# INVESTIGATION OF EXTENDED BULB ANGLE SECTIONS UNDER COMPRESSION

Part One: As Euler Columns

Part Two: As Stiffeners attached to sheet

Thesis by

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In partial fulfillment of the requirements for the Degree of Master of Science in Aeronautical Engineering.

California Institute of Technology
Pasadena, California

1937

ACKNOWLEDGMENTS:

The authors wish to thank Dr. von Kármán, Director of the Guggenheim Aeronautical Laboratory at the California Institute of Technology for the opportunity to make this research. They wish to thank Dr. E. E. Sechler under whose direction the research was carried out, for his invaluable assistance during the research and his careful review of the completed manuscript. Acknowledgment is also due Dr. A. L. Klein of the Institute and the major Aircraft Manufacturers particularly the Douglas Aircraft Company, the Northrop Corporation and the Curtiss-Wright Company, for the careful preparation of the specimens tested.

# A Study of the Properties of Various Extruded Sections Commonly Used as Stiffeners in Aircraft Construction.

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## Foreword

This is a report on work performed during the year 1936-7 in the structures laboratory of the California Institute of Technology. The work for this past year has been somewhat exploratory in nature inasmuch as it was necessary to design apparatus and to determine best testing methods. These preliminary problems have, however, largely been solved during this year, and it is felt that the continuation of the research in the coming year should make available to the designer much needed information regarding this very commonly used type of structural element.

## Purpose of the Investigation

Recent investigations have developed methods of calculation the column properties of stiffener sections made up of formed flat sheet, and further research work is being carried out on this problem by a number of laboratories. (References 1 and 2). However, one of the most important types of stiffener section used in modern airplane construction is the extruded shape, and in particular, the bulb angle is used extensively by a large number of manufacturers. For this reason, this research program was started in an effort to obtain design data on extruded sections when used as stiffeners, and the first problem was the investigation of the properties of twelve representative bulb angles.

In order to make a systematic study of a problem such as this, it is necessary to alter the variables involved individually over the range of values which are of interest. This would mean, in the case of the bulb angle, the variation of probably the following more or less independant variables

A = Length of leg containing bulb

B = Length of second leg

$T_1$ = Thickness of leg containing bulb

$T_2$ = Thickness of the second leg

R = Radius of the bulb

However, the above method of attack on the problem would mean an elaborate set of extrusion dies in order that each variable might be changed in sufficient steps in order to obtain the optimum value. It was therefore decided to choose from the lists of extruded bulb angles that were in common use at the beginning of the research, a number of representative sections covering as wide a range of the above variables as was possible. The twelve angles chosen are shown in Fig. 1A and the dimensions of these sections are given in Table I.

The problem was divided into two major fields of study, namely,

Part I--An investigation of the above twelve bulb angles when used alone as pin ended columns under concentric compression load.

Part II--An investigation of the action of the same bulb angles when attached to sheets of different thicknesses.

## Part I

An investigation of twelve typical bulb angles when used as pin ended columns under concentric compression load.

### Description of Specimens

The cross-sections tested are shown full size in Fig. 1A, and the properties of these sections are given in Table I. It should be noted that, since these sections are extruded, their dimensions may vary considerably from those given in the specifications. It was therefore necessary to check all dimensions and to recompute the section properties. From the above table it is seen that the variation from the specifications is so large in some cases that an appreciable affect on the allowable design loads would be expected. It is also noted that the deviation from the specifications is non-conservative in almost as many instances as it is conservative. All specimens were formed of 24ST aluminum alloy.

The lengths to be tested were chosen on the basis of two factors of interest to designers. First, they were chosen so as to cover the range of bulkhead spacings commonly used. Second, limits of lengths chosen were such as to insure covering both the long and short column range. For these reasons, lengths of 22, 16½, 11, and 5½ inches were selected for testing.

### Description of Apparatus

In order to obtain a true hinged end condition of the columns under test, an end fitting was constructed. A half inch ball was sunk into the base plate of this fitting and rested on a circular hardened plate which in turn lay upon the base plates of the compression testing cage. The ends of the bulb angle were clamped into the fittings in which adjustments in two directions were provided, enabling the centroid of the bulb angle to be located directly over the center of the ball.

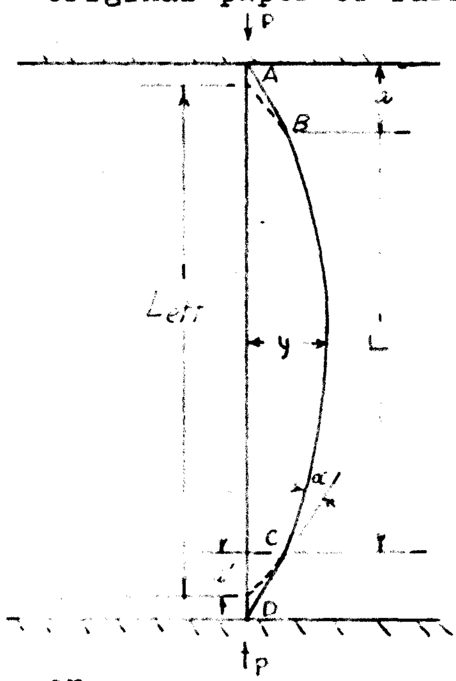
The adjustments in the fittings enabled any eccentricity present in the set-up to be removed. Two dial gauges were used to determine if any eccentricity was present. One gauge was mounted on a bracket attached to the cage, the other was held by a rigid bar mounted on flexible tabs which in turn were fixed between the circular end plates and the ends of the compression cage. The dial gauge plungers rested on the sides of the bulb angle at the midpoint of the column. Since any restraint of the column was most undesirable, it was necessary to remove the plunger main springs of the gauges, having only the hair springs acting. The photographs show clearly the construction of the end fittings and the method of setting-up the specimens.

## Testing Procedure

The bulb angle was mounted in the end fittings and placed in the compression cage. The circular bearing plates were then inserted between the balls and the base plates of the compression cage. A slight load was applied to the column in order to hold it in the machine and then it was placed approximately vertical by means of a level. The dial gauges were then attached and the load increased, a change in the readings of the gauges denoting the presence of initial eccentricity which was removed by the adjustments in the end fittings. When the load could be increased to about a third of the anticipated final load without a change in the dial gauge readings, it was assumed that all initial eccentricity had been removed. The dial gauges were then removed and the load increased steadily until failure. The tests were made on two Riehle Bros. testing machines, the longer columns in a 3,000 pound capacity unit and the shorter columns in a 30,000 pound capacity unit. The type of failure, failing load, and the description of the specimen during the loading and at failure were recorded.

<u>Length Correction</u>

Due to the fact that the end fitting is a rigid structure, a correction on the length of the column must be made. This is done in the following manner, using a method suggested by Karman and given in detail in the original paper of Parr and Beakley:




The moment equation for the beam shown is:

$$d^2y/dx^2 = -Py/EI \qquad (1)$$

the solution of which is

$$y = A\cos\sqrt{P/EI}\cdot x \qquad (2)$$

AB and CD are of infinite rigidity and hence are straight lines, therefore, at $x = L/2$

$$\tan\alpha = -dy/dx \doteq \alpha$$

$$y = a\alpha = -a\,dy/dx$$

or

$$A\cos\sqrt{P/EI}\;L/2 = -a\left[-A\sqrt{P/EI}\;\sin\sqrt{P/EI}\;L/2\right]$$

or

$$\cot\sqrt{P/EI}\;L/2 = a\sqrt{P/EI} \qquad (3)$$

which can be solved for the Euler buckling load of a column for any value of a. For a = 0, we obtain,

$$\cot \sqrt{P/EI}\ \ L/2 = 0$$

or

$$P = \pi^2 EI/L^2 \tag{4}$$

Putting

$$\sqrt{P/EI}\ \ L/2 = Z$$

equation (3) may be rewritten as

$$Z \tan Z = L/2a \tag{5}$$

and

$$\sqrt{P/EI} = 2Z/L$$

or

$$P = 4Z^2 EI/L^2 \tag{6}$$

and, comparing equation (6) with equation (4) it is seen that the effective length is given by

$$L_{eff} = \pi L/2Z \tag{7}$$

therefore:

$$a' = (L_{eff} - L)/2 = L(\pi/2Z - 1)/2$$

or, substituting from equation (5)

$$a' = a(\pi/2 - Z)\tan Z \tag{8}$$

and

$$a/L = 1/2Z \tan Z \tag{9}$$

consequently, if we take different values of Z we can calculate corresponding values of a' and a/L.

| Z | tanZ | a/L | a'/a | $L_{eff}$ |
|---|---|---|---|---|
| $\pi/2$ | $\infty$ | 0 | 1.000 | 24.24 |
| $17\pi/36$ | 11.43 | 0.029 | 0.999 | 18.73 |
| $5\pi/12$ | 3.732 | 0.102 | 0.982 | 13.21 |
| $\pi/3$ | 1.732 | 0.275 | 0.907 | 7.62 |

since

$$L_{eff} = L + 2a(a'/a)$$

Theoretical and Experimental Failure Values

Knowing the column effective length, it is now possible to proceed with a study of the theoretical curves of failure. For this type of section there are three types of failure which are possible. These are:

a. Column failure

b. Local failure

c. Torsional failure

Nearly all of the specimens tested fell into the first class, i.e. column failure. Only two specimens failed locally, and none failed in torsion. We will discuss first the column failure.

In column failure, there are two ranges of the length to radius of gyration ratio to consider. The first is the Euler or long column range and the equation for the critical stress in this range is given by

$$\sigma_c = \frac{C \pi^2 E}{\left(\frac{L_{eff}}{\rho}\right)^2} \qquad (10)$$

where C = 1.0

The second, or short column range is usually considered to be given by one of two equations; either the Johnson parabolic formula or the "Straight Line" formula. These are, respectively

$$\sigma_c = \sigma_y - \frac{\sigma_y^2 (L_{eff}/\rho)^2}{4 \pi^2 E} \qquad (11)$$

where $\sigma_y$ is taken as 43,000 lbs./sq.in., and

$$\sigma_c = 48000 - 400 (L_{eff}/\rho) \qquad (12)$$

Equations 10. 11. and 12 are shown plotted in Fig.2 and the experimental points are also shown on the same figure. It can be seen that the "Straight Line" equation gives a more conservative value for the critical stress in the short column range and also agrees slightly better with the experimental results. For this reason, it is suggested that the "Straight Line" equation be used for this type of column in the short column range. Also from Fig. 2 it can be seen that the short column range starts at an $L/\rho$ ratio of about 78.

Investigations by Howland at C.I.T. have placed the proportional limit of aluminum alloy 24ST at 19000 lbs./sq.in. Our investigations offer an opportunity to check this value, since at this stress the experimental points should separate from the Euler curve. From Fig. 2 we can see that the point of separation is at an $L/\rho$ of 78. Then, using equation 10 we get

$$\sigma_{p.l.} = \frac{\pi^2 E}{(L_{eff}/\rho)^2}$$

$$= \frac{\pi^2 \times 10.5 \times 10^6}{(78)^2}$$

$$= 17050 \text{ lbs./sq. in.}$$

which, considering the fact that the contours of the sections are not extremely accurate, and that the material has been extruded, gives a reasonable check on the value of 19000 lbs./sq.in. obtained on 24ST sheet stock.

