ANGULAR CORRELATIONS OF SUCCESSIVE NUCLEAR RADIATIONS

Thesis by

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ABSTRACT

The theory of angular distributions and correlations is described. Some new tables of Clebsch-Gordan coefficients have been calculated. In addition, particle and γ-ray radiation functions are tabulated. Distribution and correlation functions for two particles of spin 1/2 coming together to form a compound nucleus which successively emits an α-particle and then a γ-ray are tabulated for a large number of spins of the excited states involved.

Several angular distributions from the reaction $N^{15}(p,\alpha\gamma)C^{12}$ were measured using scintillation counters as detectors. The analysis of the data shows that the 12.51-Mev level of O^{16} is 2^- or greater than five, that the 12.95-Mev level of O^{16} is 2^- or greater than five, that the 13.24-Mev level of O^{16} is 4^+ or greater than five, and that the 4.43-Mev level of C^{12} is 2^+ or greater than four.

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I INTRODUCTION

At the present time one of the inportant tasks of the nuclear physicist is to obtain information about the states of the nucleus. In general the information he can obtain is whether or not a state exists, its energy, its width, its total angular momentum or spin, its parity, and if possible, any other information that would help him get a more detailed description of the state.

Angular distribution and correlation information is a very useful tool in aiding the nuclear physicist to determine some of these properties.

1. Coordinate System

Angular distributions of nuclear reactions are measured in a simple manner. Let the beam of incoming particles come in along the z axis of the polar coordinate system from the direction $\theta = \pi$, as shown in Fig. 1a. Let an incoming particle strike a target nucleus at z = 0. The combination subsequently emits the outgoing particle. When many particles are involved, we can define a probability function $W(\theta)$ that describes the relative probability of a particle being emitted into a unit solid angle located at (θ, ϕ). Clearly this function will depend only on the angle θ since the system has symmetry with respect to the azimuthal angle ϕ. To express this relation algebraically,

$$dN(\theta) = W(\theta)\, d\Omega,$$

where $dN(\theta)$ is the relative number of particles entering the solid angle $d\Omega$. $W(\theta)$ is called the angular distribution function. If $W(\theta)$ is a constant, the distribution is said to be isotropic.

Suppose that the compound nucleus* emits not one, but two successive radiations. Then one can calculate distribution functions for the two radiations. These in general are not equal at a particular angle, but their integrals are equal if the emission of one particle is always followed by the emission of the second; that is,

$$\int_{\text{sphere}} W_1(\theta)\, d\Omega = \int_{\text{sphere}} W_2(\theta)\, d\Omega ,$$

where $W_1(\theta)$ is the distribution function of the first particle, and $W_2(\theta)$ for the second. However for the purposes of this thesis, we are interested in only the relative probabilities. Therefore the distribution functions are more conveniently written disregarding constant factors. Thus the above integral relation may not in general be satisfied, and this freedom will not affect the results.

When the compound nucleus emits two successive radiations, another type of distribution function can be defined and measured experimentally. Suppose that the beam of incoming particles approaches from $\Theta = \pi$ along the Z axis, as shown in Fig. 1b. For the purposes of a generalization to be made later, the axis of the incoming particles is changed from z to Z. Let particle one be emitted in the direction Θ, $\Phi = 0$. The direction of emission

* The term "compound nucleus" will be used to describe the combination of the target nucleus and the incident particle, although the results are independent of the formation of a compound nucleus, in the strict sense of the term.

of particle two can be described in terms of Θ and Φ with respect to the Z axis. It can also be described in terms of another coordinate system that takes the direction of particle one as the z axis and the plane defined by the Z axis and the z axis as the $\phi = 0$ plane. The second coordinate system will be used since it permits calculations to be performed with more familiar mathematical functions.

The distribution function now in question is more complicated. Let particle one be emitted in a direction Θ with respect to the Z axis. Then given that particle one is emitted in this direction, what is the distribution function of particle two? Experimentally one counts particle two at a position (θ, ϕ). But in order to insure that particle two corresponds to particle one, a time coincidence device is used to count only pairs of particles that are detected so close in time to each other that the probability of counting two particles that are not a corresponding pair is either very small or at least known. Such a distribution function is commonly called a "correlation function," and the experiment is known as an "angular correlation experiment."

Notice that θ measures the angle between the particle that defines the z direction and the other particle detected. The reason the time coincidence device is not used in the distribution function measurement is that the incoming particles come from only one direction, and therefore all particles emitted are due to particles coming in along the z axis. On the other hand, in the correlation experiment there are many directions of emission of the particle that defines the z axis. If one were to generalize, a distribution function could be thought of as a correlation function between the incoming particles and the outgoing ones.

As an example, suppose an experiment is being performed by bombarding target nuclei by protons to form a compound nucleus which then emits an α-particle to leave the residual nucleus in an excited state which decays by the emission of a γ-ray. One could ask for the distribution of α-particles, the distribution of γ-rays, or the α-γ-correlation. One can think of the α-particle distribution as being a proton-α-correlation or a proton-α-distribution. The latter terminology is used by Seed and French.(1)

2. General Restrictions of the Topic

For the purposes of this thesis, a few restrictions are put on the type of distributions to be studied in order that the amount of material will not be too great to describe conveniently. The first restriction is that the outgoing particles are different from the incoming particles. Cohen(2) has treated the case of elastic scattering in which the incoming and outgoing particles are identical. Next we assume that only one level is excited at a time, or that the resonances are narrower than their separation. Finally we restrict the formation and decay of the states to the lowest possible value of the relative orbital angular momentum on the grounds that higher values are discouraged by barrier factors. The latter two restrictions will be dropped in the description of the theory, but nearly all of the calculated distributions are limited by them.

One of the results of this work is that one can find not only an agreement between a set of assignments and an experiment, but also disagreement between a fairly large class of other assignments and the experiment. This feature enables one to put certain useful limits on the result.

II DESCRIPTION OF THE THEORY

The theory of angular distributions and correlations is well treated in the literature. Here only a very brief survey of the references and a description of the results of the theory will be given for the case of different incoming and outgoing particles.

1. References

Hamilton[3] has written one of the more familiar papers on γ-γ correlations for a radio-active nucleus decaying by the emission of the two γ-rays. Falkoff and Uhlenbeck[4] have extended the theory to include correlations between any two nuclear radiations. Biedenharn, Arfken, and Rose[4a] have made a further generalization by assuming that the initial states are not randomly oriented as in the case of simple radio-active decay. This complication enables them to calculate triple cascade correlation functions and correlations in nuclear reactions in which the compound nuclei are formed in such a way that they do not behave as if they were a randomly oriented group of nuclei. Seed and French[1] have described the latter case using a different coordinate system. Their results use readily available functions and are straightforward to calculate. Their formulae and notation will be used in this thesis.

2. General Development

In order to describe the theory, the formulae will be presented, and then they will be interpreted physically. At first two simple cases will be discussed, i.e., proton capture followed by α-particle emission and then proton capture followed by γ-ray emission. The angular distribution functions will be discussed for these cases. Next the more complicated case of proton capture followed by α-particle emission leaving the residual nucleus in an excited state which in turn decays by the emission of a γ-ray will be presented. Here again the distribution functions will be discussed. Before the material about α-γ correlations is given, a description of the correlation calculations for radio-active decay will be presented. This topic will lay a foundation for a clearer understanding of the more complicated correlation functions involving an incident particle as well as the two products of the reaction.

3. (p,α) Reactions

According to Seed and French the distribution function for the proton-α reaction is

$$W(\theta_\alpha) = \sum_{\substack{j,j_z \\ J_{sz}}} \left| \sum_{\ell} (2\ell+1)^{1/2} f_r(\ell) \langle j, j_z; \ell, 0 | J_r, J_{rz} \rangle \right|^2 \times \left| \sum_{\ell'} f_s(\ell') \langle J_r, J_{rz} | J_s, J_{sz}; \ell', m' \rangle Y_{\ell'}^{m'}(\theta_\alpha, 0) \right|^2 .$$

The various quantities will be defined in the next few pages. First this expression will be simplified according to the restrictions mentioned above. The factor $f_r(\ell)$ indicates the relative importance of the various possible values of the angular momentum ℓ (in units of $\hbar$). If only the lowest value is involved in the formation of the compound nucleus, the summation over ℓ drops out. Then the $(2\ell+1)^{1/2} f_r(\ell)$ is a constant and can be ignored since we are dealing only with the relative probability of a particle being emitted in a certain direction. The same holds true for the relative angular momentum of the outgoing particle, ℓ'. If only one resonance is involved, one value of J describes the total angular momentum of the compound nucleus. This value is J_r. Finally let us assume that the residual nucleus has total angular momentum zero. Then $J_s = 0$. It turns out that

$$\langle J, J_z \mid 0, 0; \ell', m' \rangle = \delta_{J\ell'} \delta_{J_z m'} .$$

With these simplifications one obtains

$$W(\theta) = \sum_{j, j_z} \left| \left\{ g(j) \langle j, j_z; \ell, 0 \mid J, J_z \rangle \right\} \left\{ Y_J^{J_z}(\theta, 0) \right\} \right|^2 .$$

The first braces describe the formation of the compound nucleus, and the second describe the way it emits the α-particles. The factor g(j) has been inserted because more than one value of j can in general contribute to the formation of the compound nucleus. The factor g(j) describes the relative importance of the j's.

4. Channel Spin in the Formation of the Compound Nucleus

Suppose the target nucleus has a spin j'. Since the proton has a spin 1/2, these two particles can form a system of spin j'-1/2 or j'+1/2. These quantities are called the "channel spins" of the reaction. For example, suppose the spin j' of the target nucleus is 1/2; the channel spins are then 0 and 1. If both are effective in producing the compound nucleus, we must add the results of each. To write this statement algebraically, we sum over the j's. However it is not obvious how the channel spins compete. This point is discussed in a paper by Professor Christy[5]. The result is that one cannot specify the relative weights without a more detailed knowledge of the compound nucleus. If a sufficiently detailed model of the nucleus is chosen, then the channel spin ratio is determined. This topic will be treated in more detail after the treatment of angular distributions has been completed. For simplicity, g(j) is chosen to be unity for the highest value of j, A for the next lower, B for the second lowest, etc.

5. Clebsch-Gordan Coefficients

The factors $\langle j, j_z; \ell, 0 \mid J, J_z \rangle$ describe the combining of the channel spin with the relative angular momentum ℓ to form the compound nucleus of total angular momentum J. The factors are known as Clebsch-Gordan coefficients. Tables of these are given in Appendix I. They are presented in tables entitled $D_j \times D_\ell$. Van der Waerden[6] develops the theory of these coefficients, and Condon and Shortley[7] apply them to atomic spectra. Their squares represent the probability of a certain state breaking up into two other given states or, on the other hand, the probability of two certain states forming a third given state.

Consider for example the part of the $D_1 \times D_2$ table (Appendix I) shown below.

	W_3^2	W_2^2	
$U_2^2\ V_1^0$	$\sqrt{1/3}$	$\sqrt{2/3}$	⟵ $\langle 2,\ 2;\ 1,\ 0\,|\,2,\ 2\rangle$
$U_2^1\ V_1^1$	$\sqrt{2/3}$	$-\sqrt{1/3}$	

If one has a state of spin J = 2 and projection along the z axis M=2,* and if it should break up into two states, one of spin one and the other of spin two, the probability is 2/3 that it will go into the spin one state with projection zero and spin two state with projection two, while the probability is only 1/3 that it will go into spin one state with projection one and spin two state with projection one. This part of the breaking up process is purely geometrical and contains no information about the way the nucleus holds together. Therefore the terms

$$\sum_{j,\,j_z} \left| g(j) \left\langle j,\ j_z;\ \ell,\ 0 \,\middle|\, J\ \ J_z \right\rangle \right|^2$$

simply are proportional to the probability of finding the compound nucleus (of total angular momentum J) with a projection J_z. This probability is very often called "the population of the substates".

Note that in the formation of the compound nucleus the substates of a particular channel spin are equally weighted; no one of them is any more significant than the other. This is a general

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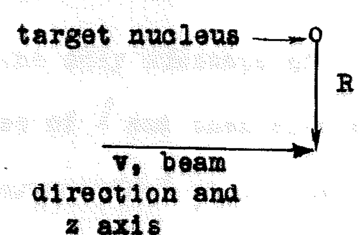
* In quantum mechanics a state of spin J has 2J + 1 possible projections along a given axis. These projections are J, J-1, J-2,...., -J + 1, -J.

property of quantum mechanical systems, namely, if a state has equal population of its substates, it is spherically symmetrical and has no preferred direction. In the case of the incoming particle and the target nucleus, both are randomly oriented, and thus the final combination of the two, the channel spin, has equally populated substates.

Now consider the relative angular momentum ℓ. Does it have a preferred direction? Obviously, since the incoming particles come from only one direction. If we choose the z axis to be the direction of the beam, the population of substates of ℓ is zero except for $\ell_z = 0$. In other words, all the particles of the beam have $\ell_z = 0$. This result is easy to understand in classical terms. Consider the beam hitting off the target nucleus by a



distance R as shown in the figure. The angular momentum, $\vec{L} = m\,\vec{R}\times\vec{v}$, is a vector pointing out of the paper toward the reader. Then it is easy to see that its z component is zero. In general, the populations of the substates of ℓ are not equal. This means that ℓ is not randomly oriented in space. This non-symmetry in ℓ is the means of introducing an asymmetry into the compound nucleus which in turn gives rise to an anisotropic angular distribution.

There is a special case of much interest, namely, when $\ell = 0$. In this case ℓ has only one substate, $\ell_z = 0$. All beam particles are in this substate. But since there is only one, all substates are equally populated, and there is no asymmetry in the compound nucleus. In general, if the substates of a compound nucleus are equally populated, there is no preferred direction and the resultant distributions are isotropic. Therefore s-wave ($\ell = 0$) reactions have isotropic distributions.

Suppose the compound nucleus has spin zero. It has one substate, $M = 0$. This state again must be one that has equal population of all of its substates. Therefore the distributions will be isotropic even though some asymmetry may have been introduced by $\ell \neq 0$.

Since $\ell_z = 0$ is the only substate of ℓ not empty, the population of the substates of ℓ and then also of J, the spin of the compound nucleus, is symmetrical about its center. For example, the relative populations of the substates M of a nucleus of spin two may be as follows:

$M = +2$	0
$M = +1$	1/2
$M = 0$	2/3
$M = -1$	1/2
$M = -2$	0

The population of +M and -M are equal for any M. Suppose $J = 1/2$. Then the substates are $M = +1/2$ and $M = -1/2$. Since the population is symmetrical, these substates are equally populated; the distributions will be symmetrical.

6. Particle Distribution Functions

Now let us consider in more detail the break-up of the compound nucleus. If the compound nucleus emits an α-particle (spin zero) leaving the residual nucleus with spin zero, it must give up all of its angular momentum as the relative angular momentum of the decay. When the particles separate with a certain relative angular momentum, their distribution is described by spherical harmonics Y_ℓ^m. That these are the proper functions is determined by the interaction Hamiltonian[4]. If a nucleus of spin J and projection M decays in this manner, the distribution is $|Y_J^M(\theta, \phi)|^2$. Thus the last factor of the formula represents the way each substate of the compound nucleus emits its particles. Tables of the Y_ℓ^m are given in Appendix II.

7. Parity

In addition to these considerations, the concept of parity is involved in understanding these nuclear reactions. Formally parity refers to the reflection properties of the wave function. If the wave function is invariant under a reflection of coordinates through the origin, the function has even parity (designated by +); if it changes sign, it has odd parity (designated by −). The nucleons all have even parity. The spherical harmonics Y_ℓ^m that describe the relative angular momentum have the parity[8] of $(-1)^\ell$. The parity of any particular nucleus in its ground state is determined by experiment if possible. Parity is conserved throughout a reaction. For example, suppose a proton (even parity) and a B^{11} nucleus (odd parity) come together with their relative angular momentum $\ell = 1$ (odd parity). The resultant state must have even parity. The same conditions hold in the decay of the nucleus. The product of the parities of the parts must equal the parity of the whole. Of particular importance is the parity change for γ-ray emission. Table I gives the change of parity for certain transitions. Suppose

Table I

Type of Radiation Multipolarity	Electric	Magnetic	Angular Momentum Carried Off
Dipole	Change	No Change	1
Quadrupole	No change	Change	2
Octupole	Change	No Change	3
16 Pole	No Change	Change	4
32 Pole	Change	No Change	5
64 Pole	No Change	Change	6

there is a γ-ray emitted from a 2^+ state leaving the nucleus in another 2^+ state. What kinds of γ-ray emission can occur? Since there is no change of parity, magnetic dipole, electric quadrupole, etc. are allowed. Actually the higher orders are not very probable (see Blatt and Weisskopf(9)). Only the magnetic dipole and electric quadrupole are detectable with present techniques. In theory, however, one can get γ radiations that carry away from

$|J-J'|$ up to $(J+J')$ units of angular momentum, where J is the spin of the initial state and J' is the spin of the final state.

8. Example of (p,α) Distribution

Let us consider a simple example. Suppose p-wave $(\ell=1)$ protons are used to bombard N^{15} (spin 1/2, odd; denoted by $1/2^-$) to form a state of total angular momentum two. What is the angular distribution of α-particles of the reaction $N^{15}(p,\alpha)C^{12}$? The channel spins are 0 and 1. The relative orbital angular momentum is 1. In the vector addition of these angular momenta, the 0 and 1 cannot give a total spin of two, but the 1 and 1 can. Thus only channel spin 1 enters into the reaction.

We first calculate the population of the substates of the compound nucleus with spin two. In order to do this, we refer to the $D_1 \times D_1$ table in Appendix I since it describes the combination of two angular momenta of one unit each. From the combination of the +1 substate of the channel spin 1 (say $U_1^{\,1}$) and the 0 substate of $\ell=1$ (say $V_1^{\,0}$) we find the population of the +1 substate of the spin two state ($W_2^{\,1}$) to be 1/2. The population of the 0 substate is 2/3; of the -1 substate, 1/2. The populations of the +2 and -2 substates are zero. Therefore the whole population distribution is just that on page 11. Note that the sum is not unity. This results from the particular type of normalization used in the tables, and does not change the physical results.

Now let the compound nucleus break up. Since α-particles (spin zero) are emitted to leave the residual nucleus in a spin zero state, the distribution of α-particles is proportional to $(Y_J^M)^2$. Therefore the +1 substate emits α-particles in the distribution $(Y_2^1)^2$. This must be multiplied by the population of the substate. Adding the results for all the substates we have

$$W(\theta) = 1/2\ (Y_2^1)^2 + 2/3\ (Y_2^0)^2 + 1/2\ (Y_2^{-1})^2$$

$$= 5/32\pi \left\{ 1/2\ (12 \cos^2\theta - 12 \cos^4\theta) + 2/3\ (2 - 12 \cos^2\theta + 18 \cos^4\theta) + 1/2\ (12 \cos^2\theta - 12 \cos^4\theta) \right\}$$

$$= 5/24\pi \quad (1 + 3 \cos^2\theta).$$

The constant factor is not very useful because we have already neglected several such factors in the initial formula of Seed and French. Thus the result is usually written just as $1+3 \cos^2\theta$.

9. Complexity Rules

Note that the $\cos^4\theta$ terms drop out. This result is due to the fact that p-wave ($\ell=1$) protons form the compound nucleus. The general rule is that if one considers the three numbers ℓ, J, ℓ' (the relative angular momentum of the break-up process, 2 in the example), twice the smallest of these is the highest power of $\cos \theta$ that appears in the result. In the example $\ell = 1$ was the smallest. Therefore the highest power of $\cos \theta$ is two, as the example illustrates.

10. γ-ray Distribution Functions

Suppose γ-rays are emitted, rather than α-particles, leaving the compound nucleus in a spin zero ground state. The distribution functions are not the same as for particles, the Y_ℓ^m, but rather they are the $X_{\ell,m}$ tabulated in Appendix III. In the example we have just considered, let the compound nucleus emit γ-rays instead of α-particles. Then

$$W(\theta) = 1/2\ (X_{2,1})^2 + 2/3\ (X_{2,0})^2 + 1/2\ (X_{2,-1})^2$$

$$= \text{constant} \times (1 + \cos^2\theta).$$

Notice again that the highest power of cos θ is two.

11. (p,αγ) Reactions

We are now in a position to consider a more complicated break-up of the compound nucleus. Suppose it emits an α-particle and then a γ-ray. What are the distributions of these particles? The spin of the excited state of the residual nucleus (after the compound nucleus has emitted the α-particle) is designated by J' and its substate by M'. The compound nucleus J, M breaks up into a residual nucleus J', M' and the relative angular momentum, say ℓ' with substate ℓ_z'. To describe the geometrical probabilities in the break-up, we choose the appropriate set of Clebsch-Gordan coefficients $\langle J, M \mid J', M'; \ell', \ell_z' \rangle$ of the table $D_{J'} \times D_{\ell'}$. Then to get the α-particle distribution we multiply these by the $Y_{\ell'}^{\ell_z'}$ and square and sum over the M'. When we sum over the M' we are saying in effect that the detector counts all α-particles in a certain direction regardless of which substate M' the residual nucleus is left in. If, perchance, more than one value of ℓ' is important in the break-up process, we will have to weight these according to their importance. This weighting factor is $f(\ell')$. All of this together is

$$W(\theta)=\sum_{j_z,M'}\left|\sum_{\ell,\ell'} f_r(\ell)\langle j,j_z;\ell,0|J_r,M\rangle f_s(\ell')\langle J_r,M|J_s',M';\ell',m'\rangle Y_{\ell'}^{m'}(\theta,\phi)\right|^2,$$

which is the formula of Seed and French. If the γ-ray distribution is desired, we ask for the population of the substates of the excited state, J', and multiply them by the γ-ray distribution functions. Thus

$$W(\theta)=\sum_{j_z,m'}\left|\sum_{\ell,\ell'} f_r(\ell)\langle j,j_z;\ell,0|J_r,M\rangle f_s(\ell')\langle J_r,M|J_s',M';\ell',m'\rangle X_{J',M'}(\theta,\phi)\right|^2,$$

also as in Seed and French.

12. Radio-active Decay Correlations

Before the angular correlation formulae for this double decay reaction are discussed, radio-active decay correlations will be described. This case will be simpler because the initial states are randomly oriented, and therefore one can choose any direction for the z axis. The problem is not so simple when the compound nucleus is formed with a non-uniform population of substates.

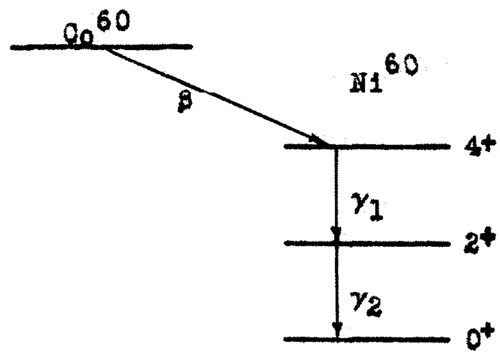
In a radio-active decay, the initial states are randomly oriented, that is, the substates are equally populated. Now let the state decay by the emission of a radiation leaving the nucleus in an excited state which decays again by the emission of a second radiation. The plan of attack is to choose certain of the first radiations that leave the excited state with a certain population distribution. From this population distribution, one can determine the angular distribution of the second radiation. Two types of decay are important from the point of view of our considerations, particle emission and γ-ray emission.

If a randomly oriented group of nuclei emit particles, their distribution will be isotropic. If the position of the particle detector determines the z axis, the previous arguments about the incoming beams show that those particles which are emitted along the z axis have $l_z = 0$ only. One can see this result from another point of view. Note the following properties of the Y_l^m 's, the distribution functions, evaluated for particles emitted along the z axis, $\theta = 0$, or $\cos\theta = 1$. The Y_l^0 's are proportional to the P_l 's which are always unity at $\cos\theta = 1$(10). For P_l^m with $m \neq 0$, one can see from the definition of the P_l^m that they are always zero when $\cos\theta = 1$ (see Appendix II for definition). Here again we see that the particles emitted along the z axis have $l_z = 0$. If γ-rays are emitted along the z axis, they have $l_z = \pm 1$ only. Knowing these properties, one can calculate the populations of the substates of the excited state by use of the Clebsch-Gordan coefficients. From this information the distribution of the second radiation can be calculated. This result is the correlation function. Algebraically it is

$$W(\theta) = \sum_{l_z\text{'s}} \left| \langle J_0, M_0 | l_1, l_{1z}; J_1, J_{1z} \rangle \langle J_1, J_{1z} | l_2, l_{2z}; J_2, J_{2z} \rangle R_{l_2}^{l_{2z}}(\theta,\phi) \right|^2$$

where l_{1_z} are the values for emission along $\theta = 0$, and the $R_l^{l_z}$'s are the radiation functions for the second radiation.

At this point an example would be useful. Let us calculate the correlation function for the two γ-rays of Co^{60}. The decay scheme is as follows[11]:



The γ-rays are electric quadrupole. Suppose the 4^+ states are randomly oriented. Its substates will be equally populated. Let each of these break up into a 2^+ state by emitting a quadrupole γ-ray. One asks for the population of substates due to the emission of a γ-ray with projection +1 or -1 along the z axis (corresponding to γ-rays emitted in this direction). To find this information, one refers to the $D_2 \times D_2$ table in Appendix I. The W_4 functions represent the initial excited state. We can let the U_2 's represent the excited state of 2^+, and the $V_2^{\pm 1}$'s represent the emission of the γ-rays. The W_4^4 state cannot emit a γ-ray along the z axis since it cannot give rise to a $V_1^{\pm 1}$ term. The W_4^3 can give a V_1^1 and leave the excited state in a +2 substate with a probability of 1/2. The W_4^2 substate gives U_2^1 with a probability of 4/7. The W_4^2 can emit γ-rays along the z axis with either +1 or -1 projection. For these cases, the emission leaves U_2^2 with a probability of 1/14 and U_2^0 with a probability of 3/7. Likewise the W_4^0 leaves both U_2^1 and U_2^{-1} with a probability of 8/35. Continuing on, W_4^{-1}

goes to U_2^0 with a probability of 3/7 and U_2^{-2} with a probability of 1/14; W_4^{-2} goes to U_2^{-1} with a probability of 4/7; and W_4^{-3} goes to U_2^{-2} with a probability of 1/2. Adding these, one obtains the populations of the substates of the excited state shown in Table II. Each of these states emits γ-rays with a distribution

Table II

Populations Due to the Emission of a V_2^1 or a V_2^{-1} γ-ray

	W_4^3	W_4^2	W_4^1	W_4^0	W_4^{-1}	W_4^{-2}	W_4^{-3}	Total
U_2^2	1/2		1/14					4/7
U_2^1		4/7		8/35				4/5
U_2^0			3/7		3/7			6/7
U_2^{-1}				8/35		4/7		4/5
U_2^{-2}					1/14		1/2	4/7

$(X_{2,m})^2$. Thus

$$W(\theta) = 4/7\ (X_{2,2})^2 + 4/5\ (X_{2,1})^2 + 6/7\ (X_{2,0})^2 + 4/5\ (X_{2,-1})^2 + 4/7\ (X_{2,-2})^2$$

$$= (\text{constant}) \times (1 + 1/8 \cos^2\theta + 1/24 \cos^4\theta),$$

a well known result[12].

Consider another example more closely related to the general correlation functions of nuclear reactions. Suppose F^{19} $(1/2^+)$ is bombarded with s-wave (ℓ=0) protons to form a 1^+ state. Only channel spin one will be important in the formation of the compound nucleus. Since s-wave protons form the state, the population of substates will be uniform. Let the state decay by emitting f-wave (ℓ= 3) α-particles to leave O^{16} in a 3^- state. Let this in turn emit electric-octupole γ-rays to leave O^{16} in its 0^+ ground state. The angular distributions of the α-particles and γ-rays will be isotropic, but the angular correlation between the α-particles and the γ-rays will not be.

To start the calculation, assume a state of spin one that breaks up into two states of total angular momentum three. Therefore the relevant numbers are obtained from the $D_3 \times D_3$ table of Clebsch-Gordan coefficients. Next the direction of emission of the α-particles is chosen to be the z axis. Thus only states V_3^0 (if V represents the relative angular momentum states) are detected. Looking at the table we see that the W_1^1 (W represents the states of the compound nucleus in this example) state decays to a U_3^1 (O^{16} state) with a probability of 3/14. The W_1^0 state does not give off α-particles in the z direction since the $\langle 1, 0 | 3, 0; 3, 0 \rangle$ coefficient is zero. The W_1^{-1} state decays to U_3^{-1} state with a probability of 3/14. Therefore the population of O^{16} substates due to α-particles emitted along the z axis is as follows:

Substate	+3	+2	+1	0	-1	-2	-3
Population	0	0	3/14	0	3/14	0	0 .

Thus the correlation function is simply proportional to $(X_{3,\pm 1})^2$, or

$$W(\theta) = 1 + 111 \cos^2\theta - 305 \cos^4\theta + 225 \cos^6\theta.$$

This distribution was measured by Arnold[13] and Barnes, French, and Devons[14]. Since θ is the angle between the z axis and the direction of γ-ray emission, it is also the angle between the two detectors. Experimentally one puts the pulses from each detector into a time-coincidence circuit that gives an output pulse only when a corresponding pair is detected. In other words, the γ-ray detector counts many γ-rays. The coincidence device allows only those counts to be registered that occur in a small time interval near the instant an α-particle is detected at $\theta = 0$.

The simplification of the last example was the fact that s-wave protons formed the compound nucleus. This gave a uniform population (no prefered direction) to the compound nucleus. Then the direction of emission of the α-particle can be chosen as the z direction. One could also have chosen the direction of emission of the γ-ray as the z axis. Then the only γ-rays detected are those from the $U^{\pm 1}$ states of the O^{16*}. Under these conditions the population of the relative angular momentum substates (given the subsequent emission of a γ-ray along the z axis) is as follows:

	Ne^{20} state	$W_1^{\,1}$	$W_1^{\,0}$	$W_1^{\,-1}$	Total
Substates of the Relative Angular Momentum	-3				0
	-2	5/28			5/28
	-1		1/28		1/28
	0	3/14		3/14	12/28
	-1		1/28		1/28
	-2			5/28	5/28
	-3				0 .

The distribution of α-particles is then

$$W(\theta) = 5/28\ (Y_3^{2})^2 + 1/28\ (Y_3^{1})^2 + 12/28\ (Y_3^{0})^2 + 1/28\ (Y_3^{-1})^2 + 5/28\ (Y_3^{-2})^2$$

$$= (\text{constant}) \times (1 + 111 \cos^2\theta - 305 \cos^4\theta + 225 \cos^6\theta).$$

Since the population of substates of the compound nucleus is uniform, we have complete freedom of choice of the z axis.

13. Reaction Correlations

The more complicated case arises if the compound nucleus is not formed by s-wave particles. The problem can be solved in two ways. One can calculate the substate population with the beam direction as the z axis and then transform this information to the new z axis, say the direction of one of the detectors. A simpler procedure is to choose the z axis initially in the direction of one of the detectors and transform the information about the incoming beam of particles to this axis. Let the beam direction be the Z axis and the direction of emission of particle one be the z axis. With respect to the Z axis, the beam is in substate $\ell_Z = 0$. With respect to the z axis, the population of substates is different unless the two axes coincide. The $\ell_Z = 0$ substate is described by the Y_ℓ^0 spherical harmonic. The following expression found in Smythe(15) is used to transform to the z axis:

$$\left[Y_\ell^0(\Theta, 0)\right]_{\theta,\phi} = (4\pi/2\ell+1)^{1/2} \sum_{m=-\ell}^{\ell} Y_\ell^m(\Theta, 0) Y_\ell^m(\theta, \phi),$$

where Θ is the angle between the axes. These waves will interfere coherently with one another. Therefore the correlation function is the same as the distribution function of particle two with two changes. The $\ell_z = 0$ wave is replaced by the above coherent summation, and the particle one is emitted along the z axis (substate 0 for nucleons, ±1 for γ-rays). Then the α-γ-correlation takes the following form:

$$W(\theta, \phi) = \sum_{j, j_z} \left| \sum_{\ell_z} \langle j, j_z; \ell, \ell_z | J, M \rangle Y_{\ell}^{\ell_z}(\Theta, 0) \langle J, M | \ell', 0; J', M' \rangle X_{J', M'}(\theta, \phi) \right|^2 .$$

This equation assumes that the α-particle is emitted with relative angular momentum ℓ' and that the ground state of the residual nucleus has spin zero. In general, this expression is a function of Θ, θ, and φ. In some cases it is a function of θ only.

In Appendix IV there are tabulated α-particle and γ-ray distributions for a target nucleus of spin 1/2 bombarded with protons of relative angular momentum ℓ to form a compound nucleus with spin J. If this state emits α-particles, the functions W_{α_0} give the angular distribution of α-particles that leave the residual nucleus in a spin zero ground state. The functions W_α give the distribution of α-particles that leave the compound nucleus with relative angular momentum ℓ' and that leave the residual nucleus in an excited state of spin J' which in turn decays by the emission of 2^J-pole γ-rays. The functions W_γ give the angular distribution of these γ-rays. A particular set of functions is designated by the four number code (ℓ, J, ℓ', J').

The functions $W_{\alpha\gamma}$ are the α-γ-correlation functions. Unless otherwise noted, they are functions of Θ, θ, and φ, but occasionally they are evaluated for $\Theta = \pi/2$.

