A DIRECT VELOCITY MEASUREMENT APPARATUS TO DETERMINE THE SPECIFIC ELECTRONIC CHARGE

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ABSTRACT

An apparatus for the precision measurement of the charge to mass ratio of the electron by a direct velocity method is described. This apparatus uses a resonant cavity as a velocity selector. A carefully collimated beam of electrons is deflected by the rapidly rotating magnetic field produced across the center of a right circular cylindrical cavity operated in two TM_{110} modes excited in time and space quadrature. Measurement is made of the beam voltages for which the beam receives no net transverse velocity from the cavity field. These voltages correspond to transit times of integral number of cycles of the field. From the cavity length and frequency the electron velocities are calculated, and from these velocities and the corresponding voltages the charge to mass ratio is obtained.

Preliminary results are presented indicating that the attainable accuracy with this instrument should be better than one part in ten thousand.

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DISCUSSION OF THE EXPERIMENT

1.0 Background

In the past few years a brilliant series of new experiments, based in large measure on the tremendous advance in microwave techniques during World War II, has effected a continuing transformation in the status of our knowledge of the fundamental atomic constants. Because of the interdependence of the various constants, the improvement has been general, and not least improved has been the charge to mass ratio of the electron. In 1947 the least squares adjustment of DuMond and Cohen, based on all the best of the then available data, yielded[1]

$$e/m = (1.75936 \pm 0.00027) \times 10^7 \text{ emu},$$

whereas the latest (November 1952) figure quoted by these analysts is[2]

$$e/m = (1.75888 \pm 0.00005) \times 10^7 \text{ emu}.$$

The errors are the standard errors of the overdetermined systems of measurements which constitute the input data.

The least squares averaging is greatly to be preferred to any individual determination as a means of stating a most probable value of an atomic constant. For the purposes of this paper, however, it will be instructive to consider the modern experiments which lead directly to e/mc, which is to say e/m in electromagnetic units. All recent determinations of this constant depend on the precision determination of a magnetic field. The reason for this situation is easily seen from the dimensionality of the constant. In a pure magnetic field a cycling electron obeys the law $eB\omega r = m\omega^2 r$

$$e/m = \omega/B \qquad (1)$$

Thus, in principle, the measurement can be made to depend for accuracy on a frequency and a magnetic field.

The modern method of standardizing magnetic fields is the proton resonance technique developed at Harvard[3]. Protons in a hydrogen rich material such as mineral oil are placed in the field to be determined, and the frequency at which they precess, i. e. that which will induce them to flip in the field, is measured by means of the energy absorbed from a radio frequency tickler coil. The precession frequency is given by

$$\omega_p = 2\mu_p B/\hbar = \gamma_p B \qquad (2)$$

where μ_p is the observed proton magnetic moment in the sample (mineral oil for instance) and γ_p is the so called proton gyromagnetic ratio. The possible resolution of such a method is greater than the precision with which a magnetic field can be absolutely measured. Thomas, Driscoll, and Hipple[4] have made an exceedingly careful determination of γ_p in absolute units at the National Bureau of Standards obtaining

$$\gamma_p(\text{oil}) = (2.67523 \pm 0.00006) \times 10^4 \ \text{sec}^{-1}\text{gauss}^{-1}$$

The stated error of 22 parts per million limits the precision of all magnetic field type measurements. Efforts are under way to improve this determination.

A direct comparison of the proton resonance frequency in oil with the electron cyclotron frequency has been made by Gardner and Purcell[5] also at Harvard. Their result is

$$\omega_e/\omega_p \text{ (oil)} = 657.475 \pm 0.008$$

a stated error of 13 parts per million. The electrons in this experiment are free and have nearly thermal velocities. Combining equations (1) and (2) we see that

$$e/m = \gamma_p \text{(oil)}\, \omega_e/\omega_p \text{ (oil)} \qquad (3)$$

$$= (1.75890 \pm 0.00005) \times 10^7 \text{ emu}$$

where the error results from combining the errors of Thomas, Driscoll, and Hipple and of Gardner and Purcell in the square.

The work of Gardner and Purcell has recently been superseded by the work of the atomic beams laboratory at Columbia University under I. I. Rabi. Following up the discovery in 1947 that the magnetic moment of the electron is not exactly one Bohr magneton as predicted by the Dirac theory[6,7] this group initiated an extensive investigation into the hyperfine structure of various atoms having one electron spectra[8,9,10]. This work culminates in the measurement by Koenig, Prodell and Kusch[11] of the Zeeman splitting in the ground state of hydrogen. From this datum they obtain directly the magnetic moment of the bound electron and hence with a relativistic correction the magnetic moment of a free electron, again in terms of μ_p(oil).

$$\mu_s/\mu_p \text{ (oil)} = 658.2288 \pm 0.0006$$

Simultaneously, on a suggestion by Schwinger[12], Karplus and Kroll[13] have calculated a radiative correction term to the electron magnetic moment. In terms of the fine structure constant and the Bohr magneton $\mu_0 = e\hbar/2m$ they obtain for the electron magnetic moment

$$\mu_s = \mu_0(1 + \alpha/2\pi - 2.973\,\alpha^2/\pi^2)$$
$$= \mu_0(1.0011454)$$

It was for the purpose of verifying this calculation that Koenig, Prodell and Kusch made their measurement. If we combine the result of Karplus and Kroll with that of Koenig, Prodell, and Kusch, we obtain

$$\mu_0/\mu_p \text{ (oil)} = 657.4757 \pm 0.0006$$

giving at once corroboration of the work of Karplus and Kroll and strength to Gardner's claim that his quoted error is conservative. Combining this value with that of Thomas, Driscoll, and Hipple we obtain

$$e/m = (1.75890 \pm 0.00004) \times 10^7 \text{ emu.}$$

This is one of the primary input data of the latest least squares adjustment, the error being entirely attributable to the experiment of Thomas, Driscoll, and Hipple.

Clearly the requirements which a determination must meet in order to be significant have changed radically since 1947 when the most accurate determination of e/m was that of Dunnington[14] with an accuracy of three parts in 10^4. In spite of this fact, the present experiment, which had its inception in 1947, appears capable of making a contribution to the general knowledge. The experiments on which the present least squares fit depends are few in number and show an inconsistency beyond that to be expected from the quoted probable errors. A larger body of evidence derived from experiments having confidence limits of error better than 100 parts per million appears greatly to be desired.

1.1 Discussion of Method

Direct velocity measurements of e/m are not new. In 1930 Perry and Chaffee[15] achieved an accuracy of about six parts in 10^4 by a method not greatly dissimilar to the present one. The method consists of accelerating electrons with a known voltage and measuring their velocity. The method offers the advantage that standards of voltage are easier to obtain than magnetic field standards. Difficulties are encountered in knowing exactly what energy is transmitted to the electrons, however, because of thermal and contact potentials, surface work functions, and space charge effects. In principle the measurement of velocity can be made in terms of a length and a time of flight, both of which are easily measured quantities. Difficulties are encountered again, however, in knowing exactly what the field configurations and electron trajectories are in the apparatus in which the velocity is determined. Furthermore, the method has characteristically low resolution because of the exceedingly small parameters on which the measurement must depend.

In the method to be discussed here many of the above objections have been overcome. Electrons originating on a very small area of a directly heated thoriated tungsten filament are accelerated to potentials of one or two thousand volts and are passed in a small, carefully collimated beam through the center of a precisely constructed radio frequency cavity operating in such a combination of modes that the electrons encounter a transverse magnetic field rotating at a frequency of about 2618 megacycles. The electrons are deflected by this field in a manner depending on the time required for an electron to cross the cavity. In particular, so called resonance velocities exist, at which velocities the electrons leave the cavity with no net velocity component transverse to the original direction of flight. The resonance velocities correspond closely to those for which the electrons traverse the cavity in an integral number of cycles of the magnetic field.

The technique of measurement is to measure as a function of beam voltage the small deflections produced by the cavity field in the neighborhood of a resonance voltage and obtain that voltage by interpolation. The measurement is performed under similar conditions of the electron source for two resonances corresponding to different numbers of cycles transit time. The velocities at the two resonances are then, without several corrections, Lf/k and Lf/n, where L is the length of the cavity, f is the frequency, and k and n are integers (in the case to be reported 3 and 4). Then non-relativistically

$$e(V_k - V_n) = ((Lf/k)^2 - (Lf/n)^2)m/2$$

$$e/m = \frac{L^2f^2}{2(V_k - V_n)} (1/k^2 - 1/n^2) \qquad (1)$$

None of the many corrections which apply are large enough to alter the general characteristics of equation (1).

One of the principal advantages of the method is immediately obvious. By taking the difference between V_k and V_n for similar conditions at the cathode the many imponderables relating to actual electron voltage cancel out. The electron gun is constructed in such a way that the fields at the filament are independent of total beam potential. Measurements are made by determining two resonance voltages in quick succession and taking the difference.

It is believed that the factor which will ultimately limit the accuracy attained with the present apparatus is the ability to resolve the resonance voltages. Many other factors affecting the possible precision must be considered, however. In the next few sections the structure of the apparatus as it affects the precision of the measurement will be described. Further chapters will discuss the problems involved in greater detail.

1.2 Factors Affecting the Length Measurement.

The length to be determined, L, is the length of the cavity traversed by the electrons. The cavity is the usual pillbox type, a right circular cylinder with plane end walls. The cylinder which forms the side walls is a ring of molybdenum of outside diameter 6.5 inches and inside diameter 5.487 inches, this being the dimension which determines the frequency. The width of the ring, i. e. the length, L, is known at present to be 1.1954 inches by micrometric methods. The end surfaces from which this dimension is obtained are ground flat and parallel with high precision, measurements with an electronic comparator indicating that they are parallel to within 0.00001 inches. Molybdenum was chosen for the ring because it provides the most advantageous combination of low thermal expansion coefficient and high thermal conductivity.

The end walls of the cavity are 1/2 inch copper plates electroplated with about 0.02 inch of chromium which has been ground and polished optically flat. The figure of the surfaces is slightly temperature dependent because of bimetallic effects. The temperature of the plates is regulated and the heat dissipated in the walls carried away by cooling water circulated through coils soldered to the backs of the plates. Great care has been taken to avoid introducing mechanical stresses into the plates with the water lines.

The contact between the end plates and the spacer ring is guaranteed by the fact that the mode in which the cavity operates is one in which currents must cross the joints. The Q of the cavity is critically dependent on the quality of these joints. Fortunately both surfaces are near optical flats, and contacts giving very close to the theoretical Q can be obtained. It seems reasonable to assert that L can be determined to better than one part in 10^5 insofar as it is determined by the physical dimensions of the cavity.

Several corrections to the cavity length must be applied. The electrons enter and leave the cavity through 1/8 inch diameter holes cut through the centers of the end plates. These holes are of necessity very precisely made. The field of the cavity at the holes has been the subject of a very difficult calculation by Professor W. R. Smythe[16]. The effect of the holes is to increase the apparent length of the cavity by about 1.5%. The skin effect is not negligible, the chromium surface having about the resistivity of gold. At the frequency used the skin depth is about 1.5×10^{-4} centimeters.

The field actually encountered by the electrons is that of two TM_{110} modes at right angles to each other excited a quarter cycle out of phase. The resulting field, while it is everywhere independent of the length variable, z, (except in the region of the end holes), is a pure magnetic field only at the center of the cylindrical wall. Off axis the magnetic field falls off slowly and electric field components make their appearance. The calculation of electron trajectories in this field is complicated. More detailed consideration will be given in Chapter 2.

1.3 Factors Affecting the Frequency Measurement

The power to drive the cavity is obtained from two Sperry type 8529 klystrons rated at 100 watts output each. Thewe two are driven by a single Sperry type SRL-6 reflex klystron through a plumbing system which provides for adjustment of phase and magnitude of excitation. The SRL-6 is powered by a highly stable supply and gives under optimum operating conditions about four watts at 2618 megacycles with a noise bandwidth of about 20 kilocycles. Power delivered to the cavity is less than 50 watts in each mode because of the difficulty of obtaining proper loading and sufficient drive for the power klystrons. A power of 50 watts corresponds to a field in the cavity of about three gauss.

Measurement of the frequency is made by beating a signal from the output of one of the power klystrons against the 2610th harmonic of a one megacycle standard crystal and measuring the difference frequency with a calibrated radio receiver. The crystal is in turn calibrated by comparison with the standard frequency signals broadcast by the National Bureau of Standards. This measurement can easily be made to any accuracy required, estimating, if necessary, the center of the output noise band. At present the reflex klystron shows a tendency to wander in frequency over about a 50 kilocycle band and must be kept on frequency by manual adjustment of the reflector voltage, the actual frequency determining parameter. Some form of automatic frequency control might be desirable.

1.4 Factors Affecting the Voltage Measurement

The feature of this method of cancelling thermal and contact potentials has already been discussed. It is felt that with these difficulties eliminated the limit of accuracy of the voltage measurement is set by the secondary standard cell bank available at this laboratory. This bank of six cells was calibrated at the Bureau of Standards in 1943, and the accuracy of the present mean of these cells can be estimated from intercomparison of the individual cells. The cells are permanently enclosed in a very fine thermal regulated box constructed by Mr. John Harris as part of his activities in setting up the standards laboratory at the Institute. It is testimony to the quality of his work that the mean still appears to be good to one part in 10^5, close to the ultimate attainable accuracy of secondary standards.

The electron beam voltage is compared with standard cells calibrated against the secondary standard bank by means of a precision voltage divider constructed also by Mr. Harris[17] and kindly lent this project by Professor J. W. DuMond. This remarkable instrument provides a voltage ratio of sufficient precision to remove it from error considerations. For further consideration of the voltage measurement the reader is referred to Mr. J. I. Lauritzen.

The voltage to accelerate the electrons is obtained from a highly stabilized supply which is continuously variable from 600 to 2300 volts. This supply is subject to slow drifts, principally thermal drifts of the reference batteries, but can be reset to about ± 0.02 volts over periods of a few minutes. Its voltage can be monitored continuously be the voltage standardizing apparatus.

Electrons to be accelerated originate from a thoriated tungsten filament constructed from 0.013 inch wire tightly bent into a hairpin and ground flat on the tip. The area from which electrons originate is limited by a molybdenum stop of diameter 0.02 inches placed about 0.005 inches in front of the tip of the hairpin. Because of the focusing action of the gun, the actual source area for the beam used is considerably smaller than the stop. Filament voltage, about two volts, is supplied by a storage battery in series with ballasting lamps. The high voltage is inserted at the midpoint of the filament potential by means of a balancing resistor. The above precautions are believed sufficient to prevent the filament supply from introducing either appreciable voltage drift or appreciable spread in voltage.

The filament is operated in the field of the first anode of the electron gun. The potential of this anode is maintained at 300 volts independent of total beam potential. The filament is operated temperature limited. By these means the spread in voltage of the electrons in the beam is kept small and constant from one resonance determination to the next.

