ON THE TORSION OF WINGS

Thesis

by

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FOREWORD

Methods of investigation of pure torsion differ in many ways from the general method of procedure for other types of stresses. First it is very difficult to subject a body to a pure torsion without superposing bending moments; second the exact determination of twisting moment is in most cases very doubtful; third, a body subjected to torsional shearing stresses in general does not deform according to a simple law, but this deformation is usually a function of the geometry of the body.

It is convenient to study the torsion of isotropic bodies under three different types:

a) Bodies of solid section where the stresses are distributed throughout the area in accordance with a certain law.

b) Bodies of cylindrical shell type where the stresses are a function of the boundary only, or in other words the thickness of the shell is small in comparison with the other dimensions, so that the assumption of uniform stress distribution over the thickness of the shell is reasonably justified.

c) Built-up sections where the individual parts can be analysed separately and by the law of superposition or the method of least work the stresses and the deformations of the whole body can be determined.

One of the problems of designing is to determine the rigidity of the given structure and at what approximate load failure will occur.

Experiments have shown that in the type b) and c) we have two distinct cases: First, it has been found that up to a certain loading the geometrical characteristics of these bodies are not changed and the deformations remain small, and when additional load is applied a sudden increase of deformation is observed accompanied by buckling. This state is generally called the first stability limit. Second, the other limiting load can be defined as that at which permanent deformation remains after the load has been removed. The above leads us directly to the problem of an elastic stability which has not yet been solved.

Lack of the exact theory by which we can predict even the first stability limit prevents an engineer from predicting at what load failure of the structure may occur.

In the type a) it is safe to assume that the failure generally will occur when the shearing stresses have reached a certain value which is characteristic of the given material, but in the types b) and c) stability is the primary cause of failure.

The aim of this paper is to give to the designer semi-empirical formulae by which he can judge the relative magnitude of the stresses and defornations of various types of airfoils under the assumption that the torsional stresses are taken by the skin alone and to show that this assumption is reasonably justified and that the order of magnitude of the computed values for deformation agrees with the order of magnitudes of experimental data. Furthermore, it should be noted that this analysis holds only up to the first stability limit.

METHODS OF INVESTIGATION

It has been shown by J.B.Wheatley (N.A.C.A. Technical Note # 366) that the best method of analysis is Prandtl's Membrane Analogy which gives the formula:

$$S\ t = \frac{M}{2\ A} \qquad (1)$$

Furthermore the angle of twist in radians is given by:

$$\theta = \frac{ML}{Gt}\ \frac{P}{(2A)^2} \qquad (2)$$

where S is the unit shearing stress; t the thickness of the shell, M the torsional moment, A the total area of the cross section, θ the angle of twist in radians per length L, G the shear mogulus and P the total perimeter of the section.

It can be seen from equation (1) that the stress is a function of the area, and from equation (2) that θ is a function of the area and the perimeter.

In order to simplify engineering computations in the case of airfoils the two variables A and P can be expressed in terms of different parameters which can be more readily obtained. In order to do this 14 of the most commonly used airfoils have been investigated, namely: Clark Y-21, Clark Y-18, Clark Y-15, Clark Y, G-387, M-6 M-12, USA-35-A, USA-35-B, USA-27, USA-5, RAF-15, N-9, and N-10. Each section has been plotted, using 100 inch chord.

The areas were measured by the planimeter and the perimeters were scaled by dividers, the tabulated results being the mean of four readings, and are correct with one per cent. Investigations were carried out:

a) for total cross section, 0 to 100% chord, table I and figure 1,

b) for the front part of the section up to 30% chord, table II and figure 2,

c) for the middle part of the section from 30% to 65% chord, table III fig.3

d) from 65% to 100% chord, table IV fig.4

TABLE I

0-100% Chord

Airfoil	h	A	k_A	A'	%err.	P_t
C-Y-21	21	1457	.694	1470	-0.9	210.1
C-Y-18	18	1249	.694	1260	-0.9	208.4
C-Y-15	15	1040	.694	1050	-0.9	206.9
C-Y	11.7	810	.693	820	-1.2	204.8
G-387	15.15	1015	.668	1060	-4.2	207.9
M-6	12	855	.712	841	+1.6	204.2
M-12	11.85	837	.706	831	+0.7	204.1
USA-35-A	18.18	1226	.675	1274	-3.8	209.8
USA-35-B	11.58	779	.673	812	-4.1	205.5
USA-27	11.04	811	.734	774	+4.8	205.0
USA-5	6.38	463	.734	448	+3.3	202.6
RAF-15	6.38	475	.745	448	+6.0	202.4
N-9	8.46	586	.693	592	-1.0	203.0
N-10	11.22	782	.700	784	+0.2	204.3
			k=.701			205.0

