INVESTIGATIONS ON THE APPLICATION OF THE HOT WIRE ANEMOMETER FOR TURBULENCE MEASUREMENTS

Thesis

by

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SUMMARY

The principle reasons for using the hot wire type of anemometer for turbulence measurements are outlined and some of the objectionable features limiting the applicability of the conventional type anemometer are pointed out. A brief historical background of the hot wire anemometer is followed by a consideration of various possibilities of improving the method of measuring the intensity of turbulence. This analysis purposes to show why such improvements can only be realized by a direct calibration of the hot wire's response to velocity fluctuations.

The consequent development of a new technique, referred to as the "Vibrator Method", is then explained. A general discussion of the results of a series of investigations using the Vibrator method is then presented and includes: a comparison with the orthodox method; the response of the hot wire to periodic fluctuations relative to the validity of the theory of hot wire lag; an extension of the method to correlation measurements using two hot wires, and other applications.

The paper concludes with a detailed description of various hot wire anemometer equipment which has been developed for use with the Vibrator method of measuring turbulence intensity.

INTRODUCTION

Contributions to the mathematical development of the theory of turbulence have been forthcoming since the early work of Osborne Reynolds, especially by Prandtl and his associates in Germany, G.I.Taylor and his co-workers in England, and von Karman, Millikan, Dryden and others in the United States. Such contributions have not yet resulted in a general solution of this extremely complex problem, but have aided materially in explaining certain turbulence phenomena and should serve as the foundation for the ultimate solution. Progress in theoretical research has been seriously handicapped by the lack of experimental verification due to difficulties in perfecting anemometers for measuring turbulence.

Although several different types of anemometers have been designed for this purpose, the one which has been generally adopted as standard is known as the "hot wire anemometer". In brief, it consists of a fine gage platinum wire, from one to several millimeters in length, fused across supporting spindles and connected to an electrical circuit for the triple purpose of 1) heating the wire by reason of its electrical resistance, 2) amplifying, and 3) measuring the heat lost by the wire due to convection, that is its response to the velocity fluctuations encountered by it in the airstream.

Among the advantages it has over other types of anemometers are the following; the size of the hot wire probe can be very small to permit explorations of minute flow regions and traverses within boundary layers of models; a high degree of sensitivity is possible by proper amplification, and can be controlled; direct measurements can be obtained of turbulence intensity with a single hot wire; correlation measurements for determining the scale of turbulence can be obtained with essentially the same equipment using two hot wires simultaneously; the apparatus can be used for measuring mean flow velocities very accurately; the possibility exists for incorporating electrical filtering systems for investigating particular frequency bands of turbulence; and the anemometer's performance should improve along with new developments in vacuum tube and electrical circuit design to a degree that is unpredictable at this early stage.

The hot wire anemometer as used at the present time by many aerodynamic laboratories has certain disadvantages, some of which are inherent, while others can be reduced in effect.

The hot wire proper is inherently fragile, it will always be quite difficult to fuse it to the spindles on the probe, and its usable life is short because its electrical and thermal responses are so easily altered by rapid ageing and accumulation of surface impurities from the airstream. The electrical equipment is inherently complex, expensive, bulky, and subject to usual unforeseen disorders.

There are three serious disadvantages which are not inherent. The majority of amplifiers in current use are inadequate for high frequency turbulence investigations. Reference to recent literature shows that very little stress has been placed upon the performance of the hot wire anemometer in regard to frequency response from 2000 to 5000 cycles per second. Also, to the writer's knowledge, all other laboratories still use the conventional type of milliameter in conjunction with a thermocouple in the amplifier's output circuit for obtaining the root mean squared values of the fluctuating voltage drop across the hot wire. With this type of output meter it is impossible to protect adequately the thermocouple from burning out due to momentary overloads without an enormous sacrifice in the meter's sensitivity. A relatively large power output stage is required in the amplifier to give full scale deflection of the pointer. However, the most serious disadvantage of the conventional hot wire anemometer is that its present design is restricted to one general method of measuring turbulence intensity which is not only extremely complicated and laboriously slow, but also very unreliable. Consequently it has been difficult to interest students in turbulence research and this has severely handicapped theoretical progress due to the lack of experimental verification.

The object of this research was the development of a new method of measuring turbulence intensity, together with the design and construction of hot wire anemometer equipment incorporating such features as to minimize the disadvantages mentioned above.

The following section sketches briefly the historical background of the hot wire anemometer, emphasizing the fundamental developments of L.V.King, and of Dryden and Kuethe in order to give a better insight into the nature of the present research.

HISTORICAL BACKGROUND OF THE HOT WIRE ANEMOMETER

The word "anemometer" is derived from the Greek words "anemos" - wind, and "metron" - measure. Thus an anemometer is an instrument for measuring wind velocities or wind pressures. It is convenient to classify anemometers according to their response to changes in wind velocity; slow response - pitot tubes, venturi meters; medium response - pressure plates and spheres; rapid response - hot wire, spark, ultra-microscope. As stated before, the hot wire anemometer consists essentially of a small gage wire supported in the wind stream and connected to an electrical circuit which supplies the medium for heating the wire and measures the heat lost by the wire due to convection, which is a function of the wind velocity.

Early experiments in thermal losses from heated bodies under various conditions dates back to the work of Dulong and Petit[2] in 1817. There followed numerous experiments concerning losses due to radiation and convection. The first experimenter to show that the convection loss in an air current is proportional to the temperature difference and to the square root of velocity, seems to have been Ser[3] in 1888. The classical work of Ayrton and Kilgour[4] (1892) and that of Petavel[5] (1898) refer to heat losses from wires.

In 1907-'09 Kennelly[6] published his results on the forced convection of heat from small copper wires. He probably was the first to suggest that the measurement of the current required to keep a wire at a given temperature (measured by its resistance) might be used as a method for measuring air velocity. His observations led to an empirical formula closely resembling that of L.V.King, which forms the basis for hot wire anemometry.

The Classical Work of L.V.King[7]

Prof. King conducted a series of experiments at McGill University, Montreal, Canada, to investigate the laws of convection of heat from small platinum wires heated by an electric current over as wide a range as possible of temperature, air velocity, and diameter in the light of the formula developed - by application of Boussinesq's transformation of Fourier's equation of heat conduction. He first considered the "mathematical theory of Heat Convection from a cylinder of any form of cross section in a Stream of Fluid". Starting with the Fourier heat equation

$$C\frac{d\theta}{dt} = \frac{\partial}{\partial x}\left(K\frac{\partial\theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial\theta}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial\theta}{\partial z}\right)$$

where θ temp. of fluid at any point (x,y,z)
C heat capacity per unit volume
K thermal conductivity

$\frac{d}{dt}$ operator .. $\partial/\partial t + u\,\partial/\partial x + v\,\partial/\partial y + w\,\partial/\partial z$

and assuming an incompressible, frictionless fluid and 2 dimentional flow and applying Boussinesq's transformation

$$\frac{\partial^2\theta}{\partial\alpha^2} + \frac{\partial^2\theta}{\partial\beta^2} = 2m\,\frac{\partial\theta}{\partial\beta}$$

where α = constant --streamlines
β = constant --equipotential lines
$2m = CV/K = s\sigma V/K$

(V--Velocity of stream at infinity,
σ--Density
s --Specific heat of fluid/unit mass)

he showed that the general equation is reduced to a form capable of solution:

$$H = -\int_0^{\beta_0} K(\partial\theta/\partial\alpha)_0\,d\beta$$ --Heat loss/unit length of cylinder

Thus he calculated the temperature distribution in a uniform stream flowing parallel to x-axis ($\alpha = 0$) when the temperature distribution or heat flux (u) is given over the interval x = 0 to x = β_0.

Assuming a constant heat flux over the boundary, he reduced this to the form:

$$H = 2\pi K\theta_0 m\beta_0 / \left[\int_0^{m\beta_0} e^{u} K_0(u)\,du\right]$$

where θ_0--temperature of cylinder above that of stream at ∞
K_0 --solution of Bessel's equation

He then developed approximate formulae for the heat loss on a cylinder from the previous equations. For instance, for a cylinder of radius -a- and taking $\beta_0 = 4a$

Expanding the denominator of the above equation

$$\int_0^{m\beta_0} e^u K_0(u)\,du + 1 = \sqrt{2\pi m\beta_0}\left[1 + \frac{1}{8m\beta_0} + \cdots\right]$$

when $m\beta_0$ is large, $1/8m\beta_0$ can be neglected, giving

$$H = K\theta_0(1+\sqrt{2\pi m\beta_0}) = K\theta_0 + 2\sqrt{\pi K s\sigma a}\,V^{1/2}\theta_0$$

or--

$$H = K\theta_0 + \lambda\sqrt{V}\,\theta_0$$

Prof. King conducted a series of experimental investigations to check the validity of the above equation in so far as its practical application to hot wire anemometry is concerned, and to obtain in absolute measure the convection constants of small platinum wires applicable for hot wire measurements. A brief outline of his investigations follows; his apparatus consisted of a whirling arm of variable radius from 54 to 264 cm, which could attain a peripheral speed of 25 mph at the point where the hot wire was supported. Measurements were made on a Kelvin double bridge with a series of hot wires ranging from 1 to 6 mils diam. and 2 to 24 cm in length. All wires were tested at the same temperature settings, ranging from 200 to 1000 deg. C. From measurements of the hot wire's current and resistance he calculated the heat loss. He also measured temperature coefficient α for platinum wire and found that the value α decreased from .00375 to .00328 for a decrease in diam. from 6 to 1 mil, and could vary from .00388 to .00188 due to the manufacturing process of wire drawing; reduction of results showed that the heat loss is a linear function of $\sqrt{\text{velocity}}$, expressed in the form $W = C + B\sqrt{V}$ where B and C are constants depending on the wire's temperature and diam. In analyzing the forced convection constant B, plotting B vs. $(\theta - \theta_0)$---(temp. diff.) the slope λ was found to be a function of wire diameter and increased slightly at large values of $(\theta - \theta_0)$. Taking $\lambda = \lambda_0 [1 + b(\theta - \theta_0)]$ he calculated $b = 8 \times 10^{-5}$. Plotting λ_0 vs. $\sqrt{a}$ he found that $\frac{\lambda_0}{\sqrt{a}} = 1.432 \times 10^{-3}$ watts. Returning to his approximate formula $\lambda_0 = \sqrt{4\pi S_0 K_0 \sigma_0 a}$ the corresponding value was $\frac{\lambda_0}{\sqrt{a}} = 1.66 \times 10^{-3}$ watts. The agreement is not bad considering that the theory neglects variations of heat conductivity etc. with temperature gradients in the neighborhood of the wire. In analyzing the free convection constant C_0 he calculated the heat loss due to radiation $E = 2\pi a \times .514 \, (\theta/1000)^{5.2}$ giving $C_0 = (C - E)$., where $C = W$ since $V = 0$.

Taking $\quad C_0 = \nu_0 (\theta - \theta_0)\left[1 + c(\theta - \theta_0)\right]$

he found that ν_0 was almost independent of wire diameter and expressed as $\nu_0 = 2.5 \times 10^{-4} \times (1 + 70a)$ --watts. Comparing this value with that in his approximate formula $K_0 = 2.37 \times 10^{-4}$ --watts, in very good agreement; conclusion - the heat loss from a hot wire in a steady airstream of mean velocity U can be expressed by the equation

$$H = (K + C\sqrt{U})(T - T_0)$$

where K -- proportionality factor allowing for free convection and radiation, C -- proportionality factor allowing for forced convection, T -- temperature of wire, T_0 -- temperature of moving fluid.

Following the classical work of King, many papers were published dealing with modifications of the electrical circuits and methods of mounting hot wires. Those dealing with the measurements of fluctuations include; measurements of gusts in natural winds by E. Huguenard, A. Magnan, A. Planiol,[8] which was one of the first investigations dealing with the lag of hot wires with a method for computing corrections; Burger's[9] method for computing lag and investigations indicating that the hot wire lag increases with an increase in hot wire diameter; A.Fage and F.C.Johansen[10] at the NPL described measurements of fluctuations behind plates, using a .025 mm diam. wire in a low velocity stream to minimize lag.

Investigations of Dryden and Kuethe[11]

In 1929 a paper was published by Dryden and Kuethe of the National Bureau of Standards which is directly responsible for the development and universal use of the compensated hot wire anemometer. During their investigations, which began in 1926, these authors verified the findings of their predecessors regarding the inability of hot wires to follow accurately very rapid velocity fluctuations encountered in airflows. It was generally conceded that some form of compensation would have to be introduced into the anemometer circuit to counteract the hot wire lag if the anemometer was to be of any practical use in turbulence measurements. The problem of correctly compensating the hot wire anemometer necessarily divides itself into two parts, the first to formulate a sound theory of lag, and the second to develop an accurate and practicable method, based upon the theory, of determining the required compensation. The notable contribution of these authors to turbulence research was their formulation of the theory of hot wire lag, the first part of this problem, which is briefly outlined below.

Defining: R, T, i - instantaneous values of resistance, temperature, current in wire

m, s, α - mass, specific heat, temp. coefficient of resistance of wire

V, p - instantaneous wind velocity, $p = 2\pi f$ (f--frequency of wind variation)

$K(T - T_0)$ - rate of heat loss from wire by radiation and free convection

$C(T - T_0)$ - rate of heat loss from wire by forced convection of wind at V

dH/dt - total rate of increase of heat energy in wire

barred values - - average values

subscript $_0$ - - conditions at room temp.

subscript $_e$ - - refer to conditions at constant speed V

Assumptions:

1- Rate of heat loss (from King's equation) does not depend on rate of variation of air speed

2- Heating current i - maintained constant

3- T - mean temp at any instant determined by R.

Basic Equation: Rate at which heat energy accumulates in wire equals rate at which electrical energy enters, less rate at which heat energy leaves.

1) $$\frac{dH}{dt} = i^2R - (K + C\sqrt{V})(T - T_o)$$

Since the increase in heat energy produces an increase in wire temperature

$$\frac{dH}{dt} = 4.2\ ms\ \frac{dT}{dt} \qquad \text{also } T - T_o = \frac{R - R_o}{R_o\alpha}$$

Thus $\frac{dT}{dt} = \frac{1}{R_o\alpha} \times \frac{dR}{dt}$ and substituting for $\frac{dT}{dt}$ and $(T - T_o)$

2) $$\frac{dH}{dt} = \frac{4.2ms}{R_o\alpha} \times \frac{dR}{dt} = i^2R - (K + C\sqrt{V}) \times \left(\frac{R_e - R_o}{R_o\alpha}\right)$$

If the cycle is performed very slowly so that $\frac{dR}{dt} = 0$, the equilibrium value R_e would be determined by taking $R = R_e$

3) $$i^2R_e - (K + C\sqrt{V}) \times \left(\frac{R_e - R_o}{R_o\alpha}\right) = 0$$

or $K + C\sqrt{V} = \frac{i^2R_eR_o\alpha}{R_e - R_o}$ and substituting in above

4) $$\frac{4.2ms}{R_o\alpha} \times \frac{dR}{dt} = \frac{i^2R_o(R_e - R)}{R_e - R_o}$$

This reduces to $R_e - R_o = \dfrac{R - R_o}{1 - \dfrac{4.2ms}{i^2R_o^2\alpha} \times \dfrac{dR}{dt}}$

This is rewritten in form

5)
$$\frac{R_e - R_o}{\bar{R} - R_o} = \frac{\dfrac{R - R_o}{\bar{R} - R_o}}{1 - M\dfrac{d}{dt}\left(\dfrac{R - R_o}{\bar{R} - R_o}\right)}$$

6) Where $M = \dfrac{4.2ms}{i^2 R_o^2 \propto}(\bar{R} - R_o) = \dfrac{4.2ms(\bar{T} - T_o)}{i^2 R_o}$ (seconds)

M - time constant giving characteristics of wire for a given heating current and operating temperature. Supposing the air speed were suddenly changed so that R_e changes from some constant value, say R_1, to $\bar{R}$. The change in R would be:

$$\bar{\bar{R}} - \bar{R} = (R_1 - \bar{R}) \cdot e^{-t/M}$$

Thus M - - is time required for $(\bar{\bar{R}} - \bar{R})$ to $= (R_1 - \bar{R}) \times 1/e$

Returning to Eq. 4) - since $(R_e - \bar{R}) << (\bar{R} - R_o)$ replace $(R_e - R_o)$ by $(\bar{R} - R_o)$ thus

7)
$$M\frac{dR}{dt} = (R_e - R)$$

The solution consists of a transient term containing $e^{-t/M}$ which soon becomes negligible and isn't considered further, and a periodic term, assuming R_e periodic.

Expanding R_e in a Fourier series

(letting $p = 2\pi \times$ fundamental frequency)

$$R_e - \bar{R} = a_1 \sin(pt) + \cdots + a_m \sin(mpt) + b_1 \cos(pt) + \cdots + b_m \cos(mpt) + \cdots$$

Assume that

$$R - \bar{R} = c_1 \sin(pt) + \cdots + c_m \sin(mpt) + d_1 \cos(pt) + \cdots + d_m \cos(mpt) + \cdots$$

Thus

$$\frac{dR}{dt} = c_1 p \cos(pt) + \cdots + m c_m p \cos(mpt) + \cdots - d_1 p \sin(pt) - \cdots - m d_m p \sin(mpt) - \cdots$$

Substituting in Eq. 7)

$$d_n = \frac{b_m - a_m M m p}{1 + M^2 m^2 p^2} \qquad c_m = \frac{a_m + b_m M m p}{1 + M^2 m^2 p^2}$$

Thus

8) $$R - \bar{R} = \sum \left\{ \frac{a_m}{\sqrt{1+M^2m^2p^2}} \sin(mpt - \tan^{-1} M m p) + \frac{b_m}{\sqrt{1+M^2m^2p^2}} \cos(mpt - \tan^{-1} M m p) \right\}$$

Now if the equilibrium value of resistance is expanded in a Fourier series, the actual value of resistance will be such that the n^{th} harmonic is reduced in amplitude in the ratio $\dfrac{1}{\sqrt{1+M^2m^2p^2}}$ and is retarded in phase by $\tan^{-1}(npM)$.

Equation 8) is the same form as that for an electrical circuit containing a resistance R_{cx} and inductance L providing $M = L/R_{cx}$. Thus if the amplified voltage variation from the hot wire circuit can be passed on to the succeeding stage with this time constant, the amplitude will be restored and the phase advanced to compensate for the wire lag.

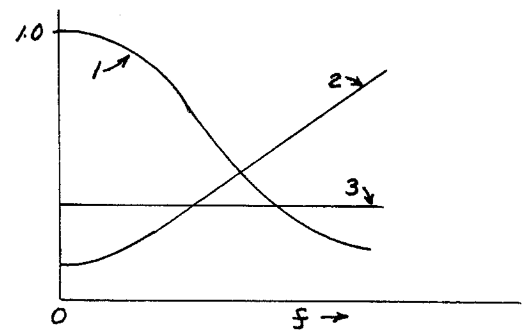
In the compensating circuit of the anemometer the value of the inductance L is fixed at a value determined for the particular circuit, and the compensating resistance R_{cx} adjustable to give the required value of M for proper compensation. Two typical examples of compensating circuits used in Galcit anemometers are shown in Figs. 21, 22

The most plausible method for finding the required compensating resistance R_{cx}, the second part of the problem previously referred to, would be to base it upon the value of the time constant M, in the Dryden and Kuethe equation

$$M = \frac{4.2ms}{i^2 R_o \alpha}(\bar{R} - R_o) = \frac{4.2s\rho(\pi d^2/4)^2}{r_r \; i^2 \alpha} \times \frac{\bar{R} - R_o}{R_o}$$

obtained from measurements of the hot wire's diameter (d), resistances (R and R_o), and current (i). In spite of the disadvantages which this procedure has been found to possess, and which were partly responsible for this research, it has been universally adopted as the "orthodox" method for finding R_{cx}.

If the correct value of M is known, the amplitude reduction characteristic $\dfrac{1}{\sqrt{1+(2\pi f M)^2}}$ due to hot wire lag can be determined as a function of frequency, as (1) in the sketch. Also, if the compensating resistance R_{cx} is correctly adjusted $\left(R_{cx} = \dfrac{L}{M}\right)$ in an amplifier whose performance is perfect, the relative output of the compensated amplifier



$\left(K_1\sqrt{R_{cx}^2+(2\pi L f)^2} = K\sqrt{1+(2\pi f M)^2}\right.$

can be determined as a function of frequency, as shown by (2). Then their products, (1) x (2) = (3), will give the correct level (K) of the anemometer (3), the overall response of the hot wire and compensated amplifier, which will be independent of frequency. The ratio of the anemometer's output voltage, measured by the output meter (constant $\times\sqrt{\overline{I^2}}$), to this level (K) will give the root mean squared value of the fluctuating voltage drop (dE) across the hot wire, which is a function of the turbulence intensity $\left(\dfrac{\sqrt{\overline{u^2}}}{U}\right)$.

The universally accepted procedure for obtaining the relationship between dE and $\frac{\sqrt{\overline{u^2}}}{U}$, together with the method outlined above for determining M, R_{cx}, and the anemometer level, comprise the "orthodox" method of measuring turbulence intensity with the hot wire anemometer, and will now be considered.

The Orthodox Method of Measuring Turbulence Intensity

A typical example illustrating this method is given in Appendix #1. A potentiometer or wheatstone bridge is used in the input circuit of the anemometer for measuring the hot wire's resistance and current. An outline of the general procedure follows:

1. Calibration of amplifier
2. Cold resistance R_o of hot wire measured by bridge
3. Potentiometer resistance determined and set for desired hot wire heating current (i)
4. Determination of Forced Convection constant (c)
   - a- Measuring hot wire resistances (R) for various wind speeds (U), keeping i = constant.

b- From King's equation $(K + c\sqrt{U})(T - T_0) = i^2 R$

and
$$T - T_0 = \frac{R - R_0}{R_0 \alpha}$$

Plot
$$K + c\sqrt{U} = \frac{i^2 R R_0 \alpha}{R - R_0} \quad \cdot vs \cdot \quad \sqrt{U}$$

c- determine slope (c)

5. Determination of Time Constant (M) for each traverse point.

From Dryden and Kuethe's equation
$$M = \frac{4.2 S \rho A^2}{K_v i^2 \alpha} \times \frac{R - R_0}{R_0}$$

a- measurements of hot wire's diameter (d)

where $A^2 = \left(\pi d^2/4\right)^2$

b- measurements of hot wire's resistance (R) for particular traverse point keeping i = constant.

6. Setting Compensation Resistance, R_{cx} , in amplifier at each traverse point

a- given from amplifier calibration data

7. Reading of Output Thermogalvanometer (I^2) at each traverse point.

8. Determination of Turbulence Intensity $\frac{\sqrt{u^2}}{U}$ for each traverse point from above measurements.

From the equation in 4

$$\frac{i^2 R R_o \alpha}{R - R_o} = K + C\sqrt{U}$$

Differentiating with respect to i, R, and U

$$\frac{2 i R R_o \alpha}{R - R_o} \times di - \frac{i^2 \alpha R_o^2}{(R - R_o)^2} \times dR = \frac{c\, du}{2\sqrt{U}}$$

Let:

$$F = \frac{i R_o^2 \alpha}{(R - R_o)^2} \qquad G = \frac{2 i R R_o \alpha}{(R - R_o)}$$

Then

$$G\, di - F i\, dR = \frac{c\, du}{2\sqrt{U}}$$

or

$$\frac{du}{U} = -\frac{2}{c\sqrt{U}}\left(F - G\frac{di}{i\, dR}\right) i\, dR$$

Let $b = -\frac{di}{i\, dR}$ and since $E = iR$

$$dE = i\, dR + R\, di$$

Then

$$i\, dR \cdot \frac{dE}{dE} = \frac{i\, dR \cdot dE}{i\, dR + R\, di} = \frac{dE}{1 + R\frac{di}{i\, dR}} = \frac{dE}{1 - Rb}$$

$$\frac{\sqrt{u^2}}{U} = \frac{2}{c\sqrt{U}}\left[F + Gb\right]\frac{dE}{1 - Rb}$$ <u>Turbulence</u> <u>Level</u> <u>Equation</u>

WHERE $\underline{du} = \sqrt{u^2}$ SINCE $dE = \sqrt{\Delta E^2} = \frac{CONST \times \sqrt{I^2}}{LEVEL}$

The value of (b) depends upon the input circuit used.

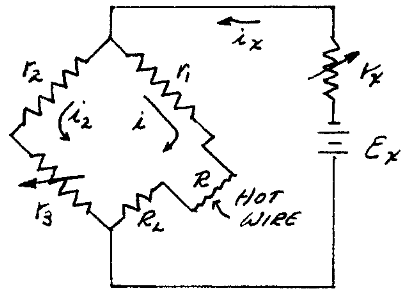
Using a Wheatstone bridge, as in Anemometer #1.

R -- resistance of hot wire

i -- current in hot wire

E -- voltage drop across hot wire

R_L - resistance of hot wire leads



$R_L + r_1 = r_4$; $r_2 + r_3 = r_5$

$i_x = i + i_2 = i\left(1 + \frac{r_4 + R}{r_5}\right)$

$r_1 = 300\,\Omega$ $r_2 = 30\,\Omega$

$r_3 = B\,\Omega$ $r_5 = 10(r_4 + R)$

$$E_x = i_x r_x + i(r_4 + R) = i\left[r_x\left(1 + \frac{r_4 + R}{r_5}\right) + (r_4 + R)\right]$$

$$\frac{r_x}{r_5} = \frac{E_x - i(r_4 + R)}{(R + r_4 + r_5)i}$$

and differentiating with respect to (i) and (R)

$$0 = -\frac{dR}{i}\left[\frac{E_x + r_5 i}{(R + r_4 + r_5)^2}\right] - \frac{E_x\, di}{i^2(R + r_4 + r_5)}$$

THUS

$$b = \frac{E_x + i r_5}{E_x(R + r_4 + r_5)} = 0.909\left(\frac{1}{300 + B} + \frac{i}{E_x}\right)$$

This method was used at the Institude by Wattendorf and Kuethe[12] during their research and the following year by the writer under the guidance of Wattendorf. The latter measurements were an attempt to extend Wattendorf's investigations of the distribution of turbulence intensity in two dimensional flow across a smooth walled channel, to the case in which the channel walls were lined with corrugated paper to simulate "roughness" effects. These investigations led to the following conclusion; that it was impossible to establish the correct turbulence levels and profile shapes by repeating measurements due to the unavoidable inaccuracies incurred by using the orthodox method of measuring turbulence. The seriousness of the situation was not limited to this particular research, since Wattendorf and Kuethe had similar experiences with this method of measurements during their hot wire investigations at the Institute. In order to utilize the advantages of the hot wire anemometer for obtaining future experimental verification of turbulence theory the Institute requested that the writer undertake the development of an improved method of measuring turbulence intensity and the design of pertinent hot wire equipment as the subject of his thesis research. The first step in this direction, which is discussed in the next section, was an analysis of the prior art in an effort to suggest possible lines of procedure.

ANALYSIS OF POSSIBILITIES OF IMPROVING THE METHOD OF MEASURING TURBULENCE INTENSITY

The orthodox method for determining the amount of compensation required to counteract the hot wire lag necessitates an evaluation of the time constant M of the hot wire, as shown by Dryden and Kuethe.[11] These authors pointed out from the equation

$$M = \frac{4.2 s \rho A^2}{r_v \alpha i^2}\left(\frac{\bar{R}-R_o}{R_o}\right); \quad A = \frac{\pi d^2}{4}; \quad \bar{T}-T_o = \frac{\bar{R}-R_o}{R_o \alpha}$$

that "the determination of the correct value of M under given conditions requires a knowledge of the diameter (d), density (ρ), temperature coefficient of resistance (α), specific heat (s), resistance at room temperature (R_o), and mean temperature of the wire ($\bar{T}$). Of these quantities, the wire diameter is the most sensitive and is the most difficult to determine accurately."

The importance of accurately estimating the value for the diameter of the wire can be seen from the above equation for M. The usual procedure is to base this value upon a series of measurements of the diameters at several sections along the wire's length made by means of a sensitive travelling microscope, or by an interferometer. In the ideal case the hot wire's diameter (d) and its heat distribution are assumed to be constant per unit length of wire. Actually the distribution of the heat along the wire is influenced by the wire's length, variations in its diameter, the condition of its surface, and the effect of the supporting spindles. An example of the rapid accumulation of dirt on the surface of a hot wire is shown in the microphotographs, and illustrates the importance of making measurements as quickly as possible. Spindle effects depend upon the design of the hot wire holder in respect to the relative masses of the hot wire and the spindles of the holder, the characteristics of the fused joints, distance between spindles, etc. An "effective diameter" should be taken for (d) in the above formula for M, which counteracts the gradient in the heat distribution. Even though the correction factors for the condition of the wire's surface and spindle effects are assumed to be negligible, such an effective diameter would be difficult to estimate because of the non-uniformity in the gage of the platinum wire. As an example, repeated measurements at this laboratory have shown variations in diameter up to 10% over a 3mm length of 0.0005" platinum wires.

Although one value is always taken for the temperature coefficient of platinum in computing M by the orthodox method, this is not in agreement with the findings of Prof. L.V.King.[7] Referring to his investigations, outlined earlier on p. 10 , his results showed possible variations of 100%.

An incorrect setting of the compensation resistance, based upon an erroneous value of M, affects the measurement of the turbulence level due to an attenuation error in the amplifier's response over the band of frequencies of the velocity fluctuations. The error in the amplitude of the fluctuations approaches the error introduced into the value of M at high frequencies because $\omega^2 M^2 \gg 1.0$ in the compensating type of amplifier whose gain characteristic varies as $\sqrt{1+\omega^2 M^2}$ where $\omega = 2\pi f$.

It is apparent that this attenuation error might be reduced by using a wire having a lower value time constant M. The above equation for M suggests two possible methods to accomplish this, either by using wires of smaller diameter, or by operating the wire at a lower heating temperature.

A convenient type of fine gage platinum wire is known as Wollaston, and consists of a very fine platinum core embedded in a coating of silver which can be etched off by nitric acid. This laboratory has used Wollaston wire of 0.001" diameter and 3-5 mm long having a 1 mm section etched at the center exposing a platinum core of approximately 0.0001" diameter. The value of M is of the order of one tenth of that for the 0.0005" platinum wire. The marked improvement in the response to turbulence fluctuations, obtained by reducing the diameter, can be seen by comparing the lag characteristics of these two wires, Fig. 1 . In high frequency turbulence measurements it is quite important to use small diameter wires in order to retain the product of $\frac{\sqrt{\overline{u^2}}}{U} \times 2\pi f M$ within the order of 0.14 and thereby justify using the first approximation $\frac{1}{\sqrt{1+(2\pi f M)^2}}$ as the magnitude of the amplitude reduction factor, shown by Dryden and Kuethe.[11]

It is evident from these curves in Fig. 1 however, that the response of even very fine wires is not sufficient to eliminate the need for electrical compensation in the amplifier. For instance, at frequencies as low as 1000 cycles per second this curve shows a 50% amplitude reduction due to lag which necessitates a corresponding increase in amplifier gain by the compensating circuit. Thus the use of small diameter hot wires does not sufficiently improve the attenuation error to eliminate the necessity for accurately determining the required amount of compensation. Reducing the wire gage increases the probable error in evaluating the effective diameter.