1.5 Factors Affecting the Resolution

It seems clear that the ultimate accuracy with which this experiment can be performed will be set by the attainable resolution. The two resonance voltages chosen lie at approximately 1154 volts and 2061 volts respectively. The difference of about 907 volts is the quantity to be resolved. Unfortunately, principally because of practical limitations imposed by the collimation, it is not possible to measure directly the point at which the deflection produced by the cavity field is zero. Small deflections must be measured over a range of a few volts each side of resonance and the resonance located by interpolation. The deflection sensitivity is exceedingly low, and for this reason the collimation of the beam is of crucial importance.

Collimation is achieved geometrically by passing the beam through two pinholes spaced nearly five feet apart. The first is in the electron gun and is of diameter 0.002 inches, while the second, of diameter 0.0035 inches, is located just beyond the exit hole of the cavity. This location eliminates from consideration small displacements of the beam suffered during passage through the cavity field. Pains are taken to see that no fields other than that of the cavity act on the beam, the entire length of flight being shielded by concentric layers of mu-metal.

The beam after leaving the second collimating pinhole drifts a distance of about 50 inches and falls on a collecting screen in which is punched a pinhole about 0.009 inches in diameter, approximately the size of the beam as determined by the collimating system. Because neither pinhole can be made infinitesimal, the beam does not have sharp edges. This fact accounts for the impossibility of determining the resonance voltage directly.

The quantity measured is the current passing through the pinhole in the collecting screen. This current is collected in a Faraday cup and measured by means of a vacuum tube electrometer having the one-half meter long scale of a mirror galvanometer for its presentation meter. The actual range of voltage variation on the grid of the electrometer tube is less than one tenth of a volt. By this means a highly linear scale which can be read with precision is obtained.

The current received through the pinhole is a very nearly linear function of the angular deflection of the beam over a range of deflections for which the uniformly illuminated portion of the spot is deflected so as to be cut by the edge of the pinhole. A more detailed discussion of this fact will be presented in a later chapter. To a first degree of approximation the angular deflection of the beam, i. e. the half angle of the cone into which the beam is fanned on a time average, is, where B is the magnitude of the magnetic field, ω is its angular frequency, and t is the time an electron is in the field,

$$\theta = \frac{2eB}{m\omega}\left|\sin\omega t/2\right| \qquad (1)$$

For B of three gauss the coefficient $eB/m\omega = 1/311$. If we let L' be the effective length of the cavity with end holes, we have for a first approximation[16]

$$L' \doteq L + 2(0.149r) = 1.214 \text{ inch} \qquad (2)$$

where r is the radius of the entrance holes, 0.0625 inch. In terms of the electron velocity, v, the time to cross the cavity is $t = L'/v$. Thus the resonance condition is $\omega L'/2v = n\pi$, the relationship used to derive 1.1 (1). Using this relationship and the energy equation $2eV = mv^2$ we obtain for the deflection sensitivity at the resonance corresponding to integer n and voltage V

$$d\theta/dV = n\pi\frac{eB}{m\omega V} \qquad (3)$$

Thus as a distance of 50 inches from the exit of the cavity the linear deflection sensitivity is

$$dr/dV = 50\frac{eBn\pi}{m\omega V}$$

$$= n/2V \quad \text{inch/volt} \qquad (4)$$

The result of plotting the current through the collector pinhole as a function of beam voltage for constant amplitude and frequency of the cavity field is a peak of current in the neighborhood of each resonance having very nearly straight sides and a rounded top. If plotted against $V^{-1/2}$, these peaks are symmetrical and their centers determine the resonance voltages.

The prediction of equation (4) is based on the assumption that the field of the cavity terminates sharply at the edge. In addition to increasing the apparent length, the end holes reduce the deflection sensitivity somewhat, particularly for the 1154 volt peak. On the basis of equation (4) we would expect the half widths of the current peaks, corresponding to deflection equal to the diameter of the beam, to be 6 volts at 1154 volts and 14 volts at 2060 volts. Actual widths for a cavity field of three gauss are about 10 volts and 13 volts.

Obviously the assertion that peaks this broad can be resolved to a tenth of a volt or better can rest only on experimental evidence. Actually peaks of exceptionally good shape are regularly obtained, the sides being straight to as good precision as the instrumentation permits. The problem of resolution reduces to one of obtaining sufficient stability of beam and cavity field to permit the accumulation of sets of points consistent with one another. In view of the fact that the desired resolution corresponds to a deflection of less than 0.0001 inch, this is perhaps not surprising. The resolution prospects are discussed in detail in the chapter on results. It now appears possible to obtain confidence limits on individual runs of ± 0.1 volts for the difference between peaks. Statistical interpretation of a number of runs can of course improve this figure.

1.6 Beam Formation and Stability Problems

The stability problems mentioned in the previous section are those of beam position and current and of cavity field amplitude and frequency. The latter two are problems limited by the available apparatus. The klystrons used are regarded by Sperry as experimental models and are used here in an application considerably different from that contemplated for them by their designers. They work exceedingly well. The requirements of this experiment are stringent, however. By careful monitoring of amplitude and frequency and good stabilization of power supplies it is possible to obtain adequate stability over sufficient periods of time to obtain data. Rapid accumulation of data is essential.

The problem of obtaining an adequate, well collimated beam which is stable in position to 0.0001 inch and whose position and current are nearly independent of voltage has been the major experimental problem accupying this group for the past three years. Difficulties arise principally from the tendency of surfaces exposed to the beam to accumulate charged insulated layers and from the tendency for slowly moving charged ions to accumulate in the beam. Both effects produce undesired focusing or defocusing effects and tend to fluctuate with periods of the order of seconds. Problems of cathode emission stability and life must also be faced. Oxide cathodes, in spite of certain advantages, have been found too short lived in the best vacuum obtainable in this system to be used. All the undesirable effects are at least indirectly due to residual gas in the system. For this reason effort has been spent on obtaining as good a vacuum as possible. Using continuous oil diffusion pumping and cold trapping a pressure of about 3×10^{-6} millimeters of mercury has been reached. The concentration of volatiles in the electron gun region is reduced by local cold trapping. It is felt that these pressures are as low as can be reached in a system having as much metal surface as this one without resort to local pumping.

Periodic cleaning is the only solution to the problem of charged layers. The cold traps in the electron gun region greatly increase the period between cleanings. These are filled only when the gun is being operated. Care is taken to have the beam strike no unnecessary surfaces. Currents in the cavity region are so low that the second collimating pinhole rarely requires cleaning.

The solution to the ion problem is to have no intense beams pass throuth regions where there are no fields to sweep out ions. In the five foot drift space after the beam leaves the first collimating pinhole, necessarily field free, total beam current cannot be made much greater than 10^{-7} amperes. In order to yield usable intensity at the second collimating pinhole the electron gun must produce a well collimated beam. The present gun produces a beam which is about one quarter inch in diameter when it falls on the viewing screen containing the entrance hole to the cavity. Since the second collimating pinhole is 0.0035 inches in diameter, the corresponding maximum beam current is 2×10^{-11} amperes. At this intensity the emission from the first collimating pinhole is about 250 amperes per square centimeter per stearadian. The small size of the beam makes it necessary to keep motion of the beam small. When the electron gun is clean, the motion of the spot is imperceptible as voltage is increased from 1100 to 2100 volts.

The present principal cause of beam instability at the collector pinhole is a combination of phenomena most of which have been mentioned. Patchy emission from the thoriated tungsten filament and fluctuating charging effects in the electron gun cause instabilities in the intensities of current delivered to various parts of the beam falling on the cavity entrance hole. In addition to causing the measured beam current to fluctuate, an effect which can be minimized by proper choice of the part of the quarter inch beam actually used, these instabilities produce fluctuations in the ion cloud and cause the position of the beam on the measuring pinhole to fluctuate. When these fluctuations reach the order of 0.001 inches, they prevent accurate placement of the beam on the pinhole and hence accurate measurements of deflections in the immediate neighborhood of the resonance. Measurements for larger deflections are less affected because the beam is blown up to larger size. This effect is seen to be dependent, in part, on the beam current used, and is another good reason for desiring to operate at less than maximum current. The amount of difficulty encountered from this effect varies from day to day and from peak to peak and accounts for the observed variations in quality of runs. The accumulation of satisfactory data is a slow process and considerably dependent on the skill of the operators.

1.7 History of the Project

The method which has been described is the outgrowth of a study undertaken by Dr. Charles H. Wilts, then a graduate student, at the request of Professor W. R. Smythe, who wished to investigate the possibility of using the microwave apparatus available after World War II. Dr. Wilts concluded that a direct velocity measurement was feasible using transverse deflection by magnetic fields in a resonating cavity. In 1947 he designed and built[13] a very successful pilot experiment demonstrating the possibilities of the method. In the same year Dr. George C. Dacey, then also a graduate student, began designing the present instrument incorporating most of the work of Dr. Wilts, but with a number of refinements designed to permit the accuracy of the experiment to be pushed to one part in 10^5 if the resolution permitted.

The author joined the project in 1948 when the instrument was in early stages of construction, assuming principal responsibility for electrical power supply and instrumentation, although rigid compartmentalization of work on the project proved to be unproductive. The great majority of construction and shop work was done by the students. The majority of construction was completed by 1950, but it soon became obvious that the early estimates of the difficulty of the beam formation problem were naive.

The project was joined in 1950 by Mr. John I. Lauritzen, who assumed responsibility for the precision voltage measurement and general analysis of the apparatus as a precision instrument. Dr. Dacey was forced to leave in 1951. He demonstrated the efficacy of the instrument[19], but it was not until over a year later that sufficient beam stability was obtained to make hopeful a measurement of modern precision. The electron gun now in use was designed and built in the fall of 1952. Work with the beam it produced has dictated a number of refinements in general supply and instrumentation which have been made. The apparatus is now a working, integrated whole. The arduous procedure of collecting and analyzing data necessary to push the measurement to the attainable limit falls to Mr. Lauritzen.

The entire work of this project has been directed by Professor W. R. Smythe without whose guidance and encouragement this work could never have been pushed through. The project is supported by joint contract with the Office of Naval Research and the Atomic Energy Commission.

THEORY OF THE VELOCITY SELECTOR

2.0 The Cavity Field

The vector potential in a right circular cylindrical cavity oscillating in the TM_{110} mode characterized by angular frequency ω and wave number $\beta = \omega/c$ is in cylindrical coordinates ρ, φ, z[20]

$$\underset{\sim}{A} = \underset{\sim}{k} 2B\beta^{-1} J_1(\beta\rho)\cos(\varphi + \psi)\cos(\omega t + \alpha) \quad (1)$$

As is shown in Appendix 1, the vector potential of a cavity excited in two such modes with arbitrary space phase angles ψ and time phase angles α can always be reduced to the form

$$\underset{\sim}{A} = \underset{\sim}{k} 2\beta^{-1} J_1(\beta\rho)(B_1 \sin\varphi\cos\omega t - B_2\cos\varphi\sin\omega t) \quad (2)$$

by proper choice of time origin and space coordinates.

Using the expansion

$$J_1(v) = v/2 - v^3/16 + v^5/384 \ldots$$

we write

$$\underset{\sim}{A} = \underset{\sim}{A}_0(1 - (\beta\rho)^2/8 + (\beta\rho)^4/192 \ldots) \quad (3)$$

where

$$\begin{aligned}\underset{\sim}{A}_0 &= \underset{\sim}{k}(B_1\rho\sin\varphi\cos\omega t - B_2\rho\cos\varphi\sin\omega t)\\ &= \underset{\sim}{k}(B_1 y\cos\omega t - B_2 x\sin\omega t) \quad (4)\end{aligned}$$

From $\underset{\sim}{A}_0$ we calculate $\underset{\sim}{B}_0 = \nabla \times \underset{\sim}{A}_0$

$$\underset{\sim}{B}_0 = \underset{\sim}{i} B_1\cos\omega t + \underset{\sim}{j} B_2 \sin\omega t \quad (5)$$

In this experiment $\beta = 0.55\ \text{cm}^{-1}$. The circularly polarized case, $B_1 = B_2$, is simpler to treat than the more general elliptically polarized case and will be considered first.

2.1 Simplified Theory

The principal agent in deflecting the electrons is the magnetic field of equation 2.0 (5). Assuming that the electrons remain sufficiently close to the axis of the cavity so that the electric field components derivable from $\underset{\sim}{A}$ can be neglected and the dependence of $\underset{\sim}{B}$ on ρ can also be neglected, Dr. Dacey has obtained equations of motion for the circularly polarized case in closed form[19]. Taking for his field

$$B_x = B\cos\omega t \qquad\qquad (1)$$
$$B_y = B\sin\omega t$$

and for initial conditions at $t = 0$

$$\dot{x} = \dot{y} = 0 \qquad\qquad (2)$$
$$\dot{z} = v$$

and observing that in the absence of electric fields the only relativistic effect which need be considered is the mass correction associated with v, he obtains by going to $\ddddot{z}$ and integrating the resulting harmonic equation

$$\dot{z} = vk^{-2}(\omega^2 + b^2\cos kt) \qquad\qquad (3)$$
$$\dot{x} = -\omega bvk^{-2}(1 - \cos kt)\cos\omega t + bvk^{-1}\sin kt\sin\omega t$$
$$\dot{y} = -\omega bvk^{-2}(1 - \cos kt)\sin\omega t - bvk^{-1}\sin kt\cos\omega t$$

where $b = eB/m$ (emu) and $k^2 = \omega^2 + b^2$. To assist in visualizing the result let us define $p = k - \omega$ and view the velocity in a system rotating with angular velocity $-p$.

Defining

$$r = \dot{x} \cos pt - \dot{y} \sin pt$$
$$s = \dot{x} \sin pt + \dot{y} \cos pt \qquad (4)$$

so that

$$\dot{x} = r \cos pt + s \sin pt$$
$$\dot{y} = - r \sin pt + s \cos pt \qquad (5)$$

we find that equations (3) reduce to

$$r = bv\omega k^{-2}(1 - \cos kt + p\omega^{-1}\sin^2 kt)$$
$$s = - bv\omega k^{-2}(\sin kt + p\omega^{-1}\sin kt \cos kt) \qquad (6)$$

We saw in section 1.5 that $eE/m\omega = b/\omega = 1/300$ so that $k^2 \doteq \omega^2(1 + 10^{-5})$ and p/ω is of the order of 5×10^{-6}. Ignoring the p/ω term we obtain for the magnitude of the transverse velocity

$$(r^2 + s^2)^{1/2} = 2bv\omega k^{-2} \left|\sin kt/2\right| \qquad (7)$$

Equation 1.5 (1) follows immediately from this equation upon assuming that the beam enters and leaves the field through sharply discontinuous boundaries.

We could have made a simple calculation based on the assumption that $\dot{z}$ is constant. Under this assumption we need merely sum the impulse given to the electrons in the x and y directions to obtain

$$\dot{x} = bv\omega^{-1}(1 - \cos\omega t)$$
$$\dot{y} = - bv\omega^{-1}\sin\omega t \qquad (8)$$

$$x - x_0 = bv\omega^{-2}(\omega t - \sin\omega t)$$
$$y - y_0 = bv\omega^{-2}(\cos\omega t - 1) \qquad (9)$$

Comparison of equations (8) with equations (6) and (5) reveals that nothing startling has been introduced by the more precise calculation. The secular time delay pt and the effective reduction in average z component of velocity could both have been estimated by averaging over the simplified trajectory if no exact solution had been found.

