EFFECT OF FOUNDATION COMPLIANCE ON THE EARTHQUAKE STRESSES IN TYPICAL TALL BUILDINGS

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ABSTRACT

This paper shows the quantitative effect that foundation compliance has on the maximum base shear force, the maximum base moment, and the fundamental period of vibration in typical tall buildings subjected to strong-motion earthquakes. A study is made of 5, 10, and 15 story building models on the Electric Analog Computer, subjecting them to the ground accelerations of actual earthquakes. The base shear forces and base moments are measured with the foundation compliance of the models being changed through a very wide range.

The properties specified for the building models are shown to be similar to the properties found in real buildings. The experimental results imply that the base shear forces and base moments in typical real buildings of 5 stories and higher during strong-motion earthquakes will be unaffected by any degree of foundation compliance that can be expected. The fundamental period of typical buildings will be increased by about 10 percent if the foundation compliance is the maximum that can be expected. It is shown, however, that even a doubling of the fundamental period does not significantly reduce the base shear forces.

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I. INTRODUCTION

In the past decade, considerable attention has been given to the field of engineering seismology - the study of earthquake stresses in engineering structures. This has included the surveillance of earthquake damage, experimental tests on physical models or actual structures, and analytical studies of the dynamic response of structures to strong-motion earthquakes. A monumental contribution to the last phase of study is the "spectrum-analyses" of 28 horizontal components of 14 strong-motion earthquakes by J. L. Alford, G. W. Housner and R. R. Martel[1]. The spectrum is essentially the influence line for maximum shear in a single-degree-of-freedom structure.

The spectrum was first constructed by use of the lengthy numerical integration method[2]. Later, Biot[3] introduced the torsion pendulum analyzer to obtain the spectrum values. Alford, Housner and Martel[1] rapidly determined the spectrum values with the electric analog computer. But in all of these procedures, a fundamental assumption is made. The base support (foundation as it is hereafter referred to) is assumed to be perfectly rigid. This is not true for actual structures built on typical foundation materials. For many years, writers have stated that yielding of the foundation will have an effect on the fundamental period of vibration and consequently on the base shear force experienced by the structure. Biot[4] deduces a rocking period of vibration for a structure with foundation yielding, thus showing a lengthening in the fundamental period. Then, referring to his design spectrum, he shows that the shear stress is reduced.

A Joint Committee of the San Francisco, Calif. Section, ASCE[5] recently discussed this effect. "It has been found that the rotation of a building on its base, in vibrating, has a very considerable effect in lengthening the natural periods of vibration of the building, especially the fundamental period. This rotation is of greater effect on a rigid building than on a flexible one. The rotation of a building on its foundation has a cushioning effect, which decreases the dynamic shears and moments throughout the building in all modes".

On the other hand, Martel, Housner and Alford[6] in discussing the above paper write, "The authors state that the rotation of a building on its base, that is, foundation yielding, has a very considerable effect upon the period of vibration of a building and reduces the dynamic shears. It is true that if appreciable foundation yielding does occur, it will affect the vibration of a building; but to the writers' knowledge there exist no reliable data that show that, for actual buildings of average type, the amount of foundation yielding that does occur has an appreciable effect on the period or on the magnitude of shear".

It is the purpose of this paper to show in a quantitative manner the actual effect that foundation yielding (hereafter called "compliance") has on the fundamental period and the maximum base shear force, the important criterion, in typical buildings subjected to strong-motion earthquakes. The problem is approached in the following manner: Specific building models are selected for study. These models are then subjected to the ground accelerations of actual earthquakes. The base shear forces are measured with the foundation compliance of the model being changed through a very wide range. This study is done electrically on the Analog Computer for convenience, although it could have been done by actually constructing the models and measuring their response mechanically. Once the above relationship is determined, it is then of interest to estimate the maximum amount of foundation compliance to be expected for buildings of the type that were modeled and thus evaluate the beneficial effect of the compliance.

II. FORMULATION OF THE FOUNDATION PROBLEM

Some insight can be gained into the effect of foundation compliance on the response of a building to earthquake ground motion by a study of a simplified model of a 1 story building. The foundation compliance can be of two types, horizontal and rotary. The rotary or "rocking" compliance depends in part on the vertical deformation constant of the soil which is considered a perfectly elastic medium. Therefore, small rocking deformations can account for some motion in the top of the model.

Both horizontal and rocking compliance can be introduced into the base support of the 1 story building and an analysis of their effect on the model's period of free vibrations is possible. In deriving the frequency equation, conditions are necessarily imposed to simplify the analysis. All springs and the support of the oscillator are considered weightless, damping is neglected, and motion is limited to small oscillations so that gravity effects are of a higher order and can be neglected.

The simplified model of a 1 story building is shown in the following sketch.

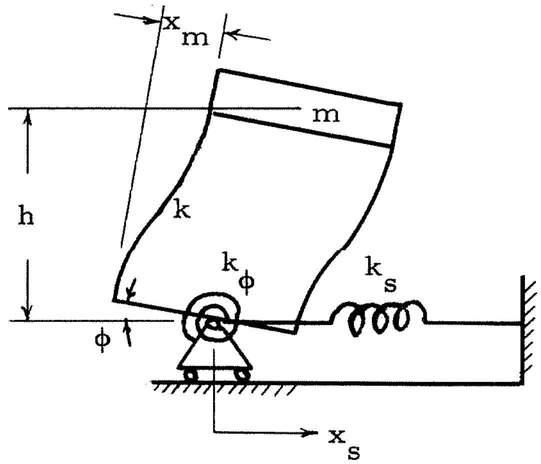
k = stiffness of the building - lb/ft

k_ϕ = stiffness of the foundation in rocking - lb-ft/radian

k_s = horizontal foundation stiffness - lb/ft

m = lumped mass of the building - lb-sec^2/ft

h = height of the lumped mass in feet



The generalized coordinates are

x_m = displacement of the mass from the equilibrium position of the vertical springs.

$h\phi$ = displacement caused by rocking compliance.

x_s = displacement of the support caused by horizontal compliance.

The kinetic energy of the mass is

$$T = \frac{1}{2} m \left[\dot{x}_m^2 + h^2 \dot{\phi}^2 + \dot{x}_s^2 + 2\dot{x}_m \dot{x}_s + 2h\dot{\phi}\dot{x}_s + 2\dot{x}_m h\dot{\phi} \right] \tag{1}$$

Terms of higher than second order have been neglected. The potential energy of the system is

$$V = \frac{1}{2} \left[kx_m^2 + k_\phi \phi^2 + k_s x_s^2 \right] \tag{2}$$

Since cross product terms appear in the kinetic energy expression, the system has dynamic coupling. Lagrange's equation for the qth coordinate is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = 0 \tag{3}$$

which gives the three equations of motion

$$\begin{aligned} \ddot{x}_m + h\ddot{\phi} + \ddot{x}_s + \frac{k}{m} x_m &= 0 \\ \ddot{x}_m + h\ddot{\phi} + \ddot{x}_s + \frac{k_\phi}{hm}\phi &= 0 \\ \ddot{x}_m + h\ddot{\phi} + \ddot{x}_s + \frac{k_s}{m} x_s &= 0 \end{aligned} \tag{4}$$

Assuming harmonic oscillations,

$$\begin{aligned} x_m &= x_m \sin\omega t \\ x_s &= x_s \sin\omega t \\ \phi &= \phi \sin\omega t \end{aligned} \tag{5}$$

the following equations for frequency and period of vibration are obtained:

$$\omega = \left[\frac{1}{1+\frac{h^2 k}{k_\phi} + \frac{k}{k_s}}\right]^{\frac{1}{2}} \sqrt{\frac{k}{m}} \tag{6}$$

$$\tau = \frac{2\pi}{\omega} = \left[1 + \frac{h^2 k}{k_\phi} + \frac{k}{k_s}\right]^{\frac{1}{2}} \cdot 2\pi \sqrt{\frac{m}{k}} \tag{7}$$

The building compliance (reciprocal of the stiffness) is $1/k$ and the rotary foundation compliance is h^2/k_ϕ. Thus the ratio of building compliance to foundation compliance, k_ϕ/h^2k, is a dimensionless parameter. Similarly, the ratio of building compliance, $1/k$, to horizontal foundation compliance, $1/k_s$, is the dimensionless parameter k_s/k. From Equations (6) and (7) it is seen that these two dimensionless parameters specify the effects of the foundation yielding on the frequency and the period of vibration. Equation (7) is plotted in Figure 1 and shows how the period of free vibrations is affected by various degrees of foundation compliance. If the ratio of building compliance to horizontal foundation compliance is reduced from infinity to 10, the period of the oscillator is increased less than 5 percent. It is rather unlikely that a value of k_s/k as low as 10 would ever be encountered in typical construction. This is substantiated by the earthquake records themselves. Most accelerometers and displacement meters for recording earthquake motion are housed in the basements of buildings. If any horizontal oscillations of the basement occurred, the instruments would record this motion as well as that of the earthquake. Studies of the records have not revealed any significant horizontal yielding. Thus the foundation problem can be simplified by neglecting the effect of horizontal compliance.

On the other hand, rotary motion of the foundation would not appear on the records of horizontal earthquake motion. The influence of this type of foundation compliance is the subject of this paper. Figure 1 shows the effect of rotary compliance on the period of the 1 story building. If the ratio of building compliance to rotatory foundation compliance (the compliance ratio)

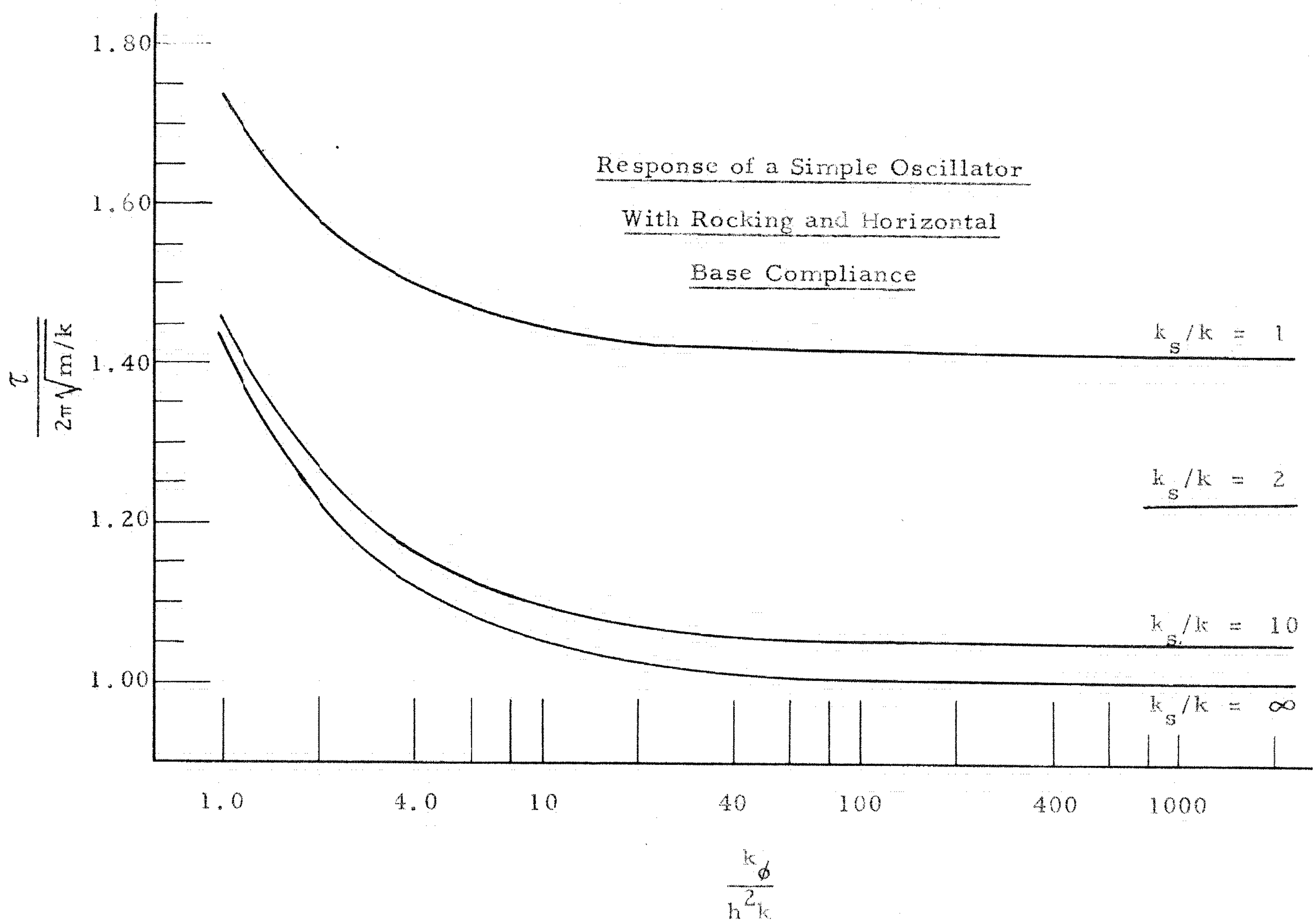

Response of a Simple Oscillator
With Rocking and Horizontal
Base Compliance
1.80
1.60
1.40
1.20
1.00
$\frac{\tau}{2\pi\sqrt{m/k}}$
$k_s/k = 1$
$k_s/k = 2$
$k_s/k = 10$
$k_s/k = \infty$
1.0
4.0
10
40
100
400
1000
$\frac{k_\phi}{h^2 k}$

FIGURE 1

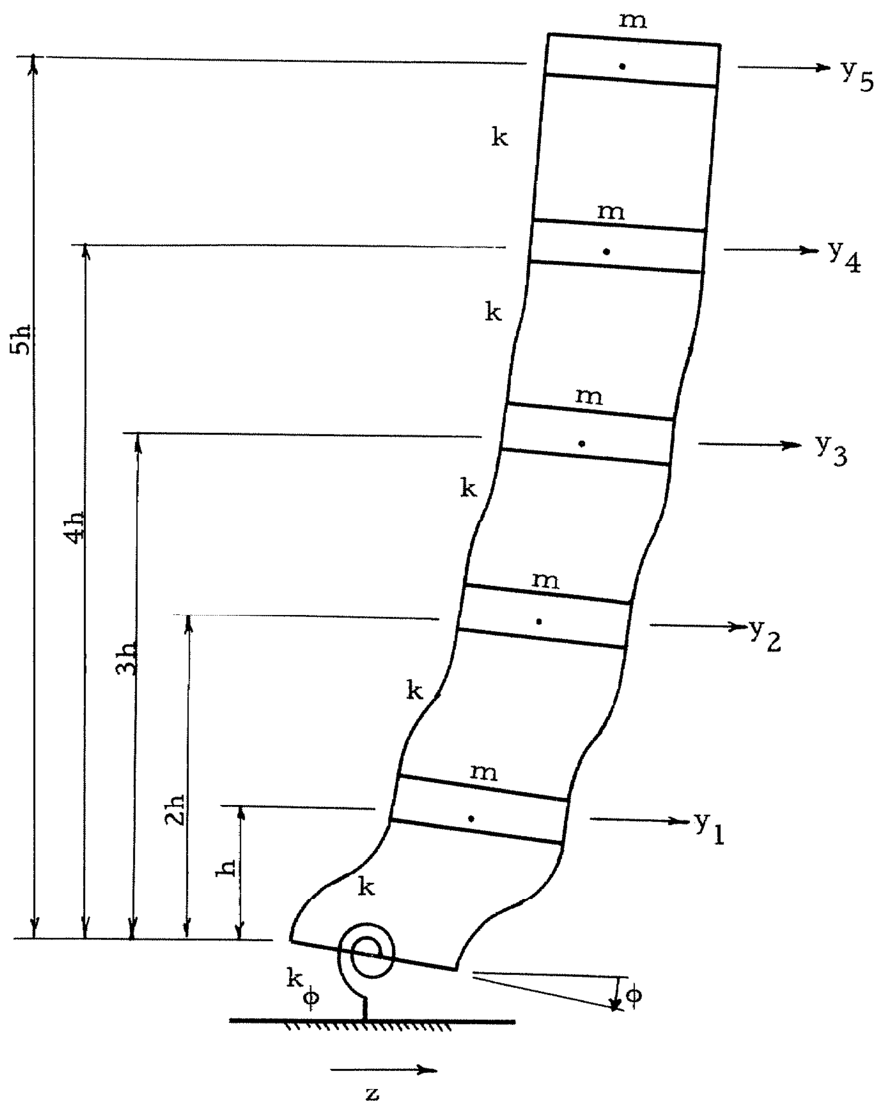
is 1, the period of vibration will be increased 40 percent. The effect of the compliance ratio on the fundamental period of multi-story building models will be presented later as will also the effect of this ratio on the base shear force in the buildings.

III. MODELING A MULTI-STORY BUILDING

It was decided to model a 5, 10, and 15 story building in this investigation because these heights are representative of many buildings on the West Coast. The model buildings are given the following characteristics:

The multi-story model building is to deflect in shear. Thus the floors of the building move parallel to each other. The mass of the building is lumped at the floor levels. The stiffness of any story is the force required to deflect one floor relative to an adjacent floor a distance of 1 foot, and this is taken to be the same for every story. The 5 story building is given a fundamental period of 0.5 seconds; the 10 story building, 1.0 seconds; the 15 story building, 1.5 seconds. It is not necessary to impose any restrictions on the dimensions of the model or its type of construction since the model is actually only a mathematical formulation and not a physical structure. The foundation rocking compliance is simulated by a torsion spring at the base where the stiffness of the spring, as in the case of the simple oscillator, is k_ϕ, the moment in pound-feet required to rotate the spring through an angle of one radian. The problem is further particularized by letting the mass of each story, the shearing stiffness of each story, and the height of each story be the same. The rotary moment of inertia of the floors about their centers of gravity is disregarded in the analysis. This excludes extraordinarily wide buildings from the study. A schematic drawing of a

5 story building model is shown in the following diagram.



Equations of motion of a 5 story building

Let y_i = absolute displacement of the i-th floor

ϕ = rotation of the base

z = absolute displacement of the ground

m = lumped mass of one story

$$\ddot{y}_5 + \frac{k}{m}(y_5 - y_4 - h\phi) = 0$$

$$\ddot{y}_4 + \frac{k}{m}(2y_4 - y_5 - y_3 - 2h\phi) = 0$$

$$\ddot{y}_3 + \frac{k}{m}(2y_3 - y_4 - y_2 - 2h\phi) = 0$$

$$\ddot{y}_2 + \frac{k}{m}(2y_2 - y_3 - y_1 - 2h\phi) = 0 \tag{8}$$

$$\ddot{y}_1 + \frac{k}{m}(2y_1 - y_2 - z - 2h\phi) = 0$$

$$\sum_{i=1}^{5} i\,\ddot{y}_i + \frac{k_\phi}{mh}\phi = 0$$

The shear force in the base spring is

$$F = k\,(y_1 - z - h\phi)$$

The moment in the torsion spring is

$$M_\phi = k_\phi\,\phi$$

The equations of motion can also be written in the following alternate form. Let

x_i = the displacement of the i-th floor with respect to the base.

$-\phi$ = rotation of the base.

$-z$ = absolute displacement of the ground.

Then the absolute displacement of the i-th floor is

$$y_i = x_i - z - ih\phi$$

The equations of motion are

$$\ddot{x}_5 - 5h\ddot{\phi} + \frac{k}{m}(x_5 - x_4) = \ddot{z}$$

$$\ddot{x}_4 - 4h\ddot{\phi} + \frac{k}{m}(2x_4 - x_5 - x_3) = \ddot{z}$$

$$\ddot{x}_3 - 3h\phi + \frac{k}{m}(2x_3 - x_4 - x_2) = \ddot{z}$$

$$\ddot{x}_2 - 2h\ddot{\phi} + \frac{k}{m}(2x_2 - x_3 - x_1) = \ddot{z} \qquad (9)$$

$$\ddot{x}_1 - h\ddot{\phi} + \frac{k}{m}(2x_1 - x_2) = \ddot{z}$$

$$\sum_{i=1}^{5} i^2 h\ddot{\phi} - \sum_{i=1}^{5} i\,\ddot{x}_i + \frac{k_\phi}{mh}\phi = -\sum_{i=1}^{5} i\,\ddot{z}$$

The shear force in the base spring is

$$F = k\,x_1$$

The moment in the torsion spring is

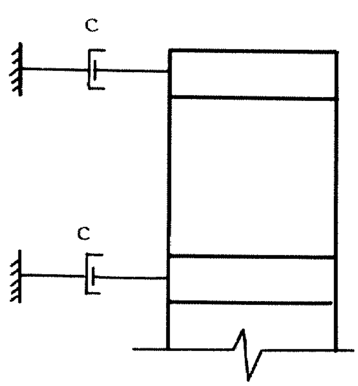
$$M_\phi = -k_\phi\,\phi$$

It is seen from Equation (9) that a force proportional to the ground acceleration is applied at each floor. On the other hand, only absolute displacements are involved in Equation (8). The equation for the moment of the floors about the base is considerably less complicated in the case of Equation (8) than (9). Equation (8) was found to be the most convenient form of the equations of motion and was used in the investigation.

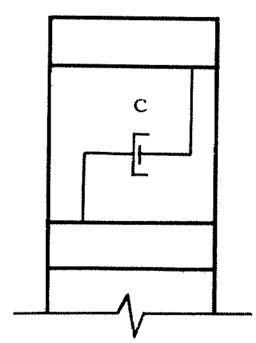
Structural Damping in the Building Models

The response of a multiple-degree-of-freedom system without damping, such as the 5 story model building, can be described in terms of the configurations taken by the system when it is vibrating at one of its natural frequencies. These configurations are called the "normal modes" of the system[8]. Rayleigh[9] suggests that it is possible to derive the normal modes of a damped system provided the damping be of a restricted mathematical form that is not generally useful for practical problems.

If the energy dissipation in the structure is caused by viscous damping, the formulation of the problem is greatly simplified. Jacobsen[10] shows this assumption is justified if the total energy dissipation is small. Two types of viscous damping can be treated easily, absolute and interfloor. Both cases are equivalent to having dashpots attached to the floors of the model, but in the first instance the dashpot is actuated by the absolute displacement of the floor whereas in the second the dashpot is actuated by the relative displacement of the floors.



Absolute Damping



Interfloor Damping

Physical reasoning shows that interfloor damping is a more reasonable way to dissipate energy in a building than absolute. But it is significant to look at both types analytically. Alford [1] has shown that absolute damping, interfloor damping, or a linear combination of both falls into the restricted mathematical form discussed by Rayleigh(9). He derives the normal modes for the system and is able to examine the damping as though each mode were a one-degree-of-freedom system. He shows that if interfloor damping is used, the percent of critical damping is proportional to the undamped natural frequency of the modes. Thus in a five-degrees-of-freedom system, if the first mode is damped 0.10 of critical, the fifth mode is damped 0.67 of critical. If absolute damping is used, the percent of critical damping is inversely proportional to the undamped natural frequency of the modes. Thus in the five-degrees-of-freedom example, if the first mode is damped 0.10 of critical, the fifth mode is only damped 0.015 of critical. If a linear combination of both types is used, it is possible to select the amount of each so that the damping in any two modes is arbitrarily specified. In this investigation, the damping in the first two modes of vibration of the models was specified to be 0.10 of critical damping.

