SOME CONSIDERATIONS IN THE APPLICATION OF A GAS TURBINE CYCLE TO THE MANUFACTURE OF NITRIC OXIDE

Thesis by

Gordon John MacLeod

In Partial Fulfillment of the Requirements
For the Degree of
Mechanical Engineer

California Institute of Technology
Pasadena, California
1953

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. S. S. Penner for his helpful guidance during the preparation of Part A of this thesis and to Dr. H. S. Tsien for his valuable criticism during the progress of this research. The author also wishes gratefully to acknowledge the worthwhile discussions he has had with Dr. W. D. Rannie and Dr. B. H. Sage. Sincere thanks are extended to Miss Ruth Winkel for the typing of this manuscript.

ABSTRACT

Certain problems associated with the application of a gas turbine cycle to the manufacture of nitric oxide are investigated. The feasibility of quenching the nitric oxide decomposition reaction with a De Laval nozzle is demonstrated. Thermochemical studies show that yields of nitric oxide approaching one percent are attainable in a gas turbine cycle. The severe turbine blade cooling problem can be surmounted with the use of either transpiration cooling or film cooling. Transpiration cooling with internal liquid vaporization is found to be the most promising method. The coolant requirements amount only to approximately one percent of the main stream mass flow per cooled-turbine-blade row. The effect of transpiration and film cooling upon cycle performance is negligible. Although all the problems associated with the application of a gas turbine cycle to the manufacture of nitric oxide appear to be surmountable, no attempt is, however, made in this study to ascertain the economical feasibility of the process. The answer to this question involves factors which are not purely technical and thus lies outside the scope of the thesis.

TABLE OF CONTENTS

Part A -- Chemical Considerations

Part B -- Engine Cycle Considerations

APPENDICES

I. SUMMARY

The simple, open-cycle gas turbine combines all the necessary elements for the production of nitric oxide into a single unit. The turbine supplies the power to drive the compressor, which, in turn, provides the high pressure necessary for expanding the combustion gases. Small amounts of nitric oxide are formed in the combustion chamber at high temperatures. The turbine nozzle provides the quenching mechanism for freezing the nitric oxide decomposition reaction. The turbine extracts a portion of the energy of the working gases. After the regenerative heat exchanger further reduces the energy of the gas stream, some extraction process, like the one development at the University of Wisconsin under Daniels, may be used to separate the nitric oxide from the combustion products.

The feasibility of quenching the nitric oxide decomposition reaction with a De Laval nozzle is demonstrated in Section III. Cooling rates of the order of 10^6 °K/sec are available in the De Laval nozzle, and these rates are sufficiently high to freeze the decomposition reaction at combustion-chamber temperatures approaching 3000°K.

Thermochemical studies, carried-out in Section IV, show that yields of nitric oxide approaching one percent are attainable without oxygen enrichment in a gas turbine cycle. Increasing the oxygen concentration by 88 percent can lead to yields of nitric oxide approaching two-and-one-half percent. Although oxygen enrichment leads to appreciable increases in the yield of nitric oxide, it is questionable whether its use is justified for the reasons discussed in Section IV.

Combustion-chamber temperatures near 2500°K are required to produce nitric oxide yields approaching one percent in the gas turbine cycle. The problems of turbine blade cooling associated with total * temperatures of 2500°K and above are quite formidable. So severe are the operating conditions that the use of unconventional methods of cooling are required. The application of transpiration and film cooling methods to the turbine blade cooling problem is considered in Section VI-B. The four cooling methods investigated are: (1) gaseous transpiration cooling, (2) liquid transpiration cooling, (3) transpiration cooling with internal liquid vaporization and (4) film cooling. All four methods were found to be comparable with one another in terms of coolant requirements. When the service-life of the turbine blades is an important consideration, calling for lower allowable wall temperatures, liquid transpiration cooling and transpiration cooling with internal liquid vaporization require coolant flow rates from one-half to one-third of those required with gaseous transpiration cooling. The vapor-lock phenomenon associated with the inherent instability of the liquid film in liquid transpiration cooling is not present in the method of transpiration cooling with internal liquid vaporization. Consequently, when turbine blade temperatures near 1200°K or lower are desired, transpiration cooling with internal liquid vaporization is the most promising of the four methods investigated. The coolant requirements for this method of transpiration cooling amount only to approximately one percent of the main stream mass flow for each cooled-turbine-blade row.

---

* "Total temperature" and "stagnation temperature" are synonymous terms.

In order to retain high turbine efficiencies as well as to prevent recovery of high static temperatures, the formation of shock waves between the turbine blades should be avoided. This means that the approach Mach number relative to the blades should be less than one. However, due to the special requirements relating to the expansion of the working gases in the application of the gas turbine cycle to the manufacture of nitric oxide, the avoidance of shock waves is impractical. Moreover, although the presence of shock waves between turbine blades leads to decreases in turbine efficiency, the effect of the shock waves on cycle performance is only moderate. Under these circumstances it is better to expand to relatively high Mach numbers in order to reduce the total temperature relative to the turbine blades even though this means an increase in the static temperature of the gases due to the shock waves. With the use of high turbine wheel speeds, which are made possible by turbine blade cooling, the ratio of the total temperature of the working gases relative to the blades to the total temperature (i.e., T_{or}/T_o) can be reduced to values approaching 0.80. Although a total temperature relative to the blades of 1500°K cannot be obtained by expanding from a combustion chamber temperature of 2500°K, the extraction of energy by the turbine will quickly reduce the relative total-temperature to 1500°K. Lower values of relative Mach number and T_{or}/T_o may be attained by counter-rotation of the turbine nozzle ring.

The effect of the coolant injection upon cycle performance is investigated in Section VI-C. A simple one-dimensional analysis discloses that the coolant injection, used in transpiration cooling or in film cooling, has a negligible effect both upon those blade rows which are directly cooled and upon that portion of the cycle downstream of the coolant-injection zone. The effect of coolant injection is negligible principally because the coolant flow rates are small.

Although all the problems associated with the application of a gas turbine cycle to the manufacture of nitric oxide appear to be surmountable, no attempt is, however, made in this study to ascertain the economical feasibility of the process. The answer to this question involves factors which are not purely technical and thus lies outside the scope of the thesis.

II. INTRODUCTION

A. Quenching Action of the De Laval Nozzle

If the pressure difference across a De Laval nozzle is large enough to yield sonic velocity at the throat, then the time-rate of decrease of temperature of a fluid particle (-DT/Dt) is very large even near the nozzle entrance section. For representative nozzles of the type used in small test rockets (-DT/Dt) is of the order of 10^6 °K/sec. These enormous cooling rates can be utilized to "freeze" chemical reactions. Chemical processes proceed at finite rates of reaction, and the speed of the reaction is an exponential function of the temperature. Therefore, if the rate of cooling is much greater than the rate of chemical reaction, it may be possible to quench the reaction in a De Laval nozzle.

In order to make the preceding statements clearer, consider the reversible chemical reaction shown in Eq. (1)

$$A + B \rightleftharpoons C + D \tag{1}$$

Suppose now that at high temperatures the reaction proceeds more rapidly to the right than to the left thereby tending to consume the reactants A and B and to yield the products C and D, and that the reverse is true at low temperatures. Suppose further that the reaction takes place in the gas phase and that chemical equilibrium has been reached in the combustion chamber. The gas mixture is then allowed to pass through a nozzle. During transit through the nozzle the gas experiences a large temperature drop. Consequently, during passage of the gas through the nozzle the reaction of Eq. (1) will tend to move to the left since it was supposed that the reaction tends to proceed to the left at low temperatures. If the cooling rate is sufficiently rapid, however, then there is insufficient time for the reaction to occur. Another way of viewing the situation is to think of the quenching action of the nozzle as inhibiting the attainment of equilibrium conditions during nozzle flow with the result that the gas composition at the exit of the nozzle is practically the same as existed at the entrance to the nozzle. The speed of a chemical reaction is an exponential function of temperature; hence, if the static temperature at the nozzle exit is sufficiently low so that the rate of reaction is essentially zero, the effect of the DeLaval nozzle has been one of "freezing" the chemical system practically at the equilibrium conditions existing in the combustion chamber.

B. Utilization of the Quenching Action of the De Laval Nozzle

The preceding discussion suggests that in order to utilize successfully the quenching action of the nozzle in the manufacture of chemicals the following conditions must be met:

1. A useful chemical product must be formed in the zone preceding the De Laval nozzle.

2. The reverse reaction must be capable of being frozen by the quenching action of the De Laval nozzle.

3. The chemical reaction leading to the desired product must be endothermic.

4. The use of the De Laval nozzle for freezing of chemical reactions must offer economic advantages over other methods of obtaining the desired chemicals.

The necessity of the first two conditions is self-evident. The question arises, however, as to what is the criterion for determining whether a given chemical reaction can be frozen at a composition approaching that of the gas mixture at the nozzle entrance. Fortunately, this criterion has been determined previously by Penner (Ref. 1)*. Nevertheless, because of the importance of the criterion in this thesis and because of the abbreviated treatment of the basic derivation in the original presentation, a complete derivation of the criterion for near-frozen flow is included in Appendix II in a form suitable for application to this study. The detailed discussion of this criterion, based upon the derivation of the basic equation obtained in Appendix II, is presented in Section III together with a representative application to the decomposition of nitric oxide.

The reason the chemical reaction must be endothermic follows from the principle of Le Chatelier, which states that when equilibrium has been reached, a change in any of the factors affecting equilibrium tends to make that reaction take place which will neutralize the effect of the change. This law then says that an endothermic reaction tends to-

---

* References are to be found in Section IX.

wards completion as the temperature is increased; that is, as heat is added. On the other hand, when the temperature is decreased, the reaction tends to reverse and reform the reactants. This reversal of reaction is exactly what we wish to prevent by the quenching action of the De Laval nozzle. The chemical reaction leading to the desired chemical compound should not be exothermic because exothermic reactions tend towards completion when the temperature is decreased. Consequently, for such reactions the quenching action of the De Laval nozzle is of no practical use.

The fourth necessary condition for the successful application of the quenching action of the nozzle relates to economic considerations. The choice of the best production method must necessarily rest upon a detailed study of all of the alternative processes. However, it is obvious that the more easily a chemical reaction can be quenched, the less important become the extremely rapid quenching rates available with the De Laval nozzle.

For practical applications of the De Laval nozzle it is of primary importance to recover the kinetic and internal energy of the exit gases. This energy may be recovered in only two ways: (1) by heat transfer to some other medium and (2) by extraction of mechanical work using a turbine. Which method or combination of methods is most suitable can be decided only after a detailed cycle analysis has been made for the chemical reaction involved. The most important variables entering the discussion will be the nozzle entrance pressure and temperature and the Mach number of the gases at the nozzle exit.

Aside from economic considerations, it is evidently necessary to reduce the energy content of the exit gases in order to prevent reheating to the stagnation temperature. Otherwise an approach to the equilibrium composition at the beginning of the process might result, thereby negating the purpose of the quenching process.

One possible but uneconomical method for energy reduction involves the injection of a cooling fluid into the jet stream. In addition to wasting the available energy, this method would greatly increase the recovery costs for extracting the desired chemical products from a greatly diluted mixture.

A second method of energy extraction, which deserves serious consideration, is the use of a turbine. The first stages of such a turbine would be of the impulse type, in which rapid energy extraction occurs with a resultant rapid decrease in stagnation temperature of the working gases. The latter stages of the turbine would be of the reaction type in which both rapid stagnation and static temperature drops occur.

The use of the De Laval nozzle in conjunction with one or a combination of these energy-extraction methods must be superior economically to other methods of obtaining the desired chemicals. But, it is clear that any discussion of the relative merits of different proposed methods must be related to a specific chemical compound. Consequently, the subsequent discussion will be confined to the manufacture of a specific chemical; namely, the manufacture of nitric oxide.

C. Quenching of the Reaction $2NO \rightleftarrows N_2 + O_2$

Nitric oxide is an extremely useful compound since it can be used as starting material for the manufacture of a large number of nitrogen compounds which find wide application in industry and agriculture. That the decomposition of nitric oxide is a relatively slow process, except at high temperatures, has been known for a long time. An indication of this fact is given by the data shown in Table I.

TABLE I

Time Required for Ten Percent Decomposition of Two Percent Nitric Oxide in the Presence of Ten Percent Oxygen and Eighty Percent Nitrogen.

T, °K	1000	1500	2000	2500
Time, sec.	10^7	90	0.27	0.032

The cooling rates required in order to freeze the reaction $2NO \rightleftarrows N_2 + O_2$ at various temperatures are discussed in Section III-B. In the formation of nitric oxide from its elements 21.6 Kcal are absorbed per mole.

In order to form some conclusion as to the economic merits of possible De Laval nozzle processes relative to existing nitrogen fixation schemes, a brief summary of the competitive methods is outlined first. The principal existing competitors of De Laval nozzle processes in the fixation of atmospheric nitrogen include the following:

1. Natural Fertilizers Ammonium salts and alkali nitrates are produced in nature through the decomposition of organic matter. Such natural fertilizers as "chile saltpetre", which provided about fifty-five percent of the world production of fixed nitrogen in 1913 and only about twenty-four percent in 1929, have largely been replaced by various processes for the more direct fixation of atmospheric nitrogen (Ref. 3, pp. 70-72).
2. Rainfall Lightning during electrical storms is responsible for the fixing of vasts amounts of nitrogen from air by a mechanism essentially identical to that described below for the electric arc process. It has been estimated (Ref. 4, p. 14) that from one to ten pounds of nitrogenous compounds are carried to the ground per acre by rainfall each year.
3. Bacterial Fixation A certain class of plants, called Legumes, in cooperation with bacteria are capable of fixing atmospheric nitrogen. Alfalfa, clover, peas, and beans are examples of legumes which are used in the practice of crop rotation to help prevent nitrogen depletion of soils by such nonlegumes as corn, cotton, tobacco and wheat (Ref. 3, p. 70).
4. Electric Arc Process The electric arc process (Ref. 5, p. 1820) is used to synthesize nitric oxide by blowing a mixture of oxygen and nitrogen through an electric arc. The

mixture temperature is raised to about 2000°C and is then rapidly cooled. This process is carried-out chiefly in Norway, where electrical power is cheap, but it is questionable whether this process can continue to compete with other methods (Ref. 6, pp. 387-389).

5. "By-product Ammonia" Ammonia is obtained from the destructive distillation of coal, which contains nitrogenous material. By-product ammonia is passed through sulfuric acid and converted to ammonium sulfate, which finds extensive use as a fertilizer. From one to one-and-half percent of the coal is converted into ammonium sulfate (Ref. 5, p. 1578).

6. Cyanamide Process Calcium cyanamide, $CaCN_2$, is produced by the reaction of pure nitrogen with calcium carbide at about 1475°K. Calcium cyanamide may then be treated with steam to yield calcium carbonate and ammonia; it may also be fused with salt and carbon to yield sodium cyanide NaCN. Calcium cyanamide is an excellant alkaline fertilizer.

7. Synthetic Ammonia Ammonia is produced by the Haber process (or by onc of its many modifications) through the direct reaction of hydrogen and nitrogen under high pressure and at temperatures in the neighborhood of 775°K. The basic raw materials are coke and air.

8. Wisconsin Process This method (Ref. 2) is used to synthesize nitric oxide from air. The air is preheated by passing it through

a pebble-bed regenerator consisting of a refractory material such as magnesium oxide. The hot air then passes to a combustion chamber, where fuel is added and burned. The resulting gas mixture is quenched in a second pebble-bed regenerator. Next, the flow of air is reversed and the cycle repeated. Temperatures in excess of 2375°K have been obtained with nitric oxide yields approximating the theoretical limit. As of March, 1953, Dr. Farrington Daniels, who developed the Wisconsin process, estimated that his method was competitive with the synthetic ammonia process, without having been perfected to the same degree. At the present time more than half of the total cost of nitrogen fixation by the Wisconsin process is tied up with recovery of nitric oxide from dilute gas mixtures, in which the concentration of NO is of the order of two percent. It is to be expected that the recovery costs will be reduced considerably as additional work is done and that ultimately the Wisconsin process may be economically superior to all of the other conventional methods of nitrogen fixation.

The first three processes, which are the principal natural sources of fixed nitrogen, are still of importance, but they are inadequate to maintain the nitrogen balance of the soil. The role of synthetic nitrogen-fixation is to help replenish man's and nature's depletion of the nitrogen of the soil and to provide raw material for the manufacture of nitrogenous compounds. The electric arc process is no longer of importance principally because of the high cost of electrical power. "By-product ammonia" will continue to be an important secondary source of fixed nitrogen. The cyanamide process is considerably more expensive than the ammonia process (Ref. 6, pp. 389-390). The cyanamide process, however, should be able to maintain itself for some time because of the value of cyanamide as a fertilizer and as a raw material for further synthesis of nitrogen-containing chemicals.

The real competition to any proposed process utilizing the DeLaval nozzle will arise from the synthetic ammonia process and from the Wisconsin process. The common ground upon which these three competitors meet is that of the manufacture of nitric acid. Nitric acid is the base for the manufacture of inorganic and organic nitrates and nitro derivatives, both of which find extensive use in the chemical industry. Ammonia is oxidized according to the reaction

$$4NH_3 + 5O_2 \rightleftharpoons 4NO + 6H_2O \qquad (2)$$

by passing a mixture of about ten percent ammonia and ninety percent of air over heated platinum gauze at $1225^{\circ}K$. The gases leaving the platinum catalyst then pass through absorption towers, where the following successive reversible reactions take place:

$$2NO + O_2 \rightleftharpoons 2NO_2 \;, \qquad (3)$$

$$3NO_2 + H_2 \rightleftharpoons 2HNO_3 + NO \;. \qquad (4)$$

About ninety-six percent of ammonia is converted to nitric acid whose concentration varies from sixty-one to sixty-five percent in comparison to the fifty to fifty-five percent concentrations obtained from the electric arc process. The chief advantages of this ammonia system are (a) the relatively low cost of producing ammonia and (b) the possibility of shipment of anhydrous ammonia to various oxidation plants located in the ultimate consumption areas. In the Wisconsin process, nitric acid is produced from nitric oxide by reactions (3) and (4) above. The nitric oxide is adsorbed on silica gel from the mixture of combusted gas and air leaving the second pebble-bed regenerator. The recovery methods are currently being improved by the group under Daniels at the University of Wisconsin. After the pilot plant stage was completed at San Jose, California, the Food Machinery and Chemical Corporation, which acquired development rights, has erected a plant at Sunflower, Kansas.

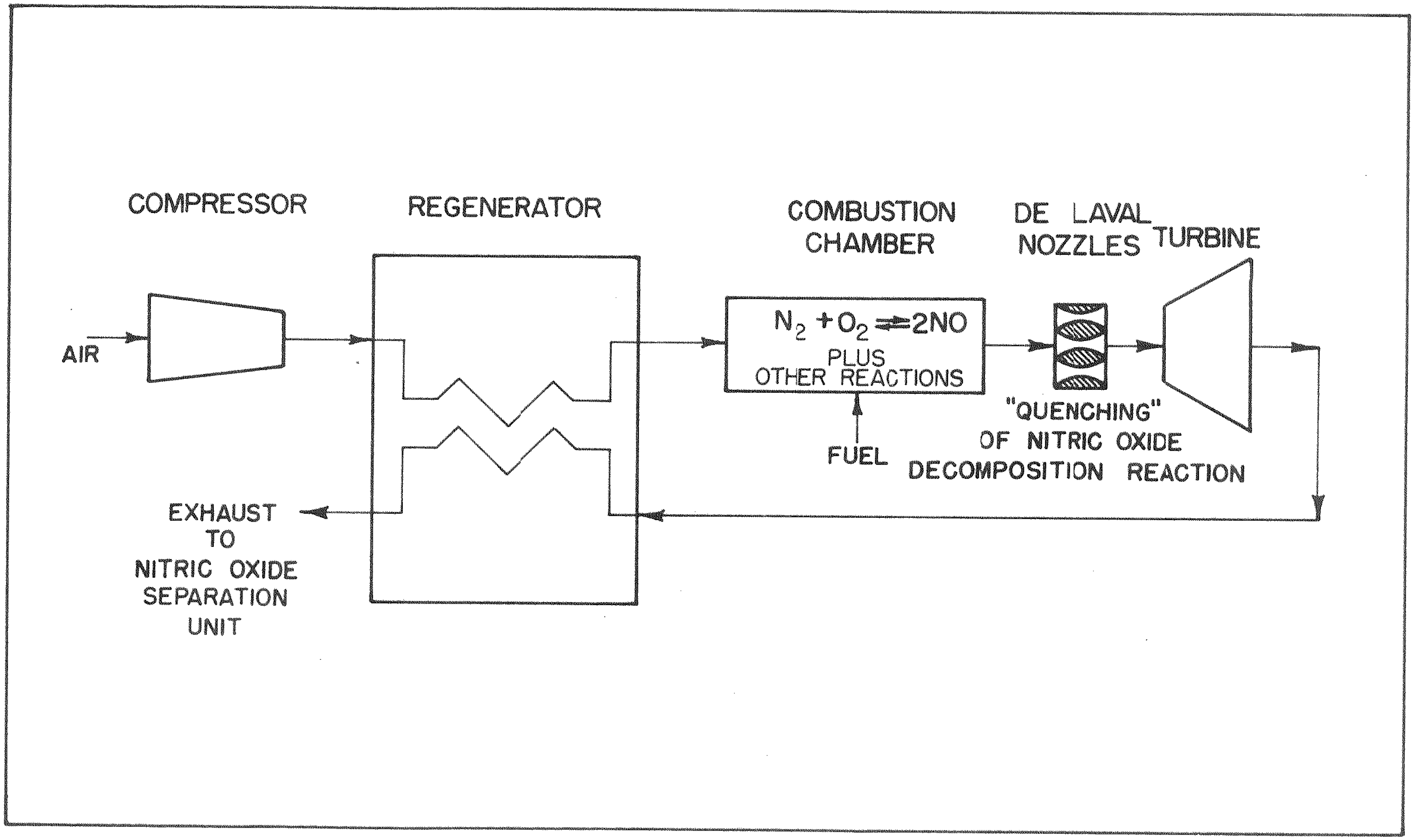
The Wisconsin process, like the turbine cycle using the De Laval nozzle, involves the problems of quenching of the nitric oxide decomposition reaction and extraction of dilute nitric oxide. At present the limiting factor in the Wisconsin process is associated with deterioration of the pebble-beds in the regenerators due to the combined effects of high temperature, temperature reversals, and erosion caused by the flowing gases. These three effects combine to limit the maximum temperature permissible in the furance, which, in turn, determines the maximum nitric oxide yields. The cost of extraction of the nitric oxide from the gas stream per pound of nitric oxide is roughly inversely proportional to the concentration of this constituent. As stated earlier in this discussion, the extraction process constitutes more than half of the total cost of the manufacture of nitric oxide. Furthermore, even if the pebble-bed deterioration problem were solved, the higher quenching rates required for further increases in maximum allowable temperature could not be met efficiently with pebble-bed quenching in its present form. The cooling rates available with pebble-bed cooling are of the order of 10^4 °K/sec (Ref. 2), which is satisfactory at 2000°K, but is inadequate at 2500°K where cooling rates of the order of 10^5 °K/sec are required.

The temperature and cooling rate limitations of the Wisconsin process may be circumvented by a DeLaval nozzle process. Theoretical cooling rates of the order of 10^6 °K/sec or greater can be obtained with suitably chosen nozzles. The real problem, of course, is connected with the energy extraction from the nozzle exit gases. Brief mention of various energy extraction methods, together with reasons why energy extraction is necessary, was made in Section II-B. In Part B of this thesis a detailed analysis is made of a cycle employing a turbine. A brief outline of the use of a gas turbine cycle will be made at this point, in order to illustrate the operation and problems involved.

The simple, open-cycle gas turbine combines all the necessary elements for the production of nitric oxide in a single unit (Cf. Fig. 1). The turbine supplies the power to drive the compressor, which, in turn, provides the high pressure necessary for expanding the combustion gases.

FIG. 1

SIMPLE OPEN-CYCLE GAS TURBINE
FLOW DIAGRAM FOR MAJOR COMPONENTS



The compression process helps to increase the temperature of the air, but the largest temperature rise is the result of combustion of fuel added in the combustion chamber. Nitric oxide is formed according to the reaction (3). Since equal numbers of moles of gas appear on both sides of this equation, the yield of nitric oxide is practically independent of pressure in the range of pressures and temperatures to be encountered in a gas turbine cycle. The turbine nozzle provides the quenching mechanism for the freezing of the nitric oxide decomposition reaction. A regenerator may also be used to enhance the efficiency of the cycle by increasing the temperature of the gases before they enter the combustion chamber. In a well-designed system an excess of power will be developed by the turbine, and consequently the gas turbine cycle is essentially a power-generating gas turbine installation. The cycle may be looked upon either as a chemical process for synthesizing nitric oxide with the excess turbine power considered as a by-product or, alternatively, as a cycle for power production with the synthetic nitric oxide as a by-product.

As was indicated earlier, very high temperatures are desirable in order to secure maximum yields of nitric oxide. Temperatures of about 2500°K are required to raise the nitric oxide yield appreciably above one percent. A turbine inlet temperature of 2500°K is roughly twice as high as can be used in conventional turbines. When it is considered that some method of turbine blade cooling is required for temperatures over 1375°K (or about 2000°F), it is apparent that the cooling problem at 2500°K will be formidable.

Combustion-chamber temperatures in the neighborhood of 2500°K lead to other problems. The initial temperature of the air prior to combustion should be above about 1000°K in order to minimize fuel consumption and to permit the utilization of atmospheric oxygen for burning of the fuel. Otherwise, little oxygen will be left to combine with the nitrogen, and the yield of nitric oxide will be low. The regenerator helps to overcome this situation by transferring heat from the exhaust gases to the incoming air.

