INVESTIGATION OF TURBULENCE IN CIRCULAR TUBES BY MEANS OF A HOT-WIRE ANEMOMETER

Thesis by

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In Partial Fulfillment of the Requirements
for the Degree of

Master of Science

California Institute of Technology
Pasadena, California

1934

INTRODUCTION

It is the purpose of this paper to present the results of some investigations of turbulent flow in straight circular tubes, conducted at the suggestion of Dr. Th. von Kármán, and to give a detailed description of the method of using the hot-wire anemometer to measure fluctuations of the air velocity in a direction parallel to the mean flow.

In 1930, Dr. von Kármán published a theory for fully developed turbulent flow in which he showed, theoretically, that there is a constant correlation between u' and v', the fluctuations in velocity parallel and perpendicular, respectively, to the mean flow. This is expressed by the relation

$$\frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}} = \text{A constant}$$

Investigations of Wattendorf and Kuethe (Ref. 3) showed that in a deep channel, the flow in which was essentially two dimensional, the distribution of u'^2 across the channel was a straight line except at the regions near the wall or at the center of the channel. This is shown in Fig. 14. It is known that the term $\overline{u'v'}$ also has a straight line distribution, because it is proportional to the shearing stress. Therefore the results of Wattendorf and Kuethe show that $\overline{u'v'}/\left(\sqrt{\overline{u'^2}}\right)^2$ is given as a straight line across the channel. This result might be expected if there was correlation between u' and v'.

This fact does not prove that Kármán's correlation theory is true. It does, however, indicate that it is probable the correlation exists. Absolute proof will have to wait upon measurements of v' which are now in progress at the Guggenheim Aeronautics Laboratory of the California Institute of Technology, referred to hereinafter as GALCIT.

The work outlined in this paper is an attempt to extend the measurement of u' to a straight circular tube, the flow in which is fully developed turbulence. It will be seen that the flow in a tube is three dimensional. A glance at Figure 13 tells the story. The distribution of u'^2 is seen to approach a straight line, but not as closely as did the distribution in the channel.

The theory involved in the recording of turbulent fluctuations in an air stream by means of the hot-wire anemometer has been fully covered in references 1, 2, 3, and 4, and will not be repeated here.

SUMMARY OF THE EXPERIMENTS

The first measurements were taken on a straight tube 38 centimeters in diameter at a point approximately 875 centimeters from the entrance with an air speed of about 3000 centimeters per second at the center of the tube. The entrance was sharp. The tunnel is the same as that described in reference 5. The distribution of u'^2 in this tunnel is shown in figure 11. The points shown are averages over a number of runs at different times. The straight line distribution is not evident. The length diameter ratio of this tube was only 23, so it was suspected that the flow was not fully developed.

Therefore an extension was put on this original tunnel. No other alterations were made. The working section was now about 1180 centimeters from the entrance, giving a length-diameter ratio of 31. The results in this tube, shown in figure 12, are very little nearer the looked-for straight line distribution than were those for the original tunnel. There were still two chances to get the desired result. First, it was possible that we still did not have fully developed turbulent flow. Kármán's universal velocity distribution curve was plotted, and the experimental points also plotted. The result is shown in figure 16. The fair agreement of the experimental points with the theoretical curve indicated that we had fully developed flow. Comparison with figure 15 shows the improvement in developement of the flow caused by the extension of the tunnel length.

The hot-wire set was designed to amplify equally any frequencies between 50 and 1000 cycles per second. It was felt that in the large tube, the frwquency might have been so low that many of the oscillations were unamplified by the set. Hence a determination of the effective frequency of oscillation was made, and it was found that the major oscillations had a frequency below 50 cycles per second. Since the frequency of turbulent oscillations is roughly proportional to the diameter of the tube, the solution was to go to a smaller sized tube. This also permitted the use of a much larger length diameter ratio in the same confines of space.

Therefore the third tunnel was constructed. It had a diameter, inside, of 11.5 centimeters, and measurements were taken at a section 600 centimeters from the sharp entrance, giving a length-diameter ratio of 52. Measurements were taken at about the same speed as in the first two tunnels, and the result is plotted in figure 13. This shows much nearer the straight line distribution of u'^2 than either of the two previous tunnels.

Unfortunately, a peculiar resonance effect set into this tunnel, and, coupled with lack of time, prevented a more comprehensive series of tests on it.

APPARATUS

The most important unit of the apparatus was, of course, the hot-wire anemometer. This has been fully described in references 3 and 4. The detailed use of the outfit will be explained in detail in a later section of this paper.

Three wind tunnels were used, all of them of circular cross-section. The first was the same tunnel as used and fully described in reference 5. The second tunnel was the same as the first with an extension of approximately 300 centimeters attached to the entrance end. The third tunnel was made from a drawn steel tube with a length of 600 centimeters from measuring section to the entrance, and 11.5 centimeters inside diameter. This third tunnel is shown in figure 23.

The velocities were measured by means of a total pressure orifice set in the wind stream and a static pressure orifice flush with the wall at the same distance from the entrance of the tube. These two orificies were connected differentially to a micro-manometer, shown in figure 25.

The power plant was the same on all three tunnels used. It consisted of a two blade wooden propeller designed for the purpose and run by a five horse power induction motor. Speed control was obtained by controlling simultaneously the frequency and voltage of the input to the motor. The electrical system was not entirely independent of other power outlets in the laboratory, and so it was rather difficult to keep a constant speed in the tunnels.

USE OF THE HOT-WIRE ANEMOMETER

The GALCIT hot-wire anemometer was designed using a compensated circuit developed by Dryden and Kuethe at the Bureau of Standards. (See Ref. 1) A complete description of the theory underlying the use of the hot wire to measure turbulence is given in Ref. 4.

The notation which will be used in this paper is as follows:

- u' = fluctuations in direction of mean flow.
- U_m = maximum velocity at the measurement section.
- i = current thru the wire.
- R = resistance of the wire. (heated with current i)
- R_o = resistance of the wire when cold.
- α = coefficient if resistivity of the wire.
- m = mass of the wire.
- s = specific heat of the wire
- M = the time constant of the wire.

Other temms will be defined as used.

We will now give the general procedure to be followed in finding the distribution of u' across a section. The steps will be numbered, and the detailed method of each operation will be discussed later in this paper.

PROCEDURE IN MAKING A u' TRAVERSE:

1. Make a velocity distribution survey across the section. Also calibrate the speed at the center of the section against static head at the wall of the tube.

2. Find R_e by passing a very small current thru the wire with the wind turned on.

3. Find the potentiometer battery voltage by comparison with the standard cell. From this, compute the setting of the Leeds and Northrup resistance box which will result in the desired current thru the wire when the potentiometer circuit is balanced.

4. Calibrate the amplifier by impressing a known voltage and frequency across the hot-wire terminals of the set.

5. Make a static calibration of the wire using the same heating current as will be used in the traverse. This operation consists of getting the wire resistance at various speeds. Use is made of step 1 to get the velocities from static head readings.

6. Make the traverse keeping the tunnel speed the same as it was when the velocity distribution was made.

a) Set the wire at required distance from the wall.

b) Check the heating current to safeguard the wire.

c) Obtain bridge balance, and at the same time keep the heating current at the desired value by using the milliammeter in the hot wire circuit.

d) After balance is obtained, check the heating current by use of the potentiometer circuit.

e) Calculate M, the time constant.

f) Set compensation resistance box to (R_c-100) where R_c is the compensation resistance.

g) Read i^2, and note the tap and hum.

h) Recheck the current thru the wire.

i) Change the wire position and repeat.

7. Check a few points on the static calibration. Also check the amplifier calibration, and R_o.

DETAILED DIRECTIONS

1. Velocity Traverse.

The procedure used, as well as the calculations and corrections, for finding the average velocity at a given point in a wind stream has been given in detail in Ref. 5. In hot-wire measurements, there are two types of traverses necessary. First is the velocity distribution. The tunnel is run at the same speed as will be used in the turbulence measurements. Second is the calibration of maximum velocity against wall pressure. A certain static orifice in the wall is chosen and used for all velocity surveys. By keeping the same head on this orifice during all runs, we are sure that our speed is always constant. It has been found that corrections of the fluid density for atmospheric conditions is not necessary in connection with the measurement of u', because the hot-wire set is not accurate enuf to warrant such pains with the velocity measurements.

2. Finding the Cold Resistance of the Wire. R_o

On the front left leg of the hot-wire set, just above the lowest shelf, there is a double throw four pole switch. This should be placed in the up position to connect the batteries to the various circuit. Right near this switch is a little toggle switch connected to one of the battery leads. When pushed back, this switch introduces resistance into the battery leads for the heating current so that a very small current is passed thru the wire, just enuf to allow the bridge circuit to give an accurate reading, but not enuf to appreciably change the resistance of the hot-wire by heating if the wind is turned on. This switch should be throw back for finding R_o.

Figure 17 shows that the table top has two panels. The top panel in the photograph is away from the operator, the lower panel near to the operator. First, the hot wire leads are connected to the terminals labeled "hot wire" in the figure 17. The far double throw double knife switch, No. 2, should be in the neutral position. Its companion switch, No. 3, should be thrown to the right. The first toggle switch from the left on the far panel should be thrown to the right. This sneds current into the wire. The galvanometer is connected into the circuit by closing switch No. 1 to the left, as shown in Fig. 17. A balance is obtained by adjusting the Decade Resistance Box, in the lower left corner of the far panel, until closing the galvanometer switch causes no deflection of the milli-voltmeter used as a galvanometer, shown in the lower left corner of Figure 17.

The next step is to find the resistance of the leads. This is done by short circuiting the hot wire itself, and cutting out the resistance in the heating current line by moving the toggle switch at the battery connection forward, and getting another bridge balance in exactly the same way as described above.

The bridge has been calibrated, and its factor has been found to be 9.71. This factor is used as follows. When the reading of the Decade Resistance Box is divided by the bridge factor, the true resistance in ohms of the circuit external to the points A-A' is given. Then, to find the cold resistance, R_o, we subtract from the first resistance obtained in this step the resistance of the leads.

3. Finding Potentiometer Battery Voltage and Setting of Leeds and Northrup Resistance Box.

The general arrangement of the switches for this operation is shown in Fig 19, with the circuit in Fig 20. Switches 2 and 3 should be in the neutral position. All toggle switches should be in the off position. The switch 1 is then thrown to the right. Balance is obtained by means of the L. and N. Resistance box (four dial). THE SWITCH No. 4 SHOULD BE DEPRESSED ONLY FOR A VERY SHORT TIME TO CHECK THE GALVANOMETER DEFLECTION. The switch No. 1 is kept closed as shown, and the switch 4 is used only to close the circuit to the galvanometer. The circuit in effect is a normal potentiometer circuit where the voltage of the standard cell is made to balance the drop across 1000 ohms plus the resistance box caused by the potentiometer battery, whose voltage is about 4volts, in series with 4000 ohms.

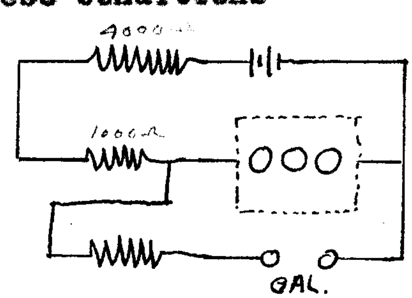
From the circuit shown, we see that

$$\text{Pot. Batt. Voltage} = 1.01865 \times \frac{4000 + \text{R.B.} + 1000}{1000 + \text{R.B.}}$$

where 1.01865 is the voltage of the standard cell and R.B. is the setting of the resistance box for balancce.

Under normal operating conditions, using the 4 volts potentiometer battery, balance will be obtained at a resistance box setting of appmoximately 400.

Now let us look at the blue print of the entire circuit, Figure 28. First, we will identify the switches. Switch 1 is shown just to the left of the input transformer. Switch 2 is shown just above the "input" terminals. Switch 3 is shown just below the bridge circuit, and switch 4 just above the bridge circuit. When using the potentiometer circuit to measure the wire current, we have switch 4 thrown to the left, switch 2 open, switch 3 to the left, and balance by closing the galvanometer switch 1 to the right. The circuit in effect under these conditions is shown at the right. Balancing the voltage drops in the two cricuits, we see that



(see next page)

we have

$$\frac{1 \times R}{5000 + R} = \frac{i}{V}$$

where R = setting of the resistance box.

i = current thru the 1 ohm resistance.

V = voltage of the potentiometer battery.

It will be noted from Fig. 28 that the 1 ohms resistance is one leg of the Wheatstone Bridge, and that the current thru it is the same as the current thru the hot-wire. Hence for a given current thru the wire, we have,

$$\text{Res. Box. Setting} = \frac{5000 \times i}{V - i}$$

where i is now the current desired thru the wire, and V remains the potentiometer battery voltage.

4. Calibration of the Amplifier

A complete means of calibrating the amplifier has been explained in Reference 4. This should be followed only when a major change has been made in the amplifier, such as the installation of new tubes or a change in the circuit. Otherwise, for normal operation, only the amplification level will change from day to day. The characteristics will remain the same. Taking advantage of this constant characteristic, the following method has been developed for finding the amplification level.

The gain of the compensated amplifier depends upon the time constant M in an inverse proportion. After a series of very careful calibrations, it was found that with M = 12.9, the output meter read 20.7 when a frequency of 200 cycles was impressed across the hot wire terminals with the bridge resistance anywhere in the normal operating range and with the output of the oscillator at 5 volts, R_c, the compensation resistance, set at 1000 ohms, with 9900 ohms in series with the oscillator, and a shunt of 0.856 ohms across the hot wire terminals.

Hence all we have to do to calibrate the amplifier level is to impose these same conditions. Our amplification will then be given by the expression:

$$\text{Amplification} = \sqrt{\frac{i^2}{20.7}} \times \frac{12.9}{M}$$

where i^2 is the reading of the output meter with the attenuator dial set at station 1, and M is the time constant which is calculated by the formula:

$$M = \frac{4.2\ m\ s\ (R - R_o)}{i^2\ R_o^2\ \alpha}$$

The terms of the above expression were defined on page 6.

5. Static Calibration of Wire

For this test, we put the wire in the center of the tunnel where we know the relation between the velocity and the static pressure at the wall. Taking about ten points on our speed range, we find the resistance of the wire for each speed. The resistance is found by the bridge circuit in the same manner as was done in finding the cold resistance. But here we have the added complication of keeping the current thru the wire constant,-usually at 0.16 amperes. This is done as follows:

The Leeds and Northrup Resistance Box is set at the value determined in step 3. Both the hot wire and the galvanometer are connected to their proper terminals. The battery switch is throw on (up position) with the resistance cut out (forward position of toggle switch). Set the bridge resistance box (Decade) at about 60 ohms. Close toggle switch below current adjusting rheostats. This completes the circuit thru the wire. Leaving switch 2 open, close switch 3 to the right as shown in Fig. 17. Throwing switch 4 to the left completes the circuit. We next obtain a balance by varying the bridge resistance box until the galvanometer shows no deflection when switch 1 is thrown to the left. While the bridge resistance is being adjusted, the current should be kept at the desired value by use of the rheostats and the milliameter. Then, after the bridge has been balanced, throw switch 1 to the right. If there is no deflection, the current thru the wire is exactly at the value for which the L. and N. resistance box has been set. If there is a deflection, the current should be varied with the rheostats until there is none. Then the bridge must be re-checked, and if an adjustment was necessary to the bridge, the current must be again checked. This playing back and forth is necessitated by the fact that changing the bridge resistace varies the current thru the wire, and changing the current thru the wire changes the resistance. When a perfect balance is obtained for both resistance and current, the reading of the bridge resistance is noted.