14. Information from Channel Spin Data

Experimentally one measures an angular distribution, correlation, or a set of them. From these data he may be able to determine the spins and parities of the states involved and the relative angular momenta. In addition, he may find the proportions of the various channel spins competing to form the compound nucleus. The paper by Professor Christy[5] treats this subject in detail. However, since the information described there is important in interpreting the experimental data to be discussed later, a brief recount will be given here for the sake of completeness.

As we have already seen, the mixing ratio of channel spins is significant whenever two or more channel spins are effective in forming the compound nucleus. The question arises about what determines the ratio of the channel spins. We have found that the ratio determines the population of substates in the compound nucleus. These in turn are proportional to the square of the magnitude of the wave functions, $\psi^*\psi$, of the states. Therefore if we have a sufficiently detailed knowledge of the wave functions, the channel spin ratio is determined. If we specify some means of the nucleons interacting with each other, we can specify enough information to determine the populations. For example, suppose we specify LS (Russell-Saunders) coupling. Then we can write the angular part of the wave functions in general. These will have equal populations of the substates. However when we stipulate that only the zero substate of the relative angular momentum enters into the reaction, some of the terms drop out, and the substates are not equally populated. For each description of the state there may be different populations and, consequently, different values of the channel spin ratio. The same holds true of jj coupling.

In Table III we have tabulated the values of the channel spin ratio for a spin 1/2 particle bombarding a $p_{1/2}$ nucleus with relative angular momenta of one, two and three units.

15. Channel Spin in Particle Emission

Note that up to this point the only particles emitted have been α-particles. Since they are spin zero particles, their use is a simplification. The more general case arises if the emitted particle does not have spin zero. However, this more general case is well treated in the framework already developed. The procedure is simply to go through the inverse of the formation using channel spin combinations, if necessary. One lets the compound nucleus break up into the relative orbital angular momentum and the channel spins. One calculates the distribution for each such channel spin and adds the results with arbitrary weight for each channel spin. A study of such reactions can lead to assignments of ratios of them and, consequently, can be of use in determining more about the wave functions.

Table III

Relative Momentum	LS Description	jj Description	Value of A	Value of x*
1	$^1P_1$		0	0.00
	$^3S_1$	$p_{1/2}$	1/2	0.33
	$^3P_1$, $^3D_1$	$p_{3/2}$	2	0.67
2	$^1D_2$		0	0.00
	$^3P_2$	$d_{3/2}$	2/3	0.40
	$^3F_2$	$d_{5/2}$	3/2	0.60
	$^3D_2$		6	0.86
3	$^1F_3$		0	0.00
	$^3D_3$	$f_{5/2}$	3/4	0.43
	$^3G_3$	$f_{7/2}$	4/3	0.57
	$^3F_3$		12	0.92
ℓ	$^1(\ell)_\ell$		0	0
	$^3(\ell-1)_\ell$	$(\ell)_{\ell-1/2}$	$\ell/\ell+1$	$\ell/2\ell+1$
	$^3(\ell+1)_\ell$	$(\ell)_{\ell+1/2}$	$\ell+1/\ell$	$\ell+1/2\ell+1$
	$^3(\ell)_\ell$		$\ell(\ell+1)$	$\ell^2+\ell/\ell^2+\ell+1$

* The quantity x is defined on page 35.

III EXPERIMENTAL APPLICATION

1. Description of Apparatus for γ-ray Distribution Experiments

One of the easiest types of distributions to measure is the γ-ray distribution. Therefore some equipment was constructed to measure them. Roughly there were two large items, the target chamber and the γ-ray detector.

A full-sized drawing of the target chamber is shown in Fig. 2. Essentially it consists of a thin-walled brass cylinder fitted with an entrance tube for the incoming particles, a quartz plate for viewing the beam, a cold trap and place for making targets, and a set of bearings on which to rotate the γ-ray detector. The top of the cylinder was fitted with a lucite plate which held the target and angular scale and which permitted viewing of the inside of the cylinder. It was connected to a conventional diffusion pump vacuum system through the entrance tube. The whole assembly was mounted on a small metal table and insulated from it with lucite. The insulation permitted the whole chamber to be connected to the current integrator while bombarding very thin foils. When using thicker targets or backing, the chamber could be grounded, and the target was then insulated from the chamber by the top plate.

The γ-ray detector was a liquid phosphor scintillation counter. The scintillator was a glass cylinder (4.5 cm diameter x 9 cm long) filled with a solution of about 1% terphenyl in xylene. One end of the cylinder was a quartz disk cemented to it to allow passage of ultra-violet as well as visible light out of the scintillator. The disk end of the cylinder was cemented with Canada Balsam to a short piece of lucite. It was flat on one end to accommodate the quartz disk, and the other end was machined to fit the end of a 5819 photomultiplier tube. This side was cemented to the tube again with Canada Balsam. Fig. 3 shows the assembly of the γ-ray detector. Fig. 4 shows the piece that holds the γ-ray detector up to the target chamber.

The type of photomultiplier tube used is very sensitive to magnetic fields. Therefore, to avoid the effect of stray fields, the phototube was shielded first with a cylinder of "μ metal," an iron designed especially for magnetic shielding purposes. Then a 1/16" wall iron tube was placed over the tube and scintillator. This second magnetic shield was also used as the light shield for the tube. Finally, a 3/16" wall iron tube was placed over the second shield. Tests were made to insure that the counter was insensitive to magnetic fields. The results showed that as the counter was moved to various angles no effect could be detected, with an upper limit of error of about 0.1%.

The output of the photomultiplier tube was fed into a pre-amplifier and then to a regular pulse amplifier having a rise time of about 0.2 microseconds. The output of the amplifier was in turn fed into a discriminator that allowed only pulses over a certain amplitude to pass through to the decade scalers.

When the system was in operation, a certain amount of charge due to the protons from the 2-Mv electrostatic accelerator were allowed to hit the target. The current integrator turned off the scalers when a predetermined amount of charge had bombarded the target. Another scintillation counter was used as a monitor. It was similar in construction to the first, except that the scintillator was a 1/2" thick by 2" diameter terphenyl crystal. It had its separate power supply, amplifier and scalers. It was kept at a fixed angle. The procedure was to make a group of several runs, each of the same amount of charge. The variable angle counter was set at one of the particular angles for each run. Then when the group was completed, the monitor counts were checked to make sure they were constant within statistical error before a group of runs was deemed significant.

2. Corrections to the Data of γ-ray Distribution Experiments

If the monitor counts were consistent, one was left with a number of γ-rays detected at the various angles. Several corrections had to be considered before the data could be made to correspond to what is calculated in a theoretical distribution.

A. Background and Absorption

First of all, there is a constant background of γ-rays from the X-rays produced in the generator. In addition, there are cosmic rays that give spurious counts. That the number of background counts is proportional to the time is a good approximation. Therefore the time of each run is measured, and then the appropriate background error is subtracted from the gross number of counts.

As γ-rays pass through material there is a certain probability that they will interact with it. It may be that as the

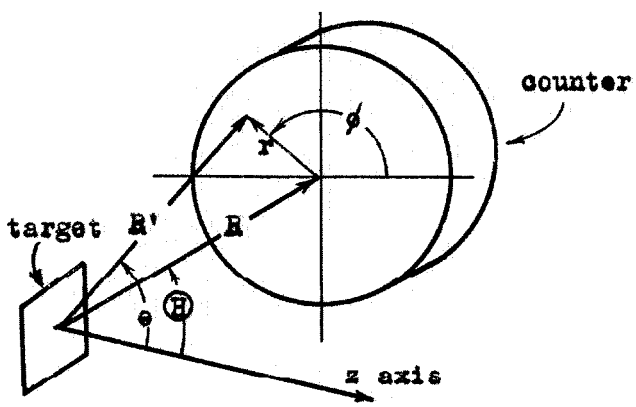
γ-rays pass from the target material to the γ-ray detector they will traverse different amounts of material at the various angles. In our experiments this is the case, and correction must be made for this effect. In order to estimate the correction, one puts an isotropic source of γ-rays at the target position and measures their angular distribution. Natural sources of Co^{60} and Th C" and the 935-kev resonance in $F^{19}(p,\alpha\gamma)O^{16}$ were used as the isotropic sources. The measured angular distribution was nearly isotropic, but at each angle a correction factor was calculated. When the datum of a particular angle was multiplied by the correction factor for that angle, the group of data gave an isotropic distribution. In no case was the correction factor more than four per cent from unity, and the variations in the correction factor corresponded very well to the variations in the amount of material the γ-rays traversed. Whenever an angular distribution is measured, the data are corrected for background and then multiplied by the absorption correction. The result is the angular distribution yet uncorrected for the fact that the solid angle of the γ-ray detector is finite.

B. Solid Angle

In order to determine the effect of the finite solid angle, a z direction is specified and a distribution with respect to this direction, $y(\theta) = 1 + a\cos^2\theta + b\cos^4\theta$. This function describes the way the rays are actually distributed, and not what the detector counts. This latter function will be $W(\Theta) =$

$1 + a' \cos^2 \Theta + b' \cos^4 \Theta$. The problem is to find the relationship between a', b', a, and b.

Next some assumptions have to be made about the γ-ray detector. It will be convenient to describe its response to γ-rays in terms of a sensitivity at a position (r, ϕ) with respect to its center which, according to the previous definition, is located at an angle Θ with respect to the z axis. Assume that the counter is uniformly sensitive out to a radius d, and beyond this it is insensitive. An experiment was performed to check the validity of this assumption by allowing γ-rays collimated into a slit by lead bricks to strike various parts of the detector. The results were in good agreement.



If the center of the γ-ray detector is R units away from the target, the yield is

$$W(\Theta) = \int_{\text{counter area}} \frac{y(\theta)\, r\, dr\, d\phi}{R'^2}$$

where R' is the distance between the target and the point (r, ϕ) on the counter. First note that

$$R'^2 = (R \cos \Theta + r \sin \Theta \cos \phi)^2 + (R \sin \Theta - r \cos \Theta \cos \phi)^2 + r^2 \sin^2\phi$$

$$= R^2 + r^2.$$

Furthermore,

$$\cos^2 e = \frac{(R \cos \Theta + r \sin \Theta \cos \phi)^2}{R^2 + r^2} .$$

If $x = r/R$,

$$W(\Theta) = \int_0^{d/R}\int_0^{2\pi} \frac{x\, d\phi\, dx}{1+x^2} + a\int_0^{d/R}\int_0^{2\pi} \frac{(\cos \Theta + x \sin \Theta \cos \phi)^2 x\, d\phi\, dx}{(1 + x^2)^2}$$
$$+ b \int_0^{d/R}\int_0^{2\pi} \frac{(\cos \Theta + x \sin \Theta \cos \phi)^4}{(1 + x^2)^3}\, x\, d\phi\, dx.$$

If $\delta = (d/R)^2$,

$$W(\Theta) = \pi\delta\Big\{(1 - \delta/2 + \delta^2/3) + a(\delta/4 - \delta^2/3) + b\, \delta^2/8$$
$$+ \cos^2 \Theta \quad a(1 - 5\delta/4 + 4\delta^2/3) + b(3\delta/2 - 13\, \delta^2/4)$$
$$+ \cos^4 \Theta \quad b(1 - 3\delta + 41\, \delta^2/8) \quad .$$

$R \approx 9$ cm for the equipment previously described, and $d \approx 2\ 1/4$ cm. Thus $\delta \approx 1/16$. Putting in these values,

$$W(\Theta) = \pi/16 \Big\{(0.970 + 0.015a + 0.008b) + (0.928a + 0.081b) \times \cos^2 \Theta + 0.831 \cos^4 \Theta\Big\},$$

or

$$a' = \frac{0.928a + 0.081b}{0.970 + 0.015a + 0.008b}$$

and

$$b' = \frac{0.831b}{0.970 + 0.015a + 0.008b} .$$

At this point one has to make an arbitrary decision about the area of comparison between experiment and theory. One can correct the data to fit the theory or one can correct the theory for the solid angle to fit the data. In this thesis the latter is done. The reason is that the experimental points can be compared with the theoretical curve more easily in this manner. The procedure is to make a least square error analysis of the data at the various angles to obtain the coefficients a' and b'. Now these are compared with a and b in one manner or the other. However the deciding factor is that in presenting the data, one frequently plots the experimental points with the theoretical curve. This automatically implies that the a and b must be used to evaluate a' and b' in order to plot the theoretical curve. Therefore the area of comparison is usually that with the data not corrected for the solid angle.

Fig. 5 shows an example of solid angle correction. If one calculates the angular distribution of γ-rays for the set of assignments $(2\ 2^-\ 1\ 2^+)$* one obtains

$$W(\theta) = (3 + 15\cos^2\theta - 16\cos^4\theta) + 6A(1 - 3\cos^2\theta + 4\cos^4\theta),$$

where A is defined on page 8. If this is put into the form

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* This notation is used to describe the units of angular momentum involved in a reaction in which a proton (spin 1/2) bombards a nucleus of spin 1/2 with relative angular momentum ℓ to form a state of spin and parity $J^{\pm}$. Then ℓ' is the relative angular momentum of the α-particle the compound nucleus emits, leaving the residual nucleus in an excited state $J'^{\pm}$ which decays by γ-ray emission to a 0^+ ground state. This set of numbers is designated by $(\ell\ J^{\pm}\ \ell'\ J'^{\pm})$. It is the same as the set on page 24 except that the parities of the excited states are indicated.

$$W(\theta) = 1 + a \cos^2 \theta + b \cos^4\theta,$$

and using as a parameter $x = A/1+A$ instead of A, the distribution coefficients a and b are

$$a = \frac{5-11x}{1+x} \quad \text{and} \quad b = \frac{-16+40x}{3(1+x)} .$$

These are plotted in Fig. 5 as the dashed curves. For each value of x there is a pair of coefficients a' and b' that are corrected for the solid angle of the detector. The curves for a' and b' are the solid curves of Fig. 5.

3. Experimental Results from the $N^{15}(p,\alpha\gamma)C^{12}$ γ-ray Distributions

The reaction $N^{15}(p,\alpha\gamma)C^{12}$ has been studied by Schardt, Fowler, and Lauritsen(16). The N^{15} and proton come together to form a state in O^{16} which emits an α-particle leaving the residual nucleus, C^{12}, in an excited state which decays by the emission of a 4.43-Mev γ-ray. They find that the reaction exhibits strong resonances at 429, 898, and 1210-kev proton energy. Figs. 6 and 7 show the angular distribution of γ-rays from this reaction at the 429 and 898-kev resonances, respectively. A titanium disk 3/8" in diameter and 0.040" thick had one surface treated with nitrogen (31% N^{15}) to make TiN, a very stable nitride. This target was bombarded with protons from the 2-Mv electrostatic accelerator of the Kellogg Radiation Laboratory. The points plotted are the measured γ-ray yields corrected for background and absorption. The solid curve in Fig. 6 (429-kev resonance) is the theoretical curve for the assignment ($2\ 2^-\ 1\ 2^+$) corrected for solid angle. The value of x (see Fig. 5) for this curve is $x = 0.82 \pm 0.04$. This value was obtained by calculating the squares of the errors and choosing the value of x that gives a minimum. The solid curve of Fig. 7 is the same except that $x = 0.58 \pm 0.03$. The parity assignments are based on N^{15} having odd parity.

Fig. 8 shows the angular distribution of γ-rays from the 1210-kev resonance. The solid curve is for the assignment $(2\ 3^-\ 1\ 2^+)$ and the dashed curve for $(3\ 4^+\ 2\ 2^+)$. As one can see, the theoretical curves (corrected for solid angle) are quite close to each other, and one cannot determine the spin and parity of the state in O^{16} involved. A calculation of the least square errors between the measured points and the theoretical curves does not give significant results. In order to resolve the question, other techniques have to be brought into play.

4. Description of the Apparatus for α-particle Distributions

The next attempt to determine the proper assignment made use of the 0 - 160° proton spectrometer of the Kellogg Radiation Laboratory to measure the angular distribution of α-particles. A drawing of the apparatus is shown in Fig. 9. The target is placed at the center of the circle marked with an arrow. It was prepared by evaporating KNO_3 (enriched to 61% N^{15}) onto a piece of copper backing material. Then the emitted α-particles traveled around the spectrometer and into a scintillation counter. Only particles of a certain energy determined by the magnetic field current could enter the counter. In this way the α-particles were separated from the protons for a large range of angles. The spectrometer could be set at various angles with respect to the proton beam.

At a particular angle both the protons and the α-particles can be detected, depending upon the magnet current. A plot of the number of particles the scintillation counter detects as a function of the field current is known as a profile.

A typical profile is shown in Fig. 10. This particular one is that measured at 40° in the laboratory coordinate system at a bombarding energy of 900-kev. The rise at the left is due to the elastically scattered protons. The dotted curve A is an estimate of the continuation of this curve and represents approximately the contribution of protons to the whole curve. Curve B represents the contribution due to several other sources. Perhaps one of the contributions is the recoil C^{12} nuclei. Some of these counts may also be due to protons that are scattered inside the spectrometer and find their way into the detector. Curve B is an estimate of what the shape of this contribution might be. Curves A and B show qualitatively what may occur. Curve C is estimated by extrapolating the left-hand rise to fit the part of the curve marked D. This curve is assumed to be quantitatively correct. The peak is due to the α-particles. Curve C represents the assumed total background.

We can also understand the shape of the α-particle part of the profile. The incident protons have an energy very close to the resonant energy. As they penetrate into the target material they lose energy, or they have an energy farther away from the resonant energy as they move deeper into the target. Therefore they produce fewer α-particles as they move deeper into the target. Since these deeper reactions have less proton energy and their α-particles have to traverse more material to get out of the target material, they enter the spectrometer with less energy. Therefore one expects the number of counts to decrease as the energy of the spectrometer is decreased. The front edge of the curve is not exactly vertical because the spectrometer does not have infinite resolution.

5. Corrections for α-particle Distributions

There are a few corrections that must be applied to the data one obtains from the profiles. First, one must subtract the assumed background. In Fig. 5 one can see that the number of counts at the peak is 3.4 after the background has been subtracted. The next correction arises from the fact that not all of the α-particles that leave the target are doubly charged. Some are singly charged and very few (at the energies we are concerned with) uncharged. The ratio of singly to doubly charged α-particles is a function of energy. Therefore at each angle one must calculate the energy of the α-particles produced and multiply the number of counts by the appropriate factor to estimate the total number of α-particles produced. The curve used to determine the ratio was prepared by Thomas[17].

The α-particles are emitted from the compound nucleus with velocities only an order of magnitude larger than the center-of-mass velocity. This fact means that the solid angle of the spectrometer appears in the center-of-mass coordinate system to vary with the angle of observation. Also the angle at which the spectrometer is set in the laboratory is not the same as it is in the center-of-mass system. The means of correcting for these effects are standard[18].

The final correction involves the fact that the surface of the target may not be perfectly smooth. In order to see the nature of the effect, simply consider a particle leaving the target surface. Suppose now that the particle is emitted at nearly a grazing angle. If the target surface is irregular, that particle may hit some projection of material from the surface and lose sufficient energy in traversing it that it is not detected. From this qualitative discussion one can see that such a correction will be important only for small angles. We desire to estimate this effect numerically.

In order to study the surface interference in more detail, we introduce the concept of a "random surface." Suppose the actual surface were divided up into a very large number of small surfaces of such a small area that the average slope that can be assigned to each one is not very much different from the actual slope at any point of the small area. Consider the group of these average slopes. It will have a certain distribution of slopes, i.e., the number of small areas (if all of equal area of projection on the zero-slope plane) with slope between m and $m + \delta m$ is proportional to

$$g(m)\,\delta m,$$

where g(m) is the distribution function. For a random surface,

$$g(m) = (1/2\pi\, \overline{m})^{1/2} \exp\,(-\, m^2/2\overline{m}^2),$$

where $\overline{m}^2$ is the mean square slope of the surface.

The next point to consider is the definition of some angles involved in the bombardment of the surface with protons. Suppose the target is set at an angle ϕ with respect to the proton beam as shown in Fig. 11. Then let the spectrometer be set at an angle ϕ as shown. Thus $\pi - 2\phi = \theta$, the angle between incident and exit particles. In

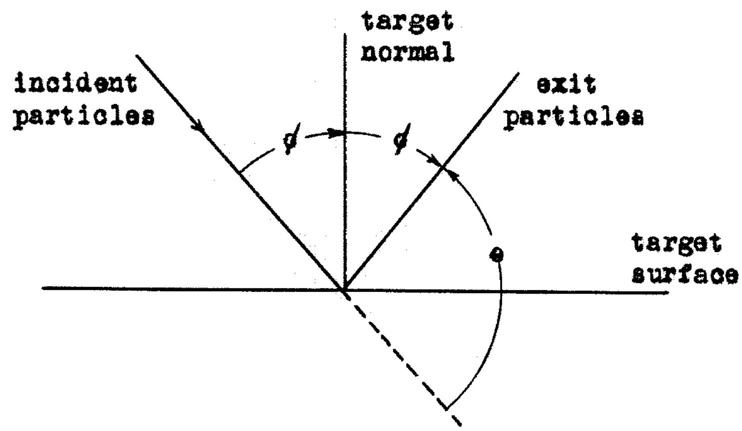


Fig. 11

all the experimental work to be reported later this condition of the target normal bisecting the angle between incident and exit particles is always fulfilled. If the exit particles are produced on a part of the surface that has a slope greater than cot ϕ, we assume they will not be counted since they will have to pass through much target material. The fraction of the surface that has a slope greater than cot ϕ is

$$f = \int_{\cot\phi}^{\infty} g(m)\, dm.$$

Next we note that not all of the surface produces particles because the incident particles do not bombard that part of the surface that has a slope algebraically less than $-\cot \phi$. The fraction of the surface that does not get bombarded is

$$\int_{-\infty}^{-\cot \phi} g(m)\, dm.$$

This quantity is equal to f if g(m) is an even function. Thus only 1 - f of the surface gets bombarded, and then only 1 - 2f of the surface emits particles that enter the spectrometer at full energy since an additional f of the surface does not permit the exit particles to escape at full energy. Thus only

$$\frac{1 - 2f}{1 - f}$$

of the particles produced get out. The correction factor is the reciprocal of this quantity.

The only quantity to be determined is $\bar{m}$. It can be determined either by experiment or perhaps by some estimates. In the present work some reasonable estimate was made on the basis of the fact that the targets were evaporated. This estimate was very rough, but a result of $\bar{m} \approx \tan^{-1} 10^0$ was obtained. That this estimate is satisfactory was determined experimentally and will be discussed later.

For the actual corrections used, g(m) was the "random surface" gaussian distribution of width $2\bar{m}$ as in the formula on the top of page 40. Some work was done using other distributions. There was not much difference (less than 0.1 in f) from the gaussian and a uniform distribution of full width $3\bar{m}/2$ or an isosceles triangle distribution with a base of width $4\bar{m}$.

In table IV values of the correction factor are given for various values of $\bar{m}$ and θ assuming g(m) is the gaussian.

Table IV

θ_{lab}	ϕ	$\tan^{-1}\bar{m}$							
		2.5°	5.0°	7.5°	10.0°	12.5°	15.0°	17.5°	20.0°
5°	87.5°	1.233	1.782	2.438	3.025	3.665	4.406	4.964	5.708
10°	85.0°	1.025	1.233	1.520	1.830	2.145	2.484	2.773	3.132
15°	82.5°	1.001	1.073	1.233	1.416	1.606	1.830	2.034	2.273
20°	80.0°	1.000	1.023	1.113	1.233	1.377	1.520	1.679	1.859
25°	77.5°	1.000	1.005	1.051	1.131	1.233	1.342	1.456	1.592
30°	75.0°	1.000	1.001	1.022	1.073	1.146	1.233	1.328	1.423
35°	72.5°	1.000	1.000	1.009	1.040	1.092	1.156	1.233	1.312
40°	70.0°	1.000	1.000	1.003	1.021	1.056	1.105	1.167	1.233

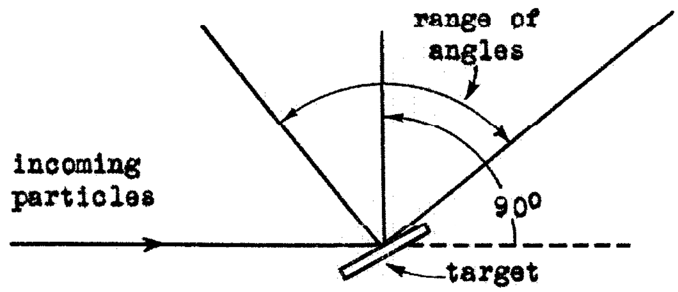
At this point the information presented thus far about surface corrections should be integrated into the general problem. This larger problem has roughly three phases, depending upon the ratio of particle penetration into the surface to what might be called a characteristic length of the surface. The latter quantity is on the order of magnitude of the length between peaks or valleys of the surface. If the penetration is much greater than the characteristic length, no correction is needed. If they are of the same order of magnitude, the present corrections suffice. If the characteristic length of the surface is large compared to the penetration, a quite different type of correction must be used. It is based on the variation of θ_1 and θ_2 in the formulae of Brown, Snyder, Fowler, and Lauritsen[18].

In order to test the accuracy of these techniques, the angular distribution of short-range α-particles was measured at the 898-kev resonance. The assignments of spins and parities are at least tentatively known from the γ-ray distribution at this resonance. The measured points corrected for the various effects just mentioned are plotted in Fig. 12. The solid curve is the theoretical curve for $x = 0.60$. It is proportional to $7 - 6 \cos^2\theta$. The points shown are corrected for surface roughness by using a value for $\overline{m}$ of $\tan^{-1} 10^0$ in accordance with our previous estimate. Since the points are in good agreement with the theoretical curve, this evidence is taken as confirming the value of $\overline{m}$ as a reasonable one. The whole procedure indicates that measurements of this sort are reliable and that the assignments for the 898-kev resonance are correct.

6. Results at the 1210-kev Resonance of $N^{15}(p,\alpha\gamma)O^{12}$

Next measurements were made at the 1210-kev resonance to try to determine the spin and parity of the state in O^{16}. The possible assignments are 3^- or 4^+ as previously considered in the γ-ray experiment. It turns out that 5^- is also a possible candidate, although the corresponding γ-ray curve does not fit the data quite as well as the other two. The theoretical curves for the various assignments are shown in Fig. 13. The solid curve is for $(2\ 3^-\ 1\ 2^+)$; the long dashed curve for $(3\ 4^+\ 2\ 2^+)$; and the short dashed curve for $(4\ 5^-\ 3\ 2^+)$. The measured points are corrected as before and plotted in Fig. 13. The only conclusion that can be drawn is that the state is not 3^-.

In order to resolve the question, another γ-ray distribution was measured. This time the distribution was measured only over a small range of angles near $\theta = 90^0$. The target was set at 30^0 with respect to the proton beam as shown below so that the γ-rays measured do not pass through any of the target material. In this way absorption correction is a minimum. At each point a large number of counts



was taken in order to minimize statistical errors. The magnitude of these errors is described by the size of the circles in Fig. 14. The actual readings corrected for background are the centers of the circles. The theoretical curves corrected for solid angle are plotted using the same designations as in Fig. 13. From these data we conclude that the state is 4^+.

IV AMBIGUITY IN SPIN AND PARITY ASSIGNMENTS

1. Two Types of Resonances

It would be of interest to investigate the limits one can put on the assignments that have been made on the basis of the measured distributions. In other words, up to this point agreement has been found with a certain set of assignments. The question under consideration is whether disagreement can be found with a large class of other assignments so some statements can be made about the ambiguity of an assignment.

It is convenient to divide all the possible assignments into two groups, depending on whether or not the O^{16} state can emit long-range α-particles leaving the C^{12} in its ground state. Actually two reactions can occur, depending on the bombarding proton energy. In one, the process can be the $N^{15}(p,\alpha\gamma)C^{12}$ reaction,

$$N^{15} + p \rightarrow (O^{16}) \rightarrow \alpha + C^{12*}$$
$$\downarrow$$
$$C^{12} + \gamma.$$

In some cases the α-particle can leave the C^{12} in its ground state,

$$N^{15} + p \rightarrow (O^{16}) \rightarrow \alpha + C^{12}.$$

This is the $N^{15}(p,\alpha)C^{12}$ reaction. Only certain excited states of O^{16} can exhibit this type of decay. The products of the decay are an α-particle (0^+), the C^{12} in its ground state (0^+), and the relative angular momentum. Therefore the excited state must have a spin and parity that corresponds to the relative angular momentum. Since the parity of the Y_ℓ^m's that describe the relative angular momentum have parity $(-1)^\ell$, the excited states that give off long-range α-particles must have parity $(-1)^J$ where J is its spin. If the parity is $(-1)^{J+1}$, the state cannot give off long-range α-particles. This type of division is very useful, for if a certain level is known to give off long-range α-particles, half of the possibilities for the excited state are excluded. On the other hand, if no long-range α-particles are observed, nothing can be said unless more is known about the assignments. However the cases in which long-range α-particles can be emitted, and are not, are rare because the energies of the decays favor long-range α-particle emission. Thus it will be convenient to assume that if none are observed the level involved is one that does not permit them. However the validity of this assumption must be checked in the light of the additional information obtained from the measurements of the angular distributions.

In the work of Schardt, Fowler and Lauritsen[16] no long-range α-particles were observed at the 429 and the 898-kev resonances. Therefore the assumption is that these are of the type with parity $(-1)^{J+1}$. The 1210-kev resonance, however, does give long-range α-particles. Thus it is of the type with parity $(-1)^{J}$.

A further distinction results from the formation of the compound nucleus. If the state has parity $(-1)^{J}$, only channel spin one enters into the formation and there is no arbitrary constant in the distributions. The distributions are completely determined by the spins and relative angular momenta. To see that this is the case, consider formation of a compound nucleus with channel spin zero. In this case $\ell = J$ (using the notation of the footnote on page 34). The parity of the N^{15} and proton combination is odd. The parity of the relative angular momentum is $(-1)^{\ell}$. Thus the state formed has parity $(-1)^{J+1}$, or it is a state that does not give long-range α-particles. If only channel spin one enters, and if only the lowest possible value of ℓ is important in the formation of the compound nucleus, $J = \ell + 1$. The parity is $(-1)^{\ell+1}$ or $(-1)^{J}$, a state that does give off long-range α-particles.

2. 1210-kev Resonance

Now let us consider the possible ambiguity of the $(3\ 4^{+}\ 2\ 2^{+})$ assignment of the 1210-kev resonance. In order to do this, one must know the angular distributions of the γ-rays for other possible assignments. First of all, half of the assignments are eliminated because the state is known to give off long-range α-particles. For the same reason there will be no arbitrary coefficients in the distributions. Next one assumes that only the lowest value of ℓ is important in the reaction. Finally assume that Fig. 8 represents a "quadrupole" distribution, that is, it contains terms including $\cos^4\theta$ but no higher power terms. If one considers cases in which the spin of the O^{16} state is up to but not always including six and in which the spin of the C^{12} excited state is up to and including four, we can calculate a table of angular distributions of the γ-rays. Then one could plot each of these and compare it with the measured points and perhaps draw some conclusions about the ambiguity. There is a simpler way to accomplish the same result. Remember that the data can be analysed to give the <u>two</u> coefficients, a' and b', of the distribution $W(\theta) = 1 + a' \cos^2\theta + b' \cos^4\theta$. These numbers a' and b' can be compared with the corresponding a' and b' of the theoretical distributions. A convenient way to do this is on a graph. Fig. 15 shows the coefficients a and b of the distributions mentioned earlier in this paragraph. The abscissa of a point is the coefficient a; the ordinate, b. The results of this experiment show that the measured distribution is near $(3\ 4^+\ 2\ 2^+)$. Looking in this region of the graph one can see that the state in C^{12} is definitely 2^+. The previous work indicates which is the correct assignment for the O^{16} state, namely, 4^+. This graph then shows that, provided the assumptions are correct, the state in O^{16} (13.24-Mev) is 4^+ or greater than 5 and that the state in C^{12} (4.43-Mev) is 2^+ or greater than 4.

3. 429 and 898-kev Resonances

Fig. 16 shows a similar plot for the angular distributions of γ-rays for the states that do not give off long-range α-particles.

These states have both channel spins zero and one forming the compound nucleus. Therefore they have an arbitrary coefficient in the distribution function. As a result any particular assignment appears as a line. The choice of the arbitrary coefficient determines a point on the line. The heavy line running diagonally along the lower left region of the graph represents a boundary line for distributions that have portions with $W(\theta)$ less than zero. If a point were below this line, the distribution function would predict a negative number of γ-rays being emitted at some range of angles. Since this does not correspond to reality, distributions below this line are excluded. The line has the equation

$$a + b + 1 = 0$$

up to the point (-2, +1). Beyond this point it has the form

$$4b - a^2 = 0.$$

A similar line also appears in Fig. 12.

The lines on the graph represent quadrupole patterns. Some of the circles indicate the intersection of two lines. For example, the ($3\ 3^+\ 2\ 2^+$) line ends at about (-0.4, 0.6); the ($2\ 2^-\ 2\ 3^-$) line continues on from this point. The circle marks the point of change. Another type of distribution is also plotted on the graph. These are octupole distributions (containing $\cos^6\theta$ terms) that have the value of x so chosen that it reduces to a quadrupole distribution. An example is the ($4\ 4^-\ 2\ 3^-$) circle near (1, 0).

