The Boundary Layer for Some Axial Symmetric Flows

Thesis

by

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SUMMARY

The following thesis consists of two sections. Part 1. deals with "The Boundary Layer for Some Axial Symmetric Flows", Part 2. with "Preliminary Experiments on the Flow between Two Circular Disks".

In Part 1. the laminar boundary layer equations for a flow impinging perpindicular to a flat circular disk are considered, assuming that the flow originates in an infinite line source, i.e. neglecting the region where the flow first contacts the disk. The velocity Q far from the disk is assumed proportional to $\frac{1}{r^p}$. If the velocity in the boundary layer q_r is taken proportional to $\frac{1}{r^p} f\left(\frac{z}{\delta}\right)$ and δ proportional to $r^{\frac{p+1}{2}}$ the boundary layer equation reduces to an ordinary differential equation. This equation is solved in three particular cases i.e. $Q =$ constant, $Q \sim \frac{1}{r}$ and $Q \sim \frac{1}{r^3}$.

The case of practical interest $Q \sim \frac{1}{r}$ is considered in some detail and velocity profiles for that case are calculated. These curves suggest the possibility in radial flow of obtaining stable profiles by accelerating the boundary layer.

In Part 2. some preliminary experimental results on the flow between two circular disks are presented.

Considerable difficulty with the entrance was encountered but by useing guide vanes a smooth out flow around the disk was obtained. The velosity profiles under such conditions showed no tendency toward a reversal of the flow.

The efficiency of the set up as a diffusor was quite satisfactory over the radial part of the flow; however for the entire set up including the entrance the efficiency is very poor.

Part 1

"The Boundary Layer for Some Axial Symmetric Flows."

In 1932 Dr. C. B. Millikan published a theory concerning the boundary layer for a figure of revolution. (1)[1] In this paper he derived the laminar boundary layer equations for an axial symmetric flow useing curvilinear coordinates.[2]

Before precceding with his analysis Millikan noted two special cases of this general equation; one in particular being the boundary layer equation for a flow impinging on a flat circular disk. Dr, von Karman suggested that in as much as this case would be of some interest in the design of radial diffusors, and draft tubes it would be worth while to consider it in more detail.

The purpose of this thesis is to do this task.

For our purpose we will consider that the flow has met the disk, has been turned through a right angle and that at some point A (Fig. 1) begins to flow radially parallel to the disk. Our problem then is to determine how rapidly from A on, the thickness of the boundary layer increases. We can simplify this problem considerably, since we choose to neglect the region from 0 to A[3], by assuming that the flow runs parallel to the disk from 0 and is of this nature for an infinite distance above the disk; i.e. we will consider the z axis as an infinite line sourse.

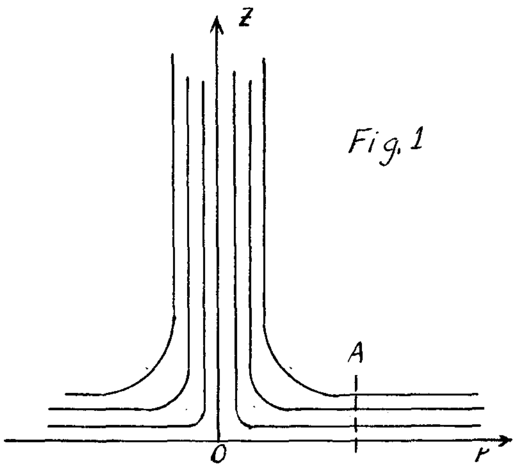


Fig. 1

1; Numbers in parenthesis refer to items in bibliography at the end of paper.

2. Millikan noted that the laminar boundary layer for such a case had been treated by Dr. E. Boltze. (2)

3. This region of the flow has been studied as a potential flow by Reich (3), and recently by W. Schach. (4)

The laminar boundary layer equation for such a case are[1] :

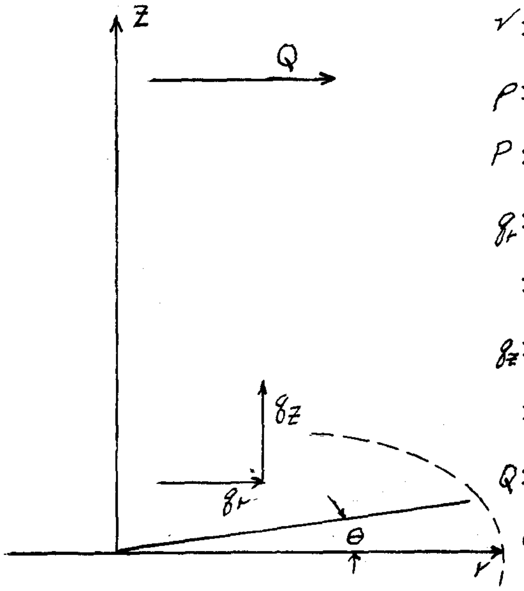
1. $$q_r \frac{\partial q_r}{\partial r} + q_z \frac{\partial q_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu \frac{\partial^2 q_r}{\partial z^2} \quad \text{and}$$

2. $$\frac{\partial P}{\partial z} = 0$$

The equation of continuity is:

3. $$\frac{\partial (r q_r)}{\partial r} + \frac{\partial (r q_z)}{\partial z} = 0$$

where



ν is the kinamatic coefficient of viscosity.

ρ is the fluid density.

P is the pressure.

q_r is the velocity within the boundary layer. in the direction of r.

q_z is the velocity within the boundary layer in the direction of z.

Q is the velocity a great distance from the disk i.e. the potential velocity.

The symmetry of the flow justifies the assumption that $q_\theta = 0$ and $\frac{\partial}{\partial \theta} = 0$.

For the physical problem under consideration the potential velocity Q must be proportional to $\frac{1}{r}$, but in our attempt to find an expression for q which will satisfy equation 1, let us write the potential velocity as proportional to $\frac{1}{r^p}$ and obtain our particular case as the special case $p = 1$. Let us also assume, following Blasius's example (5), that the velocity profiles within the boundary layer, are similar for various r's.

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1. In Millikans's notation (1) $q_r = u$, $q_z = v$, $r = x$, and $z = y$.

i.e. we write $q_r = \frac{A}{r^{n+1}} f(\frac{z}{\delta})$ where δ [1] is the boundary layer thickness at r.

Assuming $\delta \sim r^m$

4. $q_r = \frac{A}{r^{n+1}} f(\frac{z}{r^m})$ and writing $\frac{z}{r^m} = \theta$. The continuity equation requires that

4.1 $$q_z = \frac{m r^m A}{r^{n+2}} \left[\theta f(\theta) + \left(\frac{n}{m} - 1\right) \int f(\theta)\, d\theta \right].$$

Now equation 2. dictates that the pressure is independant of z and therefore will be determined by the pressure in the potential flow outside the boundary layer.

For the velocity in the potential region we write as previously noted

4.2 $Q = \frac{A}{r^p}$ where A is proportional to the sourse strnegth.

Bernoulli's equation $P + \frac{\rho}{2} Q^2 = H$ then gives

4.3 $$\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{p A^2}{r^{2p+1}} .$$

Substituting these expressions for q_r, q_z and $\frac{1}{\rho} \frac{\partial P}{\partial r}$ into equation 1. we obtain the ordinary differential equation:

5. $$\frac{\nu A}{r^{2m+n+1}} f''(\theta) - \frac{(n-m) A^2}{r^{2n+3}} f'(\theta) \int f(\theta)\, d\theta + \frac{(n+1) A^2}{r^{2n+3}} - \frac{p A^2}{r^{2p+1}} = 0$$

(where primes refer to the derivative with respect to θ .)

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1. We define the boundary layer thickness by $z = \delta$ and assume that for $z \geq \delta$ the flow is unaffected by viscosity i.e. is a potential flow. We also assume that the boundary layer is so thin compared with the demensions of the disk that the potential flow at $z = \delta$ is the same as would be given at $z = 0$ by the potential flow of a perfect fluid along the disk.

In order that there nay be a seperation of variables

5.1 $2n+3 = 2p+1 = 2m+n+1$

Expressing n and p in terms of m, and dividing by A^2 equation 5 becomes

6. $$\frac{\nu}{A} f''(\theta) - (m-2) f'(\theta) \int f(\theta) d\theta + (2m-1) f^2(\theta) - (2m-1) = 0.$$

Writing $\int f(\theta) d\theta = h(\theta)$ obtain the final form of this equation

7. $$\frac{\nu}{A} h'''(\theta) - (m-2) h''(\theta) h(\theta) + (2m-1)[h'(\theta)]^2 - (2m-1) = 0.$$

The solution of this equation in general seems practically impossible, however for special values of m it reduces to equations the solution for which are well known i. e. $m = \frac{1}{2}, 1$ or 2.

First let us look at the case of $m = \frac{1}{2}$. Conüidering equations 5.1 and 4.2 we see that $Q = A$, and therefore that the pressure is every where constant.

Equation 7. becomes immediately

8. $$\frac{2\nu}{3A} h'''(\theta) + h''(\theta) h(\theta) = 0$$

writing $\varphi = -3A\theta$ equation 8. is

8.1 $$2\nu h'''(\varphi) = h''(\varphi) h(\varphi).$$

This is the equation Blasius (6) solved when he considered the lamiar boundary layer for a two demensional flow, with no pressure drop, past a flat plate. He found a solution to be:

$$h(\varphi) = a_0 \varphi^2 + a_1 \varphi^5 + a_2 \varphi^8 + a_3 \varphi^{11} + \dots$$

where $\nu a_1 (5!) = 2 a_0^2$; $\nu^2 a_2 (8!) = 22 a_0^3$; $\nu^3 a_3 (11!) = 750 a_0^4$, ...

and $q_r = h'(\varphi)$: $q_z \doteq -A r^{-\frac{1}{2}}[4h'(\varphi) - 3h(\varphi)]$.

For the case $m=2$ the potential velocity is proportional to $\frac{1}{r^3}$ and the potential flow is that from a four dimensional sourse. Let us consider equation 6. writing $m=2$ we have:

$$9. \quad f''(\theta) = \frac{3A}{\nu}\left(f^2(\theta) - 1\right)$$

A solution of this equation is the Weierstrass elliptic function p(u).