The second type of failure, local buckling of an outstanding leg, only occurred in specimens No. 8477 and 8478 which, as can be seen from Fig. 1A have fairly long and thin outstanding members. Using the standard expression for the buckling of a plate under compressive stress (see Reference 3) we have, for the critical buckling value:

$$\sigma_{cr} = k\,\sigma_e$$

where

$$\sigma_e = \frac{\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

Now, for specimen 8477

$$a/b = \frac{7.62}{1.25} = 6.1$$

$$t = \frac{1}{2}\left[3/32 + 1/16\right] = 0.078$$

$$t/b = 0.062$$

$$\sigma_e = \frac{\pi^2 \times 10.5 \times 10^6}{12 \times .93} \times \overline{.062}^2 = 35800$$

If we assume three sides simply supported and the fourth free, we get for k

$$k \doteq 0.5$$

and

$$\sigma_{cr} = 0.5 \times 35800 = 17900 \text{ lbs./sq.in.}$$

If we assume two opposite sides simply supported, the third built in, and the fourth free, we get,

$$k = 1.33$$

and

$$\sigma_{cr} = 1.33 \times 35800 = 47600 \text{ lbs./sq.in.}$$

Now, experimentally we find a $\sigma_{cr}$ of 37400 lbs./sq.in. and therefore we see that we have neither simple support, nor rigid clamping at the side which is supported by the other leg, but as should be expected, something between the two. If we use this value of $\sigma_{cr}$, we may find the experimental value of k for this case as

$$k = 37400/35800 = 1.044$$

In a similar manner, considering specimen 8478, we have:

$$\sigma_e = 35700 \text{ lbs./sq. in.}$$

and

$$\sigma_{cr(exp)} = 34400 \text{ lbs./sq.in.}$$

Solving for k we obtain,

$$k = 34400/35700 = 0.965$$

Giving, for the two cases, a mean value of

$$k_{ave} \doteq 1.0$$

Unfortunately there were only two plate failures in this group of specimens. However, if we use the mean value of k determined above, we can check the other specimens (which failed as columns) to see if the plate failure stress is lower, thus checking the possibility of that stress being critical. In every case, the value of $\sigma_{cr}$ from a plate failure standpoint is found to be higher than the $\sigma_{cr}$ from column failure consideration.

Timoshenko (Ref. 3) gives values of k for a condition when three sides are simply supported and the fourth free, and for a condition when two opposite sides are simply supported, the third side built in, and the fourth free. For a bulb angle section, neither condition describes the actual fixity of the side at the base of the angle because, since it is attached to the other leg, it cannot be considered hinged, nor can it be considered fixed, since the latter would imply complete rigidity which is not the case. Apparently then the condition that describes the support of the side at the base of the angle lies somewhere between these two limits. The average value of k obtained above confirms this assumption.

It is therefore suggested that when bulb angle sections are used alone as columns under compression that the value of k be taken as 1.0. Further work on other sections should be done to more closely establish the exact value of this constant.

Conclusions from Part I

The material with which this part of the research problem was performed was not entirely satisfactory, due to the fact that the investigators had no control over the parameters involved. The bulb angle section, being an extrusion, had to be taken as it could be obtained from the industry. While possibly not warranted, it would be desireable from a research point of view, to have a special series of dies, thus permitting a series of specimens in which one dimension could be varied holding the others constant. This would permit a more systematic study of the effect of changes in the parameters and should enable a prediction of an optimum cross-section.

From the investigation as carried out, the following conclusions may be listed

1) Above a value of $L/\rho = 78$, the Euler curve is followed closely. See Fig. 2.

2) Below this value of $L/\rho$ the straight line formula ($\sigma = 48000 - 400\ L/\rho$) appears to approximate the experimental points more closely than the Johnson parabolic formula.

3) The proportional limit for 24ST extrusions is approximately 17000 lbs./sq.in.

4) When investigating extruded bulb angle sections used as columns for possible local plate failure, since the condition of support at the base of the angle is neither clamped nor simply supported, but an intermediate case, it is suggested that a value of $k = 1.0$ be used in the buckling equation for plate failure under compression.

5) It is seen from Table II that the allowable stresses for these sections is quite low, and that, in general, these sections would be rather inefficient when used as columns without being attached to sheet. The sheet attachment generally prevents the Euler failure of the stiffener in the direction of the least radius of gyration and will therefore tend tò increase the allowables for the combination.

## PART ONE, TABLE I

Comparison of dimensions and areas of extruded bulb angle sections as actually measured and computed, with those taken from the blue prints of the manufacturer.

As measured and computed

| Section (Alcoa No.) | A | B | R | $T_1$ | $T_2$ | Area |
|---|---|---|---|---|---|---|
| 8477 | 1.125 | 1.250 | .125 | .138 | .087 | .2774 |
| 8476 | 1.500 | .687 | .113 | .051 | .055 | .1501 |
| 8478 | 1.125 | 1.000 | .109 | .073 | .061 | .1679 |
| K-10266 | 1.000 | .688 | .095 | .059 | .065 | .1220 |
| 10265 | .875 | .500 | .094 | .052 | .056 | .0931 |
| 10282 | .750 | .500 | .067 | .047 | .042 | .0647 |
| 3046 | 1.500 | .568 | .082 | .052 | .058 | .1252 |
| 5436 | 1.500 | .996 | .156 | .104 | .079 | .2470 |
| 12224 | 1.094 | .625 | .125 | .105 | .108 | .1709 |
| K- 4200 | 1.094 | .624 | .118 | .078 | .084 | .1334 |
| K-766 | .879 | .499 | .095 | .062 | .067 | .0856 |
| 12678 | .507 | .445 | .065 | .044 | .044 | .0402 |

As taken from the blueprints of the manufacturer

| Section (Alcoa No.) | A | B | R | $T_1$ | $T_2$ | Area |
|---|---|---|---|---|---|---|
| 8477 | 1.125 | 1.250 | .125 | .125 | .0625 | .256 |
| 8476 | 1.500 | .687 | .109 | .051 | .051 | .144 |
| 8478 | 1.125 | 1.000 | .109 | .072 | .062 | .168 |
| K-10266 | 1.000 | .687 | .094 | .0625 | .0625 | .122 |
| 10265 | .875 | .500 | .094 | .051 | .051 | .090 |
| 10282 | .750 | .500 | .0625 | .040 | .040 | .057 |
| 3046 | 1.500 | .5625 | .075 | .050 | .050 | .1154 |
| 5436 | 1.500 | 1.000 | .156 | .125 | .094 | .32152 |
| 12224 | 1.094 | .625 | .125 | .1094 | .1094 | .20485 |
| K-4200 | 1.094 | .625 | .1094 | .0781 | .0781 | .15555 |
| K-766 | .875 | .500 | .094 | .0625 | .0625 | .10399 |
| 12678 | .500 | .438 | .0625 | .040 | .040 | .04476 |

# Data - Part One - Table II

| Specimen | $I_x'$ | A | $\rho^2$ | $\rho$ | $L_{eff}$ | $\left(\frac{L_{eff}}{\rho}\right)$ | $\left(\frac{L_{eff}}{\rho}\right)^2$ | $P_{Euler}$ | $P_{actual}$ | $\sigma_{theory}$ | $\sigma_{actual}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10265 | .00113 | .0931 | .01214 | .110 | 24.24 | 220 | 48400 | 199 | 196 | 2120 | 2105 |
| | | | | | 18.73 | 170 | 28900 | 334 | 335 | 3590 | 3595 |
| | | | | | 13.21 | 120 | 14400 | 671 | 615 | 7210 | 6610 |
| | | | | | 7.62 | 69.3 | 4800 | 2020 | 1540 | 21700 | 16550 |
| 10282 | .00087 | .0647 | .01343 | .116 | 24.24 | 209 | 43700 | 153 | 147 | 2365 | 2275 |
| | | | | | 18.73 | 161.5 | 26080 | 256 | 210 | 3960 | 3250 |
| | | | | | 13.21 | 114 | 13000 | 515 | 410 | 7980 | 6340 |
| | | | | | 7.62 | 65.7 | 4310 | 1552 | 1025 | 24000 | 15870 |
| 3046 | .00164 | .1252 | .01310 | .1143 | 24.24 | 212 | 45000 | 289 | 294 | 2310 | 2345 |
| | | | | | 18.73 | 164 | 26900 | 484 | 405 | 3865 | 3240 |
| | | | | | 13.21 | 115.5 | 13350 | 973 | 700 | 7770 | 5600 |
| | | | | | 7.62 | 66.6 | 4440 | 2930 | 1945# | 23400 | 15520# |
| 5436 | .01177 | .2470 | .0476 | .218 | 24.24 | 111.2 | 12380 | 2070 | 2230 | 8390 | 9030 |
| | | | | | 18.73 | 85.9 | 7370 | 3470 | 3495 | 14070 | 14140 |
| | | | | | 13.21 | 60.6 | 3670 | 6980 | 6040 | 28300 | 24450 |
| | | | | | 7.62 | 35 | 1225 | 21000 | 9605 | 85000 | 38900 |
| 12224 | .00348 | .1709 | .0204 | .143 | 24.24 | 169.5 | 28750 | 613 | 820 | 3585 | 4800 |
| | | | | | 18.73 | 131 | 17150 | 1028 | 1180 | 6010 | 6900 |
| | | | | | 13.21 | 92.4 | 8520 | 2060 | 2320 | 12070 | 13590 |
| | | | | | 7.62 | 53.3 | 2840 | 6210 | 5840 | 36350 | 34200 |

# Indicates a column failure of the bulb alone

Parameters are based on actual dimensions, not on those listed by the manufacturer.