In a gas turbine cycle, the most effective use of high combustion-chamber temperatures necessitates high pressures in the combustion chamber. High pressures are necessary for extracting the thermal energy of the working gases. As an illustration of this point, consider a turbine, having an efficiency of eighty-five percent, receiving working gases at 2500°K and twenty-five atmospheres and discharging at a back pressure of one-and-half atmospheres; the total temperature of the exhaust gases may be shown to be about 1480°K (2205°F) for $\gamma = 1.30$. On the other hand, the exit gases from the same turbine taking gases from the combustion chamber at 2500°K and ten atmospheres and discharging at the same back pressure, leave at a total temperature of 1750°K (2691°F). This difference of exit temperature represents a loss of roughly ninety hoursepower for each one lb/sec of mass flow, taking into consideration the extra compressor work required for the higher chamber pressure. On the other hand, the task of obtaining pressures of twenty-five atmospheres and above in high speed axial flow compressors in a difficult development problem.

The anticipated temperatures and pressures raise difficult problems for the design of the regenerator. Most alloys have very little strength above 1000°K (about 1350°F). Moreover, at high temperatures creep phenomena become important. Even the best high temperature alloys are limited to stresses less than approximately 5000 psi for an allowable elongation of one percent in 100,000 hours at 1150°K (1610°F) (Ref. 7, p. 443, and Ref. 8, p. 552). The combined stresses resulting from (a) the thermal stresses due to the axial temperature gradients along the direction of flow in the regenerator (Cf. Ref. 10, pp. 422-427), and (b) the normal stresses due to the pressure differential across the heat transfer surface of the regenerator, are too large for such temperature-pressure combinations as 1500°K and twenty-five atmospheres or 1750°K and ten atmospheres. In fact, the circumferential stresses arising from a pressure difference of twenty-five atmospheres across the walls of tubes are about 2500 psi (Ref. 9, p. 60). These results indicate that counterflow heat transfer cannot be used exclusively, and that some combination of the three types of heat exchangers (parallel-flow, crossflow and counterflow) must be used. Concurrent use of different types of heat exchangers leads to complications in design and requires large investments.

The preceding introduction has served (1) to describe the nature of the quenching action of the De Laval nozzle, (2) to formulate four conditions which must be met in order to utilize successfully this quenching action in the production of chemicals, (3) to indicate the applicability of the quenching mechanism to the decomposition reaction of nitric oxide, and (4) to suggest which problems should be investigated.

D. Organization of the Research Presentation

The preceding paragraphs have suggested that the research may be divided conveniently into two parts, one dealing with chemical considerations and the other with engine cycle considerations. In Part A, two important studies relating to chemical considerations are investigated. In the first analysis, presented in Section III, the criteria necessary for "freezing" a chemical reaction in a De Laval nozzle are derived and applied to the nitric oxide decomposition reaction. In the second, presented in Section IV, the thermochemical studies for the production of nitric oxide are described. The variation of the yield of nitric oxide with the amount of fuel burnt and with the amount of oxygen added in order to enrich the inlet air are investigated.

In Part B, some of the problems associated with the application of a gas turbine to the manufacture of nitric oxide are investigated. The turbine-blade cooling problem is investigated in some detail. Coolant-flow requirements are determined both for transpiration cooling and for film cooling in Section VI-B. The effect of transpiration and film cooling upon cycle performance is studied in Section VI-C.

III. CRITERION FOR NEAR-FROZEN FLOW

A. Introduction to the Basic Equation

As was indicated in the Introduction, one of the requirements for the sucessful application of the quenching action of the De Laval nozzle to the manufacture of chemicals is that the reaction products must be capable of being frozen. This means that the rate of chemical reaction must be slow enough and the rate of cooling fast enough to prevent appreciable chemical reaction during transit through the nozzle.

The criterion for determining whether a given chemical reaction can be frozen at a composition approaching that of the gas mixture in the combustion chamber has been worked out by Penner (Ref. 1) for one-dimensional isentropic expansion of a perfect gas through a De Laval nozzle. Since the treatment of the near-frozen flow criterion was very abbreviated in the original publication and because of the significance of the criterion for this thesis, a complete derivation of the criterion is given in Appendix II. The derivation makes a number of approximations and assumptions none of which are any more restrictive than the assumption of one-dimensional isentropic expansion of a perfect gas. The derivation leads to the following rather complicated equation (Cf. Eq. AII-28):

$$\frac{T_o - T'}{T_o - T} = \frac{\bar{k}_f \prod x_j^{\nu_j'} \left(\frac{p_o}{R'T_o}\frac{\rho}{\rho_o}\right)^{\Sigma \nu_j' - 1}}{\frac{\Delta H}{R T_o^2}\left(-\frac{DT}{Dt}\right)} \left[1 - \frac{K_p(T)}{K_p(T_o)}\right] \sum (\nu_k'' - \nu_k')^2 / x_k \, , \qquad (5)$$

which is applicable to chemical reactions represented by

$$\sum \nu_k' G_k = \sum \nu_k'' G_k \tag{6}$$

and in which the symbols have the following meanings:

T = static temperature

T_o = total temperature in the combustion chamber

T' = "composition temperature" corresponding to the actual gas composition at a given static temperature T

$\bar{k}_f$ = average value of the reaction rate constant for the forward reaction described in Eq. (6) over the temperature interval (T_o - T)

χ_j , χ_k = mole fraction of components j and k, respectively

ΔH = heat of reaction or the heat absorbed at constant pressure for the chemical reaction described by Eq. (6)

R, R'' = universal gas constants expressed in appropriate units

p = total pressure in the combustion chamber

ρ_o = density of gases in combustion chamber

ρ = density of gases corresponding to the static temperature T

$\left(-\frac{DT}{Dt}\right)$ = rate of decrease of static temperature with time following a gas particle

$K_p(T)$ = equilibrium constant based upon partial pressures and is to be evaluated at the static temperature T

$K_p(T_o)$ = equilibrium constant corresponding to the combustion or total temperature T_o

ν_k' , ν_k'' = coefficients multiplying the chemical symbol G in the relation representing the stoichiometry of the reaction given by Eq. (6).

Equation (5) determines the so-called "composition temperature" T' corresponding to the actual gas composition existing at the point in the expansion where the static temperature has decreased from the combustion or total temperature T_o to the static temperature T as a function of the chemical kinetics of the system and of the rate of decrease of the static temperature with time (-DT/Dt). The "composition temperature" T' is related to the actual gas composition by means of the expression for the equilibrium constant. An alternative method of interpreting Eq. (5) is to consider the temperature ratio $(T_o - T')/(T_o-T)$ as given and to determine the required rate of cooling for quenching of the reaction (i.e., -DT/Dt) as a function of this temperature ratio and of the chemical kinetics of the system. The former view-point will be used in the subsequent analysis. An expression for the rate of cooling (-DT/Dt) is derived as a function of the nozzle parameters in Appendix III for isentropic flow through a DeLaval nozzle. The expected cooling rates for De Laval nozzles are discussed in paragraph C of this section. At this point in the discussion it is sufficient to indicate that the cooling rates are of the order of 10^6 °K/sec.

B. Application to the Decomposition of Nitric Oxide

Nitric oxide decomposes according to the overall reaction

$$2NO \rightleftharpoons N_2 + O_2 \quad . \qquad (7)$$

Therefore, using Eq. (5),

$$\sum \nu_j' - 1 = 2 - 1 = 1, \tag{8}$$

$$\sum (\nu_k'' - \nu_k')^2/\chi_k = \frac{(2)^2}{\chi_{NO}} + \frac{1}{\chi_{N_2}} + \frac{1}{\chi_{O_2}}, \tag{9a}$$

and

$$\prod \chi_j^{\nu_j'} = \chi_{NO}^2 \tag{9b}$$

The rate constant k_f for the decomposition of nitric oxide according to the reaction given by Eq. (7) has been determined both experimentally and theoretically by Daniels and his co-workers (Ref. 2). The theoretical relation determined by them was found to be in good agreement with the experimental results obtained in a nitrogen fixation furnace after proper adjustment of the frequency factor. The expression for k_f reported by Daniels is

$$k_{fD} = 10^9 \exp(-70,000/RT) \ \text{atmos}^{-1} \ \text{sec}^{-1} \tag{10}$$

or

$$k_{fD} = 10^9 R'T \exp(-70,000/RT) \ \text{cc mole}^{-1} \ \text{sec}^{-1}, \tag{11}$$

where R is the gas constant equal to 1.987 cal/°K-mole and R' is the gas constant equal to 82.0 cm^3-atmos/°K-mole. More recently Wise and Frech (Ref. 11) determined the rate constant k_f for the decomposition of nitric oxide for the temperatures somewhat greater than 1000°K to be given by

$$k_{fw} = 3.1 \times 10^{15} \exp(-82{,}000/RT) \quad cm^3 \, mole^{-1} \, sec^{-1}. \qquad (12)$$

Wise and Frech obtained this expression by fitting a curve to (1) the previously available experimental data of Koerner (Ref. 12) and of Hendrickson (Ref. 13) and (2) to their own experimental results taken at temperatures below 1300°K in a quartz vessel. The rate constant k_f as evaluated from Eqs. (10) and (12) are given in Table II. Except for temperatures in the neighborhood of 2100°K, there is a significant difference between the values of k_f as given by Eqs. (10) and (12) as can be seen from Table II. In view of the fact that Daniels' rate expression is known to be in accord with results observed in the Wisconsin process for nitrogen fixation, it is preferable to use his rate data for the present applications since the conditions in the DeLaval nozzle process certainly are much closer to those existing in the pebble-bed than to those existing for purified gases in a quartz reaction vessel. Nevertheless, both Eqs. (10) and (12) will be used in subsequent calculations.

The heat of reaction ΔH for the decomposition of nitric oxide as given by Eq. (7) is 43,200 cal/mole (Ref. 14). The values of $K_p(T) = K_p'^2$ given in Table II were obtained from National Bureau of Standard reports (Ref. 15). The values of the density ratio ρ/ρ_0 for isentropic expansion may be taken from the tabulated values of Keenan and Kaye (Ref. 16, pp. 139-147).

TABLE II

Reaction Rate and Equilibrium Constants vs. Temperature

T °K	k_f atmos^{-1} sec^{-1} *		$K_p'^{\dagger} = \dfrac{(p_{NO})}{(p_{N_2})^{1/2}(p_{O_2})^{1/2}}$	$K_p'^2$
	k_{fD} Eq. (10)	k_{fW} Eq. (12)		
1000	5.13x10^{-7}	4.52x10^{-8}	8.549x10^{-5}	7.308x10^{-9}
1100	1.35x10^{-5}	1.75x10^{-7}	2.302x10^{-4}	5.299x10^{-8}
1200	1.91 x10^{-4}	3.66x10^{-5}	5.259x10^{-4}	2.766x10^{-7}
1300	1.91x10^{-3}	4.75x10^{-4}	1.055x10^{-3}	1.113 x10^{-6}
1400	1.17x10^{-2}	4.25x10^{-3}	1.918x10^{-3}	3.679x10^{-6}
1500	6.20x10^{-2}	2.84x10^{-2}	3.226x10^{-3}	1.041x10^{-5}
1600	2.75x10^{-1}	1.485x10^{-1}	5.078x10^{-3}	2.579x10^{-5}
1700	1.00	6.38x10^{-1}	7.577x10^{-3}	5.741x10^{-5}
1800	3.16	2.32	1.082x10^{-2}	1.171x10^{-4}
1900	8.85	7.35	1.487x10^{-2}	2.211x10^{-4}
2000	2.24x10^{1}	2.06x10^{1}	1.981x10^{-2}	3.924x10^{-4}
2100	5.18x10^{1}	5.26x10^{1}	2.567x10^{-2}	6.589x10^{-4}
2200	1.11x10^{2}	1.23x10^{2}	3.252x10^{-2}	1.058x10^{-3}
2300	2.23x10^{2}	2.65x10^{2}	4.035x10^{-2}	1.628x10^{-3}
2400	4.22x10^{2}	5.36x10^{2}	4.912x10^{-2}	2.413x10^{-3}
2500	7.59x10^{2}	1.02x10^{3}	5.888x10^{-2}	3.467x10^{-3}
2600	1.30x10^{3}	1.86x10^{3}		(4.80x10^{-3})**
2700	2.15x10^{3}	3.22x10^{3}		(6.55x10^{-3})
2750			8.747x10^{-2}	7.651x10^{-3}
2800	3.43x10^{3}	5.37x10^{3}		(8.88x10^{-3})
2900	5.30x10^{3}	8.61x10^{3}		(1.153x10^{-2})
3000	7.94x10^{3}	1.34x10^{4}	1.215x10^{-1}	1.476x10^{-2}

† Source for K_p' : (Ref. 15).

* To convert these units of k_f to (cm^3 mole^{-1} sec^{-1}) multiply values in the table by 82.06 times the temperature in °K.

** Figures in parenthesis were obtained from plot of $K_p'^2$ vs. temperature.

Example Consider now the combustion of a suitable fuel such as methane CH_4 at 300 psia in preheated air producing an adiabatic flame temperature of 2500°K. Suppose under these conditions that together with other constituents the combustion products include nitric oxide, nitrogen and oxygen to the extent of $\chi^{\circ}_{NO} = 0.01$, $\chi^{\circ}_{N_2} = 0.70$, and $\chi^{\circ}_{O_2} = 0.0412$, respectively, where the χ°'s are the equilibrium mole fractions. The following data may be collected for these conditions:

$$T_o = 2500^{\circ}\mathrm{K}$$

$$p = 300 \text{ psia} = 20.4 \text{ atm}$$

$$K_p'(T_o) = 5.89\times10^{-2}; \quad K_p'^{2}(T_o) = 3.47\times10^{-3}$$

$$\Delta H = 43,200 \text{ cal/mole}$$

$$a = \chi^{\circ}_{NO} = 0.01, \quad b = \chi^{\circ}_{N_2} = 0.70, \quad c = \chi^{\circ}_{O_2} = 0.0412$$

Take $(-DT/Dt) = 10^{6\,\circ}$K/sec and $\gamma = 1.30$.

The temperature ratio $(T_o - T')/(T_o - T)$ may now be calculated from Eq. (5) in order to determine the extent of the nitric oxide decomposition. Consider, for instance, the point in the expansion process where the gases have accelerated to a Mach number M of 0.5. At this Mach number and for $\gamma = 1.30$, the values of $T/T_o = 0.964$ and $\rho/\rho_o = 0.884$ may be read from the tables of Keenan and Kaye. This means that the static temperature is 0.964x2500 ≅ 2400°K. Hence,

$$\overline{k_{fD}} = 5.90\times10^{2} \text{ atmos}^{-1} \text{ sec}^{-1} = 1.25\times10^{5} \text{ liters mole}^{-1} \text{sec}^{-1};$$

$$K_p'(T) = 4.912\times10^{-2}, \quad K_p'^{2}(T) = 2.413\times10^{-3}.$$

Substituting these values into Eq. (5) leads to the result

$$\frac{T_o - T'}{T_o - T} = 4.10 \times 10^{-2} \quad ;$$

hence,

$$T_o - T' = 4.10 \times 10^{-2} (2500 - 2400) = 4.1^\circ K \ .$$

This small temperature difference means that there has been only a small amount of nitric oxide decomposition. The extent of the decomposition can be computed from an equation derived in Appendix V; viz.,

$$\xi = \frac{1 - (K_p'/K_{p_o}')^2}{2\left[1 + \frac{(b+c)}{4a} K_p'^2\right]} \quad ,$$

where $\xi = 2x/a$ is the fractional part of nitric oxide which has decomposed. At a temperature of 2495° $K_p' = 3.414 \times 10^{-3}$. Therefore, using the same initial concentrations as above (i.e., $a = 0.01$, $b = 0.20$ and $c = 0.0412$),

$$\xi = 4.95 \times 10^{-4},$$

which means that only about five hundreths of one percent of the initial nitric oxide has decomposed. If the situation is investigated at the point in the expansion where the flow has accelerated to a Mach number of 0.9, $T_o - T'$ is found to be 4.7°K as compared to 4.1°K at $M = 0.5$. Therefore, under these conditions the nitric oxide decomposition reaction has been effectively quenched. On the other hand, if the extent of the decomposition is investigated using Eq. (AII-27) for an adiabatic combustion temperature of $T_o = 3000^\circ K$ and for $a = 0.020$, $b = 0.653$ and $c = 0.415$ with $(-DT/Dt) = 10^6 \, ^\circ K/sec$, then at the point in the expansion where $M = 0.5$ (where the static temperature is $2890^\circ K$) it is found that approximately ten percent of the initial nitric oxide has decomposed.* Approximately one-half of the nitric oxide which was formed when the temperature was increased from $2890^\circ K$ to $3000^\circ K$ is decomposed in expansion to Mach number 0.5. This amount of decomposition corresponds to a flow which is approximately midway between frozen and equilibrium flow. The reason for the discrepancy between the amounts of decomposition for expansion from the two initial temperatures is principally due to the fact that the rate constant k_f increases exponentially with temperature. Consequently, the point of diminishing returns is rapidly reached in the process of increasing the combustion temperature in order to enhance the yield of nitric oxide. The point of diminishing returns can be determined only after (1) the amount of permissible decomposition has been specified, (2) the actual average value of $(-DT/Dt)$ has been computed, and (3) the actual initial concentrations of the three constituents have been determined. Each of these three considerations requires further study. The first involves economic considerations, which will not be investigated here. The second is investigated in the next section (i.e., Section III-C), and the third is studied in detail in Section IV.

---

* The effect of dissociation constituents such as atomic ozygen upon the decomposition of nitric oxide is ignored.

C. Cooling Rates in DeLaval Nozzles

An expression for (-DT/Dt) as a function of the nozzle parameters is derived in Appendix III for nozzles having sonic velocity at the throat section. The consideration of the one-dimensional isentropic expansion of a perfect gas through a De Laval nozzle (Cf. Fig. 2) whose variation of cross-sectional area may be represented by a parabolic expression in the axial coordinate led to the equation (Cf. Eq. AIII-10)

$$\left(-\frac{DT}{Dt}\right) = \frac{2\sqrt{b}}{\ell}\sqrt{R''}\;T_0^{3/2}\;f(M,\gamma)\;, \tag{13}$$

where (-DT/Dt) = rate of cooling at a section of nozzle where the Mach number is M

$$f(M,\gamma) = \left|\frac{2\sqrt{\gamma}(\gamma-1)}{\gamma+1}\;\frac{(A/A^{*}-1)^{1/2}}{A/A^{*}}\left(\frac{T}{T_0}\right)^{3/2}\frac{MM^{*2}}{1-M^{*2}}\right|\;,$$

M = Mach number

ℓ = length of nozzle from entrance section to the throat in feet

$b = (A/A^{*})_{\text{nozzle entrance}} - 1$

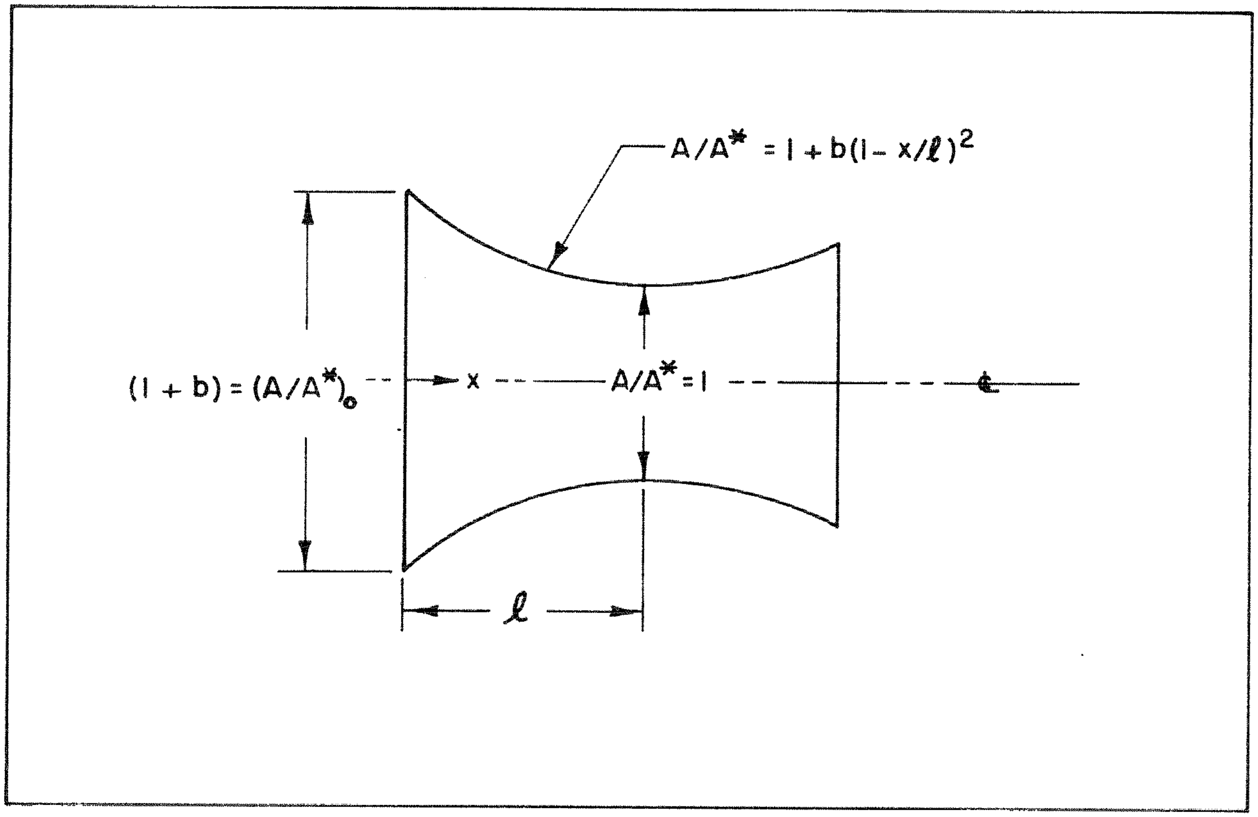
A/A^{*} = area at any cross-section divided by the nozzle throat area

$\sqrt{R''} = \sqrt{32.2\text{x}53.34\text{x}1.8} = 55.5$ ft/sec$(^{o}\text{K})^{1/2}$ in order to be consistent with units of ℓ in feet and T_o in ^{o}K

$$M^{*} = M\left(\frac{\gamma+1}{2}\,\frac{T}{T_0}\right)^{1/2}$$

FIG. 2

DE LAVAL NOZZLE
NOMENCLATURE



The values of $f(M, \gamma)$ are tabulated in Table AIII-1 as a function of Mach number in the range from $M = 0.05$ to $M = 1.50$ for $\gamma = 1.30$ and $\gamma = 1.40$. An expression for interpolating to other values of γ is given by Eq. (AIII-12).

Consider the expansion of combustion gases through a De Laval nozzle. Let the gases enter a nozzle of convergent-length $\ell = 0.10$ ft. at a total temperature $T_o = 2500^\circ K$ and at an entrance Mach number $M_o = 0.310$, which corresponds to an entrance area ratio $A/A^* = 2.000$ or to $b = A/A^* - 1 = 1.000$. For these conditions the variation of $(-DT/Dt)$ through a portion of the nozzle has been computed from Eq. (13) for $\gamma = 1.30$ and $\gamma = 1.40$, and the results are presented in Table III.

TABLE III.

The Variation of the Cooling Rate Through De Laval Nozzles as a Function of the Mach Number ($T_o = 2500^\circ K$, $\ell = 0.10$ ft., $b = 1.000$, $M_o = 0.310$).

$M \longrightarrow$		0.310	0.4	0.5	0.7	0.9	1.1	1.3	1.5
$\left(-\frac{DT}{Dt}\right) \times 10^{-6}$ $^\circ K \cdot sec^{-1}$	$\gamma = 1.30$	0.696	1.70	3.27	6.72	13.0	21.1	26.9	30.0
	$\gamma = 1.40$	0.948	2.26	4.44	8.94	16.6	26.6	32.7	36.3

The figures in Table III substantiate the previous assertion that $(-DT/Dt)$ is of the order of 10^6 °K/sec.

It should be noted that the cooling rate $(-DT/Dt)$ varies as the three-halves power of the total temperature T_o. Referring to Eq. (5)

and ignoring for the moment the variation of k_f and of the gas composition as reflected through the terms $\sum(\nu_k'' - \nu_k')^2/\chi_k$ and $\prod \chi^{\nu_j'}$ with T_o, it is clear that the temperature ratio $(T_o-T')/(T_o-T)$ varies approximately as the minus one-half power of the total temperature, since $(-DT/Dt)$ varies as the three-halves power of T_o. This remark presupposes that comparisons are made at corresponding points in the flow, i.e., ρ/ρ_o and $K_p(T)/K_p(T_o)$ are held constant. Since the change in $T_o^{-1/2}$ due to variations of T_o in the range 2500 - 3000°K is only secondary importance, the change of the temperature ratio $(T_o-T')/(T_o-T)$ is principally due to the variation in k_f and in the two terms representing composition. The rate constant varies exponentially with temperature whereas the variation in the product of the two composition terms is more nearly linear in temperature. Consequently, the assertion, made in the preceding section, that the principal variation in the temperature ratio $(T_o-T')/(T_o-T)$ with increase in total temperature T_o is due to k_f is justified.

The effects of the entrance Mach number and of the nozzle geometry upon the variation of $(-DT/Dt)$ through the nozzle may be obtained also from Eq. (13). These effects can be studied by comparing different nozzle configurations at the point in the expansion of the gases where the Mach number (or the static temperature) is the same for all nozzle configurations. If the length of the converging portion of the nozzle and the ratio of specific heats γ are held constant and if the comparisons are made at the same Mach number M and static temperature T, then from Eq. (13)

$$\left(-\frac{DT}{Dt}\right)_{M_o}\Big/\left(-\frac{DT}{Dt}\right)_{M_o=0.310} = \sqrt{b} \quad , \qquad (14)$$

where M_o is the entrance Mach number. The cooling rates for a given entrance Mach number are compared in Table IV to the cooling rates corresponding to $M_o = 0.310$ (which was selected for the sake of convenience since $b_{M_o} = 0.310 = 1.00$). For $\ell = 0.10$ ft and for $\gamma = 1.30$, the variation of the cooling rate through the nozzle may be obtained by multiplying the values in Table III by the appropriate values in Table IV. The values in Table IV indicate the advantages of large area ratios A/A* at the entrance section. Other design considerations, however, will limit the maximum practical values for the entrance area ratio.

It is apparent from Eq. (13) that the cooling rate (-DT/Dt) is inversely proportional to the nozzle convergent-length ℓ . Decreasing ℓ by one-half increases (-DT/Dt) by a factor of two.