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[1] An unpublished paper on damping in multiple-degree-of-freedom systems by J. L. Alford.

IV. EARTHQUAKE GROUND MOTION

The U. S. Coast and Geodetic Survey records of 15 strong-motion earthquakes were available for use in this study. They were:

1.	Vernon, California	March 10, 1933
2.	Vernon, California	October 2, 1933
3.	Los Angeles Subway Terminal	March 10, 1933
4.	Los Angeles Subway Terminal	October 2, 1933
5.	El Centro, California	December 30, 1934
6.	El Centro, California	May 18, 1940
7.	Helena, Montana	October 31, 1935
8.	Ferndale, California	September 11, 1938
9.	Ferndale, California	February 9, 1941
10.	Ferndale, California	October 3, 1941
11.	Santa Barbara, California	June 30, 1941
12.	Hollister, California	March 9, 1949
13.	Olympia, Washington	April 13, 1949
14.	Seattle, Washington	April 13, 1949
15.	Taft, California	July 21, 1952

The stronger acceleration components of the three most intense ground motions[1] were chosen for study. These included the N-S acceleration component of the earthquake recorded at El Centro, California on May 18, 1940, the S 80 W component of the earthquake recorded at Olympia, Washington on April 13, 1949, and the N-S component of the earthquake recorded at El Centro, California on December 30, 1934. The accelerograms for these earthquakes are shown in Figures 2 - 4. Also of immediate interest is the recent strong-motion earthquake of July 21, 1952 which had its epicenteral location about 20 miles from Tehachapi, California[7]. The nearest recording station to the center of the shock was at Taft, California, some 40 miles away. The S 69 E component of this record was also used in the study, the accelerogram appearing in Figure 5.

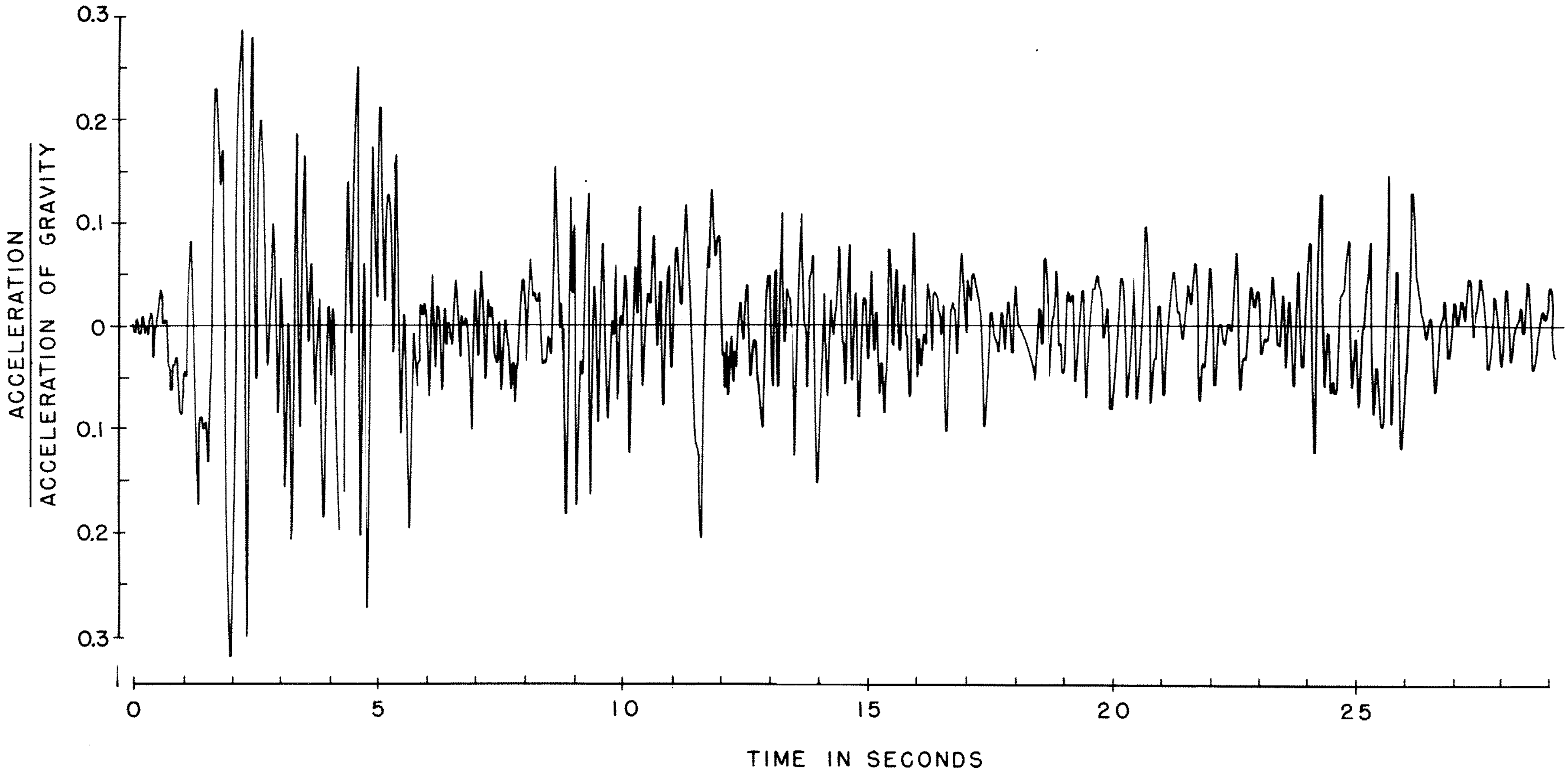


FIGURE 2. Accelerogram for El Centro, California; Earthquake of May 18, 1940. Component N-S.

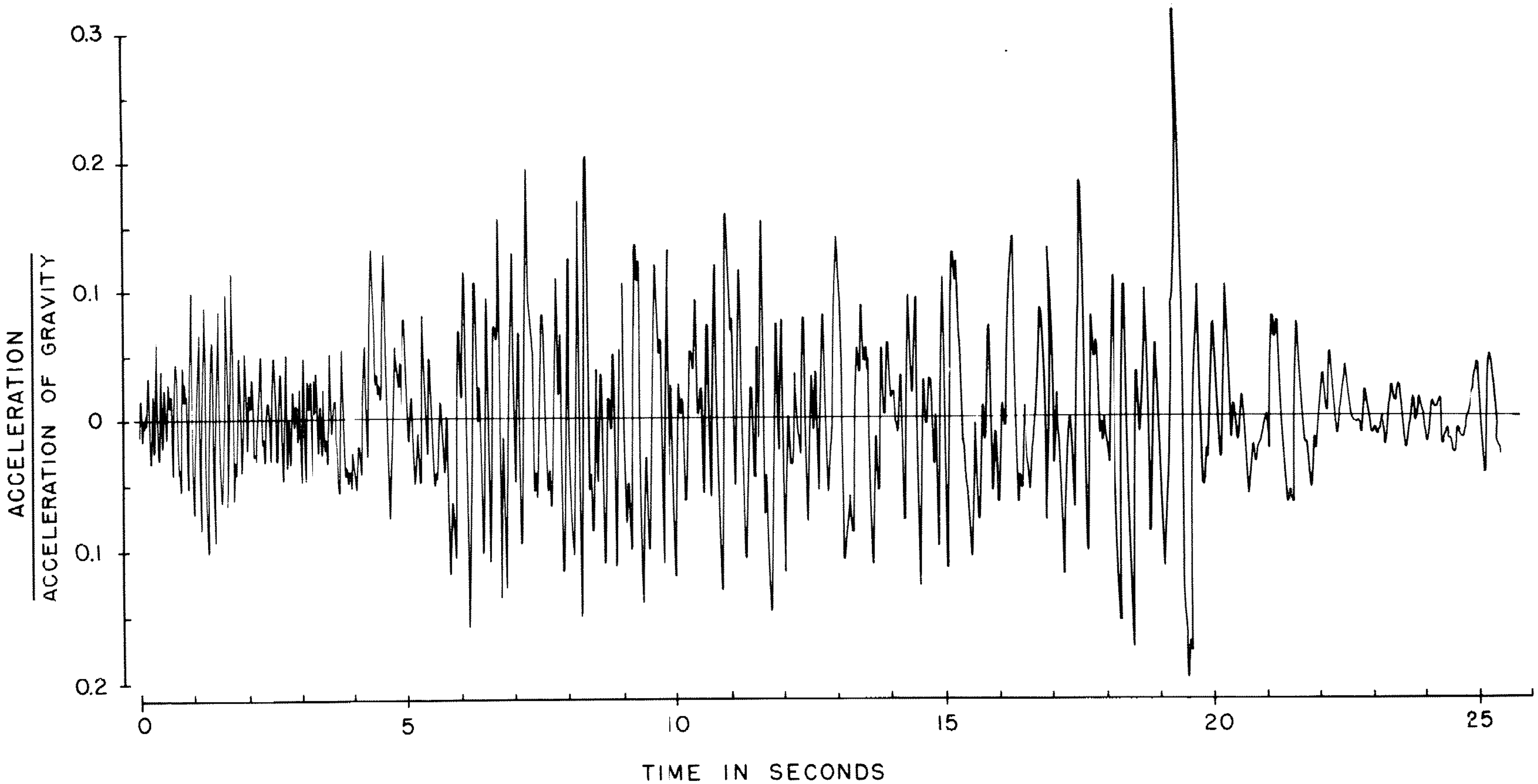


FIGURE 3. Accelerogram for Olympia, Washington; Earthquake of April 13, 1949. Component S 80 W.

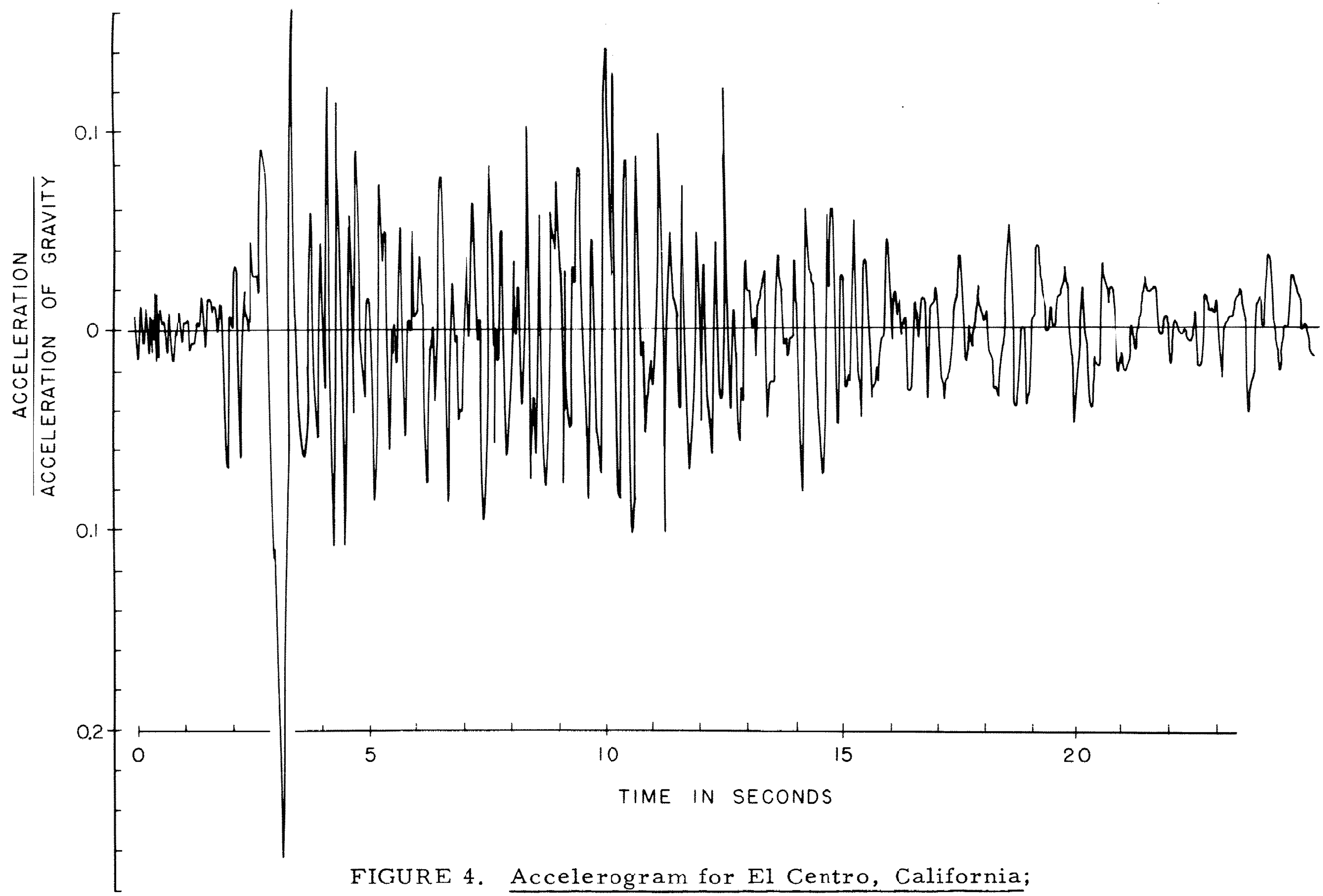


FIGURE 4. Accelerogram for El Centro, California; Earthquake of Dec. 30, 1934. Component N-S.

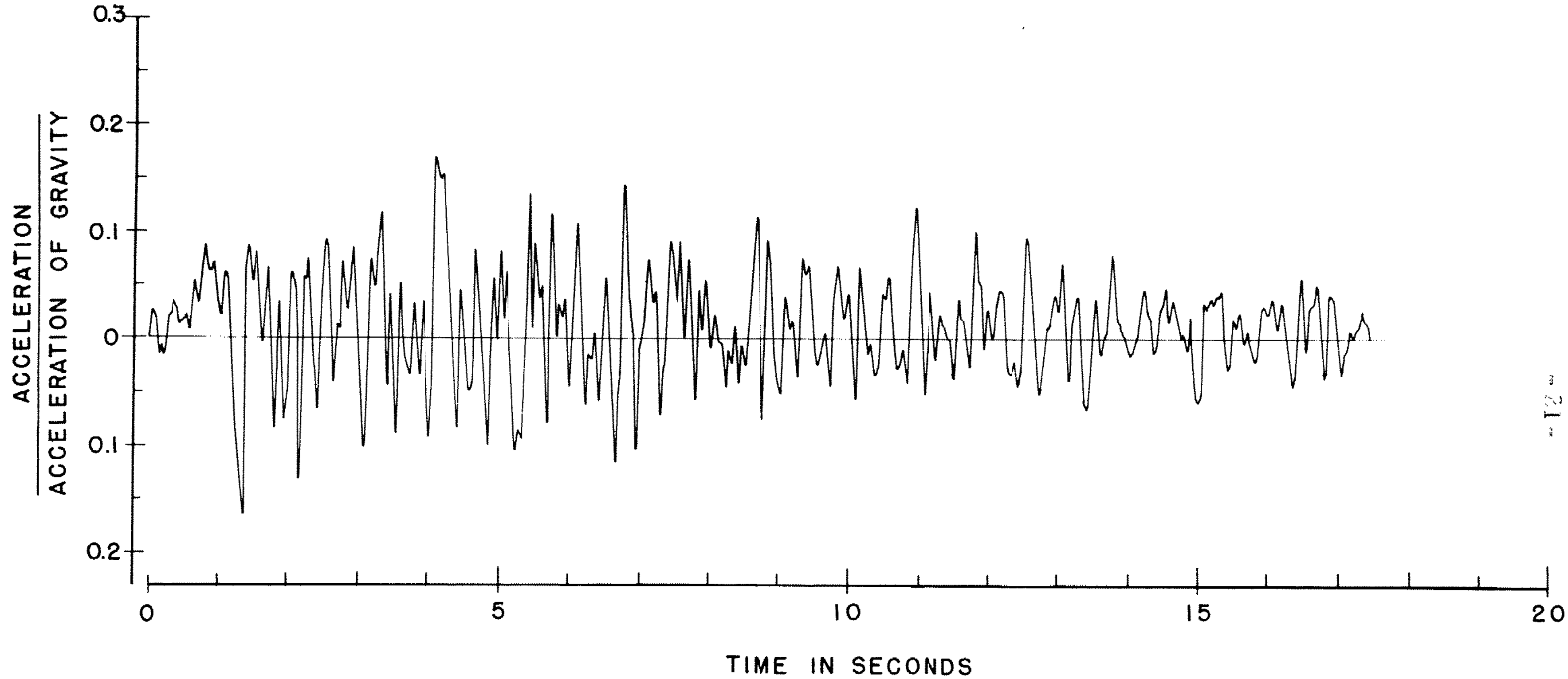


FIGURE 5. Accelerogram for Taft, California; Earthquake of July 21, 1952. Component S 69 E.

V. ANALOG COMPUTER TECHNIQUE

The measurement of actual base shear forces in building models subjected to earthquake ground motions on mechanical shaking tables would be a formidable task. However, the use of the Electric Analog Computer offers a fast, reliable method of analyzing multiple-degree-of-freedom structures with damping when subjected to the accelerations recorded during strong-motion earthquakes. This technique permits the inclusion of any desired amount of foundation compliance. The shear force in any floor can be read directly, and the effect of foundation compliance on it can be observed as a single dial is turned.

The electric analog is an electrical circuit whose components are analogous to the components of a mechanical system. Therefore, the equations that govern the electrical circuit are of the same form as the equations of motion of the mechanical system. The beauty of the technique lies in the fact that the electrical components can be changed by the turning of a dial, and that currents and voltages can be read at any point in the system. Knowing a set of proportionality factors, the corresponding forces and velocities can be computed rapidly. The analog computer and its application to earthquake problems is described in References (1) and (11).

The Nodal Analogy

The mechanical-electrical relations for the nodal analogy are outlined below.

Linear velocity, $\dot{y} = \frac{Ka}{N} e_y$

Angular velocity, $\dot{\phi} = \frac{Ka}{P_1 N} e_\phi$

Force, $F_y = \frac{K}{a} I_y$

Moment, $M_\phi = \frac{P_1 K}{a} I_\phi$

Capacitance, $C = \frac{a^2}{N^2} m$

Reciprocal Inductance, $\frac{1}{L} = a^2 k$

Reciprocal Resistance, $\frac{1}{R} = \frac{a^2}{N} c$

$$\frac{1}{L_\phi} = \frac{a^2}{P_1^2} k_\phi$$

$$\frac{L_\phi}{L} = \frac{P_1^2 k}{k_\phi}$$

Transformer ratio, $\frac{T_p}{T_s} = \frac{h}{P_1}$

$$f_{elec.} = N f_{mech.}$$

$$\tau_{elec.} = \frac{1}{N} \tau_{mech.}$$

In the above relations, $P_1 = h$ for the model buildings used.

Also,

N = time base change

a^2 = admittance base change

($\frac{1}{a^2}$ = impedance base change)

Figure 7 shows the electrical circuit used in this study. The equations of this circuit are of the same form as Equation (8). It is easily shown that it has the properties of the 5 story building model with rocking compliance. Since velocity is proportional to voltage, let the voltages at the story heights in Figure 7 be $\dot{y}_1$, $\dot{y}_2$, ... $\dot{y}_5$. Let the voltage at the base be $\dot{z}$ and the rotational part of the circuit be $\dot{\phi}$. The current across any inductance is

$$i = \frac{1}{L} \int e \, dt$$

The current in L_ϕ is

$$i = \frac{1}{L_\phi} \int \dot{\phi} \, dt = \frac{1}{L_\phi} \phi$$

Therefore,

$$M_\phi = k_\phi \phi \qquad (10)$$

which is the moment in the torsion spring. The voltage on the right side of the primary of T_1 must be $\dot{z} + h\dot{\phi}$ since the turns ratio is 1 to 1. Therefore, the voltage across L_1 must be

$$e = \dot{y}_1 - \dot{z} - h\dot{\phi}$$

and the current is

$$i = \frac{1}{L_1} \int (\dot{y}_1 - \dot{z} - h\dot{\phi}) \, dt$$

$$= \frac{1}{L_1} (y_1 - z - h\phi)$$

Therefore,

$$F = k(y_1 - z - h\phi) \qquad (11)$$

which is the shear force in the base spring.

Excitation of the Circuit

The circuit of Figure 7 requires a velocity, $\dot{z}$, at the base of the structure of the same form as the velocity of the earthquake ground motion. It is a relatively simple procedure to develop a variable voltage of the same form as the recorded earthquake accelerograms. This is done in the following manner:

A carefully scaled drawing is made of the earthquake accelerogram and wrapped around the drum of a plotting table as shown in Figure 6. A circular piece of film is placed in a photographic chamber which is attached to the upper end of the plotting table drum shaft. An operator follows the ordinate of the accelerogram with a cross-hair while the drum is revolved slowly by a motor. The vertical motion of the cross-hair is linked to a selsyn transmitter located inside the cover of the photographic chamber (lower left, Figure 6). This transmitter in turn controls the opening of a light slit. As the drum revolves, a variable-width "sound track" is exposed. Typical film records are shown at the lower right in Figure 6. In use, the film records are mounted on a constant-speed turntable in an "arbitrary function generator". The variable-width "sound track" meters the light in a photocell circuit which is in turn amplified to provide a voltage which, over a finite time interval, has the required form of the accelerogram.