A solution incorporating the first order electric field in the circularly polarized case has been obtained by Mr. Lauritzen. The analysis is necessarily much more complicated since it mixes the position of the electrons into the considerations. Inasmuch as the exact calculation of appropriate correction terms is the assigned responsibility of Mr. Lauritzen, no effort to repeat his work will be made here. Instead, the remainder of this chapter will utilize the approximation $\dot{z} = v$ to treat two effects ignored in the foregoing treatment.

The two effects most requiring consideration are the effect of the end holes and the effect of possible departure from circular polarization of the field. These effects will be treated together in the next section assuming that the field can be taken as that on the axis of the holes. A more detailed treatment of the field in the holes is given in Appendix 2.

2.2 Effect of the End Holes

A treatment similar to that described at the end of the previous section gives similar results for the elliptic case where the orbits in the $(\dot{x}, \dot{y})$ plane are ellipses instead of circles. The point of view to be adopted here is better adapted to the end hole calculation. We modify the $\underset{\sim}{A}$ of equation 2.0 (4) so that

$$A_z = f(z)(B_1 y\cos\omega t - B_2 x\sin\omega t) \tag{1}$$

$$\begin{aligned} B_x &= \partial A_z/\partial y = f(z)B_1\cos\omega t \\ B_y &= -\partial A_z/\partial x = f(z)B_2\sin\omega t \end{aligned} \tag{2}$$

For f(z) we take Smythe's value for the magnetic field on the axis of the holes. Thus f(z) is one inside the cavity away from the holes, zero outside the cavity beyond the fringing fields, and has intermediate values between.

We choose the origin of z midway between the end holes so that f(z) is a symmetric function of z and consider an electron which reaches the plane z = 0 at time $t = \tau$. We define new fixed coordinates X, Y, z such that at time τ the vector B is in the X direction. Thus if

$$B(\tau) = (B_1{}^2\cos^2\omega\tau + B_2{}^2\sin^2\omega\tau)^{1/2} \tag{3}$$

we have

$$\begin{aligned} B_X &= B_x\, B_1\cos\omega\tau/B(\tau) + B_y\, B_2\sin\omega\tau/B(\tau) \\ B_Y &= -B_x\, B_2\sin\omega\tau/B(\tau) + B_y\, B_1\cos\omega\tau/B(\tau) \end{aligned} \tag{4}$$

Putting in B_x and B_y from equation (2) we have

$$B_X = f(z)(1/B(\tau))(B_1{}^2\cos\omega t\cos\omega\tau + B_2{}^2\sin\omega t\sin\omega\tau)$$
$$B_Y = f(z)(B_1B_2/B(\tau))(\sin\omega t\cos\omega\tau - \cos\omega t\sin\omega\tau) \qquad (5)$$

B_Y is obviously a function of $(t - \tau)$. We can express B_X similarly by expanding $C_1\sin(t - \tau) + C_2\cos(t - \tau)$ and comparing coefficients. Thus

$$B_X = f(z)(B_2{}^2 - B_1{}^2)(B(\tau))^{-1}\cos\omega\tau\sin\omega\tau\sin\omega(t - \tau)$$
$$+ f(z)B(\tau)\cos\omega(t - \tau) \qquad (6)$$
$$B_Y = f(z)B_1B_2(B(\tau))^{-1}\sin\omega(t - \tau)$$

We now apply the equations of motion

$$dp_X/dt = -e\dot{z}B_Y = -evB_Y$$
$$dp_Y/dt = e\dot{z}B_X = evB_X \qquad (7)$$

where p_X and p_Y are the transverse components of momentum and we have taken account of the sign of the charge on the electron regarding e as its magnitude only. The approximation $\dot{z} = v$, $z = v(t - \tau)$ is made. If we combine equations (6) and (7) and integrate from time t_o before the electron enters the field of the cavity to time t_1 after it leaves the field of the cavity, we see that integrals of $f(z)\sin\omega(t - \tau)$ vanish as integrals of an odd function. We are left with

$$v_X(t_1) - v_X(t_o) = 0$$
$$v_Y(t_1) - v_X(t_o) = vb(\tau)\int_{t_o}^{t_1} f(z)\cos\omega(t - \tau)\,dt \qquad (8)$$

where

$$b(\tau) = B(\tau)e(1 - v^2/c^2)^{1/2}/m \qquad (9)$$

The approximation in (9) that the velocity is the same on both sides of the cavity is certainly no worse than the previous approximation. We can integrate equation (3) by parts to obtain

$$\Delta v_Y = b(\tau)v^2\omega^{-1}\int_{t_0}^{t_1} f'(v(t - \tau))\sin\omega(t - \tau)dt \quad (10)$$

For the situation of a sharply terminated field we may treat $f'(z)$ as a positive delta function at $-L/2$ and a negative delta function at $+L/2$. Equation (10) then gives

$$\Delta v_Y = -2b(\tau)v\,\omega^{-1}\sin\omega L/2v \quad (11)$$

which we can compare with equation 2.1 (7). Here $t = L/v$. The deflection pattern, instead of being circular, is now elliptical. Otherwise the expressions are similar.

Professor Smythe's calculation shows that to a high degree of approximation the axial field derivative is symmetrical about a point a distance $0.149r$ inside the end hole, where r is the radius of the hole. We thus expect the integral of equation (10) to vanish if we adjust v so that when $\omega(t - \tau) = n\pi$

$$z = v(t - \tau) = n\pi v/\omega = L/2 + 0.149r = L'/2 \quad (12)$$

This is the resonance condition offered in section 1.5.

If we express equation (8) in terms of $\theta = \Delta v_Y/v$ and take the derivative with respect to v, ignoring the very small dependence of $b(\tau)$ on v, we obtain

$$d\theta/dv = b(\tau)\int_{t_0}^{t_1}(t - \tau)f'(z)\cos\omega(t - \tau)dt \quad (13)$$

At resonance the value of $\cos\omega(t - \tau)$ passes through unity as the electron passes the points $z = \pm L'/2$. Particularly for slowly moving electrons it falls off rapidly on both sides of one of these points. The integral of $f'(z)$ must be unity, so if the function $f'(z)$ extends over very great limits on either side of $L'/2$ the value of $d\theta/dv$, and hence of $d\theta/dV$, is reduced below its maximum value obtained by letting $f'(z)$ be a delta function. This is indeed the case and accounts for the reduced sensitivity described in section 1.5.

CONSTRUCTION OF THE VACUUM TUBE

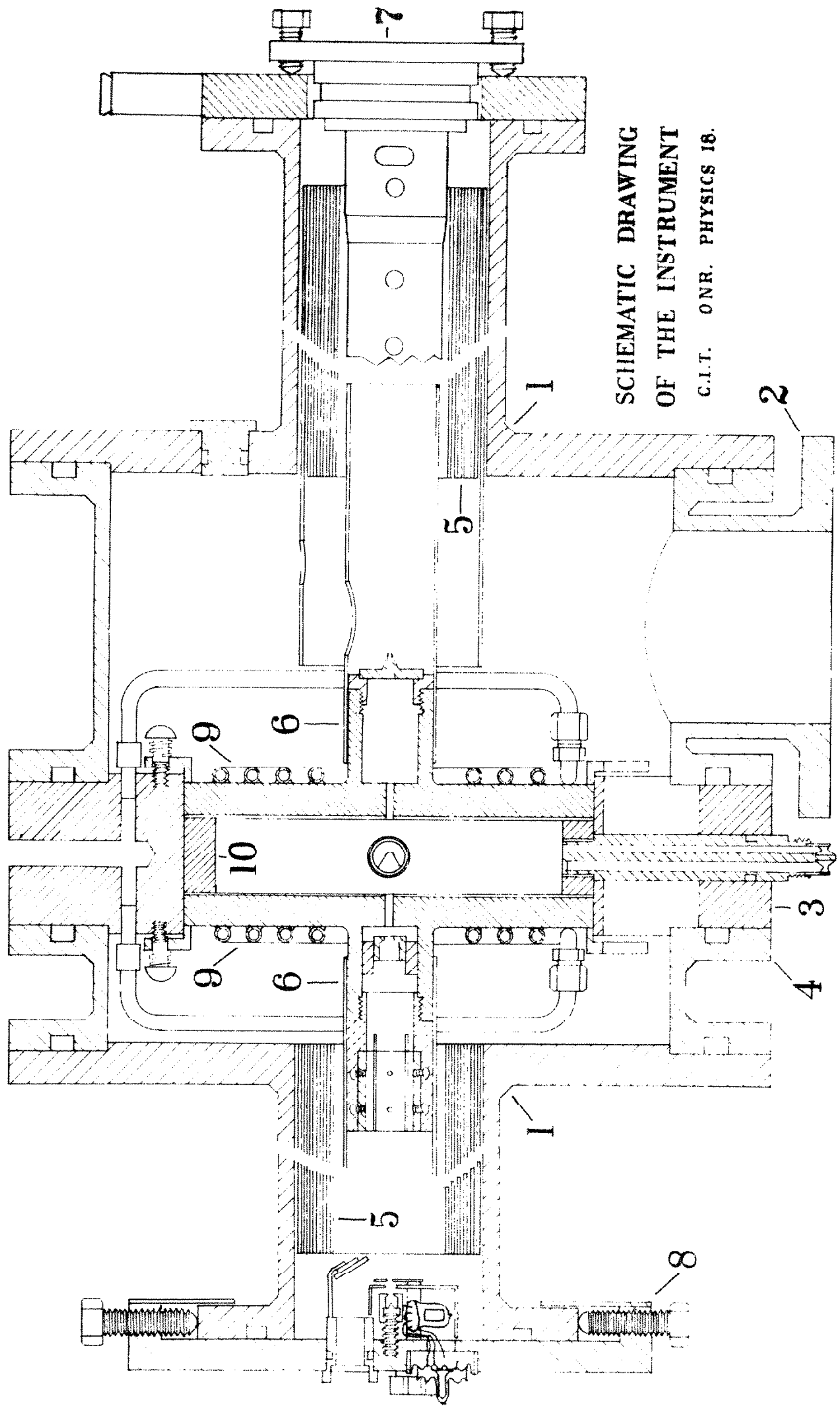
3.0 General Description

The design and construction of the vacuum tube is discussed at length by Dr. Dacey in his thesis[19]. The reader is referred to that source for details. The accompanying photograph and partially schematic drawing will assist in visualizing the structure. Numbers in the following description refer to the corresponding numbers on the diagram.

The vacuum tube consists of two horizontal, aluminum tubes (1) cantilevered by means of welded end plates from the supporting center structure. These tubes, shown foreshortened in the diagram, are 3.5 inches in diameter and four feet long. The center structure is twelve inches in diameter and consists of three pieces, the welded aluminum vacuum header (2) which supports the entire structure, the massive brass ring (3) which supports the cavity and contains access holes for water lines, vacuum pumping, and radio frequency power, and an auxiliary mounting ring (4) also of aluminum. The pieces are bolted together by hexagonally placed bolts not shown and sealed for vacuum by "O" rings in the grooves shown. The pumping tube of the vacuum header, by means of which the structure is supported, is bolted by means of the bottom flange to the supporting frame and vacuum system. These pieces, together with the end plate structures on the long tubes form the vacuum envelope.

The drift spaces inside the end tubes are shielded from electromagnetic fields by nests of alternated copper and mu-metal tubes (5). This latter material has a magnetic permeability for small fields of 50,000 or more when properly heat treated. Shielding in the cavity region is achieved with more difficulty. Small mu-metal tubes (6) shield the beam up to the cavity. Shielding for the cavity itself is by large mu-metal tubes which enclose the entire center structure, easily seen in the photograph, and by rings of mu-metal placed just inside the vacuum header and the mounting ring. The effect of the shielding is to make the operation of the instrument essentially independent of external fields. Without shielding, the Earth's field would make it impossible to pass the beam through the tube.

The electron gun and accompanying end plate structure (7) are shown separately (Chapter 7). The collector end plate (8) is movable, being propelled by large screws. The sliding joint is sealed by an "O" ring. On the plate is mounted the electrometer structure inside a shielding can. The electrometer tube and resistor are mounted inside the vacuum for protection and shielding. The position of the collector pinhole can be adjusted by sliding the plate in order to line up the instrument.



SCHEMATIC DRAWING
OF THE INSTRUMENT
C.I.T. ONR. PHYSICS 18.

3.1 Cavity Construction and Mounting

The cavity construction has already been discussed. The end plates (9) are of half inch copper plated with a heavy layer of hard chromium by the usual industrial process. The chromium has been ground and polished optically flat by the optical laboratory of the Mount Wilson Observatory. Brass tubes, 1.5 inches in diameter, were fitted precisely to the plates during construction, and all turning operations on the plates were made holding the structure by means of these tubes. In particular, the 1/8 inch end holes at the centers of the plates were drilled in register with the 6.500 inch diameter edges of the plates. Short drill rods were pressed into the holes during plating and removed only when the plates were lapped. The brass tubes are remounted and support the apertures and deflection plates associated with the cavity. The cooling coils on the backs of the end plates were soldered before turning.

The molybdenum ring (10) which forms the cavity side wall and spacer ring is made of two half rings sintered and forged by powder metallurgy methods to within 1% of the density of the element. These half rings were ground square and gold brazed together before machining. The resulting ring was then machined to the same outer diameter as the end plates and the inner diameter made

5.487 inches to fix the cavity frequency. Four symmetrically placed 5/8 inch holes were drilled through the ring to provide for power inlet. The end surfaces of the ring were finally ground and lapped to be parallel to within about 1/100,000 of an inch as verified by means of a precision comparator. The high heat conductivity, low thermal expansion coefficient, resistance to corrosion, and susceptibility to grinding operations of molybdenum make it the ideal material to use. Dr. Dacey has calculated that for a dissipation in the cavity of 200 watts the heat conducted out through the joints with the end plates is sufficient to make the expansion negligible.

Efforts to plate molybdenum with some high conductivity metal were not successful. The only process which met with success was wetting with melted gold, obviously not feasible here. Hence the lowered Q of the cavity associated with the molybdenum had to be accepted. Fortunately the major part of the cavity surface is chromium which has a conductivity nearly approaching that of gold. The theoretical Q of this cavity is estimated to be about 8,000. That the observed Q is 7,000 or more is taken as evidence that the contact between the end plates and the side ring is good. Tests made on a gold plated dummy cavity show the quality of this contact to be very critical indeed. The reduction in obtainable fields has been one of the departures from original estimates which has had to be compensated for. Improvement in collimation of the beam has gone a long way toward making good this loss. As is obvious from the considerations in Chapter 2, greatly increased fields would result in greatly perturbed electron orbits. It seems that nearly the optimum field can be reached.

Both end plates and the molybdenum ring are a very close fit in the brass ring which supports them. This ring is exceedingly massive and is made of characteristically stable navy brass. The inside surface of the ring is turned with great care. The cavity is held in place by means of pressure rings which bear against the outer edge of the end plates. A weakness of the present design is that no means is provided to position the cavity positively against tilt. The play in the fit in the brass ring has been kept so small, however, that only very slight tilt, if any, can have occurred.

