THE GROUND ROLL PHENOMENON OF APPLIED SEISMOLOGY

By

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INTRODUCTION

Definition and Description of the Ground Roll.

The seismograms of applied seismology frequently exhibit the late arrival of large amplitude, low frequency (3-30 cycles per second) oscillations. Because the arrival time of these oscillations is equal to the arrival time of the rolling motion which one may feel underfoot as one stands on the ground at the seismometer receiving the earth vibration, these oscillations are known as the ground roll, and this term may be taken to mean either this physiological sensation itself, or the oscillations corresponding to it which are observable on the seismograph record.

In the seismogram of Fig. 1, an example of the ground roll (labelled G) is recorded on the first three traces. The corresponding seismometers were all buried at the same distance from the shot point. The three seismometers were all of low natural frequency, almost critically damped, and were used with amplifiers having flat response characteristics. For comparison, the records obtained by two standard discriminatory systems are given in the last two traces. It is to be noted that the large oscillations which comprise the ground roll do not die away rapidly as do the other phases, but continue to have a sizeable amplitude for a long period of time.

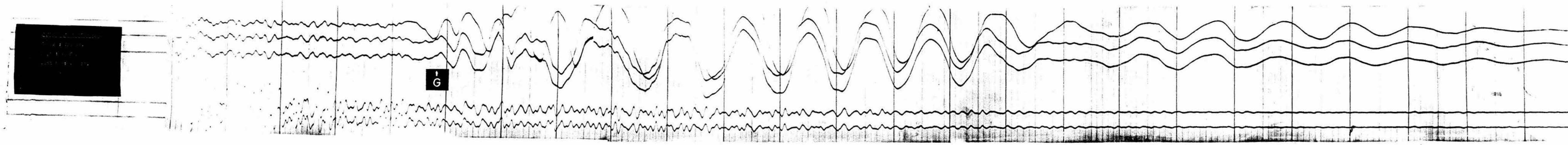

Fig. 1. Sample of Ground Roll

Also discernible from Fig. 1 is the small apparent velocity of the ground roll. Taking the observed arrival time of .57 sec., and the known distance from shot point to seismometer of 400 ft., one finds for the apparent ground roll velocity in this case only 700 ft./sec. (235 m./sec.). This small velocity is typical although it may be found to vary from place to place between the limits of 130 and 550 m./sec. For example, Angenheister,[1]* who seems to have been the first to describe ground roll phenomenon, found surface waves with velocities of 510 m./sec. in Jüterbog, Germany, at a place where the alluvium has a thickness of about 100 m.** Gutenberg, Wood and Buwalda[10] found a ground roll velocity of 550 m./sec. in the Los Angeles Basin, and velocities from 160 to 240 m./sec. in the Ventura Basin.

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* All references are given on pages 95-99.

**Approximately the first 30 m. of this alluvial layer consisted of loose sand.

At Yosemite in 1937, a seismological expedition from California Institute of Technology* found that the ground roll velocity varied from 130 to 550 m./sec.

The fact that the ground roll is a surface wave has been common knowledge for several years. As far back as 1886 Fouqué and Lévy[11] had performed seismic experiments at different depths, and found that the amplitude of low velocity waves decreased with depth. Prior to this work, a velocity of about 500 m./sec. for granite was obtained by Mallet[25] and others. This velocity is now known to be too small by a factor of almost 10. The work of Fouqué and Lévy[11] definitely established these low velocities as belonging to waves conditioned by the surface. In the earlier work of Mallet[25] only these surface waves had sufficient energy to be observed with the crude seismometer which he used. Present day investigators may now safely consider these surface waves as having been the ground roll.

It was well known from the early theoretical work of Zoeppritz[41] and others that the waves on the surface of an elastic earth should diminish rapidly with increase of depth of the origin of the waves. This led certain early seismic prospectors, who recognized the fact that the ground roll was a surface wave, to succeed in diminishing the ground roll amplitude by burying the dynamite charge at greater depths. Others, by pure coincidence, found diminishment of the ground roll amplitude accompanying deeper charge burials, which, in reality, were designed for the purpose of improvement of seismic reflections or perhaps the curtailment of explosion damage.**

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* Headed by Professors B. Gutenberg, J. P. Buwalda and C. F. Richter. The results of this expedition are as yet unpublished.

**By 1936, most seismic prospectors had recognized the possibility of taking advantage of the low frequency of the ground roll to discriminate against it by means of suitable electrical filtering in the amplifiers, or by the use of seismometers of sufficiently high natural frequency.

As a general rule it is possible to observe the ground roll in almost any locality if the shot depth is sufficiently small, even with fairly small charges of dynamite. Conversely, even if the shot depth is large, it is generally possible to secure considerable ground roll, if sufficiently large charges are used. In some localities, however, even large charges exploded on the surface fail to produce appreciable ground roll. Such localities are usually characterized by exposure of very hard geologic formations and the absence of a low velocity layer. These three factors--size of charge, shot depth, and nature of the surface material--seem to determine the <u>occurrence</u> of the ground roll.

The character of the ground roll also depends on these three factors. In addition, the character is markedly influenced by the distance of the seismometer from the shot point. Generally the number of oscillations comprising the ground roll as well as the period increases with distance from the shot point. As has been mentioned, the amplitude decreases with increase of shot depth. This is also accompanied by an increase in frequency. Generally, the periods observed will be greatest in regions of thick low velocity layers. Large charges seem to be more capable of producing the lower ground roll frequencies in such areas.

<u>The Ground Roll Problem.</u>

Since it was established quite early that the ground roll was a type of surface wave, it was logical first to seek an explanation of it in terms of either of the two well known surface waves of pure seismology; namely, Rayleigh[28] waves or Love[20] waves. But Gutenberg[9] pointed out certain objections to the application of simple Love wave or Rayleigh wave theory for an explanation of the observed waves, and he summarized his objections as follows: "Their (i.e. the ground roll) periods are of the order of .1 sec.; their wave lengths (usually between 10 and 50 meters) are too large for vibrations of a thin layer with small velocity of longitudinal waves. Their velocity seems to depend little on the elastic constants of the material and is too small for Rayleigh waves or shear waves in a thicker layer. Their amplitudes decrease very fast with distance. They may correspond to the surface waves observed by people in the epicentral region of an earthquake, and may be a gravitational type of surface wave in a viscous medium."

It is desirable to examine the objections of Gutenberg[9] so as to better understand which facts he regarded as well established, and which statements are to be regarded as plausible qualitative estimates of the conditions governing the phenomenon. That the periods of the ground roll are of the order of 0.1 sec. and their wave lengths are usually between 10 and 50 meters, may be considered as definitely true in most cases. His belief that wave lengths of this magnitude would be too large for vibrations of a thin superficial low velocity layer is based on the fact that most of the energy of a surface wave is confined to a region near the surface. The energy below a depth of about one wave length is negligible. Hence for a ground roll wave length which is large compared with the thickness of the superficial low velocity layer,* the vertical depth of penetration of most of the energy would

* The expression "weathered layer" is not considered accurate in this investigation, as the extent of true weathering is almost always uncertain. The expression "low velocity layer" will be employed. See also the next footnote.

extend well into the medium below the thin layer. One would therefore expect the velocity to be determined chiefly by the lower material. The velocity would then be too large. There is no doubt as to the validity of this reasoning. However, it must be remembered that no quantitative analysis of this objection was attempted.

The statement that the velocity seems to depend little on the elastic constants of the material is based on the fact that whereas the ground roll velocity is most frequently observed to be fairly near the value of about 300 m./sec., the elastic constants of the surface material vary to a considerable extent from region to region. The evidence for this large variation of elastic constants is based chiefly on the observation of a large variation of velocity of the compressional waves in the low velocity layer. However, it must be remembered that although very low or very high ground roll velocities were not observed as frequently as velocities near the value of 300 m./sec., still, later work, such as that done at Yosemite, has fairly well established that the ground roll may have a velocity of considerable variation. The range of velocity from 130 m./sec. to 550 m./sec. corresponds to a change in ground roll velocity by a factor of 4. The range of velocities in the low velocity layer does not correspond to a factor much in excess of this. It must be remembered, too, that the velocities referred to above usually are associated with the lower part of the low velocity layer. The upper part* almost always has a much lower velocity. The velocity of compressional waves in this very low velocity layer seldom has been

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* This upper part of the low velocity layer will be designated "very low velocity layer" to avoid confusion.

determined accurately, because in seismic prospecting a knowledge of the average velocity for the whole low velocity layer suffices for weathering corrections. On the other hand, there are some cases in which a higher ground roll velocity was obtained on a low velocity surface material than on a high velocity surface material, in support of Gutenberg's observation.

The statement that the velocity is too small for the ground roll to be Rayleigh[28] waves or shear (Love)[20] waves in a thicker layer is a logical rough estimate of the importance of the effect of an overlying low velocity stratum on the velocity of a surface wave. In Love's analysis of the propagation of shear (Love) waves in a low velocity stratum which was superposed on a high velocity substratum, he found that dispersion would occur; i.e. that the velocity of the waves would be a function of the wave length. He showed that for very small wave lengths the waves would travel with the velocity of shear waves in an infinite medium composed of the material of the low velocity stratum. As the wave length increased, the velocity increased until for very large wave lengths, the waves would have the velocity of shear waves in an infinite medium composed of the material of the high velocity substratum. Love also considered the effect of the low velocity stratum on the propagation of Rayleigh waves, and deduced entirely analogous results, although he did not calculate the specific dispersion law. He concluded that for very small wave lengths the Rayleigh waves would have the velocity of Rayleigh waves in a half space composed of the material of the low velocity stratum; as the wave length increased, the velocity increased until for very large wave lengths, the Rayleigh waves would have the velocity of Rayleigh waves in a half space composed of the material of the high velocity substratum. In connection with the ground roll, Gutenberg[9] felt that the wave length was sufficiently large for velocity to be essentially the velocity of Love waves or Rayleigh waves in high velocity material. It is immaterial whether one considers Love wave velocity or Rayleigh wave velocity since $V_R = 0.919\ V_Q$ (Poisson's ratio = 0.25), where V_R = velocity of Rayleigh waves and V_Q = velocity of shear waves. Either velocity would be larger than the observed ground roll velocity on the basis of the above deduction.

Thus it appeared several years ago that both of the accepted theories of propagation of surface waves on a plane elastic medium failed to agree with what seemed to be plausible qualitative data. In addition there was the statement that the amplitude decreased rapidly with distance from the source, which had to be answered. In seeking an explanation Gutenberg[9] suggested that the ground roll may be a gravitational type of surface wave in a viscous medium, which in view of the objections to explanations on the basis of elastic waves, seemed to be quite a reasonable hypothesis.

In addition to these problems, the oscillatory nature of the ground roll required adequate explanation. It is not immediately apparent how a disturbance of so short a duration as that of a dynamite explosion should give rise to a propagated surface wave which would continue its oscillations for so long a time.

Purpose of Investigation.

The purpose of this investigation is to examine the plausibility of various proposed explanations of the ground roll. The following hypotheses will be examined:

1. Gravitation wave in a viscous medium
2. Effect of atmosphere on Rayleigh waves
3. Surface waves on a visco-elastic medium
4. Surface waves on an elastic medium

In connection with the last hypothesis an examination will be made of the evidence furnished by seismic field records of the ground roll. Finally, a description will be given of experiments performed for the purpose of seeking a relation between the ground roll produced by explosion and the surface waves produced by a ground shaker.

VARIOUS GROUND ROLL HYPOTHESES

Gravitational Wave in a Viscous Medium.

For the deeper high velocity layers, it is well known that the laws of propagation of waves in an elastic medium are followed to a good approximation. Although these layers possess a viscosity, their rigidity is so high that it is perhaps preferable to regard them as elastic solids of high viscosity rather than as viscous fluids.

The extent of elasticity in the low velocity layer, however, is not well established. Iida[12] has performed experiments on specimens taken from the low velocity layer near Tokyo, and has found the following values of the various constants (c.g.s. units):

$$\mu = \text{rigidity} \approx 10^9$$

$$E = \text{Young's modulus} \approx 10^9$$

$$\mu' = \text{viscosity} \approx 10^5 - 10^6$$

$$\rho = \text{density} \approx 1.91$$

$$\sigma = \text{Poisson's ratio} \approx 0.31 - 0.33$$

The low value of the rigidity as well as the high value of Poisson's ratio are indicative of the fluid-like nature of the low velocity layer.

Since the low velocity layer conceivably has the properties of a viscous fluid, whereas the high velocity layers beneath are more definitely elastic, the hypothesis that the ground roll is a gravitational wave in a viscous medium must be applied only to the case of a finite depth of the viscous fluid. An infinite depth of the viscous fluid would clearly depart from physical reality. In fact, it even may be objectionable to have the depth of the fluid extend as far as the base of the low velocity layer. If the hypothesis is valid at all, it may be that it is applicable only to the situation of the depth of the viscous fluid being equal to the thickness of the very low velocity layer.

When the motion of a viscous incompressible fluid is in two dimensions, and the squares and products of the velocities are neglected, the following equations are satisfied:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right\}$$

(1)

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \nu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right\}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

where u = component of velocity of the fluid in the x direction

w = component of velocity of the fluid in the z direction

p = the pressure at the point (x, z) at the time t

g = acceleration of gravity

v = the kinematic viscosity = viscosity μ ÷ density ρ,

and the coordinates are taken as shown in Fig. 2.

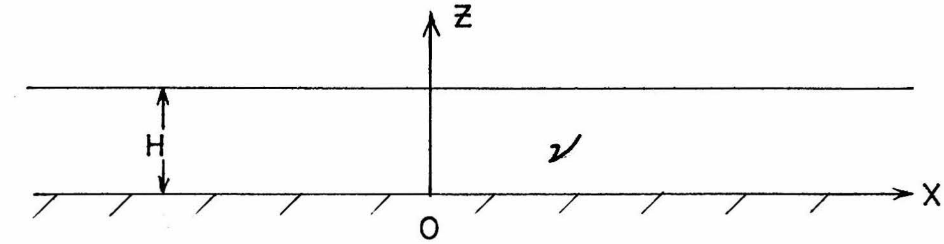


Fig. 2. Viscous Layer of Depth H on Rigid Support

It is desired to find a solution of this differential equation of the form $\phi(z)\, e^{imx + kt}$ which satisfies the boundary conditions

(a) p = constant at $z = H$

and (b) $u = 0$, $w = 0$ at $z = 0$,

where $m = 2\pi/L$, L is the wave length of the propagated wave, and k is to be determined.

Clearly, wave motion will not be possible unless k is a complex quantity whose real part is negative. For this is the only form of k which represents a train of waves whose amplitudes diminish with time.

The equation for the determination of k is readily found to be

$$(2) \qquad \left\{(k + 2\nu m^2)\cosh(mH) + \frac{gm}{k}\sinh(mH)\right\}(m^2+\alpha^2)\sinh(\alpha H) - \left\{2\nu m\alpha\cosh(\alpha H) + \frac{gm}{k}\sinh\alpha H\right\}2m^2\sinh(mH) = 0$$

where $\alpha^2 = m^2 + k/\nu$.

Although Wien,[40] Basset,[5] Arakawa[3] and Lamb[17] have discussed this equation for the case of small viscosity, it has not been considered for values of the viscosity to be expected in the seismic low velocity layer. Such values of viscosity can neither be regarded as "very small" nor as "very large." Approximations should therefore be made only for a detailed analysis.

Dividing (2) by $\sinh(\alpha H)\cdot\sinh(mH)$ it becomes

$$(3) \qquad (\alpha^2 + m^2)^2 \coth mH + \frac{gm}{\nu^2} - 4m^3\alpha\coth\alpha H = 0$$

Putting

$$(4) \qquad \left(\frac{\alpha}{m}\right)^2 = p^2 = 1 + k/\nu m^2 \text{, and}$$

(5) $$mH = \omega = \frac{2\pi H}{L}$$, (3) becomes

(6) $$p^4 + 2p^2 - 4p\frac{\coth p\omega}{\coth \omega} + \left[\frac{g\tanh \omega}{\nu^2 m^3} + 1\right] = 0$$

It is apparent that whether gravity is negligible or not depends on how the quantity $\frac{g\tanh \omega}{\nu^2 m^3}$ compares with unity. In Table I, values of $\frac{g\tanh \omega}{\nu^2 m^3}$ are computed for various values of L, H, and ν . The values of the wave length L are tabulated in the first column. The next three columns give the values of H/L, ω , and tanh ω corresponding to these values of L, for the case of H = 1 m. The fourth column gives the corresponding values of the gravitational term for $\nu = 10^5$ c.g.s. units, while the fifth column gives the values for $\nu = 10^6$ c.g.s. units. The remainder of the table gives similar results for the case of H = 10 m.

An examination of Table I shows that for H as great as 10 m., the factor tanh ω is practically unity throughout the range of wave lengths considered. For H = 1 m., however, it varies from 1.00 at L = 1 m. to 0.125 at L = 50 m. Gravity is negligible over a much larger range of wave lengths for $\nu = 10^6$ than for $\nu = 10^5$ c.g.s. units. Since Table I includes a large portion of the plausible conditions which might occur in the field, it is conceivable that the field conditions might be divided into two classes:

(A) Where the effect of gravity is negligible; even for ν as low as 10^5 c.g.s. units this will be the case for the shorter wave lengths.

(B) Where the effect of gravity is not negligible; even for ν as high as 10^6 c.g.s. units, this will be the case for the larger wave lengths.

TABLE I

Importance of Gravitational Term Under Various Conditions

L (m.)	H = 1 m.					H = 10 m.				
				$\frac{g}{\nu^2 m^3} \tanh \omega$					$\frac{g}{\nu^2 m^3} \tanh \omega$	
	H/L	ω	$\tanh \omega$	$\nu = 10^5$ c.g.s.	$\nu = 10^6$ c.g.s.	H/L	ω	$\tanh \omega$	$\nu = 10^5$ c.g.s.	$\nu = 10^6$ c.g.s.
1	1	6.28	1.00	3.95×10^{-4}	3.95×10^{-6}	10	60.28	1.00	3.95×10^{-4}	3.95×10^{-6}
2	.5	3.14	.966	3.16×10^{-3}	3.16×10^{-5}	5	31.4	1.00	3.17×10^{-3}	3.17×10^{-5}
3	.333	2.09	.969	1.04×10^{-2}	1.04×10^{-4}	3	20.9	1.00	1.07×10^{-2}	1.07×10^{-4}
6	.16	1.05	.782	6.70×10^{-2}	6.72×10^{-4}	1.6	10.5	1.00	8.56×10^{-2}	8.56×10^{-4}
10	.10	.628	.557	.220	2.20×10^{-3}	1.0	6.28	1.00	3.95×10^{-1}	3.95×10^{-3}
20	.05	.314	.304	.964	9.64×10^{-3}	.5	3.14	.996	3.16	3.16×10^{-2}
30	.0333	.209	.205	2.20	.022	.333	2.09	.969	10.4	.104
50	.02	.126	.125	6.20	.062	.200	1.26	.851	42.2	.422

<u>(A) Gravity Term Negligible</u>: If $\frac{g}{\nu^2 m^3} \tanh \omega$ is small compared with unity, equation (6) becomes

(7) $$\frac{\tanh p\omega}{\tanh \omega} = \frac{4p}{(1+p^2)^2}$$

It is interesting and of value to note that this equation is identical in form with that obtained by Lamb[17] for the symmetrical vibrations of an elastic plate. He computed the velocity of waves in such a plate for various values of ω . The velocity of such waves is given by $V^2 = (1-p^2)\,\mu/\rho$, where μ = rigidity, ρ = density. In Lamb's analysis, however, it sufficed to consider only the purely real or purely imaginary values of p. In the case under consideration, such values will not produce periodic motion. For by equation (4), k cannot be complex if p is purely real or purely imaginary, and as has been remarked earlier, unless k has an imaginary part, no periodic motion is possible. The velocity of the viscous waves should be determined by the imaginary part of k, and the damping of the waves should be determined by the real part of k. Hence in order to find the velocity and the damping of the viscous waves, it is necessary to consider the complex roots of equation (7).