# Data - Part One - Table II

| Specimen | $I_x'$ | A | $\rho^2$ | $\rho$ | $L_{eff}$ | $\left(\frac{L_{eff}}{\rho}\right)$ | $\left(\frac{L_{eff}}{\rho}\right)^2$ | $L_{eff}^2$ | $P_{euler}$ | $P_{actual}$ | $\sigma_{theory}$ | $\sigma_{actual}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8477 | .01627 | .2774 | .0586 | .242 | 24.24 | 100.2 | 10040 | 588 | 2865 | 2703 | 10330 | 9775 |
| | | | | | 18.73 | 75.7 | 5730 | 351 | 4800 | 4310 | 17300 | 15520 |
| | | | | | 13.21 | 54.7 | 2992 | 174.6 | 9640 | 7365 | 34750 | 26550 |
| | | | | | 7.62 | 31.5 | 992.3 | 58 | 29050 | 10370* | 104800 | 37400* |
| 8476 | .00333 | .1501 | .0222 | .149 | 24.24 | 162.4 | 26400 | 588 | 587 | 590 | 3910 | 3930 |
| | | | | | 18.73 | 125.7 | 15800 | 351 | 982 | 875 | 6540 | 5830 |
| | | | | | 13.21 | 88.7 | 7860 | 174.6 | 1975 | 1670 | 13170 | 11120 |
| | | | | | 7.62 | 51.1 | 2610 | 58 | 5940 | 4020 | 39550 | 26800 |
| 8478 | .00877 | .1679 | .0521 | .228 | 24.24 | 106.2 | 11290 | 588 | 1543 | 1490 | 9190 | 8870 |
| | | | | | 18.73 | 82.1 | 6740 | 351 | 2580 | 2275 | 15350 | 13540 |
| | | | | | 13.21 | 57.9 | 3355 | 174.6 | 5200 | 3875 | 31000 | 22450 |
| | | | | | 7.62 | 33.4 | 1114 | 58 | 15660 | 5770* | 93200 | 34400* |
| 10266 | .00293 | .1220 | .0240 | .155 | 24.24 | 156.3 | 24430 | 588 | 516 | 469 | 4240 | 3845 |
| | | | | | 18.73 | 120.8 | 14600 | 351 | 864 | 775 | 7080 | 6350 |
| | | | | | 13.21 | 85.2 | 7250 | 174.6 | 1737 | 1545 | 14250 | 12670 |
| | | | | | 7.62 | 49.1 | 2415 | 58 | 5223 | 2650 | 42800 | 21700 |

* Indicates plate failure

Parameters are based on actual dimensions, not on those listed by the manufacturer.

# Data - Part One - Table II

| Specimen | $I'_x$ | A | $\rho^2$ | $\rho$ | $L_{eff}$ | $\left(\frac{L_{eff}}{\rho}\right)$ | $\left(\frac{L_{eff}}{\rho}\right)^2$ | $P_{euler}$ | $P_{actual}$ | $\sigma_{theory}$ | $\sigma_{actual}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4200 | .00326 | .1334 | .0244 | .156 | 24.24 | 155 | 24000 | 575 | 621 | 4305 | 4650 |
| | | | | | 18.73 | 120 | 14400 | 963 | 955 | 7210 | 7160 |
| | | | | | 13.21 | 84.8 | 7200 | 1933 | 1900 | 14500 | 14250 |
| | | | | | 7.62 | 48.9 | 2390 | 5820 | 4280 | 43600 | 32050 |
| 766 | .00116 | .0856 | .0136 | .1165 | 24.24 | 208 | 43250 | 204 | 260 | 2385 | 3040 |
| | | | | | 18.73 | 161 | 25950 | 342 | 385 | 4000 | 4500 |
| | | | | | 13.21 | 113.3 | 12850 | 688 | 705 | 8040 | 8240 |
| | | | | | 7.62 | 65.4 | 4280 | 2007 | 2005 | 23500 | 23450 |
| 12678 | .000436 | .04023 | .0108 | .104 | 24.24 | 233 | 54300 | 77 | 107 | 1923 | 2675 |
| | | | | | 18.73 | 180 | 32400 | 129 | 155 | 3220 | 3880 |
| | | | | | 13.21 | 127 | 16140 | 258 | 305 | 6450 | 7615 |
| | | | | | 7.62 | 73.3 | 5370 | 778 | 710 | 19450 | 17750 |

Parameters are based on actual dimensions, not on those listed by the manufacturer.

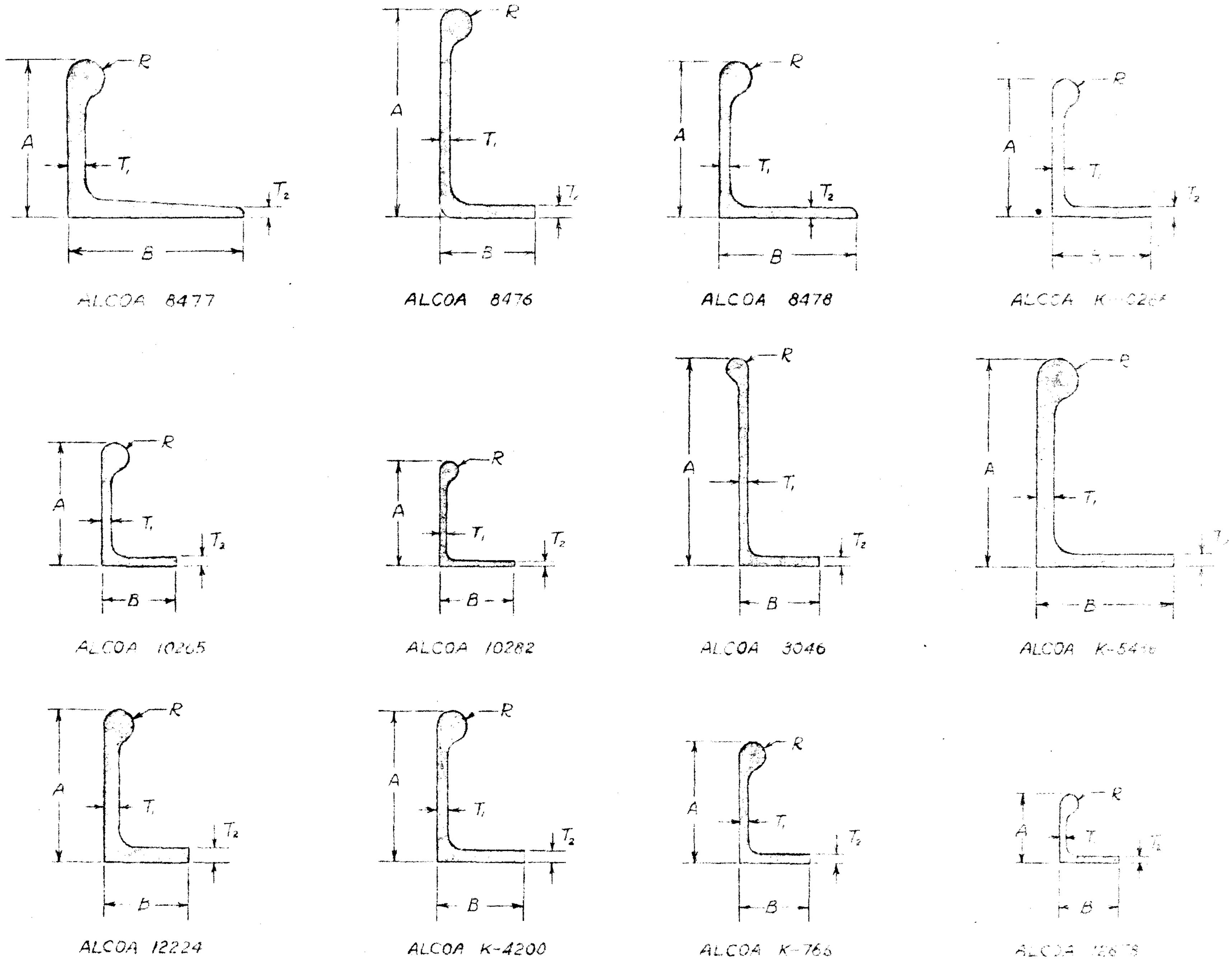

ALCOA 8477
ALCOA 8476
ALCOA 8478
ALCOA
ALCOA 10265
ALCOA 10282
ALCOA 3046
ALCOA K-54
ALCOA 12224
ALCOA K-4200
ALCOA K-766
ALCOA
A
B
R
T1
T2