The cavity water lines, of which one set is shown, are connected by means of solder bushings to inlet holes through the brass mounting ring. The lines are of soft copper tubing which has been bent into place and connected to the end plates with Imperial fittings. The tubing is bent before tightening the fitting so that its position does not change as the fitting is made. The fittings seal effectively water to vacuum with relatively slight pressure. Water from the city mains is circulated at a high speed first through the cavity coils and then through the klystron power system. The temperature is monitored as the water leaves the cavity and is found to be quite stable, remaining between 19° and 20°C.

R. F. power for the cavity is brought in through the holes in the molybdenum ring. Four 3/4 inch holes are drilled in the brass mounting ring so as to be in register with the 5/8 inch holes in the cavity. Through two of these power is brought in by means of sections of coaxial line, shorted at the surface of the cavity to form loop type couplers. The lines are sealed with glass seals and are fitted into the mounting ring with "O" ring seals so that the positions of the loops can be adjusted. One of these lines is shown in the diagram and the shorted end of the other is viewed end on.

Through the other two holes are inserted brass plugs, roughly flush with the cavity wall. By means of these plugs the frequencies of the two cavity modes can be adjusted over very narrow limits so as to make them coincide exactly. In the tips of the plugs are tiny loop couplers connected to crystal detectors. By means of these detectors the power in the cavity is monitored. These plugs are also sealed with "O" rings. The output of these detectors is a relative measurement only; they are oriented nearly parallel with the cavity field to protect them against excessive pickup. Quantitative measurement of the fields can be made best with the electron beam. A discussion of the tuning and loading problems of the cavity will be given in the next chapter.

3.2 Beam Alignment and Collimation

The second collimating pinhole is mounted in the brass turning tube on the exit side of the cavity, this tube having been bored in register with the cavity hole. This pinhole is made by punching a platinum foil with an ordinary sewing needle. The foil, 4 mils thick, is fastened to a brass backing in which has been drilled an accurately centered 1/16 inch hole. The foil is then placed on a piece of polished hardwood and punched from the back using the driving rack of a stationary drill press. The size of the hole is determined by the depth of punch. Holes with edges rolled slightly toward the beam are thus produced with excellent circularity. Centering of the punch is by eye and is verified under a microscope.

In front of the cavity is a brass viewing screen on which silver activated zinc sulfide is placed by the drying of a water suspension. The limiting stop for the cavity is about 1/16 inch in diameter and is at the front of a conical protuberance from the screen. This device helps prevent any surface charges produced by the beam from deflecting it. The inside of the hole is tapered so as not to be hit by the beam, and the hole is small enough so that no electrons strike any part of the cavity end holes. These precautions have been found to be essential. The centering of the entrance aperture is guaranteed by the brass tube affixed to the entrance side cavity end plate.

The beam from the electron gun is directed onto the viewing screen by mechanical directing of the gun, flexibility being provided by an "O" ring joint. This is the only feasible method with the possible exception of magnetic deflection, difficult in the presence of a quantity of mu-metal. Electrostatic deflection plates rapidly become loaded with electrostatically deposited residue from the gas, charge up and cause drifts. With the introduction of precision constructed electron guns, necessary for many reasons, the position of the first collimating pinhole in the gun can be relied upon to be very near the center of the drift tube. The portion of the gun in which there is electron beam is inside the shields.

With the recent introduction of the large mu-metal shields around the center structure, the position of the beam falling on the collector has become very nearly fixed. Even under the influence of changes in local magnetic field several times the magnitude of the Earth's field, induced by other apparatus in the laboratory, the position in which the beam is found on the collector varies less than 1/32 of an inch. Most striking evidence that the path of the beam is field free is the fact that the position of the beam is nearly voltage independent. In going from 1100 to 2000 volts the beam again moves less than 1/32 of an inch. The importance of this feature to the making of measurements is obvious. The line of the beam is very near to the center line predicted by the mechanical construction of the apparatus. Some sag of the cantilevered tubes is of course inevitable. Optical methods of alignment are futile because of enormous diffraction patterns.

Because of the stability of alignment it is now possible to center the beam very closely on the collector without the aid of the deflection plates, shown just beyond the collimating pinhole in the diagram. These plates, quarter segments of a tube of inside diameter 3/4 inch and length 1 1/4 inch, have now maximum voltages of about 1.5 volts, balanced against ground in pairs. These voltages are sufficient to deflect the beam about 3/32 of an inch. Maximum voltage is never used. The principal use of the plates is to trim the beam onto the pinhole over distances of the order of a few thousandths of an inch while a run is in progress.

KLYSTRON POWER SYSTEM

4.0 General Description

The power for the cavity is derived from two Sperry type 8529 klystrons, one driving each mode, which are rated at 100 watts output continuous each. These tubes are driven by a single Sperry type SRL-6 oscillator klystron through a network providing for power division and phasing. The R. F. system is shown schematically in the electrical block diagram, Figure 4.1. Although designed to drive only one power klystron, the SRL-6 has proved entirely adequate to drive two. The driving power for each 8529 is actually some fraction of a watt, while the SRL-6 delivers a rated output of five watts. An attenuating pad between the tubes absorbs the excess power and necessarily buffers the oscillator against reflections from the power klystrons. The tubes were developed by Sperry as a part of their postwar klystron research program. We are indebted to them for making their pre-production models available to us.

4.1 Oscillator and Drive Network

The SRL-6 is a reflex klystron of the internal cavity type designed for air cooling. Input power is 200 watts and output is rated at five watts. The cavity is tunable over narrow limits by means of a bellows type structure which allows for expansion of the cavity.

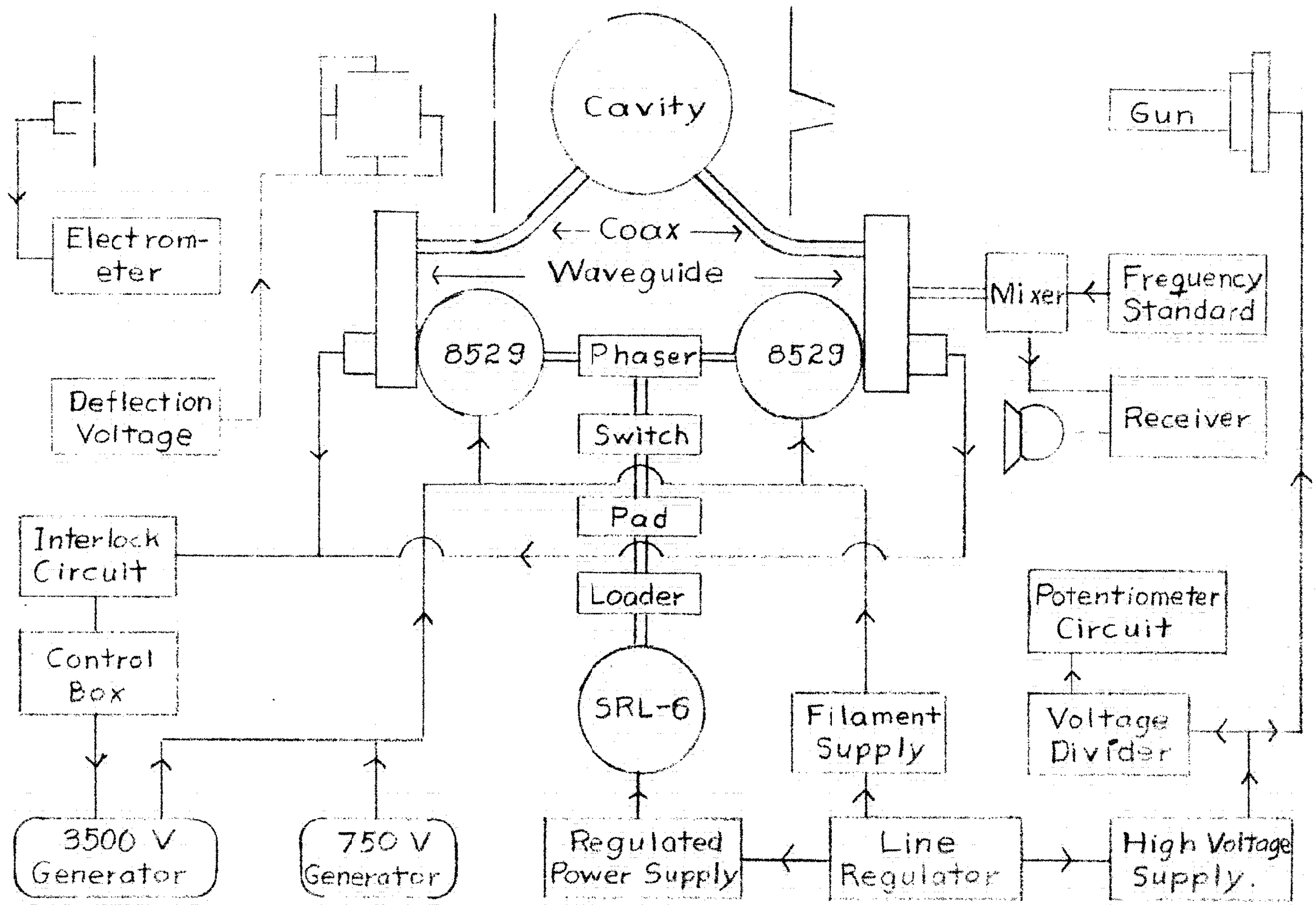


Figure 4.1 Power and Measurement Schematic

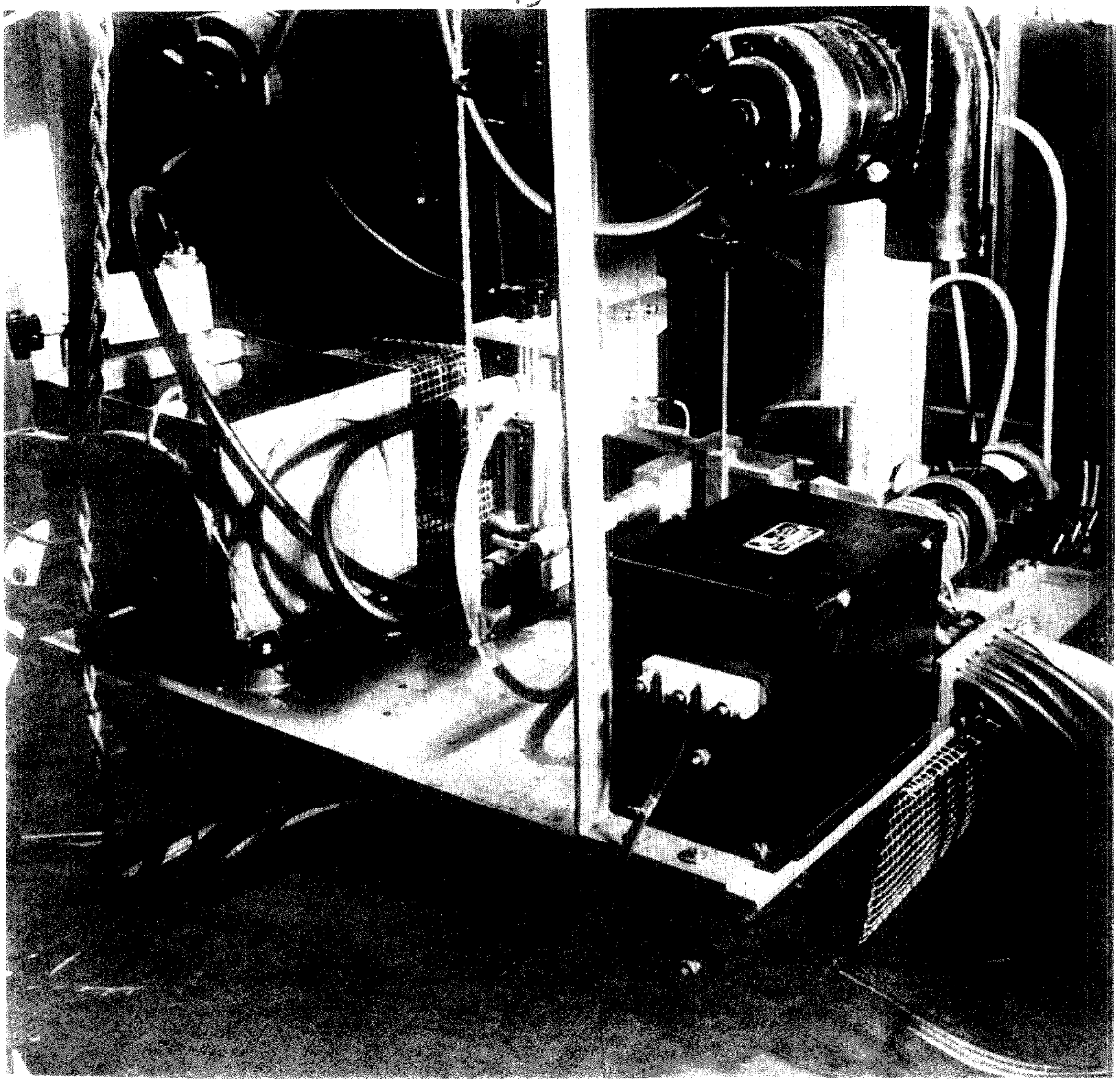


Figure 4.2 Radio Frequency System

Showing oscillator oil bath (left), low power network and power klystron (right)

The operating frequency of the cavity, 2618 megacycles, is close to the maximum for which the SRL-6 is designed. The tube normally operates in the first reflector mode. This mode employs a reflector voltage of nearly 1000 volts at this high frequency and is inefficient because of the large spacing between the grids in the cavity. It has been found advantageous, therefore, to operate on the second reflector mode, which makes the reflector about 200 volts negative with respect to the cathode. The cathode is operated at a potential of -1000 volts with respect to the grounded shell of the tube. The beam current is from 170 to 200 milliamperes. The beam power is dissipated by fins which form an integral part of the copper shell of the tube.

The ultimate frequency determining parameter is the reflector voltage. The oscillator is characteristically quite stable but is quite susceptible to microphonic and thermal gradient noise. Consequently, it has been found desirable to mount the tube in a shock mounted oil bath having water cooled walls. This treatment has greatly reduced the noise output. Further improvement has been obtained by using a storage battery to power the filament eliminating cathode modulation. By these means a noise band about 1/100,000 of the tube frequency has been obtained, that is about twenty kilocycles wide.

The tendency of the tube to show slow drifts has been mentioned. It is not known whether these drifts are due to power supply drift or to fluctuations in tube parameters, although the latter are suspected. After the tube has warmed up for about fifteen minutes, the drifts are small, rarely being as great as fifty kilocycles. The frequency is constantly monitored by means of a loud-speaker beat note and is returned by manual adjustment of the reflector voltage. In view of the necessity of monitoring constantly all the R. F. parameters, it is doubtful if the installation of automatic frequency control would greatly facilitate the taking of data.

