A Theorectical Investigation of the Distribution of Maximum Shearing Stresses in a Stiffened Flat Panel.

In Partial Fulfillment of the Requirements of the Degree of

Master of Science in Aeronautical Engineering.

by

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INTRODUCTION.

In studying stress distribution several methods have been used. The purely experimental method is one in which local stresses are measured directly by means of sensitive strain gages. The photoelastic method is the second one which gives a definite visual stress pattern to each stress system, from which the shearing stress can be read directly. There is a third one which is a purely a mathematical method. This method is based upon the assumption of continuity and the law of equilibrium for establishing the stress functions and equations derived by logical processes. The solution is then obtained by the use of boundary conditions. At the California Institute of Technology the shear characteristics of thin plates reinforced by one centrally located stiffener was investigated by the photoelastic method. The third method has been used by the author for studying the same natural problem in hopes of checking the result already obtained.

It is common practice to use panels of thin plates reinforced by longitudinal stiffeners in the construction of modern metal airplanes. So far as the strength characteristics are concerned, it is necessary to know the stress distribution of the sheets in order to obtain the strength limits of the structures. This thin plate with a stiffener is assumed to be loaded in to different ways. They are : first, uniform normal loads are applied to the stiffener at both ends; second, uniform normal load is applied to the stiffener at both end with another load uniformly distributed along the opposite edge of the plate and stiffener.

By utilizing Airy's function and Fourier's series, the author derived the stress components σ_x, σ_y and τ_{xy} for the two cases above. Then from these expressions the stress components were obtained for the principal stresses and the maximum shearing stress and their directions at any point on the thin plate were determined. The curves of the maximum shearing stress distributions were then drawn for the particular specimens tested by the photoelastic methods.

The material of the plate studied is Bakelite. The value of Poisson's ratio for it is 0.36, which was supplied by Mr. W. A. Zinzow, Research Physicist at the Bakelite Corporation, Bloomfield, N. J..

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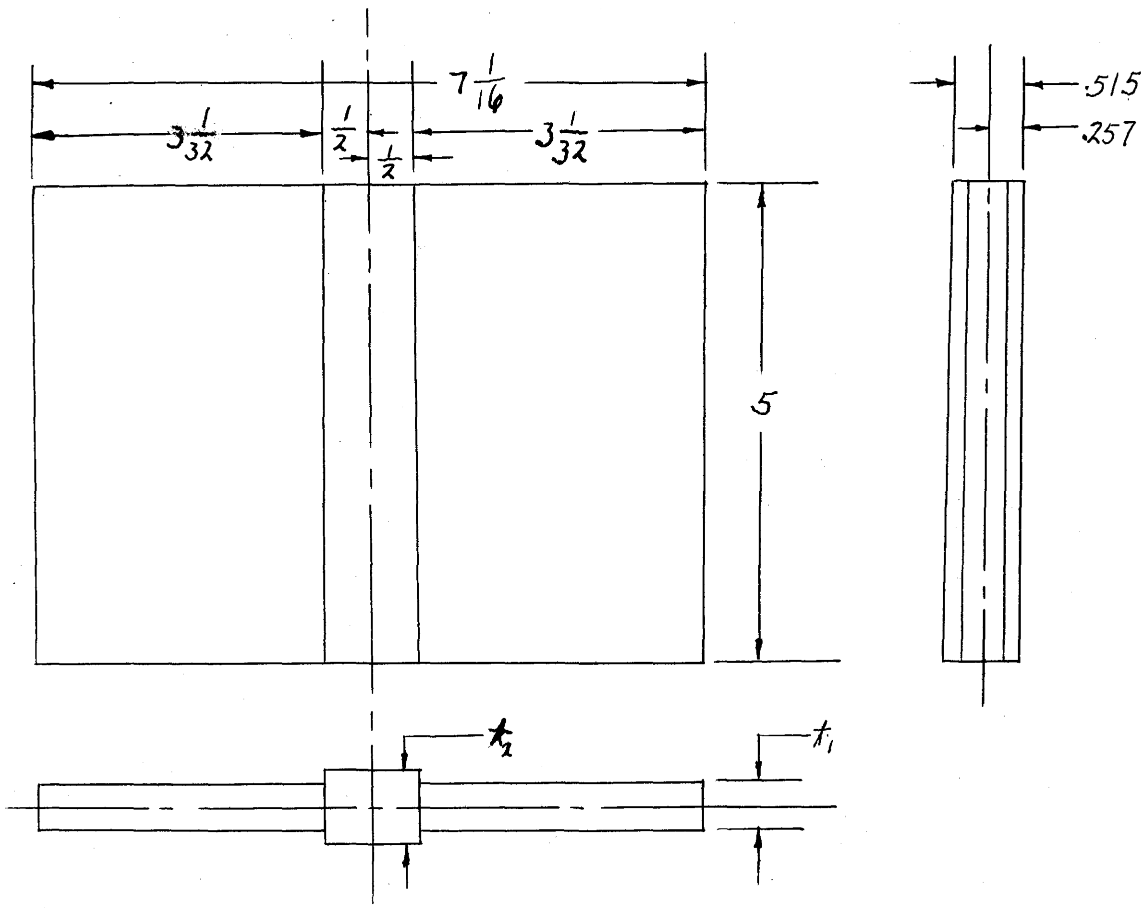
Symbols Used.

x, y	Rectangular coordinates
r, ψ	Polar coordinates
u,v	Components of the displacement parallel to the x and y axes in two dimensional doordinates.
σ_x, σ_y	Components of the normal stress parallel to the x and y axes.
σ_1, σ_2	The principal stresses.
τ	Shearing stress in rectangular coordinates
$\tau_{max.}$	Maximum shearing stress.
ϵ_x, ϵ_y	Unit elongation in x and y directions.
γ_{xy}	Shearing strain in rectangular coordinates.
σ_r, σ_θ	Radial and tangential normal stresses in polar coordinates.
$\tau_{r\theta}$	Shearing stress in polar coordinates
ϕ	Stress function
E	Modulus of elasticity in tension and compression
G	Modulus of rigidity (Modulus of Elasticity in shear).
ν	Poisson's ratio.
q	Intensity of a uniformly distributed load.

Dimensions of the Plate and Stiffener.



t_1 = 0.355 in.

P = 2,000 lbs.

ℓ = 5.0 in.

a = 3 1/32 in.

b = 1 in.

t_2 = 0.515 in.

PART ONE

Maximum shearing stress distribution in a rectangular plate re-enforced by a stiffener with uniform load applied at the two ends of the stiffener.

(a) General Assumptions.

Consider the compression of a beam of constant rectangular cross section by forces applied at the ends and parallel to one of the principal axes of the cross section. Then the shape of the compression will be as shown in the figure 1.

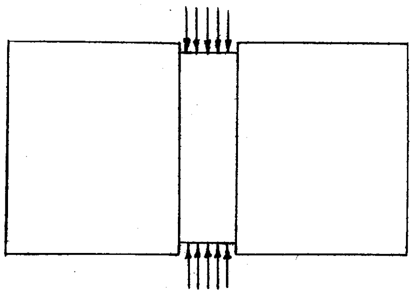


FIGURE 1.

If a plate is added, without fastening, it will be found that the plate is longer than the top of the beam in figure 1.

Thus, for the case in which the beam and plate are fastened together, there must be a shearing force applied. This shearing forcex varies from zero in the center of the beam to a maximum value at the ends. This shear is between the beam and plate.

Now we consider one half of the plate as OPQR in figure 2. The edge OP is fixed to the beam, edge QR is free and the edge OR and PQ may be either fixed or free.

