STRESSES IN METAL BEAMS WITH FLAT SHEET WEBS OF MEDIUM THICKNESS

Thesis by

Milford C. Childers

In Partial Fulfillment of the Requirements for the Degree of

Master of Science

California Institute of Technology

Pasadena, California

1936

TABLE OF CONTENTS

TABLE OF FIGURES

I. SUMMARY:

This paper develops a method by which the stresses in the component parts of a solid web beam may be computed when the dimensions of the web thickness, beam height, and distance between stiffeners are such that the web does not go into the wave state immediately upon applying a small load. The stress in the web itself is calculated by a consideration of the buckling load of the web. The load in the stiffeners and flanges is computed by assuming an effective width of sheet as acting with them. This effective width is obtained by a consideration of the buckling properties of thin sheet in shear.

The results are in the same form as the familiar Wagner beam equations except that certain terms must be modified and corrections made. The results herein obtained are not to be considered as superseding the Wagner Equations as derived in Ref. I, but supplement them to take care of cases out of the range of the Wagner assumptions.

II. NOTATION

S. - Shear load in pounds

M. - Moment at section

M_m. - Secondary bending moment in flange

Y. - Load in stiffener

E. - Modulus of Elasticity

σ_t. - Tension stress in web

σ_c. - Compression Stress in web

τ_b. - Critical buckling shear stress

τ_1. - Shear stress in web

σ_f. - Stress in flange

σ_m. - Stress in flange due to secondary moment

h. - Distance between flanges

d.- Stiffener spacing

t. - Thickness of web

a. - Long dimension of panel

b. - Short dimension of panel

c. - Distance from N. A. to outermost fiber

A_f. - Area of flange

I. - Moment of inertia of beam neglecting web

θ. - Angle of wrinkles

K. - Constant dependant on $\frac{a}{b}$

$\left(\frac{I}{c}\right)_f$. - Section modulus of flange alone

III. INTRODUCTION

It is known that experiments on thin web beams below the failure stress show considerably lower stresses than are actually calculated when using the Wagner formulae. This is especially true when a relatively thick web is used. The discrepancy is due to the fact that Wagner in his original paper, Ref. I, assumed the sheet to be very thin, while actually beams are often designed with web thicknesses which cannot be considered to lie in this range.

The formulae given by Wagner are well known but for purposes of comparison they will be given here.

Stress in Web

$$\sigma_t = \frac{2\tau}{\sin 2\theta} = \frac{2S}{ht \sin 2\theta}$$

End Load in Stiffener

$$Y = S\frac{d}{h}\tan\theta$$

Stress in Stiffener

$$\sigma_s = \frac{Y}{A_s} = \frac{S}{A_s}\frac{d}{h}\tan\theta$$

Stress in Flange

$$\sigma_f = \pm\frac{Mc}{I} - \frac{S}{2A_f}\cot\theta \pm \sigma_m$$

The derivation of these formulae may be found in Ref. I, Part I.

This paper presents methods and curves for correcting the stresses obtained by the Wagner analysis to conform more nearly with the results obtained in experiment. Some designers make arbitrary corrections based on previous test results, but as far as the author is aware there has been no theoretical work published on the subject.

IV. THEORETICAL INVESTIGATION

Stresses in Web

The following assumptions are made in the calculation of the stresses in the web:

1. The web has a finite critical buckling value which is dependent on the panel dimensions. (A panel is that portion of web included between two adjacent stiffeners and the flanges.) The validity of this assumption is obvious and needs no verification.

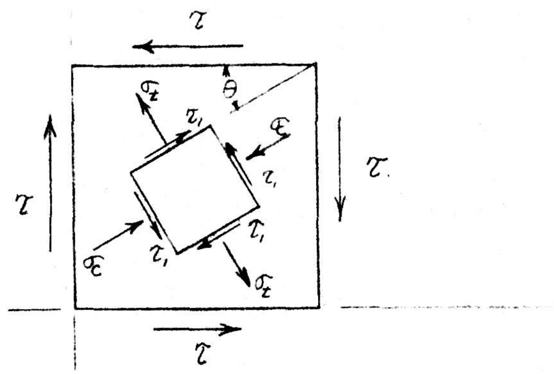
2. After buckling has occurred, the sheet is capable of carrying as shear a portion of the stress equal to the critical shear value. This means that the sheet carries a portion of its load, proportional to the critical buckling stress, as compression perpendicular to the wrinkle.

This assumption is open to some question, but the author believes it to be conservative, and the present knowledge of the subject is so incomplete that any attempt to be more exact would be unjustified.

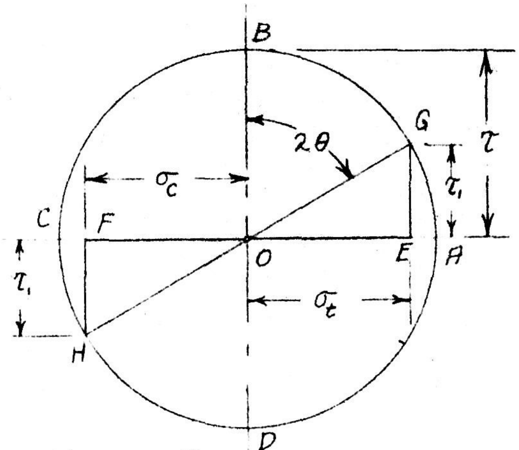
3. The shears along the wrinkle and perpendicular to the wrinkle are assumed to be equal. This is best explained by reference to the Mohr diagrams (Fig. I). The diagram I(a) illustrates the state of stress of an unbuckled sheet subjected to pure shear. O-B represents the shear applied, O-A is the principal tension stress at 45° to the applied shear, and O-C is the principal compression stress at 45° to the applied shear.

If the plane of reference is not 45° to the applied shear, the compression and tension stresses are represented by O-E and O-F