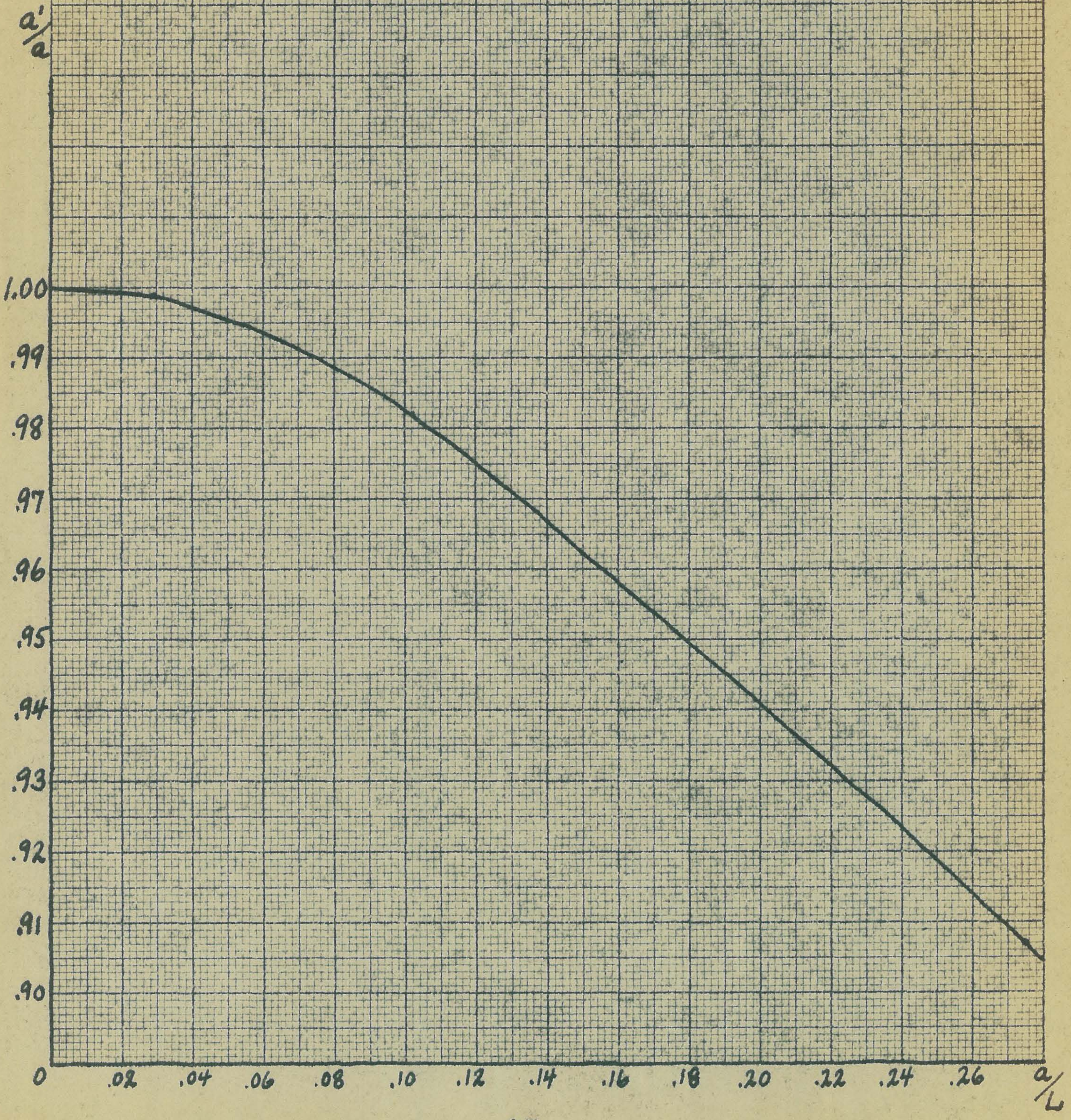
FIG. 1A

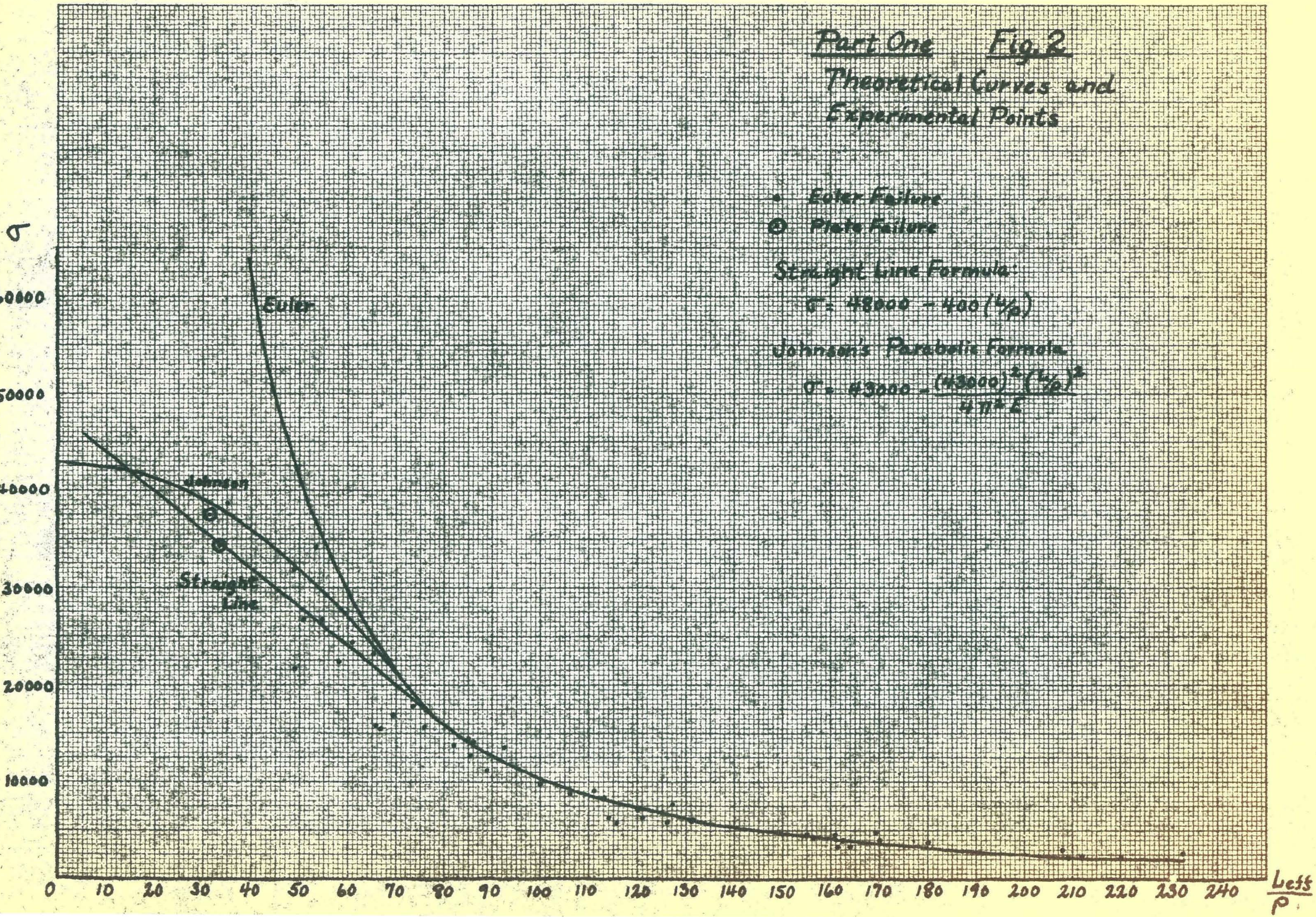
## Part One

### Fig. 1

To find $L_{eff}$.

Curve of $a'/a$ vs $a/L$




Part One Fig. 2
Theoretical Curves and
Experimental Points
• Euler Failure
⊙ Plate Failure
Straight Line Formula:
$\sigma = 48000 - 400(L/\rho)$
Johnson's Parabolic Formula
$\sigma = 43000 - \frac{(43000)^2 (L/\rho)^2}{4\pi^2 E}$
Euler
Johnson
Straight Line
σ
60000
50000
40000
30000
20000
10000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
$\frac{L_{eff}}{\rho}$

Part II--The use of bulb angles as stiffeners for sheet panels under compression.

Statement of Problem

Extrusions of various shapes riveted to sheet to give added stiffness and load carrying ability, are used extensively in aeronautical construction. Bulb angles are one of the most common types of extruded sections used and, therefore, it is of considerable importance that complete knowledge of these sections be made available to the designer. Part I of this investigation has covered the use of bulb angles as columns under compression, and this second part of the investigation will study the effect of attaching these angles to sheet, forming the usual stiffened sheet panel used widely in semimonocoque construction. The sections used will be those studied in Part I, and a variation in sheet thickness will be added to the list of variable to be studied. Inasmuch as the material of the extrusions was 24ST alloy, it was decided to use the same alloy for the sheet stock.

The following report is a summary of a partial investigation of one of the bulb angle sections tested in Part I. The method of testing has been developed, and a number of experimental observations have been obtained. These, however, have not been completely analysed, but an outline of analysis methods and of the future program to be followed is given.

Description of Panels

For the stiffener, bulb angle section Alcoa No. 10282, with a cross-sectional area of 0.0647 sq. in. and a radius of gyration of 0.27 inches, was selected. For all panels, the stiffener spacing was taken as 4 inches and the rivet spacing as 0.75 inches. This rivet spacing was so chosen that premature failure should not take place between the rivets for either the thick or thin sheet panel used. Two thicknesses of sheet were used, namely, 0.020 and 0.040 inches. Panels of two, three, and four stiffeners were made and tested, using the above spacings and thicknesses of sheet. In order to cover the current range of bulkhead spacings and at the same time extend into both the Euler long and short column ranges, the lengths of the panels chosen for test purposes were 3, 5½, 11, 16½, 22, and 27½ inches. In each panel the sheet extended 2 inches beyond each outboard stiffener, that is, the width of the panel with two stiffeners was 8 inches; 3 stiffeners, 12 inches; and 4 stiffeners, 16 inches. Table I gives a complete description of the panels, their areas, and the load carried.

Testing Procedure

In tests of this kind it is imperative that the opposite ends of the panel be parallel in order to have an even distribution of the load. With this in mind, the panels were fabricated with a plus allowance in each length, and then placed in a milling machine and milled to the lengths chosen for test purposes.

Before the panel was put in the testing machine, two extensometers were placed on the side of the sheet opposite the side to which the stiffeners were attached, and near the point of attachment of the end stiffeners of the panel. From the readings of these extensometers, the effective width of the sheet acting with the stiffener can be computed, giving a check on other methods.

The free edges of the sheet were supported by being lightly clamped in a slotted chrome moly tube. The clamping was just enough to allow the slot to keep the edge of the sheet straight, but not tight enough so that the tube would carry load through shear transfer.

Even though the panels were made so that the ends were as nearly parallel as possible, it was found to be necessary, particularly in the wider panels, to shim the ends in order to obtain an even distribution of the load. When the panel was placed in the testing machine a light load was applied and, if necessary, shims were inserted until the load was evenly distributed along the width of the panel. The load was then increased until the panel failed. In addition to the extensometer readings and failure load, the general behavior of each panel was recorded, noting in particular the first appearance of waves in both sheet and stiffener and the passage of waves though the line of rivets. This latter affect was not pronounced or consistent and failed, in many cases, to leave a permanent set in the rivet line.

<u>Experimental</u> <u>Results</u>

Table I and Figs. 1, 2, and 3 show the ultimate failing loads of the three series of panels tested. It is seen from these figures that column failure has not affected the ultimate load of the panels until the longest length had been reached. This was evident from the type of failure observed, inasmuch as failure occurred in every case due to local failure of the outstanding leg of the stiffener. From the faired curves shown in

Figs. 1, 2, and 3, it is possible to separate the load carried by the stiffener plus effective width of sheet acting with it from that carried by the side portions of sheet supported by the slotted tubes. The method of making this separation is shown in the next section. The experimental load can then be compared to the theoretical load for such a combined section acting as an Euler column and from this comparison, a value of the effective end fixity can be determined.

## Theoretical Value of Load Carried by Stiffeners plus Effective Width of Sheet.

From the work of Sechler (Reference 4) the curve of effective width as a function of stress has been replotted in Fig. 4. To cover the transition region, the two end points on the low stress and the failure curve have been taken at values of $\lambda$ corresponding to stresses of 20,000 lbs./sq. in. and 45,000 lbs./sq. in. respectively, and a straight line drawn between these two values. This method has been suggested in the above reference.

The value of the column failure load for the stiffener plus effective sheet will be calculated for two values of end fixity, namely, C = 2.0 and C = 3.0. It is first necessary to calculate the Euler failing stress for the stiffener alone, and, for the case where the stiffener will be acting with the sheet, it is assumed that the failure takes place in a direction perpendicular to the plane of the sheet material. The value of the radius of gyration of stiffener No. 10282 about an axis parallel to the sheet is

$$\rho_o = 0.27$$

from which, the Euler failing stress can be calculated as

$$\sigma_E = \frac{C \pi^2 E}{(L/\rho)^2}$$

which gives, for values of C = 2.0 and 3.0 the following failing stresses for the different lengths of stiffener used;

| L | $L/\rho$ | $\sigma_E$ (C = 2) | $\sigma_E$ (C = 3) |
|---|---|---|---|
| 3.0 | 11.1 | 1,680,000 | 2,520,000 |
| 5.5 | 20.4 | 497,000 | 746,000 |
| 11.0 | 40.8 | 124,000 | 186,000 |
| 16.5 | 61.0 | 55,600 | 83,500 |
| 22.0 | 81.5 | 31,100 | 46,800 |
| 27.5 | 102.0 | 19,900 | 29,800 |

The theoretical load will be calculated for one combination to show the method that has been used throughout. The complete tabulation is shown in Table II. Consider the 27.5 inch length with sheet thickness of 0.020 inches, and an end fixity equal to 2.0. The Euler failing stress as given above is

$$\sigma_E = 19{,}900 \text{ lbs./sq.in.}$$

and

$$\lambda = \sqrt{\frac{10.5 \times 10^6}{19{,}900}} \; \frac{.020}{4} = 0.115$$

giving, from Fig. 4

$$w_e/b = 0.265$$

or, a total width of sheet acting with the stiffener of

$$2\, w_e = 2 \times 0.265 \times 4 = 2.14 \text{ inches}$$

This new column, composed of the stiffener plus 2.14 inches of sheet acting with it, will have a new value of the radius of gyration which is given by the equation

$$(\rho/\rho_o)^2 = \frac{\sigma}{\sigma_o} = \frac{1 + \left[1 + (S/\rho_o)^2\right]\frac{\ell t}{A_o}}{\left(1 + \frac{\ell t}{A_o}\right)^2}$$

where

$A_o$ = area of stiffener alone

$\rho_o$ = rad. of gyr. of stiffener alone

$\rho$ = " " " " " plus attached sheet

$\ell$ = total width of attached sheet

t = thickness of attached sheet

S = distance from neutral axis of stiffener to the center line of the sheet

This curve is plotted in Fig. 5 for convenient use.