Having considered in brief these two cases $m = \frac{1}{2}$ & 2, we will now turn to a detailed discussion of the case $m = 1$. Here the potential velocity is proportional to $\frac{1}{r}$ which is the case we set out to treat. We have in this case $\phi = \frac{A}{r}$ and the sourse strength B therefore is: $B = 2\pi A$.

Equation 7. now becomes

$$10. \quad \frac{\nu}{A} h'''(\theta) + h''(\theta) h(\theta) + [h'(\theta)]^2 - 1 = 0$$

integrading twice with respect to θ and multiplying by $\frac{A}{\nu}$, we get

$$10.1 \quad h'(\theta) + \frac{A}{2\nu} h^2(\theta) = \frac{A}{2\nu}\theta^2 + 2a\frac{A}{2\nu}\theta + b.$$

If we write $\varphi = \sqrt{\frac{A}{2\nu}}\,\theta$ and $k(\varphi) = \sqrt{\frac{A}{2\nu}}\,h(\varphi)$ equation 10.1 becomes

$$10.2 \quad k'(\varphi) + k^2(\varphi) = \varphi^2 + a\sqrt{\frac{A}{2\nu}}\,\varphi + b.$$

Let us digress a moment before we attempt to solve this Riccatti equation. With a viscous fluid there can be no slip at the boundary that is at $z = 0$ i.e. $\theta = \varphi = 0$; $q_r = q_z = 0$. A glance at equations 4 and 4.1 shows us that this neccessitates both k and k' being zero. We see then that the intergration constant "b" of equation 10.2 is zero.

Let us write $\alpha = \varphi + c$ where $c = a\sqrt{\frac{A}{2\nu}}$ and equation 10.2 becomes

10.3 $$k'(\alpha) + k^2(\alpha) = \alpha^2 - c^2 \qquad\qquad \alpha = \sqrt{\frac{A}{2\nu}}\left(\frac{z}{r} + a\right).$$

The transformation $k = \frac{\eta'}{\eta}$ finally gives us

10.4 $$\eta''(\alpha) + (c^2 - \alpha^2)\eta(\alpha) = 0 .$$

This is a form of Hermite's equation[1] a solution of which is:

$$\eta_1 = E_n(\alpha) = \left(\sqrt{\pi}\,2^n n!\right)^{-\frac{1}{2}} e^{-\frac{\alpha^2}{2}} H_n(\alpha) \qquad c = \sqrt{2n+1}$$

where $H_n(\alpha)$ is a Hermitian polynominal of the order of n. (7.)

A second solution is obtained immediately as:

$$\eta_2 = E_n(\alpha)\int \frac{d\alpha}{E_n^2(\alpha)}$$

We have already mentioned two of the boundary conditions[2], namely

condition 1. at $z = 0;\ \theta = \varphi = 0;\ \alpha = c$: $q_r = 0$, i.e. $k' = 0$

condition 2. at $z = 0;\ \theta = \varphi = 0;\ \alpha = c$: $q_z = 0$, i.e. $k = 0$

The Riccatti equation 10.3 then gives for

$$k''(c) = 2c \quad \text{and} \quad k'''(c) = 2 .$$

The third condition is furnished in our definition of the boundary layer thickness i. e. we require that q_r approach Q at a great distance from the disk. Condition 3: for $z = \infty$ i.e. $\theta = \varphi = \alpha = \infty$

$$q_r = Q$$

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1. If $c = 0$; writing $\eta(\alpha) = \alpha^{\frac{1}{2}} Z\left(\frac{i\alpha^2}{2}\right)$ trnasforms equation 10.4 into one of Bessel type; a solution is: $Z\left(\frac{i\alpha^2}{2}\right) = 2\pi e^{\frac{\pi i}{4}}\left[\frac{\sqrt{2}}{2} J_{\frac{1}{4}}\left(\frac{i\alpha^2}{2}\right) - J_{-\frac{1}{4}}\left(\frac{i\alpha^2}{2}\right)\right].$

2. These conditions allow us to give a relation between the shearing stress τ_0 at the surface of the disk, and the integration constant "a" i.e. $\tau_0 = \mu\left(\frac{dq_r}{dz}\right)_0 = 2a\frac{\rho}{2}Q^2$

or $2a = C_f$ (the friction coefficient of the disk)

Consider now condition 3 : for $\alpha = \infty$, $g_r = \Phi$

$$\text{or} \quad \frac{g_r}{\Phi} = 1.$$

from equation 4 we see that this means that ;

$$f(\theta) = 1 \quad \text{or} \quad k'(\alpha) = 1.$$

Thus for a large value of

$$k'(\alpha) = \frac{d^2 \log \eta(\alpha)}{d\alpha^2} = 1, \text{ or}$$

10.5 $$\eta = \alpha^n e^{\frac{\alpha^2}{2}}$$

This condition at infinity excludes the possibility of η_1 satisying the physical conditions of the problem. So let us choose as a solution:

$$\eta = F\eta_1 + G\eta_2 = E_\eta(\alpha)\left[F + G\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)}\right].$$

This gives k the value,

10.6 $$k = \frac{\eta'}{\eta} = \frac{E_\eta'(\alpha)}{E_\eta(\alpha)} + \frac{1}{E_\eta^2(\alpha)\left[\gamma + \int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)}\right]}.$$

where $\gamma = \frac{F}{G}$

It will be remembered that just following equation 10.2 we used condition 1 and 2 to determine that the integration constant "b" of that equation was zero. Condition 2 now determines a relation between γ and c.

$$\frac{1}{\gamma} = -E_\eta(c)\cdot E_\eta'(c)$$

Consider 3 as we have seen, determine the nature of the function η but all of these conditions leave the integration constant c undetermined. This indeterminacy is due to the singular nature of the z axis. For $r = 0$, $z > 0$ we have g_r, g_z, Q and α all infinite. As we approach the origion itself i.e. $r = z = 0$; α remains very large; of the order of $\frac{1}{\nu}$ for $\nu \to 0$ and g_r & Q are infinite. g_z is the order $\frac{\sqrt{\nu}}{r}$ as $\nu \to 0$ and $r \to 0$. The problem becomes completely determined if we specify the drag coefficient of the disk. (i.e. the slope of the velocity profile at the wall.) or if at any r we require that δ have a given value. We have therefore a family of velocity profiles all of which satisfy the boundary conditions of our problem. i.e.

10.7
$$q_r = \frac{A}{r}\left\{\frac{E_\eta''(\alpha)}{E_\eta(\alpha)} - \left[\frac{E_\eta'(\alpha)}{E_\eta(\alpha)}\right]^2 - \frac{2E_\eta'(\alpha)}{E_\eta^3(\alpha)\left(\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)} - \frac{1}{E_\eta(c)E_\eta'(c)}\right)} - \frac{1}{E_\eta^4(\alpha)\left(\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)} - \frac{1}{E_\eta(c)E_\eta'(c)}\right)^2}\right\}$$

10.8
$$\sqrt{\frac{A}{2\nu}}\, q_z = \frac{A}{r}\left\{(\alpha - c)\left\{\frac{E_\eta''(\alpha)}{E_\eta(\alpha)} - \left[\frac{E_\eta'(\alpha)}{E_\eta(\alpha)}\right]^2 - \frac{2E_\eta'(\alpha)}{E_\eta^3(\alpha)\left(\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)} - \frac{1}{E_\eta(c)E_\eta'(c)}\right)} - \frac{1}{E_\eta^4(\alpha)\left(\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)} - \frac{1}{E_\eta(c)E_\eta'(c)}\right)}\right\} - \left(\frac{E_\eta'(\alpha)}{E_\eta(\alpha)} + \frac{1}{E_\eta^2(\alpha)\left(\int_c^\alpha \frac{d\alpha}{E_\eta^2(\alpha)} - \frac{1}{E_\eta(c)E_\eta'(c)}\right)}\right)\right\}.$$

Now our original purpose was to determine the thickness of the boundary layer as a function of r. This cannot be done exactly as by its definition the velocity q_r in the boundary layer tends asymptotically to the velocity of Q of the potential flow. However if the outer surface of the boundary layer is defined by the conditions that q_r has approached to a value differing from Q by some arbitarily small percentage, the thickness of the boundary layer can be assigned a definite value. In particualr if we define $z = \delta$ as the value of z for which q_r is within 1% of the potential velocity Q; we can express δ as a function of r. In order to do this consider the asymptotic expression for η given in equation 10.5 Expressing k in terms of η and substituting this expression for k in equation 10.3 we see that large values of α

10.9
$$k' = 1 + \frac{c^2+1}{2\alpha^2}$$

and $z = \delta$ is then

10.10 $$\delta^{1.} = \sqrt{\frac{2\nu}{A}}\left[50(c^2+1)-c\right]r.$$

The asymptotic expression for k' is rather surprizing and one's curiosity is at once aroused as to the form of these velocity profiles. At present we know only that the demensionless velocity profile ($\frac{q_r}{Q} = k'$) starts from the disk with a slope of 2 c and approaches the potential velocity Q from a region of higher velocity than that of the potential flow.

The profiles for the case c = 1 were claculated useing equation 10.7; the $\int_c e^{\alpha^2} d\alpha$ being evaluated graphically. The curve for k' thus determined is shown in Fig. 2 and 4. For other values of c approximate forms of the profiles were obtained by numerical integration of the Riccatti equation 10.3[2]. These profiles are shown in Fig. 2 to Fig. 7.

Dr. von Karman suggested an interesting physcial interpretation of these rather unusual velocity profiles. His idea being that if we should accelerate the boundary layer by forceing high velocity fluid into the flow near the disk we would obtain profiles of this type.[3]

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1. If we take $\frac{A}{\nu}$ as the Reynold Number of the flow, then $\delta \sim \frac{r}{R^{\frac{1}{2}}}$ which is similar to Blasius's result. (5) However for this case the Reynold Number of the boundary layer R_δ remains constant, i.e.

$$R_\delta = \frac{Q\delta}{\nu} = \sqrt{\frac{A}{\nu}}\cdot\sqrt{2}\cdot\left[50(c^2+1)-c\right].$$

2. The numerical integration was carried out useing Adams method(8)
3. With this interpretation the notion of a large number of stable profiles become tenable.