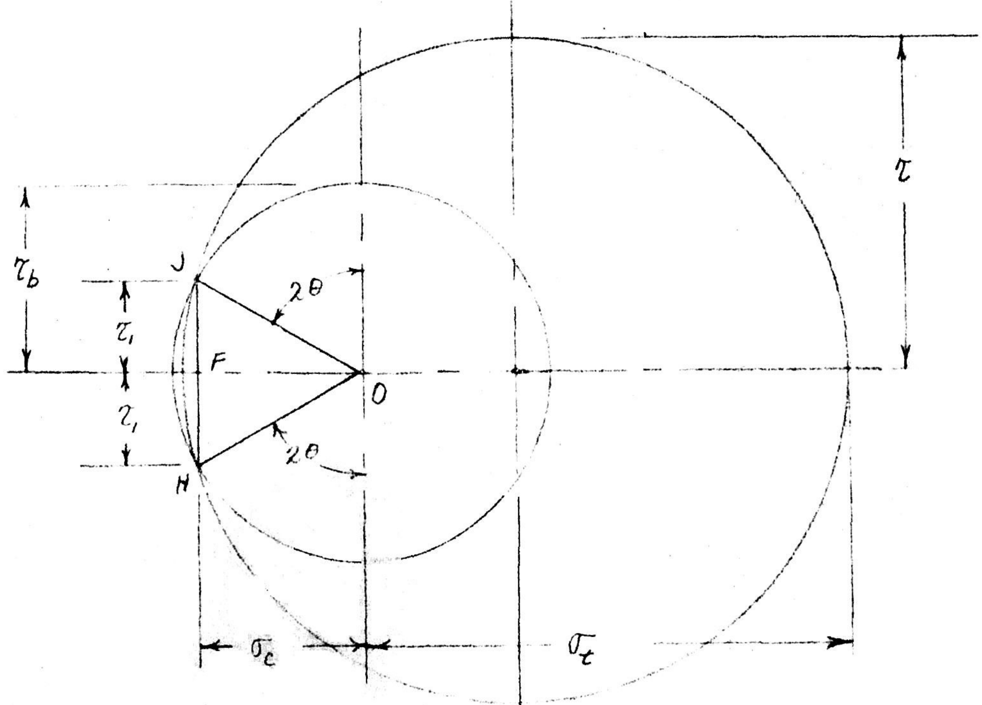
FIG 1



STRESSES IN SHEET SUBJECTED TO SHEAR



MOHR DIAGRAM $\tau \leq \tau_b$



MOHR DIAGRAM $\tau > \tau_b$, $\theta = 30°$

and the shear at this angle is G-E and F-H. If the applied shear is the buckling shear it may be said that this picture represents the sheet just at the buckle. Now if the load is increased the assumptions are that the new Mohr diagram goes through point H and has O-X as an axis. The shears perpendicular and parallel to the wrinkles are represented by F-H and F-J. Diagram K(b) illustrates this condition. The error this assumption makes is very small until θ deviates from 45° a considerable amount. At $\theta = 45^{\circ}$ there is, of course, no shear at all.

Derivation of Formulae for Tension Stress in Web.

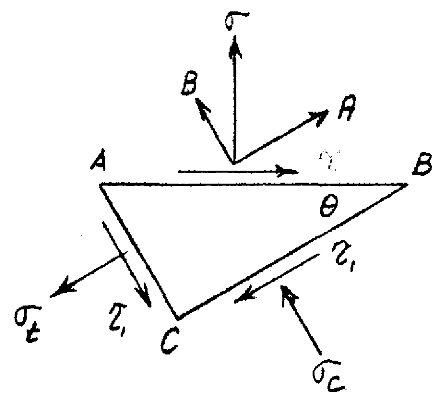


FIG. 2

The distance A-B is taken as 1,

Let AC = m

Let BC = n

$m = \sin\theta \quad n = \cos\theta$

$A = \sigma_t m + \tau_1 n$

$B = \sigma_c n + \tau_1 m$

$A = \sigma_t \sin\theta + \tau_1 \cos\theta$

$B = \sigma_c \cos\theta + \tau_1 \sin\theta$

$$\tau = A\cos\theta - B\sin\theta = (\sigma_t + \sigma_c)\sin\theta\cos\theta + \tau_1(\cos\theta - \sin\theta) \qquad (1)$$

$$\sigma = A\sin\theta + B\cos\theta = \sigma_t \sin^2\theta - \sigma_c\cos^2\theta + 2\tau_1\sin\theta\cos\theta \qquad (2)$$

if

$\tau = \tau_b$ the following conditions hold

$\sigma = 0$

$$\sigma_t = \sigma_c = \tau_b \sin 2\theta \qquad (1a)$$

$$\tau_1 = \tau_b(\cos\theta - \sin\theta) = \tau_b \cos 2\theta \qquad (2a)$$

assuming σ_c and τ_1 remain the same after buckling

$$\tau = \sigma_t \frac{\sin 2\theta}{2} + \tau_b \frac{\sin^2 2\theta}{2} + \tau_b(\cos^2\theta - \sin\theta)^2$$

$$\sigma_t = \frac{2\tau}{\sin 2\theta} - \tau_b \sin 2\theta - 2\tau_b \frac{(\cos^2\theta - \sin^2\theta)^2}{\sin 2\theta}$$

$$= \frac{2\tau}{\sin 2\theta} - \tau_b\left(\frac{2(\cos\theta - \sin\theta)}{\sin 2\theta} + \sin 2\theta\right)$$

or

$$\sigma_t = \frac{2\tau}{\sin 2\theta} - \tau_b \left(2\frac{\cos^2 2\theta}{\sin 2\theta} + \sin 2\theta \right)$$

which reduces to

$$\sigma_t = \frac{2\tau}{\sin 2\theta} - \tau_b \left(\frac{\cos^2 2\theta + 1}{\sin 2\theta} \right) \qquad (3)$$

$$\sigma_t = \frac{2\tau}{\sin 2\theta} - \tau_b f_1(\theta)$$

See Fig. 3, for $f_1(\theta)$

$$\sigma = \frac{2\tau}{\sin 2\theta} \sin^2\theta - \tau_b \left(\frac{\cos^2 2\theta + 1}{\sin 2\theta} \right) \sin^2\theta - \tau_b \sin 2\theta \cos^2\theta$$

$$+ \tau_b \sin 2\theta \cos 2\theta$$

$$\sigma = \tau \tan\theta - \tau_b \left((\cos^2 2\theta + 1)\tan\theta + \sin 2\theta \cos^2\theta - \frac{\sin 4\theta}{2} \right)$$