For the two limiting cases of $\omega = \infty$ and $\omega = 0$, the roots of (7) may be found quite easily. For $\omega = \infty$, $\tanh(p\omega)/\tanh \omega = 1$; hence (7) becomes

(8) $$p^4 + 2p^2 - 4p + 1 = 0$$

Examination of equation (7) shows that p = 1 is always a root, regardless of the value of ω . But by virtue of equation (4) it is seen that $p = 1$ corresponds to $k = 0$, or no motion. This root is thus of no importance. It is also evident that since it is p^2 that determines the value of k, it is immaterial whether $+p$ or $-p$ is considered. From equation (7) one sees that if $+p$ is a root, $-p$ is also a root. Since $p = 1$ is always a root, the quartic equation (8) may be reduced to a cubic by dividing through by $(p-1)$. The cubic obtained in this way is

$$p^3 + p^2 + 3p - 1 = 0 \tag{9}$$

and its roots are 0.2956, $(-0.647 \pm 1.715\, i)$.

If $\omega = 0$, the left member of (7) is equal to p so that (7) becomes

$$p^4 + 2p^2 - 3 = 0 \tag{10}$$

The roots of this equation are clearly $\pm 1, \pm\sqrt{3}\, i$.

These two cases are in themselves of no practical interest as they involve infinite and zero wave lengths for a finite layer thickness. However, inasmuch as the case of $\omega = 0$ is approximated by the physically conceivable case of wave length large compared to layer thickness, the consideration of the equation for $\omega = 0$ will indicate the type of wave propagation to be expected for large values of L/H. Similarly $\omega = \infty$ approximates the case of small L/H.

For $\omega = 0$, one finds from (4) that corresponding to the root $\sqrt{3}\, i$, $k = -4\, \nu m^2$. The absence of the imaginary part shows that no propagated periodic motion will be possible. A disturbance on the surface of the fluid will not be propagated along, but will simply diminish with time with the damping factor $e^{-4\left(\frac{4\pi^2\nu}{L^2}\right)t}$. For large values of L/H it appears that no propagated motion is possible. However it must be remembered that if L is large enough, the gravity term is no longer negligible. If H is as great as 1 m., large values of L/H will be possible only when L is sufficiently large for gravity not to be negligible. It must be concluded that the case of small value of ω must be considered only when the gravity term is not negligible. This case will therefore be treated later.

For $\omega = \infty$, one finds that corresponding to the roots 0.295, $(-0.647 \pm 1.715\, i)$, $k = -0.913\, \nu m^2$, $(-3.52 \pm 2.22\, i)\, \nu m^2$. Here again the real root does not correspond to propagated waves, but to a stationary disturbance which diminishes with time with a damping factor of $e^{-.913\left(\frac{4\pi^2\nu}{L^2}\right)t}$. The two complex roots, however, do correspond to propagated waves, travelling in the positive and the negative directions of x, respectively. The velocity of such waves is $V = 2.22\,(2\pi\nu/L)$ and the damping factor is $D = e^{-3.52\left(\frac{4\pi^2\nu}{L^2}\right)t}$. Thus propagated waves are possible for small values of L/H, even though gravity is negligible.

(B) <u>Gravity Term not Negligible</u>: It was found in the course of the preceding discussion that small values of ω were to be considered only when the gravity term was not negligible. If ω is very small and gravity is not negligible, equation (6) becomes

(11) $$p^4 + 2p^2 + \left[\frac{gH}{\nu^2 m^2} - 3\right] = 0$$

The roots of this biquadratic equation are

(12) $$p = -1 \pm i\sqrt{gH/\nu^2 m^2 - 4}$$

Thus in order to have propagated waves for ω very small it is necessary that the inequality

(13) $$gH > 4\nu^2 m^2 = 16\pi^2\nu^2/L^2$$

be satisfied. For in this case the value of k will be complex. The waves will be propagated with a velocity $V = \sqrt{gH - 4m^2\nu^2}$ and with a damping factor $D = e^{-2\nu m^2 t} = e^{-\frac{8\pi^2\nu t}{L^2}}$.

For large values of ω, if the gravity term is not negligible, equation (6) becomes

(14) $$p^4 + 2p^2 - 4p + (G+1) = 0, \qquad \text{where } G = g/\nu^2 m^3.$$

The discriminant of this quartic is

(15) $$\Delta = \frac{256}{3}\left\{3G^3 - 11G^2 + 54G - 33\right\}.$$

Hence $\Delta > 0$ whenever the cubic

(16) $$3G^3 - 11G^2 + 54G - 33$$

is positive. It is found that for $G < 0.66$, this is the case.

Since the coefficient of p^2 is 2 which is greater than 0, there will be no real roots of the quartic for $G > 0.66$. But for $G < 0.66$ the discriminant will be negative and there will be two real roots and two complex roots just as in the case of negligible gravity.

For the case of $G < 0.66$ a solution of the quartic was obtained for $G = 0.1$. The roots were found to be 1.132, 0.246, $(-0.639 \pm 1.75\, i)$.

Thus for a small gravity term, the complex roots are not altered appreciably from what they are for negligible gravity.

For the case of $G > 0.66$, a solution of the quartic was obtained for $G = 10$. The roots were found to be $-1.137 \pm 1.783\ i$ and $(1.137 \pm 1.189\ i)$. Thus for $G > 0.66$ two different waves are possible. For the first wave the velocity is $4.06\ \nu m$, and the damping factor is $e^{-3.80\left(\frac{4\pi^2\nu}{L^2}\right)t}$. For the second wave the velocity is $2.07\ \nu m$, and the damping factor is $e^{-1.02\left(\frac{4\pi^2\nu}{L^2}\right)t}$. The first wave thus has a velocity and a damping factor which is larger than that obtained for negligible gravity, while the second wave has a velocity and a damping which is smaller than that obtained for negligible gravity.

<u>Discussion of the Results</u>: Table II summarizes the values of the velocities and damping factors discussed above.

For $\omega = 0$, it is clear that no velocity of the waves can be greater than $\sqrt{gH}$. For if $4m^2\nu^2$ is greater than gH, the velocity would not be real, if $4m^2\nu^2$ is less than gH, the velocity will be less than $\sqrt{gH}$. But even for H = 10 m., this would give a velocity of only 10 m./sec.

For $\omega = \infty$, the velocities are all of the order of the value of $\left(\frac{2\pi\nu}{L}\right)$. Values of this quantity for various L and are given in the first three columns of Table III. Since the velocity for small (G=0.1) gravity effect or negligible (G=0) gravity effect is $2.22\left(\frac{2\pi\nu}{L}\right)$, it is evident that with a viscosity between 10^5 and 10^6 c.g.s. units, velocities of the order of the ground roll velocities

TABLE II

Summary of Velocities and Damping Factors under Various Conditions

			$\omega = 0$	$\omega = \infty$	
(A)	G Negligible	Velocity	No Wave Motion	$V = 2.22\left(\frac{2\pi\nu}{L}\right)$	
		Damping		$D = e^{-3.52\left(\frac{4\pi^2\nu}{L^2}\right)t}$	
(B)	G Not Negligible			G = .1	G = 10
		Velocity	$V = \sqrt{gH - 4\omega^2\nu^2}$	$V = 2.22\left(\frac{2\pi\nu}{L}\right)$	$V_1 = 4.06\left(\frac{2\pi\nu}{L}\right)$ $V_2 = 2.07\left(\frac{2\pi\nu}{L}\right)$
		Damping	$D = e^{-\frac{8\pi^2\nu}{L^2}t}$	$D = e^{-3.52\left(\frac{4\pi^2\nu}{L^2}\right)t}$	$D_1 = e^{-3.80\left(\frac{4\pi^2\nu}{L^2}\right)t}$ $D_2 = e^{-1.02\left(\frac{4\pi^2\nu}{L^2}\right)t}$

TABLE III

Values of $2\pi\nu/L$ and $4\pi^2\nu/L^2$ for Various L and ν

L (m.)	$\frac{2\pi\nu}{L}$ (m./sec.)		$\frac{4\pi^2\nu}{L^2}$ (sec^{-1})	
	$\nu = 10^5$ c.g.s.	$\nu = 10^6$ c.g.s.	$\nu = 10^5$ c.g.s.	$\nu = 10^6$ c.g.s.
1	62.8	628.	394.4	3944.
2	31.4	314.	98.6	986.
3	20.9	209.	43.8	438.
6	10.5	105.	10.9	109.
10	6.28	62.8	3.94	39.4
20	3.14	31.4	.99	9.86
30	2.09	20.9	.44	4.38
50	1.26	12.6	.158	1.58

are possible for small (1 to 6 m.) wave lengths. For the usual ground roll wave lengths, however, the velocity would be too small.

The damping coefficients are determined by the value of $\frac{4\pi^2\nu}{L^2}$. Values of this quantity are given in the last two columns of Table III. It is clear that for wave lengths which give velocities of the order of the ground roll velocity, the damping is too large. For example, suppose L = 6 and $\nu = 10^6$ c.g.s. units. By Table II, the damping factor will be $e^{-3.52\left(\frac{4\pi^2\nu}{L^2}\right)t}$. Hence, the wave which travels with a velocity of 2 x 105 = 210 m./sec., will be damped down to 1/e of its initial value in $\frac{1}{3.52 \times 10^9}$ = 0.00260 sec.

For the case of G = 10, by Table II the velocities will be $4.06\ m\nu$ and $2.07\ m\nu$. But since $G = g/m^3\nu^2 = 10$, the velocities are $V_1 = 4.06\sqrt{gL/20\pi}$ and $V_2 = 2.07\sqrt{gL/20\pi}$. For L as large as 100 m., the velocity of V_1 will be only 16.2 m./sec., and the value of V_2 will be even smaller. Clearly, for a large value of the gravitational term, the values of the velocity will be too small to be of any interest in the present problem.

It may be concluded that there is no possibility of any of the above waves giving a velocity and wave length as large as that of the ground roll.

Effect of Atmosphere on Rayleigh Waves.

The velocity of the ground roll ranges from 130-550 m./sec. Most frequently it is about 300 m./sec. or fairly near the velocity of sound in air (344 m./sec. at 20° C.). The early seismic prospectors observed that they felt a rolling motion underfoot more or less simultaneously with the audible noise of the dynamite blast. As a result, it became conceivable that there was some relation between the ground roll and the sound wave in air.

Angenheister[1] appears to have been the first to distinguish between the ground roll and the sound wave. He found that the period of the sound wave was about 0.01 sec. while the period of the ground roll was about 0.1 sec., and he plotted the two different travel time curves. Later Gutenberg[10] brought out the point that the ground roll was frequently observed even when the charge had been buried to such a depth that no sound was audible. Hence there was good evidence that the sound wave and the ground roll were unrelated.

On the other hand, the fact that the ground roll had approximately the velocity of sound in air remained unexplained. In attempting to establish some theoretical connection between the ground roll and the air wave, Bateman[6] discussed the influence of the atmosphere on the propagation of Rayleigh waves on a flat homogeneous isotropic earth. His investigation brought to light the existence of a "secondary Rayleigh wave," the velocity of which was found to be less than or greater than the ordinary Rayleigh wave velocity, depending on the atmospheric conditions. His theory thus offered a possible explanation of the observed range of velocity of the ground roll. Unfortunately, however, the amount of energy carried by this "secondary Rayleigh wave" could not be determined except by an analysis of the partition of wave energy at the source of the waves, a problem of considerable complexity. This analysis has never been made. Hence, the importance of this "secondary Rayleigh wave" for the problem treated here was never definitely established although its importance in other problems is not questioned. Furthermore, the severe conditions which were imposed on the atmosphere in Bateman's theory rather limited the plausibility of explaining the ground roll by means of this "secondary Rayleigh wave."

Surface Waves on a Visco-Elastic Medium.

Arakawa[2] has studied the propagation of periodic surface waves on a visco-elastic medium. His theory includes both Rayleigh and Love wave types of periodic disturbance. Sezawa[32] has made a similar study for the diffusion of tremors. However, since certain terms have been omitted in Sezawa's discussion, it is preferable to consider Arakawa's treatment.

Since the Rayleigh wave theory for a visco-elastic medium has been considered only for an homogeneous isotropic medium, it fails to correspond to the physical situation, not only because of the change of elastic constants with depth, but also because of the change of viscosity with depth. This makes it undesirable to attempt to apply the Rayleigh wave theory to the seismic results.

The Love wave theory for a visco-elastic medium, however, is based on assumptions more applicable to the surface of the earth. The variation of elastic constants and viscosity with depth is taken into account by the consideration of a stratum of constants ρ_1, λ_1,

μ_1, λ_1', μ_1' overlying a substratum of constants ρ_2, λ_2, μ_2, λ_2', μ_2' where

ρ = density

λ, μ = Lamé's constants

λ', μ' = rotational and irrotational viscosity coefficients,

and the subscripts 1, 2 correspond to the stratum and the substratum respectively.

In considering the propagation of periodic waves of the Love wave type on such a medium, Arakawa[2] defined a certain critical wave length L_L. For wave lengths less than L_L, no wave propagation was possible. He also deduced expressions for the velocity and the damping of the waves that were possible. No attempt was made to evaluate the velocity or the damping, in terms of physical constants. Since Iida[12] has found the order of magnitude of these constants for the low velocity layer, it is of interest to the ground roll problem to apply them to Arakawa's theory.

The critical wave length L_L is given by the relation

(17) $$L_L = \frac{2\pi}{\sqrt{-s^2 + 4\rho_1 \mu_1/\mu_1'^2}}$$

where s is to be determined from the equation

(18) $$\tan sH = \frac{\mu_2}{\mu_1}\left\{ \frac{f^2}{s^2}\left(1 - \frac{V_1^2}{V_2^2}\right) - \frac{V_1^2}{V_2^2} \right\}$$

where $f = 2\pi/L$ and $V_1 = \sqrt{\mu_1/\rho_1}$ $V_2 = \sqrt{\mu_2/\rho_2}$ H = layer thickness.

If (18) is written in the form

$$(19) \qquad fH = sH \left\{ \frac{V_2^2}{V_2^2 - V_1^2} \frac{\mu_1^2}{\mu_2^2} \tan^2 sH + \frac{V_1^2}{V_2^2 - V_1^2} \right\}^{1/2}$$

it is clear that as sH increases from 0 to $\pi/2$, fH increases from 0 to ∞.

Writing (17) in the form

$$(20) \qquad \frac{2\pi H}{L_L} = \sqrt{-s^2H + \frac{4\rho_1\mu_1 H^2}{\mu_1'^2}}$$

it becomes evident that s^2H^2 is negligible compared with $\frac{4\rho_1\mu_1 H^2}{\mu_1'^2}$.
For example, suppose

$\rho_1 = 2$

$\mu_1 = 10^9$

$\mu_1' = 10^6$ c.g.s. units

$H = 1$ m.

in accordance with the results of Iida.[12] The value found for will be 800. The maximum value of $(sH)^2$ is $(\pi/2)^2 = 2.47$, which is negligible compared with 800. For larger values of μ_1, or smaller values of μ_1', $(sH)^2$ will be still more unimportant. Hence

$$(21) \qquad L_L = \frac{\pi \mu_1'}{\sqrt{\rho_1 \mu_1}}$$

For the values of the physical quantities assumed, L = 0.2 m. Thus it appears that all waves in the seismic range of wave length are possible.

The velocity is given by the formula

$$(22) \qquad V = \sqrt{\mu_1/\rho_1 \left[1 + (s/f)^2\right] - \frac{\mu_1'^2}{4\rho_1^2} f^2 \left[1 + (s/f)^2\right]^2}$$

In order to compute the velocity under a typical condition, it was assumed that

$$\rho_1 = \rho_2, \qquad \mu_1/\mu_2 = 1/10 \qquad H = 10 \text{ m.} \qquad L = 30 \text{ m.}$$

It was then possible to solve the transcendental equation (18) by graphical means, and hence plot fH as a function of sH. Hence for any given value of fH, the value of s/f could be found. Substituting this value and the assumed value of $f = 2\pi/L$ into (22), the velocity could be found. For the case being considered, V was practically equal to the ordinary Love wave velocity without viscosity, V_o. For L = 1 m., the velocity was $V = 0.74\ V_o$.

Thus it seems that the presence of viscosity will reduce the velocity of ordinary Love waves in the low velocity layer, but not appreciably at the usual ground roll wave lengths.

The value of the damping coefficient is given by

$$D = e^{-\frac{f^2\left[1 + s^2/f^2\right]\mu_1' t}{2\rho_1}} \tag{23}$$

In the two examples just considered, the values of D are $e^{-6.4t}$ for L = 10 m. and $e^{-10000t}$ for L = 1 m. Thus the shorter waves will be damped out more rapidly. This could offer a possible explanation of the preponderance of low frequency in the ground roll. For a viscosity of 10^5 c.g.s. units, the value of D for L = 10 m. in the above case would be $e^{-0.64t}$, a value which is of the order of the ground roll damping.

Elastic Surface Waves.

In the introduction, certain objections to the explanation of the ground roll by means of simple elastic theory were given. For this reason, the following part of the ground roll investigation was carried out with the intention of more thoroughly examining the possibility of rigorously applying the results of theoretical investigations to the data. Before attempting to analyse the data, however, a discussion will be given of the theoretical results which might be of particular interest in connection with the ground roll.

Theoretical Background: The results of the theory of Rayleigh[28] waves on the surface of a plane homogeneous isotropic elastic half-space are well known. On such a medium, the motion of a particle will be confined to a vertical plane passing through the source of the disturbance. The particle will describe an ellipse in a retrograde sense. The ratio of the length of the major axis to the length of the minor axis of the ellipse will be 1.46 for Poisson's ratio $\sigma = 0.25$. The major axis is vertical. Thus the amplitude of the vertical component of motion is 1.46 times as great as that of the horizontal component of motion. For the same value of σ, the velocity of the Raleigh waves is 0.92 V_s, where V_s is the velocity of shear waves in the medium.

In the introduction, a discussion was given of the general nature of the dispersion which results when surface waves are propagated on a layered elastic medium. Although for Love waves, the specific dispersion relation was found in the early work of Love,[20] it was Sezawa[31] who first computed the dispersion curve for the more complicated case of Rayleigh waves. His computations were based on the assumptions that the density of the stratum ρ' was equal to the density of the sub-stratum ρ, and that Poisson's ratio was equal to 0.25 in both media. He found the dispersion curve for various values of the ratio μ'/μ, where μ', μ are the rigidities of the stratum and substratum respectively. The curves which he obtained are shown in Fig. 3. He represented the dispersion curve by plotting V/V_1 as a function of L/H, where $V_1 = \sqrt{\mu'/\rho'}$, the velocity of shear waves in an infinite medium composed of the stratum material. V is the velocity of the waves, L is the wave length, and H is the thickness of the stratum.