The point (-3, 4) is the intersection of the ($2\ 2^-\ 0\ 2^-$) and the ($2\ 2^-\ 1\ 2^+$) distributions. Along the ($2\ 2^-\ 1\ 2^+$) line there are small cross lines that indicate various values of x. At (-3, 4) the value of x is 1.0. The first cross line is x = 0.9; the second, 0.8; etc. The values measured were 0.82 ± 0.04 at 429-kev and 0.58 ± 0.03 at 898-kev. The next consideration is the region of uncertainty. It is not a circle, but rather an ellipse with its major axis having a slope of -1 and a ratio of major axis to minor axis of between five and ten, depending upon the data. Note that the points were taken every ten degrees in the angle θ. This fact means that with respect to $\cos^2\theta$, more points were taken at the ends of the range than at the middle. Therefore the region of uncertainty is not a circle, but a flat ellipse. It lies nearly along the line for the ($2\ 2^-\ 1\ 2^+$) assignment. At the point x = 0.58 the region is such that the measured distribution cannot be any other than the ($2\ 2^-\ 1\ 2^+$). This result shows that the 898-kev resonance excites a level in O^{16} (12.95-Mev) that is 2^- or greater than five. Here again the C^{12} level is 2^+ or greater than four, independent of the result at the 1210-kev resonance.

The case for the 429-kev resonance is not quite so clear. Near x = 0.82 it is slightly probable that the measured value may overlap into the region of the ($2\ 2^-\ 0\ 2^-$) line. In any case the spin of the O^{16} level (12.51-Mev) is 2^- or greater than five.

4. Channel Spin Measurements

In Table III on page 27 there are the results of some calculations of the channel spin ratio based upon some definite nuclear models assuming the N^{15} is in a $p_{1/2}$ state. The 429 and the 898-kev resonances have two units of relative angular momentum for the incoming protons. The measured values of the coefficient x are 0.82 and 0.58, respectively. These correspond best to the theoretical values of 0.86 and 0.60, respectively. The 898-kev distribution can be fit best by a value of x that agrees with either of the models; however the 429-kev distribution seems to agree only with the LS model. The reader is referred to the paper by Professor Christy(5) for more data on this point.

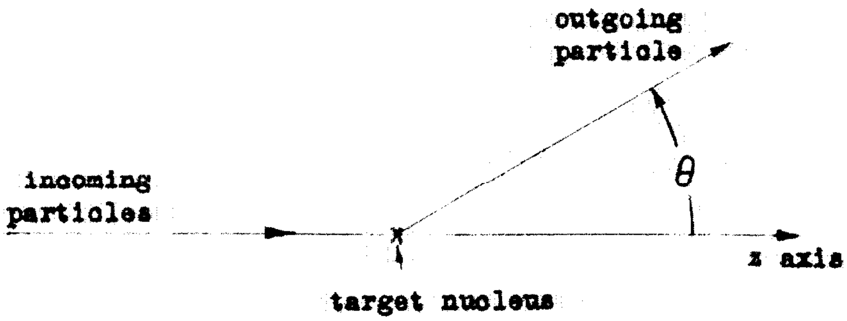

outgoing
particle
θ
incoming
particles
z axis
target nucleus

Fig. 1a

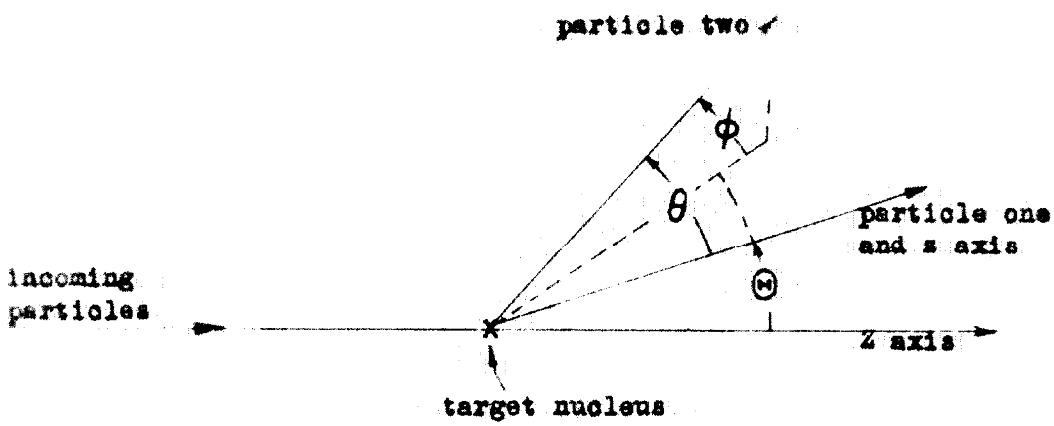

particle two
φ
θ
particle one
and z axis
Θ
incoming
particles
Z axis
target nucleus

Fig. 1b

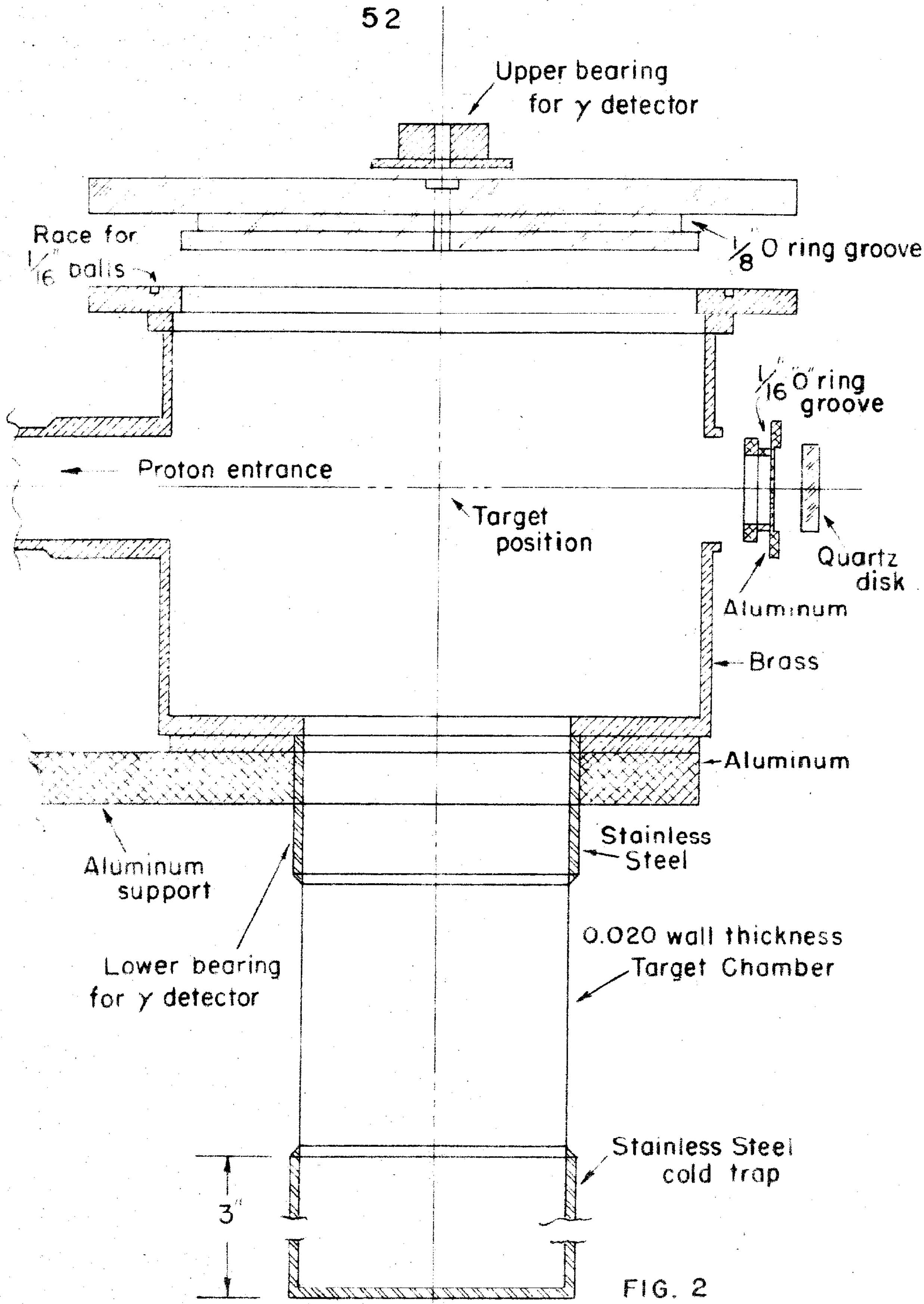

Upper bearing for γ detector
Race for 1/16" balls
1/8" O ring groove
1/16" O ring groove
Proton entrance
Target position
Quartz disk
Aluminum
Brass
Aluminum
Stainless Steel
Aluminum support
0.020 wall thickness Target Chamber
Lower bearing for γ detector
Stainless Steel cold trap
3"

FIG. 2

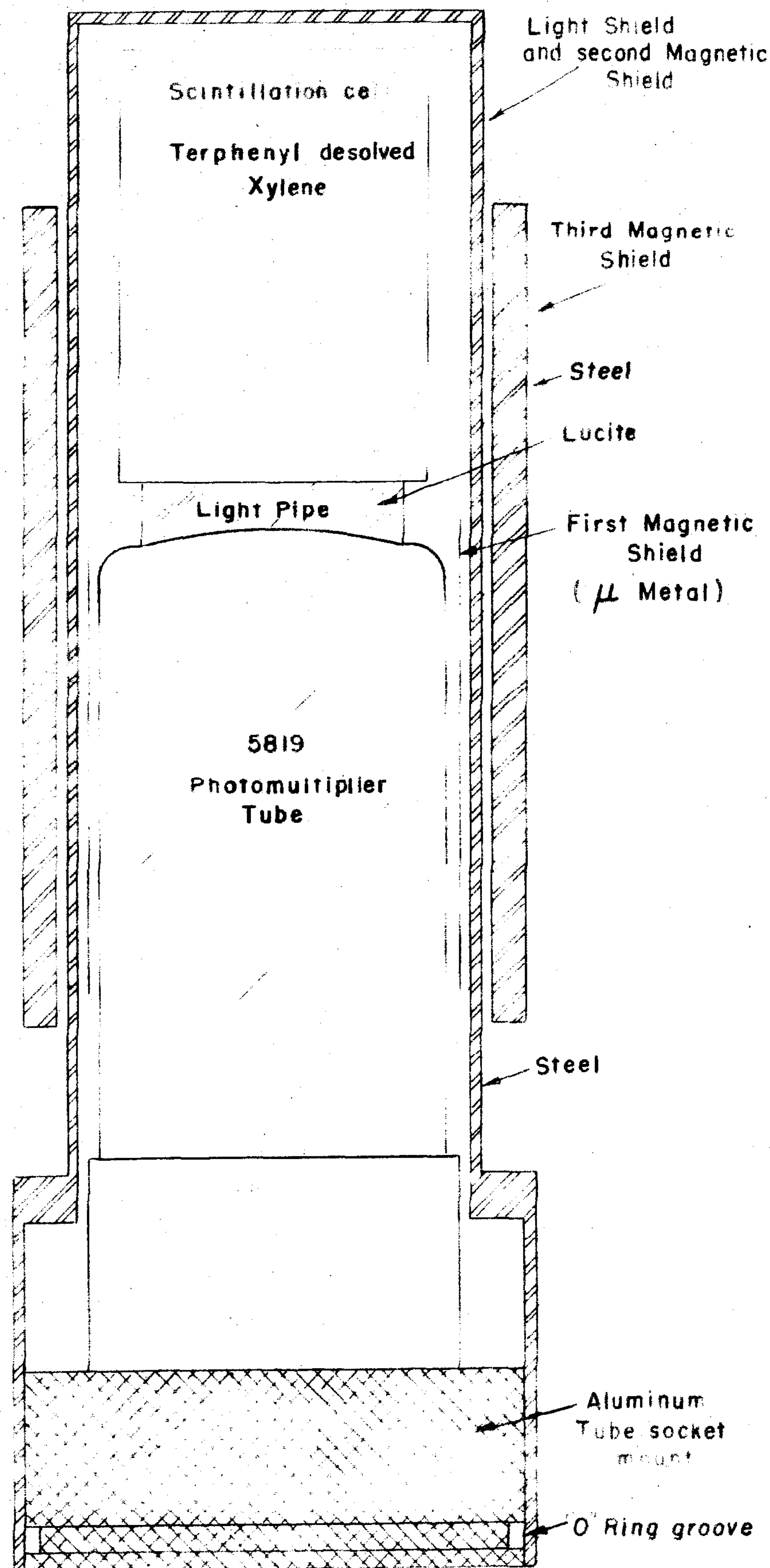

Light Shield
and second Magnetic
Shield
Scintillation ce
Terphenyl desolved
Xylene
Third Magnetic
Shield
Steel
Lucite
Light Pipe
First Magnetic
Shield
(μ Metal)
5819
Photomultiplier
Tube
Steel
Aluminum
Tube socket
mount
O Ring groove

FIG. 3

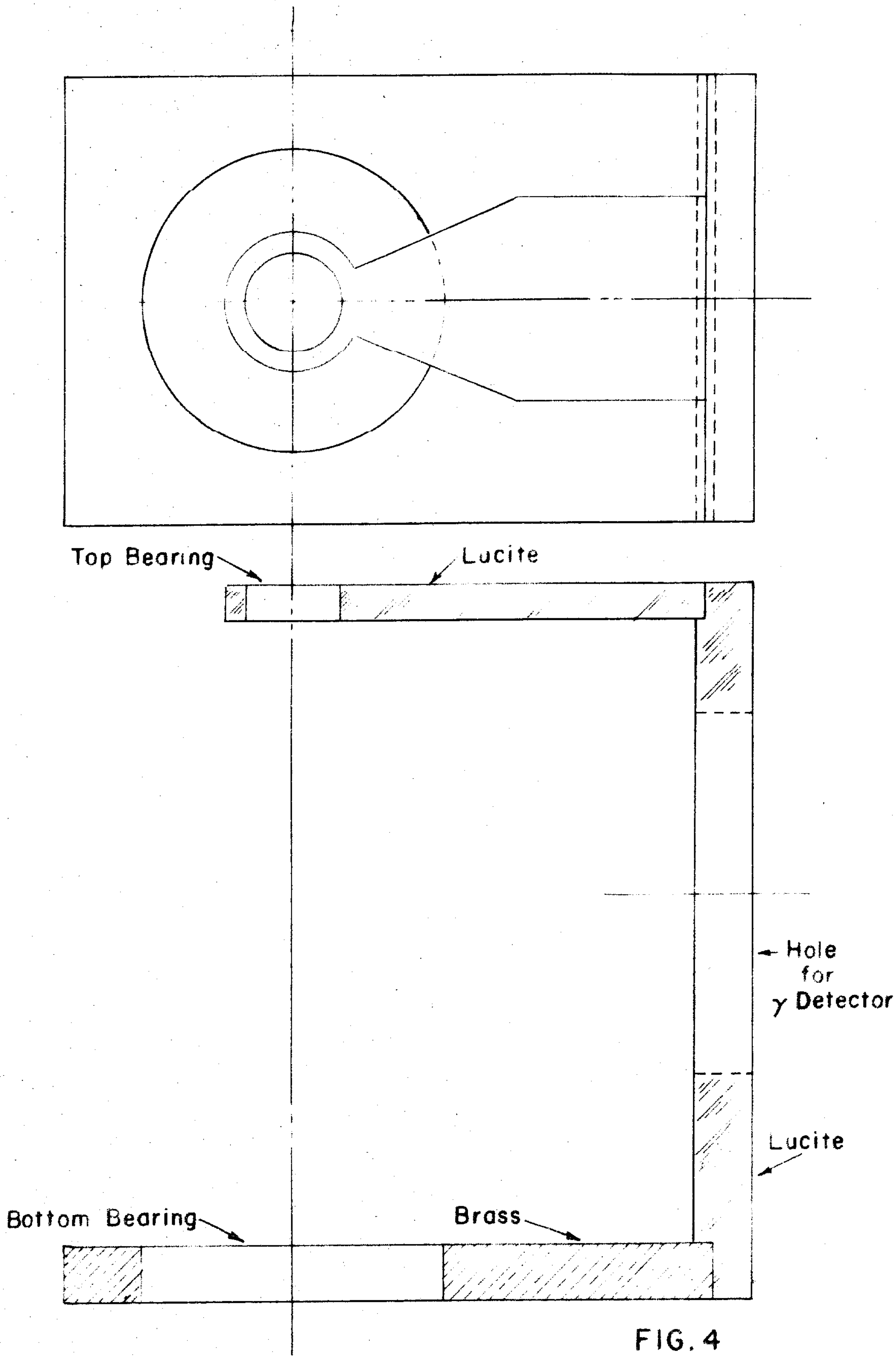

Top Bearing
Lucite
Hole
for
γ Detector
Lucite
Bottom Bearing
Brass

FIG. 4

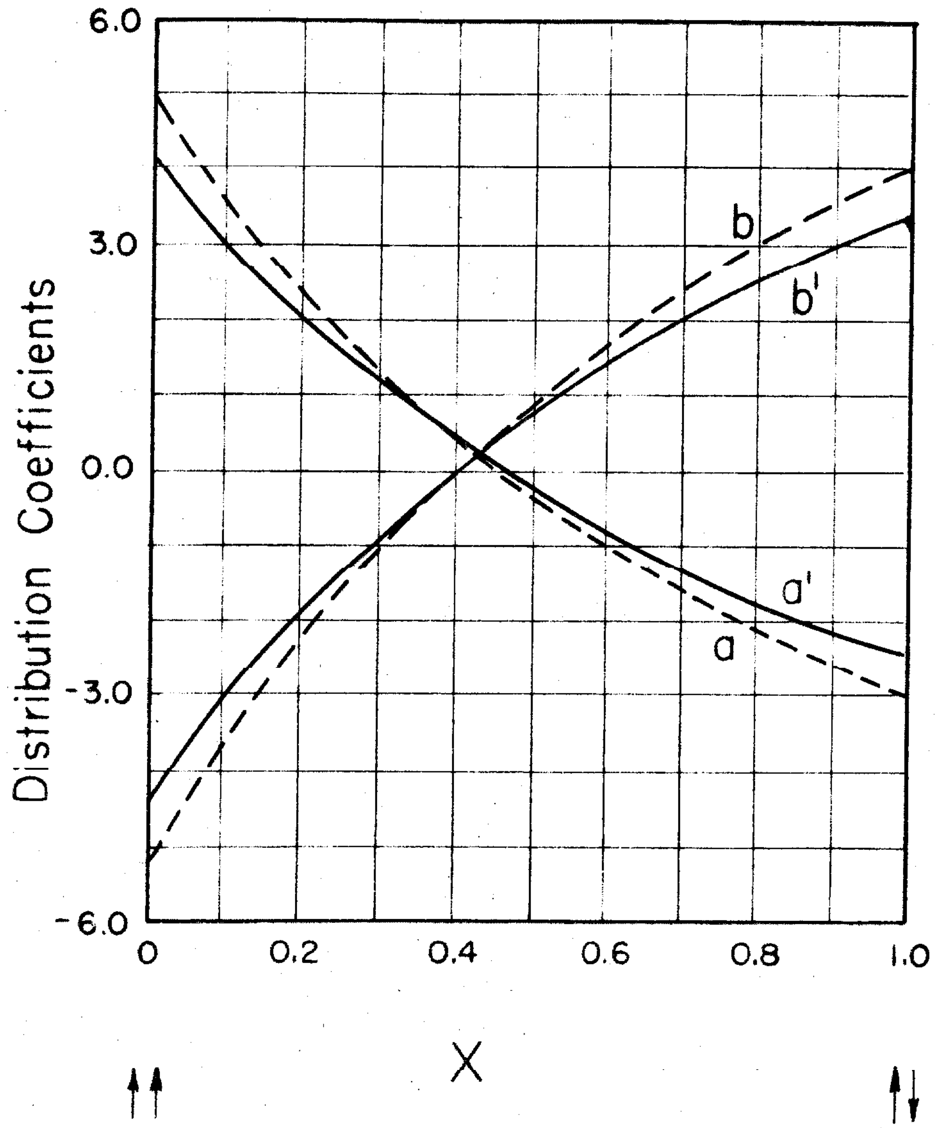

6.0
3.0
0.0
-3.0
-6.0
Distribution Coefficients
0
0.2
0.4
0.6
0.8
1.0
X
b
b'
a'
a

FIG. 5

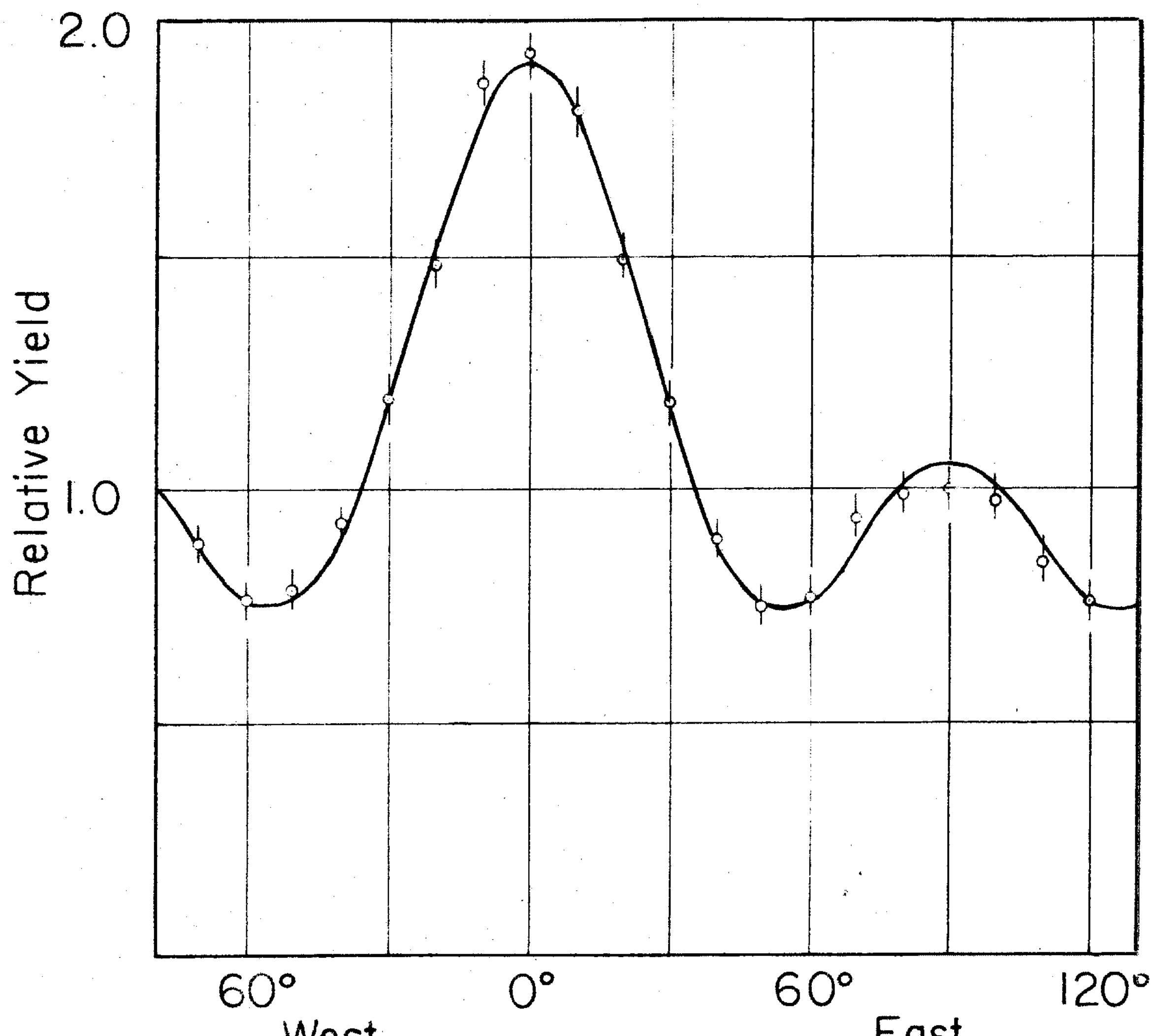


FIG. 6

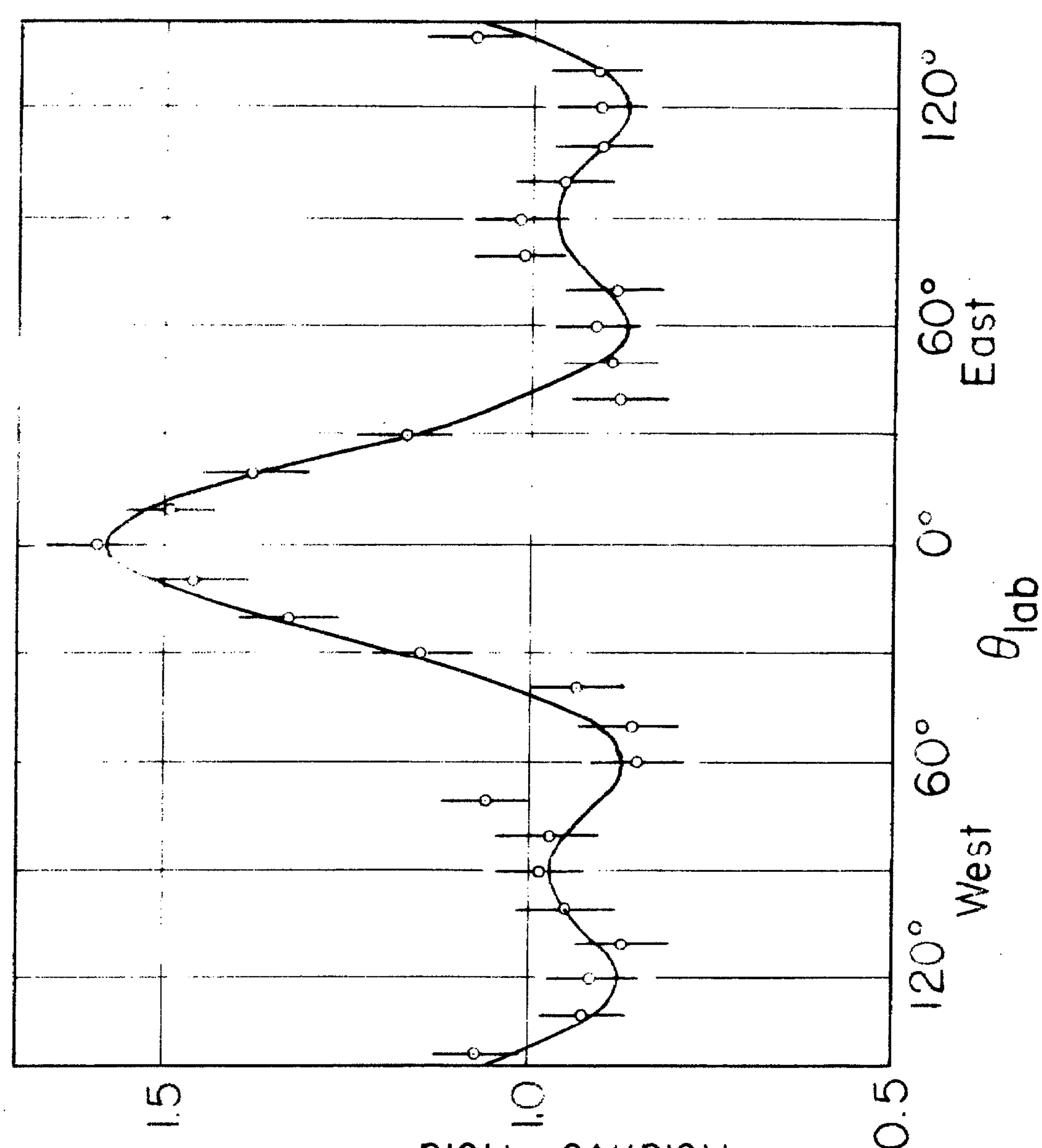


FIG. 7

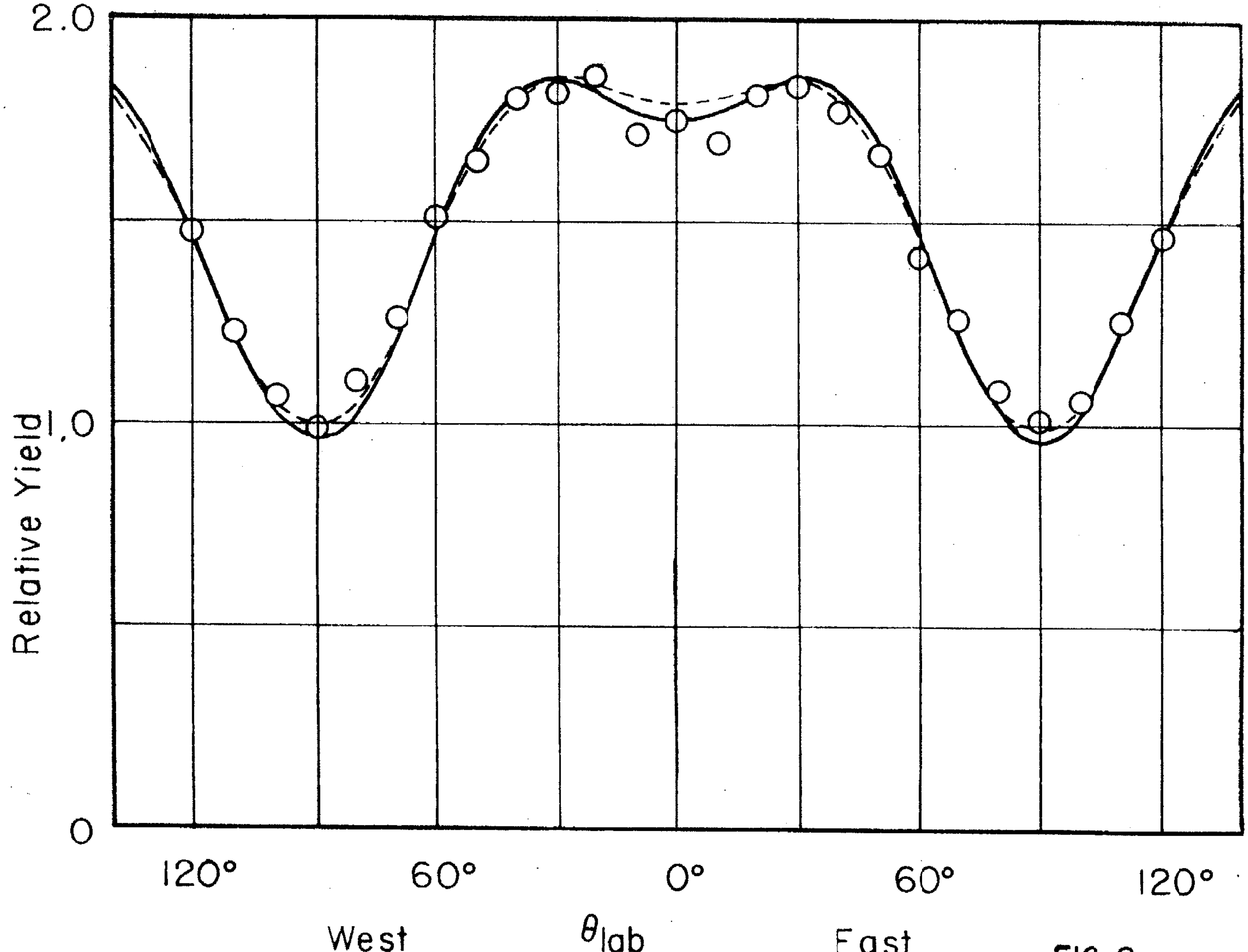


FIG. 8

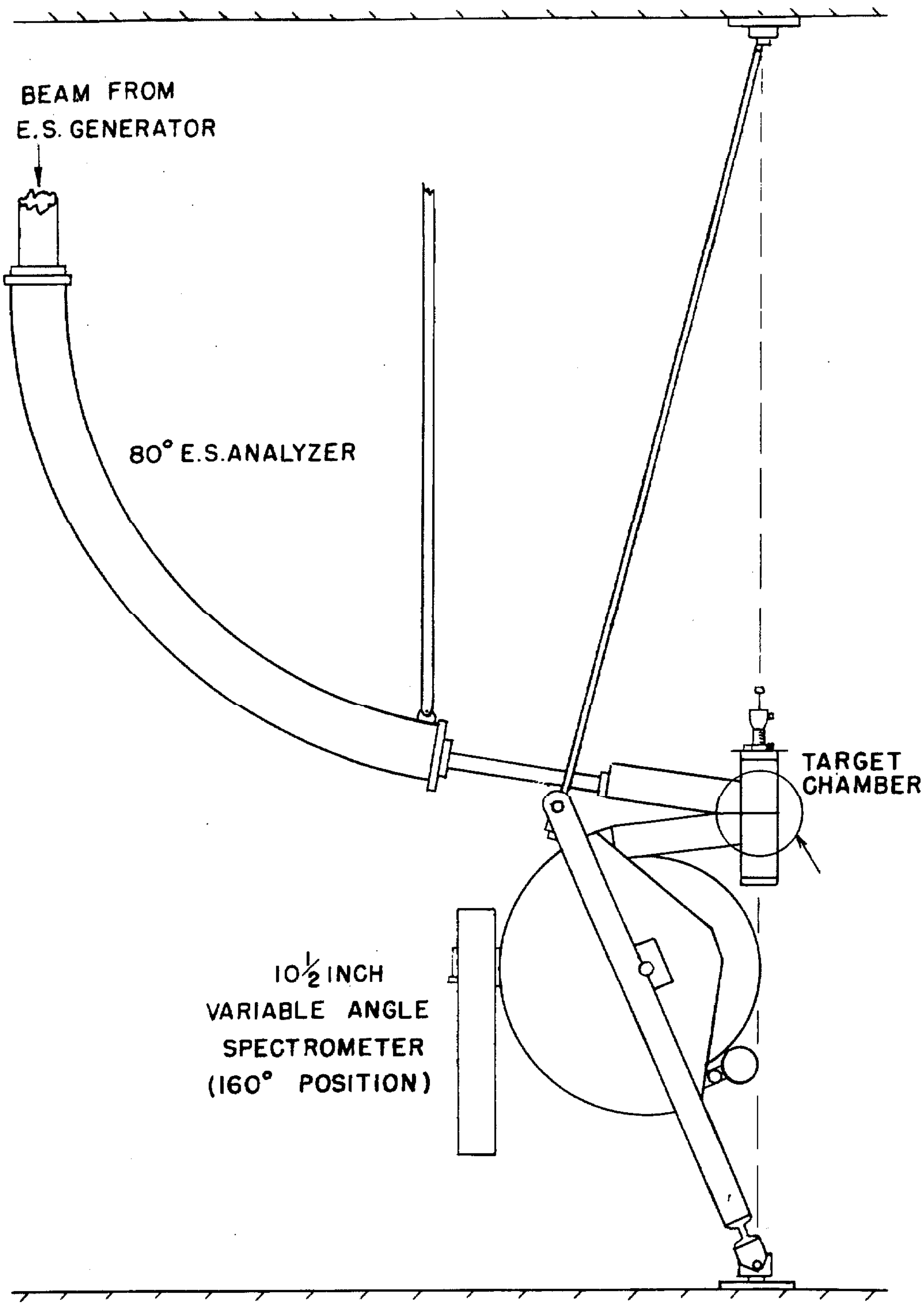

BEAM FROM
E.S. GENERATOR
80° E.S. ANALYZER
TARGET
CHAMBER
10½ INCH
VARIABLE ANGLE
SPECTROMETER
(160° POSITION)