For the case under consideration,

$t/A_o = 0.309 \qquad S/\rho_o = 1.15$

$\ell = 2.14$

and, entering Fig. 5 we find,

$$(\rho/\rho_o)^2 = \sigma/\sigma_o = 0.92$$

or, the new Euler failing stress for this combined column is

$$\sigma = 0.92\,\sigma_o = 0.92 \times 19{,}900$$

$$= 18{,}300 \text{ lbs./sq. in.}$$

This new stress will change the effective width slightly, and consequently the values of $\sigma/\sigma_o$ may change, so the process is repeated until the equation converges. In the above case, the first approximation is sufficiantly accurate, and the values of

$$2\,w_e = 2.12 \text{ inches}$$

and

$$\sigma_E = 18{,}300 \text{ lbs./sq. in.}$$

are taken for analysis purposes. It will be noticed that the sheet acting with the stiffener has been reduced from 2.14 to 2.12 inches by the change in stress, but this was insufficient to further reduce the Euler stress of the combination by an appreciable amount. A similar analysis has been carried out for all lengths of stiffener and for the two thicknesses of sheet, and the results are shown in Table II.

From the above it is possible to obtain the theoretical load for each length of stiffener plus sheet, and this is done in Table III, and the results plotted in Figs. 6 and 7. These calculated values must now be checked with the values obtained experimentally.

Experimental Values of Load Carried

In order to obtain the experimental values of the load carried by the combined column of sheet plus effective width, it is necessary to subtract from the total load carried by the panel the load carried by the two side portions supported in the slotted tubes. If we consider the panels of one length and one sheet thickness having two, three, and four stiffeners, we can write the following equations

$$P_e + 2P_s = P_1$$

$$P_e + 3P_s = P_2$$

$$P_e + 4P_s = P_3$$

In which

$P_e$ = the load carried by both side portions of the panel

$P_s$ = the load carried by each of the combined columns made up of stiffener plus effective sheet.

$P_1$, $P_2$, $P_3$, = total loads carried by the panels with two, three, and four stiffeners respectively.

We can solve the above for $P_e$ and $P_s$ since $P_1$, $P_2$, and $P_3$ are known experimentally. Doing this for each length, and for the two thicknesses of sheet, we obtain the following values:

<u>t - 0.040</u>

| | | | | | | |
|---|---|---|---|---|---|---|
| L = | 3.0 | 5.5 | 11.0 | 16.5 | 22.0 | 27.5 |
| $P_s$ = | 4400 | 4350 | 4150 | 3850 | 3350 | 2800 |

<u>t = 0.020</u>

| | | | | | | |
|---|---|---|---|---|---|---|
| L = | 3.0 | 5.5 | 11.0 | .615 | 22.0 | 27.5 |
| $P_s$ = | 2600 | 2550 | 2500 | 2400 | 2150 | 1850 |

The above values are taken from the faired curves in Figs. 1, 2, and 3, and the values shown above are plotted in Figs. 6 and 7.

<u>Discussion of Experimental Results</u>

The behavior of the panel while under test was interesting. The formation of waves, while the same in both thicknesses of sheet, was much easier to see in the 0.020 inch panels. Under a relatively low load, a slight wave was first noticed in the sheet between stiffeners. As the load was increased, the waves in the sheet became deeper, and extended closer to the stiffeners, while the outstanding leg of the stiffeners went into a wave form. Near failure, the waves in the sheet passed through the rivet line of the stiffeners and the waves in the outstanding leg became pronounced. In every panel tested, failure resulted from a plate failure of the outstanding leg of the stiffener. In the 27.5 inch panels, a tendency was noted for the panel to fail as an Euler column, however, the critical condition was still a plate failure of the outstanding stiffener leg.

It may be noted, that in Figs. 1, 2, and 3 the value of the load carried by the 11 inch panel falls below the faired curve in every case. This phenomenon has not as yet been explained, and it is suggested that several more panels of this length be tested in order to more closely check these points.

From the curves of total load vs. length for the columns made up of stiffener plus effective width of sheet (Figs. 6 and 7), it can be seen that an end fixity of 2.0 would appear to be indicated. It is therefore felt that this method of flat end testing, at least for this stiffener, and over the range of thicknesses used, leads to an end fixity of 2.0.

The check of experimental and theoretical values shown in Figs. 6 and 7 also confirms the observation made during the test that failure of the panel was not as a column but was due to local buckling, in this case, buckling of the outstanding leg of the angle. The curves indicate that only at the 27½ inch length should one expect to find any indication of column failure, and this was borne out by all of the experimental observations.

<u>Conclusions</u>

It can be concluded from the above consideration of the test work that, for panels made up of flat sheet attached tobulb angle stiffener No. 10282 and tested flat ended:

1)Failure will take place by local buckling of the outstanding leg of the angle for all lengths shorter than approximately 28 inches.

2)Failure will take place as an Euler column for lengths longer than 28 inches, using the Euler column curve with an end fixity of 2.0.

3)For any other end fixity, curves for the proper Euler failure load can be drawn in on Figs. 6 and 7 and the experimental curve will fair into them in the usual manner.

4)The scatter of some of the experimental points should be eliminated by further test work.

5)The method of calculating the point of departure from the Euler curves for various end fixities should be determined.

<u>Future Work to be Done on This Project</u>

The following is a list of problems to be carried out to complete the studies of extruded sections. As many as possible will be attempted during the coming year.

1)Check several points on the experimental curves just discussed to eliminate scatter.

2)Derive a method for determining the point where the local buckling of the outstanding leg causes a departure from the Euler column curves. This should be done for end fixities of 1, 2, 3, and 4. Work has already started on this problem.

3)Derive an equation of load vs. length to use in the local buckling range. Preliminary work has been started on this problem.

4)Carry out a similar program for the other bulb angles tested in Part I.

5)From the results obtained on the 12 bulb angles under consideration, attempt to derive some general method of analysis which will hold for all bulb angles.

6)Repeat the program for other types of extrusions such as Z sections, T sections, etc.

7)If possible, obtain the cooperation of the Alum. Co. of America towards making some type of adjustable extrusion die which would make possible independant changes in each variable, thus leading to a determination of some form of optimum section. (Contact has already been made with Alcoa and they seem willing to give at least some aid in investigating this last problem.)

## References

1--F. J. Bridget, C. C. Jerome, and A. B. Vosseller--Some New Experiments on Buckling of Thin Wall Construction--A.S.M.E., APM 56-6, August, 1934.

2--W. S. Parr and W. M. Beakley--An investigation of Duralumin Channel Section Struts Under Compression--Journ. of Aero. Sci., September, 1935.

3--S. Timoshenko--Strength of Materials, Vol. II, pg 606.

4--E. E. Sechler--Stress Distribution in Stiffened Panels Under Compression--Journ. of Aero. Sci., June, 1937.

## Part II--Table I

The following tabulation describes the panels which were tested:

| No. of stiff. | Sheet thick. | Stiff spac. | Panel len. | Panel width | Stiff. area | Sheet area | Total area | Total load | Average stress |
|---|---|---|---|---|---|---|---|---|---|
| 2 | .02 | 4 | 3 | 8 | .1294 | .16 | .2894 | 6250 | 21600 |
| 2 | .02 | 4 | 5½ | 8 | .1294 | .16 | .2894 | 6550 | 22640 |
| 2 | .02 | 4 | 11 | 8 | .1294 | .16 | .2894 | 5310 | 18350 |
| 2 | .02 | 4 | 16½ | 8 | .1294 | .16 | .2894 | 6170 | 21300 |
| 2 | .02 | 4 | 22 | 8 | .1294 | .16 | .2894 | 6160 | 21300 |
| 2 | .02 | 4 | 27½ | 8 | .1294 | .16 | .2894 | 5040 | 17400 |
| 2 | .04 | 4 | 3 | 8 | .1294 | .32 | .4494 | 9460 | 21050 |
| 2 | .04 | 4 | 5½ | 8 | .1294 | .32 | .4494 | 11080 | 24700 |
| 2 | .04 | 4 | 11 | 8 | .1294 | .32 | .4494 | 9150 | 20360 |
| 2 | .04 | 4 | 16½ | 8 | .1294 | .32 | .4494 | 10080 | 22470 |
| 2 | .04 | 4 | 22 | 8 | .1294 | .32 | .4494 | 11150 | 24800 |
| 2 | .04 | 4 | 27½ | 8 | .1294 | .32 | .4494 | 9830 | 21900 |
| 3 | .02 | 4 | 3 | 12 | .1941 | .24 | .4341 | 7860 | 18100 |
| 3 | .02 | 4 | 5½ | 12 | .1941 | .24 | .4341 | 9390 | 21630 |
| 3 | .02 | 4 | 11 | 12 | .1941 | .24 | .4341 | 7675 | 17680 |
| 3 | .02 | 4 | 16½ | 12 | .1941 | .24 | .4341 | 8915 | 20530 |
| 3 | .02 | 4 | 22 | 12 | .1941 | .24 | .4341 | 7602 | 17500 |
| 3 | .02 | 4 | 27½ | 12 | .1941 | .24 | .4341 | 6650 | 15320 |
| 3 | .04 | 4 | 3 | 12 | .1941 | .48 | .6741 | 16170 | 23970 |
| 3 | .04 | 4 | 5½ | 12 | .1941 | .48 | .6741 | 12290 | 18230 |
| 3 | .04 | 4 | 11 | 12 | .1941 | .48 | .6741 | 12870 | 19090 |
| 3 | .04 | 4 | 16½ | 12 | .1941 | .48 | .6741 | 14010 | 20780 |
| 3 | .04 | 4 | 22 | 12 | .1941 | .48 | .6741 | 14938 | 22160 |
| 3 | .04 | 4 | 27½ | 12 | .1941 | .48 | .6741 | 12240 | 18150 |
| 4 | .02 | 4 | 3 | 16 | .2588 | .32 | .5788 | 11855 | 20500 |
| 4 | .02 | 4 | 5½ | 16 | .2588 | .32 | .5788 | 11450 | 19780 |
| 4 | .02 | 4 | 11 | 16 | .2588 | .32 | .5788 | 8720 | 15060 |
| 4 | .02 | 4 | 16½ | 16 | .2588 | .32 | .5788 | 11390 | 19670 |
| 4 | .02 | 4 | 22 | 16 | .2588 | .32 | .5788 | 9985 | 17260 |
| 4 | .02 | 4 | 27½ | 16 | .2588 | .32 | .5788 | 8810 | 15220 |
| 4 | .04 | 4 | 3 | 16 | .2588 | .64 | .8988 | 18200 | 20250 |
| 4 | .04 | 4 | 5½ | 16 | .2588 | .64 | .8988 | 20230 | 22500 |
| 4 | .04 | 4 | 11 | 16 | .2588 | .64 | .6988 | 17120 | 19050 |
| 4 | .04 | 4 | 16½ | 16 | .2588 | .64 | .8988 | 18590 | 20670 |
| 4 | .04 | 4 | 22 | 16 | .2588 | .64 | .8988 | 17938 | 19960 |
| 4 | .04 | 4 | 27½ | 16 | .2588 | .64 | .8988 | 14825 | 16500 |