With these film records available for this investigation, it was only necessary to use an electrical integrator in order to

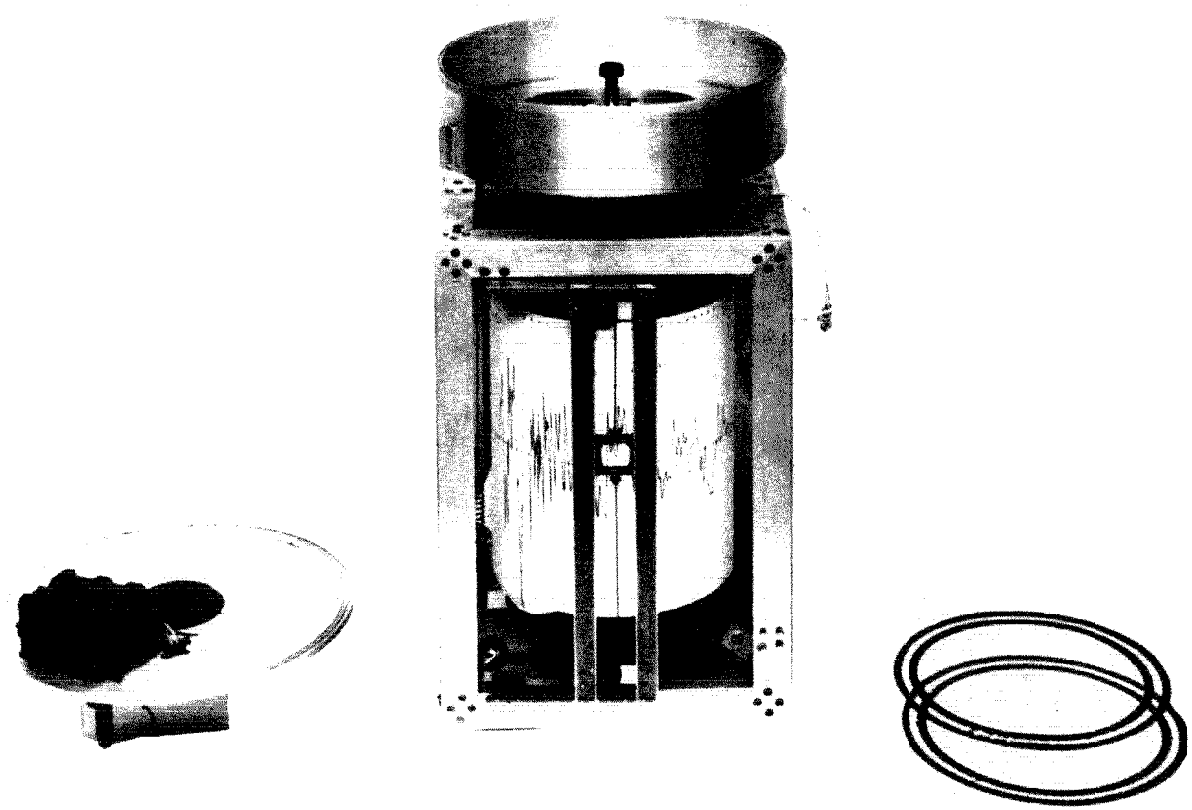


FIGURE 6. Plotting Table for Film Records

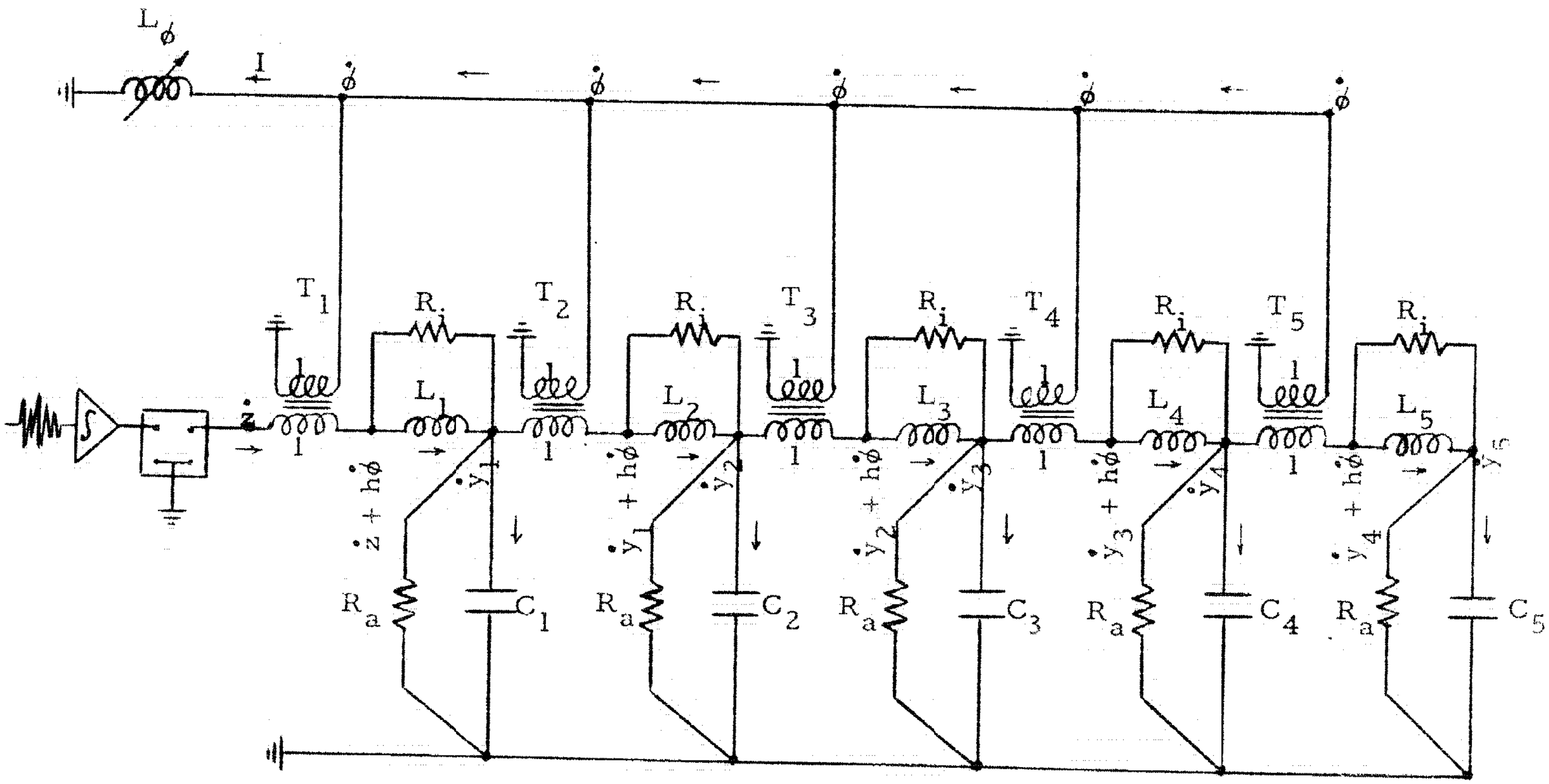


FIGURE 7. Nodal Analogy Circuit for a Five Story Structure With Foundation Compliance

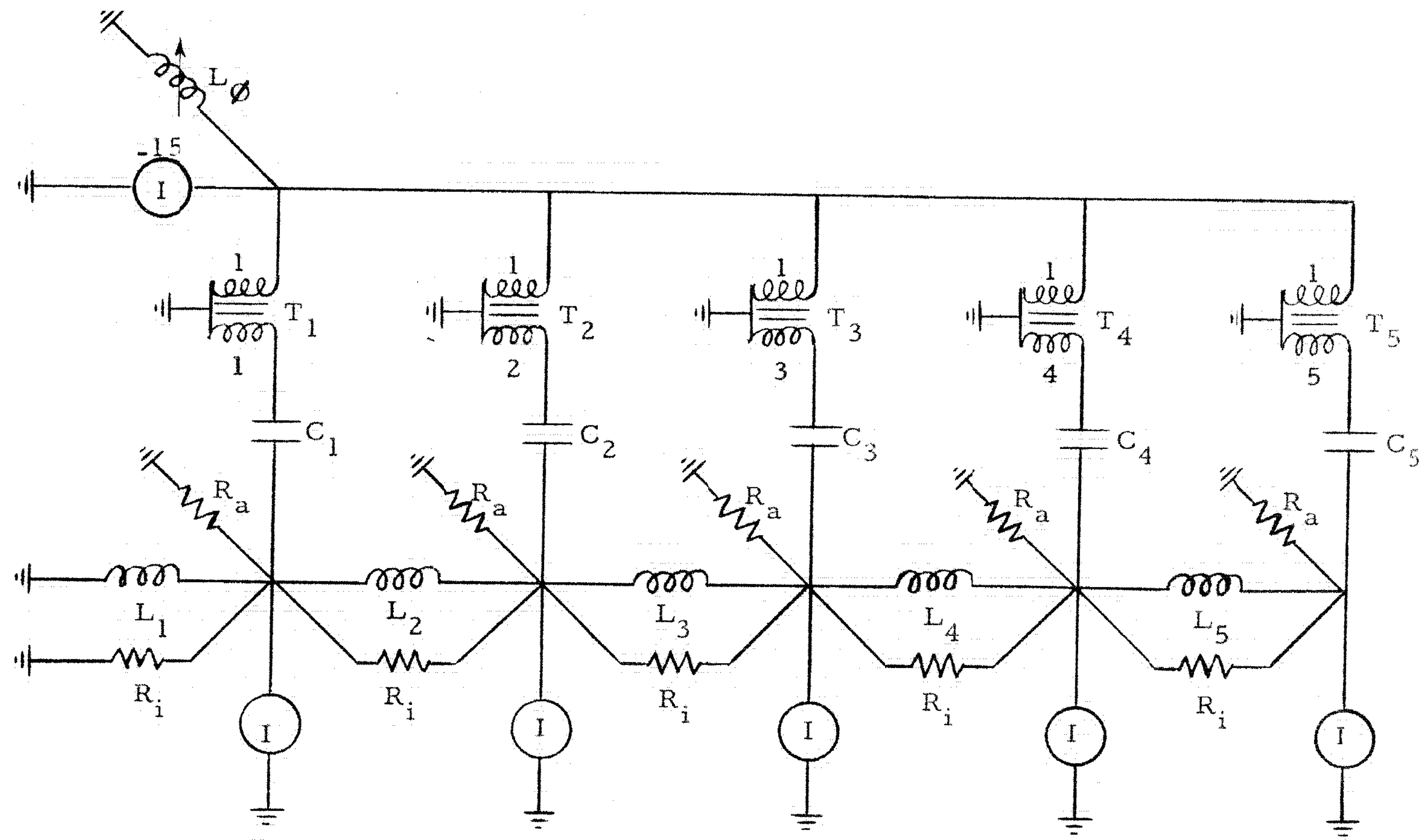


FIGURE 8. Alternate Nodal Analogy Circuit for a Five Story Building With Foundation Compliance

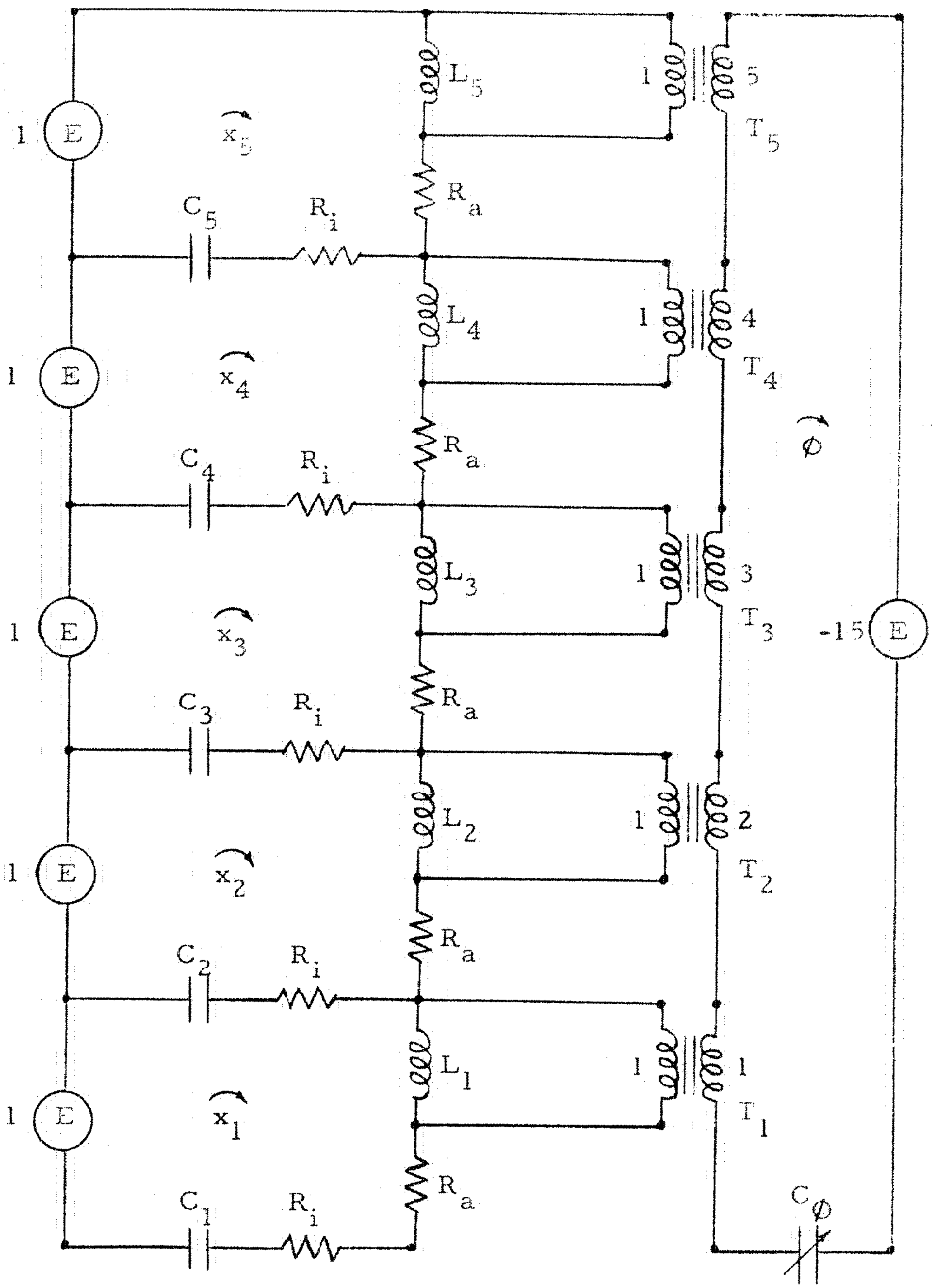


FIGURE 9. Loop Analogy Circuit for a Five Story Building With Foundation Compliance

apply a velocity of the required form to the base of the building.

The nodal-analogy circuit of Figure 7 was developed from Equation (8), and this circuit was used for the study. An alternate circuit corresponding to Equation (9) was also considered. This circuit is shown in Figure 8. It, however, has the disadvantage of requiring excitation currents at six points in the circuit. A loop analogy circuit also could have been used for Equation (9). This circuit is shown in Figure 9. It also has the disadvantage of requiring six excitation voltages, although it does have some electrical advantages over Figure 8.

VI. EXPERIMENTAL PROCEDURE

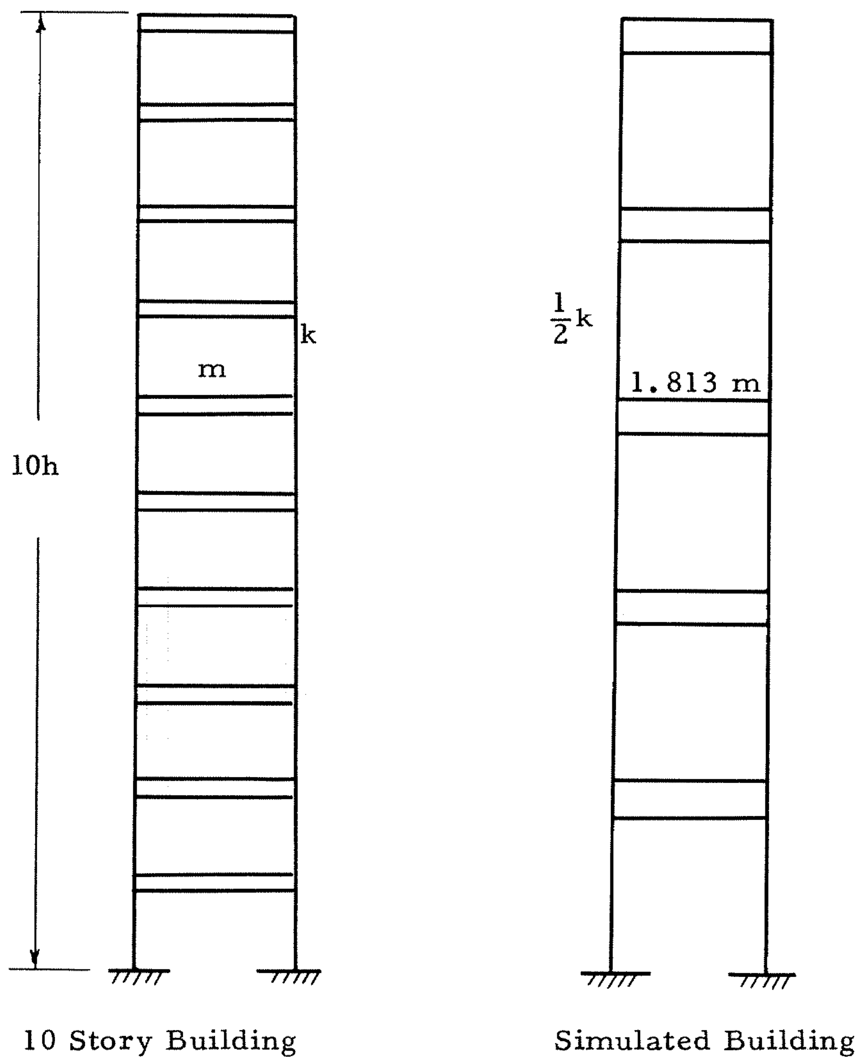
The most important properties of the building models are the frequencies of the various modes of vibration and the amount of damping. The frequencies for the 5 story building model are given in Table 1. The frequencies of the first five modes of vibration of a uniform shear beam of the same height and total mass are also tabulated for comparison. (For the uniform shear beam, k and m are stiffness and mass per unit length).

TABLE 1

Mode	Frequency of 5 Story Model	Frequency of Uniform Shear Beam
ω_1	$0.285\ \sqrt{k/m}$	$0.285\ \sqrt{k/m}$
ω_2	0.831	0.854
ω_3	1.309	1.423
ω_4	1.683	1.992
ω_5	1.919	2.561

Simulation of the 10 and 15 Story Building

Figure 7 is the exact circuit for the 5 story building. To be consistent in the modeling procedure, the 10 and 15 story buildings would have 10 and 15 lumped masses respectively. Because of the limited number of electrical elements available for the study, it was found necessary to simulate the 10 and 15 story buildings with only 5 masses. Thus Figure 7 would be the circuit for these buildings also. The following sketch shows the method used in modeling the 10 story building.



It can be shown that, if the stiffness between the masses in the simulated model is half the interfloor stiffness of the 10 mass model, the height between floors is twice the interfloor height of the 10 mass model, and the lumped mass is 1.813 times the lumped mass of the 10 mass model, then the fundamental frequency and the moment about the base of the two systems will be equal. Table 2 gives a comparison between the frequencies of the first five modes of vibration of the 10 mass model and the 5 mass model that was used in this study. The first five modes of the uniform shear beam are also tabulated for comparison.

TABLE 2

Mode	Frequency of 10 Story Model	Frequency of Simulated Model	Frequency of Uniform Shear Beam
ω_1	$0.149\sqrt{k/m}$	$0.149\sqrt{k/m}$	$0.149\sqrt{k/m}$
ω_2	0.445	0.436	0.449
ω_3	0.731	0.688	0.748
ω_4	1.000	0.884	1.047
ω_5	1.247	1.008	1.346

Since the contribution to base shear stresses from modes higher than the third is very small, the use of any one of the above three systems yields essentially the same results.

The 15 mass model building is simulated in a like manner. If the stiffness between masses in the simulated model is one third the stiffness of the model with 15 lumped masses, the height between masses is three times the interfloor height of the 15 mass model, and the lumped mass is 2.632 times the mass of the 15 mass model, the fundamental frequency and the moment about the base of the two systems will be equal. Table 3 gives a comparison between the frequencies of the first five modes of vibration of the 15 mass model and the simulated model that was used. Again the first five modes of the uniform shear beam are tabulated for comparison.

TABLE 3

Mode	Frequency of 15 Story Model	Frequency of Simulated Model	Frequency of Uniform Shear Beam
ω_1	$0.101 \sqrt{k/m}$	$0.101 \sqrt{k/m}$	$0.101 \sqrt{k/m}$
ω_2	0.303	0.296	0.304
ω_3	0.501	0.466	0.507
ω_4	0.695	0.599	0.709
ω_5	0.881	0.693	0.912

The time base change, N, for each earthquake film record is different because of a difference in the original time scales on the accelerograms. Since each of three building periods was subjected to four earthquake records, a total of 12 "runs" was made. The theoretical electrical frequencies corresponding to the five modes of vibration were calculated for the 12 runs by the formula

$$f_{elec.} = N f_{mech.} \tag{12}$$

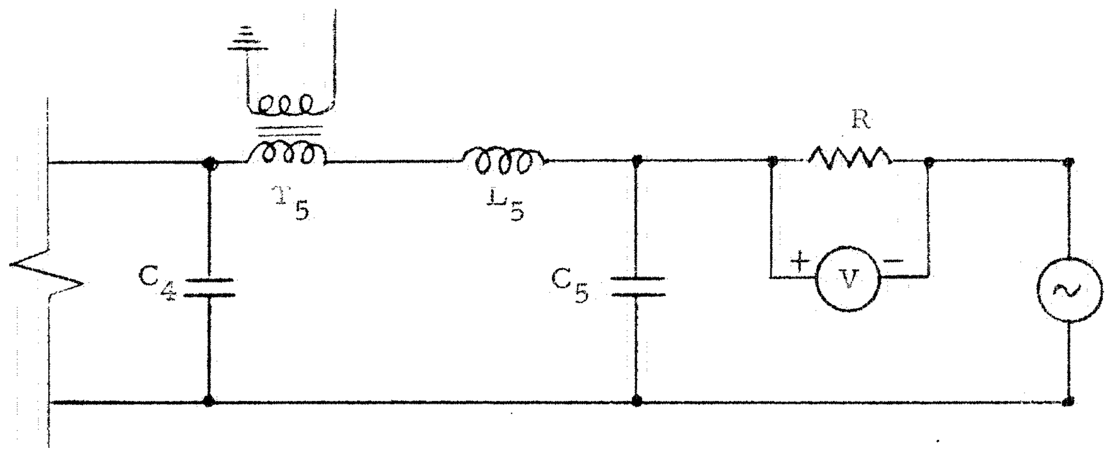
where $f_{mech.}$ for the first mode was 2 cps, 1 cps, and 0.667 cps for the 5, 10, and 15 story buildings respectively. Tables 1, 2, and 3 give the relationships for the frequencies of the other modes. The inductances and capacitances in the circuit were then adjusted so that the experimental resonant frequencies corresponded to those calculated by Equation (12). This data is available for reference in Appendix A.

Circuit Damping

It was stated earlier that a linear combination of absolute and interfloor viscous damping enables the arbitrary specification of the amount of damping in the first two modes of vibration of the building. In the nodal analogy, a resistance in parallel across a capacitance corresponds to absolute viscous damping while a resistance in parallel across an inductance corresponds to interfloor viscous damping. Figure 7 illustrates the analogy. The resistances were adjusted by trial and error until the damping in the first two modes was 0.10 of critical damping. The experimental determination of the damping in the third mode was 0.29 - 0.33 of critical damping while the damping in the fourth and fifth modes was high enough to make its measurement impossible.