The use of some other curve instead of a parabola to represent the nozzle shape will lead to somewhat different values of (-DT/Dt). For nozzles whose shape may be represented by

$$A/A^* = 1 + b\left(\frac{A}{A^*} - 1\right)^n \qquad (15)$$

(-DT/Dt) is given by Eq. (14) in Appendix III. For the case b = 1, which corresponds to an entrance Mach number of 0.310, the expression for

TABLE IV

Effect of Entrance Mach Number upon Cooling Rates in De Laval Nozzle for $\gamma = 1.3$

M_o	$(A/A^*)_o$	$\left(-\frac{DT}{Dt}\right)_{M_o} \Big/ \left(-\frac{DT}{Dt}\right)_{M_o = 0.310}$
0.05	11.721	3.274
0.10	5.885	2.210
0.15	3.952	1.718
0.20	2.994	1.412
0.25	2.426	1.194
0.30	2.054	1.025
0.31	2.000	1.000
0.35	1.793	0.8905
0.40	1.602	0.7752
0.45	1.459	0.6775
0.50	1.348	0.5899
0.55	1.261	0.5109
0.60	1.193	0.4393

(-DT/Dt) reduces to

$$\left(-\frac{DT}{Dt}\right) = \frac{n}{\ell}\left(\frac{A}{A^*} - 1\right)^{\frac{n-2}{2n}} (R'')^{1/2}\, T_o^{3/2} f(M, \gamma) . \qquad (16)$$

In the limit as n becomes very large

$$\left(-\frac{DT}{Dt}\right) \rightarrow \frac{n}{\ell}\left(\frac{A}{A^*} - 1\right)^{1/2} (R'')^{1/2}\, T_o^{3/2} f(M, \gamma) . \qquad (17)$$

This means that (-DT/Dt) is proportional to the exponent n in the expression for the area variation through the nozzle. Consequently, in order to obtain maximum values of (-DT/Dt) large values of the exponent n should be incorporated into the nozzle design.

In recapitulation, in order to maximize the cooling rate (-DT/Dt) in the converging portion of the nozzle, nozzles should be designed with (1) large entrance area-ratios A/A*, (2) small entrance-to-throat lengths ℓ, and (3) rapidly decreasing area ratios A/A*, corresponding to large values of n in Eq. (15).

D. Temperature Below Which the Nitric Oxide Decomposition Effectively Ceases

One question which has not been answered so far in the discussion is: at what temperature does the decomposition reaction of nitric oxide effectively cease? In as much as nitric oxide decomposes even at room temperature (although at an immeasurably slow rate), any criterion which is established should specify a maximum permissible rate of decomposition below which the reaction may be regarded as effectively quenched. The criterion must be based on the rates of cooling available in the downstream flow-path. For instance, suppose it were possible to reduce the total temperature of the gas from a very high temperature (greater than 3000°K say) to room temperature at the rate of 10^6 °K/sec. At this cooling rate the decomposition reaction is effectively quenched at some temperature above 2500°K, whereas at a lower cooling rate the quenching temperature would be lower. For this reason downstream cooling rates must be considered. With these preliminary statements it should be clear that the criterion could be stated in the form of an inequality; viz.,

$$\int_{t_1}^{t_1+\tau} \left(\frac{d\xi}{dt}\right) dt < (\Delta\xi)_{max} \quad , \tag{18}$$

where $(\Delta\xi)_{max}$ is the maximum permissible decomposition, and τ is the period of time required to reduce the total temperature to some value near room temperature. In order to determine the temperature under consideration from this inequality, it is necessary to know the downstream history of $d\xi/dt$.

Fortunately, because of the enormous decrease in the rates of nitric oxide decomposition with temperature, it is possible to obtain an approximate temperature below which the decomposition has effectively ceased. In Appendix IV an expression [Cf. Eq. (A IV-5)] is derived for the time required to decompose small amounts of nitric oxide under equilibrium conditions with the temperature held constant. This equation has been used to calculate the time to decompose one percent and ten percent of the initial nitric oxide concentration at various temperatures as shown in Table V. The initial concentrations of nitric oxide, nitrogen and oxygen were assumed to be such that $K_{p_o}' = a^2/bc = 3.47 \times 10^{-3}$, which corresponds to equilibrium concentrations at 2500°K. Two values of the initial concentration of nitric oxide were used as parameters. Two different sets of decomposition times are reported in the body of the table. The values not given in parenthesis were calculated using k_{f_D} from Eq. (10), and those in parenthesis were obtained using k_{f_W} from Eq. (12).

From these values of the decomposition time for equilibrium conditions it is clear that the decomposition reaction during flow may be conservatively considered to have ceased when the total temperature has been reduced to 1500°K provided that heat exchanger equipment exists downstream with sufficiently high cooling rates to reduce the total temperature to about room temperature in times of the order of ten seconds or less. This requirement is quite conservative for modern heat exchanger equipment.

A more elegant method of determining the temperature under consideration would have been to compute the average values of $(-DT/Dt)$ from Eq. (5) which is required to prevent the temperature ratio $(T_o - T')/(T_o - T)$ from exceeding a previously specified value. However, the statement that the decomposition time reaction of nitric oxide effectively ceases below 1500°K is all that is required in the practical applications which are to be considered in this thesis.

TABLE V

Decomposition Time as a Function of Temperature
for $Kp_o'^2 = 3.47\times10^{-3}$ (T_o = 2500°K)
(Seconds)

T °K	One Percent Decomposition		Ten Percent Decomposition	
	a = 0.01	a = 0.02	a = 0.01	a = 0.02
2200	0.013* (0.015)**	0.007 (0.007)	0.151 (0.168)	0.076 0.084
2100	0.024 (0.025)	0.012 (0.012)	0.272 (0.276)	0.136 (0.138)
2000	0.051 (0.048)	0.025 (0.024)	0.567 (0.522)	0.283 (0.260)
1900	0.122 (0.101)	0.061 (0.051)	1.35 (1.12)	0.675 (0.560)
1800	0.330 (0.242)	0.165 (0.121)	3.65 (2.68)	1.82 (1.33)
1700	1.03 (0.655)	0.514 (0.327)	11.3 (7.20)	5.65 (3.60)
1600	3.70 (2.00)	1.85 (1.00)	40.6 (21.9)	20.3 (11.0)
1500	16.4 (7.50)	8.19 (3.75)	180 (82.4)	89.9 (41.2)

* Values not in parenthesis: k_{f_D} from Eq. (10) was used;

** Values in parenthesis: k_{f_W} from Eq. (12) was used.

IV. THERMOCHEMICAL STUDIES FOR THE PRODUCTION OF NITRIC OXIDE

In any commerical process for the production of chemicals one of the considerations of crucial importance is the yield of the chemical products under various conditions of manufacture. As was indicated in the Introduction, nitric oxide is a commercially useful chemical product suitable for manufacture by use of the methods of the gas turbine cycle. The question naturally arises as to what are the conditions that lead to favorable yields of nitric oxide.

Since nitrogen and oxygen unite to form nitric oxide according to the reversible reaction

$$N_2 + O_2 \rightleftharpoons 2NO \quad , \tag{19}$$

several observations can be made. First, since the forward reaction is endothermic, the equilibrium is shifted to the right as the temperature is increased, according to the Principle of Le Chatelier. Furthermore, higher temperatures are desirable also because the rate of attainment of equilibrium is greatly accelerated at elevated temperatures. Second, since there is no change in the number of moles during the reaction, the direction of the reaction is unaffected by pressure. Consequently, the pressure level can be determined from other considerations. Third, since the reactants, nitrogen and oxygen, are abundantly available in the air, it is obvious that air will be the principal source of the starting materials. It may, however, be desirable under certain conditions to add extra oxygen to the air. This matter is discussed later on.

The high temperatures may be obtained in three ways: (1) inlet air may be heated in a heat exchanger; (2) fuel may be added to the inlet air which was preheated as in (1), and the mixture burnt; and finally, (3) the inlet air, which has been heated during the compression process in a gas turbine cycle, may be passed through heat transfer equipment from where the air passes to a combution chamber where fuel is added and burnt. Daniels uses the second method in his nitrogen fixation furnace (Ref. 2).

The combustion process in methods (2) and (3) above will consume oxygen in proportion to the amounts of fuel employed, thus reducing the oxygen available for nitric oxide formation. As the relative amount of fuel is increased, however, higher and higher temperatures are reached up to the point where the stoichiometric ratio of fuel and air is reached. These two processes, the higher temperatures and increased competition for the oxygen by the fuel, suggest that as the fuel-to-air ratio is increased a maximum yield of nitric oxide occurs before the stoichiometric mixture ratio is reached.

At this point the question is raised whether or not it would be advisable to add extra oxygen to the inlet air in order to increase the yield of the nitric oxide. If oxygen-enriched air is employed, the optimum combination of inlet air and inlet fuel temperatures, amount of extra oxygen to be added, etc., can only be determined by economic considerations. The most important of the economic considerations will be the relative value of the nitric oxide produced to the cost of the extra oxygen added. However, in any event, the effect of oxygen enrichment upon the yield of nitric oxide should be investigated. Accordingly, the yield of nitric oxide as a function of the fuel-to-oxygen ratio and also of the nitrogen-to-oxygen ratio (which is a measure of the amount of oxygen added) is desired. In order to keep the labor of making these calculations to a minimum, the method of Hottel, Williams and Satterfield (Ref. 17) was employed for calculating the adiabatic flame temperatures and the corresponding equilibrium concentrations.

The calculations were made for methane CH_4 burning in air or oxygen-enriched air at a pressure of 300 psia. The results of the computations are shown in (Fig. 3). The inlet temperatures of both the fuel and air were taken to be $1300^\circ K$, a temperature which represents an upper limit for the inlet temperatures obtainable because of the limitations of heat exchanger equipment. In as much as somewhat lower inlet temperatures will necessarily have to be employed in practice, the yields of nitric oxide shown in (Fig. 3) represent maximum expected values. The effect of lower inlet temperatures is indicated in Table VI, where the nitric oxide yields are contrasted at $1000^\circ K$ and $1300^\circ K$. Looking first at the case where methane is burning in air without oxygen enrichment, (Fig. 3) shows that the nitric oxide yield increases rapidly up to a value of 1.2 percent at $C/O = 0.17$, levels off, and then slowly decreases to 0.9 per cent at the stoichiometric mixture ratio of $C/O = 0.25$. For the case where the air is oxygen-enriched to the extent that $N/O = 2.0$

(which corresponds to an increase in the oxygen concentration of eighty-eight percent) (Fig. 3) shows that the nitric oxide yield increases rapidly up to a value of 2.4 percent at C/O = 0.16, and then falls rather rapidly to 1.6 percent at the stoichiometric mixture ratio of C/O = 0.25. Therefore, the maximum yield is doubled (i.e., increased from 1.2 to 2.4 percent) when the oxygen concentration is increased approximately by ninety percent. Under these conditions it is questionable in an economic sense whether oxygen-enrichment is worth-while. Furthermore, the maximum yield for N/O = 2.00 occurs at a temperature of approximately 2900°K, whereas the corresponding temperature for N/O = 3.76 is 2500°K. In Section III-B it was noted that the decomposition of nitric oxide at temperatures in the neighborhood of 3000°K is likely to be large even for the extremely large cooling rates of small sized De Laval nozzles. In summary, it may be said that although oxygen-enrichment leads to appreciable increases in the yield of nitric oxide, it is questionable whether its use is justified.

Table VI shows the effect of inlet temperature upon the yield of nitric oxide. From these figures and the fact that 1300°K represents the upper limit of inlet temperatures, it is clear that yields less than one percent are to be expected without oxygen-enrichment.

If another high-energy fuel had been used instead of methane, the results would not have been appreciably affected. In practice, the use of a liquid fuel is indicated in order to reduce the pumping work required to increase the inlet pressure to values in excess of 300 psia.

FIG. 3. YIELD OF NITRIC OXIDE vs FUEL CONSUMPTION AND OXYGEN ENRICHMENT

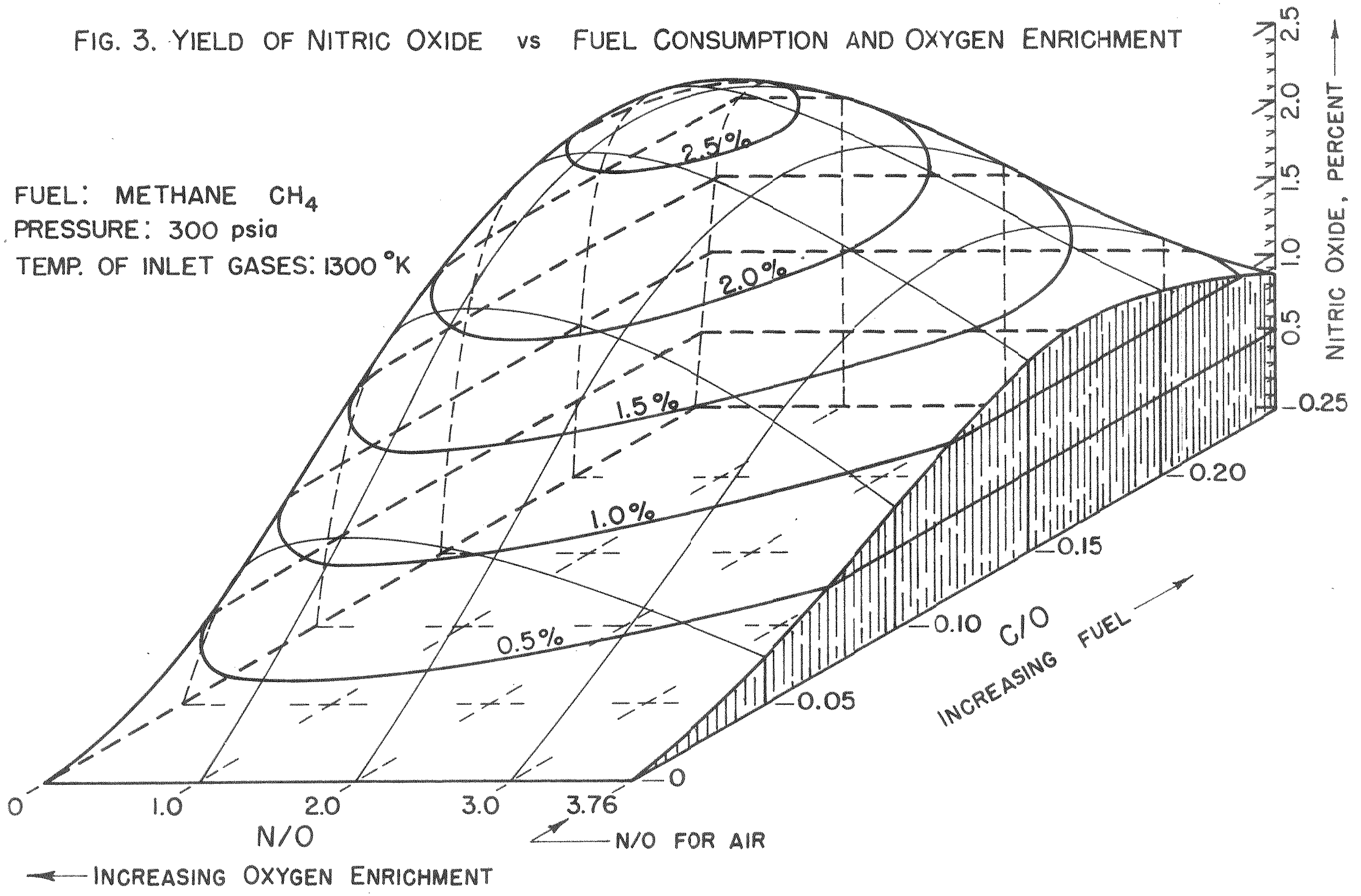


TABLE VI

Yield of Nitric Oxide in Percent for N/O = 3.76 as a Function of Gas Inlet Temperature for Various C/O Ratios.

T_{in} °K	C/O			
	0.10	0.15	0.20	0.25
1000	0.32	0.67	0.72	0.62
1300	0.75	1.2	1.2	0.89

On the other hand, the heat of vaporization would have to be supplied for liquid fuels, and the problems of increasing the inlet temperature of liquid fuels to temperatures approaching 1300°K would be formidable. In general, slightly lower yields are to be expected for liquid fuels. In summary, the maximum yields of nitric oxide to be expected for the conditions associated with the gas turbine cycle are limited to values of about one percent.

V. CONCLUSIONS FOR PART A - - CHEMICAL CONSIDERATIONS

The following conclusions evolve from the discussion concerning chemical considerations given in Section III and IV:

1. The De Laval nozzle will quench the nitric oxide decomposition reaction by limiting the decomposition to about five hundreths of one percent for combustion temperatures of 2500°K or less.
2. The De Laval nozzle is only moderately effective in freezing the decomposition reaction for combustion temperatures in the neighborhood of 3000°K.
3. The cooling rate experienced by the gas flowing through a De Laval nozzle increases very rapidly to enormous values, reaching such rates as 10^6 °K/sec when the gases have expanded to a Mach number of 0.5 and 10^7 °K/sec at a Mach number of 0.8 for typical nozzle geometries.
4. The cooling rate (-DT/Dt) in the converging portion of De Laval nozzles may be maximized by designing the nozzles with (1) large entrance area-ratios A/A^*, (2) small entrance-to-throat lengths ℓ, and (3) rapidly decreasing area ratios A/A^*.
5. The nitric oxide decomposition reaction may be considered to have ceased when the total temperature has been reduced to 1500°K, assuming the existence of heat exchanger equip-

ment downstream which is capable of reducing the temperature of the gases to room temperature in times of the order of ten seconds or less.

6. The use of oxygen-enriched air in order to increase the yield of nitric oxide is of doubtful value.
7. The maximum yields of nitric oxide to be expected for the conditions associated with the gas turbine cycle are limited to values of about one percent and will occur at fuel-to-air ratios less than stoichiometric.

VI. GAS TURBINE CYCLE

A. General Introduction

In the Introduction it was explained that the energy of the exit gases from the De Laval nozzle must be extracted in order (1) to prevent the recovery of the stagnation or combustion-chamber temperature of the gases when they are finally brought to rest and (2) to utilize the available energy of the exit gases for useful purposes because of economical considerations. The use of a turbine for the purposes of energy extraction was mentioned as one possible method. A brief discussion of the gas turbine cycle and of the associated problems was presented also. It was noted that heat exchanger limitations placed restrictions upon the attainable combustion temperatures. However, long before heat transfer equipment becomes a limiting factor in the attainment of higher combustion temperatures the problem of turbine blade cooling enters. The turbine blade cooling problem is considered in some detail from the thermodynamic point of view in the next subsection. The effect of transpiration and film cooling upon cycle performance is considered in a following subsection. The important problem of turbine blade stresses is not investigated.

Temperatures of 2500°K (4040° F) and higher are desired in the combustion chamber in order to enhance the yield of nitric oxide. In addition to the desirability of higher temperatures for the sake of increasing the yields of nitric oxide, higher operating temperatures can lead to highly significant increases in cycle performance. The advantages occurring from higher turbine inlet temperatures are well known (Cf, for instance, Ref. 18), and consequently will not be discussed here. Attention is merely called to the fact that for a given flow rate the output-power per pound of working fluid is proportional to the turbine inlet temperature.

B. Turbine Blade Cooling

The problems of turbine blade cooling associated with total temperatures of 2500°K (4040°F) and above are quite formidable. So severe are the operating conditions the use of unconventional methods of cooling the turbine blades is indicated. The various methods of turbine blade cooling are discussed next.

1. Methods of Turbine Blade Cooling

A survey of the status of turbine blade cooling problem at the end of World War II together with a summary of NACA investigations of gas turbine blade cooling to the end of 1947 was presented in a paper by Ellerbrock (Ref. 19). The results of the analytical investigations of the NACA up to the end of 1947, as reported by Ellerbrock, are summarized in Table VII. Three possible methods of turbine blade cooling by means of conductive heat transfer were investigated: (1) rim cooling, in which the temperature of the rim of the turbine wheel is reduced by the cooling fluid and the blades are cooled by heat conduction to the rim; (2) hollow-blade air cooling, in which air is forced through hollow turbine

TABLE VII

Methods of Turbine Blade Cooling

Type of Cooling	Approximate Maximum Allowable Effective Gas Temperature		$\frac{\text{Coolant Rate}}{\text{Hot Gas Rate}}$ Percent
	°F	°K	
I. Rim	1800	1255	
IIa. Hollow Blade without Inserts	2000	1360	13
IIb. Hollow Blade with Inserts	2300	1533	5
IIIa. Internal Liquid Cooling	3000	1922	
IIIb. Ceramic Coating with Internal Liquid Cooling	3500	2200	

---

* Computed from figures presented in Reference 19, assuming 1500°F as the maximum permissible effective gas temperature for turbine blades without cooling.

blades and out through the blade tips; and (3) internal liquid cooling, in which a liquid is circulated in a closed system through the passages in the turbine blades and disks. In method (2) the effect of inserts in the hollow blade passages was also investigated. In connection with internal liquid cooling the effect of ceramic coatings was also reported. From the figures in Table VII it is clear that only internal liquid cooling with ceramic coatings on the turbine blades allows an approach to temperatures of 2500°K. Objections to this method of cooling arises on the grounds that a considerable amount of energy is conducted through the blade walls to the cooling fluid. Unless a significant proportion of this energy can be recovered in an economically useable form, then this method will suffer economic disadvantages as compared with other possible methods of turbine blade cooling. Although considerable progress (including the operation of a secondary turbine by the exhaust from the rotating steam generator) has been made with internal water cooling by Schmidt in Germany (Ref. 20), it was felt that a more satisfactory solution to the turbine blade cooling problem for temperatures near 2500°K could be obtained by the methods of transpiration cooling and of film cooling. These methods do not suffer from the previously mentioned objection to internal liquid cooling. For aircraft applications the amount of cooling fluid consumed in transpiration or film cooling assumes the same vital importance as fuel consumption. For stationary, open-cycle power-plant applications, however, the amount of cooling fluid employed is of only secondary importance. In a power-plant cycle in which nitric oxide is being manufactured, the coolant injected into the main stream tends to increase the recovery costs of the nitric oxide by further diluting the concentration of the desired product. The amount of coolant required in transpiration and film cooling is, however, so small that the effect of coolant injection both upon nitric oxide recovery costs and upon cycle performance assumes only secondary importance. For this reason, therefore, only the application of transpiration and film cooling methods to the turbine blade cooling problem has been considered in this thesis.

Transpiration cooling is the cooling which is afforded to a surface by the protective layer formed when a fluid is injected through a porous medium into a hot atmosphere. If the fluid is a liquid, the cooling is called liquid transpiration cooling, or more picturesquely, sweat cooling. If the fluid is a gas, the cooling is called gaseous transpiration cooling, or alternatively, porous wall cooling. Film cooling is the cooling afforded to a surface by the protective film formed when a fluid is injected through a relatively small number of holes or slots and then is carried downstream over the surface by the main stream. Again, the coolant may be either a gas or liquid. However, liquids are used almost invariably for this purpose, and hence the term "film cooling" has come to imply liquid coolants. Film cooling resembles liquid transpiration cooling, but differs in the fact that the film must be renewed at successive points downstream as the liquid is evaporated, whereas in liquid transpiration cooling the coolant is injected more or less uniformly over the surface.

A certain amount of experimental work and theoretical study has been done on transpiration and film cooling. The principal theoretical treatments for the case of turbulent flow, which is the type of flow of most practical interest in rocket motors and in gas turbines, are those of Rannie (Ref. 21), Friedman (Ref. 22), and Crocco (Ref. 23). References to experimental work may be found in these papers. Crocco's treatment is the most general and is an extension of Rannie's approximate theory of gaseous transpiration cooling to the case in which the coolant may react chemically with the main stream gases. Crocco develops a general expression, which is valid for gaseous and liquid transpiration cooling and for film cooling. A more intensive treatment of the case for film cooling is given by Knuth (Refs. 24 and 25), who eliminates one of the basic assumptions common to all of the theoretical studies. Knuth does not make the assumption that the influence of the injected fluid is confined to the laminar sublayer, but instead considers the influence of the coolant upon the turbulent core.

Recently E. R. G. Eckert and Livingood have considered the application of a transpiration cooling to gas turbine blades. Calculations pertinent to the laminar flow region near the leading edge of the blades is presented in Reference 26. Calculations for an arbitrarily chosen transpiration-cooled gas-turbine blade are presented in Reference 27, using the Rannie-Friedman approximate theory for the turbulent regions around the blades.

It is well known that the boundary layer over the forward portion of turbine blades and airfoils is laminar for a certain distance along the blade. At a certain point along the blade and near the point of minimum pressure, transition occurs from a laminar to a turbulent boundary layer. The extent of the laminar layer is longer on the pressure side of the blade than on the suction side. The exact point of transition cannot be predicted accurately for transpiration-cooled blades because little is known as yet about the effect of the injected coolant upon the location of the transition point. Qualitatively, one would expect the injected coolant to have a destabilizing influence and to shift the transition point forward. On the other hand, cooling of the boundary layer is expected to have a stabilizing effect according to the work of Lees (Ref. 28).

The coolant requirements are greater for a turbulent boundary layer than for a laminar one because of the additional heat transported by turbulent exchange, which is present in the turbulent boundary layer but not in the laminar layer. The recent work of Giedt (Ref. 29) indicates that the level of turbulence in the gas stream flowing normal to the axis of a solid circular cylinder increased the local heat transfer coefficients between ten and twenty percent over low turbulence level values for the otherwise laminar boundary layer over the entire reach of the laminar boundary layer. In as much as a high turbulence level is to be anticipated in the main stream gases flowing past gas turbine blades, the coolant requirements can be expected to be greater than those calculated for the laminar regions on the basis of a purely laminar boundary layer.

The purpose of this section is to determine conservative estimates of the coolant requirements for both transpiration and film cooling of gas turbines operating with combustion temperatures near 2500°K. Consequently, the boundary layer around the entire length of the blades is assumed to be turbulent. This assumption not only leads to conservative estimates of coolant requirements, but also leads to simplications in the calculations. Next, the thickness of the laminar sublayer of the turbulent boundary layer is assumed to be very much smaller than the radius of curvature of the turbine blades so that the blades may be treated as flat plates. Under these two assumptions the problem reduces to Rannie's approximate theory of porous wall cooling (Ref. 21) for the case of gaseous transpiration cooling or to an extention of Rannie's theory along the lines taken by Crocco (Ref. 23) for the cases of liquid transpiration cooling and of film cooling.