$$S\,t = \frac{M}{2\,A'} = \frac{M}{2\times .701\times h\times C} = .714\,\frac{M}{h\times C} \qquad (3)$$

$$\theta = \frac{ML}{G\,t}\,\frac{P}{(2A')^2} = \frac{ML}{G\,t}\,\frac{2.05\,C}{(2\times .701\times h\times C)^2} = 1.04\,\frac{ML}{G\times t\times h^2\times C} \qquad (4)$$

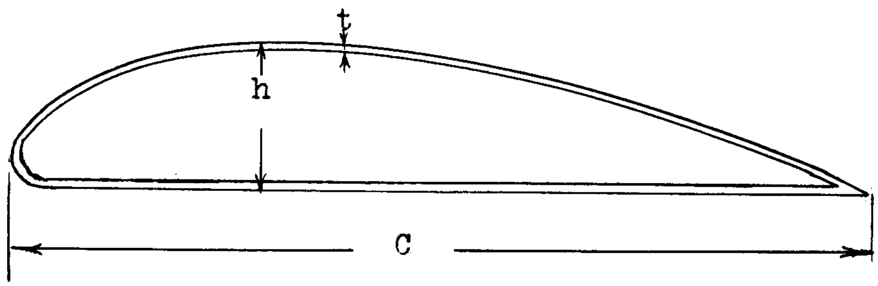


Figure 1.

h is the maximum ordinate, t is the thickness, A is the total area, k_A is the coefficient of area and is defined $A = k_A C \times h$. In our case $k_A = \frac{A}{100 \times h}$ On figure 1A the areas are plotted against h and the straight line is drown thru all the points, the slope of which come out to be .701. This value corresponds to the mean value of all the k_A's. A' is equal to k C h or .701 C$\times$h.

In each case the percentage of error in A' have been computed by the formula $\% \text{ err} = \frac{A - A'}{A'} \times 100$, which is tabulated in the column 5. P_t is the total perimeter, the mean value of which for all sections considered may be set at 2.05 C. Substituting the above values for area and the perimeter in the equations (1) and (2) we obtain the equations (3) and (4).

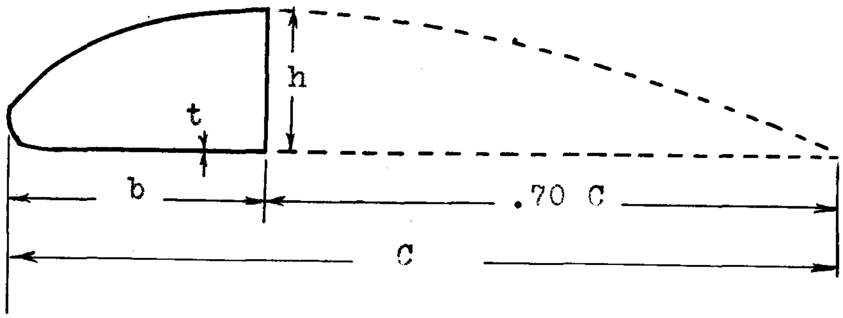
TABLE II

0-30% Chord

Airfoil	h	A	k'	A'	%err.	P_t	p'
C-Y-21	21	513	.815	517	-0.8	88.5	1.735
C-Y-18	18	441	.816	443	-0.5	84.4	1.757
C-Y-15	15	365	.815	369	-1.0	80.2	1.780
C-Y	11.7	286	.816	288	-0.7	75.2	1.800
G-387	15.15	377	.829	373	+1.0	81.3	1.800
M-6	12	284	.790	295	-2.8	75.4	1.795
M-12	11.85	281	.790	292	-2.8	75.2	1.795
USA-35-A	18.18	452	.829	447	+1.4	85.5	1.775
USA-35-B	11.58	287	.826	285	+0.7	75.8	1.820
USA-27	11.04	273	.825	272	+0.4	74.8	1.815
USA-5	6.38	162	.847	157	+3.0	68.6	1.800
RAF-15	6.38	165	.863	157	+5.0	67.9	1.865
N-9	8.46	208	.819	208	0.0	70.9	1.840
N-10	11.22	274	.815	276	-0.7	74.4	1.806
			k = .820				p = 1.805

$$S_t = \frac{M}{2A'} = \frac{M}{2 \times .820\, h \times b} = .61 \frac{M}{h \times b} \qquad (5)$$

$$\theta = \frac{ML}{Gt} \; \frac{1.805(h + b)}{(2 \times .820\, h \times b)^2} = .67 \; \frac{(h + b)}{h^2 \; b^2} \; \frac{ML}{Gt} \qquad (6)$$



Fifure 2.