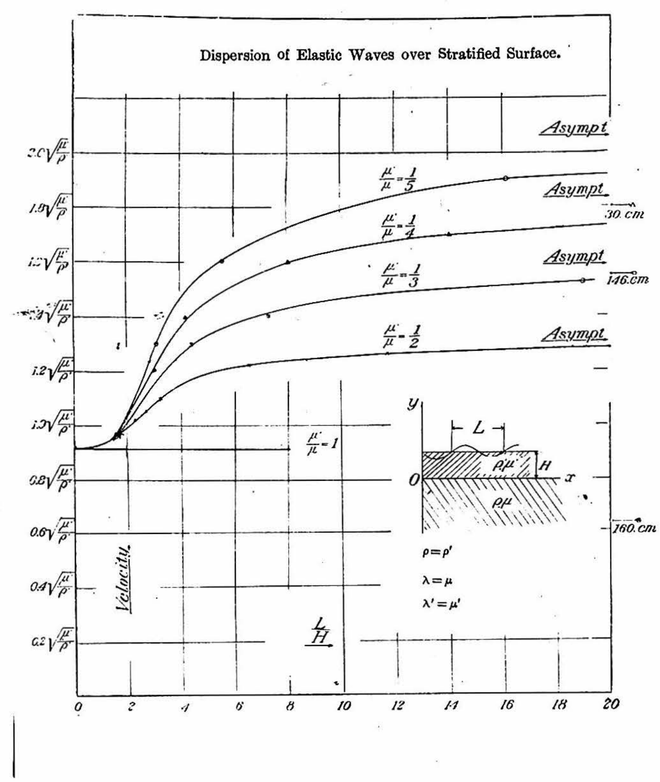
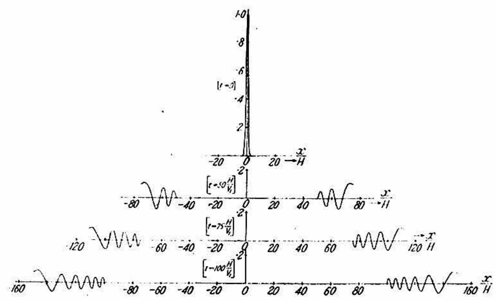


Fig. 3

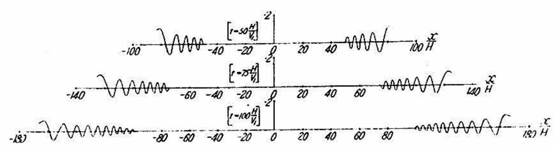
This method of representing the dispersion curve is desirable because it takes into account the possible variation of H or V_1. In attempting to interpret physical data, it may be found that the value of H does not remain constant, or perhaps the value of V_1 does not remain constant. It is seen that for this method of plotting, a single dispersion curve is possible, even with this variation, provided that the rigidity ratio remains constant.

If the curves of Fig. 3 be compared with Love wave curves obtained under the same conditions, it is found that they are very similar, in agreement with the advance predictions of Love.[20]

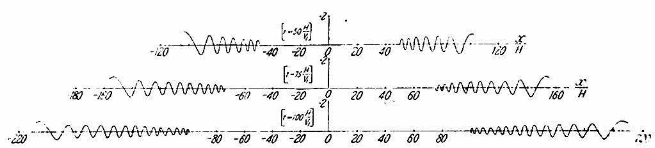
All of these investigations were made for the case of periodic waves. The more extended problem of the propagation of a pulse in a dispersive medium has been treated by Sezawa and Nishimura.[37] They show how a pulse in a dispersive medium will split up into two oscillations travelling outward. This is indicated for various μ'/μ, in Fig. 4. The number of complete oscillations to be found at any given distance is found to increase with distance. The leading and trailing parts of the disturbed portion take their positions at distances V_2t and V_1t from the origin, where V_2, V_1 are the velocities of the Rayleigh waves on half spaces composed of the material of the lower and upper media respectively, and t is the travel time of the pulse. The leading part of the oscillation will be composed of the longer wave lengths. This is to be expected from dispersion theory in which greater velocities are predicted for larger wave lengths. The most dominant length of the waves in the oscillatory part seems to be controlled by the dispersion relation, and not by the type of the



$[\mu'/\mu = 1/2.]$



$[\mu'/\mu = 1/3.]$



$[\mu'/\mu = 1/5.]$

Fig. 4

Propagation of a Pulse in a Dispersive Medium.

x = distance from source

t = time

original motion. The most common feature to be remarked in connection with the dispersed waves is that, the longer the range of the disturbed portion, the less will be the general magnitude of amplitude of the vibrating motion. Finally, the greater the value of μ/μ', the greater will be the range of the disturbed portion, and the frequency of the oscillation. The importance to the ground roll problem of this investigation is that it offers a possible explanation of oscillatory motion resulting from a single pulse; i.e. the explosion.

In his original analysis of the dispersion of Rayleigh waves, Sezawa[31] found a single dispersion curve corresponding to a given value of the rigidity ratio. Furthermore, his calculations of the velocity as a function of L/H called for a continuous, smooth curve (see Fig. 3) for every value of the ratio μ'/μ, which he employed. However, in subsequent analyses, in attempting to extend the range of values of the ratio μ'/μ to lower values, he discovered a discontinuity in the dispersion curve at a value of L/H equal to 4.622. Further work[34] indicated that there were actually two dispersion curves, as shown in Fig.5(a), for the case of $\mu'/\mu = 1/20$. For successively smaller values of the ratio μ'/μ, he found that the points O and O' of similar dispersion curves, approached each other and the discontinuities became more pronounced. In the practically ideal case of $\mu'/\mu = 1/\infty$, he found that the points O and O' coincided as shown in Fig. 5 (b).

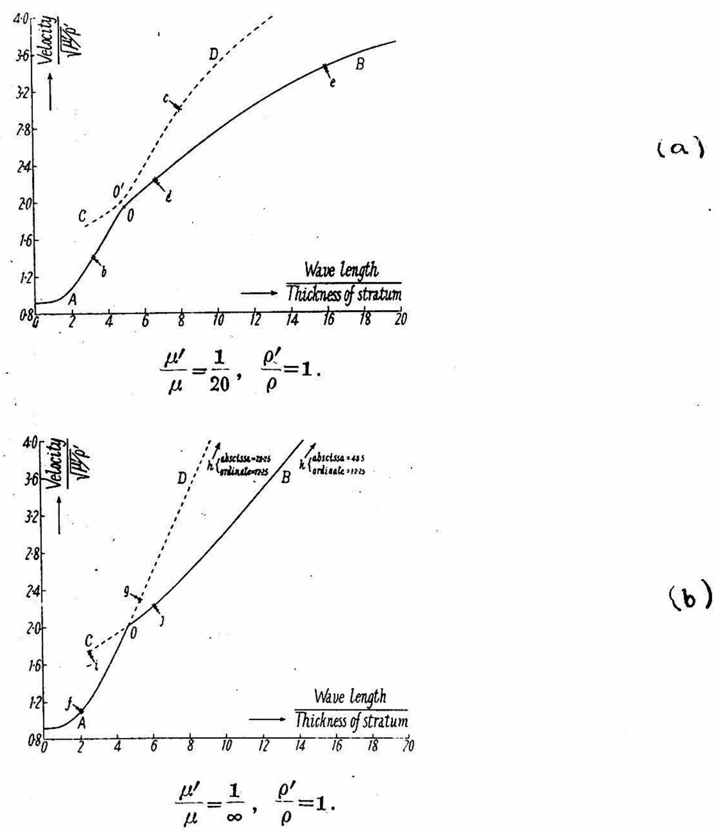


Fig. 5

Discontinuities in Rayleigh Wave Dispersion Curves

In a previous analysis of the relation between the thickness of a surface layer and the amplitudes of dispersive Rayleigh waves, Sezawa and Kanai[36] had found that the amplitude of Rayleigh waves transmitted through a stratified layer was a maximum for a certain value of L/H. It was further ascertained that there were two peaks in the resonance curve of the waves. One of these peaks represents the prevalent oscillation of Rayleigh waves, while the other concerns a mere resonance-like condition of bodily waves transmitted along the surface layer.

Now it developed that the dispersion curve AOB of Figs. 5(a), (b) corresponded to the former, while the dispersion curve CO'D corresponded to the latter. Computing the relative amplitudes on the surface of the horizontal and vertical components u, w, respectively, for both dispersion curves, Sezawa[33] obtained the resonance peaks shown in Fig. 6, where the dotted and full curves correspond with those of

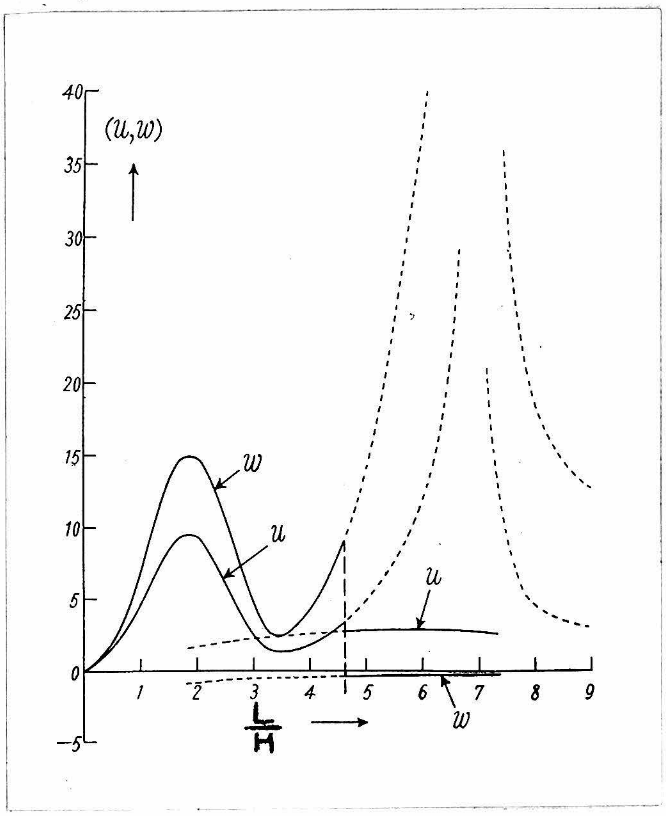


Fig. 6

Resonance of Rayleigh and Body Waves

Fig. 5. In other words, a Rayleigh wave resonance was found at L/H $\approx$ 2 and a bodily wave resonance L/H $\approx$ 7. The important significance of

the point O now becomes clear. It occurs at precisely the same value of L/H, namely 4.622, that the discontinuities of u, v occur.

It must be admitted that the entire analysis has been based on the value of $\mu'/\mu = 1/\infty$. However, if one recalls the original dispersion curves of Sezawa (Fig. 3) it will be remembered that for values of L/H somewhat smaller than 4.622, the curves are not only similar in shape, but also not extremely different in value, over a wide range of values of μ'/μ. Bearing in mind that the case corresponding to $\mu'/\mu = 1/\infty$ seems quite plausibly the limiting case of dispersion at finite values of μ'/μ, it seems reasonable to expect a somewhat similar Rayleigh resonance for the case $\mu'/\mu = 1/20$, say, and quite probably a somewhat similar body wave resonance, too.

Sezawa and Kanai[35] considered the relation between the thickness of a surface layer and the amplitudes of Love waves, and they found a resonance near the same value of L/H for which the Rayleigh wave resonance was found.

These results are in agreement with the findings of Jeffreys,[13] who predicted a maximum of amplitude of surface value of L/H corresponding to the minimum of the group velocity, provided the dispersion relation was of the type already discussed. The group velocity C is defined by the relation

$$C = V - L\frac{\partial V}{\partial L} \tag{24}$$

where V is the phase velocity corresponding to L. It can easily be shown that the minimum group velocity will occur at the value of L/H corresponding to the inflection point of the dispersion curve. The theoretical result, that this value of L/H gives a maximum amplitude, is in agreement with the result obtained without consideration of group velocity.

An examination of Fig. 6 makes it clear that the ratio of the amplitudes of the vertical and horizontal components is not a constant as in the case of a homogeneous medium, but is a function of L/H. In Fig. 7 is given the ratio of horizontal to vertical amplitude at the

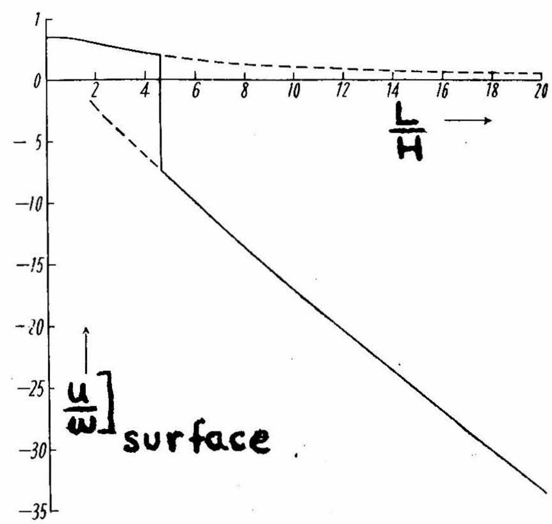


Fig. 7

Ratio of Horizontal and Vertical Amplitudes at the Surface

surface as a function of L/H. As before, full lines correspond to Rayleigh waves; dotted lines to bodily waves. It appears that for small values of L/H, $u/w = 1/1.46$ as for the case of a homogeneous medium. For values of L/H less than 4.622 the ratio has approximately this value. But for greater values of L/H this is not the case at all. The horizontal component becomes much larger. Furthermore, the motion is no longer retrograde since u/w is negative. Although this discussion has been based on the ideal case of $\mu'/\mu = 1/\infty$, Lee[22,21] has found corresponding results for constants which are physically possible.

Quite a different aspect of elastic surface wave propagation has been considered by Meissner.[26] He has considered the passive vibrations of a layer resting on an elastic half space. His analysis is similar to that for elementary Rayleigh waves and Love waves, except that the condition of zero stress on the surface is not considered to hold. Instead the stress is supposed equal to the inertial force per unit area of the vibrating layer. In the consideration of Rayleigh waves, Meissner[26] found that two different waves existed. The dispersion curves R_1 R_2 are shown in Fig. 8, together with Love wave dispersion curve Q. $L/2\pi H$ has been plotted as a function of K, where K is V_R/V_1 for the Rayleigh waves, and V_Q/V_1 for the Love waves. V_1 is the velocity of shear waves in the lower medium, and V_R, V_Q are the velocities of the passive vibrations for Rayleigh waves and Love waves respectively. L is the wave length of the waves and H is the thickness of the upper layer.

It is seen that for $L/2\pi H$ greater than 0.67 the second Rayleigh wave R_2 is not possible. As $L/2\pi H$ becomes very large, the first Rayleigh wave and the Love wave approach their respective velocities for the case of no layer.

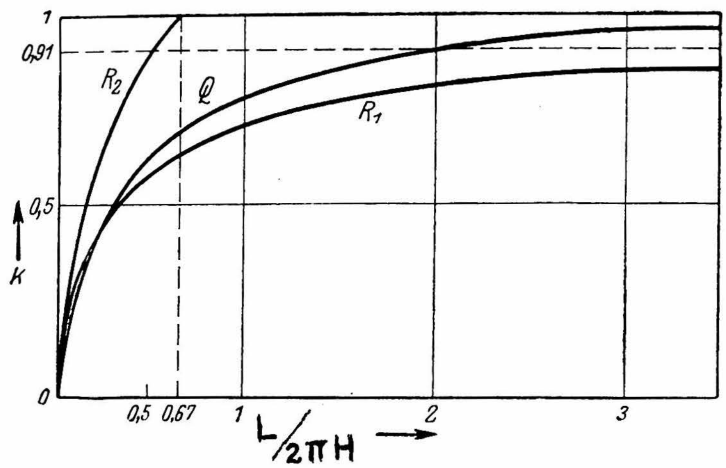


Fig. 8

Theoretical Dispersion Curve for Rayleigh and Love Waves for the Passive Vibrations of a Layer

Meissner[26] also found that for $\sigma = 0.25$ the ratio of vertical to horizontal amplitudes of the Rayleigh wave varied from 1 to 1.48 as L varied from 0 to ∞.

There is another theoretical point, apart from these considerations, which is of interest inasmuch as it gives some idea of the nature of the physical process of the formation of Rayleigh waves, and is of practical interest in the interpretation of the data. As has been mentioned, Lamb[19] found that in order to satisfy the boundary conditions on the surface of an isotropic, homogeneous, elastic half-space which is struck a sharp blow on its surface, it is necessary to add to the integral solution of the problem, another integral which amounts to the superposition of a Rayleigh wave on the other existing waves which are present.

If a disturbance occurs below the surface of such a medium at a depth which is large compared with the wave length of the longitudinal wave approaching the surface, the wave front striking the surface is essentially plane and the boundary conditions on the surface can be easily satisfied[8]. When, however, the depth of the shot is comparable to the wave length, no combination of the original and reflected waves will give zero stresses over the boundary, because of the curvature of the wave front. Nakano,[27] Banerji,[4] Coulomb,[7] and Sakai,[30] have handled the problem analytically, extending and clarifying the work of Lamb.

The results of Banerji[4] are typical and of interest. Considering the center of the disturbance to be of a dilatational nature lying at an internal point of the body, he found that near the epicentre, there appears only an irrotational wave and its amplitude is inversely proportional to the distance R from the hypocentre. At a large epicentral distance, Rayleigh waves and equivoluminal waves appear besides the irrotational wave. The amplitude of the Rayleigh waves is inversely proportional to r (r = epicentral distance). That of the equivoluminal wave is inversely proportional to R when the depth d of the hypocentre is large compared with the wave length (but small compared to r), but it is inversely proportional to r^2 when d is very small. Within the intervening region, the matter is generally complicated. It can be said approximately that Rayleigh waves do not appear within the epicentral distance defined by

$$r = \frac{v_3 d}{\sqrt{v_1^2 - v_3^2}} \tag{25}$$

and that the equivoluminal waves do not appear within the epicentral distance defined by

$$r = \frac{v_2 d}{\sqrt{v_1^2 - v_2^2}} \tag{26}$$

where v_1, v_2, and v_3 are the velocities of propagation of the irrotational, equivoluminal and Rayleigh waves respectively.

Finally, it must be remarked that no attempt was made in the preceding discussion to give a complete survey of the literature of possible importance to the ground roll problem. Only those results have been given which appear to be most pertinent to the present analysis.

The extent of the literature on elastic surface waves is enormous. For the convenience of future investigators, a bibliography is given on page 100. It is a collection of papers of general interest to the ground roll problem. These references are not discussed in this paper, but form a general background for the whole study.

Examination of Seismic Data: The results will now be given of an examination of various seismic data. Some of these data were available at the California Institute of Technology, and the remainder were obtained from the files of the Stanolind Geophysical Laboratory. The primary aim of the study is to examine the ground roll dispersion indicated by seismic records, and to see to what extent it suggests the previously considered theoretical properties. The basis of the entire analysis rests on the hypothesis that the ground roll which is observed with a vertical seismometer is the vertical component of a type of Rayleigh wave. Later, in order to test the validity of this assumption, some results will also be given of an investigation with a three-component seismometer.