It must be remembered that we have been considering a laminar flow. However one is always tempted to hope that the results obtained by considering such cases may be of some significance for the more disturbed flow which occurs in practice. If such hopes should be fulfilled it would be unquestionably desirable to obtain by some means of boundary layer acceleration velocity profiles of the type indicated by the theory.

Part 2

Some Preliminary Experiments on the Flow between Two Circular Disks.

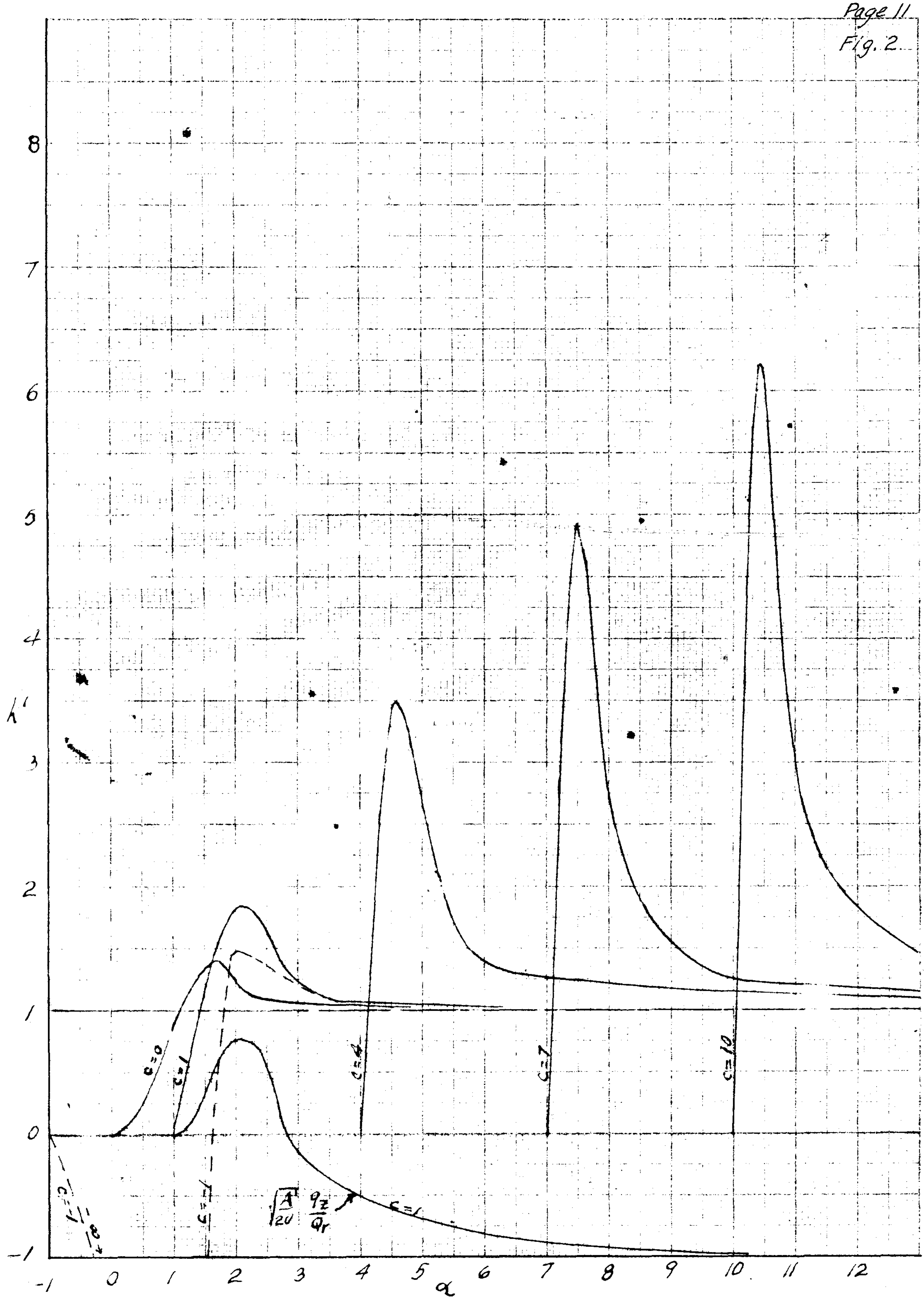


Fig. 2

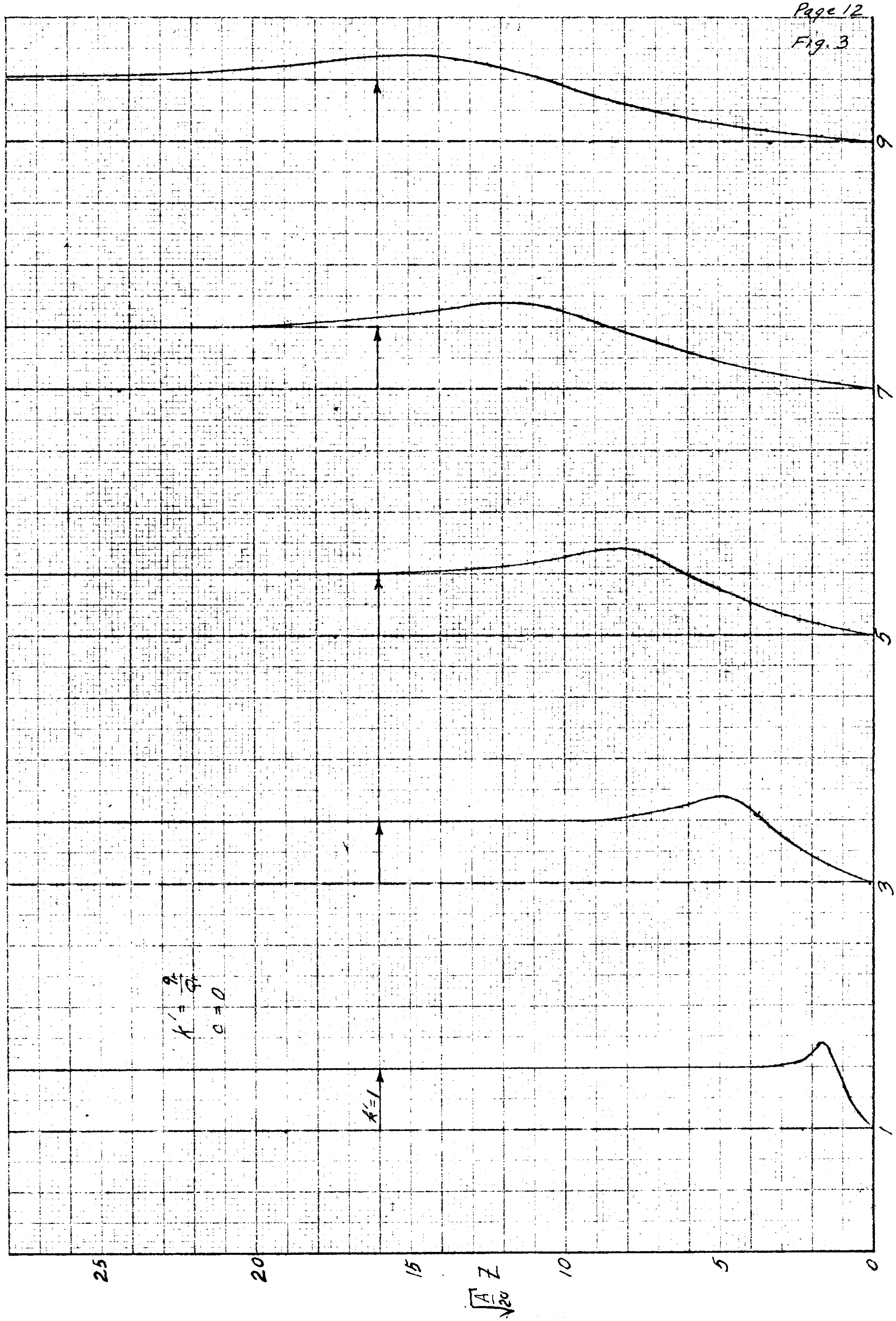


Fig. 3

Fig. 4.