$$= \tau \tan\theta - \tau_b f_2(\theta) \qquad (4)$$

See Fig. 3, for $f_2(\theta)$

if $\theta = 45^\circ$ $\quad f_1(\theta) = 1 \quad f_2(\theta) = 1.5$

So at $\theta = 45^\circ$ the equations become

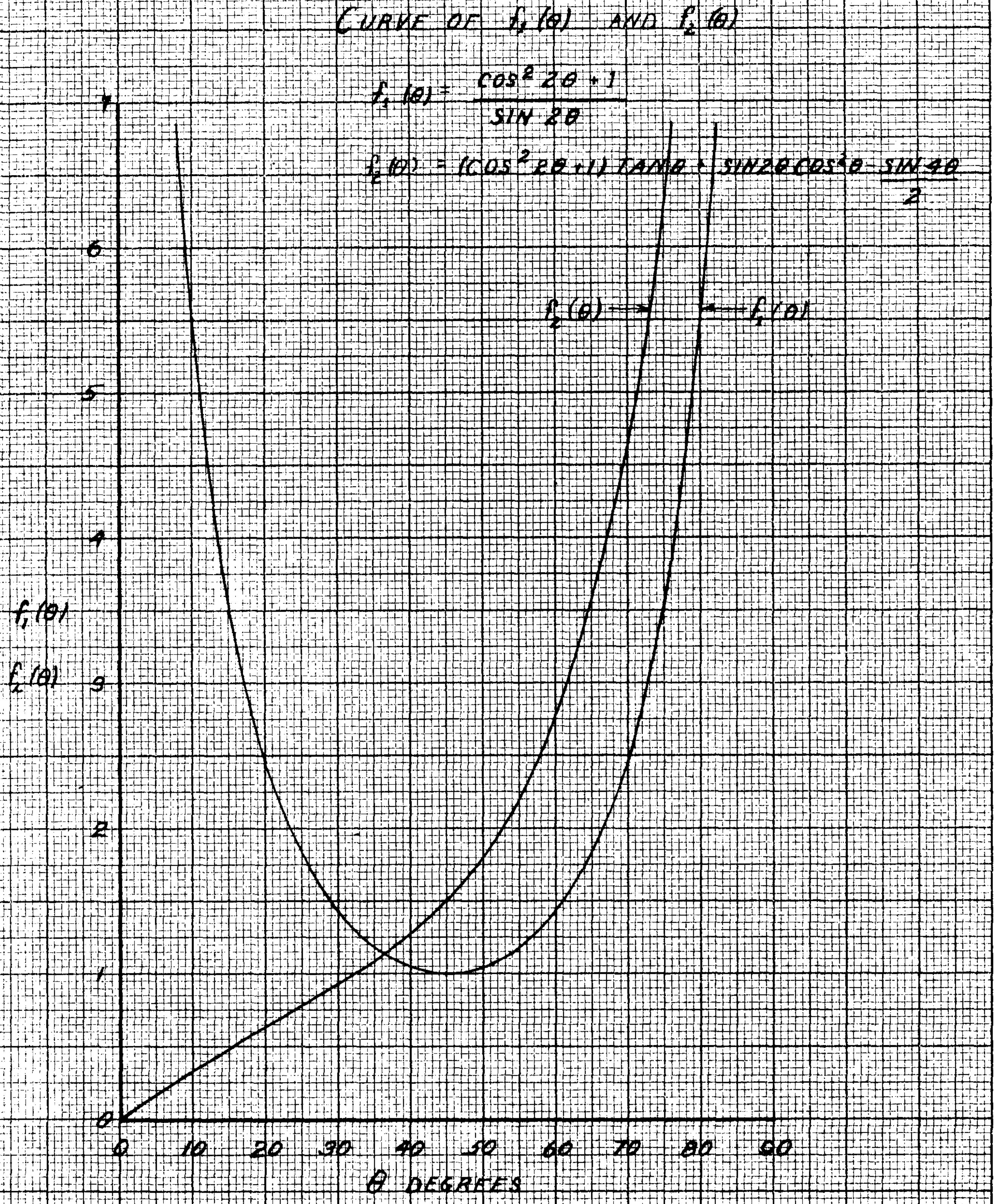
$$\sigma_t = 2\tau - \tau_b \qquad (5)$$

$$= \tau - 1.5\tau_b \qquad (6)$$

It is seen that the first term of equations (3) and (4) is identical with the Wagner equation, while the second term may be considered to be a correction factor depending on the buckling stress of the sheet and the angle of the wrinkles.

The value of τ_b has been determined theoretically and checked experimentally by Timoshenko, Ref. 2, and Southwell and Skan, Ref. 3.

FIG 3



CURVE OF f1(θ) AND f2(θ)
f1(θ) = (COS² 2θ + 1) / SIN 2θ
f2(θ) = (COS² 2θ + 1) TANθ + SIN2θ COS²θ - SIN 4θ / 2
f2(θ)
f1(θ)
f1(θ)
f2(θ)
0
1
2
3
4
5
6
7
0
10
20
30
40
50
60
70
80
90
θ DEGREES

These works have shown that the buckling stress of a flat sheet panel is

$$\tau_b = KE \left(\frac{t}{b}\right)^2$$

Substituting this expression for τ_b in equations (3) and (4) they become:

$$\sigma_t = \frac{2\tau}{\sin 2\theta} - KE\left(\frac{t}{b}\right)^2 f_1(\theta) \tag{8}$$

$$\sigma = \tau \tan\theta - KE\left(\frac{t}{b}\right)^2 f_2(\theta) \tag{9}$$

The limits between which these equations are valid are:

(a) $\tau = \tau_b$

(b) $t \longrightarrow 0$

Substituting in condition (a)

$$\sigma_t = \frac{2\tau_b}{\sin 2\theta} - \tau_b \left(\frac{\cos^2 2\theta}{\sin 2\theta} + \frac{1}{\sin 2\theta}\right)$$

$$\sigma_t = \frac{\tau_b}{\sin 2\theta} - \tau_b \frac{\cos^2 2\theta}{\sin 2\theta} = \tau_b \sin 2\theta$$

Which checks equation (1a)

For condition (b)

$$t \longrightarrow 0$$

$$\tau_b = KE\left(\frac{t}{b}\right)^2 \longrightarrow 0$$

$$\tau = \frac{S}{ht} \longrightarrow \infty$$

However, since τ_b is a function of t^2 it approaches 0 more rapidly than τ approaches infinity. So for very thin sheets the equations approach the Wagner conditions.

The equation for the diagonal tension in the web may be simplified by making the usual assumption that θ is 45° so that

$$\sigma_t = 2\tau - KE\left(\frac{t}{b}\right)^2$$

where

$$\tau = \frac{S}{ht}$$

It is convenient to look upon the second term as a correction factor to be applied to the Wagner calculations so the equation is now

$$\sigma_t = \frac{2S}{ht} - \text{Correction Factor}$$

This correction factor is plotted in Figure 5. These curves were developed as follows:

Figure 4 gives graphically the relation between K and $\frac{a}{b}$. The points for this curve were obtained from Ref. 2. Since K is a function of $\frac{a}{b}$ it is necessary to get the correction factor in terms of $\frac{a}{b}$. To do this it is necessary to consider two cases: Case 1, the distance between flanges is greater than the distances between stiffeners; Case 2, the distance between flanges is less than the distance between stiffeners.

<u>Case 1.</u>

$$\text{Correction Factor} = \tau_b = KE\left(\frac{t}{b}\right)^2$$

$$= KE\left(\frac{t}{b}\right)^2\left(\frac{a}{a}\right)^2$$

$$= KE\left(\frac{t}{a}\right)^2\left(\frac{a}{b}\right)^2$$

Thus the correction factor is a function of two parameters, $\left(\frac{a}{b}\right)^2$ and $\left(\frac{t}{a}\right)^2$. For this case set a = h and b = d, so the equation becomes

$$\tau_b = KE\left(\frac{h}{d}\right)^2\left(\frac{t}{h}\right)^2$$

or

$$\frac{\tau_b}{E} = K\left(\frac{h}{d}\right)^2\left(\frac{t}{h}\right)^2 \qquad (9)$$

This equation is plotted in Figure 4.

Case 2.