# Table II, Part Two

| Length | k = 2 | t = 0.020 | k = 3 | t = 0.020 | k = 2 | t = 0.040 | k = 3 | t = 0.040 |
|---|---|---|---|---|---|---|---|---|
| | $\sigma$ | $2w_e$ | $\sigma$ | $2w_e$ | $\sigma$ | $2w_e$ | $\sigma$ | $2w_e$ |
| 27.5 | 18300 | 2.12 | 29800 | 1.24 | 17300 | 2.42 | 25600 | 1.84 |
| 22.0 | 31100 | 1.18 | 46800 | 0.496 | 26400 | 1.80 | 46800 | 0.88 |
| 16.5 | 55600 | 0.473 | 83500 | 0.384 | 55600 | 0.84 | 83500 | 0.704 |
| 11.0 | 124,000 | 0.32 | 186,400 | 0.28 | 126,400 | 0.60 | 192,000 | 0.498 |
| 5.5 | 497,000 | 0.176 | 746,000 | 0.144 | 511,000 | 0.32 | 769,000 | 0.264 |
| 3.0 | 1,680,000 | 0.08 | 2,520,000 | 0.064 | 1,715,000 | 0.178 | 2,570,000 | 0.144 |

The high stresses corresponding to the shorter lengths have no practical meaning, being points on the steep portion of the Euler curve.

Part II--Table III

| L | $\sigma$ | $2w_e$ | $A_{sh}$ | $A_{st}$ | $A_{tot.}$ | P | |
|---|---|---|---|---|---|---|---|
| 3.0 | 2,570,000 | 0.144 | 0.0058 | 0.0647 | 0.0705 | - | |
| 5.5 | 769,000 | 0.264 | 0.0155 | " | 0.0802 | - | |
| 11.0 | 192,000 | 0.498 | 0.0199 | " | 0.0846 | 16,240 | t = 0.040 |
| 16.5 | 83,500 | 0.704 | 0.0282 | " | 0.0929 | 7,760 | C = 3.0 |
| 22.0 | 46,800 | 0.880 | 0.0352 | " | 0.0999 | 4,670 | |
| 27.5 | 25,600 | 1.840 | 0.0736 | " | 0.1383 | 3,540 | |
| 3.0 | 1,715,000 | 0.178 | 0.0071 | 0.0647 | 0.0718 | - | |
| 5.5 | 511,000 | 0.320 | 0.0128 | " | 0.0775 | - | |
| 11.0 | 126,400 | 0.600 | 0.0240 | " | 0.0887 | 11,190 | t = 0.040 |
| 16.5 | 55,600 | 0.840 | 0.0336 | " | 0.0983 | 5,460 | C = 2.0 |
| 22.0 | 26,400 | 1.80 | 0.0720 | " | 0.1367 | 3,610 | |
| 27.5 | 17,300 | 2.42 | 0.0968 | " | 0.1615 | 2,790 | |
| 3.0 | 2,520,000 | 0.064 | 0.0013 | 0.0647 | 0.0660 | - | |
| 5.5 | 746,000 | 0.144 | 0.0029 | " | 0.0676 | - | |
| 11.0 | 186,400 | 0.280 | 0.0056 | " | 0.0703 | 13,100 | t = 0.020 |
| 16.5 | 83,500 | 0.384 | 0.0077 | " | 0.0724 | 6,040 | C = 3.0 |
| 22.0 | 46,800 | 0.496 | 0.0099 | " | 0.0746 | 3,490 | |
| 27.5 | 29,800 | 1.240 | 0.0248 | " | 0.0895 | 2,670 | |
| 3.0 | 1,680,000 | 0.080 | 0.0016 | 0.0647 | 0.0663 | - | |
| 5.5 | 497,000 | 0.176 | 0.0035 | " | 0.0682 | - | t = 0.020 |
| 11.0 | 124,000 | 0.320 | 0.0064 | " | 0.0711 | 8,810 | C = 2.0 |
| 16.5 | 55,600 | 0.473 | 0.0095 | " | 0.0742 | 4,130 | |
| 22.0 | 31,100 | 1.180 | 0.0236 | " | 0.0883 | 2,750 | |
| 27.5 | 18,300 | 2.120 | 0.0424 | " | 0.1071 | 1,960 | |

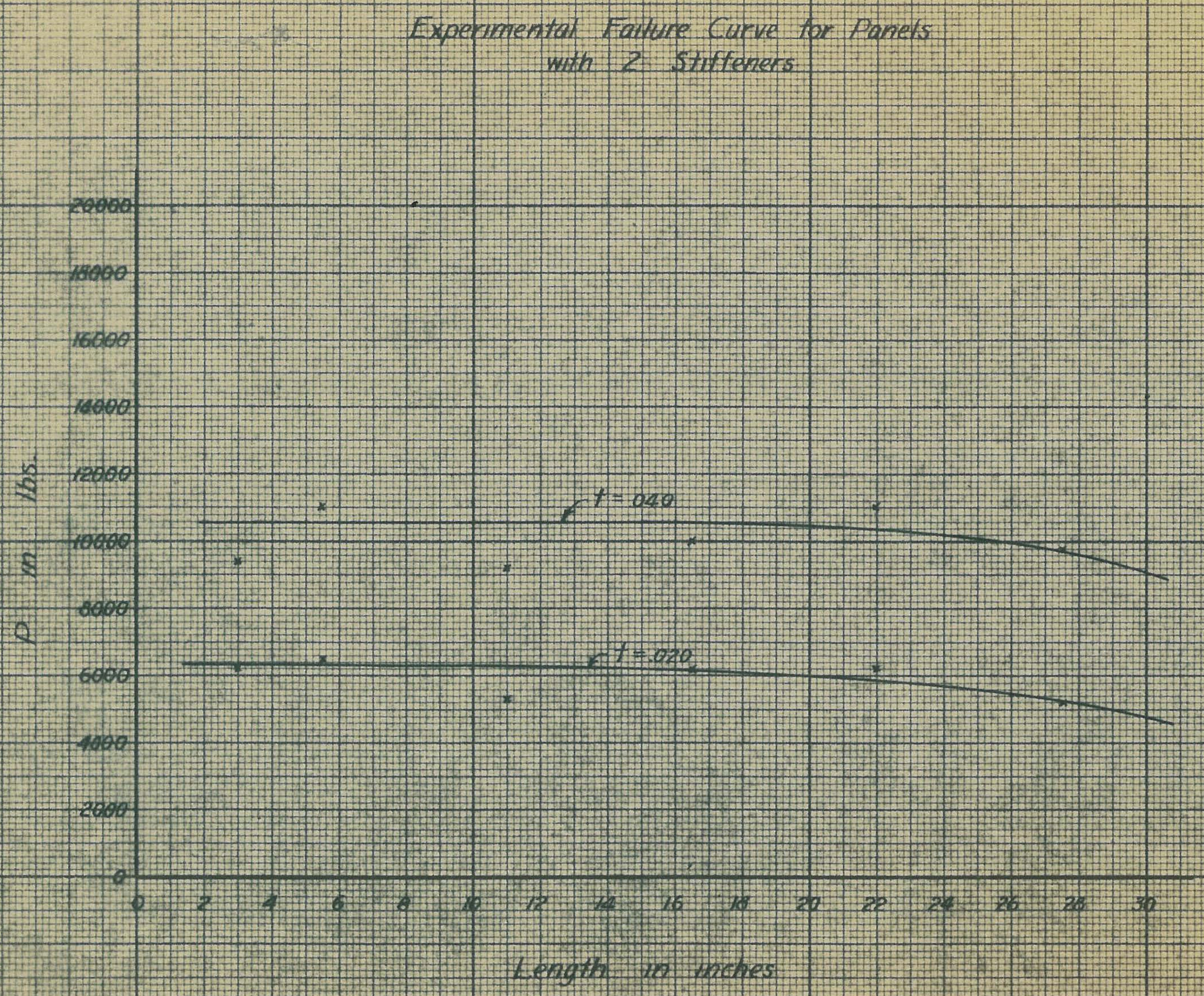

Experimental Failure Curve for Panels
with 2 Stiffeners
P in lbs.
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
t = .040
t = .020
Length in inches