FIG. 9

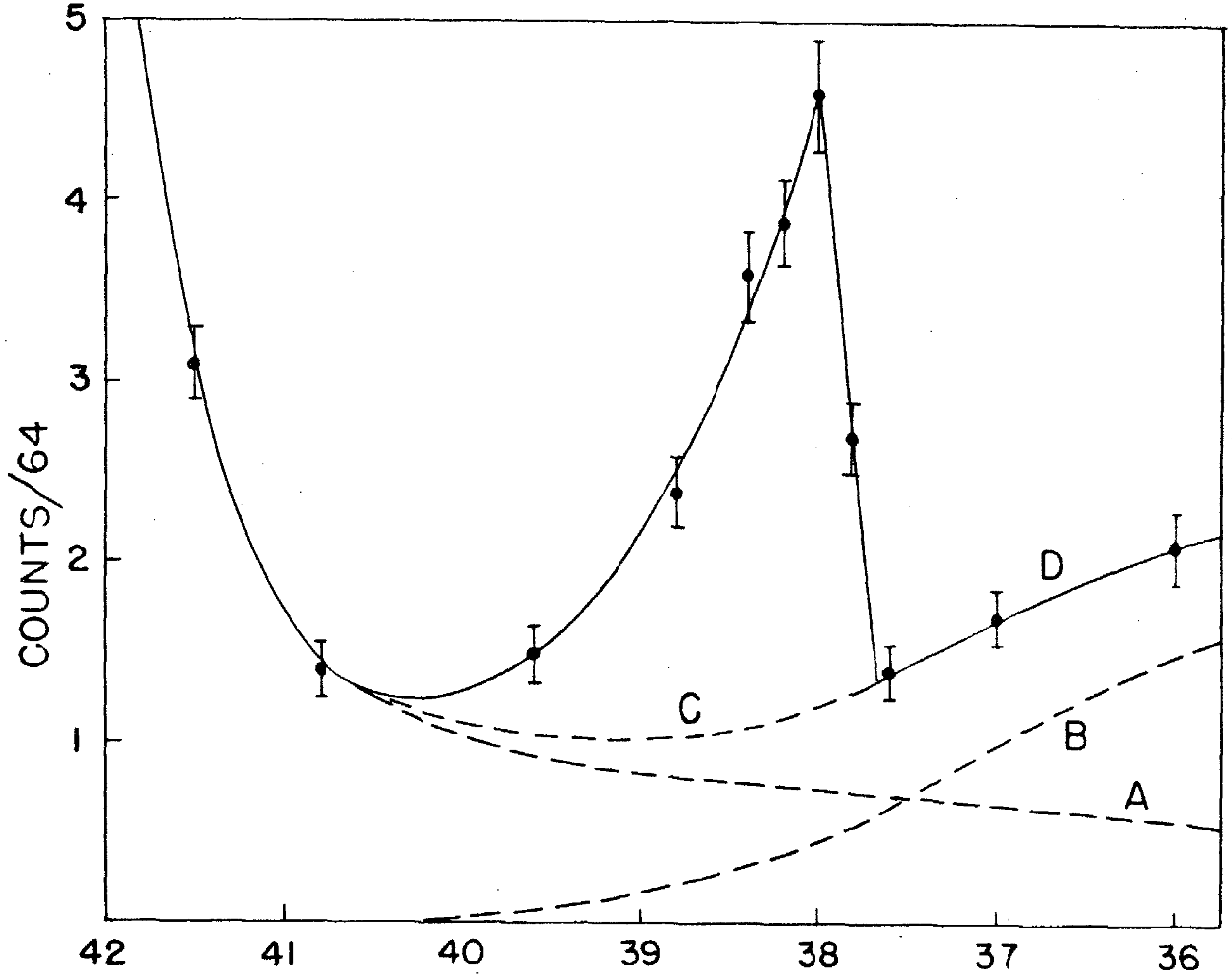

5
4
3
2
1
COUNTS/64
42
41
40
39
38
37
36
A
B
C
D
MAGNETOMETER CURRENT

FIG. 10

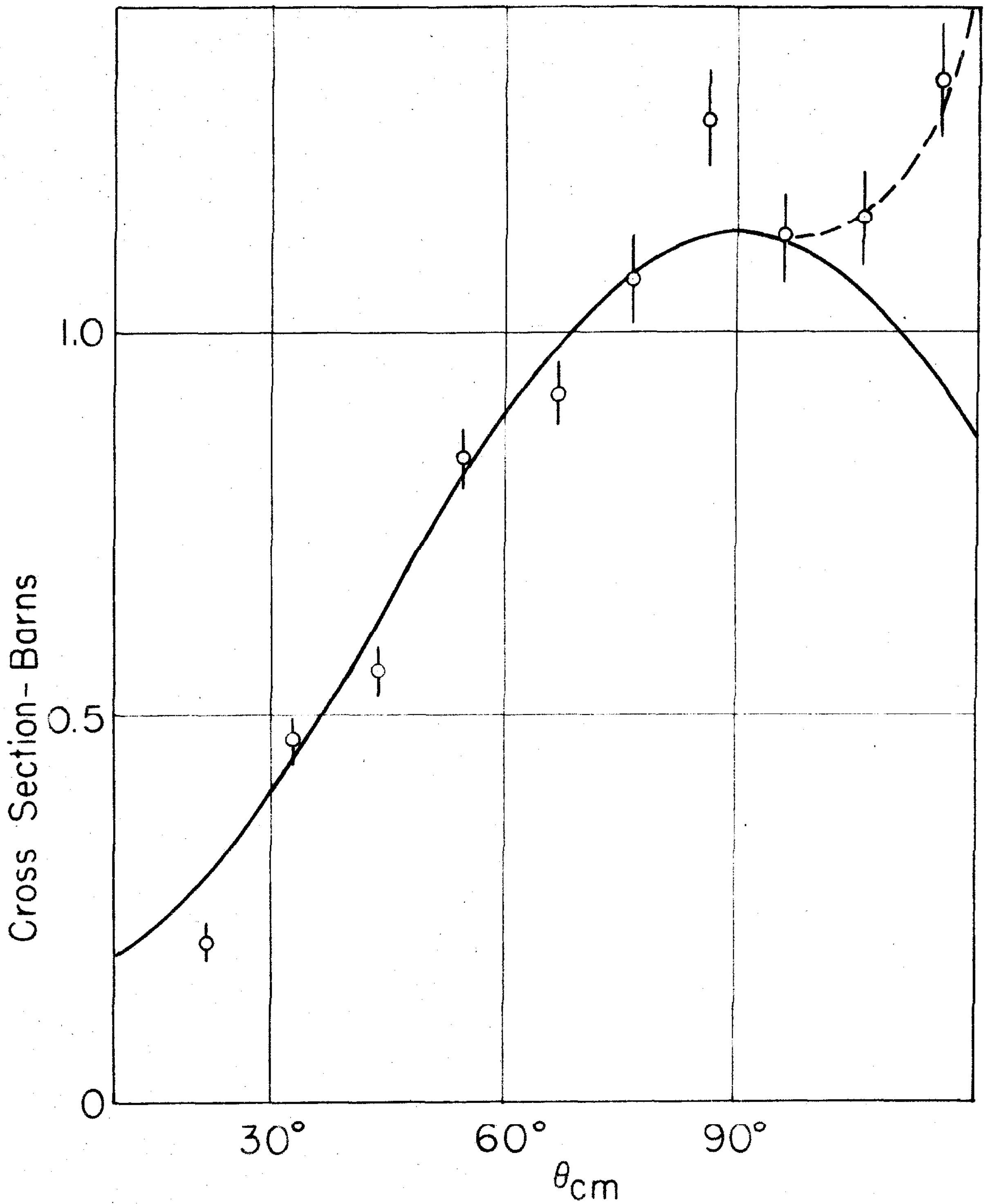


FIG. 12

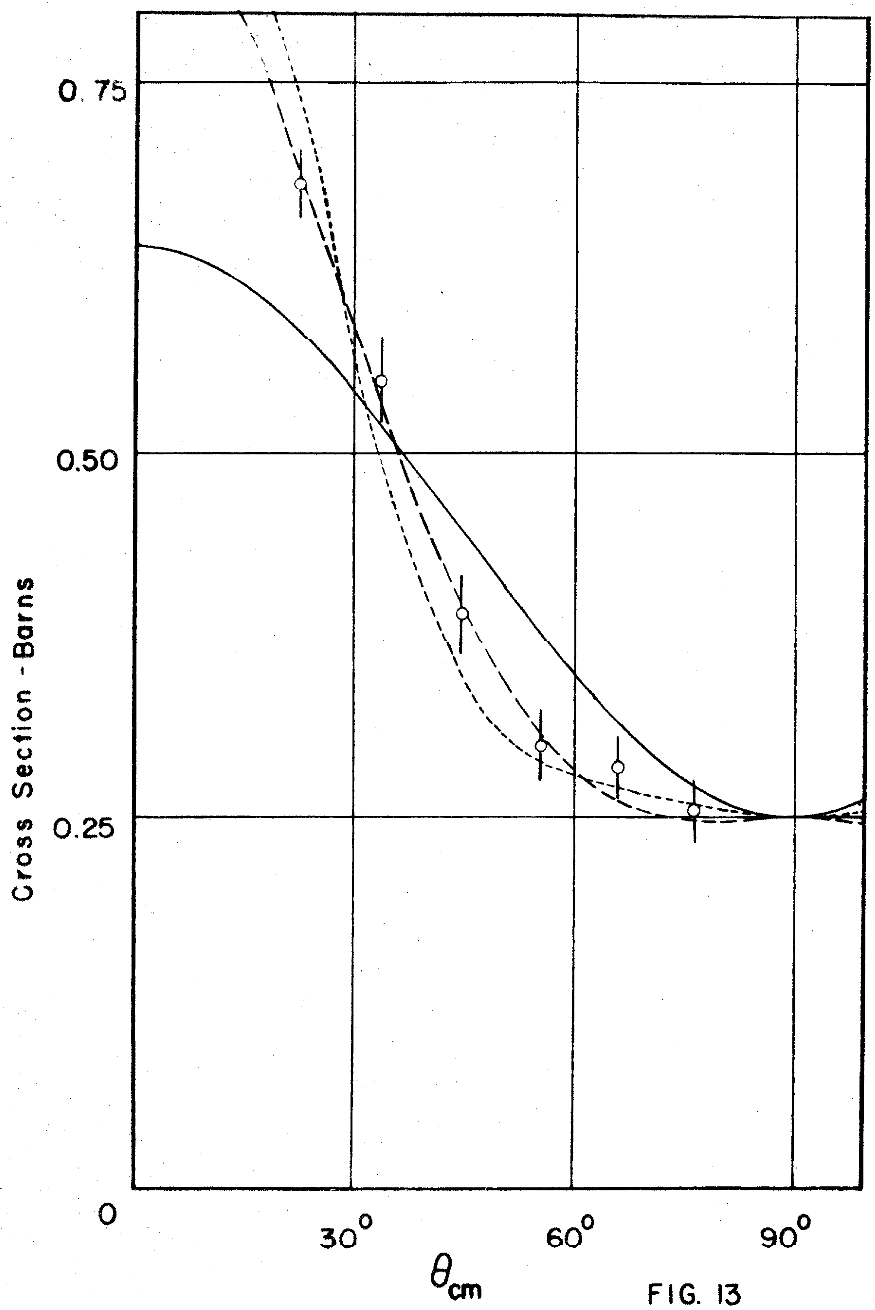


FIG. 13

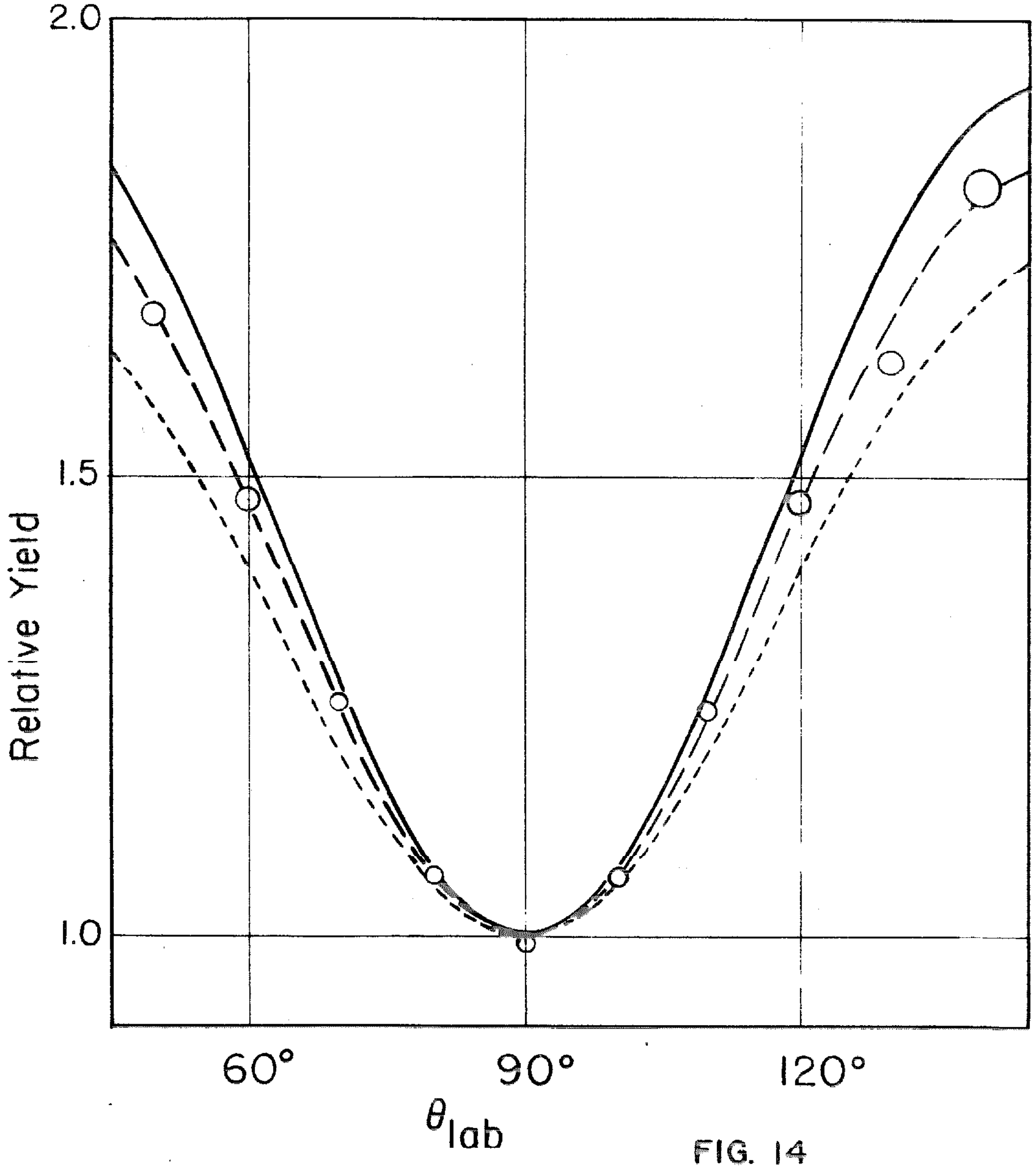
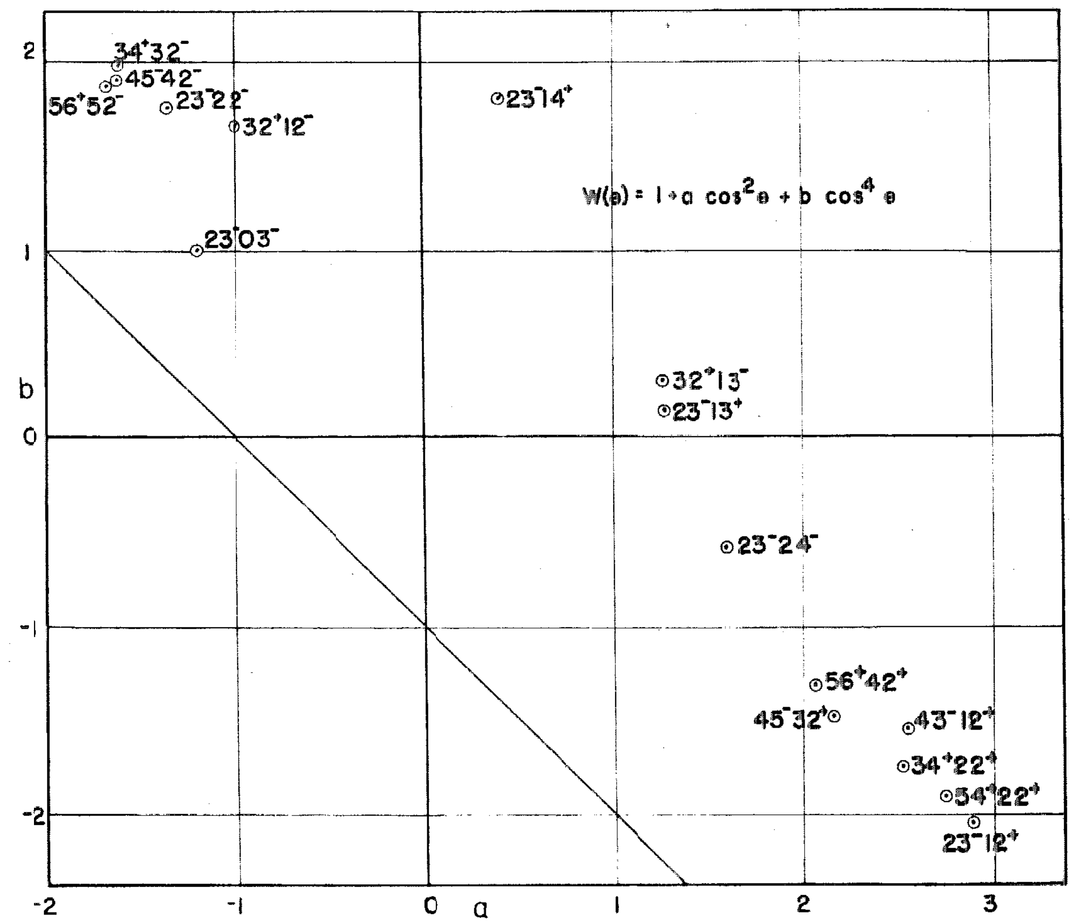
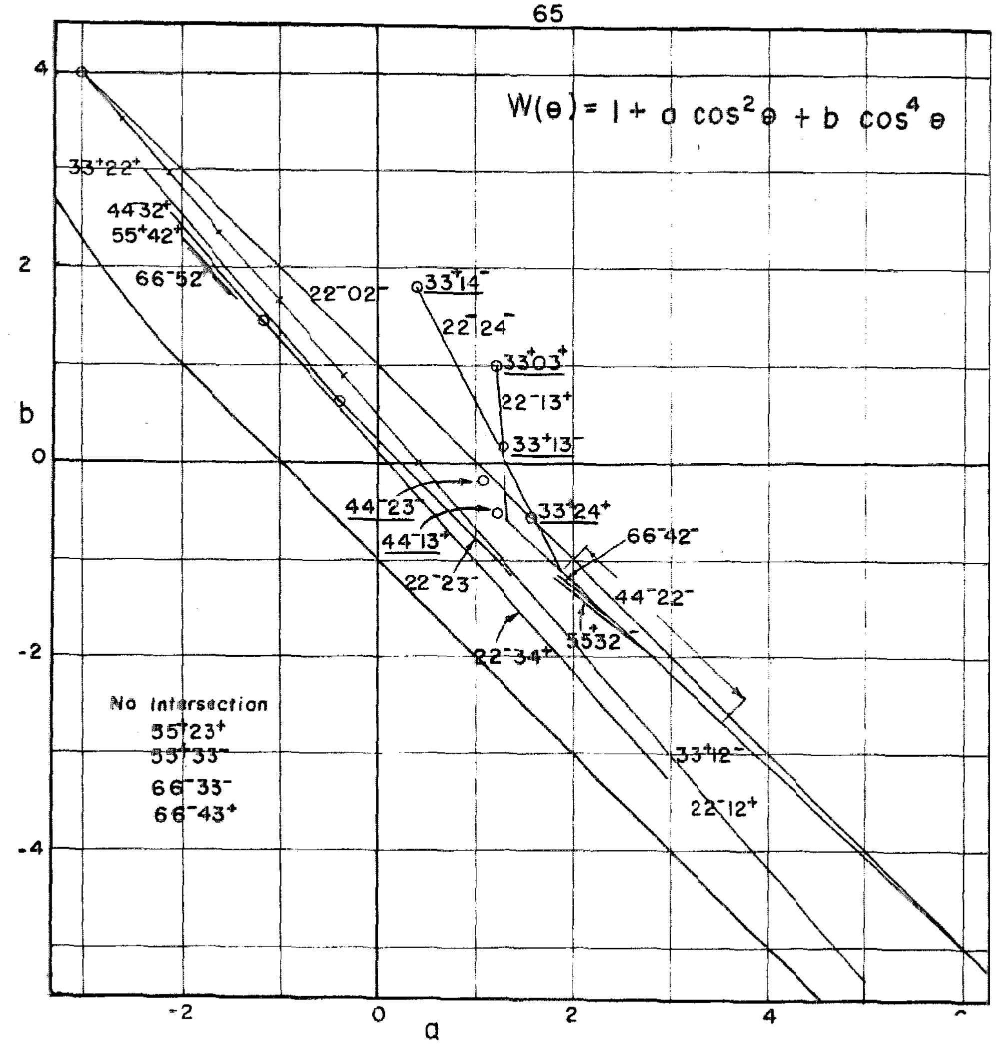


FIG. 14



W(θ) = 1 + a cos²θ + b cos⁴θ
34⁺32⁻
45⁻42⁻
56⁺52⁻
23⁻22⁻
32⁺12⁻
23⁻14⁺
23⁺03⁻
32⁺13⁻
23⁻13⁺
23⁻24⁻
56⁺42⁺
45⁻32⁺
43⁻12⁺
34⁺22⁺
54⁺22⁺
23⁻12⁺
b
a
2
1
0
-1
-2
-2
-1
0
1
2
3

FIG. 15



W(θ) = 1 + a cos²θ + b cos⁴θ
b
a
4
2
0
-2
-4
-2
0
2
4
33⁺22⁺
44⁻32⁺
55⁺42⁺
66⁻52⁻
22⁻02⁻
33⁺14⁻
22⁻24⁻
33⁺03⁺
22⁻13⁺
33⁺13⁻
44⁻23⁻
44⁻13⁺
33⁺24⁺
66⁻42⁻
22⁻23⁻
44⁻22⁻
55⁺32⁻
22⁻34⁺
33⁺12⁻
22⁻12⁺
No Intersection
55⁺23⁺
55⁺33⁻
66⁻33⁻
66⁻43⁺

FIG. 16

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APPENDIX I

Tables of Clebsch–Gordan Coefficients

The following tables are from E. Richard Cohen, Thesis, California Institute of Technology (1949):

$D_{1/2} \times D_{1/2}$	$D_{1/2} \times D_2$	$D_{3/2} \times D_{3/2}$	$D_{1/2} \times D_{7/2}$
$D_{1/2} \times D_1$	$D_1 \times D_{3/2}$	$D_{1/2} \times D_3$	$D_1 \times D_3$
$D_{1/2} \times D_{3/2}$	$D_{1/2} \times D_{5/2}$	$D_1 \times D_{5/2}$	$D_{3/2} \times D_{5/2}$
$D_1 \times D_1$	$D_1 \times D_2$	$D_{3/2} \times D_2$	$D_2 \times D_2$.

The following were calculated by this author:

$D_1 \times D_4$	$D_2 \times D_3$	$D_2 \times D_5$	$D_3 \times D_4$
$D_1 \times D_5$	$D_2 \times D_4$	$D_3 \times D_3$	$D_3 \times D_5$.

	W_1^1	W_1^0	W_0^0	W_1^{-1}
$U_{1/2}^{1/2} V_{1/2}^{1/2}$	1			
$U_{1/2}^{1/2} V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U_{1/2}^{-1/2} V_{1/2}^{1/2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U_{1/2}^{-1/2} V_{1/2}^{-1/2}$				1

$\mathfrak{D}_{1/2} \times \mathfrak{D}_{1/2}$

	$W^{3/2}_{3/2}$	$W^{1/2}_{3/2}$	$W^{1/2}_{1/2}$	$W^{-1/2}_{3/2}$	$W^{-1/2}_{1/2}$	$W^{-3/2}_{3/2}$
$U^1_1 \quad V^{1/2}_{1/2}$	1					
$U^1_1 \quad V^{-1/2}_{1/2}$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$			
$U^0_1 \quad V^{1/2}_{1/2}$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$			
$U^0_1 \quad V^{-1/2}_{1/2}$				$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
$U^{-1}_1 \quad V^{1/2}_{1/2}$				$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
$U^{-1}_1 \quad V^{-1/2}_{1/2}$						1

$$\mathfrak{D}_1 \times \mathfrak{D}_{1/2}$$

	W_2^2	W_2^1	W_1^1	W_2^0	W_1^0	W_2^{-1}	W_1^{-1}	W_2^{-2}
$U_{3/2}^{3/2} V_{1/2}^{1/2}$	1							
$U_{3/2}^{3/2} V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$					
$U_{3/2}^{1/2} V_{1/2}^{1/2}$		$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$					
$U_{3/2}^{1/2} V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$			
$U_{3/2}^{-1/2} V_{1/2}^{1/2}$				$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$			
$U_{3/2}^{-1/2} V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$	
$U_{3/2}^{-3/2} V_{1/2}^{1/2}$						$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$	
$U_{3/2}^{-3/2} V_{1/2}^{-1/2}$								1

$$\mathfrak{D}_{3/2} \times \mathfrak{D}_{1/2}$$

	W_2^2	W_2^1	W_1^1	W_2^0	W_1^0	W_0^0	W_2^{-1}	W_1^{-1}	W_2^{-2}
$U_1^1 \; V_1^1$	1								
$U_1^1 \; V_1^0$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$						
$U_1^0 \; V_1^1$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$						
$U_1^1 \; V_1^{-1}$				$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
$U_1^0 \; V_1^0$				$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$			
$U_1^{-1} \; V_1^1$				$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
$U_1^0 \; V_1^{-1}$							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U_1^{-1} \; V_1^0$							$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U_1^{-1} \; V_1^{-1}$									1

$$\mathfrak{D}_1 \times \mathfrak{D}_1$$

		$W^{5/2}_{5/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{5/2}$
U^{2}_{2}	$V^{1/2}_{1/2}$	1									
U^{2}_{2}	$V^{-1/2}_{1/2}$		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$							
U^{1}_{2}	$V^{1/2}_{1/2}$		$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$							
U^{1}_{2}	$V^{-1/2}_{1/2}$				$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$					
U^{0}_{2}	$V^{1/2}_{1/2}$				$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$					
U^{0}_{2}	$V^{-1/2}_{1/2}$						$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$			
U^{-1}_{2}	$V^{1/2}_{1/2}$						$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$			
U^{-1}_{2}	$V^{-1/2}_{1/2}$								$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$	
U^{-2}_{2}	$V^{1/2}_{1/2}$								$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$	
U^{-2}_{2}	$V^{-1/2}_{1/2}$										1

$$\mathfrak{D}_2 \times \mathfrak{D}_{1/2}$$

	$W^{5/2}_{5/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{1/2}_{1/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-1/2}_{1/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{5/2}$
$U^{3/2}_{3/2}\ V^{1}_{1}$	1											
$U^{3/2}_{3/2}\ V^{0}_{1}$		$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$									
$U^{1/2}_{3/2}\ V^{1}_{1}$		$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$									
$U^{3/2}_{3/2}\ V^{-1}_{1}$				$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$						
$U^{1/2}_{3/2}\ V^{0}_{1}$				$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$						
$U^{-1/2}_{3/2}\ V^{1}_{1}$				$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$						
$U^{1/2}_{3/2}\ V^{-1}_{1}$							$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$			
$U^{-1/2}_{3/2}\ V^{0}_{1}$							$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$			
$U^{-3/2}_{3/2}\ V^{1}_{1}$							$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$			
$U^{-1/2}_{3/2}\ V^{-1}_{1}$										$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$	
$U^{-3/2}_{3/2}\ V^{0}_{1}$										$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$	
$U^{-3/2}_{3/2}\ V^{-1}_{1}$												1

$\mathfrak{D}_{3/2} \times \mathfrak{D}_{1}$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_3^0	W_2^0	W_3^{-1}	W_2^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U_{5/2}^{5/2} V_{1/2}^{1/2}$	1											
$U_{5/2}^{5/2} V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{5}{6}}$									
$U_{5/2}^{3/2} V_{1/2}^{1/2}$		$\sqrt{\frac{5}{6}}$	$-\sqrt{\frac{1}{6}}$									
$U_{5/2}^{3/2} V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$							
$U_{5/2}^{1/2} V_{1/2}^{1/2}$				$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$							
$U_{5/2}^{1/2} V_{1/2}^{-1/2}$						$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$					
$U_{5/2}^{-1/2} V_{1/2}^{1/2}$						$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$					
$U_{5/2}^{-1/2} V_{1/2}^{-1/2}$								$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$			
$U_{5/2}^{-3/2} V_{1/2}^{1/2}$								$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$			
$U_{5/2}^{-3/2} V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{1}{6}}$	
$U_{5/2}^{-5/2} V_{1/2}^{1/2}$										$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{6}}$	
$U_{5/2}^{-5/2} V_{1/2}^{-1/2}$												1

$\mathfrak{D}_{5/2} \times \mathfrak{D}_{1/2}$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_1^1	W_3^0	W_2^0	W_1^0	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U_2^2 V_1^1$	1														
$U_2^2 V_1^0$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$												
$U_2^1 V_1^1$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$												
$U_2^2 V_1^{-1}$				$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$									
$U_2^1 V_1^0$				$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$									
$U_2^0 V_1^1$				$\sqrt{\frac{6}{15}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$									
$U_2^1 V_1^{-1}$							$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
$U_2^0 V_1^0$							$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$						
$U_2^{-1} V_1^1$							$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
$U_2^0 V_1^{-1}$										$\sqrt{\frac{6}{15}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$			
$U_2^{-1} V_1^0$										$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$			
$U_2^{-2} V_1^1$										$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$			
$U_2^{-1} V_1^{-1}$													$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
$U_2^{-2} V_1^0$													$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
$U_2^{-2} V_1^{-1}$															1