Table II is computed in the similar way as the table I, except that k' is defined $k' = \frac{A}{h \times b}$ b is used because it seem to be more logical parameter then C. P_t is the total perimeter including ordinate at 30% chord, and p' is defined by $P_t = p'(h + b)$, and p is the mean value of p'.

Again the above values are substituted in the equations (1) and (2) and we get equations (5) and (6)

TABLE III

30%-60% Chord

Airfoil	h_{30}	h_{65}	h	A	k'	A'	%err.	P_t	p'
C-Y-21	21	14.8	17.9	662	1.056	659	+0.7	106.5	2.010
C-Y-18	18	12.67	15.33	570	1.061	564	+1.0	101.3	2.010
C-Y-15	15	10.58	12.79	478	1.066	471	+1.5	96.1	2.010
C-Y	11.7	8.25	9.97	366	1.048	367	-0.2	90.4	2.010
G-387	15.18	9.84	12.51	457	1.043	471	-3.0	95.0	2.008
M-6	12	8.8	10.4	391	1.073	383	+2.0	91.1	2.003
M-12	11.85	8.8	10.3	379	1.051	379	0.0	90.9	2.005
USA-35-A	18.18	11.36	14.77	551	1.065	544	-1.4	100.0	2.005
USA-35-B	11.58	7.56	9.57	352	1.050	352	0.0	89.4	2.005
USA-27	11.04	8.64	9.84	358	1.040	362	-1.0	90.0	2.005
USA-5	6.38	4.8	5.59	197	1.007	206	-4.0	81.4	2.005
RAF-15	5.9	5.0	5.45	197	1.033	200	-1.5	81.1	2.005
N-9	8.46	5.76	7.11	264	1.062	262	+0.7	84.4	2.003
N-10	11.22	7.81	9.56	355	1.062	352	-0.7	89.3	2.003
					k = 1.051				p = 2.005

$$S_t = \frac{M}{2A'} = \frac{M}{2 \times 1.051\ h \times b} = .476 \frac{M}{h \times b} \qquad (7)$$

$$\theta = \frac{ML}{Gt} \frac{p}{(2A')^2} = \frac{ML}{Gt} \frac{2.005(h+b)}{(2 \times 1.051\ h\ b)^2} = .453 \frac{(h+b)}{h^2\ b^2} \frac{ML}{Gt} \qquad (8)$$

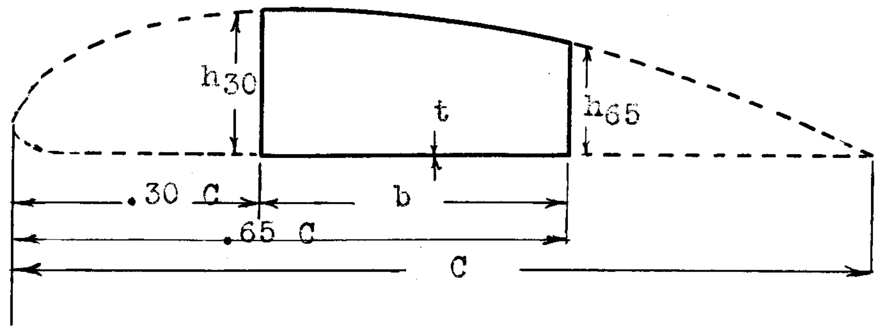


Figure 3.

Just as the tables I and II table III is computed except that h is taken as the mean value of h_{30} and h_{65} The equations obtain are (7) and (8).