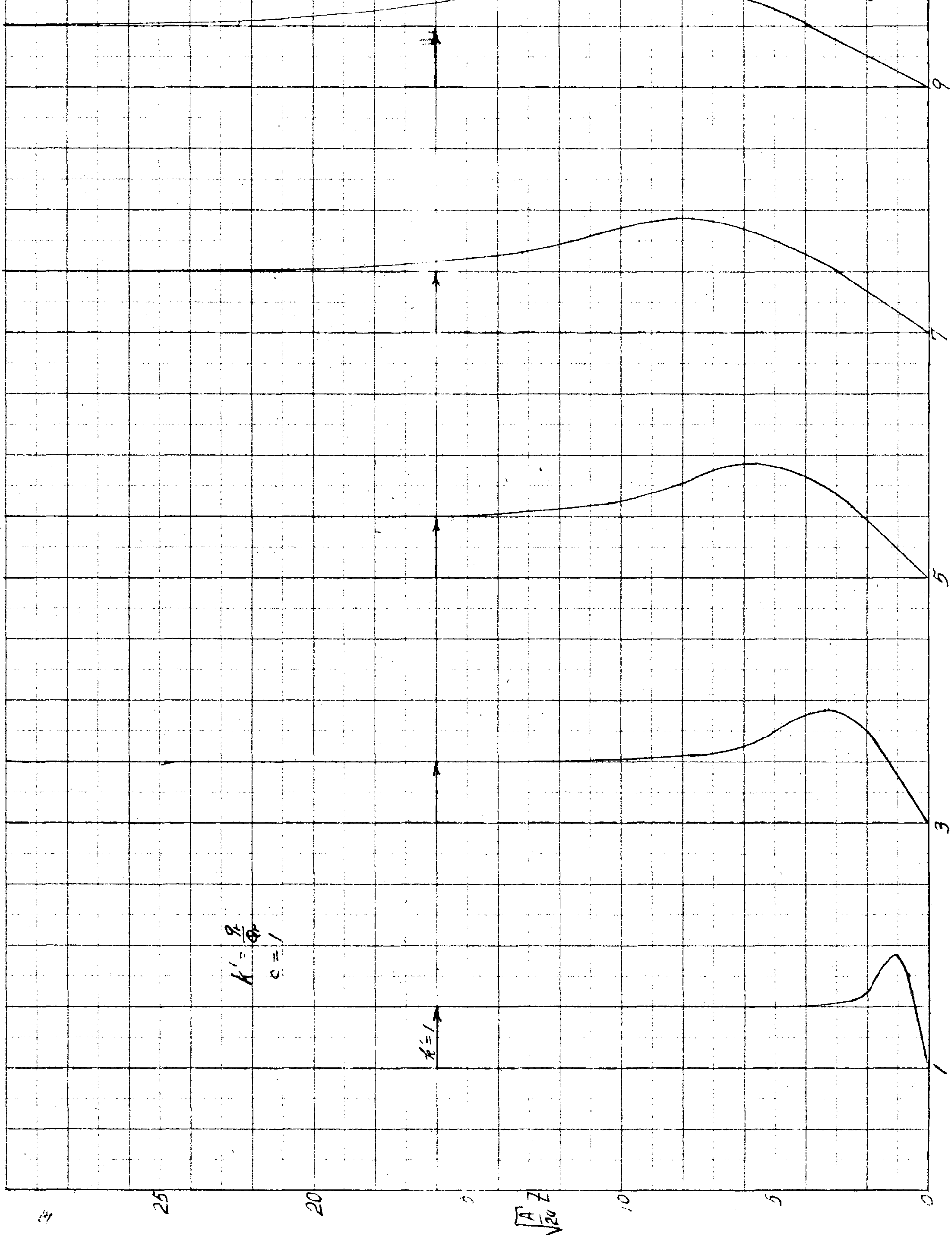


Fig. 5

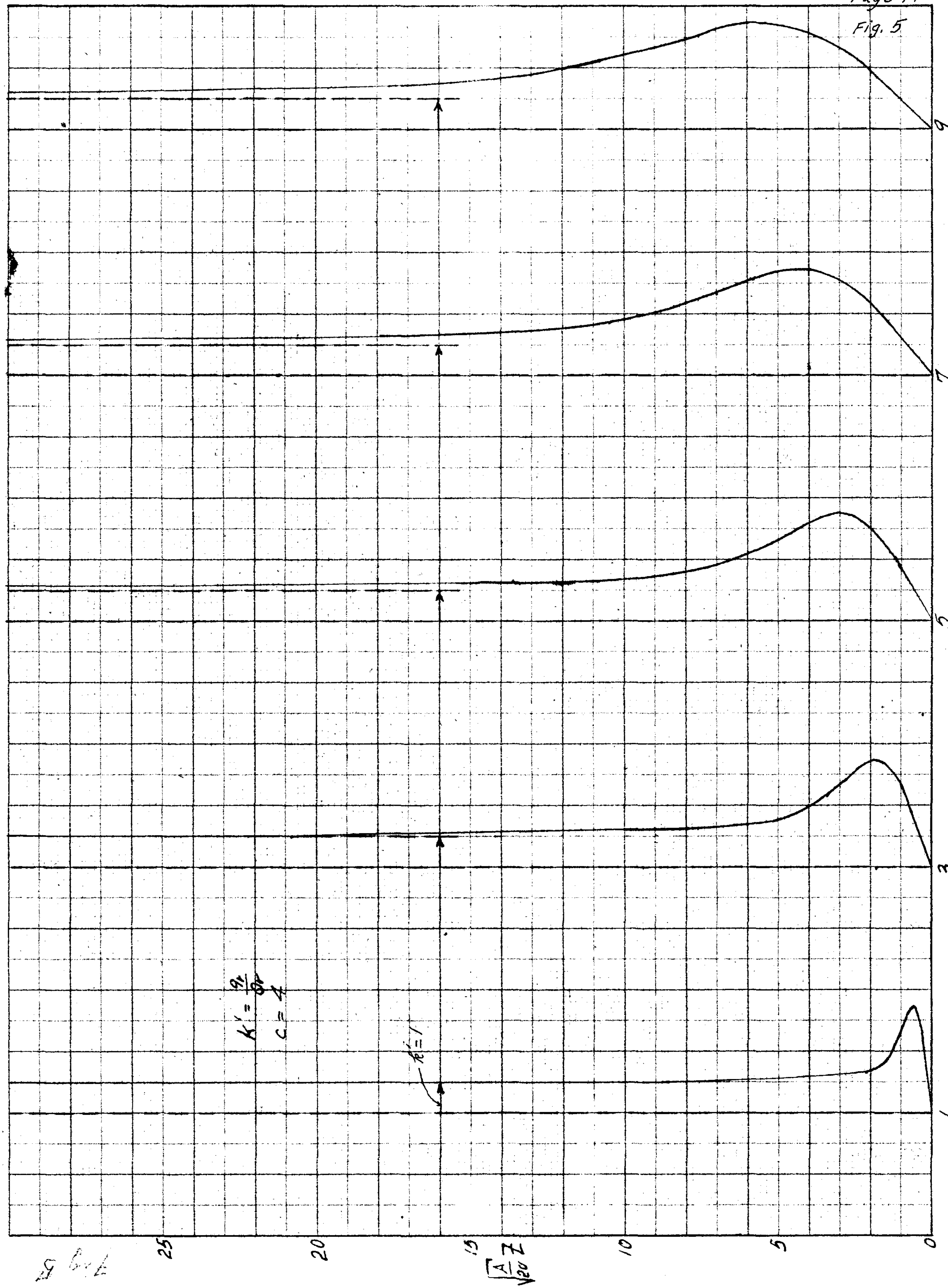
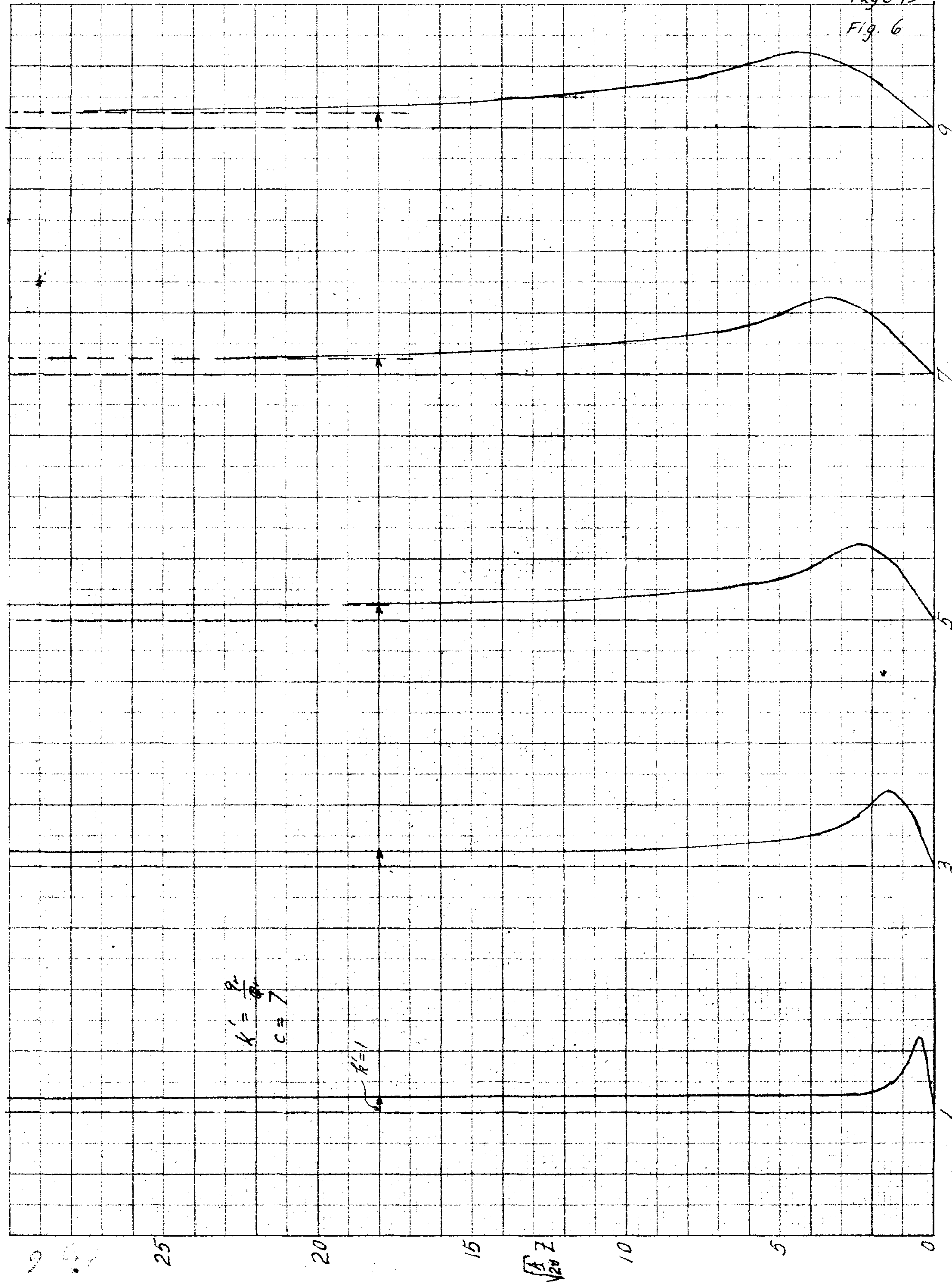