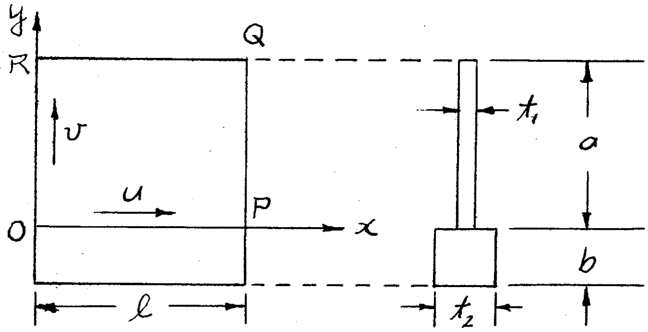


FIGURE 2.

Assumptions : For convenience we assume that

1. The lateral contraction of the beam can be neglected without much error.

2. The deformation of the elastic plate takes place in the xy plane only thus it is two-dimensional. The displacements of the particles of the plate can then be resolved into components u, v parallel to the coordinatesaxes x, y respectively.

In elasticity it is well known that the stress function has to satisfy the compatibility equation :

$$\nabla^4\phi = \frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = 0 \qquad (1)$$

The stress components are

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\phi}{\partial x^2}, \quad \tau = -\frac{\partial^2\phi}{\partial x\partial y}$$

Provided that there is no body force.

Since any stress distribution can be expressed by a Fourier's series and the accuracy of the calculations only depends upon the rapidity of the convergence of the series and the number of the terms taken, thus the assumption can be legitimately made that sine or cosine functions may be used for expressing this distribution. Furthermore we assume that the plate concerned is at rest under the action of external force as in the usual case. The resulting problem is then static, that is, independent of time.

Assume a stress function ϕ such that,

$$\phi = f_n(y) \sin \frac{n\pi x}{\ell} \qquad (2)$$

where n's are integers and f_n's are functions of y only.

This stress function means that the distribution in all sections along x is similar and only differs in amount.

By substitution, the stress components are then :

$$\sigma_x = \frac{d^2 f_n}{dy^2} \sin \frac{n\pi x}{\ell}$$

$$\sigma_y = -\left(\frac{n\pi}{\ell}\right)^2 f_n \sin \frac{n\pi x}{\ell} \qquad (3)$$

$$\tau = -\left(\frac{\pi n}{\ell}\right) \frac{df_n}{dy} \cos \frac{n\pi x}{\ell}$$

(b) Boundary Conditions.

Since the beam and plate are fastened together the displacements ~~of them~~ over the line of contact should be the same or,

$$u_{plate} = u_{beam}, \quad \text{at} \quad y = 0 \qquad (4)$$

The other boundary conditions which are self evident are :

$$v = 0 \quad \text{at} \quad y = 0 \qquad (5)$$

$$\sigma_y = 0 \quad \text{at} \quad y = a \qquad (6)$$

$$\tau = 0 \quad \text{at} \quad y = a \qquad (7)$$

From (3) and (6) :

$$f_n(y) = 0 \quad \text{at} \quad y = a \tag{8}$$

Again from elasticity we know that

$$\begin{aligned} \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{xy} &= \frac{\tau}{G} \\ \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \end{aligned} \tag{9}$$

From (5) and (9) we have

$$\frac{\partial \sigma_x}{\partial y} - \nu \frac{\partial \sigma_y}{\partial y} - \frac{E}{G}\frac{\partial \tau}{\partial x} = 0 \tag{10}$$

Substituting (3) into (10) and using the relation $G = \frac{E}{2(1+\nu)}$ we have

$$\frac{d^3 f_n}{dy^3} - \left(\frac{n\pi}{\ell}\right)^2 (2+\nu)\frac{df_n}{dy} = 0 \tag{11}$$

From (7) we have

$$\frac{df_n}{dy} = 0 \quad \text{at} \quad y = a \tag{12}$$

<u>(c) (i). To find the expression of $f_n(y)$</u>

From the compatibility equation (1) and the stress function (2), we obtain :

$$\frac{d^4 f_n}{dy^4} - 2\left(\frac{n\pi}{\ell}\right)^2 \frac{d^2 f_n}{dy^2} + \left(\frac{n\pi}{\ell}\right)^4 f_n = 0 \tag{13}$$

This differential equation is a linear one with constant coefficients, So there is a first solution $f(y) = e^{my}$. Substitution gives $m = \pm\frac{n\pi}{\ell}$ $\pm\frac{n\pi}{\ell}$; that is, it has two pairs of repeated roots, hence its complete solution is :

$$f_n(y) = A_n e^{\frac{n\pi y}{\ell}} + B_n e^{-\frac{n\pi y}{\ell}} + C_n y e^{\frac{n\pi y}{\ell}} + D_n y e^{-\frac{n\pi y}{\ell}}$$

Where An , Bn, Cn, Dn are constants which are now to be determined by the boundary conditions.

First assume an infinite large plate, i.e. $y = a \leq \infty$

Then from boundary conditions $y = a,\ f_n(y) = 0$ & $\frac{df_n}{dy} = 0$ Hence

$$f_n(y) = B_n e^{-\frac{n\pi y}{\ell}} + D_n y e^{-\frac{n\pi y}{\ell}}$$

Substituting the last expression into (11) we then have :

$$D_n = -\frac{n\pi}{\ell} B_n \frac{1+\nu}{1+\nu} \qquad \text{and hence,}$$

$$f_n = B_n e^{-\frac{n\pi y}{\ell}} \left(1 - \frac{n\pi}{\ell}\frac{1+\nu}{1-\nu} y\right) \tag{14}$$

All the solutions of the type (14) will be the solutions of (11) & (13), since the equations are linear, therefore

$$\sigma_x = \sum\left(\frac{n\pi}{\ell}\right)^2 B_n e^{-\frac{n\pi y}{\ell}} \left(\frac{3+\nu}{1-\nu} - \frac{n\pi}{\ell}\frac{1+\nu}{1-\nu} y\right) \sin\frac{n\pi x}{\ell}$$

$$\sigma_y = -\sum\left(\frac{n\pi}{\ell}\right)^2 B_n e^{-\frac{n\pi y}{\ell}} \left(1 - \frac{n\pi}{\ell}\frac{1+\nu}{1-\nu} y\right) \sin\frac{n\pi x}{\ell} \tag{3'}$$

$$\tau = \sum\left(\frac{n\pi}{\ell}\right)^2 B_n e^{-\frac{n\pi y}{\ell}} \left(\frac{2}{1-\nu} - \frac{n\pi}{\ell}\frac{1+\nu}{1-\nu} y\right) \cos\frac{n\pi x}{\ell}$$

(ii). To determine the coefficients B_n by using condition (4),

From the relations in (9) we have

$$\frac{\partial u}{\partial x} = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

Hence the displacement u, of either the plate or beam is :

$$u = \int_0^x \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) dx$$

Evaluating it at y = 0 for the plate by (3') we have then :

$$u_{plate} = -\frac{1}{E}\sum\left(\frac{n\pi}{\ell}\right) B_n \frac{3+2\nu-\nu^2}{1-\nu} \cos\frac{n\pi x}{\ell} + \frac{1}{E}\sum\left(\frac{n\pi}{\ell}\right) B_n \frac{3+2\nu-\nu^2}{1-\nu} \tag{4'}$$

Now we evaluate u for the beam as follows :

From the equilibrium condition we know that $\left(\sigma_y\right)_{beam} = \frac{t_1}{t_2}\, \sigma_{y(plate)}$

Assume that the normal stresses in the x direction are uniformly distributed over every cross-section of the beam and their value is then,

$$\sigma_x = -\frac{P - 2\int_0^x \tau\, t_1\, dx}{A}$$

where P is the total load on the end of the beam, A the area of the cross section.

