A RELATIVISTIC QUARK MODEL
WITH HARMONIC DYNAMICS

Thesis by

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ABSTRACT

A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction is constructed. This gives straight and parallel Regge trajectories and a hadron spectrum which easily can accommodate all known resonances. From the equation we derive vector and axial vector currents for baryons and mesons. The vector current matrix elements of baryons are compared to known magnetic moments, photoproduction amplitudes and inelastic electron proton cross sections. Good agreement is obtained when the theoretical results are modified by an empirical form factor. Besides this form factor, the results depend on no free parameters. Using the same form factor, we calculate radiative decay rates of vector mesons and parameters of $K_{\ell 3}$ decay, which agree fairly well with experiment. Assuming that the amplitude for emission of a pseudoscalar meson from a hadron is proportional to the divergence of the axial vector current, we calculate most of the known strong decay rates. These results depend on one new coupling constant and a smooth, empirical form factor. More than half of the calculated baryon rates agree well with experiment. The reasons for the many disagreements are discussed. All except one of the meson rates come out close to their experimental values. The angular distributions of the decay $B(1235) \rightarrow \omega\,\pi$ and $A_1(1070) \rightarrow \rho\,\pi$ are well described in this model.

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I. INTRODUCTION

An important part of experimental high energy physics is the determination of the intrinsic properties of the basic constituents of matter, going under the name elementary particles and listed in the Particle Tables (1). The main problem of theoretical high energy physics is to understand why these particles exist with certain masses, spins and parities along with their characteristic internal quantum numbers like charge, hypercharge and isospin.

More than ten years of intense studies in this field of physics have revealed two very important features of the strongly interacting particles or hadrons:

(a) Only particles with certain values of all possible combinations of quantum numbers seem to occur. For instance, no doubly charged meson has been found or a meson with $J^{PC} = 0^{--}$.

(b) When particles with the same internal quantum numbers are grouped together, they can be separated into subgroups where there exist an approximate linear relationship between their spin and mass squared.

The first feature has been successfully explained by the quark model of Gell-Mann and Zweig (2) in which one pictures the baryons as composed of three quarks and the meson as a quark-antiquark system. Evidently, one would like very much to understand also the second experimental feature in the framework of this quark model. So far no one has succeeded in doing this.

Since we believe that a complete understanding of the hadron spectrum will only be possible in a truly relativistic quantum mechanical field theory with all its built-in complexities, the problem seems to be far away from a solution. However, a great step forward would be to obtain a consistent scheme to help organize the wealth of present and future particle data. Such a scheme would not be the ultimate correct description and therefore should be kept as simple as possible as long as it satisfies basic requirements like Lorentz invariance and quantum mechanical principles.

A step in this direction has been taken by Greenberg (3) and Faiman and Hendry (4). They consider the quarks inside the hadrons as non-relativistically bound together by harmonic forces. This has two immediate desirable consequences. It admits an analytical solution of the three-quark problem in the baryons and can explain the simple relation between mass and spin in (b). Besides giving a hadron spectrum which seems to accommodate all observed particle states, this model also allows one to calculate transition rates between different states which in most cases seem to agree reasonably well with experiment as shown in reference (4) and by Copley, Karl and Obryk (5) and Walker (6).

In this work I will expose some consequences of a relativistic quark model developed by R. P. Feynman, M. Kislinger and F.Ravndal (7) based on the idea that there is something in the internal structure of hadrons which in many respects behave as if there were harmonic forces between the quarks. This may to many sound crazy and probably it is, but still I think such a model can be useful to develop a set of rules able to relate as many as possible different phenomena observed among the hadrons. These rules we derive are what we hope will have physical relevance and not the underlying picture.

This situation is similar to the first developments of the quantum mechanics of atoms where one considered the electrons to move around the nucleus in classical Kepler orbits and without radiating in conflict with electrodynamics. In spite of all the inconsistencies of this atomic model the results for the hydrogen atom were in striking agreement with experiment. The derived quantization rules were only later understood with the discovery of the wave-particle duality and modern quantum mechanics.

With this justification we will now investigate a corresponding simple set of rules for the hadrons, even if these are not based on sound, physical ideas. Hopefully, in the near future when we have a complete dynamical theory for the strong interactions, we will understand why this naive model in most cases seems to give such a coherent description of so many apparently unrelated hadronic processes.

II. HADRON STATES

In the quark model (2,8) the internal quantum numbers of the hadrons are carried by three fundamental entities called quarks. There is one quark with zero isospin, strangeness -1 and charge Q = -1/3 named "s" and two quarks with zero strangeness forming an isodoublet, the "u" quark with isospin up and Q = +2/3 and the "d" quark with isospin down and charge Q = -1/3. Together, these three quarks fill the fundamental triplet representation $\underset{\sim}{3}$ of SU(3). The antiquarks $\bar{s}$, $\bar{u}$ and $\bar{d}$ belong to the SU(3) representation $\underset{\sim}{\bar{3}}$.

The quark model in this form can easily be enlarged by assigning spin S = 1/2 to each quark. By doing this the quarks will now belong to the fundamental SU(6) representation $\underset{\sim}{6}$ and the antiquarks to $\underset{\sim}{\bar{6}}$.

Since the quarks have been given half-integer spin, one would expect them to obey Fermi-statistics. But the theorem relating spin and statistics can only be proved for free particles which can be isolated (9). It does not have to be valid for the quarks which make up the hadrons. On the contrary, to understand the observed baryon spectrum it seems necessary to let them obey symmetric Bose-statistics. This has the additional very interesting consequence that the $\Delta I = 1/2$ rule for non-leptonic, weak hyperon decays has a very simple explanation (7,10). To give the baryons anti-symmetric Fermi-statistics it is then necessary to endow them with a spinless S = 0 fermion with vacuum quantum numbers in addition to the three Bose quarks (11). An equivalent result is obtained by letting the quarks obey para-statistics as proposed by Greenberg (3). In the following we will let the quarks satisfy symmetric Bose-statistics so that only those hadron states which are completely symmetric in all their quark labels can be expected to exist.

Meson Spectrum

The mesons are thought to consist of a quark and a antiquark. In SU(3) this will correspond to nine different states which can be grouped into an octet $\underset{\sim}{8}$ and a singlet $\underset{\sim}{1}$:

$$\underset{\sim}{3} \otimes \bar{\underset{\sim}{3}} = \underset{\sim}{8} + \underset{\sim}{1} \tag{2.1}$$

Going to SU(6) space, this equation takes the form:

$$\underset{\sim}{6} \otimes \bar{\underset{\sim}{6}} = \underset{\sim}{35} + \underset{\sim}{1} \tag{2.2}$$

The two SU(6) representations $\underset{\sim}{35}$ and $\underset{\sim}{1}$ can be decomposed into SU(3) multiplets with definite spin

$$\underset{\sim}{35} = {}^{3}(\underset{\sim}{8}) + {}^{3}(\underset{\sim}{1}) + {}^{1}(\underset{\sim}{8}) \tag{2.3}$$

$$\underset{\sim}{1} = {}^{1}(\underset{\sim}{1}) \tag{2.4}$$

where upper index 3 stands for spin triplet S = 1 and 1 for spin singlet S = 0.

As in positronium, one would expect that the parity of the meson system would be $P = (-1)^{L+1}$ where L is the orbital angular momentum of the quark-antiquark system and having charge conjugation $C = (-1)^{L+S}$ when S is the total quark spin of the system.

In other words, for the lowest meson states with L = 0, the quark model predicts the existence of 36 states with negative parity, 9 with

$S = 0$ and $C = +1$, 9 with $S = 1$ and $C = -1$. All these particles have been found as seen in Table 1 in the first entry with $L = 0$.

Let us now try to extend this simple quark model in order to include states with $L > 0$. To each meson state we ascribe a wave function $\varphi = \varphi(u_a, u_b)$ where u_a and u_b are four-vector position variables for the quark and antiquark. This wave function develops in space and time according to the wave equation

$$K\varphi(u_a, u_b) = 0 \tag{2.5}$$

where the dynamical operator K is taken to be

$$K = 2(p_a{}^2 + p_b{}^2) + \frac{\Omega^2}{16}(u_a - u_b)^2 + C \tag{2.6}$$

Here p_a, p_b are four-momenta conjugate to u_a, u_b; Ω is some fixed constant while the constant C can be adjusted for the meson under consideration.

Instead of the quark momenta p_a and p_b we can use the total meson momentum $P = p_a + p_b$ together with an internal momentum ζ by the relations:

$$\begin{aligned} p_a &= \frac{1}{2}P - \frac{1}{2\sqrt{2}}\zeta \\ p_b &= \frac{1}{2}P + \frac{1}{2\sqrt{2}}\zeta \end{aligned} \tag{2.7}$$

In terms of the variables R and z conjugate to P and ζ,

$$\begin{aligned} R &= -i\frac{\partial}{\partial P} = (u_a + u_b)/2 \\ z &= -i\frac{\partial}{\partial \zeta} = (u_b - u_a)/2\sqrt{2} \end{aligned} \tag{2.8}$$

Table 1. Low-lying meson multiplets and identification of possible particle candidates.

No. of Excitations	$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	J^{PC}	Particle States			
N = 0	$[\underset{\sim}{36}, 0^-]$	$^1(\underset{\sim}{9})_0$	0^{-+}	π (138)	K (496)	η (549)	η' (958)
		$^3(\underset{\sim}{9})_1$	1^{--}	ρ (765)	K^*(892)	ω (784)	φ (1019)
N = 1	$[\underset{\sim}{36}, 1^+]$	$^1(\underset{\sim}{9})_1$	1^{+-}	B(1235)			
		$^3(\underset{\sim}{9})_0$	0^{++}	δ (966)		ϵ (700)	
		$^3(\underset{\sim}{9})_1$	1^{++}	A_1(1070)	K^*(1240)	D(1285)	
		$^3(\underset{\sim}{9})_2$	2^{++}	A_2(1300	K^*(1420)	f(1260)	f'(1514)
N = 2	$[\underset{\sim}{36}, 0^-]$	$^1(\underset{\sim}{9})_0$	0^{-+}				
		$^3(\underset{\sim}{9})_1$	1^{--}				
	$[\underset{\sim}{36}, 2^-]$	$^1(\underset{\sim}{9})_2$	2^{-+}				
		$^3(\underset{\sim}{9})_1$	1^{--}				
		$^3(\underset{\sim}{9})_2$	2^{--}				
		$^3(\underset{\sim}{9})_3$	3^{--}	g(1670)			

the quark coordinates are:

$$u_a = R - \sqrt{2}\, z \qquad (2.9)$$

$$u_b = R + \sqrt{2}\, z$$

The operator K can now be written

$$K = P^2 - \mathfrak{m}^2 \qquad (2.10)$$

where $$-\mathfrak{m}^2 = \frac{1}{2}\zeta^2 + \frac{\Omega^2}{2} z^2 + C \qquad (2.11)$$

An even simpler expression is obtained by introducing the creation and annihilation operators

$$c^+ = \sqrt{\frac{1}{2\Omega}}\, \zeta + i\sqrt{\frac{\Omega}{2}}\, z \qquad (2.12)$$

$$c = \sqrt{\frac{1}{2\Omega}}\, \zeta - i\sqrt{\frac{\Omega}{2}}\, z$$

which in equation (2.11) gives

$$-\mathfrak{m}^2 = \Omega(c^+ \cdot c) + C \qquad (2.13)$$

The operators c^+ and c are four-vectors and satisfy the fundamental commutation relation

$$\left[c_\mu, c_\nu^+\right] = -g_{\mu\nu} \qquad (2.14)$$

where $g_{\mu\nu}$ equals -1 for space and +1 for time components.

From equations (2.5) and (2.10) we see that the propagator K^{-1} for free mesons have poles at the eigenvalues M_i^2 of the operator $\mathfrak{m}^2$. Hence the meson spectrum we get is the one of a four-dimensional harmonic oscillator.

The time excited states are not physically meaningful since they can have negative norm as seen from equation (2.14). To get rid of these unwanted states, we make the extra assumption that in the rest system of the meson only spacelike states exist:

$$(P\cdot c)\ \varphi(z) = 0 \tag{2.15}$$

Having done this, we are left with a meson spectrum as shown in Figure 1. Here and in Table 1 we use the abbreviation $\underset{\sim}{36}$ for the direct SU(6) sum $\underset{\sim}{35} + \underset{\sim}{1}$ in the same way as we use the sum $\underset{\sim}{9}$, called a SU(3) nonet, for $\underset{\sim}{8} + \underset{\sim}{1}$. From equations (2.2) and (2.3) we see that each $\underset{\sim}{36}$ multiplet contains two nonets, one with quark spin S = 0 and one with S = 1. Since the total angular momentum $\underset{\sim}{J}$ of an excited meson with orbital angular momentum $\underset{\sim}{L}$ and quark spin $\underset{\sim}{S}$ is given by the ordinary addition formula

$$\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}\ , \tag{2.16}$$

we find that each excited $\underset{\sim}{36}$ level can be decomposed into three nonets with angular momentum J = L+1, L, L-1, and parity and charge conjugation $C = P = (-1)^{L+1}$ plus one nonet with J = L and $C = -P = (-1)^{L}$. This decomposition is presented in Table 1 for the first and second excited levels where each SU(3) multiplet gets a lower index equal to its total angular momentum. For example, $^{3}(\underset{\sim}{8})_{1}$ means an octet of states with quark spin S = 1 and J = 1.

At the first excited level N = 1 we see that most of the predicted states can be identified with already observed meson resonances. Going to the next excited level, we find the first radially excited multiplet,

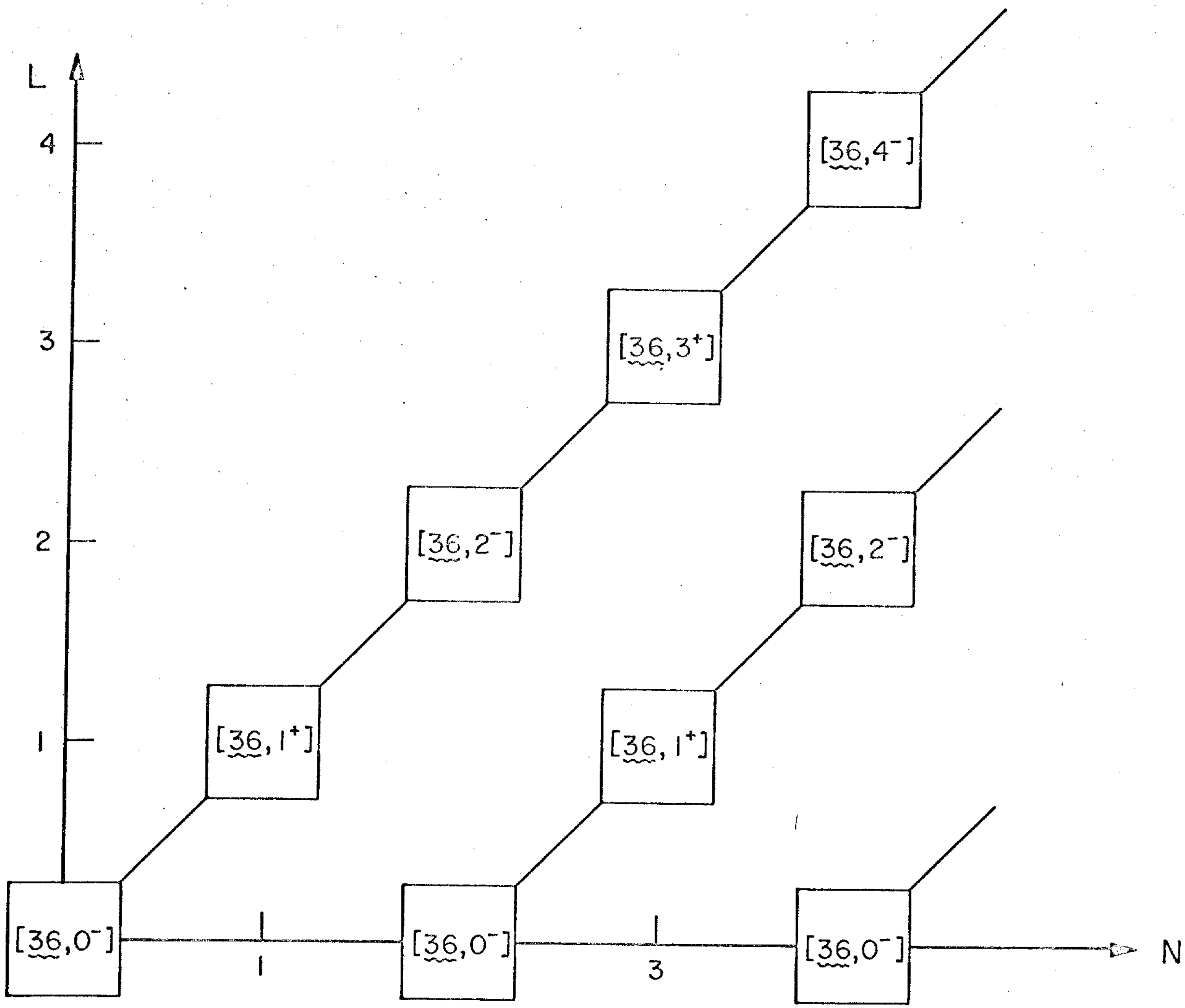

Figure 1. Lowest multiplets of the harmonic oscillator meson spectrum. L is the orbital angular momentum and N the number of excitations in each multiplet.

$\left[\underset{\sim}{36},0^-\right]$, N = 2. A possible resonance candidate belonging to this multiplet is the E(1422).

From the sequence ρ(765), A_2(1300 and g(1670) we can determine the excitation constant Ω for which we get

$$\Omega = M^2_{A_2} - M^2_\rho = M^2_g - M^2_{A_2} = 1.10 \text{ GeV}^2 \qquad (2.17)$$

According to this simple scheme the three spin triplet states δ(966), A_1(1070) and A_2(1300) should all have the same mass. This not being the case, we conclude there must be an additional spin-orbit coupling term in the meson mass operator, equation (2.13). We will not consider refinements of that kind in our model.

Baryon Spectrum

Since the baryons are composed of three quarks, they will occur only in the following SU(3) representations:

$$\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = \underset{\sim}{10} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{1} \qquad (2.18)$$

Including the quark spin, we find the possible SU(6) representations for the baryons:

$$\underset{\sim}{6} \otimes \underset{\sim}{6} \otimes \underset{\sim}{6} = \underset{\sim}{56} + \underset{\sim}{70} + \underset{\sim}{70} + \underset{\sim}{20} \qquad (2.19)$$

Each of these multiplets can be decomposed into SU(3)⊗SU(2) representations as follows:

$$\begin{aligned} \underset{\sim}{56} &= {}^4(\underset{\sim}{10}) + {}^2(\underset{\sim}{8}) \\ \underset{\sim}{70} &= {}^2(\underset{\sim}{10}) + {}^4(\underset{\sim}{8}) + {}^2(\underset{\sim}{8}) + {}^2(\underset{\sim}{1}) \\ \underset{\sim}{20} &= {}^2(\underset{\sim}{8}) + {}^4(\underset{\sim}{1}) \end{aligned} \qquad (2.20)$$

The requirement of totally symmetric baryon states makes the $\underset{\sim}{56}$ the only possible multiplet for the lowest baryons since the $\underset{\sim}{70}$ has mixed symmetry and the $\underset{\sim}{20}$ is totally anti-symmetric. Hence the ground state baryons should be found in a quark spin $S = 1/2$ octet and a $S = 3/2$ decimet. All the particles belonging to these SU(3) multiplets have been seen as shown in the first entry of Table 2.

In this theory the baryon wave function $\Psi = \Psi(u_a, u_b, u_c)$ where u_a, u_b and u_c are position four-vectors of the three quarks, will satisfy the dynamical equation

$$K\Psi(u_a,\ u_b,\ u_c) \;=\; 0 \qquad (2.21)$$

The operator K will be analogous to the one for mesons, equation (2.6),

$$K = 3(p_a{}^2+p_b{}^2+p_c{}^2) + \frac{\Omega^2}{36}\left[(u_a-u_b)^2+(u_b-u_c)^2+(u_c-u_a)^2\right]+C \qquad (2.22)$$

where the quark four-momenta p_a, p_b and p_c are conjugate to the coordinate vectors u_a, u_b and u_c. Introducing the total baryon momentum $P = p_a+p_b+p_c$ and two internal momenta ξ and η as new momentum variables by

$$\begin{aligned} p_a &= \frac{1}{3}P - \frac{1}{3}\xi \\ p_b &= \frac{1}{3}P + \frac{1}{6}\xi - \frac{1}{2\sqrt{3}}\eta \\ p_c &= \frac{1}{3}P + \frac{1}{6}\xi + \frac{1}{2\sqrt{3}}\eta \end{aligned} \qquad (2.23)$$

with their conjugate position operators

$$\begin{aligned} u_a &= R - 2x \\ u_b &= R + x - \sqrt{3}\ y \\ u_c &= R + x + \sqrt{3}\ y \end{aligned} \qquad (2.24)$$

Table 2. Low-lying baryon multiplets and identification of possible particle candidates.

No. of Excitations	Excitation	SU(6) ⊗ 0(3)	$^{2S+1}(SU(3))_J$	Particle States			
N = 0		$[\underset{\sim}{56}, 0^+]$	$^2(\underset{\sim}{8})_{1/2}$	N (938)	Σ (1192)	Λ(1115)	Ξ(1318)
			$^4(\underset{\sim}{10})_{3/2}$	Δ(1236)	Σ (1385)	Ξ(1530)	Ω (1672)
N = 1	$\underset{\sim}{a}^+$, $\underset{\sim}{b}^+$	$[\underset{\sim}{70}, 1^-]$	$^2(\underset{\sim}{1})_{1/2}$	Λ(1405)			
			$^2(\underset{\sim}{1})_{3/2}$	Λ(1520)			
			$^2(\underset{\sim}{8})_{1/2}$	N(1535)		Λ(1670)	
			$^2(\underset{\sim}{8})_{3/2}$	N(1520)	Σ (1670)	Ξ(1690)	Ξ(1820)
			$^4(\underset{\sim}{8})_{1/2}$	N(1700)			
			$^4(\underset{\sim}{8})_{3/2}$	N( )			
			$^4(\underset{\sim}{8})_{5/2}$	N(1670)	Σ (1765)	Λ(1830)	Ξ(1930)
			$^2(\underset{\sim}{10})_{1/2}$	Δ(1650)	Σ (1750)		
			$^2(\underset{\sim}{10})_{3/2}$	Δ(1670)			
N = 2	$\underset{\sim}{a}^+ \cdot \underset{\sim}{a}^+ + \underset{\sim}{b}^+ \cdot \underset{\sim}{b}^+$	$[\underset{\sim}{56}, 0^+]$	$^2(\underset{\sim}{8})_{1/2}$	N(1470)			
			$^4(\underset{\sim}{10})_{3/2}$	Δ( )			

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Table 2. Continued

No. of Excitations	Excitation	$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	Particle States		
			$^2(\mathbf{8})_{3/2}$	N( )		
			$^2(\mathbf{8})_{5/2}$	N(1688)		$\Lambda(1815)$
	$\mathbf{a}^+\mathbf{a}^+ + \mathbf{b}^+\mathbf{b}^+$	$[\mathbf{56}, 2^+]$	$^4(\mathbf{10})_{1/2}$	$\Delta(1910)$		
			$^4(\mathbf{10})_{3/2}$	$\Delta($ $)$		
			$^4(\mathbf{10})_{5/2}$	$\Delta(1890)$		
			$^4(\mathbf{10})_{7/2}$	$\Delta(1950)$	Σ (2030)	
			$^2(\mathbf{1})_{1/2}$	$\Lambda($ $)$		
	$\mathbf{a}^+\cdot\mathbf{a}^+ - \mathbf{b}^+\cdot\mathbf{b}^+$	$[\mathbf{70}, 0^+]$	$^2(\mathbf{8})_{1/2}$	N(1780)		
N= 2	$\mathbf{a}^+\cdot\mathbf{b}^+$		$^4(\mathbf{8})_{3/2}$	N( )		
			$^2(\mathbf{10})_{1/2}$	$\Delta($ $)$		
			$^2(\mathbf{1})_{3/2}$	$\Lambda($ $)$		
	$\mathbf{a}^+\mathbf{a}^+ - \mathbf{b}^+\mathbf{b}^+$	$[\mathbf{70}, 2^+]$	$^2(\mathbf{1})_{5/2}$	$\Lambda($ $)$		
	$\mathbf{a}^+\mathbf{b}^+$		$^2(\mathbf{8})_{3/2}$	N(1860)		
			$^2(\mathbf{8})_{5/2}$	N( )		

Table 2. Continued

No. of Excitations	Excitation	SU(6) ⊗ 0(3)	$^{2S+1}(SU(3))_J$	Particle States
			$^4(\underset{\sim}{8})_{1/2}$	N( )
			$^4(\underset{\sim}{8})_{3/2}$	N( )
	$\underset{\sim}{a}^+\underset{\sim}{a}^+ - \underset{\sim}{b}^+\underset{\sim}{b}^+$	$[\underset{\sim}{70}, 2^+]$	$^4(\underset{\sim}{8})_{5/2}$	N( )
	$\underset{\sim}{a}^+\underset{\sim}{b}^+$		$^4(\underset{\sim}{8})_{7/2}$	N(1990)
			$^2(\underset{\sim}{10})_{3/2}$	Δ( )
N = 2			$^2(\underset{\sim}{10})_{5/2}$	Δ( )
			$^2(\underset{\sim}{8})_{1/2}$	N( )
			$^2(\underset{\sim}{8})_{3/2}$	N( )
	$\underset{\sim}{a}^+ \times \underset{\sim}{b}^+$	$[\underset{\sim}{20}, 1^+]$	$^4(\underset{\sim}{1})_{1/2}$	Λ( )
			$^4(\underset{\sim}{1})_{3/2}$	Λ( )
			$^4(\underset{\sim}{1})_{5/2}$	Λ( )

the operator K in (2.22) can be written

$$K = P^2 - \mathcal{M}^2 \tag{2.25}$$

where

$$\mathcal{M}^2 = \frac{1}{2}(\xi^2 + \eta^2) + \frac{\Omega^2}{2}(x^2 + y^2) + C \tag{2.26}$$

In terms of the creation operators

$$a^+ = \sqrt{\frac{1}{2\Omega}}\,\xi + i\sqrt{\frac{\Omega}{2}}\,x$$

$$b^+ = \sqrt{\frac{1}{2\Omega}}\,\eta + i\sqrt{\frac{\Omega}{2}}\,y \tag{2.27}$$

and their Hermitian conjugate a and b, the baryon mass operator $\mathcal{M}^2$ in (2.26) takes the form:

$$\mathcal{M}^2 = \Omega\,(a^+\cdot a + b^+\cdot b) + C \tag{2.28}$$

The non-physical time excited states we again take out by the extra requirements

$$(P\cdot a)\psi = (P\cdot b)\psi = 0 \tag{2.29}$$

From equations (2.26) and (2.28) we see that the mass spectrum of the baryons will be the one of two independent, three-dimensional harmonic oscillators. At the first excited level we will find states with orbital angular momentum L=1, while the next level will have states with L=2,1 and 0 by adding together the two excitations, each with L=1.