2. Transpiration Cooling with a Gas

The coolant requirements for transpiration-cooled turbine blades are determined by using Rannie's approximate theory of porous wall cooling in which the coolant gases are chemically inert with respect to the main stream gases. An outline of the assumptions involved in the theory is given in Appendix VI, where an approximate expression, which is valid for small coolant flow rates, is derived. The derivation leads to the simple equation

$$\frac{w_c}{w_g} \cong L_G \frac{c_f}{2} = \frac{c_{p_m}(T_g - T_w)}{c_{p_w}(T_w - T_r)}\left(\frac{c_f}{2}\right) , \qquad (20)$$

where w_c is the coolant mass flow rate per unit area of porous wall; w_g is the main stream flow rate per unit area; T_g, T_w, and T_r are the total temperatures of the main stream, at the wall surface, and of the coolant in the reservoir on the inside of the turbine blades, respectively; c_{p_m} is an average value of specific heat at constant pressure between T_w and T_g; c_p is the average value of c_p between T_r and T_w; c_f is the skin friction coefficient ($= \tau_w/\frac{1}{2}\rho u_g^2$). A comparison of Eq. (20) with Eq. (19) of (Ref. 21) is shown in (Fig. 4). The variation of L_G for $c_{pm}/c_{pw} = 1$ over a number of temperature-combinations is shown in Table VIII. In general, L_G is less than 3.0 for most temperature combinations, and therefore the approximate expression for w_c/w_g given by Eq. (20) represents a good approximation to Rannie's more complicated equation in the range of practical interest in turbine applications. Furthermore, the approximate expression gives conservative values of the coolant requirements. In addition, Eq. (20) clearly shows the effects of the important parameters involved. The parameter, which is at the disposal of the designer, is the temperature ratio $(T_g-T_w)/(T_w-T_r)$. The coolant requirements decrease with increasing allowable turbine-blade wall temperatures. The temperature difference T_w-T_r in the denominator is an important factor and clearly indicates the advantages of reducing the temperature of the inlet coolant air. If the coolant air is bled from the high pressure end of the compressor, the use of an aftercooler in the coolant line between the compressor and the turbine would be clearly beneficial in reducing the

FIG. 4

COMPARISON OF APPROXIMATE EQUATION FOR COOLANT AIR REQUIREMENTS WITH RANNIE'S THEORY FOR GASEOUS TRANSPIRATION COOLING

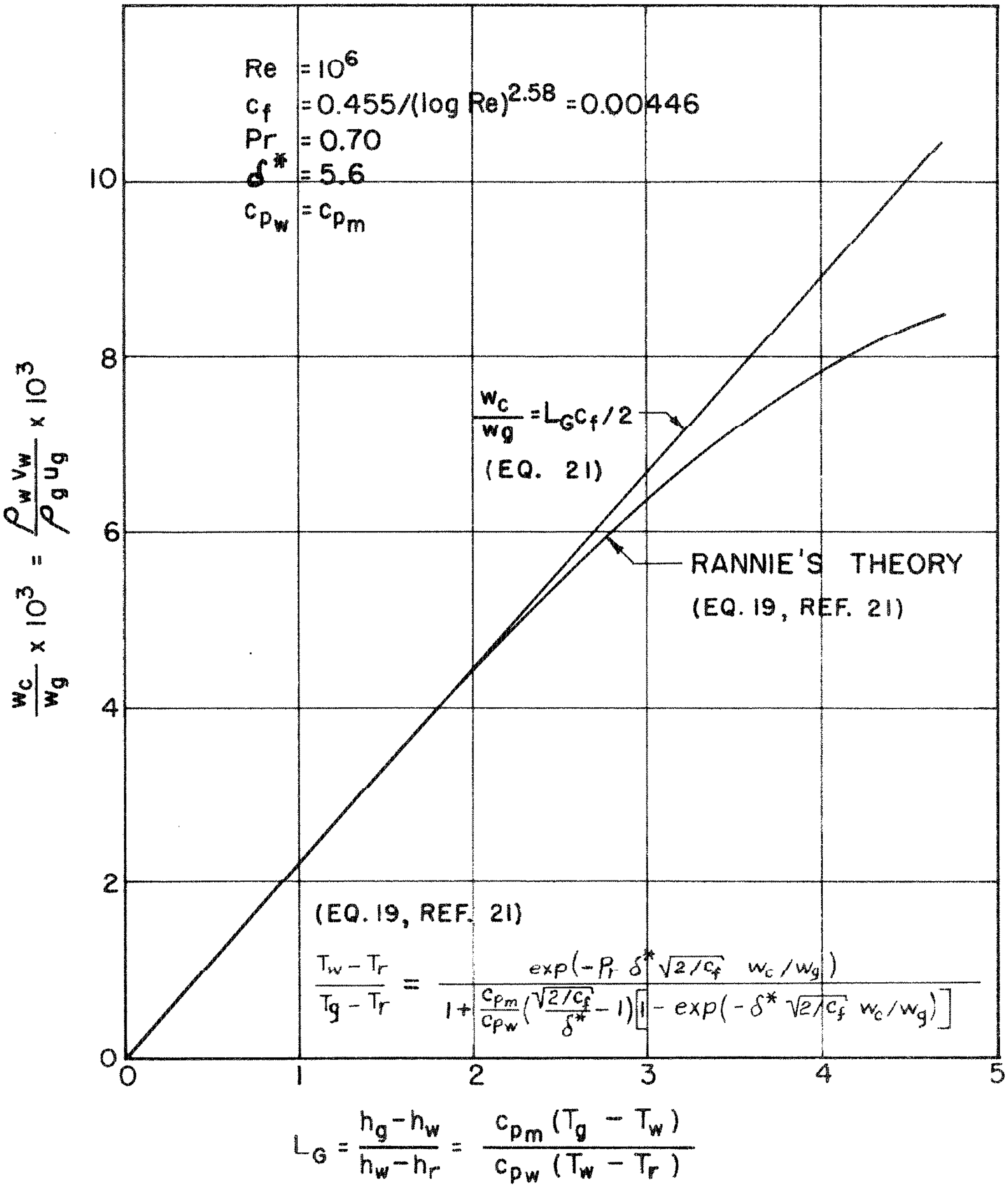


TABLE VIII

Variation of the Parameter $L_G = c_{p_m}(T_g - T_w)/c_{p_w}(T_w - T_r)$

For a Number of Temperature Combinations

$(c_{p_m}/c_{p_w} = 1)$

T_g °K	T_r °K	Wall Temperature T_w, °K		
		1150	1075	900
1400	300	0.294	0.517	0.834
"	600	0.455	0.882	1.66
1600	300	0.530	0.794	1.17
"	600	0.818	1.35	2.33
1800	300	0.765	1.07	1.50
"	600	1.18	1.82	3.00
2000	300	1.00	1.34	1.83
"	600	1.54	2.29	3.67
2200	300	1.24	1.62	2.17
"	600	1.91	2.76	4.33
2400	300	1.47	1.90	2.50
"	600	2.27	3.23	5.00

coolant consumption.

An upper limit for w_c/w_g may be obtained by employing the values of c_f for a flat plate in turbulent flow. [That c_f for a flat plate does provide an upper limit can be seen from the discussion given by Goldstein (Ref. 30, p. 406)]. For the Reynolds number ($Re_c = u_g c/\nu_g$) range from 10^5 to 10^9, Prandtl found that the formula

$$c_f = 0.455(\log_{10} Re)^{-2.53} \tag{21}$$

gives satisfactory values for c_f. [Cf. Goldstein Ref. 30, p. 365 and Fig. 110)]. The values of c_f for the above formula have been computed and tabulated by H. Schlichting (Ref. 31, p. 104). The ratio of the mass flow rates of the coolant and main stream ($W_c/W_g = \rho_c v_c A_s/\rho_g u_g A_p$) is more useful than the ratio of the mass flow rates per unit area ($w_c/w_g = \rho_c v_c/\rho_g u_g$). W_c/W_g is obtained from w_c/w_g by multiplying by the surface area of the turbine blades A_s through which the cooling fluid is injected and dividing by the passage area between the blades A_p through which the main stream gases flow; that is,

$$W = w A_s/A_p \quad . \tag{22}$$

The surface area of the turbine blades A_s may be expressed approximately as

$$A_s = 2 x s \quad ,$$

where $x_m = \frac{1}{2}(x_1 + x_2)$, x_1 is length of one curved surface of blade measured parallel to the flow, x_2 is length of the other side of blade, and s is the span of blade. The passage area A_p may be expressed as

$$A_p = sd \ ,$$

where d is the average width of passage between adjacent blades measured normal to the flow. Therefore,

$$A_s/A_p = 2(x_m/c)(c/d) \ , \tag{23}$$

where c is the chord length of blade. Vincent (Ref. 32, p. 500) gives the range of the solidity ratio c/d as 1.37 - 1.70 for turbine applications. A consideration of impulse blade shapes leads to the conclusion that x_m/c will lie between 1.0 and 1.5. Consequently, the practical limits for A_s/A_p are 3.0 and 5.0. These two figures have been used as limiting values.

The mass-flow-rate ratio W_c/W_g is tabulated as a function of the main-stream gas temperature T_g in Tables IX, X and XI for Reynolds numbers based on the blade chord of 10^5, 10^6 and 10^7, respectively. The wall temperature T_w, the reservoir temperature T_r and the area ratio A_s/A_p are employed as parameters in each of the tables.

From these tables the dependence of the coolant requirements upon the different temperatures is clearly illustrated. Referring to Table X for $Re_c = 10^6$, it can be seen that (1) in order to reduce the wall temperature T_w by 250°K from 1150°K to 900°K, the coolant flow must be doubled (other variables held constant); (2) in order to maintain the wall temperatures constant when the coolant inlet temperature T_r is increased from $300^{\circ}K$ to $600^{\circ}K$ requires that the coolant flow rate must be increased by about 150 percent; (3) in order to maintain the wall temperature constant when the main stream gas temperature T_g is increased by $400^{\circ}K$ demands that the coolant flow rate be increased by about 150 percent.

3. Transpiration Cooling with a Liquid

An approximate expression for the coolant evaporation rate for liquid transpiration cooling in turbulent flow is derived in Appendix VII by an extension of Rannie's approximate theory of porous wall cooling in a manner similiar to that employed by Crocco.* Only the case in which the coolant is chemically inert with respect to the main stream gases is considered. The assumptions involved in the theory may be found in the first part of Appendix VII. The essential difference between gaseous and liquid transpiration cooling lies in the fact that in the case of gaseous transpiration cooling the heat transfer to the porous wall is governed by the coolant flow rate, whereas in the case of liquid transpiration cooling the rate of heat transfer determines the evaporation rate. While the coolant flow rate in gaseous transpiration cooling may be selected arbitrarily over a wide range, the evaporation rate and hence the minimum coolant-injection rate is determined by the flow conditions of the main stream. Under the assumption that the evaporation rates are small, which is true for conditions of practical interest in gas

---

* The analysis presented in Appendix VII was made prior to the publication of Crocco's admirable paper.

TABLE IX

The Effect of Important Parameters Upon Coolant Requirements in Gaseous Transpiration Cooling of Turbine Blades

$c_{pm}/c_{pw} = 1; \ Re_c = \rho_g u_g c/\mu_g = 10^5$

T_w °K	T_g °K	Coolant Flow Rate W_c/W_g in Percent			
		A_s/A_p = 3.00		A_s/A_p = 5.00	
		Reservoir Temperature T_r, °K			
		300	600	300	600
1150	1400	0.31	0.48	0.52	0.81
"	1600	0.56	0.87	0.94	1.4
"	1800	0.81	1.2	1.3	2.1
"	2000	1.1	1.6	1.7	2.7
"	2200	1.3	2.0	2.2	3.4
"	2400	1.6	2.4	2.6	4.0
1025	1400	0.55	0.94	0.92	1.6
"	1600	0.85	1.4	1.4	2.4
"	1800	1.1	1.9	1.9	3.2
"	2000	1.4	2.4	2.4	4.1
"	2200	1.7	2.9	3.0	4.9
"	2400	2.0	3.4	3.4	5.7
900	1400	0.89	1.8	1.5	3.0
"	1600	1.2	2.5	2.1	4.1
"	1800	1.6	3.2	2.7	5.3
"	2000	1.9	3.9	3.8	6.5
"	2200	2.3	4.6	3.9	7.8
"	2400	2.6	5.3	4.4	8.9

TABLE X

The Effect of Important Parameters Upon Coolant Requirements in Gaseous Transpiration Cooling of Turbine Blades

$c_{p_m}/c_{p_w} = 1; \; Re_c = \rho_g u_g c/\mu_g = 10^6$

T_w °K	T_g °K	Coolant Flow Rate W_c/W_g in Percent			
		$A_s/A_p = 3.00$		$A_s/A_p = 5.00$	
		Reservoir Temperature T_r, °K			
		300	600	300	600
1150	1400	0.20	0.30	0.33	0.51
"	1600	0.35	0.56	0.59	0.91
"	1800	0.51	0.79	0.85	1.3
"	2000	0.67	1.0	1.1	1.7
"	2200	0.82	1.3	1.4	2.1
"	2400	0.98	1.5	1.6	2.5
1025	1400	0.34	0.59	0.58	0.98
"	1600	0.53	0.90	0.88	1.5
"	1800	0.72	1.2	1.2	2.0
"	2000	0.90	1.5	1.5	2.5
"	2200	1.1	1.8	1.8	3.1
"	2400	1.3	2.1	2.1	3.6
900	1400	0.56	1.1	0.93	1.8
"	1600	0.78	1.5	1.3	2.6
"	1800	1.0	2.0	1.6	3.3
"	2000	1.2	2.4	2.0	4.1
"	2200	1.4	2.9	2.4	4.8
"	2400	1.7	3.3	2.8	5.6

TABLE XI

The Effect of Important Parameters Upon Coolant Requirements in Gaseous Transpiration Cooling of Turbine Blades

$c_{p_m}/c_{p_w} = 1;\ Re_c = \rho_g u_g c/\mu_g = 10^7$

T_w °K	T_g °K	Coolant Flow Rate W_c/W_g in Percent			
		A_s/A_p = 3.00		A_s/A_p = 5.00	
		Reservoir Temperature T_r, °K			
		300	600	300	600
1150	1400	0.13	0.20	0.22	0.34
"	1600	0.23	0.37	0.40	0.61
"	1800	0.34	0.53	0.57	0.88
"	2000	0.45	0.69	0.75	1.1
"	2200	0.55	0.86	0.93	1.4
"	2400	0.66	1.0	1.1	1.7
1025	1400	0.23	0.39	0.39	0.66
"	1600	0.36	0.61	0.59	1.0
"	1800	0.48	0.82	0.80	1.3
"	2000	0.60	1.0	1.0	1.7
"	2200	0.73	1.2	1.2	2.0
"	2400	0.85	1.4	1.4	2.4
900	1400	0.37	0.75	0.62	1.2
"	1600	0.52	1.0	0.84	1.7
"	1800	0.67	1.3	1.1	2.2
"	2000	0.82	1.6	1.3	2.7
"	2200	0.97	1.9	1.6	3.2
"	2400	1.1	2.2	1.9	3.7

turbine applications, the derivation leads to the simple equation

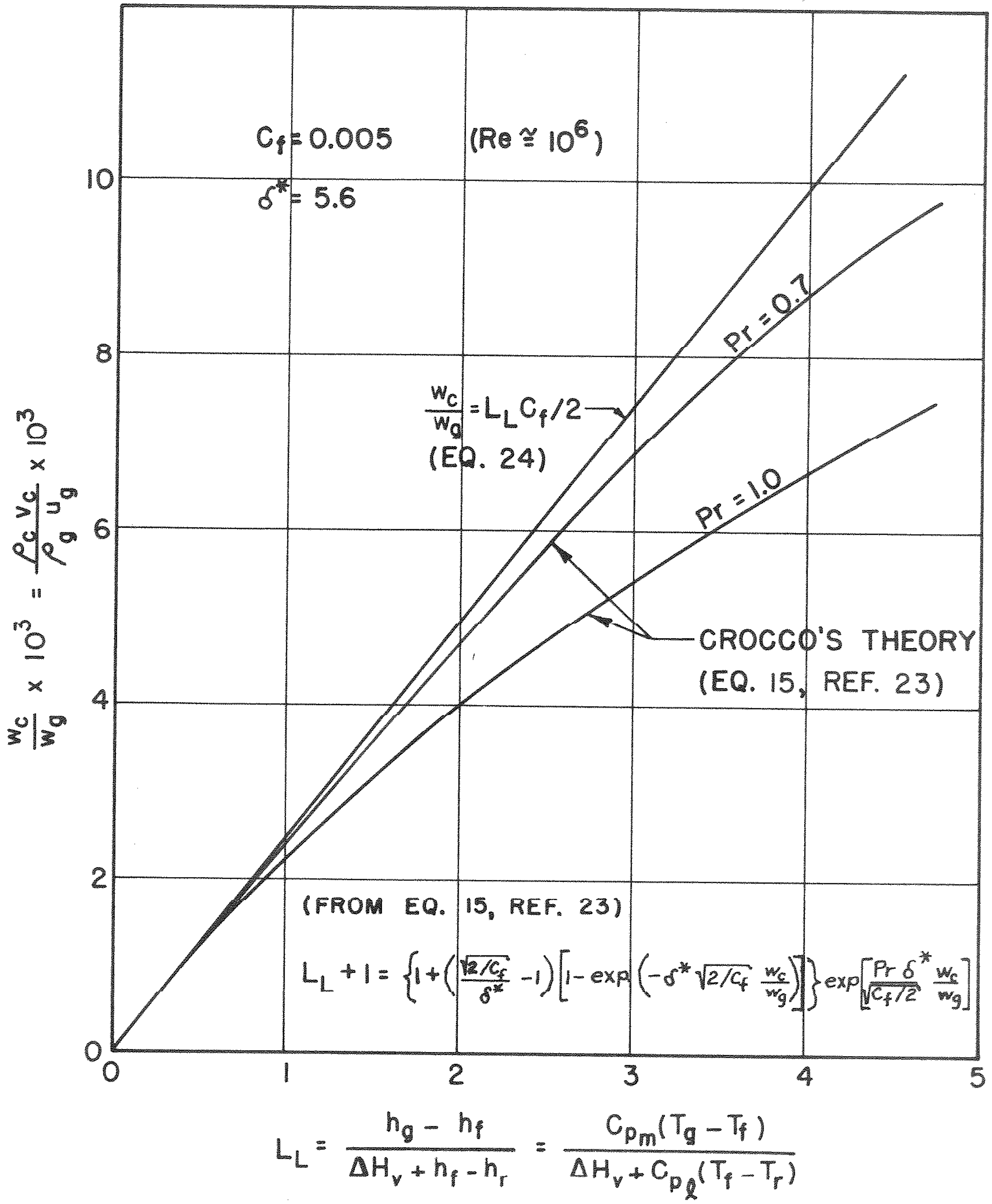
$$\frac{w_c}{w_g} = L_L \frac{c_f}{2} = \frac{c_{pm}(T_g - T_f)}{\Delta H_v + c_{p\ell}(T_f - T_r)} \left(\frac{c_f}{2}\right) , \qquad (24)$$

where c_{pm} and $c_{p\ell}$ are the average values of the specific heat at constant pressure between T_f and T_g and between T_r and T_f, respectively; T_g, T_f and T_r are the total temperatures of the main stream gases, of the liquid-gas interface, and of the liquid coolant in the reservoir, respectively; and ΔH_v is the latent heat of vaporization of the coolant under the prevailing pressure. The temperature of the liquid-gas interface for very large main stream gas temperatures is approximately equal to the boiling temperature of the coolant. Since very large working-fluid temperatures are of the principle concern of this section, the approximation is made that T_f is taken equal to the boiling temperature of the coolant under the prevailing pressure. This approximation becomes progressively poorer as the main stream temperature is reduced. For conditions under which the above approximation is not justified, the liquid-gas interface temperature may be calculated by a method outlined by Knuth (Ref. 24).

A comparison of Eq. (24) with Eq. (15) of (Ref. 23) is shown in (Fig. 5). Water is the logical coolant for stationary gas turbines, while some fuel such as gasoline or kerosene is more appropriate for aircraft gas turbines. The variation of L_L for water under a number of conditions is shown in Table XII, where the temperature of the liquid-

FIG. 5

COMPARISON OF APPROXIMATE EQUATION FOR COOLANT EVAPORATION RATE WITH CROCCO'S THEORY FOR LIQUID TRANSPIRATION COOLING



gas interface T_f is taken as the boiling temperature.

TABLE XII

Variation of the Parameter L_L for Reservoir Temperature $T_r = 300^\circ K$ and for T_f Taken Equal to the Boiling Temperature of the Coolant.

Coolant	Pressure atm.	T_g $^\circ K$	T_f $^\circ K$	ΔH_v Btu/lb	L_L
Water	1	1500	373	970	0.902
"	1	2000	373	970	1.43
"	1	2500	373	970	1.99
"	20.4	1500	487	809	0.825
"	20.4	2000	487	809	1.31
"	20.4	2500	487	809	1.82

Since the average value of the Prandtl number of water vapor for the conditions under consideration is close to 0.70, and since L_L for water is less than 2.00, the approximate expression for the coolant evaporation rate w_c/w_g given by Eq. (24) represents a good approximation to Crocco's more complicated equation in the range of L_L of practical interest in stationary gas turbine applications.

From an inspection of Eq. (24) it is apparent that the coolant should have (1) a large latent heat of vaporization and (2) a high boiling point. Increasing T_f has the double advantage of decreasing the numerator of Eq. (24) and of increasing the denominator, both of which tend to reduce the evaporation rate. Water, of course, is undoubtedly the most suitable fluid to employ.

As in the case of gaseous transpiration cooling, the area ratio A_s/A_p may be introduced. Then the coolant evaporation may be expressed as

$$\frac{W_c}{W_g} = L_L \frac{A_s}{A_p} \frac{c_f}{2} \quad . \tag{25}$$

Coolant evaporation rates as computed from Eq. (25) are presented in Table XIII for the same Reynolds numbers, main stream gas temperatures, and area rations A_s/A_p as were used in the case of gaseous transpiration cooling. Table XIII is arranged to facilitate comparision with Tables IX, X and XI. Referring to Tables X and XIII it is seen that:

1. if the allowable surface temperature T_w is 1150°K(1610°F), gaseous transpiration cooling is superior to liquid transpiration cooling, and the superiority of gaseous cooling decreases with increasing main stream gas temperature;
2. if the allowable surface temperature T_w is 1025°K (1385°F), gaseous transpiration cooling is slightly superior to liquid transpiration cooling for T_g less than 1800°K if the reservoir temperature is 300°K, and inferior if the reservior temperature is 600°K;

TABLE XIII

Evaporation Rate for Water as Coolant in Liquid Transpiration Cooling as a Function of the Important Parameters

$Re_c = \frac{\rho_g u_g c}{\mu_g}$	T_g °K	Evaporation Rate W_c/W_g in Percent	
		$A_s/A_p = 3.00$	$A_s/A_p = 5.00$
10^5	1400	0.78	1.3
	1600	1.0	1.7
	1800	1.2	2.0
	2000	1.4	2.3
	2200	1.6	2.7
	2400	1.8	3.0
10^6	1400	0.50	0.81
	1600	0.61	1.0
	1800	0.74	1.2
	2000	0.84	1.4
	2200	1.0	1.7
	2400	1.1	1.9
10^7	1400	0.33	0.55
	1600	0.41	0.68
	1800	0.50	0.83
	2000	0.58	0.97
	2200	0.67	1.1
	2400	0.77	1.3

3. if the allowable surface temperature T_w is 900°K (1160°F), (which is the case of practical interest when the service life of the turbine blades must be long), gaseous transpiration cooling is inferior to liquid transpiration cooling for both reservoir temperatures of the gaseous coolant.

The life of the turbine blades is likely to be an important consideration in a gas turbine cycle employed to manufacture nitric oxide. Furthermore, the amount of strategic metals, such as cobalt, nickel and chromium, which are required for turbines operating at high temperatures, should be kept at a minimum. For these two reasons the use of lower wall temperatures is definitely indicated. Therefore, on the basis of coolant requirements alone, the use of liquid, as opposed to gaseous transpiration cooling, is in order. Moreover, the wall surface temperatures for liquid transpiration cooling are approximately equal to the boiling temperature of the coolant. This fact is another point in favor of porous wall cooling. However, it should be noted that the figures for liquid transpiration cooling are evaporation rates, and consequently these figures represent minimum coolant requirements. A slight amount of extra coolant should be employed as a pre-cautionary measure. However, any coolant introduced in excess of the evaporation rate can not evaporate and will be carried away by the main stream. The injection of excess coolant does not directly increase the protection afforded the turbine blades, and consequently, the extra coolant injected should be kept at a minimum consistent with safety. A discussion of the behavior of the liquid-gas interface as the coolant injection rate is increased is presented by Kinney and Abramson (Ref. 33). As the coolant flow rate is increased the liquid-gas interface becomes progressively less stable. At the higher flow rates a wave-like structure appears on the liquid surface and appreciable amounts of coolant are carried-off in the form of droplets by the main stream. Such high coolant injection rates are to be avoided, since any coolant removed by the main stream in this fashion is wasted.

So far in the discussion no mention has been made of a serious disadvantage associated with liquid transpiration cooling. The evaporation of the liquid film on the porous walls exposed to the hot gases is inherently unstable. If the coolant flow should be interrupted only momentarily, the liquid will flash back through the porous wall and begin to boil on the interior of the turbine blades. Although some protection is afforded the blades by the vapor passing through the porous walls, it is unfortunately true that porous walls which are suitable for liquid transpiration cooling are not appropriate for gaseous transpiration. As will be shown in the next paragraph, the mass flow rate of the vapor through the porous wall when the liquid is boiling on the interior surface must be approximately equal to the previous mass flow rate of the liquid in order to prevent the destruction of the walls by the hot gases. Therefore, since the effective flow-area through the porous walls hasn't changed, the product (ρv) must be the same (as can be seen from the continuity equation) whether the liquid or its vapor is passing through the walls. At a pressure of twenty atmospheres the density of water is eight order of magnitudes greater than its vapor, which means that the velocity of the vapor must become eighty times greater than that for the liquid. Since the pressure differential across the walls has remained constant, there is no way of obtaining the required vapor velocities.