Fig. 1

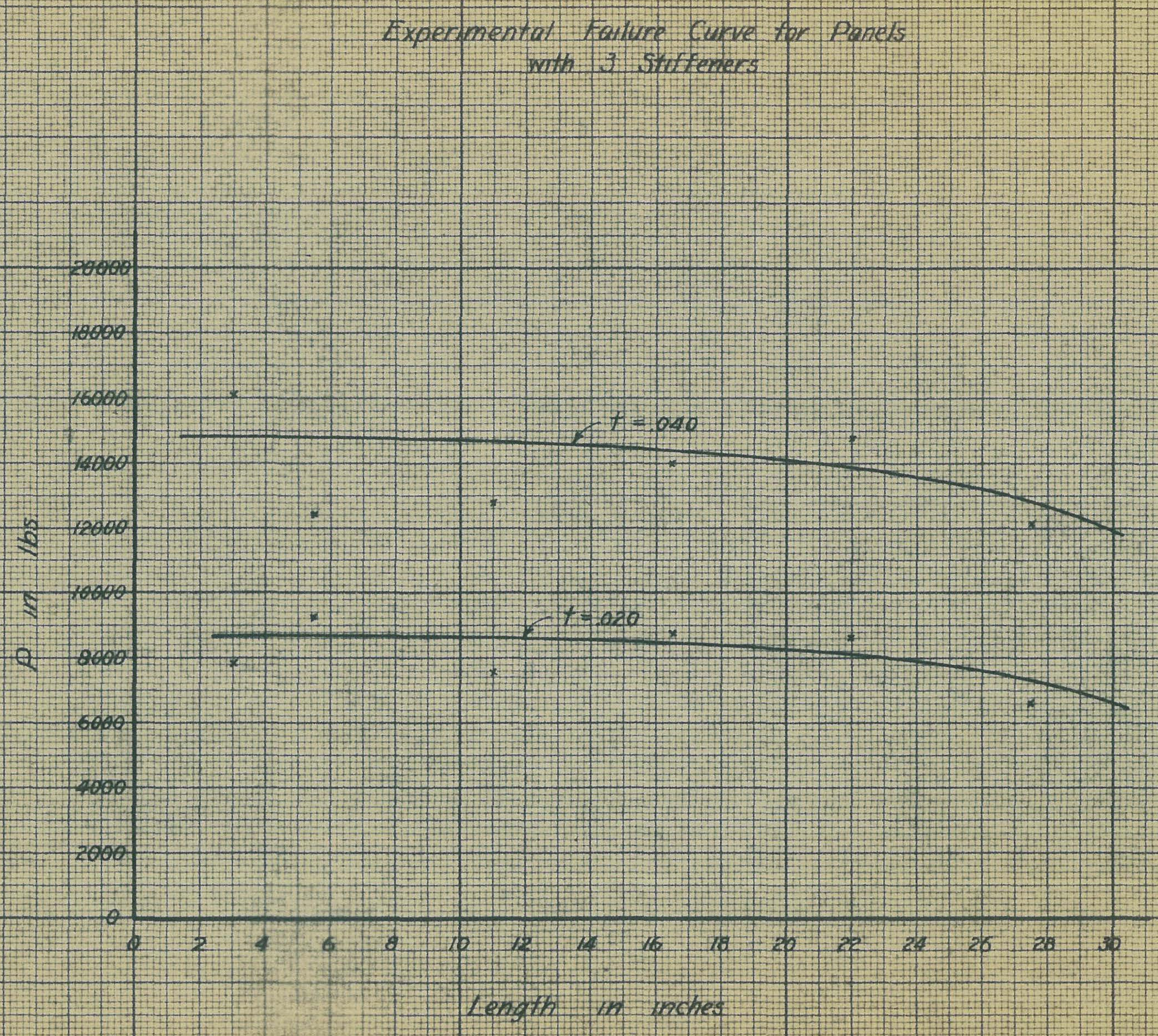

Experimental Failure Curve for Panels
with 3 Stiffeners
t = .040
t = .020
P in lbs
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Length in inches

