AN INVESTIGATION OF CHANGES IN THE MAGNETIC FIELD OF THE EARTH (MAGNETIC ANISOTROPY OF SEDIMENTARY ROCKS)

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ABSTRACT

The paper gives the results of measurements of the magnetic anisotropy of 47 samples of sedimentary rocks ranging in age from Pliocene to Silurian time. The study was undertaken to test the validity of the hypothesis that fine grained sediments containing a large number of ferromagnetic particles should exhibit a magnetic anisotropy, the direction of the maximum susceptibility being in the direction of the Earth's magnetic field existing at the time of deposition. The theory of the development of remanent moment and anisotropy in sedimentary formations has been discussed and it is claimed that the anisotropy measurements should give more reliable results than the remanent moment measurements regarding the magnetic history of the Earth. A highly sensitive Susceptibility Meter has been designed and constructed for the purpose, and the theory of measurements of the initial magnetic anisotropy in cubical rock samples has been developed. Preliminary measurements on 24 samples of Tertiary sediments show that for the past 60 million years or so, the average declination of the Earth's magnetic field has been within about 20° of the geographic north. Similar measurements on 23 samples of Silurian sediments indicate a significant difference between the Silurian and the present geomagnetic field. The effect of deformation of beds and of strong magnetic fields on anisotropy have also been studied and it is concluded that the direction of maximum susceptibility in sedimentary rocks is stable and is invariable with respect to any stray magnetic fields subsequent to deposition.

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PART I

INTRODUCTION

1. A brief summary of the earlier works on Rock Magnetism:

The magnetic properties of rocks and minerals have engaged the attention of a number of investigators for a long time. As early as 1691 Boyle found that some old bricks had a residual magnetism and behaved as permanent magnets. Since then and especially during the middle of the XIX century, the same property was observed by several Italian and French investigators to exist in igneous rocks, baked clay and clay vessels of historic time. The importance of this property was not, however, fully realised until probably 1894 when Folgheraiter (1894) pointed out that materials containing magnetic minerals on cooling from high temperatures attain a residual magnetism, the direction and intensity of which are determined by the direction and intensity of the earth's magnetic field during the time of cooling. This important discovery aroused a considerable interest in the study of residual or remanent magnetism of rocks and the physicists found a tool for tracing the magnetic history of the earth in geological epochs. For an excellent bibliography of the earlier works, the reader is referred to Konigsberger (1938) and Haalck (1942).

The physical mechanism of the development of remanent magnetism in igneous rocks, especially the effusive ones, has been studied carefully by Konigsberger (1938) and more recently and thoroughly by Nagata (1943). It is now generally believed that the ferromagnetic

property and the magnetic susceptibility are primarily due to the presence of a large number of micro-crystals, the chemical composition of which is not uniquely known, although a large part of these micro-crystals seems to consist of Magnetite (Fe_3O_4), Chromite ($FeCr_2O_4$), Pyrrhotite (Fe_xS_y) and to a lesser extent, Spinel ($MgAl_2O_4$), Martite (Fe_2O_3), and Limonite ($HFeO_2$, nH_2O); see Bersudskii (1948). Though the physical mechanism of magnetisation of these micro-crystals is not yet thoroughly understood, there is little doubt that it is developed during cooling of the rock sample in a weak magnetic field from a temperature exceeding its "Curie Point", whence it is called "Thermo-remanent" magnetisation. The direction of thermo-remanence is always found to be in the direction of the weak magnetic field. It has also been demonstrated in a number of recent lava flows in Japan that the remanent magnetisation in them agrees well with the direction of the earth's magnetic meridian; see Nagata (1943) and Harada et al (1946).

Until recently, practically all the observations for determining the earth's magnetic field in the past were confined to the rocks of igneous origin, especially the effusive ones. If the direction of the earth's magnetic vector is to be identified with that of remanent magnetic vector of the material to be observed, the position of the material must now necessarily be the same as at the time of cooling. It has, however, been observed in a great number of cases that rocks are plastic, and large scale flow is possible even below their Curie points. Thus the original direction of magnetisation in a mass influenced by the earth's magnetic field could be completely

obliterated by the subsequent flow and conclusions drawn from the measurements on them could be erroneous and misleading.

It was first suggested by McNish and Johnson (1938) that the most reliable results on the direction of the earth's magnetic field in the past should be obtained from the direction of magnetization of sedimentary deposits and not of igneous rocks. Since the bulk of the sediments consist of fragments of igneous rocks, they too ought to contain some permanently magnetised ferromagnetic minerals. These fragments, as they settle in the quiet waters of glacial lakes or deep oceans, tend to orient themselves in the direction of the earth's magnetic field existing at the time of depositon. Upon sinking to the bottom, they are locked in place by subsequent sediments and register permanently the direction of the earth's field at that time.

The suggestion of McNish and Johnson received acceptance in many laboratories especially at the Department of Terrestrial Magnetism of the Carnegie Institution of Washington where many valuable contributions have been made to the subject of rock magnetism since 1938. A few of the results obtained are worth mentioning. McNish and Johnson (1938) made measurements on two series of Pleistocene varves in Connecticut representing about 200 and 500 years of deposition respectively. In the former case the average direction of the horizontal component of remanent moment was found to be approximately 30° west of the present declination in the region, whereas, in the latter series,

the average direction was found to be 30° east of the present declination. The time gap between the two series was estimated to be a few thousand years at the most. Similar measurements made by Johnson, Murphy and Torreson (1947) on another Pleistocene varve series in New England gave somewhat different results. This series covers a period of 5000 years and it was concluded from the results of the measurements that the earth's field has not changed substantially in direction or intensity during the last 15,000 years, the range of values of the declination being between 35° east and 25° west of the present declination. In order to extend the measurements to sedimentary rocks of greater geologic age, Torreson, Murphy and Graham (1949) measured remanent moments of flat lying undisturbed sedimentary rocks mostly belonging to Tertiary age which were collected from several sites of the western United States. As a result of measurements on 99 samples they concluded that for 50 million years or so the polarity of the earth's magnetic field has been the same as now and the magnetic axis has had an average orientation that coincides with the earth's geographic axis. Measurements of magnetizations of early Paleozoic sedimentary rocks are now in progress at the Department of Terrestrial Magnetism of the Carnegie Institution of Washington.

2. <u>The Stability of Magnetism in Rocks</u>:

In using the magnetisation of sedimentary rocks as an indication of the magnetic field of the earth at the time the sediments were deposited, it is assumed that the pattern of primary magnetisation

is permanent and has not been changed with time. The stability of magnetism in rocks, in general, has been examined by a number of investigators and very different results have been obtained.

David (1904) found an evidence of the stability of magnetization for at least 2000 years in a stone wall built with blocks of basaltic rocks. Thellier (1937) investigated samples of basalt from several places and found that both the inclination and declination of remanent moment were not constant but were dependent on the influence of the weak magnetic field of the earth. Similar examples of marked instability in laboratory samples were reported by Thellier and Thellier (1944). Mercanton (1948) found that in case of several basalt specimens, the direction or intensity of magnetization did not show any change in about 8 years in excess of the probable error of determination. Ising (1942) in Sweden, as a result of several years of measurements on varved clays, came to the conclusion that the magnetization had not remained stable since deposition but had followed in the main the direction of the earth's field. Graham (1949) has given a careful consideration of the whole problem and observed that certain selected sedimentary rocks retained their magnetization over long periods. Especially in the case of some silurian beds, he was able to show that magnetization remained stable for at least 200 million years. In his studies, however, Graham encountered a case of Silurian limestone formation where the results to demonstrate the stability were not at all conclusive.

Results of the investigations as enumerated in the last paragraph show that opinions differ as to the stability of magnetism in rocks. This is, however, not unexpected. Polarisation* of magnetic minerals (which is responsible for the polarisation of the rock as a whole) is a molecular phenomenon (being due to the orientation of the domains) and hence its stability will depend on several factors such as, the physical history of the rocks, the shape and size of the grains, the Curie points and the coercivity of the grains, the strength and duration of the stray fields and probably several others. For example, Konigsberger (1938) has shown that the higher the value of Curie temperature and the smaller the grains, the greater is the magnetic stability. According to Balmemann (1938), the magnetisation of geological bodies is a function not only of the earth's field but also of the elastic deformation sustained by the bodies. Thus it is possible that though in some cases the polarization may remain stable for a long time, in many cases it may not and only in cases where stability has been properly established can we apply with any confidence the results of polarization of rocks to trace the magnetic history of the earth.

3. Purpose of the Present Investigation:

The purpose of the present investigation was to examine another characteristic feature of sedimentary rocks which might be developed

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* Polarisation and remanent magnetisation are synonymous.

in them during deposition in the earth's field and which presumably could not be modified by subsequent magnetic variations or any stray electric or magnetic fields.

It has been shown by Benedikt (1942) that when ferromagnetic powder particles mixed with clay are deposited in a magnetic field, the clay becomes 'magnetically anisotropic' or in other words, the magnetic susceptibility of the clay develops a directive property with its maximum value in the direction of the applied magnetic field. The anisotropy of the clay was explained to be due to the anisotropy of the particles themselves or groups of particles by which it was formed. A preliminary study of the blue clay of the Boston basin (about 15,000 years old) by Benedikt (1943) revealed such an anisotropy and he pointed out that this property of sediments could be utilized in the study of earth's magnetic history but the experiment was not carried further. Almost simultaneously Ising (1942) published his results of several years of measurements on varved clays in Sweden. He demonstrated the existence of anisotropy in a number of cases and explained it to be due to the unsymmetrical shape of the particles. Unfortunately, his apparatus was not sensitive enough to find the direction of maximum susceptibility to any desired degree of accuracy and so he was not able to come to any definite conclusion regarding the direction of the magnetic field at the time of deposition of the clay.

Besides the two reports mentioned, the author is not aware

of any other attempt to correlate the anisotropy of rocks with the magnetic field of the earth. Hence it was thought necessary to examine more carefully and critically this new method of attack on problems of rock magnetism.

4. Starting hypothesis and the Magnetic Anisotropy:

The following tentative hypothesis for the development of the remanent moment and of the anisotropy of susceptibility in rocks is applicable only to fine grained sediments which are supposed to have been deposited in quiet waters of glacial lakes and deep oceans.

When a bed rock containing ferromagnetic minerals is being eroded by the action of streams or glaciers, an immense number of small grains and splinters are loosened off the bed and carried by the streams, sometimes for long distances before they are finally deposited. During the settling of such a particle in water, the forces acting on it are:

1. Hydrodynamic
2. Thermal
3. Gravitational
4. Magnetic

In deep oceans and lakes, the effect of hydrodynamic forces may be supposed to be negligibly small and also, as will be shown later in the section, at ordinary temperatures the energy of thermal agitation (10^{-14} ergs) is small compared to that of the magnetic force. Thus, qualitatively speaking, it is sufficient to consider the effect of only magnetic and gravitational forces on the final orientation of the particles.

Let us suppose, in the beginning, that these small particles have little or no residual magnetism and they are isotropic (magnetically speaking) as may be expected in case of rock splinters containing dispersed magnetic minerals. It is quite probable that a fairly large proportion of these particles will have some nonspherical shape that is, one dimension of these particles may be supposed to be somewhat elongated. As these particles are carried away by streams they will tend to float with their longer dimensions in the direction of flow and the relative elongation of the particles may be increased to some extent during the time of flotation (from detachment until deposition) due to the least resistance in the direction of their longer axes. After they have been carried out in deep, quiet waters, the magnetic field of the earth will tend to orient these particles with their longest axes in the direction of the field, because the particles will have maximum magnetic flux in that direction. As they settle to the bottom they are fixed in their position by the friction against neighboring particles. Thus it is expected that there will be a preponderance of magnetic material in the direction of the magnetic field and the sediment will exhibit an anisotropy of susceptibility with its maximum value in the direction of the magnetic field of the earth.

We have assumed that the particles have little or no residual magnetism. This may not be true in many cases and the particles may have a strong remanent magnetization which will also have a directive action in the field besides the induced magnetization as discussed in the last paragraph. It appears, at first sight, that the direction

of maximum susceptibility in that case may not be in the direction of the field. However, at the detachment of the particles, the angle between the directions of the long axes and the directions of the remanent moment is random that is, the angle may have any value from 0° to 360°. During the floating time, the components of the remanent magnetization normal to the longest dimension of a particle will suffer much heavier losses than the component along it due to the greater factor of demagnetization along the transverse directions. Thus for particles having a long floating time and hence a long demagnetization time, the remanent magnetization may be supposed to coincide practically with the direction of the longest dimensions. Hence, as pointed out by Ising (1942), both the remanent as well as induced magnetization cooperate in producing an anisotropy, the direction of maximum susceptibility being in the direction of the earth's magnetic field at the time of deposition.

Let m be the mass of the particle and c the coefficient of resistance, then the equation of motion of the particle falling under gravity is:

$$m \frac{d^2z}{dt^2} + c \frac{dz}{dt} = mg \qquad (4.1)$$

Assuming that at $t = 0$, $z = 0$ and $dz/dt = 0$ the solution of eq. (4.1) can be written as:

$$z = \frac{m}{c} gt + \frac{m^2}{c^2} g \left\{ \exp(-\frac{c}{m} t) - 1 \right\} \qquad (4.2)$$

since $\frac{c}{m} t \gg 1$

eq. (4.2) reduces to,

$$s = \frac{m}{c} g \left(t - \frac{m}{c}\right)$$

or roughly,

$$s = \frac{m}{c} gt \qquad (4.3)$$

Now let $\vec{f}$ be the remanent moment of the particle and $\vec{H}$ the earth's magnetic field, the torque acting on the particle is,

$$fH \sin\varphi$$

where φ is the angle between f and H.

So far we have assumed that the particles are isotropic; if this is not the case, there will be an additional torque due to the anisotropy of the particles themselves. Let K_1 and K_2 be the susceptibilities per unit mass respectively parallel and perpendicular to $\vec{f}$. (This does not imply that the directions perpendicular to K_1 are magnetically equivalent.) Then the total moment in the direction of f is,

$$f + K_1 mH \cos\varphi$$

and the moment induced normal to f is,

$$K_2 mH \sin\varphi$$

Hence the total moment in a direction normal to the direction of the magnetic field $\vec{H}$ is, (see Fig. 1, p.19)

$$Mn = f \sin\varphi + K_1 mH \cos\varphi \sin\varphi - K_2 mH \sin\varphi \cos\varphi$$

so that the torque acting on it is,

$$MnH = fH \sin\varphi + \frac{K_1-K_2}{2} mH^2 \sin 2\varphi$$

If I be the moment of inertia of the particle and g the coefficient of resistance for rotational motion, the equation of motion of the particle due to the action of the magnetic field is,

$$I \frac{d^2\varphi}{dt^2} + g \frac{d\varphi}{dt} = -fH \sin\varphi - \frac{K_1-K_2}{2} mH^2 \sin 2\varphi \qquad (4.4)$$

since the motion of the particles is very slow,

$$I \frac{d^2\varphi}{dt^2} \ll g \frac{d\varphi}{dt}$$

Thus putting,

$$\frac{fH}{g} = a$$

$$\frac{K_1-K_2}{g} mH^2 = b \qquad (4.5)$$

and $\cos\varphi = x$

We can write eq. (4.4) in the form,

$$\frac{dx}{dt} = b\,(1-x^2)\,(x+\frac{a}{b}) \qquad (4.6)$$

Now, $(1-x^2)$ is always positive and b is taken to be positive, so that

$$\frac{dx}{dt} \text{ is positive if } x > -\frac{a}{b} \qquad (4.7)$$

$$\frac{dx}{dt} \text{ is negative if } x < -\frac{a}{b} \qquad (4.8)$$

If we assume, $a > b$

or from eq. (4.5) if $f > (K_1-K_2)mH$

we see from condition (4.7) that $x(=\cos\varphi)$ is greater than $-a/b$ for any value of φ and hence dx/dt is always positive. This means that the angle between the magnetic field H and the moment f tends to decrease with time, that is, the particle will tend to orient in the direction of the field.

If, however, $a < b$, then dx/dt is positive so long as $x > -a/b$, but when $x < -a/b$, dx/dt is negative and the particle will tend to orient in the direction diametrically opposite to the direction of $\vec{H}$. Thus, for example, if $b = 2a$, it is easily seen that the parallelism will tend to result if φ lies between -120° and $+120^\circ$ and that anti-parallelism will tend to result if φ lies between 120° and 240°.

According to Benedikt (1942), for magnetite particles having diameters lying between 10^{-2} and 10^{-4} cm., the value of $m(K_1-K_2)$ lies between 10^{-8} and 10^{-10} c.g.s. It is quite likely that f, in most sedimentary rocks, is greater than $m(K_1-K_2)$ and the anisotropy of the particles co-operate with the permanent moment in the final orientation of the particles in the direction of the magnetic field. Even if this is not the case, we can still expect that in a sediment containing a large number of magnetite particles, the anisotropy of a majority of the particles will co-operate with the permanent moment except in the exceptional case when the initial distribution of par-

ticles is such that condition (4.8) is satisfied.