It is advantageous to operate the hot wire at a lower heating temperature to obtain a smaller value of M, but this results in a sacrifice in sensitivity since $(\overline{T} - T_o) \sim (\overline{R} - R_o)$. This can be counteracted within limits by using high gain amplification. The inherent difficulties encountered in obtaining satisfactory performance with high gain amplifiers limit the possibilities of this procedure.

It can be concluded that the accuracy of the orthodox method is not materially improved by such attempts to effect a reduction in the attenuation error. The only alternative is to replace the orthodox method with one whose accuracy does not depend on an evaluation of the hot wire's effective diameter, temperature coefficient α, or on spindle effects, etc., but which gives reliable results in a comparatively short space of time.

The criterion for correct compensation is that the response of the anemometer's compensated amplifier must counteract the lag of the hot wire at all turbulence frequencies. It can be seen from the turbulence level equation on page 21 that

$$\frac{\sqrt{\overline{u^2}}}{U} = -\left[\frac{2}{c\sqrt{U}}\left(F + Gb\right)\frac{1}{1-Rb}\right] dE \qquad dE = \frac{\text{constant}\sqrt{\overline{I^2}}}{\text{level}}$$

for constant values of $\frac{\sqrt{\overline{u^2}}}{U}$, U, i, R, and with correct compensation, that dE, level, and thus I^2 remain constant at all turbulence frequencies. This suggests the possibility of using an experimental technique for determining the required compensation resistance by directly calibrating the response of the hot wire as a function of frequency. The procedure would be to measure I^2 corresponding to various values of the compensation resistance R_c^* at two or more known frequencies for a fixed, but not necessarily known, level of turbulence $\frac{\sqrt{\overline{u^2}}}{U}$, at a fixed heating current - i, and at a known value of mean flow velocity U, and from this data determine the required compensation resistance R_{cx}^* which satisfies the condition I^2 = constant. Since $\sqrt{\overline{I^2}}$ at R_{cx}^* corresponds to $\frac{\sqrt{\overline{u^2}}}{U}$, and if this value of turbulence intensity is known, then I^2 can also be calibrated as a function of the turbulence level. This procedure requires a suitable method for simulating the effect of turbulence on the hot wire, hereafter referred to as "artificial turbulence".

In 1930 Ziegler[13] suggested superposing a small alternating current, whose magnitude and frequency could be varied and measured, on the direct heating current of the hot wire, which was placed in a steady airflow of negligible turbulence. The alternating current simulated the effect of the fluctuating voltage drop (dE) across the wire, in regard to response, for small changes in the hot wire's resistance. In 1932 Mock and Dryden[14] published a very comprehensive description of their experiences using this method to verify their theory of the lag of hot wires. Extreme difficulties were encountered in the construction of an AC-DC bridge, free from inductance effects, which was suitable for such measurements. They determined the compensating resistance from a calculation of M by the orthodox method, and then demonstrated experimentally by Ziegler's method that the response of the hot wire - compensated anemometer combination was independent of frequency over a band of 25--700 cycles per second. If such difficulties as Mock and Dryden experienced in the construction and operation of the equipment could be overcome, Ziegler's method could be used for determining experimentally the required compensation resistance for the hot wire. However it is not possible to determine the intensity of turbulence corresponding to the superposed alternating current, using Ziegler's method, except by reverting to the orthodox method.

If a hot wire is mechanically vibrated sinusoidally in, and parallel to, a steady airflow of negligible turbulence, the root mean squared value of its fluctuating velocity will be proportional to the product of the frequency and the amplitude of the vibration. The wind velocity relative to the hot wire will be composed of a fluctuating component $\sqrt{\overline{u^2}}$ superposed on a steady non-fluctuating mean flow component U, both of which may be known and controllable, permitting a direct determination of the intensity of the artificial turbulence $\frac{\sqrt{\overline{u^2}}}{U}$. In 1929 Dryden and Kuethe[11] mechanically vibrated a hot wire up to 60 cps for the same purpose as that for using Ziegler's system. In 1931 H. Doetsch and P.V. Mathes[15] verified Dryden's investigations. In 1934 C. Salter and W.G. Raymer[17] constructed three different mechanical vibrators to cover a frequency band up to 270 cps for checking the performance of a compensated anemometer used in conjunction with an Einthoven oscillograph which had a distortion and a lag factor of its own in addition to that of the hot wire. To obtain vibrations from 10-20 cps they used a motor driven lever system; for vibrations at 100 cps this was replaced by a vacuum tube controlled tuning fork; and for a band of 200-270 cps they adapted a moving coil type loud speaker unit for the drive. In the two latter installations the hot wires were of necessity situated near the fairing plates of the driving mechanisms instead of in the free stream of the wind tunnel.

It is interesting to note that none of these investigators used or suggested the use of the vibrator as a means for experimentally calibrating the hot wire to obtain either the required compensation resistance or the functional relationship between the turbulence level and the output meter readings. Instead, their investigations were limited to a verification of Dryden and Kuethe's theory of the lag of hot wires, and the justification for using an inductance-resistance type of compensating network in the amplifier.

A digest of this discussion shows that either of these two systems of producing artificial turbulence can be used for determining experimentally the amount of compensation required to counteract the lag of the hot wire. Only in the second system, of sinusoidally vibrating the hot wire in a steady non-fluctuating airflow, is the value of I^2 corresponding to the turbulence level directly obtainable. Therefore it offers the distinct advantage of forming the basis of a new technique for measuring turbulence intensity, which would appear to be much more rapid and reliable than the orthodox method. The success of the "vibrator method" will depend upon the degree of precision of producing and of measuring artificial turbulence, and upon the performance of the compensated anemometer.

We will now consider this method more in detail.

DEVELOPMENT OF THE VIBRATOR METHOD OF MEASURING TURBULENCE INTENSITY

Suppose we wish to use the hot wire anemometer to measure the distribution of turbulence intensity, corresponding to a known mean velocity distribution, across the flow in a wind channel. The proposed general procedure would be:

1) with the hot wire placed in the calibration tunnel, determine by means of the vibrator equipment:

   a) the compensation resistance $R_{cx} = R_{cx}(U)$

   b) the output meter reading $I^2 = I^2\left(\frac{\sqrt{\overline{u^2}}}{U}, U\right)$

2) with the hot wire transferred to the wind channel, at each traverse point

   a) knowing the calibration function $R_{cx}(U)$, adjust the compensation resistance to R_{cx} corresponding to U

   b) read the output meter I^2

   c) from the calibration function $I^2\left(\frac{\sqrt{\overline{u^2}}}{U}, U\right)$ determine the turbulence intensity $\frac{\sqrt{\overline{u^2}}}{U}$ corresponding to I^2 and U.

The vibrator method presupposes that the mean value of the hot wire's heating current (i) and the amplifier's performance remain constant during calibration and measurements.

We will now investigate means for obtaining the calibration functions $R_{cx}(U)$ and $I^2\left(\frac{\sqrt{\overline{u^2}}}{U}, U\right)$

A) <u>The Compensation Function</u> $R_{cx}(U)$

Substituting the Time Constant equation,

$$M = \frac{4.2\,PSA^2}{r_v i^2 \alpha}\left(\frac{R-R_0}{R_0}\right) = A\left(\frac{R-R_0}{R_0}\right) \qquad \text{when } i = \text{constant}$$

into King's equation,

$$K + C\sqrt{U} = (i^2 \alpha R_0)\frac{R}{R-R_0} = B\frac{R}{R-R_0} = B\left(1+\frac{R_0}{R-R_0}\right)$$

gives

$$K + C\sqrt{U} = B\left(1+\frac{A}{M}\right)$$

or

$$\frac{1}{M} = \frac{K-B+C\sqrt{U}}{AB} = H + J\sqrt{U}$$

Thus

$$R_{cx} = N + P\sqrt{U} \qquad \text{SINCE} \quad M = L/R_{cx}$$

It should be noted that R_{cx} includes the DC resistance of the inductance r_L . Since R_{cx} is a linear function of $\sqrt{U}$, a calibration at any two velocities, U_1 and U_2 , at one value of turbulence level $\frac{\sqrt{\overline{u^2}}}{U}$ is sufficient to define R_{cx} for any value U encountered in the traverse.

The most obvious calibration procedure for R_{cx} at a given U was described on p. 29 and is graphically shown in Fig. 4 . This involves a repetition of work which can be eliminated by finding the functional relationship between R_{cx} and the ratio of the amplifier's relative output, with compensation, at any two convenient frequencies f_1 and f_2 , viz.

$$R_{cx} = R_{cx}\left(\frac{\sqrt{1+K\left(\frac{f_1}{R_{cx}}\right)^2}}{\sqrt{1+K\left(\frac{f_2}{R_{cx}}\right)^2}}\right) \quad ; \quad K=(2\pi L)^2; \quad M=L/R_{cx}$$

For a given U and $\frac{\sqrt{\overline{u^2}}}{U}$, the condition for correct compensation is:

$$\sqrt{\overline{I_c^2}} = d\mathcal{E}_a \sqrt{(R_{cx})^2+(2\pi L f_a)^2} \qquad a = 1, 2, \cdots$$

where:

$$d\mathcal{E}_a = \frac{A}{\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2}} \qquad A = \text{CONST.}$$

and $\frac{1}{\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2}}$ is the reduction in amplitude

of the root mean square of the fluctuating voltage drop dE across the hot wire due to the hot wire's response with f_a. If dE_a at f_a is impressed across the amplifier's input it may be measured with the output meter for a known constant amplification level (w), by setting $L = 0$ and $R_{cx} =$ const. (say R_{cK}). Then calling output - $\sqrt{I^2_{oa}}$

$$\frac{A}{\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2}} \times W = \underline{\sqrt{I^2_{oa}}} \qquad W = R_{cK}$$

Now

$$\sqrt{(R_{cx})^2+(2\pi L f_a)^2} = R_{cx}\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2}$$

is the response with frequency f_a of the compensating type amplifier ($L \neq 0$) for a unit voltage impressed across its input.

This reduces to

$$\sqrt{I^2_c} = \sqrt{I^2_{oa}} \times \frac{R_{cx}}{R_{cK}}\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2} \qquad a = 1, 2, \cdots$$

and for two values of a

$$\frac{\sqrt{1+K\left(\frac{f_1}{R_{cx}}\right)^2}}{\sqrt{1+K\left(\frac{f_2}{R_{cx}}\right)^2}} = \frac{\sqrt{I^2_{o2}}}{\sqrt{I^2_{o1}}}$$

we find

$$R_{cx} = R_{cx}\left(\frac{\sqrt{1+K\left(\frac{f_1}{R_{cx}}\right)^2}}{\sqrt{1+K\left(\frac{f_2}{R_{cx}}\right)^2}}\right) = R_{cx}\left(\frac{\sqrt{I_{o2}^2}}{\sqrt{I_{o1}^2}}\right)$$

Fig. 2 shows the function $R_{cx}\,(\quad)^2$ computed for two frequencies of vibration and the known value of inductance L. In these computations R_{cx} must include the DC resistance (r_L) of L, but for convenience it is plotted against the compensating resistance only, which will be referred to as R^*_{cx}. In addition, the function

$$R_{cx}\left(\frac{R_{cx}}{R_{cK}}\sqrt{1+K\left(\frac{f_1}{R_{cx}}\right)^2}\right)$$ is shown in Fig. 2 in

order to obtain $\sqrt{I_c^2}$, as explained later.

Summarizing this procedure for determining $R_{cx} = R_{cx}(U)$:

1) Keep i = constant; short circuit compensating inductance (L = 0); adjust compensating resistance to a convenient value R_{cK}
2) At mean flow velocity U_1 ,
   a) vibrate hot wire at frequency (f_1), total amplitude (Δ_1), and read output meter (I^2_{o1})
   b) repeat at (f_2) and (Δ_2), where $\Delta_2 = \frac{\Delta_1 f_1}{f_2}$ and read (I^2_{o2})

3) At mean flow velocity U_2 ,

a,b) same procedure as in 2) keeping $\frac{\sqrt{\overline{u_1^2}}}{U_1} = \frac{\sqrt{\overline{u_2^2}}}{U_2}$ for later convenience.

4) From amplifier's calibration curve $R_{cx} = R_{cx}\left(\frac{I_{02}^2}{I_{01}^2}\right)$

find R^*_{cx} corresponding to U_1 and U_2

5) Determine constants (T) and (P) in

$$R^*_{cx} = (N - r_L) + P\sqrt{U} = T + P\sqrt{U}$$

from the values of R^*_{cx} in 4).

B) <u>The Turbulence Level Function</u> $I^2\left(\frac{\sqrt{\overline{u^2}}}{U}, U\right)$

Since the root mean square of the fluctuating velocity of a sinusoidal vibration is:

$$\sqrt{\overline{u^2}} = \frac{1}{\sqrt{2}}\left(\tfrac{1}{2} \times \text{TOTAL AMPLITUDE} \times 2\pi \times \text{FREQUENCY}\right)$$

the intensity or level of artificial turbulence is:

$$\frac{\sqrt{\overline{u^2}}}{U} = 2.22 \times 10^{-3} \times \Delta \times f \times \frac{1}{U}$$

where

U --mean flow velocity (meters/second)

Δ --total amplitude of hot wire vibration (mm)

f --frequency of hot wire vibration (cps)

First let us consider a particular case:

$$I^2 = I^2\left(\frac{\sqrt{\overline{u^2}}}{U}\right) \qquad \text{at } U = \text{constant},\ R_{cx} = \text{constant}$$

From the turbulence level equation, p. 21 , it is seen that $\sqrt{\overline{I_c^2}}$ will be a linear function of $\frac{\sqrt{\overline{u^2}}}{U}$ for the turbulence range in which the variation in the mean value of the hot wire's resistance R is negligible.

Since $\sqrt{\overline{I^2}} = 0$ at $\frac{\sqrt{\overline{u^2}}}{U} = 0$, we have

$$\frac{\sqrt{\overline{u^2}}}{U} = \frac{Q}{U}\sqrt{\overline{I^2}} \qquad \text{where } Q = \frac{2.22\Delta f}{\sqrt{\overline{I_c^2}}} = \text{constant.}$$

Vibrator calibrations have verified this (Figs. 5, 14) for a range of turbulence intensities, exceeding 14%, sufficient to cover practically all types of investigations.

The value $\sqrt{\overline{I_c^2}}$ may be obtained in two ways:

1) Adjust the compensating resistance to R^*_{cx} corresponding to a given U and switch the inductance back into the compensating circuit. Read I_c^2 corresponding to convenient values of f_1 and Δ_1.
2) We have shown, p. 37 , that

$$\sqrt{\overline{I_c^2}} = \sqrt{\overline{I_{oa}^2}} \times \frac{R_{cx}}{R_{ck}} \times \sqrt{1 + K\left(\frac{f_a}{R_{cx}}\right)^2} \qquad a = 1, 2, \cdots$$

and in Fig. 2 graphed the function

$$R_{cx} = R_{cx}\left(\frac{R_{cx}}{R_{cK}}\sqrt{1+K\left(\frac{f_a}{R_{cx}}\right)^2}\right) = R_{cx}\left(\frac{\sqrt{\overline{I_c^2}}}{\sqrt{\overline{I_{01}^2}}}\right)$$

The value of $\sqrt{\overline{I_c^2}} = \sqrt{\overline{I_{01}^2}} \times \left[\frac{\sqrt{\overline{I_c^2}}}{\sqrt{\overline{I_{01}^2}}}\right]$

where $[\;]$ value is found on the second calibration curve corresponding to R^{*}_{cx} , and $\sqrt{\overline{I_{01}^2}}$ is the previous output meter reading at $f_1, \Delta_1, R_{cK}, L = 0$

Now let us consider the general case:

$$I^2 = I^2\left(\frac{\sqrt{\overline{u^2}}}{U}, U\right) \qquad \text{at } U \neq \text{ constant}$$

Since we have established $R_{cx} = R_{cx}(U)$ from a calibration at two values of U , we can determine the corresponding values of the turbulence level function directly from the amplifier calibration curves, Fig. 2 . For other values of U it will only be necessary to vibrate the hot wire at one frequency, say f_1 , to determine $\sqrt{\overline{I_c^2}}$, Q , and the required function. It is convenient to keep the turbulence level $\frac{\sqrt{\overline{u^2}}}{U} =$ constant, by adjusting the amplitude Δ in accordance with values U .

Either of the above methods can be used for determining $\sqrt{\overline{I_c^2}}$.

The above procedure can be further simplified if the external resistance in the hot wire heating circuit is sufficiently large to prevent appreciable variations in the heating current.

Thus $\quad dE = i\,dR + R\,di \doteq i\,dR \quad$ WHEN $\; di \doteq 0$

The turbulence level equation, p. 21 , becomes:

$$\frac{du}{U} = -\frac{2}{c\sqrt{U}}\left[\frac{i\,R_0^2\,\alpha}{(R-R_0)^2}\right] \times dE$$

where $\quad dE = \dfrac{\sqrt{\overline{I^2}}}{\text{LEVEL}} \quad = \quad$ constant $\times \dfrac{\sqrt{\overline{I^2}}}{R_{cx}}$

From the time constant equation, p. 35

$$\frac{1}{R-R_0} = \frac{A}{R_0} \times \frac{1}{M} = \text{CONST.} \times R_{cx}$$

Thus

$$\frac{du}{U} = \text{CONST.}\ \frac{R_{cx}}{\sqrt{U}} \times \sqrt{\overline{I^2}} = \frac{Q}{U}\sqrt{\overline{I^2}}$$

and since

$$R_{cx} = N + P\sqrt{U} \qquad \text{AND} \qquad \frac{du}{U} = \frac{\sqrt{\overline{u^2}}}{U}$$

Thus

$$\frac{\sqrt{\overline{u^2}}}{U} = S \times \left(\frac{N}{\sqrt{U}} + P\right)\sqrt{\overline{I^2}}$$

where

$$\mathcal{S} = \left(\frac{\sqrt{\overline{u^2}}}{U_1}\right)_c \times \frac{1}{\sqrt{I_c^2}} \times \left(\frac{N}{\sqrt{U_1}} + P\right)_c^{-1}$$

and subscript - c - refers to a corresponding set of values determined by vibrator calibration.

This procedure only requires a calibration at two mean velocities U, which may be summarized as follows:

1) Through 5), same as given on p. 38

6) Find constant N $\quad N = T + r_L$

7) Find $\sqrt{I_c^2}$ corresponding to R^*_{cx} at U_1 from amplifier's calibration curve $R_{cx} = R_{cx}\left(\frac{\sqrt{I_c^2}}{\sqrt{I_{01}^2}}\right)$ and output reading (I_{01}^2).

8) Determine calibration constant $\mathcal{S}$ for the known turbulence level $\left(\frac{\sqrt{\overline{u^2}}}{U_1}\right)_c = 2.22\,\Delta_1 f_1 \times \frac{1}{U_1}$ and $\sqrt{I_c^2}$ AT U_1 from the relation:

$$\mathcal{S} = \left(\frac{\sqrt{\overline{u^2}}}{U_1}\right)_c \times \frac{1}{\sqrt{I_c^2}} \times \left(\frac{N}{\sqrt{U_1}} + P\right)_c^{-1}$$

9) Then

$$\frac{\sqrt{\overline{u^2}}}{U} = \mathcal{S} \times \left(\frac{N}{\sqrt{U}} + P\right)\sqrt{I^2} = \frac{Q}{U}\sqrt{I^2}$$

where $\quad \dfrac{Q}{U} = \dfrac{\mathcal{S} \times R_{cx}}{\sqrt{U}}$

RESULTS OF INVESTIGATIONS WITH VIBRATOR EQUIPMENT

One of the first investigations with this equipment was a comparison of the vibrator and orthodox methods of measuring turbulence intensity, outlined in Appendix #1. Their results, expressed as the factor Q/U on pp. 78,81 are seen to be in excellent agreement. Further verification of the vibrator method by measurements made with the Wattendorf anemometer in 1935 is shown in the latter part of the appendix and in Fig. 10 . It is interesting to note the numerous computations involved in the older method which have been eliminated by the vibrator procedure. It should be mentioned that the total time required to calibrate the wire was less than one third the time taken to make measurements of the wire's diameter. As the latter proved to be unreliable due to excessive variations with the length of the wire, it was necessary to resort to the vibrator calibration for determining the value of the time constant M and the required compensation resistance. Fig. 7 shows that these vibrator values are correct. Although the same results could have been obtained in a much shorter time by following the calibration method given on p.38 , i.e., reading only two values of I^2 and using the amplifier's characteristics I^2_{02}/I^2_{01} vs R^*_{cx} and $\sqrt{I^2_c}/\sqrt{I^2_{01}}$ vs R^*_{cx}, this longer procedure was chosen for illustrative purposes. A study of the curves in Fig. 4 shows that the accuracy of the value R^*_{cx} is better than $\pm \frac{1}{2}\%$. The curves in Fig. 5 show that $\sqrt{I^2_c}$ is a linear function of $\frac{\sqrt{\overline{u^2}}}{U}$, as stated on p. 40, for a range of turbulence intensities beyond 16%. Its measured value shown on Fig. 4 is within $\frac{1}{2}\%$ of the mean value. Thus the output meter reading I^2_c corresponding to the required compensation resistance R^*_{cx} and turbulence level $\frac{\sqrt{\overline{u^2}}}{U}$ is sufficient to define the turbulence level function $I^2(\frac{\sqrt{\overline{u^2}}}{U}, U)$ at the mean flow velocity U.

A typical investigation of the compensation function $R_{cx}(U)$ and turbulence level function $I^2(\frac{\sqrt{\overline{u^2}}}{U}, U)$ is given in Appendix #2. Since the external resistance in the hot wire heating circuit was large, we will assume that variations in the heating current were negligible - i.e., $dE \doteq i\,dR \quad \therefore di \doteq 0$ and consider this to be an example of the case developed on p. 43. From the data sheets it is seen that the compensation resistance is correct for each mean flow velocity. This verifies the theoretical deduction that R_{cx} is a linear function of $\sqrt{U}$ and is directly proportional to $(R-R_o)^{-1}$, as shown in Fig. 11 . Since the DC resistance of the inductance is 100Ω in this case, it would be impossible to compensate below $(.43)^2 = .185$ m/s. Fig. 12 shows the variation of $(R-R_o)^{-1}$ with heating current i. Since $R_{cx} = 0$ at $(R-R_o)^{-1} = 0$ it is important to avoid excessive heating currents in order to measure turbulence levels in regions of low velocity. The theoretical minimum velocity for which King's equation is valid was shown by King[7] to be a function of the wire's diameter - i.e., $Ud = 1.87 \times 10^{-2}$ For ½ mil wire this would be 0.735 m/sec, and 3.6 m/sec for 1/10 mil Wollaston wire. The vibrator equipment offers possibilities for investigating the validity of King's expression for the limiting velocity.

The turbulence function for this particular case has been expressed on p. 42 as

$$\frac{\sqrt{\overline{u^2}}}{U} = S \times \left(\frac{N}{\sqrt{U}} + P\right) \times \sqrt{\overline{I^2}}$$

and is in excellent agreement with the measured values, as seen in the appendix. If the heating current is so adjusted that $N = 0$, this expression would give $\sqrt{\overline{I^2}} =$ const for $\frac{\sqrt{\overline{u^2}}}{U}$ at any U . This has been verified during vibrator calibrations and is advantageous because it facilitates the making of turbulence measurements.

In a previous section attention was called to some experiments made by Dryden and several others to determine the validity of theory of hot wire lag. Dryden's results showed good agreement with theory for a frequency band up to 60 cps. The writer conducted similar experiments with the vibrator, covering a frequency band up to 180 cps. The results, shown in Appendix #3, are an excellent verification of the theory even for large values of the time constant M.

The above investigations have substantiated the use of King's equation and the Dryden and Kuethe expression for the time constant M in turbulence measurements. They form the foundations for both the orthodox and vibrator methods of measuring turbulence intensity. However these investigations have shown that the latter method is to be preferred because of its increased accuracy and greater rapidity.

The spindles supporting the hot wire should be made of copper rather than of stiffer materials such as steel or phospher bronze to eliminate transverse vibrations.

Measurements have shown that these vibrations will introduce errors in the output readings I^2. A simple method for determining this error is to compare the theoretical and measured relative outputs with the compensating inductance in and out of the amplifier circuit. High frequency harmonics resulting from such vibrations are visible on the cathode ray oscilloscope output wave shape. The frequency varies with the wind velocity. Our experiments with stiff spindles have shown that the effects are not amplified by operating the vibrator, and when properly designed spindles are used the errors due to transverse vibrations are not measurable.

Correlation measurements of the scale of turbulence have been accelerated by using the vibrator for determining the relative voltage output matching factor of the two hot wires. This factor becomes more significant with shorter wires because of the increased difficulties in constructing two hot wires whose cooling responses are similar.

Preliminary investigations seem to indicate that the time constant M, and thus the required compensation, may not be independent of the length of the hot wire when the length is less than 2 mm. It is suggested that these investigations be continued as they are vitally important in their effect upon correlation measurements. One procedure would be to determine the compensating values R_{cx} corresponding to several etched lengths of the same Wollaston wire.

DESCRIPTION OF NEW AND IMPROVED HOT WIRE EQUIPMENT

Synopsis of Vibrator Equipment

Artificial turbulence is produced by a mechanical vibrator used in conjunction with a calibration wind tunnel. The vibrator is built on the principle of a vibrating string. A wire suspension system, which represents the string, supports and vibrates the hot wire along the axis of the calibration tunnel. Its ends are attached to piano-type fittings which are secured to the upper inside corners of a rigid rectangular structure which frames the entrance section of the calibration tunnel. Its lower extremity is coupled to a screw-actuated lever system to provide means for varying the initial tension in the wire suspension system, and thus its natural frequency of vibration. Inboard each fitting the wire is snugly clamped to the frame by a "stop" to fix the nodes of vibration within the straight sections of the wire branches, away from the fittings. The wire suspension is vibrated by an electromagnetic drive attached to the lower branch near the frame. The electromagnet is actuated by power-amplified impulses originating from a self-excited pick-up coil, which is located at one of the upper branches between the tunnel wall and upper stop. The magnitude of these impulses is controllable in the output circuit of an amplifier to effect adjustment of the maximum amplitude of vibration of the hot wire. Differences in phase of the impulses at the pick-up coil and at the electromagnet armature may be manually compensated by a "phase shifter" in the amplifier circuit. The electrical circuit is shown in Fig. 20 . The present calibration tunnel is of circular section, an open circuit suction type with a sharp edge entrance.

The fluctuating velocity of the hot wire is determined from measurements of the maximum amplitude and the frequency of the vibration. An optical system is used for amplitude measurements and consists of three parts; a pointer projecting downstream from the hot wire holder fitting, illuminated by a light source, and observed through a telescopic micrometer to facilitate alignment with the pointer. It is mounted on a platform attached to one of the vertical members of the wire suspension frame with the end of its draw tube positioned several inches outside the tunnel wall window. Frequency measurements are obtained with the aid of a cathode ray oscilloscope, a beat-frequency oscillator or a standard frequency source, and a frequency pick-up coil mounted on the other upper branch of the suspension wire.

Some of the above mentioned equipment will now be described in greater detail.

Description of Vibrator Equipment

Wire Suspension Systems

The rectangular frame supporting the suspension wires is fabricated from 3" x 5" x ½" steel I beams adequately bolted together. In addition there are two large angles, welded across the ends of the lower horizontal I beam and braced by two channels to the top of the frame, to prevent the structure from rocking. This construction provides a self contained unit of sufficient weight and rigidity to absorb most of the elastic energy and to maintain constant alignment of the end fittings of the wire suspension while the wire is vibrating.

Investigations of Various Wire Suspensions

Several types of wire suspensions have been used during the development of the apparatus. The first type was a single wire with a tube, representing the hot wire holder, soldered at the midpoint of the wire, perpendicular to it and to the plane of vibration. No suitable method was found to mount the tube so that it oscillated along one axis, except by exercising extreme care in the matter of balance. The second system was essentially the same as that shown in Fig. 26 replacing the single wire suspension by a Y suspension. The upper branches of the Y were of one wire joined at the mid-point to a vertical wire, differing in this respect from that in Fig. With the Y suspension it was discovered that the tube, if located near the intersection point, oscillated along an axis normal to the plane of the wires, even if a noticeable eccentricity of weight distribution was present. The natural frequency of the oscillation was changed by varying the tension in the vertical wire, which produced corresponding variations in the other branches, depending upon the geometry of the system. Changing the angle between the upper branches with proper ratios of wire lengths did not show any appreciable difference in performance. Several wire diameters were used, ranging from .016" to .073". With the present installation using a 15" diameter wind tunnel which necessitated rather long wires, a compromise was reached using a diameter of .051". With this combination the natural frequency could be varied from 30 to 250cps. This will be discussed later. One trouble encountered in this construction, which was overcome by the design of Fig. 26 , was the breakage of the upper wire at the point where the vertical wire was attached due to high stress concentration when operating at high frequencies and large amplitudes. Also the twisting of one wire around another, or at the fittings caused breakage troubles due to sharp bends and stretching of the wire.

As it was found necessary to solder loops to prevent slippage, this probably was responsible for the breakage troubles.

Two other vibrating systems were investigated having the same parallel wire suspension. This comprised a single wire, with the ends fixed to the upper fittings spaced $\frac{3}{4}$" apart, and looped around a $\frac{3}{4}$" bolt in the lower adjustable fitting to equalize tensions. A small strip was soldered across the wires half way between the upper and lower fittings to support the hot wire tube. In the first case the tube was mounted perpendicular to the strip with the plane of vibration normal to the plane of the wires. The performance was poor unless extreme care was taken to balance the tube to prevent energy transfer between the suspension wires. In the second case the tube was mounted perpendicular to and in the plane of the wires with the plane of vibration coincident with the plane of the wires. Here the performance was greatly improved, with the balance problem much less critical. However it did not compare well with the Y type of suspension.

Single Wire Y Suspension

At this stage it was decided to concentrate on the development of an improved Y suspension system, which was used extensively until 1938. The problem of wire breakage at the junction of the three branches, encountered in the previous Y installation, was eliminated by using one continuous wire. Fig. 26 shows the ends of the wire connected to the upper fittings, and the center of the wire looped around a 7/16" pin in the lower tension adjustment fitting. Thus the vertical branch of the Y is double, the wires being held in contact by binding them with a few turns of small copper wire, soldered to prevent slippage, just below the hot wire fitting and at 6" above the lower fitting pin. The frequency of vibration is changed by a hand screw which rotates a lever raising or lowering the lower fitting pin. The two upper fittings, shown in Fig. 27 , are similar to the piano type. Each is seen to consist of a rectangular block drilled to take the fitting bolt which carries the wire. The wire end is first passed thru the small lateral hole and crimped over the bolt, which is rotated by a removable rod, wrapping about two turns of wire around it. The shoulder on the bolt is drawn up against the block by the locking nut. This type of fitting has many advantages over the type previously used, in which the wire was looped around a bolt and the end twisted or ferruled and soldered. The snubbing action of the present fitting has eliminated all breakage troubles in the previous design, is much easier to rig, and is adjustable. The three fittings are so located that the plane of the suspension wires if offset $1\frac{1}{2}$" from the center line of frame. Each of the three fittings is equipped with a "stop" to assist in alignment and to establish the end node of vibration in the straight section of the wire away from the fitting. The stop consists of a small brass bar which clamps the wire against the fitting block, with the edges of each slightly rounded to prevent wire chafing.