Evaluating at $y = 0$ then we have

$$u = -\frac{Px}{EA} + \frac{2}{EA} \sum t_1 B_n \frac{2}{1-\nu} \cos\frac{n\pi x}{\ell} - \frac{1}{EA}\sum t_1 B_n \frac{2}{1-\nu} - \frac{1}{E}\frac{t_1}{t_2}\sum\left(\frac{n\pi}{\ell}\right) B_n \nu \cos\frac{n\pi x}{\ell}$$
$$+ \frac{1}{E}\frac{t_1}{t_2}\sum\left(\frac{n\pi}{\ell}\right)\nu B_n$$

It is well known mathematically that

$$x = \frac{\ell}{2} - \frac{4\ell}{\pi^2}\sum_{1,3,5}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x}{\ell} \quad \text{and} \quad \sum\frac{1}{n^2} = \frac{\pi^2}{8}$$

Using this mathematical expansion we get the following expression for u

$$u_{beam} = -\frac{1}{E}\sum\left(\frac{4t_1 B_n}{A(1-\nu)} + \frac{4\ell P}{A\pi^2 n^2} - \frac{t_1}{t_2}\left(\frac{n\pi}{\ell}\right)\nu B_n\right)$$
$$+ \frac{1}{E}\sum\left(\frac{4t_1 B_n}{A(1-\nu)} + \frac{4\ell P}{A\pi^2 n^2} - \frac{t_1}{t_2}\left(\frac{n\pi}{\ell}\right)\nu B_n\right)\cos\frac{n\pi x}{\ell} \quad (4')$$

Because of the displacements of the beam and plate are always equal, so the coefficients of the corresponding terms of expressions (4) and (4') should be identically equal. Then we get :

$$B_n = \frac{-\dfrac{\ell P(1-\nu)}{t_1\pi^2 n^2}}{1 + A\pi\left(\dfrac{3+2\nu-\nu^2}{4t_1\ell} - \dfrac{\nu-\nu^2}{4\,t_2\ell}\right)n}, \quad n = 1, 3, 5 \cdots$$

Put

$$B_n = \left(\frac{\ell}{n\pi}\right)^2 (1-\nu)\, C'_n .$$

Then: $$\sigma_x = \sum c_n' e^{-\frac{n\pi y}{l}} \left\{ 3+\nu - \frac{n\pi}{l}(1+\nu) y \right\} \sin \frac{n\pi x}{l}$$

$$\sigma_y = \sum c_n' e^{-\frac{n\pi y}{l}} \left\{ -1+\nu + \frac{n\pi}{l}(1+\nu) y \right\} \sin \frac{n\pi x}{l} \qquad (3'')$$

$$\tau = -\sum c_n' e^{-\frac{n\pi y}{l}} \left\{ 2 - \frac{n\pi}{l}(1+\nu) y \right\} \cos \frac{n\pi x}{l}$$

where $$c_n' = \frac{-\frac{P}{t_1 l}}{1 + A\pi\left(\frac{3+2\nu-\nu^2}{4t_1 l} - \frac{\nu-\nu^2}{4t_2 l}\right) n}$$

(d) The result.

It is well known that Mohr's circle can be used for determining the principal stresses and the maximum shearing stress if any set of the stress components σ_x, σ_y, τ_{xy} are given, or by the formulae :

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

For the problem concerned we have:

$$\begin{aligned}\tau_{max} = \frac{P}{t_1 l} \Big[& c_1^2 \left\{2 - \frac{\pi}{l}(1+\nu) y\right\}^2 e^{-\frac{2\pi y}{l}} + c_3^2 \left\{2 - \frac{3\pi}{l}(1+\nu) y\right\}^2 e^{-\frac{6\pi y}{l}} + \cdots \\ & + c_1 c_3 \left\{2 - \frac{\pi}{l}(1+\nu) y\right\}\left\{2 - \frac{3\pi}{l}(1+\nu) y\right\} e^{-\frac{4\pi y}{l}} \cos \frac{2\pi x}{l} \\ & + c_1 c_5 \left\{2 - \frac{\pi}{l}(1+\nu) y\right\}\left\{2 - \frac{5\pi}{l}(1+\nu) y\right\} e^{-\frac{6\pi y}{l}} \cos \frac{4\pi x}{l} \\ & + \cdots\cdots \\ & + c_3 c_5 \left\{2 - \frac{3\pi}{l}(1+\nu) y\right\}\left\{2 - \frac{5\pi}{l}(1+\nu) y\right\} e^{-\frac{8\pi y}{l}} \cos \frac{2\pi x}{l} \\ & + \cdots\cdots \Big]\end{aligned}$$

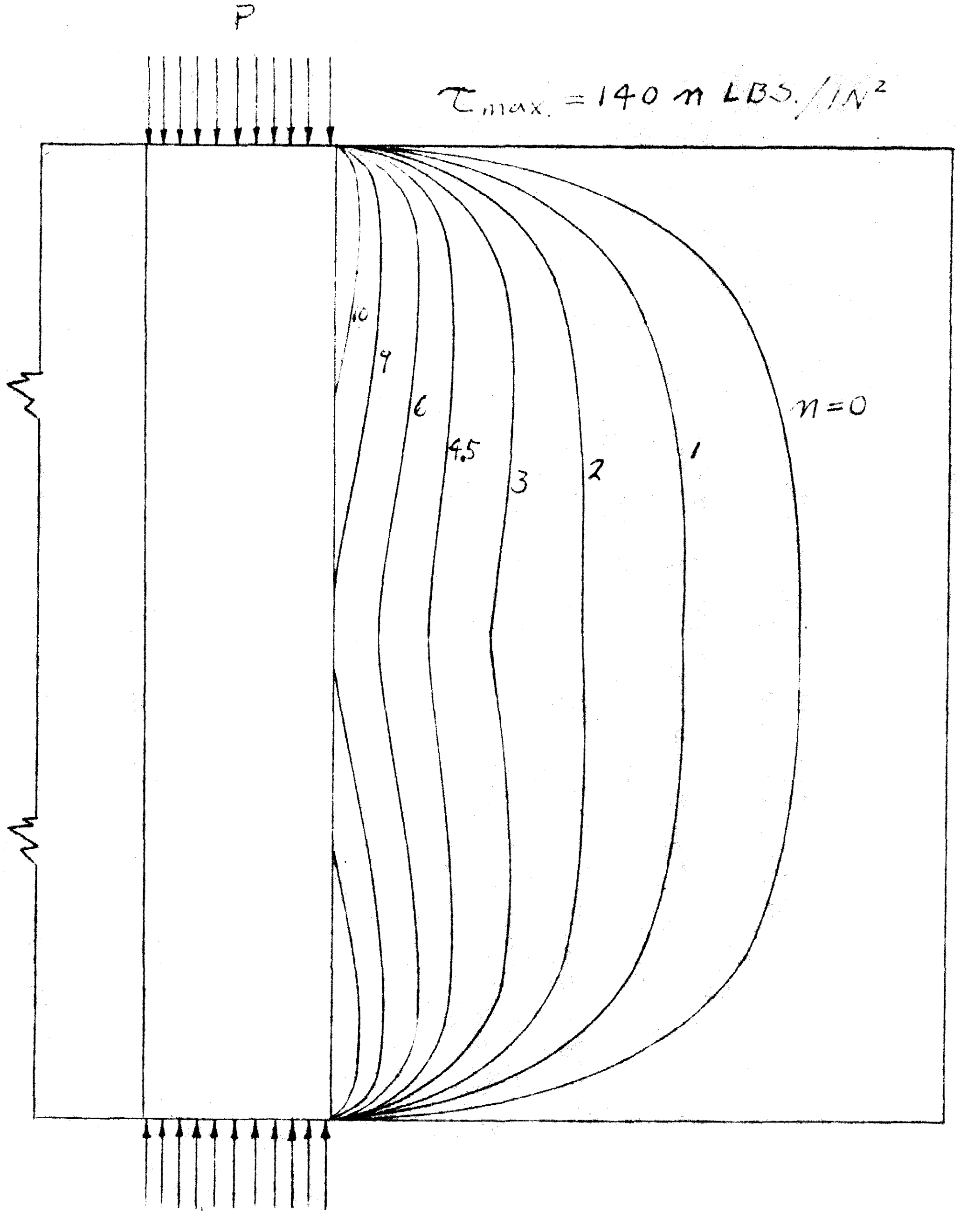
where :

$$C_n = \frac{1}{1 + A\pi\left(\frac{3+2\nu-\nu^2}{4\lambda_1 l} - \frac{\nu-\nu^2}{4\lambda_2 l}\right)n}$$