Nagata et al (1943) has ignored the effect of anisotropy of the particles and has considered the equation of motion in the form,

$$g \frac{d\varphi}{dt} = -fH \sin \tag{4.9}$$

the solution of which is,

$$\tan \frac{\varphi}{2} = \tan \frac{\varphi_0}{2} \exp \left(- \frac{h}{g} ft\right) \tag{4.10}$$

where φ_0 is the value of φ at $t = 0$.

If the number of such particles is fairly large and if p is the number of magnetized particles per unit volume of the sediment, the average magnetic moment per unit volume in the direction of the field is approximately,

$$M_{av} = pf(\cos \varphi)_{av}$$

$$\frac{pf}{\pi} \int_0^{\pi} \frac{1-\tan^2\varphi/2}{1+\tan^2\varphi/2} \cdot d\varphi_0$$

$$M_{av} = pf \tanh \left(\frac{H}{2g} ft\right) \tag{4.11}$$

Eq. (4.11) shows that the average moment in the direction of the field increases hyperbolic tangentially with increase in the time of deposition and hence (see eq. 4.3) with increase in the depth of water. Thus for a very long time of deposition ($t \to \infty$), theoretically all the particles will orient in the direction of the field and

$$M_{av} = pf \tag{4.12}$$

It is possible to derive theoretically an expression for the

effective susceptibility of a sediment and hence its anisotropy, knowing the dimensions of the particles, their initial susceptibilities and their concentration. The reader is referred to Lamson (1947) for the treatment of some ideal cases.

According to the hypothesis presented, a sedimentary deposit containing a large number of ferromagnetic grains is expected to behave as an anisotropic magnetic dipole, the axis of anisotropy and the axis of the dipole being, statistically, both parallel to the direction of the magnetic field of the earth at the time of deposition. There is, however, an important difference between the two phenomena. The polarisation or the remanent magnetisation is a molecular phenomenon whereas the anisotropy is primarily a bulk property. Thus, though the direction and intensity of the remanent moment may be changed by latter field actions, the anisotropy becomes a permanent feature of a sedimentary deposit which should be invariable with respect to any subsequent field actions.

5. Statement of the Problem:

It is hoped that the measurements of magnetic anisotropy of sedimentary deposits should give more reliable results regarding the direction of the earth's magnetic field in the past than the measurements of remanent moments as have so far been done. But the susceptibility of most sediments is a small quantity (of the order of 10^{-6} c.g.s. units and less) and the anisotropy is expected to be only

a small percent of the total susceptibility. Hence, the problem, at first, is to design and construct a highly sensitive apparatus to measure small susceptibilities to a high degree of precision and to determine the direction of maximum susceptibility with sufficient accuracy. The next step will be to correlate, if possible, the results of anisotropy measurements with those of remanent moments for the same specimens. Finally, the stability of the anisotropy of sedimentary rocks will be demonstrated by a few experiments.

6. General Theory of a Magnetically Anisotropic Medium:

In order to measure the susceptibility of some material the usual practice consists in subjecting a sample of the material to a weak magnetic field and then measuring the magnetic moment induced in it by one of the existing methods. If a test sample of such a material be introduced in a known magnetic field H, the resulting magnetic moment per unit volume is,

$$\vec{M} = \vec{M}_r + \vec{M}_i$$

where $\vec{M}_r$ is the remanent moment which in a weak field is constant and unaffected by the field and $\vec{M}_i$ is the induced moment. If the inducing field is alternating the remanent moment has no effect on the observed moment so that we have,

$$\vec{M} = \vec{M}_i$$

By definition,

$$\vec{M} = K\vec{H}' \tag{6.1}$$

where K is known as the 'Magnetic Susceptibility' of the material and is expressed in c.g.s. units per c.c. $\vec{H}'$ is the magnetizing field inside the sample and is related to the inducing field $\vec{H}$ by,

$$\vec{H}' = \vec{H} - 4\pi g K\vec{H}'$$

or

$$\vec{H}' = \frac{\vec{H}}{1+4\pi g K} \tag{6.2}$$

where the factor g expresses the demagnetization effect of the substance upon itself and depends on the geometrical shape of the sample

and its orientation with the magnetic field. In the case of sedimentary rocks K is usually a small quantity (of the order of 10^{-4} c.g.s. and less) and g is of the order of unity, hence the denominator of eq. (6.2) is sensibly equal to unity, so that eq. (6.1) can be written with sufficient accuracy as,

$$\vec{M} = K\vec{H} \tag{6.3}$$

In an isotropic medium K has the same value in all directions and the moment induced M is always in the same direction as the inducing field.

In a magnetically anisotropic medium the susceptibility K has a directive property and hence the resultant moment M will, in general, be inclined at an angle to the field H. Resolving M and H along the three orthogonal directions 1,2,3 (or X, Y, Z) which are fixed in the test sample, eq. (6.3) may be broken up into three equations, viz., (see Fig. 1)

$$\begin{aligned} M_1 &= K_{11}H_1 + K_{12}H_2 + K_{13}H_3 \\ M_2 &= K_{21}H_1 + K_{22}H_2 + K_{23}H_3 \\ M_3 &= K_{31}H_1 + K_{32}H_2 + K_{33}H_3 \end{aligned} \tag{6.4}$$

or in short,

$$M_i = \sum_{j=1}^{3} K_{ij} H_j \qquad i = 1,2,3. \tag{6.5}$$

Here, the "Normal Susceptibilities", K_{ii}, may be defined as the moment induced in the direction of $i (i = 1,2,3)$ when a field of unit strength is also acting in the same direction i; whereas, the "Cross Susceptibilities", K_{ij}, may be defined as the moment induced in the direction of i when a unit field is acting in the direction of j. It can be proved

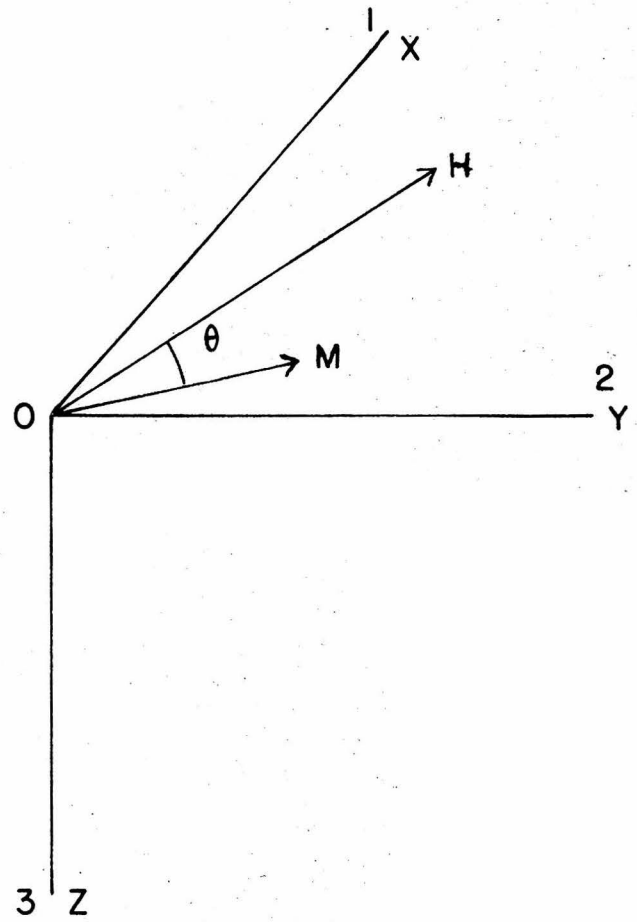

FIG. 1

from the principle of conservation of energy that for not too strong fields,*

$$K_{ij} = K_{ji}$$

Thus the susceptibility in an anisotropic medium forms a "Symmetric Tensor" characterised by six tensor components of eq. (6.4)

The resultant moment in any direction, the direction cosines of which are l_i (l_1, l_2, l_3), is the sum of the projections of three moments,

$$M_l = \sum_{i}^{3} M_i \, l_i$$

or from eq. (6.5)

$$M_l = \sum_{i}^{3} \sum_{j}^{3} l_i \, K_{ij} \, H_j \qquad (6.6)$$

If l_i be in particular the direction cosines of H, the components of H, H_j are,

$$H_j = l_j H \qquad j = 1,2,3$$

Substituting into (6.6) we get

$$M_l = H \sum_{i}^{3} \sum_{j}^{3} l_i \, l_j \, K_{ij} \qquad (6.7)$$

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*See W. R. Smythe - Static and Dynamic Electricity
- McGraw Hill Book Co. p.21, 1939.

or,

$$K = \frac{M_1}{H} = l_1^2 K_{11} + l_2^2 K_{22} + l_3^2 K_{33} + 2l_1 l_2 K_{12} + 2l_2 l_3 K_{23} + 2l_1 l_3 K_{13} \tag{6.8}$$

where K is the susceptibility in the direction l_1.

An equation of the form,

$$\sum_1^3 \sum_1^3 l_i\, l_j\, K_{ij} = \frac{p}{r^2} \tag{6.9}$$

represents a "Quadric Surface" or a second degree surface. Thus eq. (6.7) shows that for a given direction of H, the magnetic moment for that direction is pH/r^2 where r is the radius in that direction of the surface of which (6.9) is the equation. For such a surface, it is always possible to find a set of orthogonal axes such that the terms containing the cross products of the co-ordinates vanish, i.e. $K_{12} = K_{23} = K_{13} = 0$. These axes are known as the "Principal Axes" of the surface. If l_1, l_2, l_3 be the direction cosines of one of the principal axes and K is the value of susceptibility along it, then the components in 1, 2 and 3 directions of the moment are,

$$M_i = KHl_i \qquad i = 1,2,3$$

Substituting these values into eq.(6.5)

$$M_i = \sum K_{ij} H_j = H \sum K_{ij} l_j = KHl_i$$

therefore,

$$\sum K_{ij} l_j = Kl_i$$

or written in full,

$$\begin{aligned} K_{11}\, l_1 + K_{12}\, l_2 + K_{13}\, l_3 &= K\, l_1 \\ K_{12}\, l_1 + K_{22}\, l_2 + K_{23}\, l_3 &= K\, l_2 \\ K_{13}\, l_1 + K_{23}\, l_2 + K_{33}\, l_3 &= K\, l_3 \end{aligned} \tag{6.10}$$

Eliminating l_1, l_2, l_3 from eq. (6.10)

$$\begin{vmatrix} K_{11}-K & K_{12} & K_{13} \\ K_{12} & K_{22}-K & K_{23} \\ K_{13} & K_{23} & K_{33}-K \end{vmatrix} = 0 \qquad (6.11)$$

Eq. (6.11) is a cubic equation having three roots K_1, K_2, K_3. These being successively substituted in eq. (6.10) enable l_1, l_2, l_3 to be determined for each of the axes.

Let the field H be in the direction of one of these principal axes. Then the moment in that direction is,

$$M = \sum l_i M_i$$

$$= H \sum l_i l_j K_{ij}$$

but from eq. (6.10)

$$\sum K_{ij} l_j = K l_i$$

therefore,

$$M = H \sum l_i^2 K$$

$$= KH \quad \text{because} \sum l_i^2 = 1$$

Thus K_1, K_2, K_3 are the magnetic susceptibilities in the direction of the principal axes of the quadric surface eq. (6.9). They are called the "Principal Magnetic Susceptibilities" of the material of the sample.

7. Static Method of Measurements:

We have shown in Section 6 that an anisotropic medium is uniquely represented by the six coefficients of a symmetric tensor and the magnitude and direction of the maximum susceptibility can be determined in terms of these six coefficients. Hence the problem reduces to the measurements of these six coefficients, viz., K_{11}, K_{22}, K_{33}, K_{12}, K_{23} and K_{13}. Two separate methods have been developed during this investigation and are named as Static and Dynamic methods. In this section we will deal with the Static method and the Dynamic method will be described later.

Assume that the sample to be examined is in the form of a cube and the coordinate axes 1,2,3 are taken parallel to the cube faces. If the sample is placed with its 1-axis (X-axis) parallel to the magnetizing field H, we have from eq.(6.4),

$$M_1 = K_{11} H \tag{7.1}$$

assuming that H has no component normal to the 1-axis. Similarly by placing the sample such that the 2-axis and the 3 axis are respectively parallel to the field, we get,

$$M_2 = K_{22} H \quad \text{and} \quad M_3 = K_{33} H$$

If the apparatus is calibrated for samples of known susceptibility, the values of K_{11}, K_{22}, K_{33} may be determined.

Now, if the sample be rotated in the 12-plane so that the field makes an angle with the axis 1, the moment induced is,

$$M_{12} = K(\varphi_{12})H \tag{7.2}$$

where $K(\varphi_{12})$ is a susceptibility characteristic of the 12-plane, being a function of the angle φ_{12}. From eqs. (6.4) and (6.5), the moment in the direction whose direction cosines are l_1, l_2, and 0 is, ($l_1 = \cos\varphi_{12}$; $l_2 = \sin\varphi_{12}$)

$$M_{12} = l_1 M_1 + l_2 M_2$$

$$= l_1 (K_{11} H_1 + K_{12} H_2) + l_2 (K_{12} H_1 + K_{22} H_2)$$

But,

$$H_1 = l_1 H \qquad H_2 = l_2 H$$

$$\frac{M_{12}}{H} = l_1^2 K_{11} + l_2^2 K_{22} + 2l_1 l_2 K_{12} \tag{7.3}$$

comparing eqs. (7.2) and (7.3)

$$K(\varphi_{12}) = l_1^2 K_{11} + l_2^2 K_{22} + 2l_1 l_2 K_{12} \tag{7.4}$$

$K(\varphi_{12})$ is directly measured in terms of the constants of the instruments and knowing K_{11}, K_{22}, K_{12} can be calculated from eq. (7.4). Similar measurements can be made in the 23- and 13- planes. Then,

$$K(\varphi_{23}) = l_2^2 K_{22} + l_3^2 K_{33} + 2l_2 l_3 K_{23}$$

$$K(\varphi_{31}) = l_1 K_{11} + l_3 K_{33} + 2l_1 l_3 K_{13} \tag{7.5}$$

Hence, by six independent measurements on the rock sample, all the six coefficients of the cube eq. (6.11) can be determined. In actual practice, however, in any plane $K(\varphi_{ij})$ is measured for several orientations of the axes with respect to the magnetizing field and a mean value is used in calculating K_{ij} from eqs. (7.4) and (7.5). Thus the Static method consists in determining the values of susceptibilities at several angles in each of the three cube faces of the sample.

PART II

THEORY OF MEASUREMENTS

8. Dynamic Method of Measurements:

We have shown that the susceptibility in an anisotropic medium is characterised by a second degree surface or an ellipsoid whose axes determine the principal susceptibilities. Since any section of an ellipsoid is, in general, an ellipse, the measurements of susceptibility on any plane will determine an ellipse. Thus if the cubical sample be rotated continuously in any plane, one should expect to record a maximum and a minimum of susceptibility corresponding to the major and the minor axis respectively of the ellipse in that plane. If the sample be rotated through one complete revolution (i.e. through 360°), one should expect actually two maxima and two minima, because the susceptibility in the positive X-direction, say, is the same as that in the negative X-direction.

The Dynamic method of measurements consists in slowly rotating the sample in the magnetising field and recording continuously the pattern of susceptibility for one face. The procedure is then repeated for two other faces. We shall now show that it is possible to determine the magnitude and direction of the maximum susceptibility if the magnitudes and directions of the maxima and minima of susceptibilities are known for three orthogonal planes in the sample.

The equation of an ellipsoid in space with respect to a system of rectangular coordinates fixed in the sample is,

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + d = 0 \qquad (8.1)$$

The section of this ellipsoid in the XY-plane is,

$$ax^2 + by^2 + 2hxy + d = 0 \qquad (8.2)$$

If A and B are the major and minor axes of the ellipse, its equation in terms of its own axes is, (Fig.2)

$$\frac{x'^2}{A^2} + \frac{y'^2}{B^2} = 1$$

or

$$a'x'^2 + b'y'^2 + d' = 0 \qquad (8.3)$$

where

$$a'A^2 = b'B^2 = -d'$$

If eqs. (8.2) and (8.3) refer to the same ellipse, their coefficients are related by the following, *

$$a + b = a' + b'$$

$$ab - h^2 = a'\,b'$$

$$\tan 2\varphi = 2h/(a-b)$$

or

$$2a = (a' + b') + (a' - b') \cos 2\varphi$$

$$2b = (a' + b') + (a' - b') \cos 2\varphi$$

$$2h = (a' - b') \sin 2\varphi \qquad (8.4)$$

The measurements in the XY-plane give a', b' and , so that a, b, and h can be calculated from eq. (8.4).

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* See G. Salmon - A Treatise on Conic Sections
- Longmans, Green and Co., p. 157, 1929.