Damping was evaluated by measuring the steady-state driving force of an oscillator (Figure 10). At resonance, minimum force is required to drive the system. Holding the output voltage of the oscillator constant, a curve of oscillator driving force vs. frequency can be constructed. The damping in the model was evaluated from the breadth of this resonance curve for the mode being investigated. As described previously, a multiple-degree-of-freedom system tends to vibrate as a

MEASUREMENT OF OSCILLATOR DRIVING FORCE



MEASUREMENT OF BASE SHEAR FORCE

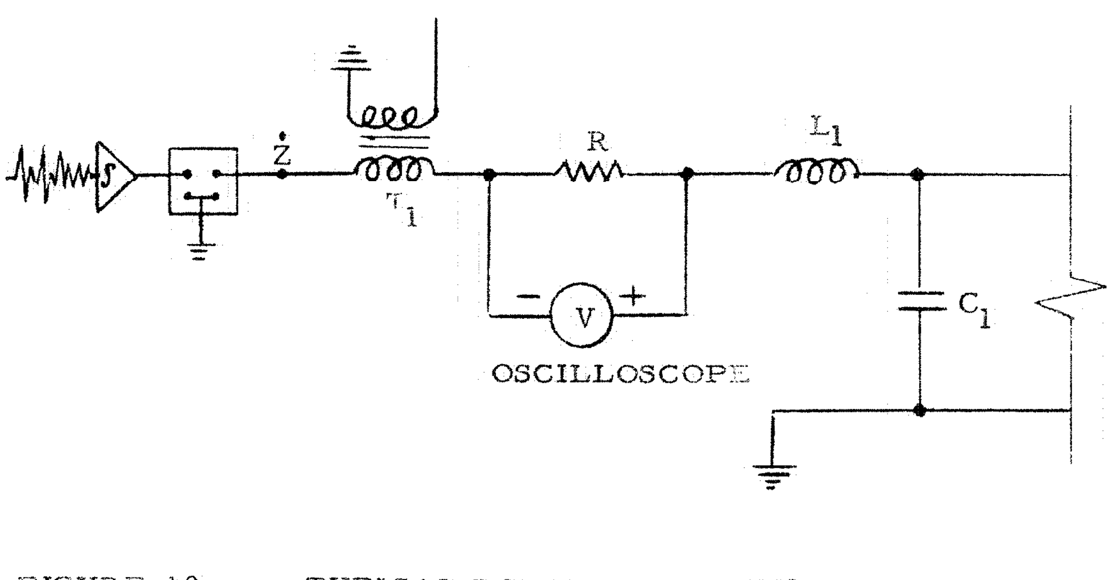


FIGURE 10. TYPICAL FORCE MEASURING CIRCUITS

single-degree-of-freedom system in a region near its resonant frequencies if the damping is small, less than 0.40 critical (12). Thus the damping is given by

$$c/c_c = \frac{\Delta f}{2 f_n} \qquad (13)$$

where

c/c_c = ratio of actual damping to critical damping

f_n = resonant frequency

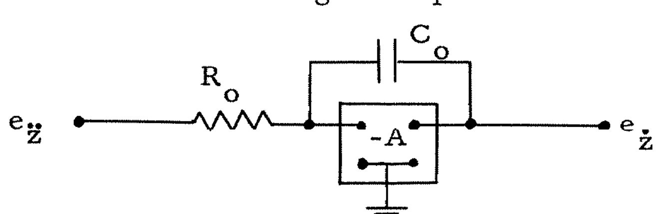
Δf = width of the force input curve at $\sqrt{2}$ times minimum input.

The values of R_i and R_a used in the experiment are recorded in Appendix A. It should be noted that these resistances are applicable to the specific computer elements used in this experiment. Since other inductances, capacitances, and especially transformers have different electrical properties, the amount of additional resistance required to give the circuit the proper damping characteristics would necessarily be different.

Circuit Calibration

The film records provide a variable voltage of the same form as the earthquake accelerograms, but it is necessary to calibrate the circuit before proceeding with the calculations. The calibration of the circuit is derived as follows.

$e_{\ddot{z}}$ = acceleration voltage at input of the integrator.



$$e_{\dot{z}} = \frac{1}{R_o C_o} \int e_{\ddot{z}} \, dt \tag{14}$$

= voltage at the base of the building.

Using the nodal analogy scale factors, it can be readily shown that

$$F = \frac{R_o C_o}{R_f C} \cdot \frac{S_s}{\ddot{z}_s} \cdot \frac{\ddot{z}}{b} \cdot jm \text{ - pounds} \tag{15}$$

$$M_\phi = \frac{R_o C_o}{R_m C} \cdot \frac{M_s}{\ddot{z}_s} \cdot \frac{\ddot{z}}{b} \cdot j'mh \text{ - lb-ft.} \tag{16}$$

where

$R_o C_o$ = integrator coefficient

C = capacitance representing the lumped mass

R_f = resistance through which the shear force current is measured (See Figure 10)

R_m = resistance through which the base moment current is measured

S_s = shear force scale divisions on the oscilloscope

M_s = moment scale divisions on the oscilloscope

$\ddot{z}$ = magnitude of ground acceleration at some point on the accelerogram in $ft/sec.^2$

$\ddot{z}_s$ = oscilloscope scale divisions for point on the accelerogram corresponding to $\ddot{z}$

b = acceleration attenuation factor from the oscilloscope

j = 1.000 for 5 story building

= 1.813 for 10 story building

= 2.632 for 15 story building

j' = 1.000 for 5 story building

= 3.626 for 10 story building

= 7.896 for 15 story building

h = height of one story in feet

m = lumped mass of one story in lb.-sec^2/ft.

Experimental Errors

The largest source of error in the experimental work is in the reading of the scale divisions on the oscilloscope. The start of the response was "zeroed" and the peak response was read. An error in this reading of 0.2 of a scale division would introduce an average error of 5 percent. Another source of error is the internal resistance of the transformers and other computer elements. In several instances, the error in the observed frequencies in comparison to the calculated frequencies was over 4 percent in the higher modes. In almost every case, the observed frequencies are on the high side, thus more closely simulating the frequencies of the 10 and 15 mass models (see Tables 2 and 3). The error in the absolute magnitude of the base shear forces and base moments could in some instances be as much as 10 percent due to the combined inaccuracies in reading and inaccuracies in the electrical components. However, the error in the relative change of the measured quantities as the foundation compliance parameter is changed is estimated to be less than 5 percent.

Foundation Damping

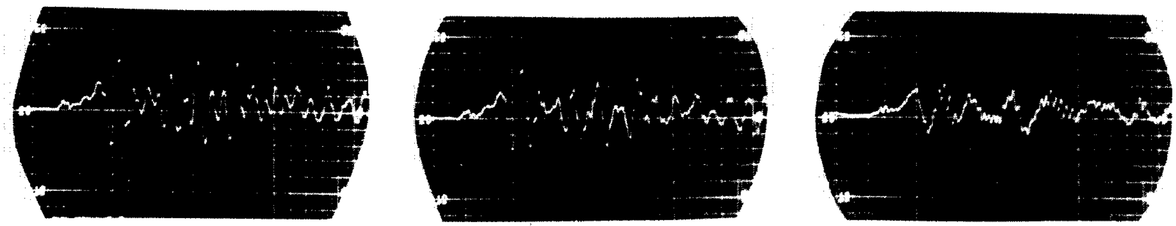
Little is known of the actual energy dissipation in the foundation soil caused by the rocking of a rigid body. It would be conservative to let the building models used in this investigation have zero damping in the foundation. This could not be achieved because of the inherent internal resistance in the transformers and the inherent resistance in the inductance L_ϕ. It is estimated that the foundation rocking motion was actually damped 0.02 of critical damping.

VII. EXPERIMENTAL RESULTS

Typical computer solutions for the base shear force in a 5, 10, and 15 story building appear in Figures 11-12, 13-14, and 15-16 respectively, where the time scale and shear force scale are as indicated. The lengthening of the period of vibration with increase in foundation compliance can be seen in these figures. In most cases, the maximum base shear occurred simultaneously with the peak acceleration force (Figures 2-5). This is especially noticeable for the 1934 El Centro earthquake and the 1949 Olympia earthquake where a large acceleration pulse occurred in the latter portion of the record. The maximum base shears are tabulated in Appendix A.

Typical computer solutions for the base moment in a 5, 10, and 15 story building appear in Figures 17-18, 19-20, and 21-22 respectively.

The effect of the foundation compliance on the maximum base shear force in a typical 5 story building is plotted in Figure 23. When the five story building is subjected to the 1940 El Centro shock, foundation compliance has a considerable effect in reducing the shear if the compliance ratio, k_ϕ/h^2k, is less than 12. In this discussion, it will be stated that the stress reduction is not significant until it is below 0.80 of the magnitude of the shear in a building on a rigid foundation. However, when the five story building is subjected to the 1934 El Centro shock, the shear stress is still unchanged at a compliance ratio of 2.5 and there is no significant



∞ 7.2 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

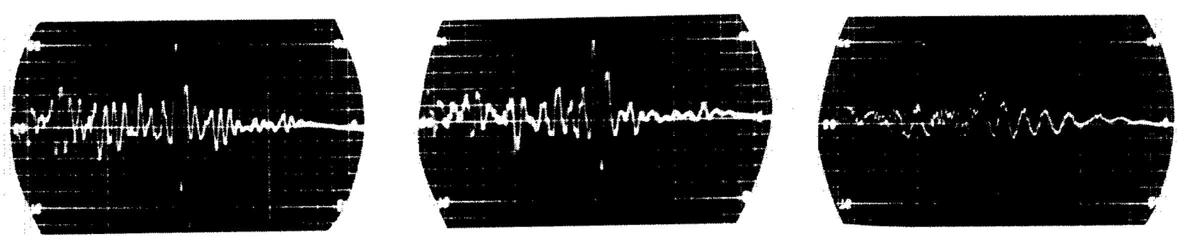
11.7 m 11.7 m 11.7 m

Base Shear Force - Pounds per Scale Division

0.52 0.52 0.52

Time Base - Seconds per Scale Division

Taft, California
Earthquake of July 21, 1952. Component S 69 E.



∞ 4.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

15.4 m 15.4 m 15.4 m

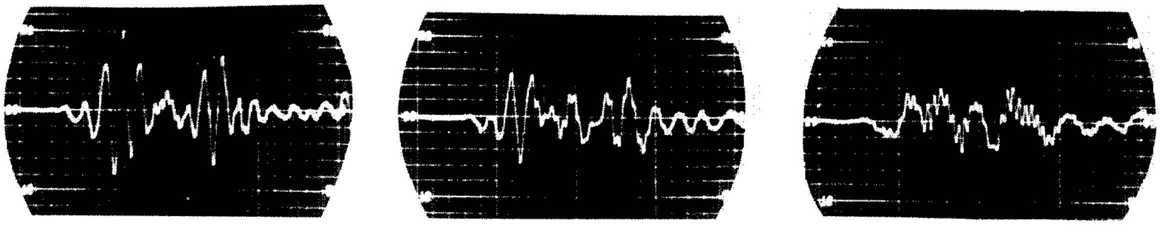
Base Shear Force - Pounds per Scale Division

0.48 0.48 0.48

Time Base - Seconds per Scale Division

Olympia, Washington
Earthquake of April 13, 1949. Component S 80 W.

FIGURE 11. Typical Computer Solutions for the Base Shear Force in a 5 Story Building



∞ 5.2 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} = \frac{k_\phi}{h^2 k}$$

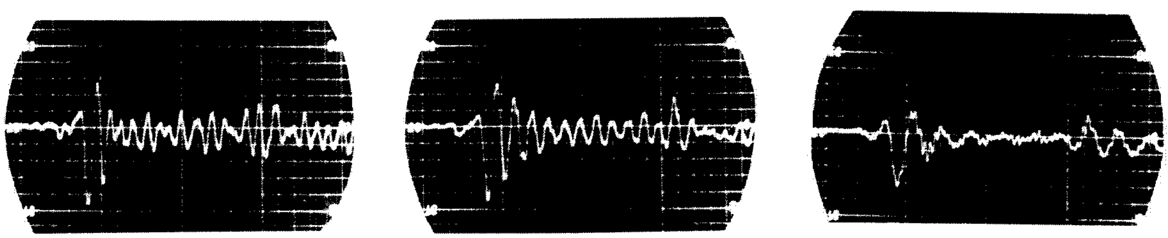
23.8 m 23.8 m 17.9 m

Base Shear Force - Pounds per Scale Division

0.46 0.46 0.46

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of May 18, 1940. Component N-S.



∞ 4.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} = \frac{k_\phi}{h^2 k}$$

14.3 m 14.3 m 14.3 m

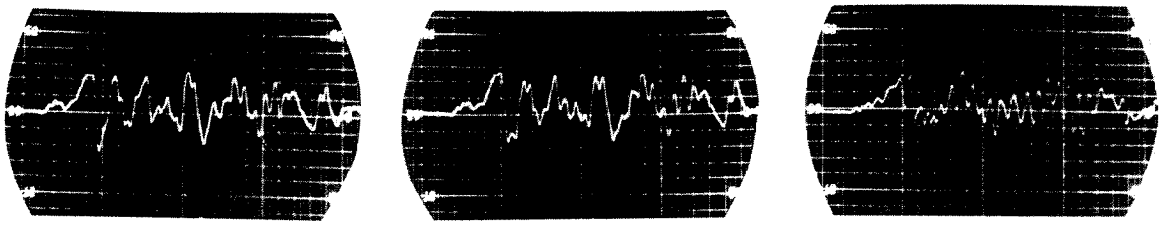
Base Shear Force - Pounds per Scale Division

0.61 0.61 0.61

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of Dec. 30, 1934. Component N-S.

FIGURE 12. Typical Computer Solutions for the Base Shear Force in a 5 Story Building



∞ 14.1 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

10.0 m 10.0 m 10.0 m

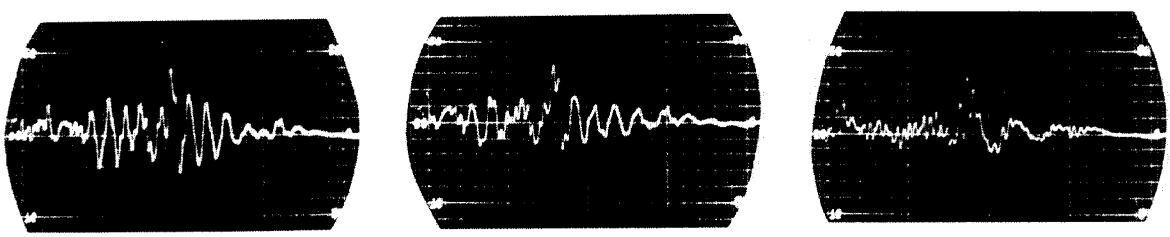
Base Shear Force - Pounds per Scale Division

0.52 0.52 0.52

Time Base - Seconds per Scale Division

Taft, California

Earthquake of July 21, 1952. Component S 69 E.



∞ 11.3 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

15.1 m 15.1 m 15.1 m

Base Shear Force - Pounds per Scale Division

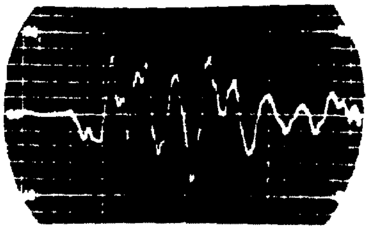
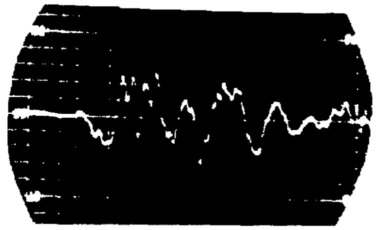
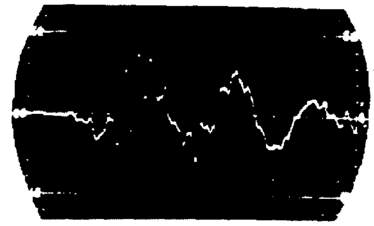
0.84 0.84 0.84

Time Base - Seconds per Scale Division

Olympia, Washington

Earthquake of April 13, 1949. Component S 80 W.

FIGURE 13. Typical Computer Solutions for the Base Shear Force in a 10 Story Building

∞ 10.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_{\phi}}{h^2 k}$$

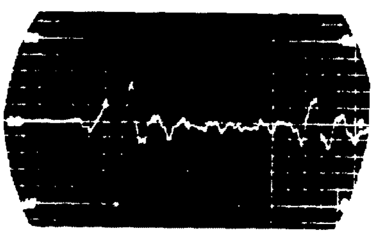
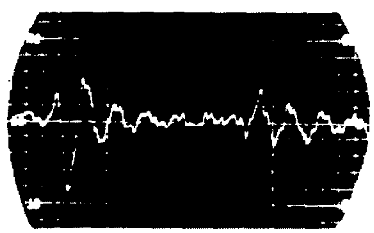
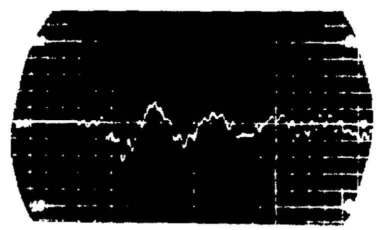
14.75 m 14.75 m 14.75 m

Base Shear Force - Pounds per Scale Division

0.46 0.46 0.46

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of May 18, 1940. Component N-S.

∞ 8.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_{\phi}}{h^2 k}$$

17.2 m 17.2 m 17.2 m

Base Shear Force - Pounds per Scale Division

0.61 0.61 0.61

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of Dec. 30, 1934. Component N-S.

FIGURE 14. Typical Computer Solutions for the Base Shear Force in a 10 Story Building

∞ 22 1.0

Building Compliance / Foundation Compliance $- \frac{k_\phi}{h^2 k}$

9.69 m 9.69 m 9.69 m

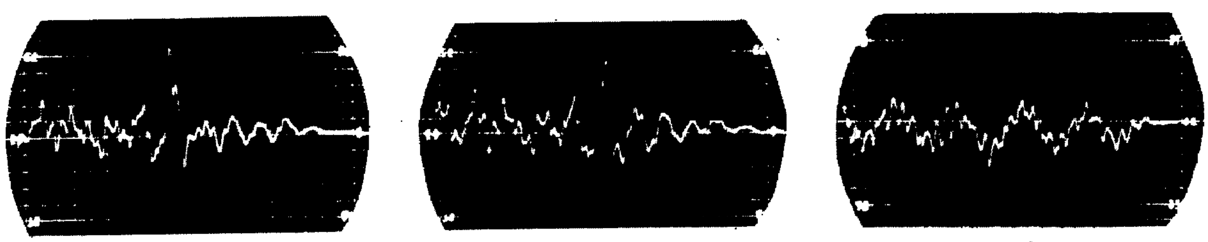
Base Shear Force - Pounds per Scale Division

0.52 0.52 0.52

Time Base - Seconds per Scale Division

Taft, California

Earthquake of July 21, 1952. Component S 69 E.



∞ 16.7 1.0

Building Compliance / Foundation Compliance $- \frac{k_\phi}{h^2 k}$

14.3 m 14.3 m 14.3 m

Base Shear Force - Pounds per Scale Division

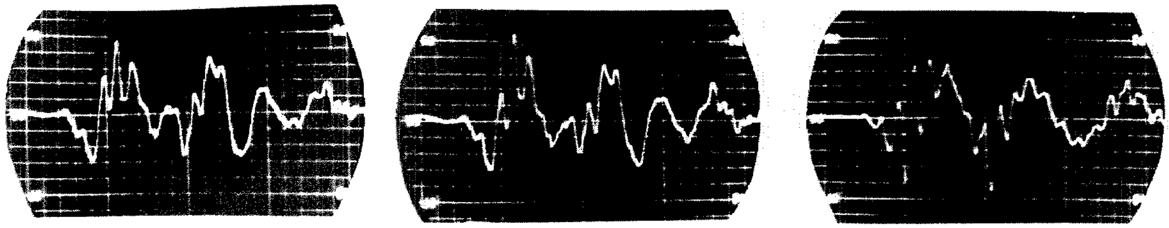
0.84 0.84 0.84

Time Base - Seconds per Scale Division

Olympia, Washington

Earthquake of April 13, 1949. Component S 80 W.

FIGURE 15. Typical Computer Solutions for the Base Shear Force in a 15 Story Building



∞ 15.4 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

14.6 m 14.6 m 14.6 m

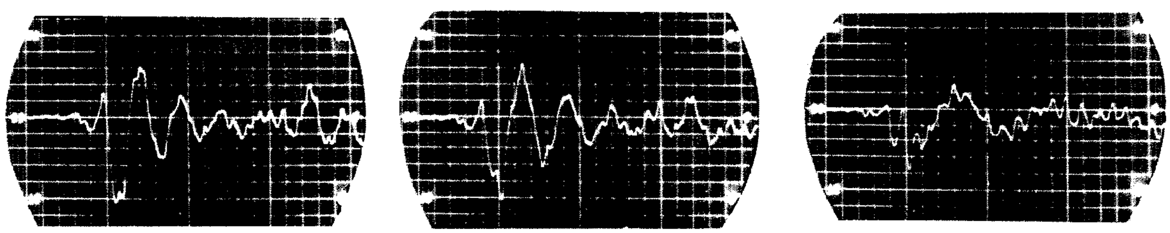
Base Shear Force - Pounds per Scale Division

0.46 0.46 0.51

Time Base - Seconds per Scale Division

El Centro, California

Earthquake of May 18, 1940, Component N-S.



∞ 13.4 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

12.8 m 12.8 m 12.8 m

Base Shear Force - Pounds per Scale Division

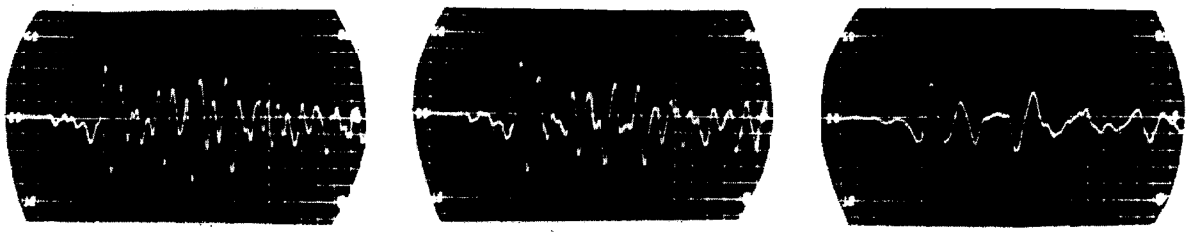
0.61 0.61 0.61

Time Base - Seconds per Scale Division

El Centro, California

Earthquake of Dec. 30, 1934, Component N-S.