The output of the SRL-6 is presented at a type N 50 ohm coaxial connector. Connected directly to this point is a double stub impedance matching network, consisting of two shorted stubs of adjustable length spaced one quarter wave length apart along the output line. By means of this tuner the oscillator is adjusted for maximum output on the frequency desired. In general the tube works best into a highly reactive load. In consequence great care must be taken to have good contact in the output connector and that no arcs occur in the resonant line between the tube and the first stub. The line on the output side of the second stub is more nearly flat. The drive network is hooked up with RG8U flexible, 50 ohm coaxial line. All fittings are type N, and air section lines are designed to have characteristic impedance of 50 ohms.

The output from the double stub network is connected to an attenuator of commercial design adjustable in 4 db steps from 0 to 20 db. The usual pad used is 8 db. Output from the attenuator goes to a switch which blocks the line with a bead of mercury very effectively sealing it. The reaction of this switch on the SRL-6 is sufficient to throw it off frequency for over thirty seconds. Use of greater attenuation in the pad improves the situation, of course, but yields insufficient drive power. Operating procedures must therefore be worked out so as to require switching of the R. F. power as seldom as possible.

From the switch the power passes through a "T" type power splitter which is movable to permit the phase of the drive to the power klystrons to be adjusted. The power klystrons are mounted with their input connectors joined by a straight coaxial line, and the "T" junction is moved along this line by means of a rack. The line is tapered at the ends to match the special Sperry input fittings and supported at the ends by polystyrene beads. Double stub networks installed at the klystron inputs were found to be ineffective and do not appear to be necessary.

4.2 Power Klystrons and Load Networks

The type 3529 klystron is a three cavity high gain amplifier klystron. Rated output is 100 watts with a drive of the order of one watt. Efficiency is nearly 10%. The amplifier is narrow band, of course, and is preset to frequency at the factory. The frequency can be tuned over a narrow range by means of paddles in the three cavities separately adjustable on flexible copper seals. Adjustments can be made only a very few times. The amplification is dependent on the frequency lineup of the three cavities, which must be trimmed after installation. Output is to a probe sealed into the vacuum under a glass dome. Coupling is made in a wave-guide shorted near the probe, specifications being provided by Sperry.

The beam of the klystron draws 300 milliamperes at 3500 volts as used in this apparatus. Voltages up to 4000 volts can be used. The cathode is an oxide coated disc type heated by electron bombardment from the back. Power for the bombarder is 30 to 100 watts, provided by a separate supply. Bombarder current is obtained from a directly heated thoriated tungsten cathode. Cooling is by means of water circulated through the tube. A flow of one half gallon per minute must be provided. The cathode structure and bombarder filament are mounted on a series of three discs sealed to glass tube sections.

Connection to the tube is made by means of the discs, and the glass is cooled by a blower. The anode and cavity structure of the tube is grounded. The tube is about two inches in diameter by about seven inches in length.

The output waveguide is slotted to permit standing wave measurements. Power is taken from the guide by a probe inserted about twelve inches from the tube probe, and the output end of the waveguide is shorted about a quarter wave beyond the probe. The position of the short is adjustable, as is the depth of insertion of the probe. The probe is connected by means of a flexible coaxial cable about four feet long to the input line of the cavity. This cable is necessarily of the high power type. A single adjustable shorted stub is connected across the line at the junction between the cable and the cavity line to help compensate for the cavity reactance. Duplicate structures are used for the two power systems.

The cavity input lines are 50 ohm air core coaxial lines about two wave lengths (nine inches) long. The vacuum glass seal is placed at the center of these lines, and they are shorted by means of a shunt which is triangular, obscuring about 100° of the open end of the line at the cavity. Even with this short an end loop the lines must be turned so as to place the loop partially parallel to the magnetic field in the cavity.

The cavity accepts power from the klystrons very well, tending to lock with the output cavities of the tubes. Considerable efforts have been spent in getting the standing wave ratio in the waveguides and flexible lines low. Slots have even been inserted in the flexible lines themselves for monitoring purposes. Decoupling at the cavity by rotation of the input loops has helped, but the extremely reactive nature of the cavity load makes the obtaining of flat lines very difficult. No difficulty has been encountered with heating of the lines, so the criterion accepted at present for proper loading is good input into the cavity.

In the interest of protecting the power klystrons, small pickup probes attached to crystal detectors have been permanently fitted in the waveguides directly across from the cavity output probes. These probes are interlocked with the main beam voltage supply by means of an amplifying and metering circuit. Fortunately no excessive fields have ever been observed at this point. The output of the detectors is a good indication of the standing wave ratio, however.

The best indication of proper loading found to date is the beam current dependence of tube output. If the beam current of a properly loaded klystron is reduced, the output, as monitored in the cavity, drops. If the klystron is improperly loaded, however, just the reverse effect is observed. This is hypothesized as being due to the rise in effective Q of the output cavity of the klystron as the loading effect of the beam passing through it is reduced. Quite startling rises in cavity power have been observed as the current is reduced, sometimes as much as six db. Using this effect as a criterion, loading of the klystrons can be greatly improved over that achieved by other methods. The operating voltage of 3500 volts still represents a compromise. The loading of the output cavity is not as heavy as that which might be achieved with a flat antenna system. Operation at 4000 volts, the tube maximum, produces no great improvement in output.

4.3 Power Supplies

The power for the main beams of the power klystrons is obtained from a twenty kilowatt generator having a variable output with a maximum of 6000 volts. The regulation of this generator is excellent after a warmup period. The voltage is controlled by a remote exciter current rheostat in the same room with the apparatus. The generator is next door. The bombarder voltage is provided by a small generator made for the purpose from two surplus dynamotors. These have been mounted on insulating blocks and reconnected to be self excited generators. Power is provided by means of an insulating pulley installed between the two generator shafts which is driven by a three phase induction motor. At present only one generator is used. This generator must be insulated for the 3500 volt beam voltage as well as its own output voltage of 750 volts. The generator is mounted in a soundproofed enclosure. Its voltage is regulated by a rheostat controlled by an insulated shaft. The regulation is not good and must be monitored. Efforts to improve this regulation are being made.

Filament power for the power klystrons is provided by two insulated transformers separately controlled by variable autotransformers. The maximum voltage is 8 volts at about 8 amperes. The life of the klystrons is limited by filament life. For this reason, great care is taken to warm the filaments slowly to avoid thermal shock and allow time for outgassing. Metering is provided for filament current, bombarder current and voltage, and beam current and voltage for each tube. The desired operating point is with a bombarder power just above that which causes the cathode to become space charge limited. This point is at about ninety watts input. Positive ion bombardment of the filament causes its temperature to be a very critical function of filament current, since heat is also obtained from the bombarder supply.

Great improvement in the stability of the entire system has been effected by the installation of a Sorenson line voltage regulator in the line supplying the klystron filaments and the regulated power supplies, described below. This regulator has been made available to this project through the generosity of Professor J. W. DuMond. The presence of this regulator makes the bombarder supply the principal source of instability in the klystron power supply system.

The power supply for the SRL-6 is extremely critical. The frequency is fixed by both the beam voltage and the reflector voltage. The most critical is the reflector voltage, which can be supplied by batteries since no current is drawn. The impedance of the reflector supply must be kept low, however, since it is of critical importance that the voltage of the reflector never have a chance to become positive with respect to the cathode. A rectifier clipper tube is connected right at the klystron to insure against such an occurrence. The low impedance also insures against dependence on the positive ion current drawn in the tube.

The high voltage regulator is a high gain feedback circuit, the most unusual feature of which is the use of a 900 volt battery as the reference voltage. This battery is open circuited on the grid of the low level amplifier tube, a 6AC7. The regulator is a pair of

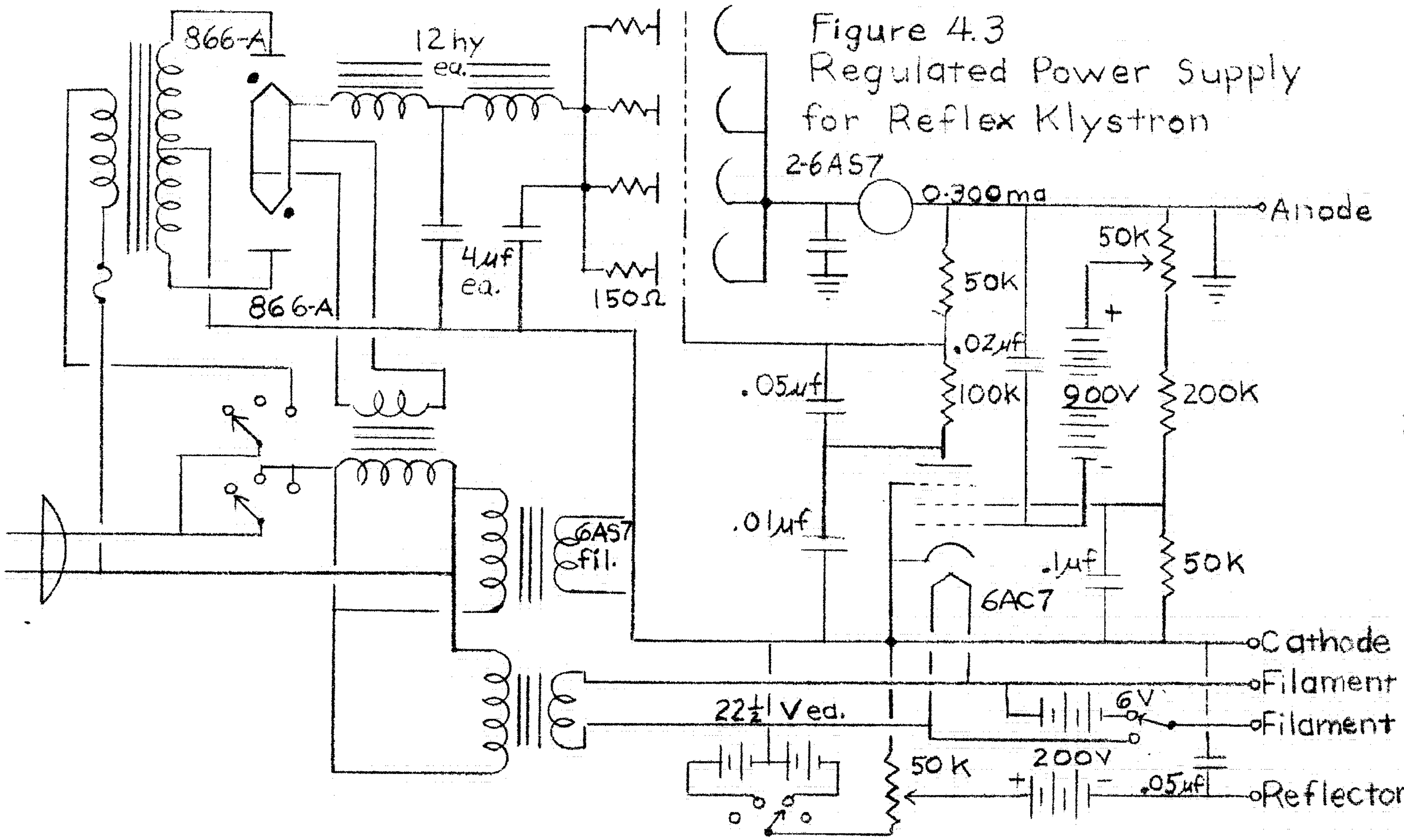


Figure 4.3
Regulated Power Supply
for Reflex Klystron

6AS7 tubes, parallel connected. Input is from a standard rectifier power supply employing two type 366 rectifiers. Feedback gain of the circuit is in excess of 500. With the Sorenson regulator suppressing cathode modulation from the 6AC7 the output is regulated and noise free to better than 0.05 volts. Temperature drifts of the batteries must of course be coped with, but these are slow. The circuit of this supply, Figure 4.3, is self explanatory.

FREQUENCY MEASURING APPARATUS

5.0 General Description

The measurement of a frequency is chiefly a problem in technique. The desired characteristics of the present apparatus were that it be simple to use, capable of constant application, and independent of drifts of component parts. The method, chosen because it is straightforward, is to beat the output of one of the power klystrons, about 2613 megacycles, against the 2610 megacycle harmonic of a one megacycle standard crystal. The harmonic is obtained by direct frequency multiplication in a series of electronic amplifiers, a method which guarantees an integer factor. The difference frequency needs to be measured with an accuracy of only about ± 0.01 megacycles to remove the frequency measurement from the error considerations of the experiment. The standard crystal, good in itself to two parts in 10^5, can be calibrated to far beyond the needed precision by comparison with the standard signals broadcast by the National Bureau of Standards over radio station WWV.

The beat frequency is monitored continuously by a Navy type RBH short wave receiver, manufactured by Scott Radio Laboratories and obtained by the Institute from war surplus. This receiver is of remarkable mechanical construction and is characterized by relatively low sensitivity and high stability. It has a remarkable dial which is free from backlash and can be read directly to the desired precision for this measurement. The dial is calibrated by means of a Navy type LM frequency meter, also obtained surplus, but this calibration is virtually unnecessary. The noise output of the klystron system is clearly audible, and it is unnecessary to use the beat frequency oscillator in the receiver. The pass band of the receiver is about five kilocycles, nicely adapted to the precision desired. Output of the receiver is presented to a loudspeaker by means of which the operator can monitor continuously the cavity frequency. During a run the klystron frequency is reset by means of the loudspeaker.

A secondary check on the frequency is desirable to guard against the possibility of the signal detected by the receiver being spurious. This check is provided by a coaxial type wavemeter, BL5554, used for general purposes in this laboratory. This meter is of the absorption type, consisting of a shorted coaxial line having one shorted end movable on a precision screw which can be read to 0.001 centimeters by means of a vernier dial. Input is by a small loop, and output from a similar loop is presented to a crystal detector. By means of this instrument the frequency can be checked to about $\pm$ 0.5 megacycles. Agreement is excellent.

The choice of 2610 as the harmonic to be used was based on the practical problems encountered in obtaining frequency multiplication. The harmonic of the crystal to be amplified must be isolated by a series of resonant circuits coupled by vacuum tube frequency multipliers. These circuits work efficiently only as doublers or triplers, particularly at frequencies above 30 megacycles where the efficiency of standard type vacuum tubes falls off rapidly. In an effort to find a suitable harmonic to use, the integers in the neighborhood of 2617 megacycles, the original target frequency, were factored into prime factors. The following table shows the result.

Table 5.1 Factorization of Integers

2603	19.137	2617	Prime
2604	31.7.3.2.2 *	2618	17.11.7.2 *
2605	521.5	2619	97.3.3.3
2606	1303.2	2620	131.5.2.2
2607	79.11.3	2621	Prime
2608	163.24	2622	23.19.3.2
2609	Prime	2623	61.43
2610	29.5.3.3.2 *	2624	41.2.2.2.2.2.2 *
2611	373.7	2625	7.5.5.5.3 *
2612	653.2.2	2626	101.13.2
2613	67.13.3	2627	71.37
2614	1307.2	2628	73.3.3.2.2
2615	523.5	2629	239.11
2616	109.3.2.2.2	2630	263.5.2

The interesting factorizations are marked with an asterisk. Of these 2618 is the least suitable by virtue of having too many large factors. 2625 offers similar difficulties. The reason for choosing 2610 will be brought out in the next section.