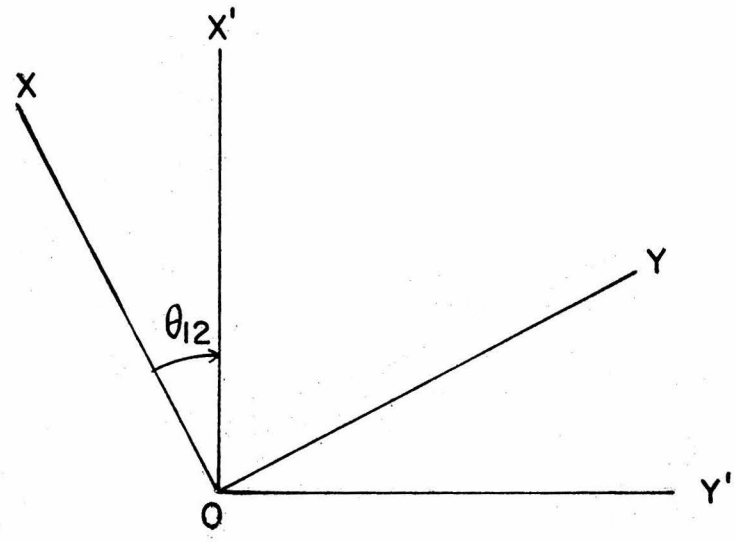


FIG. 2

Similarly, the measurements in the YZ-plane give b, c and f and those in the ZX-plane give a, c and g. Thus, from the maximum and minimum values of susceptibility and their orientations in the three orthogonal coordinate planes fixed in the sample, all the coefficients of eq. (8.1) can be found and the susceptibility ellipsoid uniquely determined.

It should be noticed that eq. (8.1) is of the same form as eq. (6.8), though the coefficients have different meanings. Hence following the same argument as in Section 6, if K is the value of a principal susceptibility having direction cosines l_1, l_2, l_3 we get equations similar to eqs. (6.10) and (6.11), viz.,

$$\begin{aligned} l_1 a + l_2 h + l_3 g &= l_1 K \\ l_1 h + l_2 b + l_3 f &= l_2 K \\ l_1 g + l_2 f + l_3 c &= l_3 K \end{aligned} \qquad (8.5)$$

and eliminating l_1, l_2, l_3 from eq. (8.5),

$$\begin{vmatrix} a-K & h & g \\ h & b-K & f \\ g & f & c-K \end{vmatrix} = 0 \qquad (8.6)$$

Eq. (8.6) written in full is,

$$\begin{aligned} K^3 - K^2(a + b + c) + K(bc + ca + ab - f^2 - g^2 - h^2) \\ - (abc + 2fgh - af^2 - bg^2 - ch^2) = 0 \end{aligned} \qquad (8.7)$$

Eq. (8.7) is a cubic equation having 3 roots K_1, K_2, K_3 which are the "Principal Susceptibilities " of the material. This equation will be hereafter known as "The Equation of Anisotropy". If K_1, K_2, K_3 are all the same, the ellipsoid of susceptibility reduces to a sphere and

the medium is isotropic. If only two of the 3 roots are the same, the ellipsoid is one of revolution and the medium is said to have an uniaxial anisotropy, whereas if all the roots are unequal, the medium has a biaxial anisotropy. For the solution of the eqs. (3.5), (3.6) see Appendix 1.

In the present chapter an attempt has been made to explain the theory of anisotropy as might be expected to exist in sedimentary deposits containing small ferromagnetic particles. We have also shown how to determine the direction and magnitude of the maximum susceptibility from the measurements on the three faces of a cubical sample of the material.

PART III

EXPERIMENTAL TECHNIQUE

9. Construction of a Sensitive Susceptibility Meter:

The present method of measuring the susceptibility of rocks is based upon the well known fact that if a ferromagnetic core be introduced into the air space of a solenoid, the self-inductance of the solenoid is increased by virtue of the increase of flux. If L_0 be the inductance of the solenoid without the core and L that with the core completely filling the air space without magnetic leakage, we can write approximately, assuming the magnetic field within is uniform,

$$L = \mu L_0$$

where μ is the "Effective Permeability" of the core. If p is the ratio of the volume of the core to that of the air space of the solenoid, then,

$$L = p\mu L_0 + (1-p)L_0$$

or

$$L = L_0\left\{1 + (\mu-1)p\right\} \tag{9.1}$$

In alternating fields, eddy currents are set up in the particles of the core which introduce a power loss depending on the particle size and frequency and the effective permeability assumes a complex form $(\mu - j\mu')$. Thus if R_e is the eddy current loss resistance, it has been shown by Burgess (1946) that,

$$L = L_0\left\{1 + \frac{\mu-1}{1+(\mu-1)g}\,p\right\}$$

and

$$R_e = \omega L_0 \frac{\mu'}{\left\{1+(\mu-1)g\right\}^2}\,p \tag{9.2}$$

where g is the demagnetization factor of the core and ω the angular frequency of the field. Expressions for μ' can be derived for special cases, thus for spherical particles of radius r, conductivity σ, permeability μ_0 and volume ratio s, Burgess (1946) has shown,

$$\mu' = \frac{18\pi}{5} \frac{s\mu_0^2 \omega r^2 \sigma}{\left\{(\mu_0+2) - s(\mu_0-1)\right\}^2} \tag{9.3}$$

In case of most fine grained sedimentary rocks both s and r are small and hence at a low frequency field (in our case 400 cycles per sec.) R_e is sensibly small. Also as we have shown in Section 6, $(\mu-1)g \ll 1$, the eq. (9.1) is approximately correct for practical purposes. If K is the volume susceptibility of the core we can write eq. (9.1) in the form,

$$\frac{L_c - L_0}{L_0} = \frac{\Delta L}{L_0} \quad 4\pi K p \tag{9.4}$$

taking $K \simeq 10^{-6}$ c.g.s., $p \simeq 0.2$ and $L_0 \simeq 10^{-2}$ henrys (see Section 11), the change of inductance expected is of the order of 10^{-8} henrys. Preliminary measurements have shown that in the case of weak samples the anisotropy may be expected to be of the order of 10^{-8} c.g.s. Hence the measuring instrument had to be designed to allow one to measure a difference of inductance of the order of 10^{-10} henrys.

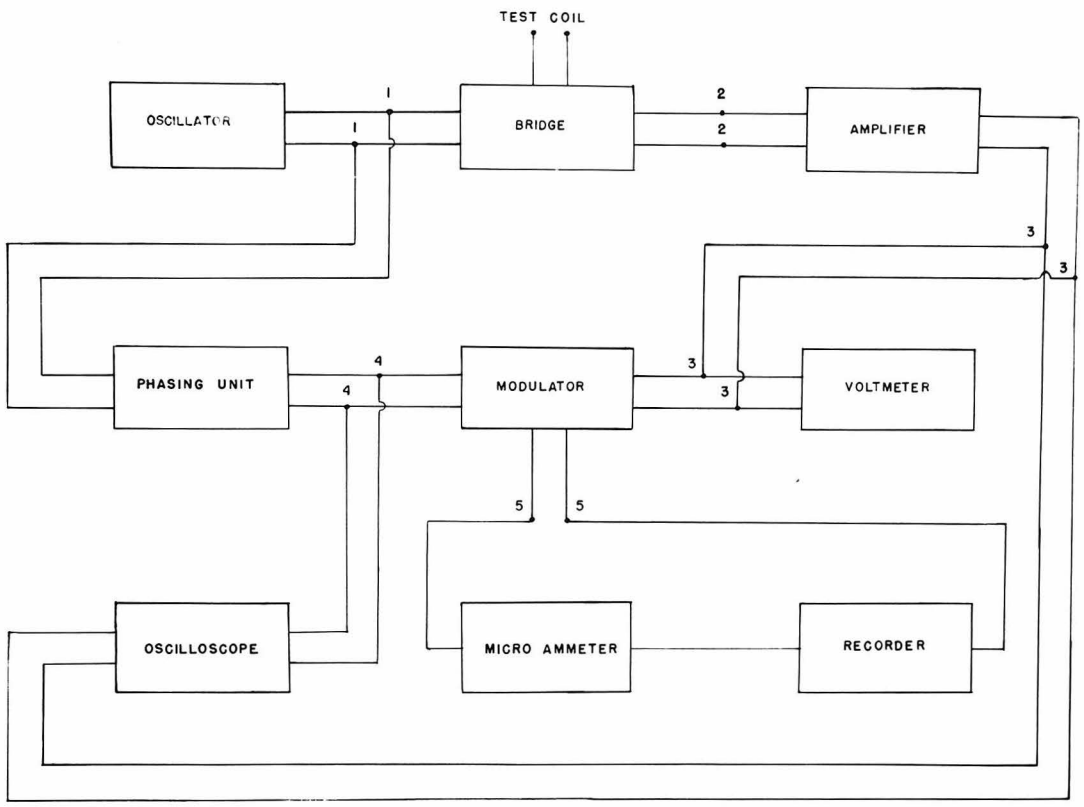
The general arrangement of the susceptibility meter is somewhat similar to that used by Lamson (1940) in measuring the magnetic properties of transformer laminations. The changes introduced in the circuits used by Lamson resulted in the increase of the sensitivity of

the meter by several hundred times. The block diagram of the susceptibility meter used in the present investigation is shown in Fig. 3 (p. 33).

The test coil forms an arm of an inductance bridge energized by an alternating voltage of a frequency of 400 cycles per second. The output of the bridge is fed into a high gain degenerative A. C. amplifier tuned sharply to a frequency of 400 cycles per second. The low impedance output of the amplifier is connected to one side of a modulator used as a phase discriminating rectifier, as will be explained later. An A. C. voltmeter across the output of the amplifier gives an indication of the noise level of the amplifier and of the off-balance of the bridge. The modulator is polarized by the same oscillator which energizes the bridge and in between the oscillator and the rectifier a phase shifting unit is introduced to get a proper phase relationship between the input of the bridge and the output of the amplifier. The measuring device is a zero-centered D. C. microammeter and a pen recorder connected in series to the output of the rectifier. For preliminary adjustments and for examining the wave character from time to time, an oscilloscope is used with its vertical plates connected to the signal output from the amplifier and with its horizontal plates connected to the polarizing voltage from the oscillator. A detail description of the parts of this susceptibility meter is given in the following sections.

10. <u>The Bridge</u>:

Of many alternating current bridge methods used for measuring



BLOCK DIAGRAM OF THE SUSCEPTIBILITYMETER

Fig. 3.

small inductances, Wien's arrangements of the Maxwell bridge is found to be most suitable, accurate and easy to manipulate for the present investigation. A description of the bridge may be found in any treatise on A. C. bridges.* Fig. 4 (p. 35) shows the circuit constants used in the present case. If Z_1, Z_2, Z_3 and Z_4 are the impedances of the branches AB, BC, CD and DA respectively, the bridge is balanced when

$$Z_1 Z_3 = Z_2 Z_4 \qquad (10.1)$$

neglecting the residual impedances of the arms of the bridge we have,

$$Z_1 = R_1 \qquad Z_2 = R_2/1 + j\omega C_2$$

$$Z_3 = R_3 \qquad Z_4 = R_4 + j\omega L$$

where ω is the angular frequency of the voltage applied. Thus (10.1) becomes,

$$R_1 R_3 = R_2 R_4 = L/c \qquad (10.2)$$

If the product $R_1 R_3$ is kept constant throughout an experiment, we get from (10.2)

$$\Delta L = \frac{L}{C}\Delta C \quad \text{and} \quad \Delta R_4 = -\frac{R_4}{R_2}\Delta R_2 \qquad (10.3)$$

Taking $L \simeq 10^{-3}$ henrys, $C \simeq 10^{-6}$ farads, if the bridge is made sensitive to a change of capacitance of 0.1 $\mu\mu$f(1×10^{-13} farad) then a change of inductance of the order of 10^{-9} henrys can be detected. It will be shown in Section 15 that the overall sensitivity of the apparatus is actually of the order of 10^{-9} henrys.

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* See B. Hague - A. C. Bridge Methods
- Isaac Pitman & Sons, Ltd. p.377, 1945.

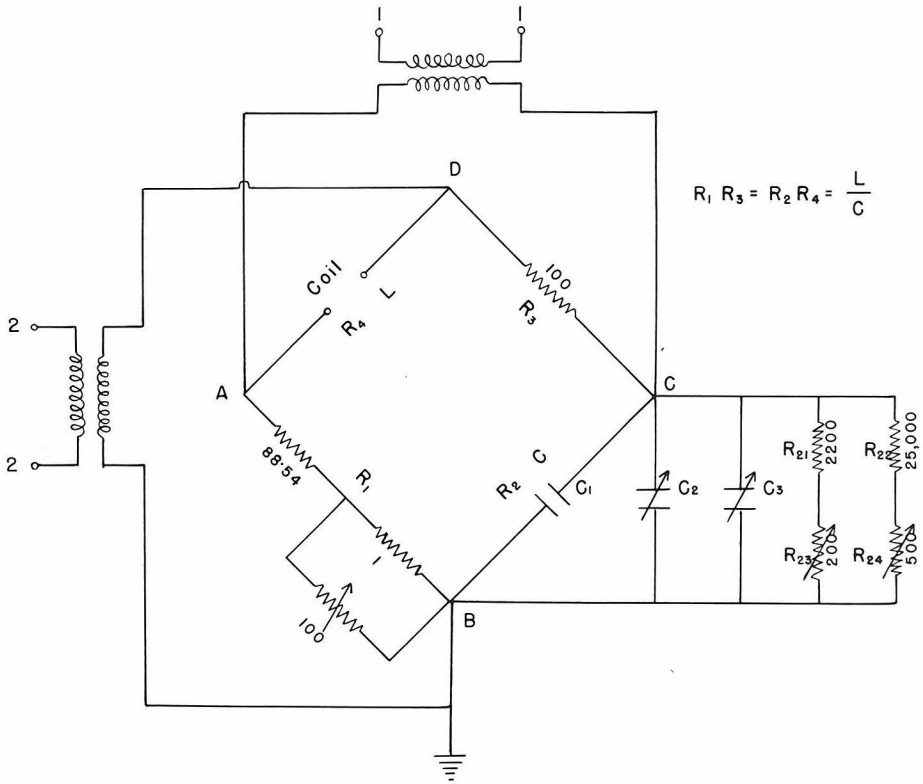


Fig. 4. The Bridge

It is seen from eq. (10.2) that the balance condition of the bridge is independent of the frequency used and, which is particularly important, the adjustments of the bridge (viz., C and R_2) are independent of each other. These two properties of the bridge make the balancing of it very convenient. Another significant advantage of this bridge over others lies in the fact, as is evident from (10.3), that a change in the reactive component, ΔL, of the unknown impedance is solely balanced by a corresponding change in capacitance C, while a change in its resistive component, ΔR_4, is balanced by a change in the resistance R_2. The importance of this particular feature will be explained in Section 13.

The fixed resistors used are standard wire wound resistors with negligible residual inductances and very small temperature coefficients (~ 0.002% per degree centigrade). The variable resistors are also standard wire wound resistors having small inductances at the low frequency used. The fixed capacitor C_1 is a standard mica condenser of 1 μf capacity having temperature coefficient less than 0.01% per degree centigrade and dissipation factor less than 0.000% at 1000 cycles per second. The capacitor C_2 consists of two decade-capacitor units connected in parallel, one having a capacity of 0.1 μf in 0.01 μf steps and the other a capacity of 0.01 μf in 0.001 μf steps. The condenser C_3 is a high precision air condenser continuously varying from 100 $\mu\mu$f to 1100 $\mu\mu$f the smallest reading on the dial being 0.2 $\mu\mu$f (1 $\mu\mu$f = 10^{-12}f). The condenser C_3 has a very small temperature coefficient, about 20 parts per million per degree centigrade.

Most of the circuit elements used were supplied by the General Radio Co., Cambridge, Massachusetts. Variables Cs and Rss are used for a rough balance of the bridge, the final accurate balance being obtained by Cs and Rss. Except the test coil, all parts of the bridge are shielded from stray electrical disturbances.

An oscillator supplying a steady A. C. voltage at 400 c.p.s. is connected to the bridge at A and C through a shielded transformer with a grounded screen between the windings. The transformer is carefully chosen to match the output impedance of the oscillator with the input impedance of the bridge. The output of the bridge BD is connected to the amplifier through a similar shielded transformer matching the low output impedance of the bridge with the high input impedance of the amplifier. The connections are all made with shielded cables.