Correction Factor = $\tau_b = KE\left(\frac{t}{h}\right)^2$

$$\tau_b = KE\left(\frac{t}{h}\right)^2\left(\frac{b}{b}\right)^2$$

$$\tau_b = KE\left(\frac{t}{b}\right)^2\left(\frac{b}{h}\right)^2$$

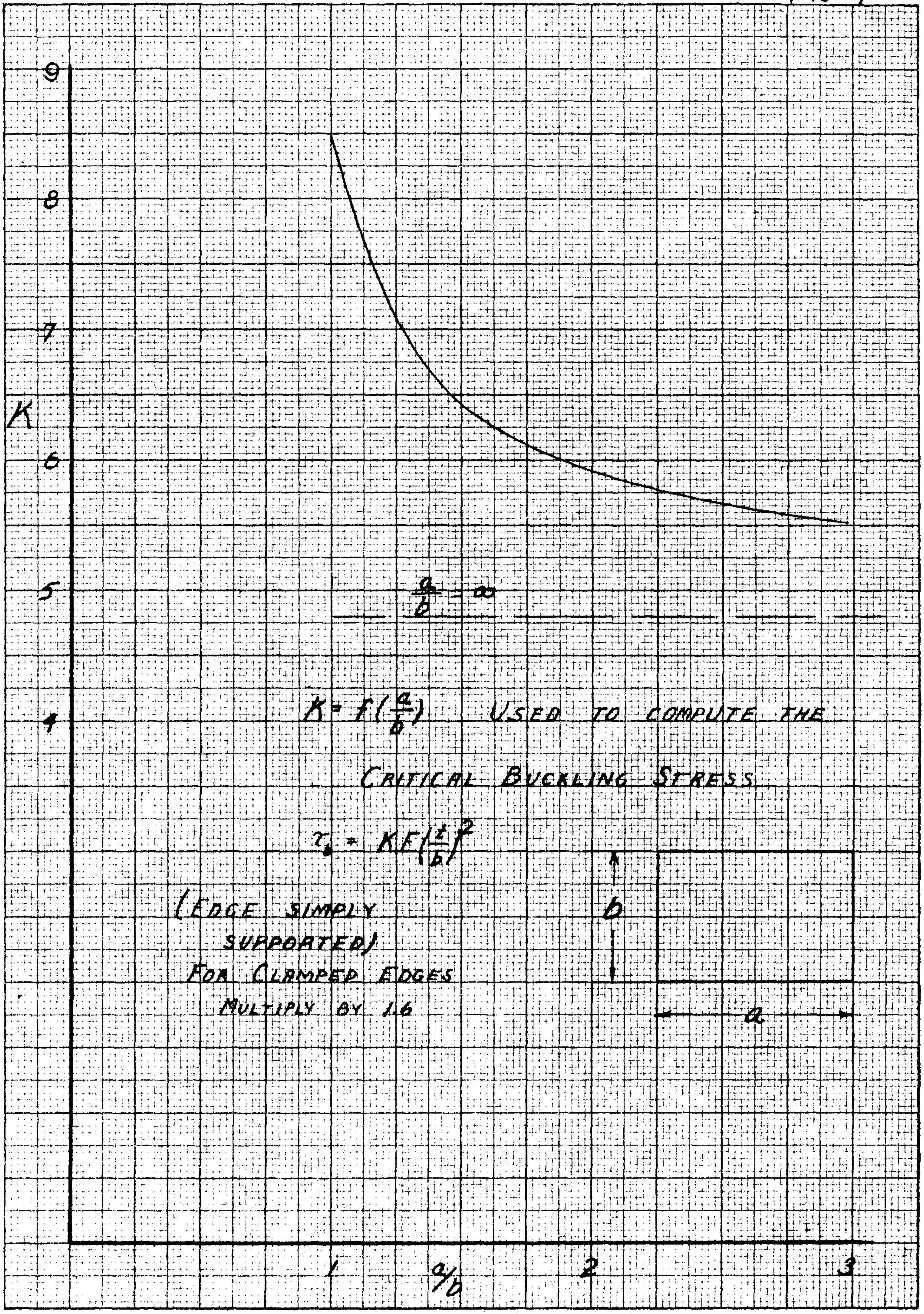
or $b = d$

$$\frac{\tau_b}{E} = K\left(\frac{t}{d}\right)^2\left(\frac{d}{h}\right)^2$$

This equation is plotted in Figure 4.

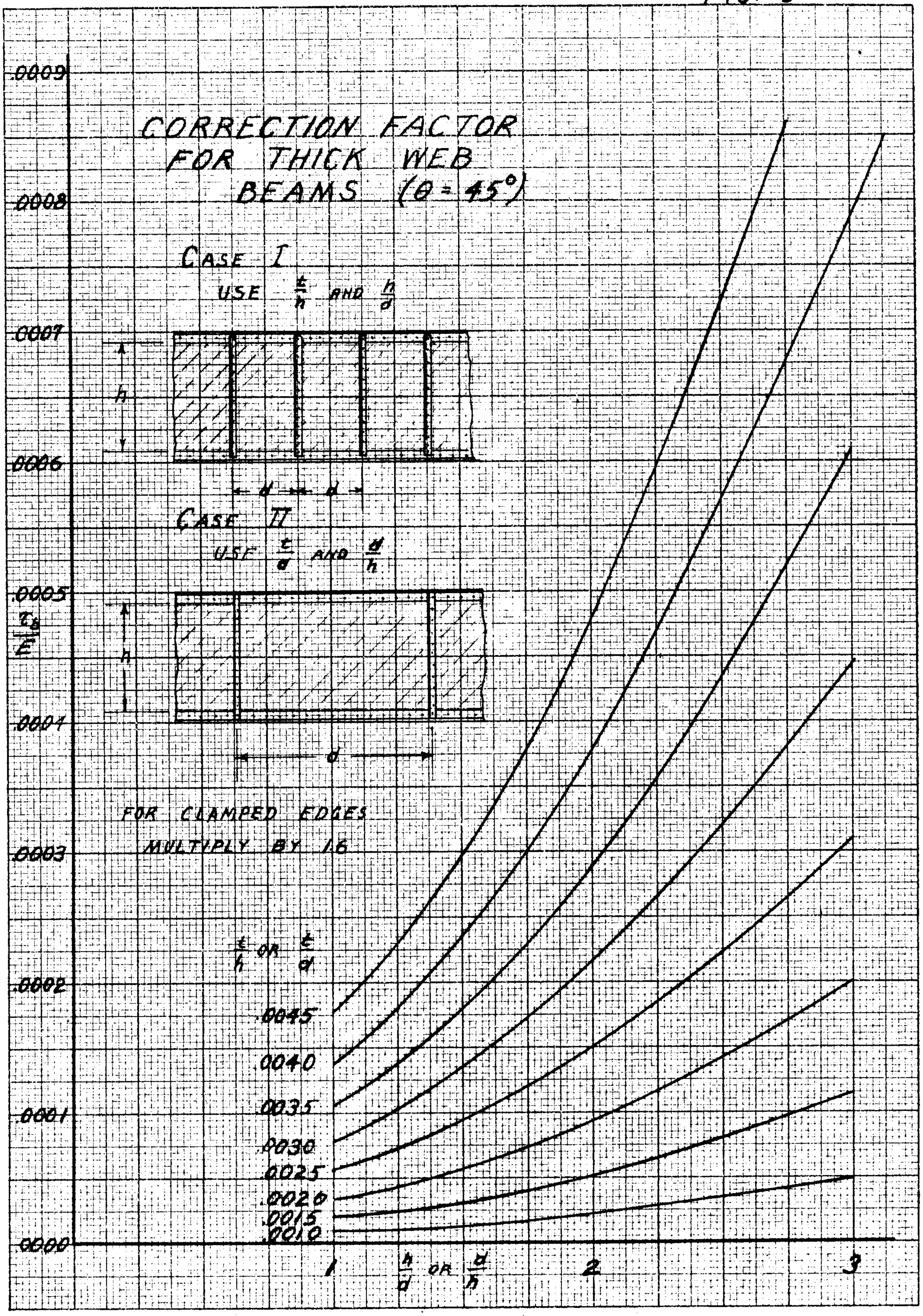
It will be noted that in figures 4 and 5 a correction factor of 1.6 must be used if the edges of the sheet are clamped. This value 1.6 has been calculated by an analysis of shear tests and is the lower limit of the values obtained. For a sheet not rigidly clamped, but more rigidly supported than simply supported, an intermediate value would be advisable.