$\mathfrak{D}_2 \times \mathfrak{D}_1$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_1^1	W_3^0	W_2^0	W_1^0	W_0^0	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U_{3/2}^{3/2} V_{3/2}^{3/2}$	1															
$U_{3/2}^{3/2} V_{3/2}^{1/2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$													
$U_{3/2}^{1/2} V_{3/2}^{3/2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$													
$U_{3/2}^{3/2} V_{3/2}^{-1/2}$				$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$										
$U_{3/2}^{1/2} V_{3/2}^{1/2}$				$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$										
$U_{3/2}^{-1/2} V_{3/2}^{3/2}$				$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$										
$U_{3/2}^{3/2} V_{3/2}^{-3/2}$							$\sqrt{\frac{1}{20}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{4}}$						
$U_{3/2}^{1/2} V_{3/2}^{-1/2}$							$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{1}{4}}$						
$U_{3/2}^{-1/2} V_{3/2}^{1/2}$							$\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{20}}$	$\sqrt{\frac{1}{4}}$						
$U_{3/2}^{-3/2} V_{3/2}^{3/2}$							$\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{1}{4}}$						
$U_{3/2}^{1/2} V_{3/2}^{-3/2}$											$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$			
$U_{3/2}^{-1/2} V_{3/2}^{-1/2}$											$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$			
$U_{3/2}^{-3/2} V_{3/2}^{1/2}$											$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$			
$U_{3/2}^{-1/2} V_{3/2}^{-3/2}$														$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U_{3/2}^{-3/2} V_{3/2}^{-1/2}$														$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U_{3/2}^{-3/2} V_{3/2}^{-3/2}$																1

$$\mathfrak{D}_{3/2} \times \mathfrak{D}_{3/2}$$

	$W_{7/2}^{7/2}$	$W_{7/2}^{5/2}$	$W_{5/2}^{5/2}$	$W_{7/2}^{3/2}$	$W_{5/2}^{3/2}$	$W_{7/2}^{1/2}$	$W_{5/2}^{1/2}$	$W_{7/2}^{-1/2}$	$W_{5/2}^{-1/2}$	$W_{7/2}^{-3/2}$	$W_{5/2}^{-3/2}$	$W_{7/2}^{-5/2}$	$W_{5/2}^{-5/2}$	$W_{7/2}^{-7/2}$
$U_3^3 \; V_{1/2}^{1/2}$	1													
$U_3^3 \; V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{6}{7}}$											
$U_3^2 \; V_{1/2}^{1/2}$		$\sqrt{\frac{6}{7}}$	$-\sqrt{\frac{1}{7}}$											
$U_3^2 \; V_{1/2}^{-1/2}$				$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$									
$U_3^1 \; V_{1/2}^{1/2}$				$\sqrt{\frac{5}{7}}$	$-\sqrt{\frac{2}{7}}$									
$U_3^1 \; V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$							
$U_3^0 \; V_{1/2}^{1/2}$						$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$							
$U_3^0 \; V_{1/2}^{-1/2}$								$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$					
$U_3^{-1} \; V_{1/2}^{1/2}$								$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$					
$U_3^{-1} \; V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$			
$U_3^{-2} \; V_{1/2}^{1/2}$										$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$			
$U_3^{-2} \; V_{1/2}^{-1/2}$												$\sqrt{\frac{6}{7}}$	$\sqrt{\frac{1}{7}}$	
$U_3^{-2} \; V_{1/2}^{1/2}$												$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{6}{7}}$	
$U_3^{-3} \; V_{1/2}^{-1/2}$														1

$\mathfrak{D}_3 \times \mathfrak{D}_{1/2}$

	$W^{7/2}_{7/2}$	$W^{5/2}_{7/2}$	$W^{5/2}_{5/2}$	$W^{3/2}_{7/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{7/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{-1/2}_{7/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-3/2}_{7/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{7/2}$	$W^{-5/2}_{5/2}$	$W^{-7/2}_{7/2}$
$U^{5/2}_{5/2}\,V^{1}_{1}$	1																	
$U^{5/2}_{5/2}\,V^{0}_{1}$		$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$															
$U^{3/2}_{5/2}\,V^{1}_{1}$		$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$															
$U^{5/2}_{5/2}\,V^{-1}_{1}$				$\sqrt{\frac{1}{21}}$	$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{3}}$												
$U^{3/2}_{5/2}\,V^{0}_{1}$				$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{9}{35}}$	$-\sqrt{\frac{4}{15}}$												
$U^{1/2}_{5/2}\,V^{1}_{1}$				$\sqrt{\frac{10}{21}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{1}{15}}$												
$U^{3/2}_{5/2}\,V^{-1}_{1}$							$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$									
$U^{1/2}_{5/2}\,V^{0}_{1}$							$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$									
$U^{-1/2}_{5/2}\,V^{1}_{1}$							$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$									
$U^{1/2}_{5/2}\,V^{-1}_{1}$										$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$						
$U^{-1/2}_{5/2}\,V^{0}_{1}$										$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$						
$U^{-3/2}_{5/2}\,V^{1}_{1}$										$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$						
$U^{-1/2}_{5/2}\,V^{-1}_{1}$													$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{1}{15}}$			
$U^{-3/2}_{5/2}\,V^{0}_{1}$													$\sqrt{\frac{10}{21}}$	$-\sqrt{\frac{9}{35}}$	$-\sqrt{\frac{4}{15}}$			
$U^{-5/2}_{5/2}\,V^{1}_{1}$													$\sqrt{\frac{1}{21}}$	$-\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{3}}$			
$U^{-3/2}_{5/2}\,V^{-1}_{1}$																$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$	
$U^{-5/2}_{5/2}\,V^{0}_{1}$																$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$	
$U^{-5/2}_{5/2}\,V^{-1}_{1}$																		1

$\mathfrak{D}_{5/2} \times \mathfrak{D}_{1}$

	$W^{7/2}_{7/2}$	$W^{5/2}_{7/2}$	$W^{5/2}_{5/2}$	$W^{3/2}_{7/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{7/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{1/2}_{1/2}$	$W^{-1/2}_{7/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-1/2}_{1/2}$	$W^{-3/2}_{7/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{7/2}$	$W^{-5/2}_{5/2}$	$W^{-7/2}_{7/2}$
$U_2^2 V_{3/2}^{3/2}$	1																			
$U_2^2 V_{3/2}^{1/2}$		$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$																	
$U_2^1 V_{3/2}^{3/2}$		$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$																	
$U_2^2 V_{3/2}^{-1/2}$				$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$														
$U_2^1 V_{3/2}^{1/2}$				$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$														
$U_2^0 V_{3/2}^{3/2}$				$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$														
$U_2^2 V_{3/2}^{-3/2}$							$\sqrt{\frac{1}{35}}$	$\sqrt{\frac{6}{35}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$										
$U_2^1 V_{3/2}^{-1/2}$							$\sqrt{\frac{12}{35}}$	$\sqrt{\frac{5}{14}}$	0	$-\sqrt{\frac{3}{10}}$										
$U_2^0 V_{3/2}^{1/2}$							$\sqrt{\frac{18}{35}}$	$-\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{5}}$										
$U_2^{-1} V_{3/2}^{3/2}$							$\sqrt{\frac{4}{35}}$	$-\sqrt{\frac{27}{70}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{10}}$										
$U_2^1 V_{3/2}^{-3/2}$											$\sqrt{\frac{4}{35}}$	$\sqrt{\frac{27}{70}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{10}}$						
$U_2^0 V_{3/2}^{-1/2}$											$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{5}}$						
$U_2^{-1} V_{3/2}^{1/2}$											$\sqrt{\frac{12}{35}}$	$-\sqrt{\frac{5}{14}}$	0	$\sqrt{\frac{3}{10}}$						
$U_2^{-2} V_{3/2}^{3/2}$											$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{6}{35}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{5}}$						
$U_2^0 V_{3/2}^{-3/2}$															$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$			
$U_2^{-1} V_{3/2}^{-1/2}$															$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$			
$U_2^{-2} V_{3/2}^{1/2}$															$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$			
$U_2^{-1} V_{3/2}^{-3/2}$																		$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$	
$U_2^{-2} V_{3/2}^{-1/2}$																		$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$	
$U_2^{-2} V_{3/2}^{-3/2}$																				1

$\mathfrak{D}_2 \times \mathfrak{D}_{3/2}$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_4^1	W_3^1	W_4^0	W_3^0	W_4^{-1}	W_3^{-1}	W_4^{-2}	W_3^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_{7/2}^{7/2} V_{1/2}^{1/2}$	1															
$U_{7/2}^{7/2} V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{7}{8}}$													
$U_{7/2}^{5/2} V_{1/2}^{1/2}$		$\sqrt{\frac{7}{8}}$	$-\sqrt{\frac{1}{8}}$													
$U_{7/2}^{5/2} V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$											
$U_{7/2}^{3/2} V_{1/2}^{1/2}$				$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$											
$U_{7/2}^{3/2} V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$									
$U_{7/2}^{1/2} V_{1/2}^{1/2}$						$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$									
$U_{7/2}^{1/2} V_{1/2}^{-1/2}$								$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$							
$U_{7/2}^{-1/2} V_{1/2}^{1/2}$								$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$							
$U_{7/2}^{-1/2} V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$					
$U_{7/2}^{-3/2} V_{1/2}^{1/2}$										$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$					
$U_{7/2}^{-3/2} V_{1/2}^{-1/2}$												$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$			
$U_{7/2}^{-5/2} V_{1/2}^{1/2}$												$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$			
$U_{7/2}^{-5/2} V_{1/2}^{-1/2}$														$\sqrt{\frac{7}{8}}$	$\sqrt{\frac{1}{8}}$	
$U_{7/2}^{-7/2} V_{1/2}^{1/2}$														$\sqrt{\frac{1}{8}}$	$-\sqrt{\frac{7}{8}}$	
$U_{7/2}^{-7/2} V_{1/2}^{-1/2}$																1

$\mathfrak{D}_{7/2} \times \mathfrak{D}_{1/2}$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_2^2	W_4^1	W_3^1	W_2^1	W_4^0	W_3^0	W_2^0	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_3^3\ V_1^1$	1																				
$U_3^3\ V_1^0$		$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$																		
$U_3^2\ V_1^1$		$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$																		
$U_3^3\ V_1^{-1}$				$\sqrt{\frac{1}{28}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{5}{7}}$															
$U_3^2\ V_1^0$				$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{21}}$															
$U_3^1\ V_1^1$				$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{1}{21}}$															
$U_3^2\ V_1^{-1}$							$\sqrt{\frac{3}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$												
$U_3^1\ V_1^0$							$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$												
$U_3^0\ V_1^1$							$\sqrt{\frac{10}{28}}$	$-\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$												
$U_3^1\ V_1^{-1}$										$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$									
$U_3^0\ V_1^0$										$\sqrt{\frac{8}{14}}$	0	$-\sqrt{\frac{3}{7}}$									
$U_3^{-1}\ V_1^1$										$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$									
$U_3^0\ V_1^{-1}$													$\sqrt{\frac{10}{28}}$	$\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$						
$U_3^{-1}\ V_1^0$													$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$						
$U_3^{-2}\ V_1^1$													$\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$						
$U_3^{-1}\ V_1^{-1}$																$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{1}{21}}$			
$U_3^{-2}\ V_1^0$																$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{21}}$			
$U_3^{-3}\ V_1^1$																$\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{5}{7}}$			
$U_3^{-2}\ V_1^{-1}$																			$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$	
$U_3^{-3}\ V_1^0$																			$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$	
$U_3^{-3}\ V_1^{-1}$																					1

$\mathfrak{D}_3 \times \mathfrak{D}_1$

	W_4^4
$U_{5/2}^{5/2} V_{3/2}^{3/2}$	1

	W_4^3	W_3^3
$U_{5/2}^{5/2} V_{3/2}^{1/2}$	$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$
$U_{5/2}^{3/2} V_{3/2}^{3/2}$	$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$

	W_4^2	W_3^2	W_2^2
$U_{5/2}^{5/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{3}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{15}{21}}$
$U_{5/2}^{3/2} V_{3/2}^{1/2}$	$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$
$U_{5/2}^{1/2} V_{3/2}^{3/2}$	$\sqrt{\frac{10}{28}}$	$-\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$

	W_4^1	W_3^1	W_2^1	W_1^1
$U_{5/2}^{5/2} V_{3/2}^{-3/2}$	$\sqrt{\frac{1}{56}}$	$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{2}}$
$U_{5/2}^{3/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{15}{56}}$	$\sqrt{\frac{49}{120}}$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{3}{10}}$
$U_{5/2}^{1/2} V_{3/2}^{1/2}$	$\sqrt{\frac{30}{56}}$	$-\sqrt{\frac{1}{60}}$	$-\sqrt{\frac{25}{84}}$	$\sqrt{\frac{1}{20}}$
$U_{5/2}^{-1/2} V_{3/2}^{3/2}$	$\sqrt{\frac{10}{56}}$	$-\sqrt{\frac{9}{20}}$	$\sqrt{\frac{27}{84}}$	$-\sqrt{\frac{1}{20}}$

	W_4^0	W_3^0	W_2^0	W_1^0
$U_{5/2}^{3/2} V_{3/2}^{-3/2}$	$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$
$U_{5/2}^{1/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$
$U_{5/2}^{-1/2} V_{3/2}^{1/2}$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$
$U_{5/2}^{-3/2} V_{3/2}^{3/2}$	$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$

	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_1^{-1}
$U_{5/2}^{1/2} V_{3/2}^{-3/2}$	$\sqrt{\frac{10}{56}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{9}{28}}$	$\sqrt{\frac{1}{20}}$
$U_{5/2}^{-1/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{30}{56}}$	$\sqrt{\frac{1}{60}}$	$-\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{3}{20}}$
$U_{5/2}^{-3/2} V_{3/2}^{1/2}$	$\sqrt{\frac{15}{56}}$	$-\sqrt{\frac{49}{120}}$	$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{3}{10}}$
$U_{5/2}^{-5/2} V_{3/2}^{3/2}$	$\sqrt{\frac{1}{56}}$	$-\sqrt{\frac{1}{8}}$	$\sqrt{\frac{5}{14}}$	$-\sqrt{\frac{1}{2}}$

	W_4^{-2}	W_3^{-2}	W_2^{-2}
$U_{5/2}^{-1/2} V_{3/2}^{-3/2}$	$\sqrt{\frac{10}{28}}$	$\sqrt{\frac{6}{12}}$	$\sqrt{\frac{1}{7}}$
$U_{5/2}^{-3/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$
$U_{5/2}^{-5/2} V_{3/2}^{1/2}$	$\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$

	W_4^{-3}	W_4^{-3}
$U_{5/2}^{-3/2} V_{3/2}^{-3/2}$	$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$
$U_{5/2}^{-5/2} V_{3/2}^{-1/2}$	$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$

	W_4^{-4}
$U_{5/2}^{-5/2} V_{3/2}^{-3/2}$	1

$\mathfrak{D}_{5/2} \times \mathfrak{D}_{3/2}$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_2^2	W_4^1	W_3^1	W_2^1	W_1^1	W_4^0	W_3^0	W_2^0	W_1^0	W_0^0	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_2^2 V_2^2$	1																								
$U_2^2 V_2^1$		$\sqrt{1/2}$	$\sqrt{1/2}$																						
$U_2^1 V_2^2$		$\sqrt{1/2}$	$-\sqrt{1/2}$																						
$U_2^2 V_2^0$				$\sqrt{3/14}$	$\sqrt{1/2}$	$\sqrt{2/7}$																			
$U_2^1 V_2^1$				$\sqrt{8/14}$	0	$-\sqrt{3/7}$																			
$U_2^0 V_2^2$				$\sqrt{3/14}$	$-\sqrt{1/2}$	$\sqrt{2/7}$																			
$U_2^2 V_2^{-1}$							$\sqrt{1/14}$	$\sqrt{3/10}$	$\sqrt{6/14}$	$\sqrt{2/10}$															
$U_2^1 V_2^0$							$\sqrt{6/14}$	$\sqrt{2/10}$	$-\sqrt{1/14}$	$-\sqrt{3/10}$															
$U_2^0 V_2^1$							$\sqrt{6/14}$	$-\sqrt{2/10}$	$-\sqrt{1/14}$	$\sqrt{3/10}$															
$U_2^{-1} V_2^2$							$\sqrt{1/14}$	$-\sqrt{3/10}$	$\sqrt{6/14}$	$-\sqrt{2/10}$															
$U_2^2 V_2^{-2}$											$\sqrt{1/70}$	$\sqrt{1/10}$	$\sqrt{4/14}$	$\sqrt{4/10}$	$\sqrt{1/5}$										
$U_2^1 V_2^{-1}$											$\sqrt{16/70}$	$\sqrt{4/10}$	$\sqrt{1/14}$	$-\sqrt{1/10}$	$-\sqrt{1/5}$										
$U_2^0 V_2^0$											$\sqrt{36/70}$	0	$-\sqrt{4/14}$	0	$\sqrt{1/5}$										
$U_2^{-1} V_2^1$											$\sqrt{16/70}$	$-\sqrt{4/10}$	$\sqrt{1/14}$	$\sqrt{1/10}$	$-\sqrt{1/5}$										
$U_2^{-2} V_2^2$											$\sqrt{1/70}$	$-\sqrt{1/10}$	$\sqrt{4/14}$	$-\sqrt{4/10}$	$\sqrt{1/5}$										
$U_2^1 V_2^{-2}$																$\sqrt{1/14}$	$\sqrt{3/10}$	$\sqrt{6/14}$	$\sqrt{2/10}$						
$U_2^0 V_2^{-1}$																$\sqrt{6/14}$	$\sqrt{2/10}$	$-\sqrt{1/14}$	$-\sqrt{3/10}$						
$U_2^{-1} V_2^0$																$\sqrt{6/14}$	$-\sqrt{2/10}$	$-\sqrt{1/14}$	$\sqrt{3/10}$						
$U_2^{-2} V_2^1$																$\sqrt{1/14}$	$-\sqrt{3/10}$	$\sqrt{6/14}$	$-\sqrt{2/10}$						
$U_2^0 V_2^{-2}$																				$\sqrt{3/14}$	$\sqrt{1/2}$	$\sqrt{2/7}$			
$U_2^{-1} V_2^{-1}$																				$\sqrt{8/14}$	0	$-\sqrt{3/7}$			
$U_2^{-2} V_2^0$																				$\sqrt{3/14}$	$-\sqrt{1/2}$	$\sqrt{2/7}$			
$U_2^{-1} V_2^{-2}$																							$\sqrt{1/2}$	$\sqrt{1/2}$	
$U_2^{-2} V_2^{-1}$																							$\sqrt{1/2}$	$-\sqrt{1/2}$	
$U_2^{-2} V_2^{-2}$																									1

$\mathfrak{D}_2 \times \mathfrak{D}_2$

	W_5^5	W_5^4	W_4^4	W_5^3	W_4^3	W_3^3	W_5^2	W_4^2	W_3^2	W_5^1	W_4^1	W_3^1	W_5^0	W_4^0	W_3^0
$U_4^4 V_1^1$	1														
$U_4^4 V_1^0$		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$												
$U_4^3 V_1^1$		$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$												
$U_4^4 V_1^{-1}$				$\sqrt{\frac{1}{45}}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{7}{9}}$									
$U_4^3 V_1^0$				$\sqrt{\frac{16}{45}}$	$\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{7}{36}}$									
$U_4^2 V_1^1$				$\sqrt{\frac{28}{45}}$	$-\sqrt{\frac{7}{20}}$	$\sqrt{\frac{1}{36}}$									
$U_4^3 V_1^{-1}$							$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{7}{20}}$	$\sqrt{\frac{7}{12}}$						
$U_4^2 V_1^0$							$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{3}}$						
$U_4^1 V_1^1$							$\sqrt{\frac{7}{15}}$	$-\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{12}}$						
$U_4^2 V^{-1}$										$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{5}{12}}$			
$U_4^1 V_1^0$										$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{5}{12}}$			
$U_4^0 V_1^1$										$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{6}}$			
$U_4^1 V_1^{-1}$													$\sqrt{\frac{2}{9}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{18}}$
$U_4^0 V_1^0$													$\sqrt{\frac{5}{9}}$	0	$-\sqrt{\frac{4}{9}}$
$U_4^{-1} V_1^1$													$\sqrt{\frac{2}{9}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{18}}$

$\mathcal{D}_4 \times \mathcal{D}_1$

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	W_5^0	W_4^0	W_3^0	W_5^{-1}	W_4^{-1}	W_3^{-1}	W_5^{-2}	W_4^{-2}	W_3^{-2}	W_5^{-3}	W_4^{-3}	W_3^{-3}	W_5^{-4}	W_4^{-4}	W_5^{-5}
$U_4^{1}V_1^{-1}$	$\sqrt{\frac{2}{9}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{18}}$		REPEATED ←										
$U_4^{0}V_1^{0}$	$\sqrt{\frac{5}{9}}$	0	$-\sqrt{\frac{4}{9}}$												
$U_4^{-1}V_1^{1}$	$\sqrt{\frac{2}{9}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{18}}$												
$U_4^{0}V_1^{-1}$				$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{6}}$									
$U_4^{-1}V_1^{0}$				$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{1}{20}}$	$\sqrt{\frac{5}{12}}$									
$U_4^{-2}V_1^{1}$				$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{5}{12}}$									
$U_4^{-1}V_1^{-1}$							$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{12}}$						
$U_4^{-2}V_1^{0}$							$\sqrt{\frac{7}{15}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{3}}$						
$U_4^{-3}V_1^{1}$							$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{7}{20}}$	$\sqrt{\frac{7}{12}}$						
$U_4^{-2}V_1^{-1}$										$\sqrt{\frac{28}{45}}$	$\sqrt{\frac{7}{20}}$	$\sqrt{\frac{1}{36}}$			
$U_4^{-3}V_1^{0}$										$\sqrt{\frac{16}{45}}$	$-\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{7}{36}}$			
$U_4^{-4}V_1^{1}$										$\sqrt{\frac{1}{45}}$	$-\sqrt{\frac{1}{5}}$	$\sqrt{\frac{7}{9}}$			
$U_4^{-3}V_1^{-1}$													$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$	
$U_4^{-4}V_1^{0}$													$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$	
$U_4^{-4}V_1^{-1}$															1

$\mathcal{D}_4 \times \mathcal{D}_1$

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	W_6^6	W_6^5	W_5^5	W_6^4	W_5^4	W_4^4	W_6^3	W_5^3	W_4^3	W_6^2	W_5^2	W_4^2	W_6^1	W_5^1	W_4^1	W_6^0	W_5^0	W_4^0
$U_5^5 V_1^1$	1																	
$U_5^5 V_1^0$		$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{5}{6}}$															
$U_5^4 V_1^1$		$\sqrt{\frac{5}{6}}$	$-\sqrt{\frac{1}{6}}$															
$U_5^5 V_1^{-1}$				$\sqrt{\frac{1}{66}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{9}{11}}$												
$U_5^4 V_1^0$				$\sqrt{\frac{10}{33}}$	$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{9}{55}}$												
$U_5^3 V_1^1$				$\sqrt{\frac{15}{22}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{1}{55}}$												
$U_5^4 V_1^{-1}$							$\sqrt{\frac{1}{22}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{36}{55}}$									
$U_5^3 V_1^0$							$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{16}{55}}$									
$U_5^2 V_1^1$							$\sqrt{\frac{6}{11}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{55}}$									
$U_5^3 V_1^{-1}$										$\sqrt{\frac{1}{11}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{28}{55}}$						
$U_5^2 V_1^0$										$\sqrt{\frac{16}{33}}$	$\sqrt{\frac{2}{15}}$	$-\sqrt{\frac{21}{55}}$						
$U_5^1 V_1^1$										$\sqrt{\frac{14}{33}}$	$-\sqrt{\frac{7}{15}}$	$\sqrt{\frac{6}{55}}$						
$U_5^2 V_1^{-1}$													$\sqrt{\frac{5}{33}}$	$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{21}{55}}$			
$U_5^1 V_1^0$													$\sqrt{\frac{35}{66}}$	$\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{24}{55}}$			
$U_5^0 V_1^1$													$\sqrt{\frac{7}{22}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{11}}$			
$U_5^1 V_1^{-1}$																$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$
$U_5^0 V_1^0$																$\sqrt{\frac{6}{11}}$	0	$-\sqrt{\frac{5}{11}}$
$U_5^{-1} V_1^1$																$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$

$\mathfrak{D}_5 \times \mathfrak{D}_1$

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	W_6^0	W_5^0	W_4^0	W_6^{-1}	W_5^{-1}	W_4^{-1}	W_6^{-2}	W_5^{-2}	W_4^{-2}	W_6^{-3}	W_5^{-3}	W_4^{-3}	W_6^{-4}	W_5^{-4}	W_4^{-4}	W_6^{-5}	W_5^{-5}	W_6^{-6}
$U_5^1 V_1^{-1}$	$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$	REPEATED ↙														
$U_5^0 V_1^0$	$\sqrt{\frac{6}{11}}$	0	$-\sqrt{\frac{5}{11}}$															
$U_5^{-1} V_1^1$	$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$															
$U_5^0 V_1^{-1}$				$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{11}}$												
$U_5^{-1} V_1^0$				$\sqrt{\frac{35}{66}}$	$-\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{24}{55}}$												
$U_5^{-2} V_1^1$				$\sqrt{\frac{5}{33}}$	$-\sqrt{\frac{7}{15}}$	$\sqrt{\frac{21}{55}}$												
$U_5^{-1} V_1^{-1}$							$\sqrt{\frac{14}{33}}$	$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{6}{55}}$									
$U_5^{-2} V_1^0$							$\sqrt{\frac{16}{33}}$	$-\sqrt{\frac{2}{15}}$	$-\sqrt{\frac{21}{55}}$									
$U_5^{-3} V_1^1$							$\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{28}{55}}$									
$U_5^{-2} V_1^{-1}$										$\sqrt{\frac{6}{11}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{55}}$						
$U_5^{-3} V_1^0$										$\sqrt{\frac{9}{22}}$	$-\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{16}{55}}$						
$U_5^{-4} V_1^1$										$\sqrt{\frac{1}{22}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{36}{55}}$						
$U_5^{-3} V_1^{-1}$													$\sqrt{\frac{15}{22}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{1}{55}}$			
$U_5^{-4} V_1^0$													$\sqrt{\frac{10}{33}}$	$-\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{9}{55}}$			
$U_5^{-5} V_1^1$													$\sqrt{\frac{1}{66}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{9}{11}}$			
$U_5^{-4} V_1^{-1}$																$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{1}{6}}$	
$U_5^{-5} V_1^0$																$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{6}}$	
$U_5^{-5} V_1^{-1}$																		1

$\mathfrak{D}_5 \times \mathfrak{D}_1$

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	W_5^5	W_5^4	W_4^4	W_5^3	W_4^3	W_3^3	W_5^2	W_4^2	W_3^2	W_2^2	W_5^1	W_4^1	W_3^1	W_2^1	W_1^1	W_5^0	W_4^0	W_3^0	W_2^0	W_1^0
$U_2^2 V_3^3$	1																			
$U_2^2 V_3^2$		$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$																	
$U_2^1 V_3^3$		$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$																	
$U_2^2 V_3^1$				$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{6}}$														
$U_2^1 V_3^2$				$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{5}{12}}$														
$U_2^0 V_3^3$				$\sqrt{\frac{2}{15}}$	$-\sqrt{\frac{9}{20}}$	$\sqrt{\frac{5}{12}}$														
$U_2^2 V_3^0$							$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{14}}$										
$U_2^1 V_3^1$							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{14}}$										
$U_2^0 V_3^2$							$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{12}{35}}$	0	$\sqrt{\frac{5}{14}}$										
$U_2^{-1} V_3^3$							$\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{27}{140}}$	$\sqrt{\frac{5}{12}}$	$-\sqrt{\frac{5}{14}}$										
$U_2^2 V_3^{-1}$											$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{35}}$					
$U_2^1 V_3^0$											$\sqrt{\frac{8}{21}}$	$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{3}{35}}$					
$U_2^0 V_3^1$											$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{3}{20}}$	$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{6}{35}}$					
$U_2^{-1} V_3^2$											$\sqrt{\frac{4}{35}}$	$-\sqrt{\frac{7}{20}}$	$\sqrt{\frac{1}{4}}$	0	$-\sqrt{\frac{2}{7}}$					
$U_2^{-2} V_3^3$											$\sqrt{\frac{1}{210}}$	$-\sqrt{\frac{3}{70}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{3}{7}}$					
$U_2^2 V_3^{-2}$																$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{7}}$
$U_2^1 V_3^{-1}$																$\sqrt{\frac{5}{21}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{8}{35}}$
$U_2^0 V_3^0$																$\sqrt{\frac{10}{21}}$	0	$-\sqrt{\frac{4}{15}}$	0	$\sqrt{\frac{9}{35}}$
$U_2^{-1} V_3^1$																$\sqrt{\frac{5}{21}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{30}}$	$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{8}{35}}$
$U_2^{-2} V_3^2$																$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{1}{7}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{7}}$

$\mathfrak{D}_2 \times \mathfrak{D}_3$

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	W_5^0	W_4^0	W_3^0	W_2^0	W_1^0	W_5^{-1}	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_5^{-2}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_5^{-3}	W_4^{-3}	W_3^{-3}	W_5^{-4}	W_4^{-4}	W_5^{-5}
$U_2^2 V_3^{-2}$	$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{7}}$															
$U_2^1 V_3^{-1}$	$\sqrt{\frac{5}{21}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{8}{35}}$															
$U_2^0 V_3^0$	$\sqrt{\frac{10}{21}}$	0	$-\sqrt{\frac{4}{15}}$	0	$\sqrt{\frac{9}{35}}$			REPEATED												
$U_2^{-1} V_3^1$	$\sqrt{\frac{5}{21}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{30}}$	$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{5}{35}}$															
$U_2^{-2} V_3^2$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{1}{7}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{7}}$															
$U_2^2 V_3^{-3}$						$\sqrt{\frac{1}{210}}$	$\sqrt{\frac{3}{70}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{3}{7}}$										
$U_2^1 V_3^{-2}$						$\sqrt{\frac{4}{35}}$	$\sqrt{\frac{7}{20}}$	$\sqrt{\frac{1}{4}}$	0	$-\sqrt{\frac{2}{7}}$										
$U_2^0 V_3^{-1}$						$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{3}{20}}$	$-\sqrt{\frac{1}{7}}$	$\sqrt{\frac{6}{35}}$										
$U_2^{-1} V_3^0$						$\sqrt{\frac{8}{21}}$	$-\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{30}}$	$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{3}{35}}$										
$U_2^{-2} V_3^1$						$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{35}}$										
$U_2^1 V_3^{-3}$											$\sqrt{\frac{1}{30}}$	$\sqrt{\frac{27}{140}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{5}{14}}$						
$U_2^0 V_3^{-2}$											$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{12}{35}}$	0	$-\sqrt{\frac{5}{14}}$						
$U_2^{-1} V_3^{-1}$											$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{14}}$						
$U_2^{-2} V_3^0$											$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{14}}$						
$U_2^0 V_3^{-3}$															$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{5}{12}}$			
$U_2^{-1} V_3^{-2}$															$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{5}{12}}$			
$U_2^{-2} V_3^{-1}$															$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{6}}$			
$U_2^{-1} V_3^{-3}$																		$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$	
$U_2^{-2} V_3^{-2}$																		$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$	
$U_2^{-2} V_3^{-3}$																				1

$\mathcal{D}_2 \times \mathcal{D}_3$

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	W^6_6	W^5_6	W^5_5	W^4_6	W^4_5	W^4_4	W^3_6	W^3_5	W^3_4	W^3_3	W^2_6	W^2_5	W^2_4	W^2_3	W^2_2	W^1_6	W^1_5	W^1_4	W^1_3	W^1_2
$U^4_4 V^2_2$	1																			
$U^4_4 V^1_2$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$																	
$U^3_4 V^2_2$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$																	
$U^4_4 V^0_2$				$\sqrt{\frac{1}{11}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{28}{55}}$														
$U^3_4 V^1_2$				$\sqrt{\frac{16}{33}}$	$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{21}{55}}$														
$U^2_4 V^2_2$				$\sqrt{\frac{14}{33}}$	$-\sqrt{\frac{7}{15}}$	$\sqrt{\frac{6}{55}}$														
$U^4_4 V^{-1}_2$							$\sqrt{\frac{1}{55}}$	$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{21}{55}}$	$\sqrt{\frac{7}{15}}$										
$U^3_4 V^0_2$							$\sqrt{\frac{12}{55}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{7}{220}}$	$-\sqrt{\frac{7}{20}}$										
$U^2_4 V^1_2$							$\sqrt{\frac{28}{55}}$	0	$-\sqrt{\frac{15}{44}}$	$\sqrt{\frac{3}{20}}$										
$U^1_4 V^2_2$							$\sqrt{\frac{14}{55}}$	$-\sqrt{\frac{7}{15}}$	$\sqrt{\frac{27}{110}}$	$-\sqrt{\frac{1}{30}}$										
$U^4_4 V^{-2}_2$											$\sqrt{\frac{1}{495}}$	$\sqrt{\frac{1}{45}}$	$\sqrt{\frac{6}{55}}$	$\sqrt{\frac{14}{45}}$	$\sqrt{\frac{5}{9}}$					
$U^3_4 V^{-1}_2$											$\sqrt{\frac{32}{495}}$	$\sqrt{\frac{5}{18}}$	$\sqrt{\frac{15}{44}}$	$\sqrt{\frac{7}{180}}$	$-\sqrt{\frac{5}{18}}$					
$U^2_4 V^0_2$											$\sqrt{\frac{56}{165}}$	$\sqrt{\frac{7}{30}}$	$-\sqrt{\frac{16}{385}}$	$-\sqrt{\frac{4}{15}}$	$\sqrt{\frac{5}{42}}$					
$U^1_4 V^1_2$											$\sqrt{\frac{224}{495}}$	$-\sqrt{\frac{7}{90}}$	$-\sqrt{\frac{243}{1540}}$	$\sqrt{\frac{49}{180}}$	$-\sqrt{\frac{5}{126}}$					
$U^0_4 V^2_2$											$\sqrt{\frac{14}{99}}$	$-\sqrt{\frac{7}{18}}$	$\sqrt{\frac{27}{77}}$	$-\sqrt{\frac{1}{9}}$	$\sqrt{\frac{1}{126}}$					
$U^3_4 V^{-2}_2$																$\sqrt{\frac{1}{99}}$	$\sqrt{\frac{7}{90}}$	$\sqrt{\frac{27}{110}}$	$\sqrt{\frac{7}{18}}$	$\sqrt{\frac{5}{18}}$
$U^2_4 V^{-1}_2$																$\sqrt{\frac{14}{77}}$	$\sqrt{\frac{16}{45}}$	$\sqrt{\frac{243}{1540}}$	$-\sqrt{\frac{1}{36}}$	$-\sqrt{\frac{20}{63}}$
$U^1_4 V^0_2$																$\sqrt{\frac{14}{33}}$	$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{221}{1540}}$	$-\sqrt{\frac{1}{12}}$	$\sqrt{\frac{5}{21}}$
$U^0_4 V^1_2$																$\sqrt{\frac{35}{99}}$	$-\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{3}{154}}$	$\sqrt{\frac{5}{18}}$	$-\sqrt{\frac{8}{63}}$
$U^{-1}_4 V^2_2$																$\sqrt{\frac{7}{99}}$	$-\sqrt{\frac{5}{18}}$	$\sqrt{\frac{30}{77}}$	$-\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{126}}$