The SU(6) content of every excited level is given by the overall symmetry requirement for the total baryon wave function. Each orbital excited state with a certain permutation symmetry in the three quark labels must be combined with one of the SU(6) multiplets $\underset{\sim}{56}$, $\underset{\sim}{70}$, or $\underset{\sim}{20}$ with the same symmetry. For example, at the second excited level we find one state with angular momentum and parity $L^P = 1^+$ which will transform like $\underset{\sim}{\xi} \times \underset{\sim}{\eta}$. Writing this product out in terms of quark momenta using equation (2.23), we get:

$$\underset{\sim}{\xi} \times \underset{\sim}{\eta} = 2\sqrt{3}\left[\underset{\sim}{p}_a \times \underset{\sim}{p}_b + \underset{\sim}{p}_b \times \underset{\sim}{p}_c + \underset{\sim}{p}_c \times \underset{\sim}{p}_a\right] \qquad (2.30)$$

This orbital state is completely anti-symmetric in the three quarks and must be combined with the anti-symmetric SU(6) multiplet $\underset{\sim}{20}$ to give the full state $\left[\underset{\sim}{20}, 1^+\right]$. In the Appendix this procedure is outlined in more detail.

At the first and second excited level this leads to the baryon multiplets

$$\begin{aligned} N=1: &\quad \left[\underset{\sim}{70},\ 1^-\right] \\ N=2: &\quad \left[\underset{\sim}{70},\ 2^+\right], \left[\underset{\sim}{56},\ 2^+\right], \left[\underset{\sim}{20},\ 1^+\right], \left[\underset{\sim}{70}, 0^+\right], \left[\underset{\sim}{56}, 0^+\right] \end{aligned} \qquad (2.31)$$

In Figure 2 we show these states together with the ones having N=3 and 4.

Now using the decomposition of SU(6) in (2.20), we present in Table 2 all the SU(3) multiplets of the three lowest levels of the baryon spectrum where we again have combined the quark spin S of each multiplet with the orbital angular momentum L to give the total angular momentum J of the SU(3) multiplet. Where possible we have identified the multiplet states with well established baryon resonances.

All the multiplets of the $\left[\underset{\sim}{70}, 1^-\right]$ except one have been seen in experiment. Only the ${}^4(\underset{\sim}{8})_{3/2}$ with a D_{13} nucleon resonance is not yet

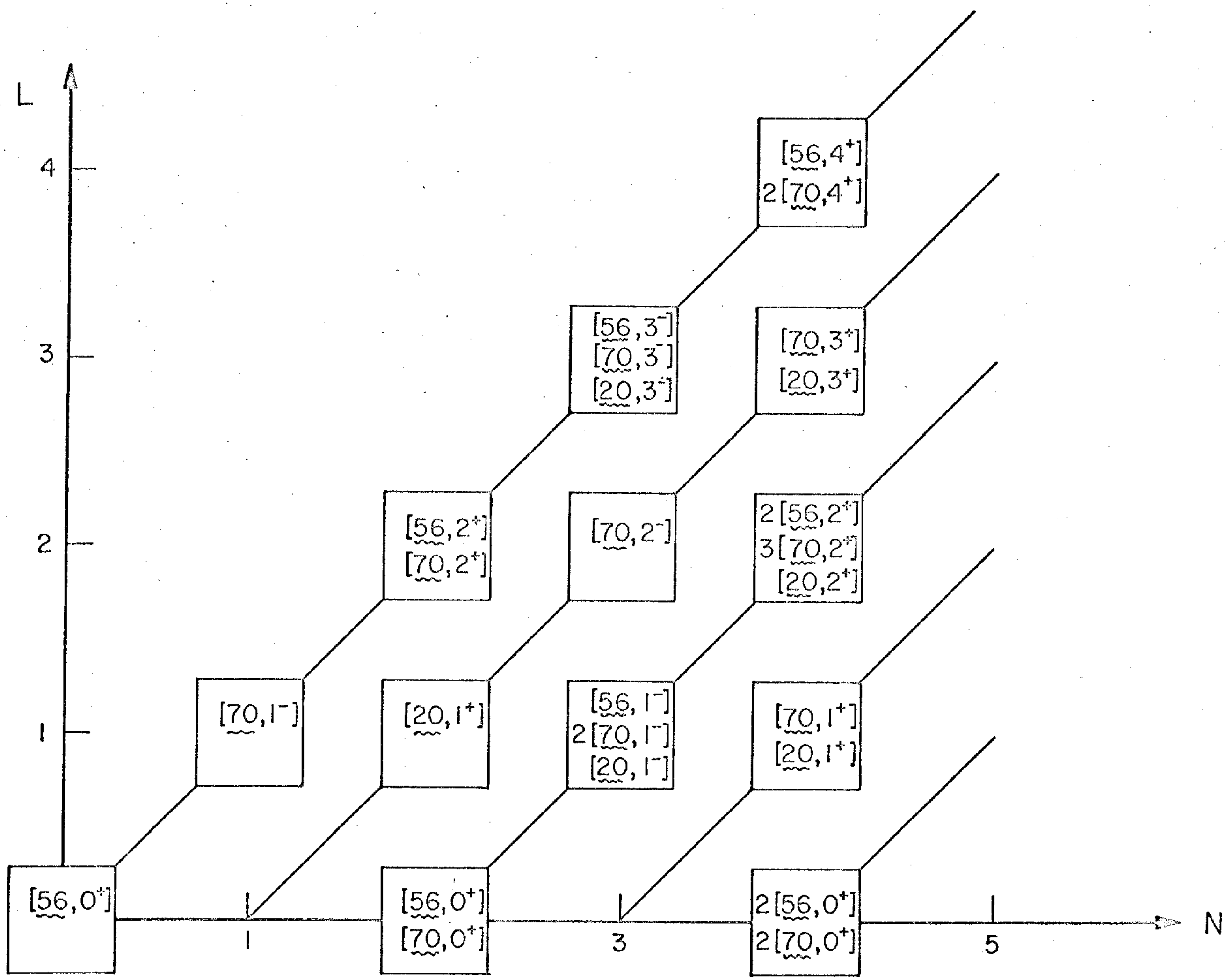

Figure 2. Lowest multiplets of the harmonic oscillator baryon spectrum. L is the orbital angular momentum and N the number of excitations in each multiplet.

positively identified, but exists in the footnotes of the Particle Tables (1) with a mass near 1700 MeV. In the $\left[\underline{56},2^+\right]$ we find the nucleon recurrence $F_{15}(1688)$ and three of the four Δ-resonances in the spin quartet decimets.

Along with the $F_{15}(1688)$ there should also be a $5/2^+$ recurrence of the $\Sigma(1192)$. A possible candidate for this would have been the $\Sigma(1910)$ if only its mass had been somewhat lower, closer to 1850 MeV. On the other hand, its mass is apparently too low to fit into the $^4(\underline{10})$ of the $\left[\underline{56},2^+\right]$ together with the $F_{35}(1890)$. So it is not very clear where this Σ-resonance belongs.

From the mass difference between the doublet or quartet states of the $\left[\underline{56},2^+\right]$ and the ground state $\left[\underline{56},0^+\right]$, we get for the excitation parameter

$$\Omega = 1.05 \text{ GeV}^2 \tag{2.32}$$

This is almost the same value as we found for the meson spectrum, and will from now on be the value we shall use both for mesons and baryons.

If we look at the nucleon resonances in the SU(3) multiplets obtained by combining L and S, we make the important observation that their masses are approximately the same. This is true for the $^2(\underline{8})$, $^2(\underline{10})$ and $^4(\underline{8})$ of the $\left[\underline{70},1^-\right]$ and for the three states of the $^4(\underline{10})$ in the $\left[\underline{56},2^+\right]$. It is not so good for the two Λ-singlets $^2(\underline{1})$ of the $\left[\underline{70},1^-\right]$. In spite of that, we conclude that the spin-orbit coupling in baryons is small, in contrast to the large LS-splitting we found for the L=1 mesons.

With this guiding principle we predict the mass of the missing P_{33} resonance in the $[56, 2^+]$ to be around 1920 MeV. The doublet partner of the $F_{15}(1688)$ is then most likely not the $P_{13}(1860)$, but rather a P_{13} state with mass closer to 1700 MeV. No such nucleon resonance has yet been established. However, a $N^*(1700)$ has been observed in diffraction production (12) and since it has a strong coupling to ΛK, it cannot be one of the $^4(8)$ states of $[70, 1^-]$ which do not couple to ΛK because of their F/D-ratio as shown in the Appendix.

The remaining, positive parity nucleon resonances in the same mass range must go into multiplets like the $[56, 0^+]$, $[70, 0^+]$, $[70, 2^+]$, and $[20, 1^+]$. Since the masses of these resonances vary from the $P_{11}(1470)$ to the $F_{17}(1990)$, the degenerate, second excited level N=2 must in some way be split by a non-harmonic term in the baryon operator K, equation (2.22).

We have no idea what this term should be, but let us make a simple guess in order to see what would happen to the degenerate levels:

$$\delta K = -\epsilon \frac{\Omega^3}{108} \left[(u_a - u_b)^4 + (u_b - u_c)^4 + (u_c - u_a)^4 \right] \qquad (2.33)$$

Since our baryon system is completely symmetric in the three quarks, the corresponding mass breaking operator can be written

$$\delta m^2 = +3\epsilon \frac{\Omega^3}{108} (u_b - u_c)^4 \qquad (2.34)$$

Using (2.24) we express the coordinate difference $u_b - u_c$ in terms of the internal position variable y:

$$\delta m^2 = +3\epsilon \frac{\Omega^3}{108} (2\sqrt{3})^4 y^4 \qquad (2.35)$$

Only the spatial components of y contribute because of (2.29). We write $\underset{\sim}{y}$ as a linear combination of $\underset{\sim}{b}$ and $\underset{\sim}{b}^+$ defined in (2.27) and get:

$$\delta m^2 = -4\epsilon\Omega \left[\tfrac{3}{2}N_0^{\ 2} + N_+^{\ 2} + N_-^{\ 2} + 2N_0N_+ \right.$$
$$+ 2N_0N_- + 4N_+N_- + \tfrac{7}{2}N_0 \qquad (2.36)$$
$$\left. + 4N_+ + 4N_- + \frac{15}{4} \right] + \text{non-diagonal terms}$$

Here we have introduced the number operators

$$N_0 = b_0^{\ +} b_0$$
$$N_+ = b_+^{\ +} b_+ \qquad (2.37)$$
$$N_- = b_-^{\ +} b_-$$

where b_0, b_+, b_- and their Hermitian conjugate are defined in the Appendix. Is is now a simple matter to calculate the diagonal matrix elements of the mass breaking operator (2.36) for the baryon orbital states also given in the Appendix. We get:

$$M^2\left[\underset{\sim}{20},1^+\right] = +4\epsilon\Omega + \text{const.}$$
$$M^2\left[\underset{\sim}{70},2^+\right] = +2\epsilon\Omega + \text{const.}$$
$$M^2\left[\underset{\sim}{56},2^+\right] = 0 + \text{const.} \qquad (2.38)$$
$$M^2\left[\underset{\sim}{70},0^+\right] = -\epsilon\Omega + \text{const.}$$
$$M^2\left[\underset{\sim}{56},0^+\right] = -6\epsilon\Omega + \text{const.}$$

These results are graphically shown in Figure 3 where we have adjusted the const. term so that the $\left[\underset{\sim}{56},2^+\right]$ remains unperturbed.

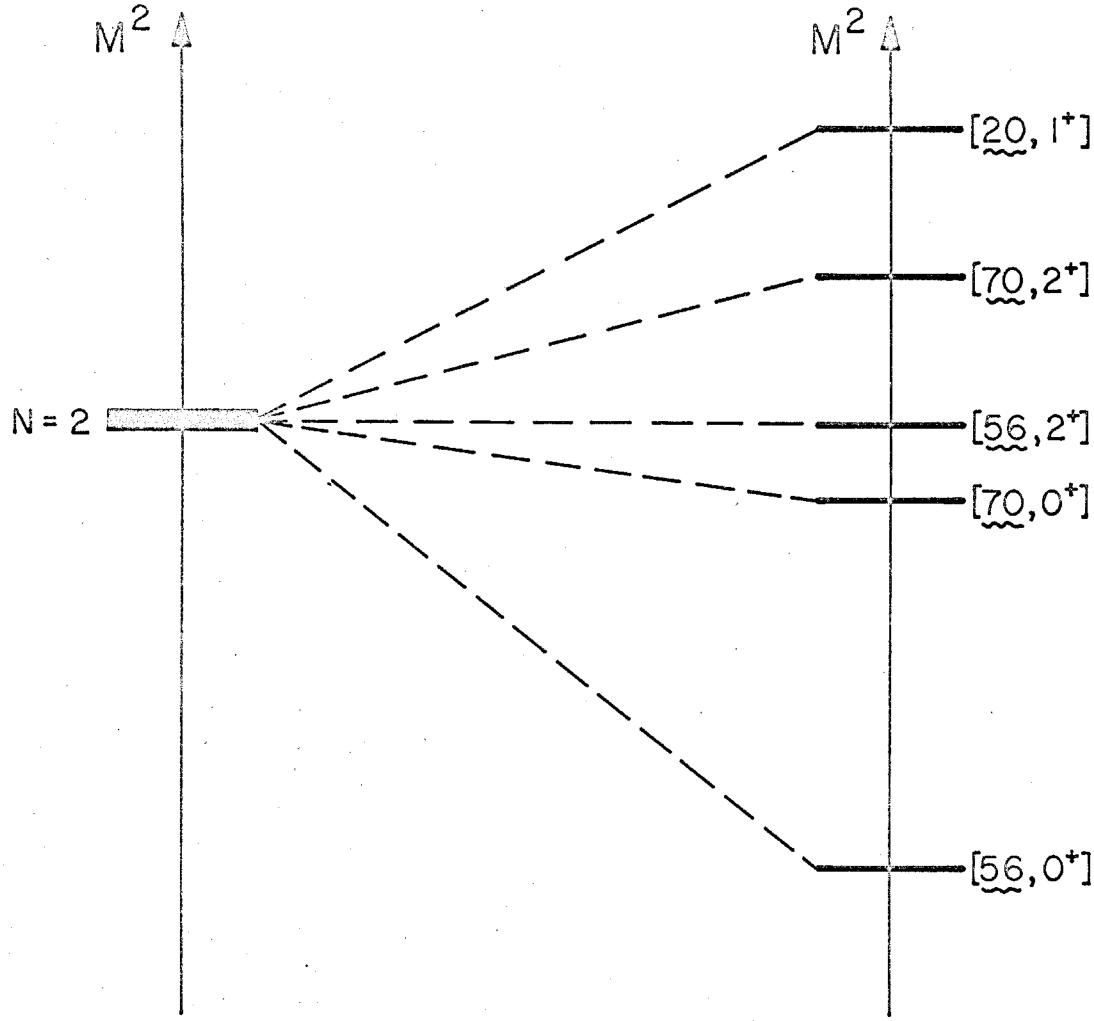

Figure 3. Splitting of second excited level of the baryon spectrum by non-harmonic term in the mass operator.

Qualitatively, the results of this perturbation calculation are that the symmetric $[\underset{\sim}{56},0^+]$ is pulled down, the anti-symmetric $[\underset{\sim}{20},1^+]$ is kicked up, the $[\underset{\sim}{70},2^+]$ should be found above the $[\underset{\sim}{56},2^+]$ and the $[\underset{\sim}{70},0^+]$ near the $[\underset{\sim}{56},2^+]$. We believe this breaking pattern is more or less independent of the exact form of the non-harmonic term in the mass operator.

Of the remaining nucleon resonances, the low-mass Roper resonance $P_{11}(1470)$ finds a natural place in the $[\underset{\sim}{56},0^+]$. A possible Δ-candidate for the same multiplet is the $P_{33}(1690)$ which exists in the data listings of the Particle Tables (1).

The $F_{17}(1990)$ fits nicely into the $^4(\underset{\sim}{8})_{7/2}$ of the $[\underset{\sim}{70},2^+]$ which is the only available octet with J = 7/2 at the N=2 level. Another possibility could have been the $^2(\underset{\sim}{8})_{7/2}$ of $[\underset{\sim}{56},4^+]$ at the N=4 level. But then it would be the doublet partner of the nucleon second recurrence H_{19} which is expected at 2200 MeV. This would mean a very big spin-orbit coupling which we so far have not observed among the baryons.

Recently a Λ-resonance, $F_{07}(2100)$, has been established (36). This is an obvious octet partner to the $F_{17}(1990)$. In our opinion, these two resonances offer the strongest evidence for a harmonic oscillator baryon spectrum with its $[\underset{\sim}{70},2^+]$ at the second excited level.

Possible multiplets for the $P_{13}(1860)$ are the $^4(\underset{\sim}{8})_{3/2}$ of the $[\underset{\sim}{70},0^+]$ or the $^2(\underset{\sim}{8})_{3/2}$ of the $[\underset{\sim}{70},2^+]$. We choose the second possibility since this will give the same SU(6)-splitting between the $^4(\underset{\sim}{8})$ and $^2(\underset{\sim}{8})$ of the $[\underset{\sim}{70},2^+]$ as in the $[\underset{\sim}{70},1^-]$.

The last well established, low-lying nucleon resonance is the $P_{11}(1780)$. From its clear evidence in η photoproduction one can conclude that it most likely belongs in the ${}^2(\underset{\sim}{8})_{1/2}$ of the $\left[\underset{\sim}{70},0^+\right]$ (13).

With these assignments we can try to check quantitatively the mass splitting predictions in (2.38). The only relation which can be unambiguously compared to experiment, is the following:

$$\left[\underset{\sim}{56},2^+\right] - \left[\underset{\sim}{56},0^+\right] = 2\left(\left[\underset{\sim}{70},2^+\right] - \left[\underset{\sim}{70},0^+\right]\right) \qquad (2.39)$$

By taking the ${}^2(\underset{\sim}{8})$ states of each multiplet, we get:

$$\begin{aligned} \text{LHS} &= 0.69 \text{ GeV}^2 \\ \text{RHS} &= 0.58 \text{ GeV}^2 \end{aligned} \qquad (2.40)$$

This reasonable agreement may be an accident, but I think it indicates that the non-harmonic term we have chosen gives a good description of the multiplet splittings in the real baryon spectrum.

Apparently, no resonances belonging to the $\underset{\sim}{20}$ at the second excited level have been seen. From (2.38) we expect these states to have masses above 2000 MeV. Since the totally anti-symmetric $\underset{\sim}{20}$ do not couple to the product of the symmetric $\underset{\sim}{56}$ and the mesonic $\underset{\sim}{35}$, the $\underset{\sim}{20}$ can only be observed as resonances in the $\left[\underset{\sim}{70},1^-\right]$ -$\underset{\sim}{35}$ mass distributions. For instance, a Σ-resonance with a large width into $\Lambda(1405)\pi$ and a negligible coupling to $\Lambda(1115)\pi$ would be a strong indication for the presence of $\left[\underset{\sim}{20},1^+\right]$ and thereby for the harmonic oscillator hadron spectrum.

III. VECTOR CURRENT MATRIX ELEMENTS

The dynamical operators K for mesons (2.6) and for baryons (2.22) do not involve the spin of the quarks. However, this can be done without changing the structure of the operators by interpreting the square of the four-momentum of quark a, p_a^2, as the Dirac operator

$$\not{p}_a^2 = (p_{a\mu}\gamma_\mu)(p_{a\mu}\gamma_\mu) = p_{a\mu}p_{a\mu} = p_a^2 \tag{3.1}$$

This enables us to define the perturbational effect caused by a vector field A_μ by the minimal coupling

$$\not{p}_a \rightarrow \not{p}_a - e_a\not{A} \tag{3.2}$$

Hence the first order perturbation from quark a in the operator K will be:

$$\delta K_a^V = e_a(\not{p}_a\not{A} + \not{A}\not{p}_a) \tag{3.3}$$

To find the matrix elements of this operator, we sandwich it between the initial and final wave functions. These must then contain Dirac spinors to describe the quarks of the particles. In the particle rest system we want these spinors to represent three quarks in the case of baryons and one quark and one antiquark for mesons. This we achieve by adding the following "Dirac equation" restrictions on the particle wave function ψ describing for instance a baryon with four-momentum P and mass M:

$$\begin{aligned} P_\mu\gamma_{a\mu}\Psi &= M\Psi \\ P_\mu\gamma_{b\mu}\Psi &= M\Psi \\ P_\mu\gamma_{c\mu}\Psi &= M\Psi \end{aligned} \tag{3.4}$$

Each quark spinor of this baryon will then have the form

$$u = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} 1 \\ \frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{P}}{E+M} \end{pmatrix} \chi \tag{3.5}$$

where E and $\underset{\sim}{P}$ are the time and space components of the baryon four-momentum P and χ is an ordinary two-component Pauli spinor for the spin 1/2 quark. These quark spinors have the normalization $\bar{u}u=1$.

Baryon Vector Current

Following the above outlined prescription we find from the baryon equation (2.22), the full perturbational effect of a vector field A_μ:

$$\delta K^V = 3 \sum_{\alpha=a}^{c} e_\alpha \left[\not{p}_\alpha \not{A}(u_\alpha) + \not{A}(u_\alpha) \not{p}_\alpha \right] \tag{3.6}$$

In the case when A_μ is the electromagnetic field, the charges e_α are +2/3 for the u quarks and -1/3 for the d and s quarks.

Let the vector field have polarization vector e_μ and carry the four-momentum q_μ. Then we can write for equation (3.6),

$$\delta K^V = 3 \sum_{\alpha=a}^{c} e_\alpha \left[\not{p}_\alpha \not{e} e^{iq\cdot u_\alpha} + \not{e} e^{iq\cdot u_\alpha} \not{p}_\alpha \right] \tag{3.7}$$

$$= J_\mu^V e_\mu$$

Hence, the vector current for baryons will be:

$$J_\mu^V = 3 \sum_{\alpha=a}^{c} e_\alpha \left[\not{p}_\alpha \gamma_\mu e^{iq\cdot u_\alpha} + \gamma_\mu e^{iq\cdot u_\alpha} \not{p}_\alpha \right] \tag{3.8}$$

Because the particle states are symmetrical in the three quarks a, b, and c, each term in the sum over α in (3.7) will give the same result. So δK^V will be 3 times the contribution from quark a alone. Moving the operator $e^{iq\cdot u_\alpha}$ to the left by

$$\not{p}_a e^{iq\cdot u_a} = e^{iq\cdot u_a}\not{p}_a + \left[\not{p}_a, e^{iq\cdot u_a}\right]$$
$$= e^{iq\cdot u_a}(\not{p}_a - \not{q}) \tag{3.9}$$

we can write equation (3.7) as:

$$J_\mu^V e_\mu = 9e_a e^{iq\cdot u_a}\left[2(p_a\cdot e) - \not{q}\not{e}\right] \tag{3.10}$$

To reduce this further, we use the standard Dirac matrices

$$\gamma_t = \beta = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad \underset{\sim}{\gamma} = \begin{pmatrix}0 & +\underset{\sim}{\sigma}\\ -\underset{\sim}{\sigma} & 0\end{pmatrix}$$

$$\underset{\sim}{\alpha} = \beta\underset{\sim}{\gamma} = \begin{pmatrix}0 & \underset{\sim}{\sigma}\\ \underset{\sim}{\sigma} & 0\end{pmatrix} \tag{3.11}$$

$$\gamma_5 = i\gamma_t\gamma_x\gamma_y\gamma_z = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

We can now write out the matrix product $\not{q}\not{e}$, putting $q = (\nu, \underset{\sim}{Q})$:

$$\not{q}\not{e} = \nu e_t - (\underset{\sim}{\sigma}\cdot\underset{\sim}{Q})(\underset{\sim}{\sigma}\cdot\underset{\sim}{e}) + \underset{\sim}{\alpha}\cdot(e_t\underset{\sim}{Q} - \nu\underset{\sim}{e}) \tag{3.12}$$

Substituting this into equation (3.10) with $p_a = (\varepsilon_a, \underset{\sim}{p}_a)$, we find

$$J_\mu^V e_\mu = 9e_a e^{iq\cdot u_a}\Big[(2\varepsilon_a - \nu)e_t - \underset{\sim}{\alpha}_a\cdot(e_t\underset{\sim}{Q} - \nu\underset{\sim}{e})$$
$$- (2\underset{\sim}{p}_a - \underset{\sim}{Q})\cdot\underset{\sim}{e} + i\underset{\sim}{\sigma}_a\cdot(\underset{\sim}{Q}\times\underset{\sim}{e})\Big] \tag{3.13}$$

We will now calculate matrix elements of this operator between an initial baryon with four-momentum P_1 and mass M and a final baryon with P_2 and m so that

$$P_1 = q + P_2 \tag{3.14}$$

The initial and final quark spinors will then be according to (3.5):

$$u_1 = \sqrt{\frac{E_1+M}{2M}} \begin{pmatrix} 1 \\ \frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{P}_1}{E_1+M} \end{pmatrix} \chi_1$$

$$\bar{u}_2 = \sqrt{\frac{E_2+m}{2m}} \; \chi_2^{+} \left(1, \; - \frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{P}_2}{E_2+m}\right) \tag{3.15}$$

So far what we have done is frame independent. To simplify the following matrix element calculations, we will from now on work in the special frame where the initial baryon is at rest:

$$\underset{\sim}{P}_1 = 0$$

$$\underset{\sim}{P}_2 = -\underset{\sim}{Q} \tag{3.16}$$

The energy carried away by the final baryon will then be

$$E_2 = (M^2 + m^2 - q^2)/2M \tag{3.17}$$

and $$\nu = (M^2 - m^2 + q^2)/2M \tag{3.18}$$

In this special frame the spinors in (3.15) take the form

$$u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi_1$$

$$\bar{u}_2 = g\chi_2^{+} \left(1, + \frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{Q}}{2mg^2}\right) \tag{3.19}$$

where $$g^2 = \frac{E_2+m}{2m} = \frac{(M+m)^2-q^2}{4Mm} \tag{3.20}$$

The contribution to the matrix element from quark b will be

$$\bar{u}_{2b}u_{1b} = g(\chi^{+}_{2b}\chi_{1b}) \tag{3.21}$$

or simply a factor g times the unit operator between the Pauli spinors. Quark c gives the same, while for the quark a we find from equation (3.13) using the spinors in (3.19) (including g^2 from b and c)

$$\begin{aligned} J^V_\mu e_\mu &= 9g^3 e^{iq\cdot u_a}\,\chi^{+}_{2a}\, e_a \Big\{(2\epsilon_a-\nu-\frac{Q^2}{2mg^2})e_t \\ &- 2\underset{\sim}{p}_a\cdot\underset{\sim}{e} + \left[\underset{\sim}{Q}\cdot\underset{\sim}{e} + i\underset{\sim}{\sigma}_a\cdot(\underset{\sim}{Q}x\underset{\sim}{e})\right](1+\frac{\nu}{2mg^2})\Big\}\,\chi_{1a} \end{aligned} \tag{3.22}$$

In the exponent of $e^{iq\cdot u_a}$ we substitute for u_a from (2.24), set R=0 and express the internal coordinate x in terms of the operators a and a^+ in (2.27):

$$\begin{aligned} \exp(iq\cdot u_a) &= e^{iq\cdot R}\,\exp(-2iq\cdot x) \\ &= \exp\left[-\sqrt{\frac{2}{\Omega}}\,(a^+-a)\cdot q\right] \end{aligned} \tag{3.23}$$

To separate this into factors involving a and a^+ only, we use the Hausdorff formula

$$e^{A+B} = e^A e^B e^{-1/2[A,B]} \tag{3.24}$$

and get $$\exp(iq\cdot u_a) = e^{q^2/\Omega}\,\exp\left[-\sqrt{\frac{2}{\Omega}}\,q\cdot a^+\right]\,\exp\left[+\sqrt{\frac{2}{\Omega}}\,q\cdot a\right] \tag{3.25}$$

The matrix element (3.22) involves the quark four-momentum p_a. This can be expressed in terms of the baryon four-momentum P_1 and the internal momentum ξ by (2.23):

$$p_a = \frac{1}{3}P_1 - \frac{1}{3}\xi$$
$$= (\frac{1}{3}M-\frac{1}{3}\xi_t, -\frac{1}{3}\underset{\sim}{\xi}) \tag{3.26}$$

Again, relating ξ to the sum of a and a^+ and using the fundamental commutation relation

$$\left[a_\mu, a_\nu^+\right] = -g_{\mu\nu} \tag{3.27}$$

we find

$$\exp(iq\cdot u_a)\xi = e^{q^2/\Omega} \exp\left[-\sqrt{\frac{2}{\Omega}}\, q\cdot a^+\right] \cdot$$
$$\left[\sqrt{\frac{\Omega}{2}}(a^+ + a) - q\right] \exp\left[+\sqrt{\frac{2}{\Omega}}\, q\cdot a\right] \tag{3.28}$$

It is now straightforward to write the baryon vector current (3.22) in its final form:

$$J^V_\mu e_\mu = 9g^3 e^{q^2/\Omega} e^{-\sqrt{\frac{2}{\Omega}}\, q\cdot a^+} e_a \Bigg\{\Bigg[\frac{2}{3}M - \frac{1}{3}\nu$$
$$- \frac{Q^2}{2mg^2} - \frac{2}{3}\sqrt{\frac{\Omega}{2}}(a_t^+ + a_t)\Bigg] e_t + \frac{2}{3}\sqrt{\frac{\Omega}{2}}\,(\underset{\sim}{a}^+ + \underset{\sim}{a})\cdot\underset{\sim}{e} \tag{3.29}$$
$$+ \underset{\sim}{Q}\cdot\underset{\sim}{e}(\frac{1}{3} + \frac{\nu}{2mg^2}) + i\underset{\sim}{\sigma}_a\cdot(\underset{\sim}{Q}\times\underset{\sim}{e})\ (1+\frac{\nu}{2mg^2})\Bigg\} e^{+\sqrt{\frac{2}{\Omega}}q\cdot a}$$

This is the expression we will use in the following to calculate baryon magnetic moments, photoproduction amplitudes and cross sections for electroproduction of nucleon resonances.

