Stochastic Bargaining Theory and Order Flow

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Dedication

To Kikuma Kato

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[1]MUDA stands for Mutiple–Unit Double Auction. This software was used to run experiments that are described in Chapter 5 of this dissertation.

and still is in many sense, beyond my cognition. I had subconsciously refused to face the presence of any negativity in humans for a long time in search of my *green house.* I was one of many caterpillars that are climbing up a mystery tower of infinite height without questioning. I believed that there would be an ideal world of happiness somewhere in the tower that is rid of any known negative feelings, such as anger, hatred, and jealousy, that human beings are prone to possess. Such an ignorance has cost me a significant amount of pain both tangibly and intangibly. I am grateful, however, to have experienced this stage, so that the damage, I believe, has been minimized. In this process of enlightenment I have learned to appreciate my life far more than I used to. Throughout the difficult times, I have been exceptionally fortunate to be surrounded by people whom I can call, with confidence, *true* friends. They are the ones who have kept my faith in human beings alive and have provided me with unconditional friendship, without which this dissertation would not have existed, and I would like to thank them here:

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Abstract

This thesis is composed of two parts, each of which reflects our attempt to describe order flow determinants in a bilateral and multilateral trading environment, respectively.

In Part I of this research, we investigate noncooperative bilateral sequential bargaining games in which the value of the asset changes stochastically according to a sequence of perfectly observable time–varying random variables. We attempt to model scientific speculations of the game participants that lead to varied length of bargaining durations. Previous studies, which have focused on the analyses of incomplete information games in interpreting bargaining delays, have shown that such delays are attributed to information asymmetry on asset values among players that results in differences in players' personal valuation of the asset. However, following the viewpoint of the Efficient Market Hypothesis, we assume in our models that there is no uneven assimilation of information of vital importance that affects the asset value once the players are at a negotiating table. Hence, one of the important features of the investigated models is that both players observe identical information regarding the future asset value, and that there is no uncertainty regarding one's opponent's preferences during the bargaining process. Despite the assumption of complete information, we argue that a delay before an agreement under certain conditions is an inevitable consequence of the stochastic component in this model.

We give game theoretic specifications for two types of bargaining games, which we call the Basic game and the Alternative game. The two games differ from each other in their timing of information arrivals with respect to players' actions. We characterize their subgame perfect equilibria that follow our particular behavioral assumptions. Characteristics

of the equilibrium outcomes of the two games are compared. We direct special attention to the study of the analytical results in comparison with those of Rubinstein (1982), Osborne and Rubinstein (1990), and Merlo and Wilson (1995). We then give statistical specifications for two types of stochastic bargaining simulations, which are the Autoregressive Binomial Model and the Generalized Wiener Process Model. Comparative statics of several variables and bargaining durations are investigated thoroughly through numerous simulation runs. Subsequently, through our research we clarify the importance of integrating stochastic concepts into the bargaining theory and its applications in search of alternative explanations for various bargaining durations.

In Part II of this research, we provide a set of experimental results in our study of order flow determinants in experimental financial markets with asymmetrically informed human subjects. The markets are organized as computerized double auctions accommodated with an order book that contains a complete set of current limit and market orders and that can be inspected by every market participant at any time during each trading period. Our empirical analysis focuses on the series of actions taken by the subjects that include quote revisions, limit order arrivals, and trades. At first, we report thorough descriptive statistics on the extracted data sets, where we do not assume any particular theory of the market microstructure. Then we show serial dependencies of order flow on the previous event type, the state of the order book, the size of bid–ask spread, and the time intervals. In so doing, we ascertain the significance of the impact of information carried in the order book.

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Part I

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Chapter 1

Introduction

Part I of this research addresses the problem of resource allocation among two parties in a dynamic context, where both parties have bargaining power, thereby significantly influencing the final outcome. The process of negotiation is modeled as a noncooperative sequential bargaining game in which the two parties alternate making offers over a share of a single divisible asset. We introduce a bilateral bargaining model in which the value of the asset changes stochastically according to a sequence of perfectly observable time-varying random variables. Then we give statistical specifications for two types of the stochastic bargaining games to generate numerical examples. We direct special attention to the duration of negotiation processes in the analysis of the simulation results.

Bargaining among parties of opposing interests constitutes a wide range of negotiation processes in economic, political, and legal spheres. To rival firms negotiating over market share, bargaining may be a process to achieve an agreement on a production level. In labor dispute, bargaining may be necessary to reach an agreement on wage levels. In the context of international trading, bargaining may indicate a negotiation over an import-export quota among nations. Yet another example can be found in a process of public policy making among opposing political parties, between executive and legislative branches, or between the authorities and the general public. Likewise, bargaining describes an intrinsic aspect in the process of solving disputes when all the participants possess non–trivial influence over final outcomes.

1.1 Bargaining Delays and Information

Bargaining may break down, that is, the negotiating parties may never come to an agreement and may quit making further efforts to reach an agreement. It is not surprising to observe such breakdowns in cases where the parties can find better outside alternatives. If the outside alternatives are not available or costly to search for, and if the parties recognize positive gains from reaching an agreement in the current negotiation, we would expect that the rational participants would prefer an agreement to a breakdown. Such an agreement is often not achieved immediately after the negotiation begins. It is frequently accomplished only after a significant delay even when the participants are aware of costs associated with the bargaining duration. This delay becomes an unavoidable source of inefficiency when similar or better terms could have been realized in the earlier stage of negotiation. On August 13, 1992, the Century Plaza Hotel in Los Angeles was crowded with bidders from the United States, Europe, and the Pacific Rim. They were there to participate in an auction for real estate assets that the American banks were trying to unload. The result was quite disappointing to the banks, for barely half the properties were sold. Was this the issue of supply–and–demand imbalance? What is it that caused the buyers to balk? Was it simply a matter of pricing? We are interested in analyzing how the bargaining participants' contention for larger gains could result in the bargaining delay.

In the related sequential bilateral bargaining literature,[1] where a seller and a buyer are bargaining over a single indivisible asset, several attempts have been made in identifying conditions that require or cause delays in reaching an agreement. The central assumption in those models was the information asymmetry among the participants. For example, let us consider a case in which a seller's valuation is common knowledge and a buyer's valuation is known only to the buyer. Suppose that they alternate in quoting asks and bids over an indivisible asset. After observing the seller's ask price, the buyer can delay to signal that his valuation is low before quoting his bid. The buyer may choose to delay even longer to convince the seller that his signal is credible (Admati and Perry (1987)). Similar results have been obtained for the case where both the seller and the buyer have private information about the value of the asset. Let us now suppose that only a seller makes an offer and a buyer responds by accepting or rejecting the offer. For example, consider a case of monopoly pricing in which the buyer's valuation is known only to the buyer. A seller naturally charges a price that is higher than the marginal cost, and a buyer will not accept the offer if his valuation is lower than the monopoly price. But the transaction would have been efficient if the buyer's valuation is higher than the marginal cost. The seller may employ a screening strategy to find out the buyer's valuation by making successively lower offers until the buyer accepts. In other words, delay becomes a part of seller's screening strategy. In such cases, however, the delay disappears as the time interval between the offers converges to zero, allowing the seller to make offers frequently (Gul, Sonnenschein,

[1] A detailed review of the literature is included in the next section.

and Wilson (1986), and Ausubel and Deneckere (1989)). In addition, if a buyer has an incentive to reject an offer that is lower than his valuation in the hope of receiving an even better offer, similar inefficiencies may result (Myerson and Satterthwaite (1983)).

Consequently, those studies have shown that delay is attributed to information asymmetry on the asset value among players, which results in differences in players' personal valuation of the asset. It is, however, unreasonable to assume that the information of vital importance which affects the value is not publicly held. Decisions and actions taken by major financial intermediaries are visible to investors, and any non–trivial information travels to major financial centers all over the world along electronic pathways immediately after it leaks out. Following the viewpoint of the Efficient Market Hypothesis, in our bargaining models we assume that there is no *uneven* assimilation of information among players once they are at a negotiating table. This assumption indicates that one player cannot take advantage of poorly informed players by identifying arbitrage opportunities resulting from informational asymmetry. Hence, one of the important features of the investigated model in the following chapters is that both parties observe identical information regarding the value of the asset and that there is no uncertainty regarding one's opponent's characteristics during a bargaining process. We conjecture that even without information asymmetry bargaining delay is generated due to the players' contention for larger gains based on their speculation in changes of the asset value in the future.

1.2 Theoretical Literature on Bargaining Delays

In Rubinstein's (1982) sequential bargaining game of complete information with an alternating offer process, it is well–known that there is a unique subgame perfect equilibrium in which the first offer is immediately accepted by a responder.[2] In other words, the agreement is reached in the first period without any delay. Such a result is not implausible due to the assumption that every participant is completely informed about all aspects of the bargaining procedure. Previously studied game–theoretic formulations incorporating incompletely informed player(s) have indicated that models with informational disparities must be investigated in order to identify main causes of delays before reaching an agreement.

A typical sequential bargaining game of incomplete information that has attracted much attention is the one with one–sided incomplete information and assumes that a seller's reservation value is common knowledge and that a buyer's valuation is his private information. Such works include those by Sobel and Takahashi (1983), Cramton[3] (1984), Rubinstein[4] (1985), Fudenberg, Levine, and Tirole (1985), Grossman and Perry (1986), Gul, Sonnenschein, and Wilson (1986), Admati and Perry (1987), Gul and Sonnenschein (1988), and Ausubel and Deneckere (1989). Rubinstein studies an alternating offer game, while the

[2]Suppose that, in Rubinstein's famous bilateral pie–sharing bargaining game, players have time preferences with constant discount factors such as δ^A for player A and δ^B for player B, where $\delta^i \in (0,1)$. Then its predicted unique subgame perfect equilibrium is the first proposer A making an offer such that

$$x^A = \left(\frac{1-\delta^B}{1-\delta^A\delta^B}, \frac{\delta^B-\delta^A\delta^B}{1-\delta^A\delta^B}\right),$$

which is accepted immediately by a responder B. Whenever we refer to the solution of Rubinstein's bargaining model later in this dissertation, we mean the above result.

[3]Cramton (1984, 1987) extends the model into a two–sided uncertainty case. Fudenberg and Tirole (1983) also studied a two–sided incomplete information case for two–period bargaining games.

[4]Rubinstein assumes that the buyer's discount factor or both his valuation and discount factor are private information.

others mainly look at a process where only the seller makes offers.[5] They usually suppose that the seller makes offers at discrete times with the interval Δ, that is, $t = 0, \Delta, 2\Delta, \cdots$, and so on. Such one–sided offer games are often referred to as screening models, since the seller quotes successively lower offers to sort the possible buyer's types. Decreasing the offer prices over time until the buyer's acception is intuitive, since the seller can infer the buyer's valuation to be low after repeated rejection by the buyer, assuming that the buyer waits for an offer smaller than his reservation value and accepts it if the cost of another delay is larger than the anticipated next lower offer. Though each model investigated in the above strategic approach specifies an extensive form that is different from each other, resulting delay can be explained by this screening strategy.

Delays generated in this manner, however, have been found to disappear as the time interval between offers becomes arbitrarily small(Gul and Sonnenschein (1988), and Gul, Sonnenschein, and Wilson (1986)). In other words, significant delays result only if offers are made infrequently or due to the agent's inability to make offers quickly. As the Coase conjecture states, one party's making offers frequently encourages its opponent to wait for an even better term for himself without a significant time loss, leading to a quick agreement with a favorable term for the opponent. Then, what would be necessary to have a significant delay in a screening model? The offer making agent has to avoid making offers frequently in order to convince its opponent that he is not going to make offers that are more favorable to the opponent than the current offer. To show that he is unlikely to compromise, he may use

[5]Grossman and Perry present numerical simulations for both alternating offer games and one–sided offer games. They show how to embed the one–sided offer equilibrium into the alternating offer process using beliefs that assign weight only to types in the interval support of the buyer's calculation. Fudenberg, Levine, and Tirole, and Ausubel and Deneckere also discuss both cases.

a significant delay before making the next offer. Hence, screening models have to assume an incentive for reputation building in order to explain significant delays observed in practice. A finding of Admati and Perry (1987) in an alternating offer framework is consistent with this argument.[6] They model an alternating offer game in which the time between offers is an endogenous strategic variable, and find that delays caused in such a model do not vanish as the time between offers approaches to zero.

In alternating offer bargaining games, strategies are more like that of two–sided signaling than one–sided screening(Admati and Perry (1987)). Delays are used as a signaling device to communicate one's type or valuation in a credible manner to his opponent. Let us first consider a one–sided uncertainty case, in which the seller's valuation is common knowledge and the buyer's valuation is either high or low. After receiving an offer from the seller, the buyer now can use a longer duration to signal that his valuation is low and finally make a counteroffer that is lower than the seller's offer. A low valuation buyer has to stall a sufficiently long time to eliminate the seller's wrong suspicion on his valuation. Hence, if the buyer has low valuation, he would have to choose a duration and a counteroffer that would have been unprofitable to him if he had high valuation. Delay here is a necessary consequence of incomplete information. In two–sided uncertainty models a similar interpretation can be provided to explain delays.

We have to notice, however, that in those models delays are generated by time intervals, leaving the relationship between delays and uncertainty unclear. Cho (1990) explicitly proved that the presence of uncertainty over the gain from trading is a necessary condition

[6] Ausubel and Deneckere (1988) show the condition to have delay even if the time interval between offers converges to zero in a seller–offer game with two–sided uncertainty.

for delay.[7] Cramton (1984) showed that the more uncertainty present, the less efficient the bargaining outcome is; that is, the bargaining results in costly delays.[8] A more detailed survey of sequential bargaining games with incomplete information is provided by Linhart, Radner, and Satterthwaite (1992) and by Osborne and Rubinstein (1990).

Though the models with asymmetric information have helped our search for the causes of costly delays, these models have not provided us with a complete list of the sources. Instead, they have indicated the necessity to investigate other sources of delays without the presence of informational disparities among the participants. Merlo and Wilson (1995) have made a creative attempt in attacking this challenge by studying a sequential bargaining game with multiple players in which both the surplus to be allocated and the identity of a person who makes an offer follow a stochastic process. Their model is that of complete information, so that any delay generated in this model is not due to informational asymmetry, but rather due to each player's speculation over the gain in the future. In fact, one of the models we investigate in the following chapters is one specific case that can be incorporated into their bargaining model. They have shown the existence of subgame perfect equilibria in their bargaining game and investigated the characteristics of stationary subgame perfect equilibria in nontransferable utility.

1.3 This Dissertation

A particular type of bargaining that we investigate is a problem of surplus sharing, where the size of surplus is known to change stochastically as time proceeds. Hence, the game

[7]It is proved in Theorem 5.4 in Cho (1990).

[8]The degree of uncertainty is measured by the amount of overlap of the supports of two uniform distributions of equal length. The uncertainty is greatest when the supports are identical.

is a discrete version of adaptive control system, in which we take a random change in the environment into account. The key feature of our bargaining model is the presence of uncertainty regarding the future value of the asset due to the stochastic factor. Each player observes identical information regarding the stochastic factor and has to make contingency plans for quoting an offer because of this uncertainty. Consequently, players employ closed–loop strategies, in which they condition their actions on the history of the game up to the current period and respond optimally to the realizations of random variables.

We introduce in our concluding remarks an idea for modeling possibilities of breakdowns explicitly in a bargaining game as a future extension to our games. This is another way of incorporating a stochastic process in explaining bargaining durations. We assume that the value of the asset in the current negotiation does not change in the short term, and that players can choose to invest their resources into the search for outside alternatives while participating in the current negotiation. The realization of better options follows a stochastic process, given each player's investment level. In such a model an agreement is generated by an increasing endogenous risk of breakdowns. Despite the assumption of complete information and the cost associated with the search, we conjecture to observe a delay caused by the presence of the potential for better outside options.

Using the results of probability theory, we expand the scope of our analysis beyond the deterministic point of view. This stochastic treatment of the dynamic process of asset value variations enables us to identify a delay before reaching an agreement as an unavoidable consequence in some bargaining situations despite the assumption of perfect information. For example, in our model families of probability distributions that typically change their forms as time progresses are reflected in the players' beliefs about asset values in the future.

Such a sequence of observable random variables helps the formation of *scientific* speculations among the players, causing them to wait for better terms. We feel the need for explicitly modeling how such contentions for larger gains, resulting from an information flow, influence bargaining durations. Therefore, we must note the importance of integrating stochastic concepts into the bargaining theory and its applications.

Sequential bargaining games usually assume the existence of some sort of impatience on the part of the participants. Such impatience has been modeled as discount factors on future payoffs or as fixed per–period costs in the existing bargaining literature. The presence of discount factors is viewed to generate an incentive to come to an early agreement. We note that discounting on payoffs over time is not considered in our models. Therefore, unlike many of previously studied bargaining models, the pressure to reach an agreement in our model is generated solely by the information flow, not by the presence of discount factors.

Though the investigated game is a specific bilateral bargaining over a divisible asset, potential scope of the model is not restricted to this pie–sharing situation. For example, a slight modification of the model can provide us with interpretation of a more general type of bargaining over an indivisible asset. Suppose that two players, a seller and a buyer, are bargaining over an indivisible asset. In each period the seller and the buyer submit an ask and a bid, respectively. Realized gains to be distributed from trade are determined by the bid–ask spread, while how it is divided is determined by the transaction price. Suppose that they can decide on the transaction price and whether or not to trade *after* observing the ask and the bid. Notice that they observe identical information regarding the size of gain, the bid–ask spread.[9] They form expectations as to what the gain in the future will

[9]Observing the identical information does not necessarily mean that the both players interpret the infor-

be after observing this bid–ask spread. Therefore, if the current gain is positive but the expected bid–ask spread in the future is larger than that in the current period, the players could have an incentive to wait until later period. Delay is not generated by information asymmetry, but by each player's speculation over the gain in the future.

Hence, our principal task in Part I of this research is to provide an alternative explanation to commonly observed phenomena of costly delays in reaching an agreement in many bargaining situations. Instead of characterizing the complete set of equilibria, we look for special types of equilibria that are consistent with specified behavioral rules. By following the language of game theory, we assume that each player, i.e., a decision maker, is *rational* and *intelligent*. By rational we mean that each player consistently acts to maximize the expected value of his own payoff and that he uses Bayes' rule to update his beliefs on the state whenever necessary. By intelligent we mean that each player knows everything that we know about the structure of the game and that he can make inferences about conditions that we can make. In summary, the main contributions of this dissertation are as follows:

1. By incorporating a notion of dynamic stochastic control into a noncooperative game, we model two types of bilateral bargaining situations with the value of the asset changing stochastically. One is called the Basic Game, and the other is called the Alternative Game. The two models differ from each other in the timing of information arrival and a player's response. By using a method of backward induction, we explicitly solve for subgame perfect equilibria for the specified decision rules to the two types of stochastic bargaining games. We demonstrate that reservation values for

mation in the same manner. In fact, there has to be a difference in interpreting the information reflecting diverse characteristics of the players in order to have any trade occurrence with indivisible assets.

proposing players differ between the Basic and the Alternative bargaining games.

2. Some properties of subgame perfect equilibria are provided through the application of Merlo and Wilson (1995)'s results. For example, under given assumptions the existence of subgame perfect equilibria in both Basic and Alternative games is shown.

3. We characterize the derived equilibria especially in comparison with Rubinstein's findings on his pie–sharing bargaining game. We give several sufficient conditions for our equilibria to be unique and thus to converge to that of Rubinstein's game. We also provide necessary and sufficient conditions for such games to have a delay before an agreement. In so doing we also discuss similarities and dissimilarities of the two games.

4. We numerically simulate such bargaining games and examine the results thoroughly through comparative statics, directing special attention to bargaining durations. We design two types of simulation models. One is called the Autoregressive Binomial Model, in which the information shock depends on a random variable that follows a binomial distribution. The other is called the Generalized Wiener Process Model, in which the information shock follows a continuous distribution. Through the analyses of the two models, especially in comparison of the Basic and the Alternative games, we show the sensitivity of bargaining outcomes to the information availability.

5. We provide a set of computerized experimental results in our study of order flow determinants in experimental financial markets with asymmetrically informed human subjects. Thorough descriptive statistics are reported, and the dependency of order flow on the previous order types and a size of order books are demonstrated through

χ^2 statistics.[10]

The rest of the dissertation is organized as follows. Part I consists of four chapters that include this introduction chapter. In Chapter 2 we describe and analyze our Basic bargaining game and an Alternative game, and characterize subgame perfect equilibria. Chapter 3 defines the Autoregressive Binomial Model and the Generalized Wiener Process Model, and provides the results of computer simulations to describe effects of parameter changes in the bargaining games analyzed in Chapter 2. We conclude Part I in Chapter 4, where we give an example of possible extension that models the effect of endogenous risk of breakdowns in bargaining. Bibliography for Part I is attached after Chapter 4. Part II consists of Chapter 5, and its detailed introduction and brief concluding remarks are contained in the chapter. Thorough descriptive statistics on the results of experimental financial markets are given in Chapter 5, and the dependency of the order flow on the previous order types and the size of order books are demonstrated. Bibliography for Part II is attached after Chapter 5. Related Appendices follow after each relevant chapter.

[10]Part II is devoted to this task. Although both parts direct their attention to a stochastic nature of information flow, the underlining models in each part are very different from one another. The models in Part I incorporate a stochastic nature of external information flow that affects bargainers' beliefs on the value of divisible assets, whereas the implicit model underlining the experiments in Part II reflects a stochastic nature of internal information assimilation process in a competitive market, where a bargaining strategy is not a primary concern of the traders. Therefore, traders' speculative behavior is interpreted from a different perspective in each part. It should also be noted that the games in Part I are those of complete information with no asymmetric information among the players, whereas players in the experiments in Part II are asymmetrically informed. Hence, bargaining durations in Part I are attributed to differences in players' beliefs, while the durations between trades in Part II reflect an information availability.

Chapter 2

Bargaining with Sequential Information Arrival

2.1 The Model

We begin with a basic sequential bilateral bargaining game of perfect information with a commonly known finite horizon. The environment is modeled as a discrete–time game. Two risk neutral players, indexed by $i \in I = \{A, B\}$, are bargaining on the partition of a single divisible asset.[1] The value of the asset in period t, i.e., the gains to be distributed if the agreement is reached in period t, is denoted by Q_t and changes stochastically over time according to a sequence of time–varying random variables $\{\delta_\tau\}_{\tau=1}^{t}$. We assume that Q_0 is positive, so that the asset is desirable for the players to begin the negotiation. The asset will be divided only after two players come to an agreement. δ_t is positive and relates the

[1] We often treat player A as female and player B as male purely for convenience' sake. Therefore, we refer A as she and B as he. We use "he" for generic individuals.

value of the asset in period $(t-1)$ to that in period t linearly; that is,

$$Q_t = \delta_t Q_{t-1}.$$

It describes a set of stochastic constraints to each player's expected payoff maximization behavior.

Let $f(\cdot)$ describe a state transition equation specifying how δ_t evolves with time:

$$\delta_{t+1} = f_t(\delta_t, \varepsilon_t).$$

It is a first order stochastic difference equation, and thus $\{\delta_\tau\}_{\tau=1}^{t}$ is a first order Markov process. Note that the current state δ_t is a sufficient statistic for predicting future states. As an argument of f_t, it is assumed that there exists an exogenous random variable, ε_t, that causes the transition from δ_t to δ_{t+1} to be stochastic, and that a sequence of such random variables, $\{\varepsilon_t\}$, is a stationary (i.i.d.) process and is independent of δ_t. We assume that the δ_t is perfectly observable by both players at the beginning of period t. Let us define a product space Ω such that $\Omega = \prod_{t=0}^{T} \Omega_t$, where

$$\delta_t \in \Omega_t$$

and

$$\delta = (\delta_0, \delta_1, \cdots, \delta_T) \in \Omega.$$

Then Ω^t defines the set of all sequences of length t of elements in a space of feasible δs of each period t. Hence, $\{\delta_\tau\}_{\tau=0}^{t} \in \Omega^t$, or simply $\delta^t \in \Omega^t$.

Let X be a product space of the sets of feasible share vectors in each t such that $X = \prod_{t=0}^{T} X_t$. Let us also define a share vector in period t as $x_t = (x_t^A, x_t^B)$ where $x_t \in X_t$. Let π_t be a payoff vector in period t, $\pi_t = (\pi_t^A, \pi_t^B)$, which will be defined for each game later in this section. A bargaining outcome, (π_t, t), describes the allocation of realized gains and the period number in which the bargaining ended. Implicitly assumed is that players care only about the resulting payoff scheme and the time of agreement, not about the history of the game that leads to the agreement. We also impose the non–negativity constraints such that

$$x_t^i \geq 0, \ \forall t,$$

where x_t^i is player i's offer in period t, or a share of Q_t that player i wants to take in period t if an agreement is reached. Note that with this formulation we allow players to make a ridiculous quote to generate a delay; that is, x_t^i could be larger than one without further restriction.

Let S^i denote the set of all strategies available to player i, i.e., S^i is the set of all sequences of strategy mappings $S^i = \{S_\tau^i\}_{\tau=0}^{T}$. In addition, we define $s_t = (s_t^A, s_t^B)$ as a strategy pair chosen by the players in period t, and $s = (s_1, s_2, \cdots, s_T)$ as a strategy combination for the entire game. The assumption of perfect information and perfect recall naturally indicates that each player knows all previous moves when one makes a decision.

Hence, an information set after period t, in general, is[2]

$$h^t = ((\delta_0, s_0), (\delta_1, s_1), \cdots, (\delta_t, s_t)) \in H^t.$$

We can think of δ_t as a piece of new information that can be either endogenous or exogenous to the system and describes the value of the asset in period t. Hence, an arrival of new information implies the beginning of another negotiation period. We consider two cases that differ in the information availability to each player when they make decisions. The first case specifies the Basic game, in which both players take some action in each period. We call the second case the Alternative game, in which only one player takes an action in each period. The Alternative game allows us to investigate a case in which a responder has a chance to observe additional information regarding the asset value before making any decision after observing a proposer's offer. These two games appear to be similar to each other, but have to be distinguished for the reason that will become clear as we proceed with our analysis and simulations. We argue that the Alternative game may allow more variations in the bargaining durations.

Before we describe detailed frameworks of the two games, we list the following assump-

[2] A precise definition of information set available for each player is slightly different between the Basic game and the Alternative game. In a Basic game, an information set for a proposer A in even t is

$$h^t = ((\delta_0, s_0), (\delta_1, s_1), \cdots, (\delta_{t-1}, s_{t-1}), \delta_t) \in H^t,$$

whereas that for a responder B in even t is

$$h^t = ((\delta_0, s_0), (\delta_1, s_1), \cdots, (\delta_{t-1}, s_{t-1}), \delta_t, s_t^A) \in H^t.$$

In an Alternative game, an information set for an action taking player in t is

$$h^t = ((\delta_0, s_0), (\delta_1, s_1), \cdots, (\delta_{t-1}, s_{t-1}), \delta_t) \in H^t.$$

tions on the players' preferences over bargaining outcomes. The following assumptions are weak enough to allow a wide variety of preferences.

Assumption 1 *Each player's preference ordering $\succeq_i$ over a bargaining outcome (π_t, t) is complete, transitive, and reflexive.*

Assumption 2 *For any $t \in T$ and any feasible $\pi_t, \hat{\pi}_t$, $(\pi_t, t) \succ_i (\hat{\pi}, t)$ if and only if $\pi^i > \hat{\pi}^i$.*

In other words, the asset is desirable for the players to engage in a negotiation process until an agreement.

Assumption 3 *For any $t, \hat{t} \in T$ and any feasible π_t, $t < \hat{t}$ implies $(\pi_t, t) \succeq_i (\pi_{\hat{t}}, \hat{t})$, with strict preference if $\pi^i > 0$.*

Hence, time is valuable in a sense that an agreement now is preferred to an agreement of the same payoff later.

Assumption 4 *Let $\{(\pi_n, t)\}_{n=1}^{\infty}$ and $\{(\hat{\pi}_n, \hat{t})\}_{n=1}^{\infty}$ be sequences of outcomes such that*

$$\lim_{n\to\infty} \pi_n = \pi \text{ and } \lim_{n\to\infty} \hat{\pi}_n = \hat{\pi}.$$

Then, $(\pi, t) \succeq_i (\hat{\pi}, \hat{t})$ if and only if $(\pi_n, t) \succeq_i (\hat{\pi}_n, \hat{t})$ $\forall n$.

Therefore, the preference ordering is continuous. Player i's preference orderings that satisfy the assumptions above can be represented by a continuous utility function u^i that is increasing in π_t^i and decreasing in t. Note that we can make analogous assumptions on the players' preferences over uncertain outcomes in the future periods in terms of their expected values. In other words, we assume that the utility function possesses the expected utility property, i.e., it is a von Neumann–Morgenstern utility function.

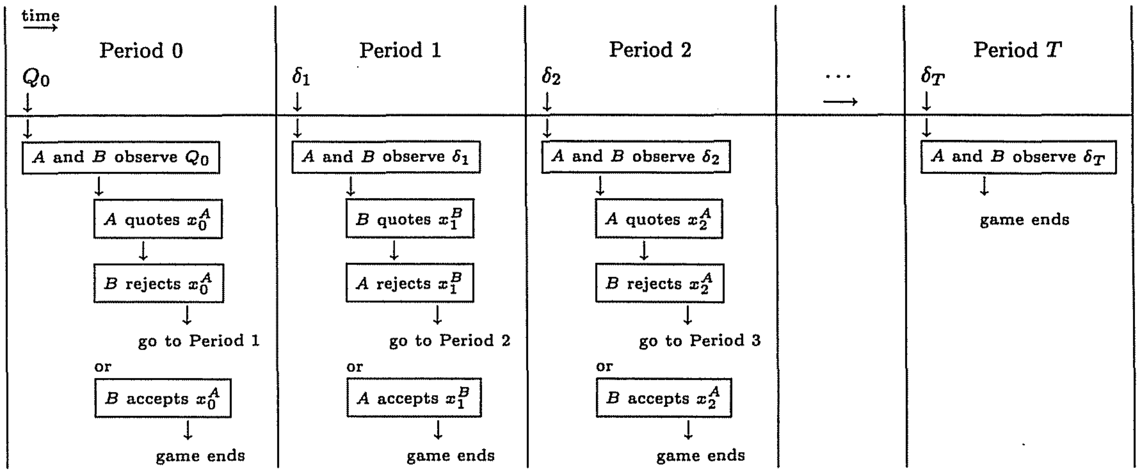

Figure 2.1: The Timing of Events (Basic Game)

2.1.1 The Basic Game

Player A starts the bargaining by making an offer $x_0^A \in X_0^A$ after both players have observed Q_0 at the beginning of period 0. In the same period player B responds to the A's offer by either accepting or rejecting it. If B accepts the offer, the bargaining terminates. If B rejects the offer, he will have a chance to make a counteroffer $x_1^B \in X_1^B$ in period 1 after both players have observed δ_1. Then A responds to the B's offer by either accepting or rejecting it. In this fashion the players alternate in making offers until one of the players accepts an offer or the exogenously predetermined final period T, where T is an even integer and common knowledge, is reached. Figure 2.1 shows the timing of events in the Basic game. Notice that A makes an offer and B responds to it by choosing either $\{accept\}$ or $\{reject\}$ in even–numbered periods, while B makes an offer and A responds to it in odd–numbered periods. The Basic game is that of perfect information in a sense that players do not move simultaneously.

When it is player i's turn to make an offer in period t, i has observed $\{\delta_\tau\}_{\tau=1}^{t}$ but has not observed $\delta_{t+1}, \delta_{t+2}, \cdots$. Thus, i has his expectation as to what the future δs will be. Taking this expectation into consideration, i chooses an offer that is optimal to him. A strategy for player i specifies the offer that i makes in period t, $x_t^i \in X_t^i$, as a function of observed δs. We define player A's strategies. Those of player B follow analogously. $\mathcal{S}^A$ is the set of all sequences of strategy mappings $S^A = \{S_\tau^A\}_{\tau=0}^T$, such that in even–numbered periods

$$S_t^A \; : \; H^t \longrightarrow X_t,$$

and in odd–numbered periods

$$S_t^A \; : \; H^t \longrightarrow \{\{accept\}, \; \{reject\}\}.$$

Note that we consider only pure strategies in our analyses.

If a responding player j accepts x_t^i, the bargaining ends and j's realized gain is $Q_t(1 - x_t^i)$ while i's realized gain is $Q_t x_t^i$. In general, i's and j's realized gains in period t are, respectively,

$$\pi_t^i = \begin{cases} Q_t x_t^i & \text{if } j \text{ accepts } x_t^i \\ 0 & \text{otherwise,} \end{cases}$$

$$\pi_t^j = \begin{cases} Q_t(1 - x_t^i) & \text{if } j \text{ accepts } x_t^i \\ 0 & \text{otherwise.} \end{cases}$$

If no agreement is reached until the final period T, A receives $\pi_T^A = Q_T x_T$ while B receives $\pi_T^B = Q_T(1 - x_T)$, where $x_T \in [0, 1]$ is a predetermined A's default share in T that is known

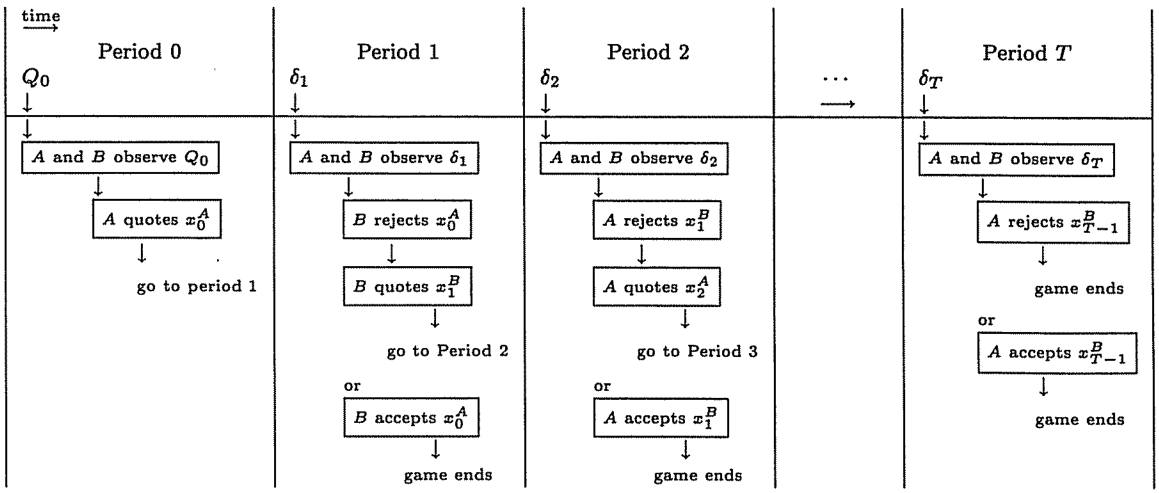

Figure 2.2: The Timing of Events (Alternative Game)

to both players before the game begins.

Let $E_t^i \pi_{t+n}^i$ denote i's conditional expected payoff in some future period $t+n$, $n \in (0, T-t)$, which is determined by a sequence of observed δs up to t and his expectation of the strategies of both his own and the opponent j. i forms his expectation on his opponent j's future strategies and his own, based on his updated expectation of the asset values in the future periods. We note that it is possible to have $E_t^i \neq E_t^j$, reflecting differences in initial priors among players. The structure of the game is common knowledge.

2.1.2 The Alternative Game

In the Alternative game player A also starts the bargaining by making an offer x_0^A in period 0 after observing Q_0. Then player B responds to the A's offer by either accepting or rejecting it *after* observing δ_1 in period 1. If B rejects the offer, then he makes a counteroffer x_1^B *before* he observes δ_2. When period 2 begins, both players observe δ_2 and A will respond to x_1^B by either accepting it or making a counteroffer x_2^A. Figure 2.2 shows the timing of events described above. Notice that A and B take an action in period t, where t is even and odd, respectively. Therefore, the Alternative game is also a game of complete and perfect information, in which exactly one player takes an action in each period.

When it is player i's turn to take an action in period t, i has observed $\{\delta_\tau\}_{\tau=1}^{t}$ but has not observed δ_{t+1}. Hence, i has his expectations as to what δ_{t+1} will be and how his opponent j will respond to his offer after observing δ_{t+1}. Based on such considerations, i chooses his action that maximizes his expected gain. Therefore, a strategy for player i specifies the action that i takes in period t, which is either to accept x_{t-1}^{j} or to quote a counteroffer x_t^i, as a function of observed δs and the opponent's immediately preceding offer x_{t-1}^{j}.

Formally, $\mathcal{S}^A$ is the set of all strategies available to player A in even–numbered periods $t(>0)$; that is, it is the set of all sequences of strategy mappings $S^A = \{S_\tau^A\}_{\tau=0}^{T}$, such that

$$S_t^A \ : \ H^t \ \longrightarrow \ \{\{accept\}, \ \{reject\}\},$$

and in the event of rejection

$$S_t^A \ : \ H^t \ \longrightarrow \ X_t.$$

Similarly, $\mathcal{S}^B$ is the set of all strategies available to player B in odd–numbered periods, and S^B is defined analogously.

If player i accepts player j's offer in period t, the bargaining terminates and i's realized gain is $Q_t(1 - x_{t-1}^{j})$ while j's realized gain is $Q_t x_{t-1}^{j}$. In general, i's and j's realized gains in period $t(>0)$ are, respectively,

$$\pi_t^i = \begin{cases} Q_t(1-x_{t-1}^j) & \text{if } i \text{ accepts } x_{t-1}^j \\ 0 & \text{otherwise,} \end{cases}$$

$$\pi_t^j = \begin{cases} Q_t x_{t-1}^j & \text{if } i \text{ accepts } x_{t-1}^j \\ 0 & \text{otherwise.} \end{cases}$$

As in the Basic game, if no agreement is reached until the final period T, player A receives $\pi_T^A = Q_T x_T$ while B receives $\pi_T^B = Q_T(1-x_T)$, where x_T is a predetermined share known to both players before the game begins. Since a proposer i's expected payoff in some future period $(t+n)$ is determined by a sequence of observed δs, the space of expected payoffs for each observed sequence $\{\delta_\tau\}_{\tau=1}^t$ is the space of continuous functions $\mathcal{C}(E_t^i \pi_{t+n}^i)$. The structure of the Alternative game is common knowledge.

2.1.3 The Equilibrium Concept

A strategy profile $\{S^A, S^B\}$ generates a path of offers and responses, which determines a payoff to each player. Let us denote player i's realized payoff that is generated by a strategy profile $\{S^A, S^B\}$ by $\pi^i(S^A, S^B)$.

Definition 1 A strategy profile $\{\hat{S}_t^A, \hat{S}_t^B\}$ is a *(pure strategy) Nash equilibrium* if and only if

1. $\hat{S}_t^i \in \mathcal{S}^i, i = A, B$;
2. $\pi_t^A(\hat{S}_t^A, \hat{S}_t^B) \geq \pi_t^A(S_t^A, \hat{S}_t^B),\ \forall S_t^A \in \mathcal{S}^A$;
3. $\pi_t^B(\hat{S}_t^A, \hat{S}_t^B) \geq \pi_t^B(\hat{S}_t^A, S_t^B),\ \forall S_t^B \in \mathcal{S}^B$.

Unfortunately, it is demonstrated that the theory cannot give a very sharp prediction in bargaining games without further refinement in the equilibrium concept. Let $\{S^A|h^t, S^B|h^t\}$ be a strategy profile induced by $\{S^A, S^B\}$ after a history $h^t \in H^t$. Then we give a definition of subgame perfect equilibrium as follows.

Definition 2 A strategy profile $\{\hat{S}^A, \hat{S}^B\}$ is a *subgame perfect equilibrium* if and only if $\{\hat{S}^A|h^t, \hat{S}^B|h^t\}$ is a Nash equilibrium in the game remaining after h^t, for all t and h^t, i.e.,

1. $\hat{S}_t^i \in \mathcal{S}^i, i = A, B$;
2. $E_t^A(\pi_t^A(\hat{S}_t^A, \hat{S}_t^B)) \geq E_t^A(\pi_t^A(S_t^A, \hat{S}_t^B)),\ \forall t, h^t, S_t^A \in \mathcal{S}^A$;
3. $E_t^B(\pi_t^B(\hat{S}_t^A, \hat{S}_t^B)) \geq E_t^B(\pi_t^B(\hat{S}_t^A, S_t^B)),\ \forall t, h^t, S_t^B \in \mathcal{S}^B$.

Subgame perfection implies that the players' strategies are best responses not only at the opening of the bargaining game, but also at any decision node. Therefore, each player's actions are optimal at every possible history. Merlo and Wilson (1995) proved the existence of subgame perfect equilibria in their stochastic sequential bargaining game.[3] In the rest of this subsection we give corollaries to their findings.

Corollary 1 *There exists a subgame perfect equilibrium in the Basic bargaining game.*

The proof follows that of Merlo and Wilson (1995) applied to our bargaining game framework. Let us first define the following and proceed with the proof. Define player i's minimum payoff to disagreement, which we shall call i's minimum reservation value, in some period t as

$$m_t^i = Q_t \underline{x_t^i} = \min\{E_t^i(\pi_\tau^i),\ \tau = t+1, \cdots, T|h^t\}.$$

[3]Also refer to Harris (1985) for the existence of pure strategy subgame perfect equilibria in sequential games of perfect information with infinite action spaces.

$\underline{x_t^i}$ is a minimum reservation share that is a share i needs to have in the present period t, given h^t, to guarantee himself a payoff exactly equal to the minimum expected continuation payoff in an equilibrium path. Similarly, define i's maximum reservation value as

$$M_t^i = Q_t\overline{x_t^i} = \max\{E_t^i(\pi_\tau^i),\ \tau = t+1, \cdots, T|h^t\}.$$

$\overline{x_t^i}$ is a maximum reservation share i needs to have in the present period t to give him a payoff exactly equal to his maximum expected continuation payoff in an equilibrium path. We write a minimum reservation value vector as $m_t = (m_t^A, m_t^B)$, and a maximum reservation value vector as $M_t = (M_t^A, M_t^B)$. By construction they are nonempty valued.

Next, define an operator on the space of feasible histories up to period t as

$$\Psi_t^i(M_t, m_t)(h^t) = \overline{\Psi_t^i} \times \underline{\Psi_t^i}(M_t, m_t)(h^t) = \left(\overline{\Psi_t^i}(M_t, m_t)(h^t),\ \underline{\Psi_t^i}(M_t, m_t)(h^t)\right),$$

where

$$\begin{aligned}\overline{\Psi_t^i}(M_t, m_t) &= Q_t \max\{1 - \underline{x_t^j}, \overline{x_t^i}\} \\ &= \max\{Q_t - m_t^j, M_t^i\},\end{aligned}$$

and

$$\begin{aligned}\underline{\Psi_t^i}(M_t, m_t) &= Q_t \max\{1 - \overline{x_t^j}, \underline{x_t^i}\} \\ &= \max\{Q_t - M_t^j, m_t^i\}.\end{aligned}$$

We can consider Ψ_t^i as a refinement on s_t^i that is a bounded measurable function. Hence, $\overline{\Psi_t^i}$ is the maximum possible payoff i can achieve in period t, while $\underline{\Psi_t^i}$ is the least payoff i can guarantee himself in period t. Let us also denote F^2 the set of bounded and measurable functions on Ω that take values in R^2, and let $p = (p^A, p^B) \in F^2$ be a feasible payoff vector.

The goal of the proof is to show that there exist extremal fixed points of $\overline{\Psi_t} \times \underline{\Psi_t}$ and that the extremal fixed points of $\overline{\Psi_t} \times \underline{\Psi_t}$ correspond to extremal subgame perfect payoffs. The corollary is proven through the series of the following lemmas, in which we first show the monotonicity and pointwise continuity of $\overline{\Psi_t} \times \underline{\Psi_t}$.

Lemma 1 *If* $m_t^1 \leq m_t^2$ *and* $M_t^1 \geq M_t^2$*, then*

$$\overline{\Psi_t^i}(M_t^1, m_t^1) \geq \overline{\Psi_t^i}(M_t^2, m_t^2),$$

and

$$\underline{\Psi_t^i}(M_t^1, m_t^1) \leq \underline{\Psi_t^i}(M_t^2, m_t^2).$$

Proof. Suppose that $m_t^1 \leq m_t^2$ and $M_t^1 \geq M_t^2$.

i) We first show $\overline{\Psi_t}(M_t^1, m_t^1) \geq \overline{\Psi_t}(M_t^2, m_t^2)$. Given $h^t \in H^t$, from the assumption it is clear that $Q_t - m_t^{i,1} \geq Q_t - m_t^{i,2}$, $Q_t - m_t^{j,1} \geq Q_t - m_t^{j,2}$, $M_t^{i,1} \geq M_t^{i,2}$, and $M_t^{j,1} \geq M_t^{j,2}$. It follows, from the definition of $\overline{\Psi_t}$, that

$$\begin{aligned} \overline{\Psi_t^i}(M_t^1, m_t^1)(h^t) &= \max\{Q_t - m_t^{j,1}, M_t^{i,1}\} \\ &\geq \max\{Q_t - m_t^{j,2}, M_t^{i,2}\} = \overline{\Psi_t^i}(M_t^2, m_t^2)(h^t), \end{aligned}$$

and

$$\begin{aligned}\overline{\Psi_t^j}(M_t^1, m_t^1)(h^t) &= \max\{Q_t - m_t^{i,1}, M_t^{j,1}\}\\ &\geq \max\{Q_t - m_t^{i,2}, M_t^{j,2}\} = \overline{\Psi_t^j}(M_t^2, m_t^2)(h^t).\end{aligned}$$

Therefore, $\overline{\Psi_t}(M_t^1, m_t^1) \geq \overline{\Psi_t}(M_t^2, m_t^2)$.

ii) Next we show $\underline{\Psi_t}(M_t^1, m_t^1) \leq \underline{\Psi_t}(M_t^2, m_t^2)$. Given $h^t \in H^t$, from the assumption it is clear that $Q_t - M_t^{i,1} \leq Q_t - M_t^{i,2}$, $Q_t - M_t^{j,1} \leq Q_t - M_t^{j,2}$, $m_t^{i,1} \leq m_t^{i,2}$, and $m_t^{j,1} \leq m_t^{j,2}$. It follows, from the definition of $\underline{\Psi_t}$, that

$$\begin{aligned}\underline{\Psi_t^i}(M_t^1, m_t^1)(h^t) &= \max\{Q_t - M_t^{j,1}, m_t^{i,1}\}\\ &\leq \max\{Q_t - M_t^{j,2}, m_t^{i,2}\} = \underline{\Psi_t^i}(M_t^2, m_t^2)(h^t),\end{aligned}$$

and

$$\begin{aligned}\underline{\Psi_t^j}(M_t^1, m_t^1)(h^t) &= \max\{Q_t - M_t^{i,1}, m_t^{j,1}\}\\ &\leq \max\{Q_t - M_t^{i,2}, m_t^{j,2}\} = \underline{\Psi_t^j}(M_t^2, m_t^2)(h^t).\end{aligned}$$

Therefore, $\underline{\Psi_t}(M_t^1, m_t^1) \leq \underline{\Psi_t}(M_t^2, m_t^2)$. This completes the proof. □

Lemma 2 *If $(u^k, w^k) \to (u, w) \in F^2 \times F^2$ pointwise, then*

$$\overline{\Psi_t} \times \underline{\Psi_t}(u^k, w^k)(h^t) \to \overline{\Psi_t} \times \underline{\Psi_t}(u, w)(h^t).$$

Proof. For $(u, w) \in F^2 \times F^2$ and $h^t \in H^t$, define

$$\|u, w\|_{h^t} = E\left(\max\{\|u\|_\infty, \|w\|_\infty\}|h^t\right),$$

where

$$\|g\|_\infty = \max\{g^i(h^t) : i = A, B\}.$$

Then, $(u^k, w^k) \to (u, w)$ pointwise implies

$$\|(u^k, w^k) - (u, w)\|_{h^t} \to 0, \ \forall h^t \in H^t.$$

Given $h^t \in H^t$, $\overline{\Psi_t} \times \underline{\Psi_t}(\cdot, \cdot)(h^t) : F^2 \times F^2 \to R^2 \times R^2$ defines a continuous function with respect to the $\|\cdot\|_{h^t}$ topology on $F^2 \times F^2$. □

Lemma 3 *There exist $M_t^*, m_t^* \in F^2$ such that*

1. $M_t^* \geq m_t^* \geq 0$.
2. $\overline{\Psi_t} \times \underline{\Psi_t}(M_t^*, m_t^*) = (M_t^*, m_t^*)$.
3. $\overline{\Psi_t} \times \underline{\Psi_t}(M_t, m_t) = (M_t, m_t) \Longrightarrow m_t^* \leq m_t, \ M_t \leq M_t^*$.