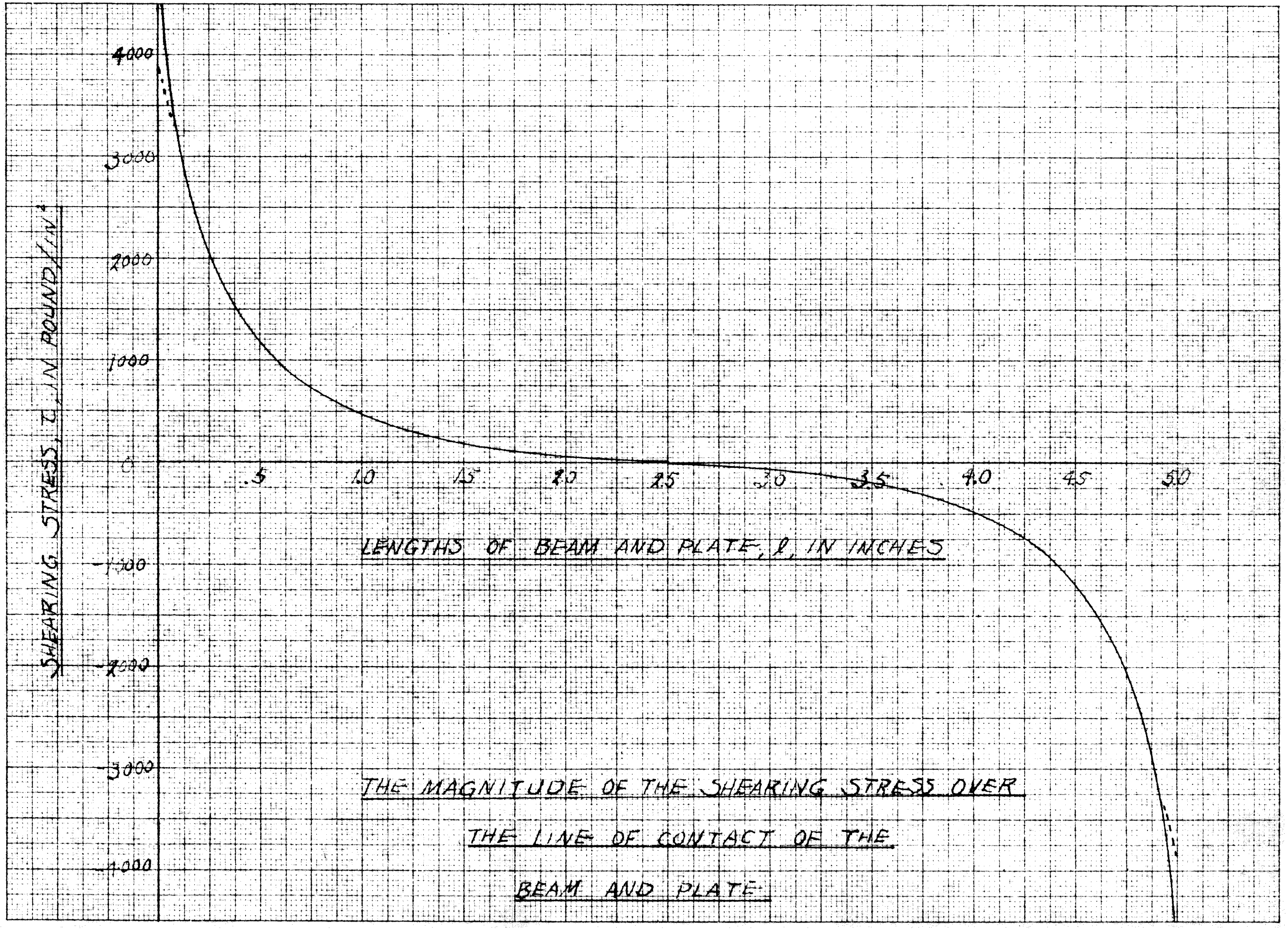
The directions of the principal stresses can be found for any particular points (x,y) by the formula :

$$\tan\ 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The direction of the maximum shearing stress for the same point lies on the plane passing this point, which makes an angle of $\pi/4$ with respect to the principal stresses, that is, on the plane bisecting the angle between the two principal stresses.