Fig. 6



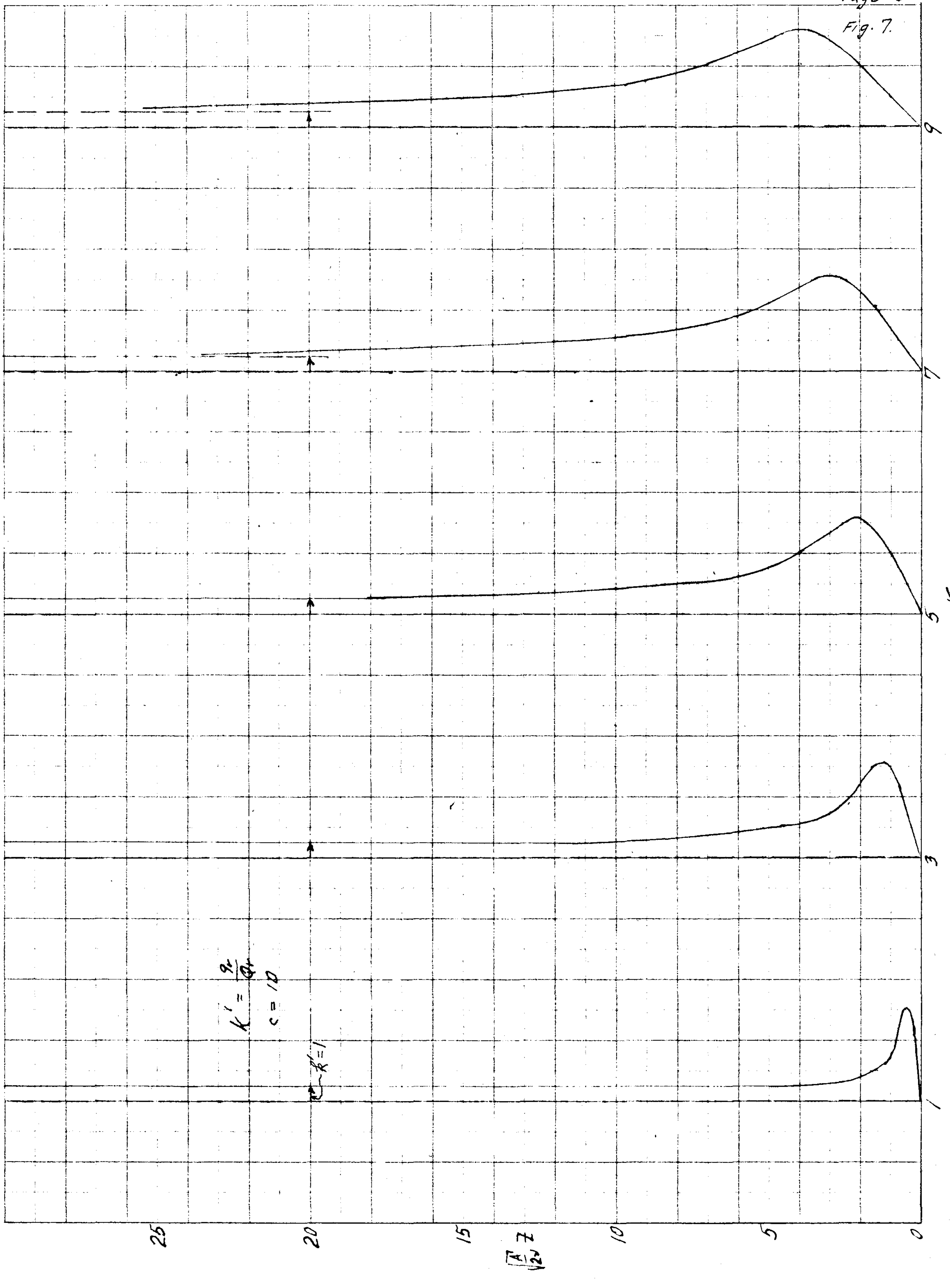


Fig. 7.

The low velocity associated with laminar flow and the accompaning lack of practical interest made it seem appropriate to make no attempt to check the preceeding theory in these preliminary experiments.

The experiments were made in a small Eiffel type wind tunnel, as shown in Fig. I

In Fig. I:

(1) A bell shaped intake.

(2) The disks (Diameter = 152 cm.)

(3) A cone to maintain contraction of the flow.

(4) An exit bell.

(5) A 12 H.P. fan unit.

The original design was for a 2.5 cm. gap between the disks and the cone or bump was built so as to have the flow continually contracting during its turn onto the disk. This neccessitated a rather sharp corner at the tunnel wall which subsequently proved quite unsatisfactory. Fig. II shows a cross section of the entrance, the original set up being with the vanes (a) and (b) removed.

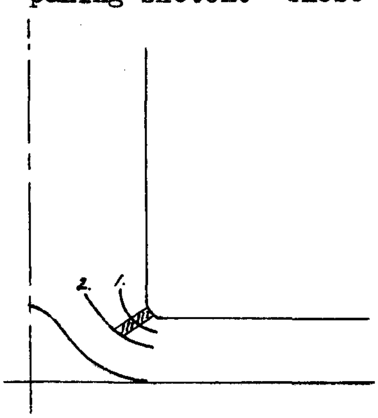
The two tubes used for measuring total head consisted: tube (1) of a piece of .32 cm. brass tubing with a right angle bend, in the end of this tube a length of hypodermic needle 0.64 mm. in diameter was carefully soldered, Tube (2), a piece of stream line brass tubing .50cm. x .08cm. with a similar needle.

The tubes were moved across the flow by a hand operated micrometer, adjustable to 0.01mm. The locations of the measuring stations on the disk are shown in Fig. I. At each station there was a small orfice in the disk, located in the same arc as the nose of the total head tube and situated about 3cm. from it, for the purpose of measuring the static pressure at the disk.

The first measurements were made useing the original set up (see Fig I) i.e. without vanes a and b (Fig II). The dynamic pressure was measured as the difference between the total pressure (useing tube 1) at z cm. from the disk[1] , and the static pressure[2] at the disk. Measurements at stations A_3 ; A_8 ; B_3 ; B_8 ; C_3 ; C_8 ; D_3 ; D_8 ; showed that the flow was satisfactory symmetrical around the disk.

The velocity profiles observed near the outside of the disk were slightly asymmetrical and some slight fluctuation of the dynamic pressure was noticable. Toward the center of the disk this fluctuation became quite violent and the velocity profiles more asymmetrical until at station A_1 ; the velocity near the disk was considerably larger than the average velocity across the section and the velocity near the tunnel wall had completely reversed. This was clearly due to the inability of the fluid to take the sharp turn at the tunnel wall.

In order to correct this, two vanes were built as shown in the accompaning sketch. These vanes were mounted to the tunnel wall at three points.



The spacers between the vanes were 2cm. x 0.5cm. stream line tubing. This arrangement proved highly unsatisfactory as the wake behind the spacers blocked the flow over a large portion of the disk. A cylindrical sheet metal tube was attached to vane (1) and this became vane (a) of Fig II.

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(1) We will call the disk the flow impinges against "the disk". The other disk "the tunnel wall."

(2) The static pressure was checked and found to be constant across the channel at a given station.

Vane (a) was thus supported in the cylindrical part of the entrance section some 20cm. ahead of the turn. This arrangement proved satisfactory and the flow out from the disk was quite smooth. However with this steady flow the wake behind the total head tube (1) became very noticable and in order to reduce this disturbance, tube (2) was constructed.

With this set up i. e. as Fig II , with vane (b) removed, useing tube (2) velocity surveys were made and are shown in Fig III. Here again the velocity reaches a maxium near the disk although now there is no reversal of the flow.

In order to get a more symmetrical velocity profile a third vane was built (vane b in Fig. II) These two vanes (a and b) were mounted so they could be moved relative to each other and to the tunnel wall. A number of vane spacings and 5 disk gaps were tried in a very rought manner (see Fig IV). In Fig. IV the remark "Good" means that to the feel the outflow is quite uniform around the perimeter of the disk. "No Good" means that there are regions in which the out flow is definitely decreased, in some cases completely reversed. "Fair " indicated that there appeared to be a slight decrease in out flow over a rather small region.

The configuration with 1.2 cm. gap and the vanes spaced equally at 0.4 cm. seemed of interest in as much as it should roughly divide the fluid equally between the two walls and the center of the channel and give a symmetric profile. Velocity profiles for this case are shown in Fig V and Fig VI for two speeds. It will be noticed that at different radii the gap between the disk was different; this was due to the variations in the surface of the disk and to the fact that the disk was sucked in toward the tunnel wall about 2mm. when the fluid was flowing at its highest velocity.

Fig III, V and VI represent curves faired through the experimental points which were obtained as indicated in the preceeding section. The coordinates being z the distance in cm. from the disk and $\frac{u}{\bar{U}}$ the ratio of the velocity at z to the velocity across the section.

$$\bar{U} = \frac{1}{G}\int_0^G u\,dz$$

Where G is the gap between the disks.

The curves at the top of these figures indicate the variation of $\bar{U}$ with r assuming $\bar{U} = \frac{B}{2\pi r}$ (where B is the source strength). B was taken as the average value of $2\pi r\bar{U}$ over the range of experimental observations. The actual value of $\bar{U}$ as measured is indicated by the experimental points. The agreement betaeen the experimental value of $\bar{U}$ and the calculated value would indicate again that no fluid enters or leaves a given region except radially.

The efficiency of the set up as a diffusor is calculated and shown on each graph as η_9' i.e. considering only the radial part of the flow from station A_1 to A_9 and neglecting the loses in the entrance, The efficiency has been defined as:

$$\eta_9' = 1 - \frac{\bar{P}_{t1} - \bar{P}_{t9}}{\bar{Q}_1}$$

Where $\bar{P}_{t1}$ is the average total pressure at station A_1

$\bar{P}_{t9}$ is the average total pressure at station A_9

$\bar{Q}_1$ is the average dynamic pressure at station A_1.

The Reynold's Number is:

$$R.N = \frac{\bar{u} r}{\nu} = \frac{B}{2\pi\nu}$$

where B the source strength

ν the kinematic coefficient of viscosity.

The velocity profiles in Fig. III are for a gap of 2.4 cm. with vane (a) set at approximately 8 mm. from the tunnel wall.