Magnetic Moments of Baryons

The coupling of a photon to spin 1/2 baryons can be written in the usual way

$$J_\mu^{EM} e_\mu = \bar{u}_2 \left[\gamma_\mu F_1 + \sigma_{\mu\nu} q_\nu F_2 \right] u_1 e_\mu \qquad (3.30)$$

where $\sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$

and $$q = P_1 - P_2 \qquad (3.31)$$

F_1 and F_2 are form factors. In the rest system of the initial baryon, equation (3.30) can be reduced to the following two-component form:

$$J_\mu^{EM} e_\mu = 2Mg^{-1} \chi_2^+ \left[e_t G_E(q^2) + i\underset{\sim}{\sigma} \cdot (\underset{\sim}{Q} \times \underset{\sim}{e}) G_M(q^2) \right] \chi_1 \qquad (3.32)$$

where the electric form factor is

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{2M} F_2(q^2) \qquad (3.33)$$

and the magnetic form factor is

$$G_M(q^2) = F_2(q^2) + \frac{1}{2M} F_1(q^2) \qquad (3.34)$$

The factor g is essentially the same as in (3.20) with M=m

$$g^2 = 1 - \frac{q^2}{4M^2} \qquad (3.35)$$

and arises here from the Lorentz-transformation of the electromagnetic coupling from the Breit-frame where q has only space-like components, to the rest system of the initial baryon.

From equation (3.32) we find for the magnetic moment of the baryon

$$\mu = eG_M(0) \tag{3.36}$$

Experimentally, the magnetic form factor of the proton is well described by the dipole form:

$$\frac{e}{\mu_N} G_M(q^2) = (1 - \frac{q^2}{0.71})^{-2} \tag{3.37}$$

We will now calculate the baryon magnetic moments μ from the general matrix element (3.29). Since we only consider the ground state, spin 1/2 baryons, all terms containing a and a^+ do not contribute. From the photon transversality condition $q \cdot e = 0$, we get

$$Q \cdot e = \nu\, e_t \tag{3.38}$$

which in equation (3.29) gives:

$$\begin{aligned} J_\mu^{EM} e_\mu &= 9g^3 e^{q^2/\Omega} e_a \left\{ \left(\frac{2}{3}M - \frac{1}{3}\nu - \frac{Q^2}{2Mg^2} \right) e_t + \left(\frac{\nu}{3} + \frac{\nu^2}{2Mg^2} \right) e_t \right. \\ &\quad \left. + i\underset{\sim}{\sigma}_a \cdot (\underset{\sim}{Q} x \underset{\sim}{e}) \left(1 + \frac{\nu}{2Mg^2} \right) \right\} \\ &= 3g^3 e^{q^{2}/\Omega}\, 2Me_a \left\{ e_t \left(1 + \frac{3q^2}{4M^2 g^2} \right) + i\underset{\sim}{\sigma}_a \cdot (\underset{\sim}{Q} x \underset{\sim}{e}) \left(\frac{3}{2M} + \frac{3\nu}{4M^2 g^2} \right) \right\} \end{aligned} \tag{3.39}$$

In Table C in the Appendix we find the matrix elements of e_a and $\underset{\sim}{\sigma} e_a$,

$$\begin{aligned} \langle e_a \rangle &= \frac{1}{3}\ (1,0) \\ \langle \underset{\sim}{\sigma} e_a \rangle &= \frac{1}{3}\ (+\frac{2}{3}, 1) \end{aligned} \tag{3.40}$$

where the numbers in the parentheses are (F,D) values. Substituting these into (3.39), we get

$$J_\mu^{EM} e_\mu = 2M \left\{ e_t (1 + \frac{3q^2}{4M^2 g^2})(1,0) \right. +$$

$$\left. i\underset{\sim}{\sigma}\cdot(\underset{\sim}{Q}\times\underset{\sim}{e})(\frac{3}{2M} + \frac{3\nu}{4M^2 g^2})(\frac{2}{3},1) \right\} g^3 e^{q^2/\Omega} \qquad (3.41)$$

Comparing this with (3.32) we find for the electric and magnetic form factors:

$$G_E(q^2) = g(1 + \frac{3q^2}{4M^2 g^2})(1,0) g^3 e^{q^2/\Omega}$$

$$= \frac{1}{g}(1 + \frac{q^2}{2M^2})(1,0) g^3 e^{q^2/\Omega} \qquad (3.42)$$

$$G_M(q^2) = g(\frac{3}{2M} + \frac{3\nu}{4M^2 g^2})(\frac{2}{3},1) g^3 e^{q^2/\Omega}$$

$$= \frac{1}{g}\,\frac{3}{2M}(\frac{2}{3},1) g^3 e^{q^2/\Omega} \qquad (3.43)$$

From this last equation we now get the magnetic moments of most interest, using the definition (3.36):

$$\begin{aligned}
\mu_{N^+} &= \frac{3e}{2M_N}(F + \frac{1}{3}D) = +3\,\frac{e}{2M_N} = +3.00\ \mu_B \\
\mu_{N^o} &= \frac{3e}{2M_N}(-\frac{2}{3}D) = -2\,\frac{e}{2M_N} = -2.00\ \mu_B \\
\mu_{\Sigma^+} &= \frac{3e}{2M_\Sigma}(F + \frac{1}{3}D) = +3\,\frac{e}{2M_\Sigma} = +2.37\ \mu_B \\
\mu_\Lambda &= \frac{3e}{2M_\Lambda}(-\frac{1}{3}D) = -1\,\frac{e}{2M_\Lambda} = -0.84\ \mu_B
\end{aligned} \qquad (3.44)$$

Here we have used the baryon-photon SU(3) couplings given in terms of

F and D as defined in the Appendix; μ_B is the Bohr magneton $e/2M_N$.

From the Particle Tables (1) we find the experimental values for these magnetic moments:

$$\mu_{N^+} = + 2.79\ \mu_B \qquad \mu_{\Sigma^+} = (+2.57\pm0.52)\mu_B$$
$$\mu_{N^o} = - 1.91\ \mu_B \qquad \mu_{\Lambda} = (-0.73\pm0.16)\mu_B \tag{3.45}$$

Besides giving the famous ratio -3/2 between the proton and neutron magnetic moments, this relativistic coupling scheme also gives unique, absolute values. SU(6) alone gives relations like $\mu_{\Sigma^+} = \mu_{N^+} = -\frac{3}{2}\mu_{N^o} = -3\mu_{\Lambda}$ without being able to take into account the different masses of the baryons. Our relations, with their explicit mass dependence, seem to be in better agreement with experiment.

From equation (3.43) we get for the proton magnetic form factor:

$$\frac{e}{\mu_N} G_M(q^2) = (1 - \frac{q^2}{4M^2})\ e^{q^2/\Omega} \tag{3.46}$$

Comparing this to the experimental form (3.37) we see that this result is completely wrong, even at very small $-q^2$. This means that our harmonic oscillator dynamics of three quarks is much too simple to adequately describe the physical structure of baryons. There must be more to it than just that. But still we think that the coupling to external agents like an electromagnetic field can be reasonably well represented by the interaction with three harmonically bound quarks when $q^2 = 0$. The analogy between these quarks and valence electrons in atoms is tempting.

So the lack of knowledge of a better theory will not make us abandon this scheme. Instead, we will compensate for it by introducing empirical form factors to try to fit experiment. That is not a non-trivial task. If we are successful, we think this model has proved its usefulness.

Consequently, from now on we will make the substitution

$$g^3 e q^2/\Omega \rightarrow G(M,q^2) \tag{3.47}$$

In order to reproduce the proton dipole form factor (3.37), the empirical form factor G must in the case of diagonal matrix elements for ground state baryons have the form

$$G_o(M,q^2) = (1 - \frac{q^2}{0.71})^{-2}(1 - \frac{q^2}{4M^2})^{1/2} \tag{3.48}$$

The ratio between the electric and magnetic form factors is independent of this choice. From equations (3.42) and (3.43) we find this to be:

$$\frac{G_E(q^2)}{G_M(q^2)} = \frac{e}{\mu}(1 + \frac{q^2}{2M^2}) \tag{3.49}$$

Experimentally, a decrease in this ratio with increasing $-q^2$ has been observed (14), but the variation is apparently not so rapid as we find. The old "scaling" hypotheses that this ratio is independent of q^2, seems to be incorrect.

Photoproduction Amplitudes

In this chapter we will calculate the amplitudes for excitation of the lowest nucleon resonances by photons. To keep the derivation most general, we will first consider the case of virtual photons with

$q^2 \neq 0$. So the process we shall investigate is

$$\gamma + N \rightarrow N^* \tag{3.50}$$

We will assume the nucleon N to be at rest in the LAB system where the photon momentum is $q = (\nu, \underset{\sim}{Q})$. The transition amplitudes will be evaluated in the rest frame of the N^* where $q = (\nu^*, \underset{\sim}{Q}^*)$. In this system $\underset{\sim}{Q}^*$ will be taken along the z-axis and the nucleon N will have $J_z = +1/2$. We add an asterisk to the photon variables in this system to distinguish them from the same variables in the LAB system.

The photon polarization vector e_μ has four components:

transverse: $e_+ = -\sqrt{\frac{1}{2}}\,(e_x + ie_y)$

$$e_- = +\sqrt{\frac{1}{2}}\,(e_x - ie_y) \tag{3.51}$$

scalar: $e_z = e_z$

longitudinal: $e_o = e_t$,

e_+ corresponds to photons with positive helicity and e_- negative helicity.

We define dimensionless current matrix elements F_μ by

$$B_\mu = e\langle N|J_\mu^{EM}|N^*\rangle = e2MF_\mu \tag{3.52}$$

Here e is the unit electrical charge,

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} \tag{3.53}$$

Current conservation

$$q_\mu \langle N|J_\mu^{EM}|N^*\rangle = 0 \tag{3.54}$$

gives a relation between the scalar and longitudinal parts of the current:

$$Q^* F_z = \nu^* F_t \qquad (3.55)$$

Now using the general expression equation (3.29) we can at once write down the three independent current matrix elements

$$\begin{aligned} F_o(q^2) &= 9G\langle N(+\tfrac{1}{2})\,|e_a S e^{-\lambda a_z}|N^*(+\tfrac{1}{2})\rangle \\ F_+(q^2) &= 9G\langle N(+\tfrac{1}{2})\,|e_a(Ta_+ + R\sigma_{a-})e^{-\lambda a_z}|N^*(+\tfrac{3}{2})\rangle \\ F_-(q^2) &= 9G\langle N(+\tfrac{1}{2})\,|e_a(Ta_- + R\sigma_{a+})e^{-\lambda a_z}|N^*(-\tfrac{1}{2})\rangle \end{aligned} \qquad (3.56)$$

corresponding to the three helicity states e_o, e_+ and e_- defined in (3.51). Here we have introduced the quantity

$$\lambda = \sqrt{\frac{2}{\Omega}}\, Q^* \qquad (3.57)$$

where

$$\begin{aligned} \left[Q^*(q^2)\right]^2 &= \nu^{*2} - q^2 \\ &= \left[(M+m)^2 - q^2\right]\left[(M-m)^2 - q^2\right]/4M^2 \end{aligned} \qquad (3.58)$$

since ν^* is the same as ν in (3.18). The expressions for S, R and T can be found directly from (3.29):

$$\begin{aligned} S = S(q^2) &= \left[\frac{2}{3}M - \frac{1}{3}\nu^* - \frac{Q^*}{2mg^2}\right]/2M \\ &= \left[\frac{2}{3}M - \frac{1}{3}\nu^* - (E_2 - m)\right]/2M \\ &= \left[3Mm + q^2 - m^2\right]/6M^2 \end{aligned} \qquad (3.59)$$

$$R = R(q^2) = \sqrt{2}\, Q^* \left(1 + \frac{\nu}{2mg^2}\right)/2M$$

$$= \sqrt{2}\, Q^* \left(1 + \frac{M^2-m^2+q^2}{(M+m)^2-q^2}\right)/2M \qquad (3.60)$$

$$= \sqrt{2}\, Q^* \frac{M + m}{(M+m)^2-q^2}$$

$$T = \frac{2}{3}\sqrt{\frac{\Omega}{2}}\frac{1}{2M} = \frac{1}{3M}\sqrt{\frac{\Omega}{2}} \qquad (3.61)$$

To check the current conservation condition equation (3.55), we write $S(q^2)$ in a slightly different form:

$$S = \left[\frac{2}{3}M\nu^* - \frac{1}{3}\nu^{*2} - \frac{Q^{*2}\nu^*}{2mg^2}\right]/2M\nu^*$$

$$= \left[\frac{1}{3}(M^2-m^2) - Q^{*2}\left(\frac{1}{3} + \frac{\nu^*}{2mg^2}\right)\right]/2M\nu^* \qquad (3.62)$$

If there are N excitations in the N^*, its wave function will contain a factor

$$|N^*\rangle \sim (a_z^+)^N|0\rangle \qquad (3.63)$$

From the expression for F_o in (3.56) we then get

$$F_o = cS\langle 0|e^{-\lambda a_z}(a_z^+)^N|0\rangle$$

$$= cS\langle 0|\ \frac{(-\lambda)^N}{N!}\ (a_z)^N(a_z^+)^N|0\rangle \qquad (3.64)$$

$$= cS(-\lambda)^N$$

where c is some numerical factor. In the same way we find for F_z:

$$
\begin{aligned}
F_z &= -c\,\frac{Q^*}{2M}\Big(\frac{1}{3} + \frac{\nu^*}{2mg^2}\Big)(-\lambda)^N \\
&\quad - \frac{c}{3M}\sqrt{\frac{\Omega}{2}}\,\langle 0|e^{-\lambda a_z}a_z^{+}(a_z^{+})^N|0\rangle \\
&= -c\,\frac{Q^*}{2M}\Big(\frac{1}{3} + \frac{\nu^*}{2mg^2}\Big)(-\lambda)^N \\
&\quad - \frac{c}{3M}\sqrt{\frac{\Omega}{2}}\,\frac{N!}{(N-1)!}\,(-\lambda)^{N-1} \\
&= -\frac{c}{2M}(-\lambda)^{N-1}\sqrt{\frac{2}{\Omega}}\left[\frac{1}{3}\Omega N - Q^{*2}\Big(\frac{1}{3} + \frac{\nu^*}{2mg^2}\Big)\right]
\end{aligned}
\tag{3.65}
$$

$$
\begin{aligned}
\frac{Q^*}{\nu^*}F_z &= +\frac{c}{2M\nu^*}(-\lambda)^N\left[\frac{1}{3}\Omega N - Q^{*2}\Big(\frac{1}{3} + \frac{\nu^*}{2mg^2}\Big)\right] \\
&= F_o - c(-\lambda)^N(M^2 - m^2 - N\Omega)/6M\nu^*
\end{aligned}
\tag{3.66}
$$

In this simple theory the mass of the N^* is given by

$$M^2 = N\Omega + m^2 \tag{3.67}$$

where m is the mass of the ground state. The last term in (3.66) will then be zero and we have current conservation.

To be consistent at this point we should also use the theoretical particle masses in the numerical calculation of the photoproduction amplitudes. This would, however, bring us into trouble with the kinematical factors entering the formulas where we must use the physical masses of the particles involved. Consequently, we will in the following use the real particle masses in all expressions. In the cases we shall consider, this makes little difference in the numerical results.

We can now calculate the amplitudes in (3.56) using the SU(6) and orbital matrix elements given in the Appendix. The results are presented in Table 3. Only the low-lying nucleon resonances with mass M<1750 MeV have been considered. We have included the not yet firmly established $D_{13}(1700)$ belonging to the $^4(\underline{8})$ of the $[\underline{70},1^-]$, the $P_{33}(1690)$ of the second excited $[\underline{56},0^+]$ and the predicted $P_{13}(1700)$ of the $[\underline{56},2^+]$. The total width Γ of each resonance is also given, for the $P_{13}(1700)$ we assume Γ = 200 MeV. We use the notation $D^+_{13}(1520)$ if the resonance is produced off protons and $D^o_{13}(1520)$ for neutron target.

Many of the amplitudes in Table 3 have been experimentally determined in photoproduction of pions by Walker (6). In this case we have real photons with q^2=0 and only $F_+(0)$ and $F_-(0)$ will contribute. The photon momentum Q^* in equation (3.58) and R in (3.60) can then be written:

$$K^* = Q^*(0) = (M^2-m^2)/2M \tag{3.68}$$

$$R(0) = \sqrt{2}\,(M-m)/2M \tag{3.69}$$

Walker uses a different wave function normalization, so to get his amplitudes A_+ and A_- from our F_+ and F_-, we must use the conversion formula:

$$\begin{aligned} A_\pm &= e\left(\frac{2M}{2E_2 2K^*}\right)^{1/2} F_\pm \\ &= e\left(\frac{2M^3}{M^4-m^4}\right)^{1/2} F_\pm \end{aligned} \tag{3.70}$$

The absolute signs of the amplitudes can be found by calculating the amplitude for the process $N^*N\pi$ of the other vertex. This we have done in Section IV where these amplitudes can be found in Table 6 .

Table 3. Photoelectric matrix elements for excitation of low-lying nucleon resonances.

$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	Resonance	Γ (MeV)	$F_+(q^2)/G$	$F_-(q^2)/G$	$F_o(q^2)/G$
$[\underset{\sim}{56},0^+]$	$^4(\underset{\sim}{10})_{3/2}$	$P^+_{33}(1236)$	120	$-\sqrt{6}\ R$	$+\sqrt{2}\ R$	0
	$^2(\underset{\sim}{8})_{1/2}$	$S^+_{11}(1535)$	120		$-\sqrt{3}T-\sqrt{\frac{3}{2}}R\lambda$	$+\sqrt{\frac{3}{2}}\ S\lambda$
		$S^o_{11}(1535)$			$+\sqrt{3}T+\sqrt{\frac{1}{6}}R\lambda$	$-\sqrt{\frac{3}{2}}S\lambda$
	$^2(\underset{\sim}{8})_{3/2}$	$D^+_{13}(1520)$	120	$+\sqrt{\frac{9}{2}}\ T$	$+\sqrt{\frac{3}{2}}T-\sqrt{3}R\lambda$	$-\sqrt{3}S\lambda$
		$D^o_{13}(1520)$		$-\sqrt{\frac{9}{2}}\ T$	$-\sqrt{\frac{3}{2}}T+\sqrt{\frac{1}{3}}R\lambda$	$+\sqrt{3}S\lambda$
	$^4(\underset{\sim}{8})_{1/2}$	$S^+_{11}(1700)$	250		0	0
$[\underset{\sim}{70},\ 1^-]$		$S^o_{11}(1700)$			$-\sqrt{\frac{1}{6}}R\lambda$	0
	$^4(\underset{\sim}{8})_{3/2}$	$D^+_{13}(1700)$	150	0	0	0
		$D^o_{13}(1700)$		$-\sqrt{\frac{9}{10}}R\lambda$	$-\sqrt{\frac{1}{30}}R\lambda$	0
	$^4(\underset{\sim}{8})_{5/2}$	$D^+_{15}(1670)$	140	0	0	0
		$D^o_{15}(1670)$		$-\sqrt{\frac{3}{5}}R\lambda$	$+\sqrt{\frac{3}{10}}R\lambda$	0
	$^2(\underset{\sim}{10})_{1/2}$	$S^+_{31}(1650)$	150		$-\sqrt{3}T+\sqrt{\frac{1}{6}}R\lambda$	$+\sqrt{\frac{3}{2}}S\lambda$

Table 3. Continued

$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	Resonance	Γ (MeV)	$F_+(q^2)/G$	$F_-(q^2)/G$	$F_o(q^2)/G$
	$^2(\underset{\sim}{10})_{3/2}$	$D^+_{33}(1670)$	240	$+\sqrt{\frac{9}{2}}\,T$	$+\sqrt{\frac{3}{2}}T+\sqrt{\frac{1}{3}}R\lambda$	$-\sqrt{3}S\lambda$
	$^2(\underset{\sim}{8})_{1/2}$	$P^+_{11}(1470)$	250		$-\sqrt{\frac{3}{4}}R\lambda^2$	$-\sqrt{\frac{3}{4}}S\lambda^2$
$[\underset{\sim}{56},0^+]$		$P^o_{11}(1470)$			$+\sqrt{\frac{1}{3}}R\lambda^2$	0
	$^4(\underset{\sim}{10})_{3/2}$	$P^+_{33}(1690)$	250	$+\sqrt{\frac{1}{2}}R\lambda^2$	$-\sqrt{\frac{1}{6}}R\lambda^2$	0
	$^2(\underset{\sim}{8})_{3/2}$	$P^+_{13}(1700)$	200	$+\sqrt{\frac{9}{10}}T\lambda$	$(+\sqrt{\frac{27}{10}}T+\sqrt{\frac{3}{5}}R\lambda)\lambda$	$-\sqrt{\frac{3}{5}}S\lambda^2$
		$P^o_{13}(1700)$		0	$-\sqrt{\frac{4}{15}}R\lambda^2$	0
$[\underset{\sim}{56},\ 2^+]$	$^2(\underset{\sim}{8})_{5/2}$	$F^+_{15}(1688)$	125	$-\sqrt{\frac{18}{5}}T\lambda$	$(-\sqrt{\frac{9}{5}}T+\sqrt{\frac{9}{10}}R\lambda)\lambda$	$+\sqrt{\frac{9}{10}}S\lambda^2$
		$F^o_{15}(1688)$		0	$-\sqrt{\frac{2}{5}}R\lambda^2$	0

In this way we arrive at the numerical photoelectric amplitudes presented in Table 4 for the well established nucleon resonances. Here we have used the form factor

$$G(M,0) = \exp\left(-\frac{K^{*2}}{\Omega}\frac{M^2}{M^2+m^2}\right) \qquad (3.71)$$

which is almost the same as the one in the non-relativistic calculations (5,6). In the mass region we have considered, it makes no substantial contribution. It ranges from 0.96 for the $P_{33}(1236)$ to 0.78 for the $F_{15}(1688)$.

The numerical values of our amplitudes in Table 4 are not very different from the non-relativistic calculations by Copley, Karl and Obryk (5) and Walker (6). Comparing the results with the existing experimental data from Walker also shown in Table 4, we find general good agreement. Even the absolute signs of the amplitudes come out right. That must be considered a triumph of this simple model. It should be stressed that our results depend on no free parameters besides the smooth form factor and the fixed constant Ω, only the physical masses of the particles enter the amplitudes.

Some of these good results are independent of the harmonic oscillator dynamics and follow from SU(6) alone. To these belong the prediction of zero amplitudes for photoproduction off protons of the $^4(\underset{\sim}{8})$ resonances in the $[\underset{\sim}{70},1^-]$, like the $D_{15}^+(1670)$. This was first observed by Moorhouse (15). The reason is that the coupling being proportional to $F + \frac{1}{3}D$, vanishes for these states since they all have F/D = -1/3. Another SU(6) result is that A_+ for $F_{15}^o(1688)$ equals zero which also agrees well with experiment.

Table 4. Calculated photoproduction amplitudes and their experimental values.

Resonance	K^* (MeV)	A_+^{calc} ($GeV^{-1/2}$)	A_+^{exp} ($GeV^{-1/2}$)	A_-^{calc} ($GeV^{-1/2}$)	A_-^{exp} ($GeV^{-1/2}$)
$P_{33}^+(1236)$	261	-0.187	-0.244	+0.108	+0.138
$D_{13}^+(1520)$	470	+0.109	+0.151	-0.034	-0.026
$D_{13}^o(1520)$		-0.109	-0.132	-0.031	
$S_{11}^+(1535)$	480			-0.156	-0.096
$S_{11}^o(1535)$				+0.108	+0.118
$D_{15}^+(1670)$	571	0	0.040	0	~0
$D_{15}^o(1670)$		-0.053		+0.038	
$S_{31}^+(1650$	558			-0.047	
$D_{33}^+(1670)$	571	+0.084		+0.088	
$P_{11}^+(1470)$	435			+0.027	
$P_{11}^o(1470)$				-0.018	
$F_{15}^+(1688)$	583	+0.059	+0.139	-0.010	~0
$F_{15}^o(1688)$		0	~0	+0.035	

Typical model dependent results are the small A_- amplitudes for $D^+_{13}(1520)$ and $F^+_{15}(1688)$. In these cases the amplitudes consist of two terms which almost cancel each other. To our knowledge this is the only model capable of explaining this experimental fact.

Only the amplitude A_+ for $F^+_{15}(1688)$ is in disagreement with experiment, being a factor 2 too small. This amplitude involves only one term. Since this resonance is the recurrence of the nucleon, we would expect the amplitudes for this particular state to be in the best agreement with experiment.

However, Walker's values are not without errors. We do not know these, but we can make an independent check of our results in electroproduction of the same nucleon resonances. Should the cross section calculated for the $F_{15}(1688)$ then be too small by a factor of 4, then Walker is right and our model will be in trouble.