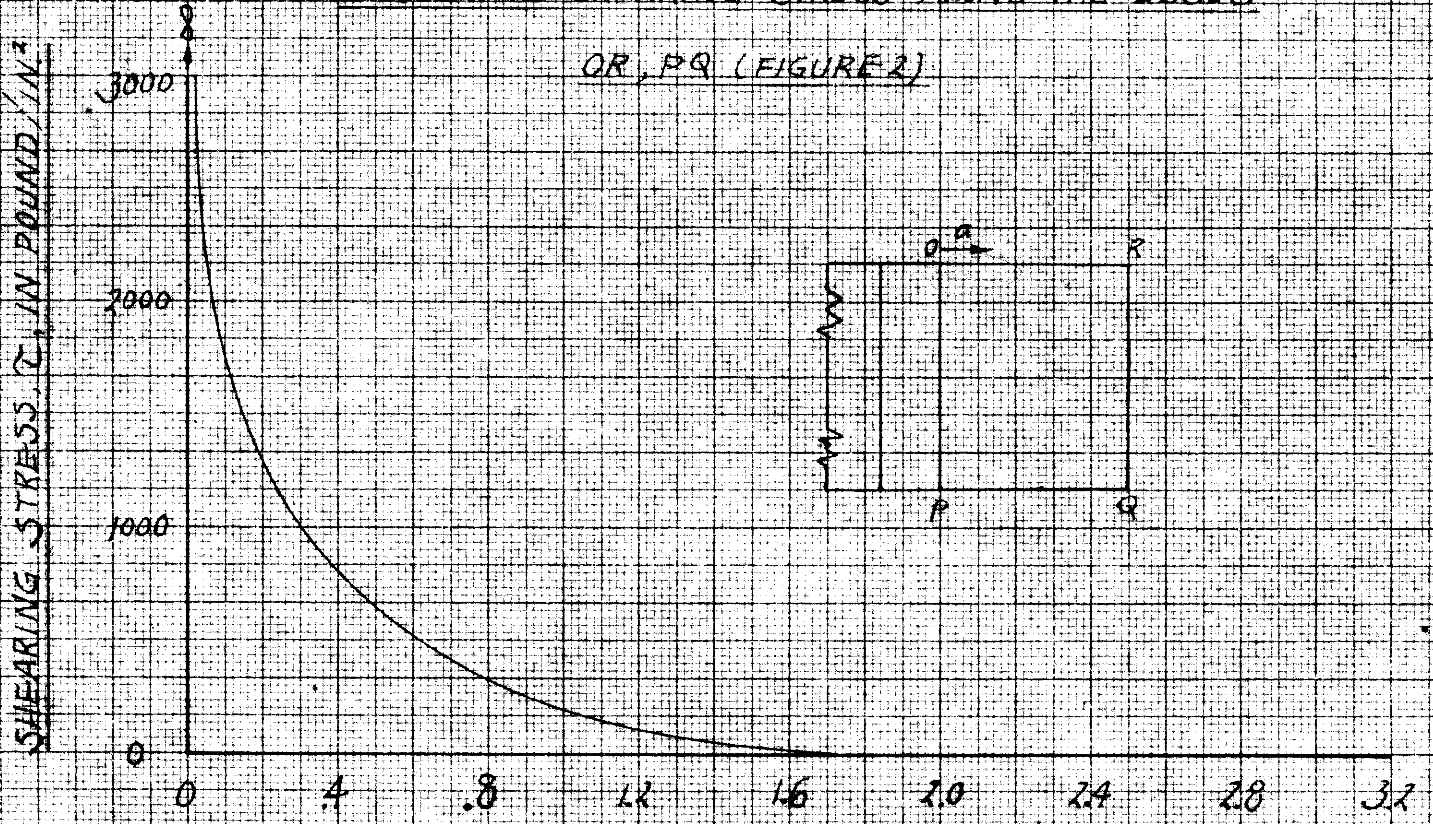


P
τmax. = 140 n LBS./IN²
10
9
6
4.5
3
2
1
n = 0
P

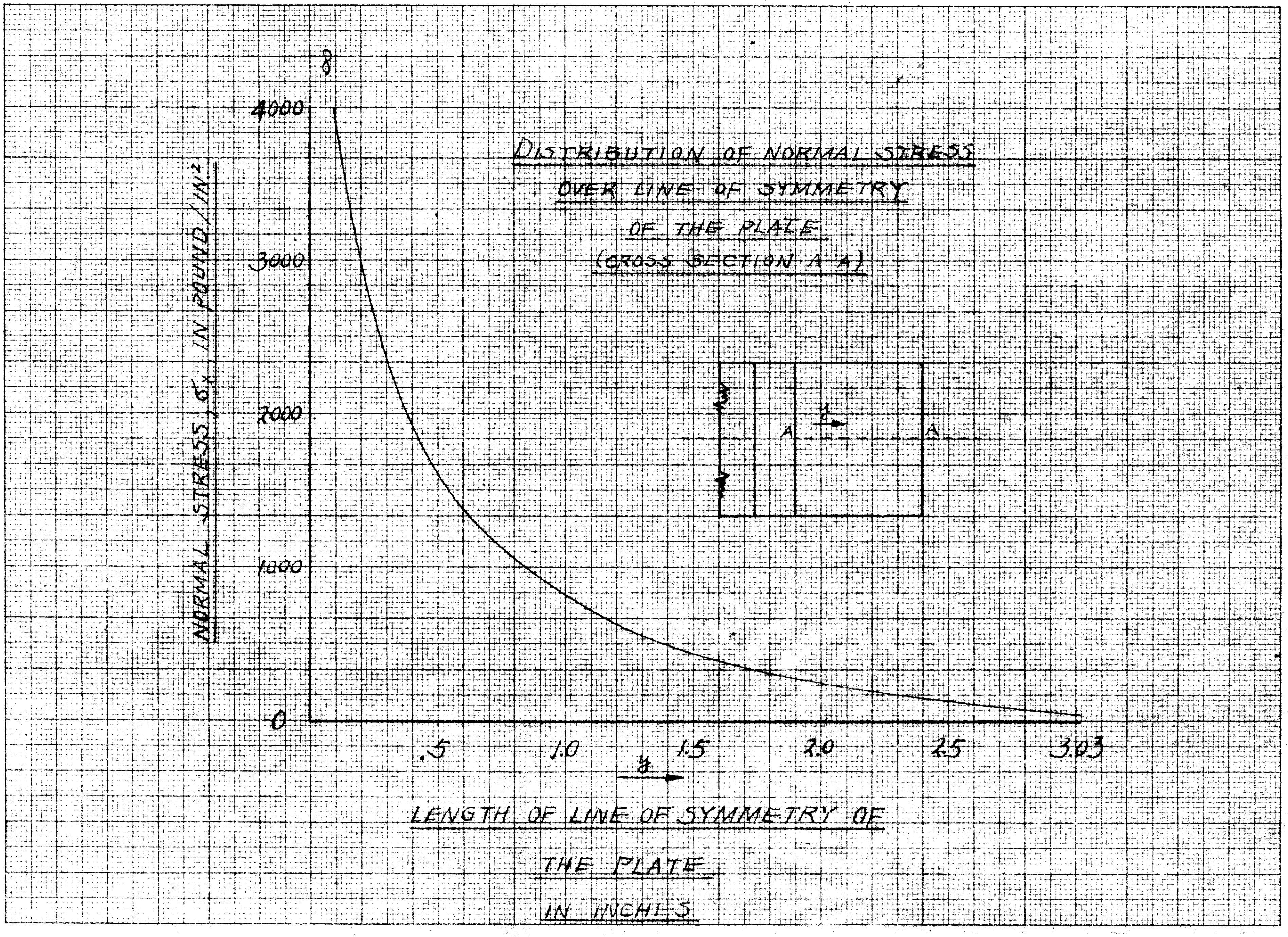

4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
SHEARING STRESS, τ, IN POUND/IN²
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
LENGTHS OF BEAM AND PLATE, l, IN INCHES
THE MAGNITUDE OF THE SHEARING STRESS OVER
THE LINE OF CONTACT OF THE
BEAM AND PLATE