FIGURE 16. Typical Computer Solutions for the Base Shear Force in a 15 Story Building



∞ 7.2 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

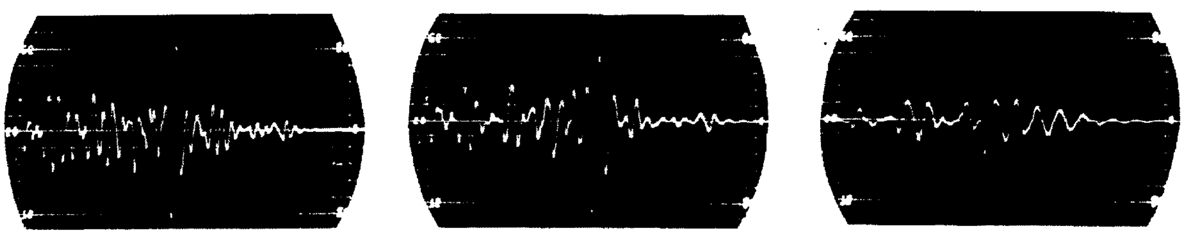
31.3 mh 31.3 mh 31.3 mh

Base Moment - Pound-feet per Scale Division

0.52 0.52 0.52

Time Base - Seconds per Scale Division

Taft, California
Earthquake of July 21, 1952. Component S 69 E



∞ 4.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

46.0 mh 46.0 mh 46.0 mh

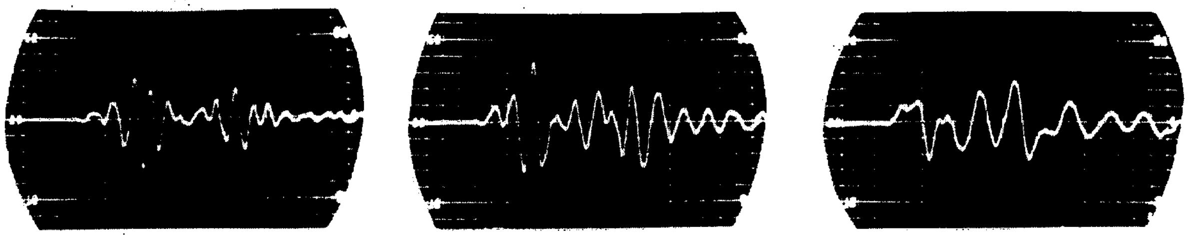
Base Moment - Pound-feet per Scale Division

0.48 0.48 0.48

Time Base - Seconds per Scale Division

Olympia, Washington
Earthquake of April 13, 1949. Component S 80 W.

FIGURE 17. Typical Computer Solutions for the Base Moment in a 5 Story Building



∞   5.2   1.0

$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$

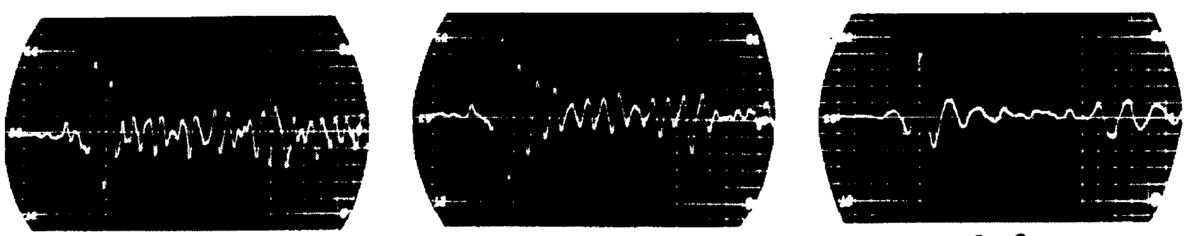
140 mh   69.7 mh   46.4 mh

Base Moment - Pound-feet per Scale Division

0.46   0.46   0.46

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of May 18, 1940. Component N-S.



∞   4.9   1.0

$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$

42.8 mh   42.8 mh   42.8 mh

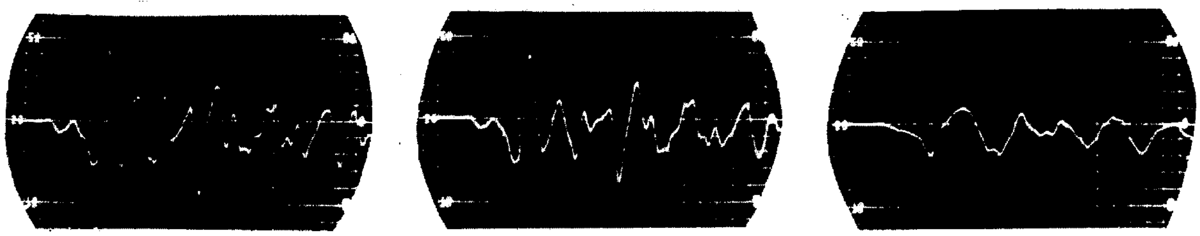
Base Moment - Pound-feet per Scale Division

0.61   0.61   0.61

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of Dec. 30, 1934. Component N-S.

FIGURE 18. Typical Computer Solutions for the Base Moment in a 5 Story Building



∞ | 14.1 | 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

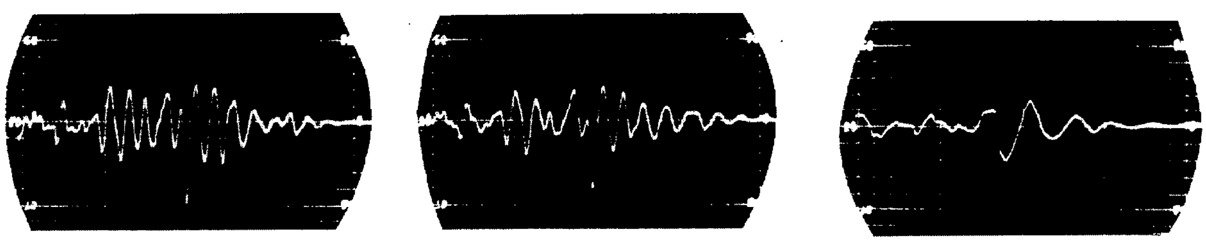
52.9 mh | 52.9 mh | 52.9 mh

Base Moment - Pound-feet per Scale Division

0.52 | 0.52 | 0.52

Time Base - Seconds per Scale Division

Taft, California
Earthquake of July 21, 1952. Component S 69 E.



∞ | 11.3 | 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

90.5 mh | 90.5 mh | 90.5 mh

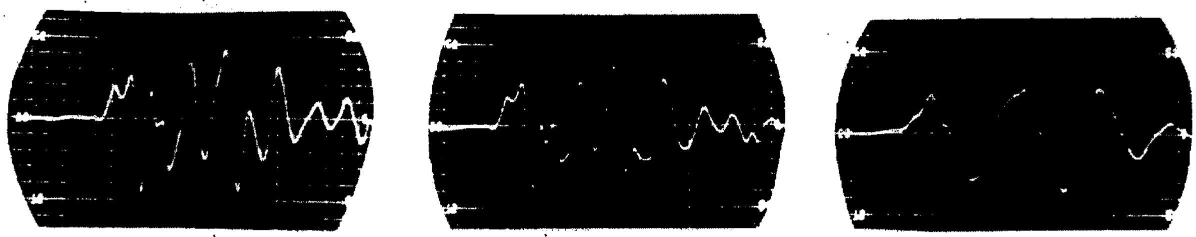
Base Moment - Pound-feet per Scale Division

0.84 | 0.84 | 0.84

Time Base - Seconds per Scale Division

Olympia, Washington
Earthquake of April 13, 1949. Component S 80 W.

FIGURE 19. Typical Computer Solutions for the Base Moment in a 10 Story Building



∞ 10.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

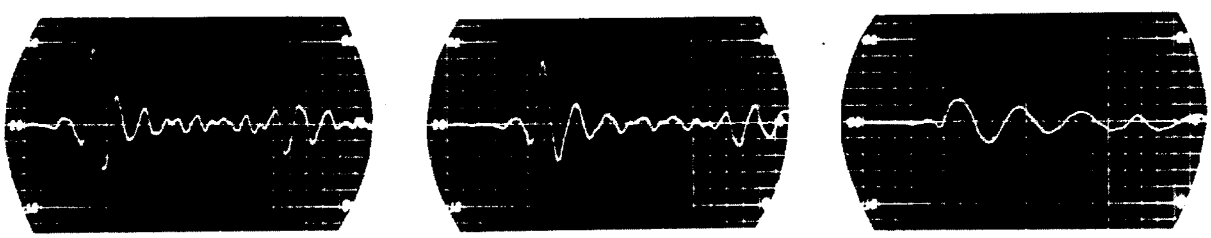
78.3 mh 78.3 mh 78.3 mh

Base Moment - Pound-feet per Scale Division

0.46 0.46 0.46

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of May 18, 1940. Component N-S.



∞ 8.9 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

138 mh 138 mh 138 mh

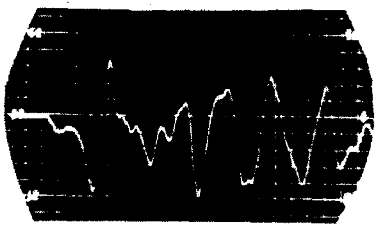
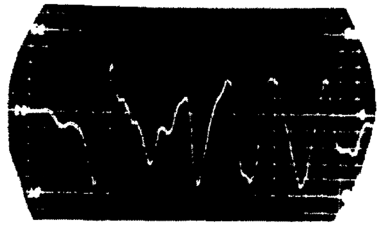
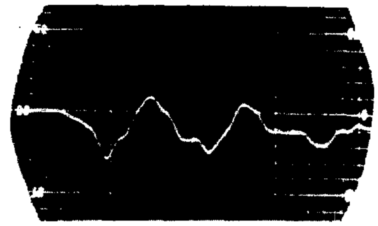
Base Moment - Pound-feet per Scale Division

0.61 0.61 0.61

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of Dec. 30, 1934. Component N-S.

FIGURE 20. Typical Computer Solutions for the Base Moment in a 10 Story Building

∞ | 22 | 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} = \frac{k_{\phi}}{h^2 k}$$

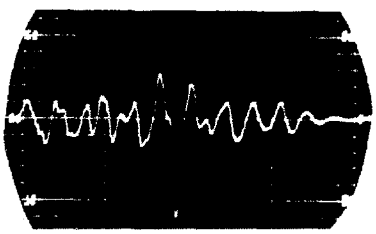
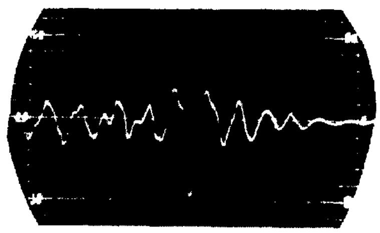
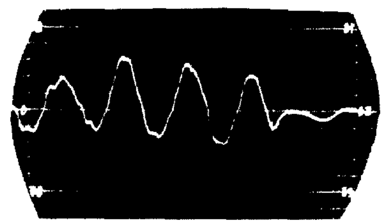
58.7 mh | 58.7 mh | 58.7 mh

Base Moment - Pound-feet per Scale Division

0.52 | 0.52 | 0.52

Time Base - Seconds per Scale Division

Taft, California
Earthquake of July 21, 1952. Component S 69 E.

∞ | 16.7 | 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} = \frac{k_{\phi}}{h^2 k}$$

128 mh | 160 mh | 63.9 mh

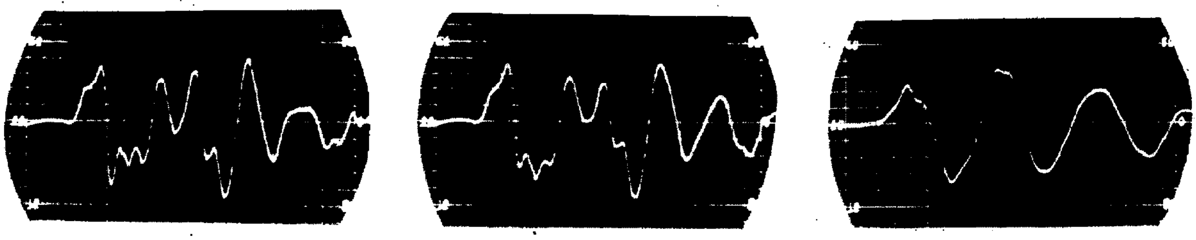
Base Moment - Pound-feet per Scale Division

0.84 | 0.84 | 0.84

Time Base - Seconds per Scale Division

Olympia, Washington
Earthquake of April 13, 1949. Component S 80 W.

FIGURE 21. Typical Computer Solutions for the Base Moment in a 15 Story Building



∞ 15.4 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

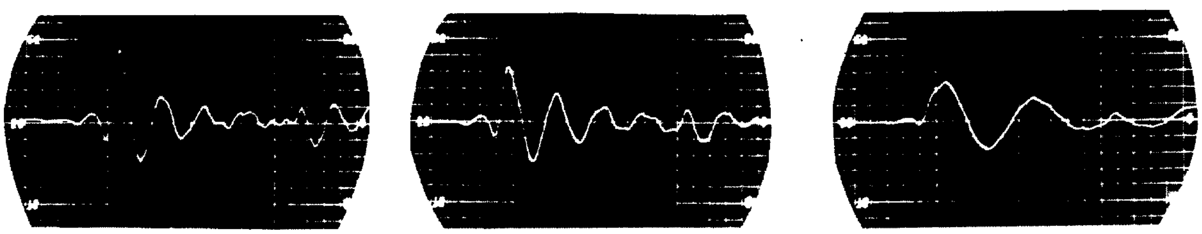
117 mh 117 mh 117 mh

Base Moment - Pound-feet per Scale Division

0.46 0.46 0.51

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of May 18, 1940. Component N-S.



∞ 13.4 1.0

$$\frac{\text{Building Compliance}}{\text{Foundation Compliance}} - \frac{k_\phi}{h^2 k}$$

193 mh 193 mh 121 mh

Base Moment - Pound-feet per Scale Division

0.61 0.61 0.61

Time Base - Seconds per Scale Division

El Centro, California
Earthquake of Dec. 30, 1934. Component N-S.

FIGURE 22. Typical Computer Solutions for the Base Moment in a 15 Story Building

reduction until the ratio becomes 1.5. The 1952 Taft earthquake also keeps the magnitude of the shear force high through a wide range of the compliance ratio. The Olympia shock magnifies the shear force slightly through part of this range.

The effect of foundation compliance on the maximum base shear force in a typical 10 story building is plotted in Figure 24. For this building, only the 1934 El Centro earthquake reduces the base shear significantly over the range of compliance ratio studied. It is interesting to note the magnification that occurs for the other three earthquakes when the compliance ratio is approximately 2 - 3. The 10 story building with a compliance ratio of 3.0 actually experiences a base shear almost 1.2 times what it experiences on a rigid foundation if subjected to the Taft earthquake. The 1940 El Centro shock has a significantly reduced effect over part of the compliance ratio range, but at 1.0, the base shear is not significantly reduced.

The effect of foundation compliance on the maximum shear force in a typical 15 story building is plotted in Figure 25. The maximum base shear force is actually magnified in the 15 story building over most of the compliance ratio range for the 1940 El Centro earthquake. The Taft and the 1934 El Centro shocks also maintain a significant base shear throughout the range.

It is of much interest to note that the same earthquake has different effects on the different height buildings. The 1934 El Centro shock permits a significant reduction in stress at very low

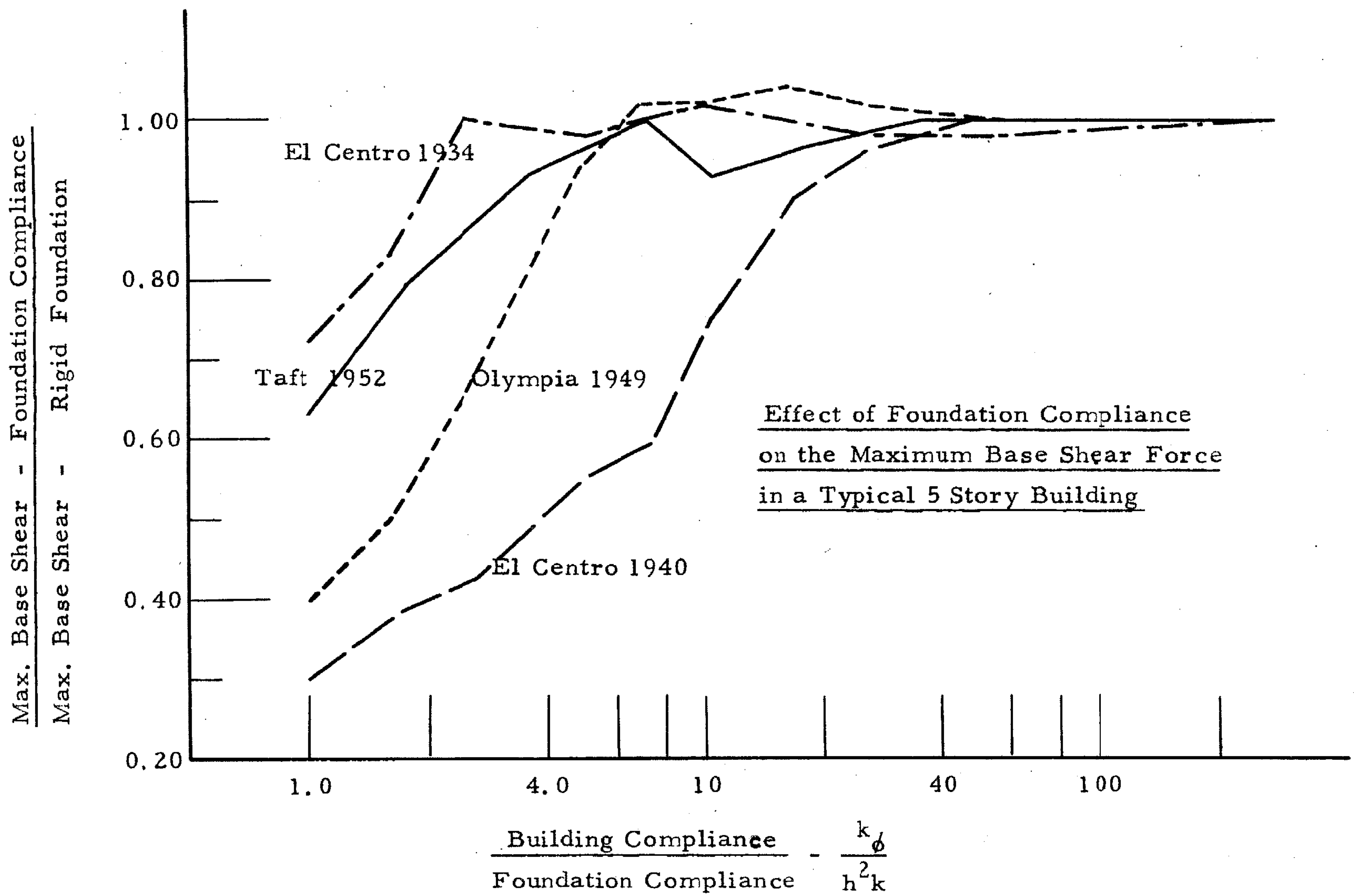

Max. Base Shear - Foundation Compliance / Max. Base Shear - Rigid Foundation
1.00
0.80
0.60
0.40
0.20
El Centro 1934
Taft 1952
Olympia 1949
El Centro 1940
Effect of Foundation Compliance
on the Maximum Base Shear Force
in a Typical 5 Story Building
1.0
4.0
10
40
100
Building Compliance / Foundation Compliance - $\frac{k_\phi}{h^2 k}$

FIGURE 23.