Proof. First we define $(M_t^k, m_t^k) = \left(\overline{\Psi_t}(M_t^{k-1}, m_t^{k-1}), \underline{\Psi_t}(M_t^{k-1}, m_t^{k-1})\right)$, and choose $(M_t^0, m_t^0) \ni m_t^0(h^t) = 0, \ M_t^0(h^t) \geq x_t, \ x_t = (x_t^A, x_t^B) \in F^2$. Then, by induction with an application of the monotonicity lemma, it is straightforward to show that $\{M_t^k\}_{k=1,2,\cdots}$ is a monotonically decreasing sequence and $\{m_t^k\}_{k=1,2,\cdots}$ is a monotonically increasing sequence, with $m_t^k \leq M_t^k \ \forall k$. Hence, (M_t^k, m_t^k) is monotonic and bounded. Then, there exists a pair of functions M_t^*, $m_t^* \in F^2$ such that $M_t^k(h^t) \downarrow M_t^*(h^t)$ and $m_t^k(h^t) \uparrow m_t^*(h^t)$ for $h^t \in H^t$. Hence, $\overline{\Psi_t} \times \underline{\Psi_t}(M_t^k, m_t^k)$ converges to (M_t^*, m_t^*) pointwise. Then, continuity implies $\overline{\Psi_t} \times \underline{\Psi_t}(M_t^k, m_t^k)$ converges to $\overline{\Psi_t} \times \underline{\Psi_t}(M_t^*, m_t^*)$. It follows that $\overline{\Psi_t}(M_t^*, m_t^*) = M_t^* \geq m_t^* = \underline{\Psi_t}(M_t^*, m_t^*)$. This proves 1 and 2.

Now let $(M_t, m_t) = \left(\overline{\Psi_t}(M_t, m_t), \underline{\Psi_t}(M_t, m_t)\right)$. Then, we have $m_t^0 \leq m_t$ and $M_t \leq M_t^0$. Hence, from the monotonicity lemma and the definition of (M_t^k, m_t^k), $m_t^1 = \underline{\Psi_t}(M_t^0, m_t^0) \leq \underline{\Psi_t}(M_t, m_t) = m_t$ and $M_t = \overline{\Psi_t}(M_t, m_t) \leq \overline{\Psi_t}(M_t^0, m_t^0) = M_t^1$. By induction with the application of the monotonicity lemma, it is straightforward to show $m_t^k \leq m_t$ and $M_t \leq M_t^k$ for $k = 1, 2, \cdots$. Therefore, $(M_t^k, m_t^k) \to (M_t^*, m_t^*)$ implies $m_t^* \leq m_t$ and $M_t \leq M_t^*$. This proves 3. □

Lemma 4 *$p_T = (p_T^i, p_T^j)$ is a subgame perfect equilibrium outcome if and only if $m_t^* \leq E_t(p_T) \leq M_t^*$.*

Proof.

i) Consider any subgame perfect payoff vector p, and suppose that $\hat{M}_t$ and $\hat{m}_t$ are supremum and infimum of subgame perfect payoffs of the game after observing h^t, respectively. By construction, we can show $p^i(h^t) \geq \underline{\Psi_t^i}(\hat{M}_t, \hat{m}_t)$ and $p^i(h^t) \leq \overline{\Psi_t^i}(\hat{M}_t, \hat{m}_t)$, which indicate $\hat{m}_t \geq \underline{\Psi_t}(\hat{M}_t, \hat{m}_t)$ and $\hat{M}_t \leq \overline{\Psi_t}(\hat{M}_t, \hat{m}_t)$. Now we construct a convergent sequence such that $(M^k, m^k) = \left(\overline{\Psi_t}(M^{k-1}, m^{k-1}), \underline{\Psi_t}(M^{k-1}, m^{k-1})\right), k = 1, 2, \cdots$, with $(M^0, m^0) = (\hat{M}_t, \hat{m}_t)$. Then, $M^1 = \overline{\Psi_t}(M^0, m^0) \geq M^0$ and $m^1 = \underline{\Psi_t}(M^0, m^0) \leq m^0$. From the monotonicity lemma, we have $M^k \geq M^{k-1}$ and $m^k \leq m^{k-1}$ for $k = 1, 2, \cdots$. Note that m^k is bounded since $\hat{m}^0$ is bounded.[4] In addition, by assumption we know that there is an upperbound

[4]Everyone is guaranteed to receive at least a payoff of 0 by not coming to an agreement in the current period.

on the feasible payoff vectors, i.e., $M^k \leq \tilde{p}$. Since $\{M^k\}$ and $\{m^k\}$ are bounded monotonic sequences, there is a $(\bar{M}, \bar{m})$ such that $(M^k, m^k) \to (\bar{M}, \bar{m})$ pointwise. Therefore, $\bar{M} \geq \hat{M} \geq \hat{m} \geq \bar{m}$. Then, from Lemma 3 we conlcude $M^* \geq \bar{M} \geq \hat{M} \geq \hat{m} \geq \bar{m} \geq m^*$.

ii) Consider a payoff vector p_τ such that $m_t^* \leq E_t(p_\tau) \leq M_t^*$. It is straightforward to show that there is a strategy profile that supports p_τ as a subgame perfect equilibrium outcome. □

Consequently, Corollary 1 has been proven. The following corollary for the Alternative bargaining game is also immediate.

Corollary 2 *There exists a subgame perfect equilibrium in the Alternative bargaining game.*

Redefine m_t and M_t in the following manner.[5] When player i is proposing an offer in a given period t, his minimum reservation value is

$$m_t^i = E_t^i(Q_{t+1})\underline{x_t^i} = \min\{E_t^i(\pi_\tau^i),\ \tau = t+2, \cdots, T|h^t\},$$

while i's maximum reservation value is

$$M_t^i = E_t^i(Q_{t+1})\overline{x_t^i} = \max\{E_t^i(\pi_\tau^i),\ \tau = t+2, \cdots, T|h^t\}.$$

Note that the reservation values when i is responding to j's offer from the previous period is the same as the definitions given in the Basic game. Hence, the definitions above are the reservation values after rejecting an opponent's offer, which incorporate the uncertainty of the asset value in the following period $(t+1)$. With these definitions, the proof of Corollary

[5]Whenever we are discussing Alternative games, we assume these definitions.

2 is analogous to that of the Basic game.

Next, we include some of the findings by Merlo and Wilson on stationary subgame perfect equilibrium. Note that a strategy profile is *stationary* if the actions depend only on the current state. Hence, a stationary subgame perfect outcome is generated by stationary subgame perfect strategy profile. We denote player i's best feasible allocation that guarantees at least p^B to player j as $BR^i(p)$, which is bounded and measurable on H^t, is continuous on $h^t \in H^t$, and exists whenever the set of feasible allocations is not empty. Let us define an operator O on the history of states such that a proposer A's equilibrium payoff is $O^A(p)(h^t) = \max\{BR^A(p)(h^t), p^A\}$ and a responder B's payoff is $O^B(p)(h^t) = p^B$. With these definitions, Theorem 1 and Theorem 8 of Merlo and Wilson (1995) can be immediately applied to our bargaining games.

Theorem 1 *(Theorem 1 of Merlo and Wilson (1995)) In Basic and Alternative bargaining games,* $p = (p^A, p^B) \in F^2$ *is a stationary subgame perfect equilibrium payoff vector if and only if* $O(p) = p$.

Theorem 2 *(Theorem 8 of Merlo and Wilson (1995))* $(M_t^{A,*}, m_t^{B,*})$ *and* $(m_t^{A,*}, M_t^{B,*})$ *are stationary subgame perfect payoff vectors.*[6]

Merlo and Wilson (1995) proves the theorem above by showing that $\overline{\Psi_t} \times \underline{\Psi_t}(M_t, m_t) = (M_t, m_t)$ implies $O(m_t^A, M_t^B) = (m_t^A, M_t^B)$.[7]

[6]Note that the monotonicity of the operator Ψ is essential for the existence of stationary subgame perfect equilibria in stochastic bargaining games.

[7]In other words, first assume $\overline{\Psi_t^A}(M_t, m_t) = M_t^A$, $\underline{\Psi_t^A}(M_t, m_t) = m_t^A$, $\overline{\Psi_t^B}(M_t, m_t) = M_t^B$, and $\underline{\Psi_t^B}(M_t, m_t) = m_t^B$. Then it is straightforward to show $O^A(m_t^A, M_t^B) = \underline{\Psi_t^A}(M_t, m_t) = m_t^A$ and $O^B(m_t^A, M_t^B) = \overline{\Psi_t^B}(M_t, m_t) = M_t^B$. Analogous argument is made for the case with $(M_t^{A,*}, m_t^{B,*})$.

In the following sections, we consider an equilibrium offer strategy such that $x_t^i = \max\{1 - \overline{x_t^j}, \overline{x_t^i}\}$. Therefore, the equilibrium payoff operator for a proposer i in the Basic game is $\Psi_t^i(x_t) = \Psi_t^i(x_t^i, x_t^j) = \max\{Q_t - M_t^j, M_t^i\}$. $Q_t - M_t^j$ is i's best payoff while guaranteeing a maximum reservation value for his opponent j. Hence, a responder j's equilibrium payoff is $\Psi_t^j(x_t) = M_t^j$. In the Alternative game, the operator is defined as $\Psi_t^i(x_t) = \Psi_t^i(x_t^i, x_t^j) = \max\{E_t^i(Q_{t+1}) - M_t^j, M_t^i\}$. This is a strategy that leads to *ex ante* Pareto optimal outcomes. We also show that the particular type of equilibrium strategies we have derived predicts stationary subgame perfect shares.

2.2 Analysis of the Basic Game

In this section we present some findings which are immediate from the formulation of the Basic bargaining game. We describe an equilibrium strategy profile derived through a method of backward induction. Recall that a strategy profile constructed by backward induction necessarily coincides with a subgame perfect equilibrium, since such an equilibrium requires the players to act optimally whenever they make decisions, i.e., choose their best responses in each period t, given an observed history h^t. We first specify behavioral assumptions and provide a fundamental algorithm for the backward–induction solution to our Basic game in words.

In the penultimate period $(T-1)$, it is player B's turn to make an offer. Consider a quote x_{T-1}^B that satisfies

C–I Accepting B's offer in period $(T-1)$ gives A at least as much as what A expects to receive in period T.

C–II In the event that A accepts B's offer in period $(T-1)$, B receives at least as much as what B expects to receive in period T.

x^B_{T-1} satisfying condition C–I is a marginal offer that guarantees A's acceptance in period $(T-1)$, while the one satisfying condition C–II grants B a payoff that motivates him not to delay untill period T. Whenever it is his turn to choose an action, B wishes to gain more than or at least as much as what he expects to receive from any future transactions. Let $\tilde{x}^B_{T-1}$ be the smallest such offer, i.e., the smallest offer that satisfies condition C–II, and let $\tilde{\tilde{x}}^B_{T-1}$ be the largest offer that satisfies condition C–I. Suppose that $\tilde{x}^B_{T-1}$ exists and consider the offers that are feasible and are greater than or equal to $\tilde{x}^B_{T-1}$. If $\tilde{\tilde{x}}^B_{T-1} \geq \tilde{x}^B_{T-1}$, then B quotes $x^B_{T-1} = \tilde{\tilde{x}}^B_{T-1}$ which will be accepted by A with certainty and will be expected to give both players at least as high a payoff as any possible payoff from future trading. If $\tilde{\tilde{x}}^B_{T-1} < \tilde{x}^B_{T-1}$, then B quotes $x^B_{T-1} \geq \tilde{x}^B_{T-1}$, knowing that his offer will be rejected by A. This is because delaying leaves B with a potential opportunity to gain a higher payoff in the future, so that B prefers delaying to having a transaction take place with his share less than $\tilde{x}^B_{T-1}$ in the current period. We assume that B will choose $x^B_{T-1} = \tilde{x}^B_{T-1}$ in such a case. Hence, under such an assumption B will quote the larger of $\tilde{\tilde{x}}^B_{T-1}$ and $\tilde{x}^B_{T-1}$. Let us also assume that if a respondent is indifferent between a guaranteed payoff in the current period and an expected payoff in the future, then he chooses to accept the current offer.

A symmetric argument applies to A's action in period $(T-2)$ when it is A's turn to make an offer. In general, the two conditions can be restated as

C–I In the event of the respondent's accepting the current offer, the respondent receives at least as much as what the respondent expects to receive in any future period.

C–II In the event of the respondent's accepting the current offer, the proposer receives at least as much as what the proposer expects to receive in any future period.

Proceeding to an initial period in this fashion leads us to find a strategy profile that is a subgame perfect equilibrium as described in the following proposition.

Proposition 1 *A strategy profile $\{\hat{S}^A, \hat{S}^B\}$ that satisfies the following conditions is a subgame perfect equilibrium of the Basic game.*

1. *In an even–numbered period t, player A makes an offer such that*

$$x_t^A = \max\left\{1 - E_t^B(\delta_{t+1} x_{t+1}^B),\ E_t^A(\delta_{t+1}\delta_{t+2} x_{t+2}^A)\right\}, \tag{2.1}$$

   and player B accepts x_t^A iff $x_t^A \leq 1 - E_t^B(\delta_{t+1} x_{t+1}^B)$, and rejects otherwise.

2. *In an odd–numbered period t, player B makes an offer such that*

$$x_t^B = \max\left\{1 - E_t^A(\delta_{t+1} x_{t+1}^A),\ E_t^B(\delta_{t+1}\delta_{t+2} x_{t+2}^B)\right\}, \tag{2.2}$$

   and player A accepts x_t^B iff $x_t^B \leq 1 - E_t^A(\delta_{t+1} x_{t+1}^A)$, and rejects otherwise.

Proof. The derivation of this equilibrium is included in Appendix 2A.1.

If an agreement is reached, the proposer's share gives him a payoff that is at least as much as the largest expected continuation payoff in future periods over a subgame perfect equilibrium,[8] plus any surplus in excess of what the players expect to receive in any following future period. For example, a surplus A can extract in an even–numbered period t is $1-(E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)+E_t^B(\delta_{t+1}x_{t+1}^B))$. Note that if $1-(E_t^A(\cdot)+E_t^B(\cdot))<0$, then there is no agreement. In other words, a negative surplus in a current period indicates that at least one player is very optimistic about the future value of the asset and that the player wants to delay until the value is maximized.[9] It is also indicative that there is an advantage to be a proposer. When an agreement is reached, a proposer's surplus in excess of his reservation value is nonnegative, whereas a responder's surplus is always zero.

It is also immediate that an equilibrium share predicted by the strategies given in Proposition 1 is stationary. Let us consider some even–numbered period t. A proof of an odd–numbered period is analogous. The reservation values for the players are written as $M_t^A = Q_tE_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)$, and $M_t^B = Q_tE_t^B(\delta_{t+1}x_{t+1}^B)$. If $1-E_t^B(\delta_{t+1}x_{t+1}^B) \geq E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)$, then there is an agreement in t. We have

$$\begin{aligned} O^A(\pi_t) &= O^A(\pi_t^A, \pi_t^B) \\ &= Q_t\max\{1-E_t^B(\delta_{t+1}x_{t+1}^B),\ E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)\} \\ &= Q_t(1-E_t^B(\delta_{t+1}x_{t+1}^B)) \\ &= Q_t - M_t^B, \end{aligned}$$

[8]For a proposer A in an even–numbered period, it is $E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)$, and for a proposer B in an odd–numbered period, it is $E_t^B(\delta_{t+1}\delta_{t+2}x_{t+2}^B)$. It can be considered as a proposer's reservation share resulting from delay.

[9]This observation is revisited again in Proposition 9.

and

$$O^B(\pi_t) = Q_t E_t^B(\delta_{t+1} x_{t+1}^B) = M_t^B.$$

Hence, $O(\pi_t) = \pi_t$. Similarly, if $1 - E_t^B(\delta_{t+1} x_{t+1}^B) < E_t^A(\delta_{t+1}\delta_{t+2} x_{t+2}^A)$, we can show $O(\pi_t) = \pi_t$. Therefore, we can conclude that the derived equilibrium satisfies the condition specified in Theorem 1.

Recall that $E_t^i(\cdot)$ is a player i's expectation conditional on the information that is available on and before period t. The proposition shows that the demand of each player depends on the expected values of both current and lagged δs. It should also be noted that in this formulation we have not excluded the possibility of having a value of x_t that exceeds one, for δs are not restricted to be less than one. The following Lemma gives a sufficient condition to guarantee x_t^i in the closed interval of zero and one.

Lemma 5 *If $\delta_t \in [0, 1)$ for all t with certainty, then $x_t^i \in [0, 1]$.*

Proof. This is immediate by following our backward induction algorithm. □

This lemma is used in the proofs of the following propositions that assume $\delta_t \in [0, 1)$. The next three propositions give sufficient conditions to have a unique subgame perfect equilibrium.

Proposition 2 *If $\delta_t \in [0, 1)$ for all t with certainty, then there is no delay before reaching an agreement.*

Proof. Suppose that $\delta_t \in [0, 1)$ for all t. What we need to show here is that player A's offer in period 0 is always accepted by player B in the period. From Proposition 1 the condition that has to be satisfied to avoid delay is

$$1 - E_0^B(\delta_1 x_1^B) \geq E_0^A(\delta_1 \delta_2 x_2^A).$$

Suppose that this is not true; that is, $1 - E_0^B(\delta_1 x_1^B) < E_0^A(\delta_1 \delta_2 x_2^A)$. Then $E_0^A(x_2^A) \leq E_0^A(\delta_1 \delta_2 x_2^A)$ since $E_0^A(x_2^A) \leq \max\{1 - E_0^B(\delta_1 x_1^B),\ E_0^A(\delta_1 \delta_2 x_2^A)\}$. This implies $E_0^A(x_2^A) \leq 0$ because $\delta_1 \delta_2 \in [0, 1)$. Then $E_0^A(\delta_1 \delta_2 x_2^A) \leq 0$. But since $\delta_1 \in [0, 1)$ and the assumption of $\delta_t \in [0, 1)$ for all t implies $x_1^B \in [0, 1]$, $1 - E_0^B(\delta_1 x_1^B) > 0$, which results in $1 - E_0^B(\delta_1 x_1^B) > E_0^A(\delta_1 \delta_2 x_2^A)$. This contradicts our assumption. Hence, we conclude that $1 - E_0^B(\delta_1 x_1^B) \geq E_0^A(\delta_1 \delta_2 x_2^A)$. □

Proposition 3 *If $\{\delta_\tau\}_{\tau=1}^t$ is non-stochastic, in particular $\delta \in [0, 1)$ and $\delta_t = \delta^t$, then the solution to the Basic game converges to that to the Rubinstein's model as $T \longrightarrow \infty$.*

Proof. We need to show that $x_0^A = \frac{1}{1+\delta}$ as $T \to \infty$. From Proposition 2 we know that

$$1 - E_0^B(\delta_1 x_1^B) \geq E_0^A(\delta_1 \delta_2 x_2^A),$$

which means $x_0^A = 1 - \delta E_0^B(x_1^B)$. In a similar fashion we can show that if period 1 is reached, then $x_1^B = 1 - \delta^2 E_1^A(x_2^A)$. In general,

$$x_t^A = 1 - \delta^{t+1} E_t^B(x_{t+1}^B)$$

and

$$x_t^B = 1 - \delta^{t+1} E_t^A(x_{t+1}^A).$$

By recursively substituting x_t^A's and x_t^B's into x_0^A, we get

$$\begin{aligned}
x_0^A &= 1 - \delta E_0^B(x_1^B) \\
&= 1 - \delta\left(1 - \delta^2 E_1^A(x_2^A)\right) \\
&= 1 - \delta\left(1 - \delta^2\left(1 - \delta^3 E_2^B(x_3^B)\right)\right) \\
&\vdots \\
&= 1 - \delta\left(1 - \delta^2\left(1 - \delta^3(\cdots(1 - \delta^T x_T))\right)\right) \\
&= \sum_{t=0}^{T-1}(-1)^t \delta^{\sum_{i=0}^t i} + \delta^{\sum_{j=0}^T j} x_T.
\end{aligned}$$

By taking a limit of the last expression, we have

$$\begin{aligned}
&\lim_{T\to\infty}\left[\sum_{t=0}^{T-1}(-1)^t \delta^{\sum_{i=0}^t i} + \delta^{\sum_{j=0}^T j} x_T\right] \\
&= \lim_{T\to\infty}\sum_{t=0}^{T-1}\left((-1)\delta^{\frac{1}{t}\sum_{i=0}^t i}\right)^t.
\end{aligned}$$

Note that by using L'Hôpital's rule,

$$\lim_{t\to\infty}\frac{\sum_{i=0}^t i}{t} = \lim_{t\to\infty}\frac{\frac{d}{dt}\sum_{i=0}^t i}{\frac{d}{dt}t} = 1.$$

Since $|\delta| < 1$, as T approaches to infinity we have the sum of a converging geometric series such that

$$x_0^A = \sum_{t=0}^{\infty}(-\delta)^t = \frac{1}{1+\delta}.$$

This completes the proof. □

Proposition 4 *If δ_t's are identically and independently distributed, and the players agree on the expected values, which is $E(\delta_t) = \delta^* \in [0,1)$, then the solution to the Basic game converges to the solution to the Rubinstein's model, provided that $T \longrightarrow \infty$.*

Proof. By using a similar reasoning to Proposition 2, we can show

$$1 - \delta^* E_0^B(x_1^B) \geq {\delta^*}^2 E_0^A(x_2^A).$$

In general,

$$x_t^A = 1 - \delta^* E_t^B(x_{t+1}^B)$$

and

$$x_t^B = 1 - \delta^* E_t^A(x_{t+1}^A).$$

Then x_0^A can be expressed as

$$\begin{aligned} x_0^A &= 1 - \delta^* E_0^B(x_1^B) \\ &= 1 - \delta^* (1 - \delta^*(\cdots(1 - \delta^* x_T))) \\ &= 1 - \delta^* + {\delta^*}^2 - {\delta^*}^3 + \cdots + {\delta^*}^T x_T \\ &= \sum_{t=0}^{T-1} (-\delta^*)^t + {\delta^*}^T x_T. \end{aligned}$$

Thus, as T approaches to infinity, we have

$$\begin{aligned} x_0^A &= \lim_{T\to\infty} \left[\sum_{t=0}^{T-1} (-\delta^*)^t + {\delta^*}^T x_T \right] \\ &= \frac{1}{1+\delta^*}. \end{aligned}$$

Hence, the proposition holds when $\delta^* \in [0, 1)$. □

The following two propositions give sufficient conditions to generate a delay in period $(T-1)$.

Proposition 5 *In period $(T-1)$, if the players' beliefs about the value of δ_T are greater than or equal to 1 with $E^A_{T-1}(\delta_T) > E^B_{T-1}(\delta_T)$, then there is delay at period $(T-1)$.*

Proof. We need to show that

$$1 - E^A_{T-1}(\delta_T x_T) < E^B_{T-1}(\delta_T(1 - x_T)).$$

Suppose that $E^A_{T-1}(\delta_T) > E^B_{T-1}(\delta_T) \geq 1$. Then we have

$$\begin{aligned} 1 - x_T E^A_{T-1}(\delta_T) &< 1 - x_T E^B_{T-1}(\delta_T) \\ &\leq E^B_{T-1}(\delta_T) - x_T E^B_{T-1}(\delta_T) \\ &= (1 - x_T) E^B_{T-1}(\delta_T). \end{aligned}$$

□

Note that we can also show that there is a delay if $E^A_{T-1}(\delta_T) \geq E^B_{T-1}(\delta_T) > 1$ in a similar manner. These results indicate that players' beliefs do not have to differ in order to generate delay if both are optimistic about the future.

Proposition 6 *In period $(T-1)$, if the players' beliefs are such that*

$$E^A_{T-1}(\delta_T) - E^B_{T-1}(\delta_T) > \frac{1}{x_T}$$

where $E^B_{T-1}(\delta_T) \in [0,1)$, *then there is delay at period* $(T-1)$.

Proof. Suppose that $E^A_{T-1}(\delta_T) - E^B_{T-1}(\delta_T) > \frac{1}{x_T}$ and $E^B_{T-1}(\delta_T) \in [0,1)$. Then we have $1 - E^B_{T-1}(\delta_T) \leq 1$ and $x_T \left[E^A_{T-1}(\delta_T) - E^B_{T-1}(\delta_T)\right] > 1$. Therefore,

$$\begin{aligned} 1 - E^B_{T-1}(\delta_T) &< x_T \left[E^A_{T-1}(\delta_T) - E^B_{T-1}(\delta_T)\right] \\ 1 - x_T E^A_{T-1}(\delta_T) &< E^B_{T-1}(\delta_T) - x_T E^B_{T-1}(\delta_T) \\ 1 - E^A_{T-1}(\delta_T x_T) &< E^B_{T-1}(\delta_T(1 - x_T)). \end{aligned}$$

□

This proposition implies that $E^A_{T-1}(\delta_T)$ is necessarily larger than 1 to generate delay under the given conditions. Propositions 5 and 6 provide sufficient conditions to result in delay in period $(T-1)$. The following proposition gives a necessary condition to have delay in period $(T-1)$, and it shows that we can also observe delay when $E^B_{T-1}(\delta_T) > E^A_{T-1}(\delta_T)$.

Proposition 7 *In period* $(T-1)$, *there is a delay only if the convex combination of* $E^B_{T-1}(\delta_T)$ *and* $E^A_{T-1}(\delta_T)$ *is larger than one.*

Proof. Under our behavioral assumptions, the necessary condition that has to be satisfied to generate delay in period $(T-1)$ is that player B quotes x^B_{T-1} that will be rejected by player A with certainty, which is

$$x^B_{T-1} = (1 - x_T) E^B_{T-1}(\delta_T),$$

where

$$1 - x_T E_{T-1}^A(\delta_T) < (1 - x_T) E_{T-1}^B(\delta_T).$$

By rearranging this inequality we have

$$x_T E_{T-1}^A(\delta_T) + (1 - x_T) E_{T-1}^B(\delta_T) > 1,$$

where the right side is a convex combination of E_{T-1}^A and E_{T-1}^B with $x_T \in [0, 1]$. □

This proposition is also intuitive in the following manner. Even if the predetermined default share x_T is 0.5, indicating that the players have equal division of the surplus in the final period, as long as the proposer B is *reasonably* optimistic about the future value of the asset, meaning that $E_{T-1}^B(\delta_T)$ is sufficiently large, B can generate a delay by making a ridiculously large quote that could be larger than one. The proposition also indicates that at least one of the players must be optimistic about the future value in order to have a delay. In other words, both $E_{T-1}^A(\delta_T)$ and $E_{T-1}^B(\delta_T)$ cannot be less than one together, since otherwise it is impossible to have the value of the convex combination larger than one.

Proposition 8 *In general, there is a delay until the final period T only if*

$$x_T E_t^A(\delta_{t+1}\delta_{t+2}\cdots\delta_T) + (1 - x_T) E_t^B(\delta_{t+1}\delta_{t+2}\cdots\delta_T) > 1 \ \forall t.$$

Proof. The proof is straightforward by using a method of backward induction with the result of Proposition 7 as its initial step. Suppose that there is a delay from some odd-numbered period $(t+1)$ through period $(T-1)$. A's strategy in period t is

$$x_t^A = \max\left\{1 - E_t^B(\delta_{t+1}x_{t+1}^B),\ E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A)\right\}.$$

Now suppose that there is a delay in period t. Then, by definition it must be true that

$$x_t^A = E_t^A(\delta_{t+1}\delta_{t+2}x_{t+2}^A) > 1 - E_t^B(\delta_{t+1}x_{t+1}^B).$$

But our induction hypothesis indicates that

$$x_{t+2}^A = x_T E_{t+2}^A(\delta_{t+3}\delta_{t+4}\cdots\delta_T),$$

and

$$x_{t+1}^B = (1 - x_T)E_{t+1}^A(\delta_{t+2}\delta_{t+3}\cdots\delta_T).$$

Therefore,

$$x_t^A = x_T E_t^A(\delta_{t+1}\delta_{t+2}\cdots\delta_T) > 1 - (1 - x_T)E_t^B(\delta_{t+1}\delta_{t+2}\cdots\delta_T).$$

$$x_T E_t^A(\delta_{t+1}\delta_{t+2}\cdots\delta_T) + (1 - x_T)E_t^B(\delta_{t+1}\delta_{t+2}\cdots\delta_T) > 1.$$

The proof for odd-numbered ts is analogous. □

The proposition also indicates that at least one of the players must be optimistic about the future values of δs in order to have a delay. Hence, both $E_t^A(\delta_{t+1}\cdots\delta_T)$ and $E_t^B(\delta_{t+1}\cdots\delta_T)$ cannot be less than one together, since otherwise it is impossible to have the value of the convex combination larger than one.

In the next proposition we restate the condition given in Proposition 1, and generalize the previous propositions.

Proposition 9 *In period t when it is i's turn to make an offer, there is a delay iff*

$$E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i) + E_t^j(\delta_{t+1}x_{t+1}^j) > 1.$$

Proof. First, suppose that there is a delay in period t. Then, $x_t^i > 1 - E_t^j(\delta_{t+1}x_{t+1}^j)$, where

$$x_t^i = \max\left\{1 - E_t^j(\delta_{t+1}x_{t+1}^j),\ E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i)\right\}.$$

But this implies

$$x_t^i = E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i) > 1 - E_t^j(\delta_{t+1}x_{t+1}^j).$$

It follows that

$$E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i) + E_t^j(\delta_{t+1}x_{t+1}^j) > 1.$$

Now let us suppose that

$$E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i) + E_t^j(\delta_{t+1}x_{t+1}^j) > 1.$$

Then

$$\begin{aligned} x_t^i &= \max\left\{1 - E_t^j(\delta_{t+1}x_{t+1}^j),\ E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i)\right\} \\ &= E_t^i(\delta_{t+1}\delta_{t+2}x_{t+2}^i) \\ &> 1 - E_t^j(\delta_{t+1}x_{t+1}^j). \end{aligned}$$

□

This proposition indicates that there is no delay unless at least one of the players is optimistic about the future values of δ. Hence, there exists a possibility of delay when a proposing player's belief $E_t^i(\delta_{t+1})$ is less than one, given that a responding player has a very optimistic view about the value of δ_{t+1}, and vice versa. Note that this proposition also indicates that a mere difference in beliefs among the players is not a sufficient condition to generate a delay.

After observing some of the results concerning the Basic model, here emerges a rather natural question not to be ignored. Since a respondent does not make his counter–offer until he observes another information in the event of his rejection, why can't he also wait to declare his acceptance or rejection until he has observed the new information? Or why shouldn't he always reject a current offer to observe another information in the next period? It is these questions that gave rise to the Alternative model. Hence, the findings in this section have served us as a prelude to our study of the Alternative game. The timing of events described in the Alternative model reflects commonly observed situations that certain information which are presently unavailable will, to varying degree, become available by the time when one's opponent makes a decision.

2.3 Analysis of the Alternative Game

Our analysis of the Alternative model proceeds by utilizing dynamic programming algorithms. We first derive the equilibrium described in the following proposition by working backwards from period T through a sequence of solutions to single stage optimization problems. Let $V^i_{t+1}(Q_t)$ represent player i's maximal expected payoff from the remaining negotiation periods including period $(t+1)$, given that the value of the asset in period t is Q_t.

By using maximal expected payoff functions, the terminal conditions are written as

$$\begin{cases} V^A_T(Q_{T-1}) = x_T E^A_t(Q_T) \\ V^B_T(Q_{T-1}) = (1-x_T)E^B_t(Q_T), \end{cases}$$

and V^i_t's describe the recurrence relation. By using such notations, we find that player i rejects x^j_{t-1} in period t if there exists x^i_t such that

$$Q_t(1-x^j_{t-1}) < V^i_{t+1}(Q_t)$$

where

$$\begin{aligned} V^i_{t+1}(Q_t) = \max_{x^i_t} \; & prob\left\{Q_{t+1}(1-x^i_t) \geq V^j_{t+2}(Q_{t+1})\right\} \\ & \times E^i_t\left(Q_{t+1}x^i_t | Q_{t+1}(1-x^i_t) \geq V^j_{t+2}(Q_{t+1})\right) \\ & + prob\left\{Q_{t+1}(1-x^i_t) < V^j_{t+2}(Q_{t+1})\right\} \\ & \times E^i_t\left(V^i_{t+2}(Q_{t+1}) | Q_{t+1}(1-x^i_t) < V^j_{t+2}(Q_{t+1})\right). \end{aligned}$$

If indeed i rejects, then i quotes such x_t^i. The condition described here states that a player would reject an offer if the maximal expected payoff from the remaining period is larger than the payoff from accepting the current offer and that in the event of his rejecting the offer, the player would quote a counter–offer with which he can expect to receive at least as much as the maximal payoff. Implicitly assumed is that each player tries to maximize his expected payoff and that one prefers trading now to delaying if no payoff improvement is expected in the future.

By using the algorithm analogous to that used in the Basic game, the equilibrium to the Alternative game is given as follows.

Proposition 10 *A strategy profile* $\{\hat{S}^A, \hat{S}^B\}$ *that satisfies the following conditions is a subgame perfect equilibrium of the Alternative game.*

1. *In an even–numbered period* t*, player* A *rejects* x_{t-1}^B *iff*

$$x_{t-1}^B > 1 - E_t^A(\delta_{t+1} x_t^A),$$

and quotes

$$x_t^A = \max\left\{1 - \frac{E_t^B(\delta_{t+1}\delta_{t+2}x_{t+1}^B)}{E_t^B(\delta_{t+1})}, \frac{E_t^A(\delta_{t+1}\delta_{t+2}\delta_{t+3}x_{t+2}^A)}{E_t^A(\delta_{t+1})}\right\}. \quad (2.3)$$

A accepts x_{t-1}^B *otherwise.*

2. *In an odd–numbered period* t*, player* B *rejects* x_{t-1}^A *iff*

$$x_{t-1}^A > 1 - E_t^B(\delta_{t+1} x_t^B),$$

and quotes

$$x_t^B = \max\left\{1 - \frac{E_t^A(\delta_{t+1}\delta_{t+2}x_{t+1}^A)}{E_t^A(\delta_{t+1})}, \frac{E_t^B(\delta_{t+1}\delta_{t+2}\delta_{t+3}x_{t+2}^B)}{E_t^B(\delta_{t+1})}\right\}. \tag{2.4}$$

B accepts x_{t-1}^A *otherwise.*

Proof. The derivation of this equilibrium is included in Appendix 2A.2.

Recall that the physical environment characterized by random variables εs is memoryless, whereas players' expectations, and thus strategies, depend on the history. Therefore, $E_t^i(\delta_{t+1}\delta_{t+2}x_{t+1}^i)$ may not be equal to $E_t^i(\delta_{t+1})E_t^i(\delta_{t+2})E_t^i(x_{t+1}^i)$. We can make a following interesting observation by examining period $(T-1)$ strategies. Suppose that period $(T-1)$ is reached, and let us consider a case in which

$$x_{T-2}^A = 1 - \frac{(1-x_T)E_{T-2}^B(\delta_T\delta_{T-1})}{E_{T-2}^B(\delta_{T-1})} > \frac{x_T E_{T-2}^A(\delta_T\delta_{T-1})}{E_{T-2}^A(\delta_{T-1})}.$$

By rearranging the inequality, we have

$$x_T\frac{E_{T-2}^A(\delta_T\delta_{T-1})}{E_{T-2}^A(\delta_{T-1})} + (1-x_T)\frac{E_{T-2}^B(\delta_T\delta_{T-1})}{E_{T-2}^B(\delta_{T-1})} \leq 1.$$

It is clear from this resulting inequality that in period $(T-2)$ there was at least one player who had a pessimistic view about the future asset value. Moreover, it is also indicative that both players might have been pessimistic in $(T-2)$, meaning that both might have speculated the value of δ to be less than one. Now consider player B's period $(T-1)$ strategies. Given the value of x_{T-2}^A as above, he rejects it if $x_{T-2}^A > 1-(1-x_T)E_{T-1}^B(\delta_T)$.

This condition can be rewritten in the following way.

$$
\begin{aligned}
1-\frac{(1-x_T)E^B_{T-2}(\delta_T\delta_{T-1})}{E^B_{T-2}(\delta_{T-1})}+(1-x_T)E^B_{T-1}(\delta_T) &> 1 \\
E^B_{T-1}(\delta_T)E^B_{T-2}(\delta_{T-1})-E^B_{T-2}(\delta_T\delta_{T-1}) &> 0.
\end{aligned}
$$

Therefore, if B speculates a higher value of δ_T after observing δ_{T-1} than he thought before observing δ_{T-1}, then he would reject x^A_{T-2} resulting in a delay until the final period. The possibility of this case is interesting, because this is the case in which both players might have not been very optimistic in period $(T-2)$, yet we may observe another delay in period $(T-1)$ if player B's view changes after observing new information δ_{T-1}. Note that such a case is not possible in the Basic bargaining game. Having no chance of observing another information before responding to a current offer in the Basic game, the other player immediately accepts such an offer that reflects *not very optimistic* views of the players.

The above observation is generalized in the following proposition.

Proposition 11 *The following conditions are sufficient to generate a delay in Alternative bargaining games.*

1. *There is a delay in an odd-numbered period t if*

$$E^B_{t-1}(\delta_t)E^B_t(\delta_{t+1}x^B_t) > E^B_{t-1}(\delta_t\delta_{t+1}x^B_t).$$

2. *There is a delay in an even-numbered period t if*

$$E^A_{t-1}(\delta_t)E^A_t(\delta_{t+1}x^A_t) > E^A_{t-1}(\delta_t\delta_{t+1}x^A_t).$$

Proof. We give a proof for the odd–numbered period. The proof for an even–numbered period is analogous. We need to look at the following two cases.

1. $x^A_{t-1} = 1 - \frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)} > \frac{E^A_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^A_{t+1})}{E^A_{t-1}(\delta_t)}$

2. $x^A_{t-1} = \frac{E^A_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^A_{t+1})}{E^A_{t-1}(\delta_t)} \geq 1 - \frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)}$.

First, consider case 1. We need to show $x^A_{t-1} > 1 - E^B_t(\delta_{t+1}x^B_t)$, which is the condition to generate a delay. Suppose

$$E^B_{t-1}(\delta_t)E^B_t(\delta_{t+1}x^B_t) > E^B_{t-1}(\delta_t\delta_{t+1}x^B_t).$$

This inequality is rearranged to

$$1 - \frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)} > 1 - E^B_t(\delta_{t+1}x^B_t).$$

But by assumption,

$$x^A_{t-1} = 1 - \frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)}.$$

Therefore,

$$x^A_{t-1} > 1 - E^B_t(\delta_{t+1}x^B_t).$$

Similarly, consider case 2, and suppose

$$E^B_{t-1}(\delta_t)E^B_t(\delta_{t+1}x^B_t) > E^B_{t-1}(\delta_t\delta_{t+1}x^B_t).$$

This inequality is rearranged to

$$1-\frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)} > 1-E^B_t(\delta_{t+1}x^B_t).$$

But by assumption

$$\begin{aligned} x^A_{t-1} &= \frac{E^A_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^A_{t+1})}{E^A_{t-1}(\delta_t)} \\ &\geq 1-\frac{E^B_{t-1}(\delta_t\delta_{t+1}x^B_t)}{E^B_{t-1}(\delta_t)}. \end{aligned}$$

Hence,

$$x^A_{t-1} > 1-E^B_t(\delta_{t+1}x^B_t).$$

□

Therefore, after observing δ_t in some odd–numbered period t, if B becomes more optimistic about the value of δ_{t+1} than he was in period $(t-1)$, then there is a delay. Note that this proposition indicates that the statement given in Proposition 2 is not true for Alternative games. In other words, the assumption of $\delta_t \in [0,1)$ alone does not guarantee an immediate agreement in Alternative games. We will observe more variations in bargaining durations in Alternative games when we run simulations in Chapter 3.

The following lemma is analogous to Lemma 1, except that we eliminate the possibility of $\delta = 0$ to avoid undefined fractions in the derived equilibrium.

Lemma 6 *If $\delta_t \in (0,1)$ for all t with certainty, then $x^i_t \in [0,1]$.*

Proof. This is immediate by following our backward induction algorithm. □

The conditions given in the next proposition, which are the same as those given in Proposition 3, guarantee an immediate agreement in Alternative games.

Proposition 12 *If $\{\delta_\tau\}_{\tau=1}^{t}$ is non–stochastic, in particular $\delta \in (0,1)$ and $\delta_t = \delta^t$, then the solution to the Alternative game converges to the solution to the Rubinstein's model as $T \to \infty$.*

Proof. First, we show that there is an immediate agreement under the given conditions. Suppose that δs are non–stochastic, $\delta \in (0,1)$, and $\delta_t = \delta^t$. Since $x_{T-1}^B = 1 - x_T$ and $\delta^T \in (0,1)$, A's strategy in period $(T-2)$ is

$$\begin{aligned} x_{T-2}^A &= \max\{1 - \delta^T(1 - x_T),\ \delta^T x_T\} \\ &= 1 - \delta^T(1 - x_T) \\ &= 1 - \delta^T x_{T-1}^B. \end{aligned}$$

Given such x_{T-2}^A, B's strategy in period $(T-3)$ is

$$\begin{aligned} x_{T-3}^B &= \max\{1 - \delta^{T-1}(1 - \delta^T(1 - x_T)),\ \delta^T \delta^{T-1}(1 - x_T)\} \\ &= 1 - \delta^{T-1}(1 - \delta^T(1 - x_T)) \\ &= 1 - \delta^{T-1} x_{T-2}^A. \end{aligned}$$

By induction, it is straightforward to show that in general

$$x_t^i = 1 - \delta^{t+2} x_{t+1}^j. \tag{2.5}$$

Hence, A's strategy in period 0 is $x_0^A = 1 - \delta^2 x_1^B$. B accepts x_0^A in period 1 if $x_0^A \leq 1 - \delta^2 E_1^B(x_1^B)$. But this is satisfied since

$$x_0^A \;=\; 1 - \delta^2 x_1^B \;\leq\; 1 - \delta^2 x_1^B \;=\; 1 - \delta^2 E_1^B(x_1^B).$$

Therefore, there is an immediate agreement.

Now we show that $x_0^A = \frac{1}{1+\delta}$ as $T \to \infty$. The equation 2.5 can be rewritten as

$$x_t^i = 1 - \delta^{t+2}\left(1 - \delta^{t+3}\left(1 - \delta^{t+4}(\cdots(1 - \delta^T(1 - x_T)))\right)\right).$$

Hence, we can express x_0^A as

$$\begin{aligned} x_0^A &= 1 - \delta^2\left(1 - \delta^3\left(1 - \delta^4(\cdots(1 - \delta^T(1 - x_T)))\right)\right) \\ &= \sum_{t=1}^{T}(-1)^{t-1}\delta^{(\sum_{i=0}^{t} i-1)} + \delta^{(\sum_{j=0}^{T} j-1)} x_T. \end{aligned}$$

By taking a limit of the last expression, we have

$$\begin{aligned} &\lim_{T\to\infty}\left[\sum_{t=1}^{T}(-1)^{t-1}\delta^{(\sum_{i=0}^{t} i-1)} + \delta^{(\sum_{j=0}^{T} j-1)} x_T\right] \\ &= \lim_{T\to\infty}\sum_{t=1}^{T}\left((-1)\delta^{\frac{1}{t-1}(\sum_{i=0}^{t} i-1)}\right)^{t-1}. \end{aligned}$$

Note that by using L'Hôpital's rule,

$$\lim_{t\to\infty}\frac{\sum_{i=0}^{t} i - 1}{t-1} = \lim_{t\to\infty}\frac{\frac{d}{dt}(\sum_{i=0}^{t} i - 1)}{\frac{d}{dt}(t-1)} = 1.$$

Since $\delta \in (0,1)$, by rewriting the sum of a converging geometric series, we have

$$x_0^A = \sum_{t=1}^{\infty}(-\delta)^{t-1} = \sum_{t=0}^{\infty}(-\delta)^t = \frac{1}{1+\delta}.$$

This completes the proof. □

The next proposition also gives conditions in which results of Alternative games converge to the Rubinstein's solution as it was observed with Basic games.

Proposition 13 *If δ_t's are identically and independently distributed, and the players agree on the expected values, which is $E(\delta_t) = \delta^* \in (0,1)$, then the solution to the Alternative game converges to the solution to the Rubinstein's model, provided that $T \longrightarrow \infty$.*

Proof. The proof proceeds in the same fashion as that of the previous proposition. Since B's strategy in period $(T-1)$ is $x_{T-1}^B = 1 - x_T$, A's strategy in period $(T-2)$ is

$$\begin{aligned} x_{T-2}^A &= \max\{1-\delta^*(1-x_T),\ \delta^* x_T\} \\ &= 1-\delta^*(1-x_T) \\ &= 1-\delta^* x_{T-1}^B. \end{aligned}$$

Similarly, B's strategy in period $(T-3)$ is

$$\begin{aligned} x_{T-3}^B &= \max\{1-\delta^*(1-\delta^*(1-x_T)),\ \delta^{*^2}(1-x_T)\} \\ &= 1-\delta^*(1-\delta^*(1-x_T)) \\ &= 1-\delta^* x_{T-2}^A. \end{aligned}$$

In general,

$$x_t^i = 1 - \delta^* x_{t+1}^j.$$

Hence, $x_0^A = 1 - \delta^* x_1^B$. B accepts x_0^A in period 1 if $x_0^A \leq 1 - \delta^* E_1^B(x_1^B)$. But this is satisfied since

$$x_0^A \;=\; 1 - \delta^* x_1^B \;\leq\; 1 - \delta^* x_1^B \;=\; 1 - \delta^* E_1^B(x_1^B).$$

Therefore, there is an immediate agreement. x_0^A can be also expressed as

$$\begin{aligned} x_0^A &= 1 - \delta^*\left(1 - \delta^*(\cdots(1 - \delta^*(1 - x_T)))\right) \\ &= 1 - \delta^* + \delta^{*^2} - \delta^{*^3} + \cdots - \delta^{*(T-1)} + \delta^{*(T-1)} x_T \\ &= \sum_{t=0}^{T-1} (-\delta^*)^t + \delta^{*(T-1)} x_T. \end{aligned}$$

Thus, as T approaches to infinity we have

$$\begin{aligned} x_0^A &= \lim_{T\to\infty} \left[\sum_{t=0}^{T-1} (-\delta^*)^t + \delta^{*(T-1)} x_T \right] \\ &= \frac{1}{1+\delta^*}. \end{aligned}$$

Hence, the proposition holds when $\delta^* \in (0,1)$. □

Consequently, if we eliminate a speculative element in the beliefs from our bargaining games in a certain way as above, then the results of both Basic and Alternative games with $T \to \infty$ conform to that of Rubinstien's model.