The reading of the bridge resistance box, when divided by 9.71, the bridge factor, gives the total resistance of the wire plus the leads. Subtracting the resistance of the leads, found in step 2. This gives us R. Subtracting R_o gives us, obviously, $R - R_o$. We then plot the points exactly as shown in Figure 7. If everything has gone correctly, the point will all fall on a straight line. We call the slope of this line C, and the intercept for zero velocity is called K. The use of these terms will appear later.

6. The Traverse

Steps a,b,c, and d have been fully covered above. The time constant is calculated from the formula given on page 13. When the proper values for the platimum wire used (diameter = 0.0127 mm) are substituted in, we have

$$M = \frac{1.94\ (R - R_o)}{i^2\ R_o} \times 10^{-4}$$

f). The value of the compensation resistance is given by the formula

$$R_c = \frac{1.87}{M}$$

In making the traverse, we get our bridge balance and calculate M. From M we find R_c, and then set the compensation resistance box, located on the second shelf of the hot wire table, to a value 100 ohms less than R_c.

g) We are now ready to read i^2 on the meter furthest to the right on the panel nearest the operator. To do this, after setting the compensation restance, both switches 2 and 3 are closed to the left. The attenuator control is set at such a value that the output meter can be read easily. (Note: The amplifier is turned on by throwing the two toggle switches below the D.C.

voltmeter on the far panel and below the 0-150 A.C. voltmeter on the near panel to the right. The far meter should read ∅ 5 volts, and the near one should be adjusted to 110 volts.)

METHOD OF MAKING CALCULATIONS

At the conclusion of the run, we have, for various positions of the wire, R, $R-R_o$, M, R_c, Tap (attenuator), and the output reading. First, we convert the output reading from i^2 to volts. The output terminals have a shunt across them to get a reading of i^2. The value of the resistance of this shunt is such that

$$\text{volts} = 0.621 \sqrt{i^2}$$

We then find the net output volts. This is the voltage output multiplied by the attenuation factor, known as a "tap factor". The factors for the various attenuator settings are:

Setting	
1:	1.00
2:	1.90
3:	3.76
4:	7.64
5:	14.90
6:	29.0
7:	56.2
8:	101.
9:	193

Next we find dE, the change in voltage drop across the wire. This equals the net output volts divided by the amplification,-the expression for which was given on page 13. Knowing the amplification level from the calibration, we find that dE is

$$dE = \frac{\text{net volts x M}}{\text{a constant}}$$

We will now derive the expression for du/u from a consideration of the bridge circuit.

The calibration equation of the wire is

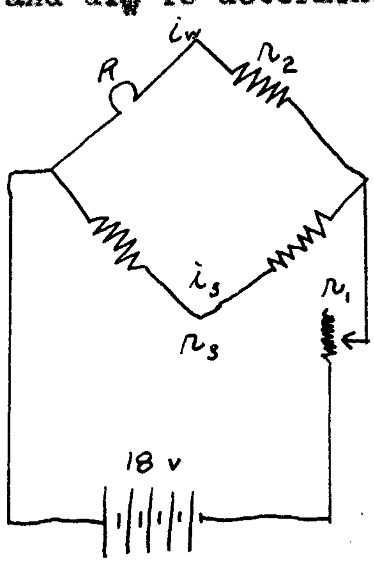
$$\frac{i_w^2 R R_0 \alpha}{R - R_0} = K + C\sqrt{u} \qquad \text{------- (1)}$$

where K is the intercept of the wire calibration curve and C is the slope. (See Figure 7) Here i_w is the current thru the wire.

Differentiating, permitting i_w and R to vary, we have

$$\frac{2 i_w R R_0 \alpha}{R - R_0}\, di_w - \frac{i_w^2 R_0 \alpha}{(R - R_0)^2}\, dR = \frac{C\, du}{2\sqrt{u}} \qquad \text{------ (2)}$$

We know dE from the output measurements as shown above, and di_w is determined by the following analysis.



The figure on the left represents schematically the bridge circuit.

Let i_w = current flowing thru hot wire branch of the bridge.

i_3 = current in lower branch.

R = resistance of the wire.

r_2 = all other resistance in the wire branch

r_3 = total resistance, lower branch.

Calling i the current flowing thru the 18 volt battery,

$$18 = i\left[r_1 + \frac{r_3(R + r_2)}{r_3 + (R + r_2)}\right] \qquad \text{--------(3)}$$

or

$$\frac{r_3}{R + r_2} = \frac{i_w}{i_3}$$

from which

$$i_3 = \frac{i_w\,(R + r_2)}{r_3}$$

But

$$i = i_w + i_3 = i_w\left[\frac{R + r_2 + r_3}{r_3}\right] \qquad \text{----- (4)}$$

Differentiating,

$$di = i_w\frac{dR}{r_3} + \left[\frac{r_3 + R + r_2}{r_3}\right]di_w \qquad \text{----- (5)}$$

Differentiating the ofiginal equation at the top of this page,

$$-\frac{18}{i^2}\,di = \frac{-(r_3R + r_3r_2)dR + (r_3 + R + r_2)r_3\,dR}{(r_3 + R + r_2)^2}$$

$$di = -\frac{i^2}{18}\,\frac{r_3^{\,2}\,dR}{(r_3 + R + r_2)^2} \qquad \text{----- (6)}$$

Substituting i_w for i according to (4), we have

$$di = -\frac{r_3^{\,2}}{18(r_3 + R + r_2)^2}\,i_w^{\,2}\,dR\,\frac{(r_3 + R + r_2)^2}{r_3^{\,2}} = -\frac{i_w^{\,2}}{18}dR \qquad \text{----- (7)}$$

Equating (7) and (5), we have

$$\frac{r_3 + r_2 + R}{r_3}\, di_w + \frac{i_w}{r_3}\, dR = -\frac{i_w^{\,2}}{18}\, dR$$

$$\frac{R + r_3 + r_2}{r_3}\, di_w = -\left[\frac{i_w}{r_3} + \frac{i_w^2}{18}\right] dR$$

$$= -\left[\frac{18 + r_3 i_w}{18 r_3}\right] i_w dR$$

from which

$$di_w = -\frac{18 + r_3 i_w}{18(r_3 + r_2 + R} i_w dR \quad \text{------(8)}$$

The final expression for dE in terms of dR is

$$dE = i_w dR + R di_w$$

$$= \left[1 + \frac{R}{18}\left(\frac{18 + r_3 i_w}{r_3 + r_2 + R}\right)\right] i_w dR \quad \text{------- (9)}$$

Let $$B = \frac{18 + r_3 i_w}{18(r_3 + r_2 + R)} \quad \text{-------------------------- (10)}$$

so that $$di_w = - B i_w dR \quad \text{-------------------------- (11)}$$

and $$dE = (1 - RB) i_w dR \quad \text{-------------------------- (12)}$$

Going back to equation (2) and substituting for di_w

$$\frac{-2 i_w R R_0 \alpha}{R - R_0} B i_w dR - \frac{i_w R_0^2 \alpha}{(R - R_0)^2} i_w dR = \frac{c\sqrt{u}}{2}\frac{du}{u}$$

$$\frac{du}{u} = -\frac{2}{c\sqrt{u}}\left[\frac{2 i_w R R_0 \alpha}{R - R_0} B + \frac{i_w R_0^2 \alpha}{(R - R_0)^2}\right]\frac{dE}{1 - RB} \quad \text{---- (13)}$$

The formula (13) gives du/u. This, multiplied by the velocity at the point considered (which was found in the velocity traverse) gives us du, the root mean square of the velocity fluctuations in a direction parallel to the mean flow.

For ease in making the calculations, remembering that $$\frac{i^2 RR_o\alpha}{R-R_o} = K + C\sqrt{u}$$

we transfrom the equation (13) to the form

$$\frac{du}{u} = \frac{2(K+C\sqrt{u})}{C\sqrt{u}}\left[\frac{K+C\sqrt{u}}{i^3R^2\alpha} + \frac{2B}{i_w}\right]\frac{dE}{1-RB}$$

For any point on the traverse, we know u from the velocity traverse. Knowing the values of K and C from the static calibration of the wire, we get the factor (K C u) which appears in the calculations.

The term B has been defined on page 19, formula (10).

SAMPLE HOT-WIRE CALCULATIONS

Let us assume we have made the velocity traverse and the static calibration of the wire. These two are perfectly stzaightforward and need no elaboration. We have found

$R_o = 3.23$ ohms Use $i = 0.160$ amperes

$R_{1ds.} = 1.55$ ohms

$\alpha = 0.0037$ (given for platimum wire)

We will use the calibration curves for the 5" tube, Figure 6. We have found, ir will be supposed, from the static calibration of the wire that

$K = 325 \times 10^{-6}$

$C = 9.5 \times 10^{-6}$

We compute an expression for the time constant,

$$M = \frac{1.94\,(R-R_o)}{i^2 R_o} = 2.35 \times 10^{-3}(R-R_o)$$

and R_c, the total compensation resistance,

$$R_c = \frac{1.87}{M} = \frac{0.796}{(R-R_o)} \times 10^3$$

When we put 5 volts at 200 cycles from the oscillator in series with 9900 hhms, across the wire terminals, with a shunt across the terminals of 0.856 ohms, and with the compensation resistance box zet at 1000 hhms, and the inductance shorted out, we found an output reading of 17.0 with the attenuator on tap 1. The amplification will therefore be

$$\text{Amplification} = \frac{12.9}{M}\sqrt{\frac{17.0}{20.7}} = \frac{11.68}{M}$$

~~= 11.68 / 20.7~~

Let us assume our wire is at a point 3.0 centimeters from the wall. Our bridge balance gives us a bridge reading of 65.0.

Then, $R + R_{1ds} = \dfrac{65.0}{9.71} = 6.69$ ohms.

hence $R = 6.69 - 1.55 = 5.14$ ohms.

and $R - R_o = 5.14 - 3.23 = 1.91$ ohms

$$M = 2.35 \times 10^{-3}(R-R_o) =$$
$$= 2.35 \times 10^{-3}(1.91) = 4.49 \times 10^{-3}$$

$$R_c = 0.796 \div (R-R_o) \times 10^3$$
$$= 0.796 \div (1.91) \times 10^3 = 420$$

Hence the compensation resistance box is set at <u>320</u> ohms, and the switches are thrown to the output. The tap used is tap 3, whose factor is known to be 3.76. The i^2 reading is found to be 84.

Converting the reading to volts,

$$\text{Output volts } 0.621\sqrt{i^2}$$
$$= 0.621\sqrt{84} = 5.69$$

The Net Output volts are found by multiplying by the attenuation factor.

$$\text{Net Output volts} = 5.69 \times 3.76 = 21.4 \text{ volts}$$

The term dE is found by dividing the net output volts by the amplification.

$$dE = 21.4 \times M \div 11.68$$
$$= 21.4 \times 4.49 \times 10^{-3} \div 11.68 = 8.30 \times 10^{-3}$$

Next we find u for our position. From figure 3 it is found to be 2975 cm. per second. The square root og this is 54.5 .

From our static calibration curve for the wire we found that $C = 9.5 \times 10^{-6}$. Then

$$C\sqrt{u} = 518 \times 10^{-6}$$

and, since $K = 325 \times 10^{-6}$, we have that

$$K + C\sqrt{u} = 843 \times 10^{-6}$$

We will use the formula (see page 20)

$$\frac{du}{u} = \left(\frac{2(K+C\sqrt{u})}{C\sqrt{u}}\right)\left\{\frac{K+C\sqrt{u}}{i^3R^2\alpha} + \frac{2B}{i_w}\right\}\frac{dE}{1-RB}$$

where we will call

$$\frac{2(K+C\sqrt{u})}{C\sqrt{u}} = A$$

and

$$\frac{K + C\sqrt{u}}{i^3R^2} = C$$

and

$$\frac{2B}{i_w} = D$$

Then

$$\frac{du}{u} = A(C + D)\frac{dE}{1 - RB}$$

Getting back to our calculations,

$$A = 2 \times 843 \div 518 = 3.25$$

$$C = 843 \times 10^{-6} \div (.16)^3 \div (5.14)^2 \div .0037$$

$$= 2.02$$

Referring now to Formula (10) on page 19, for the GALCIT bridge circuit, the constants are as follows:-

$r_g = 10.8$

$r_2 = 1.57 + (\text{resistance of the bridge resistance box.})$

Hence $B = \dfrac{18 + 10.8(i)}{18(10.8 + 1.57 + R_{box} + R)}$

$= \dfrac{18 + (10.8 \times .16)}{18(10.8 + 1.57 + 65.0 + 5.14)}$

$= 0.01325$

and $D = \dfrac{2 \times 0.01325}{0.16}$

$= 0.1655$

Then, combining,

$A(C + D) = 3.25(2.02 + .1655)$

$= 7.10$

We have $\dfrac{dE}{1-RB} = \dfrac{8.30 \times 10^{-3}}{1.0 - (5.14 \times .01325)}$

$= 8.80 \times 10^{-3}$

Therefore, finally,

$\dfrac{du}{u} = A(C+D)\dfrac{dE}{1-RB}$

$= 7.10 \times 8.80 \times 10^{-3}$

$= 0.0625$

Then, from Figure 3, we find that at our point 3 cm. from the wall,

$\dfrac{u}{u_{max}} = 0.983$

Hence $\frac{du}{u_{max}} = 6.25\% \times 0.983 = 6.15\%$

and $\left(\frac{du}{u_{max}}\right)^2 = .0615 \times .0615 = 0.00379$

Now this term (du) is nothing other than $\left(\sqrt{\overline{u'^2}}\right)$.

Hence $\frac{\overline{u'^2}}{u^2_{max}} = 0.00379$

The author wished to express his appreciation to Dr. Th. von Kármán for his guidance in the conduct of this research, to Drs. F.L. Wattendorf and A.M. Kuethe for their help in straightening out the intricacies of the hot-wire apparatus, to Dr. A.L. Klein for the design of much of the apparatus, and to Mr. K. Kitusda for providing the third hand necessary in the construction of the hot wire itself.

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2. W.C. Mock, Jr. and H.L. Dryden,-"Improved Apparatus For the Measurement of Fluctuations of Air Speed in Turbulent Flow", N.A.C.A. Technical Report No. 448, 1932.

3. F.L. Wattendorf and A.M. Kuethe,-"Investigations of Turbulent Flow by Means of the Hot-Wire Anemometer", PHYSICS, June, 1934.

4. F.L. Wattendorf,-"Preliminary Report on the GALCIT Hot-Wire Anemometer", Califnrnia Institute of Technology, 1934.

5. M.H. Nagashi,-"A Study of Fully Developed Turbulent Flow at a Very High Reynolds Number", Master's Thesis, Califnrnia Institute of Technology, 1931.