Before doing that, we will calculate one more photoelectric process

$$\Lambda(1520) \rightarrow \Lambda + \gamma \tag{3.72}$$

The amplitudes are evaluated in the Appendix, step by step, as a model example. We find:

$$\begin{aligned} F_+ &= \frac{5}{4}\sqrt{2}\, TG = 0.26 \\ F_- &= \frac{5}{4}\sqrt{\frac{2}{3}}\left[T - \sqrt{2}\, R\lambda\right] G = 0.03 \end{aligned} \tag{3.73}$$

The rate for this transition is then given by the standard relativistic rate equation, in terms of the invariant matrix elements B_+ and B_- in (3.52),

$$\Gamma = \frac{1}{8\pi}\frac{Q^*}{M^2}\frac{2}{2J+1}\left(|B_+|^2 + |B_-|^2\right)$$

$$= \frac{e^2}{4\pi}\frac{4}{2J+1}\left(|F_+|^2 + |F_-|^2\right)Q^* \qquad (3.74)$$

J is the spin of the resonance and we have multiplied by a factor 2 because the final Λ can have spin up or down. Again using the form factor (3.71) we get for the width of this decay mode:

$$\Gamma_{calc} = 0.17 \text{ MeV} \qquad (3.75)$$

The experimental value from the Particle Tables (1) is:

$$\Gamma_{exp} = (0.13 \pm 0.04) \text{ MeV} \qquad (3.76)$$

Having obtained this good result, we can now predict the rate for the following, very similar transition

$$\Lambda(1405) \rightarrow \Lambda + \gamma \qquad (3.77)$$

Since these two Λ-resonances both belong to ${}^2(\underset{\sim}{1})$ of the $\left[\underset{\sim}{70}, 1^-\right]$, the amplitude for this latter process can be obtained almost at once from (3.73) by one Clebsch-Gordan step, as shown in the Appendix:

$$F_- = \frac{5}{4}\sqrt{\frac{2}{3}}\left[\sqrt{2}\,T + R\lambda\right]\bar{G} = 0.29 \qquad (3.78)$$

This gives the predicted rate

$$\Gamma = 0.32 \text{ MeV} \qquad (3.79)$$

It should be mentioned in connection with the $\Lambda(1520)$ that the radiative decay of this resonance takes place almost entirely from the

$J_z = 3/2$ state as can be seen from (3.73). An experimental verification of this prediction would be very encouraging.

Electroproduction of Nucleon Resonances

We will assume that the inelastic electron nucleon scattering process

$$e + N \rightarrow e' + N^* \qquad (3.80)$$

proceeds through one photon exchange as in Figure 4. The initial and final electron with four-momenta $k = (E,\underset{\sim}{k})$ and $k' = (E',\underset{\sim}{k}')$, we take to be of zero mass. Then the invariant four-momentum transfer to the nucleon will be

$$q^2 = (k-k')^2 = -4EE' \sin^2 \frac{\theta}{2} \qquad (3.81)$$

where θ is the electron scattering angle. In the LAB system, the energy of the virtual photon with momentum $q = (\nu,\underset{\sim}{Q})$, will be

$$\nu = E-E' = (q\cdot p)/m \qquad (3.82)$$

assuming the target nucleon to be at rest with mass m.

As shown by Bjorken and Walecka (16) the differential cross section for this process can be written

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \left[W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right] \qquad (3.83)$$

for unpolarized initial and final nucleon states. The two invariant structure functions W_1 and W_2 are defined in terms of electromagnetic current matrix elements,

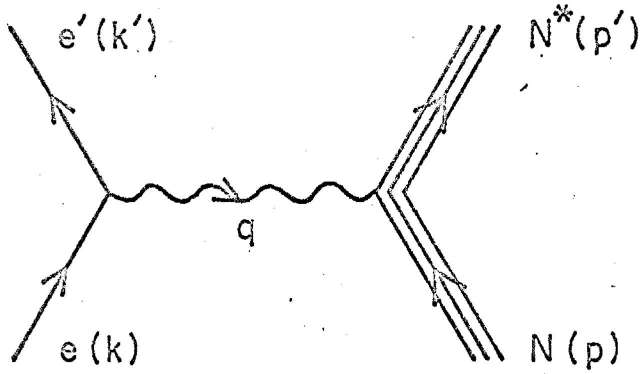

Figure 4. One-photon exchange diagram for resonance electroproduction.

$$W_{\mu\nu} = 2m \left[(p_\mu - q_\mu \frac{p\cdot q}{q^2})(p_\nu - q_\nu \frac{p\cdot q}{q^2}) \frac{W_2}{m^2} \right.$$

$$\left. + (\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}) W_1 \right] \qquad (3.84)$$

$$= \sum_n (2\pi)^3 \delta^4(p+q-p') \langle p | J_\mu^{EM} | n \rangle \langle n | J_\nu^{EM} | p \rangle$$

where the sum extends over all final states $|n\rangle$ with four-momentum p'.

With only one nucleon resonance with mass M in the final state, $W_{\mu\nu}$ takes the form:

$$W_{\mu\nu} = \langle p | J_\mu^{EM} | p' \rangle \langle p' | J_\nu^{EM} | p \rangle \delta(W^2 - M^2) \qquad (3.85)$$

Here $W^2 = (p+q)^2$ is the invariant mass of the produced N^*.

Again taking the space part of the photon four-momentum $q=(\nu^*, \underset{\sim}{Q}^*)$ in the N^* rest system along the z-axis and using the current conservation condition equation (3.55), we can express W_1 and W_2 in terms of F_o, F_+ and F_- as defined in (3.52) and (3.56):

$$W_1 = \frac{1}{2}(|F_+|^2 + |F_-|^2) \frac{M}{m} \delta(W-M)$$

$$W_2 = \frac{1}{2}\left[\frac{2q^4}{Q^{*4}}|F_o|^2 - \frac{q^2}{Q^{*2}}(|F_+|^2 + |F_-|^2)\right]\frac{m}{M}\delta(W-M) \qquad (3.86)$$

Defining partial cross sections due to the transverse and longitudinal parts of the photon by

$$\sigma_t(W) = \frac{4\pi^2\alpha}{Q^*} \frac{1}{2} (|F_+|^2 + |F_-|^2) \delta(W-M)$$

$$\sigma_\ell(W) = \frac{4\pi^2\alpha}{Q^*} (\frac{+q^2}{Q^{*2}}) |F_o|^2 \delta(W-M) , \qquad (3.87)$$

we can now write the differential cross section (3.83) as

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha}{4\pi^2} \frac{\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}} \left(\frac{-q^2}{Q}\right)\frac{1}{\varepsilon}\,(\sigma_t - \varepsilon\sigma_\ell) \tag{3.88}$$

where $$\varepsilon^{-1} = 1 + 2\left(1 - \frac{\nu^2}{q^2}\right)\tan^2\frac{\theta}{2} \tag{3.89}$$

The polarization parameter ε varies between 0 and 1.

Instead of the cross sections σ_t and σ_ℓ, experimental data are usually given by the slightly different quantities σ_T and σ_S introduced by Hand (17):

$$\sigma_T = + \frac{Q}{K}\sigma_t \quad , \qquad \sigma_S = - \frac{Q}{K}\sigma_\ell \tag{3.90}$$

Here K is the LAB energy required to produce the N^* with real photons:

$$K = (M^2 - m^2)/2m \tag{3.91}$$

Inserting σ_T and σ_S into (3.88), we get the well-known expression

$$\begin{aligned} \frac{d^2\sigma}{d\Omega dE'} &= \Gamma_T\sigma_T + \Gamma_S\sigma_S \\ &= \Gamma_T(\sigma_T + \varepsilon\sigma_S) \end{aligned} \tag{3.92}$$

with $$\Gamma_T = \frac{\alpha}{4\pi^2}\left(\frac{-K}{q^2}\right)\frac{E'}{E}\frac{2}{1-\varepsilon} \tag{3.93}$$

From (3.87) we find for σ_T and σ_S:

$$\begin{aligned} \sigma_T(W) &= \frac{4\pi^2\alpha}{K^*}\frac{1}{2}\left(|F_+|^2 + |F_-|^2\right)\frac{\Gamma/2\pi}{(W-M)^2+\Gamma^2/4} \\ \sigma_S(W) &= \frac{4\pi^2\alpha}{K^*}\left(\frac{-q^2}{Q^{*2}}\right)|F_o|^2 \frac{\Gamma/2\pi}{(W-M)^2+\Gamma^2/4} \end{aligned} \tag{3.94}$$

where we have made the substitution

$$\delta(W-M) \rightarrow \frac{1}{\pi}\frac{\Gamma/2}{(W-M)^2+\Gamma^2/4} \tag{3.95}$$

valid for an unstable resonance with a total width Γ. K^* is the quantity K evaluated in the N^* rest frame, equation (3.68).

Before we can calculate the cross sections, we have to decide upon the form factor G which enters all amplitudes. We have one serious restriction on this choice. It should in the simple case of elastic electron scattering give the form (3.48). With this in mind, we will in the following use one of the simplest possible forms

$$G(M,q^2) = (1 - \frac{q^2}{0.71})^{-2}(1 - \frac{q^2}{4M^2})^{\frac{1-N}{2}} \tag{3.96}$$

where N is the number of excitations in the N^*. This arbitrary choice seems to give a reasonable description of the present experimental data in the limited resonance region we are concerned with here.

If we in addition also want the form factor to approach the non-relativistic form (3.71) for $q^2=0$, we could instead of (3.96) use:

$$G_N(M,q^2) = (1 - \frac{q^2}{0.71})^{-2}(1 - \frac{q^2}{4M^2})^{\frac{1-N}{2}} \exp\left[-\frac{K^{*2}}{\Omega}\,\frac{M^2}{M^2+m^2}\,\frac{\Delta^2}{\Delta^2-q^2}\right] \tag{3.97}$$

This is only different from the choice $G(M,q^2)$in (3.96) when $-q^2 \ll \Delta^2$. Consequently, by letting $\Delta^2 \ll 0.5$ GeV^2, these two form factors will give the same results in the region $-q^2 > 0.5$ GeV^2 where most of the electroproduction data exist. Only future, more precise experiments for very small $-q^2$ will decide which form factor is the best choice.

We are now in the position to calculate the cross sections σ_T and σ_S for each state in Table 3 where also the amplitudes F_o, F_+ and F_- are given. The results of this numerical work are presented in Figures 5, 6, and 7 where we plot each cross section for proton targets

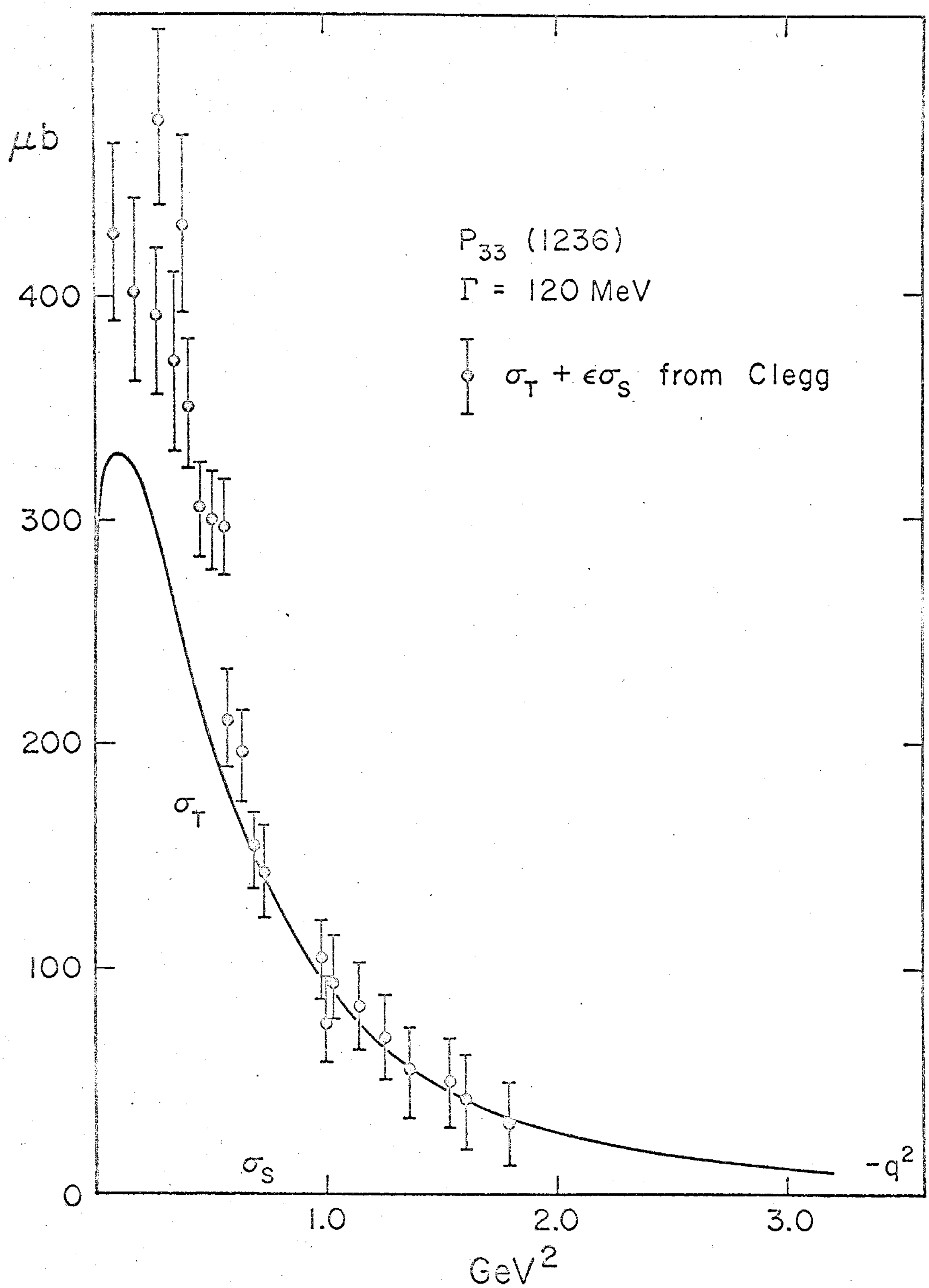

Figure 5. Resonance cross section at W = 1236 MeV with proton target. Data from reference 19.

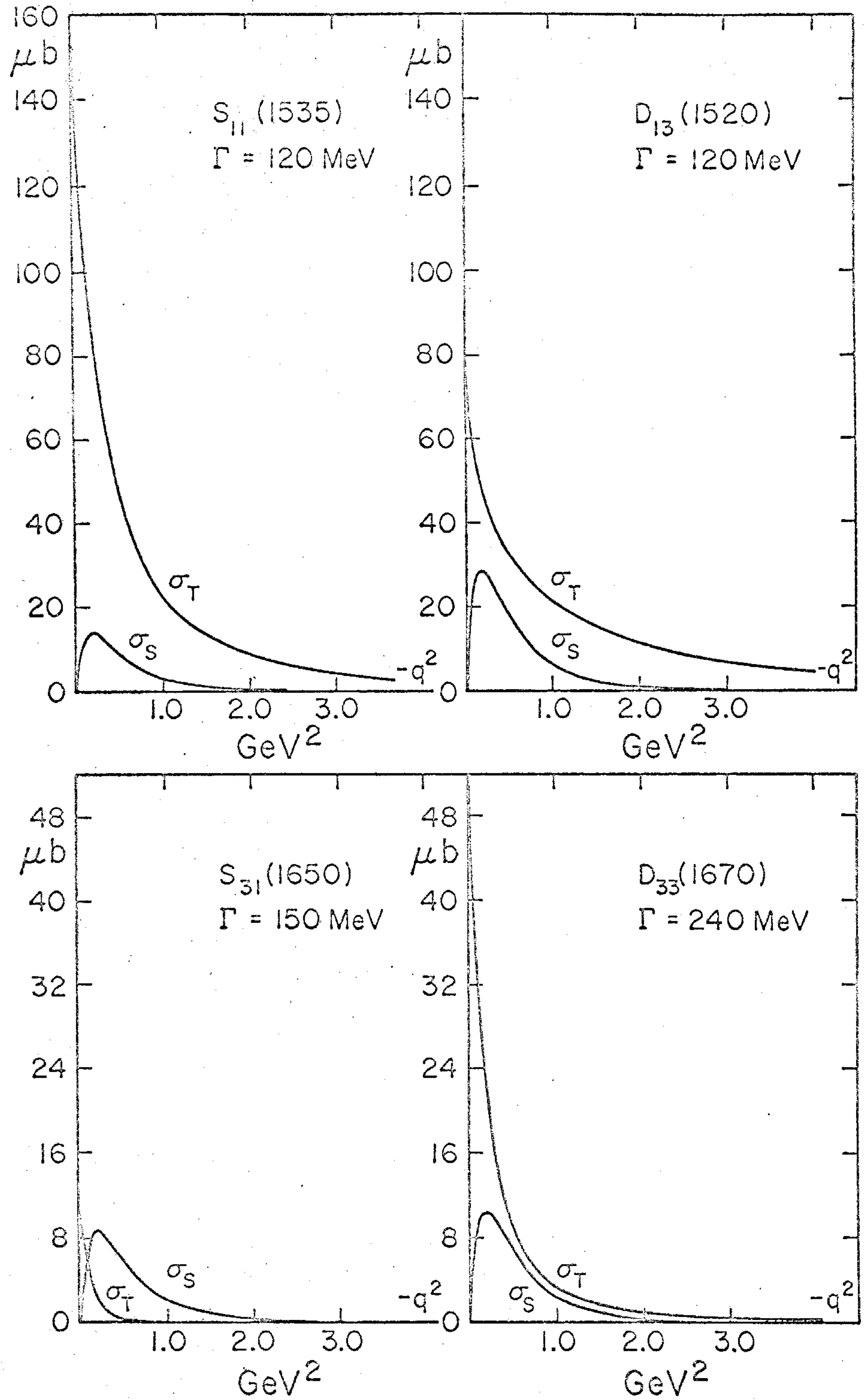

Figure 6. Transverse σ_T and scalar σ_S cross sections for the S_{11}(1535), D_{13}(1520), S_{31}(1650), and D_{33}(1670) resonances with proton target.

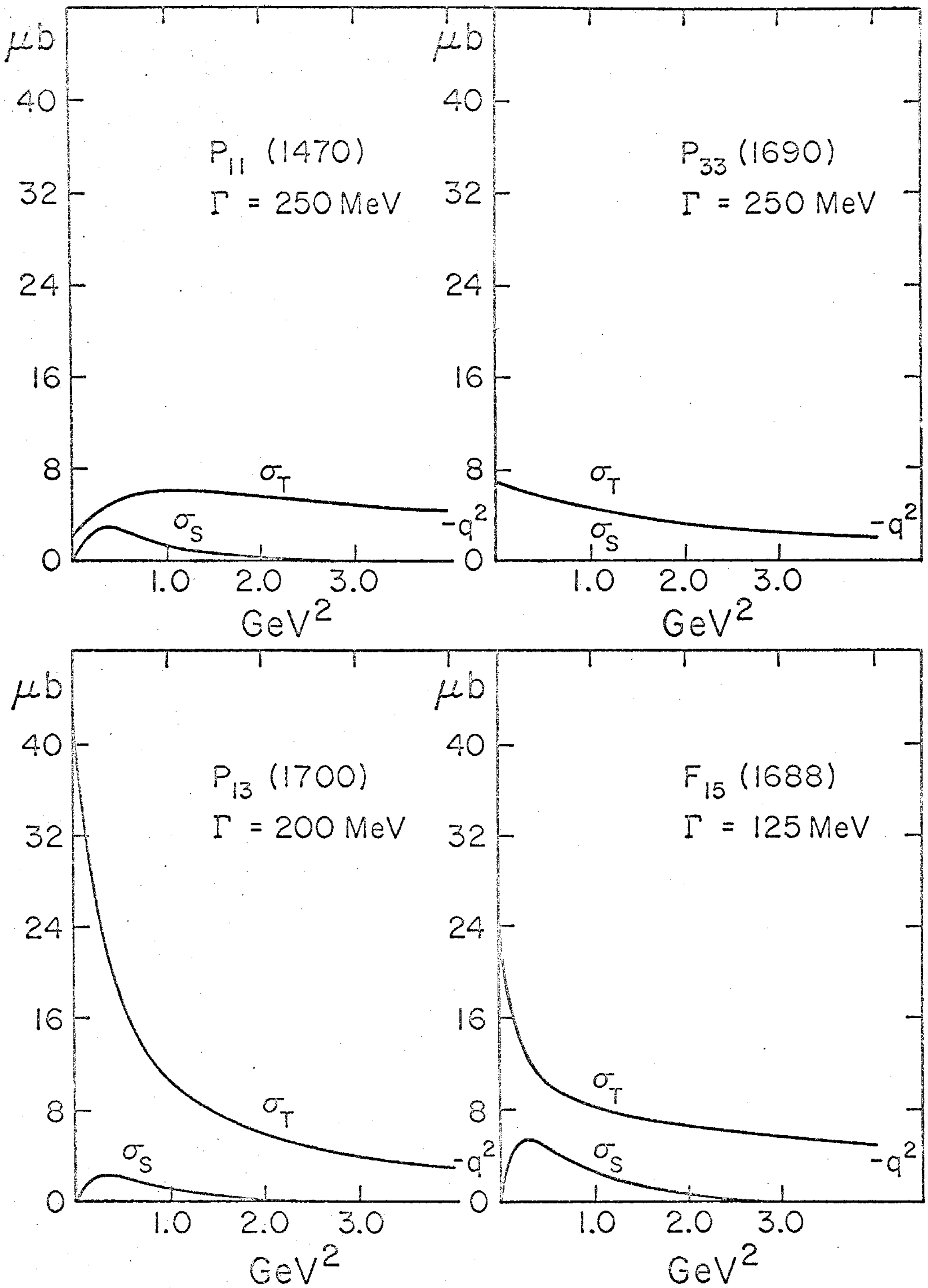

Figure 7. Transverse σ_T and scalar σ_S cross sections for the $P_{11}(1470)$, $P_{33}(1690)$, $P_{13}(1700)$, and $F_{15}(1688)$ resonances with proton target.

at the resonance peak W=M as a function of $-q^2$.

The resonances in Table 3 are grouped into three mass regions, ignoring the Roper resonance $P_{11}(1470)$ which has a small cross section as seen from Figure 7. We would expect a dominant peak in the inelastic cross section at the place of the $P_{33}(1236)$ resonance. The next bump should be found at W = 1525 MeV corresponding to the $S_{11}(1535)$ and D_{13} (1520). The remaining resonances, $S_{31}(1650)$, $D_{15}(1670)$, $D_{33}(1670)$, $F_{15}(1688)$, $P_{33}(1690)$, $S_{11}(1700)$, $D_{13}(1700)$, and $P_{13}(1700)$, are all grouped around W = 1680 MeV where also a peak in the cross section should be present. This agrees well with the experimental situation as seen from Figure 8, taken from Reference (18).

To make a quantitative comparison with experiment, we have in Figure 5 for the $P_{33}(1236)$ plotted the experimental values of $\Sigma = \sigma_T + \epsilon\sigma_S$ as evaluated by Clegg (19). The scalar cross section σ_S is zero for this resonance since it has quark spin S=3/2. For $-q^2 < 0.5$ GeV^2 we see that the theoretical values are almost 30% too small. This is in accordance with the same discrepancy we found for the photoproduction amplitudes in Table 4. For larger $-q^2$ the agreement with the experimental cross section is very good.

In Figure 9 we compare the values of Σ to the sum of the cross sections from $S_{11}(1535)$ and $D_{13}(1520)$. The agreement is good. At small $-q^2$ the largest contribution is coming from the $S_{11}(1535)$, contrary to what is usually assumed in the literature (19,20).

As already explained, the three resonances $S_{11}(1700)$, $D_{13}(1700)$ and $D_{15}(1670)$ of the $^4(\underset{\sim}{8})$ in the $[\underset{\sim}{70},1^-]$, will not contribute at the third peak for proton targets. Adding together the remaining resonances

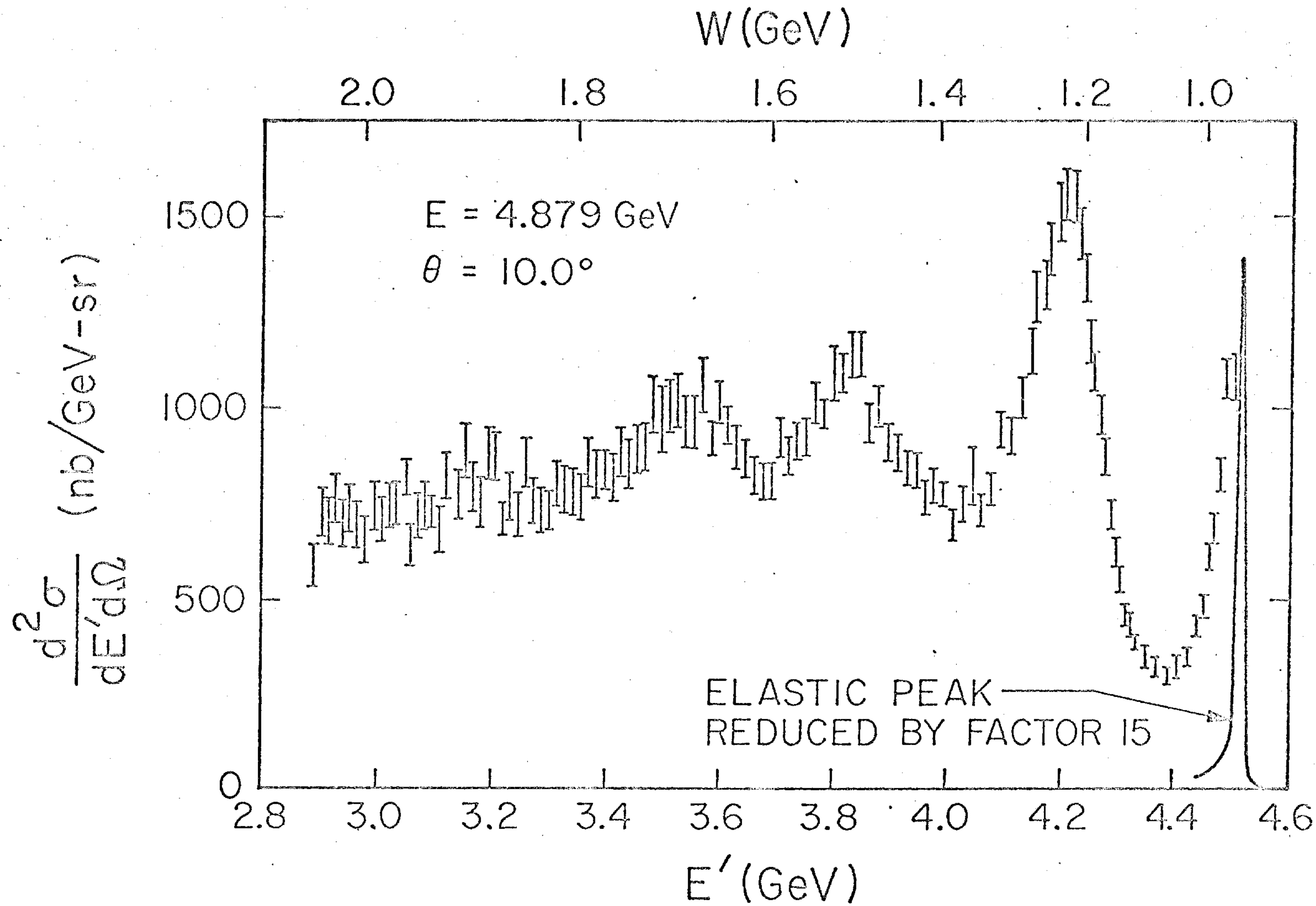

Figure 8. Inelastic electron proton cross sections at E = 4.879 GeV and $\theta = 10^{o}$ from reference 18.