FIG. 4



9
8
7
6
5
4
K
a/b = ∞
K = f(a/b) USED TO COMPUTE THE
CRITICAL BUCKLING STRESS
τ = KE(t/b)²
(EDGE SIMPLY
SUPPORTED)
FOR CLAMPED EDGES
MULTIPLY BY 1.6
b
a
1
2
3
a/b

FIG. 5


CORRECTION FACTOR
FOR THICK WEB
BEAMS (θ = 45°)
CASE I
USE t/h AND h/d
h
d
d
CASE II
USE t/d AND d/h
h
d
FOR CLAMPED EDGES
MULTIPLY BY 1.6
t/h OR t/d
.0045
.0040
.0035
.0030
.0025
.0020
.0015
.0010
τ_b/E
.0009
.0008
.0007
.0006
.0005
.0004
.0003
.0002
.0001
.0000
1
h/d OR d/h
2
3

Stiffener Stresses

The load in the stiffener is determined by using equation (6) which gives the stress in the web perpendicular to the flange.

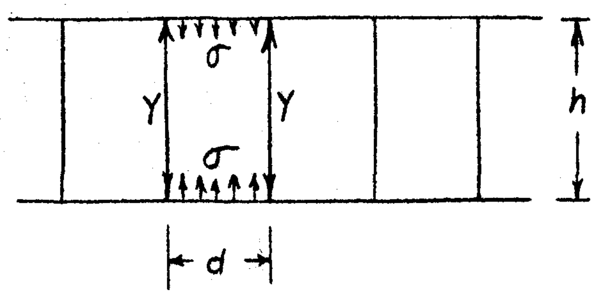


FIG. 6

For $\theta = 45^{\circ}$

$$Y = \sigma t d$$

$$Y = (\tau - 1.5\,\tau_b)\,t d \tag{11}$$

If the area of the stiffener includes an effective width of sheet acting with it, the stress in the stiffener is

$$\sigma_s = (\tau - 1.5\,\tau_b)\,\frac{t d}{A_s} \tag{12}$$

$$A_s = A_{stiffener} + A_{sheet}$$

$$A_{sheet} = 2wt\ ,$$

where w is the shear effective width as described in the next section.

Effective Width

The effective width of sheet which is assumed for the above stresses varies with the stress in the sheet and may be related to the buckling of a strip of width 2w, where w is the effective width acting.

The critical buckling stress is a strip of width 2w is

$$\tau_b' = KE\left(\frac{t}{2w}\right)^2$$

and the shear load which corresponds to τ_b' is

$$S = \tau_b' ht = KE\left(\frac{t}{2w}\right)^2 ht$$

so

$$\left(\frac{t}{2w}\right)^2 = \frac{S}{KEht}$$

$$\frac{2w}{t} = \sqrt{\frac{KEht}{S}}$$

In the limit $t \rightarrow 0$, $w \rightarrow 0$ as normally expected.

In the limit $\tau \rightarrow \tau_b$, $\tau_b = KE\left(\frac{t}{b}\right)^2$

so $$2w = t\sqrt{\frac{KE}{KE\left(\frac{t}{b}\right)^2}} = b$$

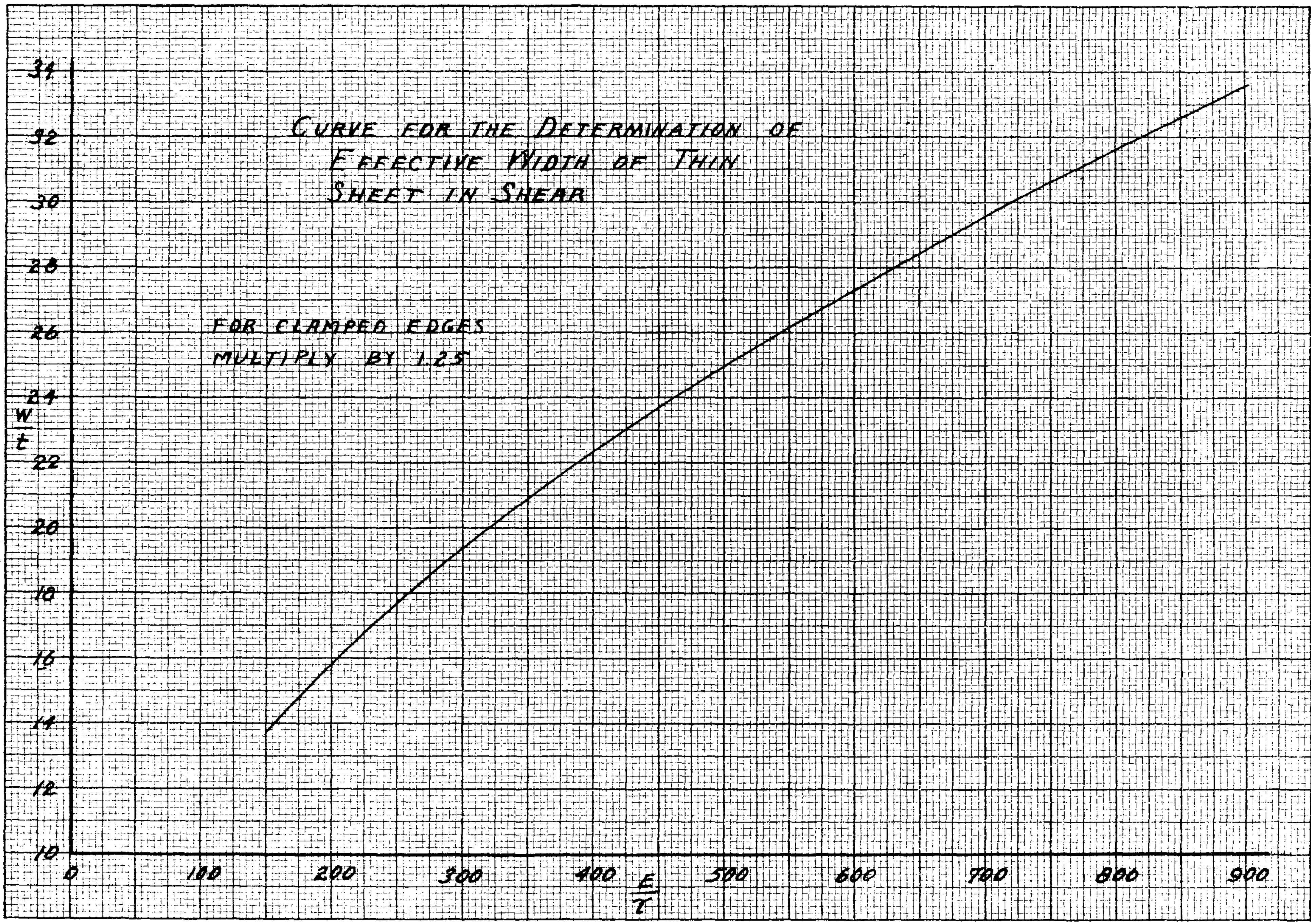
For the range in which most practical problems lie, K may be considered to have a value of 5 or 8, depending on whether the edges are clamped or not, which gives