Data from Yosemite, California

It has been mentioned in the introduction that ground roll data was obtained by a seismological expedition from California Institute of Technology to Yosemite. The purpose of this expedition was primarily to ascertain the thickness of the alluvial fill in the Yosemite gorge. Consequently, the apparatus used was designed for seismic reflection work, and was not particularly suited for the observation of the ground roll. However, because the shot depths were so small, appreciable ground roll was observed in many of the records.

A rough examination of the Yosemite ground roll data seemed to indicate some dependence of the velocity on the wave length. In a more careful analysis, it is desirable to take into account the possible variation of thickness of the low velocity layer as well as possible variation of compressional wave velocity of the low velocity layer.

The Yosemite data, however, unfortunately do not contain very accurate information on the thickness and velocity of the low velocity layer. In order to use the data for interpretation on the basis of dispersion, it is necessary to assume a value of the compressional wave velocity v_1 in the low velocity layer. From the travel time curve of a refraction profile, it was found that it took a time δ for a compressional wave to travel vertically downwards from the explosion to the base of the low velocity layer and then upwards to the surface. Hence from the assumed value of v_1 and the known value of δ, the thickness H of the low velocity layer could be calculated. The velocity of the high velocity material below the low velocity layer was known to have roughly the same value of 1600 m./sec. throughout the entire area considered.

The assumption of a value of the velocity v_1 is generally undesirable for two reasons. First, an incorrect velocity may be assumed; second, the value of v_1 may vary over the region considered. The first objection was overcome to some extent by selecting three possible values of v_1, and carrying through the complete analysis for each value. The values assumed for v_1 were 200, 400, and 600 m./sec. This range of velocity quite likely included the correct value of v_1. The second objection remains, although for the area included by the data, the nature of the upper part of the low velocity layer seemed to be the same.

The values of the wave length L were computed from the well known relation $L = VT$, where in this case V is the observed ground roll velocity and T is the observed ground roll period. The value of L/H was then computed for each L, where H is the thickness of the low velocity layer, computed as described previously. The ground roll velocity was plotted as a function of L/H for each of the three assumed values of v_1. The results are shown in Fig. 9.

It is to be noticed that the assumption of $v_1 = 200$ and 600 m./sec. gives a considerable scattering of points. No very definite curves are indicated, but the possibility of dispersion exists. For $v_1 = 400$ m./sec. there is some suggestion of two dispersion curves.

For small values of L/H, the velocities appear to approach the values 250, 200 and 150 m./sec. for the values of $v_1 = 200$, 400, and 600 m./sec. respectively. Evidently the assumption of $v_1 = 400$ m./sec. is the only one which gives the approximate Rayleigh wave velocity to be expected at small values of L/H.

Regardless of the value of v_1 which has been assumed, it is to be noted that there is a marked preponderance of points in the region of L/H between 0.5 and 4. Theoretically[33] it has been seen that a Rayleigh wave resonance should occur in this region of L/H. The large number of points obtained for L/H between 1 and 3 for $v_1 = 400$ m./sec. might be indicative of a possible Rayleigh wave resonance at L/H approximately equal to 2. This statement is based on the fact that the Rayleigh waves which exhibit such resonance will

YOSEMITE GROUND ROLL VELOCITY

(Various assumptions of V_1)

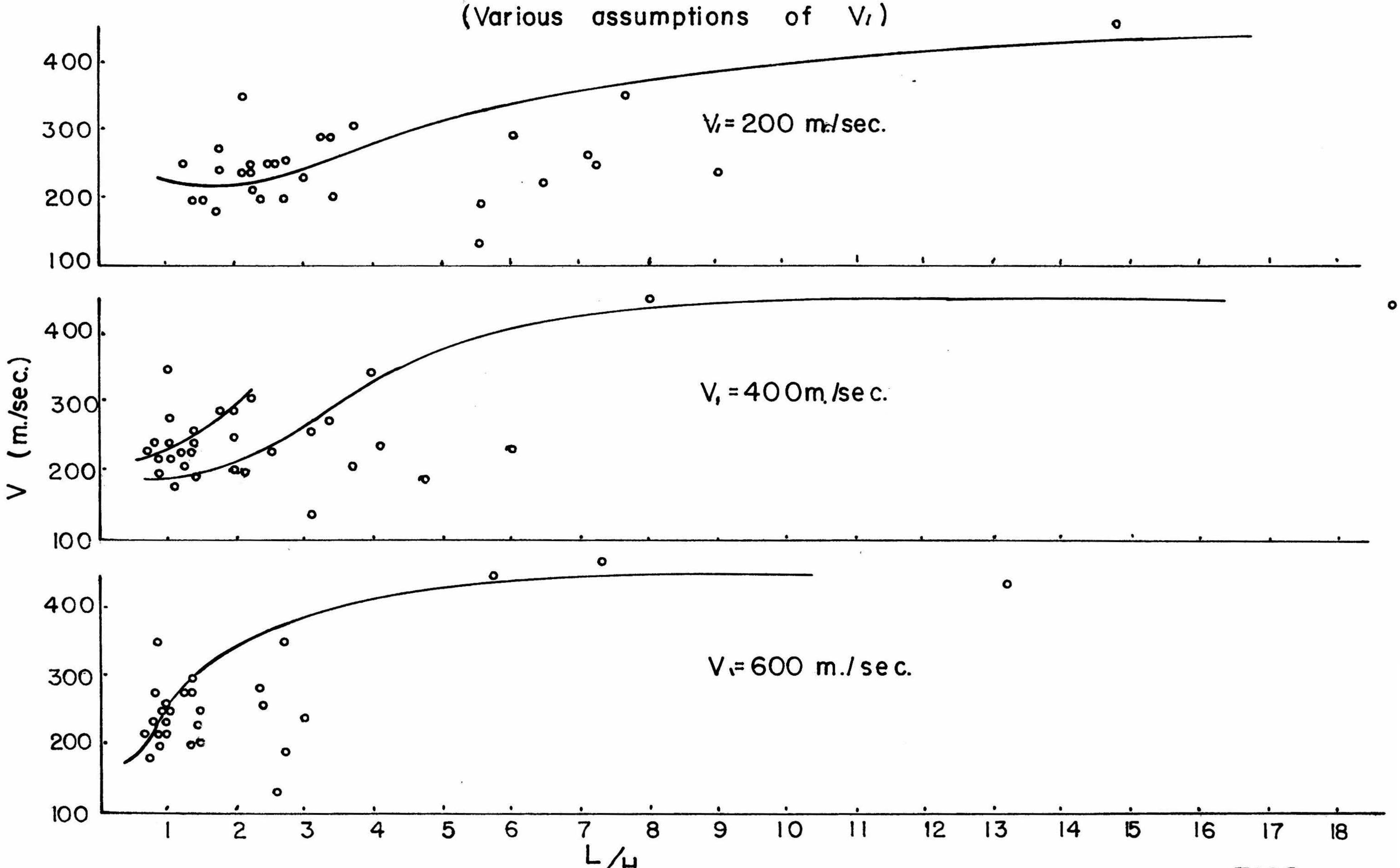


FIG9

be most easily observable. However no implication is intended that a large number of points at a particular L/H necessarily prove the existence of resonance at this L/H.

But by the same argument it would be impossible for the upper curve for v_1 = 400 m./sec. to correspond to bodily waves. The large number of points would indicate that the amplitudes were generally large, but from the theory, the bodily waves should have a small amplitude for the values of L/H less than 4.622. Hence the upper curve either would have to be considered as another Rayleigh wave dispersion curve corresponding to a slightly different rigidity ratio, or else the interpretation of the data in terms of two distinct dispersion curves would have to be abandoned.

The data are not accurate enough in certain respects to permit the drawing of very definite conclusions, but on the basis of the assumption of v_1 = 400 m./sec., the observations are not necessarily inconsistent with the theory of dispersion in a layered medium. On the other hand, there may be other causes of the observed dispersion.

Data from Fresno, California

Perhaps more impressive than the Yosemite analysis is the use of dispersion theory to explain certain anomalous variation of the ground roll velocity near Fresno, California. The data for this region are presented in the first three columns of Table IV. The second column gives the observed phase velocity V of the ground roll, and the third column gives the corresponding observed period T.

TABLE IV

Ground Roll Data from Fresno, California

Spread (ft.)	V (ft./sec.)	T (sec.)	L (ft.)	L/H	V/V_1(obs.)	V/V_1(calc.)
200-320	740	.135	100	3.0	1.1	1.28
1000-1120	1520	.12	182	5.5	2.2	2.08

From a short refraction profile the compressional wave velocity v_1 in the low velocity layer was found to be 1200 ft./sec. The thickness H of this layer was 33 ft. In the material below this layer, the compressional wave velocity v_2 was found to be 5730 ft./sec. With this information it is possible to make a rough comparison of the observations with what should be expected from dispersion on the basis of certain assumptions.

If it be assumed that Poisson's ratio is 0.25 in both upper and lower layers, and that the densities are the same, the ratio of the rigidities of the two media will be

$$\frac{\mu_1}{\mu_2} = \frac{v_1^2}{v_2^2} = \left(\frac{1200}{5730}\right)^2 = \frac{1}{23.7} \tag{27}$$

where μ_1, μ_2 are the rigidities of the upper and lower media respectively. With these assumptions the case under consideration does not differ a great deal from the theoretical case worked out by Sezawa.[34] He considered the dispersion of Rayleigh waves for the case of $\mu_1/\mu_2 = 1/20$.

In order to compare the observations with his theory, it was convenient to first calculate the values of L/H with the corresponding values of V/V_1, where $V_1 = \sqrt{\mu_1/\rho_1}$; i.e. the velocity of shear waves in the upper material. In the fourth column of Table IV the value of the wave length L is computed from the relation VT = L. The fifth column gives the value of L/H.

Since the compressional wave velocity in the upper layer is 1200 ft./sec., the shear wave velocity V_1 is $1200/\sqrt{3} = 694$ ft./sec. for Poisson's ratio = 0.25 in the upper layer. The computed values of V/V_1 are given in the sixth column. The last column indicates the theoretical values of V/V_1 for the corresponding values of L/H given in the fourth column. These values of V/V_1 correspond to the case of $\mu_1/\mu_2 = 1/20$, which is approximately the value of the ratio computed in (27). There is approximate agreement of the theoretical with the observed values of V/V_1.

On the other hand the value of L/H for the long spread is 5.5. This exceeds the value 4.622 which was found to be the value of L/H for which the amplitudes of the waves changed discontinuously to different values. By reference to Fig. 6, it is clear that for L/H greater than 4.622, the value of the amplitude of the vertical component is quite small. If the observations were extremely accurate this observation would be a strong argument against the Rayleigh wave hypothesis. But if allowance is made for a possible error of 16 per cent in the value of L/H, the value of L/H may actually have a value slightly less than 4.622. By reference to Fig. 6, it is seen that there is appreciable amplitude corresponding to any value of $L/H > 0.5$, so long as it is less than 4.622. There is, however, an equal chance that the observed value of L/H is too small by 16 per cent, which would definitely result in an inconsistency in the proposed hypothesis.

Data from Arvin and Kern, California

In studying the nature of dispersion, it would be desirable to obtain velocities corresponding to a wide range of values of L/H. The Yosemite data seemed to indicate the possibility of large values of L/H, though not as prominently as the values less than about 4. Data taken in regions near Arvin and Kern, California were selected because of the large range of values of H which were involved. It was thought that perhaps this would result in a large range of values of L/H. This, however, was not found to be the case.

The data are collected in Table V. H is the thickness of the

TABLE V

Ground Roll Data from Arvin and Kern, California

Record	H ft.	v_1 ft./sec.	v_2 ft./sec.	μ_2/μ_1	V_A ft./sec.	V ft./sec.	T sec.
1	40	1800	5900	10.8	1030	1160	.075
2	40	1800	5900	10.8	788	838	.075
3	100	2000	6300	9.9	830	970	.10
4	100	2000	6300	9.9	830	970	.05
5	100	2000	6300	9.9	1720	2480	.095
6	70	2000	6300	9.9	1760	2340	.085
7	70	2000	6300	9.9	1040	1190	.085
8	70	2000	6300	9.9	784	870	.085
9	40	1800	5900	10.8	1680	1930	.065
10	116	2300	6100	7.8	947	1120	.06
11	116	2300	6100	7.8	813	959	.06
12	40	1900	5500	8.4	2170	2600	.06

low velocity layer of compressional wave velocity v_1. The compressional wave velocity of the underlying material is v_2. In the fifth column values of the rigidity ratio μ_2/μ_1 have been computed from the values of v_1 and v_2, in the same way as was done for the Fresno data; i.e. by means of the relation $\frac{\mu_2}{\mu_1} = \frac{v_2^2}{v_1^2}$. The sixth column gives the velocity of the ground roll as determined by dividing the distance from shot hole to seismometer, by the time interval between timebreak and first arrival of the ground roll. Such a calculation will not give the correct velocity. For, as has been mentioned in the theoretical introduction, surface waves do not begin until after the compressional wave has reached the surface. It is necessary to take into account the time which elapses before the surface wave is formed. Therefore the uphole time was subtracted from the ground roll arrival time in calculating the true velocity of the surface wave. The seventh column gives the corrected ground roll velocity V.

In Fig. 10, V/V_1 has been plotted as a function of L/H where $V_1 = v_1/1.73$ = the velocity of shear waves in the low velocity layer, and the wave length L has been found from the relation L = VT. No curve has been drawn through the points plotted, but the theoretical dispersion curve has been given. It has been computed for $\mu_2/\mu_1 = 10$ on the assumption of Poisson's ratio = 0.25 in both upper and lower media, and that the densities of both media are the same.

It is seen that all of the velocities correspond to values of L/H less than 4. The larger of these values of L/H gives velocities which are greater than the theoretical Rayleigh wave velocity. The

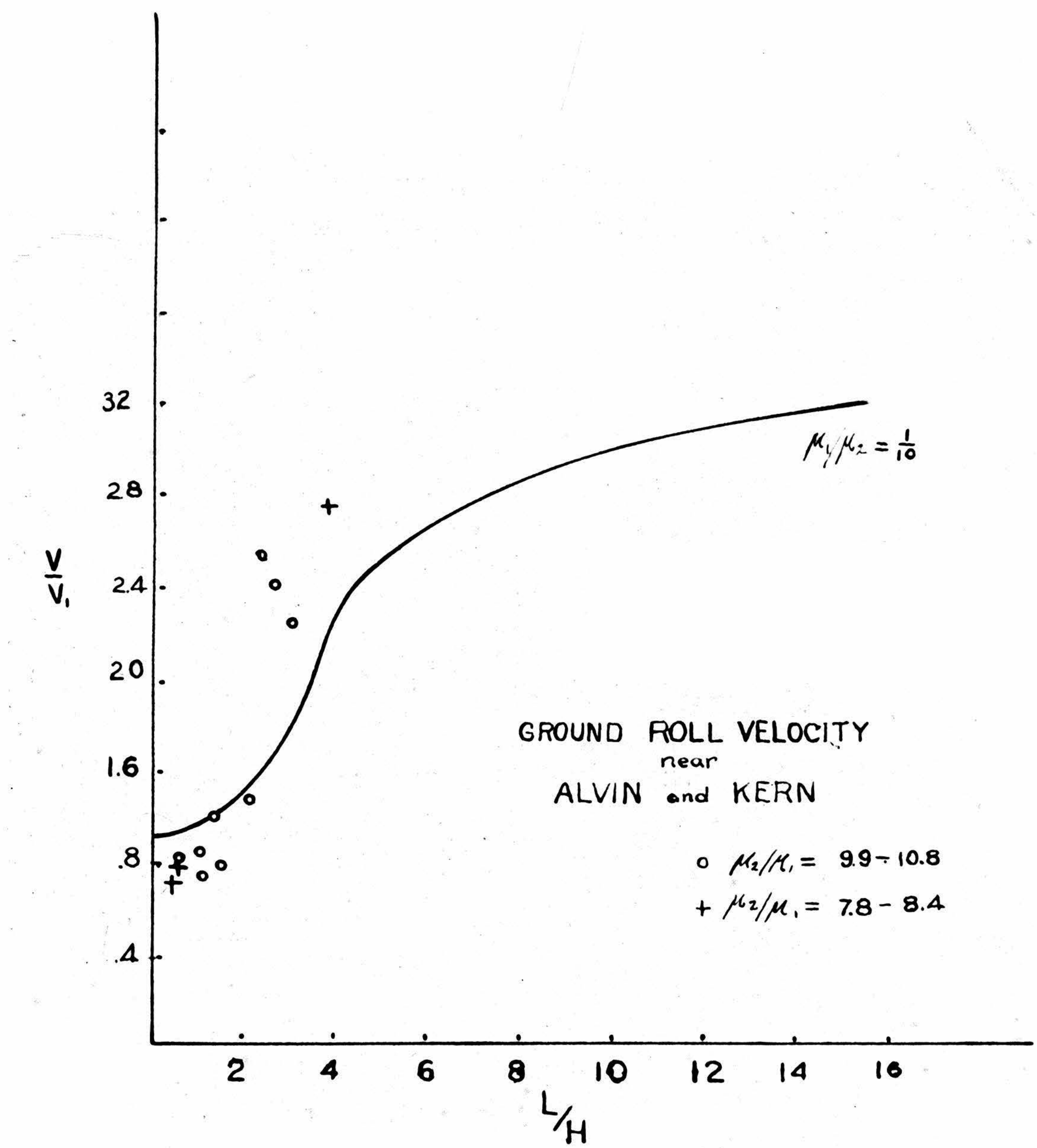

V/V₁
32
2.8
2.4
2.0
1.6
.8
.4
2
4
6
8
10
12
14
16
L/H
μ₁/μ₂ = 1/10
GROUND ROLL VELOCITY
near
ALVIN and KERN
o μ₂/μ₁ = 9.9 - 10.8
+ μ₂/μ₁ = 7.8 - 8.4

Fig. 10

smaller values of L/H correspond to velocities less than the theoretical Rayleigh wave velocity. Most of the points seem to belong to the latter group. The cluster of the points about the value of L/H = 2 is suggestive of Rayleigh wave resonance.

For the larger values of L/H, the high velocities perhaps may be associated with compressional waves in the low velocity layer. If this be assumed, there is little evidence of dispersion presented by the other points. The velocities for small L/H are generally somewhat smaller than the theoretical Rayleigh wave velocity, and the fluctuation of the value of the velocity could easily be due to slight variations of Poisson's ratio.

On the other hand, if the high velocities do correspond to Rayleigh waves, there is some indication of a different type of dispersion law than the one which is presented.

Three Components of Motion of Ground Roll

In the records of the ground roll previously considered, it was found that the ground roll velocity was sometimes approximately equal to the velocity of Rayleigh waves. This is of course approximately the velocity of Love waves too. Certain similarities have also been pointed out in the nature of dispersion in layered media of the two types of waves.

In Fig. 11 is presented a seismogram in which all three components of motion of the ground roll have been recorded. The first trace gives the vertical component, the second trace gives the horizontal longitudinal component in the direction radially out from the shot point, and the third trace gives the horizontal transverse component.