$\mathfrak{D}_4 \times \mathfrak{D}_2$

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	W_6^0	W_5^0	W_4^0	W_3^0	W_2^0
$U_4^2 V_2^{-2}$	$\sqrt{\frac{1}{33}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$
$U_4^1 V_2^{-1}$	$\sqrt{\frac{8}{33}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$
$U_4^0 V_2^0$	$\sqrt{\frac{5}{11}}$	0	$-\sqrt{\frac{20}{77}}$	0	$\sqrt{\frac{2}{7}}$
$U_4^{-1} V_2^1$	$\sqrt{\frac{8}{33}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$
$U_4^{-2} V_2^2$	$\sqrt{\frac{1}{33}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$

$\mathfrak{D}_4 \times \mathfrak{D}_2$

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	W_6^1	W_5^1	W_4^1	W_3^1	W_2^1
$U_4^1 V_2^{\bar{2}}$	$\sqrt{\frac{7}{99}}$	$\sqrt{\frac{5}{18}}$	$\sqrt{\frac{30}{77}}$	$\sqrt{\frac{2}{9}}$	$\sqrt{\frac{5}{126}}$
$U_4^0 V_2^{\bar{1}}$	$\sqrt{\frac{35}{99}}$	$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{5}{18}}$	$-\sqrt{\frac{8}{63}}$
$U_4^{\bar{1}} V_2^0$	$\sqrt{\frac{14}{33}}$	$-\sqrt{\frac{1}{15}}$	$\sqrt{\frac{289}{1540}}$	$\sqrt{\frac{1}{12}}$	$\sqrt{\frac{5}{21}}$
$U_4^{\bar{2}} V_2^1$	$\sqrt{\frac{14}{99}}$	$-\sqrt{\frac{16}{45}}$	$\sqrt{\frac{243}{1540}}$	$\sqrt{\frac{1}{36}}$	$-\sqrt{\frac{20}{63}}$
$U_4^{\bar{3}} V_2^2$	$\sqrt{\frac{1}{99}}$	$-\sqrt{\frac{7}{90}}$	$\sqrt{\frac{27}{110}}$	$-\sqrt{\frac{7}{18}}$	$\sqrt{\frac{5}{18}}$

	W_6^2	W_5^2	W_4^2	W_3^2	W_2^2
$U_4^0 V_2^{\bar{2}}$	$\sqrt{\frac{14}{99}}$	$\sqrt{\frac{7}{18}}$	$\sqrt{\frac{27}{77}}$	$\sqrt{\frac{1}{9}}$	$\sqrt{\frac{1}{126}}$
$U_4^{\bar{1}} V_2^{\bar{1}}$	$\sqrt{\frac{224}{495}}$	$\sqrt{\frac{7}{90}}$	$\sqrt{\frac{243}{1540}}$	$-\sqrt{\frac{49}{180}}$	$-\sqrt{\frac{5}{126}}$
$U_4^{\bar{2}} V_2^0$	$\sqrt{\frac{56}{165}}$	$-\sqrt{\frac{7}{30}}$	$-\sqrt{\frac{16}{385}}$	$\sqrt{\frac{4}{15}}$	$\sqrt{\frac{5}{42}}$
$U_4^{\bar{3}} V_2^1$	$\sqrt{\frac{32}{495}}$	$-\sqrt{\frac{5}{18}}$	$\sqrt{\frac{15}{44}}$	$-\sqrt{\frac{7}{180}}$	$-\sqrt{\frac{5}{18}}$
$U_4^{\bar{4}} V_2^2$	$\sqrt{\frac{1}{495}}$	$-\sqrt{\frac{1}{45}}$	$\sqrt{\frac{6}{55}}$	$-\sqrt{\frac{14}{45}}$	$\sqrt{\frac{5}{9}}$

	W_6^3	W_5^3	W_4^3	W_3^3
$U_4^{\bar{1}} V_2^{\bar{2}}$	$\sqrt{\frac{14}{55}}$	$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{27}{110}}$	$\sqrt{\frac{1}{30}}$
$U_4^{\bar{2}} V_2^{\bar{1}}$	$\sqrt{\frac{28}{55}}$	0	$-\sqrt{\frac{15}{44}}$	$-\sqrt{\frac{3}{20}}$
$U_4^{\bar{3}} V_2^1$	$\sqrt{\frac{12}{55}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{7}{220}}$	$\sqrt{\frac{7}{20}}$
$U_4^{\bar{4}} V_2^2$	$\sqrt{\frac{1}{55}}$	$-\sqrt{\frac{2}{15}}$	$\sqrt{\frac{21}{55}}$	$-\sqrt{\frac{7}{15}}$

	W_6^4	W_5^4	W_4^4
$U_4^{\bar{2}} V_2^{\bar{2}}$	$\sqrt{\frac{14}{33}}$	$\sqrt{\frac{7}{15}}$	$\sqrt{\frac{6}{55}}$
$U_4^{\bar{3}} V_2^{\bar{1}}$	$\sqrt{\frac{16}{33}}$	$-\sqrt{\frac{2}{15}}$	$\sqrt{\frac{21}{55}}$
$U_4^{\bar{4}} V_2^0$	$\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{28}{55}}$

	W_6^5	W_5^5
$U_4^{\bar{3}} V_2^{\bar{2}}$	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$U_4^{\bar{4}} V_2^{\bar{1}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$

	W_6^6
$U_4^{\bar{4}} V_2^{\bar{2}}$	1

$\mathfrak{D}_4 \times \mathfrak{D}_2$

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	W_7^7	W_7^6	W_6^6	W_7^5	W_6^5	W_5^5	W_7^4	W_6^4	W_5^4	W_4^4	W_7^3	W_6^3	W_5^3	W_4^3	W_3^3	W_7^2	W_6^2	W_5^2	W_4^2	W_3^2
$U_5^5 V_2^2$	1																			
$U_5^5 V_2^1$		$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$																	
$U_5^4 V_2^2$		$\sqrt{\frac{5}{7}}$	$-\sqrt{\frac{2}{7}}$																	
$U_5^5 V_2^0$				$\sqrt{\frac{6}{91}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{15}{26}}$														
$U_5^4 V_2^1$				$\sqrt{\frac{40}{91}}$	$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{9}{26}}$														
$U_5^3 V_2^2$				$\sqrt{\frac{45}{91}}$	$-\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{13}}$														
$U_5^5 V_2^{-1}$							$\sqrt{\frac{1}{91}}$	$\sqrt{\frac{15}{154}}$	$\sqrt{\frac{9}{26}}$	$\sqrt{\frac{6}{11}}$										
$U_5^4 V_2^0$							$\sqrt{\frac{15}{91}}$	$\sqrt{\frac{32}{77}}$	$\sqrt{\frac{6}{65}}$	$-\sqrt{\frac{18}{55}}$										
$U_5^3 V_2^1$							$\sqrt{\frac{45}{91}}$	$\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{49}{130}}$	$\sqrt{\frac{6}{55}}$										
$U_5^2 V_2^2$							$\sqrt{\frac{30}{91}}$	$-\sqrt{\frac{36}{77}}$	$\sqrt{\frac{12}{65}}$	$\sqrt{\frac{1}{55}}$										
$U_5^5 V_2^{-2}$											$\sqrt{\frac{1}{1001}}$	$\sqrt{\frac{1}{77}}$	$\sqrt{\frac{1}{13}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{7}{11}}$					
$U_5^4 V_2^{-1}$											$\sqrt{\frac{40}{1001}}$	$\sqrt{\frac{164}{770}}$	$\sqrt{\frac{49}{130}}$	$\sqrt{\frac{6}{55}}$	$-\sqrt{\frac{14}{55}}$					
$U_5^3 V_2^0$											$\sqrt{\frac{270}{1001}}$	$\sqrt{\frac{243}{770}}$	$-\sqrt{\frac{1}{390}}$	$-\sqrt{\frac{18}{55}}$	$\sqrt{\frac{14}{165}}$					
$U_5^2 V_2^1$											$\sqrt{\frac{480}{1001}}$	$-\sqrt{\frac{6}{385}}$	$-\sqrt{\frac{10}{39}}$	$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{7}{330}}$					
$U_5^1 V_2^2$											$\sqrt{\frac{30}{143}}$	$-\sqrt{\frac{28}{55}}$	$\sqrt{\frac{56}{195}}$	$-\sqrt{\frac{7}{110}}$	$\sqrt{\frac{1}{330}}$					
$U_5^4 V_2^{-2}$																$\sqrt{\frac{5}{1001}}$	$\sqrt{\frac{18}{385}}$	$\sqrt{\frac{12}{65}}$	$\sqrt{\frac{21}{55}}$	$\sqrt{\frac{21}{55}}$
$U_5^3 V_2^{-1}$																$\sqrt{\frac{90}{1001}}$	$\sqrt{\frac{11}{35}}$	$\sqrt{\frac{10}{39}}$	0	$-\sqrt{\frac{56}{165}}$
$U_5^2 V_2^0$																$\sqrt{\frac{360}{1001}}$	$\sqrt{\frac{644}{385}}$	$-\sqrt{\frac{6}{65}}$	$-\sqrt{\frac{21}{110}}$	$\sqrt{\frac{21}{110}}$
$U_5^1 V_2^1$																$\sqrt{\frac{60}{143}}$	$-\sqrt{\frac{6}{55}}$	$-\sqrt{\frac{7}{65}}$	$\sqrt{\frac{16}{55}}$	$-\sqrt{\frac{4}{55}}$
$U_5^0 V_2^2$																$\sqrt{\frac{18}{143}}$	$-\sqrt{\frac{4}{11}}$	$\sqrt{\frac{14}{39}}$	$-\sqrt{\frac{3}{22}}$	$\sqrt{\frac{1}{66}}$

$\mathfrak{D}_5 \times \mathfrak{D}_2$

	W_7^1	W_6^1	W_5^1	W_4^1	W_3^1	W_7^0	W_6^0	W_5^0	W_4^0	W_3^0	W_7^{-1}	W_6^{-1}	W_5^{-1}	W_4^{-1}	W_3^{-1}
$U_5^3 V_2^{-2}$	$\sqrt{\frac{15}{1001}}$	$\sqrt{\frac{8}{77}}$	$\sqrt{\frac{56}{195}}$	$\sqrt{\frac{21}{55}}$	$\sqrt{\frac{7}{33}}$										
$U_5^2 V_2^{-1}$	$\sqrt{\frac{160}{1001}}$	$\sqrt{\frac{27}{27}}$	$\sqrt{\frac{7}{65}}$	$-\sqrt{\frac{7}{110}}$	$-\sqrt{\frac{7}{22}}$										
$U_5^1 V_2^0$	$\sqrt{\frac{60}{143}}$	$\sqrt{\frac{1}{22}}$	$-\sqrt{\frac{27}{130}}$	$-\sqrt{\frac{3}{55}}$	$\sqrt{\frac{3}{11}}$										
$U_5^0 V_2^1$	$\sqrt{\frac{48}{143}}$	$-\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{78}}$	$\sqrt{\frac{6}{22}}$	$-\sqrt{\frac{5}{33}}$										
$U_5^{-1} V_2^2$	$\sqrt{\frac{10}{143}}$	$-\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{13}}$	$-\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{22}}$										
$U_5^2 V_2^{-2}$						$\sqrt{\frac{5}{143}}$	$\sqrt{\frac{2}{11}}$	$\sqrt{\frac{14}{39}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{7}{66}}$					
$U_5^1 V_2^{-1}$						$\sqrt{\frac{35}{143}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{1}{78}}$	$-\sqrt{\frac{2}{11}}$	$-\sqrt{\frac{8}{33}}$					
$U_5^0 V_2^0$						$\sqrt{\frac{63}{143}}$	0	$-\sqrt{\frac{10}{39}}$	0	$\sqrt{\frac{10}{33}}$					
$U_5^{-1} V_2^1$						$\sqrt{\frac{35}{143}}$	$-\sqrt{\frac{7}{22}}$	$\sqrt{\frac{1}{78}}$	$\sqrt{\frac{2}{11}}$	$-\sqrt{\frac{8}{33}}$					
$U_5^{-2} V_2^2$						$\sqrt{\frac{5}{143}}$	$-\sqrt{\frac{2}{11}}$	$\sqrt{\frac{14}{39}}$	$-\sqrt{\frac{7}{22}}$	$\sqrt{\frac{7}{66}}$					
$U_5^1 V_2^{-2}$											$\sqrt{\frac{10}{143}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{13}}$	$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{22}}$
$U_5^0 V_2^{-1}$											$\sqrt{\frac{48}{143}}$	$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{78}}$	$-\sqrt{\frac{6}{22}}$	$-\sqrt{\frac{5}{33}}$
$U_5^{-1} V_2^0$											$\sqrt{\frac{60}{143}}$	$-\sqrt{\frac{1}{22}}$	$-\sqrt{\frac{27}{130}}$	$\sqrt{\frac{3}{55}}$	$\sqrt{\frac{3}{11}}$
$U_5^{-2} V_2^1$											$\sqrt{\frac{160}{1001}}$	$-\sqrt{\frac{27}{77}}$	$\sqrt{\frac{7}{65}}$	$\sqrt{\frac{7}{110}}$	$-\sqrt{\frac{7}{22}}$
$U_5^{-3} V_2^2$											$\sqrt{\frac{15}{1001}}$	$-\sqrt{\frac{8}{77}}$	$\sqrt{\frac{56}{195}}$	$-\sqrt{\frac{21}{55}}$	$\sqrt{\frac{7}{33}}$

$\mathfrak{D}_5 \times \mathfrak{D}_2$

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	W_7^{-2}	W_6^{-2}	W_5^{-2}	W_4^{-2}	W_3^{-2}	W_7^{-3}	W_6^{-3}	W_5^{-3}	W_4^{-3}	W_3^{-3}	W_7^{-4}	W_6^{-4}	W_5^{-4}	W_4^{-4}	W_7^{-5}	W_6^{-5}	W_5^{-5}	W_7^{-6}	W_6^{-6}	W_7^{-7}
$U_5^0 V_2^{-2}$	$\sqrt{\frac{18}{143}}$	$\sqrt{\frac{4}{11}}$	$\sqrt{\frac{14}{39}}$	$\sqrt{\frac{3}{22}}$	$\sqrt{\frac{1}{66}}$															
$U_5^{-1} V_2^{-1}$	$\sqrt{\frac{60}{143}}$	$\sqrt{\frac{6}{55}}$	$-\sqrt{\frac{7}{65}}$	$-\sqrt{\frac{16}{55}}$	$-\sqrt{\frac{4}{55}}$															
$U_5^{-2} V_2^0$	$\sqrt{\frac{360}{1001}}$	$-\sqrt{\frac{64}{385}}$	$-\sqrt{\frac{6}{65}}$	$\sqrt{\frac{21}{110}}$	$\sqrt{\frac{21}{110}}$															
$U_5^{-3} V_2^1$	$\sqrt{\frac{90}{1001}}$	$-\sqrt{\frac{11}{35}}$	$\sqrt{\frac{10}{39}}$	0	$-\sqrt{\frac{56}{165}}$															
$U_5^{-4} V_2^2$	$\sqrt{\frac{5}{1001}}$	$-\sqrt{\frac{18}{385}}$	$\sqrt{\frac{12}{65}}$	$-\sqrt{\frac{21}{55}}$	$\sqrt{\frac{21}{55}}$															
$U_5^{-1} V_2^{-2}$						$\sqrt{\frac{30}{143}}$	$\sqrt{\frac{24}{55}}$	$\sqrt{\frac{56}{195}}$	$\sqrt{\frac{7}{110}}$	$\sqrt{\frac{1}{330}}$										
$U_5^{-2} V_2^{-1}$						$\sqrt{\frac{480}{1001}}$	$\sqrt{\frac{6}{385}}$	$-\sqrt{\frac{10}{39}}$	$-\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{7}{330}}$										
$U_5^{-3} V_2^0$						$\sqrt{\frac{270}{1001}}$	$-\sqrt{\frac{243}{770}}$	$-\sqrt{\frac{1}{390}}$	$\sqrt{\frac{18}{55}}$	$\sqrt{\frac{14}{165}}$										
$U_5^{-4} V_2^1$						$\sqrt{\frac{40}{1001}}$	$-\sqrt{\frac{169}{770}}$	$\sqrt{\frac{49}{130}}$	$-\sqrt{\frac{6}{55}}$	$-\sqrt{\frac{14}{55}}$										
$U_5^{-5} V_2^2$						$\sqrt{\frac{1}{1001}}$	$-\sqrt{\frac{1}{77}}$	$\sqrt{\frac{1}{13}}$	$-\sqrt{\frac{3}{11}}$	$\sqrt{\frac{7}{11}}$										
$U_5^{-2} V_2^{-2}$											$\sqrt{\frac{30}{91}}$	$\sqrt{\frac{36}{77}}$	$\sqrt{\frac{12}{65}}$	$\sqrt{\frac{1}{55}}$						
$U_5^{-3} V_2^{-1}$											$\sqrt{\frac{45}{91}}$	$-\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{49}{130}}$	$-\sqrt{\frac{6}{55}}$						
$U_5^{-4} V_2^0$											$\sqrt{\frac{15}{91}}$	$-\sqrt{\frac{32}{77}}$	$\sqrt{\frac{6}{65}}$	$\sqrt{\frac{18}{55}}$						
$U_5^{-5} V_2^1$											$\sqrt{\frac{1}{91}}$	$-\sqrt{\frac{15}{154}}$	$\sqrt{\frac{9}{26}}$	$-\sqrt{\frac{6}{11}}$						
$U_5^{-3} V_2^{-2}$															$\sqrt{\frac{45}{91}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{13}}$			
$U_5^{-4} V_2^{-1}$															$\sqrt{\frac{40}{91}}$	$-\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{9}{26}}$			
$U_5^{-5} V_2^0$															$\sqrt{\frac{6}{91}}$	$-\sqrt{\frac{5}{14}}$	$\sqrt{\frac{15}{26}}$			
$U_5^{-4} V_2^{-2}$																		$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$	
$U_5^{-5} V_2^{-1}$																		$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$	
$U_5^{-5} V_2^{-2}$																				1

$\mathfrak{D}_5 \times \mathfrak{D}_2$

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	W_6^6
$U_3^3V_3^3$	1

	W_6^5	W_5^5
$U_3^3V_3^2$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$U_3^2V_3^3$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

	W_6^4	W_5^4	W_4^4
$U_3^3V_3^1$	$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$
$U_3^2V_3^2$	$\sqrt{\frac{6}{11}}$	0	$-\sqrt{\frac{5}{11}}$
$U_3^1V_3^3$	$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$

	W_6^3	W_5^3	W_4^3	W_3^3
$U_3^3V_3^0$	$\sqrt{\frac{1}{11}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{1}{6}}$
$U_3^2V_3^1$	$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{1}{3}}$
$U_3^1V_3^2$	$\sqrt{\frac{9}{22}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{11}}$	$\sqrt{\frac{1}{3}}$
$U_3^0V_3^3$	$\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{9}{22}}$	$-\sqrt{\frac{1}{6}}$

	W_6^2	W_5^2	W_4^2	W_3^2	W_2^2
$U_3^3V_3^{-1}$	$\sqrt{\frac{1}{33}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$
$U_3^2V_3^0$	$\sqrt{\frac{8}{33}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$
$U_3^1V_3^1$	$\sqrt{\frac{5}{11}}$	0	$-\sqrt{\frac{20}{77}}$	0	$\sqrt{\frac{2}{7}}$
$U_3^0V_3^2$	$\sqrt{\frac{8}{33}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$
$U_3^{-1}V_3^3$	$\sqrt{\frac{1}{33}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$

	W_6^1	W_5^1	W_4^1	W_3^1	W_2^1	W_1^1
$U_3^3V_3^{-2}$	$\sqrt{\frac{1}{121}}$	$\sqrt{\frac{5}{84}}$	$\sqrt{\frac{15}{77}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{25}{84}}$	$\sqrt{\frac{3}{28}}$
$U_3^2V_3^{-1}$	$\sqrt{\frac{5}{44}}$	$\sqrt{\frac{9}{28}}$	$\sqrt{\frac{16}{77}}$	0	$-\sqrt{\frac{5}{28}}$	$-\sqrt{\frac{5}{28}}$
$U_3^1V_3^0$	$\sqrt{\frac{25}{66}}$	$\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{15}{154}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{3}{14}}$
$U_3^0V_3^1$	$\sqrt{\frac{25}{66}}$	$-\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{15}{154}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{3}{14}}$
$U_3^{-1}V_3^2$	$\sqrt{\frac{5}{44}}$	$-\sqrt{\frac{9}{28}}$	$\sqrt{\frac{16}{77}}$	0	$-\sqrt{\frac{5}{28}}$	$\sqrt{\frac{5}{28}}$
$U_3^{-2}V_3^3$	$\sqrt{\frac{1}{121}}$	$-\sqrt{\frac{5}{84}}$	$\sqrt{\frac{15}{77}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{3}{28}}$

$D_3 \times D_3$

	W_6^0	W_5^0	W_4^0	W_3^0	W_2^0	W_1^0	W_0^0
$U_3^3\ V_3^{-3}$	$\sqrt{\frac{1}{924}}$	$\sqrt{\frac{1}{84}}$	$\sqrt{\frac{9}{154}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{25}{84}}$	$\sqrt{\frac{9}{28}}$	$\sqrt{\frac{1}{7}}$
$U_3^2\ V_3^{-2}$	$\sqrt{\frac{3}{77}}$	$\sqrt{\frac{4}{21}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{1}{6}}$	0	$-\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{1}{7}}$
$U_3^1\ V_3^{-1}$	$\sqrt{\frac{75}{308}}$	$\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{1}{154}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{28}}$	$\sqrt{\frac{1}{28}}$	$\sqrt{\frac{1}{7}}$
$U_3^0\ V_3^0$	$\sqrt{\frac{100}{231}}$	0	$-\sqrt{\frac{18}{77}}$	0	$\sqrt{\frac{4}{21}}$	0	$-\sqrt{\frac{1}{7}}$
$U_3^{-1}\ V_3^1$	$\sqrt{\frac{75}{308}}$	$-\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{1}{154}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{1}{28}}$	$\sqrt{\frac{1}{7}}$
$U_3^{-2}\ V_3^2$	$\sqrt{\frac{3}{77}}$	$-\sqrt{\frac{4}{21}}$	$\sqrt{\frac{7}{22}}$	$-\sqrt{\frac{1}{6}}$	0	$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{1}{7}}$
$U_3^{-3}\ V_3^3$	$\sqrt{\frac{1}{924}}$	$-\sqrt{\frac{1}{84}}$	$\sqrt{\frac{9}{154}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{9}{28}}$	$\sqrt{\frac{1}{7}}$

$\mathfrak{D}_3 \times \mathfrak{D}_3$

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	W_6^1	W_5^1	W_4^1	W_3^1	W_2^1	W_1^1	W_6^2	W_5^2	W_4^2	W_3^2	W_2^2	W_6^3	W_5^3	W_4^3	W_3^3	W_6^4	W_5^4	W_4^4	W_6^5	W_5^5	W_6^6
$U_3^3V_3^2$	$\sqrt{\frac{1}{121}}$	$\sqrt{\frac{3}{54}}$	$\sqrt{\frac{15}{77}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{25}{84}}$	$\sqrt{\frac{3}{28}}$															
$U_3^2V_3^1$	$\sqrt{\frac{5}{44}}$	$\sqrt{\frac{9}{28}}$	$\sqrt{\frac{16}{77}}$	0	$-\sqrt{\frac{5}{28}}$	$-\sqrt{\frac{5}{28}}$															
$U_3^1V_3^0$	$\sqrt{\frac{25}{66}}$	$\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{15}{154}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{3}{14}}$															
$U_3^0V_3^1$	$\sqrt{\frac{25}{66}}$	$-\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{15}{154}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{3}{14}}$															
$U_3^1V_3^2$	$\sqrt{\frac{5}{44}}$	$-\sqrt{\frac{9}{28}}$	$\sqrt{\frac{16}{77}}$	0	$-\sqrt{\frac{5}{28}}$	$\sqrt{\frac{5}{28}}$															
$U_3^2V_3^3$	$\sqrt{\frac{1}{121}}$	$-\sqrt{\frac{5}{84}}$	$\sqrt{\frac{15}{77}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{3}{28}}$															
$U_3^3V_3^1$							$\sqrt{\frac{1}{33}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$										
$U_3^2V_3^0$							$\sqrt{\frac{8}{33}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$										
$U_3^1V_3^1$							$\sqrt{\frac{5}{11}}$	0	$-\sqrt{\frac{20}{77}}$	0	$\sqrt{\frac{2}{7}}$										
$U_3^0V_3^2$							$\sqrt{\frac{8}{33}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{154}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{21}}$										
$U_3^1V_3^3$							$\sqrt{\frac{1}{33}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{27}{77}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{5}{42}}$										
$U_3^3V_3^0$												$\sqrt{\frac{1}{11}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{1}{6}}$						
$U_3^2V_3^1$												$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{1}{3}}$						
$U_3^1V_3^2$												[illegible]	[illegible]	[illegible]	[illegible]						
$U_3^0V_3^3$												$\sqrt{\frac{1}{11}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{9}{22}}$	$-\sqrt{\frac{1}{6}}$						
$U_3^3V_3^1$																$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$			
$U_3^2V_3^2$																$\sqrt{\frac{6}{11}}$	0	$-\sqrt{\frac{5}{11}}$			
$\bar{U}_3^1V_3^3$																$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{11}}$			
$\bar{U}_3^3\bar{V}_3^2$																			$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$\bar{U}_3^2\bar{V}_3^3$																			$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$\bar{U}_3^3\bar{V}_3^3$																					1

$\mathfrak{D}_3 \times \mathfrak{D}_3$

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	W_7^7	W_7^6	W_6^6	W_7^5	W_6^5	W_5^5	W_7^4	W_6^4	W_5^4	W_4^4	W_7^3	W_6^3	W_5^3	W_4^3	W_3^3	W_7^2	W_6^2	W_5^2	W_4^2	W_3^2	W_2^2
$U_4^4V_3^3$	1																				
$U_4^4V_3^2$		$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$																		
$U_4^3V_3^3$		$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$																		
$U_4^4V_3^1$				$\sqrt{\frac{15}{91}}$	$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{14}{39}}$															
$U_4^3V_3^2$				$\sqrt{\frac{48}{91}}$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{35}{78}}$															
$U_4^2V_3^3$				$\sqrt{\frac{4}{13}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{26}}$															
$U_4^4V_3^0$							$\sqrt{\frac{5}{91}}$	$\sqrt{\frac{20}{77}}$	$\sqrt{\frac{28}{65}}$	$\sqrt{\frac{14}{55}}$											
$U_4^3V_3^1$							$\sqrt{\frac{30}{91}}$	$\sqrt{\frac{125}{462}}$	$-\sqrt{\frac{7}{390}}$	$-\sqrt{\frac{21}{55}}$											
$U_4^2V_3^2$							$\sqrt{\frac{6}{13}}$	$-\sqrt{\frac{2}{33}}$	$-\sqrt{\frac{8}{39}}$	$\sqrt{\frac{3}{11}}$											
$U_4^1V_3^3$							$\sqrt{\frac{2}{13}}$	$-\sqrt{\frac{9}{22}}$	$\sqrt{\frac{9}{26}}$	$-\sqrt{\frac{1}{11}}$											
$U_4^4V_3^{-1}$											$\sqrt{\frac{15}{1001}}$	$\sqrt{\frac{5}{77}}$	$\sqrt{\frac{56}{195}}$	$\sqrt{\frac{21}{55}}$	$\sqrt{\frac{7}{33}}$						
$U_4^3V_3^0$											$\sqrt{\frac{160}{1001}}$	$\sqrt{\frac{27}{77}}$	$\sqrt{\frac{7}{65}}$	$-\sqrt{\frac{7}{110}}$	$-\sqrt{\frac{7}{22}}$						
$U_4^2V_3^1$											$\sqrt{\frac{60}{143}}$	$\sqrt{\frac{1}{22}}$	$-\sqrt{\frac{27}{130}}$	$-\sqrt{\frac{3}{55}}$	$\sqrt{\frac{3}{11}}$						
$U_4^1V_3^2$											$\sqrt{\frac{45}{143}}$	$-\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{78}}$	$\sqrt{\frac{3}{11}}$	$-\sqrt{\frac{5}{33}}$						
$U_4^0V_3^3$											$\sqrt{\frac{10}{143}}$	$-\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{13}}$	$-\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{22}}$						
$U_4^4V_3^{-2}$																$\sqrt{\frac{3}{1001}}$	$\sqrt{\frac{20}{693}}$	$\sqrt{\frac{14}{117}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{35}{99}}$	$\sqrt{\frac{2}{9}}$
$U_4^3V_3^{-1}$																$\sqrt{\frac{60}{1001}}$	$\sqrt{\frac{169}{693}}$	$\sqrt{\frac{343}{1170}}$	$\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{7}{99}}$	$-\sqrt{\frac{5}{18}}$
$U_4^2V_3^0$																$\sqrt{\frac{40}{143}}$	$\sqrt{\frac{8}{33}}$	$-\sqrt{\frac{1}{95}}$	$-\sqrt{\frac{169}{770}}$	$-\sqrt{\frac{1}{66}}$	$\sqrt{\frac{5}{21}}$
$U_4^1V_3^1$																$\sqrt{\frac{60}{143}}$	$-\sqrt{\frac{1}{99}}$	$-\sqrt{\frac{128}{585}}$	$\sqrt{\frac{12}{385}}$	$\sqrt{\frac{16}{99}}$	$-\sqrt{\frac{10}{63}}$
$U_4^0V_3^2$																$\sqrt{\frac{30}{143}}$	$-\sqrt{\frac{32}{99}}$	$\sqrt{\frac{5}{117}}$	$\sqrt{\frac{15}{154}}$	$-\sqrt{\frac{49}{198}}$	$\sqrt{\frac{5}{63}}$
$U_4^{-1}V_3^3$																$\sqrt{\frac{4}{143}}$	$-\sqrt{\frac{5}{33}}$	$\sqrt{\frac{25}{78}}$	$-\sqrt{\frac{25}{77}}$	$\sqrt{\frac{5}{33}}$	$-\sqrt{\frac{1}{42}}$