$$2w = 2.24t\sqrt{\frac{E}{\tau}} \quad \text{OR} \quad 2w = 2.8t\sqrt{\frac{E}{\tau}}$$

$$w = 1.12t\sqrt{\frac{E}{t}} \qquad w = 1.4t\sqrt{\frac{E}{t}}$$

From the curve Figure 7 it may be seen that for most practical cases the effective width lies between 20 and 40 times the thickness of the sheet.

The assumption of an effective width acting with the



CURVE FOR THE DETERMINATION OF
EFFECTIVE WIDTH OF THIN
SHEET IN SHEAR
FOR CLAMPED EDGES
MULTIPLY BY 1.25
W/t
E/τ

FIG. 7

stiffeners and flanges is probably the most controversal portion of this paper. Physically an effective width does not exist, but tests on beams indicate that the portion of web adjacent to the stiffener will carry some of the load. This has led some designers to assume an arbitrary effective width based largely on the experience and test data. Other designers apply a correction factor to the column length of the stiffener. The author believes that the effective width method is the more rational and here presents a method for determining it analytically.

Stress in the Flanges

The stress in the flanges is due to three effects:

(1) Moment in the beam.

(2) Shear load in the web.

(3) Secondary bending of the flanges due to σ.

These will be discussed in the order named.

The stress due to the moment in the beam is calculated by

$$\sigma_{f_1} = M \frac{c}{I}$$

where the section modulus $\frac{I}{c}$ is computed by assuming an effective width acting with the flanges. There are two general cases to be considered:

(a) Flange on one side of the web only.

(b) Flange on both sides of the web.

If the beam is of the type (a) the section modulus of the beam is computed by neglecting all the web except a portion immediately adjacent to the rivets. (Fig. 8) If the beam is of type (b) the portion of the web included between the flanges is considered as part of the flange and all the rest of the web is neglected except a portion immediately adjacent to the flange. (Fig. 9)

The amount of web which should be included is not, strictly speaking, the effective width as computed by the preceding formula, since on one side of the beam there is direct compression in the web, caused by the bending, which will decrease the effective width. On the other side there is a direct tension which will increase the effective width, so the effective width are not symmetric, as shown in Figures 6 and 7. This condition is present on all finite thicknesses of web, but obviously as the web becomes thinner, the effect on the total modulus becomes of lesser importance. The auther is aware of no analytical method for determining how much to add and subtract, so the designer must resort to assumptions backed by tests or empirical data. If such tests are lacking, as a first assumption, no effective width is assumed on the compression side, and double the calculated on the tension side.

The stress due to the tension field is again governed by the amount of effective width chosen to be acting with the web, and is open to the some uncertainty described in the preceding paragraph. In general the Wagner equation is used, correcting for the difference in areas of the flanges. These areas are to be computed, using the assumed effective widths.

The stress due to secondary bending of the flange is

$$\sigma_m = M_s \left(\frac{c}{I} \right)_f$$

$$M_s = \frac{\sigma t d^2}{12} = (\tau - 1.5\, \tau_b) \frac{t d^2}{12} \qquad \theta = 45^\circ$$

$$\sigma_m = (\tau - 1.5\, \tau_b) \frac{t d^2}{12} \left(\frac{c}{I} \right)_f$$

This stress occurs at the stiffener. Here again the amount of sheet to add, in the computation of the section modulus of the flange, is a matter left to experience and test.

Summarizing, the total stress in the flange is

$$\sigma_f = \pm \sigma_{f_1} - \sigma_{f_2} \pm \sigma_m$$

where the sign used depends on the position of the point on the beam at which the stress is desired.