For calculation of the required pressure drop across the porous wall in order to produce the prescribed coolant flow rate, the work of Green (Ref. 34) and Ellerbrock (Ref. 35) should be consulted. Green expressed the difference of the squares of the pressures acting on both surfaces of a porous wall in terms of (1) the coolant mass flow rate, (2) the blade thickness, (3) the dynamic viscosity of the coolant at the wall, and (4) the outer wall temperature. Ellerbrock (Ref. 25) showed that the experimental data of the NACA could be correlated by plotting $(p_r^2 - p_g^2)/(\mu_{c,w}^2 \ell_w T_w)$ against $(\rho_c v_c / \mu_{c,w})$, where ℓ_w is the wall thickness.

4. Transpiration Cooling with Internal Liquid Vaporization

In order to overcome the vapor-lock phenomena associated with liquid transpiration cooling, the porosity of the walls could be increased to a value which would permit the passage of the mass flow corresponding to the evaporation rate of the liquid under the prescribed main stream flow conditions.

This manner of transpiration cooling is fundamentally identical to liquid transpiration cooling. In both cases the evaporation rate is determined by the exterior heat transfer from the main stream gases. In both cases the latent heat of vaporization is an important factor. An approximate expression for the coolant evaporation rate, corresponding to Eq. (24) may be obtained directly from the relations derived for liquid transpiration cooling. Only the boundary conditions need be changed. The slight modifications which are necessary are explained in Section 2, Appendix VII and lead to the following approximate equation for the coolant evaporation rate:

$$\frac{w_c}{w_g} = L_{LV}\frac{c_f}{2} = \frac{c_{pm}(T_g - T_w)}{\Delta H_v + c_{p\ell}(T_f - T_r) + c_{pw}(T_w - T_f)}\left(\frac{c_f}{2}\right), \quad (26)$$

where c_{pw} is the heat capacity at constant pressure of the vapor between T_f and T_w, and T_w is the temperature of the wall surface exposed to the hot gases. As was previously explained, T_f will be approximately equal to the boiling point of the liquid under the given pressure. The temperature of the wall surface T_w will be only slightly greater than T_f. The temperature difference $(T_w - T_f)$ would be roughly between 50 and 100°K for most cases of interest. Consequently, in comparing Eqs. (24) and (26), it is clear that L_{LV} is slightly less than L_L due to the presence of the term $c_{pw}(T_w - T_f)$ in the denominator of Eq. (26), which is not present in Eq. (24), and due to the fact that T_w is slightly greater than T_f. Therefore, the evaporation rate for the present case of transpiration cooling is only slightly more effective than "pure" liquid transpiration cooling. Consequently, designing the porous walls to permit stable internal evaporation of the coolant not only overcomes the vapor-lock problem, but also leads to an actual reduction in the coolant evaporation rate. Therefore, recalling that liquid transpiration cooling was superior to gaseous transpiration cooling when the service-life of the turbine blades, etc., are important considerations, it may be concluded that transpiration cooling with internal liquid vaporization is the most promising of the turbine blade cooling methods considered so far.

5. Film Cooling

Again, as in the case just discussed, film cooling is fundamentally identical to liquid transpiration cooling. In both methods the coolant requirements are determined by the evaporation rate, which in turn is controlled by the main stream flow conditions. As in the previous case, only the boundary conditions need be changed in order to obtain the expression for the coolant requirements. In order to be consistent with the assumption used in the derivation that derivatives in direction parallel to the wall are negligible in comparision with derivatives in the normal direction, the coolant is assumed to be injected at the temperature T_f, which is near the boiling point of the coolant. The heating of the coolant from the reservoir temperature T_r to T_f is neglected thereby. With these assumptions the expression for the coolant evaporation rate becomes

$$\frac{w_c}{w_g} = L_r \frac{c_f}{2} = \frac{c_{pm} (T_g - T_f)}{\Delta H_v} \left(\frac{c_f}{2} \right) . \qquad (27)$$

Comparing Eqs. (24) and (27), it may be seen that L_F is greater than L_L due to the presence of the term $c_{p_\ell}(T_f - T_r)$ in the denominator of Eq. (24), which is not present in Eq. (27). Consequently, the evaporation rate for film cooling will be somewhat greater than for liquid transpiration cooling. Actually, film cooling is a little more effective than would be indicated by Eq. (27), because the evaporation rates in the vicinity of the point of injection will be lower than the formula indicated due to the neglection of the term $c_{p_\ell}(T_f - T_r)$. Nevertheless, over most of the reach of the liquid film Eq. (27) adequately represents the coolant requirements for low evaporation rates.

In recapituation, the relative merits of cooling by gaseous transpiration, by liquid transpiration, by transpiration with internal liquid vaporization and by liquid films may be obtained through comparing the factors L_G, L_L, L_{LV} and L_F. When service conditions dictate the use of allowable wall temperatures of 900°K (1160°F) or less, L_L and L_{LV} are the smallest of the four factors. Because of the vapor-lock phenomena associated with liquid transpiration cooling, the design of porous walls to permit transpiration cooling with internal liquid vaporization is the most promising of the four cooling methods considered.

6. Miscellaneous Consideration Regarding Turbine Blade Cooling

(a) Effective Gas Temperature and Mach Number Relative to the Blades

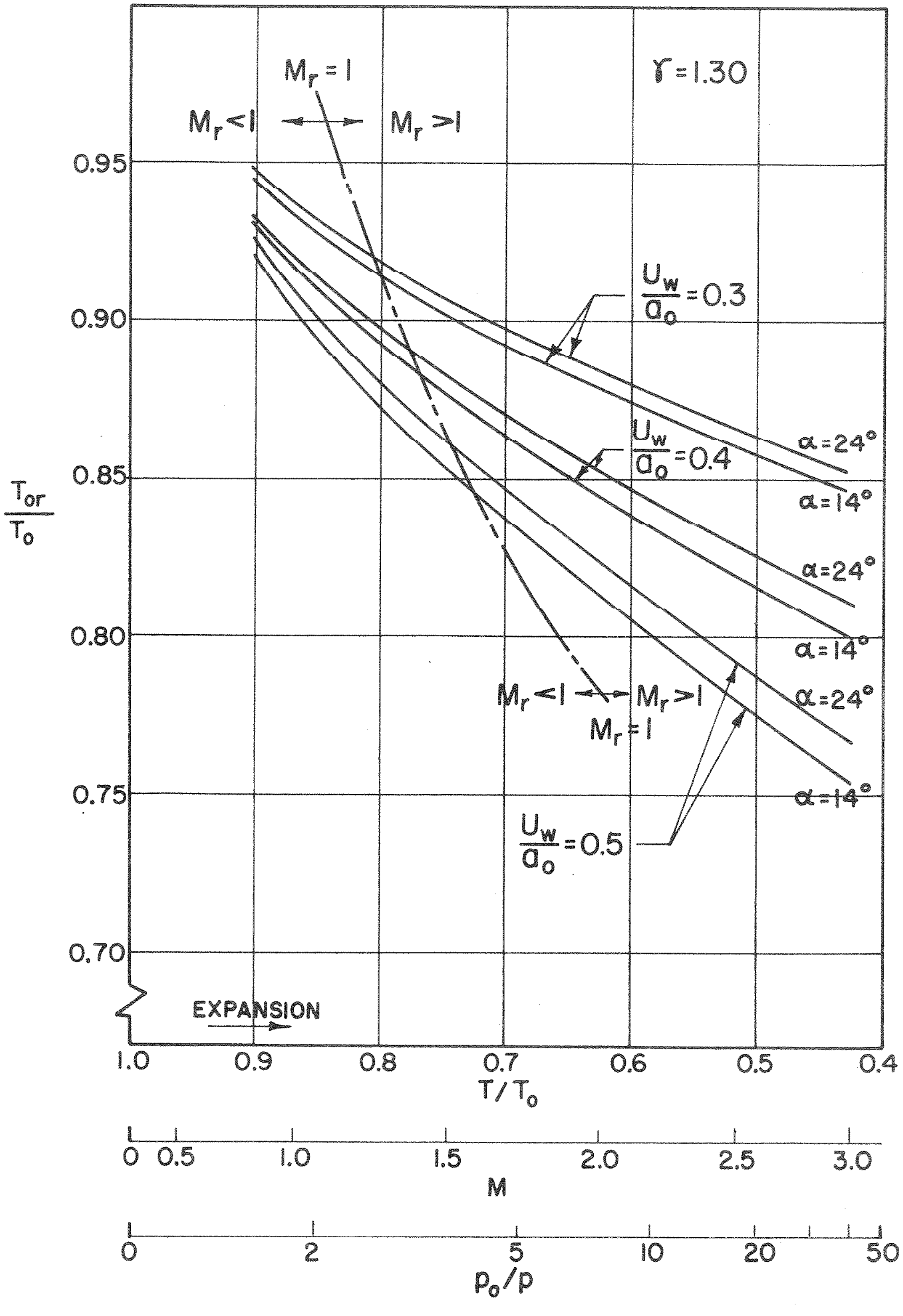
The temperature T_g employed in connection with transpiration and film cooling was the total temperature of the main stream. The corresponding temperature in turbine applications of these cooling methods is the total temperature of the main stream relative to the turbine blades. This relative total temperature will be denoted by the symbol T_{or}. It should be noted that this total temperature varies over the blade span.

In Part A of this thesis the desirability of expanding the gases to low static temperatures in order to "freeze" the nitric oxide decomposition reaction was explained. In Section III-D it was shown that the decomposition reaction during flow may be conservatively considered to have ceased when the total temperature has been reduced to $1500^{\circ}K$. In order to secure a static temperature of $1500^{\circ}K$ from a combustion-chamber temperature of $2500^{\circ}K$, an expansion to a Mach number of 1.82 for $\gamma = 1.4$ or of 2.10 for $\gamma = 1.3$ is required. The corresponding Mach numbers relative to the blades will be somewhat less. However, due to limitations on turbine wheel speeds, the Mach number relative to the blades M_r will be greater than unity for the above conditions. Unfortunately, this circumstance would cause the formation of a shock wave across the blade rows and would lead to some loss in turbine efficiency. Furthermore, the flow accelerates over a portion of the suction side of the blade. This means that the Mach number relative to the blades at the entrance to the blade row must be less than one to avoid the formation of shock waves. Under these conditions the avoidance of shock waves is impractical in the application of gas turbine cycle to the manufacture of nitric oxide. Moreover, shock waves do not have the drastic effect upon performance in turbines that they do with wings on aircraft. Under these circumstances it is better to expand to relatively high Mach numbers in order to reduce the total temperature relative to the turbine blades even though this means an increase in the static temperature of the gases due to the shock waves.

In Appendix VIII an expression for M_r at the entrance to the blade row is derived (Cf. Eq. A VIII-2). Having determined M_r, the ratio of the total temperature relative to the blades to the total temperature (i.e., T_{or}/T_o) may be computed. The results of calculations for the range of turbine parameters likely to be encountered in practice are shown in (Fig. 6). The total temperature ratio T_{or}/T_o is plotted for isentropic expansion ($\gamma = 1.30$) against the ratio of the static temperature to the total temperature T/T_o. The (absolute) Mach number relative to a fixed coordinate system M and the ratio of the total pressure to the static pressure p_o/p, corresponding to T/T_o, are indicated also along the abscissa. The ratio of the turbine wheel speed to the velocity of sound in the combustion chamber U_W/a_o and the blade entrance angle α are used as parameters. The range of practical interest for U_W/a_o and α are 0.30 - 0.50 and 14^o - 24^o, respectively. These values are used to indicate the limits of variation of T_{or}/T_o. The dash-dot line indicates a relative Mach number M_r equal to one. To the left of this line the relative Mach number is less than one and to

FIG. 6

TOTAL TEMPERATURE RELATIVE TO TURBINE BLADES AT THE ENTRANCE TO THE BLADES



the right, greater than one. In general, for M_r less than one, T_{or}/T_o is greater than 0.88, and T/T_o is greater than 0.75. For T_o equal to 2500°K, the total temperature relative to the blades T_{or} for $U_W/a_o = 0.5$ and $\alpha = 14^o$ is 2200°K. On the other hand, expansion to $M = 2.5$ reduces the total temperature relative to the blades to about 1950°K under the same conditions. Although a total temperature relative to the blades of 1500°K cannot be obtained, the extraction of energy by the turbine will quickly reduce T_{or} to 1500°K. Lower values of M_r and T_{or}/T_o may be attained by mounting the turbine nozzles upon a nozzle ring which may be rotated in the direction opposite to the rotation of the turbine wheel. Actually, this nozzle ring would resemble another turbine wheel in appearance. Although such a wheel adds energy to the main stream gases, the counter-rotation arrangement actually leads to somewhat lower Mach numbers and total temperatures relative to the turbine blades.

(b) Pressure Distribution Around Blades

The pressure distribution over the blades is not uniform and this fact has a significance for transpiration cooling. It means that the pressure differential driving the coolant through the porous wall varies over the blade surface. Consequently, unless the blade thickness and/or porosity are modified to obtain uniform flow, the coolant mass flow rate per unit area will not be uniform as assumed. Over most of the blade the porosity and the thickness may conceivably be varied to obtain the desired local flow rates. However, the leading and trailing edges of impulse blades are particularly difficult to cool directly because of the thick walls in these regions. Nevertheless, since the effect of transpiration cooling is likely to persist downstream from the last points of injection and since the turbine is continuously reducing the total temperature of the gases by extracting energy, the trailing edge is not as critical as would appear at first glance. On the other hand, the leading edge is subjected to the full total temperature of the gases relative to the blades as well as to the full total pressure relative to the blades. Therefore, at the leading edge not only are the cooling requirements most severe, but also the means of meeting these requirements are hindered by the relatively smaller pressure differential acting over a relatively thicker section of the blade.

The purpose of this thesis is not to determine the local cooling requirements, but to obtain over-all requirements and to compare the various methods of turbine blade cooling. The methods of determining coolant requirements previously presented should give conservative estimates for total coolant requirements.

(c) Total Gas Turbine Cooling Requirements

Under the high temperatures necessary in the manufacture of nitric oxide, other parts of the gas turbine in addition to the turbine blades will require cooling. No one single method of cooling will be most appropriate for all parts. Calculation of total coolant requirements must be related necessarily to specific designs. Consequently, no attempt has been made to compute the total coolant requirements for the entire gas turbine.

(d) Radial Motion of the Boundary Layer

The boundary layer is given a radial component of motion due to the action of centrifugal forces. The radial pressure gradient which exists in the main stream is appreciably smaller than the radial pressure gradient which tends to exist in the boundary layer. This situation is due to the difference in peripheral velocity between the main stream and the boundary layer. However, the pressure gradient is controlled by the flow outside the boundary layer. Consequently, since the actual pressure gradient in the boundary layer is smaller than would otherwise exist, the centrifugal forces act to impart a radial motion to the boundary layer. This radial motion has a significance in transpiration and film cooling. The radial motion is likely to be especially pronounced in liquid transpiration cool and in film cooling. This phenomenon would be benefical in cooling the blade tips. A more complete discussion of the radial motion of the boundary layer is given by Wislicenus (Ref. 36, pp. 351-354), who based his remarks on the work of Weske (Ref. 37).

C. Effect of Transpiration and Film Cooling Upon Cycle Performance

By way of emphasis let it first be clearly understood that the thermodynamical performance of the cycle is not necessarily governing. The primary function of the gas turbine here is not to generate useful output-power in the most efficient manner but to provide a means for producing commerically desirable chemicals in the most economical fashion.

In the application of the gas turbine cycle to the production of nitric oxide the recovery cost of this compound from the other products will likely account for the major portion of the production cost. Gilbert and Daniels (Ref. 2) estimate that this cost-factor is responsible for roughly seventy percent of the total manufacturing cost of the nitric oxide as produced by the Wisconsin pebble-bed process. The recovery cost in the gas turbine cycle will likely bear roughly the same ratio to the total cost as it does in the Wisconsin process. In as much as the recovery cost is very much dependent upon the percentage yield of nitric oxide, it would be advisable generally to sacrifice a little in the way of performance—if necessary—in order to gain significant increases in the yield of nitric oxide. Significant increases in yield can be obtained by increasing the combustion-chamber temperature. Higher combustion-chamber temperatures mean, however, that higher coolant flow rates for turbine cooling are required, and these higher flow rates in turn tend to reduce the available output-power from the gas turbine. At the same time, however, the reduction in the output-power due to the necessity of the increased cooling requirements will be more than offset by the power increase made available by virtue of the higher combustion-chamber temperature without adversely affecting specific fuel consumption. Consequently, the fortunate circumstance arises that turbine cooling not only leads to increases in the yield of nitric oxide but also results in higher power-output at higher thermodynamic efficiencies. At some point this process of increasing coolant flow rates in order to secure net increases in power-output and in nitric oxide yield comes to an end as in the law of diminishing returns. Although many factors are involved, one can say with some confidence that the point of diminishing returns in power-output will not occur for combustion-chamber temperatures less than $3000^{\circ}K$. On the other hand, it was shown in Section III that the De Laval nozzle is only moderately effective in freezing the decomposition reaction for combustion temperatures in the neighborhood of $3000^{\circ}K$. In addition, the development effort required to obtain combustion temperatures of $3000^{\circ}K$ is likely to be considerable. Consequently, for these reasons somewhat lower combustion temperatures will be target in the initial development program. A combustion-chamber temperature of $2500^{\circ}K$, which would produce a yield of nitric oxide of approximately one percent without resorting to oxygen-enrichment, might prove to be a suitable development target in the initial effort. Successful operation of the gas turbine at some such intermediate temperature will demonstrate both the practicability of the cooling method and the feasibility of the process for the production of nitric oxide and will at the same time furnish the necessary data and experience which will be required for the assault upon the problems of even higher combustion temperatures. Consequently, it is for these practical reasons that the following analysis will be restricted arbitrarily to gas turbines with chamber temperature near 2500°K.

The foregoing discussion was undertaken in order to show in a qualitative way the relationship between the principal factors involved. In addition, these general remarks have indicated a manner in which the analysis of the effect of transpiration and film cooling upon cycle performance may be simplified. It it can be shown that with the chamber temperature held constant coolant injection has only a small effect upon cycle performance independent of the level of the chamber temperature, then the cycle analysis may be uncoupled into two parts, which may then be treated separately without introducing an appreciable error. The first part would be the study of the effect of coolant injection upon cycle performance when the chamber temperature has been held constant. The second part would be the analysis of the effect of the variation of the chamber temperature upon cycle performance when turbine-cooling has not been used. The combined effect may then be obtained by simply adding the results of these two separate investigations. Since the second study has been thoroughly analyzed by a number of investigators, the task of the cycle analysis has been considerably reduced. Consequently, in order to complete the analysis only the first part need be actually per-formed since the results for the second part may be obtained from the literature.

A few qualitative arguments will now be advanced to show for constant combustion-chamber temperature that increases in the coolant flow-rate reduces the power-output only by small amounts. Following these qualitative arguments a simple one-dimensional study will be given to lend a little more substance to the analysis.

The reason why the effect of coolant injection for transpiration and film cooling upon cycle performance with the chamber temperature held constant is so small is due principally to the small rates of coolant flow required. As was shown in Section VI-B, a coolant mass-flow-rate of only a few percent of the working gas rate is necessary even for chamber temperatures approaching 3000°K. Moreover, certain compensating effects occur with transpiration and film cooling. Although an entropy increase occurs when the cooling and working gases mix, the mass of the gas passing through the remaining portions of the turbine downstream of the zone of coolant injection has been thereby increased, which tends to increase the available power-output. In addition, the energy loss of the main stream to the coolant fluid is not entirely lost to the gas turbine as some of this energy is recovered in the following portions of the cycle. Although there is a net reduction in the total or stagnation pressure due to the coolant injection, the stagnation-pressure loss resulting from the momentum transfer between the working and cooling gases is compensated in part by the stagnation-pressure rise which arises from the stagnation-temperature decrease. This decrease in stagnation temperature is caused by the enthalpy change undergone by the cooling fluid. (For film cooling with water the reduction in the stagnation pressure across a blade row is less than approximately one percent).

Simplified Analysis of the Effect of Coolant Injection Upon Cycle Performance When the Chamber Temperature is Held Constant.

For the case where the chamber temperature is kept constant the effect upon the gas turbine cycle of the injection of a coolant either through porous turbine blades as in transpiration cooling or through slits near the leading edge of the hollow blades as in film cooling may be analyzed in two parts. First, there is the effect of the coolant injection upon that portion of the cycle in which the fluid injection itself takes place. This portion of the cycle embraces only the first few impulse stages which demand protection against the hot gases. Second, there is the effect of the fluid injection upon the remaining portions of the cycle. These portions consist of the remaining turbine stages, the regenerator and most likely a separate low pressure turbine-generator set.

In both portions of the cycle the fact that the required rate of coolant flow is so low—of the order of a few percent of the main-stream flow— allows approximations, which simplify the analysis considerably. For this reason and for some of the other reasons previously discussed a one-dimensional analysis should be sufficient to indicate the significant factors involved and give a qualitative idea of the effect of coolant injection. Accordingly, the very useful work of Shapiro and Hawthorne (Ref. 35) will be employed, since their generalized treatment of steady one-dimensional gas flow is presented in a form very convenient for the subsequent analysis. The essentials of their work for the purposes at hand may be collected in the three following equations, which are obtained from their table of influence coefficients for constant specific heat and molecular weight:

$$\frac{dV}{V} = -\frac{1}{1-M^2}\frac{dA}{A} + \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\frac{dT'}{T'} + \frac{\gamma M^2}{2(1-M^2)}\frac{4c_f dx}{D_h} + \frac{1+\gamma M^2}{1-M^2}\frac{dW}{W}, \quad (29)$$

$$\frac{d\rho}{\rho} = +\frac{M^2}{1-M^2}\frac{dA}{A} - \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\frac{dT'}{T'} - \frac{\gamma M^2}{2(1-M^2)}\frac{4c_f dx}{D_h} - \frac{(\gamma-1)M^2}{1-M^2}\frac{dW}{W}, \quad (30)$$

$$\frac{dp'}{p'} = -\frac{\gamma M^2}{2}\frac{dT'}{T'} - \frac{\gamma M^2}{2}\frac{4c_f dx}{D_h} - \gamma M^2\frac{dW}{W}. \quad (31)$$

The notation of Shapiro and Hawthorne has been employed except for a few minor changes in order to be consistent with the notation used elsewhere in this thesis. The notation is as follows:

A = cross-sectional area

c_f = coefficient of friction of duct ($\tau_w/\frac{1}{2}\rho V^2$)

D_h = hydraulic diameter

M = Mach number

p = static pressure

T = absolute temperature

V = stream velocity

W = mass rate of flow of stream

x = distance along duct

γ = ratio of specific heats (c_p/c_v)

ρ = mass density of stream

τ_w = shearing stress on walls of duct

$(\ )'$ = refers to conditions relative to observer moving with gas.

Other notation will be explained as it is introduced.

Effect of Coolant Injection Upon the Impulse Turbine Stages in which Coolant Injection Takes Place

For an ideal impulse-turbine bucket the inlet pressure, relative velocity, density and flow area are equal to the corresponding quantities at the exit of the bucket. In any real machine, however, friction will be present. Since the pure impulse turbine is designed so that no pressure change occurs across the stage, the presence of friction results in the retardation of the relative velocity. In practice this situation is expressed by equating

$$V'_{exit} = k V'_{in} \quad ,$$

where k is called a blade-velocity coefficient and varies from 0.80 to 0.90. Since the blade entrance and exit angles are approximately equal, the decrease in relative velocity means a corresponding decrease in the axial velocity. This change in kinetic energy is converted into a increase in internal energy and is reflected by an increase in temperature and consequently by a decrease in density. Since continuity must be satisfied, the combined effect of decreases in the axial velocity component and in density means that the passage area must diverge. If the hub diameter of the turbine is maintained constant, it follows that the blade height must be increased in order to secure the proper area and consequently that the stress situation is adversely affected. This difficulty can be overcome by allowing a slight pressure drop across the turbine buckets thereby securing an acceleration in the flow which in turn permits a decrease in blade height. Off-setting this advantage, of course, is the increased leakage loss in the clearance space due to the unbalance in pressure. The same conditions leading to the desirability of designing a slight amount of reaction into impulse stages when no attempt at cooling is made are also present when cooling by fluid injection is employed.

In order to simplify the presentation of the analysis a supposition will be made which will then be verified. The supposition is that the product of relative velocity and density at a given cross-section is constant from one cross-section to another; that is, $V'\rho$ = constant or $\frac{dV'}{V'} + \frac{d\rho}{\rho} = 0$.

Under this supposition the continuity equation, which may be written in the form

$$A_p \rho V' = W \quad , \tag{32}$$

or alternatively,

$$\frac{dA_p}{A_p} + \frac{d\rho}{\rho} + \frac{dV'}{V'} = \frac{dW}{W} \tag{33}$$

reduces to

$$\frac{dA_p}{A_p} = \frac{dW}{W} \quad , \tag{34}$$

where A_p refers to the cross-section flow area of the main stream. Eq. (34) may be integrated between the entrance to the impulse blade and the exit to yield the relation

$$\frac{(A_p)_{ex}}{(A_p)_{in}} = 1 + \frac{W_c}{W_g} \quad , \tag{35}$$

where the subscript g refers to the main stream at the entrance to the turbine and subscript c refers to the injected coolant. Since W_c/W_g is of the order of a few percent, the required area increase is very small under the supposition made above. (Note that this supposition could just as well begin with the result of Eq. (34) that $\frac{dA_p}{A_p} = \frac{dW}{W}$, which would then lead to the conclusion that $\frac{dV'}{V'} + \frac{d\rho}{\rho} = 0$).