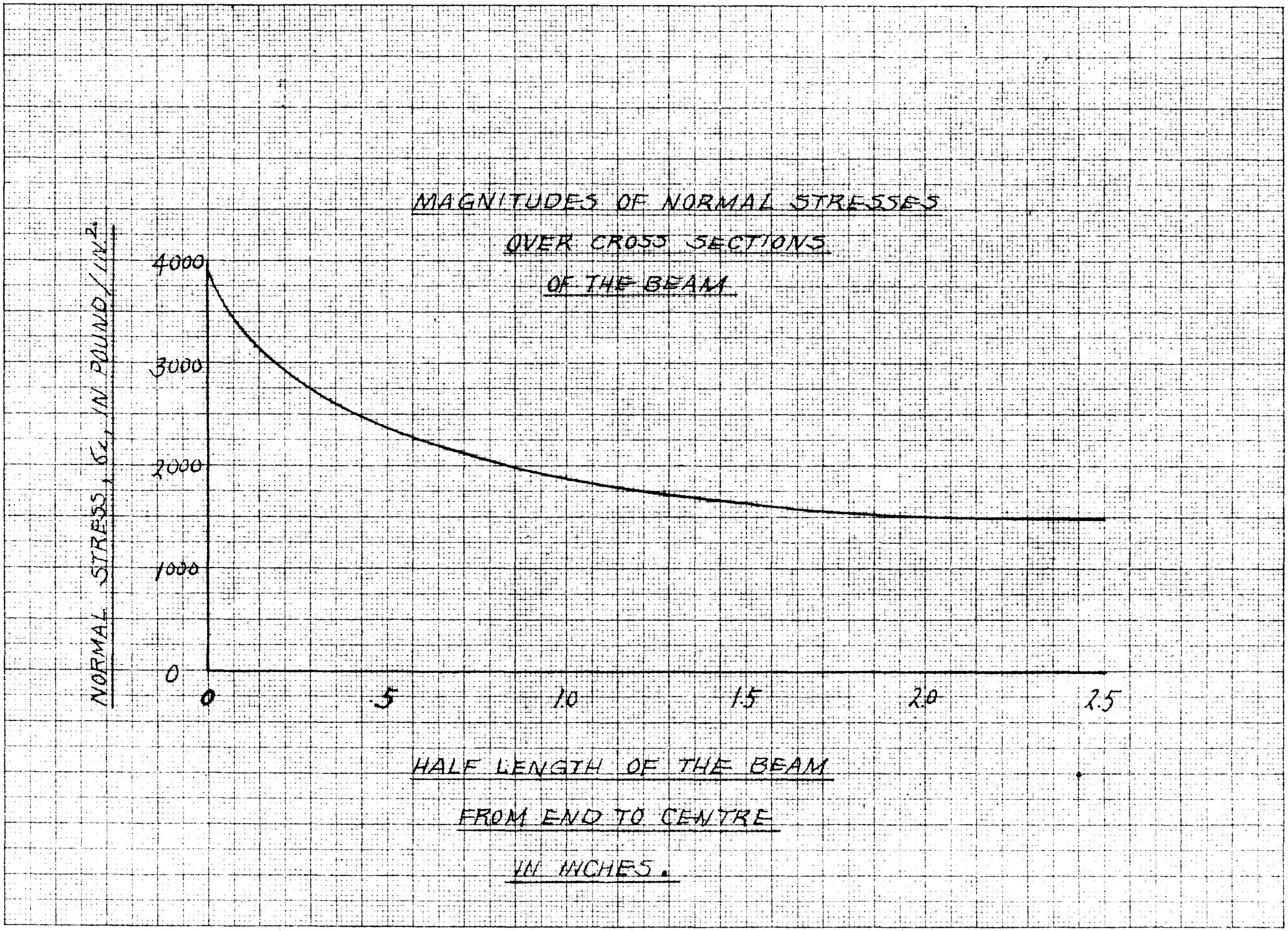
CALCULATED SHEARING STRESS ALONG THE EDGES

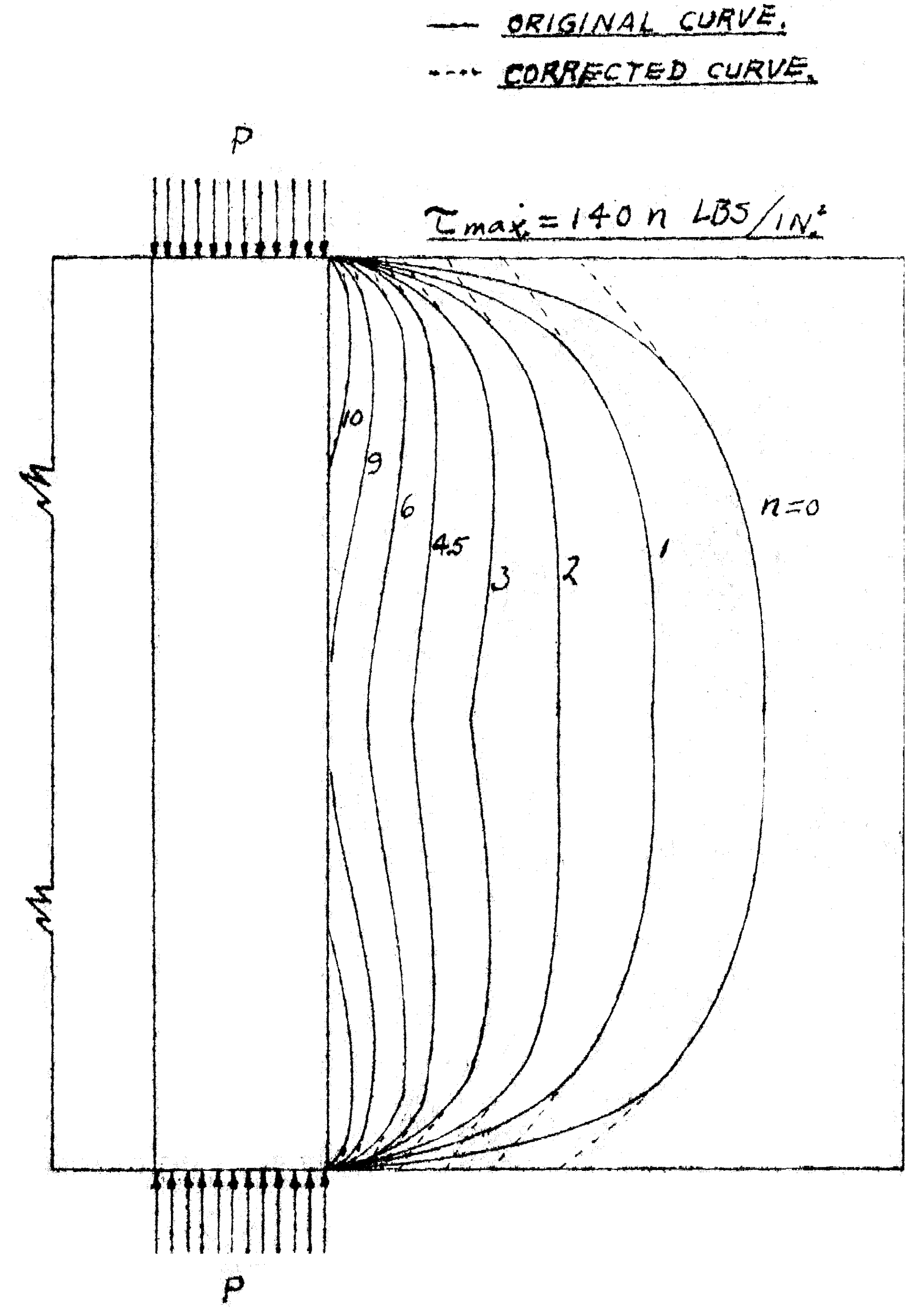
OR, PQ (FIGURE 2)



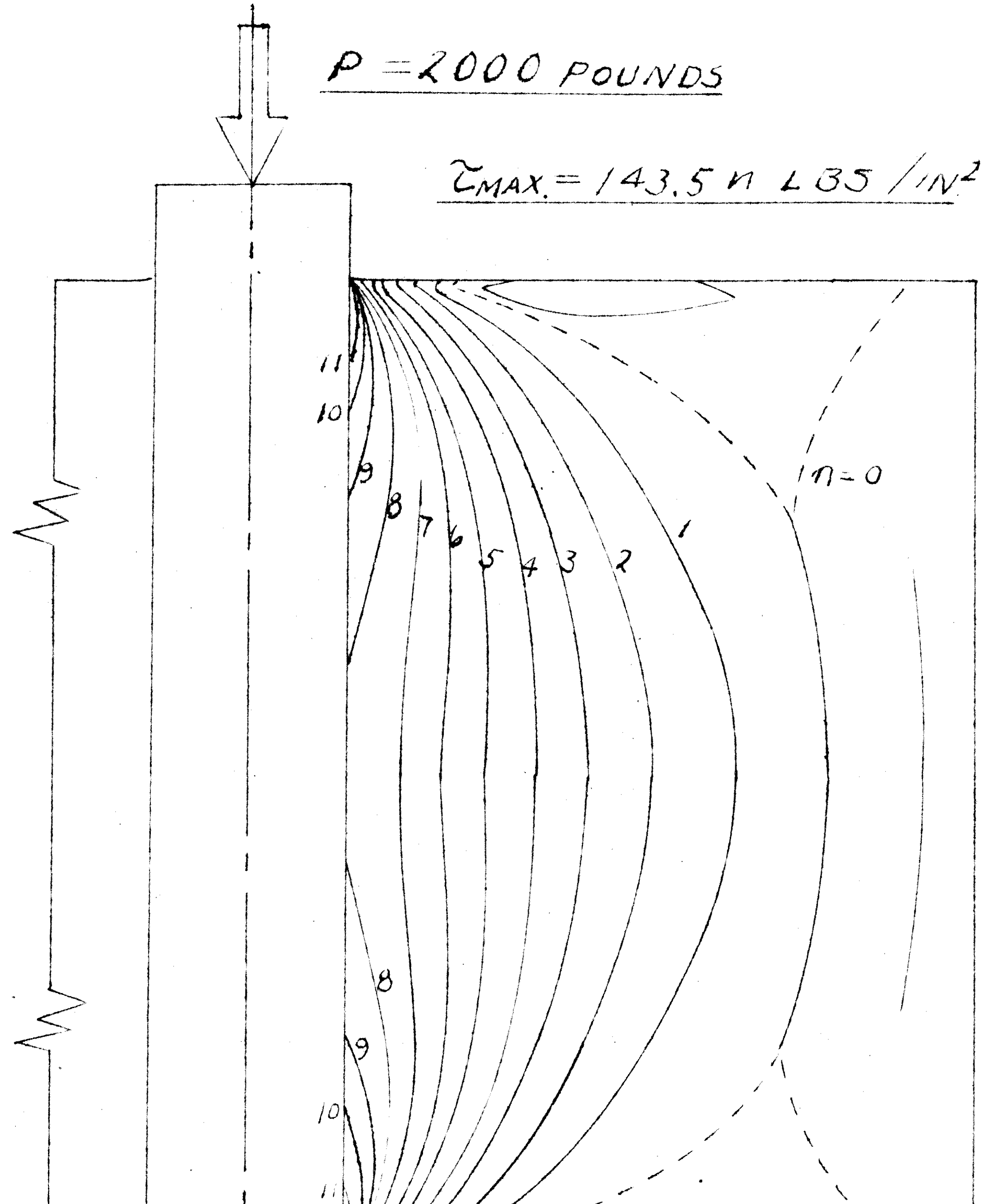
LENGTHS OF EDGES OF PLATE, a, IN INCHES.