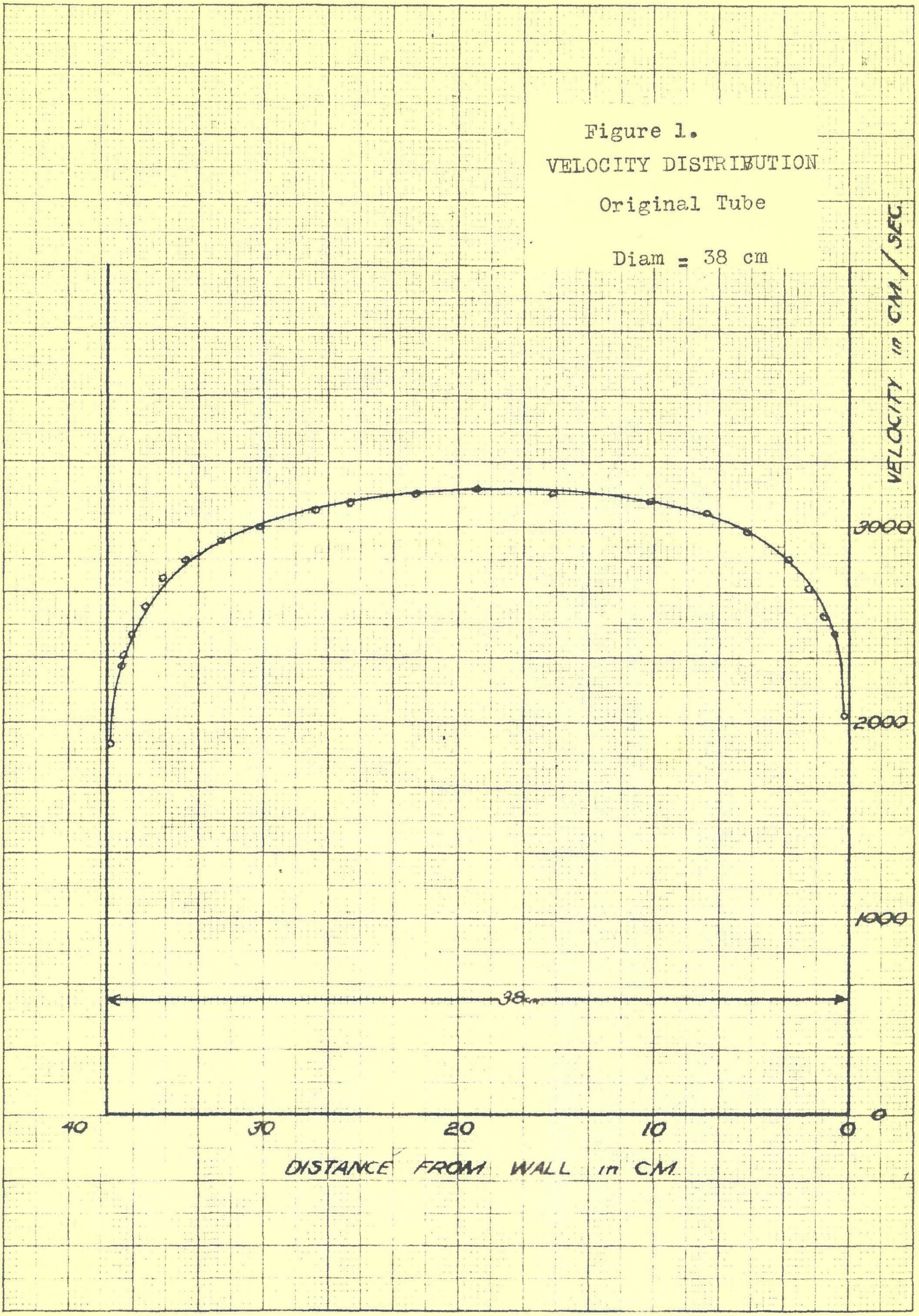


Figure 1.
VELOCITY DISTRIBUTION
Original Tube

Diam = 38 cm

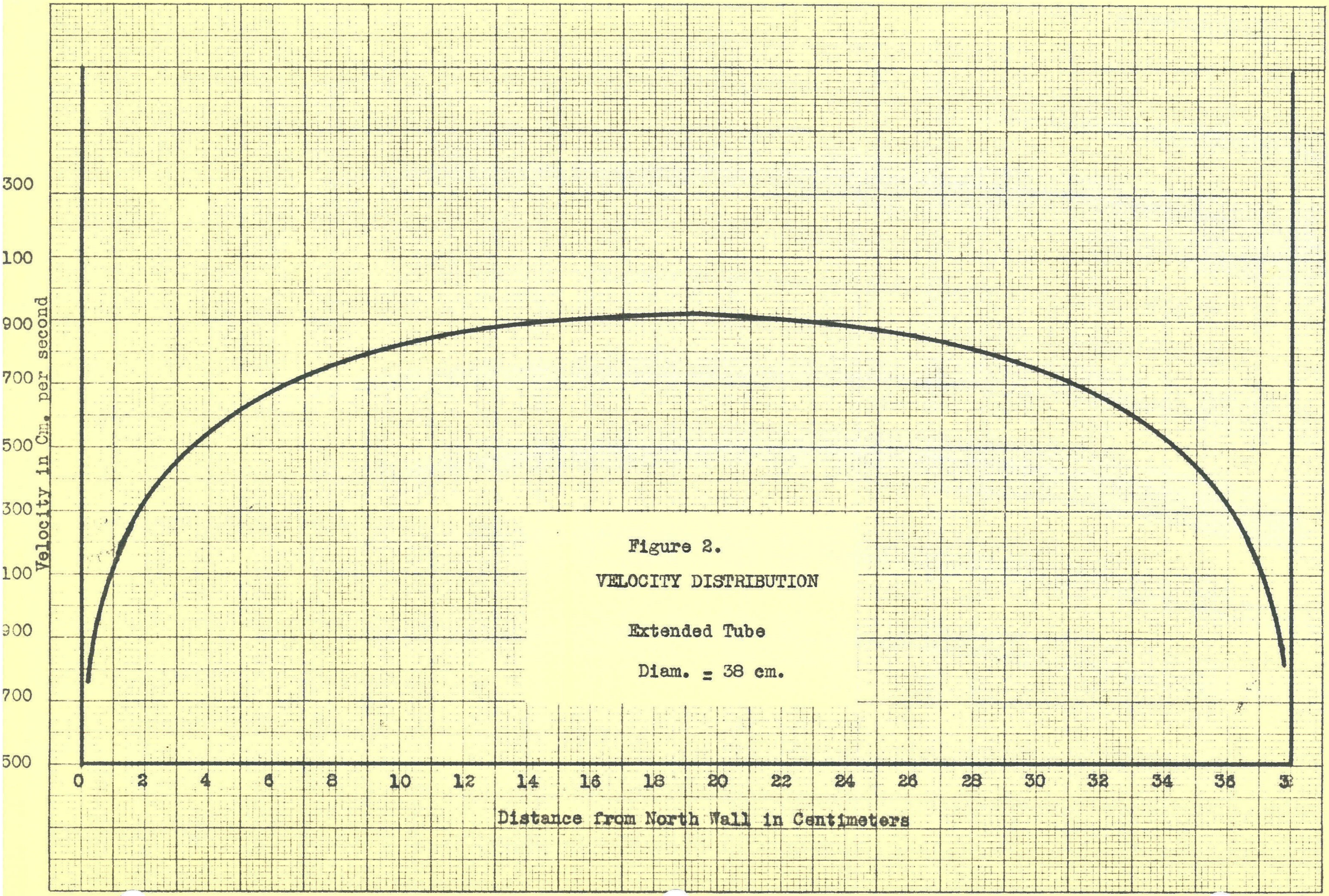


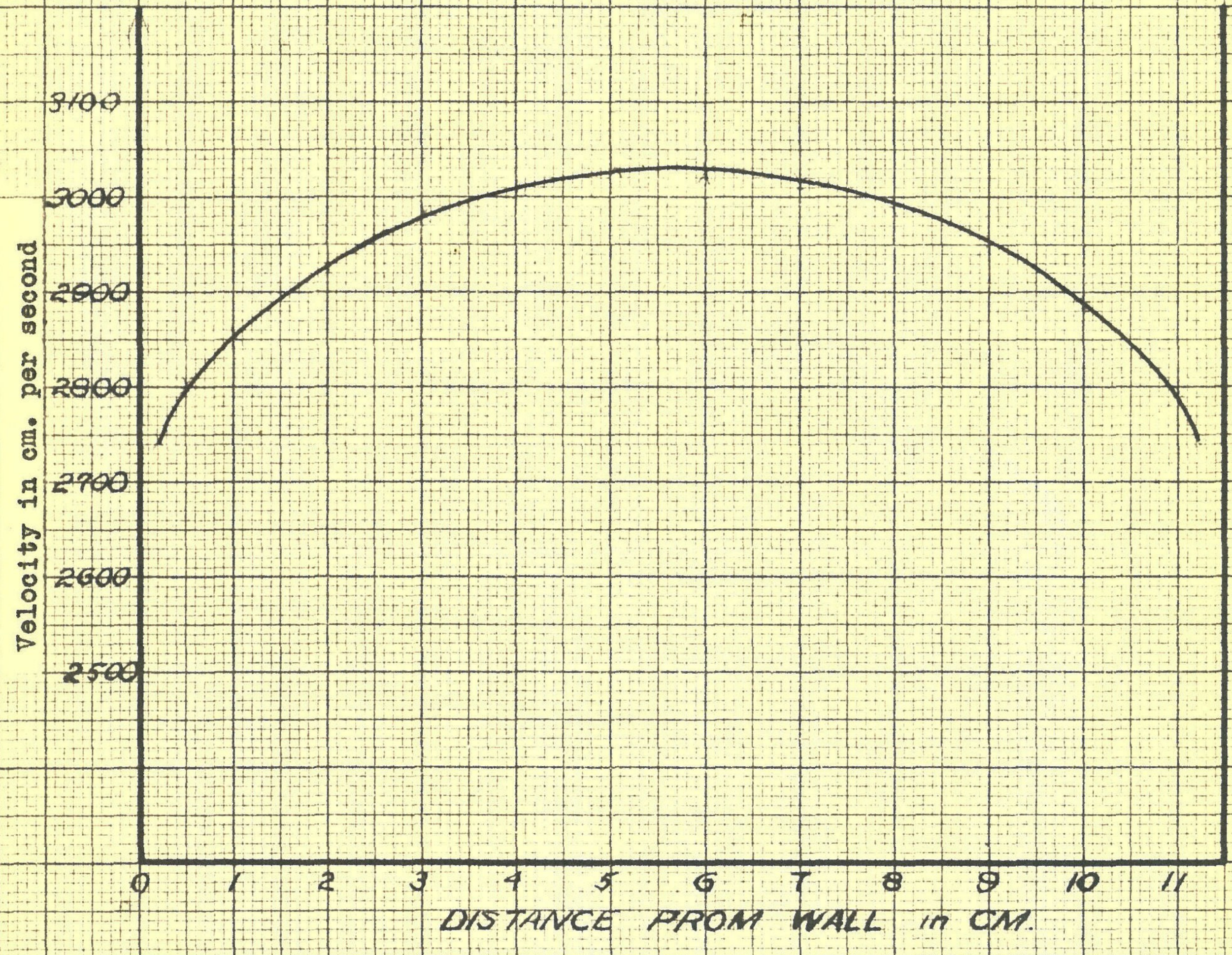
Figure 2.
VELOCITY DISTRIBUTION

Extended Tube

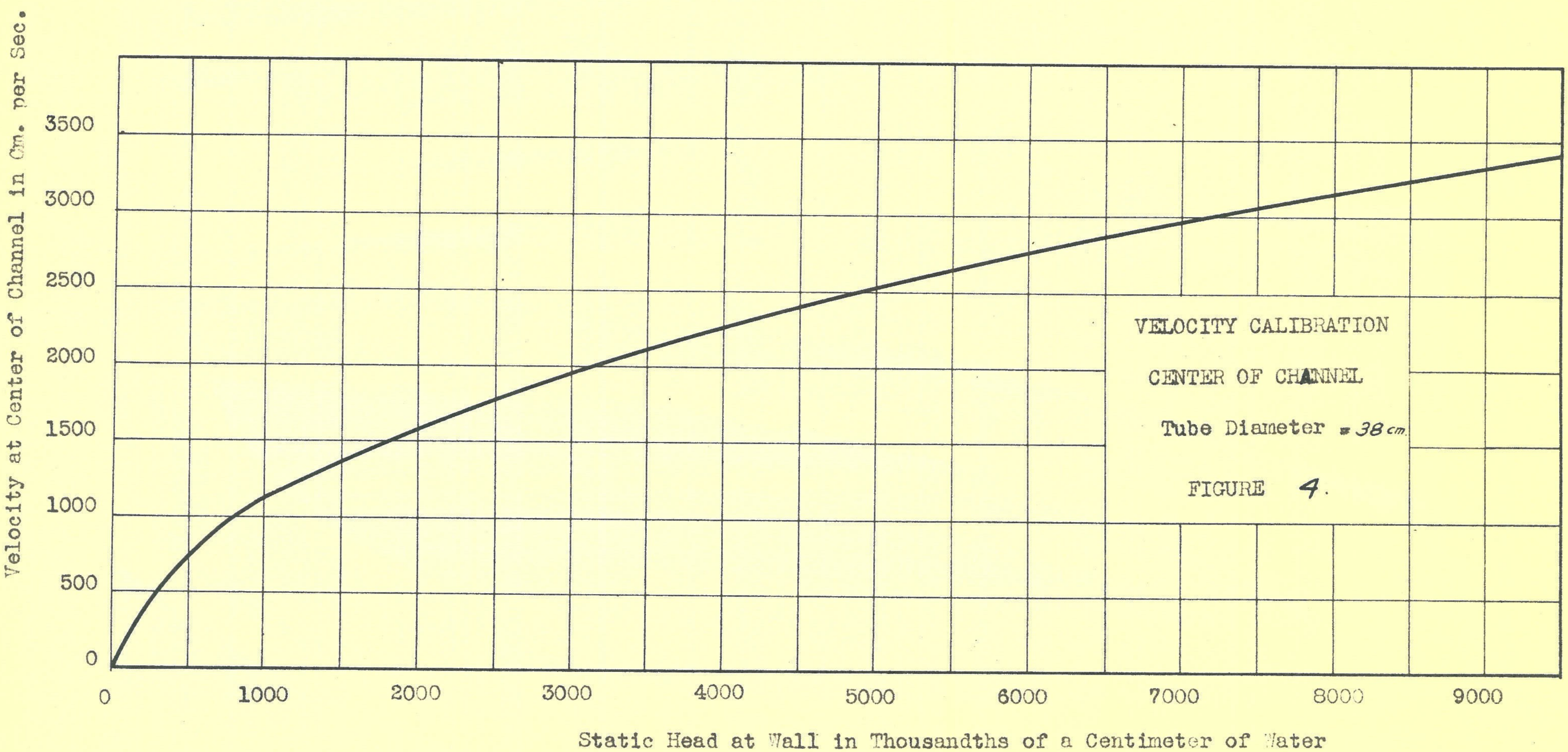
Diam. = 38 cm.