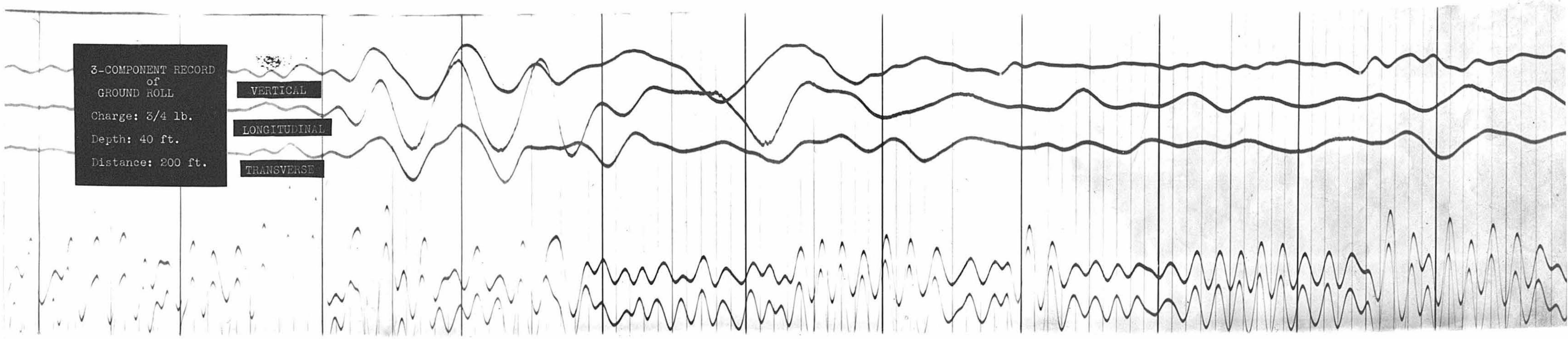


Fig. 11

Ground Roll in Three Components

It will be observed that each component occurs in appreciable magnitude. This is not always the case. Sometimes the transverse component is observed to be very small. However, the existence of a transverse component at once suggests the possibility of the presence of Love waves in the ground roll. The previous discussion has been based on the hypothesis that the vertical component of motion of the ground roll was the vertical component of Rayleigh waves. If such an assumption is valid, the longitudinal component of motion of the ground roll would have to be the longitudinal component of the Rayleigh waves. But as has been seen in the theoretical introduction, there exists a certain relation between the amplitudes of the two components. Proof that the theoretical relation exists would be an argument in favor of the proposed hypothesis. In addition, it would be an argument in favor of the hypothesis that the observed transverse component is the Love wave and not a transverse component of some other wave type, e.g. that of Uller.[38]

In order to examine the relation between the longitudinal and vertical components of motion, a diagram was constructed of the motion of a particle on the surface of the ground. A typical diagram of this kind is presented in Fig. 12. The figure was obtained by plotting the relative values of the horizontal displacement as a function of the vertical displacement at successive intervals of time. These relative displacements were obtained directly from measurements on the seismogram. The plotting was started at a time somewhat before the arrival time of the ground roll and was continued for several oscillations. Care was taken to insure that the measured displacements were plotted in the correct relation to the ground displacements. It is uncertain, however, to what extent Fig. 12 represents the true ground motion.

The numbers in Fig. 12 indicate the time. The arrows indicate the direction of particle motion. It will be noticed that the motion is not immediately retrograde, but instead is irregular at first, then goes into a forward cycle, and then becomes retrograde.

The ground roll velocity for this seismogram is 835 ft./sec. and it has a period of 0.07 sec. The thickness of the low velocity layer was 35 ft. Hence the value of L/H = 1.7. From theoretical considerations the motion should be retrograde for this value of L/H. Instead the first cycle is forward which would indicate a value of L/H larger than 4.622. The later cycles are retrograde,

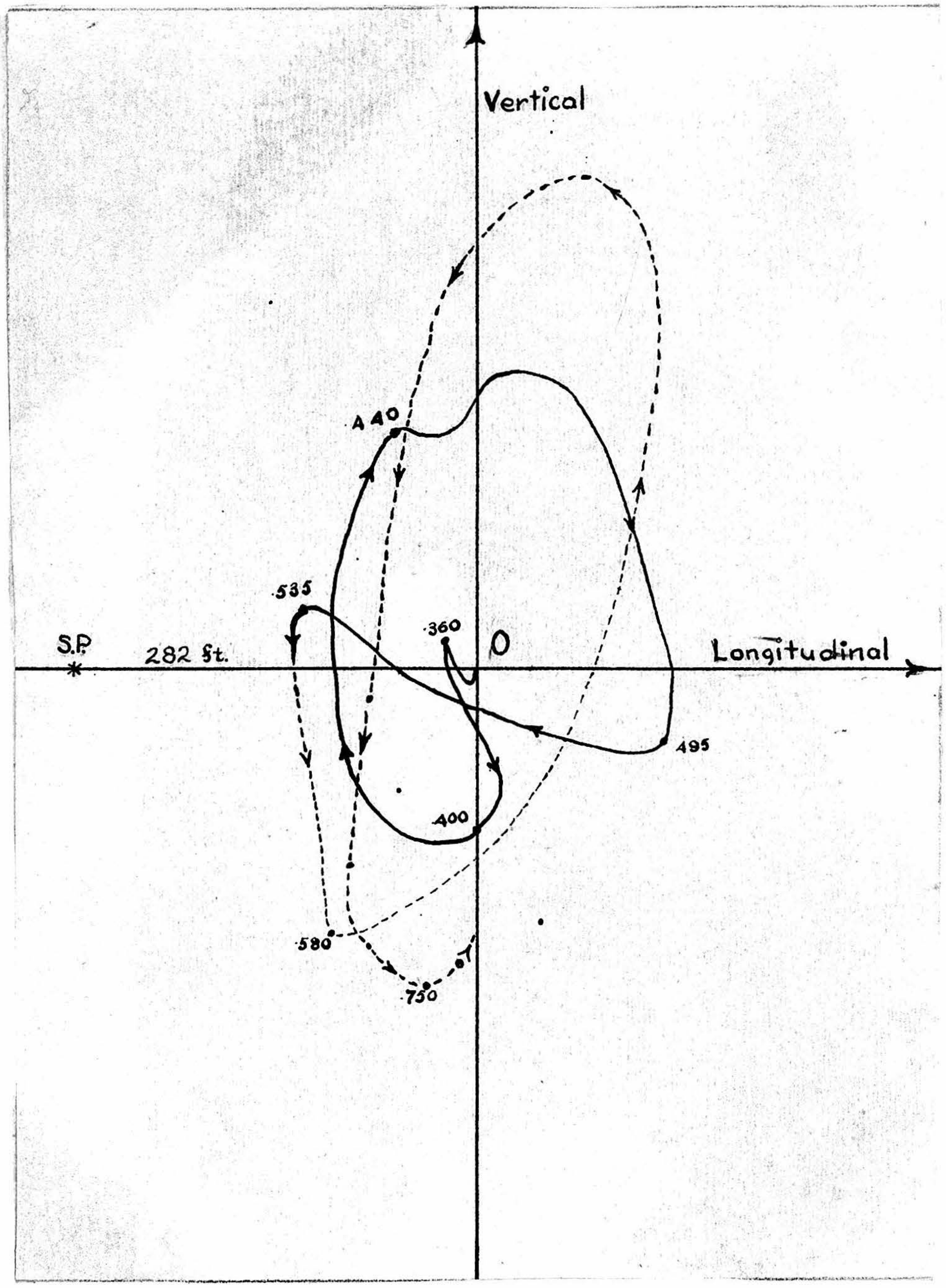


Fig. 12

Particle Motion Due to Ground Roll

but the ratio of major to minor axes of the ellipse is roughly 2.2 for this particular seismogram. The examination of the particle motion of other seismograms indicates that as a general rule the ratio is about 1.5 in agreement with Rayleigh wave theory. However, the anomalous initial forward cycle which is also usually observable, is not explanable on the basis of Rayleigh wave dispersion in a layered elastic medium, since the value of L/H is small enough to give a retrograde cycle.

Summary

Since the preceding series of investigations are of a somewhat different character than the one to follow, it is desirable at this point to summarize the results so far obtained:

1. Gravitational waves in a viscous incompressible medium are far too slow to account for the velocity of first arrival of the ground roll.
2. The importance of Bateman's secondary Rayleigh wave cannot be known without the solution of the complex problem of the partition of energy at the source of the Rayleigh waves.
3. The theory of the propagation of Love waves on the surface of a layered visco-elastic medium indicates the possibility of velocities less than the shear wave velocity obtained without viscosity. The rapid damping of very short waves is also indicated. The possibility of obtaining a damping of the order of the ground roll damping has been demonstrated.
4. An examination of seismic data on the velocity of the ground roll has indicated that the observations are not necessarily inconsistent with the theory of dispersion of Rayleigh waves in a layered elastic medium, but that there may be other causes of the observed dispersion.

Certain anomalous ground roll velocity variation near Fresno, California is possibly explanable on the hypothesis of dispersion in a layered elastic medium. For small values of L/H, the velocity of the ground roll agrees roughly with the velocity of Rayleigh waves, but these same velocities are associated with an initial anomalous forward cycle of the particle motion. Theoretically there is an inherent disadvantage in obtaining dispersion data on the ground roll by seismic means, because of the factor of possible resonance making the interpretation of the results difficult. The three component ground roll data suggest the possible co-existence of Love waves and Rayleigh waves.

RELATION OF GROUND ROLL PRODUCED BY EXPLOSION AND SURFACE WAVE PRODUCED BY GROUND SHAKER*

The purpose of this part of the investigation is to attempt to find the relation between the ground roll which is produced by an explosion, and the surface waves which are sent out when the earth receives a sinusoidal force from a ground shaker. In comparing the two phenomena, it is essential not only that the experiments be performed in the same region, but also that the underground structure of the region be well known. Accordingly, the work is divided into three parts:

1. Determination of the underground structure of the region.
2. The ground roll experiments.
3. The ground shaker experiments.

Determination of Structure of Region of Experiments.

The area selected for the experiments was one known to exhibit the ground roll. It was a relatively flat area about 10 miles east of Tulsa, Oklahoma. A rough idea of the structure of the uppermost 74 feet was obtained from the drilling log of the shot hole. The log was as follows:

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* Also called geo-oscillator, earth vibrator, earth vibration machine.

From (ft.)	To (ft.)	Formations	
0	2	Top Soil	
2	7	Yellow Clay	
7	18	Blue Shale	
18	20	Slate Breaks	Labette Shale
20	57	Blue Shale	(Penn.)
57	73	Limestone Breaks in Blue Shale	
73	74	Limestone	Ft. Scott LS. (Penn.)

In order to know the velocity of these uppermost formations, the usual procedure of running a shallow refraction profile was followed. Ten electrodynamic seismometers were used. The spreads employed, the corresponding charges of dynamite, and the shot depths may be summarized as follows:

Seismometer	1	2	3	4	5	6	7	8	9	10	Charge (lb.)	Depth (ft.)
Distance (ft.)	10	20	30	40	50	60	70	80	90	100	1/8	3
Distance (ft.)	80	100	120	140	160	180	200	220	240	260	1/8	6
Distance (ft.)	140	180	220	260	300	340	380				3/16	6
Distance (ft.)	30	35	40	45	50	55	60				1/8	6

In order to secure the fullest possible accuracy, several records were taken with each spread, and in judging arrival times on the seismograms a cylindrical hand lens was employed. All breaks were graded

"Good," "Fair," or "Poor" and averaged accordingly. Sample records are shown in Figs. 13 and 14. All travel times were referred to a 3-foot shot depth. The final travel time curve is shown in Fig. 15.

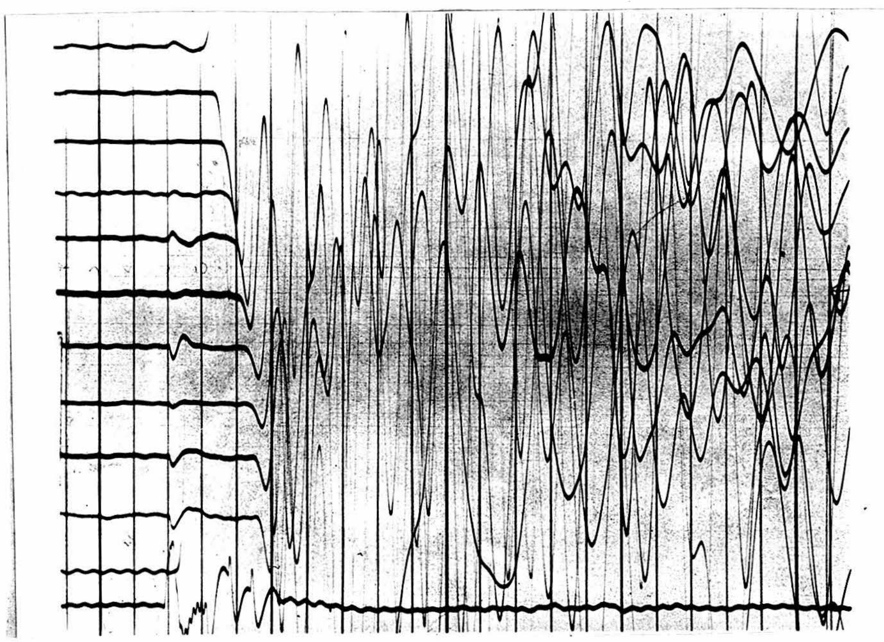

Fig. 13

Refraction Shot - 3' Depth

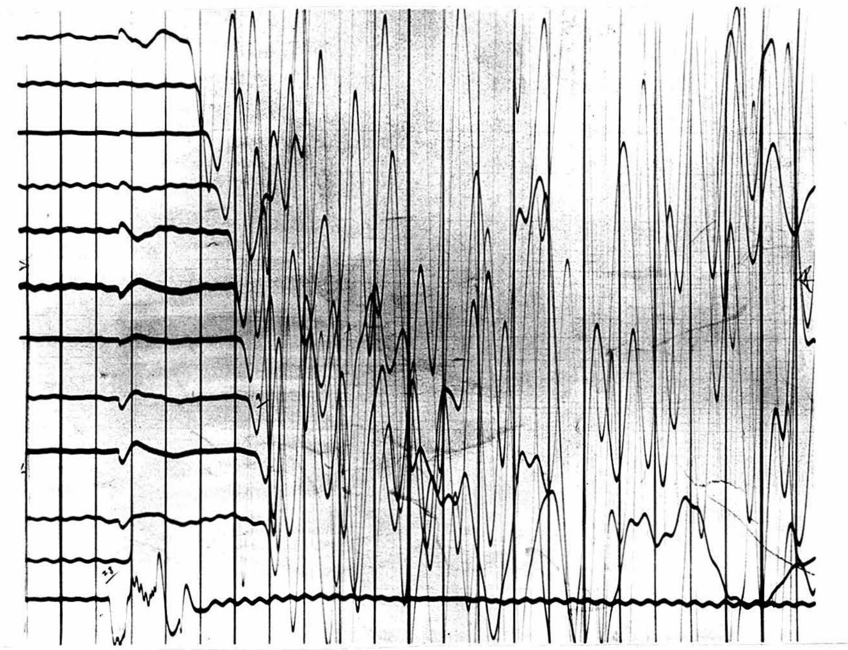

Fig. 14

Refraction Shot - 6' Depth

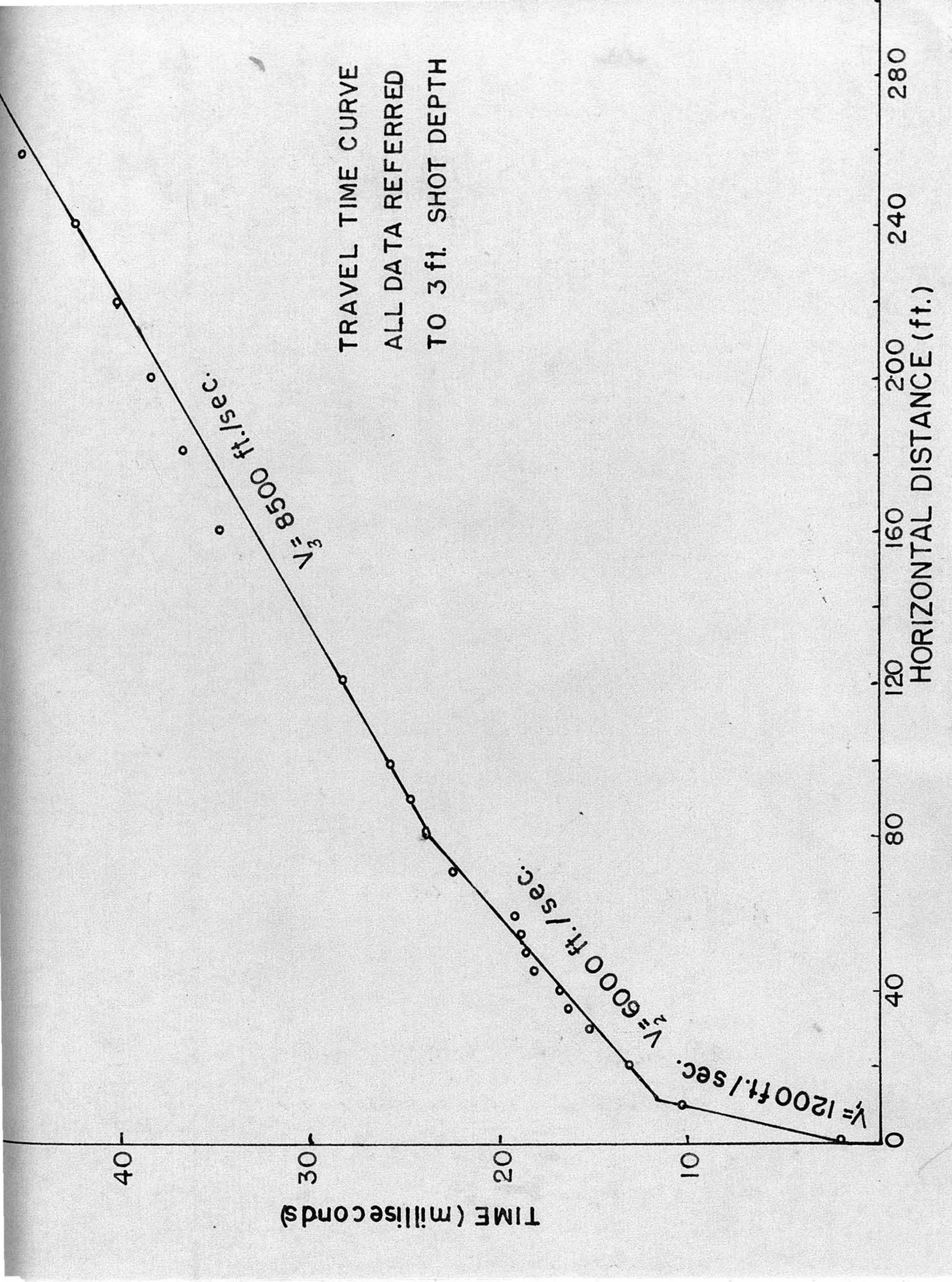
TRAVEL TIME CURVE
ALL DATA REFERRED
TO 3 ft. SHOT DEPTH
V_3 = 8500 ft./sec.
V_2 = 6000 ft./sec.
V_1 = 1200 ft./sec.
HORIZONTAL DISTANCE (ft.)
0
40
80
120
160
200
240
280
TIME (milliseconds)
10
20
30
40

Fig. 15

From the travel time curve shown in Fig. 15 it appears that three different media are involved in the refraction profile. The thickness of the first stratum is given by the well known formula

(28) $$H_1 = \frac{D_1}{2}\sqrt{\frac{v_2 - v_1}{v_2 + v_1}} + \frac{(d+h)}{2}$$

where D_1 = the abscissa of the first break in the travel time curve = 12.0 ft.

d = the depth of burial of seismometer = 0.5 ft.

h = the depth of burial of shot = 3.0 ft.

v_1 = the compressional wave velocity in the first medium = 1200 ft./sec.

v_2 = the compressional wave velocity in the second medium = 6000 ft./sec.