The next proposition gives a condition to generate a delay in a penultimate period.

Proposition 14 *There is delay in period $(T-1)$, if and only if player B is optimistic about* *the asset value in period T in such a way that* $\frac{1-x^A_{T-2}}{1-x_T} < E^B_{T-1}(\delta_T)$.

Proof. The proof is straightforward from the equilibrium conditions given in Proposition 10. In period $(T-1)$, the condition that will make player B reject x^A_{T-2} is $x^A_{T-2} > 1-(1-x_T)E^B_{T-1}(\delta_T)$, which is rearranged to give

$$\frac{1-x^A_{T-2}}{1-x_T} < E^B_{T-1}(\delta_T).$$

□

This proposition says that if B expects the asset value to increase in the next period more than the ratio of his share in the current period $(T-1)$ to his share in period T, then there will be a delay in period $(T-1)$. This sort of delay is likely to occur especially when B's expectation of period–T value in period $(T-1)$ becomes larger than his expectation in period $(T-2)$ after observing δ_{T-1}.

2.4 Notes on the Distribution of Surplus

By construction of our equilibria in the Basic bargaining game, there is an advantage of being a proposer insofar as extracting a surplus in excess of his maximum expected continuation payoff in the event of agreement. Suppose that player i proposes $x^i_t = \max\{1-\overline{x^j_t}, \overline{x^i_t}\}$ in period t. If the offer is accepted, then we know $x^i_t = 1-\overline{x^j_t} \geq \overline{x^i_t}$. Therefore, the payoff vector is $(\pi^i_t, \pi^j_t) = \left(Q_t(1-\overline{x^j_t}), Q_t\overline{x^j_t}\right) = (M^i_t+\epsilon, M^j_t)$, where $\epsilon \geq 0$. Consequently, the equilibrium was constructed in such a way that if an agreement is reached, then a proposer extracts a surplus over what he expects to receive by delaying until any future period, while a responder receives the amount exactly equal to his maximum continuation payoff.

This, however, does not mean that a proposer's payoff is always higher than that of a responder. The size of a realized payoff depends on each player's view of the future. In the example above, it can be observed that $\pi_t^i = Q_t(1-\overline{x_t^j}) > Q_t\overline{x_t^j} = \pi_t^j$ if and only if $\overline{x_t^j} < 1/2$. In other words, the proposer's payoff is larger than the responder's payoff only when the responder is sufficiently pessimistic about his future continuation payoffs. As a numerical example, let us look at Exhibit 3 that is included in Table 3.3 of Appendix 3A.1. This is the case in which player A starts the negotiation with a more pessimistic view than player B.[10] Take Path = 10 in the table, where an agreement occurs.[11] Despite the fact that A is a proposer in period 2, A receives less than B. With regard to surplus extraction, notice that $\pi_2^A > M_2^A$, whereas $\pi_2^B = M_2^B$. Hence, the proposer A has received more than her maximum reservation value, while B received exactly the same as his maximum reservation value. This is in accord with our discussion above.

In the Alternative game, it is possible to observe both players receiving positive surplus; that is, $\pi_t^A > M_t^A$ and $\pi_t^B > M_t^B$ in the same period. Suppose that player i proposes an offer such that $x_t^i = \max\{1-\overline{x_t^j}, \overline{x_t^i}\}$, and first let us consider the case in which $x_t^i = 1-\overline{x_t^j} \geq \overline{x_t^i}$. We would like to find a condition to have both $\pi_{t+1}^i > M_{t+1}^i$ and $\pi_{t+1}^j > M_{t+1}^j$ in the event of agreement in period $(t+1)$. We can observe, by simple manipulation of the inequalities, that both players have a positive surplus if and only if both $M_{t+1}^j < Q_{t+1}\overline{x_t^j}$ and $Q_{t+1}\overline{x_t^j} < Q_{t+1} - M_{t+1}^i$ are satisfied. Consequently, the necessary condition to have both inequalities satisfied is $M_{t+1}^i + M_{t+1}^j < Q_{t+1}$. In a similar fashion, we can obtain the

[10] After observing the information in period 0, player A assigns a probability 0.25 to the arrival of favorable information in period 1, whereas player B assigns a probability 0.67.

[11] Path = 10 is the path with $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$. This will be described in detail in Chapter 3.

same necessary condition in the case where $x_t^i = \overline{x_t^i} > 1 - \overline{x_t^j}$. As a numerical example, let us look at Exhibit 10 included in Table 3.10 of Appendix 3A.2. The first agreement occurs in period 1, in which $\pi_1^A = 0.596253 > 0.596184 = M_1^A$ and $\pi_1^B = 0.596279 > 0.596266 = M_1^B$, indicating a positive surplus in excess of maximum continuation payoff on both players.

2.5 Discussion

It has become clear that it matters to analyze the Basic game and the Alternative game separately.[12] In the Basic model, a proposer knows exactly what his and his opponent's payoffs will be if his offer is accepted in the current period. A responder in the Basic model also knows exactly what his and his opponent's payoffs are if he accepts the offer. On the contrary, in the Alternative model a proposer knows what his payoff in the event of acceptance will be only in terms of its expected value when he quotes his offer, while a responder knows exactly what his payoff will be if he accepts the offer. Consequently, a proposer in Alternative games cannot generate a delay with certainty when he makes an offer, since he cannot observe the next period δ that his opponent will observe before responding. But a proposer can force a delay with certainty in Basic games. For example, as indicated in Proposition 11, the assumption of $\delta_t \in (0,1)$ alone does not guarantee an immediate agreement in Alternative games, whereas it is guaranteed in Basic games as shown in Proposition 2. A bargainer's ability to observe certain information before making his decision affects his bargaining power, and proposer's bargaining power in Alternative games is not as strong as that in the Basic games in terms of manipulating durations. It is

[12] As numerical examples, it is interesting to compare Exhibits 3 and 5, or Exhibits 6 and 10 that are included in Appendix 3A.

also clear from the derived equilibria that an additional uncertainty on the proposing party affects reservation values.

Are the two games equivalent if a responding party always rejects the current offer in the Basic model to observe new information? In computing the offer–strategies the proposer's reservation values in the two games are equivalent *if* a proposer in the Basic game knows with certainty that a responder *always* rejects his offer in the current period in order to observe the next available information. This is because in such a case the proposer in the Basic game adjusts his reservation value to incorporate the additional uncertainty as if he were playing the Alternative game. Without such an anticipation of consistent rejection,[13] a proposer in the Basic game has a different reservation value from a proposer in the Alternative game as we have observed.

As it was shown, however, there are conditions with which the solution to the Basic games coincides with that to the Alternative games.[14] For example, if δ_ts are identically and independently distributed with $E(\delta_t) = \delta^* \in [0,1)$, then the solution to both games converges to a unique subgame perfect equilibrium that is the same as the Rubinstein's solution. In Rubinstein's perfect and complete information bargaining game, it is players' time preferences modeled as discounting of the asset value that gives a pressure for an early agreement. In our model the pressure comes from players' speculation on the information flow in the future. Consequently, when players speculate a series of future information shocks to be an undesirable one, such a speculation leads to an effect on bargaining outcomes that is similar to the one caused by a presence of discount factors.

[13]We note that it is not reasonable to assume such rejection strategies on the part of responder if he can achieve the highest expected payoff by accepting the current offer.

[14]Refer to propositions 3, 4, 12, and 13.

This research originally started out by introducing δ as a convenient way of capturing differences in beliefs after observing common information as a stochastic process. The models studied here have reduced the richness of the initial bargaining games by altering the interpretation of δ, and the original intent was not accomplished. However, even in this restricted framework, our analyses confirm that the introduction of stochastic components into bargaining situations is essential in describing varied bargaining durations. For example, as it was found in Proposition 3 of the Basic game and in Proposition 12 of the Alternative game, if a sequence of δs is not stochastic in addition to the assumption of $\delta_t \in (0, 1)$, then the solution converges to that of Rubinstein's bargaining model.

Appendix 2A

Derivations of Equilibria

The derivations of the equilibria given in Propositions 1 and 10 are provided. Each step of backward induction algorithm is described in detail.

2A.1 Equilibrium of the Basic Game

Proposition 1 *A strategy profile $\{\hat{S}^A, \hat{S}^B\}$ that satisfies the following conditions is a subgame perfect equilibrium of the basic game.*

1. *In an even–numbered period t, player A makes an offer such that*

$$x_t^A = \max\left\{1 - E_t^B(\delta_{t+1} x_{t+1}^B),\ E_t^A(\delta_{t+1}\delta_{t+2} x_{t+2}^A)\right\}, \tag{2A.1}$$

   and player B accepts x_t^A if $x_t^A \leq 1 - E_t^B(\delta_{t+1} x_{t+1}^B)$, and rejects otherwise.

2. *In an odd–numbered period t, player B makes an offer such that*

$$x_t^B = \max\left\{1 - E_t^A(\delta_{t+1} x_{t+1}^A),\ E_t^B(\delta_{t+1}\delta_{t+2} x_{t+2}^B)\right\}, \tag{2A.2}$$

and player A accepts x_t^B *if* $x_t^B \leq 1 - E_t^A(\delta_{t+1} x_{t+1}^A)$, *and rejects otherwise.*

Proof of Proposition 1 : The derivation of this equilibrium is immediate by using the backward–induction algorithm outlined in section 2.2. Note that subgame perfection coincides with backward induction in games of perfect information. Recall the following behavioral assumptions that each player's strategy must satisfy.

C–I In the event of the respondent's accepting the current offer, the respondent receives at least as much as what the respondent expects to receive in any future period.

C–II In the event of the respondent's accepting the current offer, the proposer receives at least as much as what the proposer expects to receive in any future period.

(i) In period $(T-1)$, it is B's turn to quote an offer. Condition C–I gives

$$\begin{aligned} Q_{T-1}(1 - x_{T-1}^B) &\geq E_{T-1}^A(Q_T x_T) \\ Q_{T-1}(1 - x_{T-1}^B) &\geq E_{T-1}^A(\delta_T Q_{T-1} x_T) \\ x_{T-1}^B &\leq 1 - x_T E_{T-1}^A(\delta_T), \end{aligned}$$

indicating that an offer that is smaller than or equal to $1 - x_T E_{T-1}^A(\delta_T)$ guarantees A's acceptance. Condition C–II gives

$$\begin{aligned} Q_{T-1} x_{T-1}^B &\geq E_{T-1}^B(Q_T(1 - x_T)) \\ Q_{T-1} x_{T-1}^B &\geq E_{T-1}^B(\delta_T Q_{T-1}(1 - x_T)) \\ x_{T-1}^B &\geq (1 - x_T) E_{T-1}^B(\delta_T), \end{aligned}$$

meaning that $(1 - x_T)E^B_{T-1}(\delta_T)$ is the smallest offer B is willing to make. Hence, player B's offer in period $(T-1)$ is

$$x^B_{T-1} = \max\left\{1 - x_T E^A_{T-1}(\delta_T),\ (1 - x_T)E^B_{T-1}(\delta_T)\right\}. \tag{2A.3}$$

A would accept x^B_{T-1} such that

$$x^B_{T-1} \leq 1 - x_T E^A_{T-1}(\delta_T),$$

since such x^B_{T-1} satisfies the condition C–I.

(*ii*) In period $(T-2)$, the largest offer A is willing to make, which satisfies condition C–I, is given as

$$x^A_{T-2} = \min\left\{1 - E^B_{T-2}(\delta_{T-1}x^B_{T-1}),\ 1 - (1 - x_T)E^B_{T-2}(\delta_T\delta_{T-1})\right\}. \tag{2A.4}$$

This is because the offer has to guarantee B a payoff at least as large as B's expected payoff in periods $(T-1)$ and that in period T. The former requires

$$\begin{aligned} Q_{T-2}(1 - x^A_{T-2}) &\geq E^B_{T-2}(Q_{T-1}x^B_{T-1}) \\ Q_{T-2}(1 - x^A_{T-2}) &\geq E^B_{T-2}(\delta_{T-1}Q_{T-2}x^B_{T-1}) \\ x^A_{T-2} &\leq 1 - E^B_{T-2}(\delta_{T-1}x^B_{T-1}), \end{aligned}$$

and the latter requires

$$\begin{aligned} Q_{T-2}(1-x^A_{T-2}) &\geq E^B_{T-2}(Q_T(1-x_T)) \\ Q_{T-2}(1-x^A_{T-2}) &\geq E^B_{T-2}(\delta_T\delta_{T-1}Q_{T-2}(1-x_T)) \\ x^A_{T-2} &\leq 1-(1-x_T)E^B_{T-2}(\delta_T\delta_{T-1}). \end{aligned}$$

The smallest offer satisfying condition C–II is

$$x^A_{T-2} = \max\left\{E^A_{T-2}(\delta_{T-1}(1-x^B_{T-1})),\ x_T E^A_{T-2}(\delta_T\delta_{T-1})\right\}, \tag{2A.5}$$

since the offer has to give A at least as much a payoff as she expects to receive in period $(T-1)$; that is,

$$\begin{aligned} Q_{T-2}x^A_{T-2} &\geq E^A_{T-2}(Q_{T-1}(1-x^B_{T-1})) \\ Q_{T-2}x^A_{T-2} &\geq E^A_{T-2}(\delta_{T-1}Q_{T-2}(1-x^B_{T-1})) \\ x^A_{T-2} &\geq E^A_{T-2}(\delta_{T-1}(1-x^B_{T-1})), \end{aligned}$$

and also at least as much as she expects to receive in period T; that is,

$$\begin{aligned} Q_{T-2}x^A_{T-2} &\geq E^A_{T-2}(Q_T x_T) \\ Q_{T-2}x^A_{T-2} &\geq E^A_{T-2}(\delta_T\delta_{T-1}Q_{T-2}x_T) \\ x^A_{T-2} &\geq x_T E^A_{T-2}(\delta_T\delta_{T-1}). \end{aligned}$$

Hence, the conditions 2A.4 and 2A.5 tell us that A makes an offer such that

$$\begin{aligned} x^A_{T-2} &= \max\left\{E^A_{T-2}(\delta_{T-1}(1-x^B_{T-1})),\ x_T E^A_{T-2}(\delta_T\delta_{T-1}),\right. \\ &\qquad \left.\min\{1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}),\ 1-(1-x_T)E^B_{T-2}(\delta_T\delta_{T-1})\}\right\}. \end{aligned} \tag{2A.6}$$

But the equation 2A.6 is simplified to

$$x^A_{T-2} = \max\left\{1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}),\ x_T E^A_{T-2}(\delta_T\delta_{T-1})\right\}, \tag{2A.7}$$

due to the reasons described in 1 and 2 below.

1. $x^B_{T-1} \geq E^B_{T-1}(\delta_T(1-x_T))$ implies

$$1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}) \leq 1-E^B_{T-2}(\delta_{T-1}E^B_{T-1}(\delta_T(1-x_T))).$$

   But by applying the law of iterated mathematical expectations to the right–hand side to condition on the information up to period $(T-2)$, this can be rewritten as

$$1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}) \leq 1-E^B_{T-2}(\delta_{T-1}\delta_T(1-x_T)).$$

   Hence,

$$\min\left\{1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}),\ 1-E^B_{T-2}(\delta_T\delta_{T-1}(1-x_T))\right\} = 1-E^B_{T-2}(\delta_{T-1}x^B_{T-1}).$$

2. $x^B_{T-1} \geq 1 - E^A_{T-1}(\delta_T x_T)$ implies

$$E^A_{T-2}(\delta_{T-1}(1 - x^B_{T-1})) \leq E^A_{T-2}(\delta_{T-1} E^A_{T-1}(\delta_T x_T)).$$

But by applying the law of iterated expectations to the right–hand side, this is written as

$$E^A_{T-2}(\delta_{T-1}(1 - x^B_{T-1})) \leq E^A_{T-2}(\delta_{T-1}\delta_T x_T).$$

Hence,

$$\max\left\{E^A_{T-2}(\delta_{T-1}(1 - x^B_{T-1})),\ E^A_{T-2}(\delta_T\delta_{T-1}x_T)\right\} = E^A_{T-2}(\delta_T\delta_{T-1}x_T).$$

(*iii*) Suppose that the equilibrium strategy prescribed in the proposition holds for periods t through $(T-1)$. We need to show that the strategies in period $(t-1)$ follow those of the equilibrium, i.e.,

$$x^j_{t-1} = \max\{1 - E^i_{t-1}(\delta_t x^i_t), E^j_{t-1}(\delta_t\delta_{t+1}x^j_{t+1})\},$$

and player i accepts x^j_{t-1} if $x^j_{t-1} \leq 1 - E^i_{t-1}(\delta_t x^i_t)$.

Condition C–I indicates that j's offer has to satisfy

$$\begin{aligned} x^j_{t-1} \ \leq\ & \min\{1 - E^i_{t-1}(\delta_t x^i_t), 1 - E^i_{t-1}(\delta_{t+1}\delta_t(1 - x^j_{t+1})), \\ & 1 - E^i_{t-1}(\delta_{t+2}\delta_{t+1}\delta_t x^i_{t+2}), 1 - E^i_{t-1}(\delta_{t+3}\delta_{t+2}\delta_{t+1}\delta_t(1 - x^j_{t+3})), \cdots\}. \end{aligned}$$

By the induction hypothesis and an application of the law of iterated mathematical expectations, we have $x^j_{t-1} \leq 1 - E^i_{t-1}(\delta_t x^i_t)$.

Condition C–II indicates that j's offer has to satisfy

$$\begin{aligned} x^j_{t-1} \geq & \max\{E^j_{t-1}(\delta_t(1-x^i_t)), E^j_{t-1}(\delta_{t+1}\delta_t x^j_{t+1}), \\ & E^j_{t-1}(\delta_{t+2}\delta_{t+1}\delta_t(1-x^i_{t+2})), E^j_{t-1}(\delta_{t+3}\delta_{t+2}\delta_{t+1}\delta_t x^j_{t+3}), \cdots\}. \end{aligned}$$

By the induction hypothesis and an application of the law of iterated mathematical expectations, we have $x^j_{t-1} \geq E^j_{t-1}(\delta_{t+1}\delta_t x^j_{t+1})$.

Consequently, player j's offer strategy in period $(t-1)$ is

$$x^j_{t-1} = \max\{1 - E^i_{t-1}(\delta_t x^i_t), E^j_{t-1}(\delta_t \delta_{t+1} x^j_{t+1})\},$$

as desired. By construction we know that player i's acceptance decision follows the strategy based on a unique reservation value, i.e., i accepts x^j_{t-1} if $x^j_{t-1} \leq 1 - E^i_{t-1}(\delta_t x^i_t)$.

Therefore, we conclude that the equilibrium strategy holds for all t.

□

2A.2 Equilibrium of the Alternative Game

Proposition 10 *A strategy profile $\{\hat{S}^A, \hat{S}^B\}$ that satisfies the following conditions is a subgame perfect equilibrium of the alternative game.*

1. *In an even–numbered period t, player A rejects x^B_{t-1} if*

$$x^B_{t-1} > 1 - E^A_t(\delta_{t+1} x^A_t),$$

and quotes

$$x^A_t = \max\left\{1 - \frac{E^B_t(\delta_{t+1}\delta_{t+2}x^B_{t+1})}{E^B_t(\delta_{t+1})}, \frac{E^A_t(\delta_{t+1}\delta_{t+2}\delta_{t+3}x^A_{t+2})}{E^A_t(\delta_{t+1})}\right\}. \tag{2A.8}$$

A accepts x^B_{t-1} otherwise.

2. *In an odd–numbered period t, player B rejects x^A_{t-1} if*

$$x^A_{t-1} > 1 - E^B_t(\delta_{t+1} x^B_t),$$

and quotes

$$x^B_t = \max\left\{1 - \frac{E^A_t(\delta_{t+1}\delta_{t+2}x^A_{t+1})}{E^A_t(\delta_{t+1})}, \frac{E^B_t(\delta_{t+1}\delta_{t+2}\delta_{t+3}x^B_{t+2})}{E^B_t(\delta_{t+1})}\right\}. \tag{2A.9}$$

B accepts x^A_{t-1} otherwise.

Proof of Proposition 10 :

By using the algorithm similar to the one used in the basic model, we derive the equilibrium of the alternative model. Hence, we work backwards from period T through a sequence of solutions to single stage optimization problems.

(i) In period $(T-1)$ after observing δ_{T-1}, it is player B's turn to respond to A's action that was taken in period $(T-2)$. B rejects x^A_{T-2} if his utility by accepting it in the current period is smaller than his expected payoff from period T; that is,

$$\begin{aligned} Q_{T-1}(1-x^A_{T-2}) &< E^B_{T-1}(Q_T(1-x_T)) \\ Q_{T-1}(1-x^A_{T-2}) &< E^B_{T-1}(\delta_T Q_{T-1}(1-x_T)) \\ x^A_{T-2} &> 1-(1-x_T)E^B_{T-1}(\delta_T). \end{aligned}$$

Then, B will quote x^B_{T-1} that satisfies

$$\begin{aligned} E^A_{T-1}(Q_T(1-x^B_{T-1})) &\geq E^A_{T-1}(Q_T x_T) \\ x^B_{T-1} &\leq 1-x_T, \end{aligned}$$

indicating that an offer that guarantees A's acceptance has to be smaller than or equal to $1-x_T$, and

$$\begin{aligned} E^B_{T-1}(Q_T x^B_{T-1}) &\geq E^B_{T-1}(Q_T(1-x_T)) \\ x^B_{T-1} &\geq 1-x_T, \end{aligned}$$

indicating that B needs to make an offer that is at least as large as $1 - x_T$ to grant himself a payoff not less than he is guaranteed to receive in period T. But B knows that it is meaningless for B to quote $x^B_{T-1} > 1 - x_T$ since it would only result in A's rejection without improving his payoff. Hence, the only offer B would make in the event of his rejecting x^A_{T-2} is $x^B_{T-1} = 1 - x_T$.

(*ii*) In period $(T-2)$, it is player A's turn to respond to B's action from the previous period. A rejects x^B_{T-3} if

$$x^B_{T-3} > \min\left\{1 - E^A_{T-2}(\delta_{T-1} x^A_{T-2}),\ 1 - x_T E^A_{T-2}(\delta_T \delta_{T-1})\right\}. \tag{2A.10}$$

This is because A would reject x^B_{T-3} if accepting it in the current period gives her a payoff strictly smaller than her expected payoff from period $(T-1)$; that is,

$$\begin{aligned} Q_{T-2}(1 - x^B_{T-3}) &< E^A_{T-2}(Q_{T-1} x^A_{T-2}) \\ x^B_{T-3} &> 1 - E^A_{T-2}(\delta_{T-1} x^A_{T-2}), \end{aligned}$$

or a payoff strictly smaller than her expected payoff from period T; that is,

$$\begin{aligned} Q_{T-2}(1 - x^B_{T-3}) &< E^A_{T-2}(Q_T x_T) \\ x^B_{T-3} &> 1 - x_T E^A_{T-2}(\delta_T \delta_{T-1}). \end{aligned}$$

Given the condition 2A.10, A will quote x^A_{T-2} such that

$$x^A_{T-2} = \max\left\{1 - \frac{(1-x_T)E^B_{T-2}(\delta_T\delta_{T-1})}{E^B_{T-2}(\delta_{T-1})}, \frac{x_T E^A_{T-2}(\delta_T\delta_{T-1})}{E^A_{T-2}(\delta_{T-1})}\right\}. \tag{2A.11}$$

The first expression in the braces is due to the condition that guarantees B's acceptance; that is, $Q_{T-1}(1-x^A_{T-2}) \geq E^B_{T-1}(Q_T(1-x_T))$. Since in period $(T-2)$ player A can infer this only in terms of her expectation given information up to period $(T-2)$, the condition is in fact,

$$\begin{aligned} E^B_{T-2}(Q_{T-1}(1-x^A_{T-2})) &\geq E^B_{T-2}(Q_T(1-x_T)) \\ x^A_{T-2} &\leq 1 - \frac{(1-x_T)E^B_{T-2}(\delta_T\delta_{T-1})}{E^B_{T-2}(\delta_{T-1})}. \end{aligned}$$

The second expression in the braces is due to the condition that gives A a payoff in period $(T-1)$ at least as much as that in period T; that is,

$$\begin{aligned} E^A_{T-2}(Q_{T-1}x^A_{T-2}) &\geq E^A_{T-2}(Q_T x_T) \\ x^A_{T-2} &\geq \frac{x_T E^A_{T-2}(\delta_T\delta_{T-1})}{E^A_{T-2}(\delta_{T-1})}. \end{aligned}$$

This condition implies

$$\begin{aligned} E^A_{T-2}(\delta_{T-1}x^A_{T-2}) &\geq E^A_{T-2}(\delta_T\delta_{T-1}x_T) \\ 1 - E^A_{T-2}(\delta_{T-1}x^A_{T-2}) &\leq 1 - E^A_{T-2}(\delta_T\delta_{T-1}x_T). \end{aligned}$$

Hence, the condition 2A.10 simplifies to

$$x^B_{T-3} > 1 - E^A_{T-2}(\delta_{T-1} x^A_{T-2}). \tag{2A.12}$$

(*iii*) In period $(T-3)$, player B rejects x^A_{T-4} if

$$\begin{aligned} x^A_{T-4} \quad > \quad & \min \left\{ 1 - E^B_{T-3}(\delta_{T-2} x^B_{T-3}),\ 1 - E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1 - x^A_{T-2})), \right. \\ & \left. 1 - (1 - x_T) E^B_{T-3}(\delta_T \delta_{T-1} \delta_{T-2}) \right\}. \end{aligned} \tag{2A.13}$$

This is because B would reject x^A_{T-4} if accepting it in the current period gives him a payoff strictly smaller than his expected payoff from period $(T-2)$; that is,

$$\begin{aligned} Q_{T-3}(1 - x^A_{T-4}) \quad &< \quad E^B_{T-3}(Q_{T-2} x^B_{T-3}) \\ x^A_{T-4} \quad &> \quad 1 - E^B_{T-3}(\delta_{T-2} x^B_{T-3}), \end{aligned}$$

or a payoff strictly smaller than his expected payoff from period $(T-1)$; that is,

$$\begin{aligned} Q_{T-3}(1 - x^A_{T-4}) \quad &< \quad E^B_{T-3}(Q_{T-1}(1 - x^A_{T-2})) \\ x^A_{T-4} \quad &> \quad 1 - E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1 - x^A_{T-2})), \end{aligned}$$

or a payoff strictly smaller than his expected payoff from period T; that is,

$$\begin{aligned} Q_{T-3}(1 - x^A_{T-4}) \quad &< \quad E^B_{T-3}(Q_T(1 - x_T)) \\ x^A_{T-4} \quad &> \quad 1 - (1 - x_T) E^B_{T-3}(\delta_T \delta_{T-1} \delta_{T-2}). \end{aligned}$$

Given the condition 2A.13, B will quote x^B_{T-3} such that

$$x^B_{T-3} = \max\left\{\frac{E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1-x^A_{T-2}))}{E^B_{T-3}(\delta_{T-2})}, \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})}, \min\{1-\frac{E^A_{T-3}(\delta_{T-1}\delta_{T-2}x^A_{T-2})}{E^A_{T-3}(\delta_{T-2})}, 1-\frac{E^A_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}x_T)}{E^A_{T-3}(\delta_{T-2})}\}\right\}. \quad (2A.14)$$

This is because x^B_{T-3} has to satisfy the condition that it gives B a payoff at least as much as that in period $(T-1)$; that is,

$$E^B_{T-3}(Q_{T-2}x^B_{T-3}) \geq E^B_{T-3}(Q_{T-1}(1-x^A_{T-2}))$$
$$x^B_{T-3} \geq \frac{E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1-x^A_{T-2}))}{E^B_{T-3}(\delta_{T-2})},$$

and a payoff at least as much as that in period T; that is,

$$E^B_{T-3}(Q_{T-2}x^B_{T-3}) \geq E^B_{T-3}(Q_T(1-x_T))$$
$$x^B_{T-3} \geq \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})}.$$

The two conditions above give

$$x^B_{T-3} \geq \max\left\{\frac{E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1-x^A_{T-2}))}{E^B_{T-3}(\delta_{T-2})}, \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})}\right\}.$$

But this simplifies to

$$x^B_{T-3} \geq \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})}, \quad (2A.15)$$

because from (*ii*)

$$\begin{aligned}
x^A_{T-2} &\geq 1 - \frac{E^B_{T-2}(\delta_T \delta_{T-1}(1-x_T))}{E^B_{T-2}(\delta_{T-1})} \\
E^B_{T-2}(\delta_T \delta_{T-1}(1-x_T)) &\geq E^B_{T-2}(\delta_{T-1}(1-x^A_{T-2})) \\
E^B_{T-3}(\delta_{T-2} E^B_{T-2}(\delta_T \delta_{T-1}(1-x_T))) &\geq E^B_{T-3}(\delta_{T-2} E^B_{T-2}(\delta_{T-1}(1-x^A_{T-2}))) \\
\frac{E^B_{T-3}(\delta_T \delta_{T-1} \delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})} &\geq \frac{E^B_{T-3}(\delta_{T-1} \delta_{T-2}(1-x^A_{T-2}))}{E^B_{T-3}(\delta_{T-2})}.
\end{aligned}$$

The last two steps are due to the fact that the information set in period $(T-3)$ is included in that in period $(T-2)$. The condition C–I that guarantees A's acceptance in period $(T-1)$ indicates that x^B_{T-3} also has to satisfy the following.

$$Q_{T-2}(1-x^B_{T-3}) \geq E^A_{T-2}(Q_{T-1} x^A_{T-2}),$$

and

$$Q_{T-2}(1-x^B_{T-3}) \geq E^A_{T-2}(Q_T x_T).$$

Since B has to infer these after observing the information up to period $(T-3)$, the first condition becomes

$$\begin{aligned}
E^A_{T-3}(Q_{T-2}(1-x^B_{T-3})) &\geq E^A_{T-3}(Q_{T-1} x^A_{T-2}) \\
x^B_{T-3} &\leq 1 - \frac{E^A_{T-3}(\delta_{T-1} \delta_{T-2} x^A_{T-2})}{E^A_{T-3}(\delta_{T-2})},
\end{aligned}$$

and the second condition becomes

$$\begin{aligned} E^A_{T-3}(Q_{T-2}(1-x^B_{T-3})) &\geq E^A_{T-3}(Q_T x_T) \\ x^B_{T-3} &\leq 1-\frac{E^A_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}x_T)}{E^A_{T-3}(\delta_{T-2})}. \end{aligned}$$

The two conditions above give the condition

$$x^B_{T-3} \leq \min\left\{1-\frac{E^A_{T-3}(\delta_{T-1}\delta_{T-2}x^A_{T-2})}{E^A_{T-3}(\delta_{T-2})},\ 1-\frac{E^A_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}x_T)}{E^A_{T-3}(\delta_{T-2})}\right\}.$$

But this simplifies to

$$x^B_{T-3} \leq 1-\frac{E^A_{T-3}(\delta_{T-1}\delta_{T-2}x^A_{T-2})}{E^A_{T-3}(\delta_{T-2})}, \tag{2A.16}$$

because from (ii)

$$\begin{aligned} x^A_{T-2} &\geq \frac{E^A_{T-2}(\delta_T\delta_{T-1}x_T)}{E^A_{T-2}(\delta_{T-1})} \\ E^A_{T-2}(\delta_{T-1}x^A_{T-2}) &\geq E^A_{T-2}(\delta_T\delta_{T-1}x_T) \\ \frac{E^A_{T-3}(\delta_{T-2}E^A_{T-2}(\delta_{T-1}x^A_{T-2}))}{E^A_{T-3}(\delta_{T-2})} &\geq \frac{E^A_{T-3}(\delta_{T-2}E^A_{T-2}(\delta_T\delta_{T-1}x_T))}{E^A_{T-3}(\delta_{T-2})} \\ 1-\frac{E^A_{T-3}(\delta_{T-2}E^A_{T-2}(\delta_{T-1}x^A_{T-2}))}{E^A_{T-3}(\delta_{T-2})} &\leq 1-\frac{E^A_{T-3}(\delta_{T-2}E^A_{T-2}(\delta_T\delta_{T-1}x_T))}{E^A_{T-3}(\delta_{T-2})}. \end{aligned}$$

From the conditions 2A.15 and 2A.16, the equation 2A.14 becomes

$$x^B_{T-3} = \max\left\{1-\frac{E^A_{T-3}(\delta_{T-1}\delta_{T-2}x^A_{T-2})}{E^A_{T-3}(\delta_{T-2})},\ \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})}\right\}. \tag{2A.17}$$

Now the condition 2A.13 simplifies to

$$x^A_{T-4} > 1 - E^B_{T-3}(\delta_{T-2}x^B_{T-3}), \tag{2A.18}$$

because the condition 2A.17 implies

$$\begin{aligned}
x^B_{T-3} &\geq \frac{E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T))}{E^B_{T-3}(\delta_{T-2})} \\
E^B_{T-3}(\delta_{T-2}x^B_{T-3}) &\geq E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T)) \\
1 - E^B_{T-3}(\delta_{T-2}x^B_{T-3}) &\leq 1 - E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T)),
\end{aligned}$$

and from (ii),

$$\begin{aligned}
x^A_{T-2} &\geq 1 - \frac{E^B_{T-2}(\delta_T\delta_{T-1}(1-x_T))}{E^B_{T-2}(\delta_{T-1})} \\
E^B_{T-2}(\delta_{T-1}(1-x^A_{T-2})) &\leq E^B_{T-2}(\delta_T\delta_{T-1}(1-x_T)) \\
E^B_{T-3}(\delta_{T-2}E^B_{T-2}(\delta_{T-1}(1-x^A_{T-2}))) &\leq E^B_{T-3}(\delta_{T-2}E^B_{T-2}(\delta_T\delta_{T-1}(1-x_T))) \\
1 - E^B_{T-3}(\delta_{T-1}\delta_{T-2}(1-x^A_{T-2})) &\geq 1 - E^B_{T-3}(\delta_T\delta_{T-1}\delta_{T-2}(1-x_T)).
\end{aligned}$$

(iv) Suppose that the equilibrium strategy prescribed in the proposition holds for periods t through $(T-1)$. We need to show that the strategies in period $(t-1)$ follow those of the equilibrium, i.e., player j rejects x^i_{t-2} if $x^i_{t-2} > 1 - E^j_{t-1}(\delta_t x^j_{t-1})$, and in the event of rejection he offers

$$x^j_{t-1} = \max\left\{1 - \frac{E^i_{t-1}(\delta_t\delta_{t+1}x^i_t)}{E^i_{t-1}(\delta_t)}, \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^j_{t+1})}{E^j_{t-1}(\delta_t)}\right\}.$$

Condition C–I indicates that j's offer has to satisfy

$$\begin{aligned} x^j_{t-1} \;\leq\; \min\Bigg\{ & 1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}x^i_t)}{E^i_{t-1}(\delta_t)}, 1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}(1-x^j_{t+1}))}{E^i_{t-1}(\delta_t)}, \\ & 1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}\delta_{t+3}x^i_{t+2})}{E^i_{t-1}(\delta_t)}, 1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}\delta_{t+3}\delta_{t+4}(1-x^j_{t+3}))}{E^i_{t-1}(\delta_t)}, \dots \Bigg\}. \end{aligned}$$

By the induction hypothesis and an application of the law of iterated mathematical expectations, we have $x^j_{t-1} \leq 1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}x^i_t)}{E^i_{t-1}(\delta_t)}$.

Condition C–II indicates that j's offer has to satisfy

$$\begin{aligned} x^j_{t-1} \;\geq\; \max\Bigg\{ & \frac{E^j_{t-1}(\delta_t\delta_{t+1}(1-x^i_t))}{E^j_{t-1}(\delta_t)}, \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^j_{t+1})}{E^j_{t-1}(\delta_t)}, \\ & \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}\delta_{t+3}(1-x^i_{t+2}))}{E^j_{t-1}(\delta_t)}, \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}\delta_{t+3}\delta_{t+4}x^j_{t+3})}{E^j_{t-1}(\delta_t)}, \dots \Bigg\}. \end{aligned}$$

By the induction hypothesis and an application of the law of iterated mathematical expectations, we have $x^j_{t-1} \geq \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^j_{t+1})}{E^j_{t-1}(\delta_t)}$.

Consequently, player j's offer strategy in period $(t-1)$ is

$$x^j_{t-1} = \max\left\{1-\frac{E^i_{t-1}(\delta_t\delta_{t+1}x^i_t)}{E^i_{t-1}(\delta_t)}, \frac{E^j_{t-1}(\delta_t\delta_{t+1}\delta_{t+2}x^j_{t+1})}{E^j_{t-1}(\delta_t)}\right\},$$

as desired. By construction we know that player j's acceptance decision follows the strategy based on a unique reservation value, i.e., j accepts x^i_{t-2} if $x^i_{t-2} \leq 1-E^j_{t-1}(\delta_t x^j_{t-1})$.

Therefore, we conclude that the equilibrium strategy holds for all t.

□

Chapter 3

Simulations of Stochastic Bargaining Games

We construct simulated negotiation processes and investigate the effects of variations in parameter values on bargaining outcomes. The structures of the games are the ones studied in Chapter 2.[1] Equilibrium strategies simulated here are the ones derived in Proposition 1 for Basic bargaining games and Proposition 10 for Alternative bargaining games. The bargaining environment is defined by utilizing statistical tools. We also attempt to provide behavioral simulation models for bargaining in the sense that we are modeling each agent's decision process based on the specified behavior rules. The behavioral assumptions made in our bargaining games help to realize such an attempt. According to the definitions of the games, the simulation is designed in such a way that the bargaining flow is governed by the initial environment, specified behavioral rules of the agents, and the information flow. Such

[1]Notations in this chapter are kept consistent with those used in Chapter 2.

simulations help us identify bargaining durations in negotiations with various parameter values and make predictions on the outcomes of similar situations in actual bargaining settings.

The factor that affects the asset value is influenced by a variable that takes a value of either one or zero in our first model, the Autoregressive Binomial Model. Such a factor is derived from a continuous distribution in our second model, the Generalized Wiener Process Model. The agent's decision is made by taking into account this newly arrived information. The behavioral rules in taking an action were given as conditions C–I and C–II in the previous chapter. They are repeated below.

C–I In the event of the respondent's accepting the current offer, the respondent receives at least as much as what the respondent expects to receive in any future period.

C–II In the event of the respondent's accepting the current offer, the proposer receives at least as much as what the proposer expects to receive in any future period.

The assumption of complete information assures that each agent has an access to the identical information as soon as it becomes available to the negotiating environment. We allow the possibility of different initial beliefs on unknown parameter values that can result from different interpretations of the same information among the players prior to the negotiation. This is to reflect a variety of speculation processes and different levels of expectations due to diversified human characteristics. Once the bargaining process begins, agents' behavior is consistent in that they use Bayes' rule whenever possible to update their conjecture on the future information flow that affects the value of the asset. We first describe in the next section the statistical specification of the binomial bargaining model in detail. Computational methods and the data structure are explained later.[2] The results of the binomial model, including findings on comparative statics, are given below. Then we investigate the continuous distribution model in the rest of the chapter.

3.1 The Binomial Distribution Model

We first simulate the case in which the information variable δs constitute a first–order autoregressive series.[3] Though we are aware that the information in actual negotiation situations may have a very complex correlation structure, we attempt to study the case of the first–order dependency structure with a stochastic factor that follows a binomial distribution as a simple approximation to the reality. Later, we will provide a specification of the model with identically and independently distributed δs.

3.1.1 The Autocorrelation Model

Let us consider a case of autocorrelated δs with perfect observation such that

$$\delta_{t+1} = \rho\delta_t + \varepsilon_{t+1}, \ |\rho| < 1,$$

where ε_ts are mutually stochastically independent[4] and

$$\varepsilon \sim binomial(1, \theta), \ \theta \in (0, 1),$$

[2] An additional description is also provided in the beginning of the simulation codes included in Appendix 3C.1 and 3C.2.

[3] We address such models as autocorrelation models or autoregressive binomial models in our research.

[4] Hence, $\{\varepsilon_t\}$ is a stationary process, in that the transition (probability) matrix is independent of time.

and

$$\theta \sim \beta eta(\alpha, \beta),$$

with positive constants α and β. We will use α_A and β_A to characterize player A and α_B and β_B to characterize player B. This will allow us to incorporate the possibility of different priors about parameter values, indicating that the two players can have different expectations on the future value of the asset. Let us for now use α and β for a generic player.

We consider ε as a message regarding the information that affects the asset value. When ε becomes available in the negotiating environment, it is instantaneously passed to both agents. Notice that we assume that the players do not know the value of θ, the probability that ε takes the value 1. Let $X = \sum_{\tau=1}^{t} \varepsilon_{\tau}$. Then $X \sim binomial(t, \theta)$. Hence, the conditional probability density function of X, given $\Theta = \theta$, is

$$f(x|\theta) = \begin{cases} \begin{pmatrix} t \\ x \end{pmatrix} \theta^{x}(1-\theta)^{t-x} & \text{if } x = 0, 1, \cdots, t \\ 0 & \text{otherwise,} \end{cases}$$

where the prior probability density function of the random variable Θ is given as

$$g(\theta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} & 0 < \theta < 1 \\ 0 & \text{otherwise.} \end{cases}$$

After observing a piece of information δ_t in period t, the players update their beliefs about the value of θ by using Bayes' rule, so that they can compute the expected value of δ_{t+1}.

The joint probability density function of X and Θ is given by $f(x|\theta)g(\theta)$ and the marginal probability density function of X is

$$h_1(x) = \begin{cases} \begin{pmatrix} t \\ x \end{pmatrix} \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+x)\Gamma(t+\beta-x)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(t+\alpha+\beta)} & \text{if } x = 0, 1, \cdots, t \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the conditional probability density function of Θ, given $X = x$, is

$$h(\theta|x) = \frac{f(x|\theta)g(\theta)}{h_1(x)}.$$

Hence, the prior mean of the distribution of Θ is expressed as

$$E_0(\theta) = \frac{\alpha}{\alpha + \beta}$$

while the posterior mean of the conditional distribution of Θ, given $X = x$, is

$$\begin{aligned} E_t(\theta|x) &= \int_0^1 \theta h(\theta|x) d\theta \\ &= \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}. \end{aligned}$$

It follows that the expected value of δ_{t+1} after observing the information in period t is computed as

$$E_t(\delta_{t+1}) = \rho\delta_t + E_t(\varepsilon_{t+1}),$$

where

$$\begin{aligned} E_t(\varepsilon_{t+1}) &= 1 \cdot E_t\left(\theta \middle| \sum_{\tau=1}^{t} \varepsilon_\tau\right) + 0 \cdot \left(1 - E_t(\theta | \sum_{\tau=1}^{t} \varepsilon_\tau)\right) \\ &= E_t\left(\theta \middle| \sum_{\tau=1}^{t} \varepsilon_\tau\right) \\ &= \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}. \end{aligned}$$

In general, the expected value of the product of lagged δs and xs are computed by

$$\begin{aligned} &E_t(\delta_{t+1} x_{t+1}) \\ &\quad = \delta_{t+1} x_{t+1}|_{\varepsilon^t, \varepsilon_{t+1}=0} \cdot pr\{\varepsilon_{t+1} = 0|\varepsilon^t\} \\ &\quad + \delta_{t+1} x_{t+1}|_{\varepsilon^t, \varepsilon_{t+1}=1} \cdot pr\{\varepsilon_{t+1} = 1|\varepsilon^t\}, \end{aligned}$$

and

$$\begin{aligned} &E_t(\delta_{t+1}\delta_{t+2}x_{t+2}) \\ &\quad = \delta_{t+1}\delta_{t+2}x_{t+2}|_{\varepsilon^t, \varepsilon_{t+1}=0, \varepsilon_{t+2}=0} \cdot pr\{\varepsilon_{t+2} = 0|\varepsilon^t, \varepsilon_{t+1} = 0\} \cdot pr\{\varepsilon_{t+1} = 0|\varepsilon^t\} \\ &\quad + \delta_{t+1}\delta_{t+2}x_{t+2}|_{\varepsilon^t, \varepsilon_{t+1}=0, \varepsilon_{t+2}=1} \cdot pr\{\varepsilon_{t+2} = 1|\varepsilon^t, \varepsilon_{t+1} = 0\} \cdot pr\{\varepsilon_{t+1} = 0|\varepsilon^t\} \\ &\quad + \delta_{t+1}\delta_{t+2}x_{t+2}|_{\varepsilon^t, \varepsilon_{t+1}=1, \varepsilon_{t+2}=0} \cdot pr\{\varepsilon_{t+2} = 0|\varepsilon^t, \varepsilon_{t+1} = 1\} \cdot pr\{\varepsilon_{t+1} = 1|\varepsilon^t\} \\ &\quad + \delta_{t+1}\delta_{t+2}x_{t+2}|_{\varepsilon^t, \varepsilon_{t+1}=1, \varepsilon_{t+2}=1} \cdot pr\{\varepsilon_{t+2} = 1|\varepsilon^t, \varepsilon_{t+1} = 1\} \cdot pr\{\varepsilon_{t+1} = 1|\varepsilon^t\}, \end{aligned}$$

where

$$
\begin{aligned}
& pr\{\varepsilon_{t+2} = 0|\varepsilon^t, \varepsilon_{t+1} = 0\} \cdot pr\{\varepsilon_{t+1} = 0|\varepsilon^t\} \\
& \quad + pr\{\varepsilon_{t+2} = 1|\varepsilon^t, \varepsilon_{t+1} = 0\} \cdot pr\{\varepsilon_{t+1} = 0|\varepsilon^t\} \\
& \quad + pr\{\varepsilon_{t+2} = 0|\varepsilon^t, \varepsilon_{t+1} = 1\} \cdot pr\{\varepsilon_{t+1} = 1|\varepsilon^t\} \\
& \quad + pr\{\varepsilon_{t+2} = 1|\varepsilon^t, \varepsilon_{t+1} = 1\} \cdot pr\{\varepsilon_{t+1} = 1|\varepsilon^t\} \\
& \quad = \left(1 - \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t + 1}\right) \cdot \left(1 - \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}\right) \\
& \quad + \left(\frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t + 1}\right) \cdot \left(1 - \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}\right) \\
& \quad + \left(1 - \frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau + 1}{\alpha + \beta + t + 1}\right) \cdot \left(\frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}\right) \\
& \quad + \left(\frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau + 1}{\alpha + \beta + t + 1}\right) \cdot \left(\frac{\alpha + \sum_{\tau=1}^{t} \varepsilon_\tau}{\alpha + \beta + t}\right) \\
& \quad = 1.
\end{aligned}
$$

ε^t is a sequence of εs up to and including period t. Note that in computing the value of δ, the order of realized εs matters.