The hot wire holder fitting is located at the intersection of the three branches of the Y , shown in Fig. 26 It consists of two parts, a small steel triangular plate having two .052" holes drilled at 90° thru which the suspension wire passes, and a clamping bar to secure the hot wire holder to the plate. The triangular plate withstands the resultant forces in the wire suspension which tend to separate the Y into a V . The vertical component of the force which tends to push the fitting downward is resisted by the small coil of wire soldered to the vertical branch under the triangular plate. This type of fitting has proven very satisfactory. It is light in weight, small, easily constructed, and readily adaptable to various hot wire holders and pitot tubes. If desired it can be weighted to obtain vibrations at low frequencies.

Single Wire X Suspension

This system, used at the present time, is comparable in performance with the Y suspension. It consists of a single wire arranged in a cross as shown in Fig. 19 . The free ends of the wire are secured to the same upper piano-type fittings (a) as before. At the lower legs of the cross the wire passes over rollers (b) which are secured to the frame. The natural frequency of vibration of the wire suspension is varied by rotating screw (c) which actuates a lever system (d) connecting a parallel linkage (e) which rests on the lower horizontal branch of the suspension. At the intersection of the cross each diagonal branch of the wire passes thru the hot wire holder fitting (f). With this arrangement there is no tendency for the wire suspension to move the fitting, as was experienced in the Y suspension. The holes in the fitting are drilled at an angle slightly different from that in the cross to insure a snug sliding fit of the suspension wire thru the fitting. The hot wire holder may be horizontally inserted in the lower part of the fitting and clamped with a set screw.

Electromagnetic Drive System

The electromagnet is mounted on a base plate in such a way that it can be properly aligned and then clamped in position. Its constructional features are shown in the photographs. The armature is a two piece laminated core-steel strip, rigidly secured to the bottom of the core frame, and drilled along its axis. An ˥ shaped piano wire has one leg slipped thru this hole and the end of the other leg secured to one branch of the suspension wires, and 1" above the lower stop. This method of transmitting the vibration of the armature to the suspension wires permits the driving mechanism to be located outside the wind tunnel. The elliptical armature coil is slipped over the armature and clamped between the two field coils.

The self exciting pick-up coil is made from the secondary winding of a transformer (10,000 Ω DC). It has a horseshoe type permanent magnet core, with one pole extended thru the coil so that its end is flush with the edge of the windings. The coil is mounted on the frame by means of an adjustable clamp, shown in the photograph. It is located at one of the upper branches of the suspension system, to avoid field effects from the electromagnet. The location is not critical provided the suspension wire approximately intersects the projection of the coil's axis at 90°. The gap between the end of the coil and suspension wire is about ½".

The amplifier, phase shifter and attenuator amplitude control are mounted in a small cabinet, as shown in the photograph. The self-exciting pick-up coil is connected thru the phase shifter to the input of the amplifier, (Fig. 20). If the wire vibrates sinusoidally, the induced alternating current is sinusoidal and is a function of the frequency. This induced current passes thru the phase shifter so that the desired phase relationship from the pick-up coil, amplifier, and armature of the electromagnet can be obtained. To secure maximum amplitude at the hot wire the suspension system must vibrate at its natural frequency and any phase shift in the electrical circuit is counteracted by adjusting the phase shifter. The phase shifter consists of a condenser type impedance bridge with a 100,000 Ω rheostat, connected in the leg adjacent to the 0.07mfd condenser, for controlling the phase. A phase reversing switch, between the bridge input and the pick-up coil, extends the range of control of the rheostat by reversing the phase 180°.

The amplifier is AC operated from a power pack, which uses a full-wave type (80) rectifier tube with condenser input filtering. The amplifier gain is sufficiently stable without the use of a line voltage stabilizing circuit, for all amplitude settings at the hot wire. The first two stages are condenser-resistance coupled with a $\frac{1}{2}$ megohm gain control potentiometer in the grid of the second tube. Voltage amplification of the current induced in the pick-up coil is obtained by using a pair of 2A-5 tubes connected in push-pull to obtain sufficient power to drive the armature in the electromagnet. The output of the push-pull amplifier may be attenuated by varying the resistance across the plates which are connected to the ends of the electromagnet armature coil, center tapped to B + from the power pack. The attenuator consists of ten 750 Ω resistors in series with a 1000 Ω vernier rheostat. The output current from the amplifier changes direction thru the armature coil each half cycle reversing the magnetic polarity of the armature strip. Since the armature strip extends thru the magnetic gap of the fixed field in the electromagnet, it is alternately attracted to north and south poles of the laminated iron core, thereby vibrating the suspension system.

The wave shape of the vibrating system is obtained by impressing the voltage, induced in a second pick-up coil, across the vertical deflecting plates of a cathode ray oscilloscope having a sweep circuit. This coil is of the same type as the self-exciting coil and is located at the other upper branch of the suspension system. The wave under normal operating conditions is undistorted and free from harmonics, being comparable to the sine wave of the beat frequency oscillator. At very low frequencies if the wire suspension is too slack, a wave distortion can usually be corrected by attaching small lead weights behind the hot wire fitting plate.

An extensive series of tests were made to investigate the possibilities of developing an electromagnetic system for measuring the fluctuating velocity of the vibrating hot wire. Since the voltage induced in a coil is proportional to the time rate of change of the flux $e \sim d\phi/dt$ it was thought possible to develop a pick-up coil whose induced voltage would be a function of the frequency $\times$ amplitude of the suspension wire vibration, or of its effective fluctuating velocity. The advantage of such a system would be the elimination of the necessity for measuring the frequency and the amplitude of vibration separately, thereby saving time in the calibration of the hot wire. Altho a great number of various types of pick-ups were constructed, not one of them gave a linear relationship between frequency $\times$ amplitude and induced voltage in the coil. Perhaps it is possible to construct a compensated amplifier to obtain this relationship, but it was not considered worthwhile, and the project was abandoned. It was decided to determine the fluctuating velocity of the hot wire from measurements of the frequency and the maximum amplitude of the vibration. The methods used at present will now be described.

Equipment for Measuring Frequency and Amplitude of Vibration

Frequency measurements are obtained by connecting a beat frequency oscillator to horizontal deflecting plates of the oscilloscope and adjusting the frequency of the oscillator or of the suspension system until the two are identical, as shown by the Lissajou image on the fluorescent screen. The usual practice is to set the oscillator at the desired frequency and then adjust the frequency of the suspension system, since the latter can be set much closer. No trouble has been experienced with the suspension maintaining its frequency over a period of several days. The pick-up coil gives the same wave shape and frequency response whether placed at the hot wire fitting or outside the tunnel. Thus with the present design the only obstructions to the wind stream are the suspension wires and hot wire holder and fitting, which have been shown by tests to be negligible.

Amplitude measurements are made by means of an optical system consisting of: a pointer which is secured to the suspension wires at the hot wire fitting plate; a light source for illuminating the pointer; and a telescopic-micrometer for observing and measuring the amplitude of the vibration of the pointer. The pointer consists of two small needles soldered together with the tip of one projecting vertically upward 1/8" thru the eye of the other, and is secured to the top of the hot wire fitting plate with the tip extending ½" downstream. The pointer is illuminated by a focusing light, AC operated, and attached to the tunnel frame. When the pointer is vibrating, the focusing light is adjusted so that the image appears as a series of illuminated horizontal lines, one above the other, with sharply defined ends. These lines are formed by the light reflecting on minute irregularities on the pointer, so that the error introduced in measurements is negligible. A small region on the inside wall of the tunnel opposite the telescope is painted black to improve the contrast between the image and its background.

The telescopic-micrometer is built with a draw tube so that the focal length can be adjusted, the present setting being 13". The objective lense gives an inverted image which is viewed thru an eye-piece having a 5 × magnification. At the image plane there are two platinum wires which serve as vertical cross hairs. The hairs are suspended to brass frames which can slide as a unit across the telescope axes by turning the left micrometer (L). The right micrometer (R) moves one frame only and is adjusted to read zero when the cross hairs are coincident. With this arrangement it is necessary to read only the right micrometer which is graduated in hundredths of a millimeter with a readable range up to 15mm. For amplitudes between 15 - 25mm, the cross hairs are set to read half the total amplitude of vibration by fixing one cross hair on the center line of the pointer when the wire is stationary. The vertical cross hairs are sufficiently illuminated by the natural light in the room to appear white, or can be made to appear as black lines by blocking off this light. A small irregularity in the vibration of the hot wire will distort the image. Several adjusting screws are incorporated in the design of the telescopic-micrometer mounting for alignment purposes. The height and the horizontal rotation of the telescope can be adjusted and clamped into position. The setting of the left micrometer can also be locked. A vernier focusing adjustment is provided to permit the base plate on which the telescope is mounted to slide fore and aft on a second base plate. The latter rests on a steel platform and can be moved around by releasing two bar clamps. The platform is also adjustable in height with respect to the suspension frame and rigidly braced, to eliminate all relative vibrational motion between the frame and telescopic-micrometer.

Calibration Wind Tunnel

The present calibration wind tunnel is a horizontal open-circuit suction type with a straight tube working section of 15" diameter. The flow is straightened by a four blade counter propeller mounted in the divergent exit section. A 7½ HP, 200 cycle variable frequency motor is directly coupled to a two blade propeller. Several entrance sections were used to compare their merits regarding free stream turbulence, freedom from gusts, and axial flow. The lowest level occurred at the entrance section where the stability was poorest, while traverses downstream showed a marked improvement in stability at the sacrifice of the free stream turbulence. Using a sharp edge entrance, formed by the straight tube of the working section, a 15" diameter 16 gauge steel shell, the turbulence level and instability of flow were found to be negligible over an axial traverse 12" downstream enclosed in an area of 6" diameter. The present suspension is located about 4" behing the entrance where the flow characteristics are very good and the turbulence level is less than 0.1%.

Hot Wire Anemometer Equipment

Hot Wire Anemometer #1

This equipment was constructed in the early part of 1936 to serve as an additional unit to the one used by F.L.Wattendorf and A.M.Kuethe[12] due to the growing interest in the type of research pioneered at this laboratory by these authors. As it was designed prior to the development of the vibrator, it incorporates circuits for measuring the resistance and the current of the hot wire, necessary for determining turbulence intensity by the orthodox technique. It can be seen from the wiring diagram, Fig. 21 , that it differs in many respects from the older equipment.

Any unbalanced DC voltage drop across the hot wire is isolated from the amplifier by having the grid of the first valve condenser-coupled to the hot wire, instead of transformer-coupled across the bridge as in the older circuit, improving the amplifier's response especially at low frequencies. The hot wire forms one leg of a Wheatstone bridge, which has a 1:10 resistance ratio. This part of the circuit is also used in conjunction with a wall-type galvanometer, for obtaining measurements of mean flow velocity. The procedure is to calibrate the deflection of the galvanometer, caused by the unbalance of the bridge, as a function of the wind speed. The total current through the bridge is indicated by a milliameter on the panel. The current through the hot wire is 10/11 of the total but may be measured more accurately by a standard make of potentiometer which can be plugged into the circuit.

The fluctuations are first attenuated and then amplified by four resistance-capacity coupled triode stages, with a variable resistance-stepped inductance compensating circuit between the second and third valves. The last stage, comprising the power amplifier, feeds into an output transformer whose secondary is connected to a thermogalvanometer. "Transformer compensation", consisting of a resistance-condenser voltage divider which attenuates high frequency impulses, is included in the grid circuit of the third valve to help counteract the inherent low frequency attenuation of the output transformer. The amplifier valves are supplied with AC across the filaments and rectified AC across the plates, with the power stage unregulated, from a single power pack.

Fig. 15 shows the performance of the amplifier with the compensating inductance short circuited. The marked improvement in the performance, obtained at both high and low frequencies by using the condenser-resistance type of transformer compensation, is shown by comparing the two curves. The relative output is constant within ± 2.5% between 35 and 6000 cycles per second, forming a very satisfactory basis for hot wire compensation. The frequency characteristic performance of the hot wire-compensated amplifier is given in Fig. 16 . The relative output is within 4% up to 4800cps, whereas the anemometer used by Wattendorf and Kuethe was not reliable above 800cps where the error in relative output reached 10%.

The amplifier is very stable in operation and has ample voltage gain for turbulence intensity research. The complete anemometer consists of four self-contained units which slide into a single metal cabinet and are interconnected at the rear by plug-ended cables. The power pack is a separate unit removed from the anemometer to eliminate stray field interference.

<u>Hot Wire Anemometer #2</u>

Because of the increasing demand for turbulence investigations it was deemed advisable to construct another hot wire anemometer in 1937. At this time we had had sufficient experience with the vibrator method to satisfy us that it was far superior to the orthodox method of measuring turbulence intensity. This permitted the deletion of such equipment incorporated in the input circuit of anemometer #1 as the Wheatstone bridge for resistance measurements and apparatus for heating current measurements. Also Dryden and Taylor had recently succeeded in obtaining correlation measurements by the simultaneous use of two hot wires, which necessitated a new type of input circuit and was desirable to incorporate in the new anemometer.

Dryden's[16] scheme, Fig. 22, was adopted because of its simplicity and the fact that it does not necessitate measurements of the hot wire's heating current or resistance. Thus it may be used for correlation or for turbulence intensity measurements by a simple switching arrangement, (Sw_1 and Sw_2). A potentiometer is used to adjust the heating currents to a constant, but not necessarily known, value. It is connected to the double bank switch (S_p). Positions (2) and (4) are for current control, while (3) and (5) may be used for determining mean velocity measurements etc. The positions of the reversing switch between the two hot wires and the amplifier's input are labelled in accordance with Dryden's notation, " M_a -- output meter reading obtained where voltages are added, and M_b -- the reading when opposed".

The input is condenser-coupled to the grid of the first tube in the amplifier. Battery grid biasing has been preferred to cathode-resistor biasing for all tubes. The first three stages of amplification utilize pentodes instead of triodes as in anemometer #1 due to their improved characteristics. The 57's, as shown in Fig. 22 , were later changed to 6C6G's as they have a higher mutual conductance. Although metal tubes proved to be highly successful in anemometer #1, they were not again used because approximately 5 of every 6 tried were either microphonic or had a high hum level.

Not being satisfied with the conventional milliameter-type of output meter because of its lack of damping, insensitivity, relatively large power output required to operate it and the fragility of its thermocouple to overloads, we found that Carl Thiele's proposal of replacing it with an ultra sensitive wall-type galvanometer eliminated all of these problems. This simplified the output circuit in two ways: it permitted the use of a small power-output tube; and it eliminated the more complicated push-pull or phase inverter circuits, as used by other laboratories, or transformer coupling used in anemometer #1. With a 56 $\mu fd.$ coupling condenser in series with 5000 Ω resistor the relative output is theoretically constant within 5% at a frequency of 10cps and 0.6% at 100cps.

Since the output is not transformer coupled, it is necessary to amplify the fluctuating voltage drop across the hot wire before attenuating in order to minimize the effect of noise in the plate circuit of the first tube. The attenuator is placed between the first and second stages to prevent overloading of the second tube. The compensating network is located in the plate circuit of this tube. The inductance in the grid circuit of the third tube corrects for slight attenuation at high frequencies caused by the coupling condensers. The amplification characteristics may be maintained by impressing a reference voltage, obtained from a standard frequency tuning fork, across the input and adjusting the level by means of a 0.1 megohm "gain control" resistor located in the grid circuit of the power tube. The output circuit has a three-way switch to accommodate a cathode ray oscilloscope, a 5 volt rectifier meter, and the thermogalvanometer. A single power pack furnishes rectified

AC voltages to the plates, automatically regulated at the first two tubes. All filaments are supplied from storage batteries.

The performance of the amplifier without compensation is shown in Fig. 17 . Its range is seen to extend beyond that of anemometer #1 and its level is constant within 2% up to 10,000 cps for common values of compensation resistance R_c^* . Even for the extreme value, $R_c^* = 9990\,\Omega$, the error is only 8% at 10,000 cps. Fig. 18 gives the performance of the hot wire - compensated amplifier combination. It is seen to be an improvement over anemometer #1 especially at high frequencies as the usable range (5% error in level) has been extended from 5000 to 7000 cps, with a slight resonance occurring at approximately 5500 cps. The operating characteristics are very stable, the wave shape is undistorted at all frequencies, and the sensitivity exceeds that of anemometer #1.

There are two anemometers of this design in current use at the laboratory. The second is operated from batteries instead of a power pack, but may be converted at some later date.

Output Meter

This instrument consists of a Leads and Northrup wall-type galvanometer, specified as type-P with a sensitivity of 1.5mm/μ volt at a mirror-to-scale distance of 1 meter, which is used with a separate heater-type thermocouple. The couple is a General Radio model 493HH with a heater resistance of 105Ω, and 10mv open circuit at 11ma. As the output of the amplifier cannot exceed 12ma with 5000Ω in series with the couple, which has 11.3Ω and a safe maximum current of 15ma, the latter is safeguarded against being burned out.

The galvanometer unit is mounted at the end of an optical bench which sets on a stable structure, as shown in the photographs. A focusing light beam is reflected by the mirror back on to a horizontal frosted glass scale mounted just below the light. A triangular shaped box encloses the scale and light source to exclude extraneous light. The scale has a range of $\pm$ 25cm and can be adjusted to zero setting by sliding the box sideways on its front support. The distance between the scale and mirror is approximately $\frac{1}{3}$ meter. The reflected image is about 1mm wide on the scale, and can be comfortably read at a considerable distance. The low resistance couple across the meter provides sufficient damping of the image to turbulence fluctuations to facilitate readings.

The wall galvanometer is also used without the thermocouple for hot wire mean flow measurements. In this case a rheostat or tapped resistors are used for obtaining the desired damping and sensitivity.

Two of these instruments have been in constant use at the laboratory since 1937. Their performances have been truly remarkable compared to that of the conventional type previously used by Wattendorf, Kuethe and the writer. In contrast to schemes developed at other laboratories, this design greatly improves the sensitivity, ruggedness, damping characteristics, convenience in taking readings, and simplifies the output circuit in the amplifier.

Standard Frequency Tuning Fork

In order to check the gain of an amplifier to insure constancy in operation the output voltage may be compared with a known input voltage. If the input voltage is stable, its use as a standard reference facilitates checking gain. When the standard input voltage is obtained from the city 110 volt AC line, attenuated by a voltage divider, the stability usually is not sufficient for use in hot wire high gain amplifiers. A beat frequency oscillator may be employed with better performance, but this procedure often is inconvenient. A simple solution of the problem is the adaptation of a vacuum tube-maintained-standard frequency tuning fork.

Such an instrument has been constructed for checking gain of the amplifiers in anemometers #1 and #2. The wiring diagram of the fork assembly is shown in Fig. 23 . It was found more feasible to drive the fork at a constant amplitude with one amplifier, and to regulate the standard voltage, obtained with a pick-up coil, by a second amplifier incorporating a tapped voltage divider at its output, than to attempt to drive the fork at different amplitudes. The fork is self-excited at its natural frequency of 100cps. The rectifier voltmeter across the primary of the voltage divider functions as the reference for the standard input voltage of the anemometer during gain checking.

The performance of the fork is very satisfactory. Its output voltage is stable to within 0.1% and its frequency variation is negligible.

APPENDIX #1

Vibrator Method vs. Orthodox Method
of
Measuring Turbulence Intensity

In order to make a simultaneous calibration of the hot wire by these two methods, measurements were taken of the hot wire's resistances, cold (R_0), and hot (R), and of the mean heating current which was maintained constant, during the vibrator calibration. The platinum hot wire used was approximately 4 mm between its supporting spindles and 0.0005" in diameter. The tests were conducted at one mean flow velocity U, and all measurements of the wire's resistances, heating currents, and percentage turbulence were made with Anemometer #1. All computations have been included in this section in order to give a better comparison of the two methods.

Vibrator Method

1) Mean Flow Velocity $U = 12.41$ meters/second.

2) Setting Compensating Resistance R^*_{cx}

R^*_c	I^2	I^2
0	42	27
1000	43	32
1500	45.5	39.5
2000	49	49
2500	54	62
3000	59	75
3500	66	91
f	141	70.5
Δ	4.4	8.8
tap	6	6

See Curves (Fig. 4)

$R^*_{cx} = 2000\Omega$

3) Calibration of Amplitude Δ as function of $\sqrt{\overline{I^2}}$

$R^*_{cx} = 2000\,\Omega$ $f = 141$ cps

Δ	I^2	Tap	$\sqrt{\overline{I^2}}$	I^2	Tap	$\sqrt{\overline{I^2}}$
6.32	100	6	10.0	29.5	7	5.43
5.57	78	"	8.83	23	"	4.8
4.60	53	"	7.28	15.5	"	3.94
3.58	32	"	5.66	9.5	"	3.08
3.15	25	"	5	7.5	"	2.74
2.72	18.5	"	4.3	67.0	5	8.19
2.05	10.5	"	3.24	38	"	6.16
1.55	6	"	2.45	22	"	4.68
1.27	78	4	8.83	14.5	"	3.81
1.06	54	"	7.36	10	"	3.16
0.87	36	"	6	7	"	2.64

See Curves Fig. 5

$m = \Delta / \sqrt{\overline{I^2}}$

Tap	m
4	0.144
5	.333
6	.632
7	1.165

4) Conversion of Thermogalvanometer readings $\sqrt{\overline{I^2}}$ to Percentage Turbulence $\sqrt{\overline{u^2}}/U$

$$\sqrt{\overline{u^2}}/U = \frac{2.22\times 10^{-3}\times\Delta\times f}{U} = \frac{2.22\times 10^{-3}\times f\times m\times\sqrt{\overline{I^2}}}{U} = \frac{Q\times\sqrt{\overline{I^2}}}{U}$$

Tap	Q/U
4	0.363×10^{-2}
5	.838 "
6	1.595 "
7	2.935 "

$U = 12.41$ m/s

$f = 141$ cps

Orthodox Method

The formula for determining the percentage turbulence with Anemometer #1 using the orthodox method is:

$$\frac{\sqrt{\overline{u^2}}}{U} = \frac{-2}{c\sqrt{U}}\times\left[F + Gb\right]\times\frac{dE}{1 - Rb}$$

where

$$F = \frac{1 R_o^2 \alpha}{(R - R_o)^2} \qquad G = \frac{2 1 R R_o \alpha}{R - R_o}$$

$$b = 0.909\left((300 + B)^{-1} + 1/E_x\right)$$

$$dE = K_b\times\sqrt{\overline{I^2}}/\text{ amplifier level}$$

1) Wire Resistance (cold)

$R_o + R(\text{leads}) = 4.93$ ohms $R(\text{leads}) = 0.09$ ohms

$R_o = 4.34$ "

2) Wire Resistance (hot)

$R + R(\text{leads}) = 7.61$ ohms

$R = 7.52$ "

3) Wire Heating Current (i)

Reference battery voltage (V)

$$V = 1.0136 \times \frac{(5020 + 2840)}{5020} = 1.594 \text{ volts}$$

Resistance of current potentiometer = 11.1 ohms

i = 1.594/11.1 = 144 milliamps

4) Determination of the Time Constant M

$$M = \frac{4.2\, s \rho A^2}{r_v\, i^2 \alpha} \times \frac{(R - R_o)}{R_o}$$

where $s = 3.2 \times 10^{-2}$ $r_v = 11.19 \times 10^{-6}$ $\rho = 21.45$

$\alpha = 3.67 \times 10^{-3}$ $i = 0.144$ amp $A = \pi d^2/4$

$R = 7.52\,\Omega$ $R_o = 4.34\,\Omega$

Measurements of the hot wire's diameter (d) showed a variation from 0.00116 cm to 0.00130 cm over a 4 mm length -- using a Bausch and Lomb microscope -- giving a variation in the value of M from 2.08×10^{-3} to 3.29×10^{-3}. Since this is excessive, it was decided to determine the effective wire diameter and M from the vibrator test value of the compensating resistance $R^*_{cx} = 2000$ ohms. A calibration of R^*_c vs $\sqrt{I^2}$ was made with the aid of a beat frequency oscillator set at a frequency of 50 cps.

The results are plotted in terms of an amplifier gain factor AF -- see Fig. 6 and Data sheet #1. Then a calibration was made of the compensated amplifier, with $R^{*}_{cx} = 2000\Omega$, giving a curve AF vs f (cps) -- see Fig. 7 and Data sheet #2. From Figs. 6 and 7 sufficient data were available to find the relationship between R^{*}_{cx} and M, shown in Fig. 8 and Data sheet #3. From this curve the correct value of the time constant was found to be $M = 2.26 \times 10^{-3}$ @ $R^{*}_{cx} = 2000\Omega$. Using this value the performance of the amplifier was computed, Fig. 7 and Data sheet #2, giving an amplifier level = 23,000. Substituting the value $M = 2.26 \times 10^{-3}$ in the equation for M, the effective diameter of the hot wire was found to be 0.00118 cm, which is within the range of the diameter measurements.

5) Coefficient of Forced Convection (c)

From King's Equation -

$$i^2 R = (K + c\sqrt{U})(\overline{T} - T_o)$$

and since $(\overline{T} - T_o) = \dfrac{R - R_o}{R_o \alpha}$

it follows $K + c\sqrt{U} = \dfrac{i^2 R R_o \alpha}{R - R_o}$

To find the value of (c) the mean flow velocity U was varied over a large range and the corresponding values of the wire's resistance were measured by the Anemometer's wheatstone bridge at a constant heating current i. The results are given in Data sheet #4, and a plot of $(K + c\sqrt{U})$ -vs- $\sqrt{U}$, Fig. 9 , shows that the value of the forced coefficient was:

$$c = 1.98 \times 10^{-4}$$

6) Percentage Turbulence for Mean Flow Velocity

$$U = 12.41 \text{ m/s}$$

$$\frac{\sqrt{\overline{u^2}}}{U} = \frac{-2}{c\sqrt{U}} \times (F + Gb) \times \frac{dE}{1 - Rb} = \frac{Q}{U}\sqrt{I^2}$$

$b = 3.49 \times 10^{-2}$ as $E = 12.2$ volts $i = .144$ amp $B = 76.1$

$G = 14.35 \times 10^{-3}$ as $R = 7.52$ $R_o = 4.84$ $\alpha = 3.67 \times 10^{-3}$

$F = 17.22 \times 10^{-4}$ and $(F + Gb) = 22.23 \times 10^{-4}$

$\frac{2}{c\sqrt{U}} = 2870$ and $\frac{1}{1 - Rb} = 1.356$

$dE = K_b\sqrt{I^2}/\text{amplifier level} = K_b\sqrt{I^2}/2.3 \times 10^4$

Tap	K	Q/U
4	9.76	0.367×10^{-2}
5	22.44	.844 "
6	42.63	1.60 "
7	78.80	2.96 "

The hot wire was removed from the calibrating tunnel, inserted in the extension holder and then mounted in a micrometer screw feed which was attached to the wall of the Wattendorf wind tunnel. A traverse was made in the straight section where the turbulence was known to be in a fully developed state. The tunnel was lined previously with corrugated paper in order to produce roughness. The scale effects at this traverse station were known from previous tests to be negligible. This fact made it possible to run a traverse at one value of M by adjusting the tunnel velocity at each new traverse position so that the mean velocity across the hot wire remained constant. Also the mean heating current through the wire was manually controlled to remain at 144 milliamps. The mean velocity across the wire $U = 12.41$ m/s corresponded to the velocity during calibration. The traverse extended from the center line to one wall of the tunnel. The intensity of turbulence was calculated from the values given in the tables of Q/U and are shown in Fig. 10 . This figure also includes data obtained from a traverse made in the same traverse station of the tunnel in 1935, which were computed from a test using the orthodox method with the hot wire anemometer set described by F.L.Wattendorf and A.M.Kuethe[12]. Computations of these data are also included.

Wattendorf - Tunnel

Hot Wire Traverse at Section #9

(Roughness #1)

Mean Velocity $U = 12.41$ m/s.

Y -- distance of hot wire to tunnel inner wall (mm).

Q/U*	Y	I^2	tap	Q/U**	$\sqrt{\overline{u^2}}/U$ % **
$.844 \times 10^{-2}$	24	28	5	$.838 \times 10^{-2}$	4.43
"	20	35	5	"	4.95
"	17	50.5	5	"	5.95
"	14	73	5	"	7.16
"	12	94.5	5	"	8.15
1.60×10^{-2}	10	35	6	1.595×10^{-2}	9.44
"	8	44.5	6	"	10.65
"	6	60	6	"	12.36
"	4	78.5	6	"	14.14
"	3	95.5	6	"	15.60
2.96×10^{-2}	$2\frac{1}{2}$	32.4	7	2.935×10^{-2}	16.70
"	$1\frac{1}{2}$	50.5	7	"	20.85

* -- Orthodox Method   Feb. 24 1937
** -- Vibrator Method   "

(See Fig. 10)

2/24/37

ANEMOMETER #1

I^2 VS. R_c^* AT 50 CPS CHOKES IN SERIES

R_c^*	I^2	TAP	$\sqrt{I^2}$	I^2	TAP	$\sqrt{I^2}$	A.F.
0	20	2	4.475				1.634×10^4
200	21	"	4.585				1.672
400	23.5	"	4.85				1.770
600	25.5	"	5.055				1.845
800	29	"	5.385				1.966
1000	31	"	5.57				2.032
1200	36.5	"	6.04				2.205
1500	44.5	"	6.67				2.435
1800	53.5	"	7.32				2.671
2000	60.0	"	7.74	13.5	3	3.68	2.82
2500				18.3	"	4.28	3.28
2900				22.5	"	4.75	3.64
3000				23.5	"	4.85	3.72
3500				29.5	"	5.43	4.16
4000				35.5	"	5.96	4.57
4500				43.5	"	6.60	5.06
5000				52.5	"	7.25	5.56
5500				61.5	"	7.85	6.02
6000				71	"	8.43	6.46

$$AF = K_r\sqrt{I^2} = 3650\sqrt{I^2} \quad \text{ON TAP 2}$$

$$7670\sqrt{I^2} \quad \text{" " 3}$$

Anemometer Response vs. Frequency $R_C = 2000\omega$

2/24/37

Anemometer #1

f_{CPS}	I^2	Tap	$\sqrt{I^2}$	$A.F. \times 10^{-4}$	$\frac{1}{\sqrt{1+\omega^2 M^2}}$	$A.L. \times 10^{-4}$
15	14.5	2	3.81			
20	22	"	4.69			
30	32.5	"	5.20			
40	44.1	"	6.64			
50	59.5	"	7.70	2.82	.814	2.30
60	—	"	—			
70	85	"	9.22			
80	100	"	10	3.310		
80	24	3	4.90	3.193		
90	29	"	5.38			
100	33.5	"	5.79	4.061	.566	2.30
120	—	"	—			
140	56.5	"	7.52			
160	72	"	8.48			
180	87	"	9.34	6.090		
180	22	4	4.69	6.180		
200	27.2	"	5.22	6.89	.332	2.29
250	41.5	"	6.44			
300	60	"	7.75	10.22	.228	2.33
350	80	"	8.95	11.80		
350	15	5	3.87	11.73		
400	19.5	"	4.42	13.59	.173	2.32
450	24	"	4.90			
500	29.5	"	5.43	16.45	.139	2.29
600	42.0	"	6.48	19.60	.117	2.29
700	57	"	7.54	22.85	.010	2.285
800	74.5	"	8.62	26.10		

$K_V \sqrt{I^2} = AF$

$f > 100$ CPS

K_V	Tap	K_f
1351	1	1
3310	2	2.45
6520	3	4.90
13,190	4	9.76
30,300	5	22.44
57,600	6	42.63
106,500	7	78.80

$f = 100$ CPS

K_V	Tap
7020	2
3450	3

$f = 50$ CPS

K_V	Tap
3650	2
7670	3

$\omega = 2\pi f$

$M = 2.26 \times 10^{-3}$

$A.F. = V(\text{output}) / V(\text{input})$

$V(\text{input}) = 1.48 \times 10^{-4} \times V_0$ (Voltage divider across oscillator $V_0 = 5V$)

$= 7.4 \times 10^{-4}$

$\therefore AF = \dfrac{K_f \sqrt{I^2}}{7.4 \times 10^{-4}} = 1351\, K_f \sqrt{I^2}$

Let $1351\, K_f = K_V$

$\therefore AF = K_V \sqrt{I^2}$

(K_V varies with attenuation)

2/24/37

R^*_{ex} vs. M, Anemometer #1, Chokes in Series.