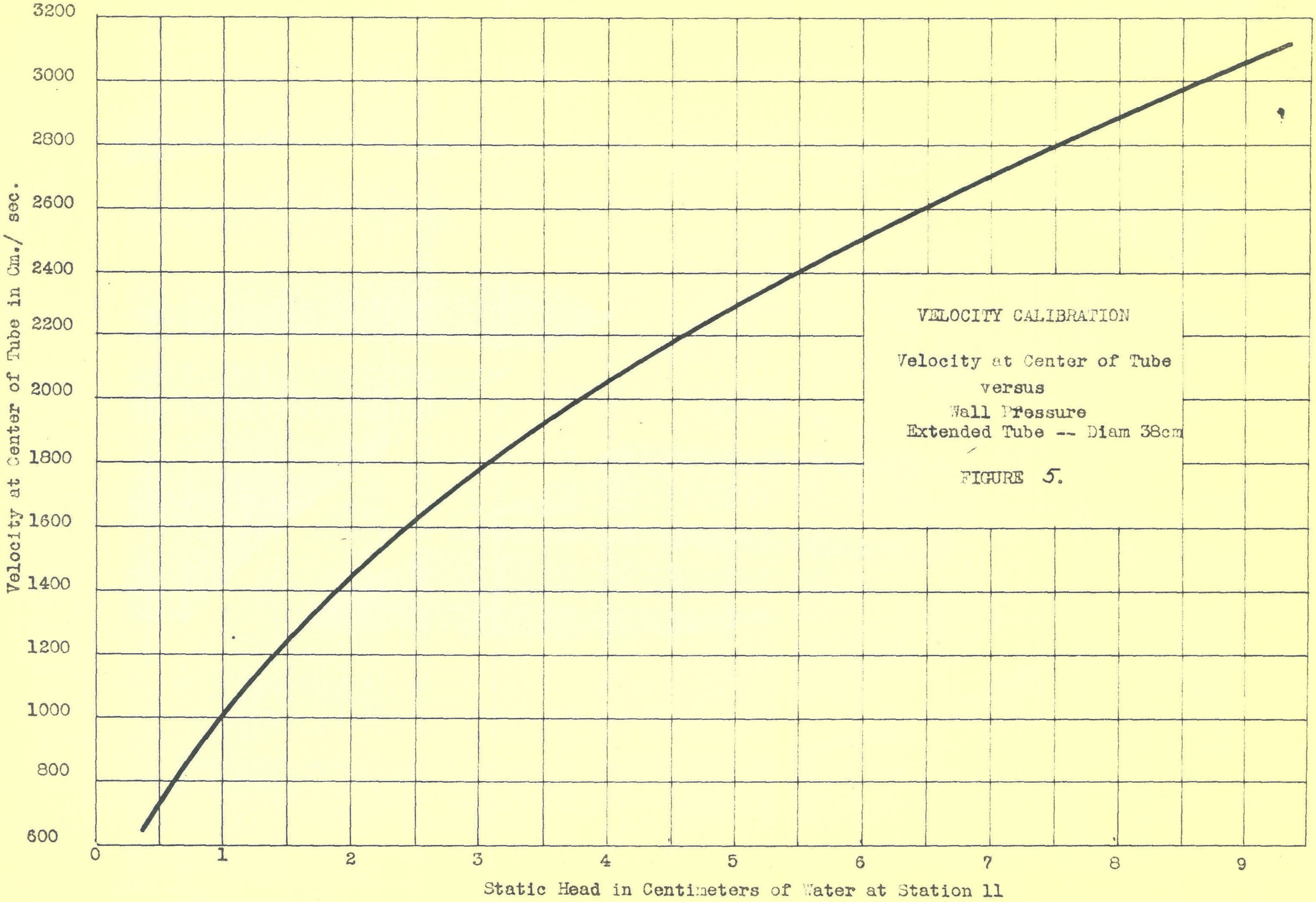
Figure 3.
VELOCITY DISTRIBUTION

5" Tube



ORIGINAL TUBE

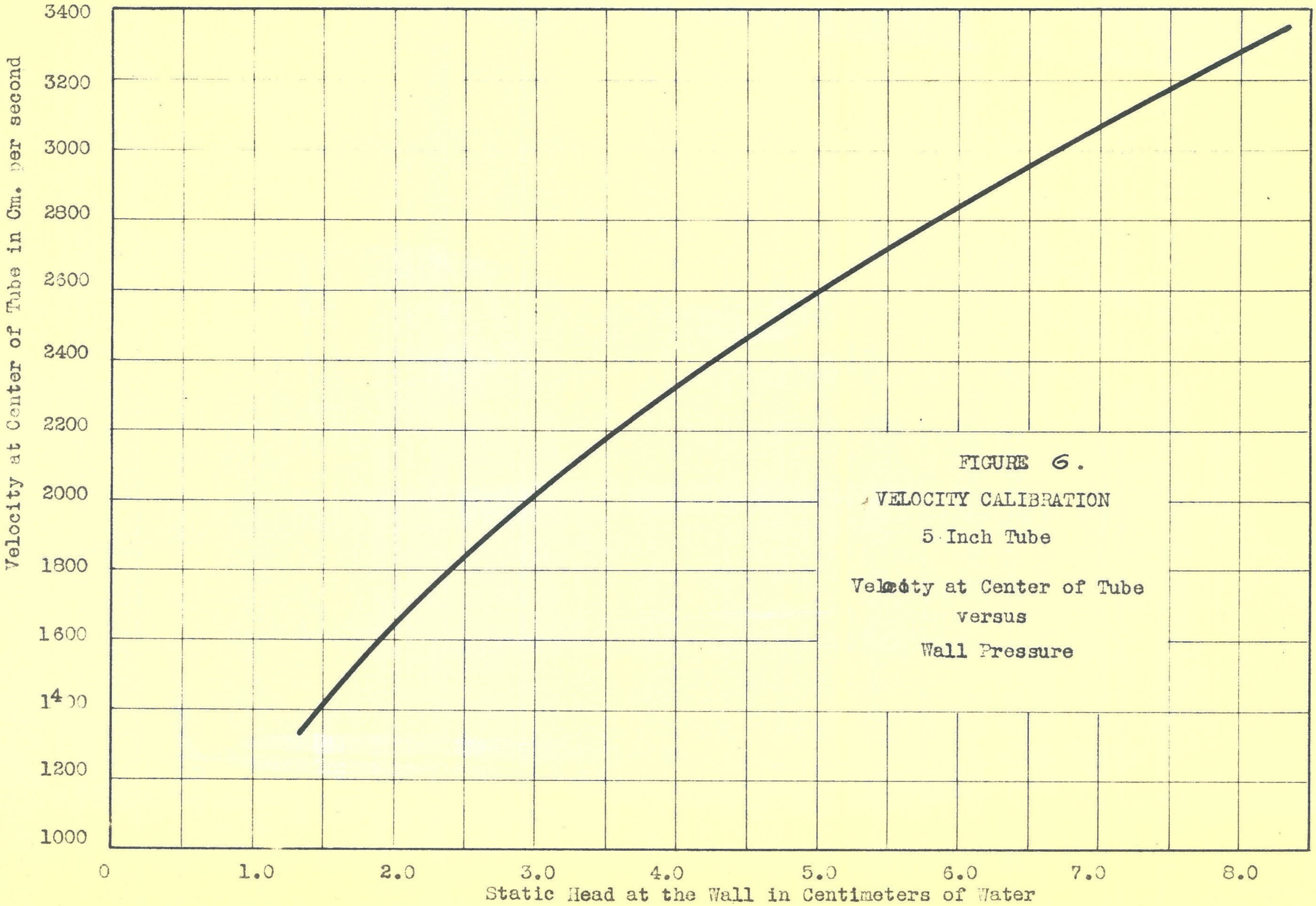




VELOCITY CALIBRATION

Velocity at Center of Tube
versus
Wall Pressure
Extended Tube -- Diam 38cm

FIGURE 5.



3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
Velocity at Center of Tube in Cm. per second
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Static Head at the Wall in Centimeters of Water
FIGURE 6.
VELOCITY CALIBRATION
5 Inch Tube
Velocity at Center of Tube
versus
Wall Pressure

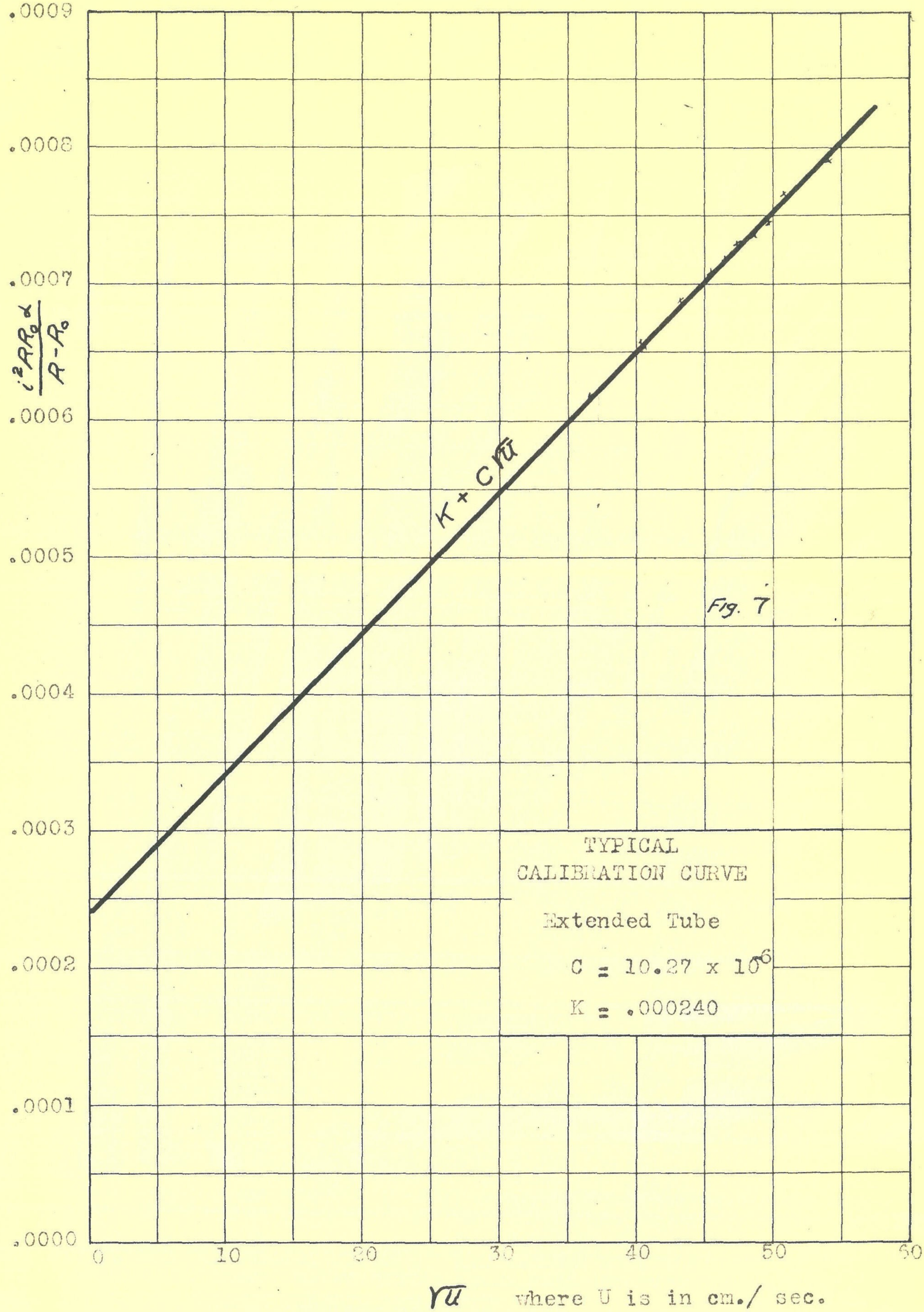

.0009
.0008
.0007
.0006
.0005
.0004
.0003
.0002
.0001
.0000
$\frac{i^2 R R_0 \alpha}{R - R_0}$
$K + C\sqrt{U}$
TYPICAL
CALIBRATION CURVE
Extended Tube
C = 10.27 x 10^-6
K = .000240
0
10
20
30
40
50
60
$\sqrt{U}$ where U is in cm./ sec.

Fig. 7

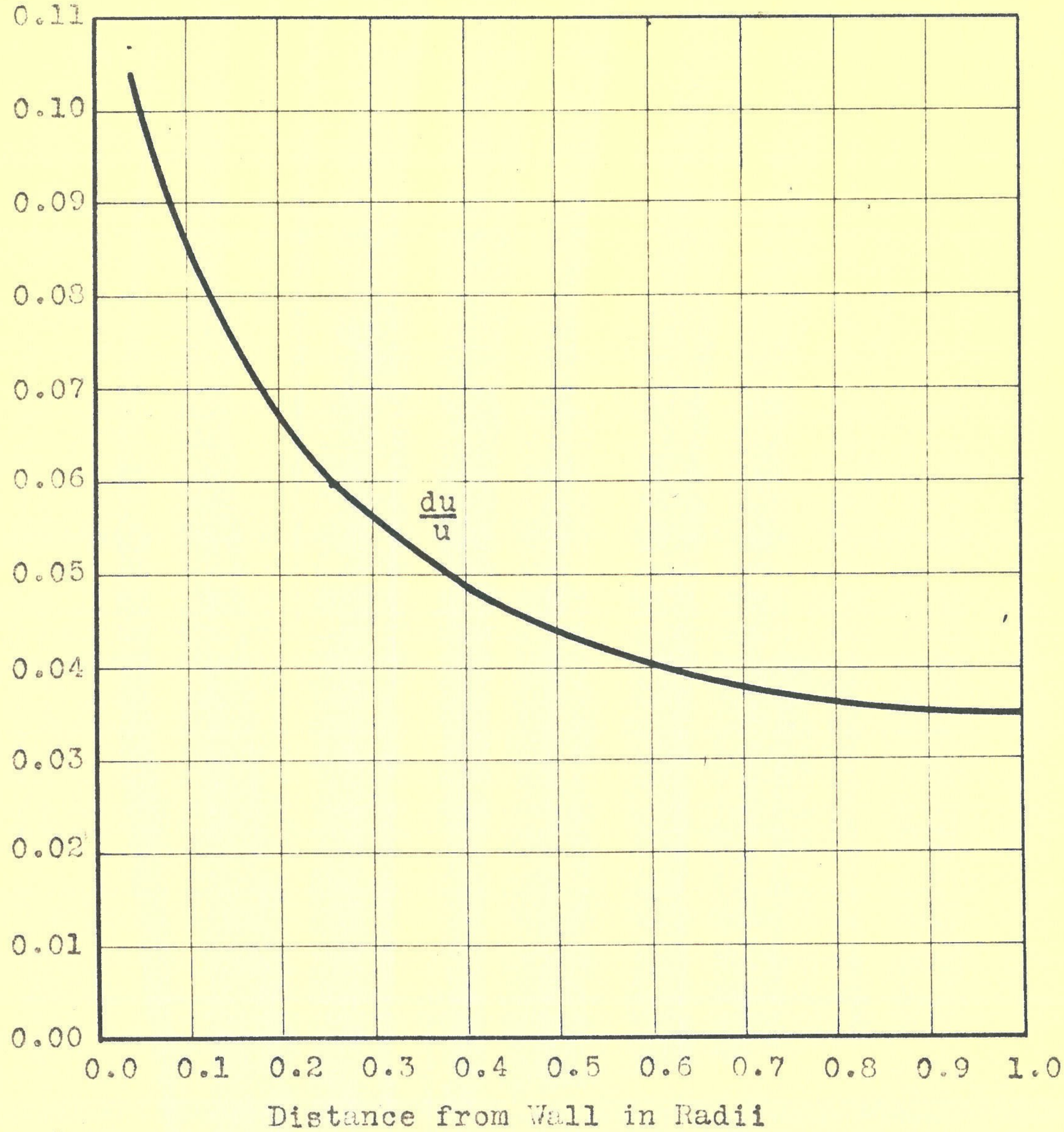


Figure 8 . RELATIVE TURBULENCE DISTRIBUTION

Original Tube Diameter = 38 cm.