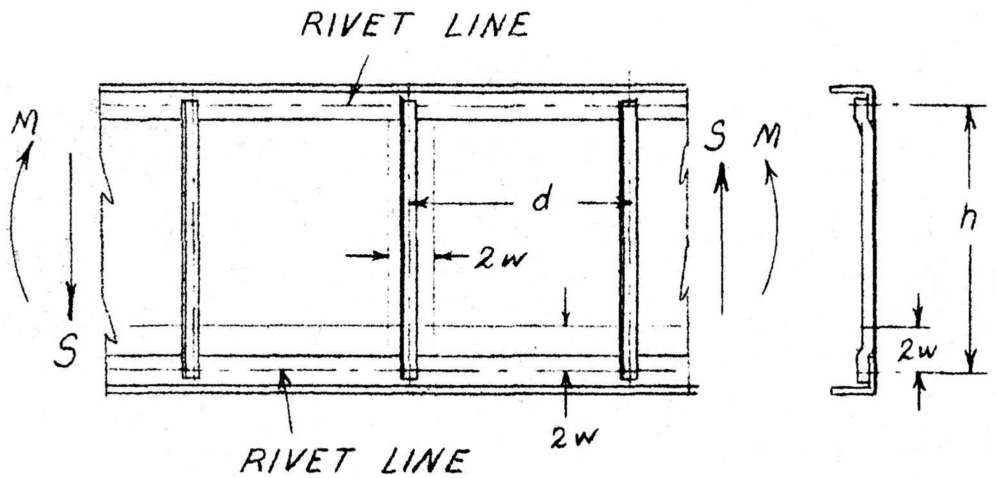

RIVET LINE
M
S
S
M
d
2w
h
2w
2w
RIVET LINE

FIG. 8

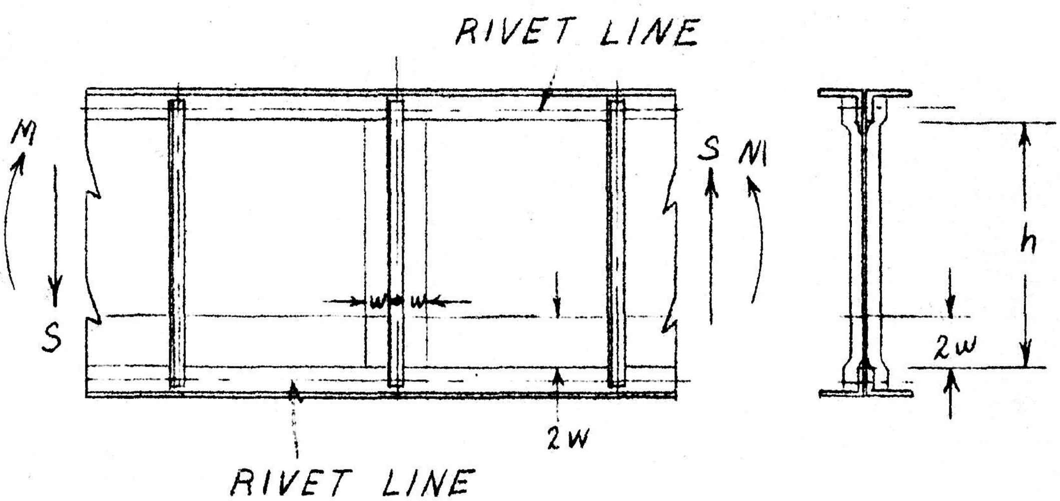

RIVET LINE
M
S
S
M
w
w
h
2w
2w
RIVET LINE

FIG. 9

V. CONCLUSIONS:

The analysis which has been presented is far from an exact solution of the problem; however, the author feels that the methods which have been presented are a step in the right direction. The results obtained are certainly more accurate than the conventional Wagner analysis. The formulae have been presented in a form familiar to engineers and should be quite easy to use.

It is unfortunate that there is so little test data available to use as a check on the theory. The effective widths obtained by the theoretical analysis are of the same order of magnitude as are actually used by some designers, who have tests to back up their assumptions. The example which follows is based on an actual beam, constructed and tested with extensometers on the stiffener. The only results available to the author are the stress readings of the stiffeners which are checked against the theory. These checked quite well.

VI. ACKNOWLEDGMENT

The author wishes to thank members of the staff of G.A.L.C.I.T.. The assistance and suggestions given by Drs. von Karman, E. E. Sechler, C. B. Millikan, and A. L. Klein, have been of great value in the preparation of this paper.

VII. REFERENCES

1. Wagner — Flat Sheet Metal Girders with very Thin Metal Webs. N.A.C.A. T.M. 604, 605, 606

2. Timoshenko — Strength of Materials, Part II.

3. Southwell-Skan — On the Stability Under Shearing Forces of a Flat Elastic Strip. Proceedings of the Royal Society, 1924, Series A, Vol. 105, No. A 733.

4. Timoshenko — Theory of Elasticity.

5. Sechler — Ultimate Strength of Thin Sheet Panels in Compression. Doctors Thesis, California Institute of Technology.

VIII. EXAMPLE

Assume a beam of the following dimensions with loads as indicated at a certain section. For comparison, the values obtained by applying the uncorrected Wagner formulae are indicated.

$h = 28.3$ inches

$d = 7.000$ "

$t = .051$ "

$S = 50,000$ lbs.

$M = 1,000,000$ in. lbs.