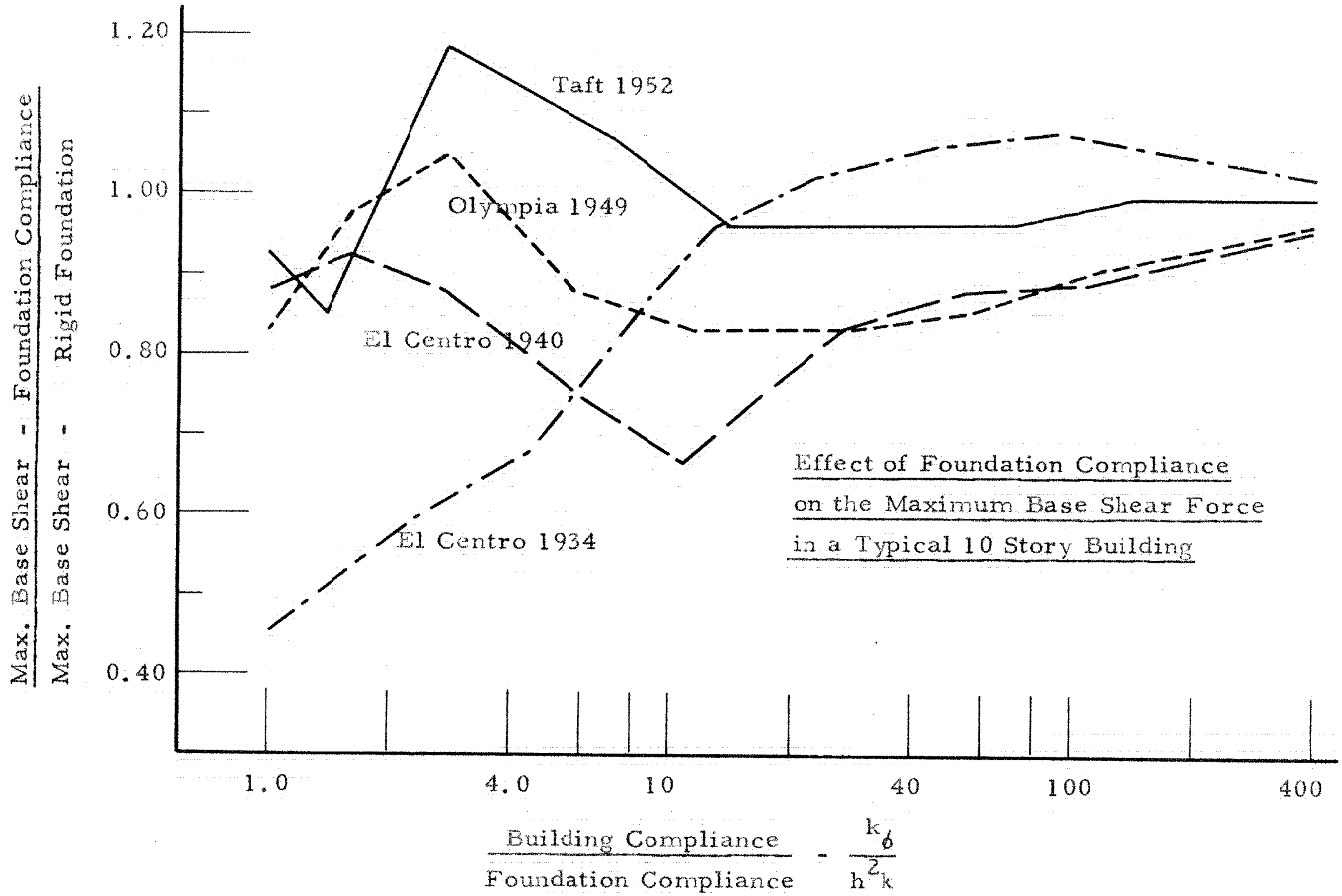
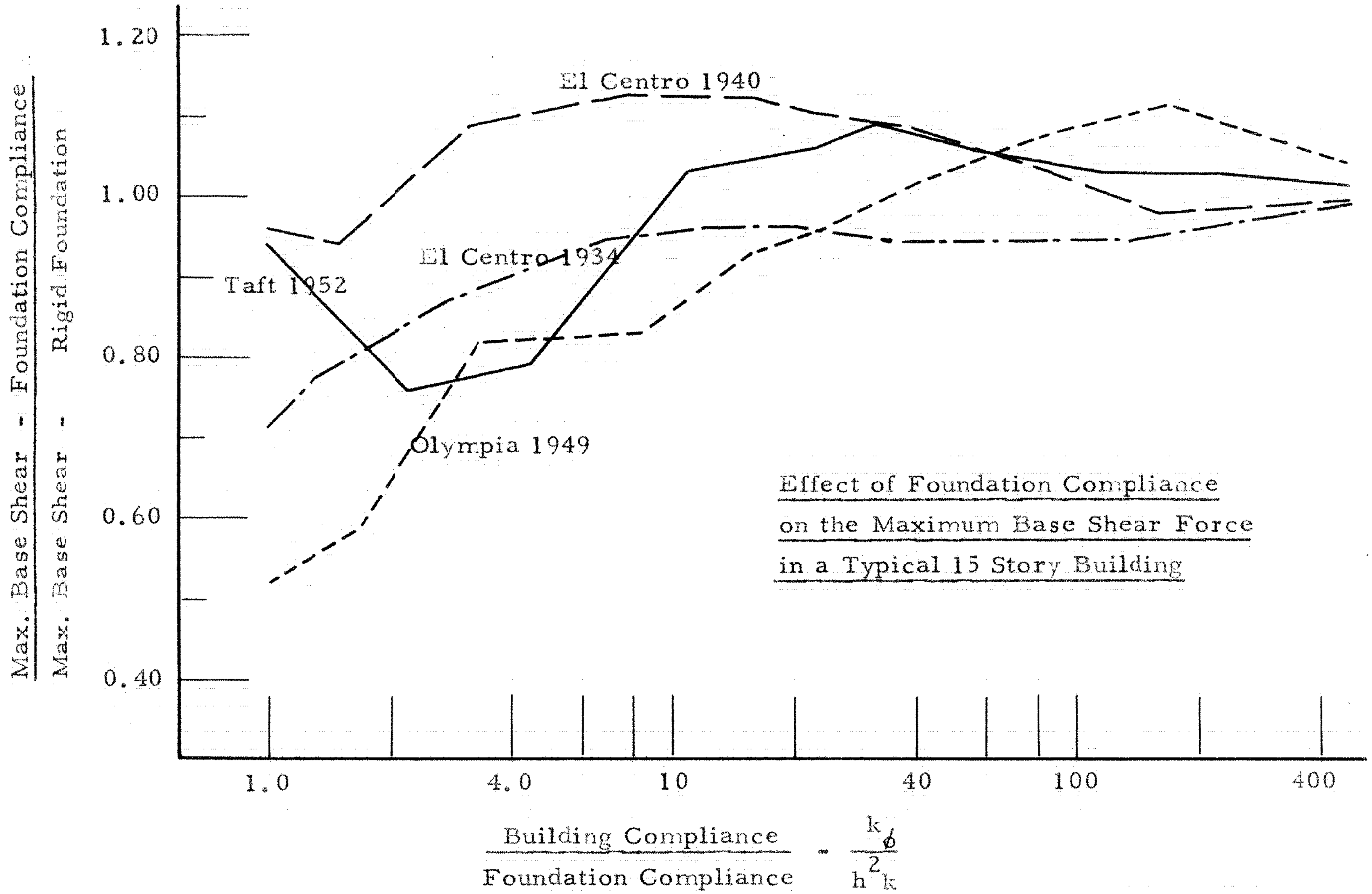

Max. Base Shear - Foundation Compliance
Max. Base Shear - Rigid Foundation
1.20
1.00
0.80
0.60
0.40
Taft 1952
Olympia 1949
El Centro 1940
El Centro 1934
Effect of Foundation Compliance
on the Maximum Base Shear Force
in a Typical 10 Story Building
1.0
4.0
10
40
100
400
Building Compliance
Foundation Compliance
- $\frac{k_\phi}{h^2 k}$

FIGURE 24.



Effect of Foundation Compliance on the Maximum Base Shear Force in a Typical 15 Story Building

FIGURE 25.

values of the compliance ratio in the 10 story building, but maintains a high stress level in the 5 and 15 story buildings. The 1940 El Centro earthquake maintains a high stress level in the 10 and 15 story buildings, but permits a significant reduction in stress in the 5 story building when the compliance ratio drops below 12. The 1949 Olympia earthquake maintains a high stress level in the 10 story building, but at low compliance ratio values, it drops below the 0.80 level in the 5 and 15 story buildings. The 1952 Taft shock maintains a very high stress level over the entire compliance ratio range in the 10 story building.

Figure 26 shows the effect of foundation compliance on the fundamental period of vibration of the typical 5, 10, and 15 story buildings. This curve was obtained experimentally by driving the top mass of the model with an oscillator and measuring the fundamental resonant frequency over a wide range of the ratio of building compliance to foundation compliance. All of the experimental points fell on the smooth curve of Figure 26. The fundamental period is increased by a factor of only 1.20 at a compliance ratio of 10. It is seen that the fundamental period of the undamped single-degree-of-freedom system (Figure 1) is increased only 1.05 at the same compliance ratio.

Figures 27, 28, and 29 show the effect of increasing the fundamental period on the maximum base shear force in a typical 5, 10, and 15 story building respectively. These curves were derived from the relationship between the fundamental period and the

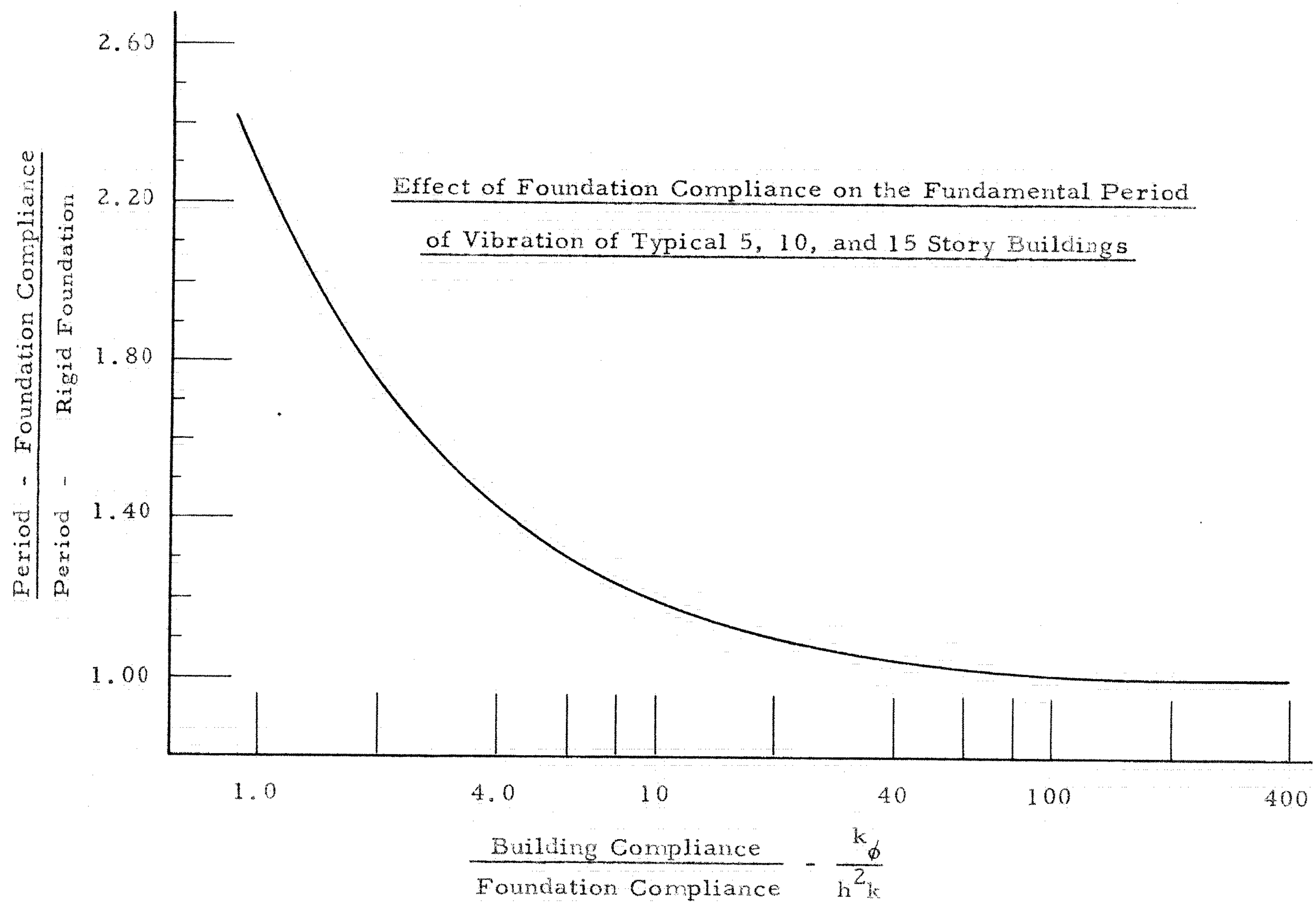

Effect of Foundation Compliance on the Fundamental Period of Vibration of Typical 5, 10, and 15 Story Buildings
Period - Foundation Compliance / Period - Rigid Foundation
2.60
2.20
1.80
1.40
1.00
1.0
4.0
10
40
100
400
Building Compliance / Foundation Compliance - $\frac{k_\phi}{h^2 k}$

FIGURE 26.

compliance ratio plotted in Figure 26. Essentially, they show the same information that is plotted in Figures 23-25. It is of interest to examine each figure in turn. Figure 27 shows that if the fundamental period of the 5 story building model is doubled, the maximum base shear force is reduced 20 percent when subjected to the 1934 El Centro earthquake, 26 percent when subjected to the Taft earthquake, and 53 percent and 64 percent for the Olympia and 1940 El Centro earthquakes respectively. Figure 28 shows that if the fundamental period of the 10 story building is doubled, the maximum base shear force is reduced only 6 percent for the Olympia and 1940 El Centro shocks, 10 percent for the Taft shock, but almost 50 percent for the 1934 El Centro earthquake. Figure 29 shows that the doubling of the fundamental period of the 15 story building causes a reduction of the maximum base shear force of 5 percent for the 1940 El Centro earthquake, 15 percent for the Taft earthquake, slightly over 20 percent for the 1934 El Centro earthquake, and 45 percent for the Olympia earthquake.

Figure 30 shows the effect of foundation compliance on the maximum base moment in a typical 5 story building. The results are similar to those for the maximum base shear force. The 1934 El Centro earthquake shows no significant reduction in the base moment over the entire range of compliance ratio examined. Figure 31 shows the effect of foundation compliance on the maximum base moment in a typical 10 story building. In this case, all four earthquakes show significant base moment reduction when

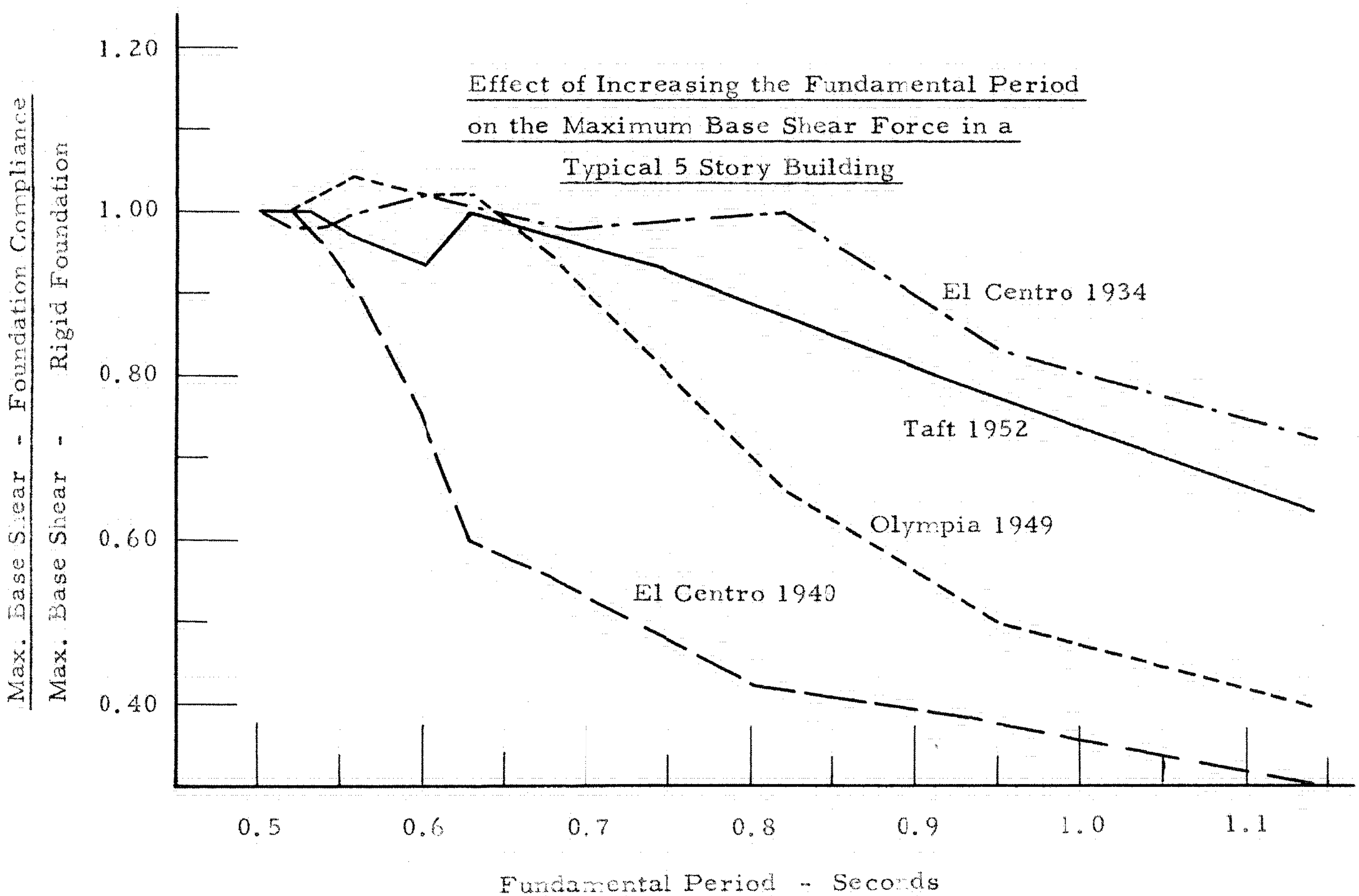

Effect of Increasing the Fundamental Period
on the Maximum Base Shear Force in a
Typical 5 Story Building
Max. Base Shear - Foundation Compliance
Max. Base Shear - Rigid Foundation
1.20
1.00
0.80
0.60
0.40
0.5
0.6
0.7
0.8
0.9
1.0
1.1
Fundamental Period - Seconds
El Centro 1934
Taft 1952
Olympia 1949
El Centro 1940

FIGURE 27.

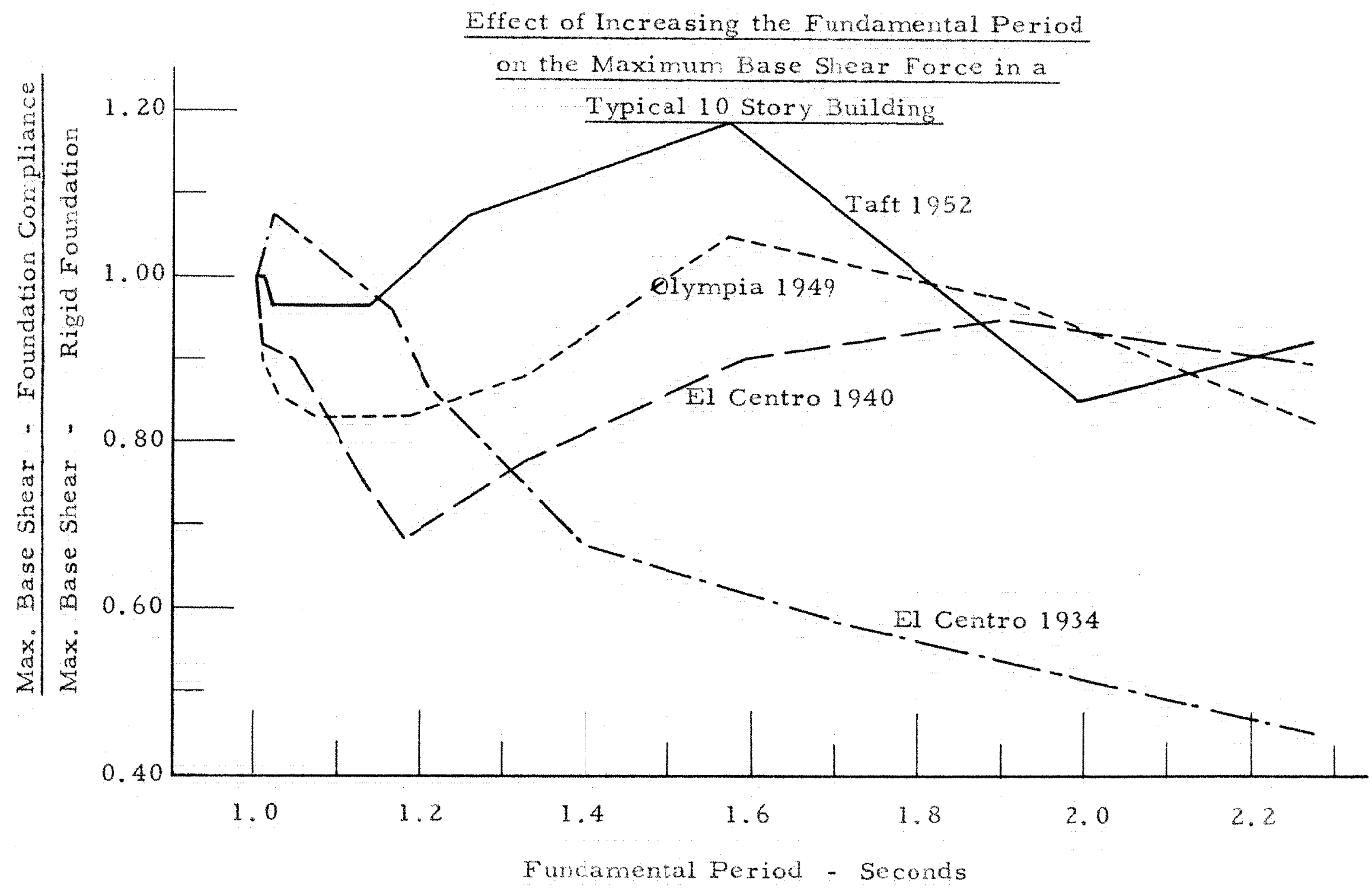

Effect of Increasing the Fundamental Period
on the Maximum Base Shear Force in a
Typical 10 Story Building
Max. Base Shear - Foundation Compliance
Max. Base Shear - Rigid Foundation
1.20
1.00
0.80
0.60
0.40
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Fundamental Period - Seconds
Taft 1952
Olympia 1949
El Centro 1940
El Centro 1934

FIGURE 28.

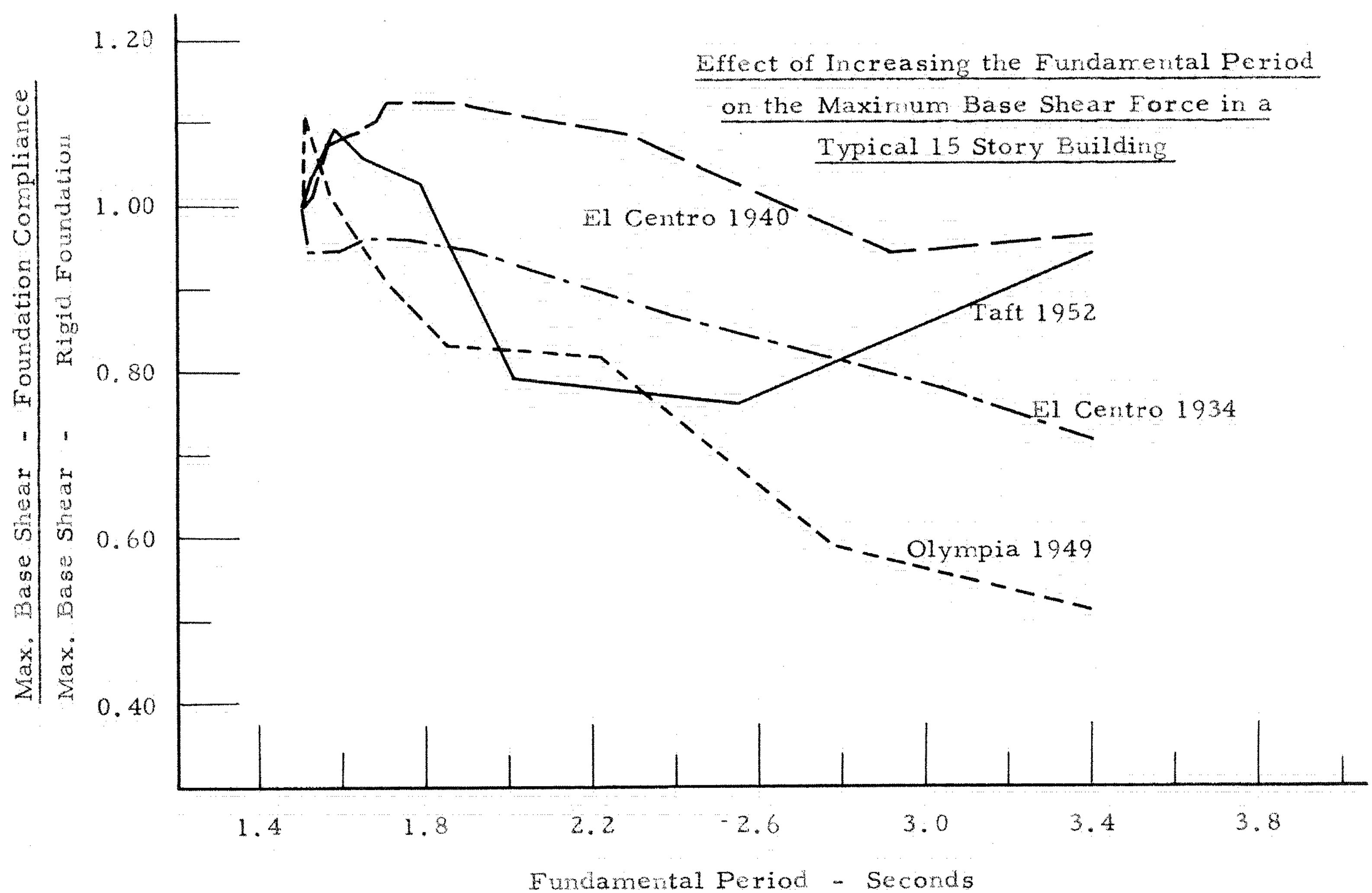

Effect of Increasing the Fundamental Period
on the Maximum Base Shear Force in a
Typical 15 Story Building
El Centro 1940
Taft 1952
El Centro 1934
Olympia 1949
Max. Base Shear - Foundation Compliance
Max. Base Shear - Rigid Foundation
1.20
1.00
0.80
0.60
0.40
1.4
1.8
2.2
2.6
3.0
3.4
3.8
Fundamental Period - Seconds

FIGURE 29.