DISTRIBUTION OF NORMAL STRESS
OVER LINE OF SYMMETRY
OF THE PLATE
(CROSS SECTION A-A)
∞
4000
3000
2000
1000
0
NORMAL STRESS, σ_x IN POUND / IN²
A
A
y
.5
1.0
1.5
2.0
2.5
3.03
y
LENGTH OF LINE OF SYMMETRY OF
THE PLATE
IN INCHES


MAGNITUDES OF NORMAL STRESSES
OVER CROSS SECTIONS
OF THE BEAM
NORMAL STRESS, σz, IN POUND/IN.²
4000
3000
2000
1000
0
0
.5
1.0
1.5
2.0
2.5
HALF LENGTH OF THE BEAM
FROM END TO CENTRE
IN INCHES.



— ORIGINAL CURVE.
---- CORRECTED CURVE.
P
$\tau_{max} = 140\,n$ LBS/IN.²
10
9
6
4.5
3
2
1
n=0
P



DISTRIBUTION OF MAXIMUM SHEARING STRESS

(FROM REFERENCE 8)

(e) Discussion

Comparing the theoretical curves of the maximum shearing stress distribution with those obtained by photoelastic method.° we find that :

1. The curvatures of the latter ones are bigger; that is the magnitudes of the maximum shearing stress distribution of the plate near the two ends of the beam decrease a little more rapidly at the points farther from the beam.

2. The curvatures of the curves obtained by the theoretical method are smaller comparatively. However the curvature increases with an increase in Poisson's ratio.

3. The magnitudes of the maximum shearing stress in the middle part of the plate obtained by these two methods closely agree.

4. On the two edges ($x = 0, \ell$) of the plate the general solution shows that τ is different from zero, but along these edges for a certain distance from the beam, τ is very small. In calculating $\tau_{max.}$ it has been assumed that on the edges τ equals zero.

5. The equation of the τ, σ_x or σ_y breaks down when it contains no factors $e^{-\frac{2\pi}{\ell}y}$ and $\cos\frac{n\pi x}{\ell}$ or $e^{-\frac{2\pi}{\ell}y}\ell\sin\frac{n\pi x}{\ell}$ simultaneously. This is the case at points y=0 and x=0 or y=0 and $x=\frac{\ell}{2}$. Since then the series which represents the τ, σ_x or σ_y is diver-

°See reference No.8

gent its value becomes infinite.

6. By an inspection of these expression (3") we can find that if ℓ approaches infinity all the values of τ , σ_x or σ_y are identically equal to zero. These are correct because at infinite distance from the points where the force is applied there is no disrtibution of stresses at all.

PART TWO

Maximum shearing stress distribution in a rectangular plate re-enforced by a stiffener with uniform load applied at the two ends of the beam and on one edge of the plate.

(a) General Assumptions.

Let a uniform normal pressure be applied along a finite length of one edge of the plate and two ends of the beam as shown in figure 4.

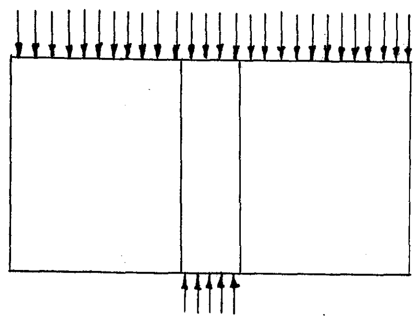


Figure 4.

The assumptions of other things and the initial shearing force applied in the section between the beam and the plate bear the same characteristics as ■ Part One.

(b) General Method

The problem stated above is dealt with by superimposing two cases:

Case (1). The uniform normal pressure acts at two ends of the beam as investigated in Part One.

Case (2). The uniform normal pressure acts at one edge of the plate. This case will now be studied.

We can assume that RQ is merely a line in an infinite plate as in figure 5.

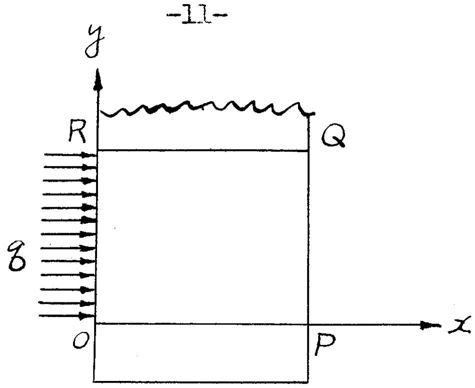


Figure 5.

The stresses at any point of the plate is with the proper units, given by the stress function*

$$\phi = \frac{q}{2\pi}\left(r^2\varphi - r'^2\varphi'\right) \qquad (15)$$

where (r, φ), $(r' \varphi')$ are polar coordinates at the origins O and R' respectively.

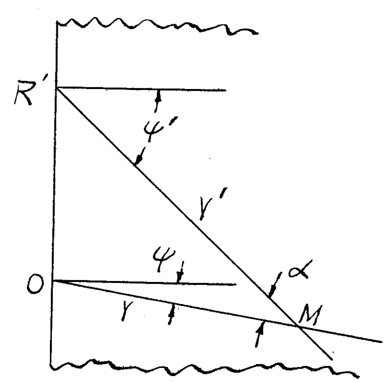


Figure 6.

We see that at any point M of the plate the first term of the stress function ϕ, that is $\phi_1 = \frac{q}{2\pi} r^2 \varphi$, at the origin O gives :

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* The solution of this problem is due to J.H. Michell, Proc. London Math. Soc., Vol. 34, p. 134, 1902.

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \psi^2} = \frac{q}{\pi}\psi$$

$$\sigma_\psi = \frac{\partial^2 \phi}{\partial r^2} = \frac{q}{\pi}\psi$$

$$\tau_{r\psi} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \psi}\right) = -\frac{q}{2\pi}$$

Similarly $\phi_2 = -\frac{q}{2\pi} r'^2 \psi'$ at the origin Q' gives

$$\sigma_r' = -\frac{q}{\pi}\psi'$$

$$\sigma_\psi' = -\frac{q}{\pi}\psi'$$

$$\tau_{r\psi}' = \frac{q}{2\pi}$$

The directions of the principal stresses bisect the angle between the radii r and r'. The magnitudes of the total principal stresses at any point M are :

$$\sigma_1 = -\frac{q}{\pi}(\alpha + \sin\alpha)$$

$$\sigma_2 = -\frac{q}{\pi}(\alpha - \sin\alpha)$$

where $\alpha = \psi' - \psi$

Hence $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)$

$$= -\frac{q}{\pi}\sin\alpha.$$

(c) Boundary Conditions

(1) The uniform normal load along OR is q, that is ,

$$\sigma_\psi + \sigma_\psi' = \frac{q}{\pi}(\psi - \psi') = q$$

It is reduced to : $\psi - \psi' = \pi$

(2) The rest of the boundary is free from the stress.

(d) Transformation of Coordinates.

For superimposing cases (1) and (2) we should have them in the same coordinate system. It is evident that this can be easily done by using a rectangular coordinate system. We therefore choose the rigin as in Figure 7.