TABLE IV

65%-100% Chord

Airfoil	h"	A in²	k'	A' in²	%err.	P_t"
C-Y-21	14.8	282	.544	290	-2.7	86.7
C-Y-18	12.67	238	.536	249	-3.6	84.1
C-Y-15	10.58	197	.532	207	-4.8	81.7
C-Y	8.25	154	.533	162	-4.9	79.3
G-387	9.84	181	.528	193	-5.8	81.2
M-6	8.8	180	.584	173	+4.9	79.3
M-12	8.8	177	.575	173	+2.0	79.3
USA-35-A	11.36	223	.560	223	0.0	83.4
USA-35-B	7.56	140	.522	148	-5.6	78.6
USA-27	8.64	180	.594	170	+5.9	79.6
USA-5	4.8	104	.620	95	+9.5	75.2
RAF-15	5.0	113	.646	98	+15.2	75.2
N-9	5.76	114	.556	113	-0.9	76.2
N-10	7.81	153	.560	153	0.0	76.6
			k = .560			

$$S_t = \frac{M}{2A} \quad \frac{M}{2 \times .56\, h \times b} = .892\, \frac{M}{h \times b} \qquad (9)$$

$$\theta = \frac{ML}{G\,t} \quad \frac{P}{(2A')^2} \quad \frac{M(2\,b+h)L}{G\,t\,h^2\,b^2} = .314\,\frac{ML}{G\,t}\left(\frac{2}{h^2 b} + \frac{1}{h\,b^2}\right) \qquad (10)$$

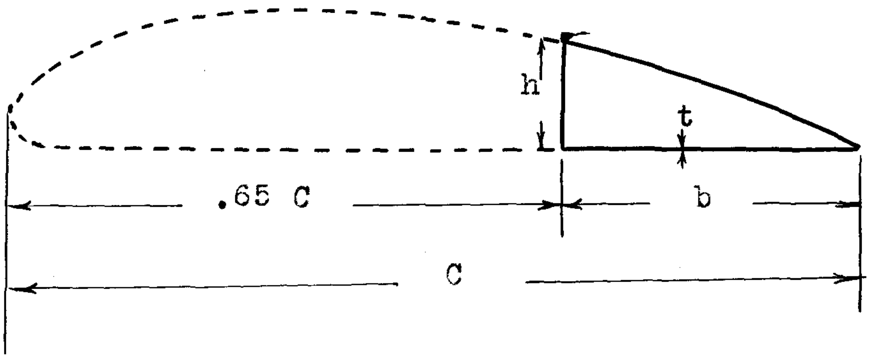


Figure 4.

Table IV.is obtained in previously described manner. But it is felt that percentage of error involve in A' is high and in practice the area of a given section can be estimted better directly then parametricly.

EXPERIMENTAL CHECK OF PRANDTL'S FORMULA

First attempt to check the validity of Prandtl's formula has been made by using experimental data published by S.von Fakla in the Luftfahrtforschung, vol.4, book 1, page 39; Munchen, 1929.

The wing tested was 150 cm. long with two spars and 20 ribs evenly spaced. The other properties of the section are given in figure 6. Curve (1) gives the angle in degrees of twist per 150 cm. of the model without the skin. The curve (2) gives the angle of twist with the paper cover over the ribs. The curve (3) gives the angle using .08 cm. plywood cover.

It can be deduced from the character of the curves that the spars and the ribs contribute very little to the regidity of the model.

If we take the following values;

$M = 10$ cm.-Kg, $C = 20$ cm. $h = 1.7$ cm

$t = .08$ cm. $G = 30000$ Kg/cm. $L = 150$ cm.

and substitute in the equation (2) we get

$$\theta^\circ = \frac{1.04 \times 10 \times 57.3 \times 150}{30000 \times .08 \times (1.7)^2 \times 20} = .642 \text{ degrees}$$

This is too low. The curve gives about 3 degrees, but with the spars and ribs included, hence we should expect slightly more than 3 degrees.

For the second set of experimental data the autor is indebted to the Northrop Aircraft Co., Burbank, Cal.

A full size wing of the Alpha plane have been tested. The wing was of monocoque constiuction having 9 ribs and longitudinal stiffeners. The plan of the wing is shown by dotted line on figure 5. At the root C-Y-18 section was.used with 100" chord. The wing has gradually taper both in the plan and the thickness so that at the tip the section was C-Y-11.7 with the chord of 66 inches.

For the purpose of this analysis it has been assumed that the wing is built-up of hollow cylinders which a constant cross section between adjacent ribs and equal to the section which corresponds to the stations # 1, # 2, # 3, etc. It is also assumed that the aileron has no effect on rigidity of the wing.