From Anemometer Calibration

$AF = 22.8 \times 10^4$ at 700 cps

Let $N = \dfrac{AF}{2\pi f} = 52.0$

Oscillator at 50 cps, $V_0 = 5V$

$\omega = 2\pi \times 50 = 100\pi$

Tap	1	2
K_V	1351	3650

$$\left[A.F. \times \frac{1}{\sqrt{1+\omega^2 M^2}}\right]_{\text{at 50 cps}} = \left[N\omega_1 \times \frac{1}{\sqrt{1+\omega_1^2 M^2}}\right]_{\text{at 700 cps}} \doteq \left[\frac{N}{M}\right]_{\text{at 700 cps}}$$

$$\therefore \left[A.F.\right]_{\text{at 50 cps}} = \frac{N}{M}\sqrt{1+\omega^2 M^2} = \frac{52}{M}\sqrt{1+(100\pi M)^2}$$

M	ωM	$\sqrt{1+(\omega M)^2}$	$52/M$	A.F.	R^*_{ex}
.0005	15.71×10^{-2}	1.012	10.4×10^4	105,200	—
.001	31.42 "	1.047	5.2 "	54,450	4880
.0015	47.13 "	1.112	3.465 "	38,500	3140
.002	62.83 "	1.181	2.60 "	30,700	2280
.0025	78.54 "	1.270	2.08 "	26,400	1780
.003	94.25 "	1.373	1.733 "	23,800	1460
.0035	109.96 "	1.486	1.485 "	22,100	1210
.004	125.66 "	1.606	1.300 "	20,900	1040

$K + c\sqrt{U}$ vs $\sqrt{U}$ 2/24/37

$$K + c\sqrt{U} = i^2 \alpha R R_0 / R - R_0$$

BAR 744.6 mm $T = 18°C$ $\rho = .1205$ $\gamma' = .824$

$$U = 1.018 \sqrt{\frac{2\gamma'}{\rho}} \sqrt{P_s} = 3.76 \sqrt{P_s} \text{ m/sec.}$$

P_s mm	U m/sec	$\sqrt{U}$	R ω	$R-R_0$ ω	$R/R-R_0$	$(K+c\sqrt{U}) \times 10^4$
1.76	4.98	2.23	8.71	3.87	2.25	8.39
3.80	7.32	2.71	8.07	3.23	2.50	9.20
5.55	8.85	2.97	7.78	2.94	2.65	9.76
10.85	12.38	3.51	7.40	2.56	2.89	10.64
16.43	15.23	3.90	7.12	2.28	3.12	11.49
17.29	15.63	3.96	7.08	2.24	3.16	11.63
19.05	16.41	4.05	7.01	2.17	3.23	11.89
21.79	17.55	4.19	6.94	2.10	3.30	12.15
23.92	18.38	4.29	6.89	2.05	3.36	12.37
28.72	20.13	4.49	6.81	1.97	3.46	12.72
33.91	21.87	4.68	6.72	1.88	3.58	13.16
38.09	23.20	4.80	6.68	1.84	3.63	13.37

$c = 1.98 \times 10^{-4}$

TESTS IN WATTENDORF CURVED CHANNEL
AT SECTION 9 ROUGHNESS #1 6/18/35
USING
WATTENDORF - KUETHE HOT-WIRE ANEMOMETER*

BAR. 743.6 mm. $\rho = .117$ $B_L + B_0 = 35.2$
T 28°C $\gamma = .806$ $B_L = 13.7$
$U = 3.71\sqrt{q}$ $B_0 = 21.5$
$R_0 = 21.5/9.64 = 2.23\,\omega$

$i = 165$ m.a. HEATING CURRENT
V_p = POTENTIOMETER BATTERY VOLTAGE
R_p = " RESISTANCE SETTING

$$V_p = \frac{5000+288}{1000+288} \times 1.01869 = 4.18V.$$

$$R_p = \frac{5000 \times .165}{4.18 - .165} = 205.5\,\omega$$

SPEED RUN AT ℄ OF CHANNEL
U = MEAN VELOCITY ACROSS HOT WIRE

$\sqrt{U \frac{cm}{sec}}$	B'	B	$R-R_0$	RR_0	$\frac{RR_0}{R-R_0}$	$M \times 10^3$	I^2	TAP	R
27.8	49.8	36.1	1.513	8.35	5.52	6.60	21	3	3.743
30.9	48.5	34.8	1.380	8.05	5.83	6.45	"		3.610
33.0	47.7	34.0	1.295	7.86	6.07	6.05	"		3.525
37.1	46.5	32.8	1.171	7.59	6.48	5.47	"		3.401
40.5	45.6	31.9	1.080	7.38	6.83	5.05	"		3.310
44.4	44.8	31.1	.995	7.19	7.23	4.65	"		3.225
46.7	44.4	30.7	.955	7.10	7.44	4.465	"		3.185

$$M = 1.042 \times 10^{-2} \left(\frac{R-R_0}{R_0}\right)$$

f	TAP	I^2	K_V	A.F
50	1	9.5	2078	6,400
100	1	35	"	12,280
200	2	37	3890	23,650
300	2	80	"	34,770
400	3	36	7580	45,500
500	3	52.5	"	54,950

AMPLIFIER CALIBRATION

* SEE "PHYSICS"
JUNE 1934

COMPUTATION OF dF VS U 6/18/35

$\sqrt{U}$ $(\frac{cm}{sec})^{1/2}$	$\frac{2}{c\sqrt{U}}$	$g \times 10^3$	$b \times 10^2$	$gb \times 10^4$	$f \times 10^4$	$gb+f$ $\times 10^4$	$\frac{1}{1-Rb}$	1/LEVEL $\times 10^4$	$dF \times 10^3$	U m/SEC	Q
27.8	7010	6.69	1.749	1.169	13.16	14.329	1.070	3.570	3.835	7.73	10.74
30.9	6300	7.07	1.787	1.263	15.82	17.083	1.069	3.489	4.01	9.55	11.51
33.0	5900	7.36	1.813	1.333	17.96	19.293	1.068	3.270	3.97	10.89	12.15
37.1	5250	7.86	1.850	1.454	21.96	23.414	1.067	2.960	3.89	13.80	13.12
40.5	4810	8.28	1.881	1.557	25.85	27.407	1.066	2.730	3.840	16.40	14.06
44.4	4386	8.76	1.910	1.672	30.46	32.132	1.065	2.515	3.775	19.71	15.01
46.7	4170	9.03	1.922	1.735	33.10	34.835	1.065	2.412	3.730	21.80	15.46

$dF = (2/c\sqrt{U})(gb+f)(1/1-Rb)(1/LEVEL)$ $\quad 2/c\sqrt{U} = 194,700/\sqrt{U}$

$c = 10.27 \times 10^{-6}$

$g = 1.211 \times 10^{-3}(RR_o/R-R_o)$ $\quad R_o = 2.23\omega$

$\left.\begin{array}{l} r_2 = 1.34+B' \\ r_3 = 10.80\,\omega \end{array}\right\} r_2+r_3 = 12.14+B'$

$b = 1.148/(12.14+R+B')$

$f = 6.06 \times 10^{-4}(R_o/R-R_o)^2 = 30.15 \times 10^{-4}/(R-R_o)^2$

COMPUTATIONS OF FACTORS

ANEMOMETER LEVEL $= 18.5/6.05 \times 10^{-3} = 3060 = N/M$

$R = 3.525\omega \quad B' = 47.7 \quad R_o = 2.23 \quad C = 10.27 \times 10^{-6}$

$b = 1.148/(12.14+3.525+47.7) = 1.148/63.365 = 1.813 \times 10^{-2}$

$g = 1.211 \times 10^{-3} \times 6.07 = 7.36 \times 10^{-3}, \quad gb = 1.333 \times 10^{-4}$

$f = 6.06 \times 10^{-4}(2.23/1.295)^2 = 17.96 \times 10^{-4}$

$S = f + gb = 19.293 \times 10^{-4}$

$P = (2/c\sqrt{U})(1/1-Rb) = 6300$

$Q = SP = 12.15$

$\frac{\sqrt{\overline{u^2}}}{U} = dE \times Q = 3.97 \times 10^{-3}$ VOLTS $, \quad dE = \frac{\text{VOLTS}}{\text{A.L.}} = \frac{\text{VOLTS}}{3060}$

Hot Wire Traverse at Section 9 6/18/35

Mean velocity U held constant across hot wire
$U = 10.89$ m/s (Scale effect negligible)
$B' = 47.7$
δ distance of wire from inner wall in m.m.

δ mm	I^2	Tap	Volts	$(\sqrt{\overline{u^2}}/U)$ in %
1.5	30	5	52.6	20.89
2.0	26	"	48.9	19.41
3.0	18	"	40.7	16.16
5.0	42	4	33.3	13.22
7.0	32	"	29.05	11.53
10.0	22	"	24.1	9.56
13.0	14.5	"	19.55	7.76
16.0	10.0	"	16.24	6.45
18.0	34	3	14.22	5.64
20.0	26	"	12.43	4.94
22.0	22	"	11.44	4.54
23.0	21	"	11.17	4.43
23.71	21	"	11.17	4.43

$$\text{Volts} = K_f \sqrt{I^2}$$

Tap	K_f
3	2.44
4	5.14
5	9.60

APPENDIX #2

Investigation of Compensation Function $R_{cx}(U)$

The object of this investigation was to determine the validity of the theory that the compensation's resistance R_{cx} is a linear function of $\sqrt{U}$, expressed on page 35 as $R_{cx} = N + P\sqrt{U}$

Anemometer #2 was used in conjunction with the wall type galvanometer. The external resistance in the hot wire heating circuit was purposely large in order that $dE \doteq i\,dR\ ,\ di \doteq 0$ and the level of turbulence $\frac{\sqrt{\overline{u^2}}}{U} = S \times \left(\frac{N}{\sqrt{U}} + P\right) \cdot \sqrt{\overline{I^2}}$ as given on page 42 .

The procedure is shown on the data sheets. The value of the calibration constant S is based upon Run I.

$$S = \left(\frac{\sqrt{\overline{u^2}}}{U_1}\right)_c \times \frac{1}{\sqrt{\overline{I_c^2}}} \times \left(\frac{N}{\sqrt{U_1}} + P\right)_c^{-1}$$

$$N = 30 \quad P = 161.5 \quad \sqrt{U_1} = 4.65$$

$$\sqrt{\overline{u^2}}/U_1 = 5.75 \quad \sqrt{\overline{I_c^2}} = 3.96$$

$$S = 8.64 \times 10^{-3}$$

Run	Turbulence Level - % Actual	Theory
I	5.75	5.75
II	5.75	5.74
III	5.75	5.76
IV	3.52	3.53

R_{CX} vs $\sqrt{U}$ 4/13/38

BAR 743.5 mm
TEMP 21.0°C
μ 0.815
$\Big\}$ ∴ $\rho = 0.120$
$q = 1.137 P_S$

$R_0 = 4.50\omega$ (COLD RES.) R = HOT RES.
LEADS = 0.32 ω
$i = 160$ ma FOR ALL RUNS

R^*_{CX} = COMPENSATION BOX SETTING. DC. RES OF CHOKE = 100 ω
R_{CX} = TRUE COMPENSATING RESISTANCE = $R^*_{CX} + 100\omega$

RUN I

$P_S = 30.0$ m $\quad R + \text{LEADS} = \dfrac{1.113\text{V}}{.160\text{A}} = 6.95\omega$, $R = 6.63\omega$

$L = 0$, $R_C = 1400$

f	Δ	I^2	TAP
70	8.0	18.3	7
140	4.0	6.25	7

$$\frac{I^2(70\text{CPS}, L=0, R_C=1400)}{I^2(140\text{CPS}, L=0, R_C=1400)} = 2.92$$

FROM CALIBRATION CURVES $R^*_{CX} = 680\,\omega$ (FIG)

$$\sqrt{I^2_C} \Big/ \sqrt{I^2(70\sim, L=0, R_C=1400)} = .925$$

$$\therefore I_C^2 = 15.65$$

$U \frac{m}{sec}$	f	Δ	I^2	TAP	R_{CX}	% TURB.	I_{TAP7}/% TURB
21.6	70	8.00	15.7	7	780	5.75	.689
"	90	6.21	"	"	"	"	"
"	110	5.09	"	"	"	"	"
"	140	4.00	"	"	"	"	"
"	160	3.50	"	"	"	"	"

$\sqrt{U} = 4.66$

AT ANY OTHER SPEED U_X

$$R_{CX} = \frac{780(R-R_0)_{U=21.6}}{(R_X - R_0)} \quad \text{SINCE } M = \frac{L}{R_{CX}} = \text{CONST}(R-R_0)$$

Run II R_{cx} vs $\sqrt{U}$ 4/13/38

$P_S = 10$ mm $R + \text{Leads} = \frac{1.215V}{.160A} = 7.60\omega$, $R = 7.28\omega$

$R_{cx} = 780 \frac{2.13}{2.78} = 598$, $R_{cx}^* \; 498\omega$

$U \frac{m}{sec}$	f	Δ	I^2	tap	R_{cx}	% turb	I_{tap7}/% turb	
12.4	70	4.59	15.3	7	598	5.75	.680	
"	90	3.57	"	"	"	"	"	$\sqrt{U} = 3.52$
"	110	2.92	"	"	"	"	"	
"	140	2.30	"	"	"	"	"	
"	160	2.01	"	"	"	"	"	

Run III

$P_S = 2.26$ mm $R + \text{Leads} = \frac{1.397V}{.160A} = 8.74\omega$, $R = 8.42\omega$

$\therefore R_{cx} = 780 \times 2.13/3.92 = 423$, $R_{cx}^* = 323\omega$

U m/sec	f	Δ	I^2	tap	R_{cx}	% turb	I_{tap7}/% turb	
5.90	70	2.18	14.7	7	423	5.75	.668	
"	70	6.0	6.8	5	"	15.80	"	$\sqrt{U} = 2.43$
"	90	4.67	"	"	"	"	"	
"	110	3.82	"	"	"	"	"	
"	140	3.00	"	"	"	"	"	
"	160	2.63	"	"	"	"	"	

Run IV

$P_S = 81.0$ mm $R + \text{Leads} = \frac{1.04V}{.160A} = 6.50\omega$, $R = 6.18\omega$

$\therefore R_{cx} = 780 \times 2.13/1.68 = 988$ $R_{cx}^* = 888\omega$

U m/sec	f	Δ	I^2	tap	R_{cx}	% turb	I_{tap7}/% turb	
35.25	70	8.00	6.0	7	988	3.52	.695	
"	90	6.21	"	"	"	"	"	
"	110	5.09	"	"	"	"	"	$\sqrt{U} = 5.93$
"	140	4.00	"	"	"	"	"	
"	160	3.50	"	"	"	"	"	

APPENDIX # 3

Hot Wire Response to Periodic Fluctuations

This investigation was an attempt to verify the theory of the lag of hot wires as formulated by Dryden and Kuethe by utilizing the Vibrator. The method used for determining the hot wire's response is given on the Data Sheets. Reference to Fig. 1 shows that a wire with a small value of M has better frequency response than if M is large. Thus any discrepancy between the actual and theoretical behavior of the wire at low frequencies would be more pronounced with larger diameter wires and higher temperatures.

Two tests were made using Anemometer #2, without the compensation inductance in the amplifier circuit, and large hot wires (0.001" diam.). By increasing the heating current, the value of the time constant $M = 6.42 \times 10^{-3}$ in one test was increased to $M = 10.76 \times 10^{-3}$ in the other. The ratios of the relative outputs at two frequencies of vibration gave the values of the time constant M. All computations and results are given in the following Data Sheets and comparison with theory is shown in Fig. 13 .

HOT WIRE RESPONSE   TEST 1   11/3/37

ANEMOMETER #2   DIAM. OF WIRE = 0.00254 CMS.
$L = 0, R_c^* = 5000\,\omega$

P_S	8.8 mm	$B + B_L$	33.7	$B_o + B_L$	13.0
BAR	747.2 mm	B_L	1.0	B_L	1.0
TEMP	22.5°C	B	32.7	B_o	12.0
γ	0.81	R	3.27 ω	R_o	1.2 ω
ρ	0.1195	i	0.536 AMP.		

f CPS	Δ mm	I^2	TAP	TAP RATIO FOR $\sqrt{I^2}$	I ON TAP 6	F	F^*
0							1.00
25							.51
50	11.79	21.9	4	3.431	16.05	.285	.285
70	8.42	11.6	4	"	11.70	.208	.207
100	5.89	21.2	5	1.768	8.14	.1445	.145
140	4.21	11.1	5	"	5.89	.1045	.105
160	3.68	8.5	5	"	5.18	.092	.092
180	3.27	21.5	6	1.00	4.64	.0824	.082

$$\overline{T} - T_o = R - R_o / R_o \alpha = 2.07 / 1.2 \times 3.67 \times 10^{-3} = 470°C$$

M = LAG CONSTANT COMPUTED FROM OUTPUT AT 50 & 180 CPS

$$\frac{16.05}{4.64} = \frac{\{1 + (2\pi \times 180 M)^2\}^{1/2}}{\{1 + (2\pi \times 50 M)^2\}^{1/2}} \quad \text{GIVES } M = 10.76 \times 10^{-3}$$

$$F^* = 1 / \{1 + (2\pi f M)^2\}^{1/2} \quad \text{FOR } M = 10.76 \times 10^{-3}$$

F = EXPERIMENTAL RESULTS REDUCED BY PLACING

$$F = \frac{(I \text{ ON TAP } 6)}{16.05} \times 0.285$$

HOT WIRE RESPONSE TEST 2 11/2/37

ANEMOMETER #2 DIAM. OF WIRE = 0.00254 CMS
$L=0, R_C^*=5000\omega$

P_S 9.0 mm.	$B+B_L$ 18.3	B_0+B_L 12.8
BAR. 748.0 mm	B_L 1.0	B_L 1.0
TEMP. 24.0°C	B 17.3	B_0 11.8
δ' 0.81	R 1.73ω	R_0 1.18ω
ρ 0.1187	i 0.36 AMP.	

f CPS	Δ m.m.	I^2	TAP	TAP RATIO FOR $\sqrt{I^2}$	I ON TAP 6	F	F^*
0							1.00
25							.705
50	11.79	12.6	6	1.00	3.55	.445	.445
70	8.42	23.0	7	1.802	2.66	.333	.333
100	5.89	12.7	7	"	1.973	.247	.248
140	4.21	20.8	8	3.231	1.41	.176	.174
160	3.68	15.9	8	"	1.234	.154	.153
180	3.27	12.4	8	"	1.09	.136	.136

$T-T_0 = 127°C$

$M = 6.42\times10^{-3}$ FROM OUTPUT AT 50 + 180 CPS

(SEE TEST 1 FOR PROCEDURE IN COMPUTATIONS)

APPENDIX #4

Performance of Hot Wire Anemometer #1

It has been shown that the response of the hot wire to periodic fluctuations is such that the fluctuating voltage drop across the wire is reduced in magnitude from its value under equilibrium conditions with the velocity fluctuations by a factor $1/\sqrt{1+(M\,2\pi f)^2}$. It is theoretically possible to incorporate an electrical compensation in the anemometer circuit in order to counteract the lag effect of the hot wire so that the voltage drop fluctuations are proportional to the velocity fluctuations. The function of the electrical compensation is to distort the response of the amplifier inversely proportional to the hot wire's response. Thus the compensated amplifier should have a gain characteristic which varies as $\sqrt{1+(M\,2\pi f)^2}$, and for a given mean flow velocity each reading of the amplifier's output meter corresponds to one value of turbulence intensity. The extent to which this is attained characterizes the performance of the compensated anemometer.

The following Data Sheets are self explanatory in regard to the method used. Although there are a great many variations of calibration procedure, the method illustrated herein was purposely chosen to show that the Vibrator Method can be relied upon to determine accurately the compensation resistance R_{cx} by extending the calibration to higher frequency bands with a beat-frequency oscillator.

PERFORMANCE OF ANEMOMETER #1

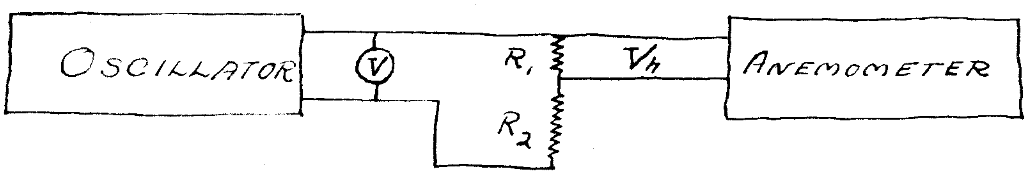
USING $L \doteq 1\frac{1}{2}h$ $r_L \doteq 55\Omega$ 7/23/37

PART 1 DETERMINATION OF R^*_{cx}

f_{cps}	Δmm	TAP	L	R^*_c	I^2
60	10.0	5	1 h	0 ω	10.3
80	7.5	"	"	"	13.1
100	6.0	"	"	"	14.9
120	5.0	"	"	"	16.5
140	4.285	"	"	"	17.6
160	3.75	"	"	"	18.0

POINTS OBTAINED USING VIBRATOR WITH .0005" PT. WIRE AT CONSTANT CURRENT AND WIND VELOCITY.

CIRCUIT FOR OSCILLATOR INPUT



$R_2 = 9999\,\omega$ $R_1 << R_2$ $\therefore R_1 + R_2 \cong 10{,}000\,\omega$

V_h SO ADJUSTED THAT FIRST I^2 COLUMN BELOW REPRODUCES VIBRATOR RESULTS. ALL VALUES IN TABLE ARE I^2.

	R^*_c								
f_{cps}	0	100	200	300	400	500	600	700	TAP
60	10.3	11.6	14.2	17.8	22.5	28.4	35.0	42.6	5
80	13.1	14.0	15.5	18.1	21.2	25.0	29.7	35.0	"
100	14.9	15.5	16.9	18.8	21.0	23.2	26.3	30.0	"
120	16.5	17.0	18.0	19.1	20.5	22.3	24.6	27.0	"
140	17.6	18.0	18.5	19.2	20.3	21.9	23.5	25.5	"
160	18.0	18.2	18.8	19.3	20.3	21.1	22.6	24.1	"

THE ACTUAL VALUES OF V_h ARE LISTED IN FIRST PART OF THE TABLE IN PART 3 OF THIS TEST.

THESE RESULTS SHOW $R^*_{cx} \doteq 345\,\omega$ FOR THE HOT WIRE AS USED

SEE FIG. 14

PERFORMANCE OF ANEMOMETER #1 (CONT.)

PART 2 COMPUTATIONS

CALCULATION OF M

FROM TABLE IN PART 3

$$\frac{V_h\ (\text{AT } 60\text{ CPS})}{V_h\ (\text{AT } 160\text{ CPS})} = \frac{4.03}{1.79}$$

SINCE R_1 & R_2 REMAINED UNCHANGED

$\therefore$ PUT

$$\frac{\{1+(2\pi\times 160\,M)^2\}^{1/2}}{\{1+(2\pi\times 60\,M)^2\}^{1/2}} = \frac{4.03}{1.79}$$

THEN $M = 3.74\times 10^{-3}$

INPUT VOLTAGES FOR ANY FREQUENCY - f

LET V = VOLTAGE ACROSS OSCILLATOR

$$V_h = \frac{R_1}{10^4}V$$

$$\frac{V_h}{(V_h)_{60CPS}} = \frac{\{1+(2\pi\times 60\times 3.74\times 10^{-3})^2\}^{1/2}}{\{1+(2\pi f\times 3.74\times 10^{-3})^2\}^{1/2}}$$

$$R_1 V\times 10^{-4} = \frac{1.73}{\sqrt{1+5.51\times 10^{-4} f^2}}\ \frac{28.1}{10^4}\ 4.03$$

$$V = \frac{6.96}{\sqrt{1+5.51\times 10^{-4} f^2}}\ \frac{28.1}{R_1} = \frac{V_h\times 10^4}{R_1}$$

WHEN V AND R_1 SATISFY THIS RELATION AND $R^*_{CX} = 345\,\omega$ THE I^2 READING SHOULD BE THE SAME FOR ALL FREQUENCIES. AT 60 CPS, $I^2 = 19.6$. THE FOLLOWING TEST SHOWS ITS VARIATIONS UP TO 6000 CPS.

PERFORMANCE OF ANEMOMETER #1 (CONT.)

PART 3 FREQUENCY CHARACTERISTICS - COMP.

$R_{cx}^{*} = 345\omega$, CORRESPONDS TO $M = 3.74 \times 10^{-3}$
TAP 5
UP TO 160 CPS V AND R_1 WERE ADJUSTED TO GIVE 19.6

f cps	R_1	V	$V/R_1 \times 10^3$	I^2	$\sqrt{I^2}/\sqrt{I^2_{60\,cps}}$
60	28.1	4.03	11.30	19.6	1.00
80	"	3.27	9.20	"	"
100	"	2.71	7.62	"	"
120	"	2.34	6.58	"	"
140	"	2.03	5.71	"	"
160	"	1.79	5.03	"	"
200	10	4.075	4.075	"	"
300	"	2.75	2.75	"	"
400	5	4.14	2.07	19.4	.996
500	4	4.15	1.658	19.2	.990
600	3	4.62	1.385	18.8	.980
700	"	3.95	1.185	18.7	.978
800	"	3.47	1.040	18.5	.972
900	"	3.08	.925	18.2	.964
1000	"	2.78	.832	18.0	.960
1500	1.5	3.70	.555	"	"
2000	"	2.78	.416	"	"
2500	1.0	3.33	.333	"	"
3000	"	2.78	.277	18.1	.962
3500	.5	4.76	.238	18.3	.966
4000	"	4.16	.208	18.5	.972
4500	"	3.69	.1846	"	"
5000	"	3.33	.1662	17.0	.932
6000	"	2.78	.1385	12.3	.792

APPENDIX #5

Performance of Hot Wire Anemometer #2

The performance of an anemometer may also be considered as the ratio of the actual relative output to the theoretical value, based upon the time constant M, as a function of frequency.

The actual relative outputs were determined by connecting a beat frequency oscillator to the input of the anemometer whose amplifier was adjusted to a desired value of compensation R^*_c. From previous measurements of the inductance L and its DC resistance r_L the value of $M = L/(R^*_{cx} + r_L)$ was found and from this the theoretical values of the relative outputs for given frequencies were computed for the condition that the amplifier's response should vary by the factor $\sqrt{(R^*_{cx} + r_L)^2 + (2\pi f L)^2}$ due to the compensating circuit.

Computations are found in the following Data Sheet and the results are shown in Fig. 18 .

CALIBRATION OF ANEMOMETER #2, COMPENSATED

$L = 2.36h$ D.C. RESISTANCE = 100ω

$M = \frac{L}{R_C}$ * INDICATES THEORETICAL VALUES.