The energy equation may be written relative to the turbine wheel in the form

$$-\left(h_g + \frac{V'^2}{2} - h_c\right)\frac{dW}{W} = \bar{c}_p \, dT_o' \quad , \tag{36}$$

where h_g and h_c are the enthalpies of the main stream gas and coolant, respectively; $\overline{c}_p$ is the appropriate mean value of the specific heat at constant pressure of the main stream; T_o' is the total temperature of the main stream relative to blades; the injection velocity of the coolant has been ignored. Eq. (36) may be rewritten in the form

$$\frac{dT_o'}{T_o'} = - \frac{\left(h_g + \frac{V'^2}{2} - h_c \right)}{\overline{c}_p \, T_o'} \frac{dW}{W} \quad . \tag{37}$$

By way of economy in notation write

$$\Delta h = h_g + \frac{V'^2}{2} - h_c \tag{38}$$

so that Eq. (37) may be shortened to

$$\frac{dT_o'}{T_o'} = - \frac{\Delta h}{\overline{c}_p T_o'} \frac{dW}{W} \quad . \tag{39}$$

The factor $4c_f \frac{dx}{D_h}$, which appears in Eqs. (29) - (31), may be rewritten in a more convenient form. The amount of cooling fluid injected in a length dx along the turbine blade may be expressed as

$$dW_g = w_c (2 s \, dx) \quad , \tag{40}$$

where w_c is the coolant mass flow rate per unit area and s (span) is the cooled length of the blade. The hydraulic diameter D_h may be expressed approximately as

$$D_h \cong \frac{4A_p}{2s(1 + d/s)} \quad , \tag{41}$$

where d is average width of passage between adjacent blades. Since the main stream flow rate per unit area w_g is given by

$$w_g = W_g / A_p \quad , \tag{42}$$

the factor $4c_f \frac{dx}{D_h}$ may be rewritten by employing Eqs. (40) and (41) as

$$4\, c_f \frac{dx}{D_h} = c_f\left(1 + \frac{d}{s}\right)\frac{w_g}{w_c}\,\frac{dW_g}{W_g} \quad . \tag{43}$$

If the expressions derived in Eqs. (34), (39) and (43) are used to replace $\frac{dA}{A}$, $\frac{dT'}{T'}$, and $4c_f \frac{dx}{D_h}$ of Eqs. (29) - (31), then the following differential equations may be written down

$$(1-M^2)\frac{dV'}{V'} = -\left(1 + \frac{\gamma-1}{2}M^2\right)\left\{\frac{\Delta h}{\bar{C}_p T_o'} - \frac{\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\left[1 + \frac{c_f}{2}\left(1+\frac{d}{s}\right)\frac{w_g}{w_c}\right]\right\}\frac{dW_g}{W_g} \tag{44}$$

$$(1-M^2)\frac{d\rho}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)\left\{\frac{\Delta h}{\bar{C}_p T_o'} - \frac{\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\left[1 + \frac{c_f}{2}\left(1+\frac{d}{s}\right)\frac{w_g}{w_c}\right]\right\}\frac{dW_g}{W_g} \tag{45}$$

$$\frac{dp_o'}{p_o'} = \frac{\gamma M^2}{2}\left\{\frac{\Delta h}{\bar{c}_p T_o'} - 2\left[1 + \frac{c_f}{2}\left(1+\frac{d}{s}\right)\frac{w_g}{w_c}\right]\right\}\frac{dW_g}{W_g} \quad . \tag{46}$$

Note that Eqs. (44) and (45) are consistant with the supposition made earlier that $dV'/V' + d\rho/\rho = 0$.

Relative Velocity Change

Interest focuses upon the effect of coolant injection upon the change in relative velocities across the blade row, because the power available from the turbine is related to this velocity change. The power-output from the turbine is equal to the power-input to the turbine by the main stream gases less the power required to pump the cooling fluid, to overcome friction and so forth. The work done on the turbine wheel can be expressed in terms of the turbine wheel speed U_w, inlet and exit relative velocities of the blades and the blade angles. Since the turbine speed, inlet relative velocity and blade angles need not be different in the cooled and uncooled turbines, enquiry is made as to whether the change in relative velocity across the blade row in the cooled turbine differs from that in the uncooled.

If Eq. (44) is divided through by $(1 - M^2)$ and is then integrated from the entrance to the bucket to the exit, the change in velocity across the blade is given by

$$\frac{V'_{ex}}{V'_{in}} = \left(1 + \frac{W_c}{W_g}\right)^{\bar{\phi}} , \tag{47}$$

or since W_c/W_g is small,

$$\frac{V'_{ex}}{V'_{in}} \cong 1 + \bar{\phi}\,\frac{W_c}{W_g} , \tag{48}$$

where
$$\phi = -\frac{\left(1 + \frac{\gamma-1}{2} M^2\right)}{1 - M^2}\left\{\frac{\Delta h}{\bar{c}_p T'_o} - \frac{\gamma M^2}{1 + \frac{\gamma-1}{2} M^2}\left[1 + \frac{c_f}{2}\left(1 + \frac{d}{s}\right)\frac{w_g}{w_c}\right]\right\} \tag{49}$$

and where the bar over the quantity indicates that the appropriate average value is to be taken. The argument against pulling the coefficient of dW_g/W_g through the integral sign thereby treating this coefficient as a constant is that this operation merely side-tracks the integration task only to introduce the problem of the determining the "appropriate average value". The actual determination of $\overline{\Phi}$ is not necessary, however, for the purpose of this study, and furthermore it would be necessary first to have prescribed the variation of Mach number through the turbine blade before $\overline{\Phi}$ could be determined. For the purposes of this investigation it suffices to say that Φ varies only slightly through the blades for the mass flow rates associated with using either air or water as coolants in turbine blade cooling. The values of and $V'ex/V'in$ as function Mach number have been computed for certain typical values of the parameters entering Eq. (49) using the maximum ratio of w_c/w_g for both air and water injection. In general, it can be said that the relative velocity across the turbine blade decreases by less than ten percent both for water injection and for air injection. This decrease in relative velocity is slightly less than that which occurs across uncooled impulse turbine blades. Therefore, as far as the relative velocity change across the buckets is concerned, the transpiration- or film-cooled impulse turbine is not significantly different from the same uncooled impulse turbine for the same combustion chamber temperature.

As indicated previously, the work done on the turbine wheel can be expressed in terms of the turbine wheel speed U, inlet and exit relative velocities, and the blade angles. Therefore, since the turbine speed, inlet relative velocity, and the blade angles need not be different and since the relative velocity change across the blade row is approximately the same in the cooled and uncooled turbine, it follows that the work done on the turbine in those blade rows in which coolant injection takes place is practically unaffected by transpiration or film cooling — as far as this simple one-dimensional analysis discloses. The work-output from these same turbine wheels will, of course, be reduced somewhat by the amount of pumping work required to inject the coolant into the main gas stream. The magnitude of this pumping work is small.

Effect of Coolant Injection upon the Portions of the Cycle Downstream of the Zone Directly Cooled

The remaining portions of the cycle can be affected principally by reductions in the stagnation pressure and the stagnation temperature and by increases in the mass flow due to the coolant injection. The decrease in the stagnation pressure means that less energy can be abstracted by the remaining work-absorbing agents in the cycle. The reduction in the stagnation temperature means that less energy may be transferred in the heat exchanger. The increase in the mass flow, on the other hand, tends to counterbalance the other two effects. The reduction in the stagnation temperature has only a secondary effect upon the rate of heat transfer in the regenerator, and for this reason the effect of the decrease in stagnation temperature upon the cycle performance is of negligible importance and will be ignored. The effects of the reduction of the stagnation pressure and the increase in the mass flow upon the power-output can be treated as a combined effect. The decrease in the stagnation pressure across the coolant injection zone needs first to be investigated, however.

Stagnation Pressure Change

If Eq. (46) is integrated from the entrance to the exit of the blade being cooled, the change in the stagnation pressure is given by

$$\frac{(p_o')_{ex}}{(p_o')_{in}} \cong 1 + \overline{\psi} \, \frac{W_c}{W_g} \quad , \tag{50}$$

where $$\psi = \frac{\gamma M^2}{2} \left\{ \frac{\Delta h}{\overline{c}_p T_o'} - 2 \left[1 + \frac{c_f}{2} \left(1 + \frac{d}{s} \right) \frac{w_g}{w_c} \right] \right\} \tag{51}$$

and where the bar over the quantity ψ indicates that the appropriate average value is to be taken. The variation of ψ with Mach number is small for air and water injection and for both high and low coolant flow rates. The values of ψ and $(p'_o)_{ex}/(p'_o)_{in}$ as a function of Mach number have been computed using the same values of the parameters as employed in the case of the relative velocity change. In general, it can be said that stagnation pressure across the turbine blade (1) decreases by less than one percent for water injection throughout the wide range of coolant flow rates investigated and (2) decreases for air injection by less than three percent. Consequently, since the reduction in the stagnation pressure across the coolant injection zone is so small for both air and water injection, the effect of the reduction of total pressure upon the portions of the cycle downstream of the injection zone is small. On the other hand, the increase in mass flow rate due to coolant injection tends to counterbalance the small reductions in total temperature and in total pressure. Therefore, it follows that the effect of coolant injection upon the portions of the cycle downstream of the zone directly cooled is negligibly small —again, as far as this simple one-dimensional analysis discloses.

In recapitulation, the coolant injection, used in transpiration cooling or in film cooling, has a negligible effect both upon those blade rows which are directly cooled and upon that portion of the cycle downstream of the coolant-injection zone.

VII. CONCLUSIONS FOR PART B -- ENGINE CYCLE CONSIDERATIONS

The following conclusions evolve from the discussion concerning engine cycle consideration given in Section VI:

1. No turbine-blade cooling method other than transpiration and film cooling appears to be adequate for combustion-chamber temperatures at 2500°K and above.
2. Gaseous transpiration cooling, liquid transpiration cooling, transpiration cooling with internal liquid vaporization, and film cooling are all comparable to one another in terms of coolant requirements, and all will provide the necessary turbine-blade protection for combustion-chamber temperature of 2500°K with coolant requirements amounting to only a few percent of the working-gas flow rate.
3. When turbine-blade temperatures near 1150°K (1610°F) are permissible, gaseous transpiration is somewhat superior to the other three cooling methods.
4. When turbine-blade temperatures of 900°K (1160°F) and lower are desired, then gaseous transpiration cooling appears to be inferior to the other three cooling methods in terms of coolant requirements; transpiration cooling with internal liquid vaporization is the most promising of them all.

5. The vapor-lock phenomena associated with liquid transpiration cooling can be surmounted with transpiration cooling with internal liquid vaporization.
6. The coolant requirements for transpiration cooling with internal liquid vaporization amount to only approximately one percent of the main stream mass flow for each cooled-turbine-blade stage.
7. The ratio of the total temperature of the working gases relative to the blades to the total temperature (i.e., T_{or}/T_o) can be reduced to values approaching 0.80 with the use of high turbine wheel speeds, which are made possible by turbine blade cooling.
8. Coolant injection in transpiration and film cooling has a negligible effect upon cycle performance.

VIII. SUMMARY CONCLUSION

Although all the problems associated with the application of a gas turbine cycle to the manufacture of nitric oxide appear to be surmountable, no attempt has been made in this study, however, to ascertain the economical feasibility of the process. The answer to this question involves factors which are not purely technical and thus lies outside the scope of the thesis. The advantages of transpiration and film cooling in turbine-blade cooling are clearly evident. Transpiration cooling with internal liquid vaporization is the most promising of the cooling methods.

IX. REFERENCES

1. Penner, S. S. -- Thermodynamics and Chemical Kinetics of One-Dimensional Nonviscous Flow Through a Laval Nozzle, J. of Chem. Phys., (July, 1951), 19, pp. 877-881.

2. Gilbert, N. and Daniels, F. -- Fixation of Atmospheric Nitrogen In a Gas Heated Furnace, Ind. Eng. Chem., (1948), 40, pp. 1719-1723.

3. Yost, Don M. and Russell, Horace Jr. -- Systematic Inorganic Chemistry of the Fifth-and-Sixth-Group Nonmetallic Elements, Prentice-Hall, Inc., New York, (1946).

4. Collings, G. H. -- Commercial Fertilizers, The Blakiston Co., Philadelphia, (1934).

5. Perry, John H. -- Chemical Engineer's Handbook, 3rd Edition, McGraw-Hill Book Company, Inc., New York, (1950).

6. Shreve, R. Norris -- The Chemical Process Industries, 1st Edition, McGraw-Hill Book Co., Inc., New York, (1945).

7. Sisco, Frank T. -- Modern Metallurgy for Engineers, 2nd Edition, Pitman Publishing Corp., New York, (1948).

8. Zucrow, M. J. -- Principles of Jet Propulsion and Gas Turbines, John Wiley and Sons, Inc., New York, (1948).

9. Timoshenko, S. and Goodier, J. M. -- Theory of Elasticity, 2nd Edition, McGraw-Hill Book Co., Inc., New York, (1951).

10. Timoshenko, S. -- Theory of Plates and Shells, McGraw-Hill Book Co., Inc., New York, (1940).

11. Wise, Henry and Frech, Maurice F. -- Kinetics of Decomposition of Nitric Oxide at Elevated Temperatures, J. Chem. Phys., (1952), 20, p. 22.

12. Koerner, W. E. -- The Thermochemistry and Decomposition Kinetics of Nitric Oxide, Ph.D. Thesis, University of Wisconsin, Madison, Wisconsin, (1949).

13. Hendrickson, W. G. -- Ph.D. Thesis, University of Wisconsin, Madison, Wisconsin, (1946).

14. Rossini, Frederick D., et.al., -- Tables of Selected Values of Chemical Thermodynamic Properties, Series I, Volume I, March 31, 1947 to June 30, 1949, National Bureau of Standards, Washington, D. C.

15. Rossini, Frederick D., et.al. -- Tables of Selected Values of Chemical Thermodynamic Properties, Series III, Volume I, March 31, 1947 to June 30, 1949, National Bureau of Standards, Washington, D. C.

16. Keenan, Joseph H. and Kaye, Joseph -- Gas Tables: Thermodynamic Properties of Air, Products of Combustion and Component Gases, Compressible Flow Functions, John Wiley and Sons, Inc., New York, (1950).

17. Hottel, H. C., Williams, G. C., and Satterfield, C. N. -- Thermodynamic Charts for Combustion Processes Parts I and II, John Wiley and Sons, Inc., New York, (1949).

18. Schey, Oscar W. -- The Advantages of High Inlet Temperature for Gas Turbines and Effectiveness of Various Methods of Cooling the Blades, ASME Paper No. 48-A-105, presented before Annual Meeting ASME (New York), (Nov. 28, - Dec. 3, 1948).

19. Ellerbrock, Herman E. -- NACA Investigations of Gas Turbine Blade Cooling, J. Aero. Sci., (Dec., 1948), 15, pp. 721-730.

20. Meyer, R. E. -- The Boundary Layer Cooling of a Flat Plate: A Contribution to a Discussion on the Cooling of Gas Turbine Blades, Aeronautical Research Council R and M , No. 2420, (1951).

21. Rannie, W. D. -- A Simplified Theory of Porous Wall Cooling, Jet Propulsion Laboratory, California Institute of Technology, Progress Report No. 4-50, (Nov. 24, 1947).

22. Friedman, Joseph -- A Theoretical and Experimental Investigation of Rocket-Motor Sweat Cooling, J. American Rocket Soc., (Dec., 1949), 79, pp. 147-154.

23. Crocco, L. -- An Approximate Theory of Porous, Sweat, or Film Cooling with Reactive Fluids, J. American Rocket Soc., (Nov.-Dec., 1952), 22, pp. 331-338.

24. Knuth, Eldon L. -- Evaporation From Liquid Films into a Turbulent Gas Stream, Jet Propulsion Laboratory, California Institute of Technology, External Publication 176, (Mar. 27, 1953).

25. Knuth, Eldon L. -- Mechanics of Film Cooling, Ph.D. Thesis, California Institute of Technology, (1953).

26. Eckert, E. R. G. and Livingood, John N. B. -- Method for Calculation of Heat Transfer in Laminar Region of Air Flow Around Cylinders of Arbitrary Cross Section (Including Large Temperature Differences and Transpiration Cooling), NACA TN 2733, (June, 1952).

27. Livingood, John N. B., and Eckert, E. R. G. -- Calculation of Transpiration-Cooled Gas-Turbine Blades, Personal communication, (Mar. 25, 1953). Paper presented at ASME Spring Meeting, Columbus, Ohio (Apr. 28-30, 1953).

28. Lees, Lester -- Stability of the Laminar Boundary Layer with Injection of Cool Gas at the Wall, Project Squid Technical Report 11, (May 20, 1948).

29. Giedt, W. H. -- Effect of Turbulence Level of Incident Air Stream on Local Heat Transfer and Skin Friction on a Cylinder, J. Aero. Sci., (Nov., 1951), 18, pp. 725-730.

30. Goldstein, S. -- Modern Developments in Fluid Dynamics, Oxford, (1938).

31. Schlichting, H. -- Lecture Series "Boundary Layer Theory" Part II - Turbulent Flows, NACA TM 1218 (Apr., 1949).

32. Vincent, E. T. -- The Theory and Design of Gas Turbines and Jet Engines, McGraw-Hill Book Co., Inc., New York, (1950).

33. Kinney, George R. and Abramson, Andrew E. -- Investigation of Annular Liquid Flow with Cocurrent Air Flow in Horizontal Tube, NACA RM E51C13, (May 25, 1951).

34. Green, Leon Jr. -- Fluid Flow Through Porous Metals, Jet Propulsion Laboratory, California Institute of Technology, Progress Report No. 4-111, (Aug. 19, 1949).

35. Ellerbrock, Herman H. Jr. -- Some NACA Investigations of Heat Transfer Characteristics of Cooled Gas-Turbine Blades. Paper presented at the General Discussion on Heat Transfer, Institute of Mechanical Engineers (London) and ASME (New York) Conference in London (Sept. 11-13, 1951).

36. Wislicenus, George F. -- Fluid Mechanics of Turbomachinery, 1st Edition, McGraw-Hill Book Company, Inc., New York, (1947).

37. Weske, J. R. -- Investigation of Blade Characteristics, Transactions ASME (July, 1944), p. 413.

38. Shapiro, A. H. and Hawthorne, W. R. -- The Mechanics and Thermodynamics of Steady, One-Dimensional Gas Flow, Transactions ASME, J. Applied Mechanics, (Dec., 1947), 14, pp. A-317 - A-336.

39. Guggenheim, E. A. -- Thermodynamics: An Advanced Treatment for Chemists and Physicists, 2nd Edition, Interscience Publishers, Inc., New York, (1950).

40. Penner, S. S. -- Calculations on the Formation and Decomposition of Nitric Oxide in Pebble Beds at High Temperatures, Part II, Ph. D. Thesis, University of Wisconsin, Madison, Wisconsin, (1946).

41. Duwez, P. and Wheeler, H. L. Jr. -- Experimental Study of Cooling by Injection of a Fluid Through a Porous Material, J. of Aero. Sci., (Sept., 1948), 15, pp. 509-521.

APPENDIX I

Nomenclature

a. Latin Letter Symbols

Symbol	Quantity
A	Area
A	Constant in Eq. (A IV-3)
A^*	Cross-sectional area at nozzle throat
a	Equilibrium concentration of nitric oxide ($=x^o_{NO}$)
a	Speed of sound
B	Constant in Eq. (A IV-3)
b	Equilibrium concentration of nitrogen ($=x^o_{N_2}$)
b	Constant in Eq. (A III-7) = $(A/A^*) - 1$
C	Constant in Eq. (A IV-3)
c	Equilibrium concentration of oxygen ($=x^o_{O_2}$)
c	Blade chord
c_f	Skin friction coefficient ($= \tau_w / \frac{1}{2} \rho_g u_g^2$)
c_k	Concentration of species k
c_p	Coefficient of specific heat at constant pressure
c_v	Coefficient of specific heat at constant volume
D	Constant in Eq. (A IV-3)
D_h	Hydraulic diameter
d	Average width of flow passage (Cf. Eq. (23))
e	Base of the natural system of logarithms

Symbol	Quantity
G	Constant in Eq. (A IV-4)
H	Enthalpy
h	Enthalpy
K_c	Equilibrium constant based on concentrations
K_p	Equilibrium constant based on partial pressures
$K_p{}^2$	Equilibrium constant = $(p_{NO})^2/(p_{N_2})(p_{O_2})$
$K_{p_o}{}^2$	Equilibrium constant = a^2/bc
k	Rate constant
k	Coefficient of thermal conductivity of vapor (Cf. A VII-5)
L_F	Quantity = $c_{p_m}(T_g-T_f)/\Delta H_v$ (Cf. Eq. (27))
L_G	Quantity = $c_{p_m}(T_g-T_w)/c_{p_w}(T_w-T_r)$ (Cf. Eq. (20))
L_L	Quantity = $c_{p_m}(T_g-T_f)/\left[\Delta H_v + c_{p_\ell}(T_f-T_r)\right]$ (Cf. Eq. (24))
L_{LV}	Quantity = $c_{p_m}(T_g-T_w)/\left[\Delta H_v + c_{p_\ell}(T_f-T_r) + c_{p_w}(T_w-T_\delta)\right]$ (Cf. Eq. (26))
L_1	Quantity defined below Eq. (A VII-6)
ℓ	Length of converging portion of De Laval nozzle
ℓ_w	Thickness of porous wall
M	Mach number
M_k	Molecular weight of species k
M^*	Quantity = $M\left(\frac{\gamma+1}{2}\ \frac{T}{T_o}\right)^{1/2}$ (Cf. Eq. A III-5)
n	Exponent in Eq. (15)
p	Pressure; partial pressure

Symbol	Quantity
q	Heat transfer per unit area per unit time
R	Universal gas constant = 1.987 cal/g-mole(^{o}K)
R'	Universal gas constant = 82.06×10^{-3} liter-atm/g-mole(^{o}K)
R''	Modified gas constant = 3.08×10^{3} ft^{2}/sec^{2}(^{o}K)
s	Blade span
T	Absolute static temperature
T_o	Absolute total temperature
T'	Composition temperature corresponding to the actual gas composition existing at the static temperature (Used in Part A of thesis)
T'	Absolute temperature relative to observer moving with the gas stream (Used in Part B of thesis)
t	Time
U_W	Peripherial speed of turbine wheel
u	Velocity
v	Velocity
$\vec{v}$	Vector velocity
W	Mass flow rate
w	Mass flow rate per unit area
x	Mole fraction; concentration of N_2 and O_2 formed during decomposition of nitric oxide
x	Rectangular Cartesian coordinate
x	Distance along duct
x_1 , x_2	See definition below Eq. (20)
y	Rectangular Cartesian coordinate

Symbol	Quantity
z	Rectangular Cartesian coordinate

b. Greek Letter Symbols

Symbol	Quantity
α	Blade entrance angle
β	Quantity = $\rho_c v_c / \mu$
β_1	Quantity = Pr β
γ	Ratio of specific heat at constant pressure to the specific heat at constant volume ($=c_p/c_v$)
Γ_k	Net rate of production of species k
δ	Laminar sublayer thickness
ε	Quantity in Eq. (A IV-4)
Λ	Quantity defined below Eq. (A VII-10)
μ	Dynamic viscosity
ν	Kinematic viscosity ($=\mu/\rho$)
ν''_k, ν'_k	Coefficients which multiply the chemical symbol G in the relation representing the stoichiometry of the reaction given by Eq. (6) or by Eq. (A II-16)
ξ	Fractional part of initial nitric oxide which has decomposed ($=2x/a$)
ρ	Density
τ	Time period (Cf. remarks following Eq. (14))
τ	Shearing stress
ϕ	Quantity defined by Eq. (49)
ψ	Quantity = $\ln (1+\varepsilon)/(1-\varepsilon) - 2\varepsilon$ (Cf. Eq. (A IV-4))
ψ	Quantity defined by Eq. (51)

c. Latin Letter Subscripts

Symbol	Quantity
b	Backward or reverse
c	Coolant
f	Forward
f	"film" or liquid-gas interface
g	Main stream
j	Component j
k	Component k
ℓ	Liquid
m	Mean value
o	Total or stagnation
p	Constant pressure
p	Passage (in connection with flow area)
r	Reservoir
r	Relative
s	Surface (in connection with area)
v	Vapor
w	Wall
W	Turbine wheel

d. Greek Letter Subscripts

δ	Laminar sublayer thickness
$\left(-\frac{DT}{Dt}\right)$	Rate of decrease of static temperature with time following the fluid

Symbol	Quantity
ΔH	Heat of reaction or the heat absorbed at constant pressure for chemical reactions
ΔH_v	Latent heat of vaporization
ΔU	Heat of reaction at constant volume
k_{fD}, k_{fW}	Reaction rate constants defined by Eqs. (10) and (12), respectively
$f(M, \gamma)$	Functional relation defined by Eq. (A III-11)

f. Superscripts and Special Signs

Sign	Meaning	Example of Application
Σ	Addition operation	$\sum_1^2 x_j = x_1 + x_2$
Δ	Change in a quantity	$\Delta \xi$ = final minus initial value of
$\overline{(\ \ )}$	Mean value	k_f = average value of k_f over the range of the independent variable considered
$(\ \ )^o$	Equilibrium value	$x^o_{N_2}$ = equilibrium mole fraction of N_2
$\prod$	Product operation	$\prod_1^2 x_j = x_1 \cdot x_2$
$\frac{D}{Dt}$	$\frac{\partial}{\partial t} + \vec{v} \cdot \nabla$, the Euler total time derivative following the fluid	$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T$
$\overrightarrow{(\ \ )}$	Vector quantity	$\vec{v}$ = vector velocity
Re_c	Reynolds number $(= u_g c / \nu_g)$	
Pr	Prandtl number	
δ^*	Constant defined by Eq. (A VI-11)	
Δh	Quantity defined by Eq. (38)	

APPENDIX II

Derivation of the Near-Frozen Flow Criterion

With a few minor changes and extensions, this derivation is merely a more detailed presentation of Penner's abbreviated treatment in the original publication (Ref. 1). Consider the one-dimensional isentropic expansion of a mixture of perfect gases through a De Laval nozzle. Let the subscript "o" represent the stagnation conditions at the nozzle entrance section.