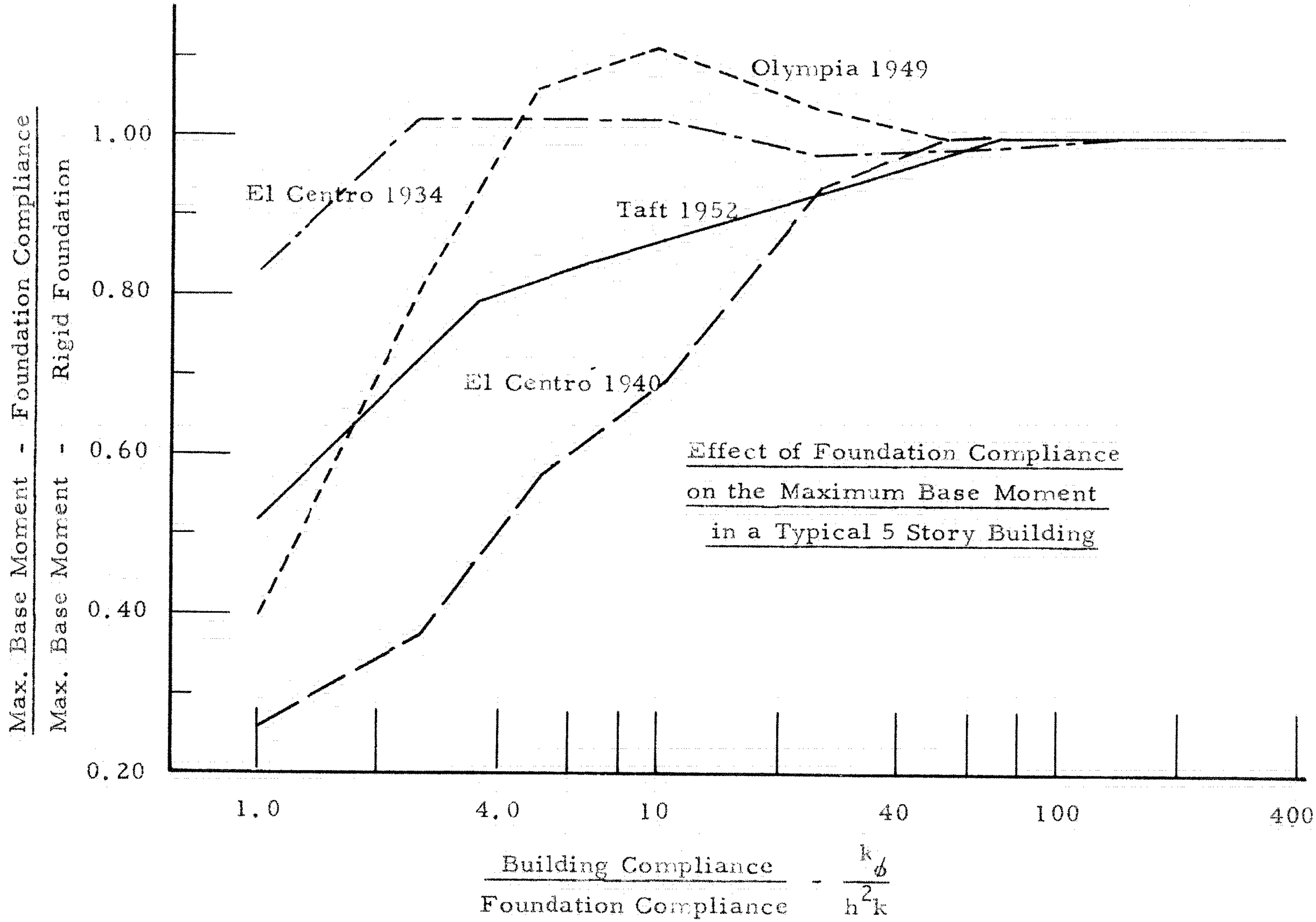

Olympia 1949
El Centro 1934
Taft 1952
El Centro 1940
Effect of Foundation Compliance
on the Maximum Base Moment
in a Typical 5 Story Building
Max. Base Moment - Foundation Compliance
Max. Base Moment - Rigid Foundation
1.00
0.80
0.60
0.40
0.20
1.0
4.0
10
40
100
400
Building Compliance
Foundation Compliance
- $\frac{k_\phi}{h^2 k}$

FIGURE 30.

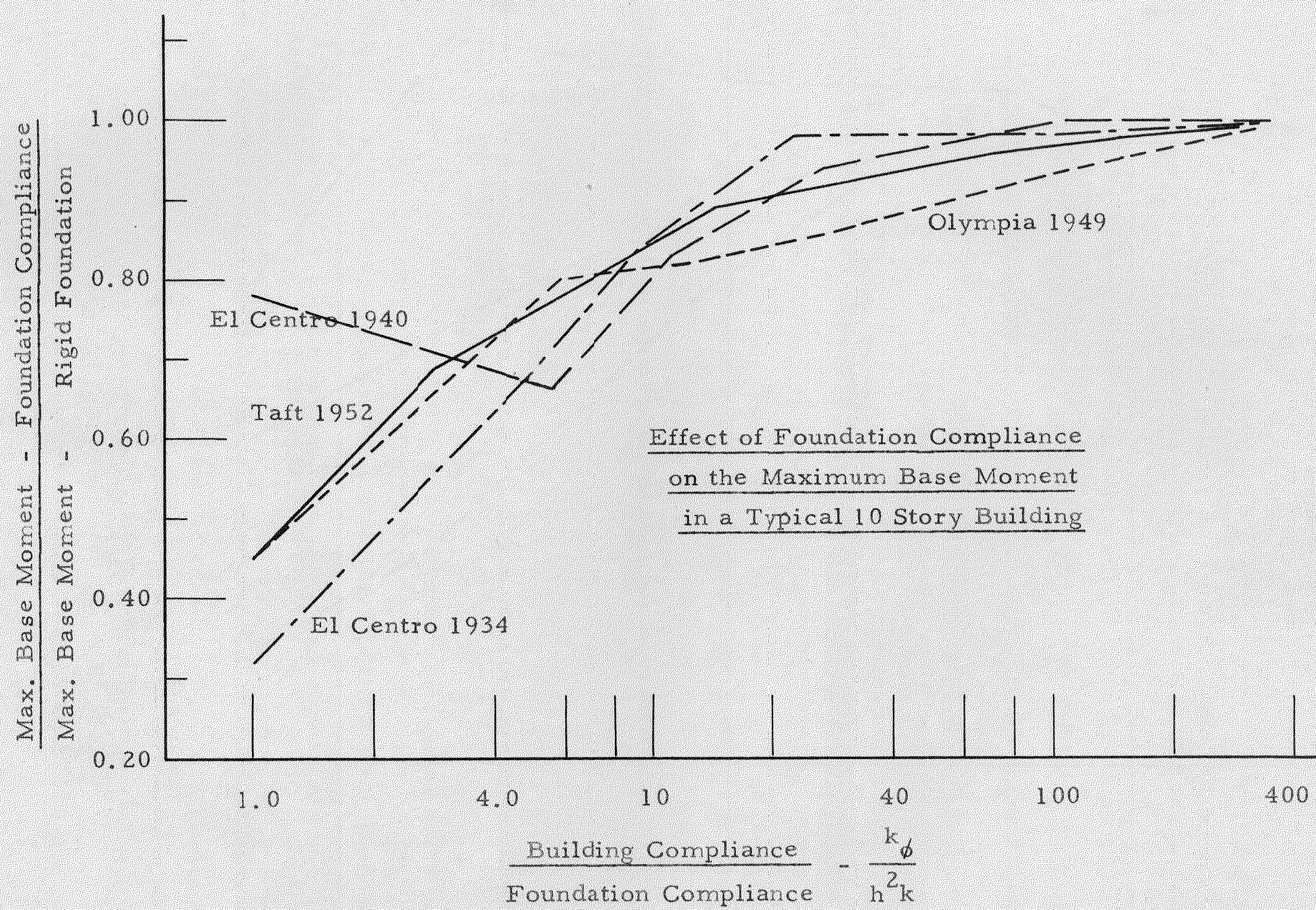

Max. Base Moment - Foundation Compliance
Max. Base Moment - Rigid Foundation
1.00
0.80
0.60
0.40
0.20
El Centro 1940
Taft 1952
El Centro 1934
Olympia 1949
Effect of Foundation Compliance
on the Maximum Base Moment
in a Typical 10 Story Building
1.0
4.0
10
40
100
400
Building Compliance
Foundation Compliance
- $\frac{k_\phi}{h^2 k}$

FIGURE 31.

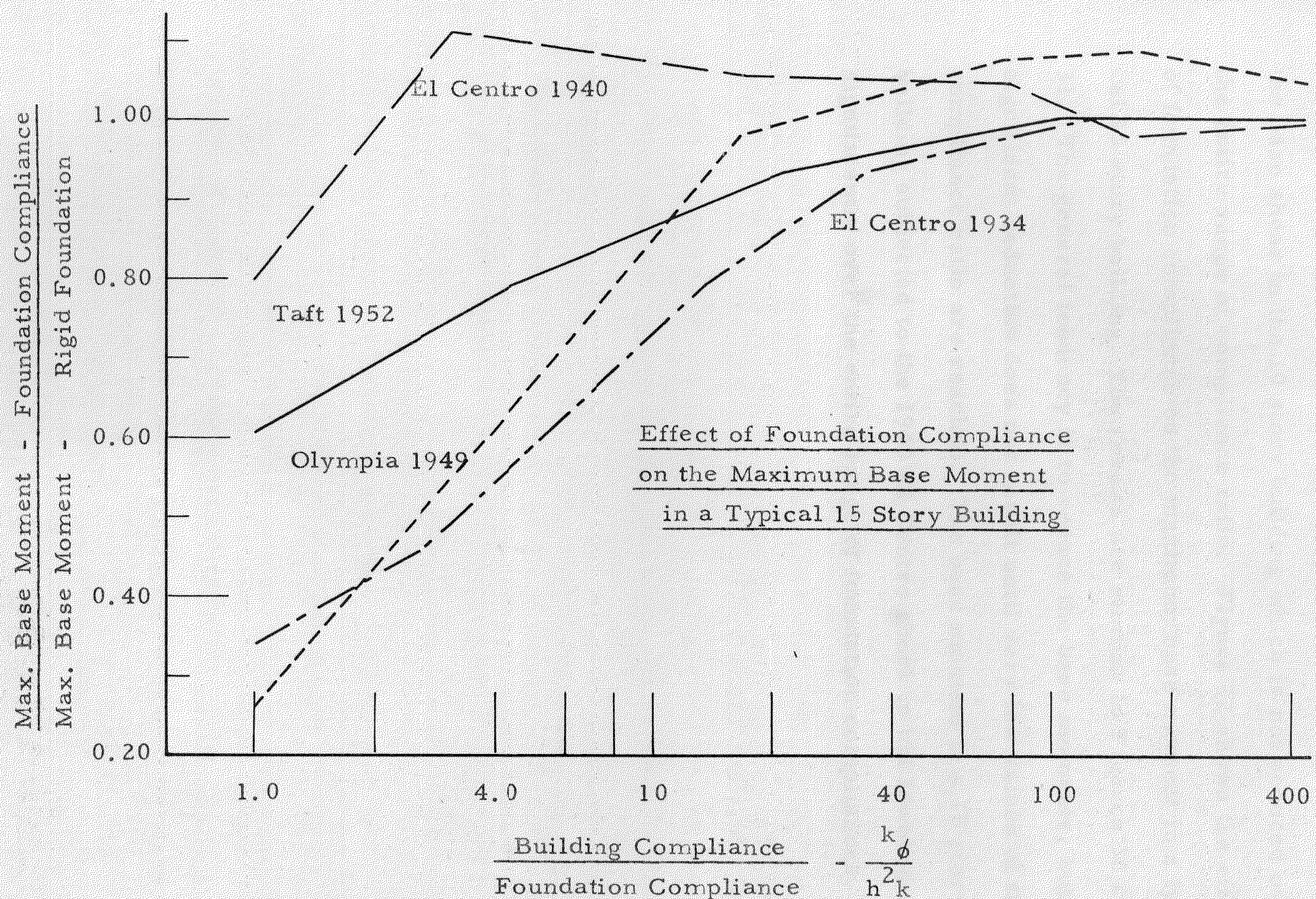

El Centro 1940
Taft 1952
Olympia 1949
El Centro 1934
Effect of Foundation Compliance
on the Maximum Base Moment
in a Typical 15 Story Building
1.00
0.80
0.60
0.40
0.20
Max. Base Moment - Foundation Compliance
Max. Base Moment - Rigid Foundation
1.0
4.0
10
40
100
400
Building Compliance / Foundation Compliance - $\frac{k_\phi}{h^2 k}$

FIGURE 32.

a compliance ratio as low as 6 is reached. This is contrary to the base shear in the 10 story building which is not reduced over the entire range of compliance ratio. Figure 32 shows the effect of foundation compliance on the maximum base moment in a typical 15 story building. The results are similar to Figures 30 and 31. The general tendency is to reduce the base moments, but significant reduction does not result until very low values of the compliance ratio are reached. The base moment in a 15 story building subjected to the 1940 El Centro shock is not lowered significantly over the entire range of foundation compliances.

VIII. PROPERTIES OF REAL BUILDINGS

The foregoing results are for the building models used in this study. These models had certain specified properties, and it is now necessary to interpret these properties in terms of actual buildings.

The Motion and Periods of Real Buildings

Observations have shown that real buildings tend to deflect in shear, that is, the floors tend to move parallel to each other and not to rotate with respect to each other. With shear deflections, the fundamental periods of buildings tend to increase proportionately with the height. If buildings deflected in bending, the fundamental periods would vary as the square of the height. It is concluded by the U. S. Coast and Geodetic Survey[13] after the measurement of the periods of 212 buildings in California that shear distortions are predominant in most Pacific Coast buildings. This study also shows that the fundamental period of vibration of the average building is approximately one tenth of a second multiplied by the number of stories. Thus the 0.5 second fundamental period of the 5 story model, the 1.0 second fundamental period of the 10 story model, and the 1.5 second fundamental period of the 15 story model are representative of the actual periods of real buildings.

In real buildings, the mass of the structure is not lumped at the floors as in the 5 story model, and certainly not lumped at every second or third floor as in the case of the 10 and 15 story models. Nor is the mass of real buildings distributed as in the case of the uniform shear beam. The real mass distribution lies somewhere in between. Since the frequencies of the lumped mass and the uniform system differ by only a small amount, as was shown, it follows that the mode shapes and frequencies of a real building must agree closely with those of the models used.

The story heights, the mass of each story, and the stiffness of each story will generally vary in a real building so that the results of this study pertain only to such structures that have approximately uniform properties from story to story.

Damping in Real Buildings

Reference (1) shows the significant effect damping has on the spectrum intensity. Housner[14] states that the experimentally measured range of damping in buildings ranges from $n = 0.07$ to $n = 0.40$, where $n = c/c_c$ is the ratio of damping to critical damping. Alford and Housner[12] report the damping in a rigid four-story reinforced concrete warehouse to be 0.07 of critical and in a building of different construction to be 0.14 of critical damping. Furthermore, they report it to be independent of the frequency in the first two modes of vibration, the only two measurable. Thus structural damping is neither interfloor viscous damping nor absolute viscous damping. In general, the more massive and monolithic the construction of a building, the less will be the damping. Therefore, the 0.07 of critical damping mentioned above is probably as small an amount of damping as can be anticipated. Average buildings will have somewhat more damping than 0.07 of critical. The damping used in this investigation, 0.10 of critical in both the first and second modes, reproduces the features of the actual damping observed in buildings, though it is probably somewhat less than would be observed in an actual building when subjected to strong-motion earthquakes.

IX. ESTIMATION OF THE COMPLIANCE RATIO FOR REAL BUILDINGS

Having established the relationship between the maximum base shear force in typical buildings having foundations with a wide range of compliance, it is necessary to estimate what the lowest actual value of the compliance ratio (ratio of building compliance to foundation compliance), k_ϕ/h^2k, might be. It must be emphasized that this is a determination of the limiting case which the writer believes to be "typical" although the method outlined is valid for any case.

The compliance ratio, k_ϕ/h^2k, has three independent variables, each having an effect on the value. The term h can be taken as a constant since there is no doubt that 12 feet is an average story height for typical buildings. The other two variables will be discussed more extensively. To lower the ratio, a typical k must be as large as possible and a typical k_ϕ must be as small as possible.

Building Stiffness, k

The stiffness of each story of the typical buildings considered is k, the force in pounds required to deflect a floor relative to an adjacent one a distance of 1 foot. The problem of determining the stiffness of actual buildings is greatly simplified by noting that the fundamental period, τ_1, is related to the stiffness and story mass. As was shown for the 5 story building, the period of vibration is related to the mass and stiffness by

$$\omega_1 = 0.285 \sqrt{k/m} = 2\pi/\tau_1 \tag{17}$$

Thus the stiffness, k, is determined when the mass per story and the fundamental period of vibration are known. The measured periods of vibration of typical 5 story buildings show that τ_1 is 0.5 seconds or greater. The value of m can be computed as follows.

Consider a typical 5 story building 50' x 50' in plan that is constructed of reinforced concrete having a density of 150 lb/ft^3 and that has the following dimensions.

Equivalent wall thickness - 6 inches

Equivalent slab thickness - 8 inches

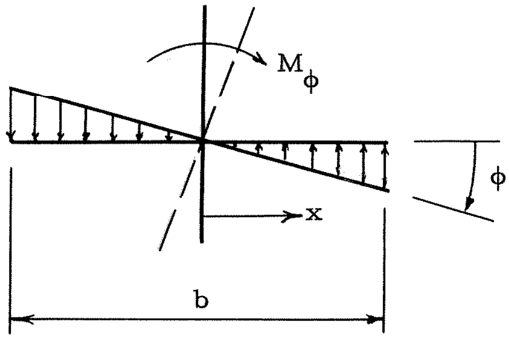
Story height - 12 feet

The dead load of each story is then 430 kips. The story load per unit area of plan is 1.19 lb/in^2. This compares to a typical value of 1.04 lb/in^2. for each 12 feet of height as given by Housner[15]. Thus the dead load for this building is on the high side, and represents a consistent estimate of the mass of a typical rigid building. Substituting into Equation (17), the stiffness is

$$k = 2.59 \times 10^7 \text{ lb/ft} \tag{18}$$

Foundation Stiffness, k_ϕ

The foundation reaction due to the rocking of a building is shown in the following sketch.



M_ϕ = base moment

ϕ = angular rotation

b = width of the building

δ = deflection of the foundation material

p_v = pressure corresponding to unit deflection of the ground

The reacting force at a distance x from the center of rocking is

$$x = \delta p_v \, dA \tag{19}$$

The reacting moment at a distance x is

$$M_x = x\delta p_v \, dA$$

For small displacements,

$$\phi = \delta / x$$

and

$$M_x = \phi x^2 p_v \, dA \tag{20}$$

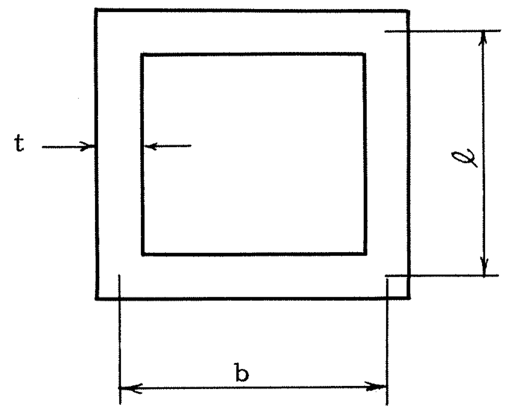
The total base moment, M_ϕ, is the integral of Equation (20). By definition, k_ϕ is the moment per radian of rotation, so the foundation stiffness is

$$k_\phi = p_v \int_{-\frac{b}{2}}^{\frac{b}{2}} x^2 \, dA \tag{21}$$

The integral of Equation (21) is the moment of inertia of the area of the foundation about its rocking axis, I_{xx}.

If the foundation material is linearly elastic in rocking, then the pressure corresponding to unit deflection, p_v, is the modulus of elasticity or the "elastic constant" of the soil. Essentially, it is the slope of the "elastic rebound" portion of the load-settlement diagram for typical soils. A medium-soft clay will have a p_v of approximately 5.19×10^5 lb/ft^3. Some 50 load-settlement curves for soils in the Los Angeles area were studied, and this value was selected as being the lower bound for soils upon which a 5 story building is likely to be erected. It is true that much softer materials are encountered, but the allowable bearing pressure for such materials is so low that it is very unlikely that a multi-story building with spread footings would be constructed on it.

To estimate the moment of inertia of the footings of the 50' x 50' building being considered, the following computations are made. Suppose, first, that the building is supported on continuous wall footings as shown in the diagram. The allowable bearing pressure is assumed to be 4000 lbs/ft^2. Since the total dead load of 5 stories is 2.15×10^6 pounds, the width of the



footing is 2.75 feet. The moment of inertia of the footing about a centroidal axis is

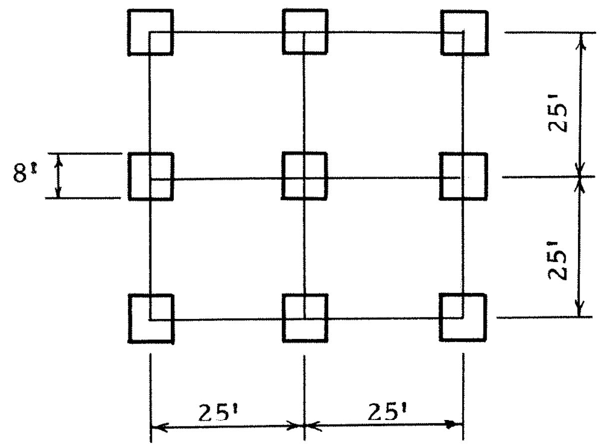
$$I_{xx} = 2.30 \times 10^5 \ \text{ft}^4 \tag{22}$$

and therefore

$$k_\phi = 1.19 \times 10^{11} \ \text{lb-ft/rad} \tag{23}$$

It can be seen from Equation (21) that the moment of inertia has a major effect on the foundation stiffness. A taller building increases the total dead load which increases the footing area and thus k_ϕ also increases. A decrease in the allowable bearing pressure also increases the footing area and k_ϕ.