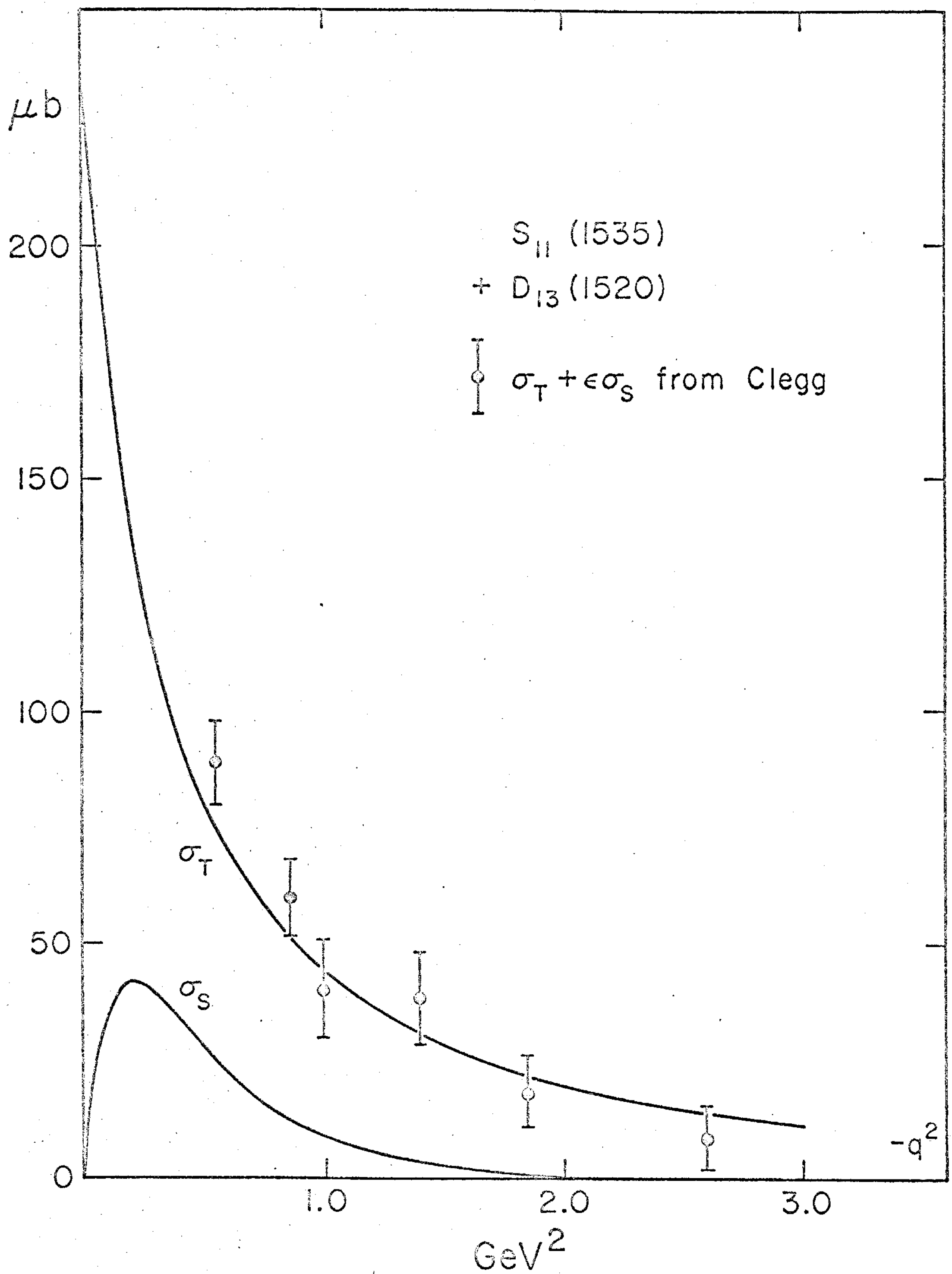

Figure 9. Total transverse σ_T and scalar σ_S cross sections at the second resonance peak W = 1525 MeV with proton target. Data from reference 19.

in this mass range as done in Figure 10, we note that the good agreement is obtained only when including the predicted resonance $P_{13}(1700)$. Without its contribution we would get a cross section approximately 30% too small, compared to the experimental data of Clegg (19).

It is generally believed that this third bump in the cross section is mainly $F_{15}(1688)$ (19,20). According to the present model, that is not the case, the $D_{33}(1670)$ being just as important. Had we tried to use the $F_{15}(1688)$ alone, we would be off by a factor of nearly 4 in the cross section. This may explain the apparent disagreement for this resonance we found for its photoproduction amplitude which experimentally is found by fits not including all the important resonances in the same mass range like the $D_{33}(1670)$ (21).

In other words, had the $F_{15}(1688)$ alone been able to explain the third peak, then this model would be in serious trouble. The same would obviously have been the case if we had obtained good agreement not taking into account the not-yet-observed $P_{13}(1700)$.

Another interesting result of this model is the small scalar cross section σ_S we find. The maximum of the ratio σ_S/σ_T at $-q^2 \simeq 0.5$ GeV^2 is 30% at the second peak and near 50% at the third peak. It is substantially smaller for all other values of $-q^2$. This agrees nicely with recent experimental results from DESY (22) where the same ratio is found to be around 20% in the resonance region.

The reason for this small scalar cross section is easily found by looking at (3.94) and (3.59). In addition to the required zero of σ_S at $q^2=0$, we see that $S=0$ for

$$-q^2 = m(3M-m) \tag{3.98}$$

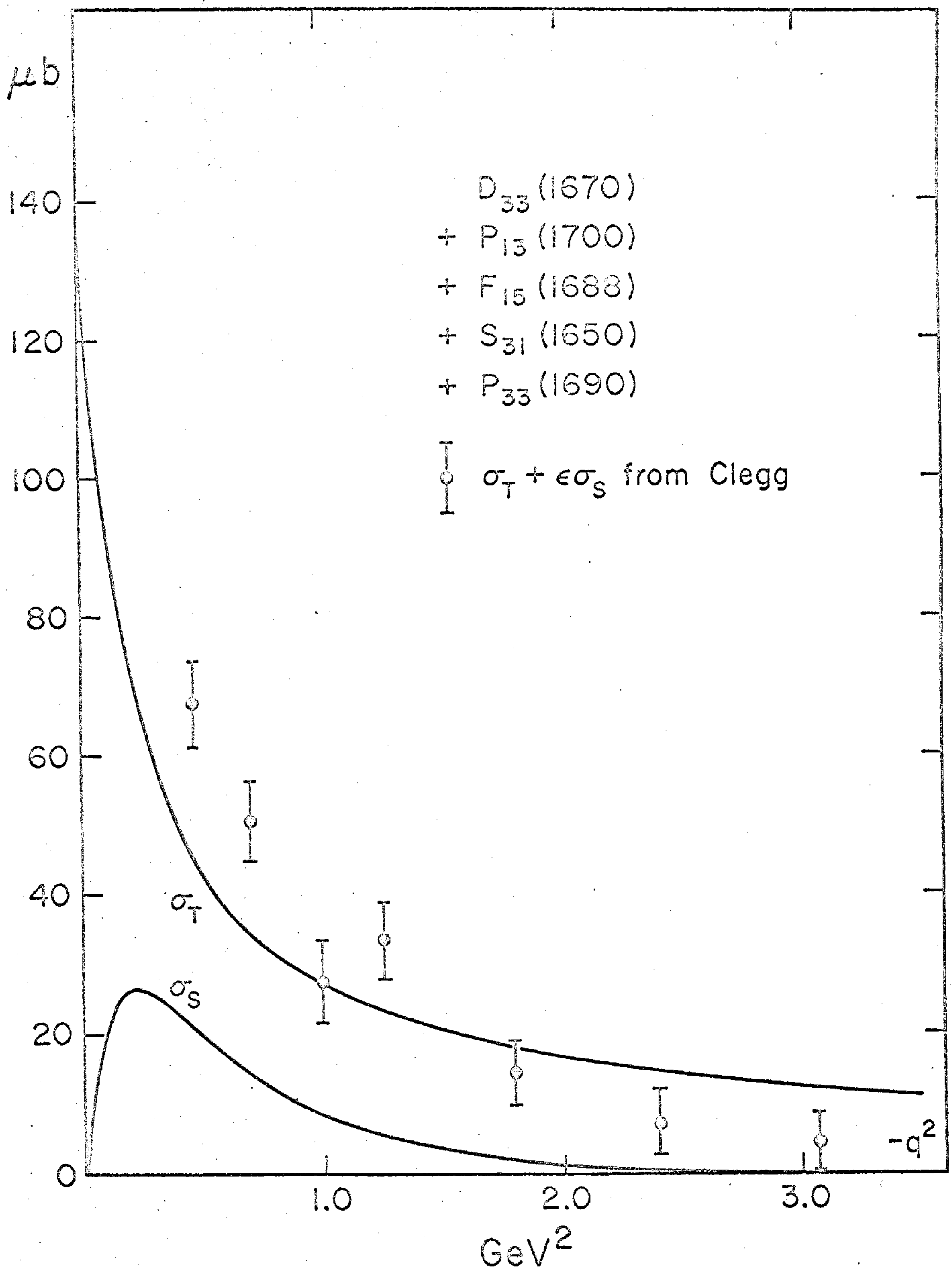

Figure 10. Total transverse σ_T and scalar σ_S cross sections at the third resonance peak W = 1680 MeV with proton target. Data from reference 19.

So σ_S will have a zero in the resonance region considered in the range $-q^2 = 3\text{-}4\ \text{GeV}^2$. This result is obviously independent of the arbitrary form factor we have used.

It should be noted that a similar non-relativistic calculation has been done by Thornber (23) who gets a large scalar cross section. But there is no reason to believe that a non-relativistic treatment of this highly relativistic process should be successful.

Meson Vector Current

The calculation of the vector current for mesons is done in the same way as for baryons. It is defined by the first order perturbational effect of a vector field:

$$\delta K^V = 2 \sum_{\alpha=a}^{b} e_\alpha \left[\not{p}_\alpha \not{e} e^{iq\cdot u_\alpha} + \not{e} e^{iq\cdot u_\alpha} \not{p}_\alpha \right] \qquad (3.99)$$

$$= J_\mu^V e_\mu$$

In the sum over α, the contribution from the a quark is the same as from the b anti-quark since we use symmetrical states and an anti-quark behaves like a quark with negative charges. Hence we only calculate the contribution from a and multiply this by 2 to get the full perturbation:

$$J_\mu^V e_\mu = 4 e_a e^{iq\cdot u_a} \left[2(p_a \cdot e) - \not{q}\not{e} \right] \qquad (3.100)$$

In this way we arrive at the meson vector current, analogous to the baryon vector current in (3.29):

$$J_\mu^V e_\mu = 4g^2 e^{q^2/2\Omega} e^{-\sqrt{\frac{1}{\Omega}} q\cdot c^+} e_a \Big\{ \Big[M - \frac{1}{2}\nu - \frac{Q^2}{2mg^2}$$

$$- \sqrt{\frac{\Omega}{4}} (c_t^+ + c_t) \Big] e_t + \sqrt{\frac{\Omega}{4}} (\underset{\sim}{c}^+ + \underset{\sim}{c}) \cdot \underset{\sim}{e} \tag{3.101}$$

$$+ \underset{\sim}{Q} \cdot \underset{\sim}{e} \left(\frac{1}{2} + \frac{\nu}{2mg^2}\right) + i \underset{\sim}{\sigma}_a \cdot (\underset{\sim}{Q} \times \underset{\sim}{e}) \left(1 + \frac{\nu}{2mg^2}\right) \Big\} e^{+\sqrt{\frac{1}{\Omega}} q\cdot c}$$

Here c and c^+ are the harmonic oscillator operators for the meson system, defined in (2.12).

Radiative Vector Meson Decays

We can test the vector current equation (101) in electromagnetic decays of vector mesons:

$$V \rightarrow P + \gamma \tag{3.102}$$

As in the same process for baryons we can calculate transition matrix elements due to the transverse parts of the photon:

$$B_\pm = e\langle P | J_\pm^{EM} | V \rangle = e 2M F_\pm \tag{3.103}$$

Only the last part of equation (3.101) will contribute to F_+ and F_- which can be written:

$$F_+ = 4GR \langle P(0) | e_a \sigma_{a-} | V(+1) \rangle \tag{3.104}$$

$$F_- = 4GR \langle P(0) | e_a \sigma_{a+} | V(-1) \rangle$$

where $$R = \sqrt{2}\, (M-m)/2M \tag{3.105}$$

We have here again introduced an empirical form factor G

$$g^2 e^{q^2/2\Omega} \rightarrow G(M,q^2) \tag{3.106}$$

and will choose this to be the same as for baryons, equation (3.71).

Using the meson wave functions in the Appendix, the matrix elements in (3.104) are easily calculated and the most important are listed in Table 5. We have only tabulated $|F_+|$ since $|F_+| = |F_-|$.

To find the corresponding transition rates we use the rate formula in (3.74), except for the extra factor of 2:

$$\begin{aligned}\Gamma &= \frac{1}{8\pi}\frac{Q}{M^2}\frac{1}{2J+1}\left(|B_+|^2 + |B_-|^2\right) \\ &= \frac{e^2}{4\pi}\frac{4}{3}|F_+|^2 Q\end{aligned} \tag{3.107}$$

Here we have set 2J+1 = 3 for the vector mesons.

Only the rate for $\omega \rightarrow \pi\gamma$ has been experimentally determined. In the Particle Tables (1) it is given as

$$\Gamma_{exp}(\omega \rightarrow \pi\gamma) = (1.1 \pm 0.1) \text{ MeV} \tag{3.108}$$

From equation (3.107) with F_+ from Table 5 we get the theoretical value:

$$\Gamma_{calc}(\omega \rightarrow \pi\gamma) = 1.9 \text{ MeV} \tag{3.109}$$

This value is somewhat too large compared to (3.108) but still within a factor 2 which we consider acceptable in this theory. The width for $\rho \rightarrow \pi\gamma$ should be equal to 1/9 of (3.109) as seen from Table 5.

We should mention that the original non-relativistic calculations of the same process by Becchi and Morpurgo (24) gave a much better

Table 5. Matrix elements for radiative decays of vector mesons.

$V \to P\gamma$	Q (MeV)	$\|F_+\|/GR$
$\omega \to \pi\gamma$	380	$\sqrt{2}$
$\omega \to \eta\gamma$	200	$1/9\sqrt{6}$
$\rho \to \pi\gamma$	370	$1/3\sqrt{2}$
$\rho \to \eta\gamma$	184	$1/3\sqrt{6}$
$K^{*+} \to K^+\gamma$	308	$1/3\sqrt{2}$
$K^{*o} \to K^o\gamma$	308	$2/3\sqrt{2}$
$\varphi \to \eta\gamma$	362	$4/9\sqrt{3}$

agreement with experiment. However, that was obtained by assuming a specific value of the quark magnetic moment in the mesons plus a dubious "relativistic" correction factor in their rate formula.

$K_{\ell 3}$ Decay

The vector current for mesons we have constructed can also be tested in the weak decay

$$K \rightarrow \pi + \ell + \nu \tag{3.110}$$

where ℓ is one of the leptons, e or μ.

According to Cabibbo's theory (25), the matrix element for this process can be written

$$B = \frac{G}{\sqrt{2}} \sin\theta \langle \pi | J_\mu^V | K \rangle \, \bar{u}_\ell \gamma_\mu (1 - \gamma_5) u_\nu \tag{3.111}$$

G is here the weak coupling constant and θ the Cabibbo angle. Only the vector current of the meson vertex will contribute since the π and K are both pseudoscalar particles. It will be a linear combination of the momentum k of the K and pion momentum p,

$$k = p + q \tag{3.112}$$

where q is the momentum of the $\ell\nu$-pair. The vector current can then be written in the form

$$\langle \pi | J_\mu^V | K \rangle = f_+(q^2)(k_\mu + p_\mu) + f_-(q^2)(k_\mu - p_\mu) \tag{3.113}$$

We will now find the form factors f_+ and f_- from the general expression (3.101). Again taking the space-part of q along the z-axis in the rest system of the K meson and considering the t- and z-components of the current, we get the two equations:

$$(f_+ + f_-)M_K + (f_+ - f_-)E_\pi \tag{3.114}$$

$$= 4G\langle\pi|e_a|K\rangle \left[M_K - \frac{1}{2}\nu - \frac{Q^2}{2m_\pi g^2}\right]$$

$$(f_+ - f_-) = 4G\langle\pi|e_a|K\rangle \left(\frac{1}{2} + \frac{\nu}{2m_\pi g^2}\right) \tag{3.115}$$

The matrix element $\langle\pi|e_a|K\rangle$ is easily found by the methods outlined in the Appendix:

$$\langle\pi|e_a|K\rangle = \frac{1}{2\sqrt{2}} \tag{3.116}$$

Then we rearrange the terms in equation (3.114),

$$2M_K f_+ - \nu(f_+ - f_-) = \sqrt{2}\, G\left[M_K - \frac{1}{2}\nu - \frac{Q^2}{2m_\pi g^2}\right], \tag{3.117}$$

substitute for $f_+ - f_-$ from equation (3.115) and solve for f_+:

$$2M_K f_+ = \sqrt{2}\, G\left[M_K - \frac{1}{2}\nu - \frac{Q^2}{2m_\pi g^2} + \frac{1}{2}\nu + \frac{\nu^2}{2m_\pi g^2}\right] \tag{3.118}$$

or:

$$f_+(q^2) = +\sqrt{\frac{1}{2}}\,\frac{(M_K+m_\pi)^2+q^2}{(M_K+m_\pi)^2-q^2}\, G(q^2) \tag{3.119}$$

The other form factor f_- is then easily found:

$$f_-(q^2) = -\sqrt{2}\,\frac{M_K{}^2-m_\pi{}^2}{(M_K+m_\pi)^2-q^2}\, G(q^2) \tag{3.120}$$

From equations (3.119) and (3.120) follows our first results, independent of the form factor G:

$$\xi = \frac{f_-(0)}{f_+(0)} = -2\,\frac{M_K - m_\pi}{M_K + m_\pi} = -1.12 \tag{3.121}$$

Polarization experiment (26) gives for the same ratio -0.94 ± 0.20 which is in accordance with our prediction.

For small q^2 we can write the form factors in the form

$$f_\pm(q^2) = f_\pm(0)\,(1 + \lambda_\pm \frac{q^2}{m_\pi^2}) \tag{3.122}$$

where
$$\lambda_+ = \frac{2m_\pi^2}{(M_K + m_\pi)^2} + \delta \tag{3.123}$$

$$= 0.094 + \delta$$

and
$$\lambda_- = \frac{m_\pi^2}{(M_K + m_\pi)^2} + \delta \tag{3.124}$$

$$= 0.047 + \delta$$

Here δ is the contribution to λ from the form factor G:

$$\delta = m_\pi^2 \left.\frac{dG}{dq^2}\right|_{q^2=0} \tag{3.125}$$

If we for G use equation (3.71), we find

$$\delta = 0.013 \tag{3.126}$$

and
$$\sqrt{2}\, f_+(0) = 0.96 \tag{3.127}$$

This last number agrees well with the experimental value (27)

$$\sqrt{2}\, f_+(0) = 0.94 \pm 0.05 \tag{3.128}$$

The value we find for λ_+ is apparently too large compared to experiment which seems to give (26) $\lambda_+ \simeq 0.04$. However, a larger value is not ruled out (28).

Using our results, we can predict the ratio between the $K_{\mu 3}$ and K_{e3} decay rates from the following formula in the Particle Tables (1),

$$\frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})} = 0.6457 + 0.1269\,\mathrm{Re}\xi + 0.0193\,|\xi|^2 + 1.390\lambda_+ + 0.476\lambda_-\mathrm{Re}\xi \tag{3.129}$$

which gives

$$\left.\frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})}\right|_{calc} = 0.650 \tag{3.130}$$

The same Particle Tables cites the experimental value

$$\left.\frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})}\right|_{exp} = 0.65 \pm 0.02 \tag{3.131}$$

This very good result finishes our investigation of $K_{\ell 3}$ decay. Our approach is very different from the standard treatment of this process (26). It gives results which are in good agreement with experiment and involves no free parameters except for the form factor G which has little influence on the results in this case.

IV. AXIAL VECTOR CURRENT MATRIX ELEMENTS

The axial vector current is obtained in the same way as the vector current. We define the perturbational effect from the presence of an axial vector field B_μ by the minimal coupling

$$\not{p}_a \rightarrow \not{p}_a - e_a\gamma_5\not{B} \qquad (4.1)$$

where the "axial charge" e_a of quark a really is the Gell-Mann matrix operator λ_a appropriate for the particular member of the axial meson nonet. This is explained in more detail in the Appendix.

Using equation (4.1) we find the first order perturbation in the dynamical operator K from quark a:

$$\delta K_a^A = e_a(\not{p}_a\gamma_5\not{B} + \gamma_5\not{B}\not{p}_a) \qquad (4.2)$$

To find the axial vector current matrix elements for baryons and mesons we proceed along the same lines as in Section III.

Baryon Axial Vector Current

In the case of baryons we get for the full first order perturbation by the axial field

$$\delta K^A = 3\sum_{\alpha=a}^{c} e_\alpha \left[\not{p}_\alpha\gamma_5\not{e}e^{iq\cdot u_\alpha} + \gamma_5\not{e}e^{iq\cdot u_\alpha}\not{p}_\alpha\right] \qquad (4.3)$$

where e_μ is the polarization vector of the field. This defines the axial vector current:

$$J_\mu^A = 3\sum_{\alpha=a}^{c} e_\alpha \left[\not{p}_\alpha\gamma_5\gamma_\mu e^{iq\cdot u_\alpha} + \gamma_5\gamma_\mu e^{iq\cdot u_\alpha}\not{p}_\alpha\right] \qquad (4.4)$$

Again we only calculate the contribution from quark a and multiply this by 3 to find the total perturbation:

$$J^A_\mu e_\mu = 9e_a \left[\not{p}_a \gamma_5 \not{e} e^{iq.u_a} + \gamma_5 \not{e} e^{iq\cdot u_a} \not{p}_a \right] \tag{4.5}$$

Using equation (3.9), this can be written

$$J^A_\mu e_\mu = 9e_a e^{iq\cdot u_a} \left[\gamma_5(\not{e}\not{p}_a - \not{p}_a\not{e}) + \gamma_5 \not{q}\not{e} \right]$$

$$= 9e_a e^{iq\cdot u_a} \left[2\gamma_5(p_a\cdot e) - 2\gamma_5 \not{p}_a \not{e} + \gamma_5 \not{q}\not{e} \right] \tag{4.6}$$

Substituting the two-component reduction of the Dirac matrix products

$$\gamma_5 \not{p}_a \not{e} = \gamma_5 \left[\varepsilon_a e_t - (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{p}_a)(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e}) \right] + \underset{\sim}{\sigma}_a \cdot (e_t \underset{\sim}{p}_a - \varepsilon_a \underset{\sim}{e})$$

$$\gamma_5 \not{q}\not{e} = \gamma_5 \left[\nu e_t - (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q})(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e}) \right] + \underset{\sim}{\sigma}_a \cdot (e_t \underset{\sim}{Q} - \nu \underset{\sim}{e}) \tag{4.7}$$

and sandwiching equation (4.6) between the initial and final spinor in equation (3.19), we get:

$$J^A_\mu e_\mu = 9g^3 e^{iq\cdot u_a} e_a \Bigg\{ \frac{\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}}{2mg^2} \Big[2(e_t \varepsilon_a - \underset{\sim}{e} \cdot \underset{\sim}{p}_a)$$

$$- 2(\varepsilon_a e_t - (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{p}_a)(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q})) + \nu e_t - (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q})(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e}) \Big] \tag{4.8}$$

$$- 2(e_t(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{p}_a) - \varepsilon_a(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e})) + e_t(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}) - \nu(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e}) \Bigg\}$$

Combining similar terms, this can be rewritten:

$$J^A_\mu e_\mu = 9g^3 e^{iq\cdot u_a} e_a \Bigg\{ (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q})(1 + \frac{\nu}{2mg^2}) e_t + \frac{(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q})}{2mg^2} \cdot$$

$$\Big[2i\underset{\sim}{\sigma}_a \cdot (\underset{\sim}{p}_a x \underset{\sim}{e}) - (\underset{\sim}{Q} \cdot \underset{\sim}{e}) - i\underset{\sim}{\sigma}_a \cdot (\underset{\sim}{Q} x \underset{\sim}{e}) \Big] + (2\varepsilon_a - \nu)(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{e})$$

$$- 2(\underset{\sim}{\sigma}_a \cdot \underset{\sim}{p}_a) e_t \Bigg\} \tag{4.9}$$

Reducing the product

$$(\underset{\sim}{\sigma}\cdot\underset{\sim}{Q})\left[-i\underset{\sim}{\sigma}\cdot(\underset{\sim}{Q}x\underset{\sim}{e})\right] = -i\underset{\sim}{Q}\cdot(\underset{\sim}{Q}x\underset{\sim}{e})+\underset{\sim}{\sigma}\cdot(\underset{\sim}{Q}x\underset{\sim}{Q}x\underset{\sim}{e})$$
$$= \underset{\sim}{\sigma}\cdot\left[\underset{\sim}{Q}(\underset{\sim}{Q}\cdot\underset{\sim}{e})-\underset{\sim}{e}Q^2\right]$$
$$= (\underset{\sim}{\sigma}\cdot\underset{\sim}{Q})(\underset{\sim}{Q}\cdot\underset{\sim}{e})-(\underset{\sim}{\sigma}\cdot\underset{\sim}{e})Q^2 \qquad (4.10)$$

and

$$(\underset{\sim}{\sigma}\cdot\underset{\sim}{Q})\left[\underset{\sim}{\sigma}\cdot(\underset{\sim}{p}_a x\underset{\sim}{e})\right] = \underset{\sim}{p}_a\cdot(\underset{\sim}{e}x\underset{\sim}{Q})+i\underset{\sim}{\sigma}\cdot(\underset{\sim}{Q}x\underset{\sim}{p}_a x\underset{\sim}{e})$$
$$= \underset{\sim}{p}_a\cdot(\underset{\sim}{e}x\underset{\sim}{Q})+i(\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_a)(\underset{\sim}{Q}\cdot\underset{\sim}{e})$$
$$- i(\underset{\sim}{\sigma}\cdot\underset{\sim}{e})(\underset{\sim}{Q}\cdot\underset{\sim}{p}_a) \qquad (4.11)$$

equation (4.9) can be simplified to:

$$J^A_\mu e_\mu = 9g^3 e^{iq\cdot u_a}\, e_a \left\{(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{Q})(1+\frac{\nu}{2mg^2})e_t\right.$$
$$+ \frac{1}{2mg^2}\left[2i\underset{\sim}{p}_a\cdot(\underset{\sim}{e}x\underset{\sim}{Q})-2(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{p}_a)(\underset{\sim}{Q}\cdot\underset{\sim}{e})+2(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{e})(\underset{\sim}{Q}\cdot\underset{\sim}{p}_a)\right]$$
$$\left.+ (2\epsilon_a-\nu-\frac{Q^2}{2mg^2})(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{e})-2(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{p}_a)e_t\right\} \qquad (4.12)$$

Here we insert equations (3.25), (3.26) and (3.28) and get the final expression for the axial vector current:

$$J^A_\mu e_\mu = 9g^3 e^{q^2/\Omega}\, e^{-\sqrt{2/\Omega}\,q.a^+}\, e_a \left\{(\underset{\sim}{\sigma}_a\cdot\underset{\sim}{Q})\left[\frac{1}{3}e_t+\frac{\nu e_t}{2mg^2}\right.\right.$$
$$\left.-\frac{2}{3}\frac{\underset{\sim}{Q}.\underset{\sim}{e}}{2mg^2}\right] + (\underset{\sim}{\sigma}_a\cdot\underset{\sim}{e})\left[\frac{2}{3}M-\frac{1}{3}\nu-\frac{1}{3}\frac{Q^2}{2mg^2}-\frac{2}{3}\sqrt{\frac{\Omega}{2}}\,(a_t+a_t^+)\right.$$
$$\left.-\frac{2}{3}\frac{Q.(a+a^+)}{2mg^2}\right]+\frac{2}{3}\sqrt{\frac{\Omega}{2}}\,\underset{\sim}{\sigma}_a\cdot(\underset{\sim}{a}+\underset{\sim}{a}^+)\left[e_t+\frac{\underset{\sim}{Q}.\underset{\sim}{e}}{2mg^2}\right]$$
$$\left.+ i\,\frac{2}{3}\sqrt{\frac{\Omega}{2}}\,(\underset{\sim}{a}+\underset{\sim}{a}^+)\cdot\frac{Qxe}{2mg^2}\right\} e^{+\sqrt{2/\Omega}\,q.a} \qquad (4.13)$$

The divergence of this current will be needed to calculate the rates of pseudoscalar meson emission from baryons. It is found by the substitution

$$e_\mu = (e_t, \underset{\sim}{e}) \rightarrow q_\mu = (\nu, \underset{\sim}{Q}) \qquad (4.14)$$

Doing this in equation (4.13), we get

$$J^A_\mu q_\mu = 9g^3 e^{q^2/\Omega} e^{-\sqrt{2/\Omega}\, q\cdot a^+} e_a \left\{ (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}) \left[\frac{1}{3}\nu + \frac{\nu^2}{2mg^2} - \frac{2}{3}\frac{Q^2}{2mg^2} \right.\right.$$

$$\left. + \frac{2}{3} M - \frac{1}{3}\nu - \frac{1}{3}\frac{Q^2}{2mg^2} \right] - \frac{2}{3}\sqrt{\frac{\Omega}{2}} (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}) \frac{\underset{\sim}{Q}\cdot(\underset{\sim}{a}+\underset{\sim}{a}^+)}{2mg^2}$$

$$\left. + \frac{2}{3}\sqrt{\frac{\Omega}{2}}\, \underset{\sim}{\sigma}_a \cdot (\underset{\sim}{a} + \underset{\sim}{a}^+)(\nu + \frac{Q^2}{2mg^2}) \right\} e^{+\sqrt{2/\Omega}\, q\cdot a} \qquad (4.15)$$

or:

$$J^A_\mu q_\mu = 6Mg^3 e^{q^2/\Omega} e^{-\sqrt{2/\Omega}\, q\cdot a^+} e_a \left\{ (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}) \cdot \right.$$

$$(1 + 3\frac{q^2}{4Mmg^2}) - \sqrt{\frac{\Omega}{2}} (\underset{\sim}{\sigma}_a \cdot \underset{\sim}{Q}) \frac{\underset{\sim}{Q}\cdot(\underset{\sim}{a}+\underset{\sim}{a}^+)}{2Mmg^2}$$

$$\left. + \sqrt{\frac{\Omega}{2}}\frac{M-m}{M}\, \underset{\sim}{\sigma}_a \cdot (\underset{\sim}{a} + \underset{\sim}{a}^+) \right\} e^{+\sqrt{2/\Omega}\, q\cdot a} \qquad (4.16)$$

We have here ignored the operators a_t and a_t^+ since they do not contribute because of the restriction (2.29).