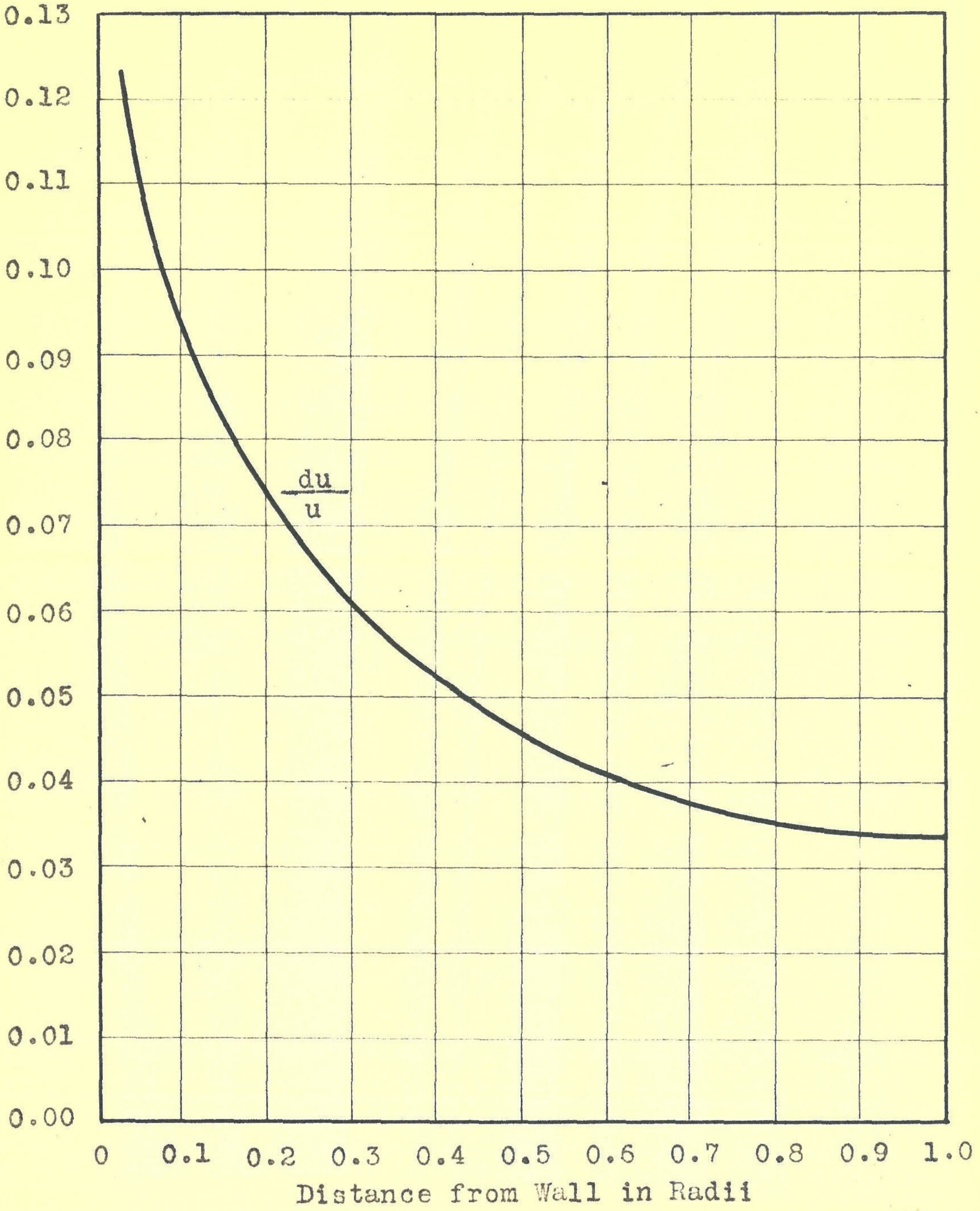


Figure 9. RELATIVE TURBULENCE DISTRIBUTION

Extended Tube Diameter = 38 cm.

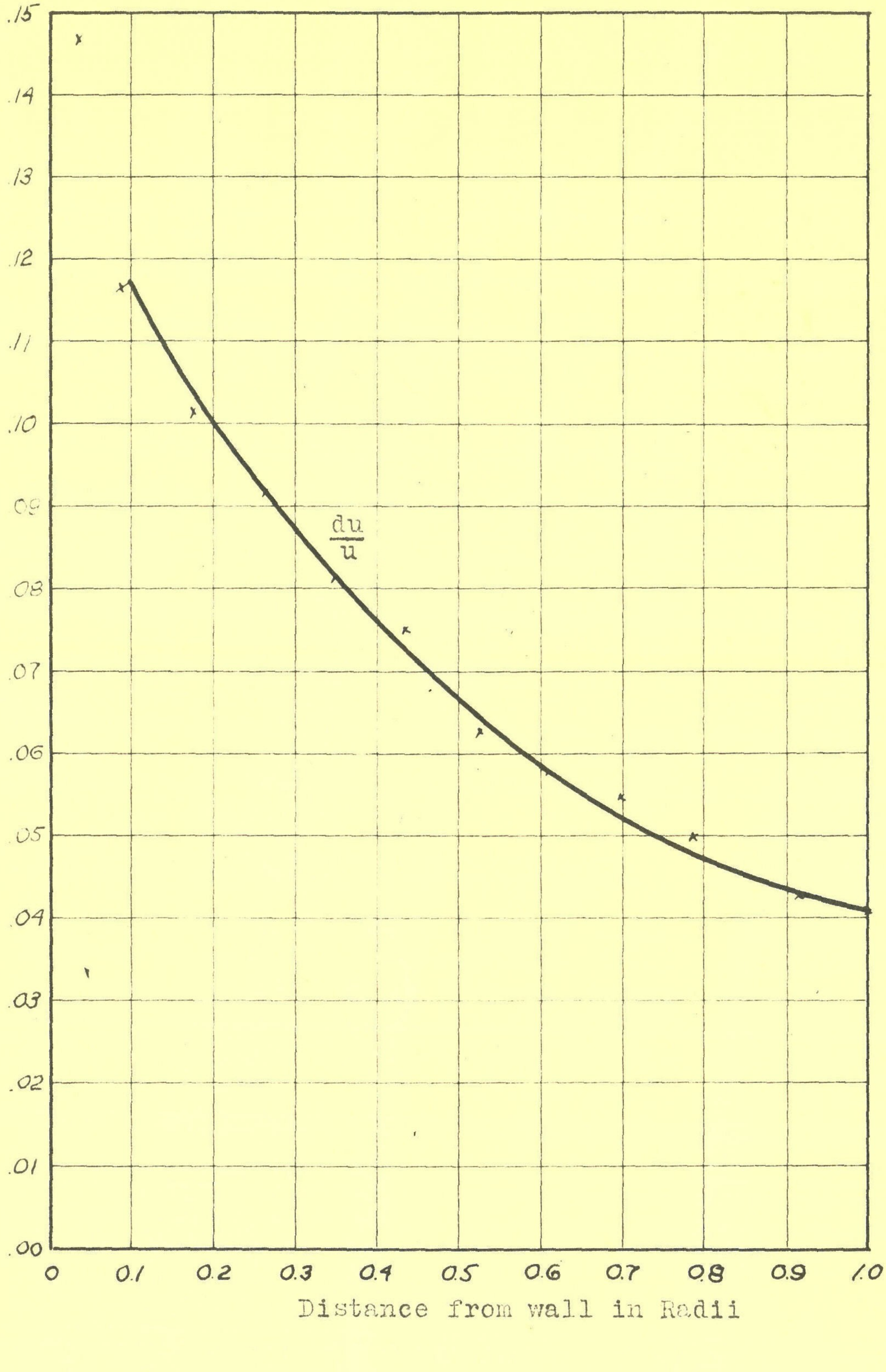


Figure 10. RELATIVE VELOCITY DISTRIBUTION

For 5 Inch Tube

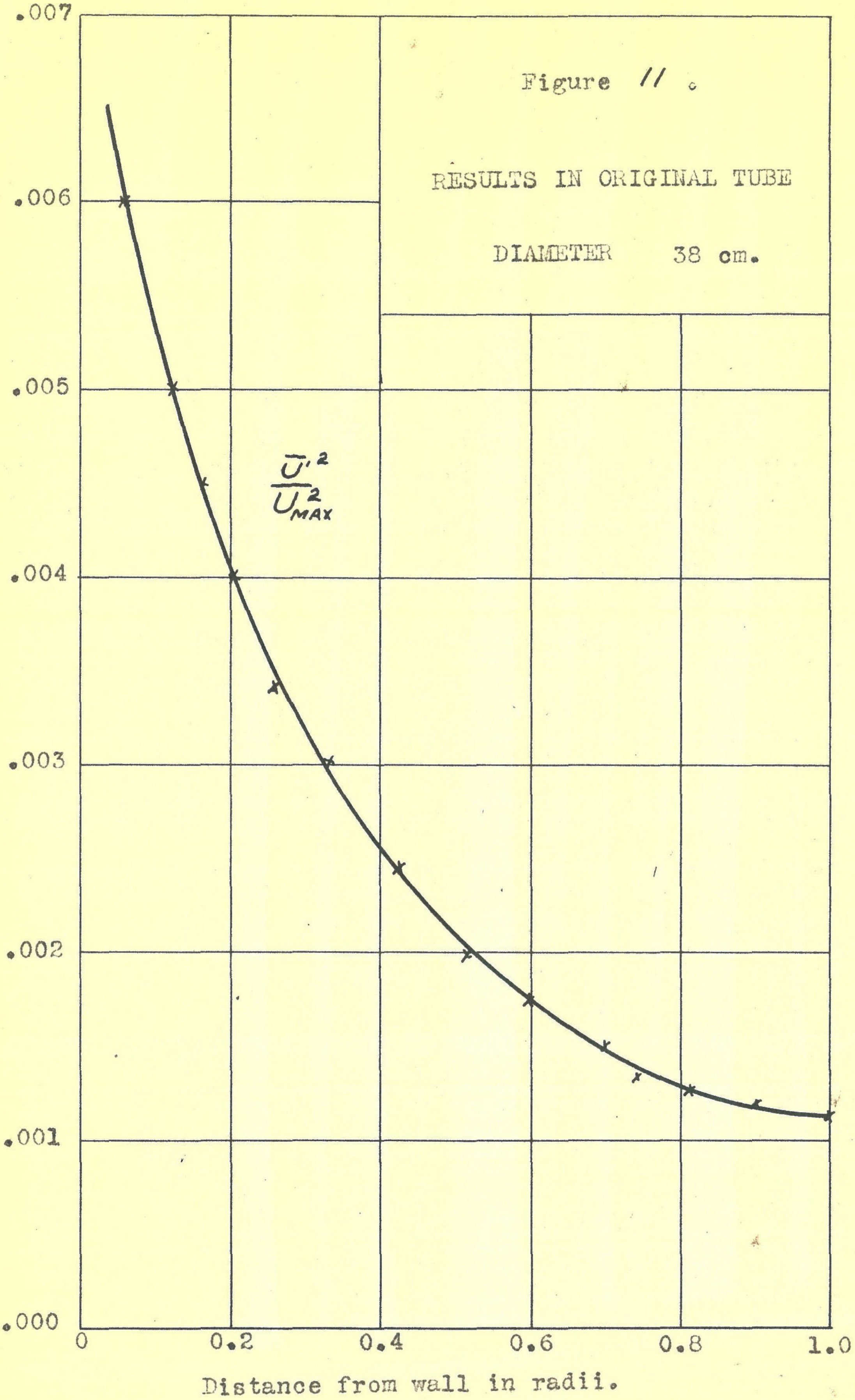


Figure 11.
RESULTS IN ORIGINAL TUBE
DIAMETER 38 cm.
$\frac{\bar{U}'^2}{U_{MAX}^2}$
.007
.006
.005
.004
.003
.002
.001
.000
0
0.2
0.4
0.6
0.8
1.0
Distance from wall in radii.

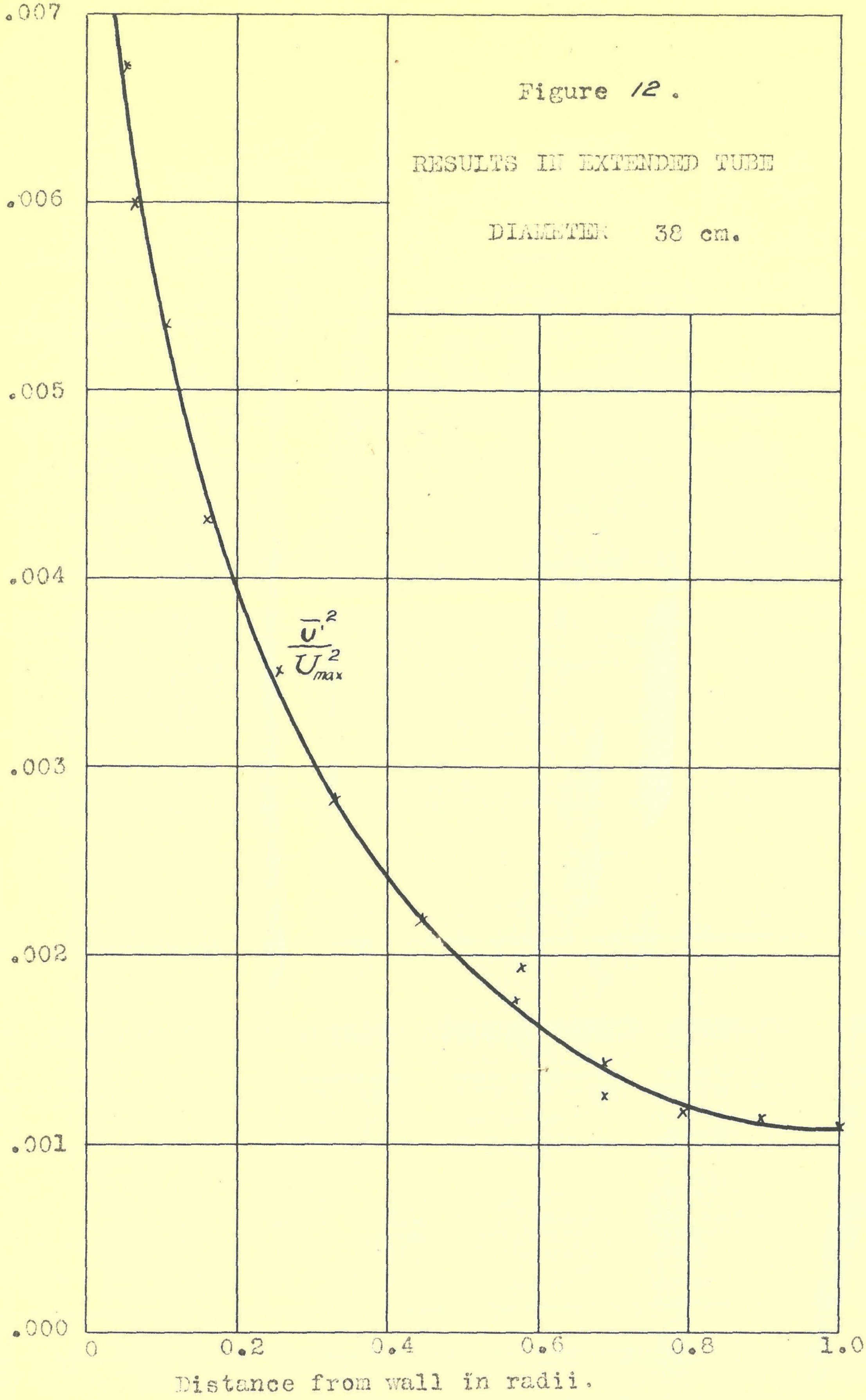


Figure 12.
RESULTS IN EXTENDED TUBE
DIAMETER 38 cm.
.007
.006
.005
.004
.003
.002
.001
.000
0
0.2
0.4
0.6
0.8
1.0
$\frac{\overline{u'}^2}{U_{max}^2}$
Distance from wall in radii.