The wake behind the vane is quite noticable and the loss of velocity due to making the turn onto the wall combined with the friction loss in flowing between the entrance tube and the vane support tube is large. The mixing along the tunnel wall is strong and the jet and wake from the vane rapidly disappear. It would seem that probably the fluid still broke away from the wall during the turn and created considerable disturbance. Also it appears that between stations A_1 and A_3 some of the fluid has left this region. However from A_3 on the flow is radial and has obtained a profile which although asymmetrical retains its form through the rest of its travels.

The curves of Fig. V and VI are for a gap of 1.2 cm. useing two vanes (a and b, of Fig II) set so as to divide the fluid approximately equally between the two walls, and the center of the channel. It appears that with this configuration the loss of velocity in making the turn onto each wall has been made about equal and by station A_5 the profile has become almost symmetrical and retains this character during the rest of the flow. The efficiency in this case shows a slight decrease which might be expected in as much as the wetted area relative to the quantity of fluid flowing over it has been increased.

From Figs. V and VI we see that there is some improvement with increase of Reynold's Number.

The preliminary nature of these results makes impossible any final statement as to the efficiency of radial diffusors. Apparently it would seem that we could expect fairly good efficiencies in the radial part of the flow at these Reynold Numbers.

The entrance section used in these experiments was very poor, the loss in total heah through the entrance to station A_1 is of the order of 50%.

In conclusion the author wishes to acknowledge his sincere appreciation of the suggestions given and the continued interest shown by Dr. von Karman, Dr. A. L. Klein, Dr. C. B. Millikan and Dr. Tollmein. Also my thaks to Mr dane for his assistance in the construction of the experimental set up.

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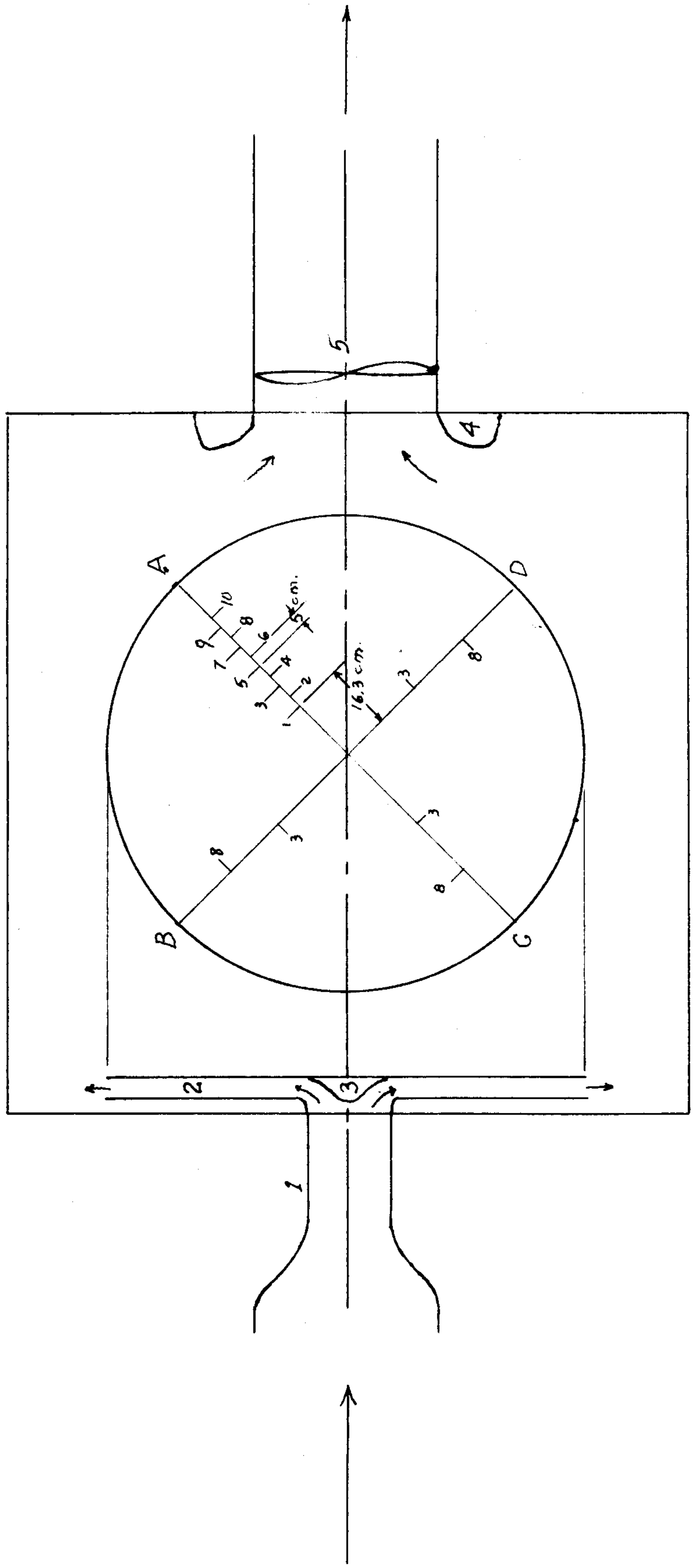
Fig. I