Pseudoscalar Meson Emission from Baryons

We will now try to calculate the transition rates for processes like

$$B^* \rightarrow B + P \qquad (4.17)$$

where B^* is some baryon resonance and P a pseudoscalar meson like π, K or η. B is a ground state baryon with no excitations. The relativistic amplitude A for this decay is assumed to be proportional to the divergence of the axial current (4.16) with constant of proportionality equal f/m_π, where m_π is the pion mass and f some unknown coupling constant:

$$A = \frac{f}{m_\pi} \langle B | \overset{A}{J_\mu} q_\mu | B^* \rangle \tag{4.18}$$

If the emitted meson has mass squared equal $q^2 = \mu^2$ and moves along the z-axis in the B^* rest system, we get for the amplitude:

$$A = f 2MF \tag{4.19}$$

where:
$$F = 3\langle B(+1/2) | H_a | B^*(+1/2) \rangle \tag{4.20}$$

The operator H_a is essentially the divergence (4.16) which in this special frame can be written:

$$H_a = G \xi e_a \left\{ \lambda D \sigma_{az} \left[1 - \lambda X a_z \right] + B \underset{\sim}{\sigma}_a \cdot \underset{\sim}{a} \right\} e^{-\lambda a_z} \tag{4.21}$$

Here we have introduced the quantities

$$\lambda = \sqrt{\frac{2}{\Omega}}\, Q$$

$$\xi = \sqrt{\frac{\Omega}{2}}\, \frac{1}{m_\pi} = 5.25$$

$$D = \frac{(M+m)^2 + 2\mu^2}{(M+m)^2 - \mu^2} \tag{4.22}$$

$$B = (M-m)/M$$

$$X = \frac{\Omega}{(M+m)^2 + 2\mu^2}$$

and again substituted an empirical form factor equal to the one previously used:

$$g^3 e^{q^2/\Omega} \rightarrow G = \exp\left(-\frac{Q^2}{\Omega}\frac{M^2}{M^2+m^2}\right) \tag{4.23}$$

For calculation purposes we must in (4.21) write out the product

$$\underset{\sim}{\sigma}_a \cdot \underset{\sim}{a} = \sigma_{az} a_z - \sqrt{2}\,\sigma_{a_+} a_+ + \sqrt{2}\,\sigma_{a_-} a_- \tag{4.24}$$

The minus in front of a_+ stems from the phase convention we use as explained in the Appendix.

The decay rates for the process (4.17) is now given by the ordinary, relativistic formula (3.74):

$$\begin{aligned}\Gamma &= \frac{1}{8\pi}\frac{Q}{M^2}\frac{2R}{2J+1}|A|^2 \\ &= \frac{f^2}{4\pi}\frac{4R}{2J+1}|F|^2 Q\end{aligned} \tag{4.25}$$

Here the extra factor 2 in 2R comes from summing over the two final states with $J_z = +1/2$ and $-1/2$. R is the inverse of the squared Clebsch-Gordan coefficient connecting the initial isospin state with the isospin of the final state. For example, if we calculate the amplitude for the decay of an excited proton into a proton and π^o, then $R = (\sqrt{3})^2 = 3$. J is the total angular momentum of the B^*.

We can determine the constant f by calculating the coupling of π^o to a ground state proton in our theory. Using our expression for the amplitude A and Table C in the Appendix, we find:

$$A(N^+N^+\pi^o) = f2M\frac{5}{3}\xi\lambda D$$

$$= \frac{f}{m_\pi} 2M \frac{5}{3} Q \tag{4.26}$$

$$= \frac{f_{NN\pi}}{m_\pi} \bar{u}_2\gamma_5 \not{q} u_1 = \frac{f_{NN\pi}}{m_\pi} 2MQ$$

The number 5/3 comes from F+D which is the SU(3) Clebsch-Gordan coefficient for the coupling of π^o to proton. In (4.26) the value of $f_{NN\pi}$ is known so we can find the coupling constant f:

$$f = \frac{3}{5} f_{NN\pi} \tag{4.27}$$

or $$\frac{f^2}{4\pi} = \frac{9}{25}\frac{f^2_{NN\pi}}{4\pi} = \frac{9}{25}\cdot 0.08 = 0.029 \tag{4.28}$$

This is the value we will use in the numerical calculations of all the decay rates.

All the necessary details concerning the evaluation of the transition amplitudes are given in the Appendix. In Table 6 we list one representative amplitude for each baryon multiplet, the $N^+ \to N^+\pi^o$ in octets, $\Delta^+ \to N^+\pi^o$ in decimets and $\Lambda \to \Sigma^+\pi^-$ in singlets. Then using the SU(3) coefficients in Table A and B in the Appendix, all the other transition amplitudes in each multiplet can be found. We have also calculated all the important decay rates which are listed in Table 6 together with their experimental values.

In some cases in the table when the theoretical result is zero because the numerical coefficient in front of the amplitude is zero for some special value of (F,D), we give the answer as O(X) where X

Table 6. Amplitudes and transition rates for pseudoscalar meson emission from baryons.

$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	F/G	Resonance	Mode	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$[\underset{\sim}{56}, 0^+]$	$^4(\underset{\sim}{10})_{3/2}$	$+\frac{4}{3}\xi\lambda D$	$\Delta(1236)$	$N\pi$	94	120
			$\Sigma(1385)$	$\Lambda\pi$	35	32
				$\Sigma\pi$	4	4
			$\Xi(1530)$	$\Xi\pi$	12	7
$[\underset{\sim}{70}, 1^-]$	$^2(\underset{\sim}{1})_{1/2}$	$+\frac{1}{2}\sqrt{2}\xi\left[\lambda^2 D(1+X)-3B\right]$	$\Lambda(1405)$	$\Sigma\pi$	56	40
	$^2(\underset{\sim}{1})_{3/2}$	$-\xi\left[\lambda^2 D(1+X)\right]$	$\Lambda(1520)$	$N\bar{K}$	7	7
				$\Sigma\pi$	12	7
	$^2(\underset{\sim}{8})_{1/2}$	$+\frac{2}{9}\sqrt{6}\xi\left[\lambda^2 D(1+X)-3B\right]$	$N(1535)$	$N\pi$	220	40
				$N\eta$	71	77
			$\Lambda(1670)$	$\Sigma\pi$	22	11
				$\Lambda\eta$	6	8
				$N\bar{K}$	415	5
	$^2(\underset{\sim}{8})_{3/2}$	$-\frac{4}{9}\sqrt{3}\xi\left[\lambda^2 D(1+X)\right]$	$N(1520)$	$N\pi$	105	60
				$N\eta$	0.2	~0.7
			$\Sigma(1670)$	$N\bar{K}$	3	
				$\Lambda\pi$	6	
				$\Sigma\pi$	49	
			$\Lambda(1690)$	$N\bar{K}$	102	16
				$\Sigma\pi$	11	21

Table 6. Continued.

$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	F/G	Resonance	Mode	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$[\underset{\sim}{70}, 1^-]$	$^2(8)_{3/2}$		$\Xi(1820)$	$\Lambda\bar{K}$	15	
				$\Xi\pi$	4	
				$\Sigma\bar{K}$	17	
	$^4(\underset{\sim}{8})_{1/2}$	$+\frac{1}{9}\sqrt{6}\xi[\lambda^2 D(1+X)-3B]$	N(1700)	$N\pi$	45	182
				$N\eta$	112	
				ΛK	0(74)	13
	$^4(\underset{\sim}{8})_{3/2}$	$\frac{-1}{45}\sqrt{30}\ \xi[\lambda^2 D(1+X)]$	N( )	$N\pi$		
	$^4(\underset{\sim}{8})_{5/2}$	$\frac{-1}{15}\sqrt{30}\ \xi[\lambda^2 D(1+X)]$	N(1670)	$N\pi$	36	60
				$N\eta$	7	< 1
				ΛK	0(0.2)	< 0.1
$[\underset{\sim}{70}, 1^-]$			$\Sigma(1765)$	$N\bar{K}$	66	53
				$\Lambda\pi$	25	17
				$\Sigma\pi$	10	~ 1
			$\Lambda(1830)$	$N\bar{K}$	0(48)	11
				$\Sigma\pi$	73	33
			$\Xi(1930)$	$\Xi\pi$	83	
				$\Lambda\bar{K}$	24	

Table 6. Continued.

$SU(6)\otimes O(3)$	${}^{2S+1}(SU(3))_J$	F/G	Resonance	Mode	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$[\underset{\sim}{70}, 1^-]$	${}^2(\underset{\sim}{10})_{1/2}$	$+\frac{1}{9}\sqrt{6}\,\xi\left[\lambda^2 D(1+X)-3B\right]$	$\Delta(1650)$	$N\pi$	25	41
			$\Sigma(1750)$	$N\bar{K}$	14	~10
				$\Lambda\pi$	9	
				$\Sigma\eta$	4	
	${}^2(\underset{\sim}{10})_{3/2}$	$-\frac{2}{9}\sqrt{3}\;\xi\left[\lambda^2 D(1+X)\right]$	$\Delta(1670)$	$N\pi$	30	31
$[\underset{\sim}{56}, 0^+]$	${}^2(\underset{\sim}{8})_{1/2}$	$-\frac{5}{18}\sqrt{3}\;\xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	N(1470)	$N\pi$	8	150
	${}^4(\underset{\sim}{10})_{3/2}$	$-\frac{2}{9}\sqrt{6}\;\xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	$\Delta($ $)$	$N\pi$		
$[\underset{\sim}{56}, 2^+]$	${}^2(\underset{\sim}{8})_{3/2}$	$+\frac{1}{9}\sqrt{15}\;\xi\left[\lambda^2 D(1+2X)-5B\right]\lambda$	N( )	$N\pi$		
	${}^2(\underset{\sim}{8})_{5/2}$	$+\frac{1}{6}\sqrt{10}\;\xi\left[\lambda^2 D(1+2X)\right]\lambda$	N(1688)	$N\pi$	64	75
				ΛK	0.07	< 0.1
				$N\eta$	0.27	< 0.6
			$\Lambda(1815)$	$N\bar{K}$	35	53
				$\Sigma\pi$	13	9
	${}^4(\underset{\sim}{10})_{1/2}$	$+\frac{4}{45}\sqrt{15}\;\xi\left[\lambda^2 D(1+2X)-5B\right]\lambda$	$\Delta(1910)$	$N\pi$	151	82

Table 6. Continued

SU(6) ⊗ 0(3)	$^{2S+1}(SU(3))_J$	F/G	Resonance	Mode	Γ_{calc}(MeV)	Γ_{exp}(MeV)
	$^4(\underset{\sim}{10})_{3/2}$	$-\frac{4}{45}\sqrt{15}\ \xi\left[\lambda^2 D(1+2X)-5B\right]\lambda$	Δ( )	Nπ		
	$^4(\underset{\sim}{10})_{5/2}$	$-\frac{4}{105}\sqrt{35}\ \xi\left[\lambda^2 D(1+2X)\right]\lambda$	Δ(1890)	Nπ	18	47
$[\underset{\sim}{56}, 2^+]$	$^4(\underset{\sim}{10})_{7/2}$	$+\frac{4}{105}\sqrt{210}\ \xi\left[\lambda^2 D(1+2X)\right]\lambda$	Δ(1950)	Nπ	103	90
				ΣK	7	5
			Σ(2030)	$N\bar{K}$	28	27
				Λπ	37	35
				Σπ	17	3
				ΞK	1	< 2
	$^2(\underset{\sim}{1})_{1/2}$	$+\frac{1}{4}\sqrt{2}\ \xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	Λ( )	Σπ		
	$^2(\underset{\sim}{8})_{1/2}$	$+\frac{1}{9}\sqrt{6}\ \xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	N(1780)	Nπ	0.5	120
$[\underset{\sim}{70}, 0^+]$	$^4(\underset{\sim}{8})_{3/2}$	$+\frac{1}{18}\sqrt{6}\ \xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	N( )	Nπ		
	$^2(\underset{\sim}{10})_{1/2}$	$+\frac{1}{18}\sqrt{6}\ \xi\left[\lambda^2 D(1+2X)-2B\right]\lambda$	Δ( )	Nπ		
$[\underset{\sim}{70}, 2^+]$	$^2(\underset{\sim}{1})_{3/2}$	$+\frac{1}{10}\sqrt{10}\ \xi\left[\lambda^2 D(1+2X)-5B\right]\lambda$	Λ( )	Σπ		

Table 6. Continued.

$SU(6) \otimes O(3)$	$^{2S+1}(SU(3))_J$	F/G	Resonance	Mode	Γ_{calc} (MeV)	Γ_{exp} (MeV)
	$^2(\underset{\sim}{1})_{5/2}$	$-\frac{1}{10}\sqrt{15}\ \xi\left[\lambda\, ^2D(1+2X)\right]\lambda$	Λ( )	$\Sigma\pi$		
	$^2(\underset{\sim}{8})_{3/2}$	$+\frac{2}{45}\sqrt{30}\ \xi\left[\lambda\, ^2D(1+2X)-5B\right]\lambda$	N(1860)	$N\pi$ ΛK	75 22	90 < 53
	$^2(\underset{\sim}{8})_{5/2}$	$+\frac{2}{15}\sqrt{5}\ \xi\left[\lambda\, ^2D(1+2X)\right]\lambda$	N( )	$N\pi$		
$\left[\underset{\sim}{70},\ 2^+\right]$	$^4(\underset{\sim}{8})_{1/2}$	$-\frac{1}{45}\sqrt{15}\ \xi\left[\lambda\, ^2D(1+2X)-5B\right]\lambda$	N( )	$N\pi$		
	$^4(\underset{\sim}{8})_{3/2}$	$+\frac{1}{45}\sqrt{15}\ \xi\left[\lambda\, ^2D(1+2X)-5B\right]\lambda$	N( )	$N\pi$		
	$^4(\underset{\sim}{8})_{5/2}$	$+\frac{1}{105}\sqrt{35}\ \xi\left[\lambda\, ^2D(1+2X)\right]\lambda$	N( )	$N\pi$		
	$^4(\underset{\sim}{8})_{7/2}$	$-\frac{1}{105}\sqrt{210}\ \xi\left[\lambda\, ^2D(1+2X)\right]\lambda$	N(1990)	$N\pi$	15	26
	$^2(\underset{\sim}{10})_{3/2}$	$+\frac{1}{45}\sqrt{30}\ \xi\left[\lambda\, ^2D(1+2X)-5B\right]\lambda$	Δ( )	$N\pi$		
	$^2(\underset{\sim}{10})_{5/2}$	$-\frac{1}{15}\sqrt{5}\ \xi\left[\lambda\, ^2D(1+2X)\right]\lambda$	Δ( )	$N\pi$		

is the value one would get for the rate if the coefficient were unity. Working backward one can then see how small the data indicate that the coefficient in fact is.

When we now compare the calculated rates with their experimental values, we should keep in mind that the amplitudes have numerical coefficients which are given by exact SU(3) symmetry within each multiplet. As can be seen from the decay $\Xi(1530) \rightarrow \Xi\pi$ in the ground state decimet, SU(3) symmetry is broken in this case and probably in many other decays where we find deviation from experiment of this order of magnitude, 20-30%.

Inspection of the results in Table 6 reveals that the best agreement between theory and experiment is found among the high-J states of the SU(6) $\otimes$ 0(3) multiplets. In these cases the B-term does not contribute and the amplitude contains only one term. Among these decays there are 32 rates which agree with experiment within a factor 2.5 and 3 rates which are off by a larger factor. These three are $\Lambda(1690) \rightarrow N\bar{K}$ of the $^2(\underset{\sim}{8})_{3/2}$, $N(1670) \rightarrow N\eta$ of the $^4(\underset{\sim}{8})_{5/2}$, both in the $[\underset{\sim}{70}, 1^-]$, and $\Sigma(2030) \rightarrow \Sigma\pi$ of the $^4(\underset{\sim}{10})_{7/2}$ in the $[\underset{\sim}{56}, 2^+]$. Since all three resonances belong to multiplets where their partners have rates in good agreement, we expect that these three discrepancies are caused by SU(3) breaking or more probably, that the experimental values are wrong.

Amplitudes for decays of low-spin resonances contain the additional B-term which only depends on the masses of the initial and final baryon, and not on Q^2 as the ordinary term. Hence, in reactions with low Q like $\Lambda(1405) \rightarrow \Sigma\pi$ or $N(1535) \rightarrow N\eta$, the B-term makes it possible to get the large, observed decay rates. That is very welcome. But in other cases it has disastrous consequences. For instance, in $\Lambda(1670) \rightarrow N\bar{K}$ the B-term is so large relative to the ordinary term that it gives a width of 415 MeV compared to the measured width of 5 MeV. On the other hand, in the decay $N(1780) \rightarrow N\pi$, the two terms almost exactly cancel each other to give a much too small rate.

This may be understood if we consider the accuracy of each term separately. In the 32 cases where we had no B-term and good agreement, the ordinary term was within a factor 1.6 of the experimental value. We then expect the B-term to be good within the same factor of 1.6. In this way the small experimental width for $\Lambda(1670) \rightarrow N\bar{K}$ could be obtained if the ordinary term was increased by a factor 1.6 and the B-term similarly reduced to give near cancellation between the two terms. The same procedure would remove the theoretical cancellation between the two terms in the amplitude for $N(1780) \rightarrow N\pi$ and thereby give a large rate as observed.

The $\Sigma(1910)$ is not included in Table 6 because it does not seem to fit into any known multiplet, as discussed in Section II. If we tried to assign it to the $^2(\underset{\sim}{8})_{5/2}$ in the $[\underset{\sim}{56}, 2^+]$, we find two of its three decay rates to be in serious disagreement with experiment. This is an additional reason not to have this state as the recurrence of the ground state $\Sigma(1192)$. Putting it into the $^4(\underset{\sim}{10})_{5/2}$ of the $[\underset{\sim}{56}, 2^+]$ along with the $\Delta(1890)$, we get better decay rates, but its mass is apparently too low to make this assignment plausible. Probably it belongs in the $[\underset{\sim}{70}, 0^+]$ or $[\underset{\sim}{70}, 2^+]$ somewhere.

The decay rates for the N(1860) and N(1990) which we in Section II concluded were good candidates for the $[\underset{\sim}{70}, 2^+]$, come out in accordance with experiment. This strengthens our belief that these assignments are correct.

Pseudoscalar Meson Emission from Mesons

To calculate the decay rates for processes like

$$M^{*} \rightarrow P+P \tag{4.29}$$

or

$$M^{*} \rightarrow V+P \tag{4.30}$$

where M^{*} is some meson resonance, V a vector meson and P a pseudoscalar meson, we represent one of the pseudoscalar mesons in (4.29) and P in (4.30) by the divergence of the axial vector current for mesons. This one is found in the same way as the divergence of the axial vector current for baryons was found. The result, analogous to equation (4.16), is:

$$\begin{aligned} J^{A}_{\mu}q_{\mu} = \; & 4M\, g^{2} e^{q^{2}/2\Omega}\, e^{-\sqrt{1/\Omega}\, q\cdot c^{+}}\, e_{a} \Big\{ (\underset{\sim}{\sigma}_{a}\cdot\underset{\sim}{Q}) \cdot \\ & (1 + 2\frac{q^{2}}{4Mmg^{2}}) - \sqrt{\frac{\Omega}{4}}\, (\underset{\sim}{\sigma}_{a}\cdot\underset{\sim}{Q})\, \frac{\underset{\sim}{Q}\cdot(\underset{\sim}{c}+\underset{\sim}{c}^{+})}{2Mmg^{2}} \\ & + \sqrt{\frac{\Omega}{4}}\, \frac{M-m}{M}\, \underset{\sim}{\sigma}_{a}\cdot(\underset{\sim}{c}+\underset{\sim}{c}^{+}) \Big\}\, e^{+\sqrt{1/\Omega}\, q\cdot c} \end{aligned} \tag{4.31}$$

Again taking the momentum $\underset{\sim}{Q}$ of the pseudoscalar meson under consideration along the z-axis in the M^{*} rest system, we can define an interaction operator H_a which is the divergence (4.31) except for some factors in front,

$$H_{a} = G\zeta e_{a} \Big\{ \rho d\sigma_{az}\big[1 - \rho x c_{z}\big] + b\underset{\sim}{\sigma}_{a}\cdot\underset{\sim}{c} \Big\}\, e^{-\rho c_{z}} \tag{4.32}$$

where
$$\rho = \sqrt{\frac{1}{\Omega}}\, Q$$

$$\zeta = \frac{\sqrt{\Omega}}{m_\pi} = 7.45$$

$$d = \frac{(M+m)^2+\mu^2}{(M+m)^2-\mu^2} \qquad (4.33)$$

$$b = (M-m)/2M$$

$$x = \frac{\Omega}{(M+m)^2+\mu^2}$$

Here μ is the mass of the emitted pseudoscalar meson, m_π the pion mass and G the empirical form factor

$$g^2 e^{q^2/2\Omega} \rightarrow G = \exp\left[-\frac{Q^2}{\Omega}\frac{M^2}{M^2+m^2}\right] \qquad (4.34)$$

The relativistic amplitude for the process (4.29) will then be

$$A_o = f2MF_o \qquad (4.35)$$

where F_o is twice the matrix element of H_a:

$$F_o = 2\langle P(0)|H_a|M^*(0)\rangle \qquad (4.36)$$

The rate of the decay (4.29) is then

$$\Gamma(M^* \rightarrow PP) = \frac{1}{8\pi}\frac{Q}{M^2}\frac{R}{2J+1}|A_o|^2$$
$$= \frac{f^2}{4\pi}\frac{2R}{2J+1}|F_o|^2 Q \qquad (4.37)$$

J is the spin of the meson resonance M^* and R has the same meaning here as in the baryon case.

For the other process (4.30) there are two independent amplitudes:

$$\begin{aligned} A_o &= f2MF_o \\ A_+ &= f2MF_+ \end{aligned} \tag{4.38}$$

The matrix elements F_o and F_+ correspond to decays of the M^* with $J_z = 0$ and $J_z = +1$:

$$\begin{aligned} F_o &= 2\langle V(0)|H_a|M^*(0)\rangle \\ F_+ &= 2\langle V(+1)|H_a|M^*(+1)\rangle \end{aligned} \tag{4.39}$$

There is also an amplitude F_- for decay from $J_z = -1$, but since $|F_+| = |F_-|$, the rate formula can be written:

$$\begin{aligned} \Gamma(M^* \to VP) &= \frac{1}{8\pi}\frac{Q}{M^2}\frac{R}{2J+1}\left(|A_o|^2 + 2|A_+|^2\right) \\ &= \frac{f^2}{4\pi}\frac{2R}{2J+1}\left(|F_o|^2 + 2|F_+|^2\right) \end{aligned} \tag{4.40}$$

We are now in the position to calculate the rates for all meson decays of the type (4.29) or (4.30). For the coupling constant f we will use the same value as for baryons, equation (4.28). In Table 7 we list the results for the L=0 and L=1 mesons where we have calculated one representative amplitude for each multiplet. The amplitudes for other decays inside the same multiplet are then given by Gell-Mann's SU(3) coefficients f_{ijk} or d_{ijk} (29) because all these couplings are pure F or D depending on the charge conjugation of the initial and final meson state.

Table 7. Amplitudes and transition rates for pseudoscalar meson emission from mesons.