The computed value of H_1 was 6.5 ft. In the above computation, v_1 and v_2 were obtained by measurement of the slopes of the first two segments of the travel time curves. The thickness T of the second layer is given by the well known formula

(29) $$T = \frac{v_2(t'' - t')}{2}$$

where t'' = the intercept on the time axis of the third segment of the travel time curve

t' = the intercept on the time axis of the second segment of the travel time curve.

The computed value of T was 11.8 ft. Hence in the region of the experiments, the underground structure was as follows:

Depth (ft.)	Velocity (ft./sec.)
0 - 6.5	1200
6.5 - 18.3	6000
Below 18.3	8500

Observation of the Ground Roll Initiated by Explosion.

Although some records of the ground roll were available for various areas in the general vicinity, it was thought advantageous to secure records of the ground roll in a particular locality whose structure was very well known. Hence ground roll records were obtained in exactly the same area as that in which the refraction profile was run.

In the experiments which were to be performed it was desirable to use an amplifier which was flat down to frequencies in the vicinity of ground roll frequency, which was known roughly to be between 10 and 20 cycles per sec. (c.p.s.). Five channels of such an amplifier were available. The response of these amplifiers was very flat. Their gain was 42 db.* from 5 to several thousand c.p.s.

The seismometers used were replicas in construction of those used in the refraction work except that they possessed springs designed to give the seismometer a natural frequency of about 20 c.p.s. Eight such seismometers were constructed and the output and response of each

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* A decibel (abbreviated db.) is the common unit of measurement of gain of amplifier. If P_1 = input power, P_2 = output power; gain (db.) = $10 \log_{10}(P_2/P_1)$.

was determined by means of a shaking table. Low pass filters were used when it was desired to cut out high frequencies.

The type of seismometer used has response to rotational as well as vertical translational motion of the ground, since the hinge line of the spring and the center of gravity of the sprung mass do not coincide. In order to eliminate the rotational response, two seismometers are connected in series and oriented so that their masses rotate in opposite directions when the ground is subjected to a given rotation. In other words the seismometers are placed "back to back," and buried in this orientation in a single trench.

Preliminary experiments were performed to see how well the four sets of seismometers matched. These seismometers had been paired together on the basis of equality of output as derived from shaking table measurements, and since three sets matched very well from this data, it was expected that when tested in the field, at least three sets would be found to match there too. This was found to be the case; furthermore, the fourth set matched the other three to a sufficient extent to be usable for the experiments.

The process of matching or bridling seismometers is performed as follows: the inputs of all amplifier channels are connected to a single seismometer and a record taken. If all the traces match, one is assured that there is no distortion difference between the different amplifier channels. Then all the seismometers are buried in a single trench and connected to their respective amplifiers and another record taken. This record shows to what extent there is bridling among the seismometers.

Page 64 missing.

An example of such a bridle is shown in Fig. 16. By means of this bridle, it is possible to observe how well the seismometers matched in general response. Likewise it is possible to note any phase shift in a particular seismometer. Such a phase shift would be the result of the particular seismometer having a slightly different natural frequency than others. On this view, one should expect the amount of the phase shift to be different at different parts of the record as a result of different incoming frequencies. Such is found to be the case. In a particular bridle seismogram, an examination was made of the relative phase shift to be expected at different times referred to the time-break. For example, the data obtained from a different record than that shown in Fig. 16 is given in Table VI.

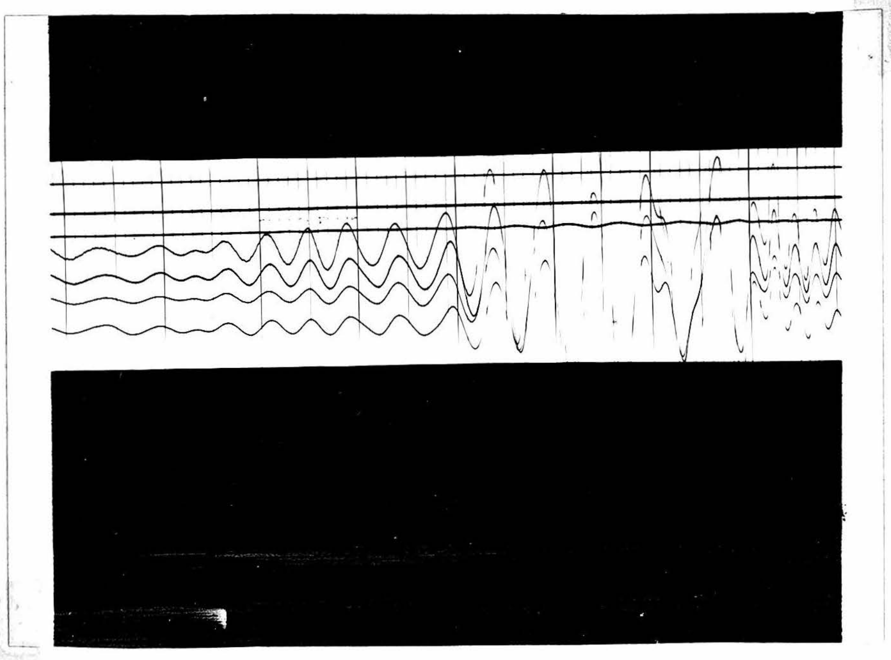


Fig. 16

Sample of Seismometer Bridle

TABLE VI

Time Delays referred to Seismometer 1

(From Record F) Seismometer No.	Time from Time-break (sec.)				
	.52	.48	.43	.38	.33
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	-.010	-.009	-.006	-.010	-.007
5	-.001	0	0	0	0

It is to be noted that in general there is but negligible phase delay, but when it does occur it is quite pronounced. From Fig. 16 it appears that there is a consistency of values of observed frequency at a particular location.

The seismometers next were placed in pairs at distances of 20, 40, 60, 80 feet from the shot point, and several records were taken. A sample record is shown in Fig. 17. This profile was repeated using different combinations of four seismometers. Only four seismometers could be used at one time, because only four low-pass filters were available.

By reference to Fig. 17 it will be noted that the first four traces are not equally spaced. In order to more conveniently carry out the analysis of velocity of the different phases, the second trace was graphically shifted upward until the four traces were equally spaced.

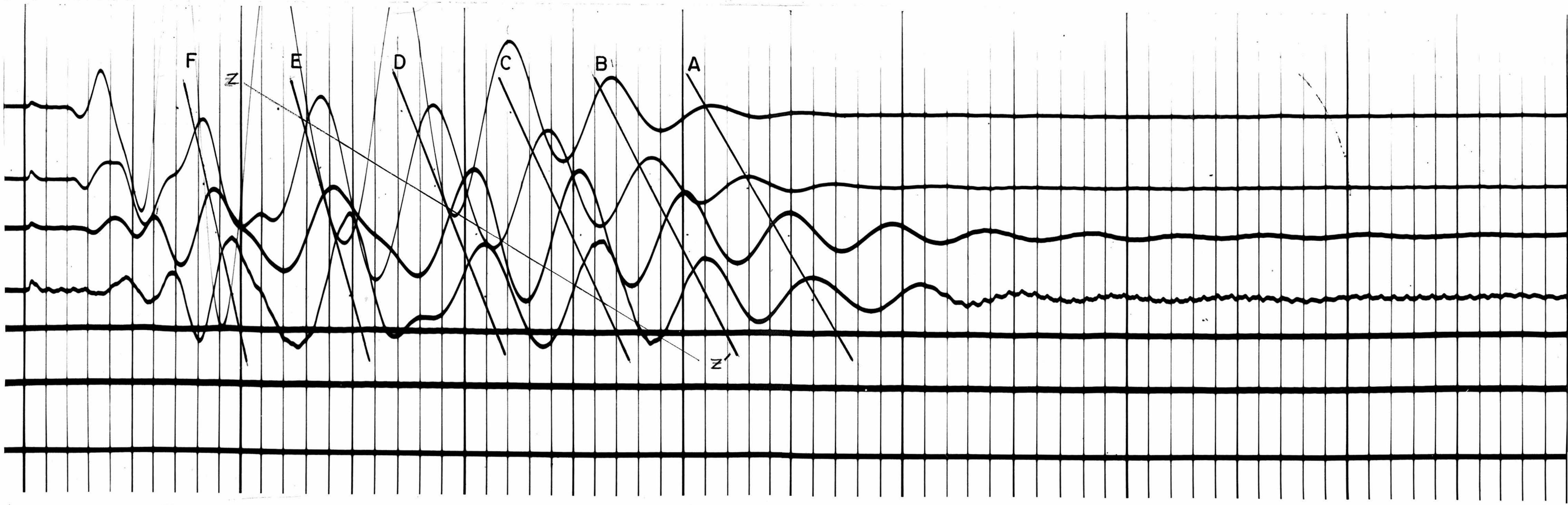


Fig. 17

Typical Ground Roll Record

Points directly below the various maxima were marked on the time axes (equalized as above described) and a straight line was drawn through the four points corresponding to each particular phase of the ground roll. The slope of these lines, AA', BB', ..., then gave the velocity of the various phases, A, B, ... Since only four points were available, a knowledge of the phase delays expectable from a particular seismometer, as previously determined from the bridling experiments, was of great help in deciding how the straight line should be drawn.

The periods of the phase were obtained by measuring the time interval between the two minima enclosing the particular phase under consideration. The results of these measurements for a typical case are shown in Table VII, and the corresponding record is shown in Fig. 17. The second column gives the time interval between the

TABLE VII

Velocities and Periods of Phases in Fig. 17

Phase	$t_{80} - t_{20}$ (sec.)	V (ft./sec.)	T_1	T_2	T_3	T_4	$T_{av.}$ (sec.)
A	.051	1180	.045	.045	.048	.050	.047
B	.042	1430	.045	.044	.047	.050	.047
C	.038	1580	.051	.049	.049	.050	.049
D	.033	1820	.049	.053	.050	.050	.050
E	.031	3150	.065	.052	.057	.053	.052
F	.025	3150	.037	.047	.047	.045	.044

arrival of a particular phase at the seismometer at 80 ft. and the arrival of this phase at the seismometer at 20 ft. In the third column the phase velocity has been computed from the relation

$$V = \frac{60}{(t_{80} - t_{20})} \tag{30}$$

In the last column of Table VII an average value of the period for a particular phase is taken. The letters in the table correspond to the letters denoting the various phases of the seismogram in Fig. 17. These data could be repeated at will with good accuracy.

The relationship between velocity and period is not clearly indicated by these data. In a general way the increase of velocity as one approaches earlier arriving phases, is indicated. Likewise an increase of period in this direction is suggested — until one arrives at the extremely early phases that are indistinguishable from the compressional waves.

At first sight, the seismogram of Fig. 17 appears to be similar to a pulse propagated in a layered elastic medium. In the theoretical discussion of the oscillations pictured in Fig. 4, it was noticed that the number of oscillations increase with the distance from the source. This appears also to be the case for the seismogram shown in Fig. 17. The general decrease in amplitude of the slower phases is also indicated.

On the other hand, an examination of the data indicates that the dispersive law which Sezawa and Nishimura[37] assumed, is not valid in this case. The wave length L of the various phases may be found from the formula L = VT. Taking the thickness H of the low velocity layer to be 6.5 ft. as determined from the refraction profile, the values of L/H may be computed. Since the experimentally found value of the compressional wave velocity in the low velocity layer was 1200 ft./sec., the shear wave velocity V_1 will be $1200/1.73 = 694$ ft./sec. for Poisson's ratio equal to 0.25. In Fig. 18, V/V_1 has been plotted as a function of L/H, for the sake of comparison with the theoretical curve of Sezawa.[34] His curve is given for approximately the correct value of the rigidity ratio in the case being considered. Clearly there is no indication of the type of dispersion curve predicted for a layered elastic medium. Instead the points all (with the exception of F which may correspond to a compressional wave) lie on a straight line.

The existence of the linear relation between values of $L/H = 8$ and $L/H = 26$ does not necessarily mean that the same relation will hold for all L/H. In particular for very small values of L/H it may be questionable that the linear relation holds. For if it did, the group velocity would be exceedingly small. By equation (24),

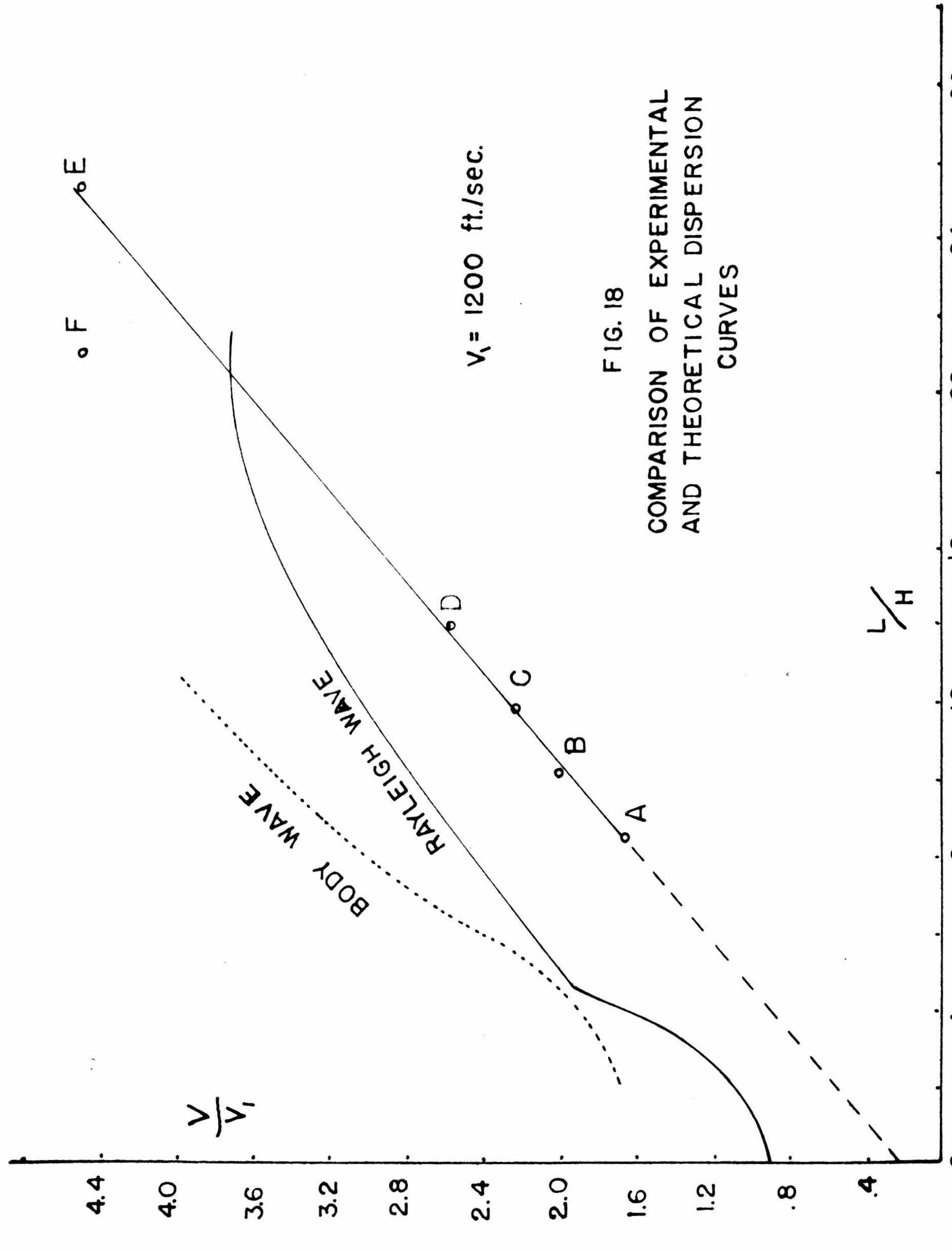
$$C = V - (L/H)\frac{\partial V}{\partial (L/H)} \tag{31}$$

where C = group velocity

V = phase velocity.

Consequently, the group velocity is equal to the phase velocity at $L/H = 0$. Thus if the velocity relation was linear all the way to $L/H = 0$, the intercept of the straight line on the velocity axis should give the group velocity. The value of V/V_1 at $L/H = 0$ in Fig. 18 is 0.22. Hence the group velocity would be only 150 ft./sec.

If the line ZZ' be drawn through the principal maxima in Fig. 17, the slope of the line ZZ' should give the group velocity. $C \approx 400$ ft./sec. is the value of the group velocity obtained in this way. This is almost



V_1 = 1200 ft./sec.
FIG. 18
COMPARISON OF EXPERIMENTAL AND THEORETICAL DISPERSION CURVES
V/V₁
L/H
RAYLEIGH WAVE
BODY WAVE
A
B
C
D
E
F
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
.8
.4
0
4
8
12
16
20
24
28

Fig. 18

three times the value obtained by assuming that a linear velocity relation holds all the way down to $L/H = 0$.

The explanation of this discrepancy will be given after the steady state experiments are described.

Observations of Surface Waves Initiated by Ground Shaker.

The sinusoidal vibration of the ground by means of a ground shaker offers a good way of investigating the dispersion of surface waves in the uppermost layers of the earth. The chief merit of this method of experimentation is that the frequency of vibration of the train of waves may be held constant.

For reasons already given, the experiments were performed in the same locality as those previously described. The frequency of the waves in the ground was known immediately from the frequency of vibration of the ground shaker. At a short distance from the ground shaker, the response of a seismometer to the periodic ground motion was observed. Then one could note the phase difference between a maximum on the repeating wave form and a periodic time-break produced by a commutator arrangement on the ground shaker. This phase difference was observed at successively increasing distances, and by plotting phase difference as a function of distance, a travel time curve was obtained. This travel time curve, being a straight line, allowed immediate computation of the velocity at the particular frequency of vibration of the ground shaker. Thus, for each frequency of the ground shaker, the corresponding velocity could be found and the dispersion curve plotted.