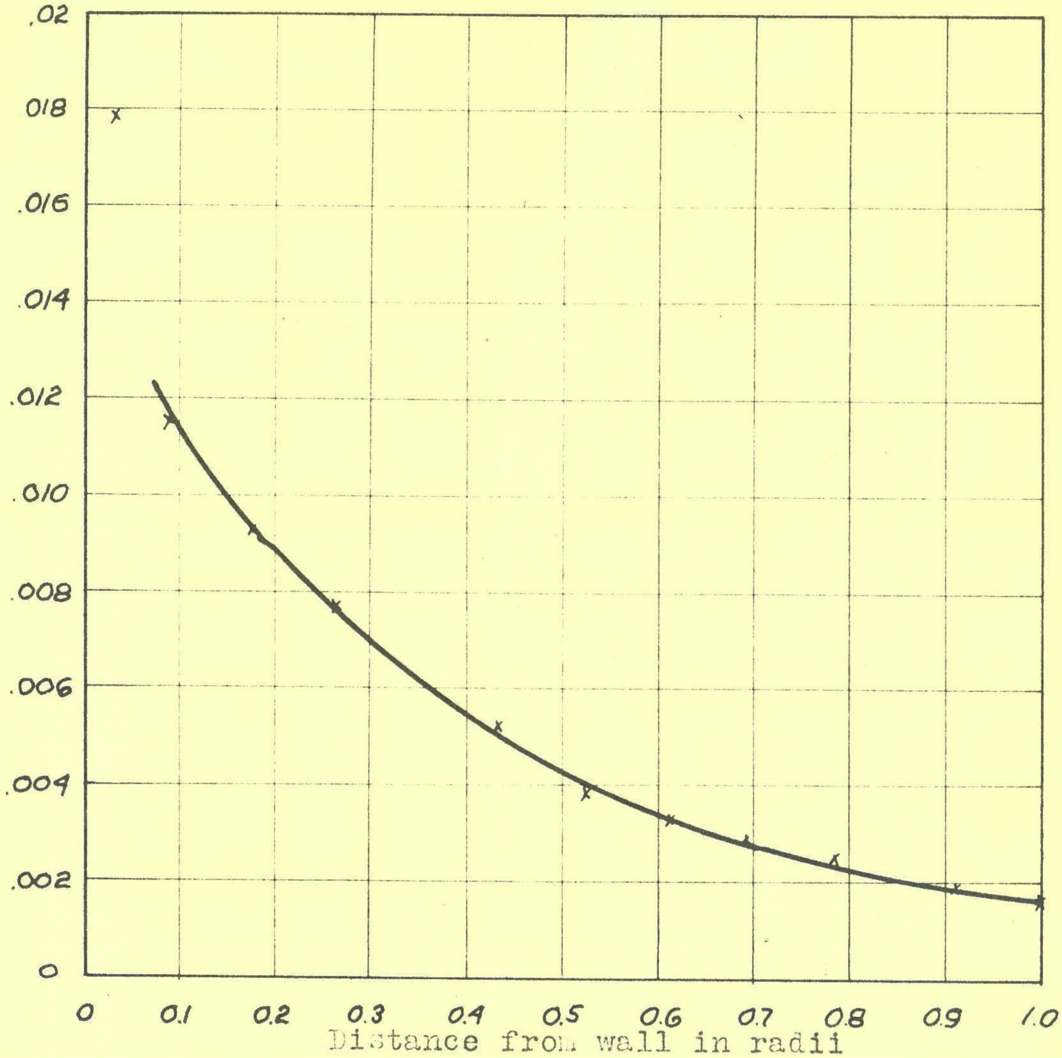


FIG 13

$\frac{u'^2}{U_m^2}$ For 5 inch Tube

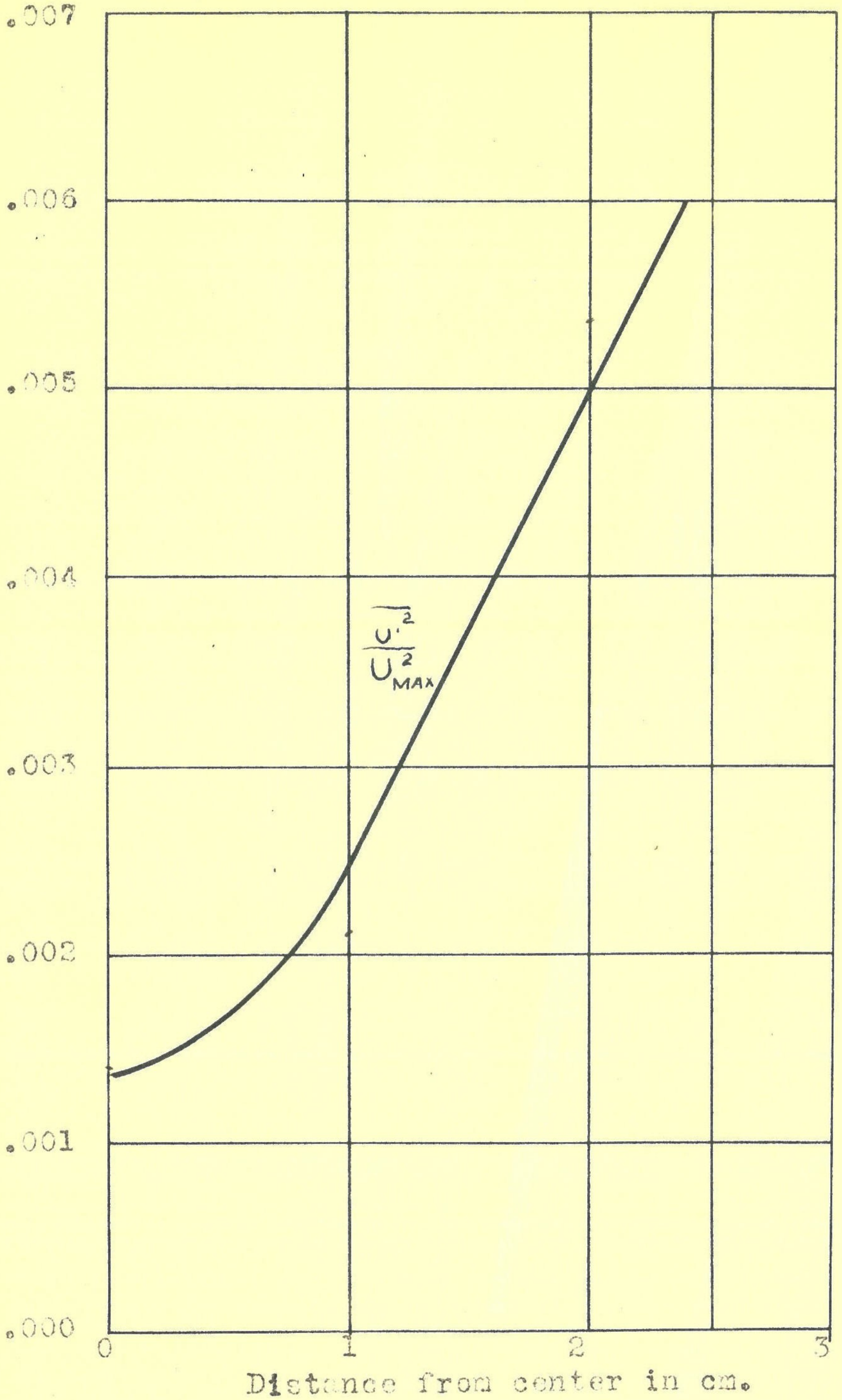


Fig. 14. Wattendorf-Kuethe Results for Straight Channel

FIGURE 15.

Kármán's Logarithmic Velocity Distribution

Original Tube-Diameter = 38 cm.

$$\frac{U_{max} - u}{\sqrt{\tau_o/\rho}} = \frac{-1}{K}\left[\log\left(1-\sqrt{\frac{y}{r}}\right) + \sqrt{y/r}\right]$$ (Theoretical Curve)

$$\frac{U_{max} - u}{\sqrt{\tau_o/\rho}} \text{ vs. } \frac{y}{r}$$ (Experimental Points)

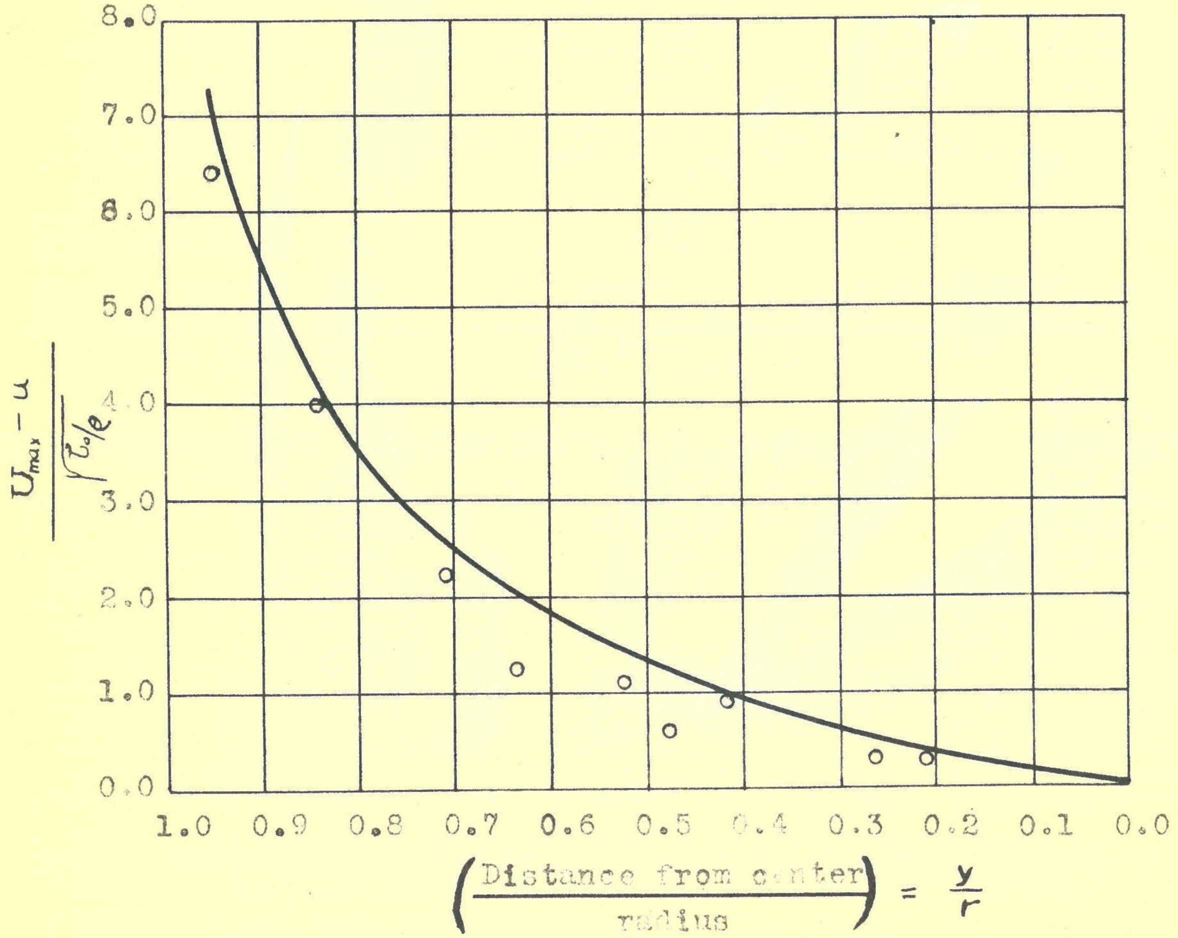


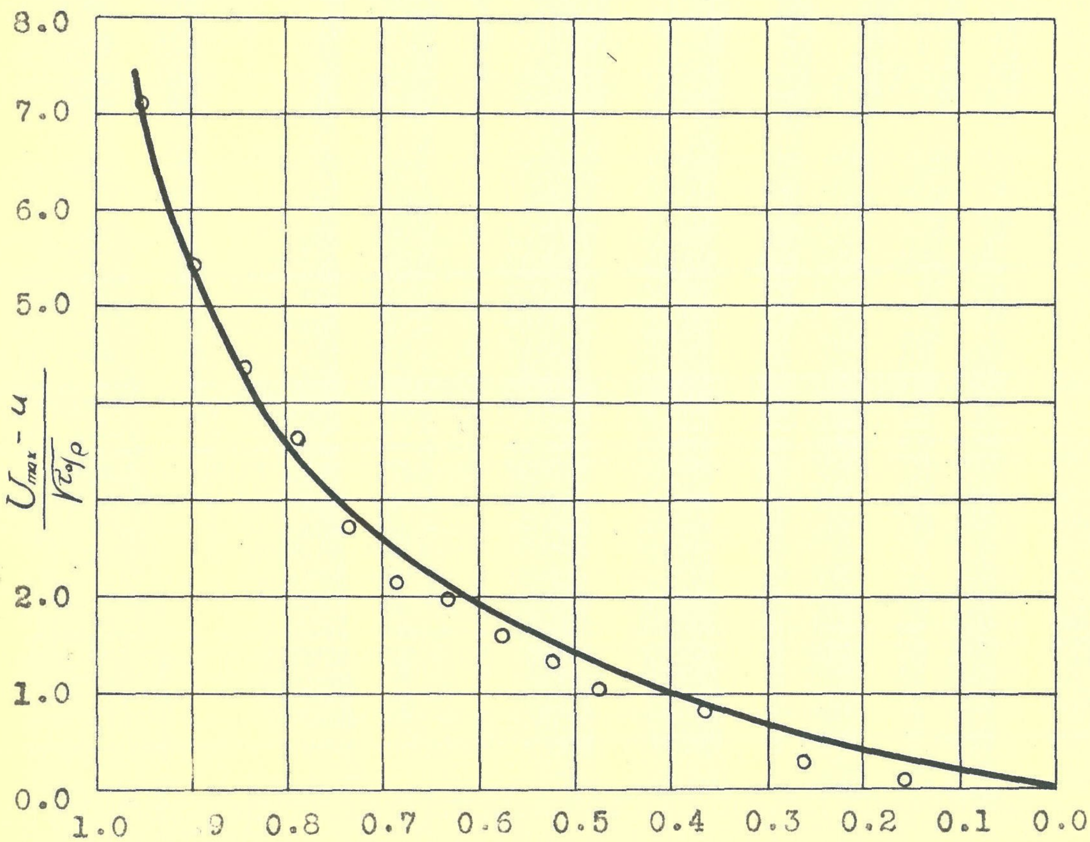
FIGURE 16.

KARMAN'S LOGARITHMIC VELOCITY DISTRIBUTION

Extended Tube -- Diameter = 38 cm.