5.1 Frequency Multiplier

The big obstacle to be overcome in any of the three available harmonics of interest is the large factor. This obstacle is met by making the factor the first to be introduced in the multiplier. Thus the present circuit goes direct from one megacycle to 29 megacycles. The one other large factor is disposed of in a jump of a factor of 15 going from the output of the multiplier to the mixer. The loss in signal level here is made up by the sensitivity of the receiver. With this general scheme there is little to choose between the three harmonics. 2610 was the first tried.

The standard crystal forms the grid tank of a tuned plate tuned grid oscillator. This circuit employs a 6SJ7 pentode with variable feedback capacity from plate to grid. The feedback is kept low to reduce drive to the crystal and promote stability. A plate supply regulated with an OD3 is used for the oscillator only. The oscillator is stable to well beyond the needs of the experiment, but a certain amount of frequency pulling is still produced as the plate tank is tuned. The crystal is a low temperature coefficient cut. Comparison of the tenth harmonic of this crystal with the ten megacycle carrier signal of WWV reveals no perceptible drift, provided ample time is allowed for circuit warmup.

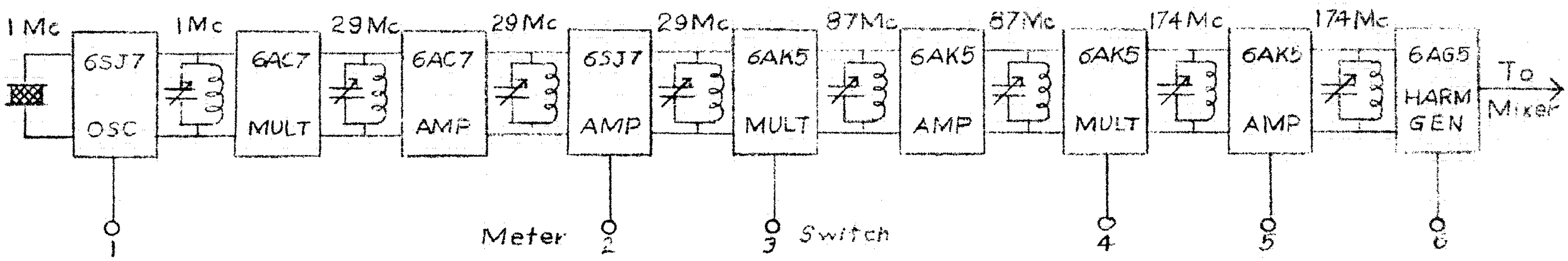
The lineup of the multiplier is shown in the accompanying block diagram. The first amplifier, being driven by a scale of 29 multiplier, runs class A. All other stages operate class C. All plate to grid coupling is capacitive with a single interstage tank circuit.

In the construction of the high frequency amplifiers, as distinguished from multipliers which have plate and grid tanks at different frequencies, considerable care was necessary to suppress parasitic oscillations, particularly in the 174 megacycle amplifier. In this circuit tank circuits consist of a half turn of a three quarter inch loop across a 15 micromicrofarad trimmer. Multiple grounds and shielding cages were used. All leads throughout the high frequency lineup were kept short.

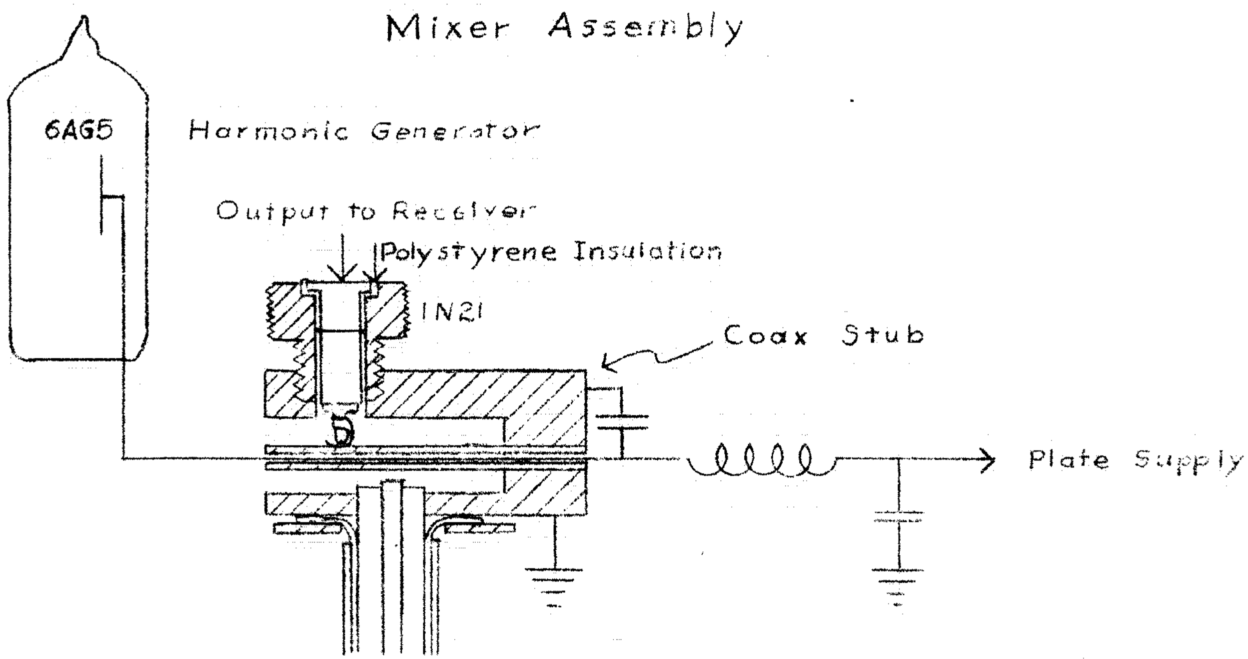
The drive to the grid of the 6AG5 harmonic generator is sufficient to cut the current in that tube to roughly a quarter of its no signal value. This drive is entirely dependent on the crystal oscillator. Transmitted parasitic oscillations would seem to be ruled out. Thus the output signal must have the fundamental periodicity of the oscillator, i. e. must be composed only of harmonics of the crystal frequency. The extent to which the desired harmonic dominates over the others is fixed by the success achieved in suppressing side harmonics in the various tank circuits in the lineup. Unfortunately, a certain amount of one megacycle modulation is superimposed on

Figure 5.1

Frequency Multiplier



Mixer Assembly



the signal because of insufficient ringing action in the resonant circuits in the 29 megacycle amplifiers. It is to guard against the spurious detection of the two side-band frequencies thus produced that the frequency check with the coaxial wavemeter must be so precise. That the signal used is the desired harmonic is confirmed both by intensity observations and by the wavemeter.

5.2 Mixer

The mixer structure is diagrammed in the accompanying drawing. Mixing is accomplished in an open quarter wave stub tuned approximately to the harmonic desired. Power bled from the power klystron is coupled capacitively to the center of the line. The pulsed output of the harmonic generator is coupled to the stub by passing the plate lead of the tubs through the center of the center conductor. A crystal rectifier, attached to the stub near the high voltage point, provides output to the receiver. The harmonic generator is capable of producing a D. C. voltage of about two hundredths of a volt in the crystal. The R. F. coupling from the klystron is adjusted to make the D. C. output voltage about a tenth of a volt. The beat frequency is then easily heard in the receiver.

BEAM FORMATION AND DETECTION

6.0 Pinhole Systems and Collimation

The function of the collimation system is to produce a beam by means of which the deflections in the neighborhood of resonance can be measured. Ideally the beam should produce a sharp edged spot on the collector. Under these circumstances, as Dacey has shown[19], by making the collector pinhole the same size as the spot, the peak of collected current against deflection would have a sharp top, rounded only by effects of thermal voltage spread. The sides of this peak are very nearly straight, their slope being given by

$$- \frac{dI}{dD} = \frac{I}{\pi R^2} (4R^2 - D^2)^{1/2} \qquad (1)$$

where I is the current collected, R is the radius of the spot and of the pinhole, and D is the deflection.

The source of equation (1) may easily be visualized by picturing the fanning out of the beam as a simple displacement at any instant of time, the direction of this displacement being the only time dependent parameter. The square root quantity is then the length of the chord of intersection of the pinhole and the spot. If the pinhole and spot are of different sizes, the expression for the length of the chord of intersection is complicated, but the relationship still holds true. The region of straight sides of the peak is now that over which the length of the chord approaches the diameter of the smaller of the two circles. The top of the peak is rounded to a width somewhat more than the difference in diameters of the circles.

As has been pointed out, a beam with sharp edges cannot be obtained with a two pinhole collimation system except by making one hole vanishingly small. Instead a beam having regions of illumination analogous to the umbral and penumbral shadows of an eclipse is produced. The umbral region is uniformly illuminated, but in the penumbral region the illumination falls off along a continuous curve. This sort of beam is the one which must be worked with.

The general analog of equation (1) is

$$-\frac{dI}{dD} = \int_{-\pi}^{\pi} i(R,\varphi,D)R\cos\varphi \, d\varphi \qquad (2)$$

where R is the radius of the collector hole, φ is the polar angle measured at the center of the hole from the line of centers of beam and spot, and $i(R,\varphi,D)$ is the intensity of the beam at the point (R,φ) on the edge of the pinhole. Analysis from this equation shows that the slope $-dI/dD$ is relatively independent of D over the range where the edge of the pinhole cuts through the umbral area on one side and is outside the beam on the other. Thus if a pinhole of diameter say midway between the umbral diameter and the penumbral diameter is used, the top is rounded to a width somewhat greater than the difference of penumbral and umbral radii.

6.1 Beam Formation

Even though the peak of current vs. deflection is symmetrical, it is highly desirable for the purpose of resolving the center that the extent of the rounded top be small. This is tantamount to requiring that the width of the penumbral area be small. By projection considerations we easily see that the diameters of the umbral and penumbral portions of the beam are respectively $|2d_2 - d_1|$ and $2d_2 + d_1$ where d_1 and d_2 are the diameters of the first and second collimating holes and the distances between holes are assumed equal.

Several interacting factors now limit possibilities:
(i) The limitation of total current between the two pinholes placed by the ion formation rate. This effect, now understood, was the principal unanticipated difficulty which impeded early beam formation system designs. Even in good vacuum this effect causes startling focusing effects in the highly collimated beams used in this experiment.
(ii) The tendency of dirt deposited ty the beam in the electron gun to close the first collimating pinhole, or to become charged and thus upset the collimating action of the gun. The problem is magnified as the collimating hole becomes smaller.

(iii) The electrostatic capacity of the electrometer collector coupled with the necessity of making rapid readings, which together require that -dI/dD be kept reasonably large. The present current sensitivity is about 10^{-13} amps/mil (1 mil = 0.001 inch) or greater. Much lower values would lead to difficulty in making readings.

(iv) The necessity of confining the beam to the axis of the cavity by a pinhole no larger than the one in use.

The ion formation limitation makes it impractical to obtain a small penumbra by making d_1 large and d_2 small. Hence we can assume the reverse, namely, that over the center of the spot the first pinhole is seen unobstructed through the second. Then the current sensitivity on the straight sides of the peak is roughly the intensity of illumination from the first pinhole times the mean diameter of the spot

$$- \frac{dI}{dD} = i_o \frac{\pi}{4} {d_1}^2 d_2 \qquad (3)$$

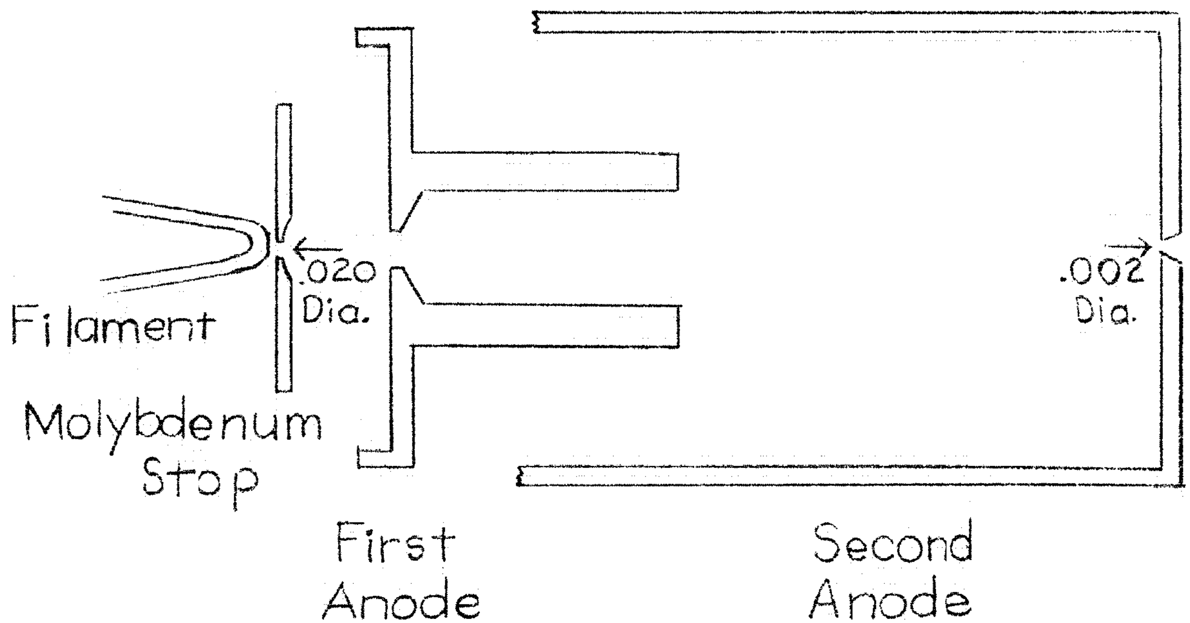
where i_o expresses the intensity with which the first pinhole is illuminated from behind by the electron gun. The width of the round top of the current deflection peak is roughly the width of the penumbral ring, d_1.

The nature of the problem is now easily seen. By reducing d_1 below its present size of 0.002 inch the top of the peak is narrowed. If, however, we compensate for the lost intensity by raising i_o, we increase the rate of deposition of charging layers on the already more easily obstructed pinhole and drastically reduce the life of the gun between cleanings.

6.2 Electron Gun

The electrode structure of the present electron gun is very simple, Figure 6.1. Collimation is achieved by acceleration in uniform fields, focusing effects of the fields being kept small. No small stops which can intercept the beam and charge up are used. All areas are swept clear of ions by the fields present. The beam size emerging from the collimating hole is fixed by the cathode structure and molybdenum stop, which stays clean by virtue of being hot. The image on the first viewing screen is a magnified image of the cathode area, in relatively sharp focus at some voltages. The concentration of heavy volatiles in the residual gas is kept low by means of two liquid air traps made of test tubes placed in the mounting plate for the gun.

Great care has been used to provide good alignment and collimation of the electrodes. Insulation is provided by fired talc insulators fitted with permanent aluminum bands and precision machined. The electrodes are supported by a precision machined aluminum tube which is externally cooled. For details the reader is referred to Mr. Lauritzen.



Electron Gun Electrode Structure
Figure 6.1 Scale × 2

The properties of the beam produced by this gun have been described. The beam varies in size on the first viewing screen from one quarter inch at 1100 volts to three-eighths of an inch at 2000 volts. There is decidedly greater intensity at 2000 volts. Motion in going from 1100 to 2000 volts is imperceptible.