Schematic diagram of set up

Fig II

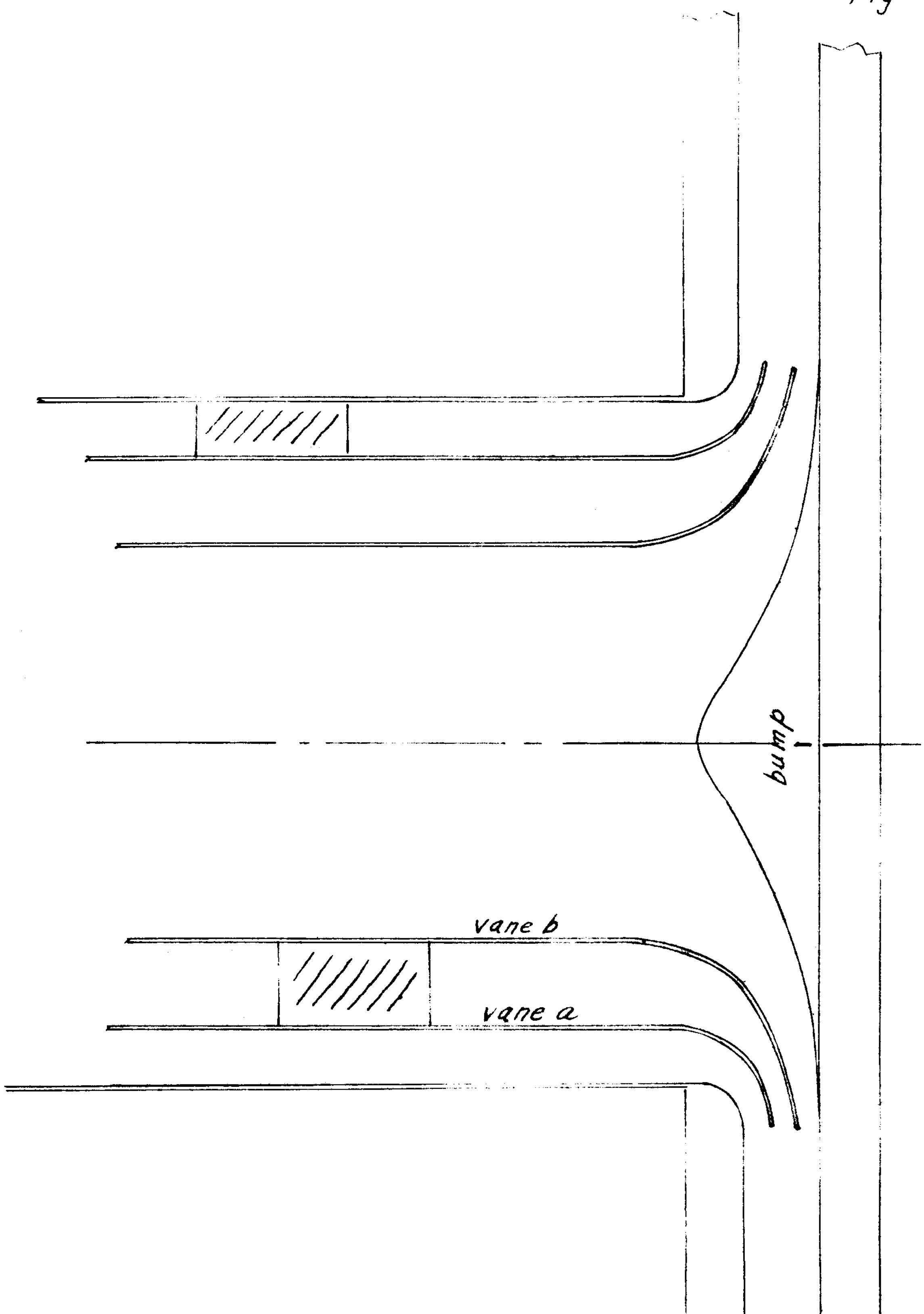


Diagram of entrance section

scale 1cm = 2cm

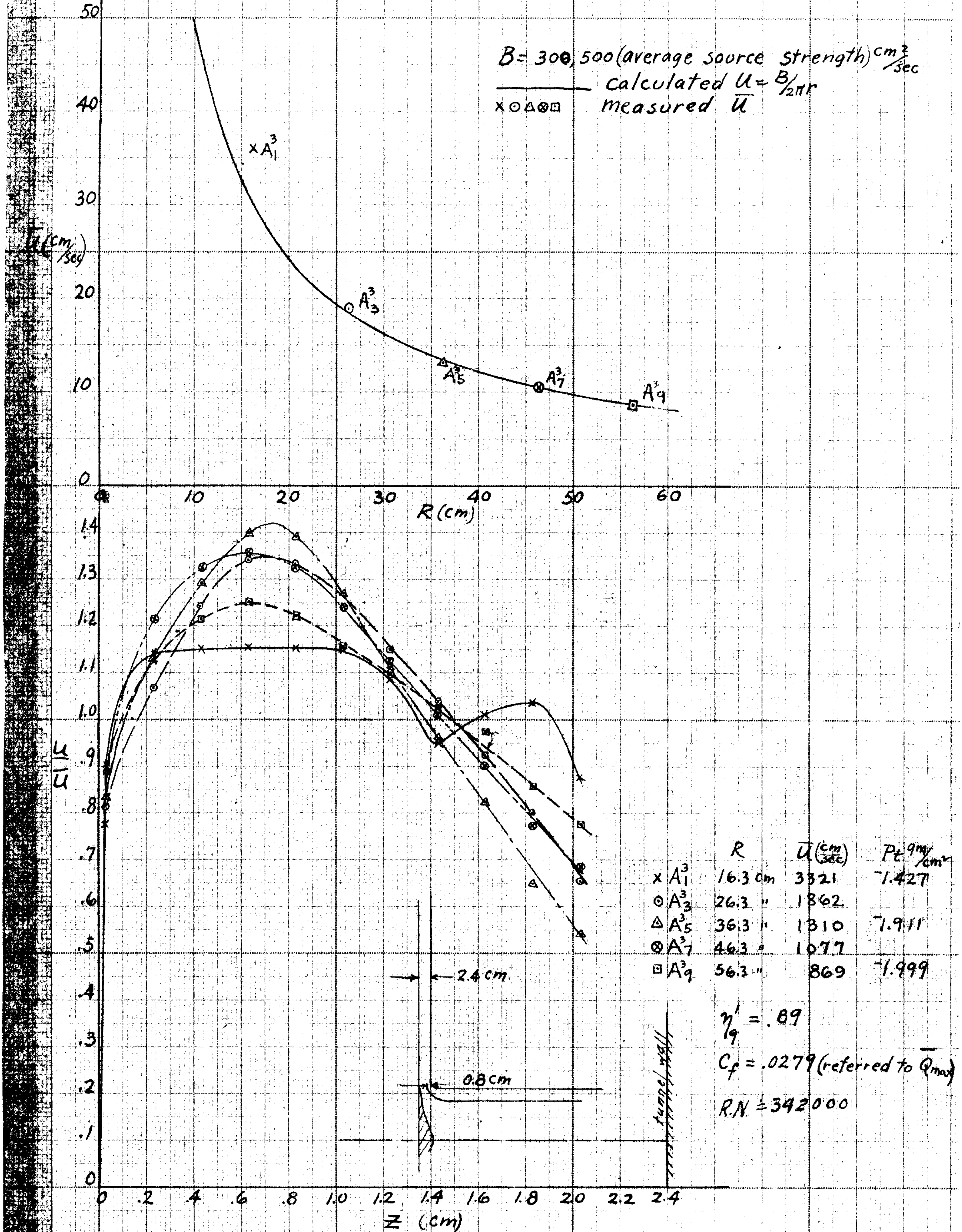
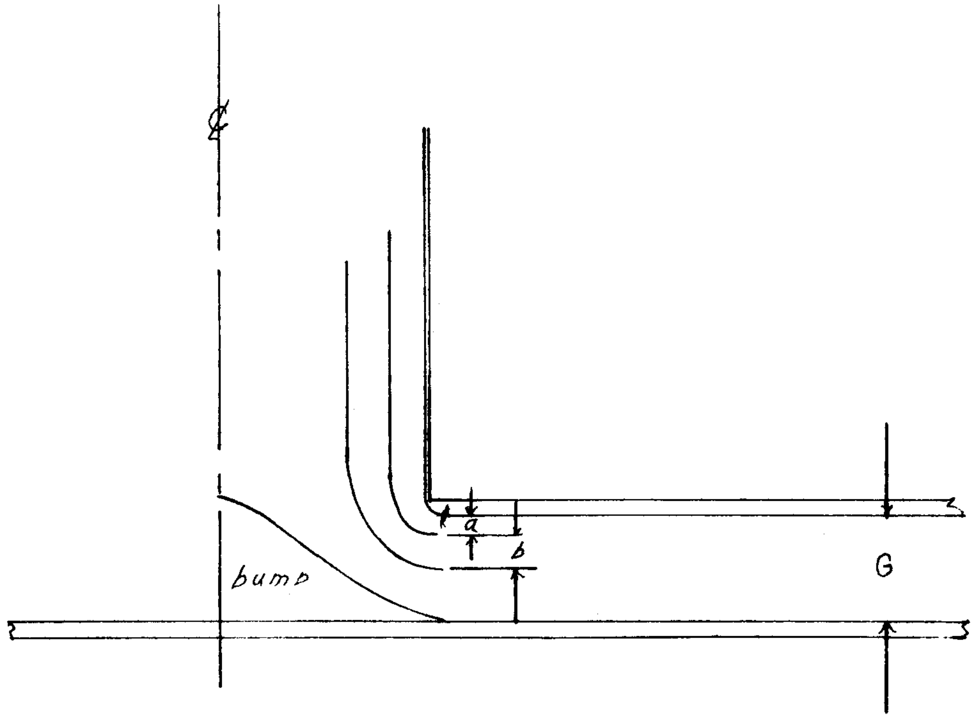

B = 300,500 (average source strength) cm²/sec
calculated U = B/2πr
measured Ū
A₁³
A₃³
A₅³
A₇³
A₉³
U (cm/sec)
R (cm)
u/Ū
Z (cm)
2.4 cm
0.8 cm
tunnel wall
R | Ū (cm/sec) | Pt gm/cm²
A₁³ 16.3 cm 3321 -1.427
A₃³ 26.3 " 1862
A₅³ 36.3 " 1310 -1.911
A₇³ 46.3 " 1077
A₉³ 56.3 " 869 -1.999
η′₉ = .89
C_f = .0279 (referred to Q̄max)
R.N. = 342000

Fig IV



bump	gap	a	b	Remark
no	2.4	no	no	no good
yes	2.4	no	no	no good
yes	2.4	0.8	no	good
yes	2.4	0.4	0.8	fair
yes	2.4	0.8	0.8	no good
yes	2.4	0.4	0.4	good
yes	2.4	0.8	0.4	no good
yes	2.4	0.8	0	good
yes	2.4	0	0	no good
yes	2.4	0	0.4	good
no	2.4	0	0	no good
no	2.4	0.8	0.8	fair
no	2.4	0.4	0.8	good
no	2.4	0	0.4	fair
no	2.4	0.4	0.4	good
no	2.4	0.8	0.4	no good

note: lengths given in centimeters

Fig. IV

bump	gap	a	b	Remarks
yes	1.2	no	no	fair
no	1.2	no	no	fair
no	1.2	0	1.0	no good
no	1.2	0.1	1.0	fair
no	1.2	0	0.4	fair
no	1.2	0.4	0.4	good
no	3.0	0	0	no good
no	3.0	0.5	0	no good
no	3.0	0	0.5	no good
no	3.0	0.5	0.5	fair
no	3.0	1.0	0.5	no good
no	3.0	0	1.0	no good
no	3.0	0.5	1.0	fair
no	3.0	1.0	1.0	no good
no	4.0	0.4	0.4	no good
no	4.0	1.0	0.4	no good
no	4.0	1.0	1.0	no good
no	4.0	1.0	1.5	no good
yes	5.0	0.4	0.4	no good
yes	5.0	0.4	1.0	no good
yes	5.0	1.0	1.0	no good
yes	5.0	1.0	1.6	no good
yes	5.0	0.8	1.6	no good