Table V was computed in the following manner:

Column 2) C" is the chord in inches

3) h" is the maximum ordinate in inches,

4) A is the total area of the cross section computed by the formula A = .694 C h (this coefficient corresponds to the Clark Y family of airfoils.

5) is the perimeter in per cent of chord. Values are obtaied from Dr. Klein's charts.

6) P" is the perimeter in inches

7) A' is the area of theaileron section. It has been found that theaileron is located at 83.9% chord and the areas were computed on the assum-

ption of triangular cross section.

8) is the effective area, and is equal to the difference between column 4) and column 7).

Table VA

Column 9) is the rib number

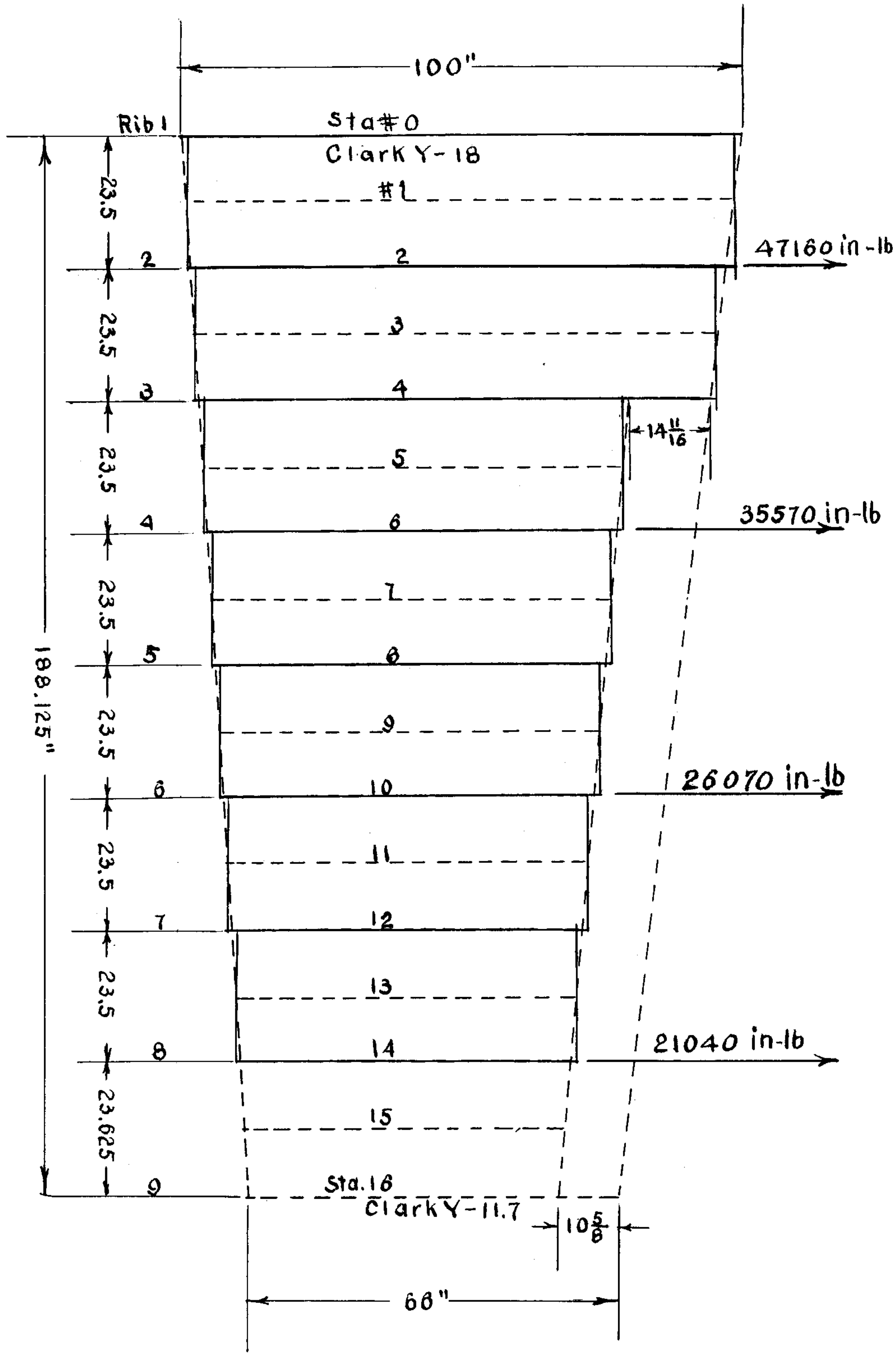
11) is the thickness of the shell. Up to the rib 3 for 74.5% chord t equals .040 in. and 125.5% chord is 0.032 in. or average t is .035 in. From rib 3 to rib 5 the average t is .0357, and from rib 5 to the tip t is .0281 inches.