The moment of inertia can be calculated for the same building on column footings spaced 25 feet on centers as shown in the following diagram. With an allowable bearing pressure of 4000 lbs/ft^2, the column footings will be approximately 8 feet square. The moment of inertia of the footings about the rocking axis will



be

$$I_{xx} = 2.43 \times 10^5 \ \text{ft}^4 \tag{24}$$

and thus

$$k_\phi = 1.26 \times 10^{11} \ \text{lb-ft/rad} \tag{25}$$

From these calculations the lower bound for the ratio of building compliance to foundation compliance is

$$\frac{k_\phi}{h^2 k} > 32 \tag{26}$$

This value is for a 5 story building having dimensions 50' x 50'. If the length and width are reduced to 40 feet, the moment of inertia of the foundation is materially affected. In this case, the width of a continuous wall footing is approximately 2.25 feet. Therefore,

$$I_{xx} = 9.63 \times 10^4 \text{ ft}^4 \tag{27}$$

$$k_\phi = 0.50 \times 10^{11} \text{ lb-ft/rad} \tag{28}$$

$$k = 1.66 \times 10^7 \text{ lb/ft} \tag{29}$$

$$\frac{k_\phi}{h^2 k} > 21 \tag{30}$$

A general expression for the compliance ratio is obtained by expressing Equation (17) in the general form

$$\frac{2\pi}{\tau_1} = \sqrt{\frac{k}{m}} \cdot \sin\left(\frac{\pi}{4n+2}\right) \tag{31}$$

where τ_1 is the fundamental period of the building and n is the number of stories. Therefore,

$$k = \frac{4\pi^2}{\sin^2\left(\frac{\pi}{4n+2}\right)} \cdot \frac{m}{\tau_1^2} \tag{32}$$

The compliance ratio is

$$\frac{k_\phi}{h^2 k} = \frac{p_v \, I_{xx} \, \tau_1^2 \, \sin^2\left(\frac{\pi}{4n+2}\right)}{4\,\pi^2\, h^2\, m} \tag{33}$$

For n greater than 5, the sine of $\pi/(4n + 2)$ can be approximated by $\pi/4n$ thus obtaining

$$\frac{k_\phi}{h^2 k} = \frac{1}{4} \left(\frac{\tau_1}{4nh}\right)^2 \cdot \frac{p_v \, I_{xx}}{m} \tag{34}$$

If similar buildings, differing only in height, are compared, the expression in the brackets in Equation (34) is the same for all n, and the mass, m, is also the same. The compliance ratio thus varies as (p_v I_{xx}). If the value of this term is dependent upon the height of the building, it will always increase with the height and will never decrease. It follows, therefore, that the taller the building, the greater will be the compliance ratio and the less effective will be the foundation yielding in mitigating earthquake stresses.

X. DISCUSSION OF RESULTS

It has been shown that the building models used in this investigation are representative of typical real buildings. Therefore, it is of interest to re-examine the results shown in Figures 23-32. Figure 23 implies that the base shear stresses during strong-motion earthquakes in a real 5 story building will be essentially unaffected by foundation compliance because it has been shown that the typical 5 story structure will not have compliance ratios less than approximately 20. Figure 24 implies that the base shear stresses in a real 10 story building will also be unaffected by foundation compliance. As was shown, the lower bound for the compliance ratio for the 10 story building will be larger than 20. Figure 25 also implies unaffected base shear stresses in a real 15 story building during earthquake motion.

Figure 26 implies that the fundamental period of typical multi-story buildings will not be increased more than 10 percent due to foundation compliance. However, for the moment let the 5 story building be on some hypothetical material soft enough to double the fundamental period. Biot[4] says that the affected period may be used to evaluate the earthquake stresses from his design spectrum. Figure 27 shows that the base shear force is only reduced 20 percent if the building is subjected to the 1934 El Centro shock. Yet Biot's curve would lead one to believe that the stresses are reduced to one half of their magnitude in a similar building on a rigid foundation. This is a consequence of his unrealistic assumption that buildings have zero damping.

If the fundamental period of the 10 story building is doubled by foundation compliance, the maximum base shear force can only be reduced 6 percent (Figure 28). If Biot's curve is extrapolated, it again suggests a shear reduction of 50 percent. In the case of the 15 story building, Figure 29, a doubling of the period does not give any appreciable stress reduction.

Figures 30-32 imply that the base moments in typical real 5, 10, and 15 story buildings subjected to earthquake motion will also be unaffected by any foundation compliance that can reasonably be expected.

XI. SUMMARY AND CONCLUSIONS

This paper shows the quantitative effect that foundation compliance has on the maximum base shear force, the maximum base moment, and the fundamental period of vibration in typical tall buildings subjected to strong-motion earthquakes. Electrical models of 5, 10, and 15 story buildings with foundation compliance were subjected to voltages of the same form as the recorded ground accelerations of actual earthquakes. The electric analog computer technique employed is completely analogous to vibrating a physical model of the building on a shaking table. It is then shown that the properties specified for the model buildings are representative of the properties of real buildings. The experimental results found for the models are thus applicable to actual structures.

The conclusions drawn from the foregoing material are subject to the restrictions discussed in detail in the body of this report. In particular they apply to typical buildings of standard construction that are 5 stories and higher. The conclusions drawn are as follows:

1. The maximum base shear force in a typical tall building subjected to earthquake ground motion will be essentially independent of the foundation compliance so long as the ratio of building compliance to foundation compliance, k_ϕ/h^2k, is greater than 1.0.

2. The maximum base moment in a typical tall building subjected

to earthquake ground motion will be essentially independent of the foundation compliance so long as the ratio of building compliance to foundation compliance, k_ϕ/h^2k, is greater than 6.0. In most cases, the base moment will be independent of foundation compliance to a value of the compliance ratio as low as 1.0.

3. The foundation stiffness, k_ϕ, is dependent on the "elastic constant", p_v, of the soil and the moment of inertia of the area of the building foundation (spread footings) about its rocking axis. The foundation stiffness increases with an increase in the total weight of the structure, increases with a decrease in allowable bearing pressure, and increases with an increase in the moment of inertia of the foundation. The width of a building materially affects the moment of inertia of the foundation and hence the foundation stiffness.

4. The lower bound for the ratio of building compliance to foundation compliance for typical tall buildings of standard construction is approximately 20.

5. The fundamental period of typical buildings is increased 10 percent when the ratio of building compliance to foundation compliance, k_ϕ/h^2k, reaches the lower bound of 20.

6. Any specific earthquake will have different effects on different height buildings. It is quite probable that two buildings of unequal height but with equal compliance ratios and equal design strengths can suffer different degrees of damage in any particular strong-motion earthquake.

XII. REFERENCES

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APPENDIX A

Earthquake Accelerogram Data

Film Record	Earthquake	Component	N	$\ddot{Z}$ ft/sec^2
30	'52 Taft	S 69 E	300	5.47
16	'49 Olympia	S 80 W	389	10.12
11	'40 El Centro	N-S	416	10.40
17	'34 El Centro	N-S	478	5.25

$\ddot{Z}$ is the peak on the Accelerogram used in calibrating the computer circuit. It corresponds to the maximum acceleration on all records except the '34 El Centro.

Calculated Electrical Frequencies

Circuit with Minimum Damping

Run	Earthquake	Bldg.	Film Record	N	f_1 cps	f_2 cps	f_3 cps	f_4 cps	f_5 cps
1	'52 Taft	15	30	300	200	583	920	1183	1347
2	'49 Olympia	15	16	389	259	757	1192	1532	1748
3	'40 El Centro	15	11	416	277	810	1277	1641	1870
4	'34 El Centro	15	17	478	319	930	1464	1883	2147
5	'52 Taft	10	30	300	300	875	1380	1775	2020
6	'49 Olympia	10	16	389	389	1135	1788	2300	2620
7	'40 El Centro	10	11	416	416	1215	1915	2460	2805
8	'34 El Centro	10	17	478	478	1395	2195	2825	3220
9	'52 Taft	5	30	300	600	1750	2760	3550	4040
10	'49 Olympia	5	16	389	778	2270	3575	4600	5240
11	'40 El Centro	5	11	416	832	2430	3830	4920	5610
12	'34 El Centro	5	17	478	956	2790	4390	5650	6440

Experimental Electrical Frequencies

Circuit with Minimum Damping

Run	Earthquake	L	C	f_1	f_2	f_3	f_4	f_5
		henries	μfarads	cps	cps	cps	cps	cps
1	'52 Taft	.220	.220	200	598	946	1215	1380
2	'49 Olympia	.167	.170	259	780	1225	1570	1800
3	'40 El Centro	.154	.160	277	840	1315	1680	1920
4	'34 El Centro	.134	.140	319	958	1500	1920	2170
5	'52 Taft	.141	.150	300	905	1415	1800	2060
6	'49 Olympia	.113	.110	389	1170	1820	2320	2620
7	'40 El Centro	.109	.100	416	1250	1930	2460	2770
8	'34 El Centro	.089	.090	478	1435	2260	2940	3360
9	'52 Taft	.072	.070	600	1800	2830	3680	4210
10	'49 Olympia	.049	.060	778	2330	3710	4820	5460
11	'40 El Centro	.052	.050	832	2470	3930	5010	5800
12	'34 El Centro	.049	.040	956	2880	4580	5900	6700

15 Story Building

Run 1

Earthquake: 1952 Taft; Component S 69 E

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.220	.220	6000	60000	7508×10^{-7}	5.0	25.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	40	20	3.3	4.8	∞	100.0	100.0	1.50	32 m	282 mh
.001	40	20	3.4	4.8	220	103.0	100.0	1.51	33 m	282 mh
.002	40	20	3.4	4.8	110	103.0	100.0	1.52	33 m	282 mh
.004	40		3.5		55	106.0		1.55	34 m	
.007	40		3.6		31.4	109.0		1.58	35 m	
.010	40	20	3.5	4.5	22.0	106.0	93.7	1.64	34 m	264 mh
.020	40		3.4		11.0	103.0		1.77	33 m	
.050	40	20	2.6	3.8	4.4	78.8	79.1	2.10	25 m	223 mh
.100	40		2.5		2.2	75.7		2.55	24 m	
.220	40	20	3.1	2.9	1.0	94.0	60.4	3.40	30 m	170 mh

15 Story Building

Run 2

Earthquake: 1949 Olympia; Component S 80 W

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.167	.170	5000	65000	7508×10^{-7}	5.5	50.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	30	10	5.4	6.0	∞	100.0	100.0	1.50	77 m	768 mh
.001	30	8	6.0	5.2	167	111.1	108.3	1.51	86 m	832 mh
.002	30	8	5.8	5.1	83.5	107.4	106.1	1.53	83 m	815 mh
.004	30		5.5		41.7	101.9		1.57	78 m	
.007	30		5.2		23.9	96.2		1.63	74 m	
.010	30	8	5.0	4.7	16.7	92.6	97.8	1.68	71 m	751 mh
.020	30		4.5		8.4	83.3		1.85	64 m	
.050	30	8	4.4	2.7	3.3	81.5	56.1	2.25	63 m	431 mh
.100	30		3.2		1.7	59.2		2.77	46 m	
.167	30	20	2.8	3.1	1.0	51.8	25.8	3.40	40 m	198 mh

15 Story Building

Run 3

Earthquake: 1940 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	ζ_s	b
.154	.160	5050	67000	7508×10^{-7}	4.9	44.7

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	40	15	4.8	4.5	∞	100.0	100.0	1.50	70 m	526 mh
.001	40	15	4.7	4.4	154	98.0	97.8	1.51	69 m	515 mh
.002	40	15	5.0	4.6	77.0	104.1	102.2	1.53	73 m	538 mh
.004	40	15	5.2	4.7	38.5	108.3	104.5	1.58	76 m	550 mh
.007	40		5.3		22.0	110.4		1.64	77 m	
.010	40	15	5.4	4.75	15.4	112.5	105.6	1.71	79 m	555 mh
.020	40		5.4		7.7	112.5		1.87	79 m	
.050	40	15	5.2	5.0	3.1	108.3	111.1	2.30	76 m	581 mh
.100	40		4.5		1.5	93.8		2.91	66 m	
.154	40	15	4.6	3.6	1.0	95.8	80.0	3.40	67 m	421 mh

Run 4

15 Story Building

Earthquake: 1934 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.134	.140	5000	67000	7508×10^{-7}	4.6	50.1

L_ϕ	R_f	R_m	S_s	M_s	L/L_ϕ	Shear Reduc.	Mom. Reduc.	Fund. Perd.	Shear Force	Base Mom.
henries	ohms	ohms	scale div.	scale div.	$\frac{k_\phi}{h^2 k}$	%	%	sec.	lbs.	ft-lbs
.000	25	5	5.3	4.2	∞	100.0	100.0	1.50	68 m	811 mh
.001	25	5	5.0	4.2	134	94.4	100.0	1.52	64 m	811 mh
.002	25		5.0		67.0	94.4		1.54	64 m	
.004	25	5	5.0	3.9	33.5	94.4	92.8	1.59	64 m	753 mh
.007	25		5.1		19.1	96.2		1.66	65 m	
.010	25	5	5.1	3.3	13.4	96.2	78.5	1.72	65 m	636 mh
.020	25		5.0		6.7	94.4		1.92	64 m	
.050	25	8	4.6	3.1	2.7	86.8	46.1	2.38	59 m	374 mh
.100	25		4.1		1.3	77.3		3.06	53 m	
.134	25	8	3.8	2.3	1.0	71.6	34.3	3.40	49 m	279 mh

10 Story Building

Run 5

Earthquake: 1952 Taft; Component S 69 E

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.141	.150	4900	67000	7508×10^{-7}	5.0	25.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	40	15	2.7	4.5	∞	100.0	100.0	1.00	27 m	238 mh
.001	40	15	2.7	4.4	141	100.0	97.8	1.01	27 m	234 mh
.002	40	15	2.6	4.3	70.5	96.3	95.6	1.02	26 m	228 mh
.004	40		2.6		35.2	96.3		1.05	26 m	
.007	40		2.6		20.1	96.3		1.10	26 m	
.010	40	15	2.6	4.0	14.1	96.3	88.9	1.14	26 m	213 mh
.020	40		2.9		7.1	107.4		1.26	29 m	
.050	40	15	3.2	3.1	2.8	118.5	68.9	1.57	32 m	165 mh
.100	40		2.3		1.4	85.2		1.99	23 m	
.141	40	15	2.5	2.0	1.0	92.6	44.5	2.27	25 m	106 mh

10 Story Building

Run 6

Earthquake: 1949 Olympia; Component S 80 W

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.113	.110	5500	70000	7508×10^{-7}	5.5	50.1

L_ϕ	R_f	R_m	S_s	M_s	L/L_ϕ	Shear Reduc.	Mom. Reduc.	Fund. Perd.	Shear Force	Base Mom.
henries	ohms	ohms	scale div.	scale div.	$\frac{k_\phi}{h^2 k}$	%	%	sec.	lbs.	ft-lbs
.000	30	10	4.1	5.0	∞	100.0	100.0	1.00	62 m	453 mh
.001	30	10	3.7	4.7	113	90.2	94.0	1.01	56 m	426 mh
.002	30		3.5		56.5	85.4		1.03	53 m	
.004	30	10	3.4	4.4	28.3	82.9	88.0	1.07	51 m	398 mh
.007	30		3.4		16.1	82.9		1.13	51 m	
.010	30	10	3.4	4.1	11.3	82.9	82.0	1.18	51 m	371 mh
.020	30	10	3.6	4.0	5.7	87.8	80.0	1.32	54 m	362 mh
.040	30		4.3		2.8	104.8		1.57	65 m	
.070	30		4.0		1.6	97.6		1.90	61 m	
.113	30	10	3.4	2.2	1.0	82.9	44.0	2.27	51 m	199 mh

10 Story Building

Run 7

Earthquake: 1940 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.109	.110	5000	60000	7508×10^{-7}	4.9	44.7

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	40	15	4.2	4.6	∞	100.0	100.0	1.00	62 m	360 mh
.001	40	15	3.75	4.6	109.0	89.3	100.0	1.01	55 m	360 mh
.002	40		3.7		54.5	88.0		1.04	55 m	
.004	40	15	3.5	4.3	27.3	83.3	93.4	1.07	52 m	336 mh
.007	40		3.1		15.6	73.8		1.13	46 m	
.010	40	15	2.8	3.8	10.9	66.6	82.6	1.18	41 m	297 mh
.020	40	15	3.2	3.0	5.5	76.1	65.3	1.33	47 m	235 mh
.040	40		3.7		2.7	88.0		1.59	55 m	
.070	40		3.9		1.6	92.8		1.90	58 m	
.109	40	15	3.7	3.6	1.0	88.0	78.2	2.27	55 m	282 mh

Run 8

10 Story Building

Earthquake: 1934 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.089	.090	5600	72000	7508×10^{-7}	4.6	50.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2k}$	Shear Reduc. %	Mom. Reduc %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	20	5	5.3	4.7	∞	100.0	100.0	1.00	91 m	648 mh
.001	20	5	5.7	4.6	89.0	107.7	97.9	1.02	98 m	634 mh
.002	20		5.6		44.5	105.7		1.04	96 m	
.004	20	5	5.4	4.6	22.3	101.9	97.9	1.09	93 m	634 mh
.007	20		5.1		12.7	96.2		1.16	88 m	
.010	20	5	4.6	3.9	8.9	86.8	83.0	1.21	79 m	538 mh
.020	20	5	3.6	3.1	4.5	68.0	66.0	1.39	62 m	428 mh
.040	20		3.1		2.2	58.5		1.69	53 m	
.089	20	5	2.4	1.5	1.0	45.3	31.9	2.27	41 m	207 mh

5 Story Building

Run 9

Earthquake: 1952 Taft; Component S 69 E

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.072	.070	5700	125000	7508×10^{-7}	5.0	25.1

L_ϕ	R_f	R_m	S_s	M_s	L/L_ϕ	Shear Reduc.	Mom. Reduc.	Fund. Perd.	Shear Force	Base Mom.
henries	ohms	ohms	scale div.	scale div.	$\frac{k_\phi}{h^2 k}$	%	%	sec.	lbs.	ft-lbs
.000	40	15	3.0	3.9	∞	100.0	100.0	0.50	35 m	122 mh
.001	40	15	3.0	3.9	72.0	100.0	100.0	0.51	35 m	122 mh
.002	40	15	3.0	3.7	36.0	100.0	94.8	0.53	35 m	116 mh
.004	40		2.9		18.0	96.6		0.56	34 m	
.007	40		2.8		10.6	93.3		0.60	33 m	
.010	40	15	3.0	3.3	7.2	100.0	84.6	0.63	35 m	103 mh
.020	40	15	2.8	3.1	3.6	93.3	79.4	0.74	33 m	97 mh
.040	40		2.4		1.8	80.0		0.91	28 m	
.072	40	15	1.9	2.0	1.0	63.3	51.3	1.14	22 m	63 mh

5 Story Building

Run 10

Earthquake: 1949 Olympia; Component S 80 W

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.049	.060	5540	88000	7508×10^{-7}	5.5	50.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	30	10	5.0	5.3	∞	100.0	100.0	0.50	77 m	244 mh
.001	30	10	5.0	5.3	49.0	100.0	100.0	0.52	77 m	244 mh
.002	30	10	5.1	5.5	24.5	102.0	103.8	0.54	79 m	253 mh
.003	30		5.2		16.3	104.0		0.56	80 m	
.005	30	10	5.1	5.9	9.8	102.0	111.3	0.60	79 m	272 mh
.007	30		5.1		7.0	102.0		0.63	79 m	
.010	30	10	4.7	5.6	4.9	94.0	105.7	0.68	72 m	258 mh
.020	30	10	3.3	4.2	2.5	66.0	79.2	0.82	51 m	193 mh
.030	30		2.5		1.6	50.0		0.95	39 m	
.049	30	10	2.0	2.1	1.0	40.0	39.6	1.14	31 m	97 mh

5 Story Building

Run 11

Earthquake: 1940 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.052	.050	6450	80000	7508×10^{-7}	4.9	44.7

L_ϕ	R_f	R_m	S_s	M_s	L/L_ϕ	Shear Reduc.	Mom. Reduc.	Fund. Perd.	Shear Force	Base Mom.
henries	ohms	ohms	scale div.	scale div.	$\frac{k_\phi}{h^2 k}$	%	%	sec.	lbs.	ft-lbs
.000	30	5	5.2	3.2	∞	100.0	100.0	0.50	124 m	447 mh
.001	30	5	5.2	3.2	52.0	100.0	100.0	0.52	124 m	447 mh
.002	30	5	5.0	3.0	26.0	96.1	93.8	0.54	119 m	419 mh
.003	30		4.7		17.3	90.4		0.56	112 m	
.005	30	10	3.9	4.4	10.4	75.0	68.7	0.60	93 m	307 mh
.007	30		3.1		7.4	59.6		0.63	74 m	
.010	30	10	2.9	3.7	5.2	55.8	57.8	0.67	69 m	258 mh
.020	30	15	2.2	3.6	2.6	42.3	37.5	0.80	52 m	168 mh
.030	30		2.0		1.7	38.5		0.93	48 m	
.052	40	15	2.1	2.5	1.0	30.3	26.0	1.14	38 m	116 mh

5 Story Building

Run 12 Earthquake: 1934 El Centro; Component N-S

L	C	R_i	R_a	R_oC_o	$\ddot{z}_s$	b
.049	.040	7000	76000	7508×10^{-7}	4.6	50.1

L_ϕ henries	R_f ohms	R_m ohms	S_s scale div.	M_s scale div.	L/L_ϕ $\frac{k_\phi}{h^2 k}$	Shear Reduc. %	Mom. Reduc. %	Fund. Perd. sec.	Shear Force lbs.	Base Mom. ft-lbs
.000	30	10	4.7	4.6	∞	100.0	100.0	0.50	67 m	197 mh
.001	30	10	4.6	4.5	49.0	97.9	97.8	0.52	66 m	193 mh
.002	30	10	4.6	4.5	24.5	97.9	97.8	0.54	66 m	193 mh
.003	30		4.7		16.3	100.0		0.56	67 m	
.005	30	10	4.8	4.7	9.8	102.0	102.1	0.60	68 m	201 mh
.007	30		4.7		7.0	100.0		0.64	67 m	
.010	30	10	4.6	4.7	4.9	97.9	102.1	0.69	66 m	201 mh
.020	30	10	4.7	4.7	2.5	100.0	102.1	0.82	67 m	201 mh
.030	30		3.9		1.6	83.0		0.95	56 m	
.049	30	10	3.4	3.8	1.0	72.4	82.6	1.14	49 m	163 mh