C, C* RELATIVE OUTPUTS BASED ON 100 CPS

	R_C = 200+100 = 300ω			R_C = 600+100 = 700ω			R_C = 1200+100 = 1300ω		
F	I^2	I RATIO	I RATIO*	I^2	I RATIO	I RATIO*	I^2	I RATIO	I RATIO*
25	2.45	1.71	1.76	5.95	1.29	1.20	7.40	1.13	1.11
50	7.20			10.60			9.50		
50	2.50	1.90	1.90	6.65	1.63	1.62	9.50	1.35	1.32
100	9.00			17.75			17.20		
100	2.20	1.97	1.96	3.05	1.84	1.86	4.20	1.65	1.64
200	8.50			10.40			11.70		
200	2.40	2.46	2.48	3.55	2.42	2.44	2.80	2.31	2.32
500	14.45			20.80			14.80		
500	5.25	1.98	2.00	5.15	1.98	1.99	5.20	1.97	1.98
1000	20.60			20.30			20.26		
1000	2.30	2.00	2.00	3.40	1.99	2.00	5.05	2.00	2.00
2000	9.20			13.40			20.20		
2000	2.55	2.10	2.00	3.30	2.10	2.00	3.75	2.07	2.00
4000	11.33			14.75			16.20		
4000	4.20	1.49	1.50	4.25	1.52	1.50	7.15	1.55	1.50
6000	9.30			9.70			17.35		
4000	4.20	1.60	1.75	4.10	1.72	1.75	3.25	1.74	1.75
7000	10.75			11.90			9.90		
4000	4.20	1.59	2.00	4.10	1.76	2.00	3.25	1.83	2.00
8000	10.65			12.60			10.80		

	$M = 7.86 \times 10^{-3}$			$M = 3.38 \times 10^{-3}$			$M = 2.36 \times 10^{-3}$		
F	C*	C	C/C*	C*	C	C/C*	C*	C	C/C*
25	.299	.308	1.03	.475	.475	1.00	.674	.656	.976
50	.526	.526	1.00	.616	.614	.995	.757	.741	.980
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
200	1.96	1.97	1.004	1.86	1.84	.990	1.64	1.65	1.005
500	4.86	4.85	.996	4.54	4.45	.985	3.80	3.82	1.003
1000	9.72	9.60	.988	9.01	8.83	.980	7.54	7.54	1.00
2000	19.44	19.20	.988	18.02	17.60	.976	14.90	14.85	1.00
4000	38.88	40.40	1.036	36.04	37.00	1.023	29.80	30.80	1.033
6000	58.40	60.00	1.029	54.10	56.20	1.040	44.70	47.70	1.070
7000	68.10	64.50	.946	63.00	63.50	1.010	52.10	53.50	1.030
8000	77.76	64.20	.830	72.08	65.10	.895	59.60	56.50	.945

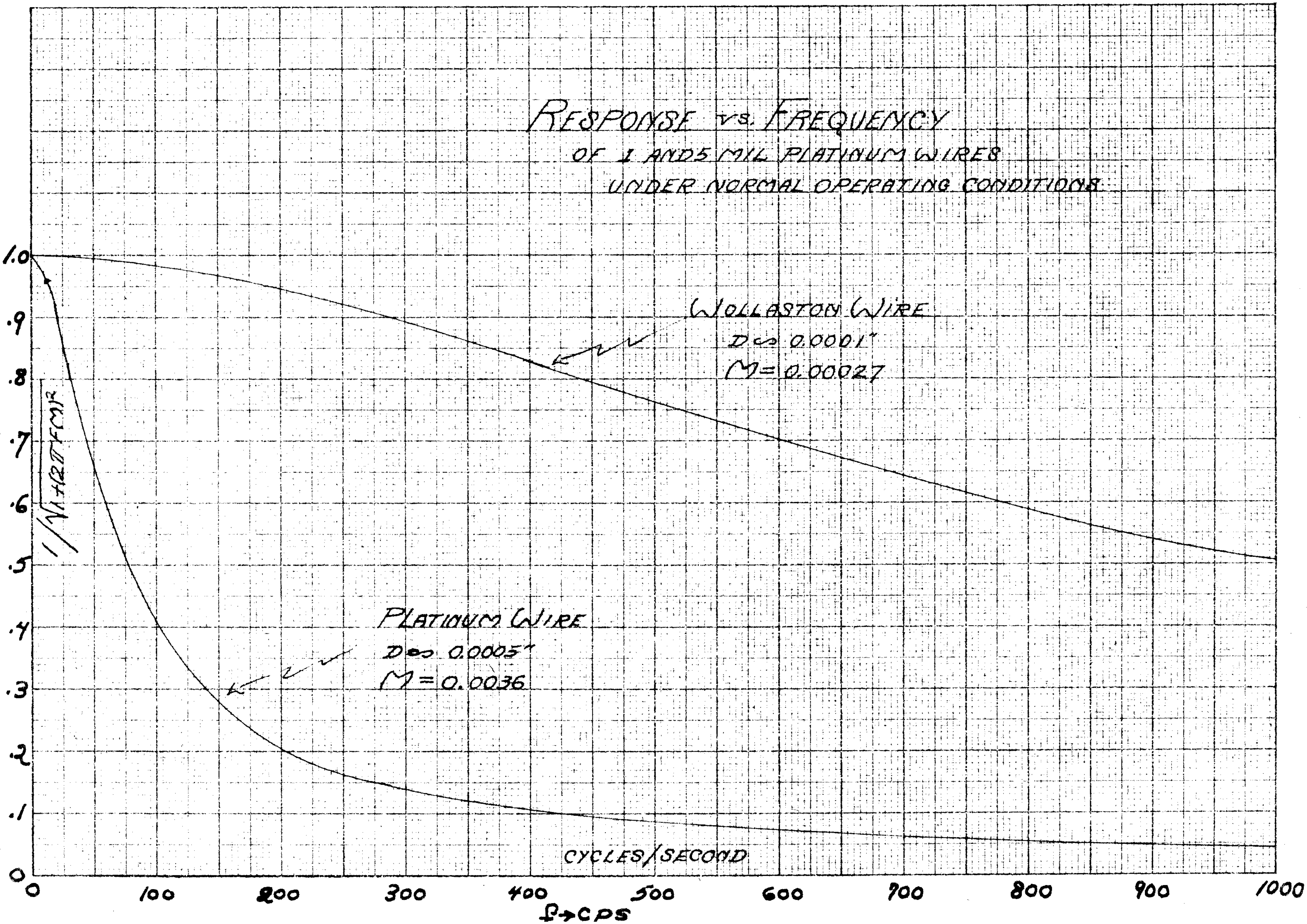
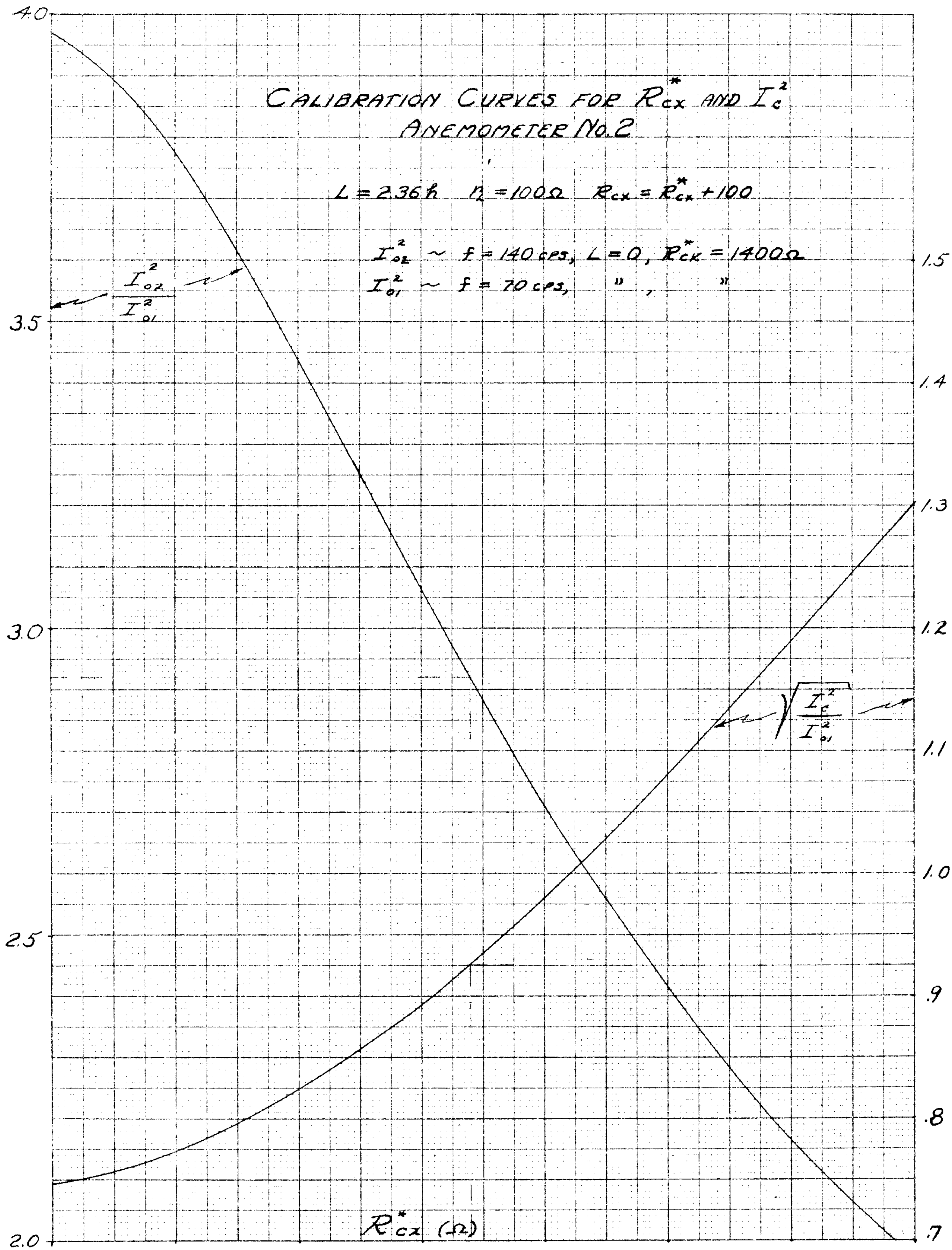

RESPONSE vs. FREQUENCY
OF 1 AND 5 MIL PLATINUM WIRES
UNDER NORMAL OPERATING CONDITIONS
WOLLASTON WIRE
D ≈ 0.0001"
M = 0.00027
PLATINUM WIRE
D ≈ 0.0005"
M = 0.0036
$1/\sqrt{1+(2\pi f M)^2}$
CYCLES/SECOND
Ω → CPS
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
100
200
300
400
500
600
700
800
900
1000

FIG-1



CALIBRATION CURVES FOR R^*_{cx} AND I^2_c
ANEMOMETER No. 2
$L = 2.36h \quad r_2 = 100\Omega \quad R_{cx} = R^*_{cx} + 100$
$I^2_{02} \sim f = 140\ cps,\ L = 0,\ R^*_{cx} = 1400\Omega$
$I^2_{01} \sim f = 70\ cps,$ " , "
$\frac{I^2_{02}}{I^2_{01}}$
$\sqrt{\frac{I^2_c}{I^2_{01}}}$
$R^*_{cx}\ (\Omega)$
4.0
3.5
3.0
2.5
2.0
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
0
200
400
600
800
1000
1200
1400

FIG-2

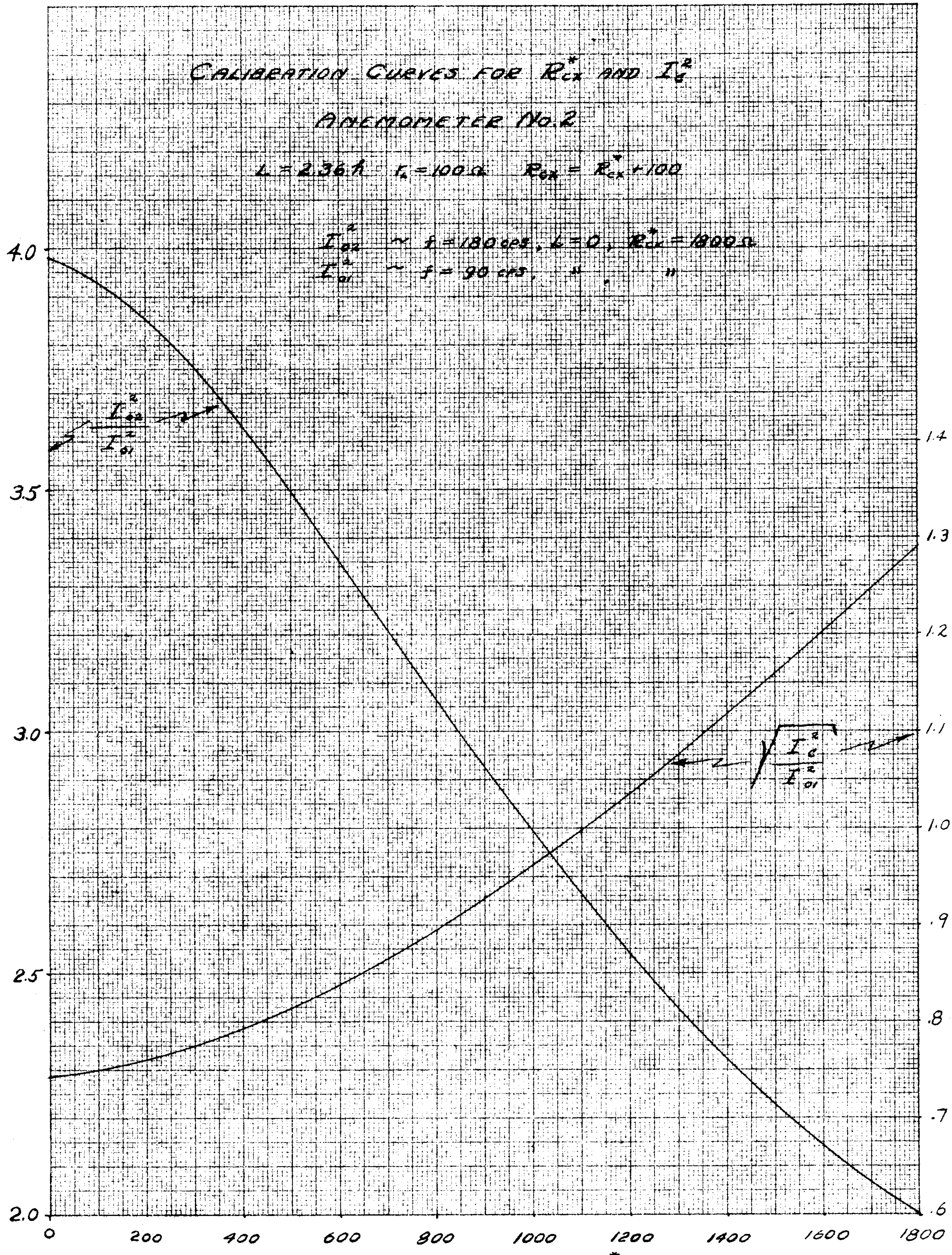

CALIBRATION CURVES FOR R_{cx}^{*} AND I_{c}^{2}
ANEMOMETER No. 2
$L = 2.36\,h$ $r_{s} = 100\,\Omega$ $R_{o2} = R_{cx}^{*} + 100$
I_{o2}^{2} ~ $f = 180$ cps, $L = 0$, $R_{cx}^{*} = 1800\,\Omega$
I_{o1}^{2} ~ $f = 90$ cps, " , "
$\frac{I_{o2}^{2}}{I_{o1}^{2}}$
$\sqrt{\frac{I_{c}^{2}}{I_{o1}^{2}}}$
4.0
3.5
3.0
2.5
2.0
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
0
200
400
600
800
1000
1200
1400
1600
1800
R_{cx}^{*} (Ω) →

FIG-3

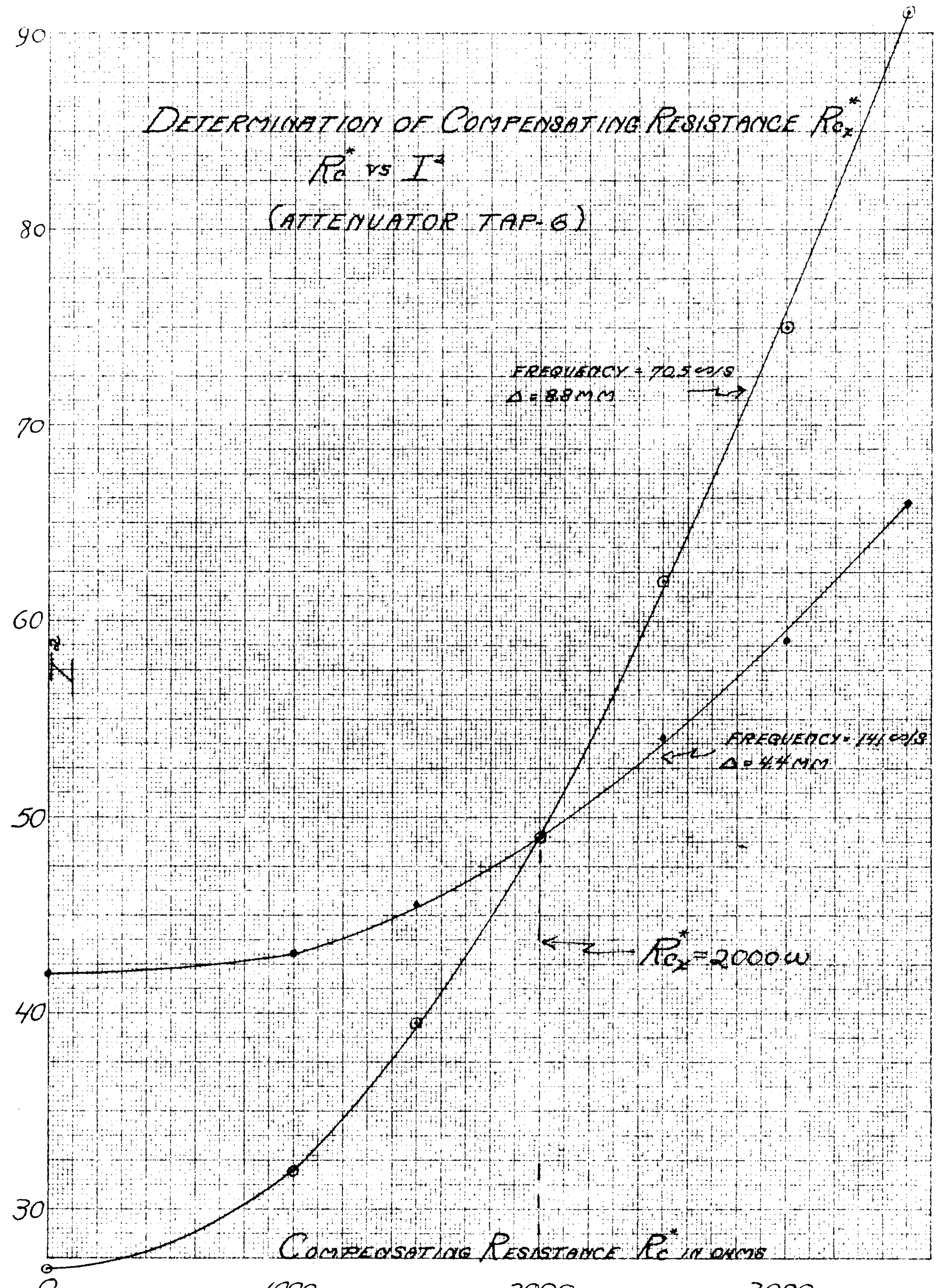
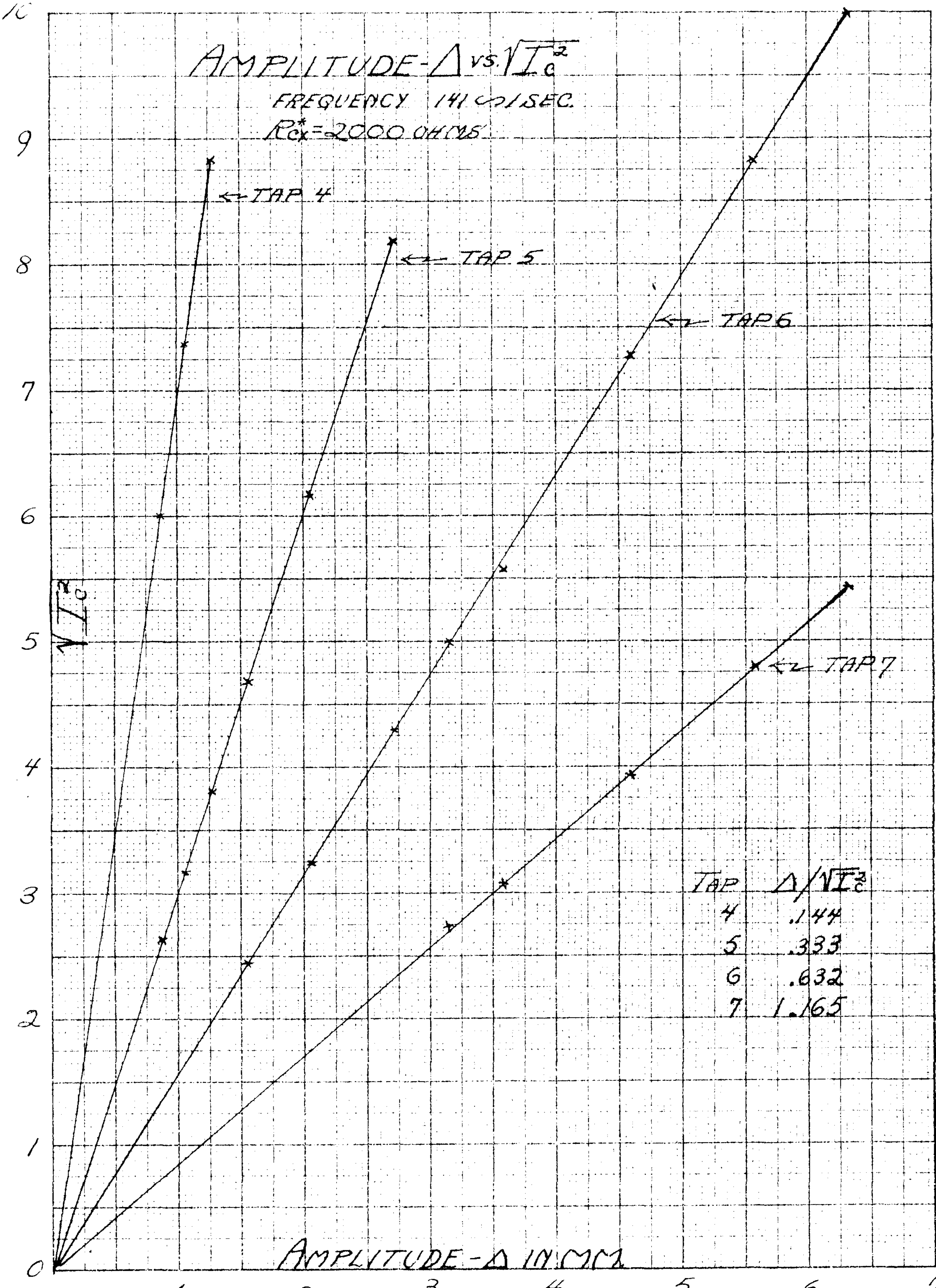

DETERMINATION OF COMPENSATING RESISTANCE $R_{c_x}^*$
R_c^* vs I^2
(ATTENUATOR TAP-6)
FREQUENCY = 70.5 ~/S
Δ = 8.8 MM
FREQUENCY = 141 ~/S
Δ = 4.4 MM
$R_{c_x}^* = 2000\omega$
I^2
90
80
70
60
50
40
30
COMPENSATING RESISTANCE R_c^* IN OHMS
0
1000
2000
3000

FIG-4



AMPLITUDE-Δ VS. √I_C²
FREQUENCY 141 ∿/SEC.
R*CX = 2000 OHMS
TAP 4
TAP 5
TAP 6
TAP 7
√I_C²
AMPLITUDE-Δ IN MM
0
1
2
3
4
5
6
7
8
9
10
TAP Δ/√I_C²
4 .144
5 .333
6 .632
7 1.165

FIG-5

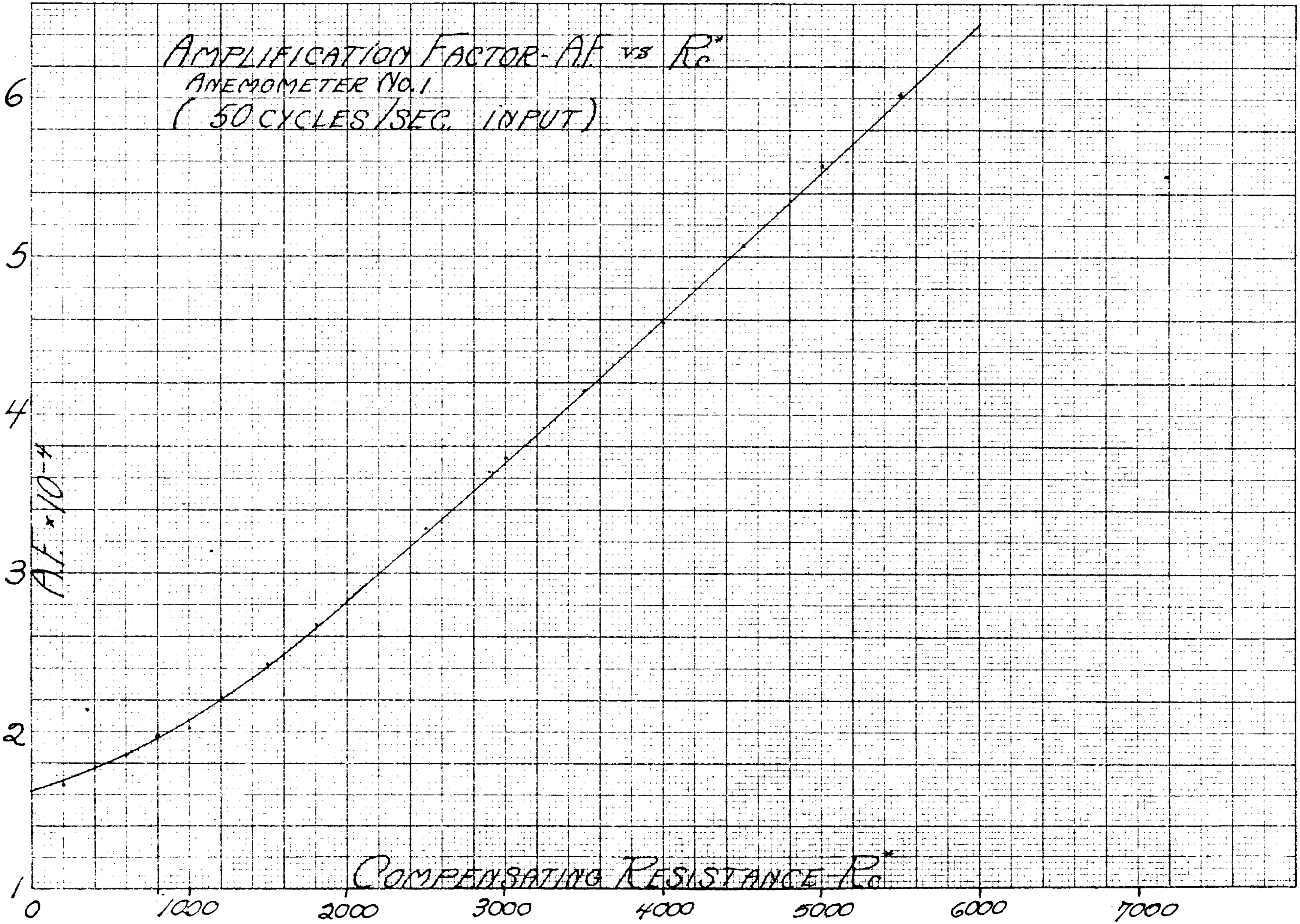
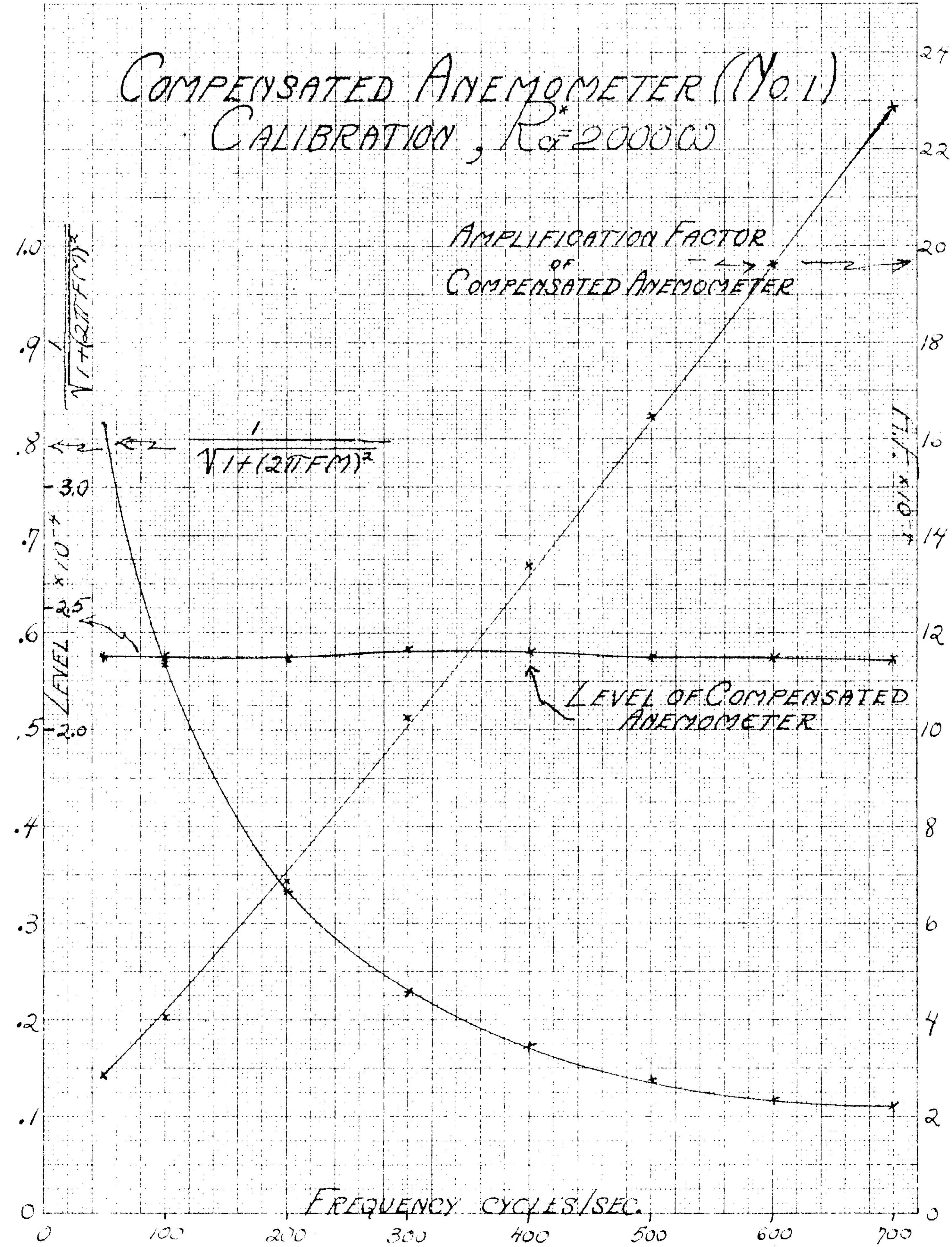

AMPLIFICATION FACTOR - AF vs R_c^*
ANEMOMETER No. 1
(50 CYCLES/SEC. INPUT)
6
5
4
3
2
1
AF × 10⁻⁴
0
1000
2000
3000
4000
5000
6000
7000
COMPENSATING RESISTANCE - R_c^*

FIG-6



COMPENSATED ANEMOMETER (No. 1)
CALIBRATION, $R_\alpha^* = 2000\,\omega$
AMPLIFICATION FACTOR OF COMPENSATED ANEMOMETER
$\frac{1}{\sqrt{1+(2\pi FM)^2}}$
LEVEL OF COMPENSATED ANEMOMETER
LEVEL $\times 10^{-7}$
$\frac{I}{i} \times 10^{-7}$
FREQUENCY CYCLES/SEC.
0
100
200
300
400
500
600
700
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
2.0
2.5
3.0
2
4
6
8
10
12
14
16
18
20
22
24

FIG-7

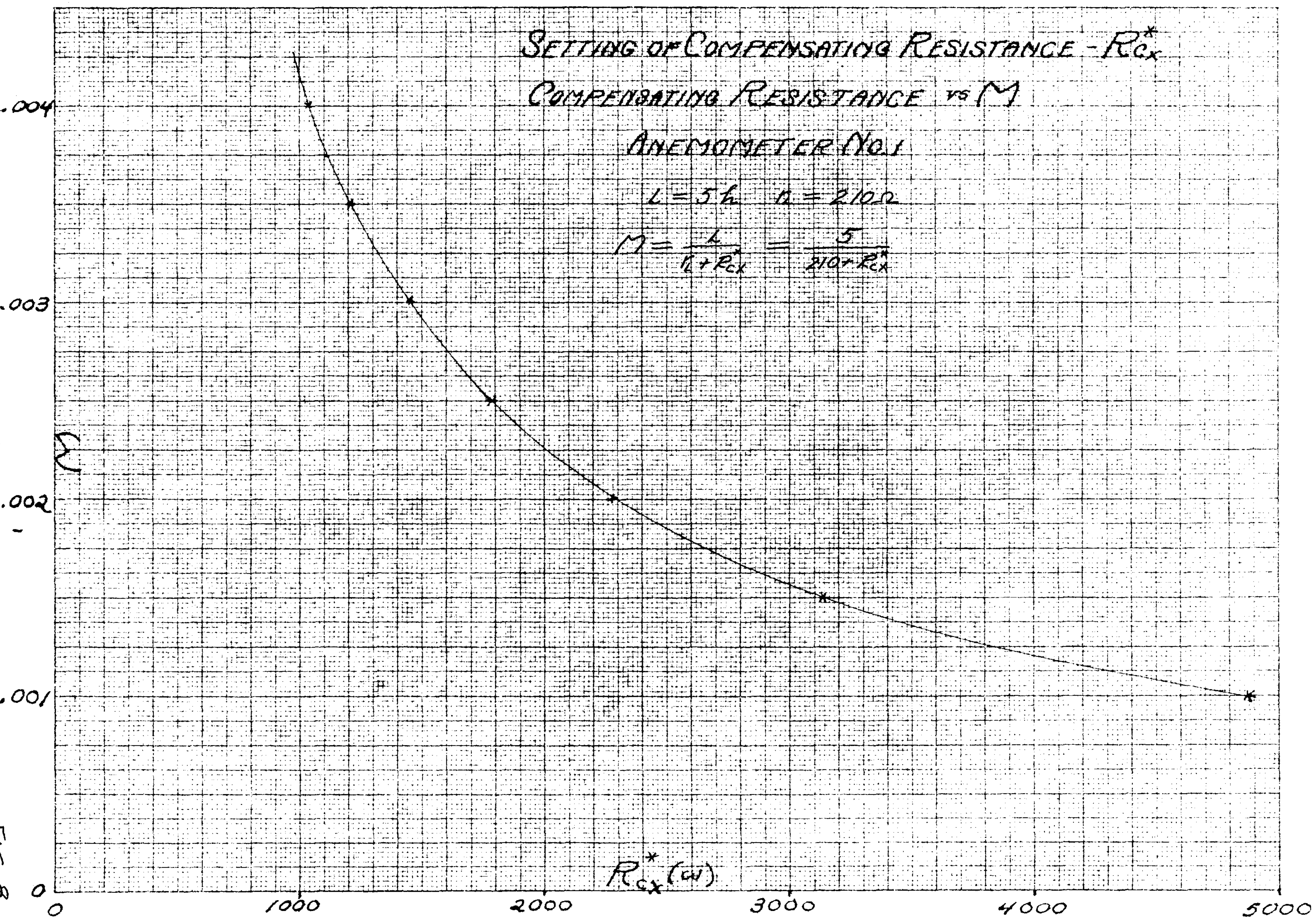

SETTING OF COMPENSATING RESISTANCE - R_{cx}^*
COMPENSATING RESISTANCE vs M
ANEMOMETER NO. 1
$L = 5h \quad R = 210\Omega$
$M = \frac{L}{R + R_{cx}^*} = \frac{5}{210 + R_{cx}^*}$
M
.004
.003
.002
.001
0
0
1000
2000
3000
4000
5000
R_{cx}^* (ω)

