INDICES OF PRINCIPAL ORDERS IN ALGEBRAIC NUMBER FIELDS

Thesis by

Melvin John Knight

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ABSTRACT

Let K be an extension of Q of degree n and $\mathfrak{O}_K$ the ring of integers of K. If θ is an algebraic integer of K and $K = Q(\theta)$, then $Z[\theta]$ is a suborder of $\mathfrak{O}_K$ of finite index. This index is called the index of θ. If k is a rational integer, the numbers θ and $\theta + k$ have equal indices. Define two numbers to be equivalent if their difference is a rational integer.

Using Schmidt's extension of Thue's Theorem it is shown that in any field of degree less than or equal to four there exist only a finite number of inequivalent numbers with index bounded by any given number. This is true for every finite extension of Q and a proof is given using a slight generalization of Schmidt's Theorem.

An application of Schmidt's Theorem to a problem on the units in a cyclic field of prime degree is given.

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