11. <u>The Test Coil and Coil Assembly:</u>

The test coil is 2.5 inches in mean diameter and 2.125 inches in length and contains 400 turns of No. 21 enameled wire wound in six layers on a non-magnetic plastic cylindrical form. The inductance and resistance of the coil as roughly measured are 0.00832 henrys and 5.48 ohms respectively. The coil is coaxially fixed at the center of a pair of Helmholtz coils (Fig. 5, p. 38) of 15.4 inches mean diameter which produces a uniform field at the center of 24 Oersteds per ampere current. This Helmholtz pair may be used, if necessary, to

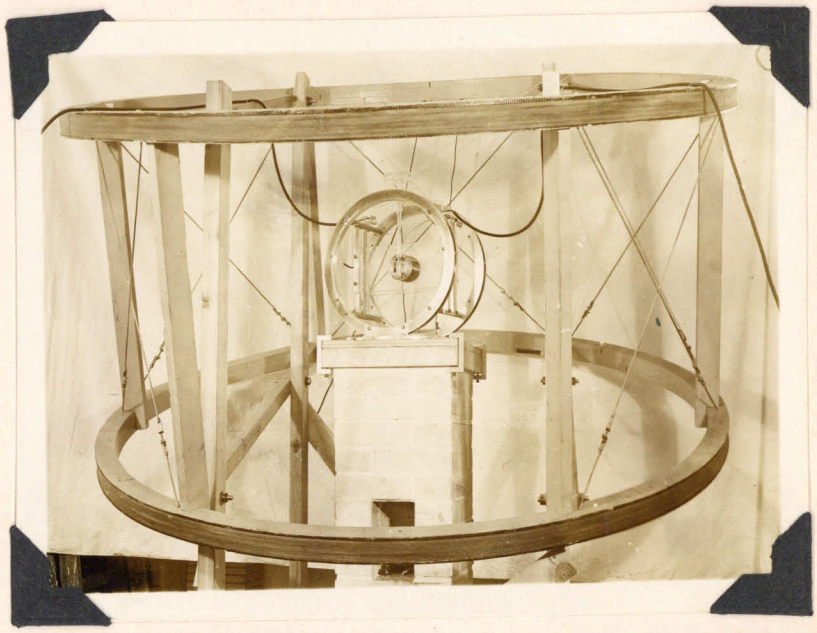


Fig. 5. The Coil Assembly

subject the samples to D. C. magnetic fields in order to study the hysterisis of the samples. This coil stands on a non-magnetic pier and the coil axis can be turned to any direction in the horizontal plane. This arrangement is necessary to minimize the stray pick-ups from electrical disturbances in the laboratory.

In order to have a region free from the earth's magnetic field at the center of the coil system, another large pair of Helmholtz coils is arranged so that its axis is in the direction of the total magnetic field of the earth in the laboratory. The mean diameter of the coils is 6 feet and the magnetic field at the center is 0.57 Oersteds per ampere current. By properly adjusting the current in the coils, it is possible to keep the magnetic field of the earth at the center of the coil system within $\pm$ 0.001 Oersteds, so that the induction effect of the earth's field on the test samples is negligible.

The "Sample Holder" is shown in Fig. 7 (p. 41) and a section of the sample holder with the test coil is shown in Fig. 6 (p. 40). The section taken is parallel to the coil axis. It consists of a pair of matched gears B and C of equal pitch fixed on to the top of a plastic plate. The gear B can be rotated by the rod D which in turn rotates the gear C in the opposite direction. Each of the gears has 36 teeth and is graduated in degrees so that the spacing between two successive teeth is ten degrees. C carries a cup which holds the cubical rock sample A. The cup can be attached to C in only one position so

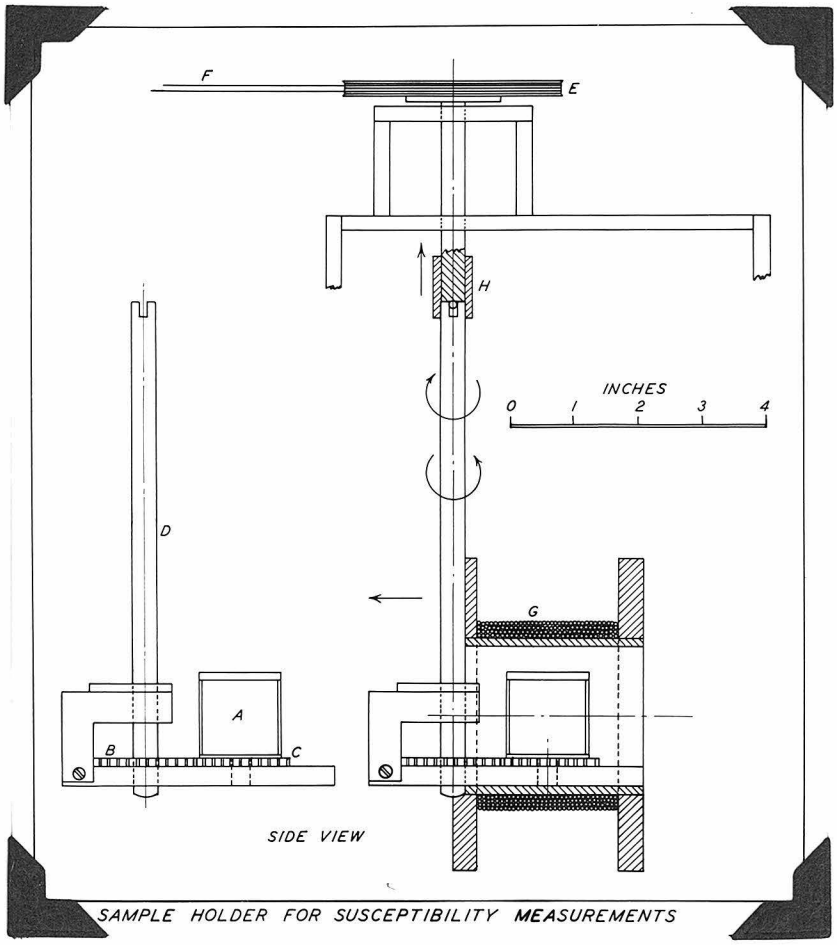

F
E
H
INCHES
0
1
2
3
4
D
G
A
B
C
SIDE VIEW
SAMPLE HOLDER FOR SUSCEPTIBILITY MEASUREMENTS

Fig. 6.

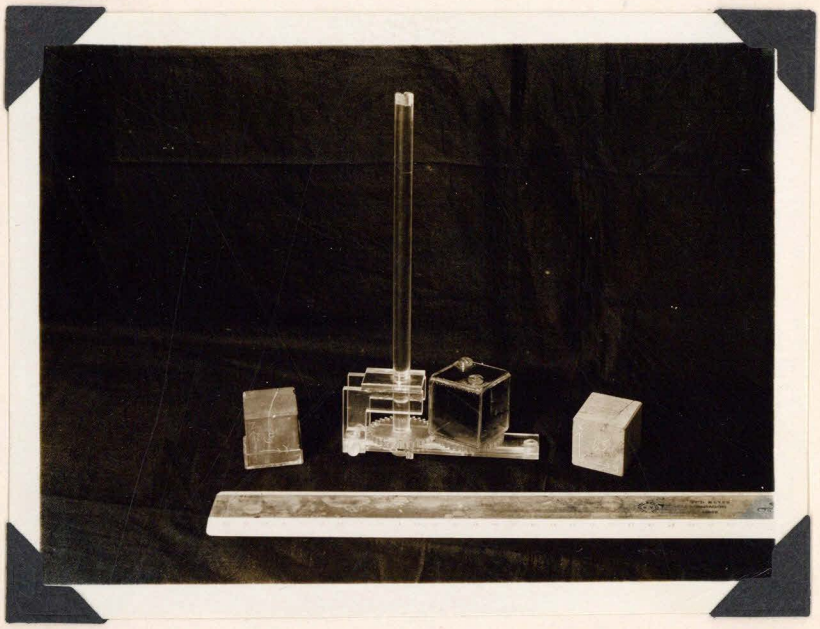


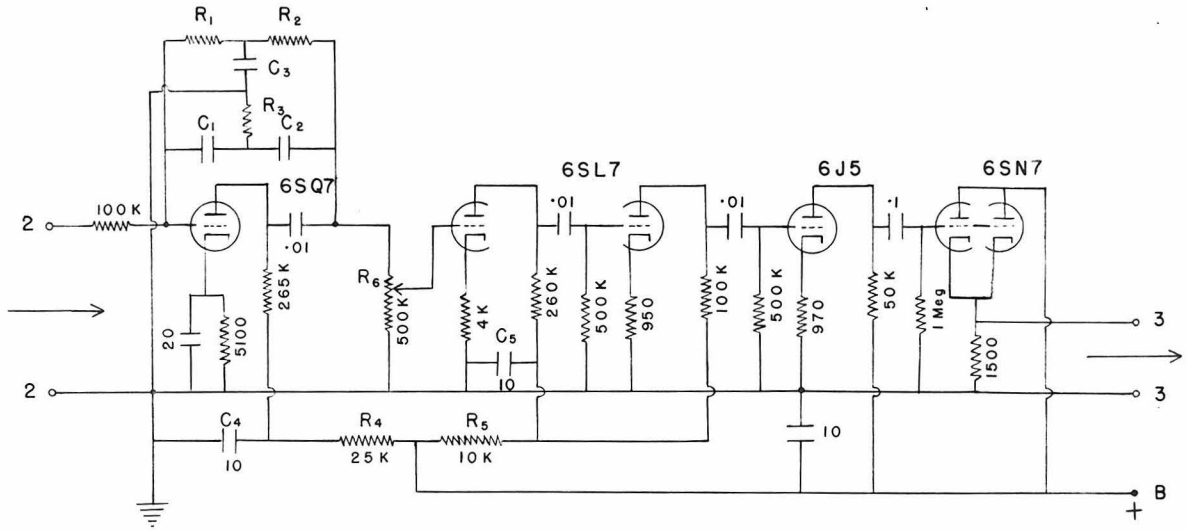
Fig. 7. The Sample Holder

that when B reads zero, the coil axis is parallel to the X-, Y-, or Z- axis of the rock sample.

The sample holder can be slid smoothly inside the test coil G on a pair of grooves. Two stops are provided on the holder such that when it is in, the center of the rock sample is always at the center of the coil. The pulley E can be attached very easily to the sample holder at H by means of a short rod. The string F couples E to another similar pulley fixed near the recording instrument about 8 feet away from the coil assembly (see Fig. 11, p.58). The second pulley near the recording instrument is graduated in degrees and can be turned into both clockwise and anti-clockwise directions. Provisions are made such that the rock sample can be turned into any direction in a plane with respect to the coil axis or can be continuously rotated through 360 degrees by means of a small synchronous motor of 1 r.p.m. attached to the pulley near the instrument. Fig. 7 (p. 41) shows the cubical box containing ferric chloride solution used as a test sample in the holder. A close look at the picture will reveal the graduation marks on the lefthand gear and also the orientation marks on the two rock samples. All the parts of the test coil and the sample holder are made of a non-magnetic plastic material called "lucite".

12. <u>The High Gain Selective Amplifier:</u>

A circuit diagram of the amplifier used in the present investigation is given in Fig. 8 (p. 43). It is a four stage resistance-



THE HIGH GAIN SELECTIVE AMPLIFIER

Fig. 8.

capacitance coupled voltage amplifier where each tube is self biased by means of the cathode resistors. The input tube 6SQ7 is a high mu triode having an amplificator factor of one hundred, the detector triode 6J5 has an amplification factor of 20 and each plate of the twin-triode tube 6SL7 has an amplification factor of 70.*

If, Z_L — load impedance

r_p — plate resistance

μ — amplification factor

then for each stage, the voltage gain without the cathode impedance is,

$$A = -\mu \frac{Z_L}{r_p+Z_L} \qquad (12.1)$$

If, however, a cathode impedance Z_k is introduced, the overall amplification becomes,**

$$A = -\mu \frac{Z_L}{r_p+Z_k(1+\mu)+Z_L} \qquad (12.2)$$

If Z_k consists of a resistance by-passed by a condenser of large capacitance ($\sim 20\mu f$), the value of Z_k is very small for alternating current and eq. (12.2) sensibly reduces to eq. (12.1). Thus in the first stage of the amplifier the cathode resistor merely produces the desired grid bias voltage drop without changing appreciably the amplification of that stage. However, this is not true in the subsequent stages where each cathode resistor produces a negative feed back current and the amplifi-

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* The author is grateful to Mr. John H. Hidy for supplying information regarding a somewhat similar circuit, now being used at Frost Geophysical Corporation, Tulsa, Oklahoma.

** See H. Reich - Theory and Applications of Electron Tubes - McGraw Hill Book Co. p.205, 1944.

cation is as given by eq. (12.2). The effective load impedance Z_L is practically equal to the resistance of the parallel combination of plate and grid leak resistances because in the mid band frequency range (400 c.p.s.) the reactances of the coupling condensers may be neglected.

In a high gain amplifier as this, the feed back resulting from B-supply regulation causes low frequency oscillation termed "motorboating". This is removed by using in each stage a 'decoupling filter' consisting of a resistance and a large capacitance (about 10 μf) viz., R_4 and C_4 in the first stage and R_5 and C_5 in the second stage.

As pointed out earlier, this amplifier is used to amplify and detect a small change of voltage developed across the bridge due to a small change of inductance in one of its arms. It is necessary to tune the amplifier to the frequency of the voltage energising the bridge so that any frequency below or above this tuned frequency is rejected by the amplifier. This has been accomplished conveniently and efficiently by using a "Twin-T net" or "Parallel-T net" as negative feed back or degenerative circuit ($R_1R_2C_3R_3C_2$) between the grid and the plate of the first tube as shown in Fig. 8 (p. 43). It was first shown by Scott (1938) that the great advantage of this circuit over other conventional degenerative circuits lies in the fact that the sensitivity of the amplifier itself is not reduced at the tuning frequency, that is, that the gain of the system at the frequency to

which it is tuned is the same as the gain of the amplifier at that frequency without the degeneration. For a detailed circuit analysis of a Twin-T net and its use as a feed back circuit, see Tuttle (1940).

It can be proved that for best performance, we must choose*

$$R_1 = R_2 = 2R_3$$

$$C_1 = C_2 = C_3/2$$

In such a case the null frequency of Twin-T is

$$f_0 = \frac{1}{2\pi R_1 C_1} \tag{12.3}$$

The feed back factor B at any frequency f is given by,

$$B = \frac{1}{1 - j\frac{4p}{p^2-1}}$$

where $p = f/f_0$.

Writing B in polar form,

$$B = |B| \exp(j\varphi)$$

we get,

$$\varphi = \tan^{-1}\frac{4p}{p^2-1}$$

and

$$B = \frac{1}{\sqrt{1 + \frac{16p^2}{(p^2-1)^2}}} \tag{12.4}$$

Eq. (12.4) is the polar equation of a circle of radius 1/2, tangent to the imaginary axis and with center at (1/2, 0). Thus it is seen that as p increases from 0 to 1 (f from 0 to f_0) φ varies from 0 to $-\frac{\pi}{2}$;

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* See G. E. Valley & H. Wallman - Vacuum Tube Amplifiers - McGraw Hill Book Co. p.387, 1948.

and as p increases from 1 to ∞ (f from f_0 to ∞), φ varies from $+\frac{\pi}{2}$ to 0. Thus a discontinuity of phase exists at p = 1.

At the frequency $f = f_0$ (p = 1) no voltage is fed back from the output of the amplifier to the input and the normal gain of the amplifier is obtained. At frequencies slightly above or below the tuning frequency, the voltage output is out of phase with that of the input and the gain of the stage is reduced. This reduction of gain is then amplified by the successive stages. Thus the high selectivity of the circuit is realized.

In the construction of the Twin-T, a set of convenient values for C_1, C_2 and C_3 ($C_1 = C_2 = C_3/2$) are first chosen and then the resistors R_1, R_2 and R_3 are simultaneously varied keeping $R_1 = R_2 = 2R_3$ until finally the balance is obtained. The stability of the net will of course depend upon the variability of the components due to temperature effects, aging and manufacturing tolerances. The components used in the present case are carefully chosen parts having very small temperature coefficients though the ideal procedure would be to use resistors and capacitors having equal and opposite temperature coefficients. The actual value of C_1 is 0.002 μf and the approximate value of R_1 = 92,000 ohms. The amplifier is tuned to 405 c.p.s. with a band width of about 10 c.p.s. The change of tuning frequency has always been within 3 c.p.s. throughout the progress of the experiment.

In order to reduce the output impedance of the amplifier, a

conventional cathode follower circuit is used in the final stage. If R_k is the cathode resistor, the voltage gain is,

$$A \quad \frac{\mu}{1+\mu} \quad \frac{R_k}{\frac{r_p}{1+\mu} + R_k}$$

Thus the voltage gain is less than unity and the impedance is reduced to approximately r_p/μ if μ is large compared to unity.

The overall gain of the amplifier is controlled by the grid leak resistor R_6 which is calibrated in terms of the sensitivity of the apparatus as a whole. The overall gain of the amplifier is found to be 1.1×10^6 and at the maximum gain the noise level of the amplifier is only 0.1 volt.

The amplifier is connected to the bridge output through a shielded transformer having grounded screen between the windings. The output of the amplifier is connected to a phase selective rectifier through an isolating transformer as explained in the next section.