For example, the expected value of lagged δs may be computed by[5]

$$\begin{aligned}
&E_t(\delta_{t+1}\delta_{t+2}) \\
&\quad = E_t\left((\rho\delta_t + \varepsilon_{t+1})(\rho\delta_{t+1} + \varepsilon_{t+2})\right) \\
&\quad = E_t\left((\rho\delta_t + \varepsilon_{t+1})(\rho^2\delta_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2})\right) \\
&\quad = \rho^3\delta_t^2 + 2\rho^2\delta_t E_t(\varepsilon_{t+1}) + \rho\delta_t E_t(\varepsilon_{t+2}) + \rho E_t(\varepsilon_{t+1}^2) + E_t(\varepsilon_{t+1}\varepsilon_{t+2}),
\end{aligned}$$

where

$$\begin{aligned}
&E_t(\varepsilon_{t+1}\varepsilon_{t+2}|\sum_{\tau=1}^{t}\varepsilon_\tau) \\
&\quad = \frac{\alpha+\sum_{\tau=1}^t \varepsilon_\tau}{\alpha+\beta+t}\cdot\left[\frac{\alpha+\sum_{\tau=1}^t \varepsilon_\tau+1}{\alpha+\beta+t+1}\cdot\frac{\alpha+\sum_{\tau=1}^t \varepsilon_\tau}{\alpha+\beta+t}\right. \\
&\qquad\left. + \frac{\alpha+\sum_{\tau=1}^t \varepsilon_\tau}{\alpha+\beta+t+1}\cdot\left(1-\frac{\alpha+\sum_{\tau=1}^t \varepsilon_\tau}{\alpha+\beta+t}\right)\right].
\end{aligned}$$

3.1.2 The I.I.D. Model

In this section we briefly outline one example of possible I.I.D. model specifications by using the statistical tools applied in the autocorrelation model in the previous section. Here the information variable δs are identically and independently distributed. The results of this I.I.D. model simulations are not reported in this version, since they do not add prominently

[5]For example, a lagged expectation of εs such as $E_1(\varepsilon_2\varepsilon_3|\varepsilon_1=1)$ is computed by

$$\begin{aligned}
E_1(\varepsilon_2\varepsilon_3|\varepsilon_1=1) &= E_1(\varepsilon_2|\varepsilon_1=1)E_1(\varepsilon_3|\varepsilon_1=1) \\
&= prob\{\varepsilon_2=1|\varepsilon_1=1\}\cdot(E_1(\varepsilon_3|\varepsilon_1=1,\varepsilon_2=1)prob\{\varepsilon_2=1|\varepsilon_1=1\} \\
&\quad + E_1(\varepsilon_3|\varepsilon_1=1,\varepsilon_2=0)prob\{\varepsilon_2=0|\varepsilon_1=1\}) \\
&= \frac{\alpha+1}{\alpha+\beta+1}\cdot\left[\frac{\alpha+2}{\alpha+\beta+2}\cdot\frac{\alpha+1}{\alpha+\beta+1}+\frac{\alpha+1}{\alpha+\beta+2}\cdot\left(1-\frac{\alpha+1}{\alpha+\beta+1}\right)\right].
\end{aligned}$$

different interpretations on the relations between bargaining durations and other parameter values from the findings on the autocorrelation model.

Let us define each δ as a sum of a prespecified constant and a variable that follows a Bernoulli trial.

$$\delta = \rho + \eta, \ \rho \in (0,1),$$

where

$$\eta \sim binomial(1, \theta), \ \theta \in (0,1),$$

and

$$\theta \sim \beta eta(\alpha, \beta),$$

with predetermined positive constants α and β. θ here is the probability that η takes the value 1, and the players have prior beliefs on the value of θ. Therefore, δ can be greater than 1 if η takes a value 1, given $\rho \in (0,1)$. The rest of the description of this model is analogous to that of the autocorrelation model.

3.1.3 Design and Data Structures

We design the autocorrelation model as a full binary tree, which is symmetric and whose depth $T+1$ is determined by an exogenously given final period number, T.[6] Hence, the tree has a total of $(2^{T+1}-1)$ nodes, with each node having a degree no larger than 2. Each

[6]We frequently use $T = 18$ as a maximum possible number of negotiations in exhibits that follow. Since $T = 18$ indicates $2^{18} = 262144$ possible *states* in the terminal period T, we think that increasing T over 18 to create a higher diversity is not necessary for our purposes. In addition, as T becomes large, the reliability of the backward induction algorithm becomes questionable due to correspondingly more involved hypotheses. Also the standard deviation of the forecast error grows as the forcast horizon increases, resulting in wider confidence intervals. The total number of nodes in the tree when $T = 18$ is $2^{19} - 1 = 524287$.

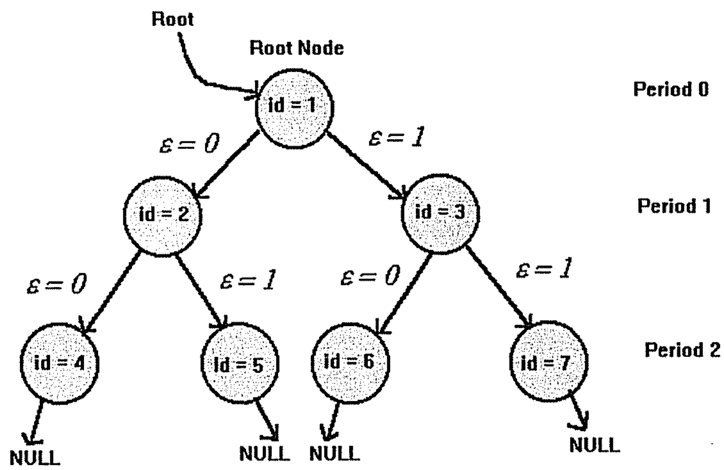

Figure 3.1: The Binary Tree of Depth 3 with Sequential Node I.D. Number

level of the tree coincides with each period t, with the first level associated with period 0. Since such a tree is *complete*[7], we can assign a unique identification number to each node from 1 to $(2^{T+1} - 1)$ in a systematic manner. Given an id number, we can find a unique path, which is defined as a sequence of observed εs, to reach a node that is identified with the id number. Hence, this numbering scheme enables us to reach any node in the tree, given an address of its root node and the id number, without using a recursive coding. At the root node the initial conditions such as the value of δ_0 and ρ are given. We use a linked representation, where each child of a parent node of period t is associated with the state that ε_{t+1} is either 0 or 1. An example of the tree with $T = 2$ is shown in Figure 3.1.

Each node in the tree is defined by data structure **node** and contains the following

[7]Consider a numbering scheme such that starting with the root node with number one we sequentially number nodes on each level from one side to the other. A binary tree with N nodes and a depth $T + 1$ is *complete* if and only if its nodes correspond to the nodes which are numbered one to N in the full binary tree of depth $T + 1$.

information.

```
struct node
{
  long unsigned int id;     /* ID number of each node, 1, 2, 3, ..., 2^(T+1) - 1. */
  unsigned int t;           /* Period number, t = 1, 2, 3, ..., T. */
  float delta;              /* Ex-post δ value, i.e., δ_t. */
  float PA0;                /* P_t^A{ε_{t+1} = 0|ε^t}. */
  float PB0;                /* P_t^B{ε_{t+1} = 0|ε^t}. */
  float PA1;                /* P_t^A{ε_{t+1} = 1|ε^t}. */
  float PB1;                /* P_t^B{ε_{t+1} = 1|ε^t}. */
  float X;                  /* Current offer. */
  float Q;                  /* Current asset value. */
  unsigned int sum_e;       /* Σ_{τ=1}^t ε_τ. */
  unsigned int R;           /* Response strategy : 1 for accept, 0 for reject. */
  unsigned int flag;        /* 0 if no trade before t, 1 otherwise. */
  struct node *left;        /* Pointer to a left child, i.e., a node with ε_{t+1} = 0 */
  struct node *right;       /* Pointer to a right child, i.e., a node with ε_{t+1} = 1 */
  struct node *parent;      /* Pointer to a parent. */
}
```

The codes for the simulation of the Basic bargaining game, along with more details on design and data structures, are included in Appendix 3C.1.[8] This binomial tree procedure is also used with the I.I.D. model.

3.1.4 Results and Comparative Statics

The Basic Game

Tables 3.1 through 3.3[9] show three simulation results of the Autoregressive Binomial model's Basic game with $T = 4$. Offers and responses are made by following the behavioral assumptions we made in the previous section. Given such equilibrium strategies, the players compute their expected payoffs from trading in the future, which also appear in the ta-

[8]The program in the appendix shows a core part of the codes, on which we have made numerous modifications to obtain various different versions to generate data sets.

[9]All the tables relevant to Chapter 3 are included in Appendix 3A. Appendix 3A.1 contains tables for Autoregressive Binomial Models, while Appendix 3A.2 contains those for Wiener Process Models.

bles. These exhibits clarify how the delays are generated by players' speculation on value increases in the future in our Basic bargaining model. Exhibit 1 provides an example of an immediate trade despite that the players have identical priors about the information flow. In this case both players are not very optimistic about the possibility of the value increase later on, as it is reflected in low probabilities they assign for the future ε to be one.[10] In period 0, player B's highest expected continuation payoff is the one in period 1, which is 0.461687. Since player A's offer in period 0 guarantees B as much as B's highest expected payoff in the future, B has no reason to reject the current offer. Hence, the acceptance occurs in period 0 with A receiving 0.738313 and B receiving 0.461687. Note that proposer A has extracted the surplus in excess of her maximum reservation payoff despite that both A and B are assumed to have the same preferences over the future.

In Exhibit 2, the players again have identical priors about the information flow. But both of them are more optimistic about the future value of the asset than in the case of Exhibit 1, so that we now observe a delay before agreement. In period 0, player B would receive nothing if he accepts the current offer,[11] while his expected payoffs in period 1, 3, and 4 are positive. Hence, B rejects the offer in period 0, and the game continues to period 1. In period 1, if the information they have observed is $\varepsilon_1 = 0$, then A would accept B's offer to receive 0.143430 that is as high as A's any expected continuation payoff in the following periods given $\varepsilon_1 = 0$. On the contrary, if they have observed $\varepsilon_1 = 1$ as new information, then both players would maintain their optimistic views and there would be another delay. The game proceeds as it is described in the table.

[10]Both players assign probability 0.25 to the event $\varepsilon_1 = 1$.

[11]In the table, this appears as a negative number, owing to the ridiculous offer player A has made to generate a delay.

Exhibit 3 is the case in which an agreement does not occur immediately despite a big difference among players' beliefs. In this exhibit, player B is more optimistic than player A about the future asset value. B assigns a probability 0.67 to the information in period 1 being 1, while A assigns a probability 0.25, indicating that B has higher incentive to delay than A does. Accepting the current offer would give B a payoff of 0.904373, which is very high compared to what the proposer A would receive, that is 0.295627. However, B foresees the possibility of even higher payoffs in future periods, that are period 1, 3, and 4. Hence, B rejects the offer in period 0, and the game continues.

In general, as the predetermined maximum length of negotiation process increases, we can observe more delay before reaching an agreement. The bargaining duration, however, is not sensitive to the change of the maximum length if both players are pessimistic about the future value of the asset. By pessimistic, we refer to the characteristics of players who assign low initial priors for the first information, ε_1, being one, and/or to the environment with low initial values of parameters such as ρ and δ_0. Exhibit 4 in Table 3.4 uses the same parameter values as Exhibit 3, except that the maximum length of time T is 2 in Exhibit 4. Player A's initial belief of $\varepsilon_1 = 1$ is 0.25, and we consider her view as pessimistic. On the other hand, relatively optimistic B assigns probability 0.67. Hence, this is the case that two players have different beliefs about the information flow, with one person being very optimistic. In Exhibit 3, we observed that the first offer made by player A in period 0 was rejected by player B, resulting in a delay. On the other hand, Exhibit 4 shows an immediate agreement in period 0. Figure 3.3 shows the relationship between the changes in T and bargaining durations with the other parameter values identical to the ones in

Exhibits 3 and 4. Z-axis indicates frequencies of agreement in each period.[12]

Similarly, if both players are optimistic, then the duration shows sensitivity to the changes in T even when the environment is not very promissing. In Figure 3.4, neither A nor B has a very pessimistic view about the future value of the asset, with A assigning probability 0.57 and B assigning 0.45 to the first information to be one. Notice also that the values of ρ and δ_0 are low to begin with. As it appears in the figure, however, as long as both players are reasonably optimistic, then the bargaining duration is sensitive to T.[13] In addition, Figure 3.5 gives another example of how sensitive the duration can be if both players are not very pessimistic and the initial environment is also promissing with large values of ρ and δ_0. The bargaining duration becomes longer as its horizon increases. For example, with $T = 6$ the first agreement can be reached in period 3, while with $T = 18$ the first possible agreement is in period 10. However, if both players are pessimistic, then we observe high agreement frequencies in earlier periods. Figure 3.6 shows such an example, in which an agreement is reached immediately in period 0 in the cases with $T = 2$ through $T = 10$.

Figures 3.7 and 3.8 show how differences in players' initial beliefs affect bargaining durations. In Figure 3.7, A has an optimistic view, while B's view is varied from optimistic

[12] All the figures relevant to Chapter 3 are included in Appendix 3B.1, 3B.2, and 3B.3. Appendix 3B.1 contains Figures 3.3 through 3.18, which are figures for Auroregressive Binomial Models. The frequency on Z-axis in the figures are computed in such a way that the total number of agreements is divided by the total number of states in a given period. We consider such a frequency as an *unconditional* frequency since it counts even unreachable nodes in the bargaining tree. Appendix 3B.2 contains Figures 3.19 through 3.34, which are also figures for Autoregressive Binomial Models. But they have *conditional* frequencies on Z-axis, in that we ignore irrelevant states or unreachable nodes in the tree. Parameter values used in the figures in Appendix 3B.2 are analogous to the ones in Appendix 3B.1. Appendix 3B.2 is provided as an additional reference. Appendix 3B.3 contains Figures 3.35 through 3.46, which are figures for Wiener Process Models studied in the next section. The frequencies used in the figures in Appendix 3B.3 are *conditional*, where we ignore unreachable nodes.

[13] Also notice that the environment with low ρ and δ_0 results in higher frequencies of agreements in a period corresponding to that in the more promising environment.

to pessimistic.[14] Frequencies of agreement show slight tendencies to increase as B's view becomes pessimistic at a given period except for period 0 and 1. In Figure 3.8, A has a pessimistic view, while B's view is varied from optimistic to pessimistic. The relationship between the frequencies of agreement and the variation in B's view are very similar to that of Figure 3.7, except that frequencies of agreement are higher in Figure 3.8 than in Figure 3.7 in any given period. Again, this is a consequence of pessimistic players' incentive to come to an early agreement.

In Figure 3.9 we provide four cases in which ρ is varied from 0.1 through 0.9 to reflect a change in the environment that influences the asset value. These four cases differ in the players' prior beliefs.[15] Case 1 is an example with optimistic A and B, Case 2 is with pessimistic A and B, Case 3 is with optimistic A and pesimistic B, and Case 4 is with pessimistic A and optimistic B. As it is clear from the figures, all of the four cases show the tendency of longer delays as ρ increases; that is, the environment becomes more promising on the value increase. For example, in Case 1 the first possible agreement can be reached in period 0 with $\rho = 0.1$, while the first possible agreement is in period 4 with $\rho = 0.6$. The figures also indicate that the frequencies of agreement in any given period decrease as ρ increases. For example, in Case 4 the frequency of agreement in period 4 with $\rho = 0.1$ is 1.0, while it is 0.0625 with $\rho = 0.7$. These findings are intuitive, since it is a hopeful speculation for a value increase that generates delays.

Figure 3.10 shows four cases in which A's predetermined default share x_T varies from

[14] The value of β_B, which appears on the x–axis labeled "Beta_B," is varied from 1 through 3.8 by the interval of 0.4. This indicates B's initial beliefs are changed from 0.67 to 0.34, i.e., the probability B assigns to the event $\varepsilon_1 = 1$ is varied from 0.67 to 0.34.

[15] In Case 1 A's prior is 0.67 and B's prior is 0.75, in Case 2 they are 0.25 and 0.29, in Case 3 they are 0.67 and 0.25, and in Case 4 they are 0.25 and 0.67, respectively.

0.1 through 0.9. The prior beliefs of the players in the four cases correspond to those in Figure 3.9. In Case 1 and 2 we do not observe much sensitivity in the bargaining duration and frequencies of agreement in a given period as the value of x_T changes. These are the cases in which players are both optimistic and are both pessimistic, respectively. In Case 3 and 4, small changes in the frequencies of agreement is observed. A is optimistic and B is pessimistic in Case 3, in which the frequencies of agreement in a given period decrease gradually and a delay becomes longer as x_T increases. Hence, this is a case that the environment that is increasingly promising to A generates a longer duration despite that a pessimistic B wants to come to an early agreement. On the other hand, A is pessimistic and B is optimistic in Case 4, in which the frequency increases and a delay becomes shorter as x_T increases. This is a case that as the environment becomes more and more unfavorable to B, an optimistic B wants to generate an early agreement. Optimistic players seem to have more control over durations than pessimistic players. Consequently, when the two players' views sufficiently differ from each other, the bargaining duration shows more sensitivity to the change in x_T.

The Alternative Game

In Alternative bargaining games, a responder has another chance to observe information that is not available when a proposer makes an offer.[16] Exhibit 5 in Table 3.5 shows the outcome of the Alternative game by using the same parameter values as the ones used in Exhibit 3 in Table 3.3. Note that an agreement in period 1 is considered as an immediate

[16] Note that the reservation values included in columns labeled "Beliefs" are the reservation values for determining the response. Recall that reservation values that a proposing party uses in Alternative games are different from those in the Basic games, due to the added uncertainty. Refer to the equilibrium concept section in Chapter 2.

agreement in Alternative games due to the structure of the game. The first offer made by A is rejected by B in Exhibit 3, since B's expected payoffs from period 1, 3, and 4 are higher than what he would receive by accepting the current offer. In Exhibit 5, however, the first offer is accepted by B in period 1 if he has observed $\varepsilon_1 = 0$. After B has observed the information in period 1, he has updated his beliefs about the future payoffs before he responds. His expected payoffs from the remaining periods, which are 0.133601 in period 2, 0.108138 in period 3, and 0.133601 in period 3, are lower than what he is guaranteed to receive in period 1 that is 0.165822. Consequently, he has no incentive to wait until later if he has observed 0 in period 1. If he has observed $\varepsilon_1 = 1$ instead, then he still maintains this optimistic view about the future value, so that the game continues.

Figures 3.11 through 3.18 show examples of Autoregressive Binomial model's Alternative game simulations and use the same parameter values corresponding to Figures 3.3 through 3.10, respectively. In Figures 3.11 through 3.14, it is observed that bargaining durations are longer as the predetermined bargaining horizon becomes larger. This finding is consistent with the finding of the Basic game.[17] It has to be noted, however, that the frequencies of agreements in a given T in Alternative games do not increase monotonically especially in the earlier periods. For example, in Figure 3.11 frequencies in some earlier even–numbered periods are lower than those in their preceding odd–numbered periods. Similar relationships are found far less frequently in the examples of Basic games. Pessimistic views of the players generate higher frequencies of agreements even in earlier periods as observed in Figure 3.14. In the promising environment with high values of ρ and δ_0, as given in Figure

[17]Also note that the frequencies of agreements in a given period in an environment with low ρ and δ_0 are higher than that in the more promising environment.

3.13, bargaining durations are very sensitive to the changes in the predetermined bargaining horizon and significant delays are generated as T increases.

Figures 3.15 and 3.16 show the relationship between the difference in priors and the frequencies of agreement that are very similar to that of the Basic game, except that each agreement frequency in the Alternative game is far lower than that in the Basic game in any given period. In other words, the structure of Alternative game is likely to generate longer bargaining durations. Such relations are also observed in Figure 3.17 in comparison to Figure 3.9, regarding the relationship between the value of ρ and the agreement frequencies.

We observe an interesting feature in the results given in Figure 3.18 on the relation between x_T and the frequencies, which are not observed in the Basic game's counterpart given in Figure 3.10. In all of the four cases we observe increasing trade frequencies in even–numbered periods and decreasing frequencies in odd–numbered periods as x_T increases. This may attribute to B's incentive to come to an early agreement, so that B makes an offer that is acceptable to A before reaching the final period. That means that B finds a current payoff higher than the payoff he expects to receive when the predetermined default share is realized.

3.2 The Continuous Distribution Model

In this section we provide another example, in which the state evolves according to a continuous distribution. In order to model the behavior of the asset value in our bargaining games, we consider a stochastic process that is frequently used to study stock prices. It is a particular type of Markov stochastic process used in physics to describe the motion of a particle that is referred to as Brownian motion.[18]

3.2.1 The Geometric Brownian Motion (Wiener Process) Model

Let us suppose that a sequence of asset values, $\{Q_\tau\}_{\tau=0}^{T}$, can be represented by a generalized Wiener process. In other words, the asset value can be described by an expected drift rate and a variance rate. Such a process can be expressed as[19]

$$\frac{dQ_{t-1}}{Q_{t-1}} = \mu_t dt + \sigma\varepsilon_t\sqrt{dt}, \tag{3.1}$$

where

$$\varepsilon \sim N(0,1).$$

μ is the expected rate of value increase per unit time, and σ is the volatility of the value and we assume that it is a constant. Hence, the second term on the right–hand side of the equation $\sigma\varepsilon\sqrt{dt}$ is the stochastic component of the value change. We assume that σs are exogenously determined and known to the players. It is also assumed that ε_ts are independent. In the model a length of each period, *dt*, is specifically incorporated in computing the asset value changes. But we note that the bargaining duration in terms of the number of negotiation periods is not sensitive to a change in the length of the interval in our bargaining games.

[18]For detailed introduction of the Wiener process, refer to Chapter 2 in Krylov (1995).
[19]The equation 3.1 is a widely used model of stock price behavior, where μ is referred to as the expected rate of return and σ is referred to as the stock price volatility.

Let us define ΔQ_{t-1} as a change in the asset value in a small time interval Δt such that

$$\Delta Q_{t-1} = Q_t - Q_{t-1}.$$

Then the discrete version of the equation 3.1 can be written as[20]

$$\frac{\Delta Q_{t-1}}{Q_{t-1}} = \mu_t \Delta t + \sigma_t \varepsilon_t \sqrt{\Delta t}.$$

It follows that the information shock δ_t is expressed by

$$\begin{aligned} \delta_t &= \frac{Q_t}{Q_{t-1}} \\ &= \frac{Q_{t-1} + \Delta Q_{t-1}}{Q_{t-1}} \\ &= 1 + \mu_t \Delta t + \sigma_t \varepsilon_t \sqrt{\Delta t}. \end{aligned}$$

We also assume that μs are independently and identically distributed random variables with

$$\mu \sim N(\theta, \varphi)$$

and the value of μ_t is linearly related to the value of ε_t.

Note that the value of θ is unknown to the players, whereas the value of φ is known. Each player has a prior belief on θ that can be expressed in terms of a normal distribution, so that

$$\theta \sim N(\theta_0^A, \varphi_0^A)$$

[20]Note that with this formulation we consider a fixed length of time interval for each period, which is not the case in the binomial model examples given in the earlier section.

and

$$\theta \sim N(\theta_0^B, \varphi_0^B)$$

characterize player A and B, respectively. We use θ_t and φ_t to indicate a generic player's beliefs in period t. The players observe μ_t in period t, and use it to compute the information shock δ_t and to update their beliefs on the value of θ by using Bayes' rule for formulating their strategies. The initial probability density function, conditional on the information available before the game begins, is given by

$$\begin{aligned} p(\theta|\mu_{-1}) &= \frac{p(\theta)\, p(\mu_{-1}|\theta)}{\int p(\theta)\, p(\mu_{-1}|\theta)\, d\theta} \\ &= \frac{1}{\sqrt{2\pi\varphi_0}} exp\left(-\frac{(\theta-\theta_0)^2}{2\varphi_0}\right) \end{aligned}$$

where μ_{-1} indicates the initial information that a player possesses to form his prior beliefs. The posterior probability density function is given by

$$\begin{aligned} p(\theta|\mu^t) &= \frac{p(\theta|\mu^{t-1})\, p(\mu_t|\theta,\ \mu^{t-1})}{\int p(\theta|\mu^{t-1})\, p(\mu_t|\theta,\ \mu^{t-1})\, d\theta} \\ &= \frac{1}{\sqrt{2\pi\varphi_t}} exp\left(-\frac{(\theta-\mu_t)^2}{2\varphi_t}\right). \end{aligned}$$

It follows that the posterior density is

$$\theta|\mu^{t-1} \sim N(\theta_t, \varphi_t)$$

where the posterior mean is a weighted mean of a prior mean and an observed value with the weights being proportional to their respective precisions (i.e., the respective reciprocals of the variances),

$$\theta_t = \theta_{t-1}\frac{1/\varphi_{t-1}}{1/\varphi_{t-1}+1/\varphi} + \mu_t\frac{1/\varphi}{1/\varphi_{t-1}+1/\varphi},$$

and the posterior variance is expressed as

$$\varphi_t = \frac{1}{1/\varphi_{t-1}+1/\varphi}.$$

After updating the beliefs θ_t and φ_t in period t, the players compute their expected payoffs in the future periods.[21]

3.2.2 Design and Data Structures

The bargaining process is stored in a linked list, in which two types of data structures coexist. One structure defined by **node** is frequently referred to as a base list in our program, and each node carries the following information.

[21] For example, after observing μ_t in period t, player A computes θ_t^A and φ_t^A. Then she computes expected information shocks such as

$$E_t^A(\delta_{t+1}) = 1 + E_t^A(\mu_{t+1})\Delta t = 1 + \theta_t^A\Delta t$$

and

$$E_t^A(\delta_{t+2}) = 1 + E_t^A(\mu_{t+2})\Delta t = 1 + \theta_t^A\Delta t.$$

The second equation follows immediately from

$$E_t^A(\mu_{t+2}) = E_t^A(\theta_{t+1}^A) = \theta_t^A\frac{1/\varphi_t}{1/\varphi_t+1/\varphi} + E_t^A(\mu_{t+1})\frac{1/\varphi}{1/\varphi_t+1/\varphi} = \theta_t^A.$$

It follows that A's expected valuation of the asset is expressed as

$$E_t^A(Q_{t+n}) = (1+\theta_t^A\Delta t)^n Q_t.$$

```
struct node
{
 unsigned int t;          /* Period number, t = 1, 2, 3, ..., T. */
 float delta;             /* Ex-post δ value, i.e., δ_t. */
 float thetaA;            /* A's prior on the mean of θ. */
 float thetaB;            /* B's prior on the mean of θ. */
 float PhiA;              /* A's prior on the variance of θ. */
 float PhiB;              /* B's prior on the variance of θ. */
 float X;                 /* Current offer strategy. */
 float Q;                 /* Current asset value. */
 unsigned int R;          /* Response strategy : 1 for accept, 0 for reject. */
 float Apay;              /* A's payoff if trade now. */
 float Bpay;              /* B's payoff if trade now. */
 struct node *next;       /* Pointer to its child node. */
 struct node *past;       /* Pointer to its parent node. */
 struct node *strat;      /* Pointer to a strategy list. */
}
```

This list is used to store the offer and response strategies that the players have decided to take after observing the information available up to the current period.

The other list is defined by a structure **snode**, and we often refer to the list as a strategy list. Each snode carries the following information.

```
struct snode
{
 unsigned int t;          /* Period number, t = 1, 2, 3, ..., T. */
 float EAdelta;           /* A's expected value of δ. */
 float EBdelta;           /* B's expected value of δ. */
 float EAQ;               /* A's expected value of the asset. */
 float EBQ;               /* A's expected value of the asset. */
 float X;                 /* Offer strategy. */
 unsigned int R;          /* Response strategy : 1 for accept, 0 for reject. */
 float EApay;             /* A's expected payoff in the future period. */
 float EBpay;             /* B's expected payoff in the future period. */
 struct node *next;       /* Pointer to its child node. */
 struct node *past;       /* Pointer to its parent node. */
 struct node *base;       /* Pointer to a base list. */
}
```

This list stores the information such as expected payoffs in the future that are necessary for the players to compute their strategies in the current period. This list is connected to

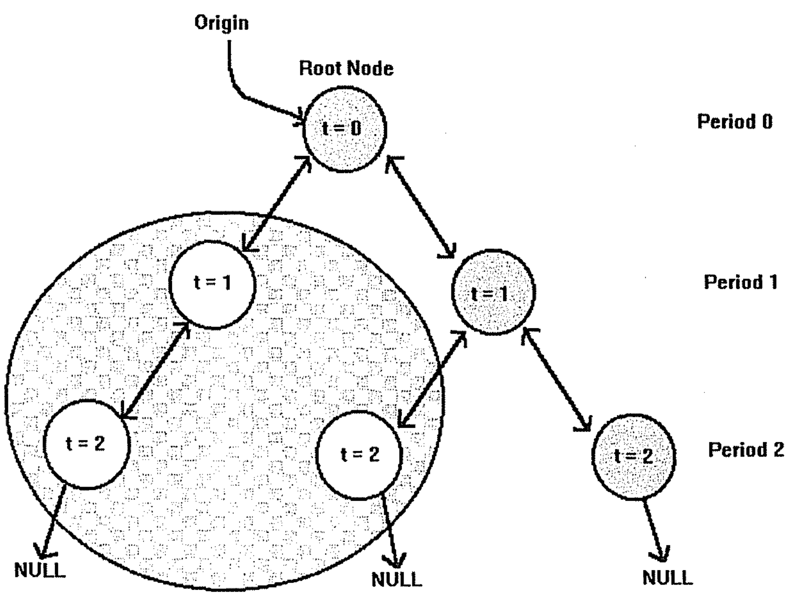

Figure 3.2: Design of Wiener Process Model with $T = 2$

the base list with a pointer *base*, along with a pointer *strat* pointed from a node in the base list. Figure 3.2 shows a case of $T = 2$. The shaded nodes in the figure constitute the base list, while the nodes surrounded by a big circle constitute the strategy list.[22]

3.2.3 Results and Comparative Statics

Tables 3.6 through 3.12 in Appendix 3A.2 show several random sample runs, each of which is associated with an idum number[23] that generates a unique sequence of pseudo–random numbers. They contain the information such as how the asset value and the players' beliefs evolve from the opening of the negotiation untill the final period. Figures 3.35 through

[22]The core part of the codes for Alternative games is included in Appendix 3C.2, which also provides more detailed description of the design. Primary input variables are also described in detail in the appendix.

[23]An idum number is an integer number given to a subroutine that generates a sequence of pseudo–random numbers. Each idum number is associated with a particular sequence of random numbers, so that it helps us to reproduce a specific sequence of events by feeding the code with the same idum number.

3.46 in Appendix 3B.3 show the results of simulation runs, each of which consists of 10,000 random sample runs. In all of the figures, we assume that $x_T = 0.5$, $\sigma = 0.06$, $\Delta t = 0.01$, $Q_0 = 1.2$, and $\varphi = 0.0004$.

Exhibits 6 through 8, contained in Tables 3.6 through 3.8, are examples on the process of Basic game. All three examples use the same input parameter values, except for an idum number that generates a sequence of random numbers. Exhibit 6 is a case in which there is an immediate agreement. After observing an information in period 0, both players know that the value of the information shock on the asset value has been 0.994825 and that the value now is equal to 1.193791. Based on this observation they update their beliefs on the true value of θ, and compute their expected payoffs in the future. If B accepts A's offer 0.499918, he knows with certainty that his payoff in the current period would be 0.596994. His computed expected payoffs in any future period is lower than this payoff. Hence, B has no incentive to reject the offer, resulting in an immediate acceptance. A then receives a payoff 0.596797 that is also higher than any of her expected payoffs in the future. Note that the proposer A receives a positive surplus 0.000139 in excess of her maximum continuation payoff 0.596658.

In Exhibit 7 we observe a delay until the final period. If B accepts A's offer in period 0, A receives 0.600693 and B receives 0.600329. These payoffs are higher than what they end up with in Exhibit 6. However, B speculates even higher payoffs in the future and hence there is a delay. In a similar fashion, the bargaining continues until the default period is reached. Indeed, both players receive payoffs that are higher than any payoffs they have gotten in earlier periods. It is interesting to note that we observe a delay in period 4 in which the information shock has turned out to be less than 1. This reflects that players accumulate knowledge, so that there has to be a significant decrease in the asset value to prevent a delay if the value has been increasing for several consecutive periods. Exhibit 8 is the case of one period delay, and a similar interpretation can be given to this case as above.

Exhibits 10 through 13, contained in Tables 3.10 through 3.13, are examples of Alternative bargaining games. We use the same input parameter values in these exhibits as in the Basic game exhibits described above, i.e., Exhibits 10, 11, 12, and 13 are given the same idum numbers as Exhibits 6, 7, 8, and 9, respectively. In Exhibit 10 the first offer made by A is accepted by B in period 1 with payoffs that are lower than what they would have received if they were playing an equivalent Basic game.[24] Exhibit 11 shows a delay until the final period and therefore the payoffs players receive are the same as what they would have received in an equivalent Basic game.[25] In Exhibit 12 the first offer made by A is accepted by B in period 1. A difference in the players' payoffs in this case is smaller than a difference in the case of a Basic game. Exhibit 13 gives yet another example of the Alternative game, in which we observe a delay until a penultimate period. An interesting observation is made in $t = 3$, in comparison to the result of Exhibit 9. The expected values of δs in the remaining periods are less than 1. As Proposition 2 in the previous chapter indicates, there is an acceptance of a current offer in such a case in the Basic game. In Exhibit 13, however, there is a rejection to cause a delay in $t = 3$ despite that the expected values of δs in all the remaining periods are less than 1.

Figures 3.35 and 3.36 describe how the change in the value of θ influences bargaining durations in Basic games. Both players have optimistic priors on the value of θ in the first

[24] Refer to Exhibit 6.
[25] Refer to Exhibit 7.

figure, while both have low priors in the second figure. Y–axis takes the number of first occurrences of agreement in a specified period out of 10,000 simulation runs. In other words, unreachable nodes in the bargaining game tree are ignored. For example, in Figure 3.35 almost all of the agreements are reached immediately in period 0 with $\theta = -0.05$, while most of the agreements are not reached until the final period as θ approaches to 0.06. The two figures appear to be almost identical, despite the significant differences in the priors. This is due to the fact that once the players have observed the first piece of information in period 0, by using Bayes' rule both can update their beliefs that will approach to the true value fairly quickly. For instance, A's belief on the mean of θ is updated into a negative number after observing the first information. Since θ characterizes μ that is the expected rate of value increase, it is intuitive to observe an immediate agreement if players conjecture the value of θ to be negative. Figures 3.37 and 3.38 are the Alternative game's analogue to Figures 3.35 and 3.36, respectively. Alternative games generate more delays even when the value of θ is very low.[26] Moreover, in Alternative games we observe more variations in bargaining durations when θ is low than in Basic games.

Figures 3.39 and 3.40 show the relationship between the difference between the players' beliefs and the frequencies of agreements. In the first figure B has a pessimistic prior on the value of θ, while A's belief is varied from 0.5 through 6.0. In the second figure B has an optimistic prior, while A's prior is varied from 0.5 through 6.0. Both figures show almost identical relationships, indicating that the agreement frequencies are not very sensitive to the difference in priors. In these examples nearly half of the simulations result in an immediate agreement and nearly a quarter of them result in a delay until the final period.

[26] Recall that an agreement in period 1 is considered as an immediate agreement in Alternative games.

A relatively low value of θ encourages an early agreement, while once there is a delay, then players tend to wait until the final period. Figures 3.41 and 3.42 are an Alternative game's analogue to Figures 3.39 and 3.40. They show low frequencies of an immediate agreement and high frequencies of delays until the final period, despite that they are given the same initial conditions as the Basic game's counterpart. Again, the frequencies do not show a significant sensitivity to the change in the size of differences among players' priors.

Figures 3.43 through 3.46 show the relationship between the change in the predetermined bargaining horizon and the agreement frequencies. The frequencies show only a slight sensitivity to the change in T. But the difference between the Alternative games and the Basic games appear to be clear in these figures. In Alternative games only less than a quarter of 10,000 sample runs result in an immediate agreement, while nearly half of them show an immediate agreement in Basic games. In general, in both Autoregressive Binomial Models and Wiener Process Models the Alternative games show more delays than the Basic games with the same parameter values. The fact that one player can observe another piece of information before responding to a current offer expands the players' expectation of value increases, causing higher frequencies of delays.

3.3 Summary and Discussion

Analyzing simulation results helps us capture non–monotonic relations among correlated variables and subtle changes in durations, and is useful in identifying a general trend in case of such non–monotonic relations where analytical solutions are frequently not straightforward to interpret. We also believe that the differences in comparative statics between

Basic and Alternative games are adequately demonstrated through our simulations. We summarize our findings on comparative statics as follows.

- Autoregressive Binomial Model

  1. As T increases, i.e., as the bargaining horizon becomes longer, the occurrence of the first agreement is observed in a later period. The sensitivity of the duration is significant especially when players have optimistic initial priors on the future asset values. [For unconditional frequencies refer to Figures 3.3 through 3.6 (Basic Game) and Figures 3.11 through 3.14 (Alternative Game). For conditional frequencies refer to Figures 3.19 through 3.22 (Basic Game) and Figures 3.27 through 3.30 (Alternative Game).]

  2. As players' initial priors become more pessimistic, we observe higher frequencies of agreement in a given period. Moreover, we observe agreements in earlier periods as players become pessimistic. If we look at the conditional frequencies in Figure 3.24, for example, it is clear that first agreements occur immediately in the very first period when both players are sufficiently pessimistic. Note that the difference in players' initial priors does not seem to have much influence over bargaining durations. [For unconditional frequencies refer to Figures 3.7 and 3.8 (Basic Game) and Figures 3.15 and 3.16 (Alternative Game). For conditional frequencies refer to Figures 3.23 and 3.24 (Basic Game) and Figures 3.31 and 3.32 (Alternative Game).]

  3. As ρ increases, i.e., as the bargaining environment becomes more promising on the asset value increase in the future for both players, then there are longer

bargaining durations before we observe the first agreement. If players have pessimistic priors on information shocks at the same time when ρ is low, then we observe an immediate agreement. [For unconditional frequencies refer to Figure 3.9 (Basic Game) and Figure 3.17 (Alternative Game). For conditional frequencies refer to Figure 3.25 (Basic Game) and Figure 3.33 (Alternative Game).]

4. As x_T increases, i.e., as A's predetermined default share in period T increases, then in Basic games the bargaining durations are not very sensitive when both players have similar priors about information shocks. If A is optimistic and B is pessimistic, we observe a subtle tendency to have a longer duration as x_T increases, while if A is pessimistic and B is optimistic, we observe a subtle tendency to come to an early agreement. On the other hand, in Alternative games we observe higher sensitivity of durations than we can observe in Basic games. With larger x_T we observe higher frequencies of agreement in a given even–numbered period and lower frequencies in a given odd–numbered period. We observe the similar tendency regardless of players' priors, except that there is higher frequencies of agreement in earlier periods with one or more pessimistic players. [For unconditional frequencies refer to Figure 3.10 (Basic Game) and Figure 3.18 (Alternative Game). For conditional frequencies refer to Figure 3.26 (Basic Game) and Figure 3.34 (Alternative Game).]

- Wiener Process Model

1. As θ increases, i.e., as the expected rate of value increase becomes larger, the first agreement is reached in a later period. It is clearly demonstrated that the

variances in durations until the first agreement is larger in Alternative games than in Basic games. [Refer to Figures 3.35 and 3.36 (Basic Game) and Figures 3.37 and 3.38 (Alternative Game).]

2. The frequencies of agreement do not show much sensitivity to the change in T as in the case of Autoregressive Binomial Models. But how differently Basic games and Alternative games predict are reflected clearly in our findings. [Refer to Figures 3.43 and 3.44 (Basic Game) and Figures 3.45 and 3.46 (Alternative Game).]

3. The frequencies of agreement do not show much sensitivity to the size of differences in players' initial priors. But how differently Basic games and Alternative games predict are reflected clearly in our findings. [Refer to Figures 3.39 and 3.40 (Basic Game) and Figures 3.41 and 3.42 (Alternative Game).]

We conclude our discussion by listing several extensions that can be made to our simulations. We can endogenize the time interval between information arrivals to investigate the physical length of bargaining durations, instead of measuring the duration by the number of periods. Incorporating an exogenously determined varied length of time interval between information shocks is another potential extension. For example, we may suppose that it occurs at some rate per unit time, so that the frequency of information arrivals in a given period length follows the Poisson distribution. Modeling a risk averse player's behavior is also another extension. We may characterize a risk averse player with a strategy of compromising with a lower share than that of a risk neutral counterpart after observing undesirable information to avoid any delay, or with a higher share only after observing a series of desirable information to generate a delay. In addition, players' uncertainty about the value of the asset at a certain time in the future depends on how far they are looking ahead. We may model such uncertainty by using a variance as its measure.

Appendix 3A

Tables for Chapter 3

3A.1 Autoregressive Binomial Models

Table 3.1 : Exhibit 1 : Autoreg. Binomial Model (Basic Game)
$[T = 4,\ \alpha_A = \alpha_B = 1,\ \beta_A = \beta_B = 3,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Table 3.2 : Exhibit 2 : Autoreg. Binomial Model (Basic Game)
$[T = 4,\ \alpha_A = \alpha_B = 2,\ \beta_A = \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Table 3.3 : Exhibit 3 : Autoreg. Binomial Model (Basic Game)
$[T = 4,\ \alpha_A = 1,\ \alpha_B = 2,\ \beta_A = 3,\ \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Table 3.4 : Exhibit 4 : Autoreg. Binomial Model (Basic Game)
$[T = 2,\ \alpha_A = 1,\ \alpha_B = 2,\ \beta_A = 3,\ \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Table 3.5 : Exhibit 5 : Autoreg. Binomial Model (Alternative Game)
$[T = 4,\ \alpha_A = 1,\ \alpha_B = 2,\ \beta_A = 3,\ \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Table 3.1

Exhibit 1 : Autoregressive Binomial Model (Basic Game)
$T=4,\ \alpha_A=\alpha_B=1,\ \beta_A=\beta_B=3,\ x_T=0.5,\ \rho=0.5,\ \delta_0=0.5,\ Q_0=1.2$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in odd t, B's response in even t	π_t^A	π_t^B
At $t=0$								
	$E(\pi_1)$	0.138313	$E(\pi_1)$	0.461687	0.615261	**accept**	**0.738313**	**0.461687**
	$E(\pi_2)$	0.329378	$E(\pi_2)$	0.128122				
	$E(\pi_3)$	0.151179	$E(\pi_3)$	0.310071				
	$E(\pi_4)$	0.267448	$E(\pi_4)$	0.267448				
At $t=1$								
If Path = 0	$E(\pi_2)$	0.053873	$E(\pi_2)$	0.043627	0.820423	**accept**	**0.053873**	**0.246127**
	$E(\pi_3)$	0.023372	$E(\pi_3)$	0.043972				
	$E(\pi_4)$	0.030748	$E(\pi_4)$	0.030748				
If Path = 1	$E(\pi_2)$	1.155892	$E(\pi_2)$	0.381608	0.738913	reject	0.391631	1.108369
	$E(\pi_3)$	0.534600	$E(\pi_3)$	1.108369				
	$E(\pi_4)$	0.977548	$E(\pi_4)$	0.977548				
At $t=2$								
If Path = 00	$E(\pi_3)$	0.002883	$E(\pi_3)$	0.005711	0.847703	**accept**	**0.031789**	**0.005711**
	$E(\pi_4)$	0.002883	$E(\pi_4)$	0.002883				
If Path = 01	$E(\pi_3)$	0.105327	$E(\pi_3)$	0.197016	0.421363	reject	0.142210	0.195290
	$E(\pi_4)$	0.142210	$E(\pi_4)$	0.142210				
If Path = 10	$E(\pi_3)$	0.230844	$E(\pi_3)$	0.374625	0.600400	**accept**	**0.562875**	**0.374625**
	$E(\pi_4)$	0.265634	$E(\pi_4)$	0.265634				
If Path = 11	$E(\pi_3)$	0.990234	$E(\pi_3)$	2.208984	0.839146	reject	2.045419	0.392081
	$E(\pi_4)$	2.045419	$E(\pi_4)$	2.045419				

(Continued on next page)

Exhibit 1 : Autoregressive Binomial Model (Basic Game)

$T = 4,\ \alpha_A = \alpha_B = 1,\ \beta_A = \beta_B = 3,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2$

At $t = 3$								
If Path = 000	$E(\pi_4)$	0.000204	$E(\pi_4)$	0.000204	0.912946	**accept**	**0.000204**	**0.002140**
If Path = 001	$E(\pi_4)$	0.016275	$E(\pi_4)$	0.016275	0.591518	**accept**	**0.016275**	**0.023568**
If Path = 010	$E(\pi_4)$	0.053817	$E(\pi_4)$	0.053817	0.716518	**accept**	**0.053817**	**0.136026**
If Path = 011	$E(\pi_4)$	0.318996	$E(\pi_4)$	0.318996	0.604911	reject	0.208348	0.318996
If Path = 100	$E(\pi_4)$	0.064741	$E(\pi_4)$	0.064741	0.779018	**accept**	**0.064741**	**0.228228**
If Path = 101	$E(\pi_4)$	0.667419	$E(\pi_4)$	0.667419	0.542411	reject	0.563049	0.667419
If Path = 110	$E(\pi_4)$	0.826669	$E(\pi_4)$	0.826669	0.582589	**accept**	**0.826669**	**1.153800**
If Path = 111	$E(\pi_4)$	3.264169	$E(\pi_4)$	3.264169	0.738839	reject	1.153800	3.264169
At $t = 4$								
If Path = 0000	N/A				Default		0.000037	0.000037
If Path = 0001							0.001208	0.001208
If Path = 0010							0.010583	0.010583
If Path = 0011							0.030505	0.030505
If Path = 0100							0.026697	0.026697
If Path = 0101							0.121619	0.121619
If Path = 0110							0.205994	0.205994
If Path = 0111							0.469666	0.469666
If Path = 1000							0.022888	0.022888
If Path = 1001							0.169373	0.169373
If Path = 1010							0.403748	0.403748
If Path = 1011							1.018982	1.018982
If Path = 1100							0.402283	0.402283
If Path = 1101							1.392517	1.392517
If Path = 1110							2.001892	2.001892
If Path = 1111							4.210876	4.210876

Path identifies a sequence of observed ε's up to period t. For example, Path = 011 indicates a sequence of $\varepsilon_1 = 0$, $\varepsilon_2 = 1$, and $\varepsilon_3 = 1$. In the last period, i.e., $t = 4$, payoffs are computed by a predetermined default share, which is $x_T = 0.5$ in this exhibit.