12) Twisting moments applied to the wing are shown on figure 5. M is the total moment in in-lbs. at given section.

13) θ is the angle in radians and was obtaied by using equation (2)

14) is the total angle of twist for a given section.

It can be seen that the total deformation at the rib 8 is .005230 radians. Since there is no momemt between rib 8 and rib 9 the same deflection is assumed for the tip. The chord at the tip is 66 inches long, therefore the deflection should be .005230 x 66 or .345 inches. The experimental value is .70 inches.



100"
Rib 1
Sta#0
Clark Y-18
#1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Sta.16
Clark Y-11.7
23.5
23.625
188.125"
14 11/16
10 5/8
66"
47160 in-lb
35570 in-lb
26070 in-lb
21040 in-lb

Fig 5.

TABLE V

1	2	3	4	5	6	7	8
Sta.	C"	h"	A	P%C	P"	A'	A_e
0	100.00	18.0	1250	207.8	207.8	--	1250
1	97.88	17.6	1183	207.5	203.1	--	1183
2	95.75	17.2	1094	207.3	198.4	--	1094
3	93.62	16.8	1022	207.0	193.7	--	1022
4	91.50	16.4	952	206.9	189.2	--	952
5	89.37	16.0	886	179.6	160.6	42.0	844
6	87.25	15.6	824	179.1	156.2	39.4	785
7	85.12	15.2	769	178.8	152,0	36.9	732
8	83.00	14.8	708	178.4	148.0	34.3	674
9	80.87	14.4	658	178.1	144.0	31.8	626
10	78.75	14.0	602	177.7	139.8	29.3	573
11	76.62	13.6	555	177.3	135.7	26.7	548
12	74.50	13.2	508	177.0	131.8	24.2	484
13	72.37	12.8	465	176.7	127.8	21.6	443
14	70.25	12.4	424	176.3	123.9	19.1	405
15	68.12	12.0	386	176.0	119.9	16.1	370
16	66.00	11.7	353	175.7	115.9	14.0	339

TABLE VA

9	10	11	12	13	14	15
Rib	$(2A_e)^2$	t"	L"	M"-#	θ rad.	θ_t rad.
1						
2	5598000	.035	23.5	130840	.000930	.000930
3	4178000	.035	23.5	82680	.000695	.001625
4	2849000	.0357	23.5	82680	.000877	.002502
5	2144000	.0357	23.5	47110	.000629	.003131
6	1568000	.0281	23.5	47110	.001032	.004163
7	1201000	.0281	23.5	21040	.000568	.004731
8	785000	.0281	23.5	21040	.000499	.005230
9			23.625			.005230