The shakers used in this work were of two types. The first shaker (hereafter called Shaker A) to be used was already available at the Stanolind Laboratory. It was of the Kelley[15] pendular variety, and is shown in Fig. 19. The electric motor A was 110 A.C. single phase

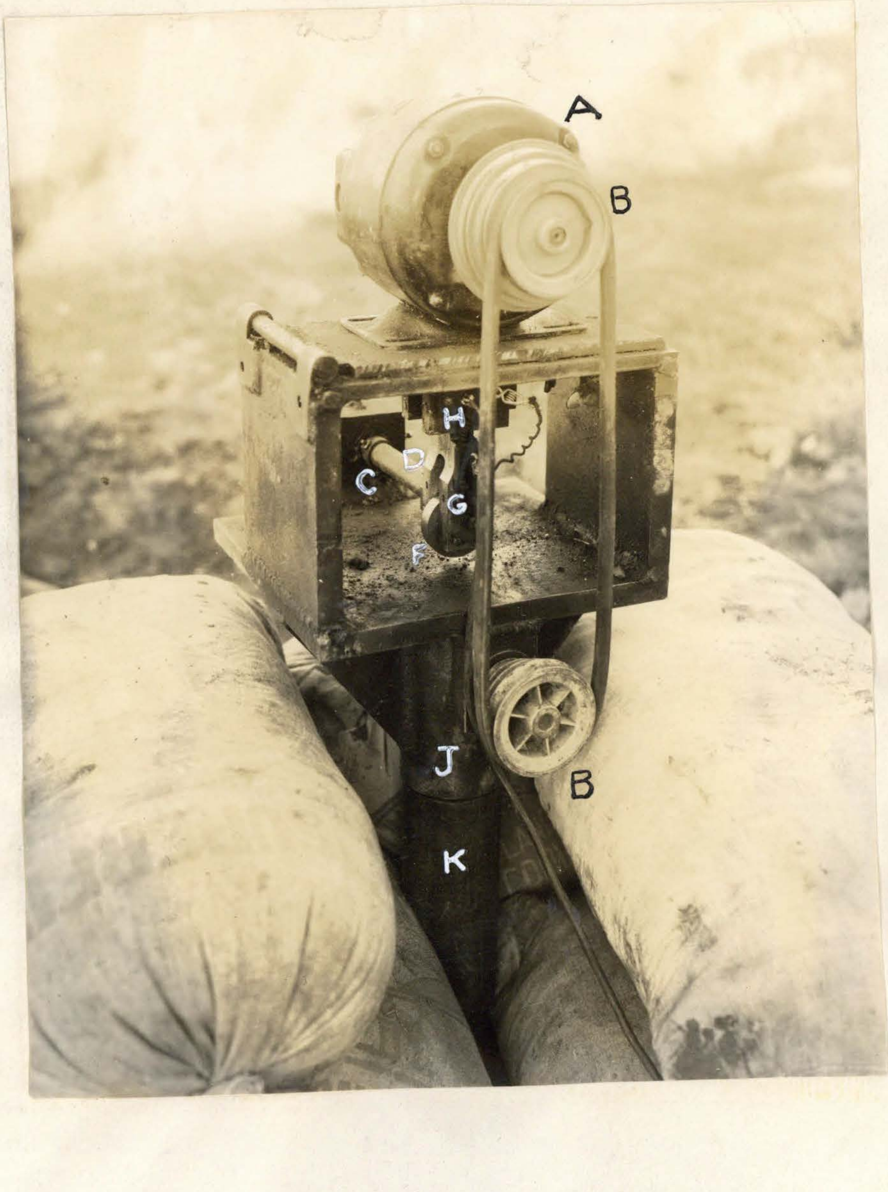


Fig. 19

Shaker A - Kelley Pendular Type

1725 R.P.M. induction motor. The belts and pulleys B made possible frequencies of about 100, 80, 60, 50, 40, 30 cycles per sec., of the shaft C. The latter was coupled to a shaft E on the pendulum by means of a rubber hose D. The shaft E rotated an eccentric mass F which supplied the necessary force. Most of the other details of the apparatus are apparent from the photograph. The force was applied to the ground through a flat disc 1 ft. in diameter. With this apparatus satisfactory sinusoidal motion could be obtained.

This type of shaker worked very satisfactorily for frequencies of 60, 80, 100 c.p.s. For lower frequencies, however, the inherent vibration of the electric motor at 30 c.p.s. masked the vibrations of the pendular mass. When various attempts to eliminate this background vibration of the motor failed, it was concluded that the only satisfactory course of procedure lay in the design of a shaker capable of providing larger forces at lower frequencies. Accordingly a new shaker, employing the same general principle as that of the U. S. Coast & Geodetic Surveys,[39] was designed and constructed.

This second shaker (hereafter called Shaker B) is shown in Fig. 20. One of the shafts, A is driven by coupling through a flexible shaft to a portable motor and pulley system capable of delivering frequencies of 12, 22, 39, 70 c.p.s. The rotation of this shaft causes the rotation in the opposite direction of the shaft C by means of the gears B. Eccentric masses D_1, D_2, D_3, D_4, are fixed to shafts so that their relative position adjusted to make vertical components of force to add and horizontal components of force to subtract.

Fig. 20

Shaker B - U.S. Coast & Geodetic Surveys Type

Details of the dynamics of this shaker are best understood by reference to Fig. 21, in which the shaker is schematically represented. In (a), the position of the weights is such that components of force F_V on the two shafts will add. (c) and (d) show the next two 90 degree positions of the weights. It is evident that one should obtain a

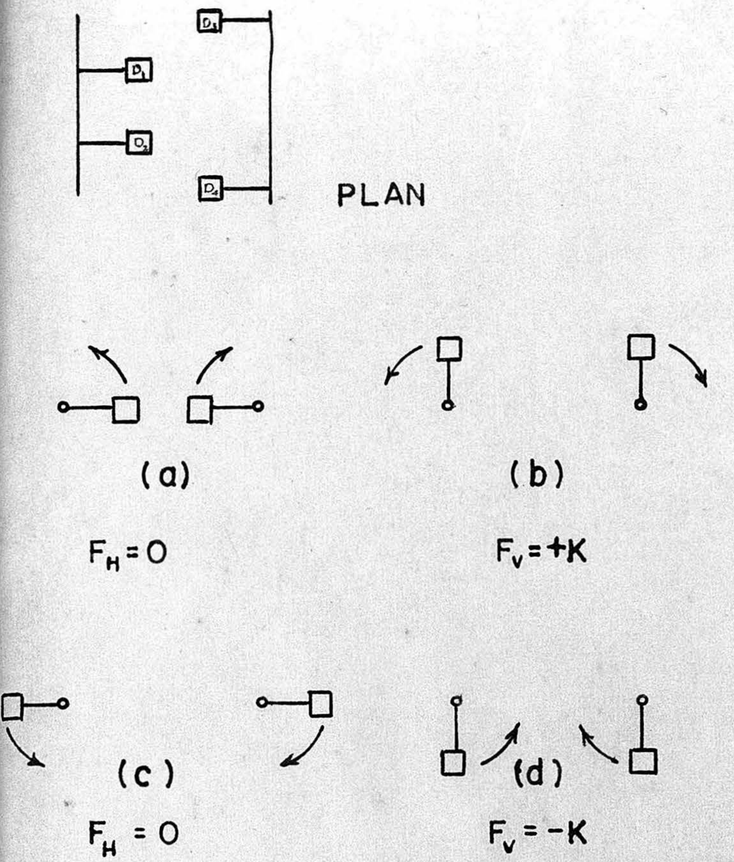


Fig. 21

Schematic Diagram of Manner of Horizontal Force Cancellation

sinusoidal vertical force and a zero horizontal force with such a shaker.

It was necessary to have the shaker weighted down to the extent of having the force of gravity exceed the force supplied by the shaker; otherwise, the application of the upward force of the shaker would cause the base plate to completely lose its contact with the ground, thus badly distorting the wave form. Such distortion seemed to be most serious at low frequencies. Hence for use with the low frequency shaker (Shaker B) a large disc mounting (21" in diameter) was used and loaded down with six 100-pound bags of sand. Since the maximum force developed was only 396 pounds, fully sufficient weighting was supplied.

Both of these shakers produce forces which are proportional to the square of the frequency of vibration. In order to avoid excessive forces at high frequencies in the use of Shaker B, part of the rotating mass may be removed from each of the four weights. This will cut down the force by a factor of 9. Hence the force at 30 c.p.s. with the masses removed is the same as the force at 10 c.p.s. without the masses removed. The Shaker B was designed to produce a force of 96 pounds at 10 c.p.s. Shaker A produced the same force at 60 c.p.s.

Mounted on the drill rod of the shaker was a low sensitivity seismometer with a natural frequency of 18 c.p.s., and which possessed a peak at this frequency but otherwise exhibited a frequency response which was flat to several hundred cycles per second, as can be seen from its response characteristic shown in Fig. 22. This seismometer

Fig. 22

as well as the others to be described were connected by leads to the switches in the recording truck.

The seismometers used to detect the wave motion at various distances from the shaker were of two types. They were identical in general construction to the seismometers used in the previously described work. However, the springs and the masses used were such that one seismometer had a natural frequency of 20 c.p.s. and the other a natural frequency of 60 c.p.s. The former seismometer was the same type as that used in the ground roll experiments with explosions. The latter had the same frequency response as the seismometer used in the refraction profile. The seismometer with a natural frequency of 20 c.p.s. was used exclusively with Shaker B. The seismometer with a natural frequency of 60 c.p.s. was used exclusively with Shaker A.

In the truck means were provided for switching the output from any seismometer into an amplifier which in turn was connected with the vertical plates of an oscilloscope. This arrangement was very satisfactory for the higher frequencies where there was no difficulty in synchronizing the wave form on the fluorescent screen of the oscilloscope. The procedure is well known of adjusting the sweep circuit of an oscilloscope to synchronize a periodic voltage. When the synchronization is obtained, the repeating wave form appears stationary on the screen of the oscilloscope. At the lower frequencies a different technique, to be described later, was employed.

The use of a commutator circuit for producing the periodic time-breaks has been mentioned earlier. Clearly the phase of these

Page 79 missing.

time-breaks is immaterial in computing the phase differences. It was decided arbitrarily to have the time-breaks appear at the instant of application of maximum force to the ground. For this purpose a commutator ring and brush was arranged in connection with the eccentric shaft so that in the position of maximum applied force, contact was made between commutator ring and brush. The circuit closed by this action consisted of a battery induction coil and variable resistance arranged as shown in Fig. 23. By varying the resistance R, the size of the pulse, which appeared superposed on the wave, could be altered as desired.

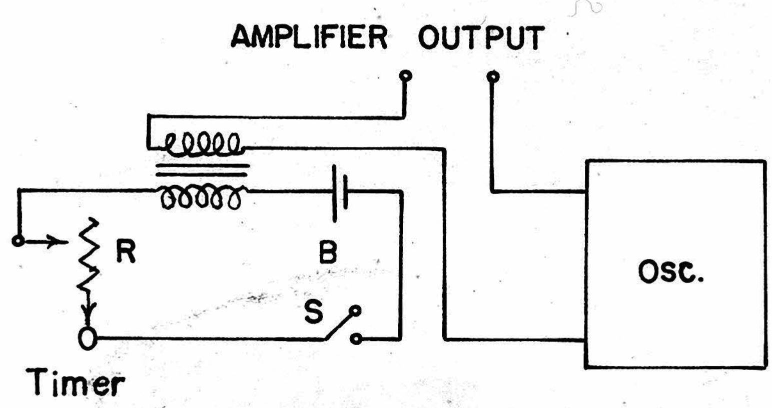


Fig. 23

Reference Time-break Circuit

With the oscilloscope sweep circuit in operation the output of the low sensitivity seismometer which was attached to the shaker was observed. This was mainly for the purpose of being assured that the vertical motion into the ground was of a satisfactory sinusoidal nature. When it was not, it was found that more firmly packing the ground supporting the shaker, or perhaps oiling various parts of the mechanism was all that was required to make the motion sinusoidal.

The profile line was usually selected to be a radial line out from the shaker, in a direction chosen at will. To test for azimuthal effect, several profiles in various directions had to be run.

At a given position of the field seismometer on the profile line, the following procedure was employed: the output of the seismometer was observed on the oscilloscope in the same manner as for the shaker seismometer. However, at first, the motor was run with the belt connecting to the pendulum shaft C removed. The observed pattern was then a result of extraneous forces produced in the apparatus (inherent noise) plus the actual background disturbance existing in the field (field noise). In order to get the response to more or less purely vertical motion, the amplitude of the observed noise was first diminished until the oscilloscope exhibited only the horizontal line. This could be done either by means of the amplifier control or by the oscilloscope control. In general the amplifier control was kept sufficiently low in setting to avoid over-driving.

Next the belt was replaced, and the oscillogram was again observed. Generally several oscillations were visible on the screen, and by means of the various frequency controls of the oscillograph, the number of oscillations was reduced to two or three. The amplitude was also adjusted until a conveniently sized oscillogram was obtained.

The timing mechanism produced a small vertical line which was superimposed on the oscillogram. By means of the variable resistance the size of the line was adjusted so as to give the clearest time mark with the least distortion of the oscilloscope wave form. When this was accomplished, the entire oscillogram was traced onto onion skin paper. A sample tracing is shown in Fig. 24. The vertical line

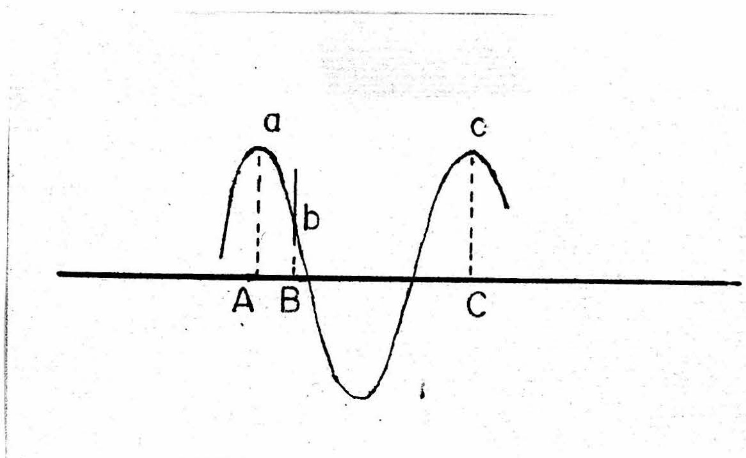


Fig. 24

Sample Oscillogram Tracing

at b is the marker which appears simultaneously with application of maximum force downward by the shaker. The phase difference between the arrival time t_a of the wave crest a, and the instant t_b of application of maximum force downward by the shaker is given by

$$\phi = 2\pi f (t_b - t_a) \tag{32}$$

where f = frequency. Hence by plotting $\phi/2\pi f$ as a function of the seismometer distance D, a travel time curve is obtained. Actually, it was more convenient to plot $\phi/2\pi$ instead of $\phi/2\pi f$. For since $\phi/2\pi$ = AB/AC, it was only necessary to place transparent coordinate paper over the oscillogram and measure AB and AC in arbitrary units. As far as the travel time curve is concerned, it is immaterial what portion of the sinusoid ac is used as a phase reference point, since it is found that the travel time curve is a straight line, and only the slope of this straight line is needed to obtain the velocity. However, it is obviously necessary to consistently choose the same reference point at each distance. Likewise care must be exercised in the measurements of ab as it increases through the value ac. For when ab has increased to the value ac, $\phi/2\pi$ is equal to unity, but its value might erroneously be taken as zero. For example, if ab had already increased to a value greater than ac and Fig. 24 was obtained, unity would have to be added to the value of AB/AC. In general each time the distance D is increased by a wave length, the integer to be added to AB/AC must be increased by unity.

From the slope of the straight line obtained, the velocity of the wave may be computed according to the formula $v = \frac{D}{\phi/2\pi} \cdot f$. Then if velocity is plotted as a function of frequency, the dispersion curve is obtained.

The type of tracings obtained in running a typical profile are shown in Fig. 25. The time-break is represented by the vertical line AA', and in order to show the change of relative position of time-break and crest, the crests are successively numbered. For convenience all traces have been reduced to a common scale. The progression of the waves is clearly indicated.

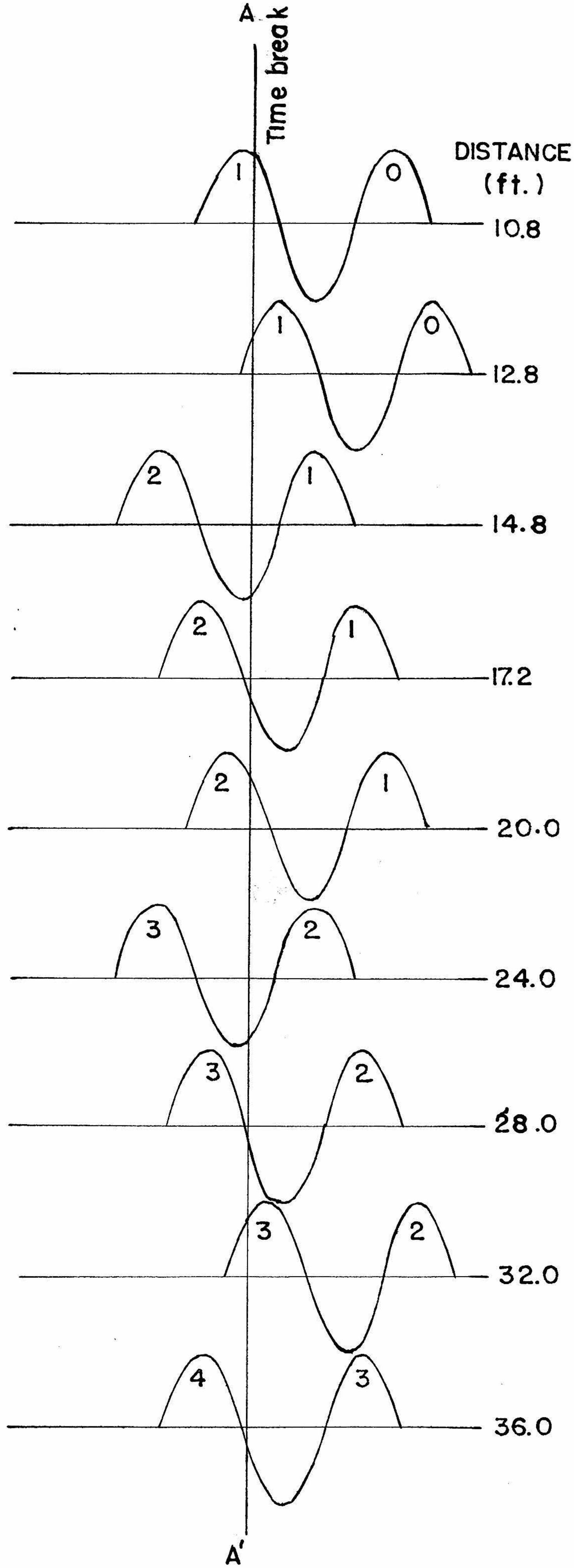


FIG. 25
OSCILLOSCOPE TRACINGS
(FOR f = 60 c.p.s.)