Table 3.2

Exhibit 2 : Autoregressive Binomial Model (Basic Game)
$T=4,\ \alpha_A=\alpha_B=2,\ \beta_A=\beta_B=1,\ x_T=0.5,\ \rho=0.5,\ \delta_0=0.5,\ Q_0=1.2$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t=0$								
	$E(\pi_1)$	-0.690032	$E(\pi_1)$	1.790032	1.482448	reject	1.778938	-0.578938
	$E(\pi_2)$	1.778938	$E(\pi_2)$	-0.341438				
	$E(\pi_3)$	0.390801	$E(\pi_3)$	1.785293				
	$E(\pi_4)$	1.775662	$E(\pi_4)$	1.775662				
At $t=1$								
If Path = 0	$E(\pi_2)$	0.143430	$E(\pi_2)$	0.044070	0.521899	**accept**	**0.143430**	**0.156570**
	$E(\pi_3)$	0.062490	$E(\pi_3)$	0.142354				
	$E(\pi_4)$	0.133601	$E(\pi_4)$	0.133601				
If Path = 1	$E(\pi_2)$	2.596692	$E(\pi_2)$	-0.534192	1.737842	reject	-1.106763	2.606763
	$E(\pi_3)$	0.554956	$E(\pi_3)$	2.606763				
	$E(\pi_4)$	2.596692	$E(\pi_4)$	2.596692				
At $t=2$								
If Path = 00	$E(\pi_3)$	0.007976	$E(\pi_3)$	0.009368	0.750195	**accept**	**0.028132**	**0.009368**
	$E(\pi_4)$	0.008474	$E(\pi_4)$	0.008474				
If Path = 01	$E(\pi_3)$	0.117004	$E(\pi_3)$	0.275339	0.766602	reject	0.258728	0.078772
	$E(\pi_4)$	0.258728	$E(\pi_4)$	0.258728				
If Path = 10	$E(\pi_3)$	0.288391	$E(\pi_3)$	0.567078	0.561914	reject	0.526794	0.410706
	$E(\pi_4)$	0.526794	$E(\pi_4)$	0.526794				
If Path = 11	$E(\pi_3)$	0.643811	$E(\pi_3)$	3.286658	1.348372	reject	3.286658	-0.849158
	$E(\pi_4)$	3.286658	$E(\pi_4)$	3.286658				

(Continued on next page)

Exhibit 2 : Autoregressive Binomial Model (Basic Game)

$T = 4,\ \alpha_A = \alpha_B = 2,\ \beta_A = \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2$

At $t = 3$								
If Path = 000	$E(\pi_4)$	0.000427	$E(\pi_4)$	0.000427	0.817708	**accept**	**0.000427**	**0.001917**
If Path = 001	$E(\pi_4)$	0.020544	$E(\pi_4)$	0.020544	0.515625	reject	0.019299	0.020544
If Path = 010	$E(\pi_4)$	0.074158	$E(\pi_4)$	0.074158	0.609375	**accept**	**0.074158**	**0.115686**
If Path = 011	$E(\pi_4)$	0.381775	$E(\pi_4)$	0.381775	0.723958	reject	0.145569	0.381775
If Path = 100	$E(\pi_4)$	0.096130	$E(\pi_4)$	0.096130	0.671875	**accept**	**0.096130**	**0.196838**
If Path = 101	$E(\pi_4)$	0.813904	$E(\pi_4)$	0.813904	0.661458	reject	0.416565	0.813904
If Path = 110	$E(\pi_4)$	1.062439	$E(\pi_4)$	1.062439	0.536458	reject	0.918030	1.062439
If Path = 111	$E(\pi_4)$	3.842712	$E(\pi_4)$	3.842712	0.869792	reject	0.575257	3.842712
At $t = 4$								
If Path = 0000	N/A				Default		0.000037	0.000037
If Path = 0001							0.001208	0.001208
If Path = 0010							0.010583	0.010583
If Path = 0011							0.030505	0.030505
If Path = 0100							0.026697	0.026697
If Path = 0101							0.121619	0.121619
If Path = 0110							0.205994	0.205994
If Path = 0111							0.469666	0.469666
If Path = 1000							0.022888	0.022888
If Path = 1001							0.169373	0.169373
If Path = 1010							0.403748	0.403748
If Path = 1011							1.018982	1.018982
If Path = 1100							0.402283	0.402283
If Path = 1101							1.392517	1.392517
If Path = 1110							2.001892	2.001892
If Path = 1111							4.210876	4.210876

Path identifies a sequence of observed ε's up to period t. For example, Path = 011 indicates a sequence of $\varepsilon_1 = 0$, $\varepsilon_2 = 1$, and $\varepsilon_3 = 1$. In the last period, i.e., $t = 4$, payoffs are computed by a predetermined default share, which is $x_T = 0.5$ in this exhibit.

Table 3.3

Exhibit 3 : Autoregressive Binomial Model (Basic Game)

$T=4,\ \alpha_A=1,\ \alpha_B=2,\ \beta_A=3,\ \beta_B=1,\ x_T=0.5,\ \rho=0.5,\ \delta_0=0.5,\ Q_0=1.2$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in odd t, B's response in even t	π_t^A	π_t^B
At $t=0$								
	$E(\pi_1)$	-0.243497	$E(\pi_1)$	1.832446	0.246356	reject	0.295627	0.904373
	$E(\pi_2)$	0.295627	$E(\pi_2)$	0.326980				
	$E(\pi_3)$	0.111788	$E(\pi_3)$	1.798102				
	$E(\pi_4)$	0.267448	$E(\pi_4)$	1.775662				
At $t=1$								
If Path = 0	$E(\pi_2)$	0.049873	$E(\pi_2)$	0.103000	0.833756	**accept**	**0.049873**	**0.250127**
	$E(\pi_3)$	0.019186	$E(\pi_3)$	0.147093				
	$E(\pi_4)$	0.030748	$E(\pi_4)$	0.133601				
If Path = 1	$E(\pi_2)$	1.032887	$E(\pi_2)$	0.438969	1.749071	reject	-1.123606	2.623606
	$E(\pi_3)$	0.389594	$E(\pi_3)$	2.623606				
	$E(\pi_4)$	0.977548	$E(\pi_4)$	2.596692				
At $t=2$								
If Path = 00	$E(\pi_3)$	0.002883	$E(\pi_3)$	0.010711	0.714369	**accept**	**0.026789**	**0.010711**
	$E(\pi_4)$	0.002883	$E(\pi_4)$	0.008474				
If Path = 01	$E(\pi_3)$	0.084401	$E(\pi_3)$	0.283476	0.421363	reject	0.142210	0.195290
	$E(\pi_4)$	0.142210	$E(\pi_4)$	0.258728				
If Path = 10	$E(\pi_3)$	0.182016	$E(\pi_3)$	0.579633	0.381724	**accept**	**0.357867**	**0.579633**
	$E(\pi_4)$	0.265634	$E(\pi_4)$	0.526794				
If Path = 11	$E(\pi_3)$	0.700963	$E(\pi_3)$	3.304930	0.839146	reject	2.045419	0.392081
	$E(\pi_4)$	2.045419	$E(\pi_4)$	3.286658				

(Continued on next page)

Exhibit 3 : Autoregressive Binomial Model (Basic Game) $T=4,\ \alpha_A=1,\ \alpha_B=2,\ \beta_A=3,\ \beta_B=1,\ x_T=0.5,\ \rho=0.5,\ \delta_0=0.5,\ Q_0=1.2$								
At $t=3$								
If Path = 000	$E(\pi_4)$	0.000204	$E(\pi_4)$	0.000427	0.912946	**accept**	**0.000204**	**0.002140**
If Path = 001	$E(\pi_4)$	0.016275	$E(\pi_4)$	0.020544	0.591518	**accept**	**0.016275**	**0.023568**
If Path = 010	$E(\pi_4)$	0.053817	$E(\pi_4)$	0.074158	0.716518	**accept**	**0.053817**	**0.136026**
If Path = 011	$E(\pi_4)$	0.318996	$E(\pi_4)$	0.381775	0.723958	reject	0.145569	0.381775
If Path = 100	$E(\pi_4)$	0.064741	$E(\pi_4)$	0.096130	0.779018	**accept**	**0.064741**	**0.228228**
If Path = 101	$E(\pi_4)$	0.667419	$E(\pi_4)$	0.813904	0.661458	reject	0.416565	0.813904
If Path = 110	$E(\pi_4)$	0.826669	$E(\pi_4)$	1.062439	0.582589	**accept**	**0.826669**	**1.153800**
If Path = 111	$E(\pi_4)$	3.264169	$E(\pi_4)$	3.842712	0.869792	reject	0.575257	3.842712
At $t=4$								
If Path = 0000	N/A				Default		0.000037	0.000037
If Path = 0001							0.001208	0.001208
If Path = 0010							0.010583	0.010583
If Path = 0011							0.030505	0.030505
If Path = 0100							0.026697	0.026697
If Path = 0101							0.121619	0.121619
If Path = 0110							0.205994	0.205994
If Path = 0111							0.469666	0.469666
If Path = 1000							0.022888	0.022888
If Path = 1001							0.169373	0.169373
If Path = 1010							0.403748	0.403748
If Path = 1011							1.018982	1.018982
If Path = 1100							0.402283	0.402283
If Path = 1101							1.392517	1.392517
If Path = 1110							2.001892	2.001892
If Path = 1111							4.210876	4.210876

Path identifies a sequence of observed ε's up to period t. For example, Path = 011 indicates a sequence of $\varepsilon_1=0$, $\varepsilon_2=1$, and $\varepsilon_3=1$. In the last period, i.e., $t=4$, payoffs are computed by a predetermined default share, which is $x_T=0.5$ in this exhibit.

Table 3.4

Exhibit 4 : Autoregressive Binomial Model (Basic Game)

$T = 2,\ \alpha_A = 1,\ \alpha_B = 2,\ \beta_A = 3,\ \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in odd t, B's response in even t	π_t^A	π_t^B
At $t = 0$	$E(\pi_1)$	0.153750	$E(\pi_1)$	0.771250	0.357292	**accept**	**0.428750**	**0.771250**
	$E(\pi_2)$	0.228750	$E(\pi_2)$	0.718750				
At $t = 1$								
If Path = 0	$E(\pi_2)$	0.048750	$E(\pi_2)$	0.093750	0.837500	**accept**	**0.048750**	**0.251250**
If Path = 1	$E(\pi_2)$	0.768750	$E(\pi_2)$	1.031250	0.687500	reject	0.468750	1.031250
At $t = 2$								
If Path = 00	N/A				Default		0.018750	0.018750
If Path = 01							0.168750	0.168750
If Path = 10							0.468750	0.468750
If Path = 11							1.218750	1.218750

Path identifies a sequence of observed ε's up to period t. For example, Path = 01 indicates a sequence of $\varepsilon_1 = 0$ and $\varepsilon_2 = 1$. In the last period, i.e., $t = 2$, payoffs are computed by a predetermined default share, which is $x_T = 0.5$ in this exhibit.

Table 3.5

Exhibit 5 : Autoregressive Binomial Model (Alternative Game)
$T=4,\ \alpha_A=1,\ \alpha_B=2,\ \beta_A=3,\ \beta_B=1,\ x_T=0.5,\ \rho=0.5,\ \delta_0=0.5,\ Q_0=1.2$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t=0$								
	$E(\pi_1)$	0.268355	$E(\pi_1)$	0.608015	0.447259	N/A	N/A	
	$E(\pi_2)$	-0.078533	$E(\pi_2)$	1.776552				
	$E(\pi_3)$	0.268355	$E(\pi_3)$	0.824834				
	$E(\pi_4)$	0.267448	$E(\pi_4)$	1.775662				
At $t=1$								
If Path = 0	$E(\pi_2)$	0.028027	$E(\pi_2)$	0.133601	0.712539	**accept**	**0.134178**	**0.165822**
	$E(\pi_3)$	0.031958	$E(\pi_3)$	0.108138				
	$E(\pi_4)$	0.030748	$E(\pi_4)$	0.133601				
If Path = 1	$E(\pi_2)$	-0.398216	$E(\pi_2)$	2.596692	1.259002	reject	0.670889	0.829111
	$E(\pi_3)$	0.977548	$E(\pi_3)$	1.183183				
	$E(\pi_4)$	0.977548	$E(\pi_4)$	2.596692				
At $t=2$								
If Path = 00	$E(\pi_3)$	0.004395	$E(\pi_3)$	0.008474	0.511402	**accept**	**0.010780**	**0.026720**
	$E(\pi_4)$	0.002883	$E(\pi_4)$	0.008474				
If Path = 01	$E(\pi_3)$	0.142210	$E(\pi_3)$	0.207801	0.470359	reject	0.097018	0.240482
	$E(\pi_4)$	0.142210	$E(\pi_4)$	0.258728				
If Path = 10	$E(\pi_3)$	0.265634	$E(\pi_3)$	0.480154	0.438724	reject	-0.242814	1.180314
	$E(\pi_4)$	0.265634	$E(\pi_4)$	0.526794				
If Path = 11	$E(\pi_3)$	2.045419	$E(\pi_3)$	1.417525	0.639350	reject	-0.631318	3.068818
	$E(\pi_4)$	2.045419	$E(\pi_4)$	3.286658				

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Exhibit 5 : Autoregressive Binomial Model (Alternative Game)

$T = 4,\ \alpha_A = 1,\ \alpha_B = 2,\ \beta_A = 3,\ \beta_B = 1,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2$

At $t = 3$								
If Path = 000	$E(\pi_4)$	0.000204	$E(\pi_4)$	0.000427	0.500000	**accept**	**0.001199**	**0.001145**
If Path = 001	$E(\pi_4)$	0.016275	$E(\pi_4)$	0.020544	0.500000	reject	0.020376	0.019468
If Path = 010	$E(\pi_4)$	0.053817	$E(\pi_4)$	0.074158	0.500000	**accept**	**0.089295**	**0.100549**
If Path = 011	$E(\pi_4)$	0.318996	$E(\pi_4)$	0.381775	0.500000	reject	0.248041	0.279303
If Path = 100	$E(\pi_4)$	0.064741	$E(\pi_4)$	0.096130	0.500000	**accept**	**0.128532**	**0.164436**
If Path = 101	$E(\pi_4)$	0.667419	$E(\pi_4)$	0.813904	0.500000	reject	0.539836	0.690632
If Path = 110	$E(\pi_4)$	0.826669	$E(\pi_4)$	1.062439	0.500000	reject	1.266212	0.714257
If Path = 111	$E(\pi_4)$	3.264169	$E(\pi_4)$	3.842712	0.500000	reject	2.824626	1.593343
At $t = 4$								
If Path = 0000	N/A				Default		0.000037	0.000037
If Path = 0001							0.001208	0.001208
If Path = 0010							0.010583	0.010583
If Path = 0011							0.030505	0.030505
If Path = 0100							0.026697	0.026697
If Path = 0101							0.121619	0.121619
If Path = 0110							0.205994	0.205994
If Path = 0111							0.469666	0.469666
If Path = 1000							0.022888	0.022888
If Path = 1001							0.169373	0.169373
If Path = 1010							0.403748	0.403748
If Path = 1011							1.018982	1.018982
If Path = 1100							0.402283	0.402283
If Path = 1101							1.392517	1.392517
If Path = 1110							2.001892	2.001892
If Path = 1111							4.210876	4.210876

Path identifies a sequence of observed ε's up to period t. For example, Path = 011 indicates a sequence of $\varepsilon_1 = 0$, $\varepsilon_2 = 1$, and $\varepsilon_3 = 1$. In the last period, i.e., $t = 4$, payoffs are computed by a predetermined default share, which is $x_T = 0.5$ in this exhibit.

3A.2 Generalized Wiener Process Models

Table 3.6 : Exhibit 6 : Wiener Process Model (Basic Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -1]$

Table 3.7 : Exhibit 7 : Wiener Process Model (Basic Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -3]$

Table 3.8 : Exhibit 8 : Wiener Process Model (Basic Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -1000]$

Table 3.9 : Exhibit 9 : Wiener Process Model (Basic Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -555]$

Table 3.10 : Exhibit 10 : Wiener Process Model (Alternative Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -1]$

Table 3.11 : Exhibit 11 : Wiener Process Model (Alternative Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -3]$

Table 3.12 : Exhibit 12 : Wiener Process Model (Alternative Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -1000]$

Table 3.13 : Exhibit 13 : Wiener Process Model (Alternative Game)
$[T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ \theta = 0.0014,\ \varphi = 0.0004,\ idum = -555]$

Table 3.6

Exhibit 6 : Wiener Process Model (Basic Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -1$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t = 0$								
$\delta_0 = 0.994825$	$E(\pi_1)$	0.596658	$E(\pi_1)$	0.596994	0.499918	**accept**	**0.596797**	**0.596994**
$Q_0 = 1.193791$	$E(\pi_2)$	0.596658	$E(\pi_2)$	0.596855				
	$E(\pi_3)$	0.596520	$E(\pi_3)$	0.596855				
	$E(\pi_4)$	0.596520	$E(\pi_4)$	0.596717				
	$E(\pi_5)$	0.596381	$E(\pi_5)$	0.596717				
	$E(\pi_6)$	0.596381	$E(\pi_6)$	0.596578				
At $t = 1$								
$\delta_1 = 0.998946$	$E(\pi_2)$	0.596185	$E(\pi_2)$	0.596266	0.500068	**accept**	**0.596185**	**0.596348**
$Q_1 = 1.192532$	$E(\pi_3)$	0.596103	$E(\pi_3)$	0.596266				
	$E(\pi_4)$	0.596103	$E(\pi_4)$	0.596185				
	$E(\pi_5)$	0.596022	$E(\pi_5)$	0.596185				
	$E(\pi_6)$	0.596021	$E(\pi_6)$	0.596103				
At $t = 2$								
$\delta_2 = 1.001174$	$E(\pi_3)$	0.596910	$E(\pi_3)$	0.596988	0.499982	**accept**	**0.596945**	**0.596988**
$Q_2 = 1.193932$	$E(\pi_4)$	0.596910	$E(\pi_4)$	0.596954				
	$E(\pi_5)$	0.596876	$E(\pi_5)$	0.596954				
	$E(\pi_6)$	0.596876	$E(\pi_6)$	0.596920				

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Exhibit 6 : Wiener Process Model (Basic Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-1$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t=3$ $\delta_3=1.010030$ $Q_3=1.205907$	$E(\pi_4)$ 0.603055 $E(\pi_5)$ 0.602979 $E(\pi_6)$ 0.603055	$E(\pi_4)$ 0.602928 $E(\pi_5)$ 0.603079 $E(\pi_6)$ 0.603079	0.500104	reject	0.602828	0.603079
At $t=4$ $\delta_4=0.998918$ $Q_4=1.204602$	$E(\pi_5)$ 0.602295 $E(\pi_6)$ 0.602350	$E(\pi_5)$ 0.602363 $E(\pi_6)$ 0.602363	0.500041	reject	0.602350	0.602252
At $t=5$ $\delta_5=1.004063$ $Q_5=1.209497$	$E(\pi_6)$ 0.604784	$E(\pi_6)$ 0.604789	0.500033	reject	0.604708	0.604789
At $t=6$ $\delta_6=0.996627$ $Q_6=1.205417$	N/A		Default		0.602709	0.602709

Table 3.7

Exhibit 7 : Wiener Process Model (Basic Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -3$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in odd t, B's response in even t	π_t^A	π_t^B
At $t = 0$								
$\delta_0 = 1.000852$	$E(\pi_1)$	0.600229	$E(\pi_1)$	0.600887	0.500152	reject	0.600693	0.600329
$Q_0 = 1.201023$	$E(\pi_2)$	0.600693	$E(\pi_2)$	0.600515				
	$E(\pi_3)$	0.600415	$E(\pi_3)$	0.600887				
	$E(\pi_4)$	0.600693	$E(\pi_4)$	0.600701				
	$E(\pi_5)$	0.600600	$E(\pi_5)$	0.600887				
	$E(\pi_6)$	0.600693	$E(\pi_6)$	0.600887				
At $t = 1$								
$\delta_1 = 1.005739$	$E(\pi_2)$	0.604334	$E(\pi_2)$	0.603748	0.500378	reject	0.603501	0.604415
$Q_1 = 1.207916$	$E(\pi_3)$	0.603834	$E(\pi_3)$	0.604415				
	$E(\pi_4)$	0.604334	$E(\pi_4)$	0.604081				
	$E(\pi_5)$	0.604167	$E(\pi_5)$	0.604415				
	$E(\pi_6)$	0.604334	$E(\pi_6)$	0.604415				
At $t = 2$								
$\delta_2 = 1.002991$	$E(\pi_3)$	0.605587	$E(\pi_3)$	0.606097	0.500240	reject	0.606054	0.605474
$Q_2 = 1.211528$	$E(\pi_4)$	0.606054	$E(\pi_4)$	0.605786				
	$E(\pi_5)$	0.605898	$E(\pi_5)$	0.606097				
	$E(\pi_6)$	0.606054	$E(\pi_6)$	0.606097				

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Exhibit 7 : Wiener Process Model (Basic Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-3$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t=3$ $\delta_3=1.008932$ $Q_3=1.222349$	$E(\pi_4)$ 0.611477 $E(\pi_5)$ 0.611267 $E(\pi_6)$ 0.611477	$E(\pi_4)$ 0.611082 $E(\pi_5)$ 0.611502 $E(\pi_6)$ 0.611502	0.500268	reject	0.610847	0.611502
At $t=4$ $\delta_4=0.997205$ $Q_4=1.218933$	$E(\pi_5)$ 0.609460 $E(\pi_6)$ 0.609609	$E(\pi_5)$ 0.609622 $E(\pi_6)$ 0.609622	0.500117	reject	0.609609	0.609324
At $t=5$ $\delta_5=1.002529$ $Q_5=1.222015$	$E(\pi_6)$ 0.611077	$E(\pi_6)$ 0.611082	0.500061	reject	0.610933	0.611082
At $t=6$ $\delta_6=1.003518$ $Q_6=1.226314$	N/A		Default		0.613157	0.613157

Table 3.8

Exhibit 8 : Wiener Process Model (Basic Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-1000$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t=0$								
$\delta_0=0.999042$	$E(\pi_1)$	0.599282	$E(\pi_1)$	0.599591	0.499977	reject	0.599397	0.599453
$Q_0=1.198850$	$E(\pi_2)$	0.599397	$E(\pi_2)$	0.599499				
	$E(\pi_3)$	0.599328	$E(\pi_3)$	0.599591				
	$E(\pi_4)$	0.599397	$E(\pi_4)$	0.599545				
	$E(\pi_5)$	0.599374	$E(\pi_5)$	0.599591				
	$E(\pi_6)$	0.599397	$E(\pi_6)$	0.599591				
At $t=1$								
$\delta_1=0.991538$	$E(\pi_2)$	0.594241	$E(\pi_2)$	0.594322	0.500094	**accept**	**0.594241**	**0.594464**
$Q_1=1.188705$	$E(\pi_3)$	0.594098	$E(\pi_3)$	0.594322				
	$E(\pi_4)$	0.594098	$E(\pi_4)$	0.594180				
	$E(\pi_5)$	0.593956	$E(\pi_5)$	0.594180				
	$E(\pi_6)$	0.593956	$E(\pi_6)$	0.594038				
At $t=2$								
$\delta_2=1.000038$	$E(\pi_3)$	0.594264	$E(\pi_3)$	0.594397	0.499982	**accept**	**0.594353**	**0.594397**
$Q_2=1.188750$	$E(\pi_4)$	0.594264	$E(\pi_4)$	0.594307				
	$E(\pi_5)$	0.594175	$E(\pi_5)$	0.594307				
	$E(\pi_6)$	0.594175	$E(\pi_6)$	0.594218				

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Exhibit 8 : Wiener Process Model (Basic Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -1000$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t = 3$ $\delta_3 = 1.004875$ $Q_3 = 1.194545$	$E(\pi_4)$ 0.597252 $E(\pi_5)$ 0.597236 $E(\pi_6)$ 0.597236	$E(\pi_4)$ 0.597276 $E(\pi_5)$ 0.597276 $E(\pi_6)$ 0.597260	0.500017	**accept**	**0.597252**	**0.597293**
At $t = 4$ $\delta_4 = 1.003111$ $Q_4 = 1.198261$	$E(\pi_5)$ 0.599124 $E(\pi_6)$ 0.599138	$E(\pi_5)$ 0.599151 $E(\pi_6)$ 0.599151	0.500006	reject	0.599138	0.599123
At $t = 5$ $\delta_5 = 0.999591$ $Q_5 = 1.197771$	$E(\pi_6)$ 0.598889	$E(\pi_6)$ 0.598894	0.500007	reject	0.598877	0.598894
At $t = 6$ $\delta_6 = 1.006852$ $Q_6 = 1.205978$	N/A		Default		0.602989	0.602989

Table 3.9

Exhibit 9 : Wiener Process Model (Basic Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-555$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t=0$								
$\delta_0=0.997595$	$E(\pi_1)$	0.598427	$E(\pi_1)$	0.598654	0.499918	**accept**	**0.598459**	**0.598654**
$Q_0=1.197114$	$E(\pi_2)$	0.598427	$E(\pi_2)$	0.598622				
	$E(\pi_3)$	0.598394	$E(\pi_3)$	0.598622				
	$E(\pi_4)$	0.598394	$E(\pi_4)$	0.598589				
	$E(\pi_5)$	0.598361	$E(\pi_5)$	0.598589				
	$E(\pi_6)$	0.598361	$E(\pi_6)$	0.598557				
At $t=1$								
$\delta_1=1.001631$	$E(\pi_2)$	0.599551	$E(\pi_2)$	0.599539	0.500082	reject	0.599434	0.599632
$Q_1=1.199066$	$E(\pi_3)$	0.599481	$E(\pi_3)$	0.599632				
	$E(\pi_4)$	0.599551	$E(\pi_4)$	0.599585				
	$E(\pi_5)$	0.599527	$E(\pi_5)$	0.599632				
	$E(\pi_6)$	0.599551	$E(\pi_6)$	0.599632				
At $t=2$								
$\delta_2=1.001411$	$E(\pi_3)$	0.600318	$E(\pi_3)$	0.600479	0.500047	reject	0.600436	0.600322
$Q_2=1.200758$	$E(\pi_4)$	0.600436	$E(\pi_4)$	0.600400				
	$E(\pi_5)$	0.600396	$E(\pi_5)$	0.600479				
	$E(\pi_6)$	0.600436	$E(\pi_6)$	0.600479				

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Exhibit 9 : Wiener Process Model (Basic Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -555$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_t^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in odd t B's response in even t	π_t^A	π_t^B
At $t = 3$ $\delta_3 = 0.994211$ $Q_3 = 1.193807$	$E(\pi_4)$ 0.596880 $E(\pi_5)$ 0.596858 $E(\pi_6)$ 0.596858	$E(\pi_4)$ 0.596904 $E(\pi_5)$ 0.596904 $E(\pi_6)$ 0.596882	0.500020	**accept**	**0.596880**	**0.596927**
At $t = 4$ $\delta_4 = 1.000582$ $Q_4 = 1.194502$	$E(\pi_5)$ 0.597234 $E(\pi_6)$ 0.597234	$E(\pi_5)$ 0.597258 $E(\pi_6)$ 0.597247	0.499995	**accept**	**0.597245**	**0.597258**
At $t = 5$ $\delta_5 = 0.992396$ $Q_5 = 1.185419$	$E(\pi_6)$ 0.592680	$E(\pi_6)$ 0.592685	0.500025	**accept**	**0.592680**	**0.592739**
At $t = 6$ $\delta_6 = 1.004370$ $Q_6 = 1.190599$	N/A		Default		0.595300	0.595300

Table 3.10

Exhibit 10 : Wiener Process Model (Alternative Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-1$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t=0$						
$\delta_0=0.994825$	$E(\pi_1)$ 0.596797	$E(\pi_1)$ 0.596855	0.499989	N/A	N/A	
$Q_0=1.193791$	$E(\pi_2)$ 0.596658	$E(\pi_2)$ 0.596855				
	$E(\pi_3)$ 0.596658	$E(\pi_3)$ 0.596716				
	$E(\pi_4)$ 0.596520	$E(\pi_4)$ 0.596717				
	$E(\pi_5)$ 0.596520	$E(\pi_5)$ 0.596578				
	$E(\pi_6)$ 0.596381	$E(\pi_6)$ 0.596578				
At $t=1$						
$\delta_1=0.998946$	$E(\pi_2)$ 0.596184	$E(\pi_2)$ 0.596266	0.500027	**accept**	**0.596253**	**0.596279**
$Q_1=1.192532$	$E(\pi_3)$ 0.596185	$E(\pi_3)$ 0.596185				
	$E(\pi_4)$ 0.596103	$E(\pi_4)$ 0.596185				
	$E(\pi_5)$ 0.596103	$E(\pi_5)$ 0.596103				
	$E(\pi_6)$ 0.596021	$E(\pi_6)$ 0.596103				
At $t=2$						
$\delta_2=1.001174$	$E(\pi_3)$ 0.596944	$E(\pi_3)$ 0.596954	0.500001	reject	0.596933	0.596999
$Q_2=1.193932$	$E(\pi_4)$ 0.596910	$E(\pi_4)$ 0.596954				
	$E(\pi_5)$ 0.596910	$E(\pi_5)$ 0.596920				
	$E(\pi_6)$ 0.596876	$E(\pi_6)$ 0.596920				

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Exhibit 10 : Wiener Process Model (Alternative Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -1$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t = 3$ $\delta_3 = 1.010030$ $Q_3 = 1.205907$	$E(\pi_4)$ 0.602904 $E(\pi_5)$ 0.603055 $E(\pi_6)$ 0.603055	$E(\pi_4)$ 0.603079 $E(\pi_5)$ 0.603004 $E(\pi_6)$ 0.603079	0.500069	reject	0.602954	0.602953
At $t = 4$ $\delta_4 = 0.998918$ $Q_4 = 1.204602$	$E(\pi_5)$ 0.602350 $E(\pi_6)$ 0.602350	$E(\pi_5)$ 0.602308 $E(\pi_6)$ 0.602363	0.500020	reject	0.602217	0.602385
At $t = 5$ $\delta_5 = 1.004063$ $Q_5 = 1.209497$	$E(\pi_6)$ 0.604784	$E(\pi_6)$ 0.604789	0.500000	reject	0.604773	0.604724
At $t = 6$ $\delta_6 = 0.996627$ $Q_6 = 1.205417$	N/A		Default		0.602709	0.602709

Table 3.11

Exhibit 11 : Wiener Process Model (Alternative Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -3$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in even t, B's response in odd t	π_t^A	π_t^B
At $t = 0$								
$\delta_0 = 1.000852$	$E(\pi_1)$	0.600693	$E(\pi_1)$	0.600422	0.500126	N/A	N/A	
$Q_0 = 1.201023$	$E(\pi_2)$	0.600322	$E(\pi_2)$	0.600887				
	$E(\pi_3)$	0.600693	$E(\pi_3)$	0.600608				
	$E(\pi_4)$	0.600508	$E(\pi_4)$	0.600887				
	$E(\pi_5)$	0.600693	$E(\pi_5)$	0.600794				
	$E(\pi_6)$	0.600693	$E(\pi_6)$	0.600887				
At $t = 1$								
$\delta_1 = 1.005739$	$E(\pi_2)$	0.603668	$E(\pi_2)$	0.604415	0.500303	reject	0.604111	0.603805
$Q_1 = 1.207916$	$E(\pi_3)$	0.604334	$E(\pi_3)$	0.603915				
	$E(\pi_4)$	0.604001	$E(\pi_4)$	0.604415				
	$E(\pi_5)$	0.604334	$E(\pi_5)$	0.604248				
	$E(\pi_6)$	0.604334	$E(\pi_6)$	0.604415				
At $t = 2$								
$\delta_2 = 1.002991$	$E(\pi_3)$	0.606054	$E(\pi_3)$	0.605630	0.500180	reject	0.605397	0.606131
$Q_2 = 1.211528$	$E(\pi_4)$	0.605743	$E(\pi_4)$	0.606097				
	$E(\pi_5)$	0.606054	$E(\pi_5)$	0.605941				
	$E(\pi_6)$	0.606054	$E(\pi_6)$	0.606097				

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Exhibit 11 : Wiener Process Model (Alternative Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -3$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t = 3$ $\delta_3 = 1.008932$ $Q_3 = 1.222349$	$E(\pi_4)$ $E(\pi_5)$ $E(\pi_6)$	0.611057 0.611477 0.611477	$E(\pi_4)$ $E(\pi_5)$ $E(\pi_6)$	0.611502 0.611292 0.611502	0.500178	reject	0.611394	0.610955
At $t = 4$ $\delta_4 = 0.997205$ $Q_4 = 1.218933$	$E(\pi_5)$ $E(\pi_6)$	0.609609 0.609609	$E(\pi_5)$ $E(\pi_6)$	0.609473 0.609473	0.500058	reject	0.609249	0.609684
At $t = 5$ $\delta_5 = 1.002529$ $Q_5 = 1.222015$	$E(\pi_6)$	0.611077	$E(\pi_6)$	0.611082	0.500000	reject	0.611079	0.610936
At $t = 6$ $\delta_6 = 1.003518$ $Q_6 = 1.226314$	N/A				Default		0.613157	0.613157

Table 3.12

Exhibit 12 : Wiener Process Model (Alternative Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-1000$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer: x_t^A in even t, x_t^B in odd t	Response: A's response in even t, B's response in odd t	π_t^A	π_t^B
At $t=0$								
$\delta_0=0.999042$	$E(\pi_1)$	0.599397	$E(\pi_1)$	0.599476	0.499981	N/A	N/A	
$Q_0=1.198850$	$E(\pi_2)$	0.599305	$E(\pi_2)$	0.599591				
	$E(\pi_3)$	0.599397	$E(\pi_3)$	0.599522				
	$E(\pi_4)$	0.599351	$E(\pi_4)$	0.599591				
	$E(\pi_5)$	0.599397	$E(\pi_5)$	0.599568				
	$E(\pi_6)$	0.599397	$E(\pi_6)$	0.599591				
At $t=1$								
$\delta_1=0.991538$	$E(\pi_2)$	0.594241	$E(\pi_2)$	0.594322	0.500028	**accept**	**0.594330**	**0.594376**
$Q_1=1.188705$	$E(\pi_3)$	0.594240	$E(\pi_3)$	0.594180				
	$E(\pi_4)$	0.594098	$E(\pi_4)$	0.594180				
	$E(\pi_5)$	0.594098	$E(\pi_5)$	0.594038				
	$E(\pi_6)$	0.593956	$E(\pi_6)$	0.594038				
At $t=2$								
$\delta_2=1.000038$	$E(\pi_3)$	0.594353	$E(\pi_3)$	0.594307	0.500024	reject	0.594342	0.594408
$Q_2=1.188750$	$E(\pi_4)$	0.594264	$E(\pi_4)$	0.594307				
	$E(\pi_5)$	0.594264	$E(\pi_5)$	0.594218				
	$E(\pi_6)$	0.594175	$E(\pi_6)$	0.594218				

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Exhibit 12 : Wiener Process Model (Alternative Game)

$T = 6,\ \theta_0^A = 1.2,\ \theta_0^B = 0.8,\ \varphi_0^A = 0.5,\ \varphi_0^B = 0.05,\ x_T = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2,\ idum = -1000$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t = 3$ $\delta_3 = 1.004875$ $Q_3 = 1.194545$	$E(\pi_4)$ 0.597252 $E(\pi_5)$ 0.597252 $E(\pi_6)$ 0.597236	$E(\pi_4)$ 0.597276 $E(\pi_5)$ 0.597260 $E(\pi_6)$ 0.597260	0.500007	reject	0.597301	0.597244
At $t = 4$ $\delta_4 = 1.003111$ $Q_4 = 1.198261$	$E(\pi_5)$ 0.599138 $E(\pi_6)$ 0.599138	$E(\pi_5)$ 0.599137 $E(\pi_6)$ 0.599151	0.500003	reject	0.599123	0.599139
At $t = 5$ $\delta_5 = 0.999591$ $Q_5 = 1.197771$	$E(\pi_6)$ 0.598889	$E(\pi_6)$ 0.598894	0.500000	reject	0.598889	0.598882
At $t = 6$ $\delta_6 = 1.006852$ $Q_6 = 1.205978$	N/A		Default		0.602989	0.602989

Table 3.13

Exhibit 13 : Wiener Process Model (Alternative Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-555$

	Beliefs				Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$		$E_t^B(\pi_{t+n}^B)$		Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t=0$								
$\delta_0=0.997595$	$E(\pi_1)$	0.598459	$E(\pi_1)$	0.598622	0.499946	N/A	N/A	
$Q_0=1.197114$	$E(\pi_2)$	0.598427	$E(\pi_2)$	0.598622				
	$E(\pi_3)$	0.598427	$E(\pi_3)$	0.598589				
	$E(\pi_4)$	0.598394	$E(\pi_4)$	0.598589				
	$E(\pi_5)$	0.598394	$E(\pi_5)$	0.598557				
	$E(\pi_6)$	0.598361	$E(\pi_6)$	0.598557				
At $t=1$								
$\delta_1=1.001631$	$E(\pi_2)$	0.599458	$E(\pi_2)$	0.599632	0.500066	reject	0.599468	0.599598
$Q_1=1.199066$	$E(\pi_3)$	0.599551	$E(\pi_3)$	0.599562				
	$E(\pi_4)$	0.599504	$E(\pi_4)$	0.599632				
	$E(\pi_5)$	0.599551	$E(\pi_5)$	0.599609				
	$E(\pi_6)$	0.599551	$E(\pi_6)$	0.599632				
At $t=2$								
$\delta_2=1.001411$	$E(\pi_3)$	0.600436	$E(\pi_3)$	0.600361	0.500035	reject	0.600300	0.600458
$Q_2=1.200758$	$E(\pi_4)$	0.600357	$E(\pi_4)$	0.600479				
	$E(\pi_5)$	0.600436	$E(\pi_5)$	0.600440				
	$E(\pi_6)$	0.600436	$E(\pi_6)$	0.600479				

(Continued on next page)

Exhibit 13 : Wiener Process Model (Alternative Game)

$T=6,\ \theta_0^A=1.2,\ \theta_0^B=0.8,\ \varphi_0^A=0.5,\ \varphi_0^B=0.05,\ x_T=0.5,\ \theta=0.0014,\ \varphi=0.0004,\ \sigma=0.06,\ Q_0=1.2,\ idum=-555$

	Beliefs		Equilibrium Strategies		Payoffs if accept x_{t-1}^i	
	$E_t^A(\pi_{t+n}^A)$	$E_t^B(\pi_{t+n}^B)$	Offer x_t^A in even t x_t^B in odd t	Response A's response in even t B's response in odd t	π_t^A	π_t^B
At $t=3$ $\delta_3=0.994211$ $Q_3=1.193807$	$E(\pi_4)$ 0.596880 $E(\pi_5)$ 0.596880 $E(\pi_6)$ 0.596858	$E(\pi_4)$ 0.596905 $E(\pi_5)$ 0.596882 $E(\pi_6)$ 0.596882	0.500007	reject	0.596946	0.596861
At $t=4$ $\delta_4=1.000582$ $Q_4=1.194502$	$E(\pi_5)$ 0.597245 $E(\pi_6)$ 0.597234	$E(\pi_5)$ 0.597247 $E(\pi_6)$ 0.597247	0.500002	reject	0.597243	0.597259
At $t=5$ $\delta_5=0.992396$ $Q_5=1.185419$	$E(\pi_6)$ 0.592680	$E(\pi_6)$ 0.592685	0.500000	**accept**	**0.592712**	**0.592708**
At $t=6$ $\delta_6=1.004370$ $Q_6=1.190599$	N/A		Default		0.595300	0.595300

Appendix 3B

Figures for Chapter 3

3B.1 Autoregressive Binomial Models I

Unconditional Frequencies

Figure 3.3 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Figure 3.4 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.5 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2]$

Figure 3.6 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.7 : Autoreg. Binomial Model (Basic Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.8 : Autoreg. Binomial Model (Basic Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.9 : Autoreg. Binomial Model (Basic Game) $[\rho = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.10 : Autoreg. Binomial Model (Basic Game) $[x_T = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.11 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Figure 3.12 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.13 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2]$

Figure 3.14 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.15 : Autoreg. Binomial Model (Alternative Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.16 : Autoreg. Binomial Model (Alternative Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.17 : Autoreg. Binomial Model (Alternative Game) $[\rho = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.18 : Autoreg. Binomial Model (Alternative Game) $[x_T = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.3: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2)$

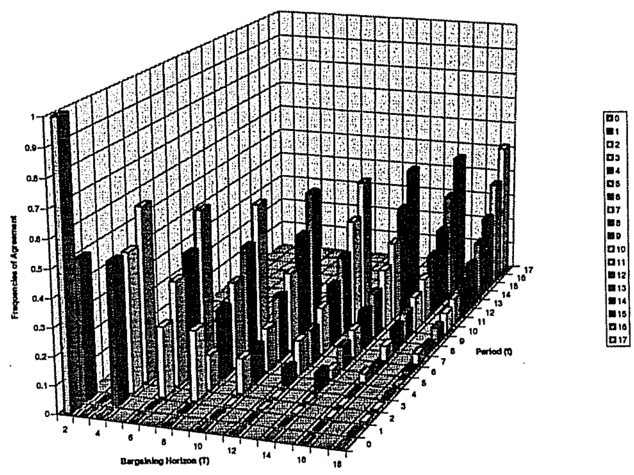


Figure 3.4: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2)$

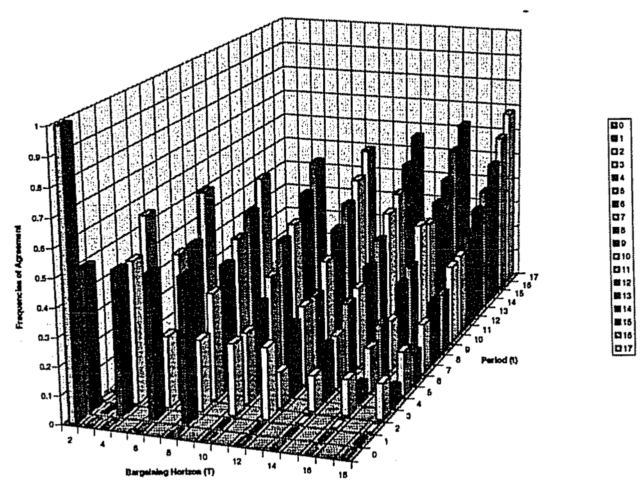


Figure 3.5: Autoregressive Binomial Model (Basic Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2$)

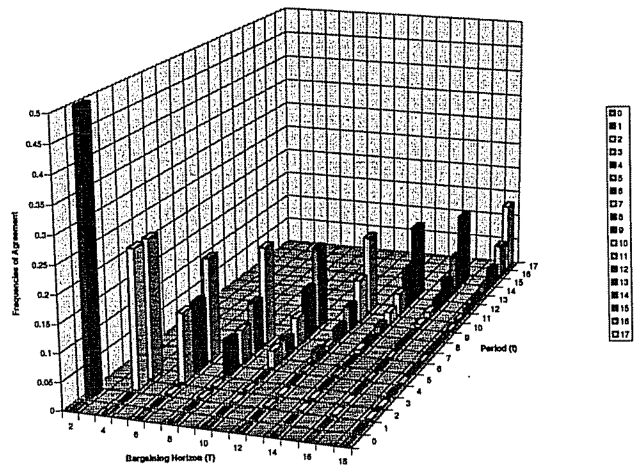


Figure 3.6: Autoregressive Binomial Model (Basic Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2$)

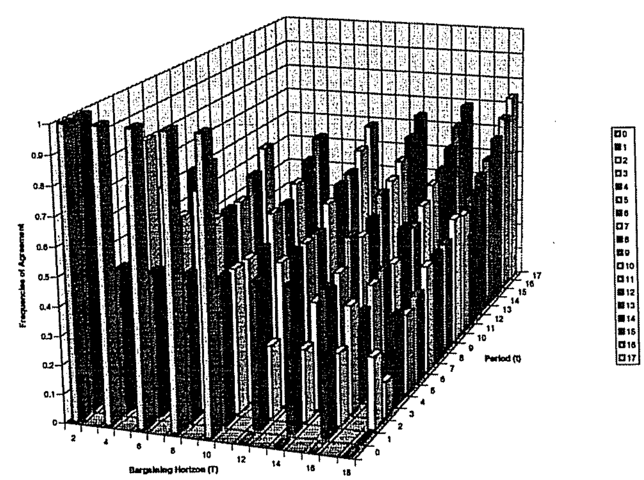


Figure 3.7: Autoregressive Binomial Model (Basic Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

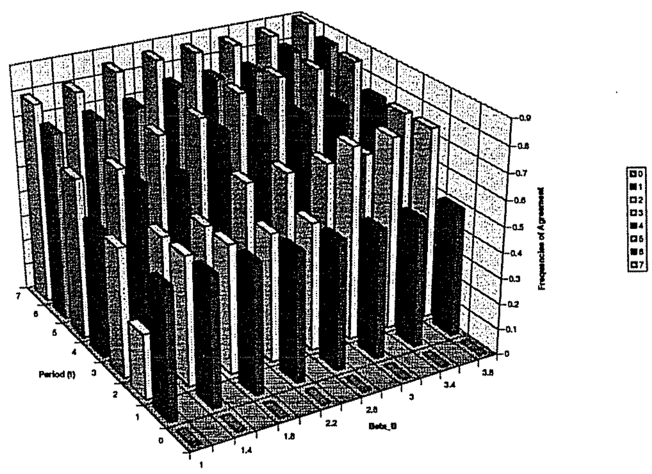


Figure 3.8: Autoregressive Binomial Model (Basic Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

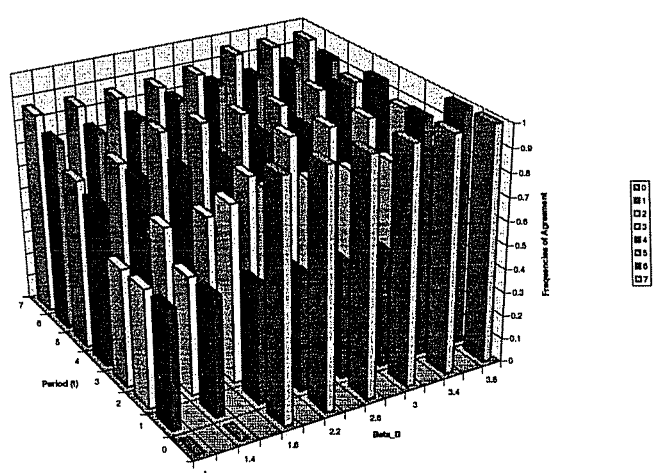
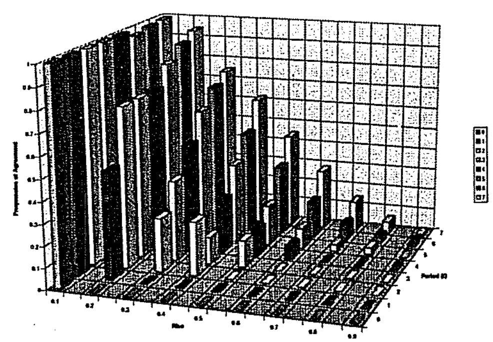
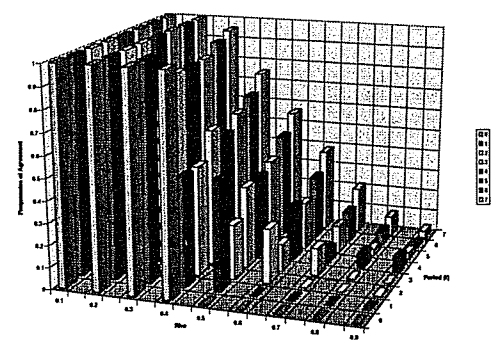