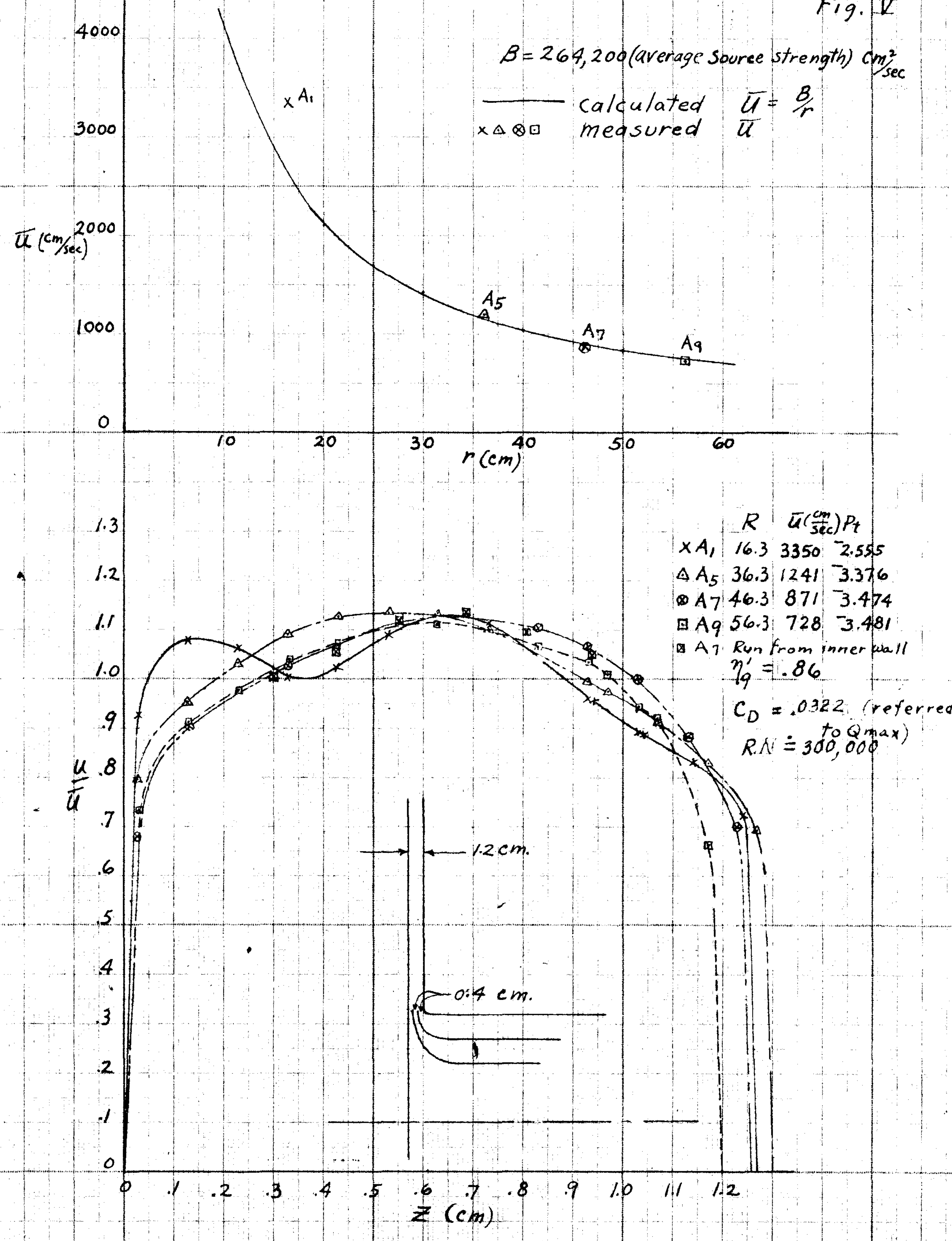
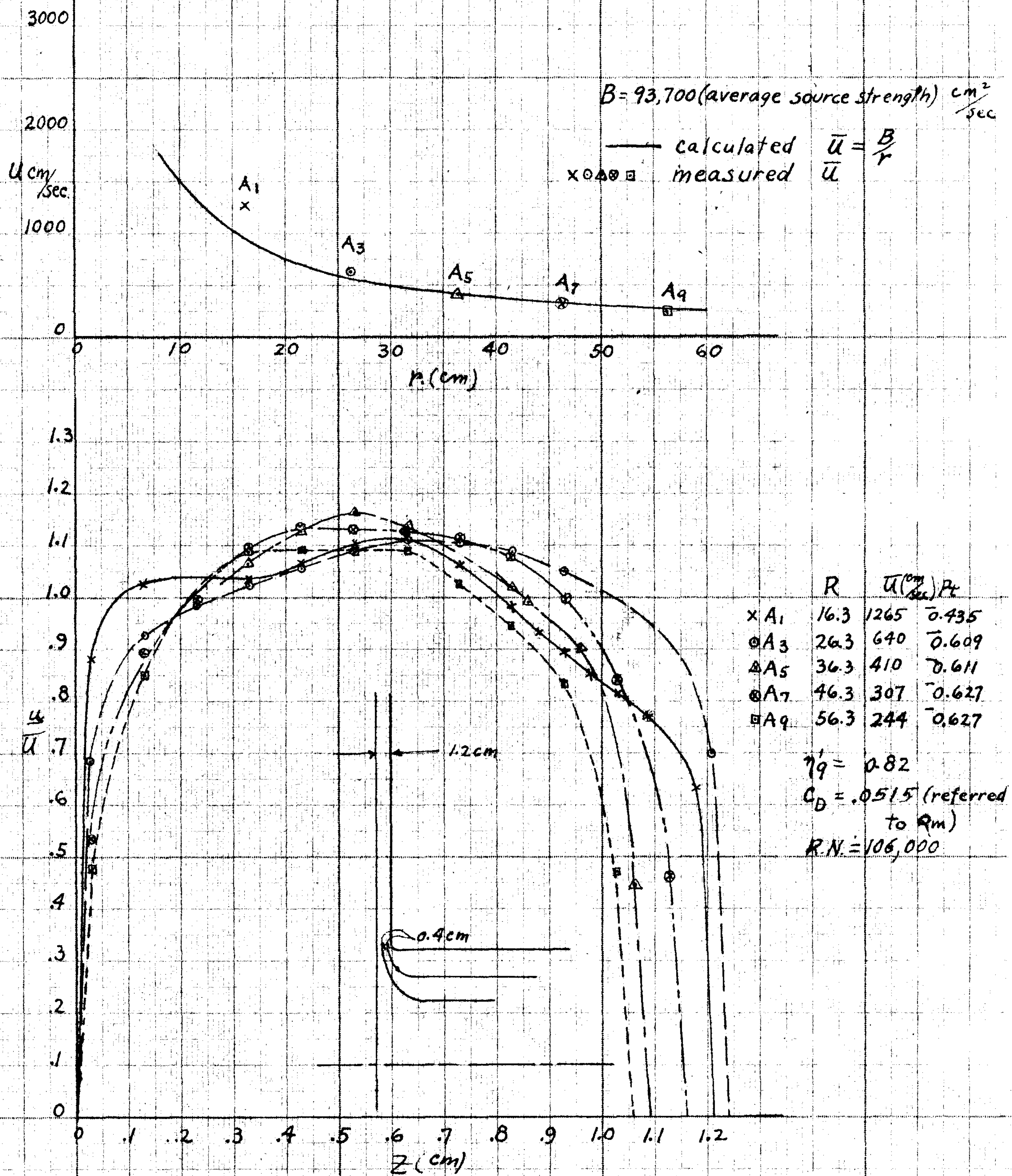
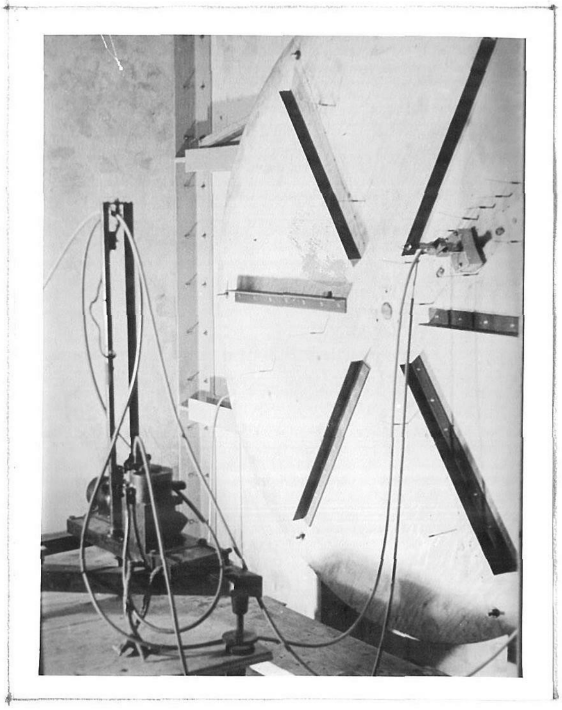
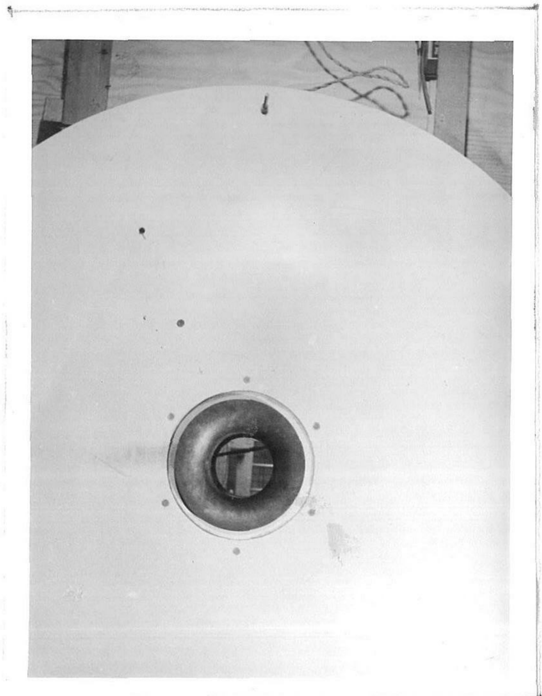

B = 264,200 (average Source strength) cm²/sec
calculated Ū = B/r
× △ ⊗ ⊡ measured Ū
Ū (cm/sec)
4000
3000
2000
1000
0
A1
A5
A7
A9
10
20
30
40
50
60
r (cm)
R Ū(cm/sec) Pt
× A1 16.3 3350 ‾2.555
△ A5 36.3 1241 ‾3.376
⊗ A7 46.3 871 ‾3.474
⊡ A9 56.3 728 ‾3.481
⊠ A7 Run from inner wall
η'9 = .86
CD = .0322 (referred to Qmax)
RN ≐ 300,000
u/Ū
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1.2 cm.
0.4 cm.
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
Z (cm)

Fig. VI

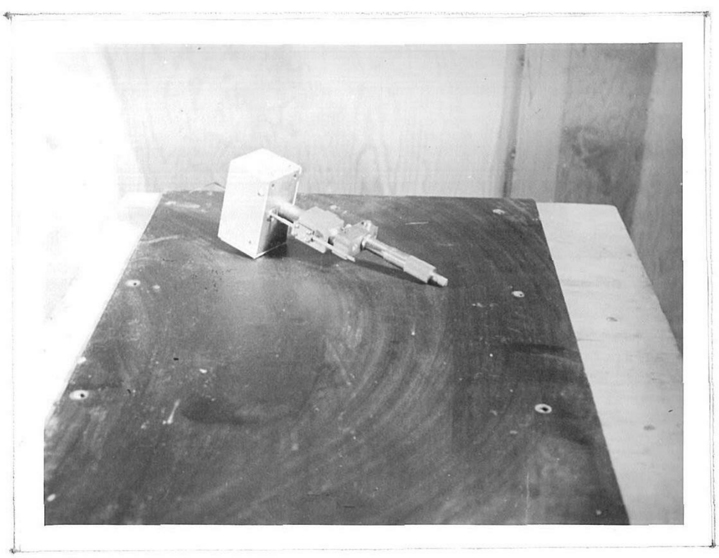

B = 93,700 (average source strength) cm²/sec
——— calculated $\bar{U} = \frac{B}{r}$
× ⊙ △ ⊗ ⊡ measured $\bar{U}$
U cm/sec.
3000
2000
1000
0
A1
A3
A5
A7
A9
0 10 20 30 40 50 60
r (cm)
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
u/Ū
1.2 cm
0.4 cm
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2
Z (cm)
R Ū (cm/sec) Pt
× A1 16.3 1265 -0.435
⊙ A3 26.3 640 -0.609
△ A5 36.3 410 -0.611
⊗ A7 46.3 307 -0.627
⊡ A9 56.3 244 -0.627
η'q = 0.82
C_D = .0515 (referred to Rm)
R.N. = 106,000



Experimental Set Up



Tunnel Wall with
Vane (a) and (b)



Micrometer with tube (2)