(Theoretical Curve) $\frac{U_{max} - u}{\sqrt{\tau_o/\rho}} = \frac{-1}{K}\left[\log\left(1-\sqrt{\frac{y}{r}}\right) + \sqrt{\frac{y}{r}}\right]$

$\frac{U_{max} - u}{\sqrt{\tau_o/\rho}}$ vs. $\frac{y}{r}$ (Experimental Points)



$\frac{\text{Distance from center}}{\text{radius}} = \frac{y}{r}$

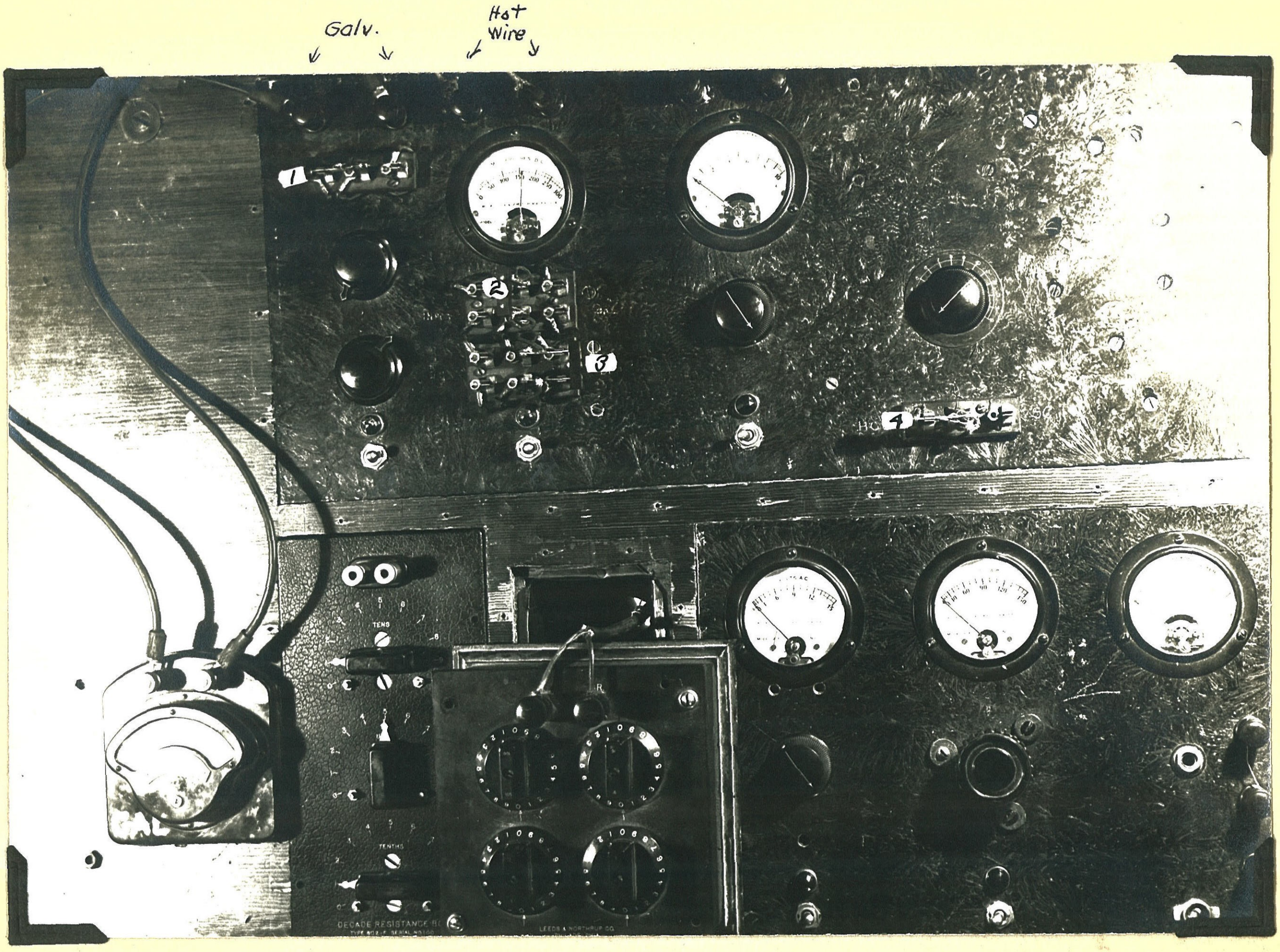


Fig. 17. SWITCH ARRANGEMENT FOR CIRCUIT SHOWN BELOW

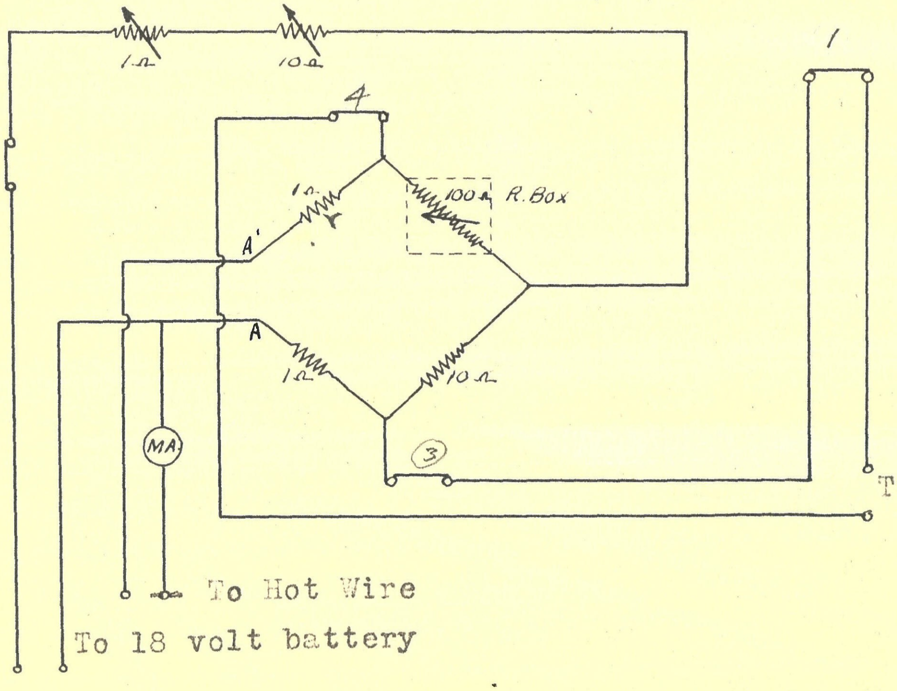


Figure 18. Circuit for measuring wire resistance.