$\mathfrak{D}_4 \times \mathfrak{D}_3$

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	W_7^1	W_6^1	W_5^1	W_4^1	W_3^1	W_2^1	W_1^1	W_7^0	W_6^0	W_5^0	W_4^0	W_3^0	W_2^0	W_1^0	W_7^{-1}	W_6^{-1}	W_5^{-1}	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_1^{-1}
$U_4^4 V_3^{-3}$	$\sqrt{\frac{1}{3003}}$	$\sqrt{\frac{1}{231}}$	$\sqrt{\frac{1}{39}}$	$\sqrt{\frac{1}{11}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$														
$U_4^3 V_3^{-2}$	$\sqrt{\frac{16}{1001}}$	$\sqrt{\frac{289}{2772}}$	$\sqrt{\frac{121}{468}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{7}{99}}$	$-\sqrt{\frac{1}{36}}$	$-\sqrt{\frac{1}{4}}$														
$U_4^2 V_3^{-1}$	$\sqrt{\frac{20}{143}}$	$\sqrt{\frac{125}{396}}$	$\sqrt{\frac{1849}{16380}}$	$-\sqrt{\frac{12}{385}}$	$-\sqrt{\frac{20}{99}}$	$-\sqrt{\frac{5}{252}}$	$\sqrt{\frac{5}{28}}$														
$U_4^1 V_3^0$	$\sqrt{\frac{160}{429}}$	$\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{361}{2730}}$	$-\sqrt{\frac{81}{770}}$	$\sqrt{\frac{5}{66}}$	$\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{5}{42}}$														
$U_4^0 V_3^1$	$\sqrt{\frac{50}{143}}$	$-\sqrt{\frac{25}{198}}$	$-\sqrt{\frac{121}{1638}}$	$\sqrt{\frac{27}{154}}$	$\sqrt{\frac{1}{198}}$	$-\sqrt{\frac{25}{126}}$	$\sqrt{\frac{1}{14}}$														
$U_4^{-1} V_3^2$	$\sqrt{\frac{16}{143}}$	$-\sqrt{\frac{11}{36}}$	$\sqrt{\frac{625}{3276}}$	0	$-\sqrt{\frac{16}{99}}$	$\sqrt{\frac{7}{36}}$	$-\sqrt{\frac{1}{28}}$														
$U_4^{-2} V_3^3$	$\sqrt{\frac{4}{429}}$	$-\sqrt{\frac{3}{44}}$	$\sqrt{\frac{75}{364}}$	$-\sqrt{\frac{25}{77}}$	$\sqrt{\frac{3}{11}}$	$-\sqrt{\frac{3}{28}}$	$\sqrt{\frac{1}{84}}$														
$U_4^3 V_3^{-3}$								$\sqrt{\frac{1}{429}}$	$\sqrt{\frac{1}{44}}$	$\sqrt{\frac{5}{52}}$	$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{1}{12}}$							
$U_4^2 V_3^{-2}$								$\sqrt{\frac{7}{143}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{30}{273}}$	$\sqrt{\frac{15}{154}}$	$-\sqrt{\frac{1}{66}}$	$-\sqrt{\frac{4}{21}}$	$-\sqrt{\frac{1}{7}}$							
$U_4^1 V_3^{-1}$								$\sqrt{\frac{35}{143}}$	$\sqrt{\frac{35}{132}}$	$\sqrt{\frac{1}{1092}}$	$-\sqrt{\frac{27}{154}}$	$-\sqrt{\frac{5}{66}}$	$\sqrt{\frac{5}{84}}$	$\sqrt{\frac{5}{28}}$							
$U_4^0 V_3^0$								$\sqrt{\frac{175}{429}}$	0	$-\sqrt{\frac{20}{91}}$	0	$\sqrt{\frac{2}{11}}$	0	$-\sqrt{\frac{4}{21}}$							
$U_4^{-1} V_3^1$								$\sqrt{\frac{35}{143}}$	$-\sqrt{\frac{35}{132}}$	$\sqrt{\frac{1}{1092}}$	$\sqrt{\frac{27}{154}}$	$-\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{5}{84}}$	$\sqrt{\frac{5}{28}}$							
$U_4^{-2} V_3^2$								$\sqrt{\frac{7}{143}}$	$-\sqrt{\frac{7}{33}}$	$\sqrt{\frac{80}{273}}$	$-\sqrt{\frac{15}{154}}$	$\sqrt{\frac{1}{66}}$	$\sqrt{\frac{4}{21}}$	$-\sqrt{\frac{1}{7}}$							
$U_4^{3} V_3^3$								$\sqrt{\frac{1}{429}}$	$-\sqrt{\frac{1}{44}}$	$\sqrt{\frac{5}{52}}$	$-\sqrt{\frac{5}{22}}$	$\sqrt{\frac{7}{22}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{1}{12}}$							
$U_4^2 V_3^{3}$															$\sqrt{\frac{4}{429}}$	$\sqrt{\frac{3}{44}}$	$\sqrt{\frac{75}{364}}$	$\sqrt{\frac{25}{77}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{3}{28}}$	$\sqrt{\frac{1}{84}}$
$U_4^1 V_3^{-2}$															$\sqrt{\frac{16}{143}}$	$\sqrt{\frac{11}{36}}$	$\sqrt{\frac{625}{3276}}$	0	$-\sqrt{\frac{16}{99}}$	$-\sqrt{\frac{7}{36}}$	$-\sqrt{\frac{1}{28}}$
$U_4^0 V_3^{-1}$															$\sqrt{\frac{50}{143}}$	$\sqrt{\frac{25}{198}}$	$-\sqrt{\frac{121}{1638}}$	$-\sqrt{\frac{27}{154}}$	$\sqrt{\frac{1}{198}}$	$\sqrt{\frac{25}{126}}$	$\sqrt{\frac{1}{14}}$
$U_4^{-1} V_3^0$															$\sqrt{\frac{160}{429}}$	$-\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{361}{2730}}$	$\sqrt{\frac{81}{770}}$	$\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{5}{42}}$	$-\sqrt{\frac{5}{42}}$
$U_4^{-2} V_3^1$															$\sqrt{\frac{20}{143}}$	$-\sqrt{\frac{125}{396}}$	$\sqrt{\frac{1849}{16380}}$	$\sqrt{\frac{12}{385}}$	$-\sqrt{\frac{20}{99}}$	$\sqrt{\frac{5}{252}}$	$\sqrt{\frac{5}{28}}$
$U_4^{-3} V_3^2$															$\sqrt{\frac{16}{1001}}$	$-\sqrt{\frac{289}{2772}}$	$\sqrt{\frac{121}{468}}$	$-\sqrt{\frac{3}{11}}$	$\sqrt{\frac{7}{99}}$	$\sqrt{\frac{1}{36}}$	$-\sqrt{\frac{1}{4}}$
$U_4^{-4} V_3^3$															$\sqrt{\frac{1}{3003}}$	$-\sqrt{\frac{1}{231}}$	$\sqrt{\frac{1}{39}}$	$-\sqrt{\frac{1}{11}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$

$\mathfrak{D}_4 \times \mathfrak{D}_3$

	W_7^{-2}	W_6^{-2}	W_5^{-2}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_7^{-3}	W_6^{-3}	W_5^{-3}	W_4^{-3}	W_3^{-3}	W_7^{-4}	W_6^{-4}	W_5^{-4}	W_4^{-4}	W_7^{-5}	W_6^{-5}	W_5^{-5}	W_7^{-6}	W_6^{-6}	W_7^{-7}
$U_4^{1}V_3^{-3}$	$\sqrt{\frac{4}{143}}$	$\sqrt{\frac{5}{33}}$	$\sqrt{\frac{25}{78}}$	$\sqrt{\frac{25}{77}}$	$\sqrt{\frac{5}{33}}$	$\sqrt{\frac{1}{42}}$															
$U_4^{0}V_3^{-2}$	$\sqrt{\frac{30}{143}}$	$\sqrt{\frac{32}{99}}$	$\sqrt{\frac{5}{117}}$	$-\sqrt{\frac{15}{154}}$	$-\sqrt{\frac{49}{198}}$	$-\sqrt{\frac{5}{63}}$															
$U_4^{-1}V_3^{-1}$	$\sqrt{\frac{60}{143}}$	$\sqrt{\frac{1}{99}}$	$-\sqrt{\frac{128}{585}}$	$-\sqrt{\frac{12}{385}}$	$\sqrt{\frac{16}{99}}$	$\sqrt{\frac{10}{63}}$															
$U_4^{-2}V_3^{0}$	$\sqrt{\frac{40}{143}}$	$-\sqrt{\frac{8}{33}}$	$-\sqrt{\frac{1}{195}}$	$\sqrt{\frac{169}{770}}$	$-\sqrt{\frac{1}{66}}$	$-\sqrt{\frac{5}{21}}$															
$U_4^{-3}V_3^{1}$	$\sqrt{\frac{60}{1001}}$	$-\sqrt{\frac{169}{693}}$	$\sqrt{\frac{343}{1170}}$	$-\sqrt{\frac{3}{55}}$	$-\sqrt{\frac{7}{99}}$	$\sqrt{\frac{5}{18}}$															
$U_4^{-4}V_3^{2}$	$\sqrt{\frac{3}{1001}}$	$-\sqrt{\frac{20}{693}}$	$\sqrt{\frac{14}{117}}$	$-\sqrt{\frac{3}{11}}$	$\sqrt{\frac{35}{99}}$	$-\sqrt{\frac{2}{9}}$															
$U_4^{0}V_3^{-3}$							$\sqrt{\frac{10}{143}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{13}}$	$\sqrt{\frac{5}{22}}$	$\sqrt{\frac{1}{22}}$										
$U_4^{-1}V_3^{-2}$							$\sqrt{\frac{48}{143}}$	$\sqrt{\frac{5}{22}}$	$-\sqrt{\frac{1}{78}}$	$-\sqrt{\frac{3}{11}}$	$-\sqrt{\frac{5}{33}}$										
$U_4^{-2}V_3^{-1}$							$\sqrt{\frac{60}{143}}$	$-\sqrt{\frac{1}{22}}$	$-\sqrt{\frac{27}{130}}$	$\sqrt{\frac{3}{55}}$	$\sqrt{\frac{3}{11}}$										
$U_4^{-3}V_3^{0}$							$\sqrt{\frac{160}{1001}}$	$-\sqrt{\frac{27}{77}}$	$\sqrt{\frac{7}{65}}$	$\sqrt{\frac{7}{110}}$	$-\sqrt{\frac{7}{22}}$										
$U_4^{-4}V_3^{1}$							$\sqrt{\frac{15}{1001}}$	$-\sqrt{\frac{8}{77}}$	$\sqrt{\frac{56}{195}}$	$-\sqrt{\frac{21}{55}}$	$\sqrt{\frac{7}{33}}$										
$U_4^{-1}V_3^{-3}$												$\sqrt{\frac{3}{13}}$	$\sqrt{\frac{9}{22}}$	$\sqrt{\frac{7}{26}}$	$\sqrt{\frac{1}{11}}$						
$U_4^{-2}V_3^{-2}$												$\sqrt{\frac{5}{13}}$	$\sqrt{\frac{2}{33}}$	$-\sqrt{\frac{11}{39}}$	$-\sqrt{\frac{3}{11}}$						
$U_4^{-3}V_3^{-1}$												$\sqrt{\frac{30}{91}}$	$-\sqrt{\frac{125}{462}}$	$-\sqrt{\frac{7}{390}}$	$\sqrt{\frac{21}{55}}$						
$U_4^{-4}V_3^{0}$												$\sqrt{\frac{5}{91}}$	$-\sqrt{\frac{20}{77}}$	$\sqrt{\frac{28}{65}}$	$-\sqrt{\frac{14}{55}}$						
$U_4^{-2}V_3^{-3}$																$\sqrt{\frac{4}{13}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{5}{26}}$			
$U_4^{-3}V_3^{-2}$																$\sqrt{\frac{48}{91}}$	$-\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{35}{78}}$			
$U_4^{-4}V_3^{-1}$																$\sqrt{\frac{15}{91}}$	$-\sqrt{\frac{10}{21}}$	$\sqrt{\frac{14}{39}}$			
$U_4^{-3}V_3^{-3}$																			$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$	
$U_4^{-4}V_3^{-2}$																			$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$	
$U_4^{-4}V_3^{-3}$																					1

$\mathfrak{D}_4 \times \mathfrak{D}_3$

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	W^8_8	W^7_8	W^7_7	W^6_8	W^6_7	W^6_6	W^5_8	W^5_7	W^5_6	W^5_5	W^4_4	W^4_7	W^4_6	W^4_5	W^4_4	W^3_8	W^3_7	W^3_6	W^3_5	W^3_4	W^3_3
$U^5_5 V^3_3$	1																				
$U^5_5 V^2_3$		$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$																		
$U^4_5 V^3_3$		$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$																		
$U^5_5 V^1_3$				$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{25}{56}}$	$\sqrt{\frac{3}{7}}$															
$U^4_5 V^2_3$				$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{7}}$															
$U^3_5 V^3_3$				$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{27}{56}}$	$\sqrt{\frac{1}{7}}$															
$U^5_5 V^0_3$							$\sqrt{\frac{1}{28}}$	$\sqrt{\frac{75}{364}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{30}{91}}$											
$U^4_5 V^1_3$							$\sqrt{\frac{15}{56}}$	$\sqrt{\frac{35}{104}}$	0	$-\sqrt{\frac{36}{91}}$											
$U^3_5 V^2_3$							$\sqrt{\frac{27}{56}}$	$-\sqrt{\frac{9}{728}}$	$-\sqrt{\frac{2}{7}}$	$\sqrt{\frac{20}{91}}$											
$U^2_5 V^3_3$							$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{81}{182}}$	$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{91}}$											
$U^5_5 V^{-1}_3$											$\sqrt{\frac{3}{364}}$	$\sqrt{\frac{25}{364}}$	$\sqrt{\frac{18}{77}}$	$\sqrt{\frac{36}{91}}$	$\sqrt{\frac{42}{143}}$						
$U^4_5 V^0_3$											$\sqrt{\frac{10}{91}}$	$\sqrt{\frac{30}{91}}$	$\sqrt{\frac{15}{77}}$	$-\sqrt{\frac{6}{455}}$	$-\sqrt{\frac{252}{715}}$						
$U^3_5 V^1_3$											$\sqrt{\frac{135}{364}}$	$\sqrt{\frac{45}{364}}$	$-\sqrt{\frac{10}{77}}$	$-\sqrt{\frac{64}{455}}$	$\sqrt{\frac{168}{715}}$						
$U^2_5 V^2_3$											$\sqrt{\frac{36}{91}}$	$-\sqrt{\frac{12}{91}}$	$-\sqrt{\frac{6}{77}}$	$\sqrt{\frac{27}{91}}$	$-\sqrt{\frac{14}{143}}$						
$U^1_5 V^3_3$											$\sqrt{\frac{3}{26}}$	$-\sqrt{\frac{9}{26}}$	$\sqrt{\frac{4}{11}}$	$-\sqrt{\frac{2}{13}}$	$\sqrt{\frac{3}{143}}$						
$U^5_5 V^{-2}_3$																$\sqrt{\frac{1}{728}}$	$\sqrt{\frac{125}{8008}}$	$\sqrt{\frac{6}{77}}$	$\sqrt{\frac{20}{91}}$	$\sqrt{\frac{105}{286}}$	$\sqrt{\frac{7}{22}}$
$U^4_5 V^{-1}_3$																$\sqrt{\frac{25}{728}}$	$\sqrt{\frac{1445}{8008}}$	$\sqrt{\frac{24}{77}}$	$\sqrt{\frac{64}{455}}$	$-\sqrt{\frac{21}{1430}}$	$-\sqrt{\frac{7}{22}}$
$U^3_5 V^0_3$																$\sqrt{\frac{75}{364}}$	$\sqrt{\frac{1215}{4004}}$	$\sqrt{\frac{1}{77}}$	$-\sqrt{\frac{242}{1365}}$	$-\sqrt{\frac{63}{715}}$	$\sqrt{\frac{7}{33}}$
$U^2_5 V^1_3$																$\sqrt{\frac{75}{182}}$	$\sqrt{\frac{15}{2002}}$	$-\sqrt{\frac{18}{77}}$	$-\sqrt{\frac{1}{1365}}$	$\sqrt{\frac{343}{1430}}$	$-\sqrt{\frac{7}{66}}$
$U^1_5 V^2_3$																$\sqrt{\frac{15}{52}}$	$\sqrt{\frac{147}{572}}$	0	$\sqrt{\frac{8}{39}}$	$-\sqrt{\frac{11}{52}}$	$\sqrt{\frac{5}{132}}$
$U^0_5 V^3_3$																$\sqrt{\frac{3}{52}}$	$-\sqrt{\frac{135}{572}}$	$\sqrt{\frac{4}{11}}$	$-\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{572}}$	$-\sqrt{\frac{1}{132}}$

$\mathfrak{D}_5 \times \mathfrak{D}_3$

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	W^2_8	W^2_7	W^2_6	W^2_5	W^2_4	W^2_3	W^2_2
$U^5_5 V^{-3}_3$	$\sqrt{\frac{1}{8008}}$	$\sqrt{\frac{15}{8008}}$	$\sqrt{\frac{1}{77}}$	$\sqrt{\frac{5}{91}}$	$\sqrt{\frac{45}{286}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{5}{11}}$
$U^4_5 V^{-2}_3$	$\sqrt{\frac{15}{2002}}$	$\sqrt{\frac{11}{182}}$	$\sqrt{\frac{15}{77}}$	$\sqrt{\frac{27}{91}}$	$\sqrt{\frac{24}{143}}$	0	$-\sqrt{\frac{3}{11}}$
$U^3_5 V^{-1}_3$	$\sqrt{\frac{675}{8008}}$	$\sqrt{\frac{315}{1144}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{1}{1365}}$	$-\sqrt{\frac{243}{1430}}$	$-\sqrt{\frac{7}{66}}$	$\sqrt{\frac{5}{33}}$
$U^2_5 V^0_3$	$\sqrt{\frac{300}{1001}}$	$\sqrt{\frac{180}{1001}}$	$-\sqrt{\frac{8}{231}}$	$\sqrt{\frac{529}{2730}}$	$\sqrt{\frac{3}{715}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{5}{66}}$
$U^1_5 V^1_3$	$\sqrt{\frac{225}{572}}$	$-\sqrt{\frac{15}{572}}$	$-\sqrt{\frac{2}{11}}$	$\sqrt{\frac{9}{130}}$	$\sqrt{\frac{1849}{20020}}$	$-\sqrt{\frac{9}{44}}$	$\sqrt{\frac{5}{154}}$
$U^0_5 V^2_3$	$\sqrt{\frac{27}{143}}$	$\sqrt{\frac{45}{143}}$	$\sqrt{\frac{2}{33}}$	$\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{240}{1001}}$	$\sqrt{\frac{4}{33}}$	$-\sqrt{\frac{5}{462}}$
$U^{-1}_5 V^3_3$	$\sqrt{\frac{15}{572}}$	$-\sqrt{\frac{81}{572}}$	$\sqrt{\frac{10}{33}}$	$-\sqrt{\frac{25}{78}}$	$\sqrt{\frac{675}{4004}}$	$-\sqrt{\frac{5}{132}}$	$\sqrt{\frac{1}{462}}$

	W^1_8	W^1_7	W^1_6	W^1_5	W^1_4	W^1_3	W^1_2
$U^4_5 V^{-3}_3$	$\sqrt{\frac{1}{1144}}$	$\sqrt{\frac{81}{8008}}$	$\sqrt{\frac{4}{77}}$	$\sqrt{\frac{2}{13}}$	$\sqrt{\frac{81}{286}}$	$\sqrt{\frac{7}{22}}$	$\sqrt{\frac{2}{11}}$
$U^3_5 V^{-2}_3$	$\sqrt{\frac{27}{1144}}$	$\sqrt{\frac{1083}{8008}}$	$\sqrt{\frac{64}{231}}$	$\sqrt{\frac{8}{39}}$	$\sqrt{\frac{3}{286}}$	$-\sqrt{\frac{7}{66}}$	$-\sqrt{\frac{8}{33}}$
$U^2_5 V^{-1}_3$	$\sqrt{\frac{45}{286}}$	$\sqrt{\frac{55}{182}}$	$\sqrt{\frac{5}{77}}$	$-\sqrt{\frac{9}{130}}$	$-\sqrt{\frac{128}{715}}$	0	$\sqrt{\frac{5}{22}}$
$U^1_5 V^0_3$	$\sqrt{\frac{105}{286}}$	$\sqrt{\frac{15}{286}}$	$\sqrt{\frac{5}{33}}$	$-\sqrt{\frac{14}{195}}$	$\sqrt{\frac{1083}{10010}}$	$\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{40}{231}}$
$U^0_5 V^1_3$	$\sqrt{\frac{189}{572}}$	$-\sqrt{\frac{75}{572}}$	$-\sqrt{\frac{2}{33}}$	$\sqrt{\frac{7}{39}}$	$\sqrt{\frac{3}{4004}}$	$\sqrt{\frac{9}{44}}$	$-\sqrt{\frac{4}{77}}$
$U^{-1}_5 V^2_3$	$\sqrt{\frac{63}{572}}$	$-\sqrt{\frac{13}{44}}$	$\sqrt{\frac{2}{11}}$	0	$\sqrt{\frac{625}{4004}}$	$\sqrt{\frac{9}{44}}$	$-\sqrt{\frac{4}{77}}$
$U^{-2}_5 V^3_3$	$\sqrt{\frac{3}{286}}$	$-\sqrt{\frac{21}{286}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{25}{78}}$	$\sqrt{\frac{75}{286}}$	$-\sqrt{\frac{7}{66}}$	$\sqrt{\frac{1}{66}}$

	W^0_8	W^0_7	W^0_6	W^0_5	W^0_4	W^0_3	W^0_2
$U^3_5 V^{-3}_3$	$\sqrt{\frac{1}{286}}$	$\sqrt{\frac{9}{286}}$	$\sqrt{\frac{4}{33}}$	$\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{143}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{2}{33}}$
$U^2_5 V^{-2}_3$	$\sqrt{\frac{8}{143}}$	$\sqrt{\frac{32}{143}}$	$\sqrt{\frac{3}{11}}$	$-\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{5}{143}}$	$-\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{3}{22}}$
$U^1_5 V^{-1}_3$	$\sqrt{\frac{35}{143}}$	$\sqrt{\frac{35}{143}}$	0	$-\sqrt{\frac{7}{39}}$	$-\sqrt{\frac{11}{182}}$	$\sqrt{\frac{5}{66}}$	$\sqrt{\frac{15}{77}}$
$U^0_5 V^0_3$	$\sqrt{\frac{56}{143}}$	0	$-\sqrt{\frac{7}{33}}$	0	$\sqrt{\frac{180}{1001}}$	0	$-\sqrt{\frac{50}{231}}$
$U^{-1}_5 V^1_3$	$\sqrt{\frac{35}{143}}$	$-\sqrt{\frac{35}{143}}$	0	$\sqrt{\frac{7}{39}}$	$-\sqrt{\frac{11}{182}}$	$-\sqrt{\frac{5}{66}}$	$\sqrt{\frac{15}{77}}$
$U^{-2}_5 V^2_3$	$\sqrt{\frac{8}{143}}$	$-\sqrt{\frac{32}{143}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{5}{143}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{3}{22}}$
$U^{-3}_5 V^3_3$	$\sqrt{\frac{1}{286}}$	$-\sqrt{\frac{9}{286}}$	$\sqrt{\frac{4}{33}}$	$-\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{143}}$	$-\sqrt{\frac{7}{33}}$	$\sqrt{\frac{2}{33}}$

$\mathfrak{D}_5 \times \mathfrak{D}_3$

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THIS SECTION REPEATED FROM PAGE 2

	W_8^0	W_7^0	W_6^0	W_5^0	W_4^0	W_3^0	W_2^0
$U_5^3 V_3^{-3}$	$\sqrt{\frac{1}{286}}$	$\sqrt{\frac{9}{286}}$	$\sqrt{\frac{4}{33}}$	$\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{143}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{2}{33}}$
$U_5^2 V_3^{-2}$	$\sqrt{\frac{8}{143}}$	$\sqrt{\frac{32}{143}}$	$\sqrt{\frac{3}{11}}$	$-\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{5}{143}}$	$-\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{3}{22}}$
$U_5^1 V_3^{-1}$	$\sqrt{\frac{35}{143}}$	$\sqrt{\frac{35}{143}}$	0	$-\sqrt{\frac{7}{39}}$	$-\sqrt{\frac{11}{182}}$	$\sqrt{\frac{5}{66}}$	$\sqrt{\frac{15}{77}}$
$U_5^0 V_3^0$	$\sqrt{\frac{56}{143}}$	0	$-\sqrt{\frac{7}{33}}$	0	$\sqrt{\frac{180}{1001}}$	0	$-\sqrt{\frac{150}{231}}$
$U_5^{-1} V_3^1$	$\sqrt{\frac{35}{143}}$	$-\sqrt{\frac{35}{143}}$	0	$\sqrt{\frac{7}{39}}$	$-\sqrt{\frac{11}{182}}$	$-\sqrt{\frac{5}{66}}$	$\sqrt{\frac{15}{77}}$
$U_5^{-2} V_3^2$	$\sqrt{\frac{8}{143}}$	$-\sqrt{\frac{32}{143}}$	$\sqrt{\frac{3}{11}}$	$\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{5}{143}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{3}{32}}$
$U_5^{-3} V_3^3$	$\sqrt{\frac{1}{286}}$	$-\sqrt{\frac{9}{286}}$	$\sqrt{\frac{4}{33}}$	$-\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{143}}$	$-\sqrt{\frac{7}{33}}$	$\sqrt{\frac{2}{33}}$

	W_8^{-1}	W_7^{-1}	W_6^{-1}	W_5^{-1}	W_4^{-1}	W_3^{-1}	W_2^{-1}
$U_5^2 V_3^{-3}$	$\sqrt{\frac{3}{286}}$	$\sqrt{\frac{21}{286}}$	$\sqrt{\frac{7}{33}}$	$\sqrt{\frac{25}{78}}$	$\sqrt{\frac{75}{286}}$	$\sqrt{\frac{7}{66}}$	$\sqrt{\frac{1}{66}}$
$U_5^1 V_3^{-2}$	$\sqrt{\frac{63}{572}}$	$\sqrt{\frac{13}{44}}$	$\sqrt{\frac{2}{11}}$	0	$-\sqrt{\frac{625}{4004}}$	$-\sqrt{\frac{9}{44}}$	$-\sqrt{\frac{4}{77}}$
$U_5^0 V_3^{-1}$	$\sqrt{\frac{189}{572}}$	$\sqrt{\frac{75}{572}}$	$-\sqrt{\frac{2}{33}}$	$-\sqrt{\frac{1}{39}}$	$\sqrt{\frac{9}{4004}}$	$\sqrt{\frac{25}{132}}$	$\sqrt{\frac{25}{231}}$
$U_5^{-1} V_3^0$	$\sqrt{\frac{105}{286}}$	$-\sqrt{\frac{15}{286}}$	$-\sqrt{\frac{5}{33}}$	$\sqrt{\frac{14}{195}}$	$\sqrt{\frac{1083}{10010}}$	$-\sqrt{\frac{5}{66}}$	$-\sqrt{\frac{40}{231}}$
$U_5^{-2} V_3^1$	$\sqrt{\frac{45}{286}}$	$-\sqrt{\frac{55}{182}}$	$\sqrt{\frac{5}{77}}$	$\sqrt{\frac{9}{130}}$	$-\sqrt{\frac{128}{715}}$	0	$\sqrt{\frac{5}{22}}$
$U_5^{-3} V_3^2$	$\sqrt{\frac{27}{1144}}$	$-\sqrt{\frac{1083}{8008}}$	$\sqrt{\frac{64}{231}}$	$-\sqrt{\frac{8}{39}}$	$\sqrt{\frac{3}{286}}$	$\sqrt{\frac{7}{66}}$	$-\sqrt{\frac{8}{33}}$
$U_5^{-4} V_3^3$	$\sqrt{\frac{1}{1144}}$	$-\sqrt{\frac{81}{8008}}$	$\sqrt{\frac{4}{77}}$	$-\sqrt{\frac{2}{13}}$	$\sqrt{\frac{81}{286}}$	$-\sqrt{\frac{7}{22}}$	$\sqrt{\frac{2}{11}}$

	W_8^{-2}	W_7^{-2}	W_6^{-2}	W_5^{-2}	W_4^{-2}	W_3^{-2}	W_2^{-2}
$U_5^1 V_3^{-3}$	$\sqrt{\frac{15}{572}}$	$\sqrt{\frac{81}{572}}$	$\sqrt{\frac{10}{33}}$	$\sqrt{\frac{25}{78}}$	$\sqrt{\frac{675}{4004}}$	$\sqrt{\frac{5}{132}}$	$\sqrt{\frac{1}{462}}$
$U_5^0 V_3^{-2}$	$\sqrt{\frac{27}{143}}$	$\sqrt{\frac{45}{143}}$	$\sqrt{\frac{2}{33}}$	$-\sqrt{\frac{5}{78}}$	$-\sqrt{\frac{240}{1001}}$	$-\sqrt{\frac{4}{33}}$	$-\sqrt{\frac{5}{462}}$
$U_5^{-1} V_3^{-1}$	$\sqrt{\frac{225}{572}}$	$\sqrt{\frac{15}{572}}$	$-\sqrt{\frac{2}{11}}$	$-\sqrt{\frac{9}{130}}$	$\sqrt{\frac{1849}{20020}}$	$\sqrt{\frac{9}{44}}$	$\sqrt{\frac{5}{154}}$
$U_5^{-2} V_3^0$	$\sqrt{\frac{300}{1001}}$	$-\sqrt{\frac{180}{1001}}$	$-\sqrt{\frac{8}{231}}$	$\sqrt{\frac{529}{2730}}$	$\sqrt{\frac{3}{715}}$	$-\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{5}{66}}$
$U_5^{-3} V_3^1$	$\sqrt{\frac{675}{8008}}$	$-\sqrt{\frac{315}{1144}}$	$\sqrt{\frac{7}{33}}$	$-\sqrt{\frac{1}{1365}}$	$-\sqrt{\frac{243}{1430}}$	$\sqrt{\frac{7}{66}}$	$\sqrt{\frac{5}{33}}$
$U_5^{-4} V_3^2$	$\sqrt{\frac{15}{2002}}$	$-\sqrt{\frac{11}{182}}$	$\sqrt{\frac{15}{77}}$	$-\sqrt{\frac{27}{91}}$	$\sqrt{\frac{24}{143}}$	0	$-\sqrt{\frac{3}{11}}$
$U_5^{-5} V_3^3$	$\sqrt{\frac{1}{8008}}$	$-\sqrt{\frac{15}{8008}}$	$\sqrt{\frac{1}{77}}$	$-\sqrt{\frac{5}{91}}$	$\sqrt{\frac{45}{286}}$	$-\sqrt{\frac{7}{22}}$	$\sqrt{\frac{5}{11}}$

$\mathfrak{D}_5 \times \mathfrak{D}_3$

	W_8^{-3}	W_7^{-3}	W_6^{-3}	W_5^{-3}	W_4^{-3}	W_3^{-3}
$U_5^{0}V_3^{-3}$	$\sqrt{\frac{3}{52}}$	$\sqrt{\frac{135}{572}}$	$\sqrt{\frac{4}{11}}$	$\sqrt{\frac{10}{39}}$	$\sqrt{\frac{45}{572}}$	$\sqrt{\frac{1}{132}}$
$U_5^{-1}V_3^{-2}$	$\sqrt{\frac{15}{52}}$	$\sqrt{\frac{147}{572}}$	0	$-\sqrt{\frac{8}{39}}$	$-\sqrt{\frac{11}{52}}$	$\sqrt{\frac{5}{132}}$
$U_5^{-2}V_3^{-1}$	$\sqrt{\frac{75}{182}}$	$-\sqrt{\frac{15}{2002}}$	$-\sqrt{\frac{18}{77}}$	$\sqrt{\frac{1}{1365}}$	$\sqrt{\frac{343}{1430}}$	$\sqrt{\frac{7}{66}}$
$U_5^{-3}V_3^{0}$	$\sqrt{\frac{75}{364}}$	$-\sqrt{\frac{1215}{4004}}$	$\sqrt{\frac{1}{77}}$	$\sqrt{\frac{242}{1365}}$	$-\sqrt{\frac{63}{715}}$	$-\sqrt{\frac{7}{33}}$
$U_5^{-4}V_3^{1}$	$\sqrt{\frac{25}{728}}$	$-\sqrt{\frac{1445}{8008}}$	$\sqrt{\frac{24}{77}}$	$-\sqrt{\frac{64}{455}}$	$-\sqrt{\frac{21}{1430}}$	$\sqrt{\frac{7}{22}}$
$U_5^{-5}V_3^{2}$	$\sqrt{\frac{1}{728}}$	$-\sqrt{\frac{125}{8008}}$	$\sqrt{\frac{6}{77}}$	$-\sqrt{\frac{20}{91}}$	$\sqrt{\frac{105}{286}}$	$-\sqrt{\frac{7}{22}}$

	W_8^{-4}	W_7^{-4}	W_6^{-4}	W_5^{-4}	W_4^{-4}
$U_5^{-1}V_3^{-3}$	$\sqrt{\frac{3}{26}}$	$\sqrt{\frac{9}{26}}$	$\sqrt{\frac{4}{11}}$	$\sqrt{\frac{2}{13}}$	$\sqrt{\frac{3}{143}}$
$U_5^{-2}V_3^{-2}$	$\sqrt{\frac{36}{91}}$	$\sqrt{\frac{12}{91}}$	$-\sqrt{\frac{6}{77}}$	$-\sqrt{\frac{27}{91}}$	$-\sqrt{\frac{14}{143}}$
$U_5^{-3}V_3^{-1}$	$\sqrt{\frac{135}{364}}$	$-\sqrt{\frac{45}{364}}$	$-\sqrt{\frac{10}{77}}$	$\sqrt{\frac{64}{455}}$	$\sqrt{\frac{168}{715}}$
$U_5^{-4}V_3^{0}$	$\sqrt{\frac{10}{91}}$	$-\sqrt{\frac{30}{91}}$	$\sqrt{\frac{15}{77}}$	$\sqrt{\frac{6}{455}}$	$-\sqrt{\frac{252}{715}}$
$U_5^{-5}V_3^{1}$	$\sqrt{\frac{3}{364}}$	$-\sqrt{\frac{25}{364}}$	$\sqrt{\frac{18}{77}}$	$-\sqrt{\frac{36}{91}}$	$\sqrt{\frac{42}{143}}$

	W_8^{-5}	W_7^{-5}	W_6^{-5}	W_5^{-5}
$U_5^{-2}V_3^{-3}$	$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{81}{182}}$	$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{91}}$
$U_5^{-3}V_3^{-2}$	$\sqrt{\frac{27}{56}}$	$\sqrt{\frac{9}{728}}$	$-\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{20}{91}}$
$U_5^{-4}V_3^{-1}$	$\sqrt{\frac{15}{56}}$	$-\sqrt{\frac{35}{104}}$	0	$\sqrt{\frac{36}{91}}$
$U_5^{-5}V_3^{0}$	$\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{75}{364}}$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{30}{91}}$