$[\underset{\sim}{36}, 0^-]^3(\underset{\sim}{9})_1$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$\rho(765)$	$\pi^+\pi^o$	$2\ \zeta\rho d$		157	125
$\varphi(1019)$	K^+K^-	$\sqrt{2}\ \zeta\rho d$		10	3.2
$K^*(892)$	$K^+\pi^o$	$1\ \zeta\rho d$		66	51
	π^oK^+	$1\ \zeta\rho d$		159	51
$[\underset{\sim}{36}, 1^+]^1(\underset{\sim}{9})_1$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
B(1235)	$\omega^o\pi^+$	$-2\ \zeta\left[\rho^2 d(1+x)-b\right]$	$+2\ \zeta b$	85	102
$[\underset{\sim}{36}, 1^+]^3(\underset{\sim}{9})_0$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$\delta(966)$	$\eta^o\pi^+$	$+\frac{2}{3}\left[\rho^2 d(1+x)-3b\right]$			
$[\underset{\sim}{36}, 1^+]^3(\underset{\sim}{9})_1$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$A_1(1070)$	$\rho^+\pi^o$	$+2\sqrt{2}\ \zeta b$	$-\sqrt{2}\ \zeta\left[\rho^2 d(1+x)-2b\right]$	161	95
$[\underset{\sim}{36}, 1^+]^3(\underset{\sim}{9})_2$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
$A_2(1300)$	$\rho^+\pi^o$		$\sqrt{2}\ \zeta\left[\rho^2 d(1+x)\right]$	66	64
	$\eta\ \pi$	$\frac{2}{3}\sqrt{2}\ \zeta\left[\rho^2 d(1+x)\right]$		22	16
	$\pi\ \eta$	$\frac{2}{3}\sqrt{2}\ \zeta\left[\rho^2 d(1+x)\right]$		45	16
	K^+K^-	$\frac{1}{3}\sqrt{6}\ \zeta\left[\rho^2 d(1+x)\right]$		17	10

Table 7. Continued

$[36,1^+]\ ^3(9)_2$	Mode	F_o/G	F_+/G	Γ_{calc}(MeV)	Γ_{exp}(MeV)
f(1260)	$\pi^+\pi^-$	$\frac{2}{3}\sqrt{6}\zeta\left[\rho^2 d(1+x)\right]$		244	145
	K^+K^-	$\frac{1}{3}\sqrt{6}\zeta\left[\rho^2 d(1+x)\right]$		13	~5
f'(1514)	K^+K^-	$\frac{2}{3}\sqrt{3}\zeta\left[\rho^2 d(1+x)\right]$		103	52
	$\bar{K}K^*$ $+K\bar{K}^*$		$\sqrt{2}\ \zeta\left[\rho^2 d(1+x)\right]$	15	7
$K^*(1420)$	$K^{*+}\pi^o$		$\frac{1}{2}\sqrt{2}\zeta\left[\rho^2 d(1+x)\right]$	22	34
	$\rho^o K^+$		$\frac{1}{2}\sqrt{2}\zeta\left[\rho^2 d(1+x)\right]$	8	8
	ω K		$\frac{1}{2}\sqrt{2}\zeta\left[\rho^2 d(1+x)\right]$	2	4
	$K^+\pi^o$	$\frac{1}{3}\sqrt{6}\zeta\left[\rho^2 d(1+x)\right]$		87	48
	$\pi^o K^+$	$\frac{1}{3}\sqrt{6}\zeta\left[\rho^2 d(1+x)\right]$		140	48
	K η	$\frac{1}{3}\sqrt{2}\zeta\left[\rho^2 d(1+x)\right]$		5	~2
	η K	$\frac{1}{3}\sqrt{2}\zeta\left[\rho^2 d(1+x)\right]$		4	~2

In the decay (4.29) we can replace only one of the two pseudoscalar mesons by the divergence of the axial vector current, the other pseudoscalar meson being a quark-antiquark system. When the two mesons have different masses as in $K^* \to K\pi$, the result depends on which meson is replaced by the divergence. This would not happen if we used some mean mass for the pseudoscalar octet. In Table 7 we have therefore listed both cases where a mode like $K\pi$ means that the π is replaced by the axial divergence, always the last particle.

All the calculated rates except one are in reasonable agreement with their experimental values (30) when in decays of the type (4.29) we only consider the case where the meson of smallest mass is replaced by the divergence. πK modes have much larger widths than the corresponding $K\pi$ modes. The only bad case is $\varphi \to K\bar{K}$ which is off by a factor 3. Hopefully, when this theory is made symmetrical, the πK modes will be reduced to agree with experiment and thereby also the rate for $\varphi \to K\bar{K}$.

The presence of the b-term in the interaction operator (4.32) is crucial for the angular distribution of the decays $A_1(1070) \to \rho\pi$ and $B(1235) \to \omega\pi$. To show that, let us first consider the $J_z=0$ state of the A_1. Since this meson is a L=1 ρ-meson, its wave function can be written

$$|A_1;0\rangle = \sqrt{\frac{1}{2}}|\rho(+1)\rangle|L_z = -1\rangle - \sqrt{\frac{1}{2}}|\rho(-1)\rangle|L_z = +1\rangle \qquad (4.41)$$

Without the b-term we see that the amplitude for decay into $\rho\pi$ from this state would be zero since the ρ in the final state has L=0. The A_1 would therefore only decay from the state $J_z = \pm 1$.

Similarly, we see that the B meson which is a L=1 π would not decay from $J_z = \pm 1$ if the b-term was absent,

$$|B;\pm 1\rangle = |\pi(0)\rangle |L_z = \pm 1\rangle \quad (4.42)$$

since the final ω has L=0. So the B could only decay from J_z=0.

With the b-term from our model we find for the ratios between the two helicity amplitudes in Table 7 for the two 1^+ mesons:

$$\frac{F_\pm}{F_o}\Big|_{A_1} = 1 - \rho^2 d(1+x)/2b = 0.76 \quad (4.43)$$

$$\frac{F_o}{F_+}\Big|_{B} = 1 - \rho^2 d(1+x)/b = 0.19 \quad (4.44)$$

The experimental situation concerning these two ratios is somewhat uncertain. For the A_1(1070) values between 0.5 and 0.9 are found (31) while for the B(1235) experiment seem to give ratios from 0.2 to 0.7 (31). However, in reference (31) a careful analysis of all available experimental data for these decays has been performed with the result:

$$\frac{F_\pm}{F_o}\Big|_{A_1} \text{ (C-R)} \approx 0.8 \quad (4.45)$$

$$\frac{F_o}{F_+}\Big|_{B} \text{ (C-R)} \approx 0.2 \quad (4.46)$$

These values agree very well with those we find from our model and demonstrates the necessity of a b-like term in the interaction operator for pseudoscalar meson emission.

V. CONCLUDING REMARKS

We have developed a relativistic quark model with harmonic dynamics in order to explain the straight, parallel meson and baryon Regge trajectories. That the slopes of the meson trajectories also approximately equal the baryon slopes is not understood in the framework of this model but has been taken as an experimental input.

In that way both the meson and baryon spectrum is uniquely given by the excitation constant Ω. Only a few of the predicted meson multiplets have been established so more resonances are needed to verify the harmonic oscillator character of the meson spectrum. On the other hand, for the baryons we have shown that there is strong evidence at the present time for the typical harmonic oscillator multiplets $[\underset{\sim}{70}, 0^+]$ and $[\underset{\sim}{70}, 2^+]$ at the second excited level.

To determine the transition rates between different states in this scheme we constructed vector and axial vector current operators. The vector current matrix elements could be compared with photon induced reactions like photoproduction and electroproduction. The results were found to be good when we introduced an empirical form factor instead of the one following from the model. For the transition $\omega \to \pi\gamma$ we were off by a factor 1.8 in the decay rate. This discrepancy could be due to the large breaking of gauge invariance in this particular case.

An independent check of the vector current operator was done in calculating the meson vertex in $K_{\ell 3}$ decay. The result was surprisingly good. However, this was not really a test of the harmonic dynamics since it only involves ground state mesons, but rather of the prescription used to construct the current.

The magnitudes of the axial vector current matrix elements do not seem to be in the same good agreement with experiment as we found for the vector current. This can be seen at once by calculating the axial vector constant for neutron beta decay for which we get 5/3 instead of the experimental value 1.23. So this matrix element is off by nearly 40%.

Deviations of the same order of magnitude were found when we calculated the rates for pseudoscalar meson emission from baryons and mesons by using a coupling proportional to the divergence of the axial vector current and modulating the matrix elements by an ad hoc smooth form factor. In many cases the transition amplitudes for this process involved two terms, one ordinary term and one B-term as we called it. Since these two terms always come in with opposite signs, the full amplitude is extremely sensitive to the magnitudes of each term. When these are only accurate to within a factor 1.6, we can understand why many of these amplitudes come out completely wrong. However, the presence of the B-term was necessary to explain many of the low-Q transitions among the baryons and also for the angular distributions of the decays $A_1(1070 \rightarrow \rho\pi$ and $B(1235) \rightarrow \omega\pi$ which are very well described in this model.

Our relativistic model explains all the successes of the somewhat controversial non-relativistic quark model as reviewed in reference (8). One assumption made in those calculations, that the quark mass equals 1/3 the nucleon mass, can be understood in our model. From (3.44), we found for the proton magnetic moment

$$\mu_{N^+} = 3\mu_B \qquad (5.1)$$

while a non-relativistic calculation will give

$$\mu_{N^+} = \mu_Q \tag{5.2}$$

where μ_Q is the quark magnetic moment

$$\mu_Q = g \frac{e}{2m_Q} \tag{5.3}$$

Assuming the quark g-factor to equal one as for a point particle and comparing (5.1) and (5.2), we find that the quark mass m_Q must be 1/3 of the proton mass M.

With this value for the quark mass the non-relativistic calculations of the photoproduction amplitudes in references (5) and (6) are very similar to our results. The reason is that the relativistic effects do not show up in this limited resonance mass region. However, the electroproduction of the same nucleon resonances is highly relativistic and our results are different and better than the non-relativistic results in reference (23).

Many of the baryon decay rates have also been calculated in the non-relativistic quark model. Faiman and Hendry (4) calculated pion emission from nucleon resonances using the non-relativistic form of the quark-meson interaction

$$f\bar{u}_2\gamma_5 u_1 \overset{NR}{=} f(\underset{\sim}{\sigma}\cdot\underset{\sim}{Q}) \tag{5.4}$$

where index 1 and 2 refer to the quark before and after the meson emission and $\underset{\sim}{Q} = \underset{\sim}{p}_1 - \underset{\sim}{p}_2$. This interaction has also recently been used by Faiman (32) to calculate the decay rates for strange baryons. Since the interaction (5.4) is just the ordinary term of our relativistic emission operator (4.21), the results of references (4) and (32) differ little from ours where we do not have the B-term.

It should be mentioned here that in the non-relativistic harmonic oscillator quark model one automatically gets a similar form factor as the one we were forced to introduce, equation (3.71), in order to get meaningful results.

Non-relativistically the B-term is obtained from the divergence of the quark axial vector current

$$\frac{f}{m_\pi} \bar{u}_2 \gamma_5 \not{q} u_1 \overset{NR}{=} \frac{f}{m_\pi}\left[(1 + \frac{\nu}{2m_Q})(\underset{\sim}{\sigma}\cdot\underset{\sim}{Q}) - \frac{\nu}{m_Q}(\underset{\sim}{\sigma}\cdot\underset{\sim}{p}_1)\right] \qquad (5.5)$$

where $q = (\nu,\underset{\sim}{Q})$ and $\underset{\sim}{p}_1$ is the quark momentum in the initial state. Comparing this to the interaction operator (4.21) for baryons, we see that in this non-relativistic limit

$$\nu = M-m \qquad (5.6)$$

and $$m_Q(\text{baryon}) = \frac{1}{3}M \qquad (5.7)$$

Similarly, from (4.32) we get for the quark mass in mesons

$$m_Q(\text{meson}) = \frac{1}{2} M \qquad (5.8)$$

So using the non-relativistic interaction (5.5) with the quark mass from (5.7), we can understand why that will give results similar to those of our relativistic model, at least in the limited resonance region where $M < 2$ GeV. That has been done by Mitra (33). But since in his model there is no harmonic oscillator dynamics, he is not able to relate the all over coupling constants for multiplets at different excitation levels.

Our model suffers from some serious shortcomings like lack of unitarity, the non-symmetrical three-meson vertex and the necessity of an empirical form factor. In spite of that, we think that this work has successfully demonstrated the very many physical consequences of this simple quark model which is able to relate phenomena ranging from straight Regge trajectories, through photoproduction amplitudes and angular distributions in A_1 and B decays to $K_{\ell 3}$ form factors. We hope it will turn out to be a useful scheme for organizing hadron data and predicting results of future experiments.

However, the most important outcome of this investigation is the realization that this naive model seems to give a realistic description of nature at the most fundamental level and to try to understand what there is in the dynamics of hadrons which in so many respects can be represented by a simple quark model with harmonic interaction.

APPENDIX

We will first discuss the operators which are useful for constructing states with definite angular momentum in a 3-dimensional harmonic oscillator. Then we shall write down the SU(6) wave functions for the mesons and calculate some typical transition rates according to our model.

Next we will describe a useful way to treat the symmetries of the three-quark baryon systems. This will enable us to calculate all SU(6) and orbital matrix elements of interest. Some examples are then given in detail.

Harmonic Oscillator Operators

Let us consider the mass squared operator of our model in the particle rest system where we can ignore all time-variables

$$\mathcal{M}^2 = \frac{1}{2}\underset{\sim}{P}^2 + \frac{\Omega^2}{2}\underset{\sim}{X}^2 + C \qquad (A.1)$$

where $P_i = (P_x, P_y, P_z)$ and $X_i = (x, y, z)$. We can introduce the creation operators

$$a_i^{+} = \sqrt{\frac{1}{2\Omega}}\, P_i + i\sqrt{\frac{\Omega}{2}}\, X_i \qquad (A.2)$$

and their Hermitian conjugate annihilation operators

$$a_i = \sqrt{\frac{1}{2\Omega}}\, P_i - i\sqrt{\frac{\Omega}{2}}\, X_i \qquad (A.3)$$

These satisfy the fundamental commutation relation

$$\left[a_i,\, a_j^{+}\right] = \delta_{ij} \qquad (A.4)$$

The operators (A.2) create excitations along the x_i-axis. Instead of these, it is more convenient to define operators which creates states with definite angular momentum:

$$a_+^+ = -\sqrt{\frac{1}{2}}\,(a_x^+ + ia_y^+)$$

$$a_-^+ = +\sqrt{\frac{1}{2}}\,(a_x^+ - ia_y^+) \qquad \text{(A.5)}$$

$$a_o^+ = a_z^+$$

It is easily seen that a_+^+ acting on the ground state gives a state with angular momentum L=1 and $L_z = +1$. a_-^+ will then give the $L_z = -1$ state and a_o^+ the state with $L_z=0$. The corresponding annihilation operators are then from (A.5):

$$a_+ = -\sqrt{\frac{1}{2}}\,(a_x - ia_y)$$

$$a_- = +\sqrt{\frac{1}{2}}\,(a_x + ia_y) \qquad \text{(A.6)}$$

$$a_o = a_z$$

The only non-vanishing commutators are now

$$\left[a_+, a_+^+\right] = 1$$

$$\left[a_-, a_-^+\right] = 1 \qquad \text{(A.7)}$$

$$\left[a_o, a_o^+\right] = 1$$

which can be checked using (A.4).

In many cases it is convenient to have the angular momentum raising and lowering operators

$$L_+ = L_x + iL_y$$
$$L_- = L_x - iL_y \qquad (A.8)$$

which now can be written

$$L_+ = \sqrt{2}\,(a_o a_+^+ + a_o^+ a_-)$$
$$L_- = \sqrt{2}\,(a_o a_-^+ + a_o^+ a_+) \qquad (A.9)$$

Using (A.7) we find

$$\left[L_+, L_-\right] = 2L_z \qquad (A.10)$$

where $$L_z = a_+^+ a_+ - a_-^+ a_- \qquad (A.11)$$

Mesons

The meson SU(3) and SU(6) wave functions are easily found and given in reference (8). For the pseudoscalar mesons they are:

$$\pi^+ = u\bar{d} \qquad \pi^o = \sqrt{\tfrac{1}{2}}\,(u\bar{u} - d\bar{d})$$
$$K^+ = u\bar{s} \qquad K^o = d\bar{s}$$
$$\eta = \sqrt{\tfrac{1}{6}}\,(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad (A.12)$$
$$\eta' = \sqrt{\tfrac{1}{3}}\,(u\bar{u} + d\bar{d} + s\bar{s})$$

To find the full SU(6) wave function for these, we just multiply the states (A.12) by the spin S=0 state of two spin 1/2 objects

$$|0,0\rangle = \sqrt{\tfrac{1}{2}}\,(|+\,-\rangle - |-\,+\rangle) \qquad (A.13)$$

The vector mesons can be in three different spin states:

$$|1,+1\rangle = |++\rangle$$

$$|1,0\rangle = \sqrt{\tfrac{1}{2}}(|+-\rangle + |-+\rangle)$$

$$|1,-1\rangle = |--\rangle$$

For the ω and φ mesons we will use the un-mixed quark states with SU(3) content:

$$\omega = \sqrt{\tfrac{1}{2}}(u\bar{u} + d\bar{d})$$

$$\varphi = s\bar{s} \tag{A.14}$$

Let us now calculate the very simple decay

$$K^{*+} \rightarrow K^{+} + \pi^{o} \tag{A.15}$$

We must first write the initial and final meson state in a symmetric form. Only the J_z=0 state of the initial K^* can decay:

$$\begin{aligned} K^{*+}(0) &= \frac{1}{2}\left[u^{+}\bar{s}^{-} + u^{-}\bar{s}^{+} + \bar{s}^{-}u^{+} + \bar{s}^{+}u^{-}\right] \\ K^{+}(0) &= \frac{1}{2}\left[u^{+}\bar{s}^{-} - u^{-}\bar{s}^{+} + \bar{s}^{-}u^{+} - \bar{s}^{+}u^{-}\right] \end{aligned} \tag{A.16}$$

The charge e_a in the emission operator (4.32) will then be the isospin operator τ_{az} corresponding to the π^o. This operator has the obvious properties:

$$\begin{aligned} \tau_z u &= +u & \qquad \tau_z \bar{u} &= -\bar{u} \\ \tau_z d &= -u & \qquad \tau_z \bar{d} &= +\bar{d} \\ \tau_z s &= 0 & \qquad \tau_z \bar{s} &= 0 \end{aligned} \tag{A.17}$$

We then act with H_a in (4.32) on the initial state and get:

$$H_a|K^{*+}(0)\rangle = G\zeta\rho d\frac{1}{2}\,(u^+\bar{s}^- - u^-\bar{s}^+) \qquad (A.18)$$

Projecting this onto the final state, we find for the transition amplitude (4.36):

$$F_o = 2\langle K^+(0)|H_a|K^{*+}(0)\rangle$$

$$= 2G\zeta\rho d\frac{1}{4}(1+1) = G\zeta\rho d \qquad (A.19)$$

This is the amplitude found in Table 7. A slightly more difficult calculation is necessary for the decay

$$A_1^+ \rightarrow \rho^+ + \pi^o \qquad (A.20)$$

The A_1 is an excited ρ with L=1. Its $J_z = +1$ state can then be written

$$|A_1^+(+1)\rangle = \sqrt{\frac{1}{2}}\,|\rho^+(+1)\rangle|1,0\rangle - \sqrt{\frac{1}{2}}\,|\rho^+(0)\rangle|1,+1\rangle$$

where $\quad |1,0\rangle = c_o^+|0\rangle \qquad$ (A.21)

and $\quad |1,+1\rangle = c_+^+|0\rangle$

We first calculate the SU(6) matrix elements which are easily found by the same method as in the first example:

$$\langle\rho^+(+1)|e_a\sigma_{az}|\rho^+(+1)\rangle = +1$$

$$\langle\rho^+(0)|e_a\sigma_{a-}|\rho^+(+1)\rangle = +\sqrt{\frac{1}{2}} \qquad (A.22)$$

The orbital matrix elements are also simple:

$$\langle 0|c_o e^{-\rho c_o}|1,0\rangle = \langle 0|c_o e^{-\rho c_o} c_o^{+}|0\rangle$$
$$= +1$$
$$\langle 0|c_{\pm} e^{-\rho c_o}|1,\pm 1\rangle = \langle 0|c_{\pm} e^{-\rho c_o} c_{\pm}^{+}|0\rangle$$
$$= +1$$

$$\langle 0|e^{-\rho c_o}|1,0\rangle = \langle 0|e^{-\rho c_o} c_o^{+}|0\rangle$$
$$= -\rho \qquad (A.23)$$

Using (A.21) with (A.22) and (A.23) we find for the matrix element of the b-term in (4.32):

$$\langle \rho^{+}(+1)|e_a(\sigma_{az}c_z - \sqrt{2}\,\sigma_{a+}c_+)|A_1^{+}(+1)\rangle$$
$$= \sqrt{\frac{1}{2}}\cdot 1 - \sqrt{2}\,(-\sqrt{\frac{1}{2}})(+\sqrt{\frac{1}{2}}) = +\sqrt{2} \qquad (A.24)$$

For the amplitude F_+ in (4.39) we then get:

$$F_+ = 2G\zeta\left[-\rho^2 d(1+x)\sqrt{\frac{1}{2}} + b\sqrt{2}\right]$$
$$= -\sqrt{2}\,G\zeta\left[\rho^2 d(1+x) - 2b\right] \qquad (A.25)$$

The A_1 can in our model also decay from the state $J_z=0$:

$$|A_1^{+}(0)\rangle = \sqrt{\frac{1}{2}}\,|\rho^{+}(+1)\rangle|1,-1\rangle - \sqrt{\frac{1}{2}}|\rho^{+}(-1)\rangle|1,+1\rangle$$

In this case only the b-term will contribute. We already have all the matrix elements necessary to calculate this contribution:

$$\langle \rho^{+}(0)|e_a(\sqrt{2}\,\sigma_{a-}c_- - \sqrt{2}\,\sigma_{a+}c_+)|A_1^{+}(0)\rangle$$
$$= (+\sqrt{2})(+\sqrt{\frac{1}{2}})(+\sqrt{\frac{1}{2}}) - \sqrt{2}\,(-\sqrt{\frac{1}{2}})(+\sqrt{\frac{1}{2}}) = +\sqrt{2}$$

This is the same result as (A.24) as it should be when we take the matrix element of a scalar operator. The corresponding transition amplitude F_o will then be from (4.39):

$$F_o = +2\sqrt{2}\, G\zeta b \tag{A.26}$$

This is the way all amplitudes in Table 7 have been found.

Symmetries of the Three-Quark System

If an object can be in one of a number of conditions x, y, z, ... we can, when we have three such objects, form states of four kinds of symmetry which we call S, α, β, A; symmetric, mixed symmetric α,β and antisymmetric A.

$$|S\rangle = |xyz\rangle_S$$
$$= \frac{1}{\sqrt{6}}(|xyz\rangle + |xzy\rangle + |yxz\rangle + |yzx\rangle + |zxy\rangle + |zyx\rangle)$$

$$|\alpha\rangle = |xyz\rangle_\alpha$$
$$= \frac{1}{2\sqrt{3}}(|xyz\rangle + |xzy\rangle + |yxz\rangle + |yzx\rangle - 2|zxy\rangle - 2|zyx\rangle)$$

$$|\beta\rangle = |xyz\rangle_\beta$$
$$= \frac{1}{2}(|xyz\rangle - |xzy\rangle + |yxz\rangle - |yzx\rangle)$$

$$|A\rangle = |xyz\rangle_A$$
$$= \frac{1}{\sqrt{6}}(-|xyz\rangle + |xzy\rangle - |yzx\rangle + |yxz\rangle - |zxy\rangle + |zyx\rangle) \tag{A.27}$$

where $|zxy\rangle$ means the first object is in state z, the second in x and the third in y. If x and y are the same state, y = x, we must replace $|xyz\rangle + |yxz\rangle$ by $\sqrt{2}|xyz\rangle$. If all three are the same, only the S state survives as $|xxx\rangle_S = |xxx\rangle$. The state α has been chosen to be symmetric in the last two objects, state β is antisymmetric.

If we combine two states of these kinds, say $|1\rangle$ and $|2\rangle$, we can recompose states of varying symmetry by the following rules:

$$|1\rangle_S|2\rangle_S = +|\rangle_S \qquad |1\rangle_S|2\rangle_\alpha = +|\rangle_\alpha$$

$$|1\rangle_S|2\rangle_\beta = +|\rangle_\beta \qquad |1\rangle_S|2\rangle_A = +|\rangle_A$$

$$|1\rangle_A|2\rangle_S = +|\rangle_A \qquad |1\rangle_A|2\rangle_\alpha = +|\rangle_\beta$$

$$|1\rangle_A|2\rangle_\beta = -|\rangle_\alpha \qquad |1\rangle_A|2\rangle_A = +|\rangle_S$$

(A.28)

$$\sqrt{\tfrac{1}{2}}\,(+|1\rangle_\alpha|2\rangle_\alpha + |1\rangle_\beta|2\rangle_\beta) = |\rangle_S$$

$$\sqrt{\tfrac{1}{2}}\,(-|1\rangle_\alpha|2\rangle_\alpha + |1\rangle_\beta|2\rangle_\beta) = |\rangle_\alpha$$

$$\sqrt{\tfrac{1}{2}}\,(+|1\rangle_\alpha|2\rangle_\beta + |1\rangle_\beta|2\rangle_\alpha) = |\rangle_\beta$$

$$\sqrt{\tfrac{1}{2}}\,(-|1\rangle_\alpha|2\rangle_\beta + |1\rangle_\beta|2\rangle_\alpha) = |\rangle_A$$

SU(6) Baryon States and Matrix Elements

When we have three quarks with spin 1/2, the combined system will have spin 3/2 or 1/2. To find the appropriate states we can use ordinary Clebsch-Gordan coefficients or (A.27) with x, y and z equal to + or - corresponding to quark spin up or down:

$$
\begin{aligned}
|\tfrac{3}{2}, +\tfrac{3}{2}\rangle_S &= |{+}{+}{+}\rangle_S \\
|\tfrac{3}{2}, +\tfrac{1}{2}\rangle_S &= |{+}{+}{-}\rangle_S \\
|\tfrac{3}{2}, -\tfrac{1}{2}\rangle_S &= |{+}{-}{-}\rangle_S \\
|\tfrac{3}{2}, -\tfrac{3}{2}\rangle_S &= |{-}{-}{-}\rangle_S
\end{aligned}
\qquad (A.29)
$$

Total spin 1/2 can be obtained in two ways, first adding two quark spins together to give spin 0 so that the third quark gives total spin 1/2, or first adding two quark spins together to give spin 1 and then add this to the third quark spin to give a total spin 1/2. These two ways correspond to states with α and β symmetry:

$$
\begin{aligned}
|\tfrac{1}{2}, +\tfrac{1}{2}\rangle_\alpha &= +|{+}{+}{-}\rangle_\alpha \\
|\tfrac{1}{2}, -\tfrac{1}{2}\rangle_\alpha &= -|{-}{-}{+}\rangle_\alpha \\
|\tfrac{1}{2}, +\tfrac{1}{2}\rangle_\beta &= +|{+}{+}{-}\rangle_\beta \\
|\tfrac{1}{2}, -\tfrac{1}{2}\rangle_\beta &= -|{-}{-}{+}\rangle_\beta
\end{aligned}
\qquad (A.30)
$$

The state $|\ \rangle_A$ cannot be constructed from only the two spin states, up or down.

When we consider the quark SU(3) states u, d and s, then the combined symmetric state $|\ \rangle_S$ will be a decimet $|\underset{\sim}{10}\rangle_S$, α and β will be octet states $|\underset{\sim}{8}\rangle_\alpha$ and $|\underset{\sim}{8}\rangle_\beta$ and $|\ \rangle_A$ will be a SU(3) singlet $|\underset{\sim}{1}\rangle_A$ with the Λ-quantum numbers. For instance, an α octet proton will have the SU(3) state $|N^+\rangle_\alpha = |uud\rangle_\alpha = \sqrt{\frac{1}{6}}(|udu\rangle + |uud\rangle - 2|duu\rangle)$ while a Δ^+ of a decimet will be $|\Delta^+\rangle_S = |uud\rangle_S = \sqrt{\frac{1}{3}}(|udu\rangle + |uud\rangle + |duu\rangle)$.