In Table VIII are given data taken from such tracings. The distance of the seismometer from the shaker is given in the first column. In the three double columns the data are recorded for the

TABLE VIII

Travel Time Curve Data

D(ft.)	$(n + AB/AC)$	$= \phi/2\pi$	$(n + AB/AC)$	$= \phi/2\pi$	$(n + AB/AC)$	$= \phi/2\pi$
			Frequency			
	60 c.p.s.		80 c.p.s.		109 c.p.s.	
2.3	-0.7/19	-0.0368	8/18	0.444	0	0
4.5	0.5/18.5	0.0270	11/18	0.612	5.5/21.5	0.256
6.6	8/20.5	0.371	1+1/19.5	1.051	16/17.5	0.916
8.7	14/25	0.561	1+5/13.5	1.371	1+6.7/19.5	1.343
10.8	18.5/20	0.929	1+13/20	1.650	1+14/18	1.799
12.8	1+45/18.5	1.243	2	2.000	2+4/18	2.222
14.8	1+8/20	1.400	2+4.5/18	2.250	2+7/18.5	2.378
17.2	1+12.5/20	1.625	2+10/15	2.667	3	3.000
20	1+22/25	1.880	3+1.5/19	3.026	3+7/22	3.318
24	2+8/21.5	2.371	3+13/18.5	3.703	3+12.5/18.5	3.676
28	2+16.5/21	2.786	4+8/20	4.440	5+16/21	5.764
32	3+3.5/21	3.166	4+17/19	4.897	6+14.5/22	6.660
36	3+14/21	3.666	5+13/20	5.650	7+1/21	7.048
40	4+1.5/21	4.072	6+2.5/19.5	6.128	7+19/21	7.906

frequencies 60, 80, 109 c.p.s. respectively. The first part of each double column indicates the value of the phase difference as obtained from superposition of the oscillograph tracing on coordinate paper. The fractions which give the ratio AB/AC are preceded by the proper additive integer n. In the second part of the double column, the phase difference is expressed in decimals.

The travel time curves for these data are given in Fig. 26. The decrease of velocity with frequency is to be noted. At such high frequencies, however, this decrease of velocity is not very large.

At low frequencies the oscilloscope was unsatisfactory, since it would not give a sufficiently low sweep frequency. For this reason, it was necessary to use a somewhat different method of measurement of phase at low frequencies.

One procedure employed was to feed the seismometer output of the field seismometer into the vertical plates of the oscilloscope in the same manner as before, but instead of employing the sweep circuit, a reference seismometer was used and its output fed into the horizontal plates of the oscilloscope. From measurements made on the ellipse which generally resulted, it was possible to compute[24] the phase difference between the two seismometers. However, because of the difficulties encountered on securing an undistorted wave form at the lower frequencies, it was almost impossible to obtain an ellipse regular enough in shape to permit phase measurements. Accordingly, it was decided to take records of the vibrations in the usual seismic manner, i.e. as described previously in connection with ground roll observations.

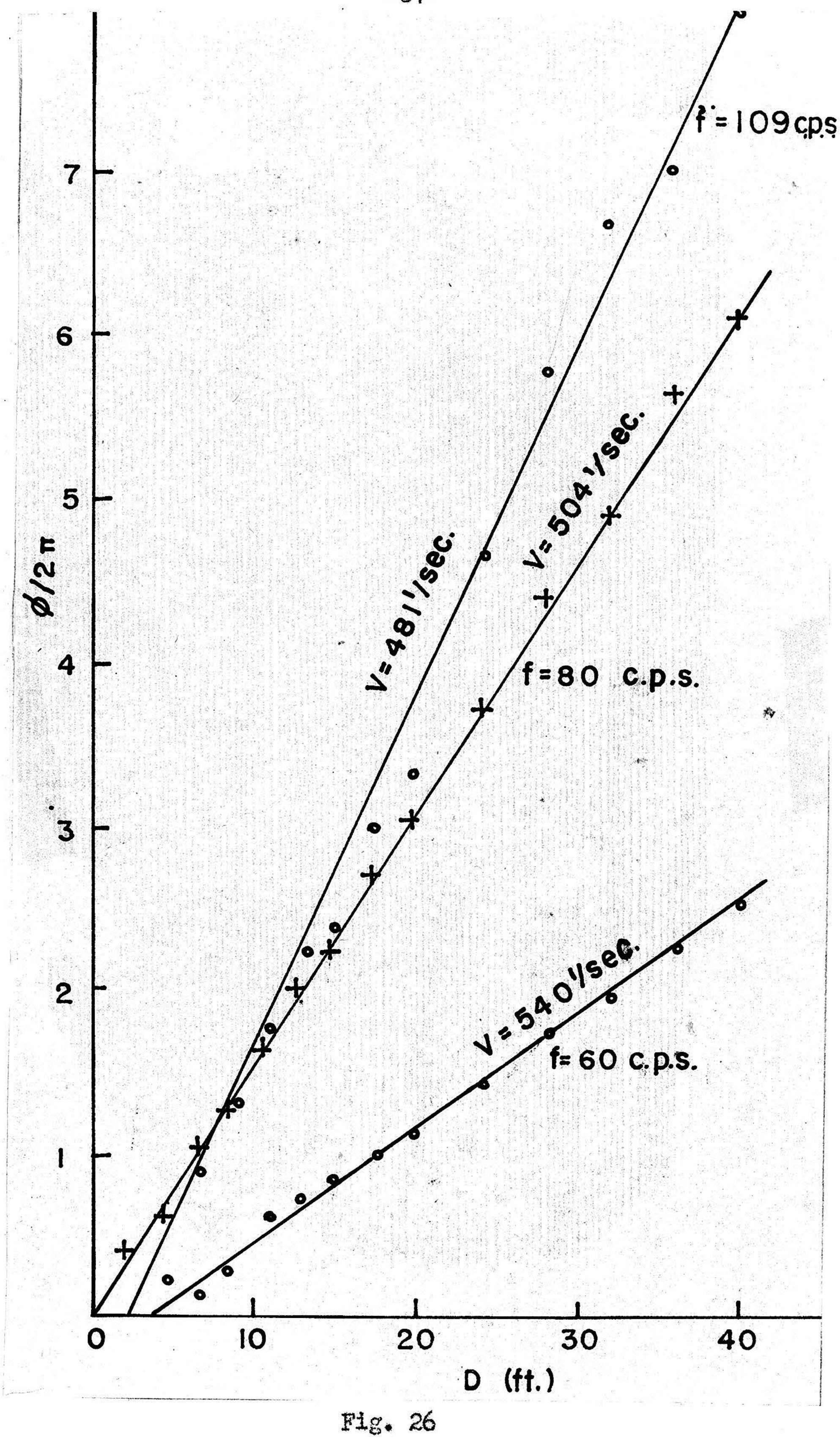


Fig. 26

Curves for Determining Velocity at Various Frequencies

There are several advantages to taking seismic recordings instead of tracing oscillograms. In the first place, greater accuracy is possible, since the personal error involved in tracing the oscillogram is removed. Secondly, the wave form may be carefully analyzed, and the cause of various undesirable small vibrations can be found more expediently. Thirdly, the timing lines provide a means of accurately knowing the frequency of the vibration as well as enabling the direct measurement of travel time instead of phase.

It was not found possible to secure amplitudes sufficiently large compared with the background noise to satisfactorily record the 10 c.p.s. vibration. The lower frequencies employed were, therefore, restricted to 20 and 40 c.p.s.

The procedure employed in running the profile with the satisfactory low frequencies possible with Shaker B, was exactly the same as that employed with Shaker A at higher frequencies, with the exception that oscillograph recording was employed in the former. It was possible to obtain a rough synchronization of the 20 c.p.s. wave on the oscilloscope, and the placement of the seismometer on the ground was altered to give what appeared to be the best wave shape, before the seismometer output was switched over to the camera recording system. A typical seismogram is shown in Fig. 27. The time marker is on the first trace and the seismometer output is on the second trace.

Velocities of 1000 and 700 ft./sec. were obtained for frequencies of 20 and 40 c.p.s., respectively.

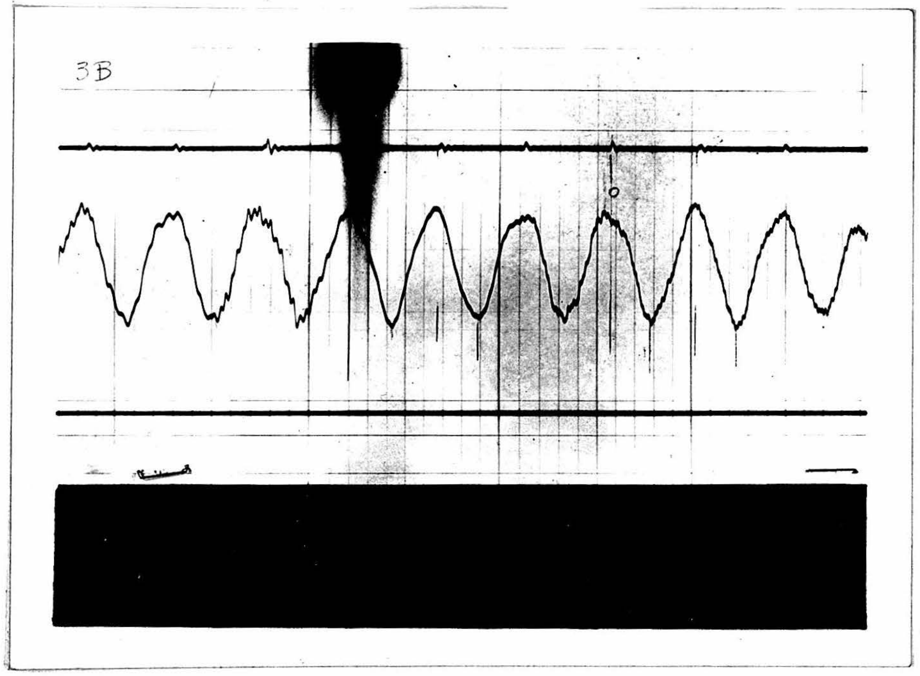


Fig. 27

Seismogram of Surface Waves (20 c.p.s.)

Plotting velocity as a function of the frequency, the dispersion curve shown in Fig. 28 was obtained. In Fig. 29 the same curve has been plotted with the velocity as a function of L/H, and the previous ground roll data obtained from the explosion experiments has been added. As was mentioned in the discussion of the ground roll record, the value of V which is obtained from extrapolation of the straight line to L/H = 0 results in a value of the group velocity C which is too low by a factor of almost 3. It will be seen presently that the ground shaker results offer a possible explanation of this discrepancy.

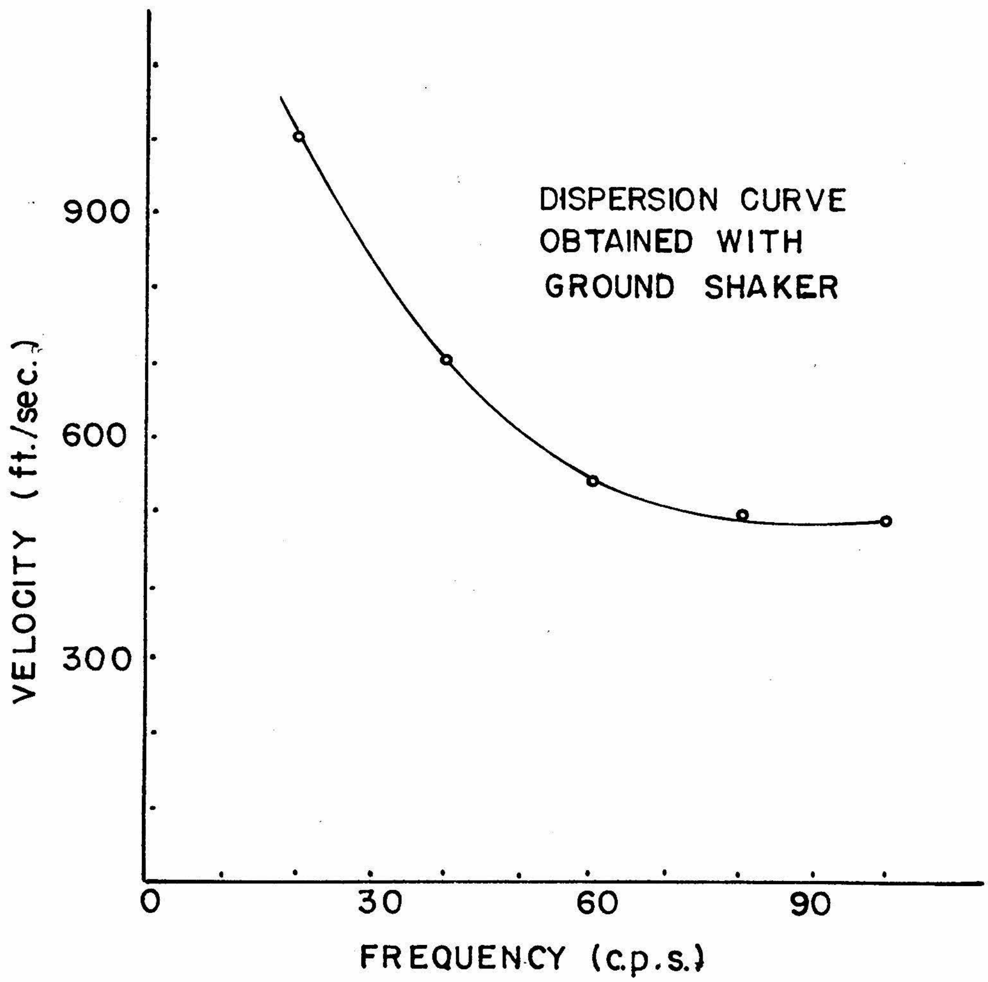

DISPERSION CURVE
OBTAINED WITH
GROUND SHAKER
VELOCITY (ft./sec.)
900
600
300
0
30
60
90
FREQUENCY (c.p.s.)

Fig. 28

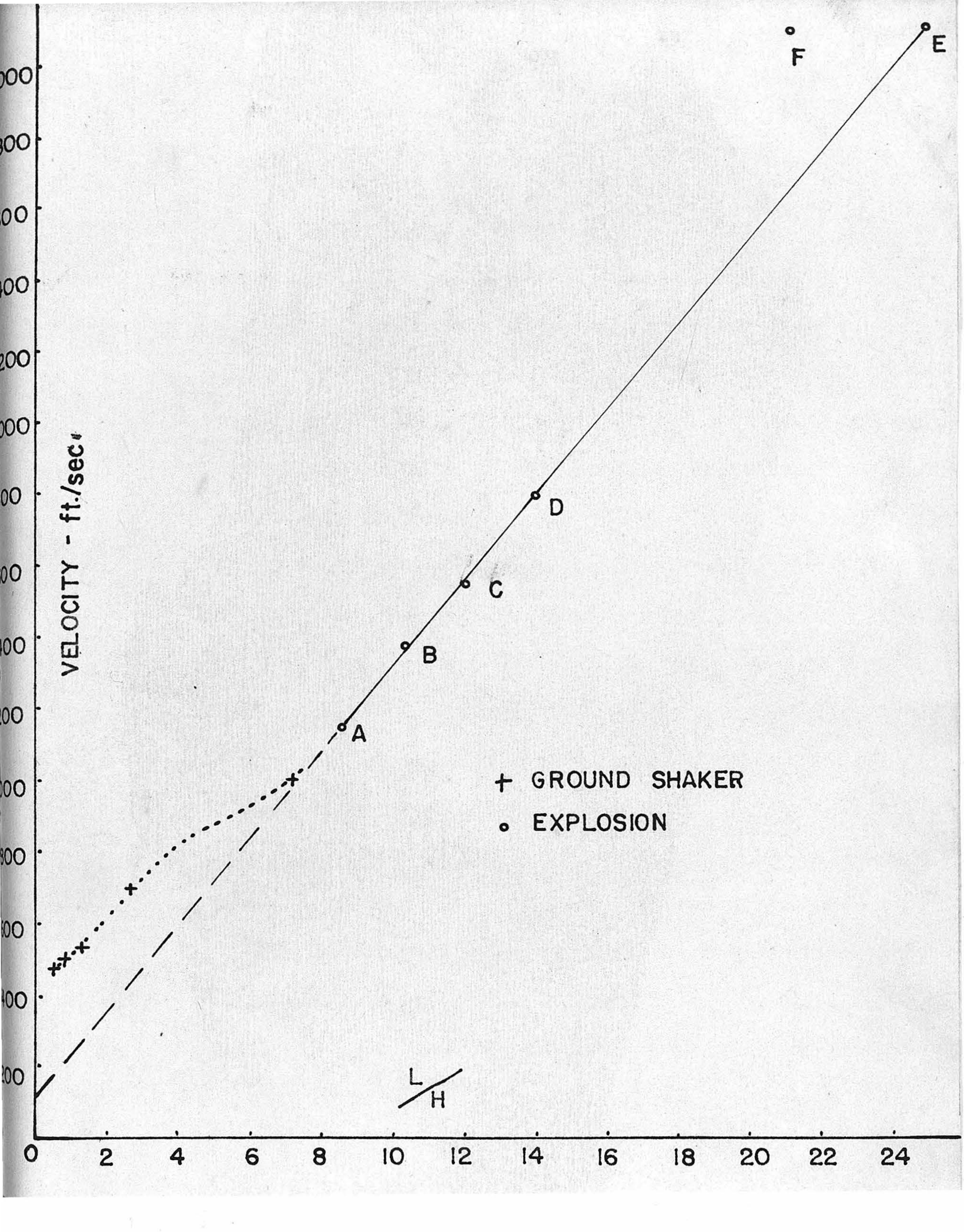

VELOCITY - ft./sec.
F
E
D
C
B
A
+ GROUND SHAKER
o EXPLOSION
L
H
0
2
4
6
8
10
12
14
16
18
20
22
24

Fig. 29.

It is observed that values of L/H obtained with the ground shaker are generally less than about 8. Similarly, the values of L/H obtained from the explosion experiments are generally greater than 8. But at the value of L/H equal to 7.2 the two dispersion curves seem to come together. This would mean that the ground roll and the waves from the ground shaker would have a velocity of 1100 ft./sec. To be able to know definitely how the two curves come together would require points for L/H larger than 7.2 in the ground shaker dispersion curve. The junction point of the two curves corresponds to a frequency of about 20 C.P.S. of the shaker. Corresponding values of L/H would require even lower frequencies in the ground shaker. Unfortunately the force at these low frequencies was insufficient to enable satisfactory profiles to be run.

It is to be observed that regardless of whether the ground shaker points are interpreted in the way suggested in Fig.29, or whether they are interpreted as a straight line, or a smooth curve tangent to the ground roll curve, an important fact is that the value of the velocity will be about 400 ft./sec. at L/H = 0. In other words the interpretation of the ground shaker data as being continuous with the explosion data enables one to remove the previously found discrepancy in the value of the group velocity.

The fact that the ground roll data are made consistent by the assumption that the surface waves from the ground roll are of the same type with different L/H , and the fact that the curves seem to join together to form a single curve at a certain L/H is indicative of the close relation between the two phenomena.

Summary

1. Neither the ground roll data nor the ground shaker data shows indication of obeying Sezawa's theoretical dispersion curve on the above assumptions, and using the data obtained from a refraction profile.

2. The general group velocity relation has been found to connect the two different types of observations.

3. The range of L/H obtained with the shaker was small compared with that obtained by explosion; the nature of the curves shows that the same velocity would be obtained for each method if the value of L/H were the same.

4. For the large values of L/H obtained with explosion, it is found that a linear relation between velocity and L/H obtains.

5. Only by assuming that the dispersion curve for small L/H obtained with the shaker is part of the same curve obtained with explosion, can the observed group velocity of the ground roll be explained.

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