FIG. 2

Experimental Failure Curve for Panels
with 4 Stiffeners

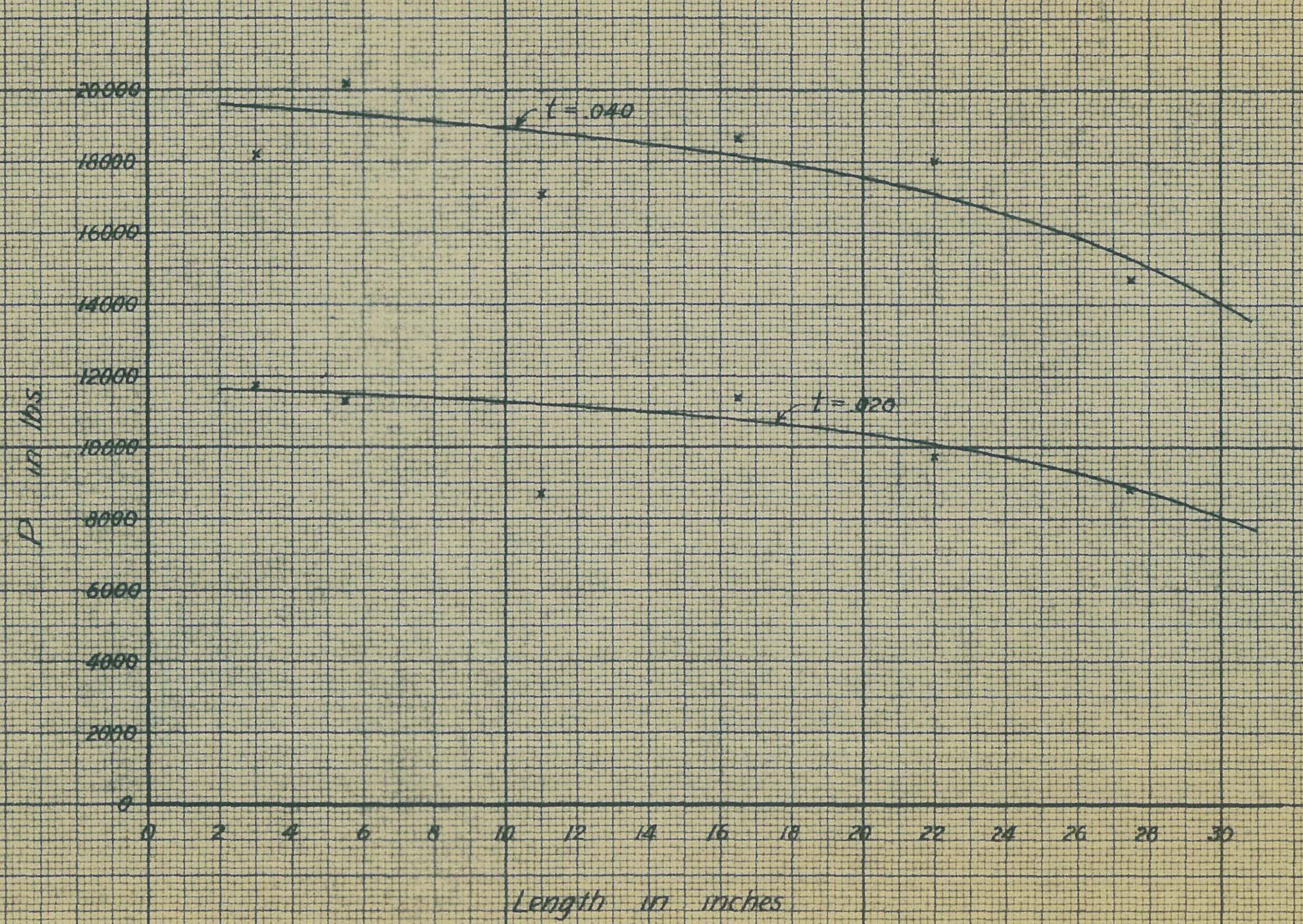



Fig. 3

Effective Width as a Function of λ

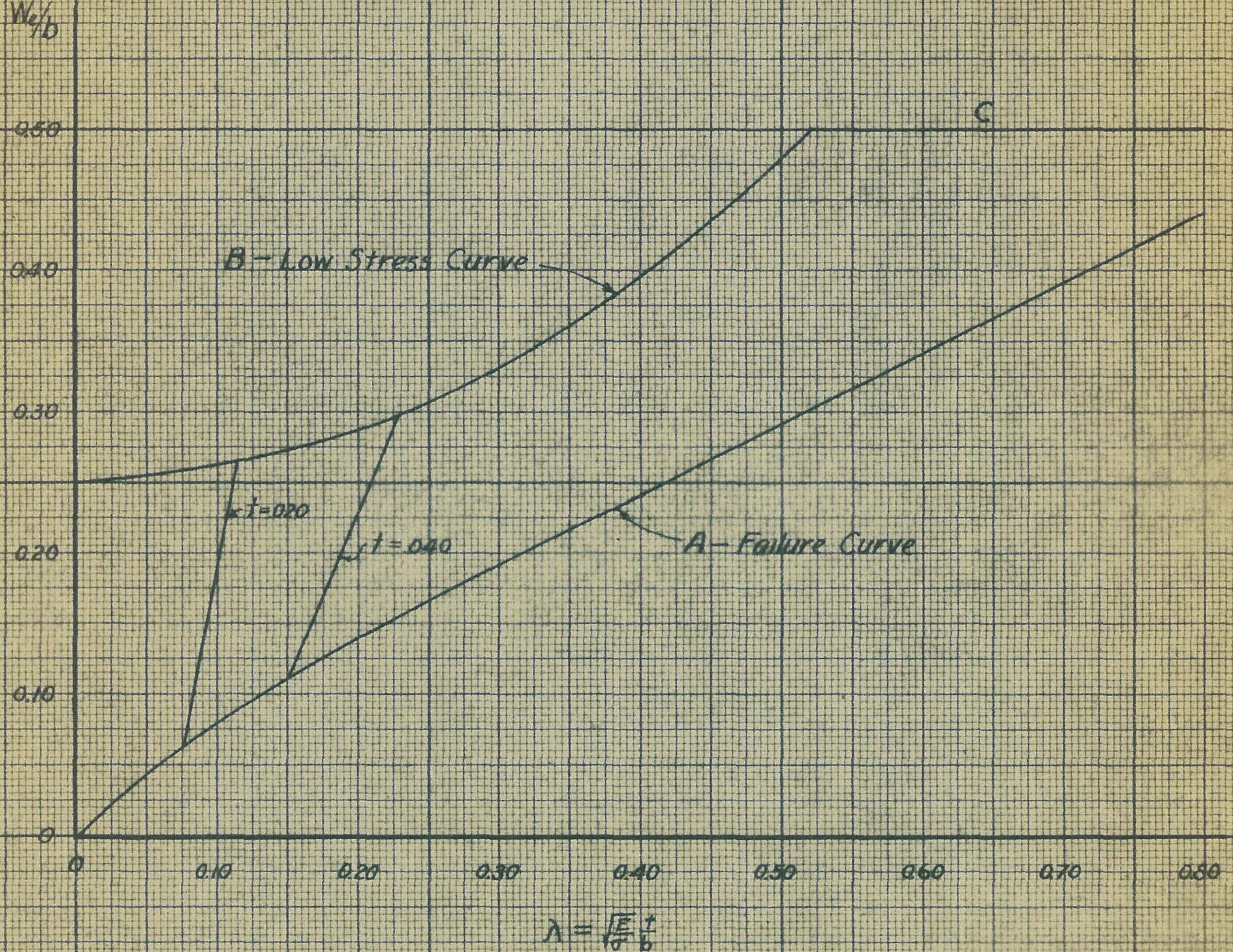



Figure 4

# CURVE FOR THE DETERMINATION OF $\left(\frac{\rho}{\rho_0}\right)^2 = \frac{\sigma}{\sigma_0}$



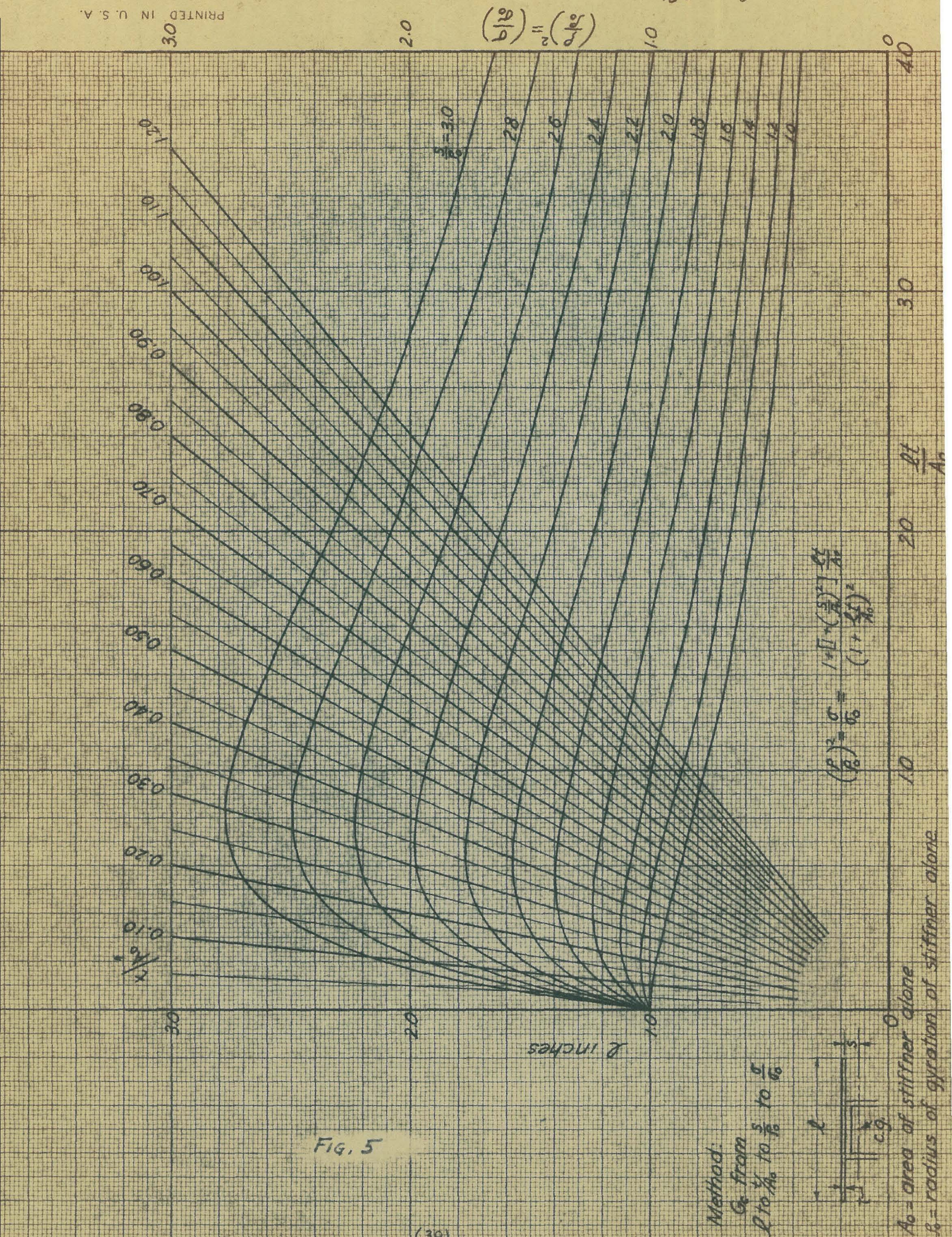
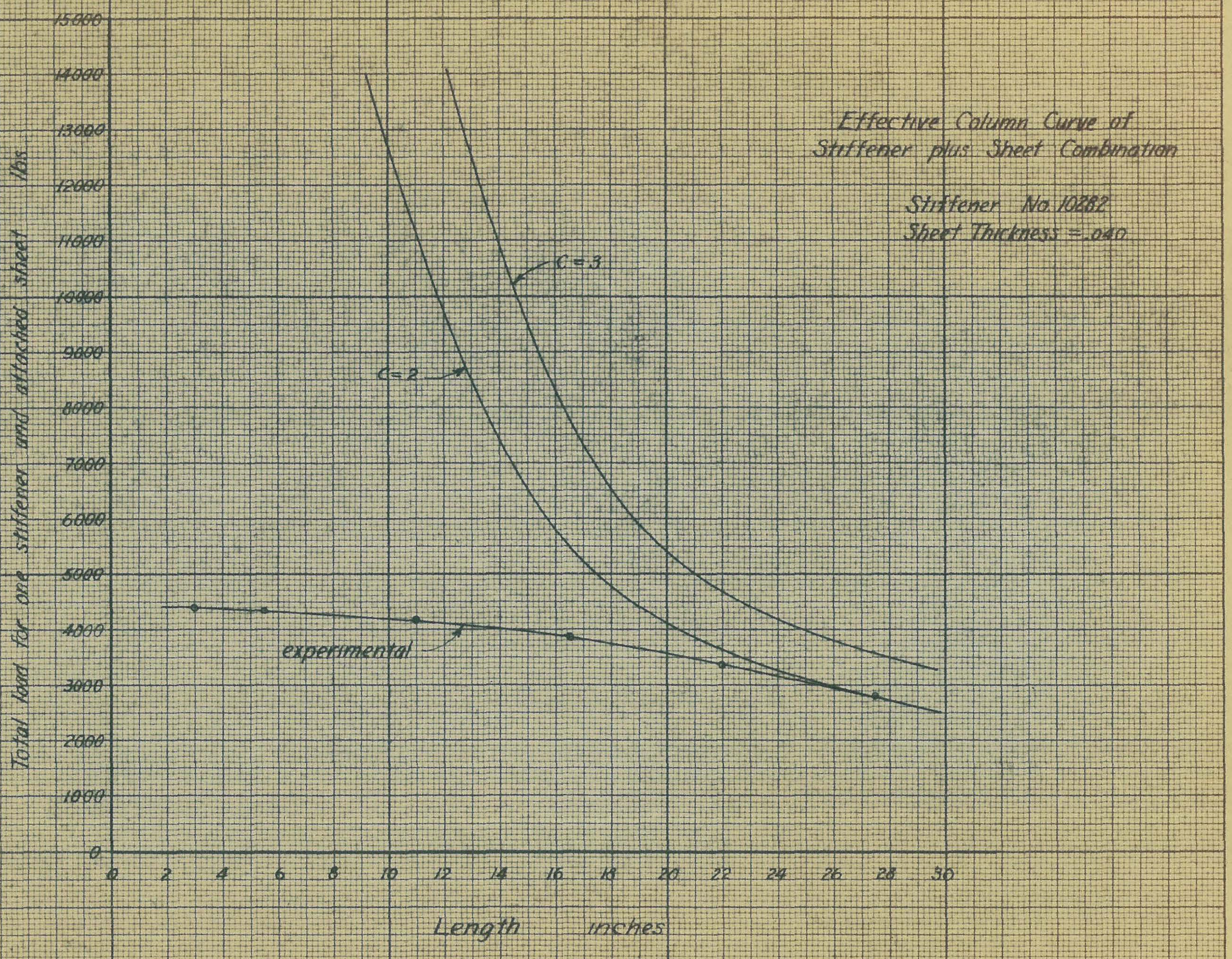
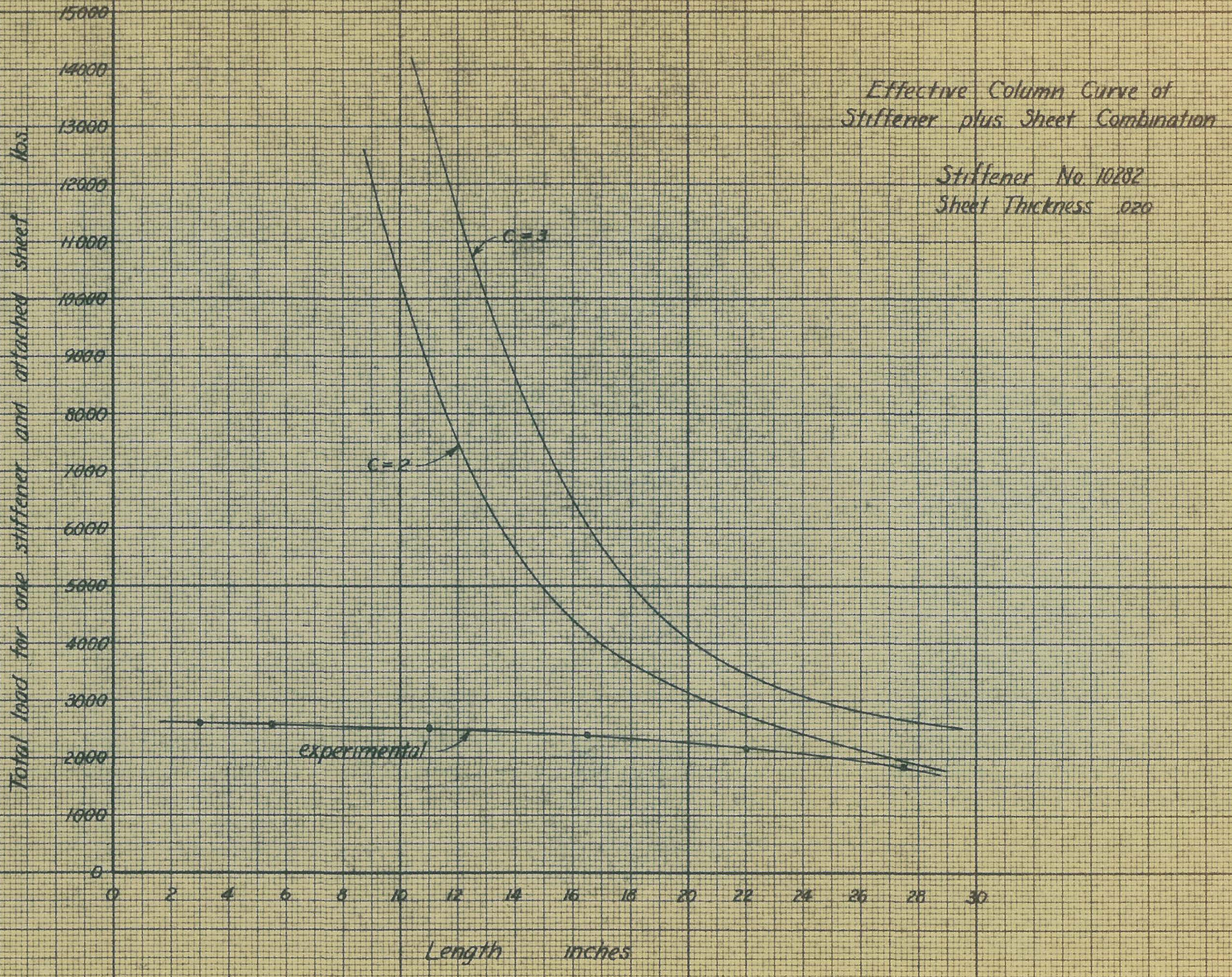



Fig. 5



Effective Column Curve of
Stiffener plus Sheet Combination
Stiffener No. 10282
Sheet Thickness = .040
C = 3
C = 2
experimental
Total load for one stiffener and attached sheet lbs.
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Length inches


Fig. 6



Effective Column Curve of
Stiffener plus Sheet Combination
Stiffener No. 10282
Sheet Thickness .020
Total load for one stiffener and attached sheet lbs.
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
C = 3
C = 2
experimental
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Length inches


FIG. 7.