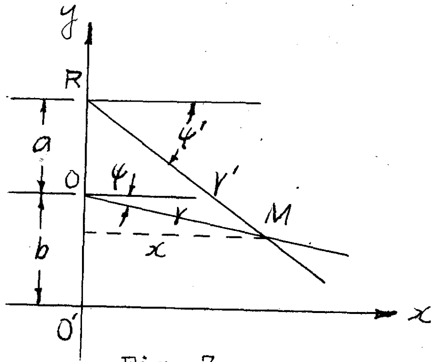


Fig. 7.

From this figure we have.

$$x = r\cos\varphi$$
$$b - y = r\sin\varphi$$
$$x = r'\cos\varphi'$$
$$a + b - y = r'\sin\varphi'$$

Substituting them into the stress function (15), we then have :

$$\phi = \frac{g}{2\pi}\left\{\left[x^2+(b-y)^2\right]\tan^{-1}\frac{b-y}{x} - \left[x^2+(a+b-y)^2\right]\tan^{-1}\frac{a+b-y}{x}\right.$$

Put b = o for satisfying our case, then

$$\sigma_x = \frac{g}{\pi}\left\{\tan^{-1}\frac{-ax}{x^2-y(a-y)} - \frac{xy}{x^2+y^2} - \frac{x(a-y)}{x^2+(a-y)^2}\right\}$$
$$\sigma_y = \frac{g}{\pi}\left\{\tan^{-1}\frac{-ax}{x^2-y(a-y)} + \frac{xy}{x^2+y^2} + \frac{x(a-y)}{x^2+(a-y)^2}\right\}$$
$$\tau_{xy} = \frac{g}{\pi}x^2\left\{\frac{1}{x^2+(a-y)^2} - \frac{1}{x^2+y^2}\right\} *$$

* See appendix

$$\sigma_{1,2} = -\frac{\gamma}{\pi}\left[\tan^{-1}\frac{ax}{x^2-y(a-y)} \pm \frac{ax}{\sqrt{a^2x^2+\{x^2-y(a-y)\}^2}}\right]$$

$$\tau_{max} = -\frac{\gamma}{\pi}\frac{ax}{\sqrt{a^2x^2+\{x^2-y(a-y)\}^2}}$$

(e) Result for τ_{max}

In superimposing cases (1) and (2), we have the required result :

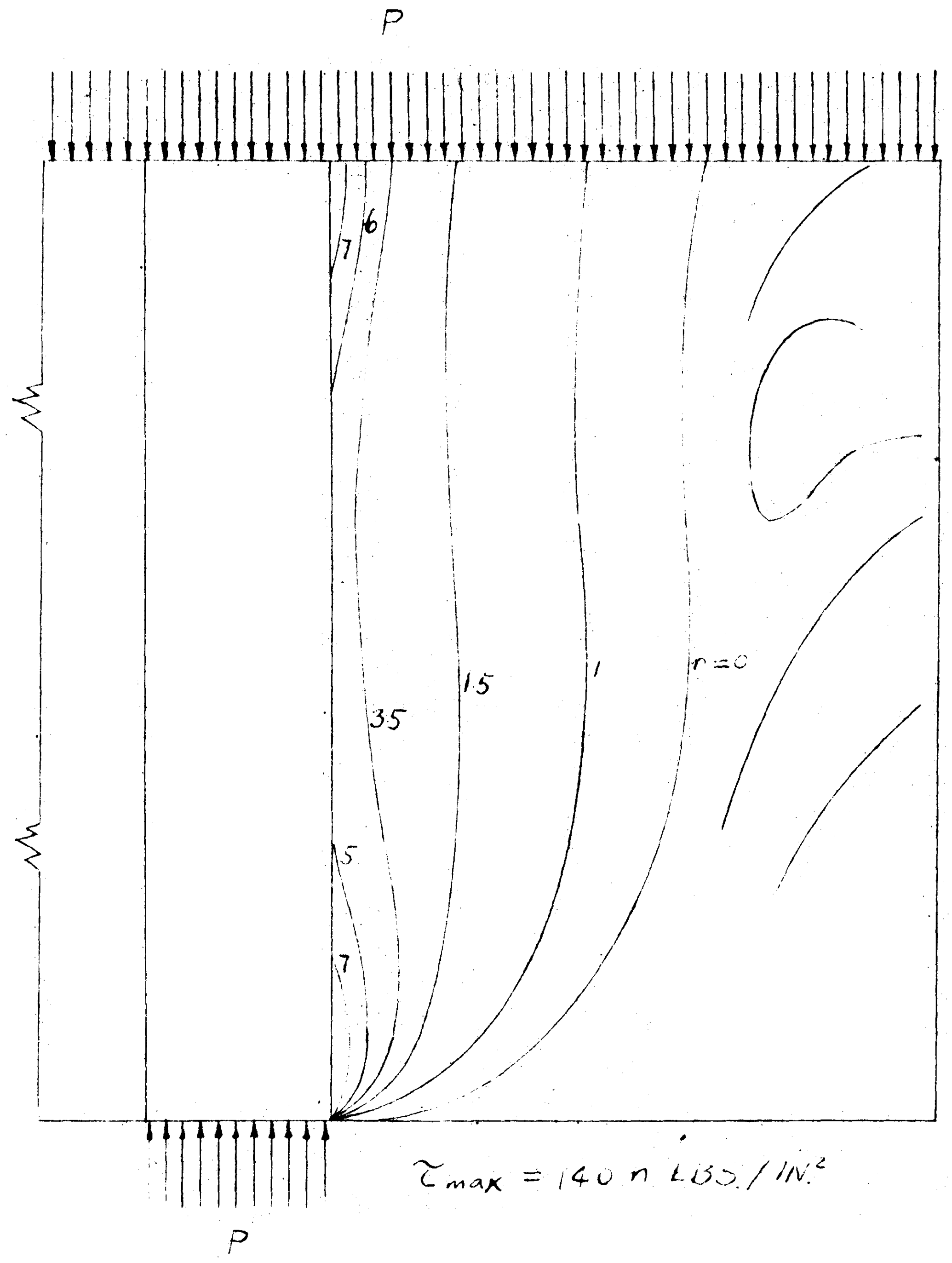
$$\tau_{max} = \tau_{max.(part\,1)} - \frac{\gamma}{\pi}\frac{ax}{\sqrt{a^2x^2+\{x^2-y(a-y)\}^2}}$$

(f) Discussion.

1. As a whole the magnitude of the maximum shearing stress obtained is smaller than that of the maximum shearing stress obtained by the photo-elastic method.*

2. The maximum shearing stress of the upper part of the plate as drawn in the curves is larger near the edge and smaller for the lower part of the plate around the beam, than that obtained experimentally.

* See Reference 8.


P
6
7
1.5
1
n=0
.35
.5
.7
P
$\tau_{max} = 140\, n$ LBS./IN.^{2}

Appendix.

Since it is mathematically true that :

$$\frac{x^2+(a+b-y)^2}{x^2+(a+b-y)^2}-\frac{x^2+(b-y)^2}{x^2+(b-y)^2}=0$$

$$\therefore\quad \frac{x^2}{x^2+(a+b-y)^2}-\frac{x^2}{x^2+(b-y)^2}=-\frac{(a+b-y)^2}{x^2+(a+b-y)^2}+\frac{(b-y)^2}{x^2+(b-y)^2}$$

Hence there are three forms for expressing the shearing stress of the second problem, these are :

$$\tau_{xy}=-\frac{g}{\pi}x^2\left\{\frac{1}{x^2+(a+b-y)^2}-\frac{1}{x^2+(b-y)^2}\right\}$$

$$\tau_{xy}=-\frac{g}{\pi}\left\{-\frac{(a+b-y)^2}{x^2+(a+b-y)^2}+\frac{(b-y)^2}{x^2+(b-y)^2}\right\}$$

and

$$\tau_{xy}=-\frac{g}{2\pi}\left\{\frac{x^2-(a+b-y)^2}{x^2+(a+b-y)^2}-\frac{x^2-(b-y)^2}{x^2+(b-y)^2}\right\}$$

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