For compressible fluids the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \tag{1}$$

which may also be written

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0 . \tag{2}$$

For a mixture of compressible fluids a smiliar equation is valid for each constitutent, except that a term accounting for the production of chemical species must be added to the right-hand side of the equation; viz.,

$$\frac{D\rho_k}{Dt} + \rho_k \nabla \cdot \vec{v}_k = \Gamma_k M_k , \tag{3}$$

where the product $\Gamma_k M_k$ represents the net rate of production by chemical reaction of species k in units of mass per unit volume per unit time.

The density of a component is related to its concentration through the relation

$$\rho_k = M_k \; c_k \quad . \tag{4}$$

Since M_k is a constant for each component independent of time and position, the substitution of Eq. (4) into Eq. (3) yields the relation

$$\frac{D c_k}{Dt} + c_k \nabla \cdot \vec{v_k} = \Gamma_k \tag{5}$$

If it is assumed that each constitutent of the fluid moves with approximately the mean mass-weighted velocity of the gas stream (i.e., if the diffusion velocity of each component is small in comparison with the mean mass-weighted velocity of the gas stream), then Eq. (5) takes the form

$$\frac{D c_k}{Dt} + c_k \nabla \cdot \vec{v} = \Gamma_k \quad , \tag{6}$$

which may also be written

$$\frac{D(\ln c_k)}{Dt} + \nabla \cdot \vec{v} = \frac{\Gamma_k}{c_k} \quad . \tag{7}$$

But from Eq. (2)

$$\nabla \cdot \vec{v} = -\frac{D(\ln \rho)}{Dt} \quad ; \tag{2}$$

hence, Eq. (7) may be written

$$\frac{D(\ln c_k)}{Dt} - \frac{D(\ln \rho)}{Dt} = \frac{\Gamma_k}{c_k} \quad . \tag{8}$$

Introducing the isentropic relation between the density and temperature; viz.,

$$\rho = \text{Constant} \times T^{1/(\gamma-1)}$$

in the form

$$\frac{D(\ln \rho)}{Dt} = \frac{1}{\gamma-1} \frac{D(\ln T)}{Dt} \tag{9}$$

into Eq. (8), yields the continuity equation

$$\frac{D(\ln c_k)}{Dt} = \frac{\Gamma_k}{c_k} + \frac{1}{\gamma-1} \frac{D(\ln T)}{Dt} . \tag{10}$$

Consider reversible chemical reaction which may be represented by the relationship

$$\sum \nu_k' G_k \overset{k_f}{\underset{k_b}{\rightleftarrows}} \sum \nu_k'' G_k . \tag{11}$$

From the principle of mass action the net rate of production of species k in moles per unit volume per unit time may be written

$$\Gamma_k = (\nu_k'' - \nu_k')\left[k_f \prod c_j^{\nu_j'} - k_b \prod c_j^{\nu_j''}\right] , \tag{12}$$

which may also be written

$$\Gamma_k = (\nu_k'' - \nu_k') k_f \prod c_j^{\nu_j'} \left\{1 - \frac{K_c(T)}{K_c(T_0)} \frac{K_c(T_0)}{K_c(T')}\right\} , \tag{13}$$

where

$$K_c(T) = k_b/k_f = \prod c_j^{\circ\,\nu_j'} \Big/ \prod c_j^{\circ\,\nu_j''} \quad , \tag{14}$$

and

$$K_c(T') = \prod c_j^{\nu_j'} \Big/ \prod c_j^{\nu_j''} \quad . \tag{15}$$

The last expression is to be considered as a defining equation for the so-called "composition temperature" T', which corresponds to the temperature at which the actual combination of concentrations on the right-hand side of the equation would exist at equilibrium. On the other hand, $K_c(T)$ is the usual equilibrium constant based upon equilibrium concentrations existing at the temperature T.

The concept of "near-frozen flow" is now introduced in order to simplify Eq. (13). If the rate of cooling is sufficiently rapid to "freeze" the chemical reaction in the sense discussed in the Introduction, then the "composition temperature" T' will be approximately equal to the stagnation or total temperature T_o at the entrance section of the nozzle. Consequently, a Taylor expansion of the equilibrium constant $K_c(T')$ about $K_c(T_o)$ can be made, retaining only the first few terms as an approximation; viz.,

$$K_c(T') = K_c(T_o) - \left(\frac{dK_c}{dT}\right)_{T_o}(T_o - T') - \frac{1}{2}\left(\frac{d^2K_c}{dT^2}\right)_{T_o}(T_o - T')^2 + \dots, \tag{16}$$

which may also be written

$$\frac{K_c(T')}{K_c(T_o)} = 1 - \left(\frac{d\ln K_c}{dT}\right)_{T_o}(T_o - T') - \left(\frac{d^2K_c}{dT^2}\right)_{T_o}\frac{(T_o - T')^2}{2K_c(T_o)} + \dots, \tag{17}$$

The equilibrium constant K_c is related to the heat of reaction at constant volume ΔU for the chemical reaction by the relation due to van't Hoff (Ref. 39, p. 265)

$$\frac{d \ln K_c}{dT} = \frac{\Delta U}{RT^2} \tag{18}$$

Therefore, Eq. (17) may be rewritten after some algebraic manipulation as

$$\frac{K_c(T')}{K_c(T_0)} = 1 - \frac{\Delta U}{RT_0^2}(T_0 - T')\left\{1 - \left[\frac{1}{2}\frac{\Delta U}{RT_0^2} - \frac{1}{T_0}\right](T_0 - T')\right\} + \cdots \tag{19}$$

If the values, $\Delta U = 50,000$ cal/mole, $T_0 = 2500^\circ K$, and $(T_0 - T') = 25^\circ K$, may be taken as representative, then $\Delta U(T_0 - T')/RT_0^2$ is of the order of 0.1, and $\left\{\left[\frac{1}{2}\frac{\Delta U}{RT_0^2} - \frac{1}{T_0}\right] \times (T_0 - T')\right\}$ is of the order of 0.05. For all practical purposes the third term of Eq. (19) may be neglected, and for some purposes even the second term of Eq. (19) could be dropped. For the sake of generality the second term will be retained. The approximation for the reciprocal of Eq. (19); viz.,

$$\frac{K_c(T_0)}{K_c(T')} \cong 1 + \frac{\Delta U}{RT_0^2}(T_0 - T') \tag{20}$$

is justified.

By taking the Eulerian logarithmic derivative of Eq. (15), the relation

$$\frac{D}{Dt}[\ln K_c(T')] = -\sum\left\{(\nu_k'' - \nu_k')\frac{D \ln c_k}{Dt}\right\} \tag{21}$$

is obtained. Substituting Eq. (20) into Eq. (13) and then subsituting the resulting expression for Γ_k into Eq. (10), and finally introducing the subsequent expression for $\frac{D(\ln c_k)}{Dt}$ into Eq. (21), leads to the result

$$\frac{D \ln K_c(T')}{Dt} = -k_f \prod c_j^{\nu_j'} \left\{1 - \frac{K_c(T)}{K_c(T_0)}\left[1 + \frac{\Delta U}{RT_0^2}(T_0 - T')\right]\right\} \sum \left[\frac{(\nu_k'' - \nu_k')^2}{c_k}\right] - \frac{1}{\gamma - 1}\,\frac{D \ln T}{Dt} \sum (\nu_k'' - \nu_k') \;. \tag{22}$$

The expression on the left-hand side of Eq. (22) may be rewritten using Eq. (18) as

$$\frac{D \ln K_c(T')}{Dt} = \frac{\Delta U}{RT_0^2}\left(\frac{T_0}{T'}\right)^2 \frac{dT'}{dT}\,\frac{DT}{Dt} \;.$$

But $(T_0 - T')/T_0$ is of the order of 25/2500 or 0.01; hence,

$$\frac{D \ln K_c(T')}{Dt} \cong \frac{\Delta U}{RT_0^2}\,\frac{dT'}{dT}\,\frac{DT}{Dt} \;. \tag{23}$$

A good approximation to dT'/dT is given by

$$\frac{dT'}{dT} \cong \frac{T_0 - T'}{T_0 - T} \;. \tag{24}$$

Substituting this expression into Eq. (23) and introducing the resulting expression into Eq. (22), leads to the result

$$\frac{T_0 - T'}{T_0 - T} = \frac{\bar{k}_f \prod c_j^{\nu_j'}}{\frac{\Delta U}{RT_0^2}\left(-\frac{DT}{Dt}\right)} \left\{1 - \frac{K_c(T)}{K_c(T_0)}\left[1 + \frac{\Delta U}{RT_0^2}\,\frac{T_0 - T'}{T_0 - T}\,(T_0 - T)\right]\right\} \sum (\nu_k'' - \nu_k')^2/c_k + \frac{\sum (\nu_k'' - \nu_k')}{\frac{\Delta U}{RT_0^2}(\gamma - 1)T} \;, \tag{25}$$

from which the expression for $(T_o - T')/(T_o - T)$ is obtained; viz.,

$$\frac{T_o - T'}{T_o - T} = \frac{\dfrac{\overline{k}_f \prod c_j^{\nu_j'}}{(-DT/Dt)} \left[1 - \dfrac{K_c(T)}{K_c(T_o)}\right] \sum \left[\dfrac{(\nu_k'' - \nu_k')^2}{c_k}\right] + \dfrac{\Sigma(\nu_k'' - \nu_k')}{(\gamma - 1)T}}{\dfrac{\Delta U}{R T_o^2} \left\{1 + \dfrac{\overline{k}_f \prod c_j^{\nu_j'}}{(-DT/Dt)} \dfrac{K_c(T)}{K_c(T_o)} (T_o - T) \sum \left[\dfrac{(\nu_k'' - \nu_k')^2}{c_k}\right]\right\}} . \qquad (26)$$

Each factor, aside from DT/Dt, in the expression on the right of this last equation is either a constant or a known function of the temperature. The rate constant $\overline{k}_f$ is to be evaluated at the average static temperature T. The concentrations should be taken at the "composition temperature" T'. However, the error introduced in taking the concentrations at the total temperature T_o at the entrance section of the nozzle instead of at T' is small and leads to conservative estimates of $(T_o - T')/(T_o - T)$. Moreover, neglecting the second term in the braces in the denominator further increases the conservativeness of the determination of $(T_o - T')/(T_o - T)$. A discussion of the calculation of (-DT/Dt) is given in Appendix III.

For chemical reactions in which the number of moles of reactants is equal to the number of moles of products $\Sigma(\nu_j'' - \nu_j') = \Sigma \nu_j'' - \Sigma \nu_j' = 0$. Consequently, for this case Eq. (26) reduces to

$$\frac{T_o - T'}{T_o - T} = \frac{\overline{k}_f \prod x_j^{\nu_j'} \left(\dfrac{p_o}{R T_o} \dfrac{\rho}{\rho_o}\right)^{\Sigma \nu_j' - 1} \left[1 - \dfrac{K_p(T)}{K_p(T_o)}\right] \sum \left[\dfrac{(\nu_k'' - \nu_k')^2}{x_k}\right]}{\dfrac{\Delta H}{R T_o^2} \left(-\dfrac{DT}{Dt}\right) \left\{1 + \dfrac{\overline{k}_f \prod x_j^{\nu_j'} \left(\dfrac{p_o}{R T_o} \dfrac{\rho}{\rho_o}\right)^{\Sigma \nu_j' - 1}}{(-DT/Dt)} \dfrac{K_p(T)}{K_p(T_o)} (T_o - T) \sum \left[\dfrac{(\nu_k'' - \nu_k')^2}{x_k}\right]\right\}} \qquad (27)$$

where use has been made of the fact that $\Delta H = \Delta U$, $K_c = K_p$ and $c_k = (p/RT)x_k = (p_o/RT_o)(\rho/\rho_o)x_k$ for the reactions under consideration; i.e., in which $\Sigma(\nu_j'' - \nu_j') = 0$. (Ref. 39, p. 265).

If the second term on the left of Eq. (19) had been neglected, the second term in the braces in the denominator would not have appeared. Therefore, under the approximation that $\Delta U(T_o - T')/RT_o^2 \cong 0$, the expression for $(T_o - T')/(T_o - T)$ is

$$\frac{T_o - T'}{T_o - T} \cong \frac{\overline{k}_f \prod \chi_j^{\nu_j'} \left(\frac{p_o}{RT_o}\frac{\rho}{\rho_o}\right)^{\Sigma \nu_j - 1}}{\frac{\Delta H}{RT_o^2}\left(-\frac{DT}{Dt}\right)} \left[1 - \frac{K_p(T)}{K_p(T_o)}\right] \sum \left[\frac{(\nu_k'' - \nu_k')^2}{\chi_k}\right] \quad (28)$$

for the case $\Sigma(\nu_j'' - \nu_j') = 0$. The neglect of the second term in the braces in the denominator of Eq. (27) leads to conservative estimates for $(T_o - T')/(T_o - T)$, as does the evaluation of the concentrations at the total temperature T_o.

Since $\Sigma(\nu_j'' - \nu_j') = 0$ for the decomposition of nitric oxide (i.e., for the reaction $2NO \rightleftarrows N_2 + O_2$), Eqs. (27) and (28) are applicable. Eq. (28) is used in preference to Eq. (27) in calculating the actual numerical values for the temperature ratio $(T_o - T')/(T_o - T)$ for sake of ease in calculations. The error introduced by employing Eq. (28) instead of Eq. (27) is small and leads to conservative values for the temperature ratio $(T_o - T')/(T_o - T)$.

APPENDIX III

Derivation of (-DT/Dt) as a Function of the Nozzle Parameters

Consider the one-dimensional isentropic expansion of a perfect gas through a De Laval nozzle with sonic velocity at the nozzle throat section. Since the temperature field through the nozzle is a function of the axial coordinate x alone and is not a function of time,

$$\left(-\frac{DT}{Dt}\right) = -U\frac{dT}{dx} = -Ma\frac{dT}{dA}\frac{dA}{dx} \quad . \tag{1}$$

Since $a = \sqrt{\gamma RT}$, Eq. (1) may be rewritten for γ equal to a constant as

$$\left(-\frac{DT}{Dt}\right) = -\frac{M\sqrt{\gamma R}\,T^{3/2}}{\ell}\left(\frac{T}{T_o}\right)^{1/2}\frac{d(T/T_o)}{d(A/A^*)}\;\frac{d(A/A^*)}{d(x/\ell)} \quad , \tag{2}$$

where the superscript asterisk indicates the nozzle throat section and where ℓ is the length of the nozzle from the entrance section to the throat.

The following relations hold for isentropic flow (Ref. 16, p. 209):

$$T/T_o = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1} \tag{3}$$

$$A/A^* = (1/M)(M/M^*)^{(\gamma+1)/(\gamma-1)} \tag{4}$$

$$M^* = M\left(\frac{\gamma+1}{2}\frac{T}{T_o}\right)^{1/2} \quad . \tag{5}$$

An expression for $d(T/T_o)/d(A/A^*)$ can be derived from Eqs. (3), (4) and

(5) by performing the indicated differentiation and by simplifying the resulting equation; viz.,

$$\frac{d(T/T_o)}{d(A/A^*)} = \frac{2(\gamma-1)}{\gamma+1} \; \frac{T/T_o}{A/A^*} \; \frac{M^{*2}}{1-M^{*2}} \; . \tag{6}$$

The quantities A/A^*, T/T_o and M^* have been calculated and tabulated as a function of Mach number for various values of γ by Keenan and Kaye (Ref. 16, pp. 139-147).

The area ratio A/A^* can be expressed as

$$A/A^* = 1 + b(1 - x/\ell)^2 \; , \tag{7}$$

where b is the area ratio A/A^* at the nozzle entrance section minus unity $\left[\text{i.e., } b = (A/A^*)_o - 1\right]$, for De Laval nozzles whose cross-sectional shape can be represented by a parabola. For such nozzles

$$\frac{d(A/A^*)}{d(x/\ell)} = \pm 2\sqrt{b}\,(A/A^* - 1)^{1/2} \; , \tag{8}$$

where the minus sign is taken for x/ℓ less than one and the plus sign for x/ℓ greater than one.

Substituting the results of Eqs. (6) and (8) into Eq. (2) leads to the equation

$$\left(-\frac{DT}{Dt}\right) = \frac{2\sqrt{b}}{\ell}\sqrt{R''}\,T_o^{3/2} \left| \frac{2\sqrt{\gamma}(\gamma-1)}{\gamma+1} \; \frac{\left(\frac{A}{A^*} - 1\right)^{1/2}}{A/A^*} \left(\frac{T}{T_o}\right)^{3/2} \frac{M M^{*2}}{1-M^{*2}} \right| , \tag{9}$$

or

$$\left(-\frac{DT}{Dt}\right) = \frac{2\sqrt{b}}{\ell} \sqrt{R''} \; T_o^{3/2} \; f(M,\gamma) \; , \tag{10}$$

where

$$f(M,\gamma) = \left| \frac{2\sqrt{\gamma}(\gamma-1)}{\gamma+1} \; \frac{\left(\frac{A}{A^*}-1\right)^{1/2}}{A/A^*} \left(\frac{T}{T_o}\right)^{3/2} \frac{MM^{*2}}{1-M^{*2}} \right| . \tag{11}$$

The function f(M, γ) may be computed directly using the tabulated values of Keenan and Kaye. The values of f(M, γ) are given in Table AIII-1 for $\gamma = 1.3$ and $\gamma = 1.4$. Since the dependence of A/A^*, T/T_o and M^* upon γ is small, a good approximation for interpolating to other values of γ is given by the expression

$$f(M,\gamma) = \left[\frac{\sqrt{\gamma}(\gamma-1)}{\gamma+1}\right]\left[\frac{\sqrt{\gamma_1}(\gamma_1-1)}{\gamma_1+1}\right]^{-1} f(M,\gamma_1) . \tag{12}$$

When the conditions at the nozzle entrance section and the geometry of the nozzle are specified, the rate of temperature decrease experienced by the gas (-DT/Dt) can be calculated from Eq. (1) with the aid of Table AIII-1 and Eq. (12).

For nozzles whose variation of cross-sectional area may be represented by an expression of the form

$$A/A^* = 1 + b\left(1 - x/\ell\right)^n , \tag{13}$$

the equation for (-DT/Dt) is

$$\left(-\frac{DT}{Dt}\right) = \frac{n\,b^{1/n}}{\ell}\left(\frac{A}{A^*}-1\right)^{\frac{n-2}{2n}} \sqrt{R''}\, T_o^{3/2} f(M,\gamma) . \tag{14}$$

For the case b = 1, which corresponds to an entrance Mach number of 0.310, Eq. (14) reduces to

TABLE AIII-1

M	$f(M, \gamma) \times 10^3$	
	$\gamma = 1.3$	$\gamma = 1.4$
0.05	0.0120	0.0166
0.10	0.130	0.179
0.15	0.512	0.707
0.20	1.34	1.84
0.25	2.76	3.84
0.30	5.00	6.83
0.35	8.15	11.1
0.40	12.2	16.5
0.45	17.5	23.5
0.50	23.7	31.9
0.55	31.4	44.0
0.60	39.4	52.8
0.65	48.5	64.4
0.70	58.8	77.4
0.75	69.6	91.2
0.80	80.5	105.
0.85	93.3	120
0.90	106	135
0.95	119	150
1.00	132	163
1.10	154	191
1.20	175	215
1.30	193	235
1.40	208	249
1.50	218	261

$$\left(-\frac{DT}{Dt}\right) = \frac{n}{\ell} \left(\frac{A}{A^*} - 1\right)^{\frac{n-2}{2n}} \sqrt{R''}\; T_o^{3/2} f(M,\gamma) . \qquad (15)$$

In the limit as n becomes very large

$$\left(-\frac{DT}{Dt}\right) \longrightarrow \frac{n}{\ell}\left(\frac{A}{A^*} - 1\right)^{1/2} \sqrt{R''}\; T_o^{3/2} f(M,\gamma) . \qquad (16)$$

APPENDIX IV

Decomposition Time of Nitric Oxide

The decomposition of nitric oxide proceeds according to the reaction

$$2NO \rightleftharpoons N_2 + O_2 \quad . \tag{1}$$

If the reformation of nitric oxide is considered, the rate expression may be written

$$-\frac{d(a-2x)}{dt} = k_f(a-2x)^2 - k_b(b+x)(c+x) \ , \tag{2}$$

where

x = concentration of N_2 and O_2 formed during decomposition
a = initial concentration of NO
b = initial concentration of N_2
c = initial concentration of O_2

Equation (2) may be integrated directly to yield the result

$$t = \frac{2}{k_f D} \ln \left| \frac{(Aa\xi + B - D)(B + D)}{(Aa\xi + B + D)(B - D)} \right| , \tag{3}$$

where

$\xi = 2x/a$ = the fraction of nitric oxide decomposed
$A = 4 - K_p'^2$
$B = -[4a + (b + c)K_p'^2]$

$$C = a^2 - bc\,K_p'^2$$

$$D = \sqrt{B^2 - 4AC}$$

$$K_p'^2 = k_b / k_f$$

$\overline{K}_f$ = average value of rate constant between t = 0 and t = t .

The decomposition time t has been calculated by Penner (Ref. 40), using Eq. (3), for a wide range of temperatures and for many different combinations of the initial concentrations a, b, and c. The results of these calculations have been conveniently presented in graphical form so that the time for a given concentration of nitric oxide to decompose to a given fraction of its initial value may be read directly.

It is possible, however, to simplify Eq. (3) considerably when the decomposition time is desired only for small amounts of nitric oxide decomposition. Under the approximation that $K_p'^2/4$ may be neglected in comparison with unity, Eq. (3) may be simplified to

$$t = \frac{\xi\left[1 + \psi(\varepsilon)\right]}{a\,\overline{k}_f\left\{1 - \left(K_p'/K_{p_0}'\right)^2 - \left[1 + (b+c)K_p'^2/4a\right]\xi\right\}}, \tag{4}$$

provided that ε is less than one, where

$$\psi(\varepsilon) = \ln\left[(1+\varepsilon)/(1-\varepsilon)\right] - 2\varepsilon$$

$$\varepsilon = \frac{D\xi}{4a\left\{1 - \left(K_p'/K_{p_0}'\right)^2 - \left[1 + (b+c)K_p'^2/4a\right]\xi\right\}}$$

$$D = B^2 - 4AC \cong 4(bc)^{1/2}\,G\,K_p'$$

$$G = \left\{1 + \left[1 + 2(b+c)/a\right]K_{p_0}'^2/4\right\}^{1/2}$$

$$K_{p_0}'^2 = a^2/bc$$

The variation of $\psi(\varepsilon)$ with ε is shown in Table AIV-1. For most cases of interest ε is less than 0.3, which means that the omission of $\psi(\varepsilon)$ in Eq. (4)

TABLE AIV-1

ε	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\psi(\varepsilon)$	0.0005	0.0055	0.0195	0.0459	0.0986	0.1863	0.3352

leads to an error of less than two percent. Furthermore, except for temperatures in the neighborhood of 3000°K the inequality $\left[(b+c)K_p'^2/4a \ll 1\right]$ holds; therefore, Eq. (4) simplifies to

$$t \cong \frac{\xi}{a\,\overline{k_f}\left\{1-\left(K_p'/K_{p_o}'\right)^2-\xi\right\}} \quad . \tag{5}$$

APPENDIX V

Derivation of the Dependence of Nitric Oxide Decomposition upon the Equilibrium Constant

From the principle of mass action and for the case that the number of moles of reactants and the products are equal, the equilibrium constant may be written, using the same nomenclature as was used in Appendix IV, (as long as ideal mixtures obtain) as

$$K_p'^2 = (a - 2x)^2/(b + x)(c + x) \quad , \tag{1}$$

or

$$(4 - K_p'^2)x^2 - [4a + (b + c)K_p'^2]x + (a^2 - bcK_p'^2) = 0. \tag{2}$$

But $K_p'^2 << 4$; hence by neglecting $K_p'^2$ in comparison with 4, Eq. (2) becomes to a good approximation

$$\left(\frac{2x}{a}\right)^2 - 2\left[1 + \frac{(b + c)}{4a} K_p'^2\right]\left(\frac{2x}{a}\right) + \left[1 - \left(\frac{K_p'}{K_{p_o}'}\right)^2\right] = 0. \tag{3}$$

Letting $\xi = 2x/a$, which represents the fractional part of the nitric oxide decomposed, it follows that

$$\xi = \left[1 + \frac{(b + c)}{4a} K_p'^2\right]\left\{1 - \left[1 - \frac{1 - (K_p'/K_{p_o}')^2}{[1 + (b + c)K_p'^2/4a]}\right]^{1/2}\right\}, \tag{4}$$

where the minus sign is taken in front of the square root sign so that $\xi = 0$ when $K_p'^2 = a^2/bc$ initially. For small changes in the equilibrium constant due to small changes in temperature $(K_p'/K_{p_o}')^2 \cong 1$;

hence,

$$\xi \cong \frac{1 - (K_p'/K_{p_0}')^2}{2\left[1 + (b+c)K_p'^2/4a\right]} \; . \qquad (5)$$

The extent of the decomposition of nitric oxide for small changes in the equilibrium constant resulting from small changes in temperature may be calculated from Eq. (5).

APPENDIX VI

Derivation of an Expression for the Coolant Flow Rate Required to Maintain the Surface of Porous Walls Below a Specified Temperature with Gaseous Transpiration Cooling

The discussion in Section VI-B indicated that one promising method of turbine blade cooling is to employ hollow turbine blades which have porous walls through which a coolant is forced. Rannie (Ref. 21), using the data of Duwez and Wheeler (Ref. 41), pointed out that the cooling fluid, which is forced through the porous walls and into the boundary layer, acts as a protective film shielding the surface of the porous walls against the hot main stream gases. In keeping with the findings of Duwez and Wheeler, it is assumed that the amount of cooling fluid required to maintain the surface of the turbine blades below a specified temperature will be small. The cooling fluid may be either a gas or a liquid. For the sake of simplicity in the presentation of the analysis, the case of gaseous transpiration cooling is first investigated, deferring the more complicated study of liquid transpiration cooling until Appendix A VII.