6.3 Voltage Supply

The filament voltage for the electron gun is supplied by a storage battery in series with two type 1133 auto lamps in parallel. The current drawn is six amperes. The beam voltage, applied to the center tap of a balancing resistor across the filament, is obtained from the power supply whose circuit is shown in the accompanying diagram, Figure 6.2 The diagram is self-explanatory in most details. The most important features are the use of a 600 volt battery as reference and a reference voltage divider consisting of all Koboloy resistors operated at 1/10 rated power or less. Both battery and resistors are thermally insulated, the resistors in a convection cooled box; no thermal regulation is used.

The temperature coefficient of the battery is about 10^{-4} per degree centigrade. For this reason the circuit shows a slow drift tendency, but the high thermal inertia of the batteries and insulating box prevents rapid changes. The voltage vernier is a ten turn Helipot having a range

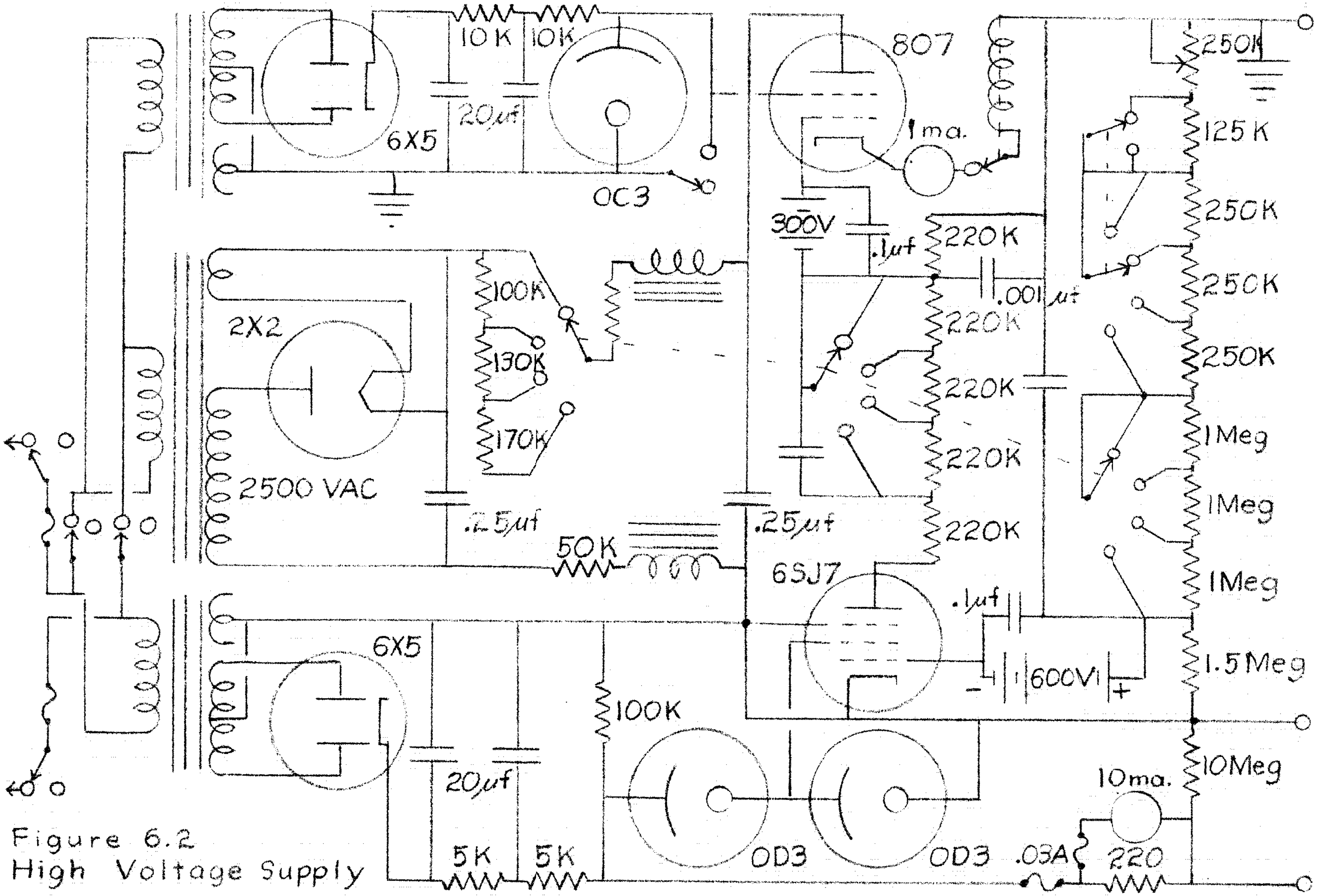


Figure 6.2
High Voltage Supply

of 100 volts. By means of this potentiometer the voltage can be set to a precision of ± 0.02 volts subject to absolute calibration by the voltage measuring apparatus.

The supply, in addition to affording beam voltage for the gun, puts out one to two milliamperes, its principal load, to the one megohm precision voltage divider. An additional output of 300 volts with respect to the cathode is provided for the first anode of the gun.

The supply is protected against overloads by a sensitive relay, shunted to 7 milliamperes, in the cathode circuit of the 807 regulator circuit. Additional protection is obtained with sensitive fuses in the first anode circuit.

6.4 Electrometer

The beam current passing through the collector pinhole falls into a Faraday cup and is transmitted to the grid of a type CK 571-AX sub-miniature electrometer pentode. The grid is returned through a 10^{10} ohm resistor and it is across this resistor that the signal voltage is developed. Both tube and resistor are inside the vacuum.

The tube is one member of a balanced circuit, balanced against drift by means of a filament rheostat. The output of this circuit is read by a Leeds and Northrup Type P mirror galvanometer which has a sensitivity of 0.004 μ a/mm and a period of six seconds. This long period is a real

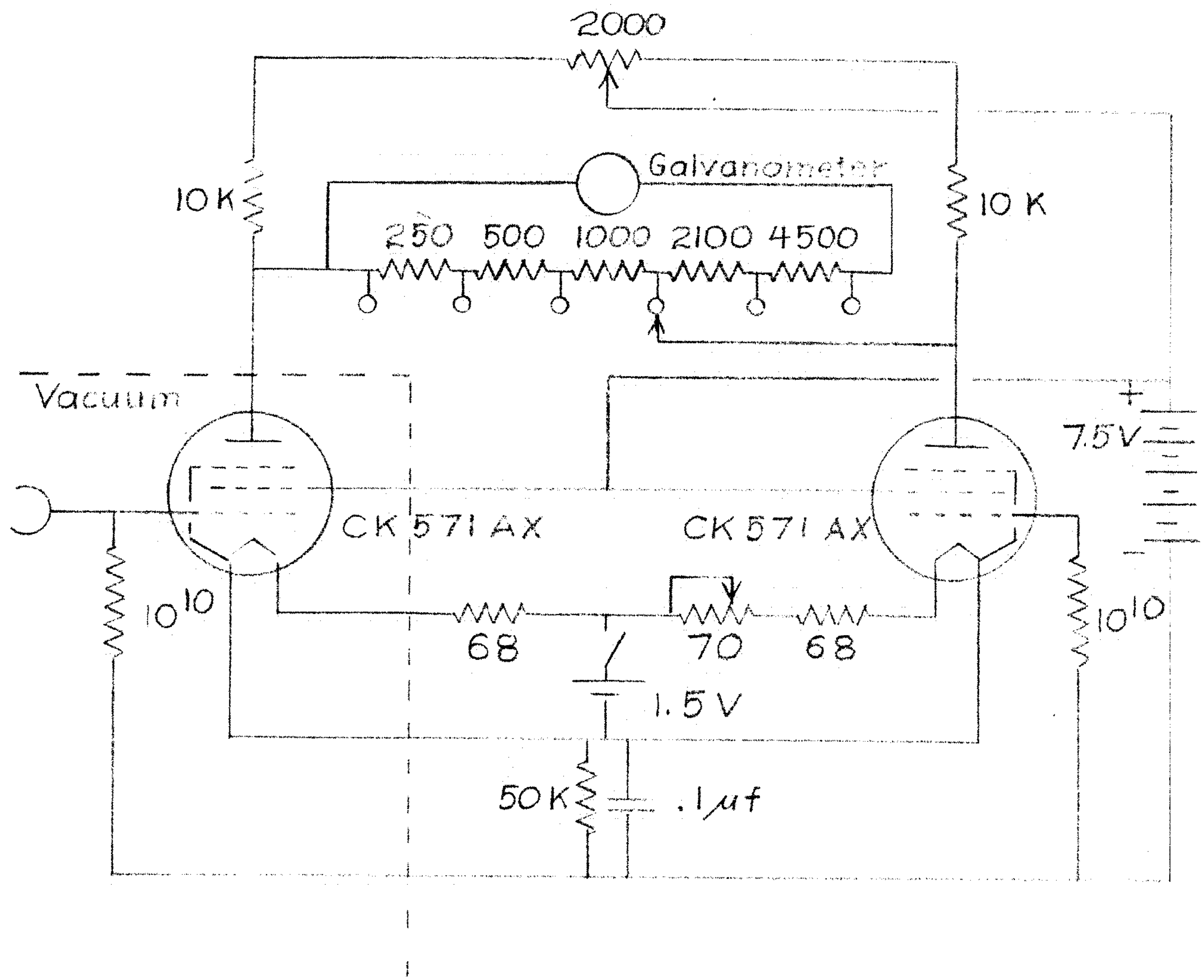


Figure 6.3 Electrometer Circuit

impediment to the rapid collection of data, but the other properties of the galvanometer make it desirable to use.

PRELIMINARY RESULTS

7.0 General Discussion

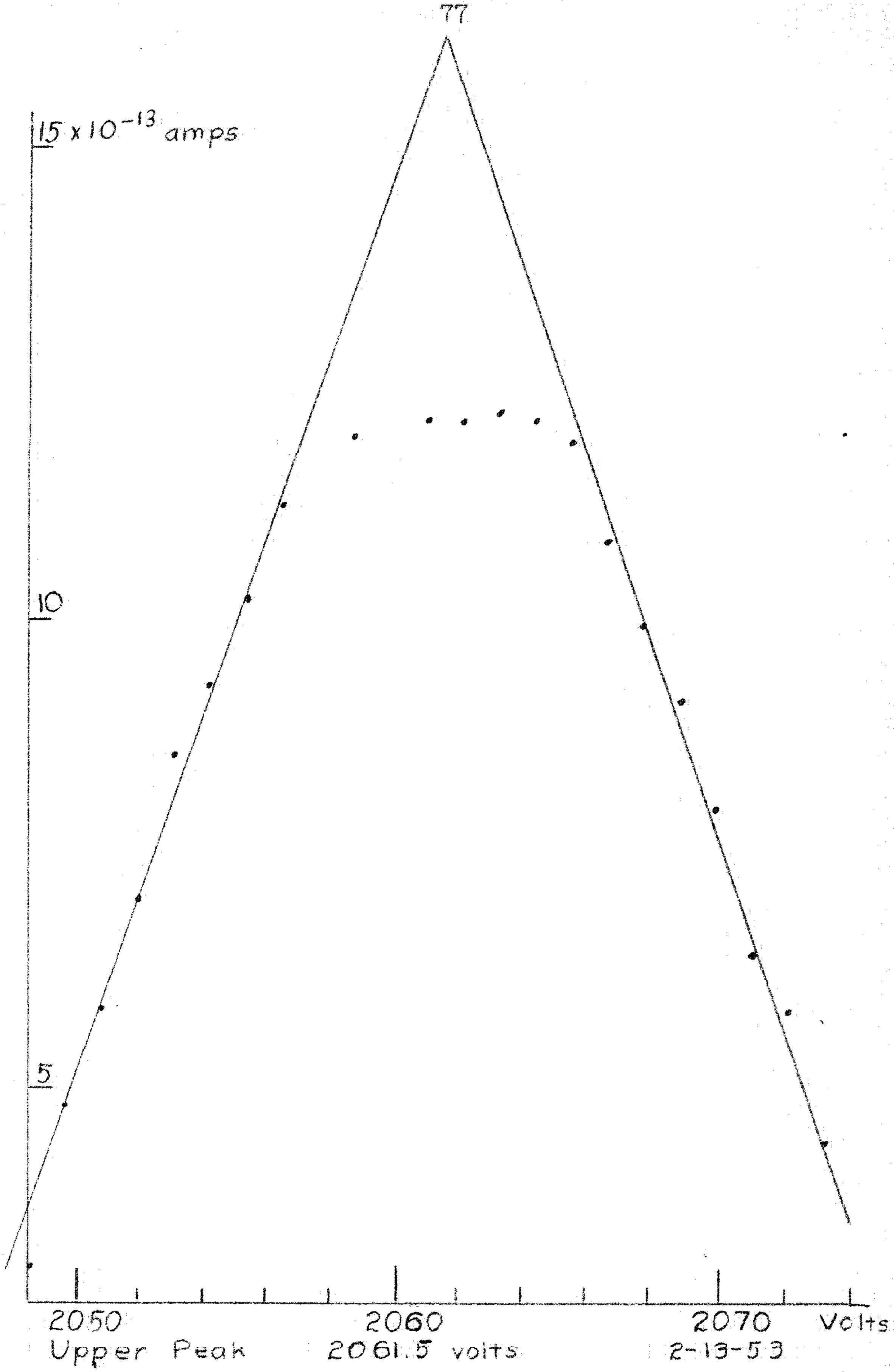
The results available for the preparation of this report, prepared against a time limit, are preliminary only. The data so far are encouraging, particularly in view of the difficulties experienced when they were taken. The following chart gives the center voltages of the peaks now accumulated, together with error limits based on the apparent consistency of the data yielding each peak.