FIG-8

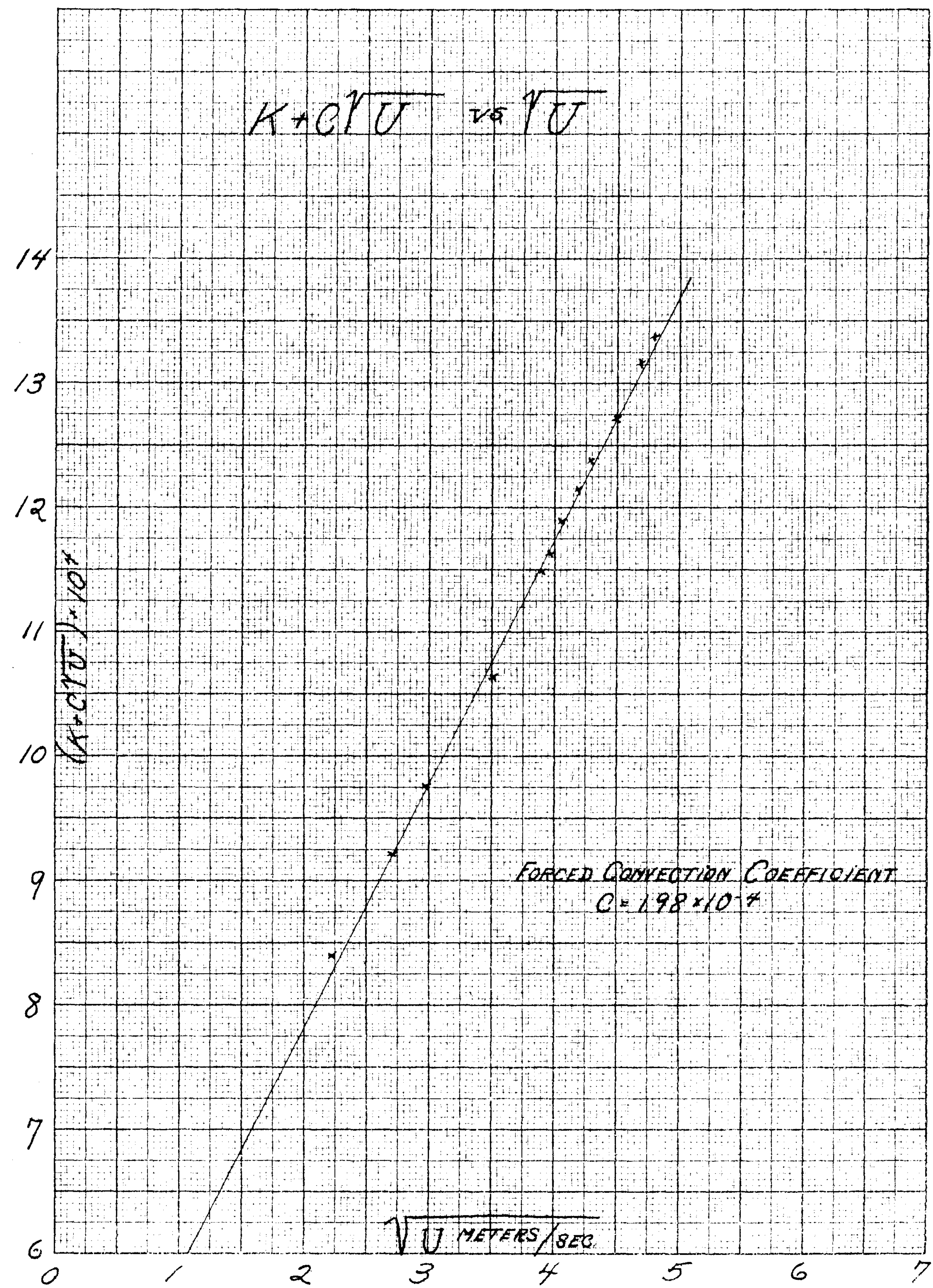
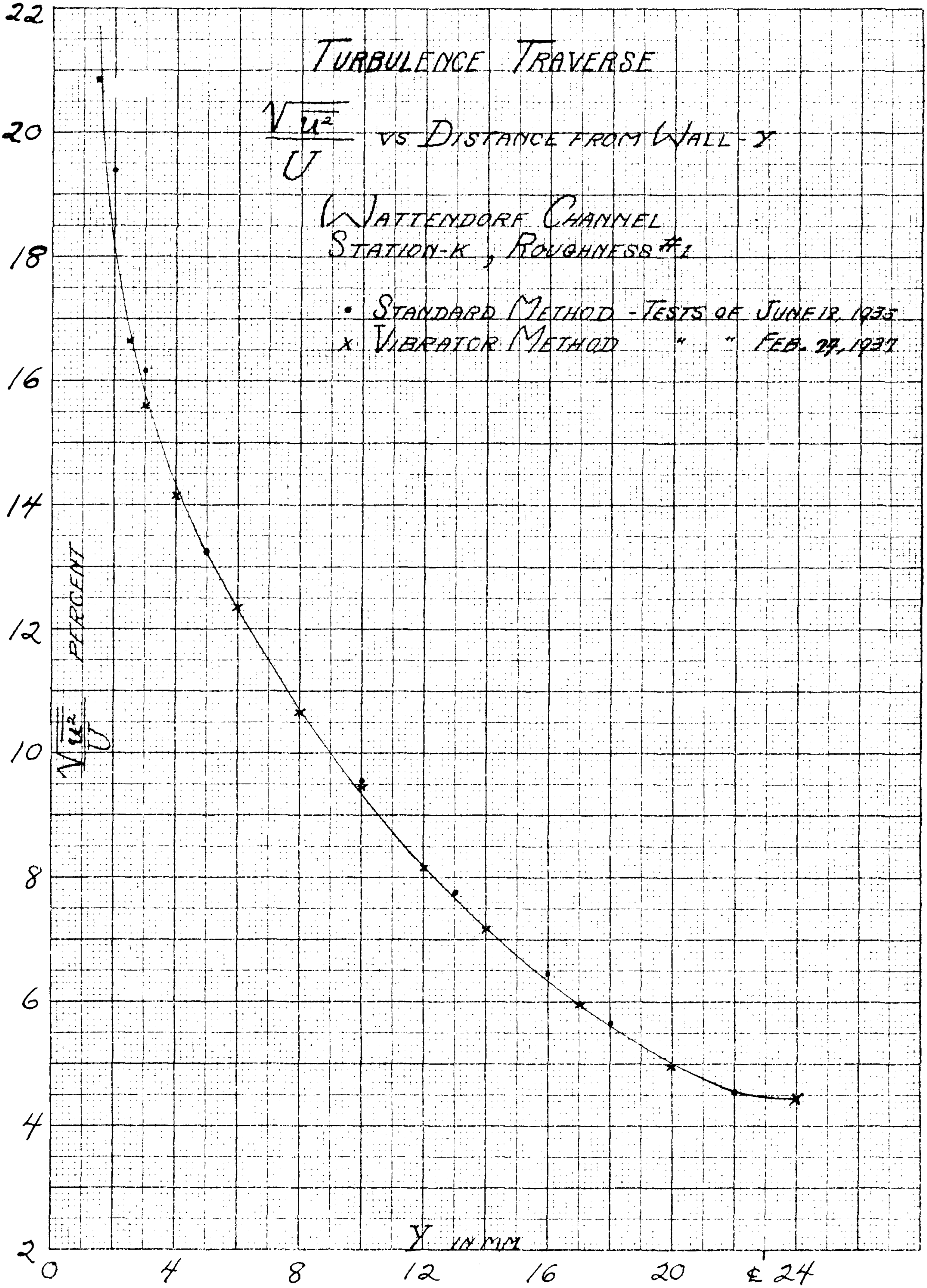
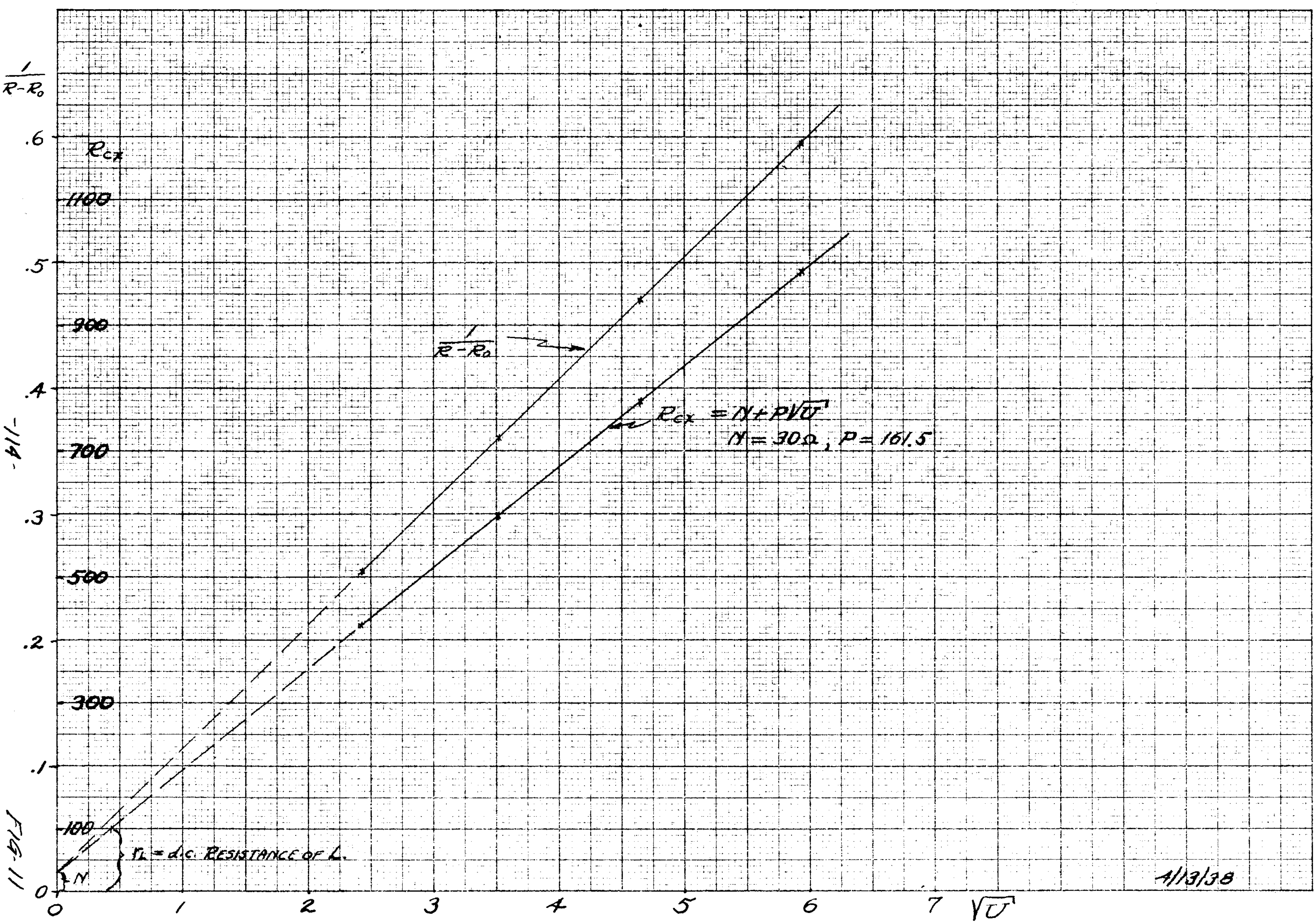

$K+C\sqrt{U}$ vs $\sqrt{U}$
$(K+C\sqrt{U}) \times 10^7$
14
13
12
11
10
9
8
7
6
0
1
2
3
4
5
6
7
$\sqrt{U}$ METERS/SEC
FORCED CONVECTION COEFFICIENT
$C = 1.98 \times 10^{-4}$

FIG-9



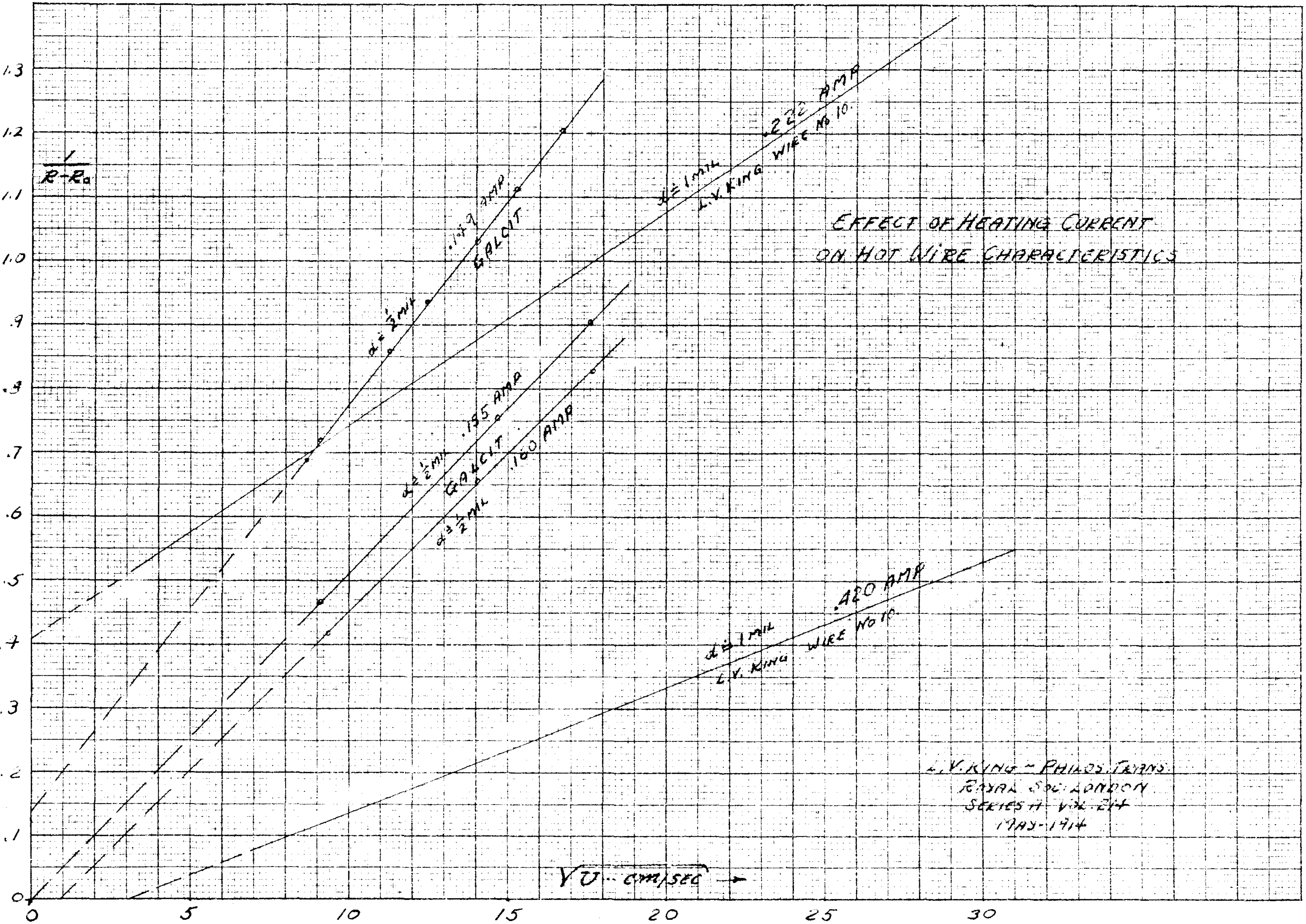
TURBULENCE TRAVERSE
$\frac{\sqrt{\overline{u^2}}}{U}$ VS DISTANCE FROM WALL - Y
WATTENDORF CHANNEL
STATION-K , ROUGHNESS #1
• STANDARD METHOD - TESTS OF JUNE 12, 1935
x VIBRATOR METHOD " " FEB. 27, 1937
$\frac{\sqrt{\overline{u^2}}}{U}$ PERCENT
Y IN MM
22
20
18
16
14
12
10
8
6
4
2
0
4
8
12
16
20
℄ 24

FIG-10

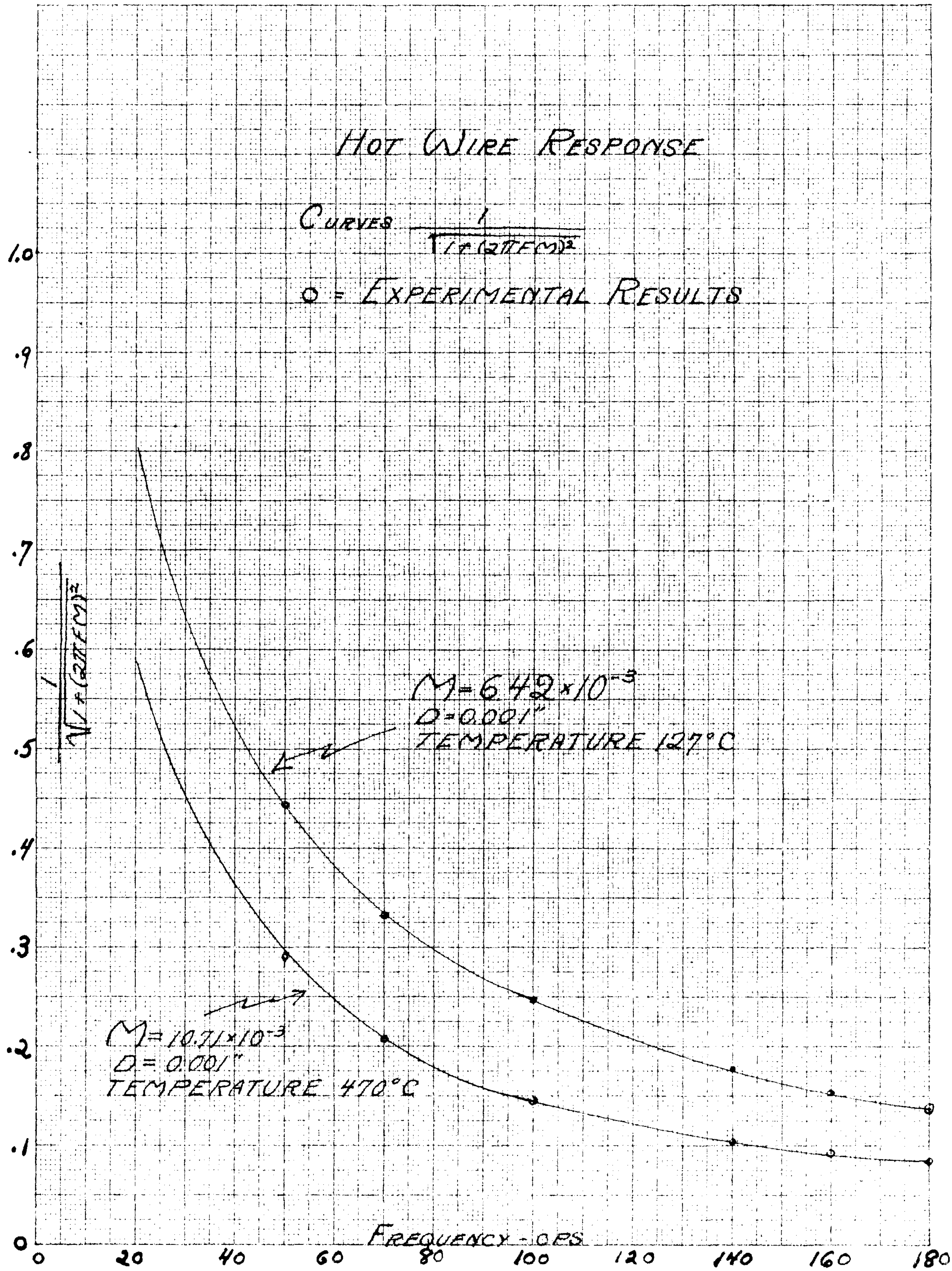


$\frac{1}{R-R_o}$
.6
.5
.4
.3
.2
.1
0
R_{cx}
1100
900
700
500
300
100
$\frac{1}{R-R_o}$
$R_{cx} = N + P\sqrt{U}$
$N = 30\Omega$, $P = 161.5$
r_L = d.c. RESISTANCE OF L.
N
0
1
2
3
4
5
6
7
$\sqrt{U}$
4/13/38

FIG-11


EFFECT OF HEATING CURRENT
ON HOT WIRE CHARACTERISTICS
$\frac{1}{R-R_0}$
$\sqrt{U}$ .. CM/SEC →
.222 AMP
d = 1 MIL
L.V. KING WIRE No 10.
.149 AMP
GALCIT
d = ½ MIL
.185 AMP
GALCIT
.160 AMP
.420 AMP
L.V. KING WIRE No 10
L.V. KING ~ PHILOS. TRANS.
ROYAL SOC. LONDON
SERIES A VOL. 214
MAY-1914
0
5
10
15
20
25
30
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3

FIG~12



HOT WIRE RESPONSE
CURVES $\frac{1}{\sqrt{1+(2\pi f M)^2}}$
○ = EXPERIMENTAL RESULTS
$M = 6.42 \times 10^{-3}$
$D = 0.001''$
TEMPERATURE 127°C
$M = 10.71 \times 10^{-3}$
$D = 0.001''$
TEMPERATURE 470°C
$\frac{1}{\sqrt{1+(2\pi f M)^2}}$
FREQUENCY - CPS
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
0 20 40 60 80 100 120 140 160 180

FIG-13

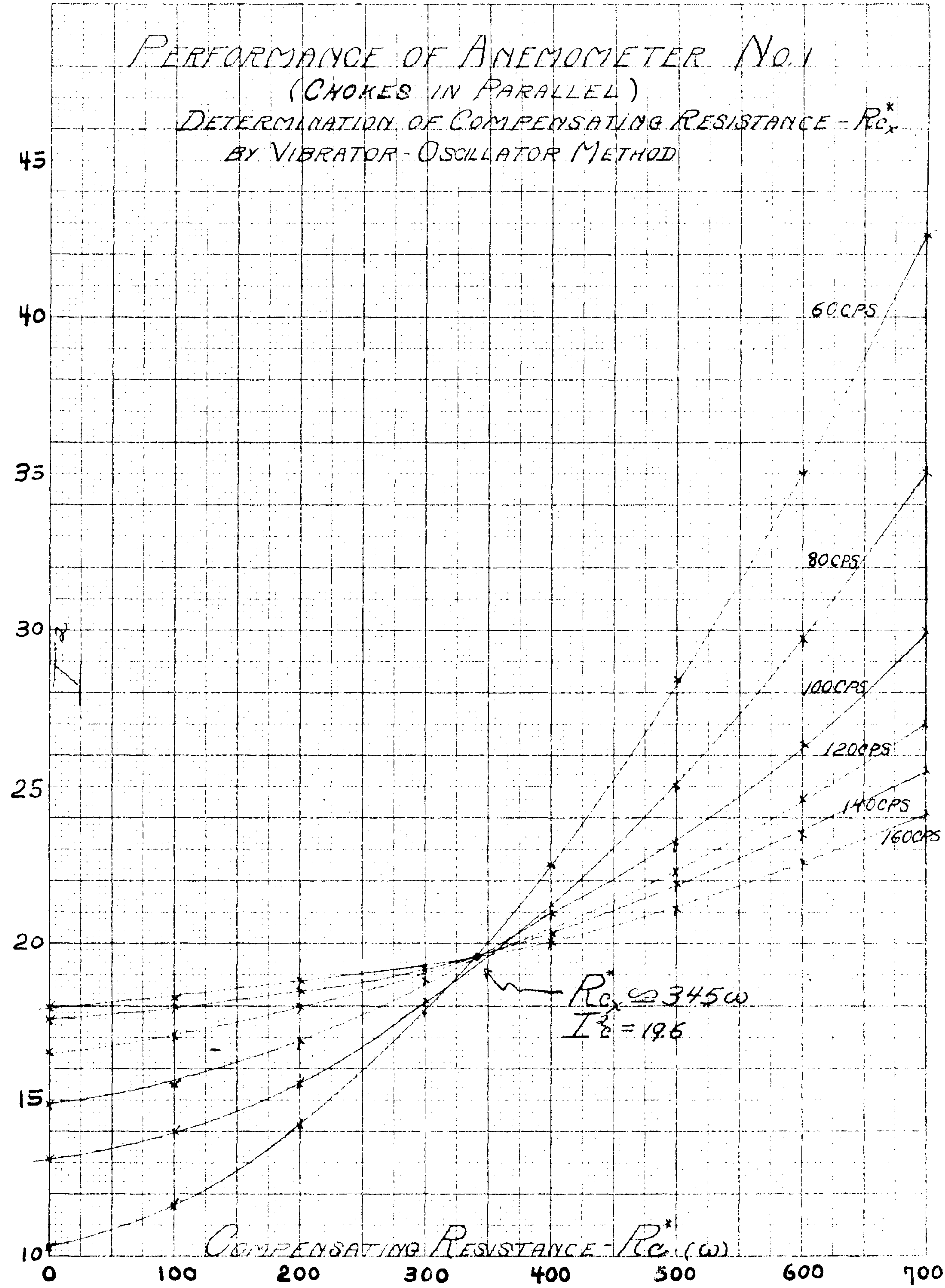
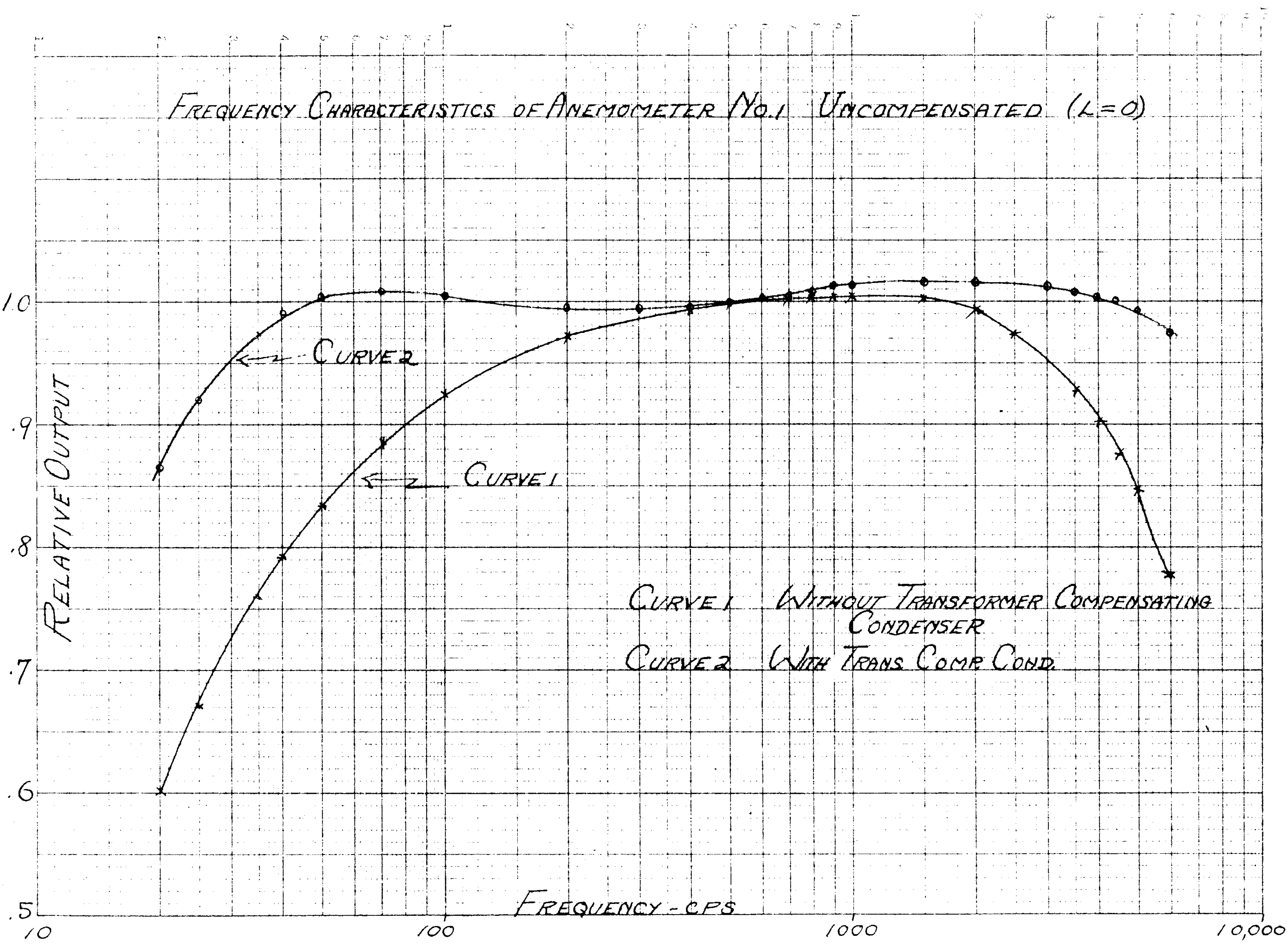
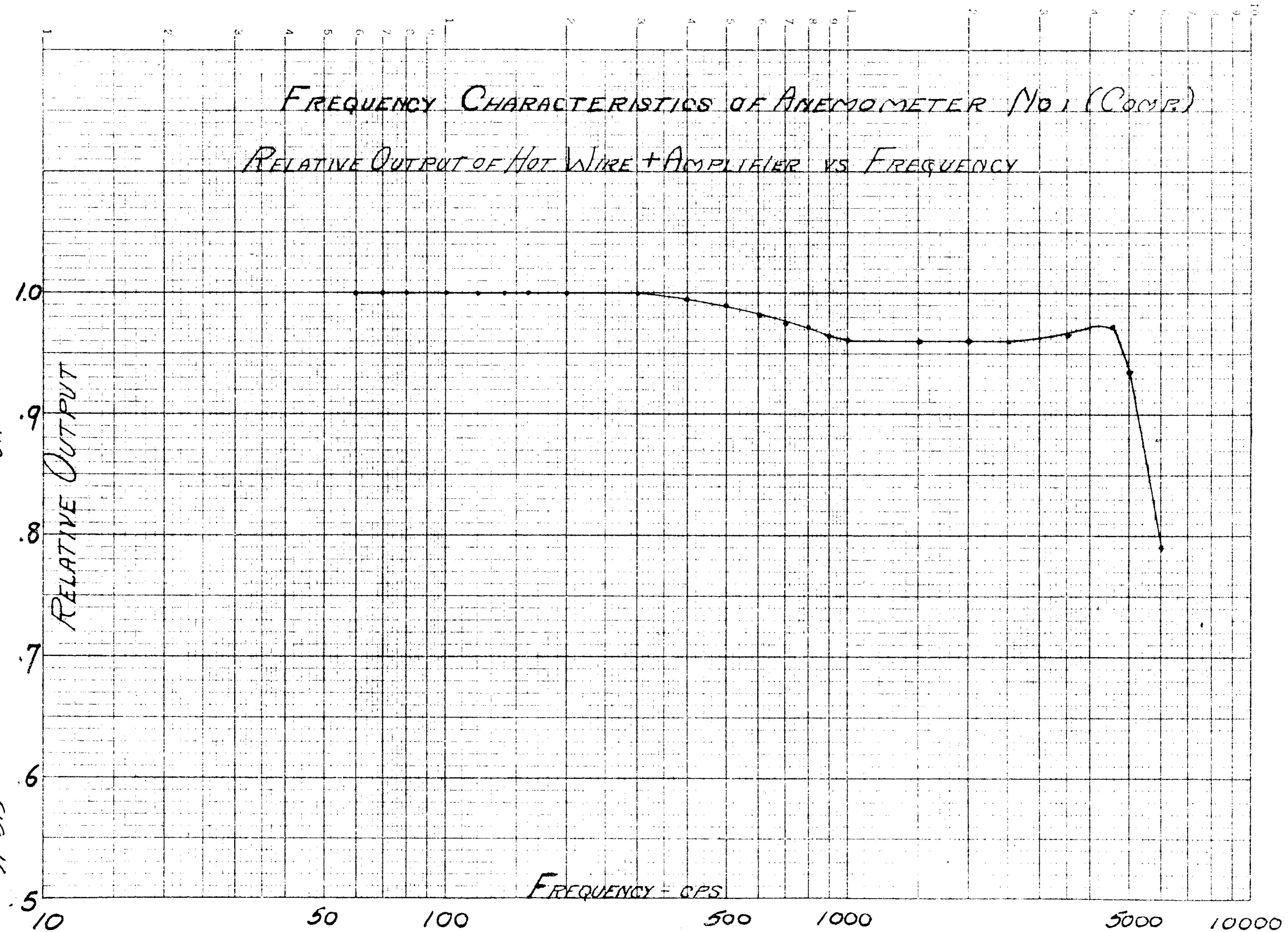