Figure 3.9: Autoregressive Binomial Model (Basic Game) [$\rho = 0.1 \rightarrow 0.9$]
($T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0$)

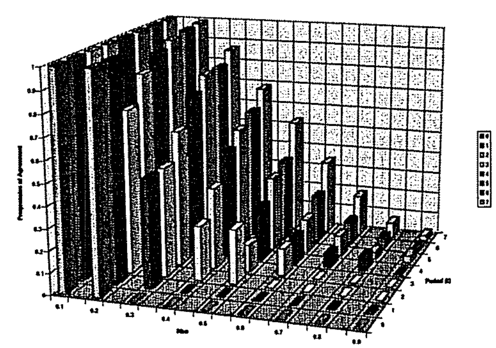
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

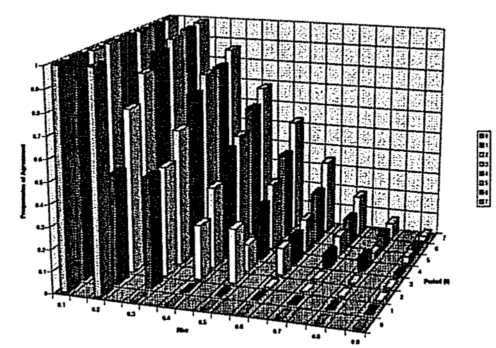
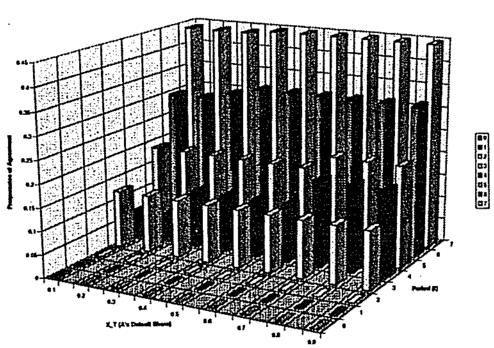
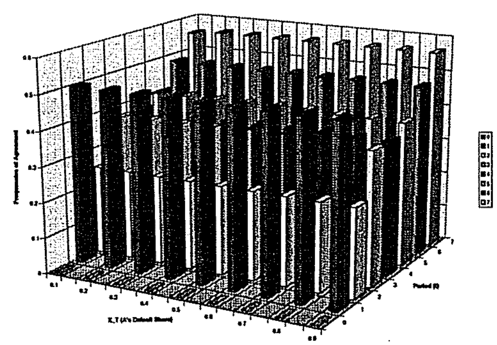


Figure 3.10: Autoregressive Binomial Model (Basic Game) $[x_T = 0.1 \rightarrow 0.9]$
$(T = 8,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0)$

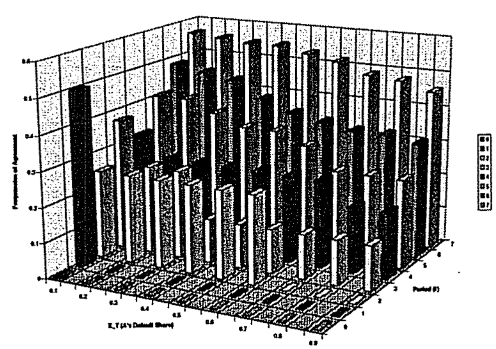
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

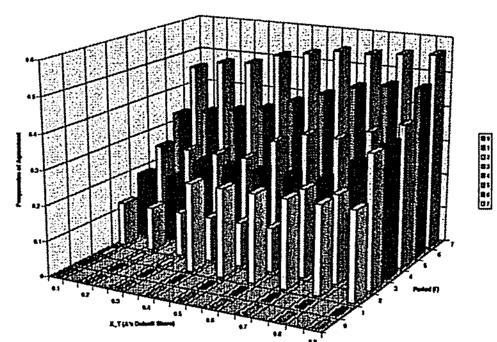


Figure 3.11: Autoregressive Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2)$

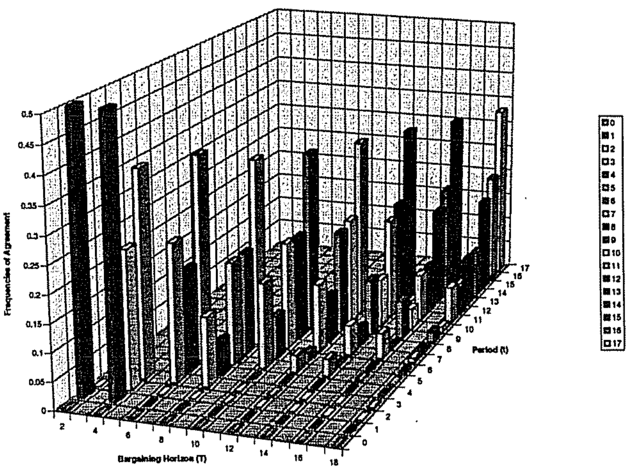


Figure 3.12: Autoregressive Binomial Model (Alternative Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2)$

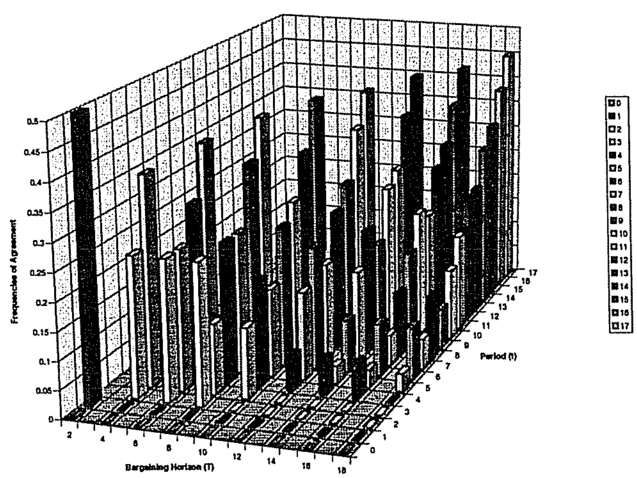


Figure 3.13: Autoregressive Binomial Model (Alternative Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2$)

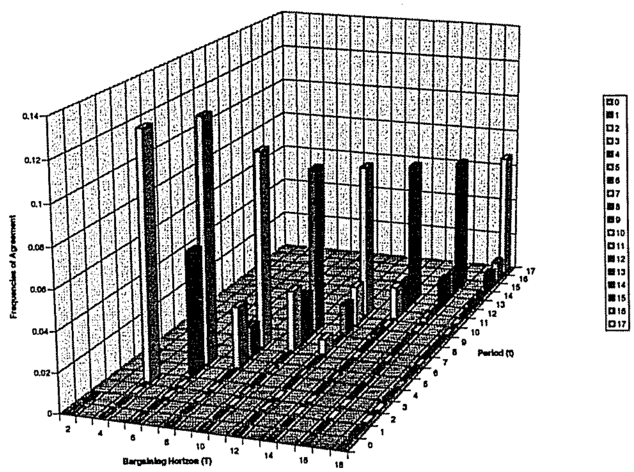


Figure 3.14: Autoregressive Binomial Model (Alternative Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2$)

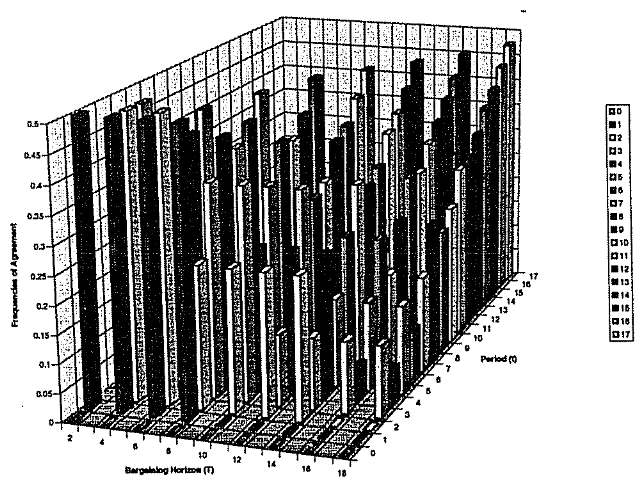


Figure 3.15: Autoregressive Binomial Model (Alternative Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

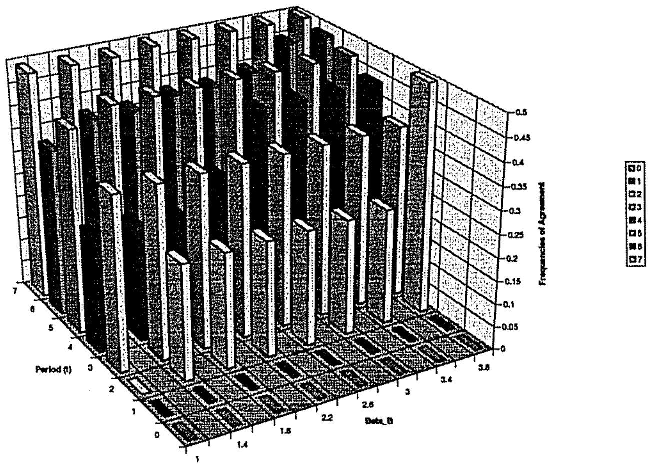


Figure 3.16: Autoregressive Binomial Model (Alternative Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

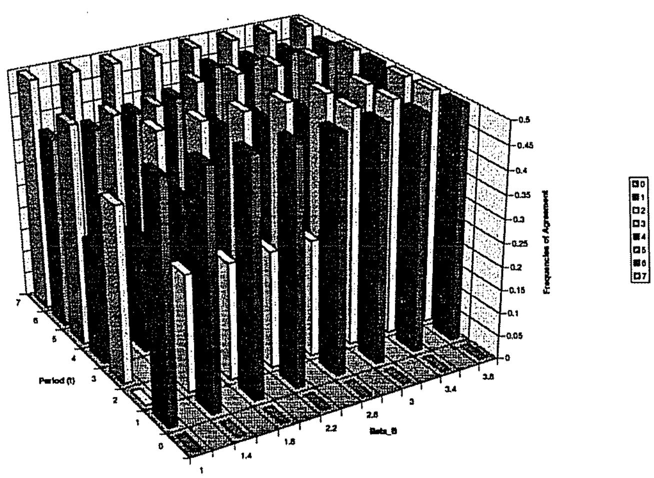
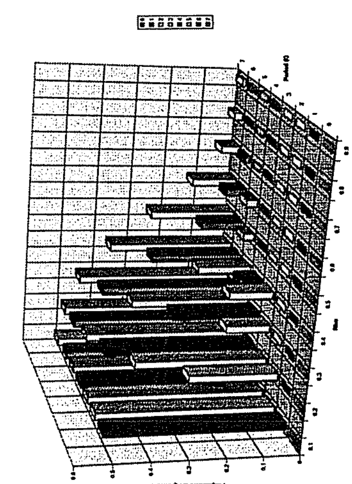
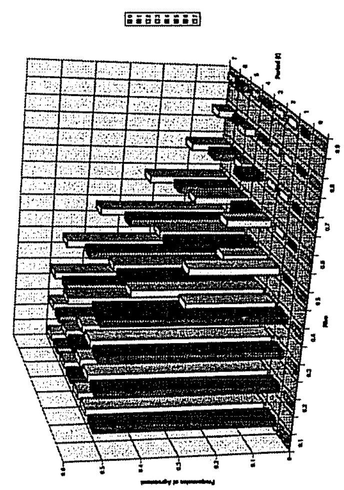


Figure 3.17: Autoregressive Binomial Model (Alternative Game) [$\rho = 0.1 \rightarrow 0.9$]
($T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0$)

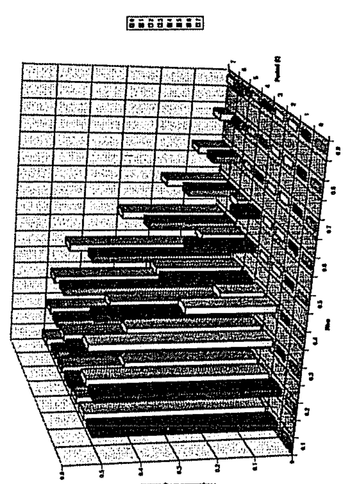
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

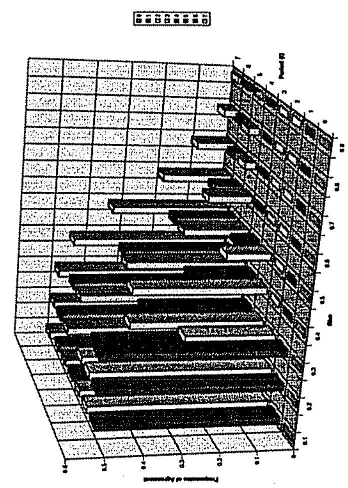
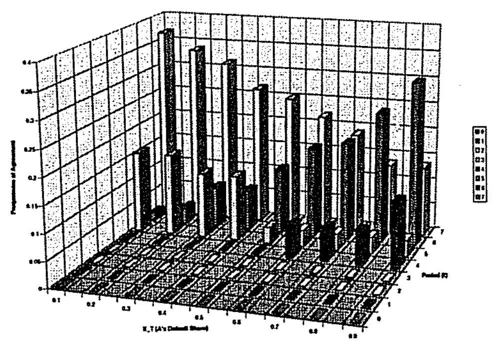
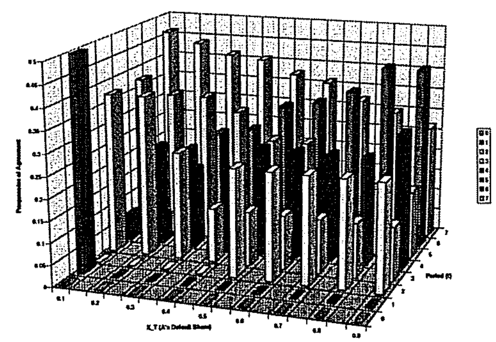


Figure 3.18: Autoregressive Binomial Model (Alternative Game) $[x_T = 0.1 \to 0.9]$
$(T = 8,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0)$

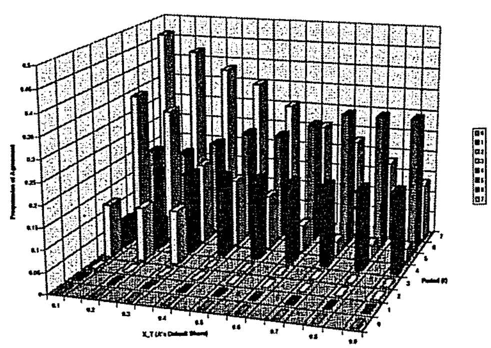
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



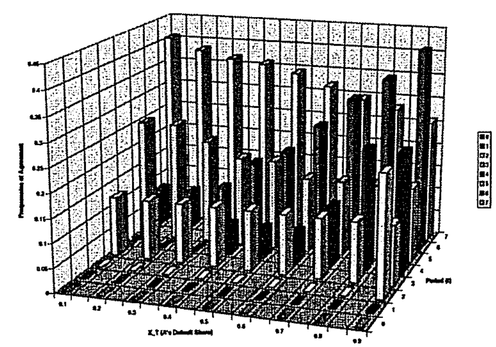
Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$



3B.2 Autoregressive Binomial Models II

Conditional Frequencies

Figure 3.19 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Figure 3.20 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.21 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2]$

Figure 3.22 : Autoreg. Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.23 : Autoreg. Binomial Model (Basic Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.24 : Autoreg. Binomial Model (Basic Game) $[\beta_B = 1.0 \rightarrow 3.8]$
$[T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.25 : Autoreg. Binomial Model (Basic Game) $[\rho = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.26 : Autoreg. Binomial Model (Basic Game) $[x_T = 0.1 \rightarrow 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.27 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \to 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2]$

Figure 3.28 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \to 18]$
$[\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.29 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \to 18]$
$[\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2]$

Figure 3.30 : Autoreg. Binomial Model (Alternative Game) $[T = 2 \to 18]$
$[\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2]$

Figure 3.31 : Autoreg. Binomial Model (Alternative Game) $[\beta_B = 1.0 \to 3.8]$
$[T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.32 : Autoreg. Binomial Model (Alternative Game) $[\beta_B = 1.0 \to 3.8]$
$[T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0]$

Figure 3.33 : Autoreg. Binomial Model (Alternative Game) $[\rho = 0.1 \to 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.34 : Autoreg. Binomial Model (Alternative Game) $[x_T = 0.1 \to 0.9]$
$[T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0]$
Case 1 : $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$
Case 2 : $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$
Case 3 : $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$
Case 4 : $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

Figure 3.19: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0,\ x_T = 0.5,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.2)$

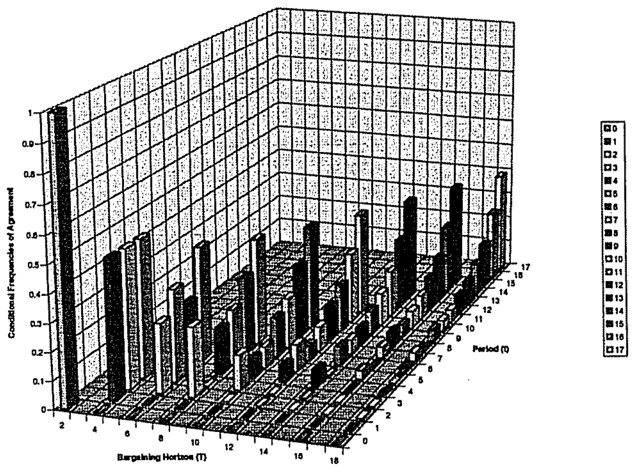


Figure 3.20: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.3,\ \alpha_B = 1.5,\ \beta_A = 1.0,\ \beta_B = 1.8,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2)$

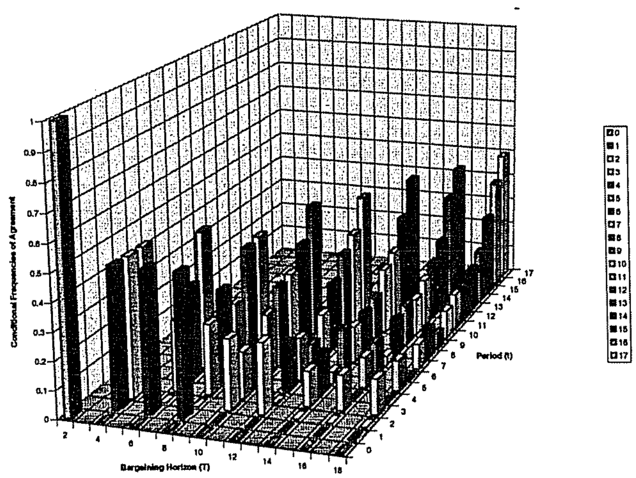


Figure 3.21: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2)$

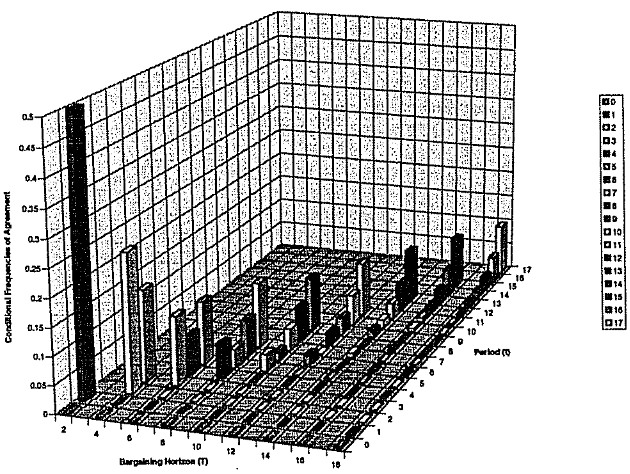


Figure 3.22: Autoregressive Binomial Model (Basic Game) $[T = 2 \rightarrow 18]$

$(\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2)$

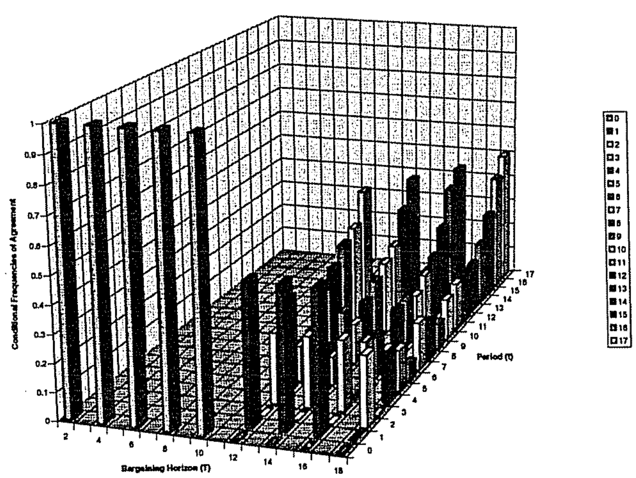


Figure 3.23: Autoregressive Binomial Model (Basic Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

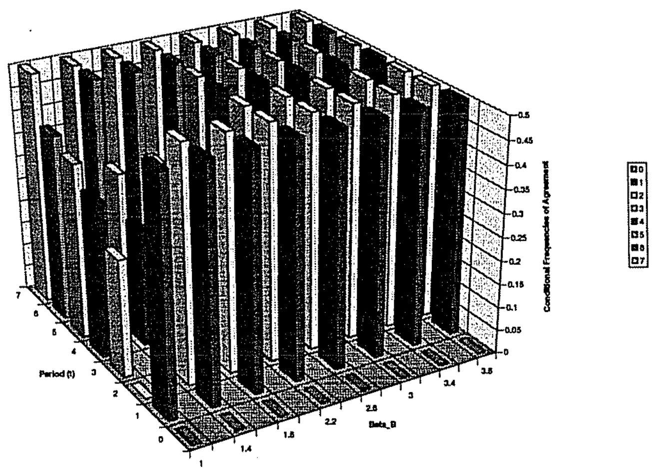


Figure 3.24: Autoregressive Binomial Model (Basic Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

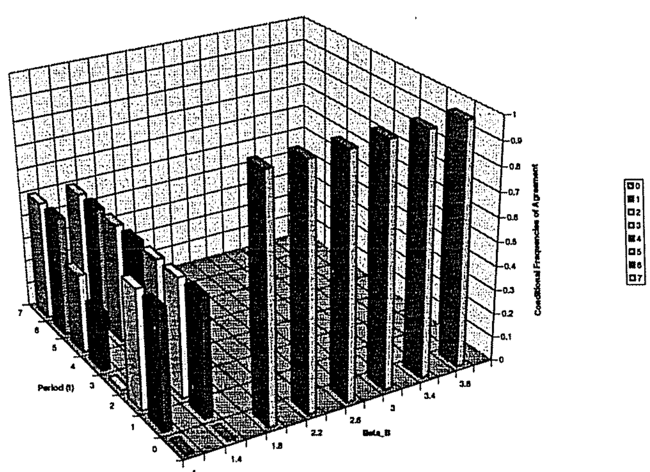
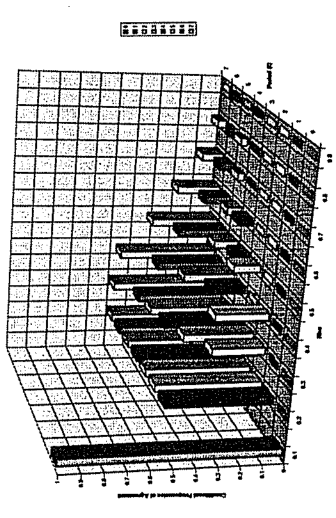
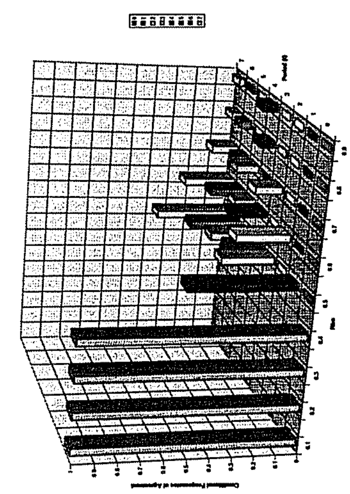


Figure 3.25: Autoregressive Binomial Model (Basic Game) [$\rho = 0.1 \rightarrow 0.9$]
($T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0$)

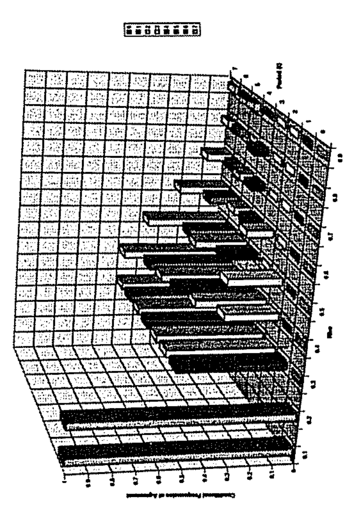
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

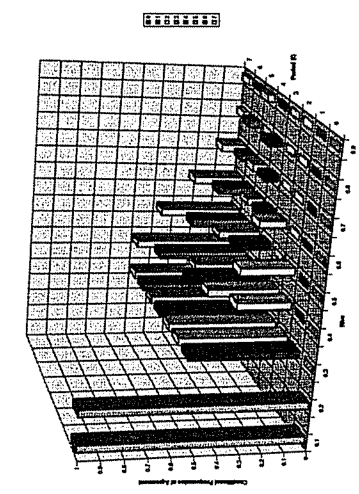
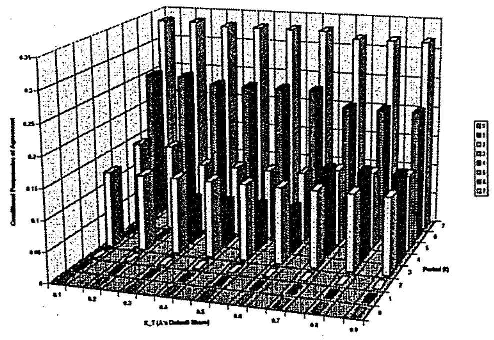
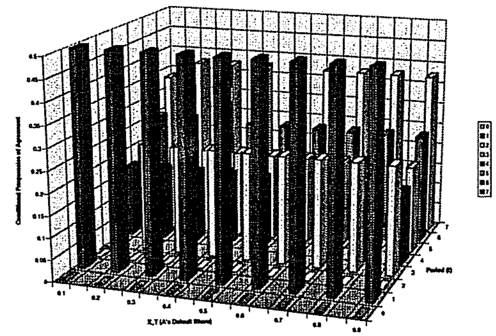


Figure 3.26: Autoregressive Binomial Model (Basic Game) $[x_T = 0.1 \to 0.9]$
$(T = 8,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0)$

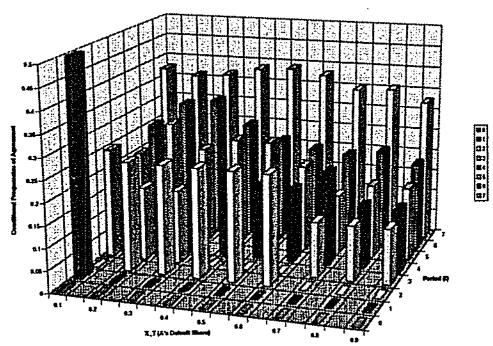
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

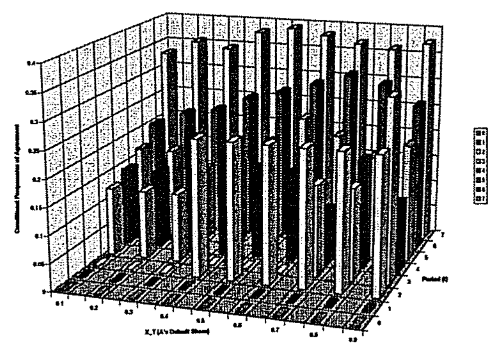


Figure 3.27: Autoregressive Binomial Model (Alternative Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.0$, $\alpha_B = 2.0$, $\beta_A = 3.0$, $\beta_B = 1.0$, $x_T = 0.5$, $\rho = 0.5$, $\delta_0 = 0.5$, $Q_0 = 1.2$)

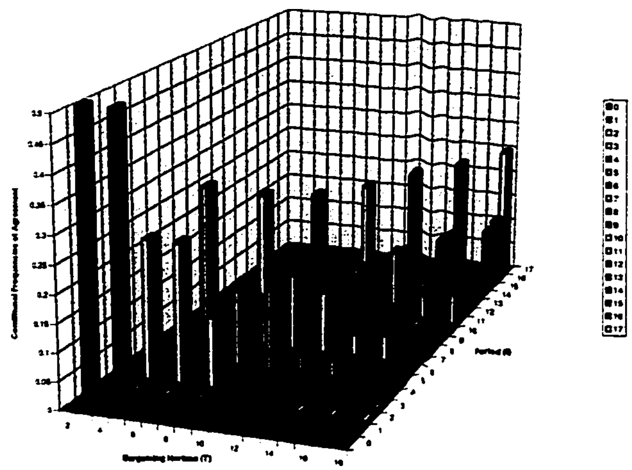


Figure 3.28: Autoregressive Binomial Model (Alternative Game) [$T = 2 \rightarrow 18$]

($\alpha_A = 1.3$, $\alpha_B = 1.5$, $\beta_A = 1.0$, $\beta_B = 1.8$, $x_T = 0.5$, $\rho = 0.4$, $\delta_0 = 0.4$, $Q_0 = 1.2$)

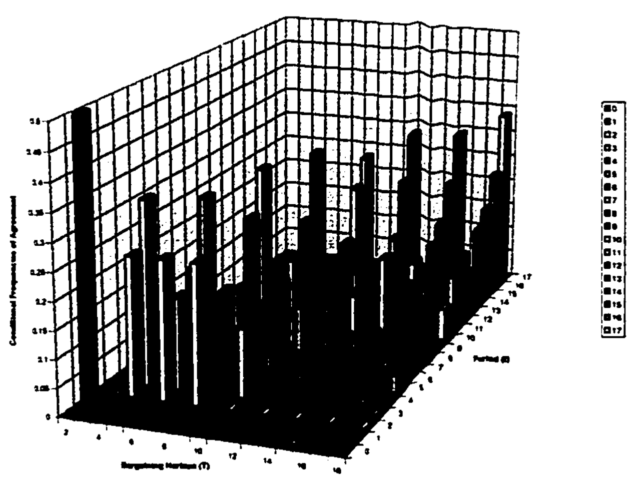


Figure 3.29: Autoregressive Binomial Model (Alternative Game) $[T = 2 \to 18]$

$(\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 1.5,\ \beta_B = 2.0,\ x_T = 0.4,\ \rho = 0.7,\ \delta_0 = 0.8,\ Q_0 = 1.2)$

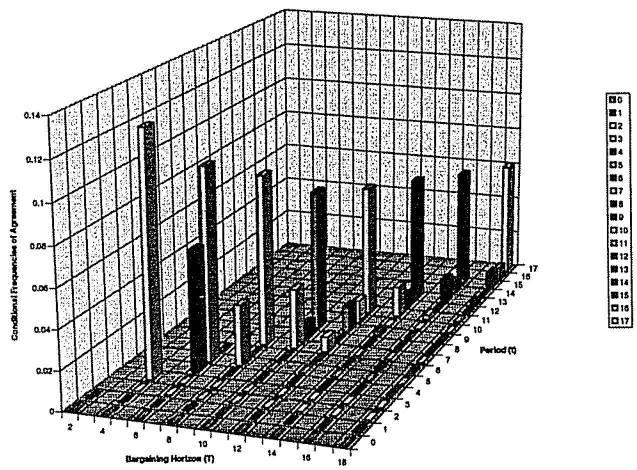


Figure 3.30: Autoregressive Binomial Model (Alternative Game) $[T = 2 \to 18]$

$(\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 3.0,\ x_T = 0.5,\ \rho = 0.4,\ \delta_0 = 0.4,\ Q_0 = 1.2)$

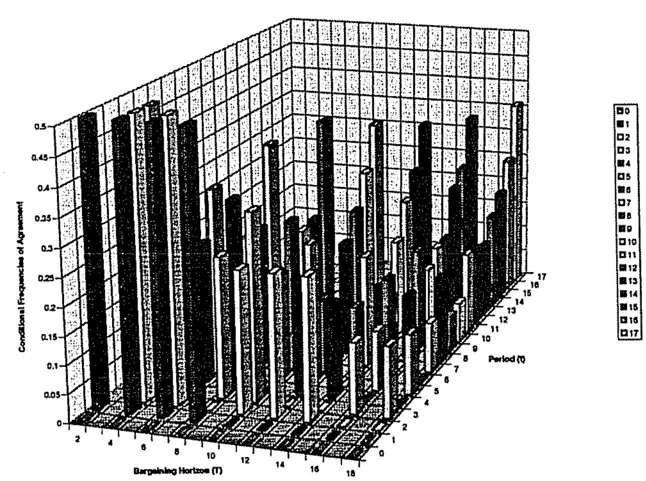


Figure 3.31: Autoregressive Binomial Model (Alternative Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 2.0,\ \alpha_B = 2.0,\ \beta_A = 1.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

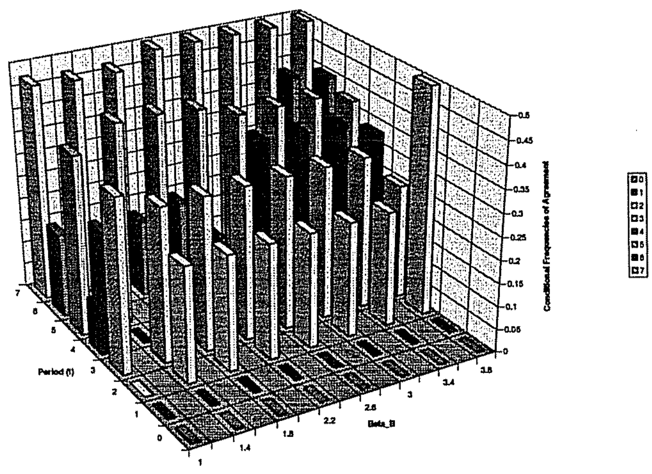


Figure 3.32: Autoregressive Binomial Model (Alternative Game) [$\beta_B = 1.0 \rightarrow 3.8$]

($T = 8,\ \alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ x_T = 0.3,\ \rho = 0.3,\ \delta_0 = 0.3,\ Q_0 = 1.0$)

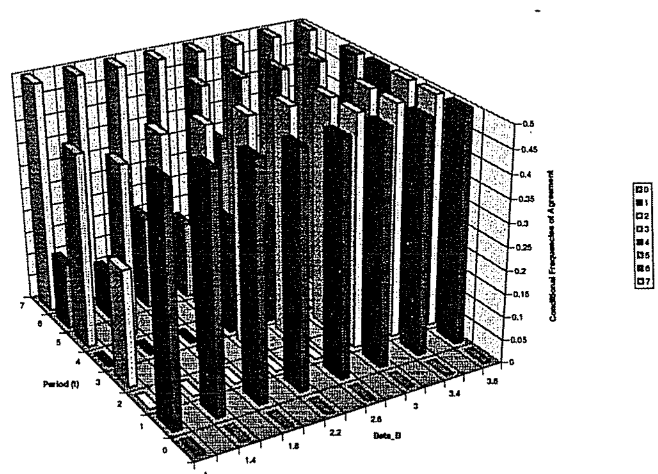
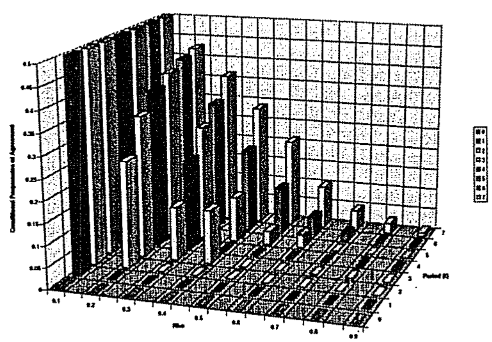
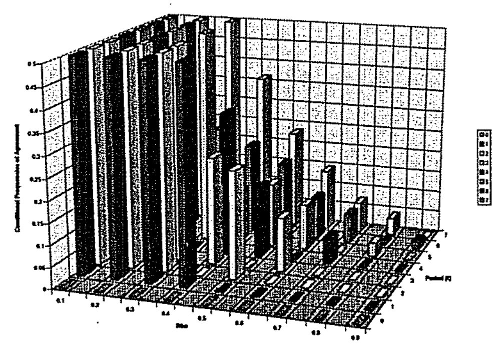


Figure 3.33: Autoregressive Binomial Model (Alternative Game) [$\rho = 0.1 \rightarrow 0.9$]
($T = 8,\ x_T = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0$)

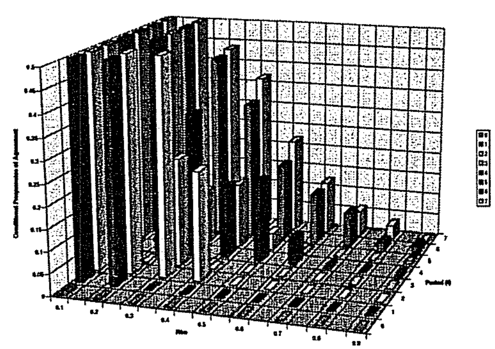
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$

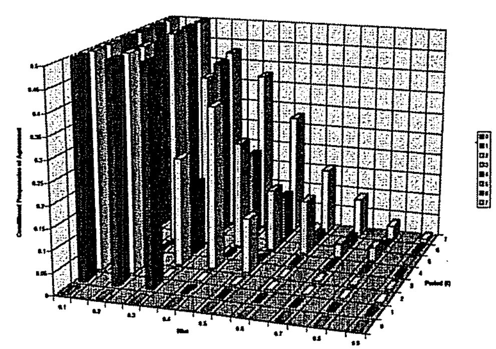
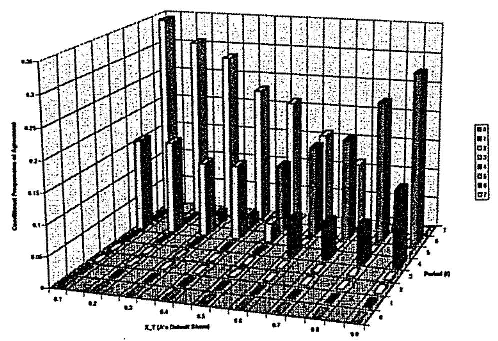
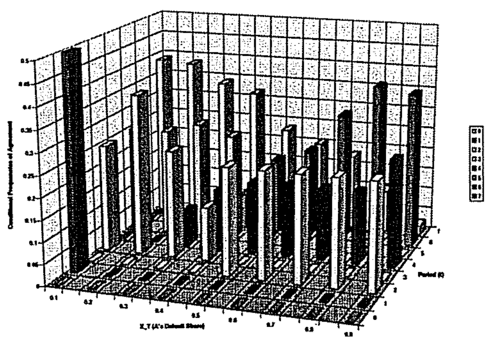


Figure 3.34: Autoregressive Binomial Model (Alternative Game) [$x_T = 0.1 \rightarrow 0.9$]
($T = 8,\ \rho = 0.5,\ \delta_0 = 0.5,\ Q_0 = 1.0$)

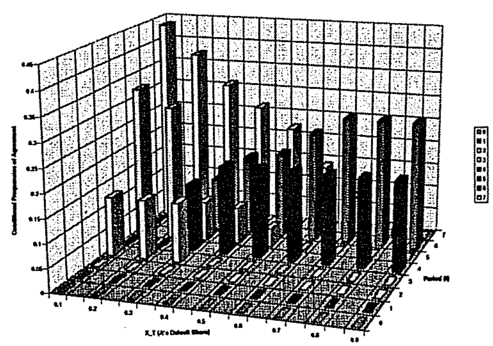
Case 1: $\alpha_A = 2.0,\ \alpha_B = 3.0,\ \beta_A = 1.0,\ \beta_B = 1.0$



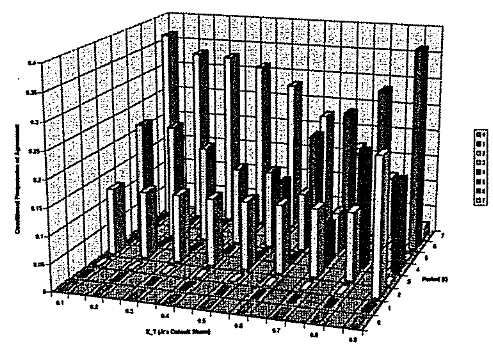
Case 2: $\alpha_A = 1.0,\ \alpha_B = 1.0,\ \beta_A = 3.0,\ \beta_B = 2.5$



Case 3: $\alpha_A = 2.0,\ \alpha_B = 1.0,\ \beta_A = 1.0,\ \beta_B = 3.0$



Case 4: $\alpha_A = 1.0,\ \alpha_B = 2.0,\ \beta_A = 3.0,\ \beta_B = 1.0$



3B.3 Generalized Wiener Process Models

Conditional Frequencies

Figure 3.35 : Wiener Process Model (Basic Game) $[\theta = -0.05 \rightarrow 0.06]$
$[\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.36 : Wiener Process Model (Basic Game) $[\theta = -0.05 \rightarrow 0.06]$
$[\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.37 : Wiener Process Model (Alternative Game) $[\theta = -0.05 \rightarrow 0.06]$
$[\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.38 : Wiener Process Model (Alternative Game) $[\theta = -0.05 \rightarrow 0.06]$
$[\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.39 : Wiener Process Model (Basic Game) $[\theta_A = 0.5 \rightarrow 6.0]$
$[\theta_B = 0.5,\ \varphi_A = 1.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.40 : Wiener Process Model (Basic Game) $[\theta_A = 0.5 \rightarrow 6.0]$
$[\theta_B = 6.0,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.41 : Wiener Process Model (Alternative Game) $[\theta_A = 0.5 \rightarrow 6.0]$
$[\theta_B = 0.5,\ \varphi_A = 1.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.42 : Wiener Process Model (Alternative Game) $[\theta_A = 0.5 \rightarrow 6.0]$
$[\theta_B = 6.0,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.43 : Wiener Process Model (Basic Game) $[T = 2 \rightarrow 12]$
$[\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.44 : Wiener Process Model (Basic Game) $[T = 2 \rightarrow 12]$
$[\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.45 : Wiener Process Model (Alternative Game) $[T = 2 \rightarrow 12]$
$[\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.46 : Wiener Process Model (Alternative Game) $[T = 2 \rightarrow 12]$
$[\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2]$

Figure 3.35: Wiener Process Model (Basic Game) [$\theta = -0.05 \rightarrow 0.06$]

($\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

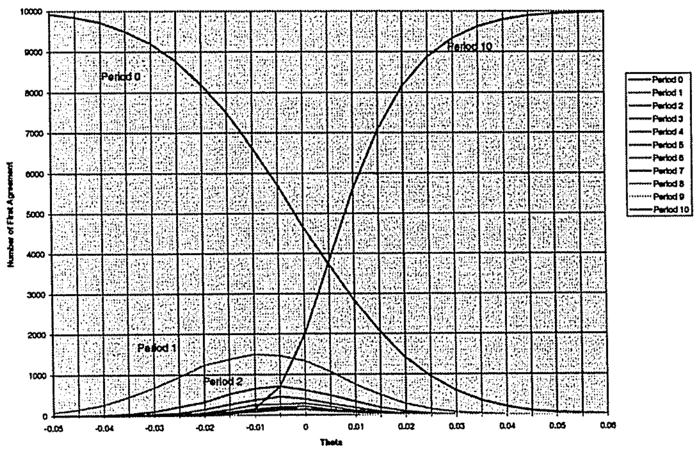


Figure 3.36: Wiener Process Model (Basic Game) [$\theta = -0.05 \rightarrow 0.06$]

($\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

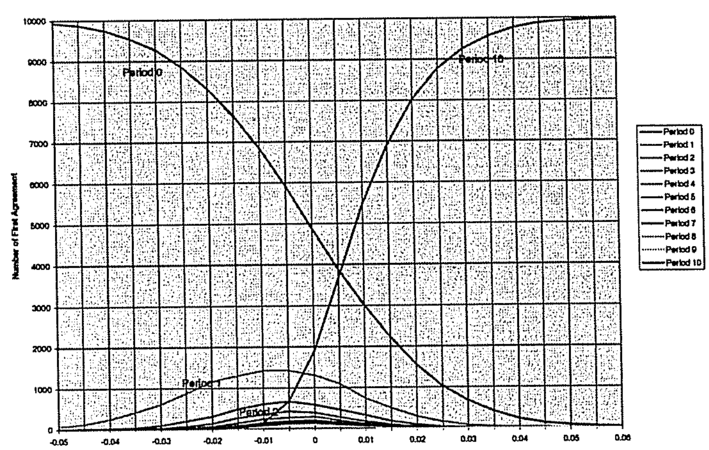


Figure 3.37: Wiener Process Model (Alternative Game) [$\theta = -0.05 \rightarrow 0.06$]

($\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

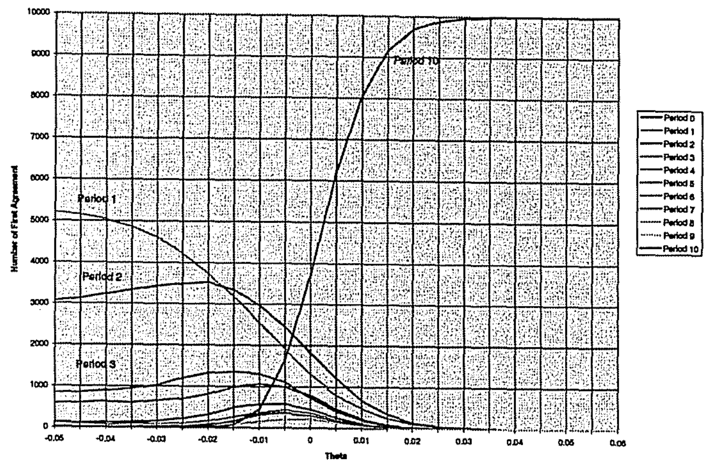


Figure 3.38: Wiener Process Model (Alternative Game) [$\theta = -0.05 \rightarrow 0.06$]

($\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

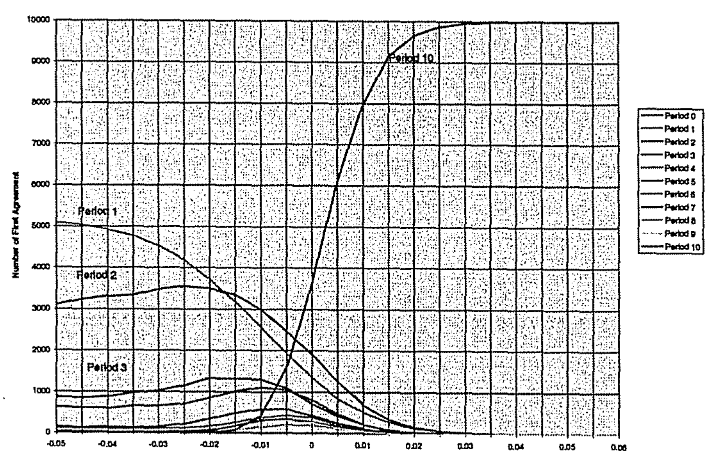


Figure 3.39: Wiener Process Model (Basic Game) [$\theta_A = 0.5 \rightarrow 6.0$]