13. The Phase Selective Rectifier:

The total output e.m.f. of an alternating current bridge is, in general, complex, the phase difference between the input and the output voltages being determined by the bridge constants. The two mutually perpendicular vector components of the output e.m.f. are controlled individually by the direct or reactive balance and the quadrature or resistive balance. Thus in the present bridge (Fig. 4, p.35)

the direct and quadrature bridge controls are respectively C and R_2 in the arm BC. The customary technique in bridge balancing requires an alternate to and fro manipulation of the direct and quadrature bridge controls, setting to successively decreasing minima until a sufficiently precise complete balance is obtained. However, by suitable equipment and proper adjustment of phase, it is possible to balance the bridge separately and independently for the direct and quadrature components; thus, for example, the direct component can be precisely balanced without balancing accurately the quadrature component. Since in our present investigation we are interested only in measuring the change of inductance, the recording meter is made sensitive only to the off-balance due to a change of reactance in the bridge, the effect of off-balance due to a change of resistance in the bridge being negligibly small. This is accomplished by using a phase shifting device in conjunction with a so-called "Ring Modulator" or "Doubly Balanced Modulator" circuit, which is used widely in carrier telephone systems. Such a circuit used as a phase selective rectifier is shown in Fig. 9 (p.50).

It consists of four cyclically connected rectifiers. The rectifiers used in the present case are balanced crystal rectifiers of IN40 (Sylvania) type. Four resistances of 400 ohms each and two variable resistances of 200 ohms each are introduced into the bridge as shown in the figure. One side of the bridge AC and the other side BD are connected respectively to the secondaries of two isolating and matched transformers T_2 and T_1. The primary of T_2 is connected to the oscillator (which energizes the inductance bridge) through a phase

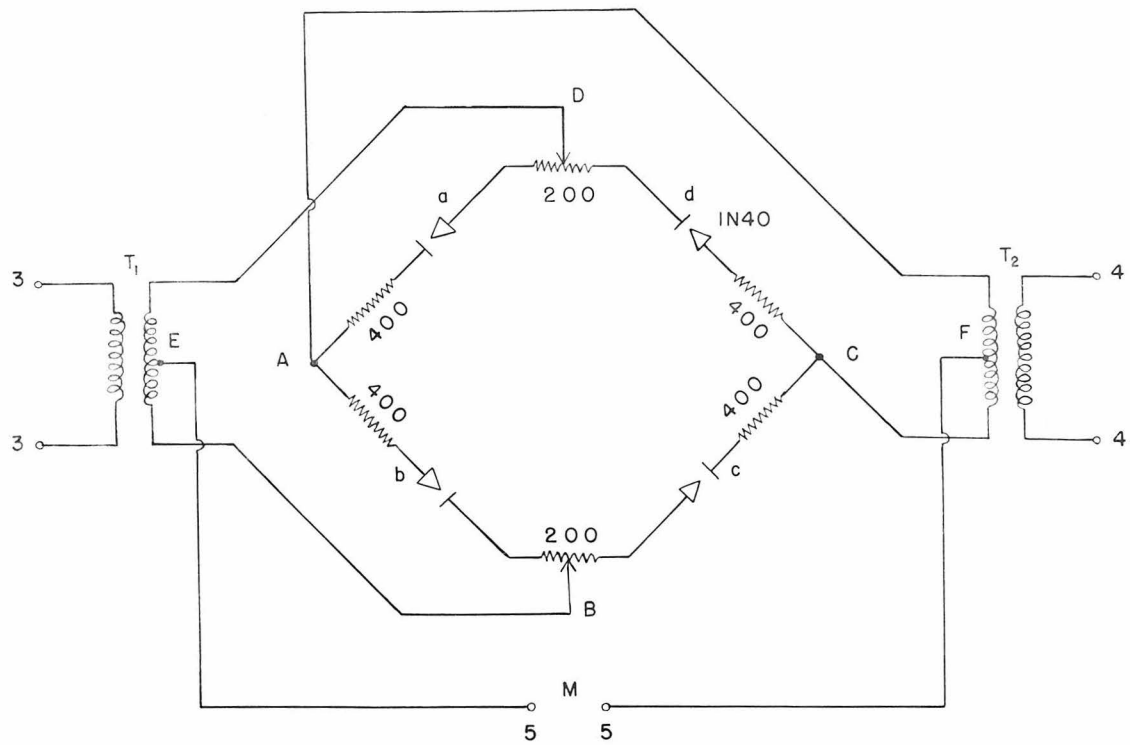

Fig. 9. The Modulator

shifting device as will be explained later in this section. The primary of T_1 is connected to the cathode output of the amplifiers. The central taps of the secondaries of T_1 and T_2 are connected to the zero centered direct current microammeter. A circuit analysis of this modulator has been given by Caruthers (1938). The action may be explained qualitatively in the following way.

After the bridge is properly balanced, (a voltage from the oscillator being impressed between the points A and C, and the amplifier being disconnected), in successive half cycles either the path ABC or the path CDA will be conducting but the potentials at E and F will be equal throughout so that no current will flow through the microammeter M. Likewise if the oscillator be disconnected and a voltage from the amplifier be impressed between B and D, again no current will flow in M. If the amplifier and oscillator voltages both exist together, the current through M will depend upon the phase difference between the two voltages.

We shall assume that the oscillator voltage (applied through the phasing unit at 44) is much larger than the amplifier voltage (applied at 33) so that the current in the modulator is dominantly controlled by the oscillator voltage. Suppose in the first place, both these voltages are in phase and the points A and D are simultaneously positive in the first half cycle. It is easily seen that in this case*

---

*The current can only flow through the rectifiers in the direction of the arrows.

the rectifier b carries more current than either of the rectifiers a and c and the non-linear variation of current with voltage causes the resistance of b to fall. The potential of B then approaches that of A causing potential of F higher than that of E and a current will flow through M in the direction from F to E. Similarly, if A and D are simultaneously negative during the second half cycle, the rectifier d will carry more current than either a or c, the potential of C approaches that of D and the current will flow in the same direction from F to E through M. Thus when the voltages are in phase, the current always flows in the same direction (from F to E) through M and the amount of current is evidently proportional to the amplifier voltage.

Now, suppose the two voltages are out of phase so that when A is positive, D is negative. In this case the rectifier c is more conductive than either b or d, the potential of B approaches that of C and the potential of F becomes lower than that of E so that a current flows through M in the direction from E to F. The direction of the current remains the same (from E to F) during the next half cycle when A is negative and D is positive.

This shows that in the Modulator of the type described, the current through the microammeter depends on the phase difference between the polarising voltage (i.e. the oscillator voltage) and the voltage to be measured (i.e. the amplifier voltage). This phase difference can be adjusted when the oscillator voltage is fed through a "Phasing Unit" (Fig. 3, p. 33). A simple form of such phase shifting

unit is shown in Fig. 10 (p.54). It was first described by Turner and McNamara (1930).

The phase shifting unit essentially consists of two equal fixed capacitors C_1 and C_2 and two adjustable resistors R_1 and R_2 which are at all times equal to each other but which may be varied jointly by a single control knob. If exp(j t) is the voltage of the oscillator applied to A and C, the voltage at B and D without any load will be,

$$e_B - e_D = -\left(R - \frac{1}{j\omega C}\right) \frac{\exp(j\omega t)}{\left(R + \frac{1}{j\omega C}\right)}$$

or

$$e_B - e_D = \frac{1 - j\omega CR}{1 + j\omega CR} \exp(j\omega t) \qquad (13.1)$$

Here $R_1 = R_2 = R$ and $C_1 = C_2 = C$

We can write eq. (13.1) in the form,

$$e_B - e_D = \exp\left\{j(\omega t - 2\varphi)\right\} \qquad (13.2)$$

where

$$\tan 2\varphi = \frac{2\omega CR}{1 - \omega^2 C^2 R^2} \qquad (13.3)$$

or

$$\varphi = \tan^{-1} \omega CR$$

It is seen from eq. (13.2) that the output voltage has the same magnitude as the input voltage but its phase has changed by an amount 2φ where φ is given by eq. (13.3). Thus when $R_1 = R_2 = 0$, the input and output voltages are identical whereas if $R_1 = R_2 = 1/\omega C$ then the output voltage will be in quadrature with the e.m.f. of the oscillator. Finally if $R \gg \frac{1}{\omega C}$ then the output voltage will be nearly

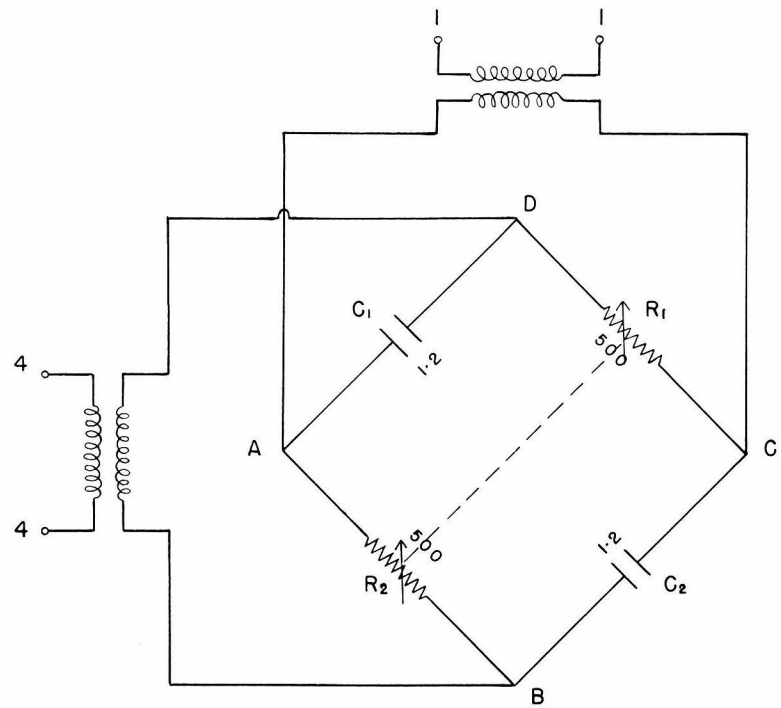

Fig. 10. The Phasing Unit

180° out of phase with the oscillator e.m.f. Thus a simultaneous adjustment of R_1 and R_2 provides a progressive phase shift of nearly 180° of the e.m.f. of the oscillator polarizing the modulation bridge. Two isolating and matched transformers are connected to the input and the output of the phase shifting unit as shown in Fig. 10 (p. 54).

Thus by means of the phase shifting unit it is possible to adjust the polarizing voltage of the modulator so that it is either in phase or 90° out of phase with the amplifier voltage. In actual practice a D. C. microammeter is used as the recording instrument and the phase of the polarizing voltage is varied until, at the balance point, the meter becomes insensitive to any change of resistive component in the bridge. Thereafter, the meter registers only any change in reactance in the bridge and hence can be calibrated in terms of known values. Thus an exact balance of the resistive component is not necessary; the off-balance of the bridge due to a change of resistance can be read in an A. C. voltmeter used across the amplifier output as shown in Fig. 3 (p. 33) and if necessary can be rectified from time to time.

14. Sampling Technique:

All the rock samples used in the present investigation were borrowed from the collection (made in the years 1948 and 1949) of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. The sampling and cubing operations used for these rocks

have been described in detail by Torreson et al (1949) and Graham (1949). For the sake of continuity we give here an outline of the methods used.

The rock samples used belong to fine grained sedimentary deposits which have no obvious signs of metamorphism, recrystallisation or stream bedding. After a site is chosen, a small unweathered surface is selected on the rock wall. The surface may be either the top or bottom of a slab projecting slightly from the main body. All orientations are measured with a Brunton compass. In case of horizontal beds a line with an arrow is scribed on the surface with a diamond pencil to represent the direction of geomagnetic north. In case of inclined or folded beds, a horizontal line is scribed on the surface, and its strike and dip are noted. The oriented sample is then carefully broken away from the rock wall with chisel and hammer or a crowbar. For coring, the sample is set in quick-setting plaster such that the surface with the orientation marks is horizontal. The coring bit then cuts a cylinder (1-5/8 inch diameter and about 2 inches long) having its axis normal to the horizontal bedding of the rock. A 3cm. cube is then cut from the core using diamond saws such that the edges are parallel to the two orientation marks (viz., the strike and dip) and normal to the orienting surface. The strike is usually designated as X-axis, the remaining edges being marked as Y- and Z- axes as in a right handed system of coordinates. The measurements are carried out relative to these edges.

For poorly consolidated rocks, the cube samples are prepared

rather laboriously by sandpaper and hacksaw. Having obtained a satisfactory cube as well as possible, a coating of lacquer is applied to prevent disintegration. It is claimed that in spite of these elaborate methods of coring and cubing, no noticeable change in the magnetic properties of the samples was observed.

15. Details of Measurements:

We have described in the Sections 9 to 13 the design procedures and the operation of the various parts of the susceptibility meter used in the present investigation for measuring the anisotropy of sedimentary rocks. The general arrangement of the apparatus without the coil assembly is shown in Fig. 11 (p. 58). In the present section we shall discuss a few important details of the actual measurements.

The apparatus was calibrated by ferric chloride solution whose susceptibility is accurately known for different concentrations (see Section 16). The solutions were kept in small lucite cubical cups whose internal volumes were equal to those of the rock samples to be examined. These cups could be very conveniently attached to the sample holder as shown in Fig. 7 (p. 41). In order to find out if the shape of the test samples had any effect on their susceptibilities, susceptibilities of ferric chloride solutions were measured for different orientations of their axes with respect to the axis of the test coil. No change of susceptibility was noticed and its effect, if any, was certainly much smaller than the sensitivity of the apparatus.

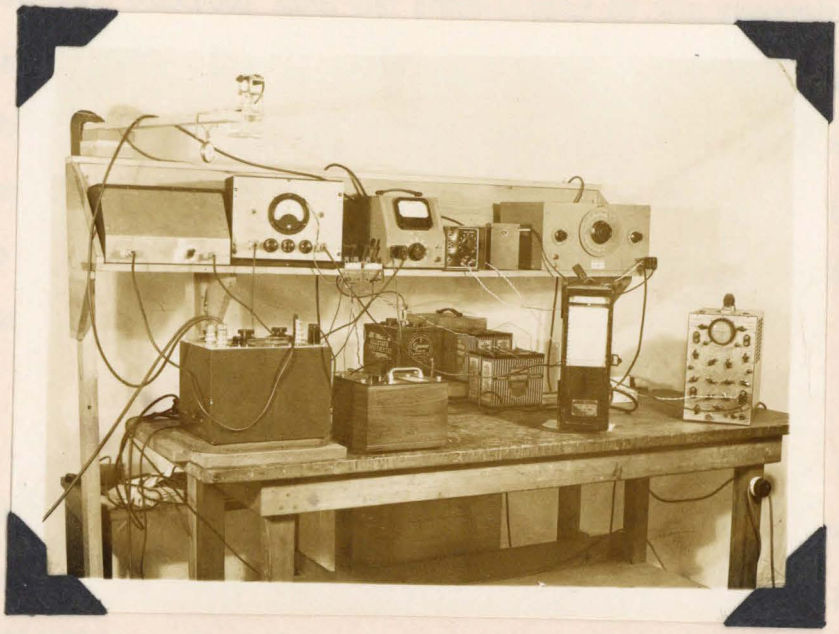

Fig. 11. The Susceptibility Meter

Hence the change of susceptibility measured for rock samples was due to magnetic anisotropy and not due to their cubical shapes.

The inducing magnetic field in the test coil was kept as low as possible in order to be certain not to disturb the initial conditions in the samples. The magnetic field used in the present investigation was only 0.5 gauss which is comparable to the present magnetic field. It was, however, realised that the sensitivity of the apparatus could be increased several times by increasing the magnetic field to about 4 gauss without introducing any serious error in the measurements. All the measurements were made in an 'Earth's field free' region (see Fig. 5, p. 38), so that any effect of the Earth's magnetic field on the susceptibility of the test samples was eliminated.

The susceptibility meter was designed to be able to measure a susceptibility difference of 10^{-8} c.g.s. per c.c. Unfortunately, the maximum sensitivity could not be used in any case due to some instrumental difficulties. The minimum susceptibility that could be determined with sufficient accuracy was only 10^{-7} c.g.s. per c.c. and in some cases 5×10^{-8} c.g.s. per c.c. When the sensitivity was raised higher, the drifting of the balance point of the bridge made it extremely difficult, if not impossible, to make any measurement with desirable accuracy. This was probably mostly due to the changes of impedances of the test coil and other components of the bridge, caused by fluctuations of temperature inside the laboratory. It seems this can be avoided and the full sensitivity of the meter realized in practice by conducting

the experiment in a temperature controlled room. Owing to this practical limitation encountered in the investigation, measurements of anisotropy were restricted to rock samples having total susceptibilities greater than 5×10^{-5} c.g.s. per c.c. on the supposition that the anisotropy in any case was at least one per cent of the total susceptibility.

We have described in Part II, two methods of measuring the anisotropy of rock samples. Actually, it is easily seen that the Dynamic method of measurements is equivalent to a large number of Static measurements. In case of rock samples having susceptibilities of the order of 10^{-4} c.g.s. per c.c., the Dynamic method was found to be convenient, quick and accurate. As pointed out before, in this method the bridge was first properly balanced with the sample inside the test coil, having one of its axes parallel to the coil axis, and then the sample was slowly rotated in one plane by means of a synchronous motor. The pattern of susceptibility was recorded on a pen recorder. The experiment was then repeated for the other two planes. In most cases the positions of the maximum and minimum of susceptibility in any plane could be determined within a few degrees and the direction of the maximum susceptibility in space could be determined within ten degrees. In case of samples having susceptibilities less than 10^{-4} c.g.s. per c.c. the Dynamic method was found to be less accurate, because, due to the drifting of the balance point, (at higher sensitivity) the positions of maximum and minimum could not be determined within a few degrees. In these cases, the Static method, though laborious, had to be used. This

method consists in the actual measurements of susceptibility at different orientations of the sample with respect to the coil axis. The direction of maximum susceptibility in these cases could be determined with an uncertainty somewhat higher than ten degrees. The present paper reports the results of measurements of only those samples when the directions were thought to be correct within twenty degrees, though in a great majority of cases the directions were probably correct within ten degrees.