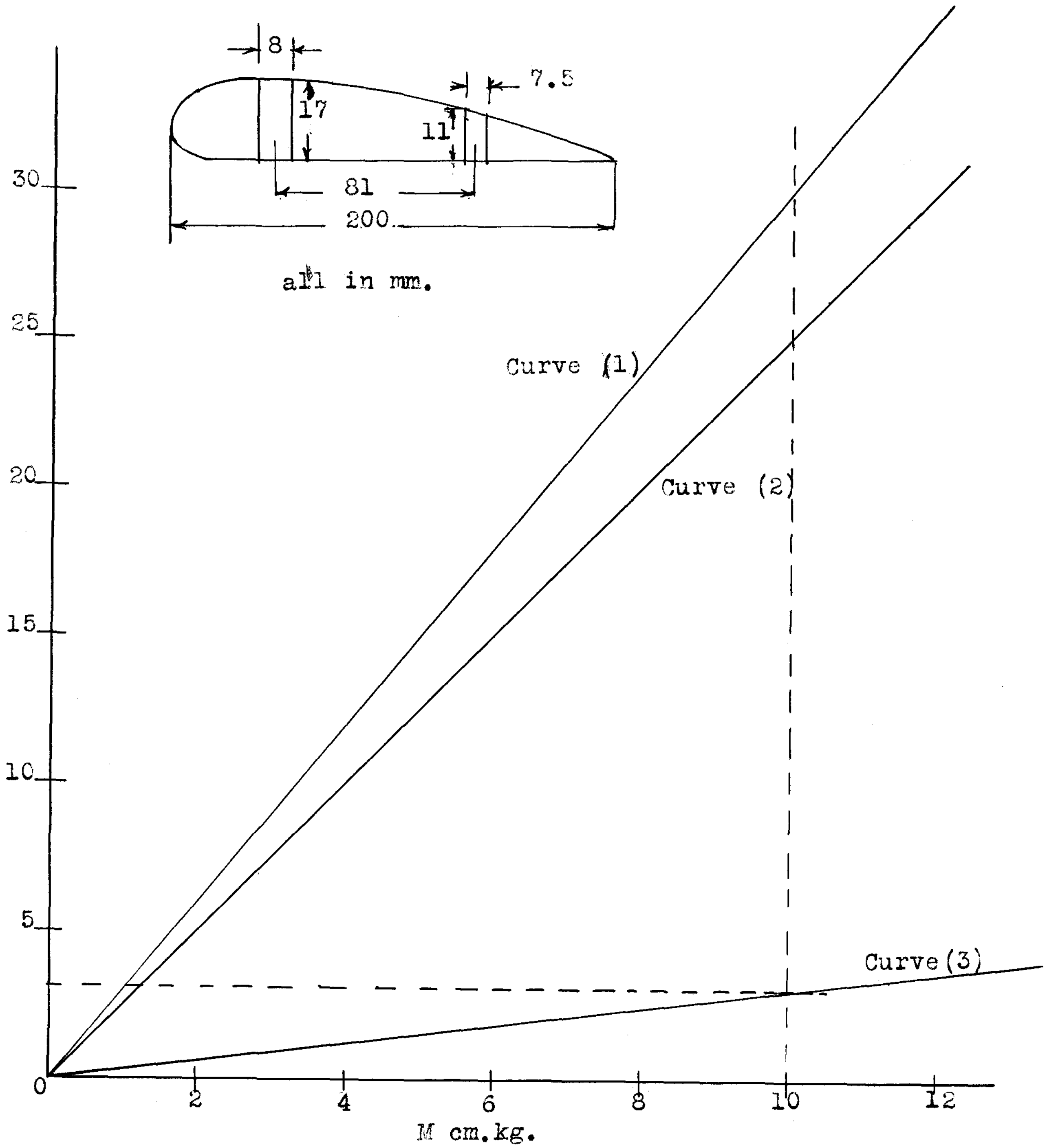

8
7.5
17
11
81
200
all in mm.
30
25
20
15
10
5
0
2
4
6
8
10
12
Curve (1)
Curve (2)
Curve (3)
M cm.kg.

Figure 6

DISCUSSION OF RESULTS

All computed values of deflection are much smaller then those which have been found experimentaly. The author feels that data obtained by Fakla are not reliable. In his report there is a lack of description of the set up and the manner in which the experiments were performed. All that can be inferred is that the testing model was in a horizontal position, that the loads were applied at one edge of the free end and that the other edge was attached to the celing by means of a spring balace. Such a set up implies that the elastic center is at mid-point of the section. It is however known that this is not generally the case for airfoils. There is then a posibility that a vertical component of force was present when the couple was applied. Farthermore the question of fixed end is not discussed properly, and it is not known haw rigidly the model was attached. And finally the dimensions of the model would seem to be out of proportion. A specimen which is 150 cm. long and only 20 cm. wide and of 1.7 cm. maximum thickness and made out of wood, propably is too flexible for this experiment.

On this bases it is very doubtful Fakla's data can be used as an experimental check of Prandlt's formula.

In the case of the Northrop wing the calculated value is about 50% smaller than the observed value. But in Northrop report it is stated that due to the set up of the experiment it is possible that from 25% to 50% of the observed deflection is in the supporting jig and not in the wing. There is another possibility of experimental error. Two equal forces in opposite directio were applied at the leading and the trailing edge respectively. Hence the assumption that the elastic center is at the mid-point of the section is also made in this case. Due to this method of loading we can not be certain that the two forces are exactly equal, therefore a linear deflection may occur in bending, which would greatly increase the apparent angular twist. This would be further aggrivated by the fact that readings were taken at rib 9 in place of rib 8 for which computations were made.