	W_8^{-6}	W_7^{-6}	W_6^{-6}
$U_5^{-3}V_3^{-3}$	$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{27}{56}}$	$\sqrt{\frac{1}{7}}$
$U_5^{-4}V_3^{-2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{7}}$
$U_5^{-5}V_3^{-1}$	$\sqrt{\frac{1}{8}}$	$-\sqrt{\frac{25}{56}}$	$\sqrt{\frac{3}{7}}$

	W_8^{-7}	W_7^{-7}
$U_5^{-4}V_3^{-3}$	$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$
$U_5^{-5}V_3^{-2}$	$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$

	W_8^{-8}
$U_5^{-5}V_3^{-3}$	1

$\mathfrak{D}_5 \times \mathfrak{D}_3$

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Spherical Harmonics and their Squares

APPENDIX II

Part I Definition

Spherical harmonics, Y_ℓ^m, are defined in Schiff[18] as follows:

$$Y_\ell^m(\theta,\phi) = \left[\frac{2\ell + 1}{4\pi}\,\frac{(\ell - |m|)!}{(\ell + |m|)!}\right]^{1/2} P_\ell^{|m|}(\theta)\; e^{im\phi},$$

where[19]

$$P_\ell^m(x) = (-1)^m\,(1 - x^2)^{m/2}\,\frac{d^m}{dx^m}\,P_\ell(x)$$

and[20]

$$P_\ell(x) = \frac{1}{2^\ell\,\ell!}\,\frac{d^\ell}{dx^\ell}\,(x^2 - 1)^\ell\,.$$

Part II Table of Spherical Harmonics and their Squares

$Y_0^0 = (4\pi)^{-1/2}$ $\qquad |Y_0^0|^2 = (4\pi)^{-1}$

$Y_1^0 = (3/8\pi)^{1/2}\;(2)^{1/2}\cos\theta$ $\qquad |Y_1^0|^2 = (3/8\pi)\;2\cos^2\theta$

$Y_1^{\pm1} = (3/8\pi)^{1/2}\;\sin\theta\; e^{\pm i\phi}$ $\qquad |Y_1^{\pm1}|^2 = (3/8\pi)\;(1 - \cos^2\theta)$

$Y_2^0 = (5/32\pi)^{1/2}\;(2)^{1/2}\;(3\cos^2\theta - 1)$

$Y_2^{\pm1} = (5/32\pi)^{1/2}\;(12)^{1/2}\;\sin\theta\cos\theta\; e^{\pm i\phi}$

$Y_2^{\pm2} = (5/32\pi)^{1/2}\;(3)^{1/2}\;\sin^2\theta\; e^{\pm 2i\phi}$

$$|Y_2^0|^2 = (5/32\pi)\;\;2(1 - 6\cos^2\theta + 9\cos^4\theta)$$

$$|Y_2^{\pm1}|^2 = (5/32\pi)\;\;12(\cos^2\theta - \cos^4\theta)$$

$$|Y_2^{\pm2}|^2 = (5/32\pi)\;\;3(1 - 2\cos^2\theta + \cos^4\theta)$$

$$Y_3^0 = (7/64\pi)^{1/2}\, 2(5\cos^2\theta - 3)\cos\theta$$

$$Y_3^{\pm1} = (7/64\pi)^{1/2}\,(3)^{1/2}\,(5\cos^2\theta - 1)\sin\theta\, e^{\pm i\phi}$$

$$Y_3^{\pm2} = (7/64\pi)^{1/2}\,(30)^{1/2}\sin^2\theta\cos\theta\, e^{\pm 2i\phi}$$

$$Y_3^{\pm3} = (7/64\pi)^{1/2}\,(5)^{1/2}\sin^3\theta\, e^{\pm 3i\phi}$$

$$\left|Y_3^0\right|^2 = (7/64\pi)\ 4(9\cos^2\theta - 30\cos^4\theta + 25\cos^6\theta)$$

$$\left|Y_3^{\pm1}\right|^2 = (7/64\pi)\ 3(1 - 11\cos^2\theta + 35\cos^4\theta - 25\cos^6\theta)$$

$$\left|Y_3^{\pm2}\right|^2 = (7/64\pi)\ 30(\cos^2\theta - 2\cos^4\theta + \cos^6\theta)$$

$$\left|Y_3^{\pm3}\right|^2 = (7/64\pi)\ 5(1 - 3\cos^2\theta + 3\cos^4\theta - \cos^6\theta)$$

$$Y_4^0 = (9/512\pi)^{1/2}\,(2)^{1/2}\,(35\cos^4\theta - 30\cos^2\theta + 3)$$

$$Y_4^{\pm1} = (9/512\pi)^{1/2}\,(40)^{1/2}\,(7\cos^2\theta - 3)\sin\theta\cos\theta\, e^{\pm i\phi}$$

$$Y_4^{\pm2} = (9/512\pi)^{1/2}\,(20)^{1/2}\,(7\cos^2\theta - 1)\sin^2\theta\, e^{\pm 2i\phi}$$

$$Y_4^{\pm3} = (9/512\pi)^{1/2}\,(280)^{1/2}\sin^3\theta\cos\theta\, e^{\pm 3i\phi}$$

$$Y_4^{\pm4} = (9/512\pi)^{1/2}\,(35)^{1/2}\sin^4\theta\, e^{\pm 4i\phi}$$

$$\left|Y_4^0\right|^2 = (9/512\pi)\ 2(9 - 180\cos^2\theta + 1110\cos^4\theta - 2100\cos^6\theta + 1225\cos^8\theta)$$

$$\left|Y_4^{\pm1}\right|^2 = (9/512\pi)\ 40(9\cos^2\theta - 51\cos^4\theta + 91\cos^6\theta - 49\cos^8\theta)$$

$$\left|Y_4^{\pm2}\right|^2 = (9/512\pi)\ 20(1 - 16\cos^2\theta + 78\cos^4\theta - 112\cos^6\theta + 49\cos^8\theta)$$

$$\left|Y_4^{\pm3}\right|^2 = (9/512\pi)\ 280(\cos^2\theta - 3\cos^4\theta + 3\cos^6\theta - \cos^8\theta)$$

$$\left|Y_4^{\pm4}\right|^2 = (9/512\pi)\ 35(1 - 4\cos^2\theta + 6\cos^4\theta - 4\cos^6\theta + \cos^8\theta)$$

$$Y_5^0 = (11/1024\pi)^{1/2}\ 2(63\cos^4\theta - 70\cos^2\theta + 15)\cos\theta$$

$$Y_5^{\pm 1} = (11/1024\pi)^{1/2}\ (30)^{1/2}\ (21\cos^4\theta - 14\cos^2\theta + 1)\sin\theta\ e^{\pm 1i\phi}$$

$$Y_5^{\pm 2} = (11/1024\pi)^{1/2}\ (840)^{1/2}\ (3\cos^2\theta - 1)\cos\theta\ \sin^2\theta\ e^{\pm 2i\phi}$$

$$Y_5^{\pm 3} = (11/1024\pi)^{1/2}\ (35)^{1/2}\ (9\cos^2\theta - 1)\ \sin^3\theta\ e^{\pm 3i\phi}$$

$$Y_5^{\pm 4} = (11/1024\pi)^{1/2}\ (630)^{1/2}\ \cos\theta\ \sin^4\theta\ e^{\pm 4i\phi}$$

$$Y_5^{\pm 5} = (11/1024\pi)^{1/2}\ (63)^{1/2}\ \sin^5\theta\ e^{\pm 5i\phi}$$

$$\left|Y_5^0\right|^2 = (11/1024\pi)\quad 4(225\cos^2\theta - 2100\cos^4\theta + 6790\cos^6\theta - 8820\cos^8\theta + 3969\cos^{10}\theta)$$

$$\left|Y_5^{\pm 1}\right|^2 = (11/1024\pi)\quad 30(1 - 29\cos^2\theta + 226\cos^4\theta - 826\cos^6\theta + 1029\cos^8\theta - 441\cos^{10}\theta)$$

$$\left|Y_5^{\pm 2}\right|^2 = (11/1024\pi)\ 840(\cos^2\theta - 8\cos^4\theta + 22\cos^6\theta - 24\cos^8\theta + 9\cos^{10}\theta)$$

$$\left|Y_5^{\pm 3}\right|^2 = (11/1024\pi)\quad 35(1 - 21\cos^2\theta + 138\cos^4\theta - 295\cos^6\theta + 261\cos^8\theta - 81\cos^{10}\theta)$$

$$\left|Y_5^{\pm 4}\right|^2 = (11/1024\pi)\ 630(\cos^2\theta - 4\cos^4\theta + 6\cos^6\theta - 4\cos^8\theta + \cos^{10}\theta)$$

$$\left|Y_5^{\pm 5}\right|^2 = (11/1024\pi)\quad 63(1 - 5\cos^2\theta + 10\cos^4\theta - 10\cos^6\theta + 5\cos^8\theta - \cos^{10}\theta)$$

APPENDIX III

γ-Ray Radiation Functions and their Squares

The vector describing the electric and magnetic fields are given[22] in terms of vector spherical harmonics,

$$\bar{X}_{\ell,m}(\theta,\ \phi) = \frac{-i\ \bar{r}\ x\ \bar{\nabla} Y^m(\theta,\ \phi)}{[\ell(\ell-1)]^{1/2}} .$$

Vectors are designated by the bar. For electric multipole transitions the magnetic vector of the radiation is equal to this vector times a function of the radius. For magnetic multipole transitions the electric vector of the radiation is equal to this vector times a function of the radius. The vectors are calculated assuming $\bar{r}$, $\bar{\theta}$, and $\bar{\phi}$ are unit vectors in their respective directions. The square of the magnitude of $\bar{X}_{\ell,m}$ is $Z_{\ell,m}$.

$\bar{X}_{0,0} = 0$ $\qquad Z_{0,0} = 0$

$\bar{X}_{1,0} = (3/16\pi)^{1/2}\ (2)^{1/2}\ i\bar{\phi}\ \sin\theta \qquad Z_{1,0} = (3/16\pi)2(1-\cos^2\theta)$

$\bar{X}_{1,\pm1} = \mp(3/16\pi)^{1/2}\ (\bar{\theta} \pm i\bar{\phi}\ \cos\theta)e^{\pm i\phi} \qquad Z_{1,\pm1} = (3/16\pi)\ (1+\cos^2\theta)$

$\bar{X}_{2,0} = (5/16\pi)^{1/2}\ (6)^{1/2}\ i\bar{\phi}\ \cos\theta\ \sin\theta$

$\bar{X}_{2,\pm1} = \mp(5/16\pi)^{1/2}\ (\bar{\theta}\ \cos\theta \pm i\bar{\phi}\ [2\cos^2\theta - 1]\ )\ e^{\pm i\phi}$

$\bar{X}_{2,\pm2} = \mp(5/16\pi)^{1/2}\ (\bar{\theta}\ \sin\theta \pm i\bar{\phi}\ \sin\theta\ \cos\theta)\ e^{\pm 2i\phi}$

$$Z_{2,0} = (5/16\pi)\,6(\qquad \cos^2\theta - \cos^4\theta)$$

$$Z_{2,\pm1} = (5/16\pi)\ \ (1 - 3\ \cos^2\theta + 4\ \cos^4\theta)$$

$$Z_{2,\pm2} = (5/16\pi)\ \ (1 \qquad\qquad - \cos^4\theta)$$

$$\bar{X}_{3,0} = (7/256\pi)^{1/2}\ (12)^{1/2}\ i\bar{\phi}\ (5\cos^2\theta - 1)\sin\theta$$

$$\bar{X}_{3,\pm1} = \mp(7/256\pi)^{1/2}\ (\bar{\theta}\,[5\cos^2\theta - 1] \pm i\bar{\phi}\,[15\cos^2\theta - 11]\cos\theta)e^{\pm i\phi}$$

$$\bar{X}_{3,\pm2} = \mp(7/256\pi)^{1/2}\ (10)^{1/2}\ (\bar{\theta}\ 2\sin\theta\cos\theta \pm i\bar{\phi}\,[3\cos^2\theta - 1]\sin\theta)\ e^{\pm2i\phi}$$

$$\bar{X}_{3,\pm3} = \mp(7/256\pi)^{1/2}\ (15)^{1/2}\ (\bar{\theta}\sin^2\theta \pm i\bar{\phi}\sin^2\theta\cos\theta)\ e^{\pm3i\phi}$$

$$Z_{3,0} = (7/256\pi)\ 12(1 - 11\cos^2\theta + 35\cos^4\theta - 25\cos^6\theta)$$

$$Z_{3,-1} = (7/256\pi)\ (1 + 111\cos^2\theta - 305\cos^4\theta + 225\cos^6\theta)$$

$$Z_{3,-2} = (7/256\pi)\ 10(1 - 3\cos^2\theta + 11\cos^4\theta - 9\cos^6\theta)$$

$$Z_{3,-3} = (7/256\pi)\ 15(1 - \cos^2\theta - \cos^4\theta + \cos^6\theta)$$

$$\bar{X}_{4,0} = (9/256\pi)^{1/2}\ (20)^{1/2}\ i\bar{\phi}\ (7\cos^2\theta - 3)\cos\theta\sin\theta$$

$$\bar{X}_{4,\pm1} = \mp(9/256\pi)^{1/2}\ (\bar{\theta}\,[7\cos^2\theta - 3]\cos\theta \pm i\bar{\phi}\,[28\cos^4\theta - 27\cos^2\theta$$
$$+ 3])e^{\pm i\phi}$$

$$\bar{X}_{4,\pm2} = \mp(9/256\pi)^{1/2}\ (2)^{1/2}(\bar{\theta}\,[7\cos^2\theta - 1]\sin\theta \pm i\bar{\phi}\,[14\cos^2\theta - 8]$$
$$\cos\theta)e^{\pm2i\phi}$$

$$\bar{X}_{4,\pm3} = \mp(9/256\pi)^{1/2}\ (7)^{1/2}(3\bar{\theta}\,[\cos^2\theta - 1]\cos\theta \pm i\bar{\phi}\,[4\cos^4\theta - 5\cos^2\theta$$
$$+ 1])e^{\pm3i\phi}$$

$$\bar{X}_{4,\pm4} = \mp(9/256\pi)^{1/2}\ (14)^{1/2}(\bar{\theta}\,[\cos^2\theta - 1]\sin\theta \pm i\bar{\phi}\,[\cos^2\theta - 1]\cos\theta$$
$$\sin\theta)e^{\pm4i\phi}$$

$$Z_{4,0} = (9/256\pi)\ 20(9 \cos^2\theta - 51 \cos^4\theta + 91 \cos^6\theta - 49 \cos^8\theta)$$

$$Z_{4,\pm1} = (9/256\pi)\ (9 - 153 \cos^2\theta + 885 \cos^4\theta - 1463 \cos^6\theta + 784 \cos^8\theta)$$

$$Z_{4,\pm2} = (9/256\pi)\ 2(1 + 49 \cos^2\theta - 225 \cos^4\theta + 371 \cos^6\theta - 196 \cos^8\theta)$$

$$Z_{4,\pm3} = (9/256\pi)\ 7\ (1 - \cos^2\theta + 15 \cos^4\theta - 31 \cos^6\theta + 16 \cos^8\theta)$$

$$Z_{4,\pm4} = (9/256\pi)\ 14(1 - 2 \cos^2\theta + 2 \cos^6\theta - \cos^8\theta)$$

$$\bar{X}_{5,0} = (11/2048\pi)^{1/2}\ (60)^{1/2}\ i\bar{\phi}[21 \cos^4\theta - 14 \cos^2\theta + 1]$$

$$\bar{X}_{5,\pm1} = \mp(11/2048\pi)^{1/2}\ (2)^{1/2}(\bar{\theta}[21 \cos^4\theta - 14 \cos^2\theta + 1] \pm i\bar{\phi}[105 \cos^4\theta - 126 \cos^2\theta + 29] \cos\theta)\ e^{\pm i\phi}$$

$$\bar{X}_{5,\pm2} = \mp(11/2048\pi)^{1/2}\ (56)^{1/2}(2\bar{\theta}[3 \cos^2\theta - 1] \cos\theta \pm i\bar{\phi}[15 \cos^4\theta - 12 \cos^2\theta + 1]) \sin\theta\ \ e^{\pm 2i\phi}$$

$$\bar{X}_{5,\pm3} = \mp(11/2048\pi)^{1/2}\ (21)^{1/2}(\bar{\theta}[9 \cos^2\theta - 1] \pm i\bar{\phi}[15 \cos^2\theta - 7] \cos\theta)\ \sin^2\theta\ \ e^{\pm 3i\phi}$$

$$\bar{X}_{5,\pm4} = \mp(11/2048\pi)^{1/2}\ (42)^{1/2}(4\bar{\theta} \cos\theta \pm i\bar{\phi}[5 \cos^2\theta - 1])\ \sin^3\theta\ e^{\pm 4i\phi}$$

$$\bar{X}_{5,\pm5} = \mp(11/2048\pi)^{1/2}\ (105)^{1/2}\ (\bar{\theta} \pm i\bar{\phi} \cos\theta)\ \sin^4\theta\ \ e^{\pm 5i\phi}$$

$$Z_{5,0} = (11/2048\pi)\ 60\ (1 - 29 \cos^2\theta + 226 \cos^4\theta - 826 \cos^6\theta + 1029 \cos^8\theta - 441 \cos^{10}\theta)$$

$$Z_{5,\pm1} = (11/2048\pi)\ 2\ (1 + 813 \cos^2\theta - 7070 \cos^4\theta + 21378 \cos^6\theta - 26019 \cos^8\theta + 11025 \cos^{10}\theta)$$

$$Z_{5,\pm2} = (11/2048\pi)\ 56(1 - 21 \cos^2\theta + 170 \cos^4\theta - 474 \cos^6\theta + 549 \cos^8\theta - 225 \cos^{10}\theta)$$

$$Z_{5,\pm 3} = (11/2048\pi)\ \ 21(1 + 29\cos^2\theta - 190\cos^4\theta + 514\cos^6\theta - 579\cos^8\theta + 225\cos^{10}\theta)$$

$$Z_{5,\pm 4} = (11/2048\pi)\ \ 42(1 + 3\cos^2\theta + 10\cos^4\theta - 58\cos^6\theta + 69\cos^8\theta - 25\cos^{10}\theta)$$

$$Z_{5,\pm 5} = (11/2048\pi)\ 105(1 - 3\cos^2\theta + 2\cos^4\theta + 2\cos^6\theta - 3\cos^8\theta + \cos^{10}\theta)$$

APPENDIX IV

Tables of Angular Distributions and Correlations

An assignment is designated by four numbers according to the definition of page 34. The distribution of long-range α-particles is designated by α_0, of the short-range α-particles by α, of the γ-rays by γ, and of the correlation of the short-range α-particles and the γ-rays by $\alpha\gamma$. The coordinates are the same as those defined in the introduction (pp. 1 - 3).

0011	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 - \cos^2\theta$
0022	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$\cos^2\theta - \cos^4\theta$
0033	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 - 11\cos^2\theta + 35\cos^4\theta - 25\cos^6\theta$
0044	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$9\cos^2\theta - 51\cos^4\theta + 91\cos^6\theta - 49\cos^8\theta$

0055	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 - 29\cos^2\theta + 226\cos^4\theta - 826\cos^6\theta + 1029\cos^8\theta - 441\cos^{10}\theta$
0101	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	1
0111	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 + \cos^2\theta$
0112	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 + \cos^2\theta$
0122	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$1 - 3\cos^2\theta + 4\cos^4\theta$
0123	γ	1
	α_0	1
	α	1
	$\alpha\gamma$	$5 + 6\cos^2\theta + 5\cos^4\theta$

0133 γ 1

α_0 1

α 1

$\alpha\gamma$ $1 + 111\cos^2\theta - 305\cos^4\theta + 225\cos^6\theta$

0134 γ 1

α_0 1

α 1

$\alpha\gamma$ $9 - 9\cos^2\theta + 39\cos^4\theta - 7\cos^6\theta$

0144 γ 1

α_0 1

α 1

$\alpha\gamma$ $9 - 153\cos^2\theta + 855\cos^4\theta - 1463\cos^6\theta + 784\cos^8\theta$

0145 γ 1

α_0 1

α 1

$\alpha\gamma$ $13 + 44\cos^2\theta - 710\cos^4\theta + 364\cos^6\theta - 147\cos^8\theta$

0155 γ 1

α_0 1

α 1

$\alpha\gamma$ $1 + 813\cos^2\theta - 7070\cos^4\theta + 21378\cos^6\theta - 26019\cos^8\theta + 11025\cos^{10}\theta$

1011 γ 1

α_0 1

α 1

$\alpha\gamma$ $1 - \cos^2\theta$

1022	γ	1
	a_0	1
	a	1
	$a\gamma$	$\cos^2\theta - \cos^4\theta$
1033	γ	1
	a_0	1
	a	1
	$a\gamma$	$1 - 11\cos^2\theta + 35\cos^4\theta - 25\cos^6\theta$
1044	γ	1
	a_0	1
	a	1
	$a\gamma$	$9\cos^2\theta - 51\cos^4\theta + 91\cos^6\theta - 49\cos^8\theta$
1055	γ	1
	a_0	1
	a	1
	$a\gamma$	$1 - 29\cos^2\theta + 226\cos^4\theta - 826\cos^6\theta + 1029\cos^8\theta - 441\cos^{10}\theta$
1101	γ	$(1 + \cos^2\theta) + 2A(1 - \cos^2\theta)$
	a_0	$(1 - \cos^2\theta) + 2A\cos^2\theta$
	a	1
	$a\gamma$	$(1 + 2A) + (1 - 2A)(\sin\Theta\sin\theta\cos\phi + \cos\Theta\cos\theta)^2$
1111	γ	$(3 - \cos^2\theta) + 2A(1 + \cos^2\theta)$
	a_0	$(1 - \cos^2\theta) + 2A\cos^2\theta$
	a	$(1 + \cos^2\theta) + 2A(1 - \cos^2\theta)$
	$a\gamma$	$\cos^2\Theta(1 + \cos^2\theta) + \sin^2\Theta\{(\cos^2\theta\cos^2\phi + \sin^2\phi) + 2A(\cos^2\theta\sin^2\phi + \cos^2\phi)\}$

1112 γ $(3 - \cos^2\theta) + 2A\,(1 + \cos^2\theta)$

a_0 $(1 - \cos^2\theta) + 2A\,(\cos^2\theta)$

a $(7 - \cos^2\theta) + 2A\,(3 + \cos^2\theta)$

$a\gamma$ $\cos^2 \Theta\,(1 - 3\cos^2\theta + 4\cos^4\theta) + \sin^2\Theta\{(\cos^2\phi + 2A\sin^2\phi)$

$\times(1 - 4\cos^2\theta + 4\cos^4\theta) + \cos^2\theta(\sin^2\phi + 2A\cos^2\phi)\}$

1122 γ $(7 + 3\cos^2\theta) + 2A\,(5 - 3\cos^2\theta)$

a_0 $(1 - \cos^2\theta) + 2A\cos^2\theta$

a $(1 + \cos^2\theta) + 2A\,(1 - \cos^2\theta)$

$a\gamma$ $(1 - 3\cos^2\theta + 4\cos^4\theta) + (2A - 1)\sin^2\Theta\,\{(1 - 4\cos^2\theta + 4\cos^4\theta)$

$\times\sin^2\phi + \cos^2\theta\cos^2\phi\}$

1123 γ $(23 - 9\cos^2\theta) + 2A\,(7 + 9\cos^2\theta)$

a_0 $(1 - \cos^2\theta) + 2A\cos^2\theta$

a $(5 - \cos^2\theta) + 2A\,(2 + \cos^2\theta)$

1133 γ $(13 + 9\cos^2\theta) + 2A\,(11 - 9\cos^2\theta)$

a_0 $(1 - \cos^2\theta) + 2A\cos^2\theta$

a $(1 + \cos^2\theta) + 2A\,(1 - \cos^2\theta)$

1202 γ $1 + \cos^2\theta$

a_0 $1 + 3\cos^2\theta$

a 1

1211 γ $47 - 21\cos^2\theta$

a_0 $1 + 3\cos^2\theta$

a $13 + 21\cos^2\theta$

1212 γ $7 + 3\cos^2\theta$

α_0 $1 + 3\cos^2\theta$

α $7 - 5\cos^2\theta$

1221 γ $9 + 5\cos^2\theta$ $\alpha\gamma$ $(\cos^2\phi + \cos^2\theta - \cos^2\phi\cos^2\theta)$

α_0 $1 + 3\cos^2\theta$ $+\cos^2\Theta\,(1 - \cos^2\phi + \cos^2\phi\cos^2\theta)$

α $1 + \cos^2\theta$

2202 γ $(1 - 3\cos^2\theta + 4\cos^4\theta) + 6A\,(1 - \cos^2\theta)\cos^2\theta$

α_0 $6(1 - \cos^2\theta)\cos^2\theta + A\,(1 - 6\cos^2\theta + 9\cos^4\theta)$

α 1

2212 γ $(3 + 15\cos^2\theta - 16\cos^4\theta) + 6A\,(1 - 3\cos^2\theta + 4\cos^4\theta)$

α_0 $6(1 - \cos^2\theta)\cos^2\theta + A\,(1 - 6\cos^2\theta + 9\cos^4\theta)$

α $(5 - 3\cos^2\theta) + 6A\,(1 - \cos^2\theta)$

$\alpha\gamma$ $4(-4\cos^2\phi + 3)\cos^2\phi\cos^4\theta + (32\cos^2\phi - 27)\cos^2\phi\cos^2\theta$

$+(-16\cos^4\phi + 15\cos^2\phi + 1) + 6A\{4(\cos^2\phi - 1)\cos^2\phi\cos^4\theta$

$\Theta = \pi/2$ $+(-8\cos^4\phi + 8\cos^2\phi - 1)\cos^2\theta + (4\cos^4\phi - 4\cos^2\phi + 1)\}$

2213 γ $2(9 + 12\cos^2\theta - 5\cos^4\theta) + 3A(5 + 6\cos^2\theta + 5\cos^4\theta)$

α_0 $6(1 - \cos^2\theta)\cos^2\theta + A\,(1 - 6\cos^2\theta + 9\cos^4\theta)$

α $(34 + 16\cos^2\theta) + 9A(1 - \cos^2\theta)$

2223 γ $2(8 - 3\cos^2\theta + 5\cos^4\theta) + A\,(13 + 18\cos^2\theta - 15\cos^4\theta)$

α_0 $6(1 - \cos^2\theta)\cos^2\theta + A\,(1 - 6\cos^2\theta + 9\cos^4\theta)$

α $8(2 - 3\cos^2\theta + 3\cos^4\theta) + 3A\,(5 - 2\cos^2\theta + 3\cos^4\theta)$

2224	γ	$2(8 + 15\cos^2\theta - 9\cos^4\theta) + 3A(5 + 2\cos^2\theta + 9\cos^4\theta)$
	a_0	$6(1 - \cos^2\theta)\cos^2\theta + A(1 - 6\cos^2\theta + 9\cos^4\theta)$
	a	$2(15 + 6\cos^2\theta - \cos^4\theta) + 3A(9 + 6\cos^2\theta + \cos^4\theta)$
2234	γ	$(125 - 147\cos^2\theta + 180\cos^4\theta) + 6A(14 + 41\cos^2\theta - 45\cos^4\theta)$
	a_0	$6(1 - \cos^2\theta)\cos^2\theta + A(1 - 6\cos^2\theta + 9\cos^4\theta)$
	a	$(3 - 3\cos^2\theta + 2\cos^4\theta) + 3A(1 - \cos^4\theta)$
2303	γ	$5 - 6\cos^2\theta + 5\cos^4\theta$
	a_0	$1 - 2\cos^2\theta + 5\cos^4\theta$
	a	1
2312	γ	$27 + 78\cos^2\theta - 55\cos^4\theta$
	a_0	$1 - 2\cos^2\theta + 5\cos^4\theta$
	a	$23 + 36\cos^2\theta$
2313	γ	$33 + 42\cos^2\theta + 5\cos^4\theta$
	a_0	$1 - 2\cos^2\theta + 5\cos^4\theta$
	a	$10 - 9\cos^2\theta$
2314	γ	$5 + 2\cos^2\theta + 9\cos^4\theta$
	a_0	$1 - 2\cos^2\theta + 5\cos^4\theta$
	a	$2 + \cos^2\theta$
2322	γ	$31 - 42\cos^2\theta + 55\cos^4\theta$
	a_0	$1 - 2\cos^2\theta + 5\cos^4\theta$
	a	$35 + 162\cos^2\theta - 165\cos^4\theta$

$x = \cos^2\theta$

2324	γ	$79 + 126x - 45x^2$
	α_0	$1 - 2x + 5x^2$
	α	$21 - 6x - 11x^2$
3202	γ	$1 + 6x - 5x^2$
	α_0	$1 - 2x + 5x^2$
	α	1
3212	γ	$3 - 3x + 5x^2$
	α_0	$1 - 2x + 5x^2$
	α	$7 - 6x$
3213	γ	$81 + 102x + 25x^2$
	α_0	$1 - 2x + 5x^2$
	α	$31 + 12x$
3303	γ	$(1 + 111x - 305x^2 + 225x^3) + 12A(1 - 11x + 35x^2 - 25x^3)$
	α_0	$21(1 - 11x + 35x^2 - 25x^3) + 14A(13x - 60x^2 + 50x^3)$
	α	1
3312	γ	$(9 + 12x - 5x^2) + 6A(1 + 6x - 5x^2)$
	α_0	$21(1 - 11x + 35x^2 - 25x^3) + 14A(13x - 60x^2 + 50x^3)$
	α	$(7 + 9x) + 6A(1 + 2x)$
3313	γ	$(123 - 831x + 2765x^2 - 2025x^3) + 12A(1 + 111x - 305x^2 + 225x^3)$
	α_0	$21(1 - 11x + 35x^2 - 25x^3) + 14A(13x - 60x^2 + 50x^3)$
	α	$(13 - 9x) + A(1 - x)$

$x = \cos^2\theta$

3314	γ	$3(11 + 17x - 3x^2 + 7x^3) + 4A(9 - 9x + 39x^2 - 7x^3)$
3322	γ	$(8 - 3x + 5x^2) + 2A(5 - 12x + 15x^2)$
	α_0	$21(1 - 11x + 35x^2 - 25x^3) + 14A(13x - 60x^2 + 50x^3)$
	α	$(13 + 18x - 15x^2) + 6A(1 + 14x + 15x^2)$
	$\alpha\gamma$	$(96\{x - 1\}^2\{\cos^4\phi - \cos^2\phi\} + \{-x^2 - 24x + 25\})$ $+ 12A(\{1 - \cos^2\phi\}\{2x - 1\}^2 + x\cos^2\phi)$ $\quad \Theta = \pi/2$
3422	γ	$112 + 285x - 195x^2$
	α_0	$9 + 45x - 165x^2 + 175x^3$
	α	$229 - 30x + 585x^2$
3432	γ	$79 - 129x + 156x^2$
	α_0	$9 + 45x - 165x^2 + 175x^3$
	α	$29 - 69x + 387x^2 - 315x^3$
4422	γ	$5(8 + 15x - 9x^2) + 2A(17 + 60x - 45x^2)$
	α_0	$2(9x - 51x^2 + 81x^3 - 49x^4) + A(9 - 18x + 1110x^2 - 2100x^3 + 1225x^4)$
	α	$(9 - 22x + 45x^2) + 4A(25 - 82x + 153x^2)$
4432	γ	$(125 - 147x + 180x^2) + 20A(7 - 15x + 18x^2)$
	α_0	$2(9x - 51x^2 + 81x^3 - 49x^4) + A(9 - 18x + 1110x^2 - 2100x^3 + 1225x^4)$
	α	$3(91 + 301x - 315x^2 + 147x^3) + 840A(5 - 35x + 277x^2 - 105x^3)$
4532	γ	$1327 + 2868x - 1947x^2$
	α	$741 + 573x - 1929x^2 + 3751x^3$
4542	γ	$129 - 210x + 245x^2$
5422	γ	$11 + 30x - 21x^2$

5532	γ	$2(30 + 57x - 35x^2) + 5A(11 + 30x - 21x^2)$	$x = \cos^2\theta$
	α_0	$5(3 - 29x + 266x^2 - 826x^3 + 1029x^4 - 441x^5)$ $+ 7A(65x - 600x^2 + 1940x^3 - 2520x^4 + 1134x^5)$	
	α	$(147 - 9x + 57x^2 + 317x^3) + 24A(5 + 13x - 45x^2 + 55x^3)$	
5542	γ	$2(21 - 30x + 35x^2) + 15A(3 - 6x + 7x^2)$	
	α_0	$5(3 - 29x + 266x^2 - 826x^3 + 1029x^4 - 441x^5)$ $+ 7A(65x - 600x^2 + 1940x^3 - 2520x^4 + 1134x^5)$	
	α	$(225 - 972x - 2330x^2 - 8316x^3 + 3969x^4)$ $- 15A(9 + 180x - 1634x^2 + 2268x^3 - 1323x^4)$	
5642	γ	$723 + 1491x - 952x^2$	
	α	$10755 + 756x + 53298x^2 - 144386x^3 + 138915x^4$	