PART IV

RESULTS AND DISCUSSIONS

16. Calibration of the Susceptibility Meter:

Measurements of susceptibility by the present Susceptibility Meter are not absolute, hence it is necessary to calibrate the Meter for samples of known susceptibility. Aqueous ferric chloride solutions of different concentrations were found to be most suitable for the purpose. The Smithsonian Physical Tables (Ed. 8, P. 475, 1933) gives a relation between the percentage by weight (p) of $FeCl_3$ and the 'Mass Susceptibility' as follows:

$$K_m = (p/100)H + (1 - p/100)H_0 \qquad (16.1)$$

Where H is the mass susceptibility of pure $FeCl_3$, given as 90×10^{-6}, and H_0 is the mass susceptibility of water, given as -0.79×10^{-6}. To obtain the volume susceptibility, this must be multiplied by the density of the solution used, which may be found in any chemical handbook.

It has been shown in Sections 9 and 10 that if a ferromagnetic sample be introduced into the core of the test coil which forms an arm of the bridge, the bridge becomes unbalanced, the change of inductance of the coil being proportional to the susceptibility of the sample. This change of inductance can be completely balanced by a change of capacitance in the opposite arm of the bridge (see Fig. 4, p. 35). Hence values of susceptibility can be expressed in terms of the changes of capacitance and vice versa. Table I (p. 63) gives the results of measurements on six concentrations of $FeCl_3$ solution and

Fig. 12 (p. 44) shows the calibration curve where the volume susceptibility is plotted against the change of capacitance.

TABLE I

Calibration Chart

$FeCl_3$ Soln.	$K \times 10^6$ c.g.s./c.c.	$C \times 10^{12}$ Farads
5%	3.90	2.7
10%	9.00	9.2
20%	20.48	23.8
30%	34.12	41.3
40%	50.20	61.8
50%	68.92	85.7

The results of the measurements can be expressed by the formula,

$$K = 0.783\,\Delta C + 1.82 \qquad (16.2)$$

where K, the volume susceptibility, is expressed in 10^{-6} c.g.s. per c.c. and ΔC, the change of capacitance in 10^{-12} farads. It is seen from eq. (16.2) that if $K = 0$, then $\Delta C = -2.33$, or in other words, if the sample holder (see Fig. 7, P. 41) without any sample in it be slid into the test coil, there is a negative change of inductance which shows that the material of the sample holder behaves as a dia-magnetic substance. This dia-magnetic effect of the sample holder was found to be constant throughout the investigation and the constant term in eq. (16.2) may be regarded as a constant of the apparatus.

17. Anisotropy of Tertiary Sediments:

Torreson, Murphy and Graham (1949) have reported the results of measurements of magnetic polarisation on 99 samples of flat lying,

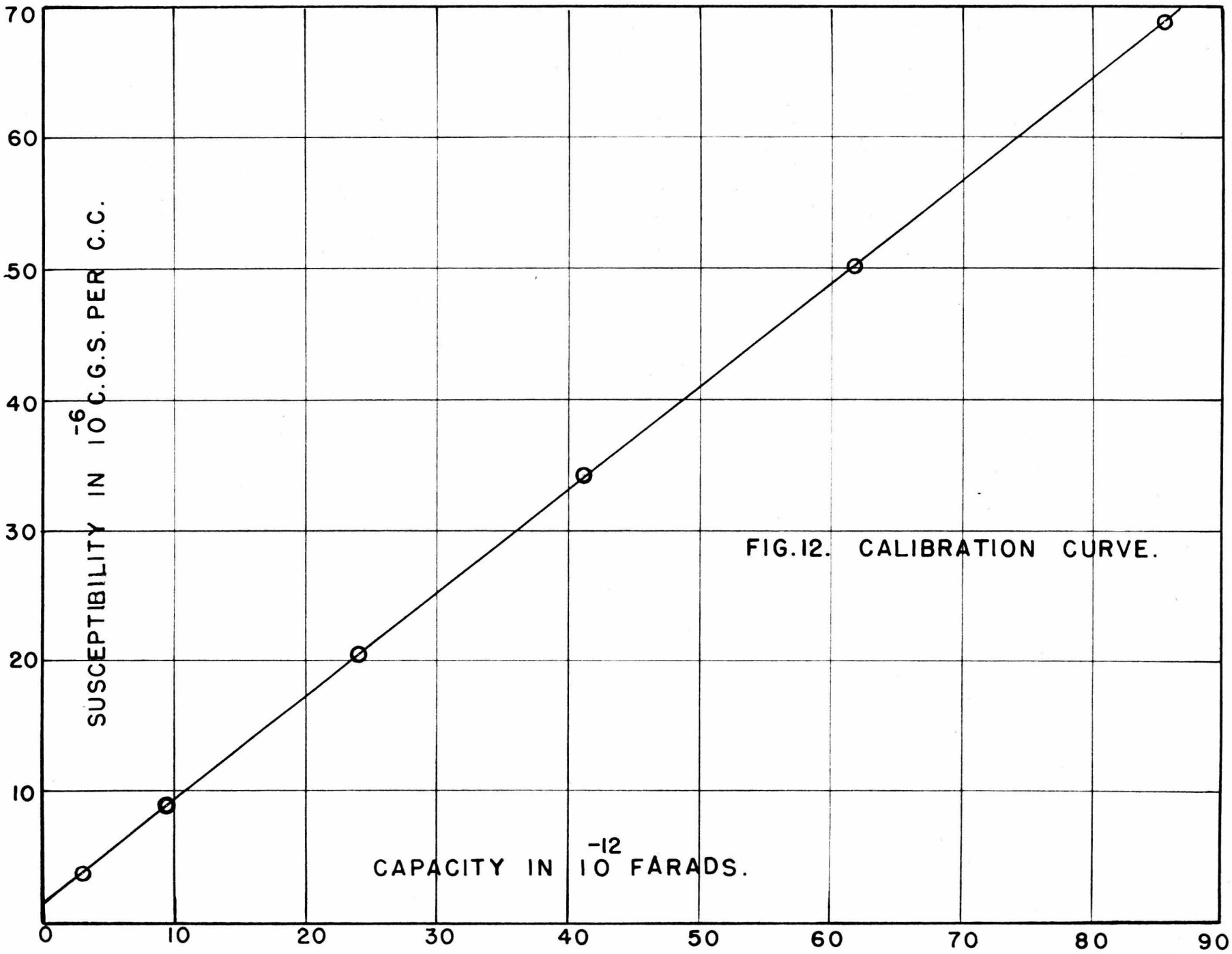

FIG. 12. CALIBRATION CURVE.

has a direction in space, it does not have a sense of direction, or in other words, the value of susceptibility along any line is unchanged if the line is rotated through 180°. On the other hand the remanent moment (or polarization) in a rock sample has both direction and sense. In accordance with the custom of thinking of the Earth's magnetic field as dipping into the globe (in the northern hemisphere), the inclinations of remanent moment are considered positive when the north seeking side is dipping downwards and they are negative when the south seeking side is downwards.

In order to visualise the angular relationships of directions in space, the directions of both maximum susceptibility and remanent moment are presented as points in the "Equal Area Projection" of Schmidt (1925). The usefulness of this projection in magnetic work has been discussed by Graham (1949). For consistency, the lower hemisphere has been chosen for projection and the cardinal points on the plane of projection (horizontal plane) are chosen as geographic north, south, east and west.

We shall discuss in the next few pages the results of anisotropy and polarisation measurements of some specific formations studied during the investigation.

a. Pliocene Time: Glenns Ferry, Idaho.

About one mile east of Glenns Ferry, Idaho, there is an excellent exposure of a flat lying undisturbed sedimentary formation of

Lower Pliocene or Upper Miocene age, Fig. 13 (p. 68). This has been described by Lindgren (1896) as fine grained fresh water lake deposits and has been named Payette formation. Samples were taken from a vertical distance of 80 ft. at an altitude of about 2,500 ft. Though 13 samples were available for measurements, only 4 were found to be sufficiently strong to enable anisotropy measurements. The results are given in Table II and Table III (pp. 79 & 80), the samples being marked from 1 to 4. The values of maximum susceptibility range from 44.7 to 74.61 with an average value of 61.23×10^{-6} c.g.s. per c.c. The average values of anisotropy in the two principal planes are av. $A_1 = 1.6$ and av. $A_2 = 4.5$. The average direction of maximum susceptibility is given by

$$D_i = 355^\circ \pm 7^\circ \quad \text{and} \quad I_i = 66^\circ \pm 5^\circ$$

The average direction of remanent moment in these samples is,

$$D_r = 361^\circ \pm 6^\circ \quad \text{and} \quad I_r = 65^\circ \pm 2^\circ$$

The direction of the maximum susceptibility agrees fairly well with that of the remanent moment. The direction of the present magnetic field at the site is, $D = 17^\circ$ and $I = 70^\circ$. Hence if the direction of maximum susceptibility can be taken as the direction of the magnetic field during the Pliocene time, we see that the present declination is at least 10° east of the declination in Pliocene time.

In order to find out if there is any relation between the moments induced (M_i) in the samples by the present Earth's field and their remanent moments (M_r), the values of the ratio M_i / M_r are given

Fig. 13. Payette Formation, Glenns Ferry, Idaho

in Table III (p. 80). M_i is the product of the maximum susceptibility and the total intensity of the Earth's magnetic field, the average value of which has been taken to be 0.56 gauss. Values of M_r are taken from the measurements of Torreson et al (1949). In the case of the Glenns Ferry samples, the ratio has an average value of 1.1.

b. Miocene Time: Badlands, South Dakota.

Samples of Miocene age were collected from the Badlands, South Dakota (Fig. 14, p. 70). The formation is a fine grained undisturbed sediment known as Arikaree sand stone; see Osborn (1909), Tables II and III (pp. 79 & 80), give the results of measurements on five samples (numbered 5 to 9). The average direction of the maximum susceptibility is given by,

$$D_i = 13^\circ \pm 8^\circ \quad \text{and} \quad I_i = 64^\circ \pm 4^\circ$$

The direction of the present magnetic field at the locality is, $D = 14^\circ$ and $I = 72^\circ$. The direction of remanent moment in these samples varies considerably from sample to sample. It is also interesting to notice that the ratio of induced to permanent moment in these samples is much higher than in any case examined during the investigation. It is not safe at this stage to draw any definite conclusion regarding the discrepancies in susceptibility and polarisation measurements in this case. It, however, appears that the relatively small values of remanent moments and the scattering of directions might be due to the disturbing influence of some stray magnetic field on the magnetisation of the sediments after they were deposited.

Fig. 14. Arikaree Formation, Badlands, South Dakota

c. Miocene Time: Selah, Washington.

Samples of Miocene age were collected from an exposure about one mile west of the town of Selah (Yakima), Washington, (Fig. 15,p. 72). The formation is known as Ellensberg formation and has been described by Smith (1903). It consists of river gravels and sands overlain and interbedded with many feet of fine grained, largely volcanic sediments. The samples were taken from a gravel pit. The vertical extension of sampling being about 20 ft.

The samples (numbered 10 to 20) have in general high values of susceptibility and also relatively high remanent moments, the ratio of induced to remanent moment being about 2.5 on an average. The nature of anisotropy in three orthogonal planes of a sample as found by actual measurements is shown in Fig. 16, p. 73). It is seen from Table III (p. 80) that the direction of maximum susceptibility varies considerably from sample to sample, although in 7 out of a total of 11 samples, the declination varies between 357° and 19° with an average value, $D_i = 6° \pm 6°$, the average value of inclination being $I_i = 44° \pm 7°$. The direction of polarization of these samples is also quite scattered, although in 6 samples, the average value of declination is 360°. The direction of the present magnetic field of the Earth at the site is, $D = 22°$ and $I = 70°$.

The scattering of the direction of maximum susceptibility and also of the direction of polarization in these samples suggests the

Fig. 15. Ellensberg Formation, Selah, Washington

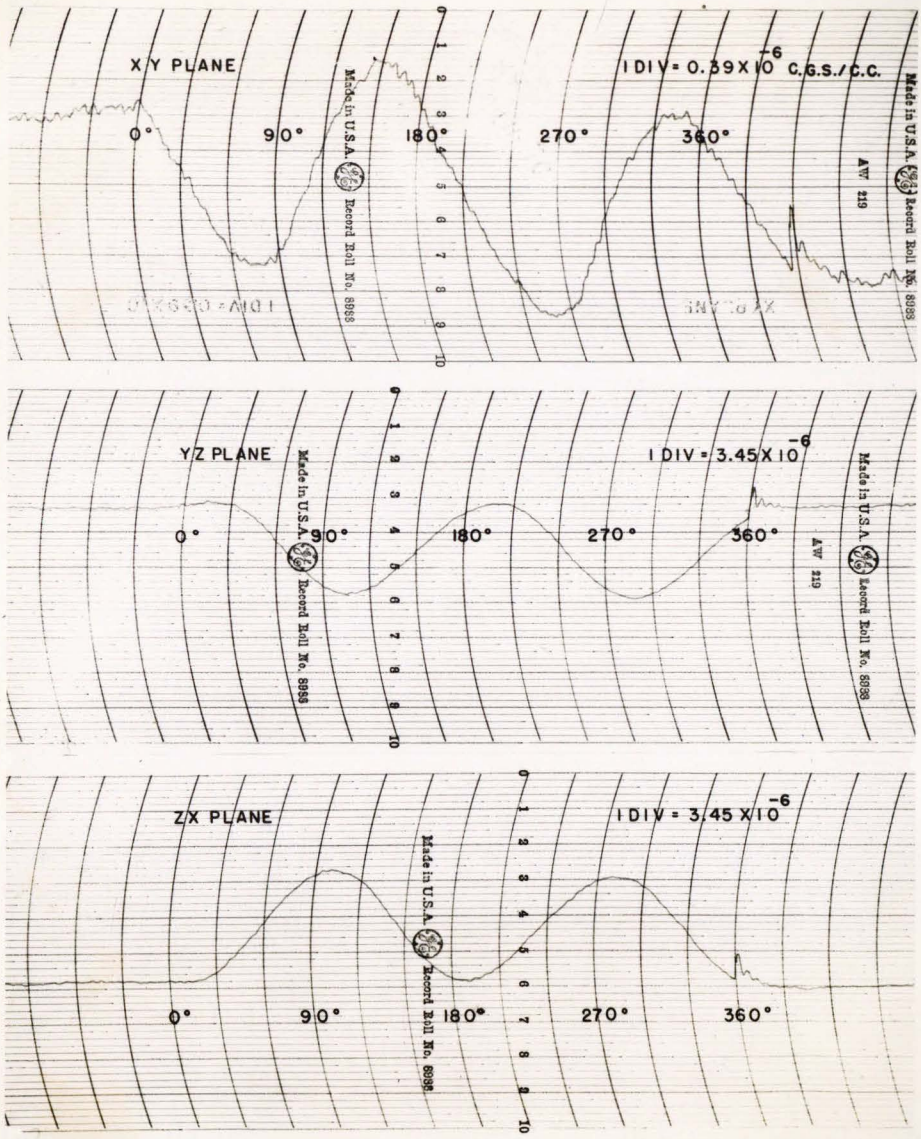

Fig. 16. The Anisotropy of Sample No. 7
Selah, Washington

existence of disturbing influences at the time of deposition of this formation. Since this formation has been described by Smith (1903) as of probable fluviatite origin, it is possible that shallow water, currents and comparatively larger size of particles made it difficult for a large number of particles to orient in the magnetic field of the Earth existing at that time.

d. Eocene Time: Anvil Points, Colorado.

Near the town of Anvil Points, Colorado, there is an excellent exposure of Green River formation of middle Eocene age described by Sears & Bradley (1925), Fig. 17. (p.75). Seven oriented samples were collected from a vertical distance of about 3 ft. at an elevation of about 8000 ft. Tables II and III (pp. 79 & 80) give the results of measurements on four of these samples (numbered 21 to 24). Fig. 18, (p. 76) is a typical example of anisotropy on three cube faces of one of these samples.

The average value of the direction of maximum susceptibility is,

$$D_i = 360^\circ \pm 8^\circ \qquad I_i = 46^\circ \pm 13^\circ$$

The average value of the direction of polarisation in these samples is,

$$D_r = 350^\circ \pm 8^\circ \qquad I_r = 63^\circ \pm 7^\circ$$

The direction of the present magnetic field of the Earth is,

$$D = 15^\circ \qquad I = 67^\circ$$

The ratio of induced to remanent moment in these samples has an average value of 1.6.