Combining SU(3) and spin according to the multiplication rules (A.28), we get the baryon SU(6) wave functions. The symmetric states $|\ \rangle_S$ will now constitute a symmetric $\underset{\sim}{56}$:

$$|\underset{\sim}{56}\rangle_S: \quad \begin{aligned} {}^4(\underset{\sim}{10}) &= |\tfrac{3}{2}\rangle_S |\underset{\sim}{10}\rangle_S \\ {}^2(\underset{\sim}{8}) &= \sqrt{\tfrac{1}{2}}\,(+|\tfrac{1}{2}\rangle_\alpha |\underset{\sim}{8}\rangle_\alpha + |\tfrac{1}{2}\rangle_\beta |\underset{\sim}{8}\rangle_\beta) \end{aligned} \tag{A.31}$$

Similarly we find two $\underset{\sim}{70}$ with mixed symmetry:

$$|\underset{\sim}{70}\rangle_\alpha: \quad \begin{aligned} {}^4(\underset{\sim}{8})_\alpha &= |\tfrac{3}{2}\rangle_S |\underset{\sim}{8}\rangle_\alpha \\ {}^2(\underset{\sim}{8})_\alpha &= \sqrt{\tfrac{1}{2}}(-|\tfrac{1}{2}\rangle_\alpha |\underset{\sim}{8}\rangle_\alpha + |\tfrac{1}{2}\rangle_\beta |\underset{\sim}{8}\rangle_\beta) \\ {}^2(\underset{\sim}{10})_\alpha &= |\tfrac{1}{2}\rangle_\alpha |\underset{\sim}{10}\rangle_S \\ {}^2(\underset{\sim}{1})_\alpha &= -|\tfrac{1}{2}\rangle_\beta |\underset{\sim}{1}\rangle_A \end{aligned} \tag{A.32}$$

$$|\underset{\sim}{70}\rangle_\beta: \quad \begin{aligned} {}^4(\underset{\sim}{8})_\beta &= |\tfrac{3}{2}\rangle_S |\underset{\sim}{8}\rangle_\beta \\ {}^2(\underset{\sim}{8})_\beta &= \sqrt{\tfrac{1}{2}}(+|\tfrac{1}{2}\rangle_\alpha |\underset{\sim}{8}\rangle_\beta + |\tfrac{1}{2}\rangle_\beta |\underset{\sim}{8}\rangle_\alpha) \\ {}^2(\underset{\sim}{10})_\beta &= |\tfrac{1}{2}\rangle_\beta |\underset{\sim}{10}\rangle_S \\ {}^2(\underset{\sim}{1})_\beta &= +|\tfrac{1}{2}\rangle_\alpha |\underset{\sim}{1}\rangle_A \end{aligned} \tag{A.33}$$

The last $\underset{\sim}{20}$ antisymmetric states are:

$$|\underline{20}\rangle_A: \qquad {}^4(\underline{1})_A = +|\tfrac{3}{2}\rangle_S |\underline{1}\rangle_A$$

$${}^2(\underline{8})_A = \sqrt{\tfrac{1}{2}}(-|\tfrac{1}{2}\rangle_\alpha |\underline{8}\rangle_\beta + |\tfrac{1}{2}\rangle_\beta |\underline{8}\rangle_\alpha) \qquad (A.34)$$

To calculate SU(6) matrix elements we have to know the vector charges e_a in (3.56) or the axial change in (4.21) or (4.32). In the simplest case with photons acting through the electromagnetic current, e_a will just be the diagonal quark electrical charge operator

$$e_a(\gamma) = \begin{pmatrix} +2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix} \qquad (A.35)$$

When we have mesons acting through the divergence of the axial vector current, e_a will similarly be the 3x3 Gell-Mann matrix operator (29) λ_i for the i-th number of the pseudoscalar octet, acting on quark a. For instance, when we have the emission of π^o, then e_a will be the diagonal operator

$$e_a(\pi^o) = \lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \qquad (A.36)$$

Emission of η^o will then correspond to the charges

$$e_a(\eta^o) = \lambda_8 = \sqrt{\tfrac{1}{3}} \begin{pmatrix} +1 & & \\ & +1 & \\ & & -\sqrt{2} \end{pmatrix} \qquad (A.37)$$

In the same way we get the non-diagonal operator for a K^+

$$e_a(K^+) = \frac{1}{2}(\lambda_4 - i\lambda_5) = v_- \qquad (A.38)$$

with the properties

$$\begin{aligned} v_-|u\rangle &= |s\rangle \\ v_-|d\rangle &= 0 \\ v_-|s\rangle &= 0 \end{aligned} \qquad (A.39)$$

Matrix elements of diagonal charges are easily found using our SU(3) wave functions for N^+ and Δ^+:

$$\begin{aligned} {}_\alpha\langle \underset{\sim}{8}|e_a|\underset{\sim}{10}\rangle_S &= \frac{1}{3}\sqrt{2}(e_u-e_d) \\ {}_\alpha\langle \underset{\sim}{8}|e_a|\underset{\sim}{8}\rangle_\alpha &= \frac{1}{3}(e_u+2e_d) \\ {}_\beta\langle \underset{\sim}{8}|e_a|\underset{\sim}{8}\rangle_\beta &= e_u \\ {}_\beta\langle \underset{\sim}{8}|e_a|\underset{\sim}{10}\rangle_S &= 0 \\ {}_\alpha\langle \underset{\sim}{8}|e_a|\underset{\sim}{8}\rangle_\beta &= 0 \end{aligned} \qquad (A.40)$$

Similarly we find for spin matrix elements between states with $S_z = +\frac{1}{2}$:

$$\begin{aligned} {}_\alpha\langle \tfrac{1}{2}|1|\tfrac{3}{2}\rangle_S &= 0 \\ {}_\alpha\langle \tfrac{1}{2}|\sigma_z|\tfrac{3}{2}\rangle_S &= +\frac{2}{3}\sqrt{2} \\ {}_\alpha\langle \tfrac{1}{2}|1|\tfrac{1}{2}\rangle_\alpha &= +1 \\ {}_\alpha\langle \tfrac{1}{2}|\sigma_z|\tfrac{1}{2}\rangle_\alpha &= -\frac{1}{3} \\ {}_\beta\langle \tfrac{1}{2}|1|\tfrac{1}{2}\rangle_\beta &= +1 \\ {}_\beta\langle \tfrac{1}{2}|\sigma_z|\tfrac{1}{2}\rangle_\beta &= +1 \end{aligned} \qquad (A.41)$$

Matrix elements of σ_+ and σ_- can then be found using the Wigner-Eckart theorem.

Let us now calculate as an example the matrix element of $e_a\sigma_{az}$ between a N^+ and a Δ^+ in the $\underset{\sim}{56}$. From (A.31) we first get:

$$\langle N^+|e_a\sigma_{az}|\Delta^+\rangle = \sqrt{\frac{1}{2}}\left\{ {}_{\alpha}\langle\tfrac{1}{2}|\sigma_z|\tfrac{3}{2}\rangle_S \; {}_{\alpha}\langle\underset{\sim}{8}|e_a|\underset{\sim}{10}\rangle_S \right.$$

$$\left. + \; {}_{\alpha}\langle\tfrac{1}{2}|\sigma_z|\tfrac{3}{2}\rangle_S \; {}_{\beta}\langle\underset{\sim}{8}|e_a|\underset{\sim}{10}\rangle_S \right\}$$

Using (A.40) and (A.41) we then find:

$$\langle N^+|e_a\sigma_{az}|\Delta^+\rangle = \sqrt{\frac{1}{2}}\left\{(+\frac{2}{3}\sqrt{2})\ \left[+\frac{1}{3}\sqrt{2}(e_u-e_d)\right]\right\}$$

$$= +\frac{2}{9}\sqrt{2}\ (e_u-e_d)$$

When e_a corresponds to an electromagnetic interaction, we get from (A.35):

$$\langle N^+|e_a(\gamma)|\Delta^+\rangle = +\frac{2}{9}\sqrt{2}\ (\frac{2}{3}+\frac{1}{3}) = +\frac{2}{9}\sqrt{2}$$

For π^o we get from (A.36):

$$\langle N^+|e_a(\pi^o)|\Delta^+\rangle = +\frac{2}{9}\sqrt{2}\ (1+1) = +\frac{4}{9}\sqrt{2} \qquad \text{(A.42)}$$

When we have matrix elements of octet operators between two octets, we can express the result as a linear combination of the two SU(3) reduced matrix elements F and D. These we take from Gasiorowicz (34) and Feynman (35) and list them in Table A. As soon as we know the F and D values, we can then calculate all the matrix elements of interest.

Table A. SU(3) matrix elements for octet-octet transitions.

Transition	SU(3) Matrix Element
$N^+ \rightarrow N^+ \pi^0$	$F + D$
$N^+ \rightarrow N^+ \eta^0$	$\sqrt{3}\ (F - 1/3\ D)$
$N^+ \rightarrow \Lambda^0 K^+$	$\sqrt{3}\ (F + 1/3\ D)$
$N^+ \rightarrow \Sigma^+ K^0$	$\sqrt{2}\ (F - D)$
$\Lambda^0 \rightarrow \Lambda^0 \eta^0$	$2/3\ \sqrt{3}\ D$
$\Lambda^0 \rightarrow \Sigma^+ \pi^-$	$2/3\ \sqrt{3}\ D$
$\Lambda^0 \rightarrow N^+ K^-$	$\sqrt{3}\ (F + 1/3\ D)$
$\Sigma^+ \rightarrow \Sigma^+ \pi^0$	$2\ F$
$\Sigma^+ \rightarrow \Lambda^0 \pi^+$	$2/3\ \sqrt{3}\ D$
$\Sigma^+ \rightarrow N^+ \bar{K}^0$	$\sqrt{2}\ (F + D)$
$\Sigma^+ \rightarrow \Sigma^+ \eta^0$	$2/3\ \sqrt{3}\ D$
$\Xi^- \rightarrow \Xi^- \pi^0$	$F - D$
$\Xi^- \rightarrow \Sigma^0 K^-$	$\sqrt{2}\ (F + D)$
$\Xi^- \rightarrow \Lambda^0 K^-$	$\sqrt{3}\ (F - 1/3\ D)$
$\Xi^- \rightarrow \Xi^- \eta^0$	$\sqrt{3}\ (F + 1/3\ D)$
$N^+ \rightarrow N^+ \gamma$	$F + 1/3\ D$
$\Sigma^+ \rightarrow \Sigma^+ \gamma$	$F + 1/3\ D$
$N^0 \rightarrow N^0 \gamma$	$-\ 2/3\ D$
$\Xi^0 \rightarrow \Xi^0 \gamma$	$-\ 2/3\ D$
$\Sigma^- \rightarrow \Sigma^- \gamma$	$-\ F + 1/3\ D$
$\Xi^- \rightarrow \Xi^- \gamma$	$-\ F + 1/3\ D$
$\Lambda^0 \rightarrow \Lambda^0 \gamma$	$-\ 1/3\ D$
$\Sigma^0 \rightarrow \Sigma^0 \gamma$	$+\ 1/3\ D$
$\Sigma^0 \rightarrow \Lambda^0 \gamma$	$\sqrt{1/3}\ D$

For an octet operator between a decimet and an octet, there is only one reduced matrix element. We can then relate all other matrix elements to this single one using Table B where we have set the $\Delta^+ N^+ \pi^o$ matrix element equal to 1. To find the absolute values, we have to perform a calculation similar to the one which led to (A.42).

These results are listed in Table C where the final baryon state is an octet proton in the $\underset{\sim}{56}$ with spin up. For an initial decimet state we take a Δ^+ and the matrix element given is $\Delta^+ N^+ \pi^o$. So the value in (A.42) is found in this table.

For an initial singlet Λ, the matrix element given is for decay into $N^+ K^-$. With an initial octet we have in Table C listed the corresponding (F,D) values. These we have found by calculating two diagonal matrix elements like $N^+ N^+ \pi^o$ and $N^+ N^+ \eta^o$ in the way already described, and then expressing these in terms of F and D as found in Table A.

Baryon Orbital States and Matrix Elements

The ground state $|0\rangle$ has no orbital excitations and is therefore completely symmetric. Combining this with the symmetric SU(6) state $|\underset{\sim}{56}\rangle_S$, we get the complete baryon ground state:

$$\left[\underset{\sim}{56}, 0^+\right] = |\underset{\sim}{56}\rangle_S |0\rangle \qquad (A.43)$$

At the first excited level, there are two orbital excited states, one with α-symmetry,

$$|1\rangle_\alpha^1 = a^+|0\rangle \qquad (A.44)$$

and the other with β-symmetry

$$|1\rangle_\beta^1 = b^+|0\rangle \qquad (A.45)$$

Table B. SU(3) matrix elements for decimet-octet transitions.

Transition	SU(3) Matrix Element
$\Delta^{+} \rightarrow N^{+} \pi^{o}$	1
$\Delta^{+} \rightarrow \Sigma^{o} K^{+}$	1
$\Delta^{+} \rightarrow N^{+} \gamma$	1/2
$\Sigma^{+} \rightarrow \Sigma^{+} \pi^{o}$	1/2
$\Sigma^{+} \rightarrow \Sigma^{+} \eta^{o}$	$1/2 \sqrt{3}$
$\Sigma^{+} \rightarrow \Lambda^{o} \pi^{+}$	$1/2 \sqrt{3}$
$\Sigma^{o} \rightarrow N^{+} K^{-}$	1/2
$\Sigma^{o} \rightarrow \Xi^{-} K^{+}$	1/2
$\Xi^{-} \rightarrow \Xi \pi^{o}$	1/2
$\Xi^{-} \rightarrow \Xi \eta^{o}$	$1/2 \sqrt{3}$
$\Xi^{-} \rightarrow \Lambda^{o} K^{-}$	$1/2 \sqrt{3}$
$\Xi^{-} \rightarrow \Sigma^{o} K^{-}$	1/2
$\Omega^{-} \rightarrow \Xi^{o} K^{-}$	$1/2 \sqrt{6}$

Table C. SU(6) matrix elements with $N^+(+1/2)$ belonging to $\underset{\sim}{56}$ in the final state. The numbers in parentheses are (F,D) values.

Multiplet B	$\langle P,+\frac{1}{2}\|e_a\|B,+\frac{1}{2}\rangle$	$\langle P,+\frac{1}{2}\|e_a\sigma_{a-}\|B,+\frac{3}{2}\rangle$	$\langle P,+\frac{1}{2}\|e_a\sigma_{az}\|B,+\frac{1}{2}\rangle$	$\langle P,+\frac{1}{2}\|e_a\sigma_{a+}\|B,-\frac{1}{2}\rangle$
${}^2(\underset{\sim}{8})\ \underset{\sim}{56}_S$	$+\frac{1}{3}\ (+1,0)$		$+\frac{1}{3}\ (+\frac{2}{3},1)$	$+\frac{1}{3}\ (+\frac{2}{3},1)$
${}^4(\underset{\sim}{10})\underset{\sim}{56}_S$	0	$-\frac{2}{9}\sqrt{6}$	$+\frac{4}{9}\sqrt{2}$	$+\frac{2}{9}\sqrt{2}$
${}^2(\underset{\sim}{1})\ \underset{\sim}{70}_\alpha$	$+\frac{1}{3}\sqrt{3}$		$+\frac{1}{3}\sqrt{3}$	$+\frac{1}{3}\sqrt{3}$
${}^2(\underset{\sim}{8})\ \underset{\sim}{70}_\alpha$	$+\frac{1}{2}\ (+\frac{1}{3},1)$		$+\frac{1}{6}\ (+\frac{5}{3},1)$	$+\frac{1}{6}(+\frac{5}{3},1)$
${}^4(\underset{\sim}{8})\ \underset{\sim}{70}_\alpha$	0	$-\frac{1}{6}\sqrt{3}(-\frac{1}{3},1)$	$+\frac{1}{3}(-\frac{1}{3},1)$	$+\frac{1}{6}(-\frac{1}{3},1)$
${}^2(\underset{\sim}{10})\underset{\sim}{70}_\alpha$	$+\frac{2}{3}$		$-\frac{2}{9}$	$-\frac{2}{9}$

The superscript 1 in the orbital state means that there is N=1 excitation in it. Combining these orbital states with the SU(6) states with the same symmetry, (A.32) and (A.33) according to the rule (A.28) we get the complete first excited baryon state:

$$[\underset{\sim}{70},1^-] = \sqrt{\tfrac{1}{2}}\left(|\underset{\sim}{70}\rangle_\alpha |1\rangle_\alpha^1 + |\underset{\sim}{70}\rangle_\beta |1\rangle_\beta^1\right) \qquad (A.46)$$

When N=2, we have two excitations of the type (A.44) and (A.45). These can be combined to give total angular momentum L=2,0,1 according to (A.28):

$$\begin{aligned}
|2\rangle_S^2, |0\rangle_S^2 &= \sqrt{\tfrac{1}{2}}\left(+|1\rangle_\alpha^1 |1\rangle_\alpha^1 + |1\rangle_\beta^1 |1\rangle_\beta^1\right) \\
|2\rangle_\alpha^2, |0\rangle_\alpha^2 &= \sqrt{\tfrac{1}{2}}\left(-|1\rangle_\alpha^1 |1\rangle_\alpha^1 + |1\rangle_\beta^1 |1\rangle_\beta^1\right) \\
|2\rangle_\beta^2, |0\rangle_\beta^2 &= \sqrt{\tfrac{1}{2}}\left(+|1\rangle_\alpha^1 |1\rangle_\beta^1 + |1\rangle_\beta^1 |1\rangle_\alpha^1\right) \\
|1\rangle_A^2 &= \sqrt{\tfrac{1}{2}}\left(-|1\rangle_\alpha^1 |1\rangle_\beta^1 + |1\rangle_\beta^1 |1\rangle_\alpha^1\right)
\end{aligned} \qquad (A.47)$$

Combining these orbital states with the appropriate SU(6) states, we can form the five multiplets, $[\underset{\sim}{56},2^+]$, $[\underset{\sim}{56},0^+]$, $[\underset{\sim}{70},2^+]$, $[\underset{\sim}{70},\ 0^+]$ and $[\underset{\sim}{20},1^+]$. For instance

$$[\underset{\sim}{70},2^+] = \sqrt{\tfrac{1}{2}}\left(|\underset{\sim}{70}\rangle_\alpha |2\rangle_\alpha^2 + |\underset{\sim}{70}\rangle_\beta |2\rangle_\beta^2\right)$$

and $$[\underset{\sim}{20},1^+] = |\underset{\sim}{20}\rangle_A |1\rangle_A^2 \qquad (A.48)$$

The angular momentum of each excitation in (A.47) is combined using ordinary Clebsch-Gordan coefficients. For example, using

$$|2,0\rangle = \sqrt{\tfrac{1}{6}}\left(|+1\rangle|-1\rangle + |-1\rangle|+1\rangle + 2|0\rangle|0\rangle\right) \qquad (A.49)$$

we get

$$|2,0\rangle^2_\beta = \sqrt{\frac{1}{6}}\,(a^+_+ b^+_- + a^+_- b^+_+ + 2a^+_z b^+_z)\,|0\rangle$$

and

$$|2,0\rangle^2_S = \sqrt{\frac{1}{6}}\,(a^+_+ a^+_- + a^+_z a^+_z + b^+_+ b^+_- + b^+_z b^+_z)\,|0\rangle \tag{A.50}$$

Similarly we find:

$$|0,0\rangle^2_S = \sqrt{\frac{1}{12}}\,(2a^+_+ a^+_- - a^+_z a^+_z + 2b^+_+ b^+_- - b^+_z b^+_z)\,|0\rangle \tag{A.51}$$

We can now easily calculate the orbital matrix elements. Let us do a few examples:

$$\langle 0|e^{-\lambda a_z}|2,0\rangle^2_S = \sqrt{\frac{1}{6}}\,\langle 0|e^{-\lambda a_z} a^+_z a^+_z|0\rangle$$

$$= \sqrt{\frac{1}{6}}\,\frac{(-\lambda)^2}{2!}\,\langle 0|a_z a_z a^+_z a^+_z|0\rangle = +\frac{1}{6}\sqrt{6}\,\lambda^2$$

$$\langle 0|a_z e^{-\lambda a_z}|2,0\rangle^2_S = \sqrt{\frac{1}{6}}\,\langle 0|a_z e^{-\lambda a_z} a^+_z a^+_z|0\rangle$$

$$= \sqrt{\frac{1}{6}}\,(-\lambda)\langle 0|a_z a_z a^+_z a^+_z|0\rangle = -\sqrt{\frac{2}{3}}\,\lambda \tag{A.52}$$

All the relevant orbital matrix elements are found in this way and are listed in Table D.

Examples

To show how to use the machinery outlined in this Appendix, we will now calculate two typical transition amplitudes. First, let us consider the process

$$\Lambda(1520) \to \Lambda(1115) + \gamma \tag{A.53}$$

Table D. Orbital matrix elements.

Multiplet B	$\langle 0 \vert e^{-\lambda a_z} \vert B,0\rangle$	$\langle 0 \vert a_+ e^{-\lambda a_z} \vert B,+1\rangle$	$\langle 0 \vert a_z e^{-\lambda a_z} \vert B,0\rangle$	$\langle 0 \vert a_- e^{-\lambda a_z} \vert B,-1\rangle$
N=0 $L=0_S$	+1	0	0	0
N=1 $L=0_\alpha$	$-\lambda$	0	0	0
N=2 $L=2_S$	$+\frac{1}{6}\sqrt{6}\,\lambda^2$	$-\sqrt{\frac{1}{2}}\,\lambda$	$-\sqrt{\frac{2}{3}}\,\lambda$	$-\sqrt{\frac{1}{2}}\,\lambda$
N=2 $L=2_\alpha$	$-\frac{1}{6}\sqrt{6}\,\lambda^2$	$+\sqrt{\frac{1}{2}}\,\lambda$	$+\sqrt{\frac{2}{3}}\,\lambda$	$+\sqrt{\frac{1}{2}}\,\lambda$
N=2 $L=0_S$	$-\frac{1}{6}\sqrt{3}\,\lambda^2$	0	$+\sqrt{\frac{1}{3}}\,\lambda$	0
N=2 $L=0_\alpha$	$+\frac{1}{6}\sqrt{3}\,\lambda^2$	0	$-\sqrt{\frac{1}{3}}\,\lambda$	0
N=2 $L=1_A$	0	0	0	0

The initial Λ belongs to the $^2(\underset{\sim}{1})_{3/2}$ of the $\left[\underset{\sim}{70},1^-\right]$. It can decay from the states with $J_z = +3/2$ and $J_z = -1/2$. These we find by adding L=1 together with the quark spin S = 1/2 of the singlet Λ_o:

$$\Lambda_{3/2}(+\tfrac{3}{2}) = |^2\Lambda_1(+\tfrac{1}{2})\rangle|1,+1\rangle^1_\alpha \tag{A.54}$$

$$\Lambda_{3/2}(-\tfrac{1}{2}) = \sqrt{\tfrac{2}{3}}\,|^2\Lambda_1(-\tfrac{1}{2})\rangle|1,0\rangle^1_\alpha + \sqrt{\tfrac{1}{3}}\,|^2\Lambda_1(+\tfrac{1}{2})\rangle|1,-1\rangle^1_\alpha$$

Then we need the SU(6) matrix elements

$$\langle\Lambda_8(+\tfrac{1}{2})|e_a|^2\Lambda_1(+\tfrac{1}{2})\rangle = \frac{5}{18}$$

$$\langle\Lambda_8(+\tfrac{1}{2})|e_a\sigma_{a+}|^2\Lambda_1(-\tfrac{1}{2})\rangle = \frac{5}{18} \tag{A.55}$$

The orbital matrix elements we find in Table D. In this way we get for the two amplitudes F_+ and F_- in (3.56):

$$F_+ = 9G\left[\frac{5}{18}T\right]\sqrt{\frac{1}{2}} = \frac{5}{4}\sqrt{2}\,TG$$

$$F_- = 9G\left[\sqrt{\frac{1}{3}}\left(\frac{5}{18}T\right) + \sqrt{\frac{2}{3}}\left(\frac{5}{18}R\right)(-\lambda)\right]\sqrt{\frac{1}{2}}$$

$$= \frac{5}{4}\sqrt{\frac{2}{3}}\,(T - \sqrt{2}\,R\lambda)\,G \tag{A.56}$$

These are the two amplitudes given in (3.73). The extra factor $\sqrt{1/2}$ in (A.56) comes from (A.46), only the α-symmetric term contributes when the initial state is $\underset{\sim}{70}$.

The doublet partner of $\Lambda(1520)$ is the $\Lambda(1405)$ with total spin $J = 1/2$. This one can also decay by emitting a photon:

$$\Lambda(1405) \rightarrow \Lambda(1115) + \gamma \tag{A.57}$$

In this case there will only be one amplitude, for decay from $J_z = -1/2$:

$$\Lambda_{1/2}(-\tfrac{1}{2}) = \sqrt{\tfrac{1}{3}}\, |^2\Lambda_1(-\tfrac{1}{2})\rangle |1,0\rangle^1_\alpha - \sqrt{\tfrac{2}{3}}\, |^2\Lambda_1(+\tfrac{1}{2})\rangle |1,-1\rangle^1_\alpha \qquad (A.58)$$

The result is similar to (A.56) except for these different CG-coefficients:

$$F_- = \frac{5}{4}\sqrt{\frac{2}{3}}\,(\sqrt{2} + R\lambda)\, G \qquad (A.59)$$

Our last example will be the decay

$$N^+(1700) \rightarrow N^+ + \pi^o \qquad (A.60)$$

The initial nucleon belongs to the $^4(\underset{\sim}{8})_{1/2}$ of the $\left[\underset{\sim}{70},1^-\right]$. It decays from the $J_z = +1/2$ state which we again construct using ordinary CG-coefficients:

$$N_{1/2}(+\tfrac{1}{2}) = \sqrt{\tfrac{1}{2}}\,|^4N_8(+\tfrac{3}{2})\rangle |1,-1\rangle^1_\alpha - \sqrt{\tfrac{1}{3}}\,|^4N_8(+\tfrac{1}{2})\rangle |1,0\rangle^1_\alpha$$

$$+ \sqrt{\tfrac{1}{6}}\, |^4N_8(-\tfrac{1}{2})\rangle |1,+1\rangle^1_\alpha \qquad (A.61)$$

In the meson emission operator (4.21) we can now calculate all the matrix elements using Tables C and D. First of all we get for the SU(3) matrix element

$$F + D = -\frac{1}{3} + 1 = +\frac{2}{3}$$

so that the transition amplitude will be:

$$F = 3G\xi\,\frac{2}{3}\left\{\lambda D(-\sqrt{\tfrac{1}{3}})(+\tfrac{1}{3})(-\lambda)(1+X)\right.$$

$$\left. + B\left[(-\sqrt{\tfrac{1}{3}})(+\tfrac{1}{3}) - \sqrt{2}(+\sqrt{\tfrac{1}{6}})(+\tfrac{1}{6}) + \sqrt{2}(+\sqrt{\tfrac{1}{2}})(-\tfrac{1}{6}\sqrt{3})\right]\right\}\sqrt{\frac{1}{2}}$$

We have again multiplied by $\sqrt{1/2}$ because the initial state belongs to a $\underset{\sim}{70}$. Adding terms together, we get the final answer for the transition amplitude which can be found in Table 6:

$$F = + \frac{1}{9}\sqrt{6}\, G\xi \left\langle \lambda^2 D(1+X) - 3B \right\rangle \tag{A.62}$$

All the amplitudes are calculated in this way.

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