If the coolant flow rate is very low, it is reasonable to agree with Rannie's assumption that the injected gas has very little influence on the heat convection and momentum transfer in the turbulent stream and that its effects are largely confined to the laminar sublayer. If the laminar sublayer thickness is very much smaller than the radius of curvature of the turbine blades, it can be assumed that as far as the analysis of the laminar sublayer is concerned the turbine blade behaves as a flat plate. Only the case in which the coolant gas is chemically inert with respect to the combustion gases is considered. With these assumptions the problem reduces to one investigated by Rannie (Ref. 21).

In the analysis of the laminar sublayer Rannie assumes steady flow and that all derivatives with respect to the direction of flow are negligible compared to derivatives with respect to the coordinate normal to the plate. The pores of the wall are assumed to be sufficiently small and numerous that flow of the injected gas is uniform and continuous. Rannie further assumed that the coolant gas has the same physical properties as the main stream gas and that the physical properties are constant through the laminar sublayer. This assumption is not necessary provided that appropriate average values can be chosen to represent the various properties in the different regions.

Consider the flow situation and the nomenclature shown in (Fig. 7). The velocity parallel and normal to the wall are denoted by u and v, respectively. Since steady state is assumed and since all derivatives in the x-direction are neglected in comparison to those in the y-direction, the continuity equation reduces to

$$\frac{d(\rho v)}{dy} = 0 \quad ; \tag{1}$$

therefore,

$$\rho v = \rho_c v_c = constant \quad . \tag{2}$$

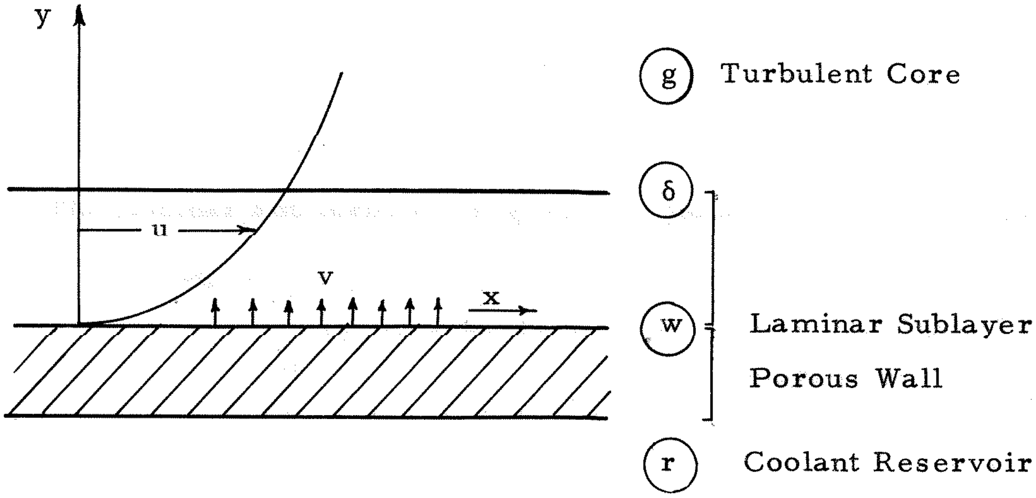


Fig. 7

By equating the force in the x-direction to the rate of change of momentum in this direction, the velocity distribution through the sublayer is obtained as

$$u = \frac{1}{\beta}\left(\frac{\mu_w}{\bar{\mu}}\right)\left(\frac{du}{dy}\right)_w \left(e^{\beta y} - 1\right) , \qquad (3)$$

where $\beta = \rho_c v_c / \bar{\mu}$, $\bar{\mu}$ = appropriate average dynamic viscosity between $y = 0$ and $y = y$, ρ = density, and where the subscripts refer to the various horizontal planes indicated in (Fig. 7). By considering an energy balance in a element of space located in the laminar sublayer, the temperature distribution is obtained as

$$\frac{T - T_w}{T_\delta - T_w} = \frac{e^{\bar{\beta}_1 y} - 1}{e^{\bar{\beta} y} - 1} , \qquad (4)$$

where T = temperature, $\bar{\beta}_1 = \bar{P_r}\,\bar{\beta}$ and $\bar{P_r}$ = appropriate average Prandtl number between $y = 0$ and $y = y$.

The laminar and turbulent regions are joined by employing the Reynolds analogy in the form

$$\frac{q_\delta}{c_{p_\delta}(T_g - T_\delta)} = \frac{\tau_\delta}{u_g - u_\delta} , \qquad (5)$$

where q is the heat transfer rate per unit area towards the wall, τ is the shearing stress, and c_{p_δ} is the average specific heat at constant pressure between T_δ and T_g. By combining Eqs. (3) and (4) with (5) and after some manipulation, an expression for the temperature of the wall T_w as a function of the flow rate, the velocity distribution, and the sublayer thickness is found in the form

$$\frac{T_w - T_r}{T_g - T_r} = \frac{e^{-\bar{\beta}_1 \delta}}{1 + \frac{c_{pw}}{c_{p_\delta}}\left(\frac{u_g}{u_\delta} - 1\right)\left(1 - e^{-\bar{\beta}\delta}\right)} , \qquad (6)$$

where c_{p_w} is the average specific heat at constant pressure between T_r and T_δ.

By supposing that $\bar{\beta}\delta$ and $\bar{\beta}_1\delta$ are small, an approximate expression can be developed for the temperature ratio given by Eq. (6) by expanding the exponential terms in power series. It turns out that the resulting approximate expression reproduces Eq. (6) very closely even for moderately large values of $\bar{\beta}\delta$. By making an assumption regarding the thickness of the laminar sublayer, the derived approximate expression for Eq. (6) is then in a form such that the ratio of the coolant flow rate per unit area $w_c = \rho_c v_c$ to the main stream flow rate per unit area $w_g = \rho_g u_g$ may be solved for in terms of the allowable temperature difference across the porous wall and the local skin friction coefficient c_f. Before this program is executed, however, expressions for u_g/u_δ and $\bar{\beta}\delta$ are required.

From Eq. (3) the velocity at the interface between the sublayer and the turbulent zone is given by

$$u_\delta = \frac{1}{\bar{\beta}} \frac{\mu_w}{\bar{\mu}} \left(\frac{du}{dy}\right)_w \left(e^{\bar{\beta}\delta} - 1\right) \quad . \tag{7}$$

Recalling that the local skin friction coefficient c_f is defined as

$$c_f = \tau_w / \left(\tfrac{1}{2} \rho_g u_g^2\right) \tag{8}$$

and noting that

$$\tau_w = \mu_w \left(\frac{du}{dy}\right)_w \quad , \tag{9}$$

the velocity at the interface u_δ may be rewritten by substituting these expressions in Eq. (7); viz.,

$$\frac{u_\delta}{u_g} = \frac{c_f}{2}\,\frac{w_g}{w_c}\left(e^{\bar{\beta}\delta} - 1\right) . \tag{10}$$

An expression for $\bar{\beta}\delta$ is now derived. Under the assumption that the laminar sublayer thickness is not changed because of fluid injection, it is supposed that the sublayer thickness is given by the same type of relation which has been found to hold for pipes; viz.,

$$\delta\,\frac{\rho}{\mu}\left(\frac{\tau_w}{\rho}\right)^{1/2} = \delta^{*} \tag{11}$$

where δ^* is a constant defined by Eq. (11). The value usually taken for δ^* in isothermal, incompressible turbulent boundary layers is 5.6. The dynamic viscosity μ and density ρ are to be evaluated at points in the laminar sublayer appropriate for the validity of Eq. (11). The subsequent analysis will show that only approximate values of μ and ρ are required when the coolant flow rates are small. Using Eqs. (8) and (11), the sublayer thickness may be written as

$$\delta = \frac{\delta^{*}\mu}{w_g}\left(\frac{\rho_g}{\rho}\right)^{1/2}\left(\frac{c_f}{2}\right)^{-1/2} ;$$

hence,

$$\bar{\beta}\delta = \frac{\mu}{\bar{\mu}}\left(\frac{\rho_g}{\rho}\right)^{1/2}\left(\frac{c_f}{2}\right)^{-1/2}\frac{w_c}{w_g}\,\delta^{*} . \tag{12}$$

By substituting Eqs. (10) and (12) into Eq. (6), by expanding the exponential terms of the resulting expression in a power series, and by neglecting square and higher terms of $\bar{\beta}\delta$, an expression for w_c/w_g may be obtained in the form

$$\frac{w_c}{w_g} \cong \frac{L_G\left(\frac{c_f}{2}\right)}{1-(1-\bar{P}_r)\delta^*\left(\frac{\mu}{\bar{\mu}}\right)\left(\frac{\rho_g}{\rho}\right)^{1/2}\left(\frac{c_f}{2}\right)^{1/2}} , \qquad (13)$$

where

$$L_G = \frac{c_{p_m}(T_g - T_w)}{c_{p_w}(T_w - T_r)} = \frac{h_g - h_w}{h_w - h_r} , \qquad (14)$$

and c_{p_w} is the average value of c_p between T_w and T_g and h_g, h_w and h_r are the total enthalpies at stations g, w and r, respectively. Assuming that $\rho = \rho_g$ and $\mu = \bar{\mu}$ in Eq. (13), and taking $\overline{Pr} = 0.7$, $\delta^* = 5.6$ and $c_f/2 = 2.5 \times 10^{-3}$ as representative of the parameters, then the denominator of Eq. (13) is

$$1 - (1 - \bar{P}_r)\delta^*(c_f/2) \simeq 1 - 0.084 = 0.916$$

Consequently, since Eq. (13) is only an approximate expression, the second term in the denominator of Eq. (13) is neglected in comparision to unity. Under this approximation the expression for the coolant flow rate simplifies to

$$\frac{w_c}{w_g} \cong L_G \frac{c_f}{2} . \qquad (15)$$

Coolant requirements calculated from Eq. (15) are conservative in comparison with values determined from the exact expression. (Cf. Fig. 4, p. 56). This fact further justifies the approximation used in simplifying Eq. (13) to Eq. (15).

APPENDIX VII

Derivation of Expressions for Coolant Evaporation Rates for (1) Liquid Transpiration Cooling, (2) Transpiration Cooling with Internal Liquid Vaporization and (3) Film Cooling.

1. Liquid Transpiration Cooling

As in the case of gaseous transpiration cooling, the coolant flow rate is assumed to be low enough so that the injected fluid has very little influence upon the turbulent stream and its effects are confined largely to the laminar sublayer. Again, steady state is assumed, and all derivatives with respect to the direction of flow are taken to be negligible compared to derivatives with respect to the coordinate normal to the porous walls. The coolant is assumed to be inert with respect to the combustion gases. The coolant is assumed to be injected uniformally and continuously over the surface. The resulting liquid film layer is assumed to be stable.

Consider the flow situation and nomenclature shown in (Fig. 8). Again, the velocity parallel and normal to the wall are denoted by u and v, respectively.

Mass Balance Since steady state is assumed and since all derivatives in the x-direction are neglected in comparison to those in the y-direction, the continuity equation reduces to that given by Eq. (A VII-1), and hence Eq. (A VII-2) applies in this case also.

Momentum Balance By equating the force in the x-direction to the

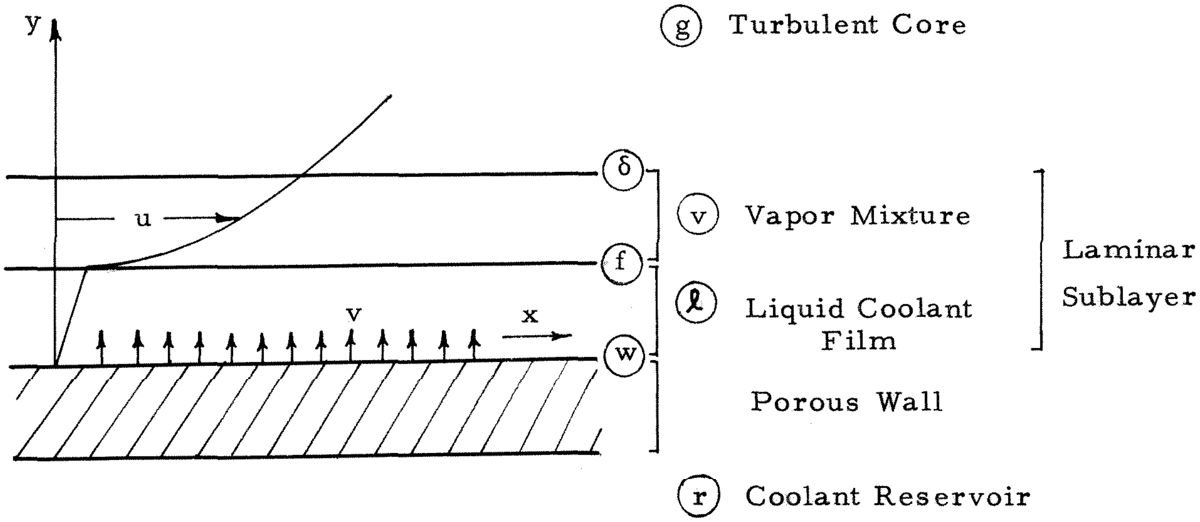


Fig. 8

rate of change of momentum in the same direction, the velocity distribution through the sublayer may be obtained. The derivation begins by writing

$$\tau - \tau_f = \rho_c v_c (u - u_f) \quad , \tag{1}$$

where the subscripts refer to various horizontal planes indicated in (Fig. 8). Using Newton's law of shearing stress that $\tau = \mu \frac{du}{dy}$ and letting $\beta_v = \rho_c v_c / \mu_v$, Eq. (1) may be rewritten

$$\frac{du}{dy} - \frac{\mu_f}{\mu_v}\left(\frac{du}{dy}\right)_f = \beta_v (u - u_f) \quad . \tag{2}$$

Equation (2) may be directly integrated between the limits y_f and some point y in the region ⓥ to yield the relation

$$u - u_f = \frac{1}{\overline{\beta}_v} \frac{\mu_f}{\overline{\mu}_v} \left(\frac{du}{dy}\right)_f \left(e^{\overline{\beta}_v (y - y_f)} - 1\right), \tag{3}$$

where $\overline{\beta}_v$ is the appropriate average value of β_v between $y = y_f$ and $y = y$. Letting $u' = u - u_f$ and $y' = y - y_f$, Eq. (3) may be rewritten

$$u' - \frac{1}{\overline{\beta}_v} \frac{\mu_f}{\overline{\mu}_v} \left(\frac{du}{dy}\right)_f \left(e^{\overline{\beta}_v y'} - 1\right) . \tag{4}$$

Energy Balance By considering an energy balance in an element of space located in region ⓥ in the laminar sublayer, the temperature distribution may be obtained in a manner similiar to that used to obtain the velocity distribution. Since all derivatives in the x-direction are assumed to be negligible in comparison to derivatives in the y-direction (which excludes, of course, transfer of heat and transfer of energy by convection in the x-direction), all the energy which passes through the laminar sublayer does so by the mechanism of heat conduction. This energy is obsorbed in increasing the enthalpy of the cooling fluid which is moving away from the porous wall in the y-direction. The enthalpy increase of the cooling fluid consists of three parts: (1) that resulting from the increase of the coolant temperature from the reservoir temperature T_r to the temperature of the liquid-gas interface T_f, i.e., $c_{p_\ell}(T_f - T_r)$ (2) that due to the heat of vaporization ΔH_v of the liquid, and (3) that resulting from the increase of the temperature of the vapor from T_f to T, i.e., $c_{p_v}(T-T_f)$. Therefore, using Fourier's law of heat conduction, the energy balance leads to the relation

$$q = \rho_c v_c \left[c_{p_\ell}(T_f - T_r) + \Delta H_v + c_{p_v}(T - T_f) \right] = k \frac{dT}{dy} , \qquad (5)$$

where q is the heat transfer rate per unit area towards the wall, c_{p_ℓ} is the average value of heat capacity at constant pressure of the liquid between T_r and T_f, c_{p_v} is the average value of the heat capacity at constant pressure of the vapor mixture between T_f and T, and k is the coefficient of thermal conductivity of the vapor at the temperature T. In analogy to the derivation of the velocity distribution, Eq. (5) may be integrated directly between the limits y_f and y to yield the equation

$$\frac{T - T_f}{T_f - T_r} = L_1 \left(e^{\beta_1 y'} - 1 \right) , \qquad (6)$$

where

$$L_1 = \frac{c_{p\ell}}{c_{pv}} \left[1 + \frac{\Delta H_v}{c_{p_\ell}(T_f - T_r)} \right]$$

$$\beta_{1v} = Pr_v \beta_v = Pr_v \rho_c v_c / \mu_v$$

Pr_v = Prandtl number of the vapor = $c_{p_v} \mu_v / k_v$

and

$\overline{\beta}_{1v}$ = appropriate average value of β_{1v} between $y = y_f$ and $y = y$.

Coolant Evaporation Rate Again in analogy with the analysis of gaseous transpiration cooling, the laminar and turbulent regions are joined by employing Reynolds' analogy in the form given by Eq. (A VI-5); viz.,

$$\frac{q_{\delta'}}{c_{p_{\delta'}}(T_g - T_{\delta'})} = \frac{\tau_{\delta'}}{u_g - u_{\delta'}} , \qquad \text{(A VI-5)}$$

where $c_{p_{\delta'}}$ is the average specific heat at constant pressure of the vapor between $T_{\delta'}$ and T_g. From Eqs. (5) and (6) expressions for the temperature and the heat transfer rate at the sublayer interface $y = \delta$ may be written as

$$T_{\delta'} = T_f + (T_f - T_r) L_1 (e^{\overline{\beta}_{1v} \delta'} - 1) , \qquad (7)$$

and

$$q_{\delta'} = c_{p_v} \rho_c v_c (T_f - T_r) L_1 e^{\overline{\beta}_{1v} \delta'} . \qquad (8)$$

From Newton's law of shearing stress and Eqs. (2) and (4) the shearing stress at the laminar sublayer interface may be written

$$\tau_{\delta'} = \frac{u'_\delta \rho_c v_c e^{\overline{\beta}_v \delta'}}{e^{\overline{\beta}_v \delta'} - 1} . \qquad (9)$$

By combining Eqs. (7), (8) and (9) with Eq. (A VI-5) and after some manipulation, an expression for temperature ratio $(T_f - T_r)/(T_g - T_r)$ is obtained; viz.,

$$\frac{T_f - T_r}{T_g - T_r} = \frac{1}{1 + L_1 \Lambda e^{\overline{\beta_{1v}}\delta'}\left[1 + \frac{1 - e^{-\overline{\beta_1}\delta'}}{\Lambda}\right]} , \qquad (10)$$

where

$$\Lambda = \left(\frac{c_{p_v}}{c_{p_{\delta'}}}\right)\left(\frac{u_g}{u'_\delta} - 1\right)\left(1 - e^{-\overline{\beta_v}\delta'}\right) .$$

Equation (10) corresponds to Eq. (AVI-6), which was derived for gaseous transpiration cooling. The temperature ratio $(T_f - T_r)/(T_g - T_r)$, the dimensionless enthalpy term L_1, and the heat capacity ratio (c_{p_v}/c_{p_δ}) are presumed to be known. The velocity ratio u_g/u_δ is determined from Eq. (4) by introducing the local skin friction coefficient c_f just as was done in the case of gaseous transpiration cooling. Consequently, Eq. (10) determines implicitly the quantity $\overline{\beta_{1v}}\ \delta' = \overline{\beta_v}\ \overline{Pr_v}\delta'$ when the values of T_r, T_f and T_g are given. Therefore, since the dimensionless coolant flow rate or evaporation rate $w_c/w_g = \rho_c v_c/\rho_g u_g$ is related to $\overline{\beta_v}\delta'$ through an expression similiar to Eq. (AVI-13), the required coolant flow rate is essentially determined by Eq. (10). The analysis now proceeds as in the case of gaseous transpiration cooling by supposing that $\overline{\beta_v}\delta'$ and $\overline{\beta_{1v}}\ \delta'$ are small and by expanding the exponential terms in series. The resulting expression may then be solved for the coolant evaporation rate w_c/w_g. First, however, expressions for u_g/u'_δ and $\overline{\beta_v}\delta$ are derived. From Eqs. (AVI-8) and from Eq. (4), it may be shown that

$$\frac{u'_\delta}{u_g} = \frac{c_f}{2}\frac{w_g}{w_c}\left(e^{\bar{\beta}_v\delta'} - 1\right) \quad . \tag{11}$$

In a manner analogous to that for gaseous transpiration cooling, it may be shown that

$$\bar{\beta}_v\delta' = \frac{\mu}{\bar{\mu}_v}\left(\frac{\rho_g}{\rho}\right)^{1/2}\left(\frac{c_f}{2}\right)^{-1/2}\frac{w_c}{w_g}\,\delta^* \quad . \tag{12}$$

By substituting Eqs. (11) and (12) into Eq. (10), by expanding the exponential terms of the resulting expression in a power series, and by neglecting square and higher terms of $\bar{\beta}_v\delta'$, an expression for w_c/w_g may be obtained in the form

$$\frac{w_c}{w_g} \cong \frac{L_L(c_f/2)}{1-(1-\overline{Pr})\delta^*\left(\frac{\mu}{\bar{\mu}_v}\right)\left(\frac{\rho_g}{\rho}\right)^{1/2}\left(\frac{c_f}{2}\right)^{1/2}} \ , \tag{13}$$

where

$$L_L = \frac{c_{p_m}(T_g - T_f)}{\Delta H_v + c_{p_\ell}(T_f - T_r)} = \frac{h_g - h_f}{\Delta H_v + h_f - h_r} \ , \tag{14}$$

and c_{p_m} is the mean value of c_p between T_f and T_g and h_g, h_f and h_r are the total enthalpies at the stations g, f, and r, respectively. Using the same approximation involved in simplifying Eq. (13) to Eq. (15) in Appendix VI, the present Eq. (13) reduces to

$$\frac{w_c}{w_g} = L_L\frac{c_f}{2} \quad . \tag{15}$$

2. Transpiration Cooling with Internal Liquid Vaporization

The case is now considered where a liquid is evaporating on the reservoir side of the porous wall and where the vapor passes through the porous walls as in gaseous transpiration cooling. This case closely resembles liquid transpiration cooling, and the approximate expression for the coolant evaporation rate may be obtained directly from that for liquid transpiration cooling previously derived. Only the boundary conditions need be changed. The vertical coordinate is no longer taken from the liquid surface but from the wall. This means that Eq. (5) must be modified to

$$q = \rho_c v_c \left[c_{p_\ell}(T_f - T_r) + \Delta H_v + c_{p_w}(T_w - T_f) + c_{p_v}(T - T_w) = k \frac{dT}{dy}, \quad (16)$$

where c_{p_w} is the average value of c_p for the vapor between T_f and T_w, and c_{p_v} is the average value for c_p between T_w and T. This change in the expression for q leads to a slight modification in the equation for the coolant evaporation rate; viz.,

$$\frac{w_c}{w_g} = L_{LV} \frac{c_f}{2} \quad , \qquad (17)$$

where

$$L_{LV} = \frac{c_{p_m}(T_g - T_w)}{\Delta H_v + c_{p_\ell}(T_f - T_r) + c_{p_w}(T_w - T_f)} . \qquad (18)$$

3. Film Cooling

Again, film cooling closely resembles liquid transpiration cooling, and the approximate expression for the coolant evaporation rate may be obtained directly from that for liquid transpiration cooling. As in the previous case only the boundary conditions need be changed. Since derivatives in the x-direction are neglected, the entrance effect of warming the liquid film from the entrance temperature to the final temperature of the liquid-gas interface T_f is ignored. This assumption amounts to assuming that the fluid enters at the film temperature T_f. With these modifications the expression for the coolant evaporation rate becomes

$$\frac{w_c}{w_g} = L_F \frac{c_f}{2} \quad , \tag{19}$$

where

$$L_F = \frac{c_{pm}(T_g - T_f)}{\Delta H_v} \quad . \tag{20}$$

APPENDIX VIII

Calculation of Total Temperature of the Main Stream Gases at the Entrance to the First Turbine Blade Row Based on the Velocity Relative to the Blade.

Let T_o = total temperature of the gases in the combustion chamber,

T = static temperature of the gases at entrance to blade row,

T_{or} = total temperature of the gases at entrance to blade row relative to the turbine blades,

M = Mach number at entrance to blade row relative to fixed nozzle ring based on the static temperature T,

M_r = Mach number at the entrance to blade row relative to turbine blades based on static temperature T,

a = velocity of sound at the temperature T,

a_o = velocity of sound at the temperature T_o in the combustion chamber,

U_W = turbine wheel speed,

α = blade entrance angle.

The Mach number diagram for the conditions at the entrance to the first blade row is shown in (Fig. 9). From (Fig. 9) it follows that

$$M_r^2 = M^2 + \left(\frac{U_W}{a_o}\right)^2\left(\frac{a_o}{a}\right)^2 - 2M\left(\frac{U_W}{a_o}\right)\left(\frac{a_o}{a}\right)\cos\alpha , \qquad (1)$$

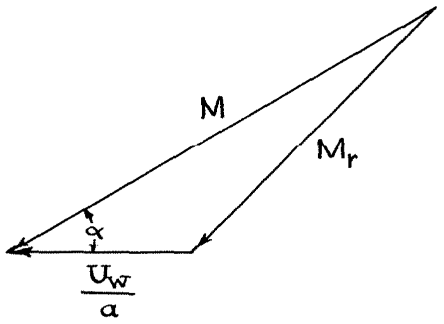


Fig. 9

or

$$M_r^2 = M^2 + \left(\frac{U_W}{a_o}\right)^2 \left(\frac{T_o}{T}\right) - 2M\left(\frac{U_W}{a_o}\right)\left(\frac{T_o}{T}\right)^{1/2} \cos\alpha \,. \qquad (2)$$

The total temperatures and the static temperatures are related through the equations

$$T_o = T\left(1 + \frac{\gamma - 1}{2} M^2\right) , \qquad (3)$$

and

$$T_{or} = T\left(1 + \frac{\gamma - 1}{2} M_r^2\right) \,. \qquad (4)$$

When the quantities T/T_o, γ, U_W/a_o and α are specified, the relative Mach number M_r may be computed from Eqs. (2) and (3). Having determined M_r, the total temperature ratio T_{or}/T_o may be calculated from Eq. (4). The range of practical interest for U_W/a_o and α are 0.30 - 0.50 and 14° - 24°, respectively. (Fig. 6) has been computed according to this scheme for $\gamma = 1.30$.