Fig. 17. Green River Formation, Anvil Points, Colorado

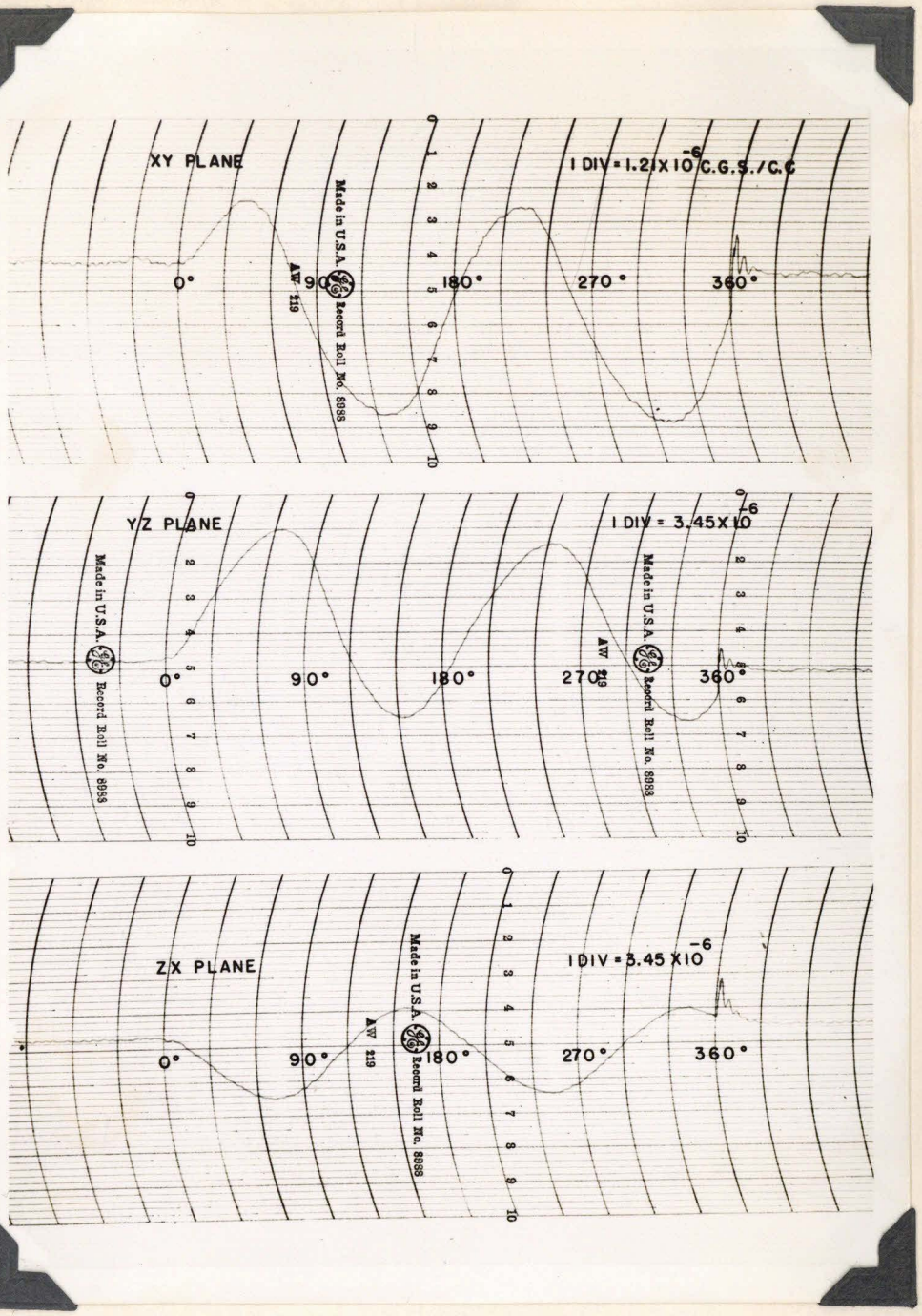

Fig. 18. The Anisotropy of Sample No. 23
Anvil Points, Colorado

The results of measurements on all the 24 samples of Tertiary time are plotted on an equal area projection of Fig. 19 (p.78). For comparison, the results of polarization measurements on the same samples are given in Fig. 20 (p. 78). In the figures, the concentric circles represent 30° intervals of inclination, increasing values being measured from the periphery toward the center. Values of declination are read from geographic north, clockwise through 360°. The extent of the direction of the present magnetic field is also shown in the figures.

It is seen from Fig. 19 (p. 78) that in a majority of cases the declination lies within 15° of the geographic north and the inclination lies between 50° and 65°. It is premature at the present moment to draw any definite conclusion regarding the geomagnetic field in Tertiary time, because the data are too few and statistical fluctuations are quite large. But it can be said that the average declination of the geomagnetic field in the last 60 million years or so has been within 15° of the geographic north. It may be remarked at this point that though the declination of maximum susceptibility in fine grained sediments may represent faithfully the declination of the geomagnetic field existing at the time of deposition, its inclination may not represent the geomagnetic inclination because the action of gravity and compaction with time may tend to diminish the inclination of the particles.

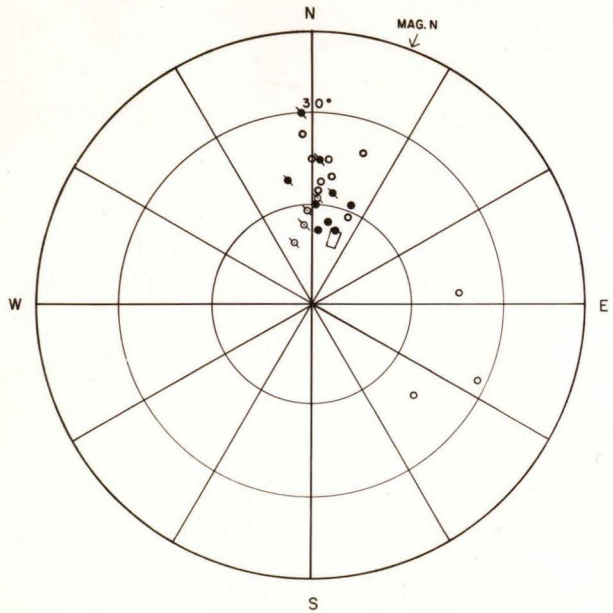

Fig. 19. Direction of Maximum Susceptibility in Tertiary Sediments

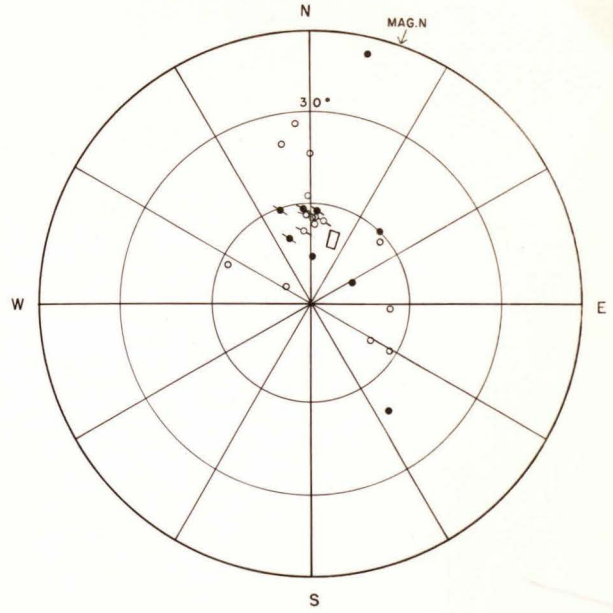

Fig. 20. Direction of Polarization in Tertiary Sediments

- Glenns Ferry
- Badlands
- Selah
- Anvil Points
- Extent of the present magnetic field

TABLE II

Anisotropy in Tertiary Sediments

Locality	Sample No.	$K_1 \times 10^6$	$K_2 \times 10^6$	$K_3 \times 10^6$	$A_1\%$	$A_2\%$
Glenns Ferry (Pliocene)	1.	44.70	44.08	42.65	1.4	4.6
	2.	74.61	72.72	70.70	2.5	5.2
	3.	71.84	71.16	69.61	0.9	3.1
	4.	53.78	52.97	52.09	1.5	5.0
Badlands (Miocene)	5.	175.22	173.25	164.63	1.1	6.0
	6.	70.97	69.30	69.35	1.6	2.3
	7.	64.62	63.78	61.13	1.3	5.4
	8.	77.53	76.44	73.96	1.4	4.6
	9.	76.34	74.35	72.95	2.6	4.4
Selah (Miocene)	10.	292.27	290.22	281.16	0.7	3.8
	11.	296.24	294.28	288.87	0.7	2.5
	12.	203.71	201.61	191.83	1.0	5.8
	13.	249.78	248.97	239.49	0.3	4.1
	14.	160.35	158.48	147.59	1.2	8.0
	15.	194.87	192.53	183.37	1.2	5.9
	16.	214.47	212.75	204.60	0.8	4.6
	17.	148.18	147.33	138.29	0.6	6.7
	18.	156.34	153.84	142.74	1.6	8.7
	19.	142.67	140.53	131.68	1.5	7.7
	20.	204.07	202.03	190.40	1.0	6.7
Anvil Points (Eocene)	21.	293.24	284.45	275.66	3.0	6.0
	22.	526.02	520.06	497.06	1.0	5.5
	23.	229.56	220.58	205.40	3.9	10.5
	24.	441.15	430.12	400.56	2.5	9.2

TABLE III

Direction of Maximum Susceptibility and Polarisation in Tertiary Sediments

Sample No.	Max. Susc.		Polarisation		$\frac{M_i}{M_r}$
	D_i°	I_i°	D_r°	I_r°	
1.	355	66	3	64	1.4
2.	3	58	9	65	1.2
3.	350	62	355	68	1.0
4.	344	71	357	63	1.9
5.	16	65	13	6	25.8
6.	2	60	2	76	4.3
7.	5	68	64	76	7.5
8.	18	67	144	50	9.6
9.	22	58	45	60	7.1
10.	3	56	3	64	1.9
11.	9	51	4	62	2.3
12.	86	45	93	66	2.4
13.	4	53	359	57	1.7
14.	132	48	306	81	4.1
15.	115	43	121	69	2.6
16.	0	46	350	40	1.8
17.	23	52	295	62	3.3
18.	19	41	48	62	2.6
19.	357	38	355	34	2.4
20.	7	45	360	44	1.8
21.	357	30	342	69	1.0
22.	3	46	342	60	1.2
23.	11	56	1	62	3.0
24.	349	52	356	61	1.1

13. Anisotropy of Silurian Sediments:

a. Silurian Time: Pinto, Maryland.

In order to extend the study of anisotropy to sedimentary rocks of remote geologic times, measurements were made on samples of Silurian time which are supposed to be about 350 million years old.

In connection with his study of the stability of rock magnetism, Graham (1949) selected a fold in the Rose Hill formation of Silurian time near the town of Pinto, Maryland, Fig. 21 (p. 82). According to Swartz (1923), who has described in detail the geology of this region, the folding of these Silurian beds took place toward the end of the Paleozoic era, about 200 million years ago. Polarization measurements were made on 20 samples taking from a single massive bed that varies in thickness from two to four inches. By graphically restoring the bed point by point to the horizontal position (which was the original position prior to the time of folding), Graham found that the directions of polarizations became parallel to one another (within statistical fluctuations); see Fig. 22 (p. 83). The results were explained by saying that with the folding of the bed, the polarizations at various points in the fold (which were originally parallel) were rotated correspondingly and since then remained unchanged for 200 million years. Graham further argued that since the polarizations in this case have been proved to remain stable for the past 200 million years, they may be considered likely to have done so for the 150 million years

Fig. 21. Folded Rose Hill Formation,
Pinto, Maryland

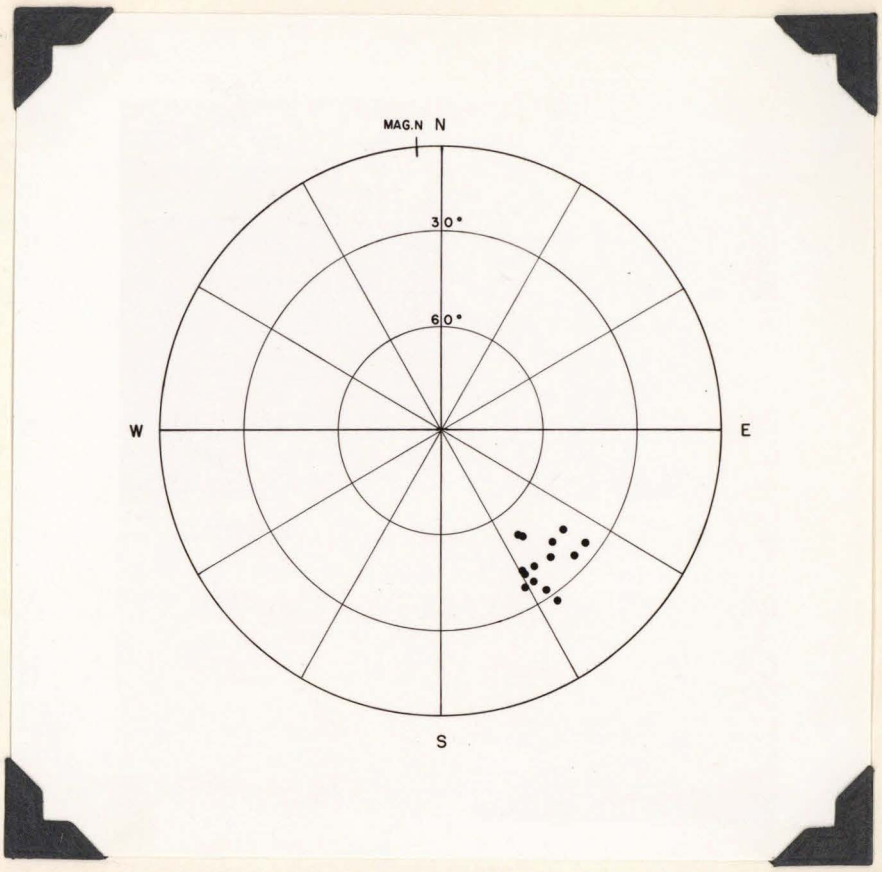

Fig. 22. Polarization of Folded Rose Hill (Silurian) Formation corrected for Tilt of Beds

prior to their deformation.

In order to find out if the direction of polarizations in these samples actually represent the direction of the geomagnetic field at the time of deposition and also to examine the effect of deformation of beds on magnetic anisotropy, measurements of susceptibility were made on the same samples. The results are given in Tables IV and V (pp. 89 & 90). The samples are arranged in three groups, the first group of five being taken from the west limb of the anticline, the second group of five from the east limb of the anticline (or the west limb of the syncline) and the third group of four from the east limb of the syncline; see Fig. 21, (p. 82). The values of susceptibility vary within narrow limits, the maximum deviation being only 12 x 10^{-} c.g.s. The anisotropies A_1 and A_2 in the two principal planes are also quite appreciable, except in case of samples No. 25, 26 and 27. Table V (p. 90) gives the values of the observed directions of maximum susceptibility and also the directions when the bed was restored to the horizontal position. Fig. 23 and Fig. 24 (p.85) are the plots of observed and corrected directions of maximum susceptibility. The average direction of maximum susceptibility, when corrected for tilt of beds is given by,

$$D_1 = 148^\circ \pm 5^\circ \qquad I_1 = 54^\circ \pm 6^\circ$$

The average direction of corrected polarisation of these samples is given by, (see Fig. 22, p. 83)

$$D_r = 142^\circ \pm 7^\circ \qquad I_r = -40^\circ \pm 7^\circ$$

The direction of the present magnetic field at the site is,

$$D = 355^\circ \qquad I = 71^\circ$$

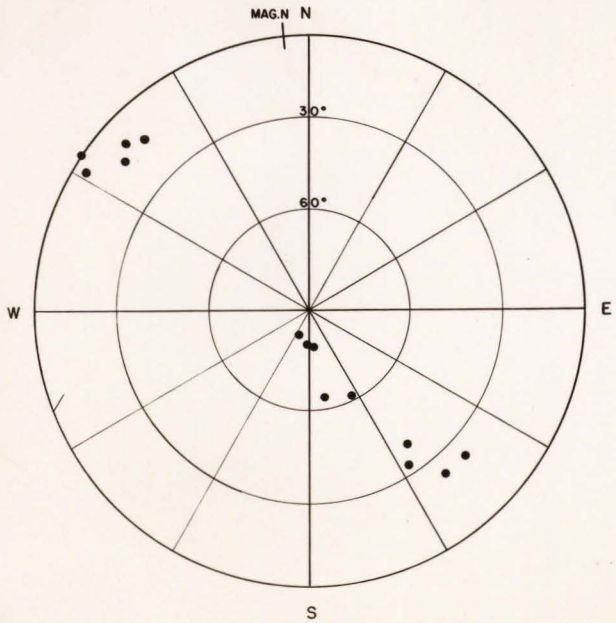

Fig. 23. Observed Direction of Maximum Susceptibility in Folded Rose Hill (Silurian) Formation

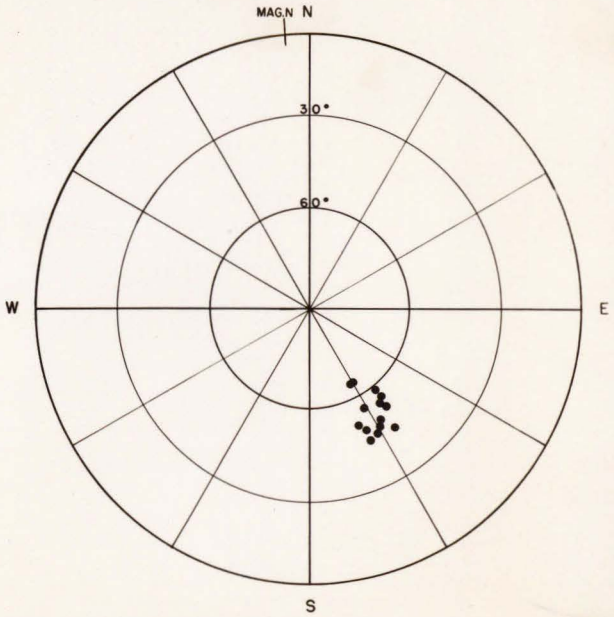

Fig. 24. Direction of Maximum Susceptibility in Folded Rose Hill Formation Corrected for Tilt of Beds

The direction of maximum susceptibility agrees well with the direction of polarization within statistical fluctuations. It is interesting to notice that the inclinations of polarizations are negative. According to the convention adopted this means that in this formation the particles were oriented with their south seeking poles dipping downwards instead of north seeking poles as in the case of Tertiary sediments. As was pointed out before, it is impossible to determine the sense of direction from susceptibility measurements. But the close agreement between the susceptibility and polarization measurements and the demonstration of the stability of the directions over long periods of time suggest that the polarizations in these samples probably represent the direction of the geomagnetic field during Silurian time.