PERFORMANCE OF ANEMOMETER NO. 1
(CHOKES IN PARALLEL)
DETERMINATION OF COMPENSATING RESISTANCE - $R_{c_x}^*$
BY VIBRATOR - OSCILLATOR METHOD
60 CPS
80 CPS
100 CPS
120 CPS
140 CPS
160 CPS
$R_{c_x}^* \cong 345\omega$
$I_2^2 = 19.6$
COMPENSATING RESISTANCE - R_c^* (ω)
10
15
20
25
30
35
40
45
0
100
200
300
400
500
600
700

FIG-14



FREQUENCY CHARACTERISTICS OF ANEMOMETER NO.1 UNCOMPENSATED (L=0)
CURVE 2
CURVE 1
CURVE 1 WITHOUT TRANSFORMER COMPENSATING CONDENSER
CURVE 2 WITH TRANS COMP COND.
RELATIVE OUTPUT
1.0
.9
.8
.7
.6
.5
FREQUENCY - CPS
10
100
1000
10,000

FIG-15



FREQUENCY CHARACTERISTICS OF ANEMOMETER NO 1 (COMP.)
RELATIVE OUTPUT OF HOT WIRE + AMPLIFIER VS FREQUENCY
RELATIVE OUTPUT
1.0
.9
.8
.7
.6
.5
FREQUENCY - CPS
10
50
100
500
1000
5000
10000

FIG-16

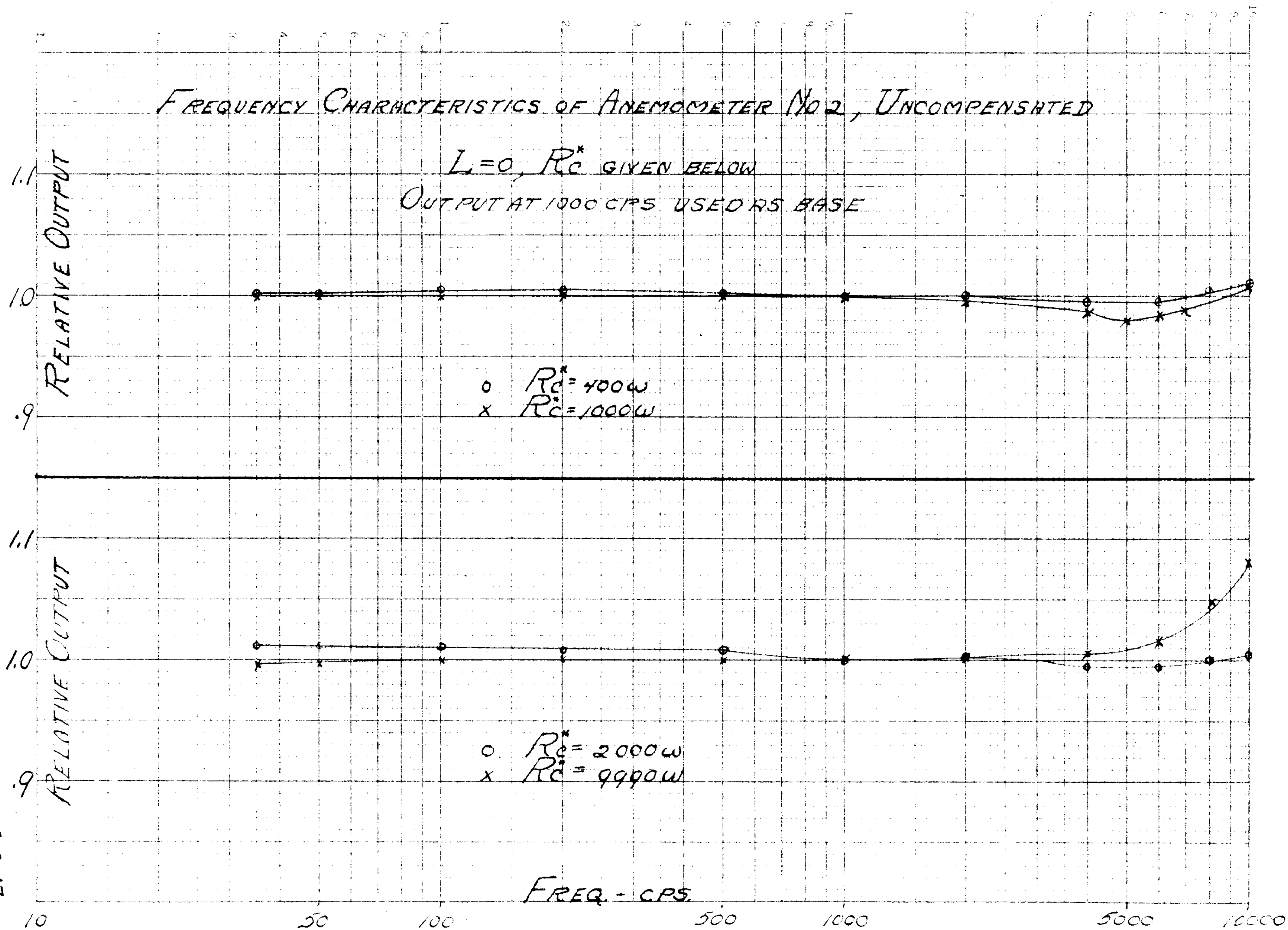

FREQUENCY CHARACTERISTICS OF ANEMOMETER No 2, UNCOMPENSATED
L=0, R_c^* GIVEN BELOW
OUTPUT AT 1000 CPS USED AS BASE
RELATIVE OUTPUT
1.1
1.0
.9
o R_c^* = 400ω
x R_c^* = 1000ω
RELATIVE OUTPUT
1.1
1.0
.9
o R_c^* = 2000ω
x R_c^* = 9990ω
FREQ. - CPS.
10
50
100
500
1000
5000
10000

FIG-17

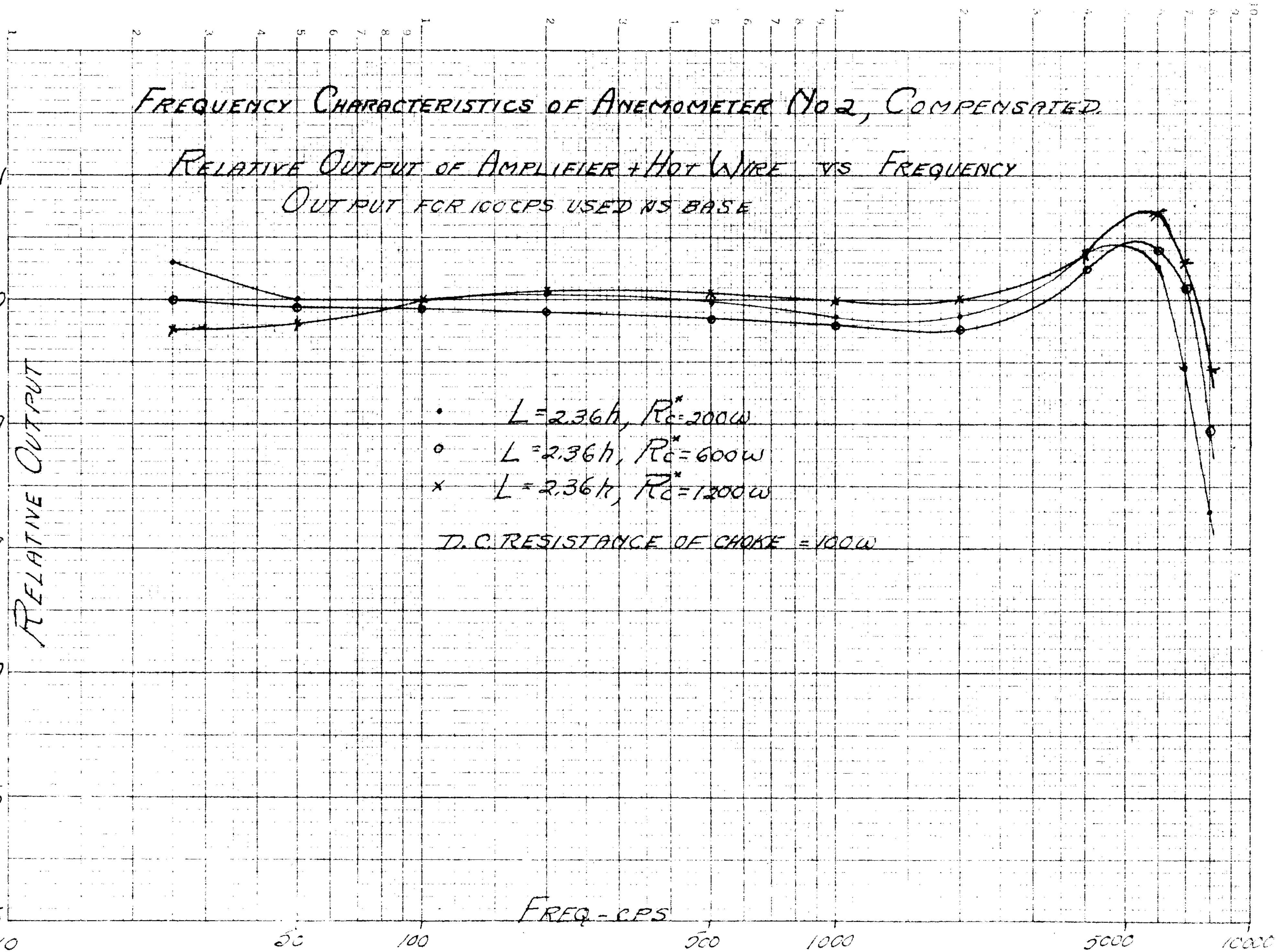
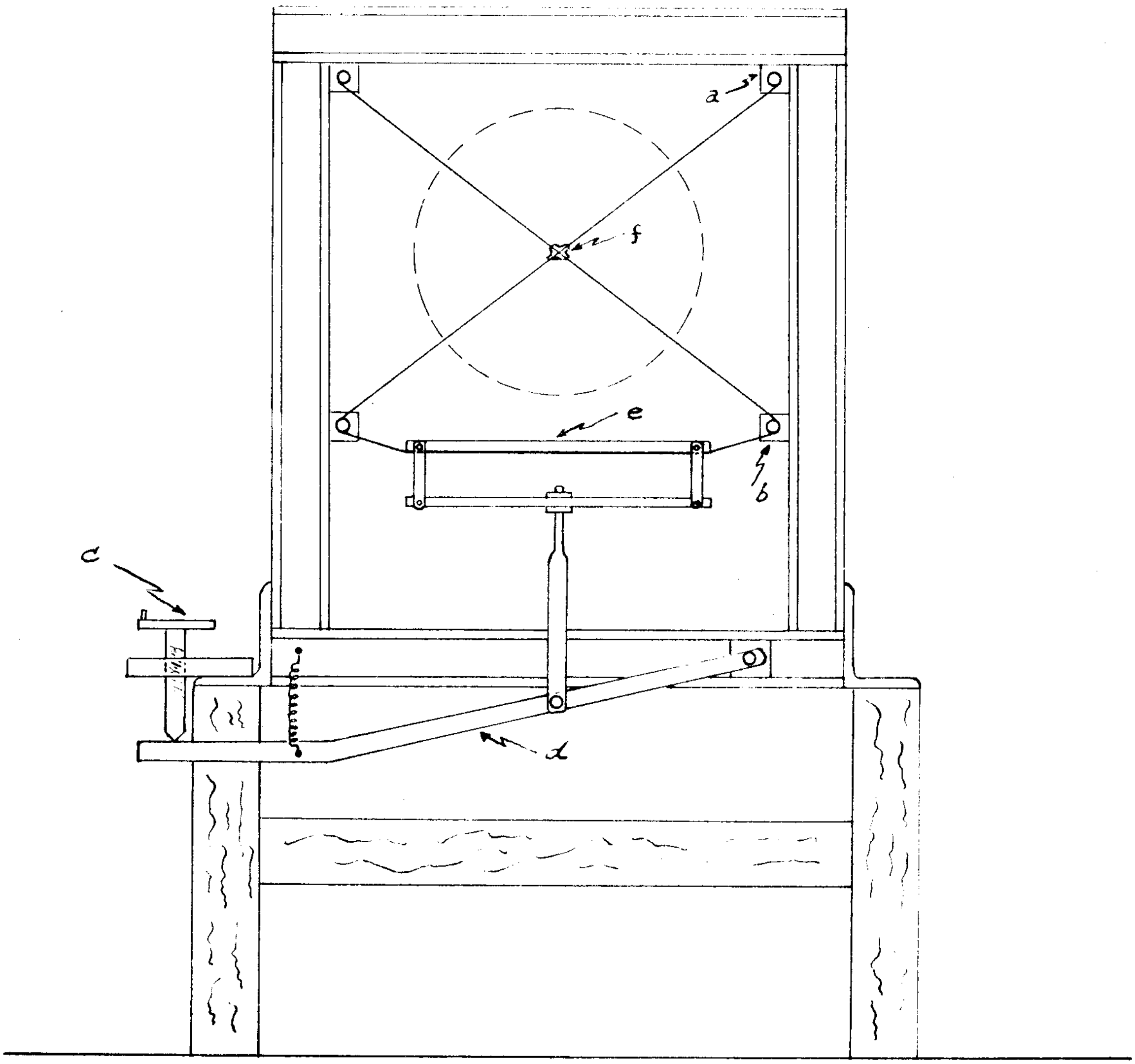

FREQUENCY CHARACTERISTICS OF ANEMOMETER No 2, COMPENSATED.
RELATIVE OUTPUT OF AMPLIFIER + HOT WIRE VS FREQUENCY
OUTPUT FOR 100CPS USED AS BASE
• L=2.36h, Rc*=200ω
o L=2.36h, Rc*=600ω
× L=2.36h, Rc*=1200ω
D.C RESISTANCE OF CHOKE = 100ω
RELATIVE OUTPUT
FREQ - CPS
1.1
1.0
.9
.8
.7
.6
.5
10
50
100
500
1000
5000
10000

FIG-18



SINGLE WIRE X SUSPENSION ON

HOT WIRE VIBRATOR

FIG-19

VIBRATOR CIRCUIT

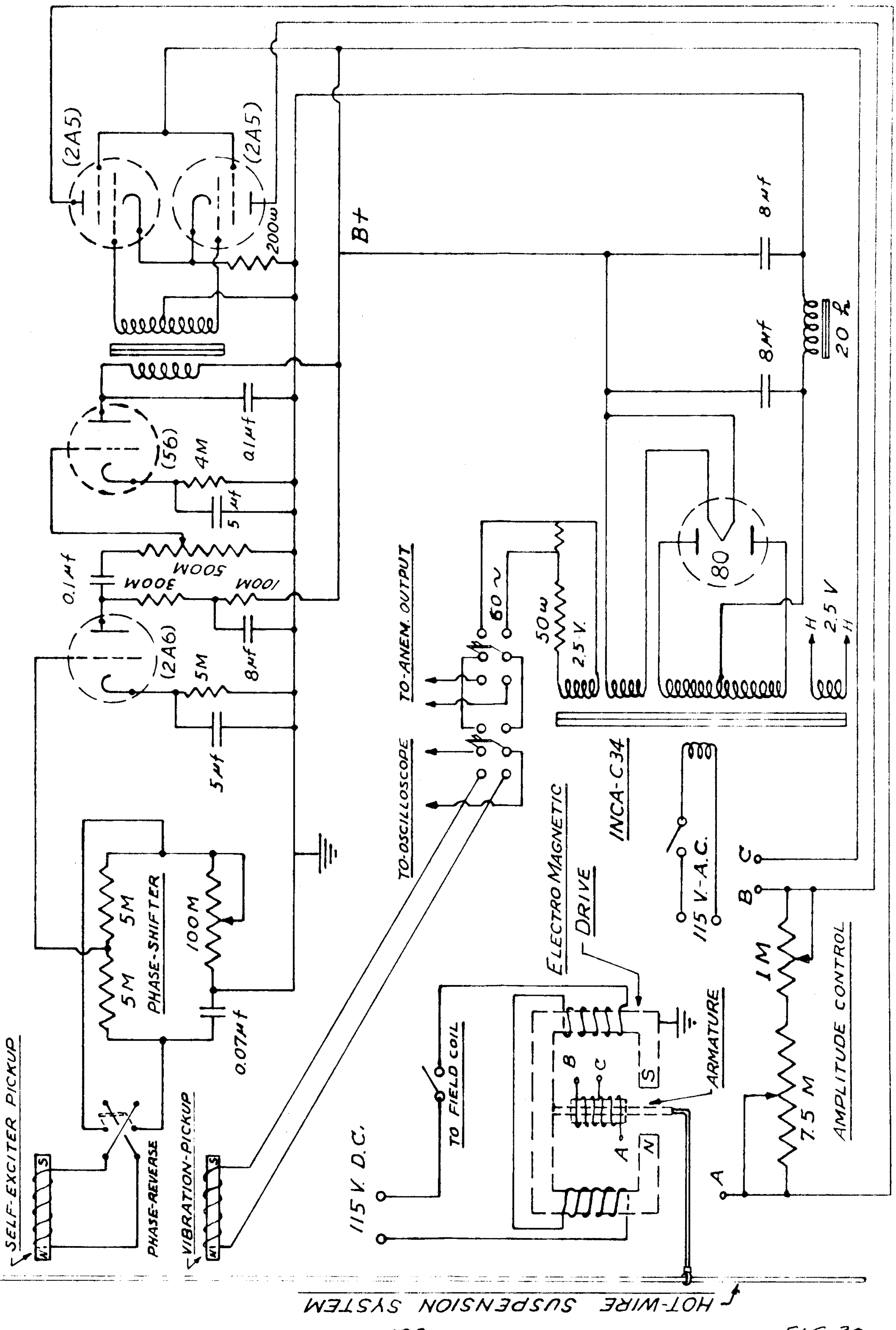
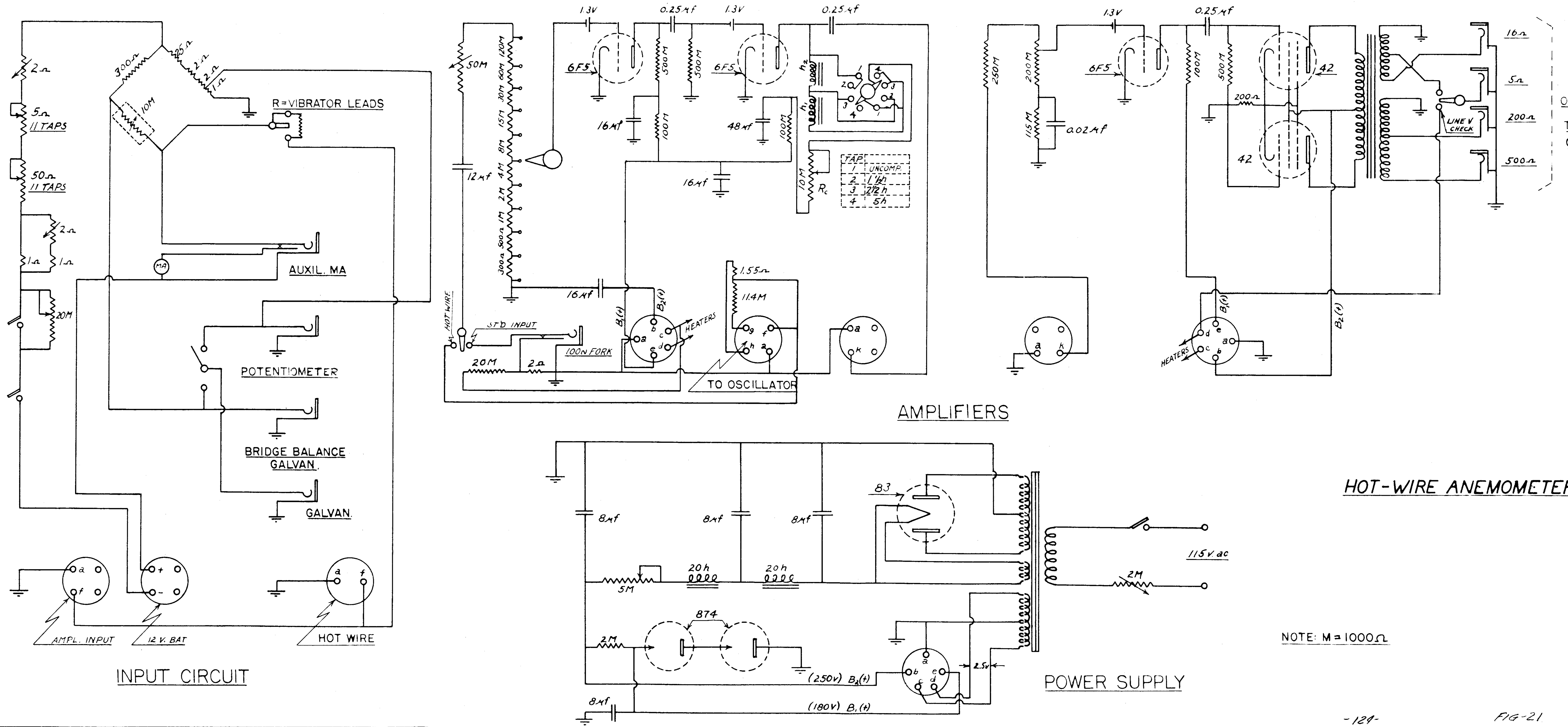


FIG-20



HOT-WIRE ANEMOMETER - #1

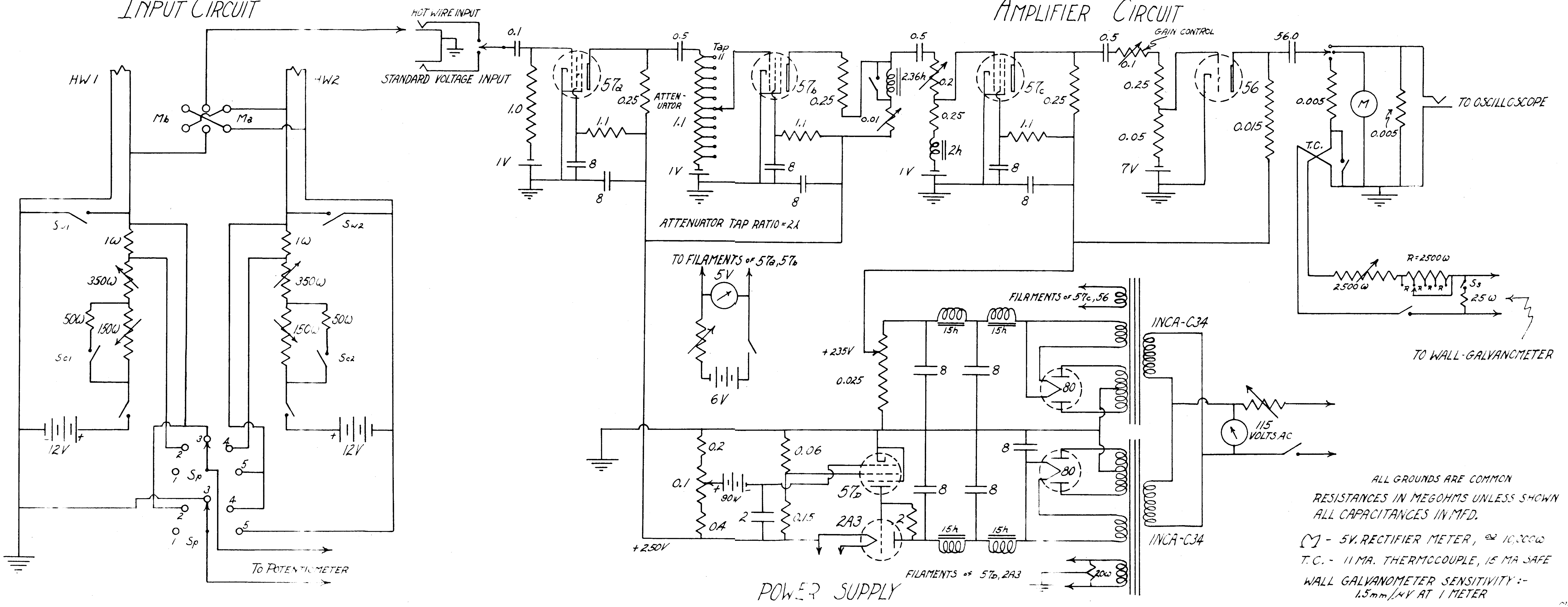
NOTE: M = 1000Ω

FIG-21

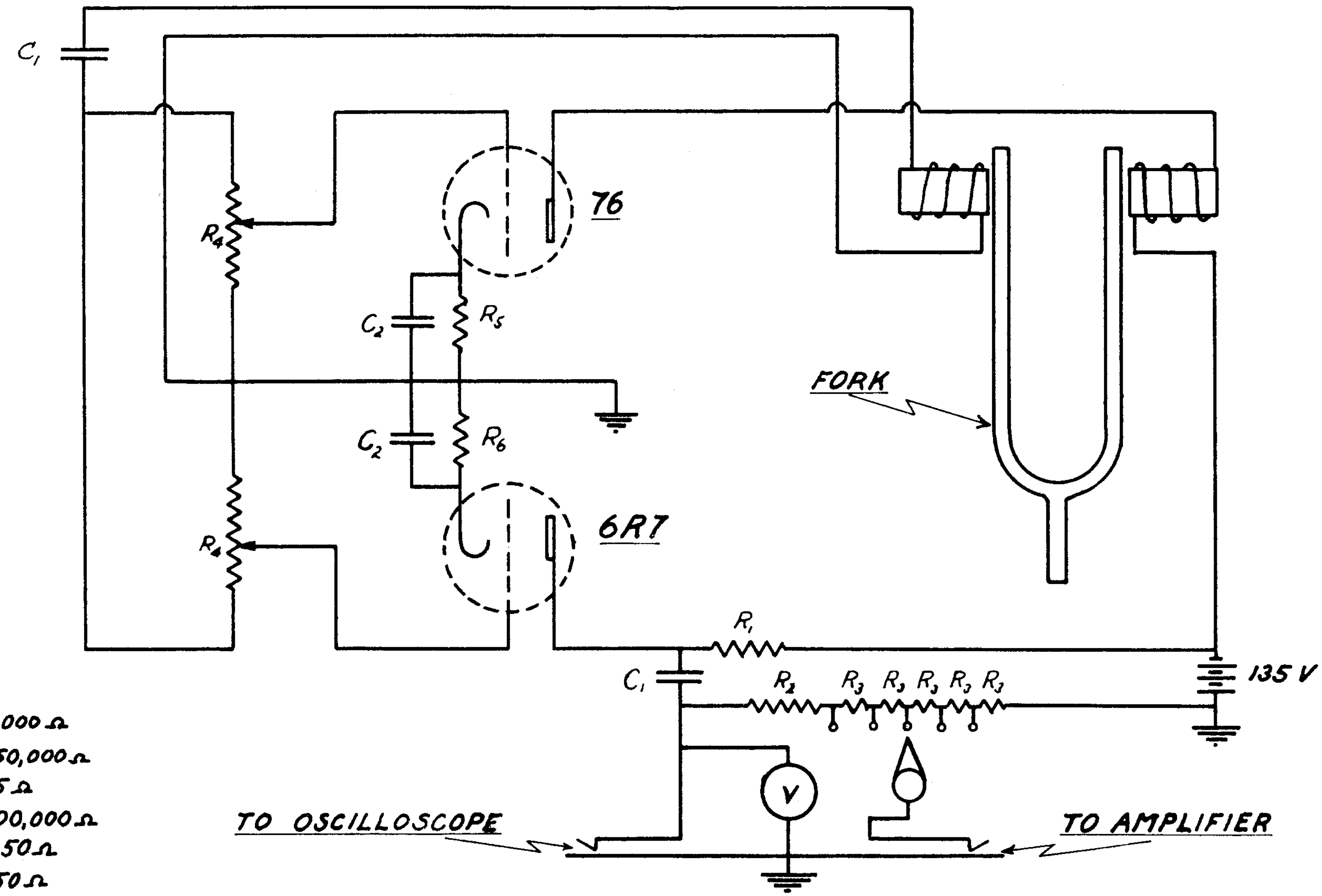
MAY 1938

FIG-22

ANEMOMETER No. 2



MAY 1938



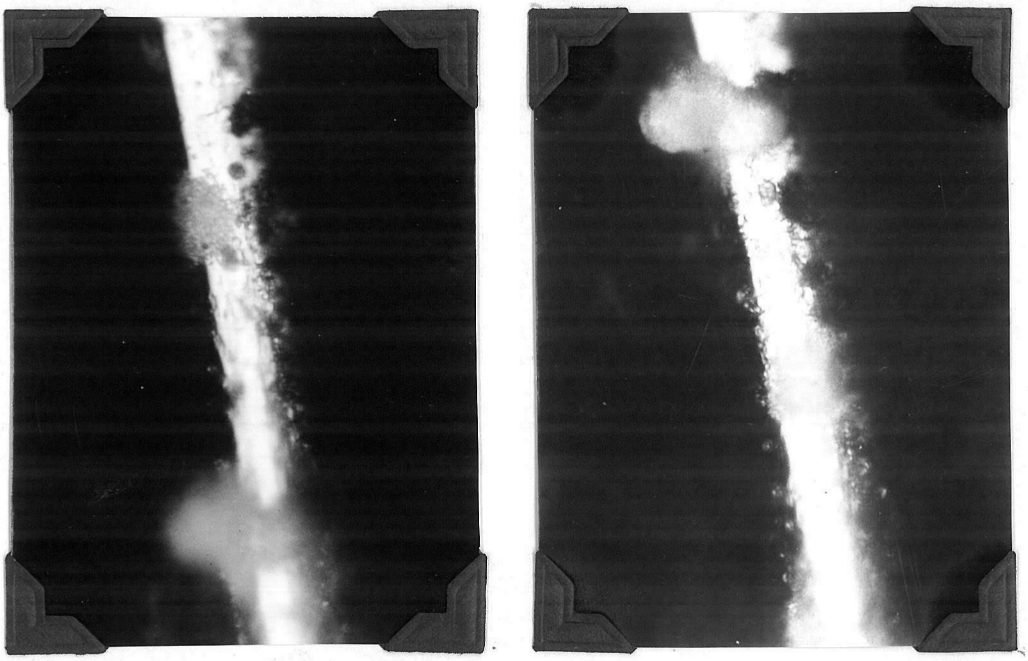
R_1 - 12000 Ω
R_2 - 250,000 Ω
R_3 - 25 Ω
R_4 - 500,000 Ω
R_5 - 1250 Ω
R_6 - 750 Ω
C_1 - 0.1 μf
C_2 - 10. μf

