-\frac{|\alpha_k|^2+\alpha_p|^2-2\alpha_k^*\alpha_p}{2}} + \int dq dp f(p)^* f(q) e^{-ipx} e^{-\frac{|\alpha_q|^2+\alpha_p|^2-2\alpha_p^*\alpha_q}{2}} \\
&= \int dk dp f(k)^* f(p) e^{ipx} e^{-\frac{|\alpha_k|^2+\alpha_p|^2-2\alpha_k^*\alpha_p}{2}} + \int dk dp f(p)^* f(k) e^{-ipx} e^{-\frac{|\alpha_k|^2+\alpha_p|^2-2\alpha_p^*\alpha_k}{2}}
\end{aligned}$$

For simplicity, let us assume both $f(k)$ and α_k are real.

$$\bar{\phi(x)} = \int dk dp f(k) f(p)(e^{ipx} + e^{-ipx}) e^{-\frac{(\alpha_k-\alpha_p)^2}{2}}$$

Clearly, once the integrations are carried out, the only variable left is x, as desired.

To see more clearly, if all amplitudes α_p are equal, this further reduces to

$$\begin{aligned}\bar{\phi(x)} &= 2[\int dk f(k)] \int dp f(p)(e^{ipx} + e^{-ipx}) \\ &= const(\mathscr{F}_f[x] + \mathscr{F}_f[-x])\end{aligned}$$

where $\mathscr{F}_f[x]$ are Fourier transformations. This result depends on x, and if $f(p)$ are chosen wisely, Eqn 6.10 can surely be localized around some space-time region. This means Eqn 6.10 will have the properities of a desired wavepacket.

6.3 The physical meaning of momentum eigenstates

One potential objection the readers may raise is, you theory of quantum plane wave and wave packet relies on the fact that you choose to measure the field operator $\phi(x)$ to get the wave-like oscillating behaviors. If one measure some other operators, such as

$$O(x) = \int dp a_p^\dagger a_p cos[px] \tag{6.11}$$

, one can also show that $O(x)$ oscillates in space-time or the corresponding wave-packets localize in space-time.

It is true that the oscillatory part comes from the observable, not from the state. But it does not mean that any observable makes sense. Let us step back a little to look at the quantum to classical correspondence. Quantum states ρ correspond to a unit region in the classical phase-space. Quantum operators O correspond to a classical observable. In classical physics, when we say the EM wave is a plane-wave, we mean that if we measure the EM field amplitude at different locations in space-time, the results are correlated in ways like $cos[\vec{p} \cdot \vec{x} - \omega t]$. The direct correspondence in quantum physics is that if we measure the EM *field operator* at different locations in space-time, the results are correlated in ways like $cos[\vec{p} \cdot \vec{x} - \omega t]$. It is thus natural to measure the field operators, which directly correspond to field amplitudes. On the other hand, the observable 6.11 is arbitrary, without a clear physical meaning. While it is true that the $a_p^\dagger a_p$ may be related to excitation number (density), there is no reason to multiply it by $cos[px]$, if not for the sole objective to make it look like a plane-wave. But for the field operator, the e^{ipx} comes directly from field quantization, which is natural and necessary.

One may further argue that if we quantize the field in a different way, such as

$$\phi(x) := \int \tilde{dp} a_p a_p^\dagger e^{ipx} + a_p^\dagger a_p e^{-ipx} \tag{6.12}$$

, we will have the desired oscillation when measuring $\langle p|\phi(x)|p\rangle$. But this quantization scheme is not the quantum field theory we are working on, and may not be self-consistent for various reasons.

The physical meaning of momentum eigenstates are clearer in Heisenberg picture. They provides a common reference frame for operators. Operators evolve and accumulate phases, and these phases are relative to momentum eigenstates, which span the free Hilbert space.

We can construct the Hilbert space of quantum field theory as follow. We choose a foliation of space-time labeled by a reference system (x, t). At each time slice t, we associate a Hilbert space $\mathscr{H}(t)$. For now let us focus on the free theory, which furnishes a Fock space structure. There is a natural isomorphism between the Hilbert spaces defined on t and $t + \tau$. $|p\rangle \in \mathscr{H}(t)$ is identified with $|p\rangle \in \mathscr{H}(t + \tau)$ (up to a global phase). We do not label $|p\rangle$ with time. One reason is that all $|p\rangle$ in different Hilbert spaces are identified with each other, since there is a natural isomorphism between the Hilbert spaces defined on different time slices. The other reason is the time-energy uncertainty. Since $|p\rangle$ are eigenstates of the Hamiltonian, it has a definite energy (density). If we label $|p, t\rangle$ as some quantum mechanics textbook did, when doing the Lorentz transformation, we will get into trouble.

But what about time-energy uncertainty in this setting? If we say $|p\rangle$ lies in a Hilbert space defined on a time-slice t, does it mean that $|p\rangle$ is on this precise time t? If so, how do we explain the fact that it has a precise time and energy together?

Let us look at space-momentum dimension. $|p\rangle$ has a precise momentum (density), but it is completely delocalized in space. This means the configuration of every point in space are the same. In other words, $|p\rangle$ describes the situation in the whole time-slice.

In analogy, $|p\rangle$ has a precise energy (density), so it must be completely delocalized in time. This means that at every point in time, the configuration should be the same. This also means that, $|p\rangle$ should be stationary in time. If we know we are in the state $|p\rangle$ at some time, we will be in $|p\rangle$ at any time in the future, and we were in $|p\rangle$ at any time in the past.

This actually makes sense, since $|p\rangle$ is the eigenstate of the Hamiltonian. Its evolution will only acumulate a global phase, which is not physically measurable.

This is also consistent with Lorentz covariance. $|p\rangle$ spans the whole space. When we do a Lorentz covariance, we will mix space and time. As space runs from $-\infty$ to $+\infty$, time should do the same as well.

We thus run into trouble in non-unitary cases, since in those cases, $|p\rangle$ will evolve non-unitarily, thus will not stay as $|p\rangle$ in general, unless in the very special case that $|p\rangle$ is the stable state of the Lindblad equation.

Chapter 7

CONCLUDING REMARKS

This thesis explored the possible ways of constructing a non-unitary quantum field theory that satisfies the Lorentz symmetry. Due to the conceptual, mathematical, and physical inconsistencies, as analyzed in the previous chapters, I have to conclude that non-unitary quantum field theory is fundamentally inconsistent with the Lorentz symmetry.

I would like to point out that these inconsistencies are fundamental and cannot be overlooked. As physicists, we are used to making various approximations. But here, the issues are different. We are not trying to make approximations to perform some useful computation. We are trying to build a fundamental theory aiming at solving fundamental problems. the Lorentz symmetry has to be exact. The definitions have the be clear. The assumptions have to make sense. Some physicists acknowledged the difficulties I pointed out, yet they want to ignore these issues and move on as if they don't exist. I could not agree with this attitude.

I had thought that a Lorentz covariant non-unitary quantum field theory can be constructed, and presented our results at a few conferences. In one of them, I met Professor Sandip Trivedi, who happened to be one of John's first PhD students at Caltech. After I told him what I was working on, he revealed that John had suggested that he work on this problem back in the 1980s; however, he declined. I realized John wanted to construct a non-unitary quantum field for so many years. In some sense, I feel regretted that I may have disappointed him, since as I have shown, this does not work. But on the other hand, I feel relived, since my research gives a conclusive answer to this old question. This would help to clarify the misunderstandings in this field for so many years. Even though the final result is not positive and encouraging as we anticipated, we gained a small yet deeper understanding of the physical nature. This makes my time and efforts meaningful and rewarding.

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