Although it is not within the scope of this study to explain the reversal of polarity of these sediments, a few remarks are worth mentioning. The abnormal polarisation in this case may be explained to be due to several causes, viz., the reversal of polarity of the geomagnetic field in Silurian time, the movement of the crust since Silurian time, some stray magnetic fields and probably several others. It was also shown in Section 4 (p. 13) that if the magnetic anisotropy of the particles is much larger than their remanent moments, then under certain conditions of sedimentation the particles may orient with their moments 180° away from the direction of the existing magnetic field. Intensive work over wide regions is necessary before any definite conclusion can be made.

b. Silurian Time: Mays Gap, West Virginia.

To examine the regional consistency of the Rose Hill formation, samplings were made about one and a half miles north of Maysville, West Virginia. The geology of this region has been described in West Virginia Geological Survey Bull., (Vol. 14, p. 60, 1941). Samples cover a vertical distance of about 50 ft. The results of susceptibility measurements are given in Table VI (p. 91), the samples being arranged from the top of the exposure towards the bottom. Fig. 25 (p. 88) is a plot of the directions of maximum susceptibility. The average direction of maximum susceptibility is given by,

$$D_1 = 169^\circ \pm 8^\circ \qquad I_1 = 38^\circ \pm 6^\circ$$

The declination in this case is larger than that of the Rose Hill formation at Pinto, Maryland.

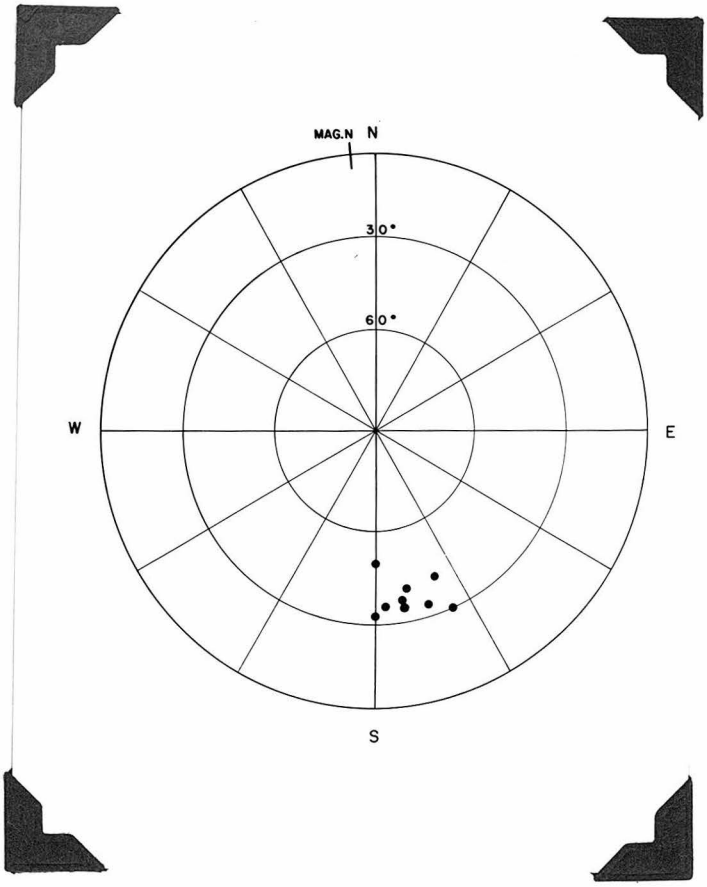

Fig. 25. Maximum Susceptibility
of Rose Hill Formation
Mays Gap, West Virginia

TABLE IV

Anisotropy of Silurian Sediments at Pinto, Maryland

Sample No.	$K_1 \times 10^6$	$K_2 \times 10^6$	$K_e \times 10^6$	$A_1\%$	$A_2\%$
30.	54.89	53.40	51.95	2.7	5.4
31.	53.53	52.25	50.58	2.4	5.5
32.	54.66	53.62	53.14	1.9	2.8
33.	53.52	52.71	51.95	1.5	2.9
34.	49.56	48.80	47.88	1.5	3.4
25.	54.52	54.19	53.32	0.6	2.2
26.	55.62	55.37	54.26	0.4	2.4
27.	56.77	56.40	55.31	0.7	2.6
28.	55.53	54.92	53.81	1.1	3.1
29.	56.74	56.25	55.31	0.9	2.5
35.	48.12	47.90	46.24	0.5	3.5
36.	58.61	57.73	56.38	1.5	3.8
37.	59.29	58.28	56.99	1.7	3.9
38.	60.75	59.46	58.61	2.1	3.5

TABLE V

Directions of Maximum Susceptibility in Silurian Sediments at Pinto, Maryland

Sample No.	Observed Directions		Corrected Directions		$\frac{M_1}{M_2}$
	D°	I°	D°	I°	
25.	304	0	140	55	2.6
26.	301	5	151	64	2.9
27.	309	15	142	52	3.2
28.	316	17	149	48	3.3
29.	312	12	147	50	2.7
30.	181	80	141	58	2.1
31.	170	63	157	51	1.5
32.	172	79	149	64	1.7
33.	153	61	144	45	1.4
34.	200	82	143	54	1.5
35.	144	39	157	46	1.6
36.	147	33	155	45	2.2
37.	140	24	155	49	1.8
38.	133	23	151	55	2.4

TABLE VI

Anisotropy and the Direction of the Maximum Susceptibility of Silurian Sediments at Mays Gap, West Virginia

Sample No.	$K_1 \times 10^6$	$K_2 \times 10^6$	$K_3 \times 10^6$	$A_1\%$	$A_2\%$	D_1°	I_1°
39.	70.06	67.84	66.96	3.2	4.4	163	34
40.	70.03	67.72	66.95	3.3	4.4	180	50
41.	81.50	79.06	77.83	3.0	4.5	175	36
42.	79.05	76.52	75.89	3.2	4.0	156	30
43.	53.20	52.40	51.55	1.5	3.1	171	38
44.	51.07	50.20	49.54	1.7	3.0	158	42
45.	51.32	50.48	49.66	1.6	3.2	169	41
46.	49.48	48.62	48.01	1.7	3.0	180	33
47.	54.05	53.15	52.28	1.7	3.3	171	35

19. Effect of Strong Magnetic Fields on Anisotropy:

According to the hypothesis put forward in Section 4 (p. 15), the remanent moment expected to develop in a sedimentary formation is a molecular phenomenon whereas the anisotropy is primarily a bulk property. To demonstrate this, three samples (Nos. 3, 4 and 5) were subjected to a strong magnetic field (about 200 gauss) along their X-, Y- and Z- axes respectively. Measurements of susceptibility and remanent moment were made both before and after the action of the magnetic field. No noticeable change either in the volume of anisotropy or in the direction of maximum susceptibility was observed in any case although each sample was found to be strongly polarized in the direction of this magnetic field. This shows that the anisotropy in sedimentary rocks is invariable with respect to any magnetic field subsequent to deposition and hence the inferences derived from susceptibility measurements may be regarded as more reliable than those based upon polarization measurements of the same samples.

20. Concluding Remarks:

The results of the present investigation should be regarded as of preliminary nature. The chief object of this work has been to develop and demonstrate the usefulness of a new method of attacking the problems of rock magnetism. The importance of this method can not be overemphasized. We have been able to show the superiority of this type of measurement over the existing method of polarisation

measurements in tracing the magnetic history of the Earth. Besides giving an indication of the direction of the geomagnetic field at the time of deposition, the measurement of anisotropy of a rock sample provides further a measure of the amount of magnetic material (taking part during the process of sedimentation) which is responsible for the total susceptibility of the material. The method, however, suffers from an inherent difficulty, namely, that the direction of maximum susceptibility does not give the polarity of the geomagnetic field and for this we have to depend on the measurements of polarisations. It is felt that in order to obtain a better and clearer picture of the history of rock magnetism (and indirectly the history of the geomagnetic field), it is necessary to combine the results of anisotropy and polarisation measurements. The anisotropy in most sedimentary rocks is a small quantity and the sensitivity of the present experimental method is still too low to cover a large number of sediments. The author hopes to improve upon the experimental technique and widen the limits of applicability of this proposed method of the study of rock magnetism.

APPENDIX I

Solution of the "Equation of Anisotropy":

We have shown in Sections 7 and 8 that the principal susceptibilities in a rock specimen are given by,

$$K^3 - K^2(a + b + c) + K\,(bc + ca + ab - f^2 - g^2 - h^2)$$
$$- (abc + 2fgh - af^2 - bg^2 - ch^2) = 0 \qquad (1)$$

Where the coefficients a, b, c, f, g, h are known from actual measurements.

The eq. (1) is a cubic equation having three roots K_1, K_2, K_3 which can be proved to be all real.* To put it in a standard form we set,

$$K = x + \tfrac{1}{3}(a + b + c) \qquad (2)$$

Then eq. (1) reduces to,

$$x^3 + qx - r = 0 \qquad (3)$$

Where,

$$q = \tfrac{1}{3}(a + b + c)^2 - (bc + ca + ab - f^2 - g^2 - h^2)$$

$$r = \tfrac{2}{27}(a + b + c)^3 - \tfrac{1}{3}(a + b + c)(bc + ca + ab - f^2 - g^2 - h^2)$$
$$+ (abc + 2fgh - af^2 - bg^2 - ch^2) \qquad (4)$$

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* See G. Salmon - An Analytical Geometry of Three Dimensions.
- Longmans, Green & Co. p. , 19 .

Eq. (3) can be solved by the use of trigonometrical tables. Thus let us put in eq. (3)

$$x = ny \tag{5}$$

then,

$$y^3 - \frac{q^2}{n^2} y - \frac{r}{n^3} = 0 \tag{6}$$

If α denotes any angle, we know,

$$\cos^3\alpha - \frac{3}{4}\cos\alpha - \frac{1}{4}\cos 3\alpha = 0 \tag{7}$$

Assuming $y = \cos\alpha$ in eq. (6) and then comparing eq. (5) and eq. (6), we got,

$$\frac{q}{n^2} = \frac{3}{4} \qquad \text{or} \qquad n = 2\sqrt{\frac{q}{3}} \tag{8}$$

and

$$\cos 3\alpha = \frac{4r}{n^3} = \frac{r}{2}\sqrt{\frac{27}{q^3}} \tag{9}$$

Cos α can be determined from eq. (9) by the use of trigonometrical tables. Thus the three roots of eq. (6) are,

$$\cos\alpha; \qquad \cos\left(\alpha + \frac{2\pi}{3}\right); \qquad \cos\left(\alpha + \frac{4\pi}{3}\right)$$

Hence, from eq. (2), eq. (5) and eq. (8), the three values of susceptibilities are given by,

$$K = \frac{1}{3}(a + b + c) + 2\sqrt{\frac{q}{3}}\cos\left(\alpha + \frac{s\pi}{3}\right) \tag{10}$$

where $s = 0, 2$ or 4.

Now, if l_1, l_2, l_3 be the direction cosines of one of the principal susceptibilities, we have shown in Section 8, eq. (5) that,

$$l_1 a_o + l_2 h + l_3 g = 0$$
$$l_1 h + l_2 b_o + l_3 f = 0 \qquad (11)$$
$$l_1 g + l_2 f + l_3 c_o = 0$$

where,

$$a_o = a-k, \quad b_o = b-k, \quad c_o = c-k$$

Eliminating l_2 between the first two equations and l_1 between the last two equations of eq. (11), we get,

$$l_1 = \frac{b_o g - fh}{h^2 - a_o b_o} l_3$$

$$l_2 = \frac{c_o h - fg}{b_o g - fh} l_3 \qquad (12)$$

If ρ be any radius vector x $l_1 = \rho$ etc., so that eq. (12) can be written as,

$$x = mz$$
$$y = nz \qquad (13)$$

Where,

$$m = \frac{b_o g - fh}{h^2 - a_o b_o}; \qquad n = \frac{c_o h - fg}{b_o g - fh} \qquad (14)$$

Eq. (13) represents a straight line in space passing through the origin of coordinates, the direction cosines of which are,

$$\frac{m}{\sqrt{1 + m^2 + n^2}} \qquad \frac{n}{\sqrt{1 + m^2 + n^2}} \qquad \frac{1}{\sqrt{1 + m^2 + n^2}} \qquad (15)$$

It is customary in the literature of terrestrial magnetism to express the directions of magnetic vector in terms of the Declination and Inclination. As shown in Fig. 26 (p.97), let OM be the direction of the maximum susceptibility in the rock sample. Then the direction cosines of OM are $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ and D and I are respectively

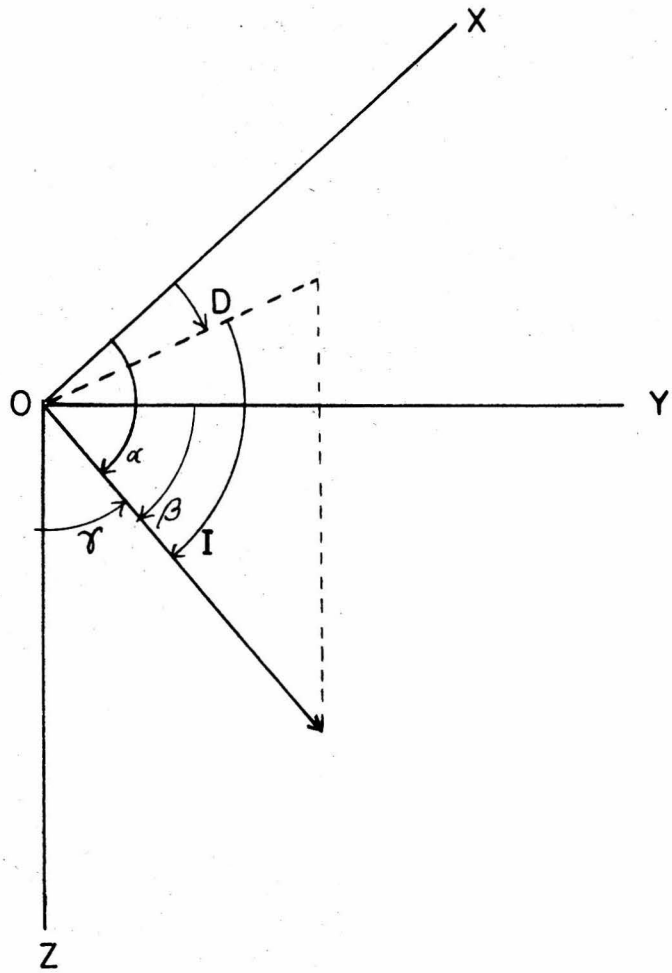

FIG. 26

the declination and the inclination of OM with respect to the axes of the cube sample.

It is easily seen from the Fig. 25 (p.97) that,

$$\cos\alpha \cos D = \cos I$$

$$\cos\beta \sin D = \cos I$$

$$\cos\gamma = \sin I$$

Therefore,

$$\tan D = \frac{\cos\alpha}{\cos\beta}$$

$$\sin I = \cos\gamma$$

Hence, using eq. (15), D and I are given by,

$$\tan D = \frac{m}{n}$$

$$\sin I = \frac{1}{\sqrt{1 + m^2 + n^2}} \qquad (16)$$

where m and n are known from eq. (14).

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