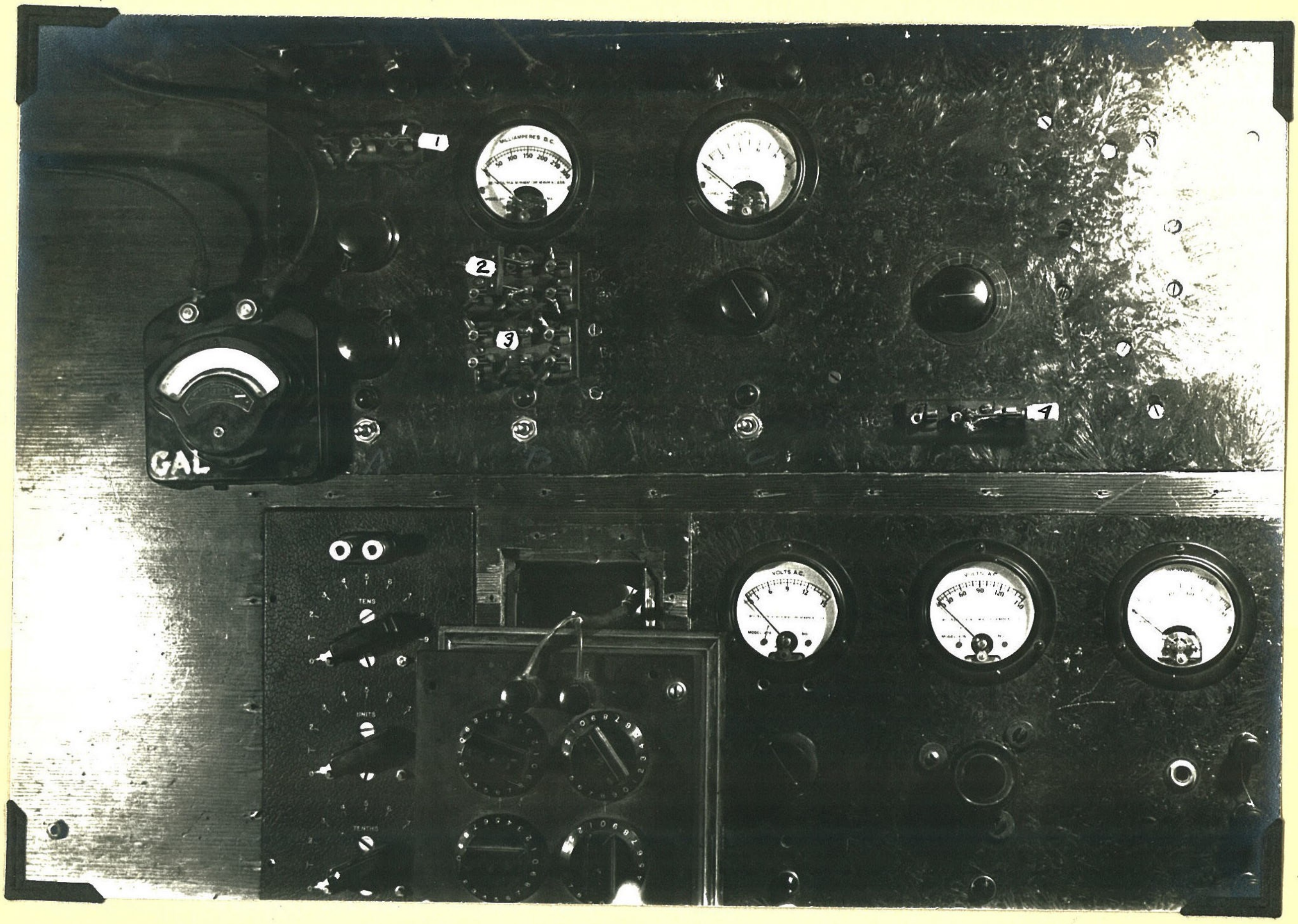


Fig. 19. SHOWING SWITCH ARRANGEMENT FOR CIRCUIT SHOWN BELOW

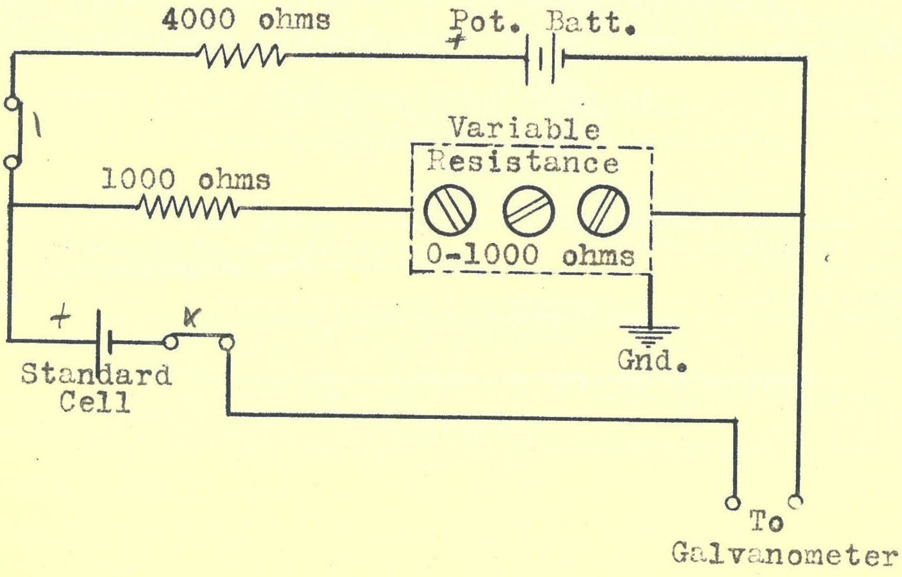


Figure 20. Potentiometer Circuit

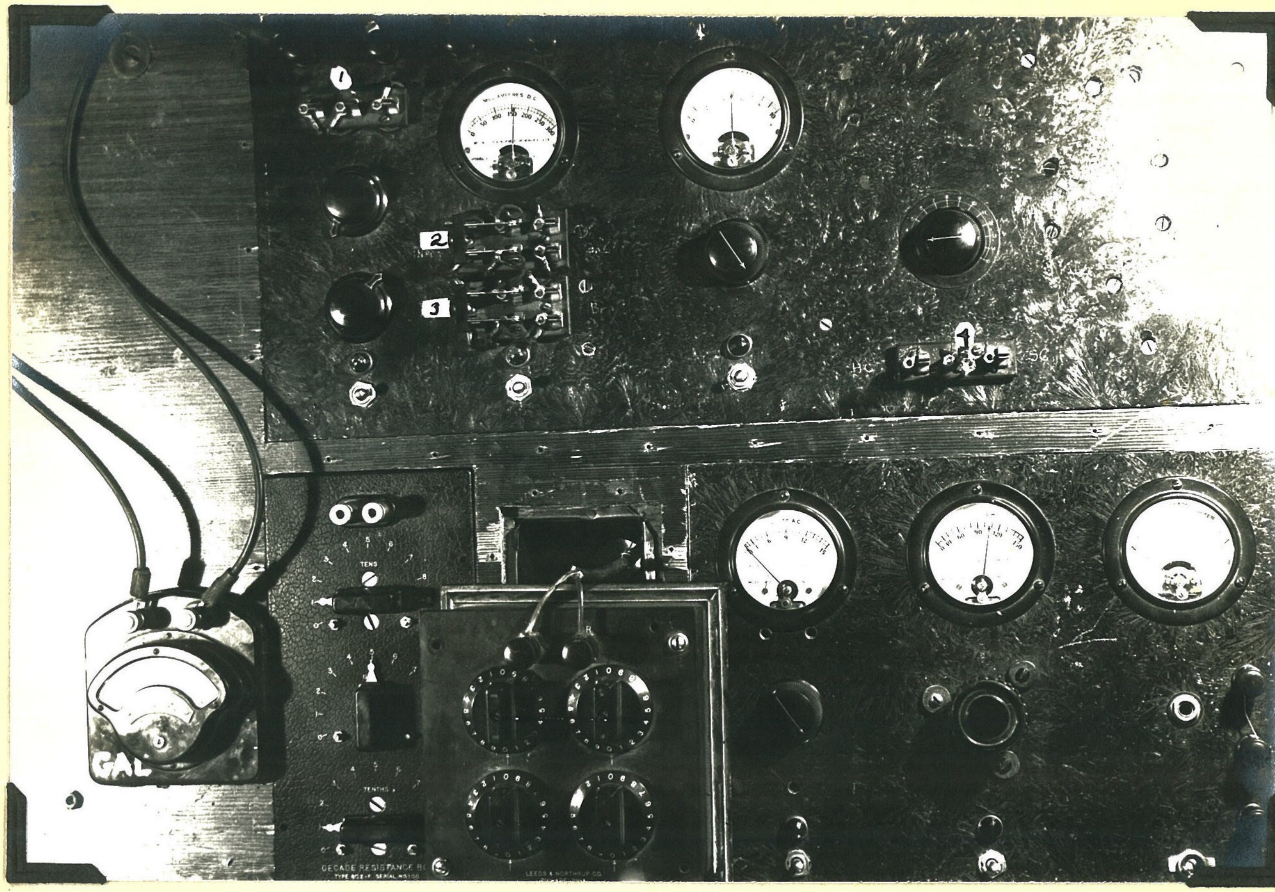


Fig. 21. SHOWING SWITCH ARRANGEMENT FOR CIRCUIT BELOW

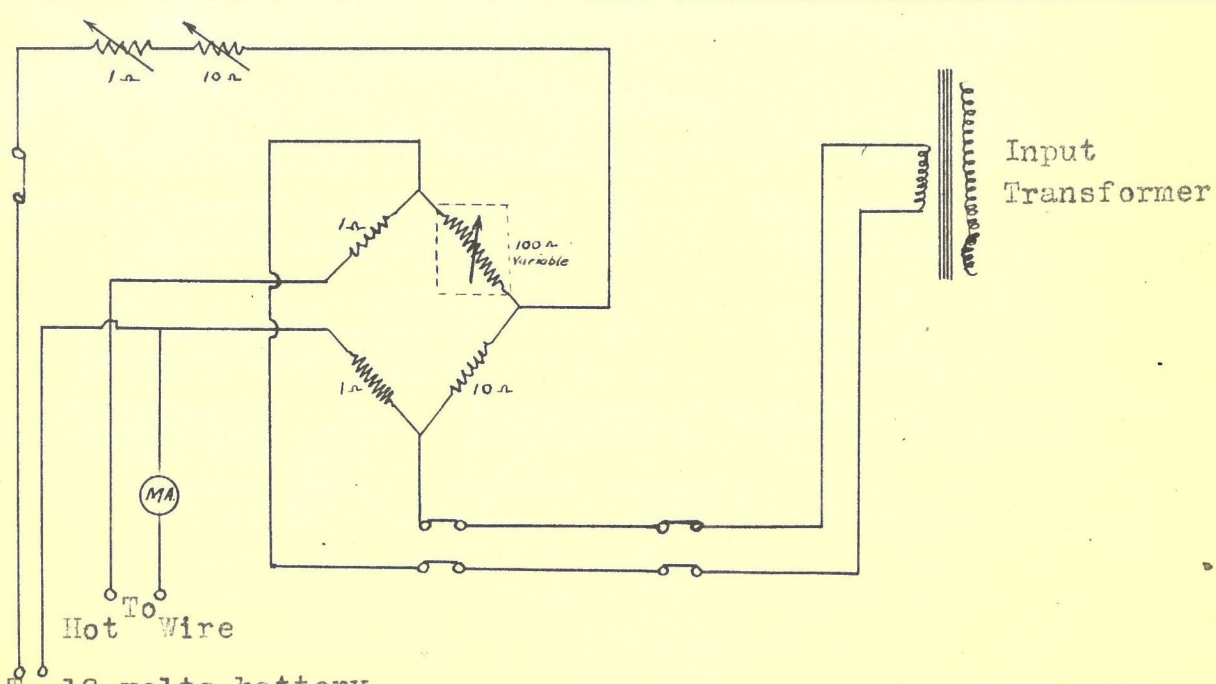


Figure 22. Primary Circuit Feeding into Amplifier

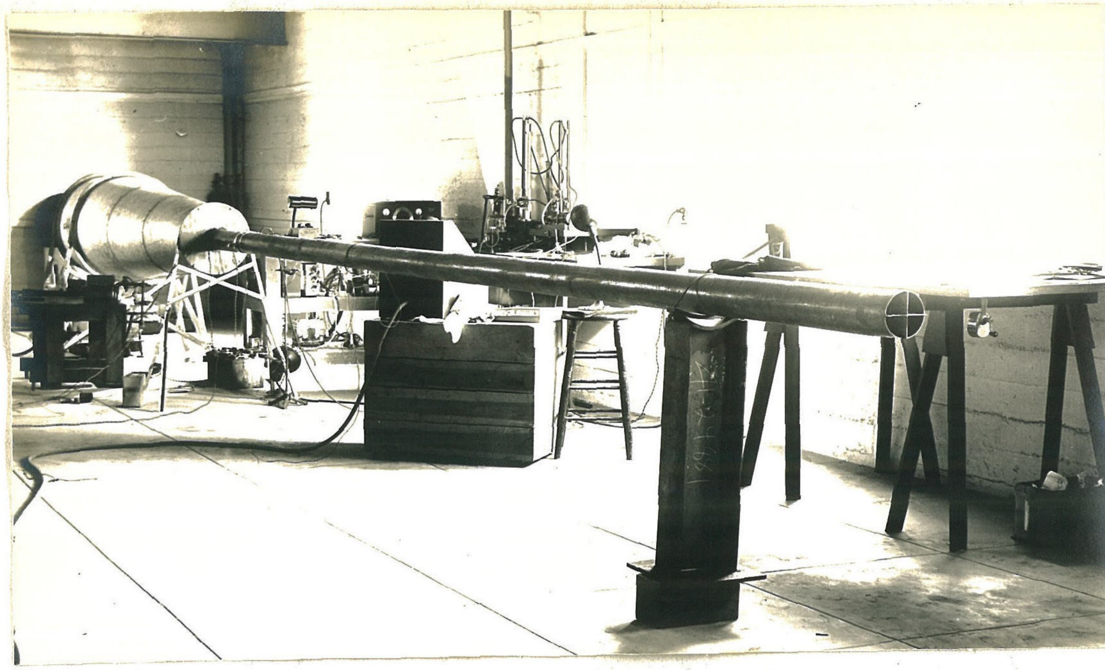


Fig. 23. SHOWING SET-UP OF 5 INCH TUBE

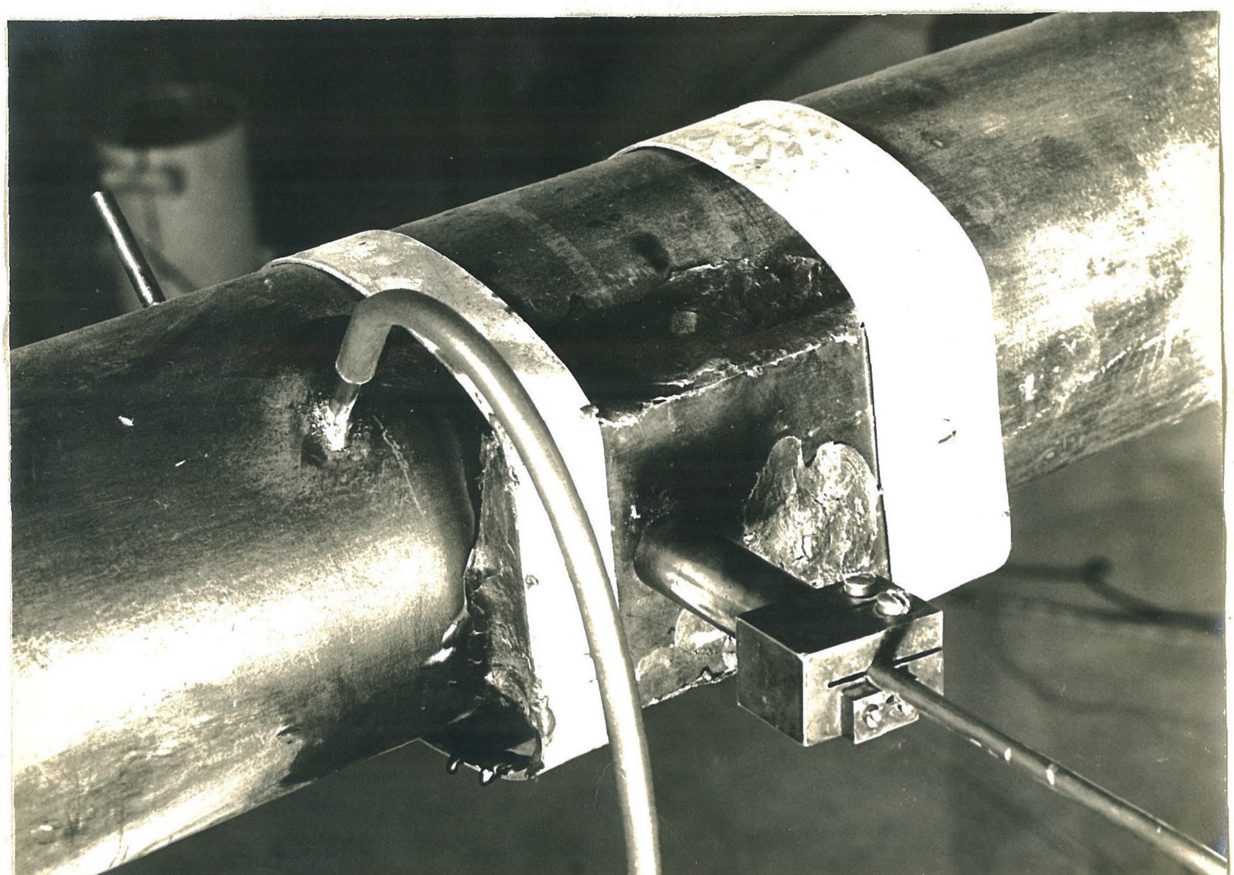
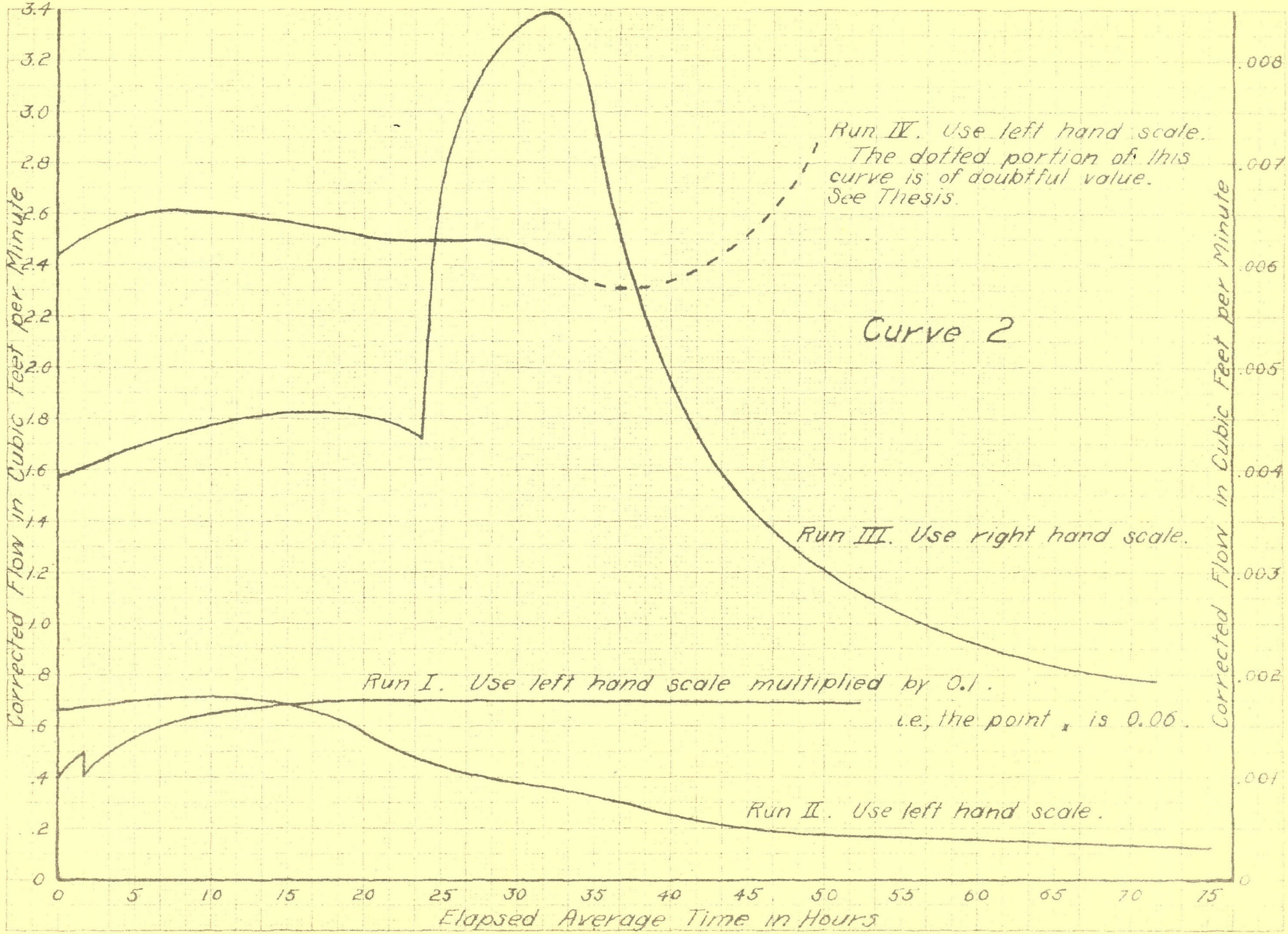
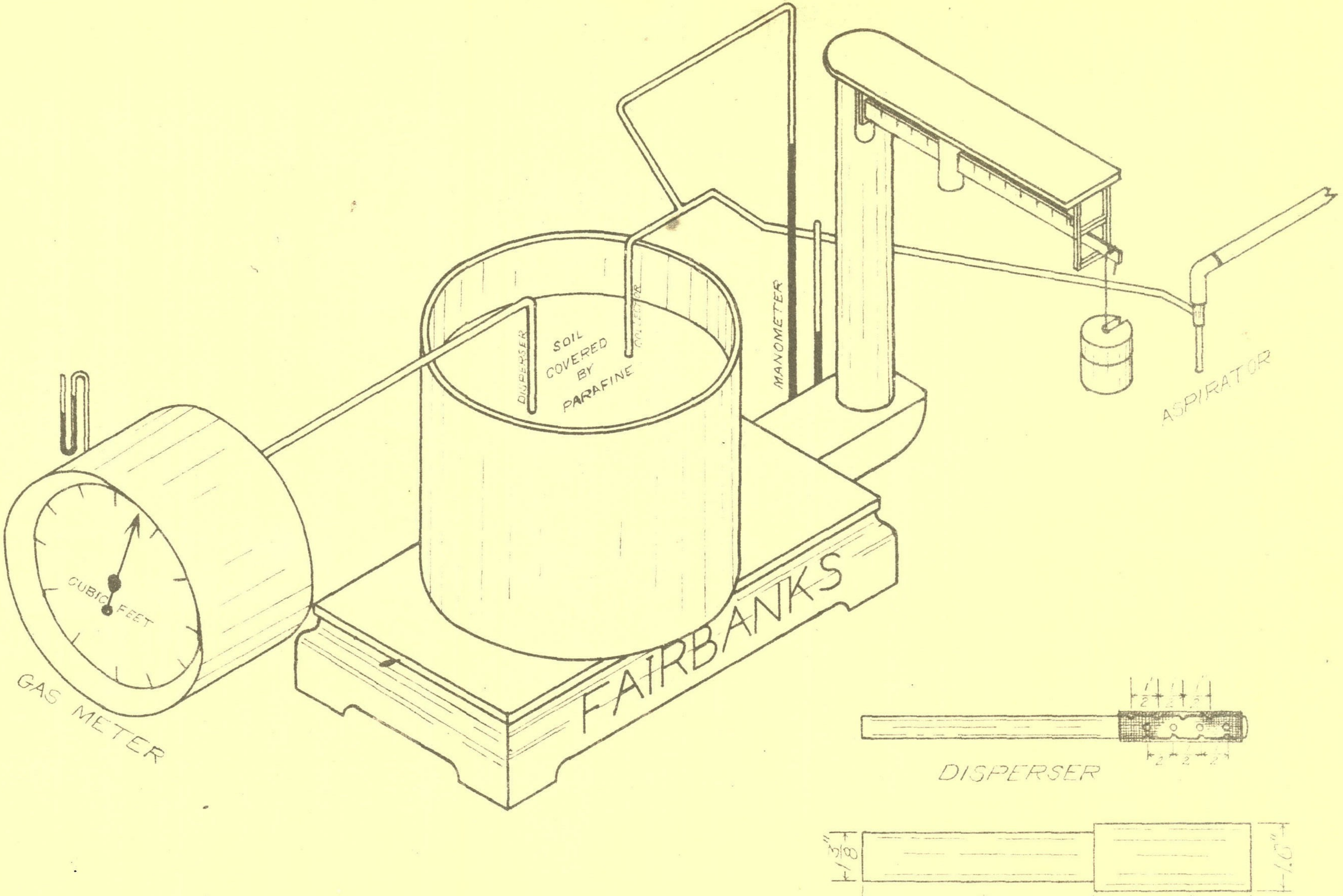


Fig. 24 SHOWING METHOD OF MOUTING HOT WIRE IN TUBE


Curve 2
Run IV. Use left hand scale.
The dotted portion of this curve is of doubtful value.
See Thesis.
Run III. Use right hand scale.
Run I. Use left hand scale multiplied by 0.1.
i.e, the point x is 0.06.
Run II. Use left hand scale.
Corrected Flow in Cubic Feet per Minute
Corrected Flow in Cubic Feet per Minute
Elapsed Average Time in Hours
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
0
.001
.002
.003
.004
.005
.006
.007
.008
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75



ISOMETRIC SKETCH OF APPARATUS

Fig. 1