($\theta_B = 0.5,\ \varphi_A = 1.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

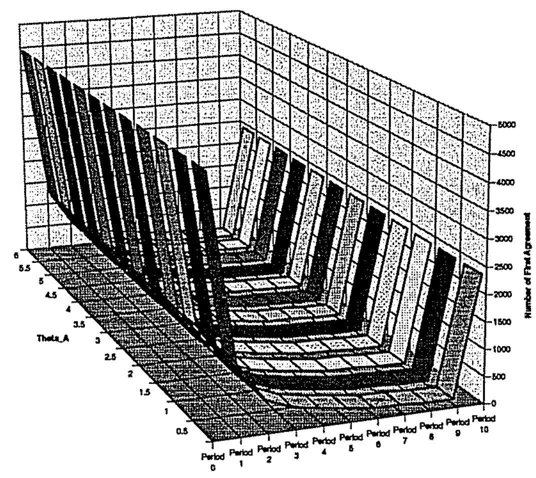


Figure 3.40: Wiener Process Model (Basic Game) [$\theta_A = 0.5 \rightarrow 6.0$]

($\theta_B = 6.0,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

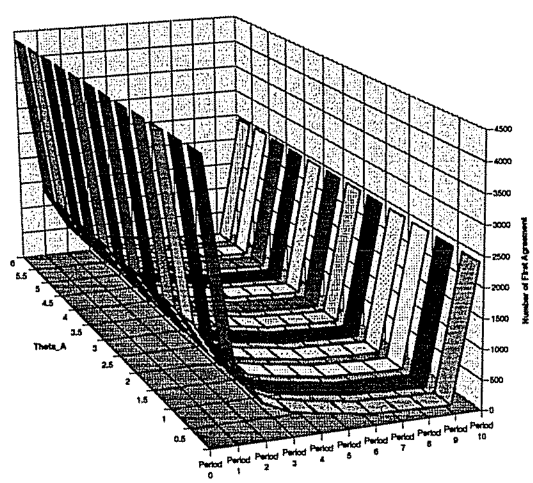


Figure 3.41: Wiener Process Model (Alternative Game) [$\theta_A = 0.5 \rightarrow 6.0$]

($\theta_B = 0.5,\ \varphi_A = 1.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

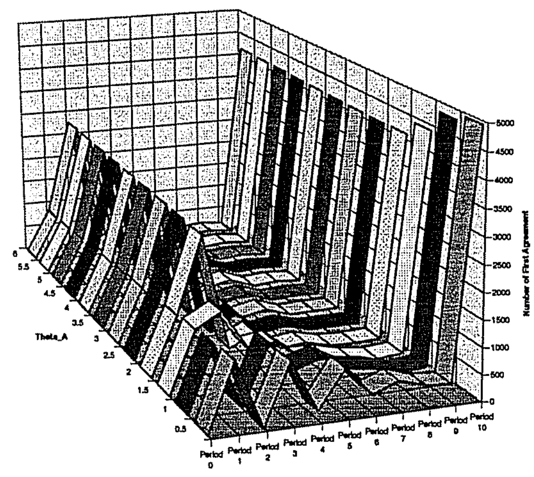


Figure 3.42: Wiener Process Model (Alternative Game) [$\theta_A = 0.5 \rightarrow 6.0$]

($\theta_B = 6.0,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2$)

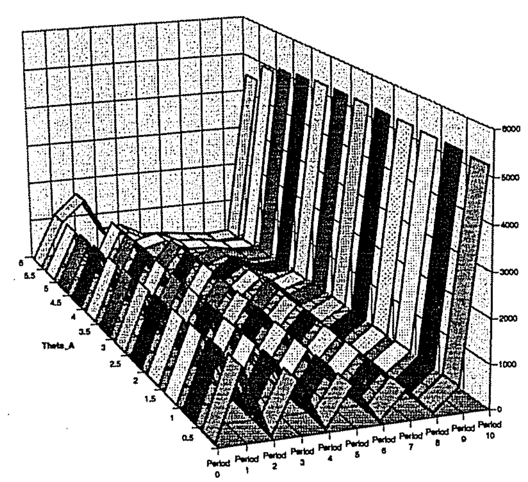


Figure 3.43: Wiener Process Model (Basic Game) $[T = 2 \rightarrow 12]$

$(\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2)$

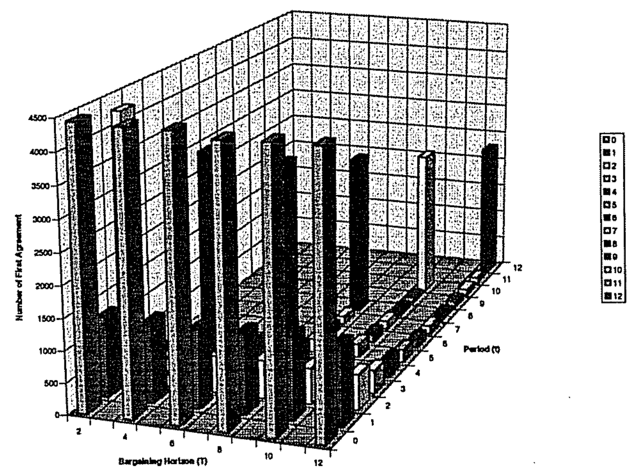


Figure 3.44: Wiener Process Model (Basic Game) $[T = 2 \rightarrow 12]$

$(\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2)$

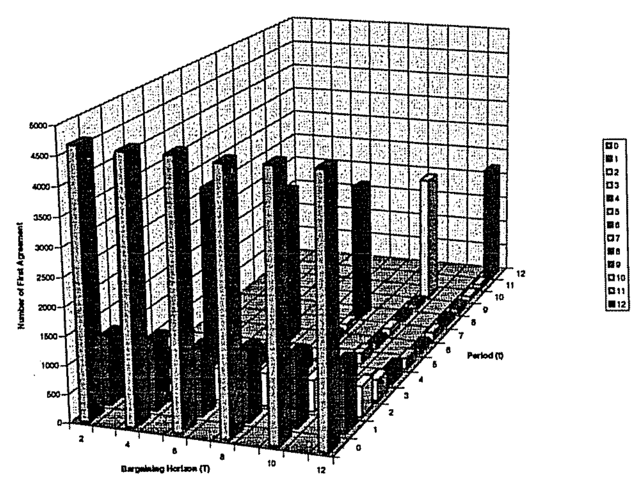


Figure 3.45: Wiener Process Model (Alternative Game) $[T = 2 \rightarrow 12]$

$(\theta_A = 5.5,\ \theta_B = 5.5,\ \varphi_A = 1.5,\ \varphi_B = 1.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2)$

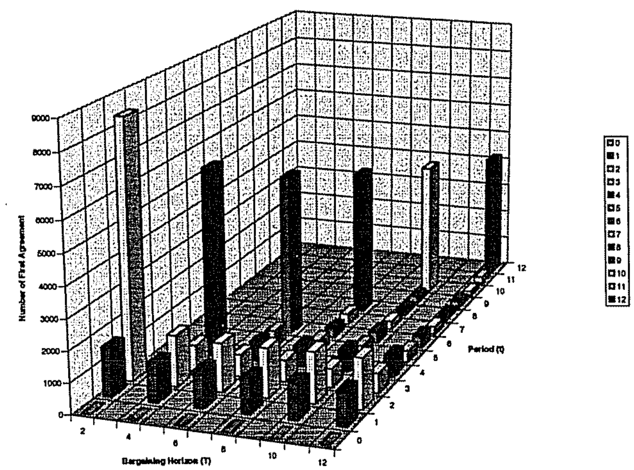
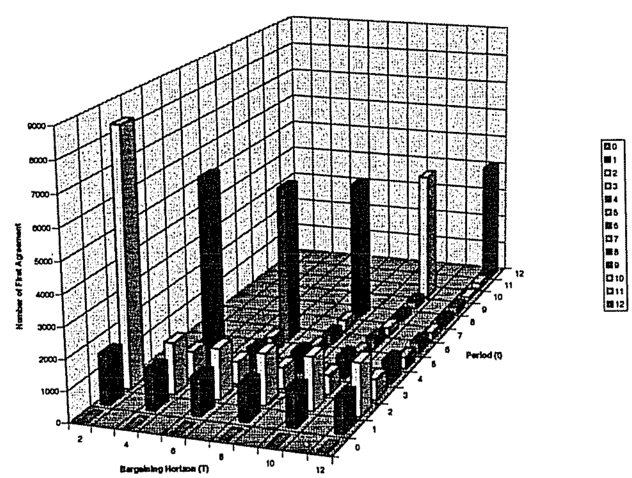


Figure 3.46: Wiener Process Model (Alternative Game) $[T = 2 \rightarrow 12]$

$(\theta_A = 0.5,\ \theta_B = 0.5,\ \varphi_A = 0.5,\ \varphi_B = 0.5,\ \theta = 0.0014,\ \varphi = 0.0004,\ \sigma = 0.06,\ Q_0 = 1.2)$



Appendix 3C

Simulation Codes

3C.1 Autoregressive Binomial Models

```
ABBM1.c        Fri Apr  7 03:16:14 1995        1

#include <stdio.h>
#include <malloc.h>
#include <math.h>
#include <stdlib.h>


/*       ABBM1.c : Autoregressive Binomial Bargaining Model One          */

/* ================================================================== */
/* discription of the program "ABBM1.c" ============================= */
/* ================================================================== */

/*    ABBM1.c simulates a special case of bilateral bargaining game     */
/* that is studied in Chapter 2 and Chapter 3 of my Ph.D. dissertation */
/* at California Institute of Technology.  The detailed behavioral     */
/* assumptions of the game participants are given in the chapters, and */
/* we do not repeat them here.  The particular game that is simulated  */
/* is the one called "Basic Game" of "Autoregressive Binomial          */
/* Bargaining Model" in the dissertation.  Its "Alternative Game" is   */
/* simulated in another version of this program, which can be obtained */
/* from the author.  Hence, this version contains a very basic part of */
/* the simulation process, upon which we have made numerous            */
/* modifications and extensions to create different versions of the   */
/* program and have generated various types of data sets.              */

/*                          DATA STRUCTURES                             */
/*    The bargaining process is described by a full binary tree with   */
/* its depth equaling to an exogenously predetermined bargaining       */
/* horizon minus one period.  The tree is stored in a linked structure */
/* "node," which is defined below.  Each level in the tree describes   */
/* each trading period, with the first level corresponding to Period   */
/* 0.  A left child of each node is associated with a state in which a */
/* currently observed value of epsilon is zero, whereas a right child  */
/* is associated with epsilon one.  Each node in the tree is given an  */
/* unique id number, with its root node corresponding to one.  The id  */
/* number is incremented by one from the left node to the right node   */
/* in each level.  This numbering scheme enables us to reach any node  */
/* in the tree, given the address of the root node and the id number,  */
/* without using a recurrsive coding.  In addition, each node can also */
/* be identified by an uniquely associated "path."  A "path" is a      */
/* sequence of observed epsilons upto the current period, and the      */
/* sequence is stored in a linked list structure "binary," which is    */
/* defined below.  We have accomodated each node in both "node" and    */
/* "binary" with a pointer that enables us to move in the list in any  */
/* direction we wish, hoping to have some flexibility in future        */
/* extensions.                                                          */

/*                              DESIGN                                  */
/*    After reading primary input values, we first create a root node  */
/* of the bargaining tree and store the initial information by calling */
/* a subroutine plant_first_node() from main().  Then we add one node  */
/* at a time by calling plant_tree() to create a full tree.  We call   */
/* point_to_parent() to add a pointer to each node that points to its  */
/* parent node.  calc_node() computes variable values to be stored in  */
/* each node that are necessary for the computation of players'        */
/* bargaining strategies.  calc_strategy() computes each player's      */
/* offer strategies and response strategies at each node.  At this     */
/* point, we have a full binary tree with each node containing         */
/* information that describe the bargaining process.  Consequently,    */
/* optional subroutines can be accomodated to the program afterwards   */
/* to generate various output.                                          */
```

```
/*                     FUNDAMENTAL NOTATIONS                          */
/*    Notations are kept consistent with the ones used in the         */
/* dissertation whenever it is appropriate and possible.  For example, */
/* alpah_A in the dissertation is denoted as a_A in this program.     */
/* Likewise, alpha_B = a_B, beta_A = b_A, and beta_B = b_B.  Other    */
/* Greek letters are replaced with English writings, such as rho and  */
/* delta.                                                             */

/*                     PRIMARY INPUT VARIABLES                        */
/*    In this version, primary input variables are read interactively */
/* by calling a subroutine get_par() in main().                       */
/* Such input variables are :                                         */
/*      T : unsigned int (positive integer)                           */
/*          Predetermined number of maximum bargaining periods        */
/*      a_A : float (positive constant)                               */
/*            alpha_A, a parameter for Beta distribution              */
/*            that characterizes player A's prior                     */
/*      a_B : float (positive constant)                               */
/*            alpha_B, a parameter for Beta distribution              */
/*            that characterizes player B's prior                     */
/*      b_A : float (positive constant)                               */
/*            beta_A, a parameter for Beta distribution               */
/*            that characterizes player A's prior                     */
/*      b_B : float (positive constant)                               */
/*            beta_B, a parameter for Beta distribution               */
/*            that characterizes player B's prior                     */
/*      X_T : float (0 <= X_T <= 1)                                   */
/*            Predetermined player A's default share in Period T      */
/*      rho : float (|rho| < 1)                                       */
/*            Describes the autocorrelation of delta                  */
/*      delta0 : float                                                */
/*               Initial information that is available in Period 0    */
/*      Q0 : float (positive real)                                    */
/*           Initial value of the asset in Period 0                   */

/*                        BASIC OUTPUT                                */
/*    We have included in this version, as an example of output, a    */
/* subroutine trade_freq().  trade_freq() computes unconditional and  */
/* conditional frequencies of agreement in each trading period, and   */
/* writes the results in formatted output files, ta.dat and tb.dat.   */
/* ta.dat contains information such as T, X_T, rho, and t (current    */
/* period) along with computed unconditional and conditional          */
/* frequencies.  tb.dat contains information such a_A, a_B, b_A, b_B, */
/* and t along with the computed frequencies.  Unconditional          */
/* frequencies are computed by dividing (the number of nodes in which */
/* a response strategy indicates an acceptance) by (the total number  */
/* of nodes in the level (trading period)).  Conditional frequencies  */
/* are computed by dividing (the number of reachable nodes in which a */
/* response strategy indicates an acceptance) by (the total number of */
/* reachable nodes in the level).  A node is 'reachable' if there has */
/* been no trade upto the current period.  If, for example, there has */
/* been an agreement in period 0, then none of the nodes in the tree  */
/* other than the root node is reachable.  Hence, the conditional     */
/* frequencies cannot be computed for the rest of the nodes, and in   */
/* such a case we give 0 frequency as its output.                     */

/*                      RELATED VERSIONS                              */
/*    There are other versions of ABBM1.c that contain modules to     */
/* simulate "Alternative Game" and to generate other output files.    */

/*      NOTE:  main() is defined at the very end of the program.      */
```

```
/* ================================================================ */
/* Defining messages =============================================== */
/* ================================================================ */

#define DISPLAY1 "BARGAINING SIMULATION PROGRAM : ABBM1.c \n"
#define ENDMESSAGE1 "END OF BARGAINING SIMULATION PROGRAM : ABBM1.c \n"

/* ================================================================ */
/* Defining macros ================================================= */
/* ================================================================ */

#define space2 printf("\n\n")
#define space3 printf("\n\n\n")
#define LINE1 printf("********************** \n")

#define out2(fp, a1, a2) fprintf(fp, "%u %u \n", a1, a2)
#define out3(fp, a1, a2, a3) fprintf(fp, "%f %f %u \n", a1, a2, a3)

#define sqr(x) (x*x)  /* sqr gives a squared value of the argument */
#define max(A, B) ((A) > (B) ? (A) : (B))
                       /* max gives a larger number of the given two numbers */

/* ================================================================ */
/* Defining data structure (declaration of global variables) ========= */
/* ================================================================ */

/* "node" specifies each node in a full binary tree that describes     */
/* the stochastic bargaining process.  Each node contains information  */
/* necessary to compute players' strategies as it appears below.       */

   struct node
   {
      long unsigned int id; /* id of each node: 1, 2, ..., 2^(T+1)-1   */
      unsigned int t;              /* period number t = 0, 1, 2, ..., T */
      float delta;                                   /* ex-post delta */
      float PA0;               /* A's prior of e = 0 in the next period */
      float PA1;               /* A's prior of e = 1 in the next period */
      float PB0;               /* B's prior of e = 0 in the next period */
      float PB1;               /* B's prior of e = 1 in the next period */
      int sum_e;                  /* sum of e's upto the current period */
      float X;                 /* player i's offer in the current period */
      unsigned int R;     /* player j's respose: 1 = accept, 0 = reject */
      float Q;                     /* asset value in the current period */
      int flag;          /* 0 if no trade before t, 1 if trade before 1 */
      struct node *left;        /* left child, i.e., a node with e = 0 */
      struct node *right;      /* right child, i.e., a node with e = 1 */
      struct node *parent;                 /* pointer to its parent node */
   };

/* ================================================================ */

/* "binary" stores a value of e, 0 or 1, in each period, the sequence  */
/* of which is stored in a linked list and is used to identify a path  */
/* to each node in the bargaining tree.                                */

   struct binary
   {
      int num;
      struct binary *next;
      struct binary *reverse;
   };

/* ================================================================ */
```

```
   FILE *fa;                                   /* Formatted output file */
   FILE *fb;                                   /* Formatted output file */

/* ================================================================== */
/* ================================================================== */
/* =============== INPUT/MISCELLANEOUS SUBROUTINES =================== */
/* ================================================================== */
/* ================================================================== */

/* ================================================================== */
/* display ========================================================== */
/* ================================================================== */
/* displays the current program name.                                 */

void display()

{
   space3;
   printf(DISPLAY1);
   space3;

   return;

}  /* End of display() */

/* ================================================================== */
/* open_files ======================================================= */
/* ================================================================== */
/* opens files to write the output.                                   */

void open_files()

{

   fa = fopen("ta.dat", "w");
   fb = fopen("tb.dat", "w");

   return;

}  /* End of open_files() */

/* ================================================================== */
/* close_files ====================================================== */
/* ================================================================== */
/* closes the output files that have been opened in open_files().     */

void close_files()

{

   fclose(fa);
   fclose(fb);

   return;

}  /* End of close_files() */

/* ================================================================== */
/* end_message ====================================================== */
/* ================================================================== */
/* prints the ending message on the monitor.                          */

void end_message()
```

```
{
   space3;
   printf(ENDMESSAGE1);
   space3;

   return;

}  /* End of end_message() */

/* ================================================================= */
/* power =========================================================== */
/* ================================================================= */
/* raise base to n-th power ; n >= 0 */

long int power(base, n)

   long int base, n;

{
   long int i, p;

   p = 1;
   for(i=1; i<=n; ++i)
      p = p * base;
   return p;

} /* End of power() */

/* ================================================================= */
/* get_par ========================================================= */
/* ================================================================= */
/* get_par() is called in main().                                    */
/* get_par() function reads input parameter values interactively.    */

void get_par(T, a_A, a_B, b_A, b_B, X_T, rho, delta0, Q0)

   unsigned int *T;
   float *a_A, *a_B, *b_A, *b_B, *X_T, *rho, *delta0, *Q0;

{

   printf("Enter T (even integer) : ");
   scanf("%d", T);
   printf("Enter a_A (positive real) : ");
   scanf("%f", a_A);
   printf("Enter a_B (positive real) : ");
   scanf("%f", a_B);
   printf("Enter b_A (positive real) : ");
   scanf("%f", b_A);
   printf("Enter b_B (positive real) : ");
   scanf("%f", b_B);
   printf("Enter X_T (positive real) : ");
   scanf("%f", X_T);
   printf("Enter rho (|real| < 1) : ");
   scanf("%f", rho);
   printf("Enter delta0 (real) : ");
   scanf("%f", delta0);
   printf("Enter Q0 (positive real) : ");
   scanf("%f", Q0);

   space3;

   return;
```

```
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}  /* End of get_par() */

/* ================================================================ */
/* ================================================================ */
/* =================== CONSTRUCTING BINARY TREE ===================== */
/* ================================================================ */
/* ================================================================ */


/* ================================================================ */
/* plant_first_node ================================================ */
/* ================================================================ */
/* creates the origin node of the binary tree.                       */

struct node *plant_first_node(p, id, a_A, a_B, b_A, b_B, X_T, rho, delta0, Q0)

   struct node *p;
   long unsigned int id;
   float a_A, a_B, b_A, b_B, X_T, rho, delta0, Q0;

{

   p = (struct node *)calloc(1, sizeof(struct node));
   p->left = NULL;
   p->right = NULL;
   p->id = id;
   p->delta = delta0;
   p->PA1 = a_A / (a_A + b_A);
   p->PA0 = 1 - p->PA1;
   p->PB1 = a_B / (a_B + b_B);
   p->PB0 = 1 - p->PB1;
   p->sum_e = 0;
   p->X = 0.0;
   p->Q = Q0;
   return p;

} /* End of plant_first_node() */

/* ================================================================ */
/* plant_tree ====================================================== */
/* ================================================================ */
/* plant_tree() constructs a full binary tree by adding one node each */
/* time.  Each node added is assigned an unique id number that enables */
/* us to reach any node we wish, given the address of the origin and  */
/* a "path" that is defined later.  The id numbers are used throughout */
/* this program.                                                       */

struct node *plant_tree(origin, id, path)

   struct node *origin;
   long unsigned int id;
   struct binary *path;

{
   struct node *p, *temp;

   p = origin;
   while (path != NULL)
   {
      if (path->num == 0)
      {
         if (p->left != NULL)
            p = p->left;
```

```
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         else
         {
            temp = (struct node *)calloc(1, sizeof(struct node));
            temp->left = NULL;
            temp->right = NULL;
            temp->id = id;
            p->left = temp;
         }
      }
      else
      {
         if (p->right != NULL)
            p = p->right;
         else
         {
            temp = (struct node *)calloc(1, sizeof(struct node));
            temp->left = NULL;
            temp->right = NULL;
            temp->id = id;
            p->right = temp;
         }
      }
      path = path->reverse;
   } /* End of while */

   return origin;

} /* End of plant_tree() */

/* ================================================================= */
/* find_path ======================================================= */
/* ================================================================= */
/* Given an id number of a node, find_path() finds a unique sequence   */
/* of observed epsilons upto the current period.  Such a sequence is   */
/* stored in a structure "binary."                                     */

struct binary *find_path(Z)

   long unsigned int Z;

{
   struct binary *first, *last, *now, *past, *list[30], *rt;
   long int counter;

   counter = 1;
   while (Z >= 1)
   {
     now = (struct binary *)calloc(1, sizeof(struct binary));
     list[counter]=now;
     now->num = (Z%2);
     Z =(int)(Z/2);
     now->next = NULL;
     first = now;
     if (counter == 1)
     {
        last = now;
        last->reverse = NULL;
     }
     else
     {
        past->next = now;
        now->reverse = past;
     }
     past = now;
```

```
    counter = counter + 1;
  }

  rt = first->reverse;
  counter --;
  while(counter > 0)
  {
    free(list[counter]);
    counter--;
  }

  return rt;

} /* End of find_path() */

/* ================================================================ */
/* find_period ==================================================== */
/* ================================================================ */
/* find_period() is called in calc_node() with an id number.        */
/* Given an id number of a node in the tree, find_period() returns  */
/* a period number to which the node with the given id number belongs. */

unsigned int find_period(id)

   long unsigned int id;

{
   long unsigned int counter;
   struct binary *path;

   counter = 0;
   path = find_path(id);
   while (path != NULL)
   {
      counter = counter + 1;
      path = path->reverse;
   }
   return counter;

} /* End of find_period() */

/* ================================================================ */
/* goto_node ====================================================== */
/* ================================================================ */
/* Provided the address of the root node, an id number and a path   */
/* associated with a particular node in concern, goto_node() finds the */
/* address of the node.                                             */

struct node *goto_node(root, id, path)

   struct node *root;
   long unsigned int id;
   struct binary *path;

{
   struct node *p;

   p = root;
   while (path != NULL)
   {
      if (path->num == 0)
         p = p->left;
      else
         p = p->right;
```

```
      path = path->reverse;
   }
   return p;

} /* End of goto_node() */

/* ================================================================ */
/* point_to_parent ================================================ */
/* ================================================================ */
/* accomodate each node with a pointer that points back to its parent  */
/* node.                                                             */

void point_to_parent(root, T)

   struct node *root;
   unsigned int T;

{
   struct node *current_node;
   struct binary *path;
   long unsigned int i;

   for (i=1; i <= (power(2,T) - 1); i++)
   {
      path = find_path(i);
      current_node = goto_node(root, i, path);
      current_node->left->parent = current_node;
      current_node->right->parent = current_node;
   }
   return;

} /* End of point_to_parent() */

/* ================================================================ */
/* ================================================================ */
/* =================== BARGAINING STRATEGIES ====================== */
/* ================================================================ */
/* ================================================================ */

/* ================================================================ */
/* calc_node ====================================================== */
/* ================================================================ */
/* computes basic information to be stored in each node, given primary */
/* input values.                                                     */

void calc_node(root, T, rho, a_A, a_B, b_A, b_B)

   struct node *root;
   unsigned int T;
   float rho, a_A, a_B, b_A, b_B;

{
   struct node *p;
   struct binary *path;
   long unsigned int i;

   p = root;
   for (i=2; i <= (power(2,T+1) - 1); i++)
   {
      path = find_path(i);
      p = goto_node(root, i, path);
      p->sum_e = compute_sum_e(i);
      p->t = find_period(i);
      p->delta = rho * p->parent->delta + e_past(i);
```

```
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      p->PA1 = (a_A + p->sum_e) / (a_A + b_A + p->t);
      p->PA0 = 1 - p->PA1;
      p->PB1 = (a_B + p->sum_e) / (a_B + b_B + p->t);
      p->PB0 = 1 - p->PB1;
      p->Q = p->parent->Q * p->delta;
   }
   return;

} /* End of calc_node() */

/* ================================================================= */
/* compute_sum_e =================================================== */
/* ================================================================= */
/* sums a sequence of epsilons upto the current node, given its id     */
/* number.                                                             */

int compute_sum_e(Z)

   long unsigned int Z;

{
   struct binary *moving;
   int e;

   moving = find_path(Z);

   if (moving != NULL)
   {
     e = moving->num;
     while (moving->reverse != NULL)
     {
       moving = moving->reverse;
       e = e + moving->num;
     }
   }
   else
     e = 0;

   return e;

} /* End of compute_sum_e() */

/* ================================================================= */
/* e_past ========================================================== */
/* ================================================================= */
/* Given an id number, returns the latest observed value of e. */

int e_past(Z)

   long unsigned int Z;

{
   struct binary *moving;
   int i;

   moving = find_path(Z);

   if (moving == NULL)
     i = 0;
   else
   {
     while (moving->reverse != NULL)
       moving = moving->reverse;
     i = moving->num;
```

```
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   }

   return i;

} /* End of e_past() */

/* ================================================================ */
/* calc_strategy ================================================== */
/* ================================================================ */
/* computes players' offer and response strategies at each node.      */

void calc_strategy(root, T, X_T)

   struct node *root;
   unsigned int T;
   float X_T;

{
   struct binary *path;
   float arg1, arg2;
   struct node *p; /* points to a current node */
   long unsigned int lastnode_id; /* the largest id number in the tree */
   long unsigned int i; /* id counter */

   lastnode_id = power(2, T+1) -1;
   for (i = lastnode_id; i >= 1; i--)
   {
      path = find_path(i);
      p = goto_node(root, i, path);

      if (p->t == T)
      {
       p->X = X_T;
       p->R = 1;
      }
      else if (p->t == (T-1))
      {
       arg1 = 1 - (X_T * (p->PA0 * p->left->delta + p->PA1 * p->right->delta));
       arg2 = (1 - X_T) * (p->PB0 * p->left->delta + p->PB1 * p->right->delta);
       p->X = max(arg1, arg2);
       if (p->X <= arg1)
          p->R = 1; /* A accepts B's offer in T-1. */
       else
          p->R = 0; /* A rejects B's offer in T-1. */
      }
      else if (p->t == (T-2))
      {
       arg1 = 1 - (p->PB0 * p->left->delta * p->left->X
              + p->PB1 * p->right->delta * p->right->X);
       arg2 = X_T * (p->PA0 * p->left->PA0 * p->left->delta
              * p->left->left->delta + p->PA0 * p->left->PA1
              * p->left->delta * p->left->right->delta
              + p->PA1 * p->right->PA0 * p->right->delta
              * p->right->left->delta + p->PA1 * p->right->PA1
              * p->right->delta * p->right->right->delta);
       p->X = max(arg1, arg2);
       if (p->X <= arg1)
          p->R = 1; /* B accepts A's offer in T-2. */
       else
          p->R = 0; /* B rejects A's offer in T-2. */
      }
      else if ((p->t % 2) == 0) /* strategy for even numbered periods */
      {
       arg1 = 1 - (p->PB0 * p->left->delta * p->left->X
```

```
            + p->PB1 * p->right->delta * p->right->X);
    arg2 = p->PA0 * p->left->PA0 * p->left->delta * p->left->left->delta
           * p->left->left->X + p->PA0 * p->left->PA1 * p->left->delta
           * p->left->right->delta * p->left->right->X
           + p->PA1 * p->right->PA0 * p->right->delta
           * p->right->left->delta * p->right->left->X
           + p->PA1 * p->right->PA1 * p->right->delta
           * p->right->right->delta * p->right->right->X;
    p->X = max(arg1, arg2);
    if (p->X <= arg1)
       p->R = 1; /* B accepts A's offer. */
    else
       p->R = 0; /* B rejects A's offer. */
   }
   else if ((p->t % 2) == 1) /* strategy for odd numbered periods */
   {
    arg1 = 1 - (p->PA0 * p->left->delta * p->left->X
           + p->PA1 * p->right->delta * p->right->X);
    arg2 = p->PB0 * p->left->PB0 * p->left->delta * p->left->left->delta
           * p->left->left->X + p->PB0 * p->left->PB1 * p->left->delta
           * p->left->right->delta * p->left->right->X
           + p->PB1 * p->right->PB0 * p->right->delta
           * p->right->left->delta * p->right->left->X
           + p->PB1 * p->right->PB1 * p->right->delta
           * p->right->right->delta * p->right->right->X;
    p->X = max(arg1, arg2);
    if (p->X <= arg1)
       p->R = 1; /* A accepts B's offer. */
    else
       p->R = 0; /* A rejects B's offer. */
   }

  } /* End of for (i = lastnode_id; i >= 1; i--) */

  return;

 } /* End of calc_strategy() */

/* ================================================================= */
/* trade_flag ====================================================== */
/* ================================================================= */
/* If a node in concern is reachable, i.e., there has been no trade   */
/* in a path upto the node, then flag receives 0.  If a node is not   */
/* reachable, flag receives 1.                                         */

void trade_flag(root, T)

   struct node *root;
   unsigned int T;

{
   struct binary *path;
   struct node *temp, *p;
   long unsigned int i;
   int addR; /* sum of R upto the previous period */

   for (i=1; i<=(power(2, T+1) -1); i++)
   {
     path = find_path(i);
     p = goto_node(root, i, path);
     if (p->parent == NULL)
       p->flag = 0;
     else
     {
```

```
      temp = p;
      addR = 0;
      while (temp->parent != NULL)
      {
        addR = addR + temp->parent->R;
        temp = temp->parent;
      }
      if (addR == 0)
        p->flag = 0;
      else
        p->flag = 1;
    } /* End of else */
  } /* End of for (i=1; i<=(power(2, T+1) -1); i++) */

  return;

} /* End of trade_flag() */

/* ================================================================= */
/* ================================================================= */
/* ====================== OUTPUT SUBROUTINES ======================== */
/* ================================================================= */
/* ================================================================= */

/* ================================================================= */
/* trade_freq ====================================================== */
/* ================================================================= */
/* computes and outputs unconditional and conditioanl frequencies of   */
/* agreement in each trading period.  Output files, ta.dat and tb.dat  */
/* are created.                                                        */

void trade_freq(root, T, X_T, rho, a_A, a_B, b_A, b_B)

   struct node *root;
   unsigned int T;
   float X_T, rho, a_A, a_B, b_A, b_B;

{
   unsigned int i;
   unsigned int f1; /* number of agreement in t (unconditional) */
   unsigned int f2; /* number of agreement in t (if no trade upto t-1) */
   long unsigned int j;
   long unsigned int f1total; /* total number of nodes in concern for freq */
   long unsigned int f2total; /* total number of nodes in concern for confreq */
   float freq, confreq;
   struct binary *path;
   struct node *p;

   i = 0; /* period counter */
   j = 0; /* id counter */
   for (i=0; i <= (T-1); i++)
   {
      f1 = 0;
      f2 = 0;
      f1total = power(2, i);
      f2total = 0;
      for (j = power(2,i); j <= (power(2, i+1) - 1); j++)
      {
         path = find_path(j);
         p = goto_node(root, j, path);
         if (p->R == 1)
           f1 = f1 + 1;

         if (p->flag == 0)
```

```
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          {
            f2total = f2total + 1;
            if (p->R == 1)
              f2 = f2 + 1;
          }
        }
        freq = (f1 * 1.0) / (f1total * 1.0);
        confreq = (f2 * 1.0) / (f2total * 1.0);
        fprintf(fa, "%d %f %f %d %f %f \n", T, X_T, rho, i, freq, confreq);
        fprintf(fb, "%f %f %f %f %d %f %f \n", a_A, a_B, b_A, b_B, i, freq, confreq);
     }

     return;

} /* End of trade_freq() */

/* ================================================================ */
/* display_tree ==================================================== */
/* ================================================================ */
/* displays basic information stored in each node of the binary tree.  */

void display_tree(root, T)

   struct node *root;
   unsigned int T;

{
   struct binary *path;
   struct node *temp;
   long unsigned int i;

   for (i=1; i <= (power(2, T+1) - 1); i++)
   {
      path = find_path(i);
      temp = goto_node(root, i, path);
      printf("id = %d, period = %d \n", temp->id, temp->t);
      printf("sum_e = %d, delta = %f \n", temp->sum_e, temp->delta);
      printf("PA0 = %f, PA1 = %f \n", temp->PA0, temp->PA1);
      printf("PB0 = %f, PB1 = %f \n", temp->PB0, temp->PB1);
      printf("Q = %f \n", temp->Q);
      printf("X = %f \n", temp->X);
      printf("R = %d \n", temp->R);
      LINE1;
   }
   return;

} /* End of display_tree() */

/* ================================================================ */
/* ================================================================ */
/* ================================================================ */
/* =========================== MAIN =============================== */
/* ================================================================ */
/* ================================================================ */
/* ================================================================ */

main()
{

/* ================================================================ */

int T;                      /* the maximum number of periods in the game */
long unsigned int i;                                       /* id counter */
float a_A, a_B, b_A, b_B;             /* a_A = alpha_A in the paper */
```

```
                                /* Likewise, b_A = beta_A, and so on. */
float X_T;      /* player A's predetermined share in the final period T */
float rho;                                                /* |rho| < 1 */
float delta0;                                  /* delta_0 in period 0 */
float Q0;                                          /* Q_0 in period 0 */
struct node *root;            /* root node (origin) of the binary tree */
struct binary *path;
    /* path from the root node to a given node in the sequence of e's */

/* =============================================================== */

   display();

   open_files();
   get_par(&T, &a_A, &a_B, &b_A, &b_B, &X_T, &rho, &delta0, &Q0);

   root = NULL;
   root = plant_first_node(root, 1, a_A, a_B, b_A, b_B, X_T, rho, delta0, Q0);
   for (i=2; i <= (power(2, T+1) - 1); i++)
   {
      path = find_path(i);
      root = plant_tree(root, i, path);
   }
   point_to_parent(root, T);
   calc_node(root, T, rho, a_A, a_B, b_A, b_B);
   calc_strategy(root, T, X_T);
   trade_flag(root, T);

   trade_freq(root, T, X_T, rho, a_A, a_B, b_A, b_B);
   display_tree(root, T);
   close_files();

   end_message();

}  /* End of main() */

/* ====================== End of "ABBM1.c" ======================== */
/* =============================================================== */
/* =============================================================== */
/* =============================================================== */
/* =============================================================== */
/* =============================================================== */
```

3C.2 Generalized Wiener Process Models

```
WPBM2.c        Sun Apr  9 01:25:25 1995        1

#include <stdio.h>
#include <malloc.h>
#include <math.h>
#include <stdlib.h>


/*            WPBM2.c : Wiener Process Bargaining Model Two             */

/* ================================================================== */
/* discription of the program "WPBM2.c" ============================= */
/* ================================================================== */

/*    WPBM2.c simulates a special case of bilateral bargaining game     */
/* that is studied in Chapter 2 and Chapter 3 of my Ph.D. dissertation */
/* at California Institute of Technology.  The detailed behavioral     */
/* assumptions of the game participants are given in the chapters, and */
/* we do not repeat them here.  The particular game that is simulated  */
/* is the one called "Alternative Game" of "Generalized Wiener Process */
/* (or Brownian Motion) Model" in the dissertation.  "Basic Game" of   */
/* this model is simulated in another version of this program, which   */
/* can be obtained from the author.  Hence, this version contains a    */
/* very basic part of the simulation process of the Wiener Process     */
/* model.                                                              */

/*                  DATA STRUCTURES AND DESIGN                         */
/*    The bargaining process is stored in a linked list, in which two  */
/* types of data structures coexist.  One structure defined by "node"  */
/* below is frequently referred to as 'a base list' in this program.   */
/* This list begins with its origin corresponding to period 0, and     */
/* each node carries the information such as players' offer and        */
/* response strategies in each period.  The other structure defined by */
/* "snode" is frequently referred to as 'a strategy list,' that        */
/* contains the players' updated beliefs over future values of the     */
/* asset given the information upto the current period.  The           */
/* information stored in this list is used to compute what strategy to */
/* take in the current period.  A pointer 'strat' points to 'snode' in */
/* this strat list of each period from 'node' of the corresponding     */
/* period, while a pointer 'base' points from the strategy list to its */
/* corresponding 'node' in the base list.  In addition, each node is   */
/* accomodated with a pointer to point back to its parent node, which  */
/* adds the program flexibility.                                       */
/*    In this version, each simulation run consists with 10,000 sample */
/* runs.  In other words, given initial values, a sequence of random   */
/* numbers associated with each idum number is generated for 10,000    */
/* times.  idum takes a negative integer.  We give an initial value    */
/* of idum = -1 in main() as an example, that is decremented by 1      */
/* down to -10,000.                                                    */

/*                   FUNDAMENTAL NOTATIONS                             */
/*    Notations are kept consistent with the ones used in the          */
/* dissertation whenever it is appropriate and possible.  For example, */
/* a Greek letter 'theta' with a subscript 'A' in the dissertation is  */
/* written thetaA in this program.  Likewise, PhiA, PhiB, sigma, and   */
/* so on.                                                              */

/*                  PRIMARY INPUT VARIABLES                            */
/*    In this version, the initail values of variables are read        */
/* interactively by calling a subroutine get_par() in main().          */
/* We give initial values to the following variables.                  */
/*     T : unsigned int (positive integer)                             */
/*         Predetermined number of maximum bargaining periods          */
/*     thetaA : float                                                  */
/*            Player A's prior belief on the mean of theta             */
```

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/*      thetaB : float                                                 */
/*              Player B's prior belief on the mean of theta           */
/*      PhiA : float                                                   */
/*            Player A's prior belief on the variance of theta         */
/*      PhiB : float                                                   */
/*            Player B's prior belief on the variance of theta         */
/*      theta : float                                                  */
/*             Mean of a normal distribution that describes mu, the    */
/*             expected rate of value increase                         */
/*      Phi : float                                                    */
/*           Variance of a normal distribution that describes mu       */
/*      sigma : float                                                  */
/*             Predetermined volatility of the asset value             */
/*      delta_t : float                                                */
/*               Predetermined time interval of information arrival    */
/*      initialQ : float (positive real)                               */
/*                Initial value of the asset                           */
/*      X_T : float (0 <= X_T <= 1)                                    */
/*           Predetermined player A's default share in Period T        */

/*                          BASIC OUTPUT                               */
/*    We have included in this version, as an example of output, a     */
/* subroutine out_freq() and out_exp_delta().  out_freq() writes       */
/* frequencies of first agreement in each period, that are computed in */
/* calc_freq().  'Frequency' here is a number of agreement out of      */
/* 10,000 simulation runs, given that there has been no trade upto a   */
/* period in concern.
/*    out_exp_delta() writes in an output file the expected values of  */
/* delta that are stored in a strategy list.                           */

/*                        RELATED VERSIONS                             */
/*    There are other versions of WPBM2.c that contains modules to     */
/* simulate "Basic Game" and to generate other output files.           */


/*      NOTE:  main() is defined at the very end of the program.       */

/* ================================================================== */
/* Defining DISPLAY messages ======================================== */
/* ================================================================== */

#define DISPLAY1 "BARGAINING SIMULATION PROGRAM : WPBM2.c \n"
#define ENDMESSAGE1 "END OF BARGAINING SIMULATION PROGRAM : WPBM2.c \n"

/* ================================================================== */
/* Defining constants =============================================== */
/* ================================================================== */

#define MINIDUM -10000 /* Minimum number idum takes */
#define ITR 0.0005 /* iteration interval for the value of theta */

/* ================================================================== */
/* Defining constants for random() ================================== */
/* ================================================================== */
/* Refer to "Numerical Recipes in C : Second Edition" (p280) for this  */
/* random number generating function.                                  */

#define IA 16807
#define IM 2147483647
#define AM (1.0/IM)
#define IQ 127773
#define IR 2836
#define NTAB 32
#define NDIV (1+(IM-1)/NTAB)
```

```
#define EPS 1.2e-7
#define RNMX (1.0-EPS)

/* =============================================================== */
/* Defining macros =============================================== */
/* =============================================================== */

#define space2 printf("\n\n")
#define space3 printf("\n\n\n")

#define out3(fp, a1, a2, a3) fprintf(fp, "%f %f %u \n", a1, a2, a3)

#define sqr(x) (x*x)  /* sqr gives a squared value of the argument */
#define max(A, B) ((A) > (B) ? (A) : (B))
                      /* max gives a larger number of the given two numbers */

/* =============================================================== */
/* Defining data structure (declaration of global variables) ========= */
/* =============================================================== */

/* "node" specifies a node in a linked list (base list), in which      */
/* players' strategies are stored along with the following data.      */

struct node
{
   unsigned int t;                    /* period number t = 0, 1, ..., T */
   float delta;                                        /* ex-post delta */
   float thetaA;                    /* A's prior on the mean of theta */
   float thetaB;                    /* B's prior on the mean of theta */
   float PhiA;                  /* A's prior on the variance of theta */
   float PhiB;                  /* B's prior on the variance of theta */
   float Q;                          /* asset value in the current period */
   float X;                       /* offer strategy in the current period */
   unsigned int R;            /* response strategy in the current period */
   float Apay; /* A's payoff if agreement occurs in the current period */
   float Bpay; /* B's payoff if agreement occurs in the current period */
   struct node *next;                           /* pointer to its child */
   struct node *past;                          /* pointer to its parent */
   struct snode *strat;                  /* pointer to a strategy list */
};