V - RECTIFIER VOLTMETER

TUNING FORK CIRCUIT

FREQUENCY 100 CPS

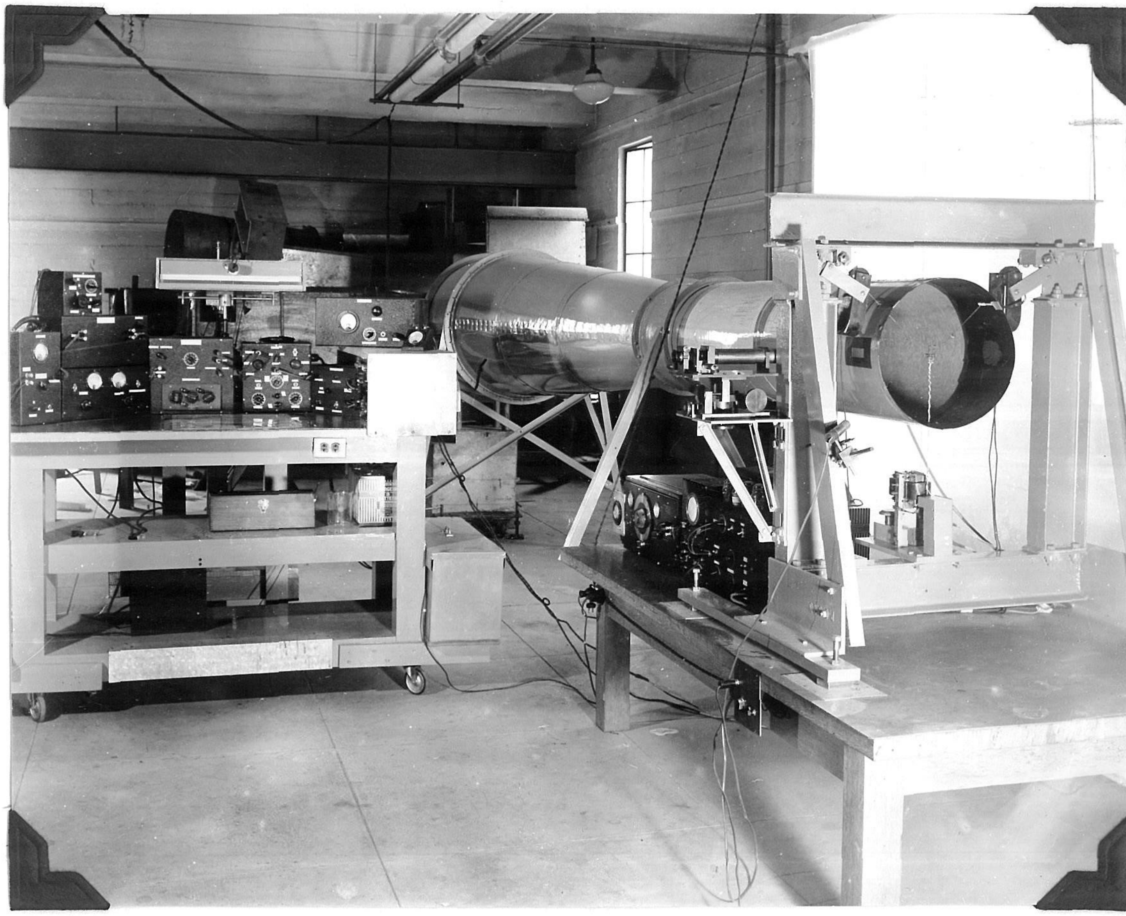
FIG-23



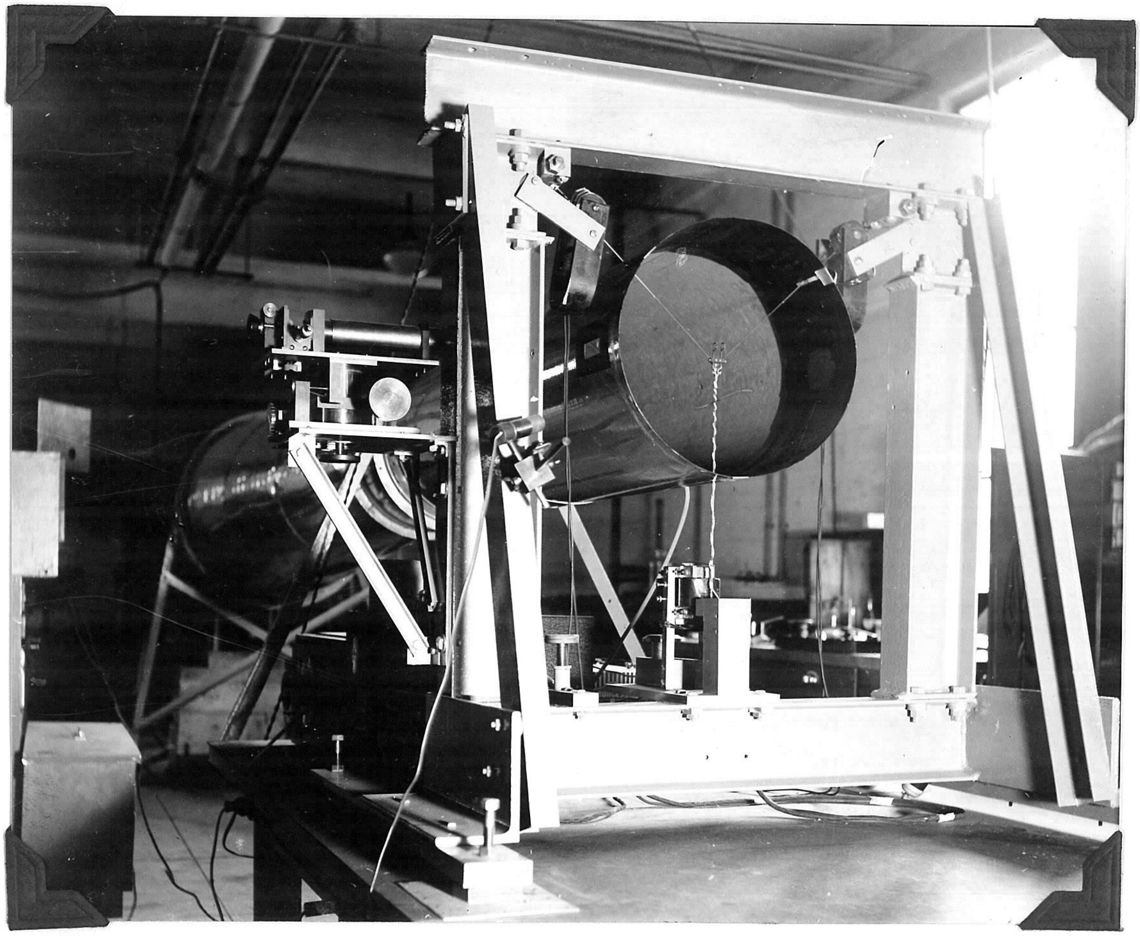
Hot Wire Specimens After 8 Hours Of Service
(Magnification x 250)

Note- The Accumulation of Surface Impurities
From The Airstream

FIG-24

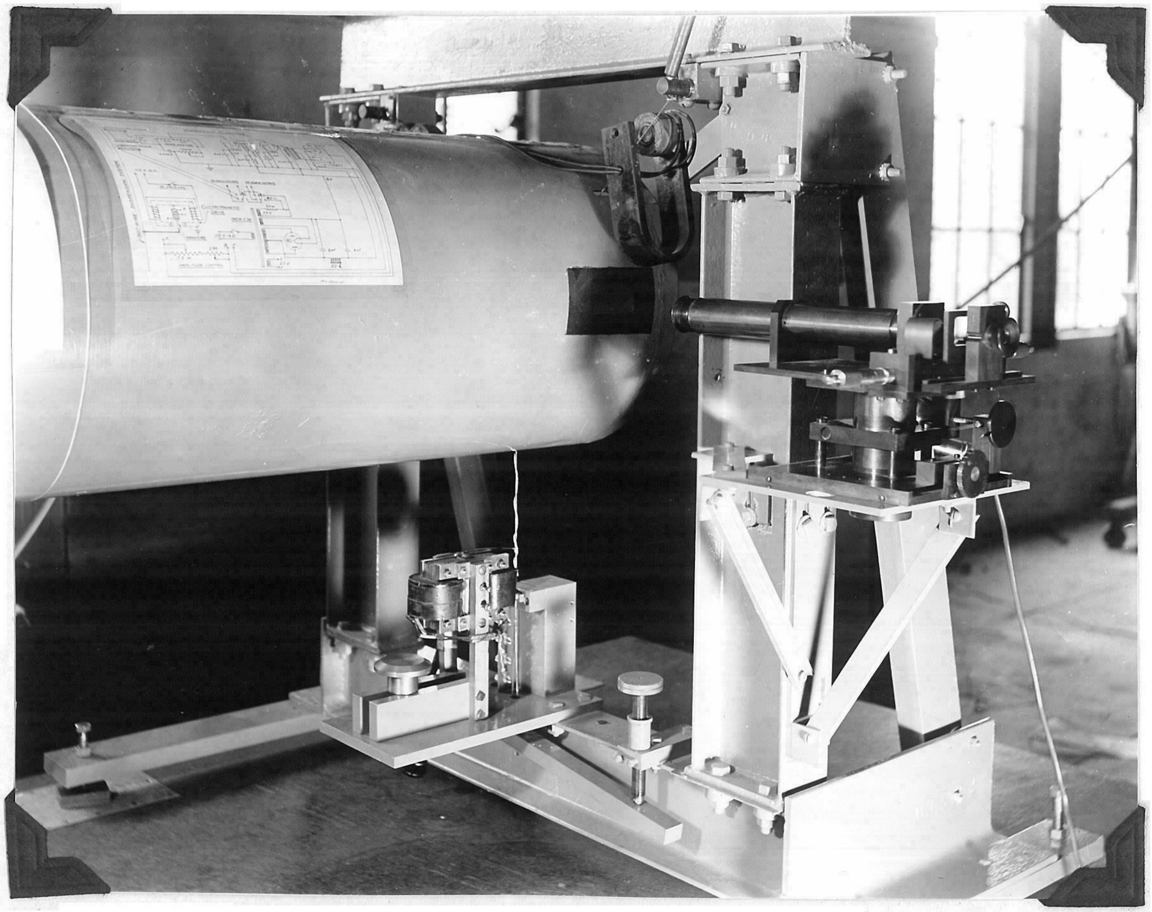


Assembly of Vibrator Equipment
And Anemometer No. 2



Vibrator For Calibrating Hot Wires

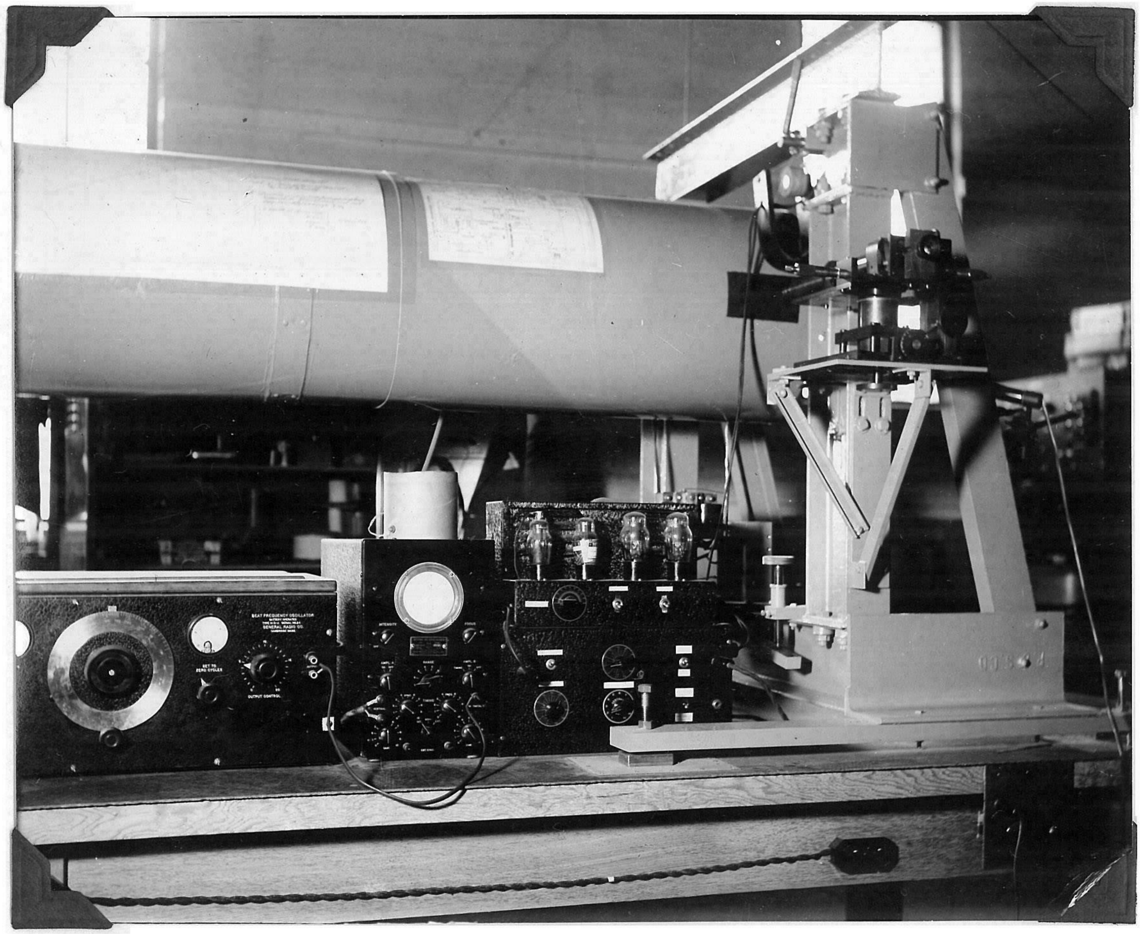
Note- Two "Correlation" Hot Wires Mounted
On Single Wire Y Suspension



Rear View of Vibrator

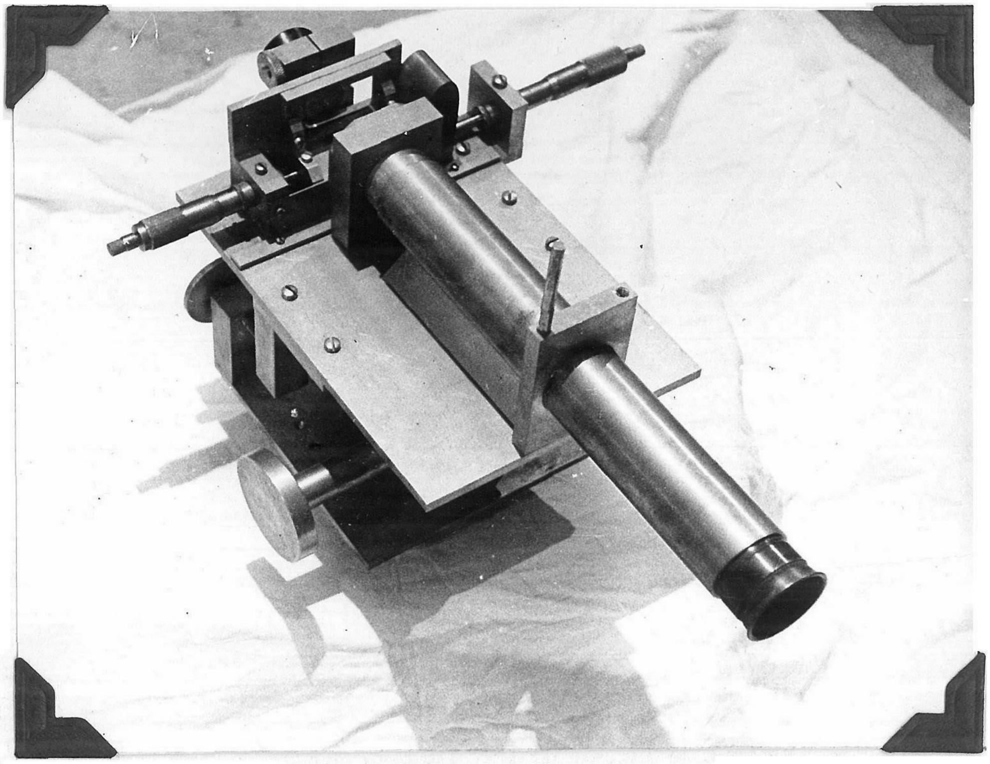
Note- Electromagnetic Drive System
Frequency Adjustment
Self-Excited Pick-up Coil
And Telescopic Micrometer

FIG- 27

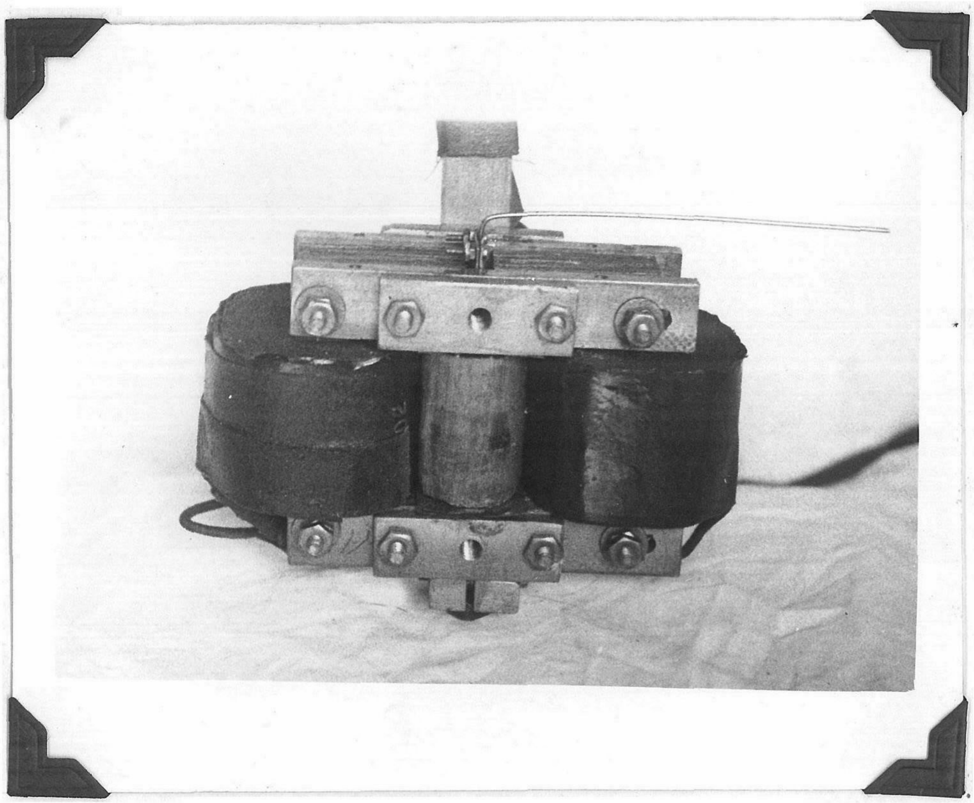


Apparatus For Controlling And Measuring
Amplitude And Frequency of Vibration

Note- Beat Frequency Oscillator, Cathode Ray Oscilloscope,
Amplifier For Adjusting Amplitude of Vibration

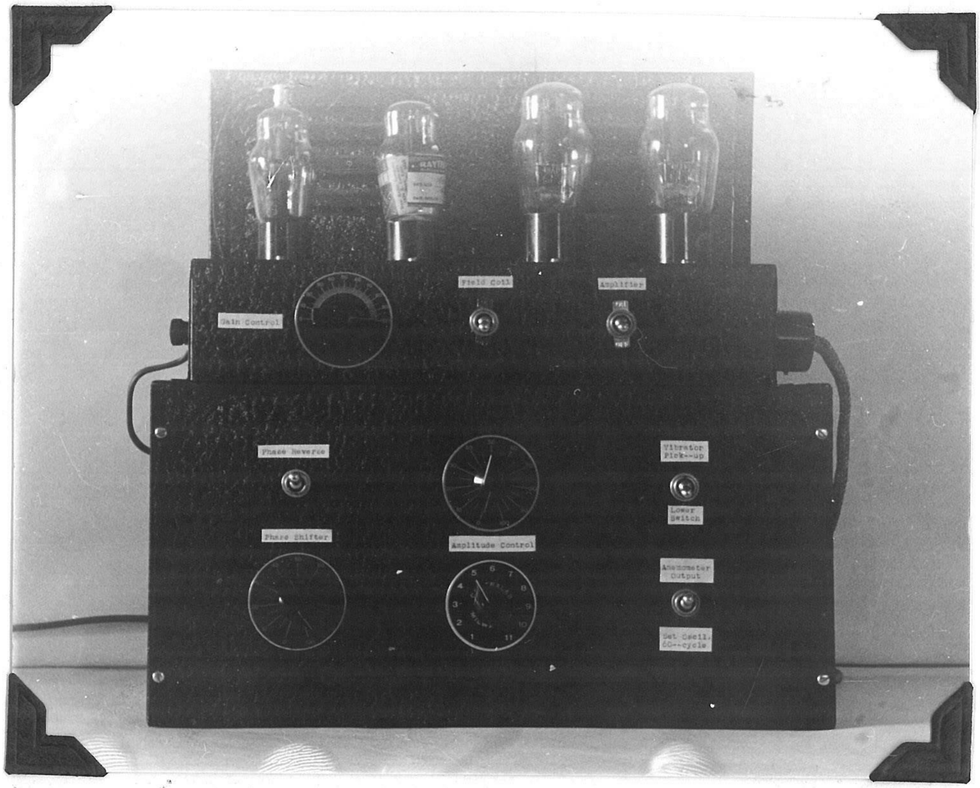


Telescopic Micrometer
For Measuring Amplitudes of Vibration

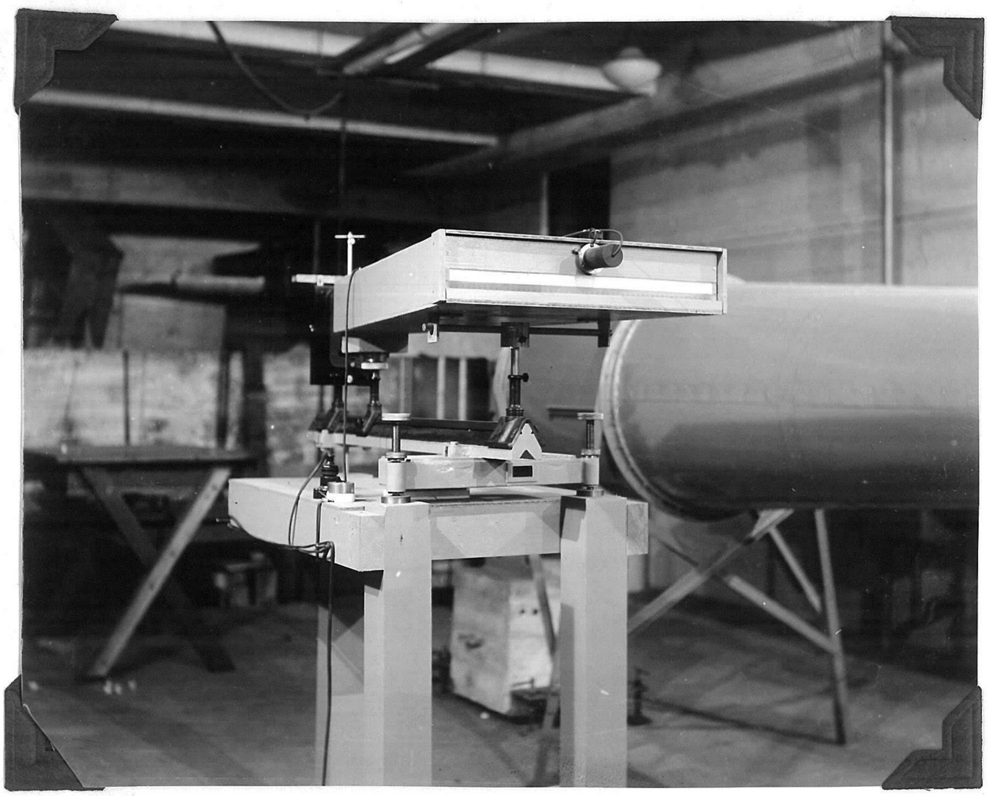


Armature And Field Coils Of
Electromagnetic Drive

FIG-29,30



Vibrator Control Unit



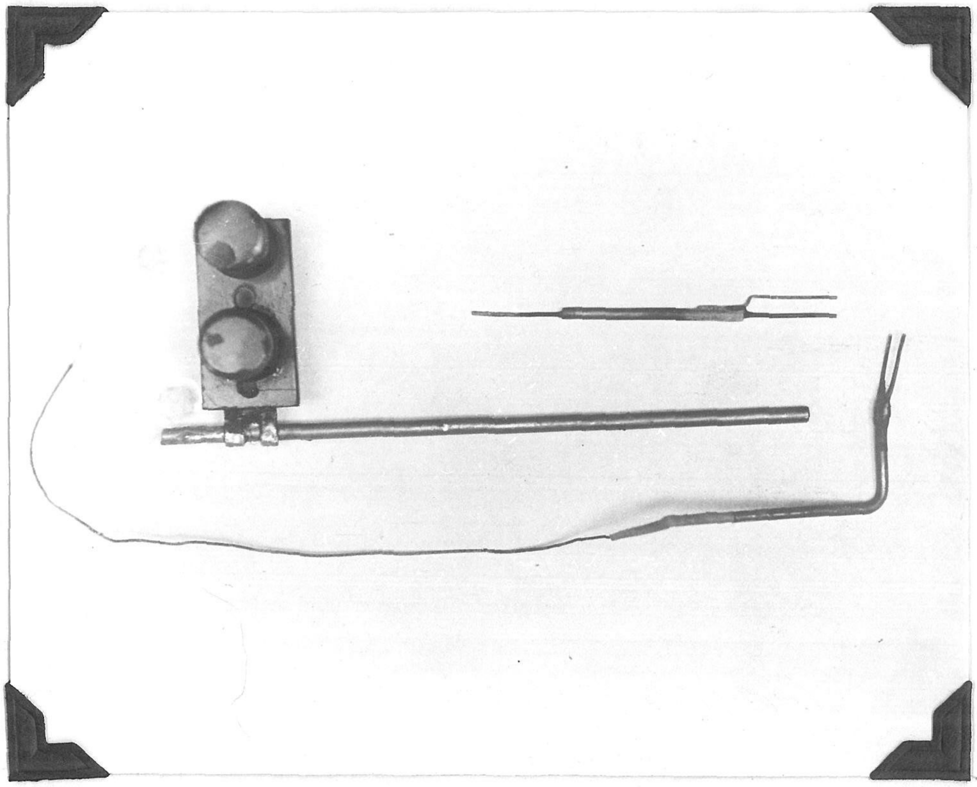
Wall Type Thermogalvanometer

FIG-31, 32

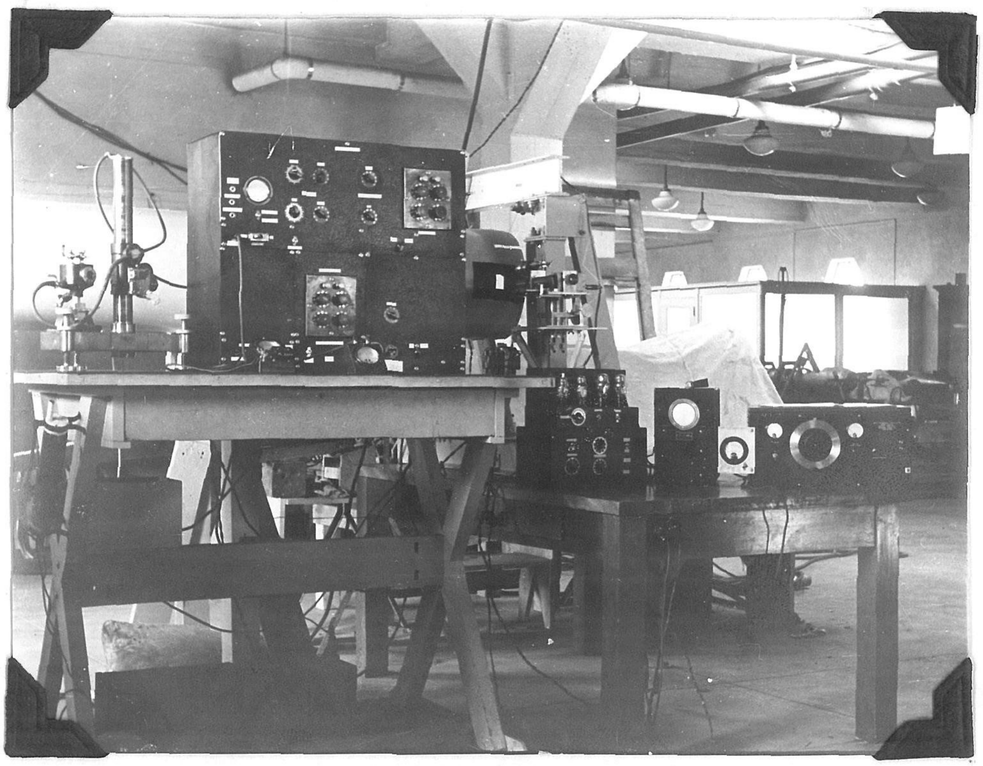
Hot Wire Anemometer No. 2

Note- Wall Type Thermogalvanometer

FIG-33



Typical Hot Wire Holders
Note- Detachable Feature For Vibrator Calibration



Assembly of Vibrator Equipment
and Anemometer No. 1