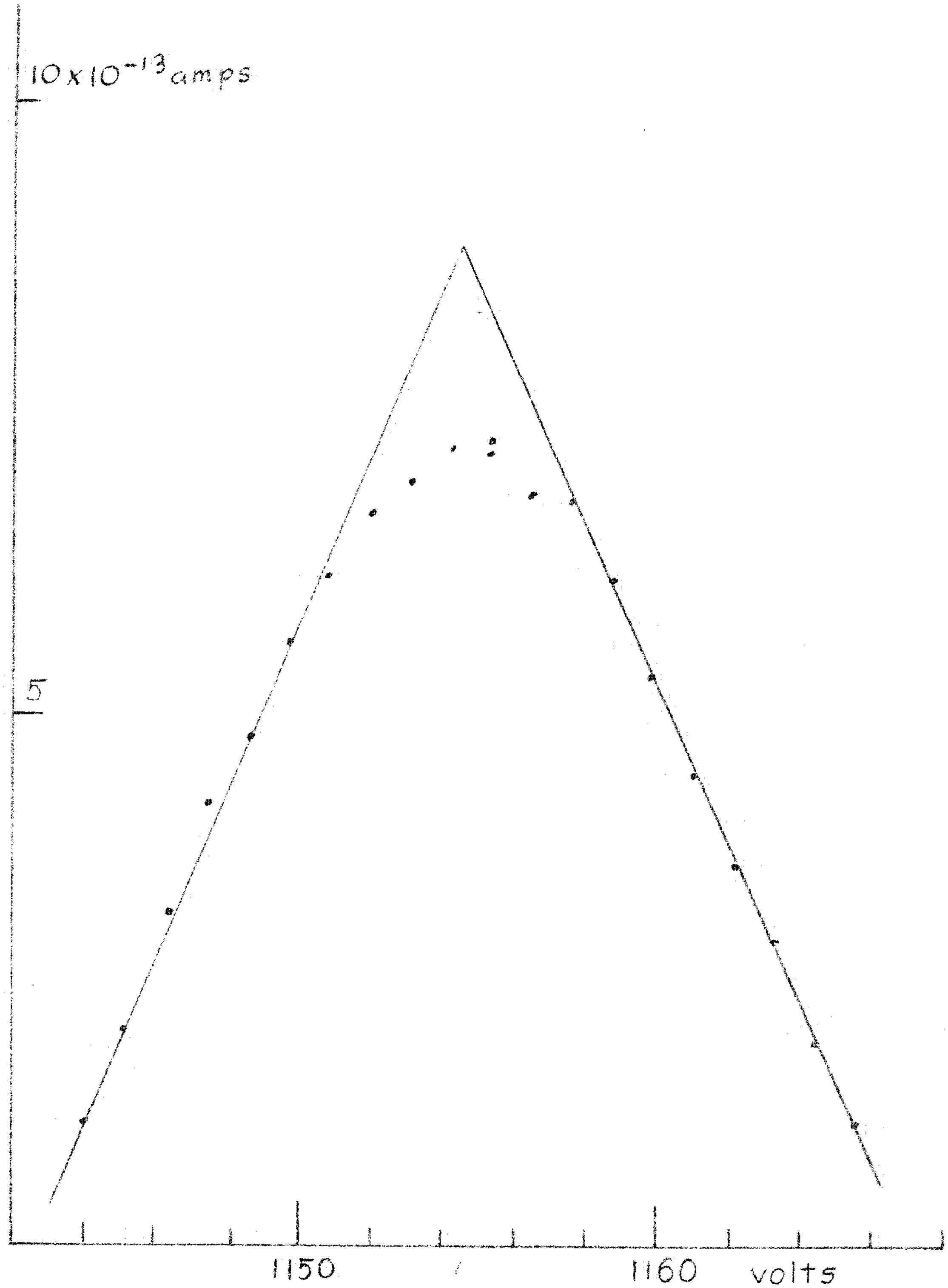
Date	Lower	Upper	Difference	Remarks
1-27	1155.6 ± 1	2060 ±.5	904.4	Very wide peaks
1-30	1154 ± 1	2057 ± 1	903	R. F. unstable
2-4	1153.3 ±.5	2058 ± 2	905	R. F. unstable
2-6	1153.6 ±.5	2060 ± 1	906.4	Electrometer drift
2-10	1154.4 ±.3	2059.6 ±.5	905.2	
2-13	1154.5 ±.2	2061.0 ± 1	906.5	
		2061.5 ±.2	907	Best so far
3-4	1153.9 ± 1	2056.4 ±.2	903.2	Data taken after
	1153.2 ±.3	2057.1 ±.2	903.9	
		2057.5 ±.2	904.3	short pumpdown

The discrepancies among the voltages are beyond those explainable by the possible inconsistency of the data. There can be no doubt that real shifts of apparent resonance position have been observed. On the basis of a number of observed phenomena the inconsistency is believed to be due to the presence in the beam of secondary electrons ejected from the cathode by positive ion bombardment. As Dacey shows[19], the presence of such electrons moves the center voltage and contributes to the rounding of the top of the peak but does not affect the straightness of the sides. Steps to correct this difficulty are being taken.

The prospect for accuracy is best predicted from the quality of the curves. The two best curves obtained on 2-13-53 are reproduced on the following two pages. These are the best peaks obtained so far, although the peaks of 3-4-53 are comparable. When these peaks were taken, difficulty was still being experienced with the electrometer stability. This difficulty has been corrected by improved shielding. There appears to be excellent prospect of fitting curves with a confidence of ± 0.1 volts when the improvements now being made are completed.

If the accepted value of e/m and a frequency of 2613.5 megacycles are put into equation 1.1 (1) corrected for the relativistic mass shift during acceleration and the value of the length from section 1.5, 1.214 inches, is used, the desired value of the voltage difference is found to be about 910 volts. This result is in satisfactory agreement with observation, particularly if allowance is made for the fact that the secondary electron phenomenon depresses the value of this difference. For more detailed consideration of the entire accuracy question the reader is referred to Mr. Lauritzen and, of course, to Professor Smythe.


77
15 x 10^-13 amps
10
5
2050
2060
2070
Volts
Upper Peak
2061.5 volts
2-13-53



Lower Peak 1154.5 volts 2-13-53

APPENDIX 1. Cavity Polarization

The general form of the vector potential of two TM_{110} modes superimposed with arbitrary space and time phase is from equation 2.0 (1)

$$\underset{\sim}{A} = \underset{\sim}{k}2\beta^{-1}J_1(\beta\rho)\Big\{B_I\cos(\varphi + \Psi_I)\cos(\omega t + \alpha_I) + B_{II}\cos(\varphi + \Psi_{II})\cos(\omega t + \alpha_{II})\Big\} \qquad (1)$$

We treat the factor in brackets by vector methods. Define unit vectors $\underset{\sim}{m}$, $\underset{\sim}{n}$, $\underset{\sim}{o}$ having arguments $-\Psi_I$, $-\Psi_{II}$, and φ respectively. Then if

$$\underset{\sim}{r} = \underset{\sim}{m}\,B_I\cos(\omega t + \alpha_I) + \underset{\sim}{n}\,B_{II}\cos(\omega t + \alpha_{II}) \qquad (2)$$

we have

$$\underset{\sim}{r}\cdot\underset{\sim}{o} = B_I\cos(\varphi + \Psi_I)\cos(\omega t + \alpha_I) + B_{II}\cos(\varphi + \Psi_{II})\cos(\omega t + \alpha_{II})$$

If $\underset{\sim}{m}$ and $\underset{\sim}{n}$ are not perpendicular, we make them so by resolving $\underset{\sim}{n}$ along $\underset{\sim}{m}$ and $\underset{\sim}{p}$ where the argument of $\underset{\sim}{p}$ is $\pi/2 - \Psi_I$. The resulting expression can be written

$$\underset{\sim}{r} = \underset{\sim}{m}\,B'_I\cos(\omega t + \alpha'_I) + p\,B'_{II}\cos(\omega t + \alpha'_{II}) \qquad (3)$$

It is known from the theory of Lissajous figures that the path of the tip of the vector r is an ellipse. We can choose new vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ oriented along the major and minor axes of the ellipse.

$$\underset{\sim}{r} = -\underset{\sim}{i}\,B_2\sin(\omega t + \alpha) + \underset{\sim}{j}\,B_1\cos(\omega t + \alpha)$$

where the signs depend on the direction of rotation. If we now measure φ from the vector $\underset{\sim}{i}$ and set $\alpha = 0$ by proper choice of time origin, we get

$$\underset{\sim}{r}\cdot\underset{\sim}{o} = -B_2\cos\varphi\sin\omega t + B_1\sin\varphi\cos\omega t \qquad (4)$$

APPENDIX 2. Field in the End Holes of the Cavity

The general $\underset{\sim}{A}$ of the problem must satisfy the wave equation and have zero divergence. Since we expect the components of $\underset{\sim}{A}$ to oscillate with angular frequency ω, we can write the wave equation in terms of $\beta = \omega/c$

$$\nabla^2 A = -\beta^2 A \qquad (1)$$

This equation is satisfied by the series

$$\underset{\sim}{A} = \underset{\sim}{A}_0 + \underset{\sim}{A}_1 + \underset{\sim}{A}_2 \cdot \cdot \cdot \qquad (2)$$

if

$$\nabla^2 \underset{\sim}{A}_0 = 0 \qquad (3)$$

$$\nabla^2 \underset{\sim}{A}_{n+1} = -\beta^2 A_n \qquad (4)$$

$$\nabla \cdot \underset{\sim}{A}_n = 0 \qquad (5)$$

From this series we obtain

$$\underset{\sim}{B} = \underset{\sim}{B}_0 + \underset{\sim}{B}_1 + \underset{\sim}{B}_2 \cdot \cdot \cdot \qquad (6)$$

$$\nabla^2 \underset{\sim}{B}_0 = 0 \qquad (7)$$

$$\nabla^2 \underset{\sim}{B}_{n+1} = -\beta^2 B_n \qquad (8)$$

$$\nabla \cdot \underset{\sim}{B}_n = 0 \qquad (9)$$

The series for $\underset{\sim}{A}$ is the analog of the series obtained in section 2.0 for the field of the cavity away from the end holes

$$\underset{\sim}{A} = \underset{\sim}{A}_0(1 - (\beta\rho)^2/8 + (\beta\rho)^4/192 \cdot \cdot \cdot) \qquad 2.0\ (3)$$

We will try to construct an $\underset{\sim}{A}_0$ suitable in the end holes which connects with the $\underset{\sim}{A}_0$ of section 2.0

$$\underset{\sim}{A}_0 = \underset{\sim}{k}(B_1 y \cos\omega t - B_2 x \sin\omega t) \qquad 2.0\ (4)$$

We have at our disposal f(z) as calculated by Professor Smythe for a uniform magnetic field fringing into a round hole in an impermeable plane boundary of the field where f(z) is the magnitude of the field on the axis of the hole, being one on the field side, zero deep in the hole, and having intermediate values between. Thus as in section 2.2 we desire for our $\underset{\sim}{A}_o$ on the axis of the holes

$$A_z = f(z)(B_1 y\cos\omega t - B_2 x\sin\omega t) \qquad (10)$$

where we drop the subscript $_o$ for the remainder of the treatment.

To simplify description of the elliptical field we use rotating coordinates aligned with $\underset{\sim}{B}$. Thus, defining

$$B(t) = (B_1{}^2\cos^2\omega t + B_2{}^2\sin^2\omega t)^{1/2} \qquad (11)$$

as the magnitude of the field where f(z) = 1, we choose unit vectors and coordinates

$$\begin{aligned} \underset{\sim}{m} &= \underset{\sim}{i}(B_1\cos\omega t)/B(t) + \underset{\sim}{j}(B_2\sin\omega t)/B(t) \\ \underset{\sim}{n} &= -\underset{\sim}{i}(B_2\sin\omega t)/B(t) + \underset{\sim}{j}(B_1\cos\omega t)/B(t) \end{aligned} \qquad (12)$$

$$\begin{aligned} \beta &= \underset{\sim}{m}\cdot\underset{\sim}{r} = x(B_1\cos\omega t)/B(t) + y(B_2\sin\omega t)/B(t) \\ \alpha &= \underset{\sim}{n}\cdot\underset{\sim}{r} = -x(B_2\sin\omega t)/B(t) + y(B_1\cos\omega t)/B(t) \end{aligned} \qquad (13)$$

in terms of which

$$\underset{\sim}{B} = f(z)\ B(t)\ \underset{\sim}{m} \qquad (14)$$

$$A_z = f(z)\ B(t)\,\alpha \qquad (15)$$

To obtain the fields where f(z) = 1 we follow Smythe and use a scalar potential for B.

$$B = -\nabla\Omega \qquad (16)$$

$$\nabla^2\Omega = 0 \qquad (17)$$

We have at our disposal an important symmetry property of

Ω. At large distances from the hole on the field side Ω must reduce to

$$-\Omega = B(t)\beta = B(t)\rho\sin\varphi \qquad (18)$$

where now $\rho^2 = \beta^2 + \alpha^2$ and φ is the appropriate angle. The boundary conditions are independent of φ. Hence in general

$$-\Omega = B(t)\sin\varphi F(z,\rho) \qquad (19)$$

We expand $F(z,\rho)$ in terms of the various order derivatives of $f(z)$ multiplied by appropriate powers of ρ

$$-\Omega = B\sin\varphi\sum_{n=0}^{\infty} C_n f^{(n)}(z)\rho^{n+1} \qquad (20)$$

with $C_o = 1$. Applying equation (17)

$$\left\{\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right\}\Omega = 0$$

we obtain

$$0 = B\sin\varphi\sum_{n=0}^{\infty} C_n\left\{f^{(n)}(n+1)^2\rho^{n-1} + f^{(n+2)}\rho^n - f^{(n)}\rho^{n-1}\right\}$$

Collecting coefficients of $f^{(n)}(z)\rho^{n-1}$ we get

$$C_n(n+1)^2 - C_n + C_{n-2} = 0 \qquad n = 2, 3, \cdots$$

$$C_o = 1 \qquad C_1 = 0$$

The resulting set of coefficients are those of $2J_1(v)$.

$$-\Omega = B\sin\varphi\left\{f(z) - f''(z)\rho^3/3 + f''''(z)\rho^5/192 \cdot\right\}$$

$$= B(t)\left\{f(z)\beta - f''(z)\beta(\alpha^2 + \beta^2)/8 \cdot \cdot\right\} \qquad (22)$$

We now apply equation (16) to obtain $\underset{\sim}{B}$

$$B_\alpha = -B(t)f''(z)\alpha\beta/4 \cdot \cdot \cdot$$

$$B_\beta = B(t)f(z) - B(t)f''(z)(\alpha^2 + 3\beta^2)/3 \cdot \cdot \quad (23)$$

$$B_z = B(t)f'(z)\beta - B(t)f'''(z)(\alpha^2\beta + \beta^3)/3 \cdot \cdot$$

We obtain $\underset{\sim}{A}$ by inspection. It is not difficult to verify that the only expansions for $\underset{\sim}{A}$ which give $\underset{\sim}{B}$ in the form of equations (23) and satisfy $\nabla \cdot \underset{\sim}{A} = 0$ and $\nabla^2 \underset{\sim}{A} = 0$ are

$$A_\alpha = 0 \quad (24)$$

$$A_\beta = -B(t)f'(z)\alpha\beta + B(t)f'''(z)(\alpha\beta^3/3 + \alpha^3\beta/24) \cdot$$

$$A_z = B(t)f(z)\alpha - B(t)f''(z)(3\alpha\beta^2/3 + \alpha^3/24) \cdot \cdot$$

If it is desired to allow for the possibility that the electrical axis of the cavity may not coincide with the axis of the holes, the simplest way to expand the theory is to write in place of equation (15)

$$A_z = f(z)B(t)(\alpha - \alpha_a) \quad (25)$$

where α_a is the time dependent α coordinate of the electrical center of the cavity in terms of the coordinate system still centered in the end holes. The resulting equations analogous to (24) contain in addition to the terms in (24) terms in

$$A_\beta : \; B(t)f'(z)\alpha_a\beta - B(t)f'''(z)\alpha_a\beta^3/6 \cdot \cdot$$

and in $\quad A_z : \; -B(t)f(z)\alpha_a + B(t)f''(z)\alpha_a\beta^2/2 \cdot \cdot \quad (26)$

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