Assume $= 45^{\circ}$

Assume sheet clamped in flanges and between stiffeners

$(\ )_w$ = Wagner value

1. Tension Stress in Web

$$\sigma_t = \frac{2S}{ht} - \tau_b$$

$$\frac{t}{h} = .0018$$

$$\frac{h}{b} = 4.05$$

$$K = 5.25$$

$$\tau_b = 2800 \times 1.6 = 4280$$

$$\sigma_t = 69,400 - 4280 = 65,120 \text{ lbs./ins.}^2$$

$$(\sigma_t)_w = 69,400 \text{ lbs./ins.}^2$$

2. Effective Width Calculation

$$\tau = \frac{S}{ht} = 34,700 \text{ lbs./ins.}^2$$

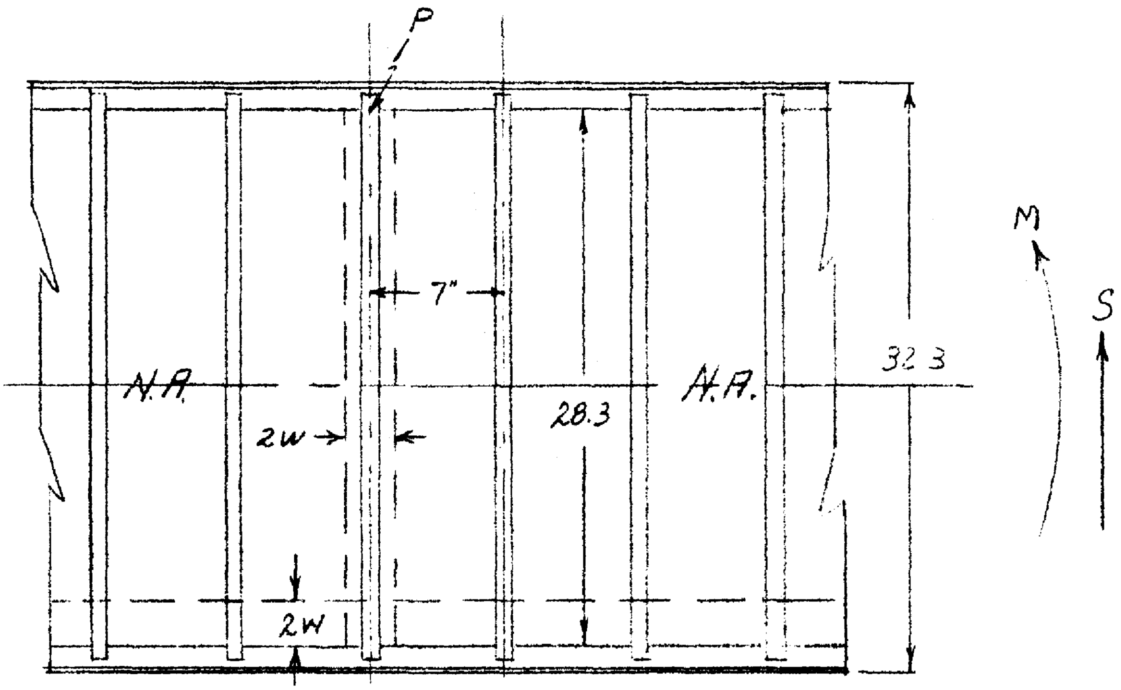
$$\frac{E}{\tau} = 288$$

$$\frac{w}{t} = 19 \times 1.25 = 23.7 \text{ (From Fig. 7)}$$

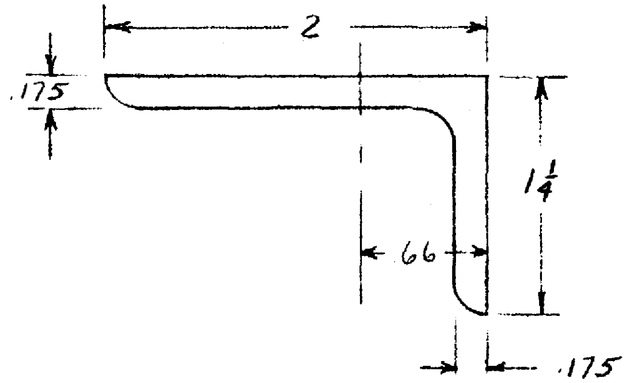
$$w = 23.7 \times .051 = 1.21$$

3. Stiffener Design

Fig. 10



(a) Dimensions of Beam



(b) Flange Detail

$$\sigma_{stiff.} = (\tau \tan \theta - 1.5\tau_b) \frac{th}{A_{stiff.}}$$

Assume the area of the stiffener alone is .414.

Total area of stiffener plus effective width of sheet.

$$A_{stiff.} = .414 + 2 \times 1.2 \times .051 = .414 + .121 = .535$$

$$\sigma_{stiff.} = (34{,}700 - 6450)\ .051 \times \frac{7}{.535} = 18{,}900 \text{ lbs./ins.}^2$$

$$(\sigma_{stiff.})_w = 30{,}500$$

At this point it might be well to discuss the result. In the first place the beam which has been chosen as an example is actually a beam which has been built and tested. The only data from this test which is available to the author are some stress readings on the stiffener. Just where on the stiffener the readings were taken is not known. The result obtained was

$$\sigma_{stiff.} = 16{,}000 \text{ lbs./ins.}^2$$

The agreement is close; and if it is further assumed that the sheet is clamped between two stiffeners of .207 area each, and the sheet between the stiffeners acts in addition to the effective width, the agreement is better. Assuming the base of the stiffeners to be 3/4", the area of sheet between the stiffeners is

$$.121 + .75 \times .051 = .121 + .038 = .159 \text{ Sq. In.}$$

and the total area is

$$.414 + .159 = .573 \text{ Sq. In.}$$

$$\sigma_{stiff.} = \frac{10{,}090}{.573} = 17{,}600 \text{ lbs./ins.}^2$$

4. Flange Stress

Nothing is known about the flanges used on the actual beam so it will be necessary to assume a section for purposes of calculation.

(a) Assumptions

(1) The flanges are identical top and bottom.

(2) There is a flange on each side of the web.

(3) The flanges are angles.

(4) The dimensions of the angles are as shown.

Area = (2.00 + 1.125) .175 = 3.125 x .175 = .55 ins.^{2}

$$= \frac{2 \times .175 \times 1 + 1.075 \times .175 \times .088}{.55}$$

$$= \frac{.350 + .015}{.55} = \frac{.365}{.55}$$

$= .664$

$$I_{base} = 2 \times .175 \times 1 + 1.075 \times .175 \times .087^2 + \frac{.175 \times 2^3}{12}$$

$$= .350 + .0014 + .1167 = .468$$

$$I_{y-y} = .468 - .55 \times .664^2 = .468 - .242 = .226$$

(b) $\sigma_{f_1} = \frac{Mc}{I}$

Calculation of section properties of beam.

Area upper flange = Area lower flange

$2 \times .55 + 2 \times .051 = 1.21$

Area effective width = 2.4 x .051 = .122

$$c = \frac{1.21 \times .66 + 1.21 \times 30.14 + .122 \times 27.6}{2.42 + .122} = 16$$

$$I_{na} = 1.21 \times 15.3^2 + 1.21 \times 14.14^2 + .122 \times 11.6^2 = 531$$

Stress at inner portion of flange (Point P)

$$\sigma_{f_1} = \frac{10^6 \times 14}{531} = -26{,}300 \text{ lbs./ins.}^2$$

(c) $\sigma_{f_2} = \frac{S}{2 \times A}$

$$\sigma_{f_2} = \frac{50,000}{2 \times 2.54} = -9,850 \text{ lbs./ins.}^2$$

(d) $$\sigma_m = \left(M \frac{c}{I}\right)_f$$

$$\sigma_m = (\tau - 1.5\tau) \frac{td^2}{12} \left(\frac{c}{I}\right)_f$$

Computation of $\left(\frac{c}{I}\right)_f$

Using the two angles plus included sheet assume neutral axis of flange plus included sheet is .66 " from top.

$$I_{base} = 2 \times .468 + .102 \times 1 + \frac{.051 \times 2^3}{12} = 1.072 \text{ ins.}^4$$

$$I_{y-y} \; 1.072 - 1.21 \times .66^2 = .535 \text{ ins.}^4$$

$$c_f = 2 - .66 = 1.33$$

$$\left(\frac{c}{I}\right)_f = \frac{1.33}{.535} = 2.51$$

$$\sigma_m = 28.270 \times \frac{.051 \times 49}{12} \times 2.51 = -14,750$$

$$\sigma_f = -\sigma_{f_1} - \sigma_{f_2} - \sigma_m$$

$$\sigma_f = -(26,300 + 9,850 + 14,750)$$

$$\sigma_f = -50,900 \text{ lbs./ins.}^2 \qquad \text{(Point P)}$$

The corresponding Wagner stresses are

$$\sigma_{f_1} = \frac{Mc}{I} = -\frac{10^6 \times 14.25}{2 \times 1.21 \times 15.58^2} = -24,200$$

$$\sigma_{f_2} = \frac{Q}{2F_h} = \frac{50,000}{2 \times 2.42} = -10,300$$

$$\sigma_m = \frac{Q}{h} \frac{d^2}{12} \left(\frac{c}{I}\right)_f = \frac{50,000 \times 49}{28.3 \times 12} \times \frac{1.33}{.452} = -21,200$$

$$\sigma_{f_w} = -55,700 \text{ lbs./ins.}^2 \qquad \text{(Point P)}$$

Summary

	Modified			Experiment	
	Wagner	Stress	% of Wagner	Stress	% Wagner
Stress in Web	+69,400	+65,120	94%		
Stress in Stiff.	-30,500	-18,900 -17,600	62% 57.7%	16,000	52.5%
Stress in Flange	-55,700	-50,900	91.3%		