/* =============================================================== */

/* "snode" specifies a node in a linked list (strategy list), in which */
/* expected values of various variables are stored.                   */

struct snode
{
   unsigned int t;                    /* period number t = 0, 1, ..., T */
   float EAdelta;                        /* A's expected value of delta */
   float EBdelta;                        /* B's expected value of delta */
   float EAQ;                        /* A's expected value of the asset */
   float EBQ;                        /* B's expected value of the asset */
   /* offer strategy in the future, based on the available information */
   /* upto the current period */
   float X;
   /* response strategy in the future, based on the available          */
   /* information upto the current period */
   unsigned int R;
   float EApay;             /* A's expected payoff in the future period */
   float EBpay;             /* B's expected payoff in the future period */
   struct snode *next;                          /* pointer to its child */
   struct snode *past;                         /* pointer to its parent */
   struct node *base;                          /* pointer to a base list */
```

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};

/* =================================================================== */

/* "freq" specifies a node in a linked list, in which trading          */
/* frequencies of each period is stored.  Also refer to calc_freq()    */
/* and out_freq().                                                     */

struct freq
{
   unsigned int t;                    /* period number t = 0, 1, ..., T */
   unsigned int i;                                /* frequency counter */
   struct freq *next;                            /* pointer to its child */
};

/* =================================================================== */

   FILE *fa;                                  /* Formatted output file */
   FILE *fb;                                  /* Formatted output file */

/* =================================================================== */
/* =================================================================== */
/* =============== INPUT/MISCELLANEOUS SUBROUTINES =================== */
/* =================================================================== */
/* =================================================================== */

/* =================================================================== */
/* display =========================================================== */
/* =================================================================== */
/* displays the current program name.                                  */

void display()

{
   space3;
   printf(DISPLAY1);
   space3;

   return;

} /* End of display() */

/* =================================================================== */
/* open_files ======================================================== */
/* =================================================================== */
/* opens files to write the output.                                    */

void open_files()

{

   fa = fopen("ta.dat", "w");
   fb = fopen("tb.dat", "w");

   return;

}  /* End of open_files() */

/* =================================================================== */
/* close_files ======================================================= */
/* =================================================================== */
/* closes the output files that have been opened in open_files().      */

void close_files()
```

```
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{

   fclose(fa);
   fclose(fb);

   return;

}  /* End of close_files() */

/* ============================================================== */
/* end_message ================================================== */
/* ============================================================== */
/* prints the ending message.                                     */

void end_message()

{

   space3;
   printf(ENDMESSAGE1);
   space3;

   return;

}  /* End of end_message() */

/* ============================================================== */
/* power ======================================================== */
/* ============================================================== */
/* raise base to n-th power : n >= 0.                             */

float power(base, n)

   float base;
   long int n;

{
   long int i;
   float p;

   p = 1.0;
   for(i=1; i<=n; ++i)
      p = p * base;

   return p;

} /* End of power() */

/* ============================================================== */
/* get_par ====================================================== */
/* ============================================================== */
/* get_par() is called in main().                                 */
/* get_par() reads input data (parameter values) interactively.   */

void get_par(T, theta, Phi, X_T, delta_t, sigma, initialQ, thetaA, thetaB, PhiA, PhiB)

   unsigned int *T;
   float *theta, *Phi, *X_T, *delta_t, *sigma, *initialQ;
   float *thetaA, *thetaB, *PhiA, *PhiB;

{

   printf("Enter T (even int) : ");
```

```
   scanf("%d", T);
   printf("Enter theta (real) : ");
   scanf("%f", theta);
   printf("Enter Phi (real) : ");
   scanf("%f", Phi);
   printf("Enter X_T (positive real) : ");
   scanf("%f", X_T);
   printf("Enter delta_t (positive real) : ");
   scanf("%f", delta_t);
   printf("Enter sigma (positive real) : ");
   scanf("%f", sigma);
   printf("Enter initialQ (positive real) : ");
   scanf("%f", initialQ);
   printf("Enter thetaA (real) : ");
   scanf("%f", thetaA);
   printf("Enter thetaB (real) : ");
   scanf("%f", thetaB);
   printf("Enter PhiA (real) : ");
   scanf("%f", PhiA);
   printf("Enter PhiB (real) : ");
   scanf("%f", PhiB);

   return;

}  /* End of get_par() */

/* ================================================================= */
/* random ========================================================== */
/* ================================================================= */
/* "Minimal" random number generator of Park and Miller with Bays-     */
/* Durham shuffle and added safeguards.  Returns a uniform random      */
/* deviate between 0.0 and 1.0 (exclusive of the endpoint values).     */
/* Call with idum, a negative integer to initialize; thereafter, do    */
/* not alter idum between succesive deviates in a sequence.  RNMX      */
/* should approximate the largest floating value that is less than 1.  */
/* (Taken from p280 of "Numerical Recipes in C" : Second edition)      */

float random(flag, idum)

   int flag;
   long *idum;
{
   int j;
   long k;
   static long iy=0;
   static long iv[NTAB];
   float temp;

if (flag== 1)
{
   if (*idum <= 0 || !iy)
   {
     if (-(*idum) < 1)
       *idum = 1;
     else
       *idum = -(*idum);
     for (j=NTAB+7;j>=0;j--)
     {
       k=(*idum)/IQ;
       *idum=IA*(*idum-k*IQ)-IR*k;
       if (*idum < 0) *idum += IM;
       if (j < NTAB) iv[j] = *idum;
     }
     iy = iv[0];
```

```
   }
}

   k = (*idum)/IQ;
   *idum = IA * (*idum-k*IQ)-IR*k;
   if (*idum < 0)
     *idum += IM;
   j = iy/NDIV;
   iy = iv[j];
   iv[j] = *idum;
   if ((temp=AM*iy) > RNMX )
     return RNMX;
   else
     return temp;

} /* End of random() */

/* ================================================================= */
/* stnormal ========================================================= */
/* ================================================================= */
/* Refer to "Numerical Recipes in C : Second edition."                */
/* Given idum, it returns a value from a standard normal distribution. */

float stnormal(flag, idum)

   int flag;
   long *idum;

{

   static int iset=0;
   static float gset;
   float fac, rsq, v1, v2;

   if (iset == 0)
   {
     do
     {
       v1 = 2.0*random(flag, idum)-1.0;
       flag = 0;
       v2 = 2.0*random(flag, idum)-1.0;
       rsq = v1 * v1 + v2 * v2;
     } while ((rsq >= 1.0) || (rsq == 0.0));

     fac = sqrt(-2.0*log(rsq)/rsq);
     gset=v1*fac;
     iset=1;
     return v2*fac;
   }
   else
   {
     iset = 0;
     return gset;
   }

} /* End of stnormal() */

/* ================================================================= */
/* normal =========================================================== */
/* ================================================================= */
/* stn = a value taken from a standardized normal distribution (i.e., */
/* a normal distribution with a mean 0 and standard deviation 1).     */
/* Returns a value converted from N(0,1) to N(theta, Phi).            */
```

```
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float normal(stn, theta, Phi)

   float stn, theta, Phi;

{
   float stdiv;
   float n;

   stdiv = sqrt(Phi);
   n = theta + (stdiv * stn);
   return n;

} /* End of normal() */

/* ================================================================ */
/* calc_delta ====================================================== */
/* ================================================================ */
/* Called in build_base_list() to compute an ex-post delta values that */
/* is stored in each node in the base list.                         */

float calc_delta(e, mu, sigma, delta_t)

   float e, mu, sigma, delta_t;

{
   float d;

   d = 1 + mu*delta_t + sigma*e*sqrt(delta_t);
   return d;

} /* End of calc_delta() */

/* ================================================================ */
/* calc_Q ========================================================== */
/* ================================================================ */
/* Called in build_base_list() to compute an ex-post asset value that  */
/* is stored in each node in the base list.                         */

float calc_Q(e, mu, sigma, delta_t, preQ)

   float e, mu, sigma, delta_t, preQ;

{
   float tempQ;
   float d;

   d= 1 + mu*delta_t + sigma*e*sqrt(delta_t);
   tempQ = d * preQ;
   return tempQ;

} /* End of calc_Q() */

/* ================================================================ */
/* ================================================================ */
/* ============== CONSTRUCTING THE BARGAINING LIST ================= */
/* ================================================================ */
/* ================================================================ */

/* ================================================================ */
/* build_base_list ================================================= */
/* ================================================================ */
/* Construct the base list.                                         */

struct node *build_base_list(origin, idum, T, theta, Phi, initialQ, delta_t, sigma, thetaA
```

```
, thetaB, PhiA, PhiB)

   struct node *origin;
   long idum;
   int T;
   float theta, Phi, initialQ, delta_t, sigma;
   float thetaA, thetaB, PhiA, PhiB;

{
   struct node *p, *temp;
   int t;
   int flag;
   float e, mu;

   flag = 1;
   origin = (struct node *)calloc(1, sizeof(struct node));
   origin->next = NULL;
   origin->past = NULL;
   origin->strat= NULL;
   origin->t = 0;
   e = stnormal(flag, &idum);
   mu = normal(e, theta, Phi);
   origin->delta = calc_delta(e, mu, sigma, delta_t);
   origin->Q = calc_Q(e, mu, sigma, delta_t, initialQ);
   origin->thetaA = (thetaA * ((1/PhiA)/((1/PhiA) + (1/Phi)))) + (mu * ((1/Phi)/((1/PhiA)
+ (1/Phi))));
   origin->thetaB = (thetaB * ((1/PhiB)/((1/PhiB) + (1/Phi)))) + (mu * ((1/Phi)/((1/PhiB)
+ (1/Phi))));
   origin->PhiA = 1/((1/PhiA) + (1/Phi));
   origin->PhiB = 1/((1/PhiB) + (1/Phi));

   flag = 0;

   p = origin;
   for (t=1; t<=T; t++)
   {
     temp = (struct node *)calloc(1, sizeof(struct node));
     temp->next = NULL;
     temp->past = p;
     temp->strat= NULL;
     temp->t = t;
     e = stnormal(flag, &idum);
     mu = normal(e, theta, Phi);
     temp->delta = calc_delta(e, mu, sigma, delta_t);
     temp->Q = calc_Q(e, mu, sigma, delta_t, temp->past->Q);
     temp->thetaA = (temp->past->thetaA * ((1/temp->past->PhiA)/((1/temp->past->PhiA) + (1
/Phi)))) + (mu * ((1/Phi)/((1/temp->past->PhiA) + (1/Phi))));
     temp->thetaB = (temp->past->thetaB * ((1/temp->past->PhiB)/((1/temp->past->PhiB) + (1
/Phi)))) + (mu * ((1/Phi)/((1/temp->past->PhiB) + (1/Phi))));
     temp->PhiA = 1/((1/temp->past->PhiA) + (1/Phi));
     temp->PhiB = 1/((1/temp->past->PhiB) + (1/Phi));
     p->next = temp;
     p = p->next;
   }
   return origin;

} /* End of build_base_list() */

/* ================================================================ */
/* build_strat_list ================================================ */
/* ================================================================ */
/* Construct the strategy list.                                     */
/* P1 points through the base list made in build_base_list()        */
```

```
struct node *build_strat_list(origin, T, delta_t)

   struct node *origin;
   int T;
   float delta_t;

{

   struct node *P1;
   struct snode *P2, *temp;
   float EAdelta, EBdelta, Qt;
   int t;

   P1 = origin;
   while (P1->next != NULL)
   {
     EAdelta = 1 + P1->thetaA * delta_t;
     EBdelta = 1 + P1->thetaB * delta_t;
     Qt = P1->Q;
     for (t = P1->t + 1; t <= T; t++)
     {
       temp = (struct snode *)calloc(1, sizeof(struct snode));
       temp->next = NULL;
       temp->t = t;
       temp->EAdelta = EAdelta;
       temp->EBdelta = EBdelta;
       temp->EAQ = Qt * (power(EAdelta, (t - P1->t)));
       temp->EBQ = Qt * (power(EBdelta, (t - P1->t)));

       if ((t - P1->t) == 1)
       {
         P1->strat = temp;
         temp->base = P1;
       }
       else
       {
         P2->next = temp;
         temp->past = P2;
       }

       P2 = temp;

     } /* End of for (t = P1->t + 1; t <= T; t++) */

     P1 = P1->next;

   } /* End of while (P1->next != NULL) */

   return origin;

} /* End of build_strat_list() */

/* ================================================================= */
/* calc_offer_strategy ============================================= */
/* ================================================================= */
/* Calculates offer strategies, X, to be stored in the base list and  */
/* the strategy list.                                                 */

struct node *calc_offer_strategy(origin, T, X_T)

   struct node *origin;
   int T;
   float X_T;
```

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```
{
   int i, count;
   struct node *P1;
   struct snode *P2;
   float arg1, arg2;

   P1 = origin;
   while (P1->next != NULL)
   {
     count = T-1;
     while (count >= 0)
     {
       P2 = P1->strat;
       for (i = P1->strat->t; i <= count; i++)
         P2 = P2->next;

/* compute offer strategies in the strategy list */

       if (P2->t == T)
       {
         P2->X = X_T;
       }
       else if (P2->t == (T-1))
       {
         P2->X = 1 - X_T;
       }
       else if (P2->t == (T-2))
       {
         arg1 = 1 - ((1 - X_T) * P2->next->next->EBdelta);
         arg2 = X_T * P2->next->next->EAdelta;
         P2->X = max(arg1, arg2);
       }
       else if ((P2->t % 2) == 0)
       {
         arg1 = 1 - (P2->next->next->EBdelta * P2->next->X);
         arg2 = P2->next->next->EAdelta * P2->next->next->next->EAdelta * P2->next->next->
X;
         P2->X = max(arg1, arg2);
       }
       else if ((P2->t % 2) == 1)
       {
         arg1 = 1 - (P2->next->next->EAdelta * P2->next->X);
         arg2 = P2->next->next->EBdelta * P2->next->next->next->EBdelta * P2->next->next->
X;
         P2->X = max(arg1, arg2);
       }

       count = count - 1;

     } /* End of while (count >= 0) */

/* compute offer strategies in the base list */

     if (P1->t == (T-1))
       P1->X = 1 - X_T;
     else if (P1->t == (T-2))
     {
       arg1 = 1 - ((1 - X_T) * P1->strat->next->EBdelta);
       arg2 = P1->strat->next->EAdelta * X_T;
       P1->X = max(arg1, arg2);
     }
```

```
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     else if ((P1->t % 2) == 0)
     {
       arg1 = 1 - (P1->strat->next->EBdelta * P1->strat->X);
       arg2 = P1->strat->next->EAdelta * P1->strat->next->next->EAdelta * P1->strat->next-
>X;
       P1->X = max(arg1, arg2);
     }
     else if ((P1->t % 2) == 1)
     {
       arg1 = 1 - (P1->strat->next->EAdelta * P1->strat->X);
       arg2 = P1->strat->next->EBdelta * P1->strat->next->next->EBdelta * P1->strat->next-
>X;
       P1->X = max(arg1, arg2);
     }


     P1 = P1->next;

   } /* End of while (P1->next != NULL) */

   P1->X = X_T;

   return origin;

} /* End of calc_offer_strategy() */

/* =============================================================== */
/* exp_payoff ==================================================== */
/* =============================================================== */
/* Computes expected payoffs if an offer in a period in concer is      */
/* accepted.  The computed expected payoffs are stored in the strategy */
/* list and the base list.                                             */

struct node *exp_payoff(origin, T, X_T)

   struct node *origin;
   int T;
   float X_T;

{
   struct node *P1;
   struct snode *P2;

   P1 = origin;
   while (P1->next != NULL)
   {
     P2 = P1->strat;

     if ((P2->t % 2) == 0)
     {
       P2->EApay = (1 - P1->X) * P2->EAQ;
       P2->EBpay = P1->X * P2->EBQ;
     }
     else
     {
       P2->EApay = P1->X * P2->EAQ;
       P2->EBpay = (1 - P1->X) * P2->EBQ;
     }


     if (P2->next != NULL)
       P2 = P2->next;

     while (P2->next != NULL)
```

```
    {
      if ((P2->t % 2) == 0)
      {
        P2->EApay = (1 - P2->past->X) * P2->EAQ;
        P2->EBpay = P2->past->X * P2->EBQ;
      }
      else
      {
        P2->EApay = P2->past->X * P2->EAQ;
        P2->EBpay = (1 - P2->past->X) * P2->EBQ;
      }

      P2 = P2->next;

    } /* End of while (P2->next != NULL) */

    P2->EApay = X_T * P2->EAQ;
    P2->EBpay = (1 - X_T) * P2->EBQ;

    if (P1->t == 0)
    {
      P1->Apay = 0.0;
      P1->Bpay = 0.0;
    }
    else if ((P1->t % 2) == 0)
    {
      P1->Apay = (1 - P1->past->X) * P1->Q;
      P1->Bpay = P1->past->X * P1->Q;
    }
    else if ((P1->t % 2) == 1)
    {
      P1->Apay = P1->past->X * P1->Q;
      P1->Bpay = (1 - P1->past->X) * P1->Q;
    }

    P1 = P1->next;

  } /* End of while (P1->next != NULL) */

  P1->Apay = X_T * P1->Q;
  P1->Bpay = (1 - X_T) * P1->Q;

  return origin;

} /* End of exp_payoff() */

/* ================================================================= */
/* calc_resp_strategy ============================================== */
/* ================================================================= */
/* Computes response strategies in the base list.                     */
/* R = 0 if reject, R = 1 if accept.                                  */

struct node *calc_resp_strategy(origin, T, X_T)

  struct node *origin;
  int T;
  float X_T;

{
  struct node *P1;
  struct snode *P2;
  float arg;

  P1 = origin;
```

```
   while (P1->next != NULL)
   {
     if (P1->t == 0)
       P1->R = 0;
     else if ((P1->t % 2) == 0)
     {
       arg = 1 - (P1->strat->EAdelta * P1->X);
       if (P1->past->X <= arg)
         P1->R = 1;
       else
         P1->R = 0;
     }
     else
     {
       arg = 1 - (P1->strat->EBdelta * P1->X);
       if (P1->past->X <= arg)
         P1->R = 1;
       else
         P1->R = 0;
     }
     P1 = P1->next;
   }
   P1->R = 1;

   return origin;

} /* End of calc_resp_strategy() */

/* ================================================================ */
/* ================================================================ */
/* ================== OUTPUT RELATED SUBROUTINES =================== */
/* ================================================================ */
/* ================================================================ */

/* ================================================================ */
/* init_first ===================================================== */
/* ================================================================ */
/* Initialize a linked list pointed by a pointer "first" with a data   */
/* structure "freq" that will be used to store the number of first     */
/* agreement in each trading period out of 10,000 sample runs.         */

struct freq *init_first(first, T)

   struct freq *first;
   int T;

{
   struct freq *temp1, *temp2;
   int t;

   first = (struct freq *)calloc(1, sizeof(struct freq));
   first->t = 0;
   first->i = 0;
   first->next = NULL;
   temp1 = first;
   t = 1;
   while (t <= T)
   {
     temp2 = (struct freq *)calloc(1, sizeof(struct freq));
     temp2->t = t;
     temp2->i = 0;
     temp2->next = NULL;
     temp1->next = temp2;
     temp1 = temp2;
```

```
    t = t + 1;
  }

  return first;

} /* End of init_first() */

/* ================================================================= */
/* calc_freq ======================================================= */
/* ================================================================= */
/* By moving through the base list, it counts the number of first    */
/* agreement in each period out of 10,000 sample runs, and store the */
/* result in a list with the data structure "freq."                  */

struct freq *calc_freq(first, origin)

  struct freq *first;
  struct node *origin;

{
  struct node *P1;
  struct freq *P2;

  P1 = origin;
  P2 = first;
  while (P1->R != 1)
  {
    P1 = P1->next;
    P2 = P2->next;
  }
  P2->i = P2->i + 1;

  return first;

} /* End of calc_freq() */

/* ================================================================= */
/* out_freq ======================================================== */
/* ================================================================= */
/* Writes the information stored in a structure "freq" in an output  */
/* file "tb.dat."                                                    */

void out_freq(first, theta)

  struct freq *first;
  float theta;

{
  struct freq *temp;

  temp = first;
  while (temp->next != NULL)
  {
    fprintf(fb, "theta = %f, t = %d, freq = %d \n", theta, temp->t, temp->i);
    temp = temp->next;
  }
  fprintf(fb, "theta = %f, t = %d, freq = %d \n", theta, temp->t, temp->i);

} /* End of out_freq() */

/* ================================================================= */
/* out_exp_delta =================================================== */
/* ================================================================= */
/* Writes expected values of delta stored in the strategy list into an */
```

```
/* output file "ta.dat."                                              */

void out_exp_delta(origin, T)

   struct node *origin;
   int T;

{
   struct node *temp;
   struct snode *temp2;
   int t;

   temp = origin;
   while (temp != NULL)
   {
     temp2 = temp->strat;
     for (t = temp->t +1; t <= T; t++)
     {
       fprintf(fa, "t = %d, EAQ= %f, EBQ= %f \n", temp2->t, temp2->EAQ, temp2->EBQ);
       fprintf(fa, "t = %d, EAdelta= %f, EBdelta= %f \n", temp2->t, temp2->EAdelta, temp2-
>EBdelta);
       fprintf(fa, "\n");
       temp2 = temp2->next;
     }
     temp = temp->next;
   }

   return;

} /* End of out_exp_delta() */

/* ================================================================== */
/* ================================================================== */
/* ================================================================== */
/* =========================== MAIN ================================= */
/* ================================================================== */
/* ================================================================== */
/* ================================================================== */

main()
{

/* ================================================================== */

int T;                    /* the maximum number of periods in the game */
long idum;      /* seed number to generate a sequence of random numbers */
/* Refer to the description of the input variables in the beginning of */
/* this program for the followings.                                     */
float delta_t, sigma;
float theta, Phi;
float thetaA, thetaB, PhiA, PhiB;
float X_T;
float initialQ;

struct node *origin;        /* pointer to a root node of the base list */
struct freq *first;    /* pointer to a root node of the frequency list */

/* ================================================================== */

   display();
   open_files();
   get_par(&T, &theta, &Phi, &X_T, &delta_t, &sigma, &initialQ, &thetaA, &thetaB, &PhiA, &
PhiB);
```

```
   first = NULL;
   first = init_first(first, T);

   idum = -1;
   while (idum >= MINIDUM)
   {
     origin = NULL;
     origin = build_base_list(origin, idum, T, theta, Phi, initialQ, delta_t, sigma, theta
A, thetaB, PhiA, PhiB);
     origin = build_strat_list(origin, T, delta_t);
     origin = calc_offer_strategy(origin, T, X_T);
     origin = exp_payoff(origin, T, X_T);
     origin = calc_resp_strategy(origin, T, X_T);
     first = calc_freq(first, origin);
     idum = idum - 1;
   }

   out_freq(first, theta);
   out_exp_delta(origin, T);
   end_message();
   close_files();

}  /* End of main() */

/* ====================== End of "WPBM2.c" ========================= */
/* ================================================================== */
/* ================================================================== */
/* ================================================================== */
/* ================================================================== */
```

Chapter 4

Concluding Remarks on Part I

Incorporating stochastic elements into sequential bargaining games has been proven to provide us with an alternative way of describing various bargaining durations. Despite the assumption of complete information, in our games it is not uncommon to observe delays before the first agreement. The comparison of the Basic and the Alternative games showed us the sensitivity of the durations to the timing of information arrivals and players' actions. In addition, our equilibrium strategies predict differences in reservation values of an offer–making player between the Basic and the Alternative games. Simulation outputs have confirmed many of our analytical findings and conjectures with regard to comparative statics results.

By construction, however, neither of our bargaining games explains breakdowns[1] in the current negotiation. It is not new to us, however, to observe a bargaining breakdown even if there are positive gains for both parties from a potential agreement. Such breakdowns

[1]By breakdown we mean that the bargaining parties never come to an agreement and leave the negotiation table without any transaction among themselves.

can leave one or both parties with zero payoff and become a source of inefficiency. In addition to explaining various bargaining durations, we need to construct a model that incorporates such possibilities. We conclude Part I by including the following idea as a potential extension that will enhance our understanding of bargaining durations in more complicated situations.

We would like to model delays and eventual negotiation breakdowns explicitly in the presence of complete information, where players can choose at their own cost to search for outside options that can be realized stochastically. Consider a negotiation process where both parties see positive gains from an agreement at the same time when each player knows that his opponent is constantly searching for a better outside option. Suppose that both the value of the asset in the current negotiation and of the outside option they are looking for are common knowledge and positive constant. Furthermore, let us assume that there is a cost associated with the search for the outside option, and the option always gives a higher utility than the agreement in the current bargaining to the party that has found it. This sort of model can be motivated by the following simple example. Suppose that person A is looking for an apartment to rent, hoping to move in very soon, and is currently negotiating with a condominium owner B who wants to sublet. We can reasonably assume that the value of the asset in concern is constant, since the value of the condominium or other apartments may not fluctuate much in a short term. In the meantime A is still looking for a better deal elsewhere and B is hoping to find someone who wants to rent it with the price he is asking for. Naturally, the more vigorously one looks for an outside option, the more likely the person finds a better option. If neither finds a better option within a reasonable length of time, then they compromise with each other and settle with the price that is determined by an exogenous factor such as information through reliable real estate agents. If one finds a better option, then the current negotiation breaks down. For example, if A finds *a deal of the century*, she receives the value of the outside option higher than the agreement in the current negotiation, whereas B is left with no payment.

According to Rubinstein and Osborne (1990), with the assumption of complete information, opting out is not a credible threat, and thus an outside option has no effect on the bargaining outcome. We conjecture that the introduction of stochastic realization of outside options can result in varied bargaining durations. We need to model an endogenous risk of breakdowns in our bargaining games. By introducing a decision variable such as each player's search intensity of outside options, we will have a bargaining model in which an outside option becomes stochastically available to players. In a discrete time model, for example, at the beginning of each negotiation period both players first compute their reservation values and decide on their level of investment into the search to maximize expected payoffs. For example, let $V_t^i(\cdot)$ be a player i's continuation payoff after period t begins, so that both players decide on their investment levels to maximize V_t^i and V_t^j. Then, one player quotes a price, to which the other responds. In other words, i quotes an offer x_t^i such that $x_t^i = \max\{V_t^i/Q_t, 1-(V_t^j/Q_t)\}$, which j accepts if $x_t^i = 1-(V_t^j/Q_t)$ or rejects if $x_t^i = V_t^i/Q_t > 1-(V_t^j/Q_t)$. The game ends if the responder accepts the offer or if the responder rejects and at least one player finds a better option by the end of the period. If the responder rejects and no one finds a better option by the end of the period, the next negotiation period $(t+1)$ opens. The game continues until a predetermined time horizon is reached. Note that an agreement is generated by an increasing endogenous risk of breakdown, while the potential for finding a better outside option may cause a delay. We shall solve for Nash equilibrium level of investment and may find conditions to guarantee a unique investment choice process. We shall also check conditions for the uniqueness of players' contingency plans or equilibrium payoffs. Results should be compared to those of Rubinstein and Osborne (1990), which predicts unique subgame perfect equilibrium with the presence of outside options. Running simulations to study comparative statics, especially with respect to bargaining durations, is also recommended.

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Part II

Information and Order Flow in Experimental Markets

Chapter 5

Order Flow in Experimental Financial Markets

5.1 Introduction

The efficient market hypothesis, which has acquired widespread support in the fields of economics and finance, states that financial markets with significant informational asymmetries such as securities exchanges are said to be efficient if the prices of the securities traded fully convey available information. In rational expectation settings, the informational efficiency of prices is achieved since the model predicts that the prices reflect all relevant private information about the asset value, provided that the market is in perfect competition (Milgrom and Stokey (1982)). In other words, a market that provides an efficient mechanism for information dissemination resolves uncertainty among rational traders. It has been successfully demonstrated in the related experimental economics literature that laboratory asset markets disseminate privately held information efficiently (Forsythe, Palfrey, and Plott (1982), Plott and Sunder (1982), and Plott and Sunder (1988)).

The existing market microstructure literature that investigates the information dissemination and aggregation properties of markets in experimental context has mainly been constructed within a restricted framework in which traders are allowed to submit only market orders. Limit orders, along with market orders, constitute an intrinsic part of financial market trading. For instance, in a market such as the Paris Stock Exchange, the Tokyo Stock Exchange, or the Toronto Stock Exchange, every market participant observes both market and limit orders entered by other members, so that one can utilize the information in estimating the demand and supply for a particular stock one is interested in trading. In other words, the traders condition their beliefs not only on the transaction prices, but also on the more detailed order flow that can be found in the limit order book. Consequently, the list of limit orders is part of a source for predicting the future stock prices, thereby influencing transaction outcomes. In a specialist market such as the New York Stock Exchange, where investors submit limit orders to the specialist who is the only person having an access to the list of all the limit orders, the specialist utilizes the information for promoting effective execution of orders.

Likewise, it is a common practice for investors to submit limit orders in the operation of stock market. In the computerized simulated markets Bollerslev and Domowitz (1993) investigated, the size of the order book is shown to be positively related to the amount of information available, for the price volatility decreases as the book length grows.[1] Hence, we may conjecture that the system provided with a limit order book carries more information both implicitly and explicitly about the market, and the effects of allowing the traders to

[1]This result is in accordance with Kyle(1985)'s theoretical findings.

have an access to such information should not be ignored. We attempt to investigate how the information conveyed in the order book affects the order flow in experimental financial markets with asymmetrically informed traders.

In this chapter the markets are organized as computerized double auctions accommodated with an order book that contains a complete list of current limit and market orders.[2] In practice, investors can submit limit orders in the operation of the stock market. In our experimental market all the participants can submit both market and limit orders. The list of the orders can be inspected by every market participant at any time during each market period. All of the trades are executed at outstanding quotes in the book. Our empirical analysis of the experimental data sets focuses on the series of actions taken by the subjects that include quote revisions, limit order arrivals, and trades. Players' actions are identified as events. The state of the book is updated immediately after the occurrence of each event. In the analysis of order flow, we report the presence of serial dependencies of order arrivals on the previous event type, the state of the order book, the size of the bid–ask spread, and the time intervals. The method of the analysis follows that of Biais, Hillion, and Spatt (1993).[3]

We seek to provide an empirical analysis of the acquired data in an attempt to understand the order flow dynamics and to identify the determinants of the order flow by creating a market that reflects several essential features of financial markets in a controlled laboratory environment. We are particularly interested in how the traders interpret various

[2]The software we used is called MUDA, Multiple–Unit Double Auction, and has been developed at the California Institute of Technology. We describe our market organization in the next section. For more details of this software, refer to Plott (1991).

[3]Biais *et al.* uses summary statistics to characterize the order book and contingency tables to analyze the determinants of the order flow in the analysis of data from the Paris Bourse.

states of the order book and how they typically respond to certain information they extract from the order book. In summary, our tasks in this chapter are i) to observe the interaction of subjects' behavior and the information conveyed in the transaction prices and the limit order book, ii) to ascertain the significance of the impact of information carried in the order book, iii) to empirically examine the determinants of the order flow, and iv) to compare the results from the data acquired in the computerized laboratory financial markets with the previous findings in the literature of order flow analyses, especially in comparison with those of Biais *et al.* (1993).

The rest of the chapter is organized as follows. In section 5.2, we describe the organization of our experimental financial market along with the details of the experimental design. In section 5.3, we give selected descriptive statistics on the acquired data sets. Section 5.4 contains the analyses of the determinants of the order flow, such as the previous event type, the state of the order book, the size of bid–ask spread, and the time intervals. We give brief concluding remarks and several ideas for a future research in section 5.5.

5.2 Experimental Procedure

All of the experiments were conducted in the Laboratory of Experimental Economics and Political Science at the California Institute of Technology. The experiments were run on a set of computers operated in a local area network. The subjects were undergraduate students of various majors and backgrounds at the California Institute of Technology, and were recruited by the announcement of an invitation to participate in an economics experiment. They were told that the experiment would not require any prior knowledge of economics or computers and that they would be paid the amount they have earned through their decision making in cash at the end of the experiment. We particularly recruited those who have never participated in an experiment of the similar environment before.

Once the subjects were in the laboratory, each person was randomly seated at a terminal where he/she was given a packet of instructions along with a subject identification number.[4] The content of the packet is described in detail in the section below where the market environments are discussed. The subjects were read the instructions by the experimenter, and several examples were given on a board to enhance their understanding of the rules. Approximately thirty minutes were spent for the instructional purposes. Then a practice period, Period 0, was run for 7 minutes to accustom the subjects to the rules and the environment. The procedure that is specific to each market environment is also included below. We ran 15 periods in each experimental session, in which each peirod lasted for 5 minutes. The subjects were aware of the length of each period, but were not told how many periods would be run. After each session has ended, each subject was paid in cash before leaving the laboratory.

5.2.1 Market Organization

All of the markets were organized as a computerized double auction accommodated with an open limit order book. The subjects can submit market and limit orders at any time during a trading period through their terminals. The subjects' actions are transparent in that everyone has an access to observe everyone else's action at any time during a trading peirod, including the activities in the order book. The order book shows the market and

[4] A copy of the instruction packet used in the actual experiments is included in Appendices 5D and 5E.

limit bid and ask quotes along with the associated quantities and the subject identification numbers. The price priority rule is enforced over the time priority rule. The size of the book can be considered infinite in that the complete list of the limit orders were listed in our experiments, though there is a physical limit. We consider the lowest limit order on the sell–side as the standing market ask and the highest limit order on the buy–side as the standing market bid. Hence, orders entered as limit orders automatically become standing market orders when they are the lowest or highest on each side of the book. The limit orders do not have a prespecified lifetime; that is, the subjects are allowed to keep or revise the orders as they wish. Hence, an unexecuted limit order remains in the book until it is deleted by a person who has entered it. However, the limit order book is cleared at the end of each trading period; that is, any orders remaining at the closing of a trading period are not carried over to the next period.

Transactions can occur only at the current standing quotes, which eliminates the possibility of transactions at prices strictly within the bid–ask spread. Along with the data in which we can keep track of each subject's choice of actions, this feature is a significant advantage of our experimental markets in the analysis of the order flow, since we can readily identify each trade as buyer–initiated or seller–initiated.[5]

[5] In the analysis of data from stock exchanges, we need to employ certain methods to classify the direction of each transaction. But since many transactions occur within the bid–ask spread and since we do not know who exactly is a seller or a buyer, it is difficult to classify every transaction accurately just by looking at the transaction prices data. For such classification methods, refer, for example, to Blume, MacKinlay, and Terker (1989).

5.2.2 Experimental Design

The experimental design described in this section is summarized in Tables 5.1a, 5.1b, 5.1c, and 5.1d. There are two types of market environments, which we often refer to as the market 1 environment and the market 2 environment. The four data sets analyzed and reported here are indexed as 042393b:Market1, 042393b:Market2, 042393c:Market1, and 042393c:Market2, where 042393 indicates that the experiment was run on April 23, 1993. The market 1 environment simulates a financial market with no specialist, where assets of uncertain values are traded in the presence of asymmetric information among subjects. The market 2 environment is a simple competitive market design often used in testing a competitive behavior of subjects characterized by symmetric demand and supply schedules. In each experiment, there are eight subjects, half of which is identified as type I and the other half is type II in the market 1 environment, while half is a seller and the other half is a buyer in the market 2 environment. Each packet of instructions contained the materials that identify the subjects with these types along with a copy of the general instruction.

Market 1 environment

In this hypothetical asset market, three subjects are randomly chosen before the beginning of each period to become insiders in market 1.[6] The insiders are given an opportunity to observe an *ex post* liquidation value of the risky asset. Assuming anonymity, each subject does not know who is informed or uninformed other than about himself/herself. There is no exogenous arrival of information regarding the value of the asset in the middle of each trading period, so that the information revelation is endogenous. Consequently, in

[6]We refer to the informed subjects as insiders despite that other interpretations are possible.

such a market environment, uninformed traders have access to only the market–generated information.

The asymmetric information in the market is generated in the following manner. Each period is associated with one of three *states*, X, Y, and Z, which is known to only the insiders.[7] The value of the asset varies from person to person, and from state to state. Therefore, the asset value to each subject is defined by one of the state–dependent types, type I and type II, which specify the asset's dividend value for a given *state*. These types are given in Table 5.2a. The dividend values follow those used by Forsythe and Lundholm (1990) in their investigation of information aggregation properties of experimental markets. The state of each period was determined randomly by the experimenter before the experiment sessions by using a random number table. After each trading period has ended, the state of the past period becomes public information, and the subjects compute their profits for the period.

Market 2 environment

The market 2 environment is described by a set of demand and supply schedules illustrated in Figure 5.1. It is a symmetric market with respect to supply and demand, in which each trader is classified as either a buyer or a seller and is provided with a table containing one's own reservation values for units one trades during a given trading period.[8]

[7]Refer to Table 5.1b.
[8]Refer to Tables 5.2b through 5.2d.

5.3 Descriptive Summary

Figures 5.2a through 5.2d show the obtained time series of transaction prices from the experiments. Several summary statistics are included in Tables 3a through 3d in order to provide some intuition for the acquired data sets. The trading volume in the tables is the number of units traded in each period, which is equivalent of the number of transactions since in our experiment the subjects are not allowed to place multiple–unit orders. Note that it is not necessarily equal to the total number of contracts outstanding at the end of each trading period. This indicates the existence of a trader who has taken both a long position and a short position within the same period.[9] An action taken by the subjects is identified as an event that belongs to one of ten event types. These event types are described and explored in detail in the following sections. But briefly, on the buyers' side they are "take ask," which results in immediate trading at the standing ask price, "new bid > standing bid," meaning a buyer overbidding the current standing bid, "new bid = standing bid," meaning a buyer entering a new bid equal to the current standing bid that will be recorded in the limit order book, "new bid < standing bid," meaning a buyer entering a new bid lower than the current standing bid that will also be recorded in the book accordingly, and "cancel bid," meaning a buyer cancelling a bid he has entered previously. The event types on the sellers' side are defined analogously. The price change is defined as a difference between transaction prices at transaction times t and $t-1$. The bid–ask spread is expressed in francs that is a difference between the lowest ask quote and the highest bid quote, and is

[9]This is commonly observed in real exchanges. For example, the volume of trading in some commodity futures contracts in a day can be larger than its open interest at the end of the day, reflecting a large number of day trades.

updated whenever a new event occurs. The time between events is a time elapsed in seconds between two consecutive events. Additional descriptive statistics on the market activities, such as the number of bids or asks, the size of the bid–side or ask–side spread in the order book, and the size of the bid–side or ask–side depth are included in Figures 5.3a through 5.4d.

5.3.1 Trading Activities and Bid–Ask Spreads

We include variables such as the trading volume and the number of events for each period as direct measures of the level of trading activities. A glance over the number of events in each market indicates that the subjects are generally more active in the markets with information asymmetry than in the market 2 environment, where the equilibrium trading volume predetermined by the experimenter has proven to prevail by previous researchers. This simply confirms the conjecture that the speculators make a market active and that an active market attracts speculators. This indicates the positive relation between the trading volume and the absorptive capacity of the market, or the liquidity of the market.[10]

The previous literature has found that the level of trading activities is a determinant of the bid–ask spread. McInish and Wood (1992) study the intraday patterns of bid–ask spreads in the NYSE data and find that the size of bid–ask spread is significantly inversely related to the number of trades and the number of shares per trade.[11] Our data do not necessarily contradict their findings, but it is not clear whether we can conclude that it is true in the analysis of the data on the inter–period basis. In Table 5.3a both the trading

[10]For example, see Kyle (1985) and Admati and Pfleiderer (1988).

[11]Copeland and Galai (1983) show that the bid–ask spread is inversely related to the frequency of trading. For the analysis of data for intervals of a day or longer, see Tinic and West (1972), and Benston and Hagerman (1974).

volume and the event frequency are highest in period 12 where the mean bid–ask spread is 56.11 francs, whereas in period 1, where the event frequency is lowest and the trading volume is relatively low, the mean bid–ask spread is 104.02 francs. But in period 2, where the trading volume is lowest and the event frequency is relatively low, the mean bid–ask spread is 40.86 francs with a low standard deviation, which is smaller than 56.11 francs. We also find such inconsistencies in the other three data sets.[12] Accordingly, we find that it is difficult to conclude that the level of activities is a significant determinant of bid–ask spreads in our data sets. This inconsistency with the prior work, however, certainly does not mean that our laboratory markets have produced uninterpretable data. The key argument made by previous researchers as a reason of the inverse relationship between the trading activities and the spread size is the economies of scale in transactions costs, i.e., an increase in trading activities results in lowering the trading costs due to the economies of scale, which in turn leads to a smaller spread.[13] In the absence of transactions costs, we need to seek for another interpretation. Moreover, there is empirical evidence that the pattern of differences in bid–ask spreads across days of a week is not stable over time compared to their intraday patterns.[14] Since our notion of a period supposedly corresponds to a day, this instability might have contributed to the observed inconsistency.

McInish and Wood (1992) also find the crude reverse J–shape pattern in the analysis of the minute–by–minute bid–ask spreads. Figures 5.7a1 and 5.7a2 shows two examples of the intra–period patterns of bid–ask spreads that approximately follow a reverse J–

[12] Along with Tables 5.3a through 5.3d, refer also to Figures 5.3a through 5.3d, which show the average bid and ask in francs and the average number of bids and asks computed for each period.

[13] An alternative interpretation is due to Ho and Stoll (1983) in inventory control models of a dealership market, where increasing trading volume may lead to a larger spread if dealers are put in an undesired inventory position.

[14] This statement was noted by McInish and Wood (1992).

shape pattern. The spreads are large in the early stage of the given period, then they become smaller in the middle, followed by slightly larger spreads near the end of the period. Examples in which the pattern observed is very different from the reverse J–shape pattern are given in Figures 5.7b1 and 5.7b2. In period 10 of 042393c data, the spread becomes smaller almost monotonically as time advances. Since the market is designed as that of asymmetric information, we can interpret it as follows. The private information was revealed fairly early in the period that led to a gradual but permanent information adjustment and eventually was conveyed into prices by the end of the period. Hence, there is no informational shock near the end of the period nor speculation of higher risk. Another example included in Figure 5.7b2 is the pattern in period 12 of 042393c data. The data shows the spread widening later in the period followed by a succession of very large spreads. This observation can be interpreted that the private information was successfully concealed by the insiders in the early stage and that there was an informational impact later in the period.[15] In fact, we can observe in the time series graphed in Figure 5.2b that the transaction prices begin to rapidly move up to a price level predicted by the rational expectations model in the last half of the period. Likewise, the spread size depends on how the information is assimilated into the market price reflecting the insiders' strategies that may not be obvious to the others.

[15]By using NYSE data McInish and Wood (1992) show that there is a direct relationship between spreads and the amount of information comimg to the market.

5.3.2 Inferences Concerning Variances

In the next observation we deal with inferences concerning variances of the selected variables. The standard deviation of transaction prices are higher in the market 1 environment than in the market 2 environment in period–by–period comparison, except for a few periods. This result is consistent with previous theoretical and empirical findings. In market 2 each trader has no uncertainty about the asset value since one's reservation value is predetermined and given to him in the beginning of the experiment, while in market 1 there exists uncertainty regarding the *ex post* liquidation value among the subjects except for three randomly chosen insiders. Hence, there is no necessity of speculation nor information extraction from prices in the market 2 environment, whereas the information that is available to the insiders in the market 1 environment may not be fully absorbed into prices especially in the early stage of each period. The large price volatility increases speculative profits by increasing the chances of buying low and selling high in an asymmetric information market where speculating traders condition their actions on the prices. In Kyle's (1985) insider trading model, he finds that the price volatility decreases as information is conveyed into prices. The empirical investigation conducted by Bollerslev and Domowitz (1993) reports the negative monotonic relation between the amount of available information and the price variability.[16] The findings on the standard deviation of price changes also fall in line with this argument. The standard deviations of bid–ask spreads are also higher in the market 1 environment than in the market 2 environment. This is consistent with Bollerslev *et al.*'s finding that the standard deviation of transaction prices and that of bid–ask spreads follow similar patterns.

[16] In the Bollerslev *et al.*'s simulated markets, the level of available information is modeled as a varied length of the electronic order book, in which the longer book size indicates more information available in the market.

The above observations are supported formally by the following statistical arguments. Since it is not doubtful whether the assumption of normality is appropriate for the distribution of such variables by observing their skewness and kurtosis, we can employ the ratio of the variances as a test statistic for testing the hypotheses

$$(A) \quad H_0 \; : \; \sigma^2_{tr1} = \sigma^2_{tr2} \; ; \; H_a \; : \; \sigma^2_{tr1} > \sigma^2_{tr2}$$

$$(B) \quad H_0 \; : \; \sigma^2_{pc1} = \sigma^2_{pc2} \; ; \; H_a \; : \; \sigma^2_{pc1} > \sigma^2_{pc2}$$

$$(C) \quad H_0 \; : \; \sigma^2_{ba1} = \sigma^2_{ba2} \; ; \; H_a \; : \; \sigma^2_{ba1} > \sigma^2_{ba2},$$

where σ^2_{tr1}, σ^2_{pc1}, and σ^2_{ba1} are the variance of transaction prices, price changes, and bid–ask spreads in Market 1, respectively, and the others are defined analogously. The data from 042393b and 042393c are pooled by market environment.[17] The test statistics and the critical regions are shown in Table 5.4. We find the results in favor of the alternative hypothesis for all three cases; in other words, there is sufficient evidence to doubt the equality of the variances of these variables between the two market environments. Hence, we can conclude that the data support the contention that there is more variability in the transaction prices, the price changes, and the bid–ask spreads in markets with information asymmetries.

[17]The standard deviation of transaction prices in the market 1 environment is calculated over the transaction prices pooled across 15 periods in Market 1 of both 042393b and 042393c data sets. The standard deviations of price changes and of bid–ask spreads are defined similarly.

5.3.3 Other Findings on the Descriptive Statistics

With insiders present in the market 1 environment, the rational expectations model and the prior information model predict different transaction prices for three different states X, Y, and Z, while they predict the same price for any state in the market without insiders.[18] As it can be observed in Tables 5.3a and 5.3b, the mean transaction prices differ in period–by–period comparison, whereas it is not obvious among the mean price changes and the mean bid–ask spreads. We perform an analysis of variance to test whether the differences among the means of selected descriptive variables in the three states in the market 1 environment are significant. The null hypotheses are

$$(A)\ H_0 \ : \ \mu_{trX} = \mu_{trY} = \mu_{trZ}$$

$$(B)\ H_0 \ : \ \mu_{pcX} = \mu_{pcY} = \mu_{pcZ}$$

$$(C)\ H_0 \ : \ \mu_{baX} = \mu_{baY} = \mu_{baZ},$$

where μ_{trX}, μ_{pcX}, and μ_{baX} are the mean transaction price, the mean price change, and the mean bid–ask spread in X–state periods, respectively, and the others are defined similarly. The alternative hypothesis to each null hypothesis is that the μ's are not all equal. The ANOVA tables are included in Tables 5.5a and 5.5b. The null hypothesis is rejected for the mean of transaction prices in both 042393b and 042393c data sets at any reasonable level of significance. In other words, the transaction price series fluctuate about different levels in different states. Therefore, the series is *nonstationary* in the mean. This result can easily be inferred from the time series plot in Figures 5.2a and 5.2b, in which both inter–

[18]Refer to Table 5.1d.

and intra–period series show a time trend and wander away from a fixed horizontal level. Nonstationarity can be caused by a shift in the influence of periodic factors. In inter–period time series, it is the change in the information on the terminal value of the asset that becomes available to three traders at the beginning of each trading period. In intra–period time series, it may reflect an uneven assimilation of information throughout a given trading period. Hence, this result confirms the existence of insiders in the markets according to the rational expectations model's and the prior information model's predictions. The null hypothesis cannot be rejected for the mean price change, indicating that the observed difference between the two means is not significant. Hence, one cannot conclude that the means of price changes vary among different states. In addition, we can say that the process resulted by taking successive differences of transaction prices is *stationary*, indicating that the original series could be a homogeneous nonstationary process of order one. It suggests a potential application of Autoregressive integrated moving average (ARIMA) models, that are frequently used in the the analysis of capital markets, to our experimental time series. These results should be visited again when we deal with the specification of time series models. The difference in the mean bid–ask spreads is significant at the 0.05 level of significance in both 042393b and 042393c data, but is not at the 0.01 level in 042393b data.

The unconditional frequencies of ten event types are computed for the data pooled across 15 periods and included in Table 5.6, and are also computed for each period of the four data sets and included in Tables 5.7a through 5.7d.[19] The data in the tables will be compared to the frequencies that are conditional on several variables in the next section, where we turn our attention more to the determinants of the order flow. Before moving

[19]Hence, each column in the tables sums to 100.

on to the analysis of the order flow, however, we shall determine whether there in fact is a relationship between the state of the market and the event frequencies. The states of the market we look at here are the X, Y, and Z states in the market 1 environment that are defined in section 2. We test the null hypothesis concerning proportions,

$$H_0 \ : \ P_{iX} = P_{iY} = P_{iZ}, \ for \ i = 1, \dots, 10,$$

where P_{ij} is the unconditional frequency of event i in state j, with $\sum_{i=1}^{10} P_{ij} = 100$ for each of the three states. The alternative hypothesis is that the P's are not equal for at least one event type. This is equivalent of testing an existence of a dependence between the proportion of certain event type and the state of the market. We compute the following statistic for the test.

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}, \ with \ E_{ij} = \frac{(\sum_{i=1}^{r} N_{ij})(\sum_{j=1}^{c} N_{ij})}{N_{total}},$$

where O_{ij} is the observed and E_{ij} is the expected frequency of event i in state j, and r and c are the number of event types and states, respectively. The results included in Table 5.8b along with the contingency table in Table 5.8a clearly show that the null hypothesis has to be rejected in both 042393b and 042393c data sets at any reasonable level of significance. Hence, we conclude that there is a dependence between the frequencies of event types and the state of the market, so that the probability of certain event's occurrence is not the same for the three different states. We note that this result should be referred when the evidence of information dissemination in the market is investigated.

5.4 Analysis of the Order Flow

As it was mentioned earlier, we have differentiated subjects' actions into ten categories, and identify them by different event types. These event types are used as indices of the direction and the level of aggressiveness of an action.[20] The direction indicates whether a trade is buyer– or seller–initiated, and with our data sets we can classify the direction accurately. Event 1, for example, is "take ask," which results in an immediate trading at the current standing ask quote and thus is the most aggressive type of action taken by a buyer. Event 2 is the second most aggressive action by a buyer, which is "new bid > standing bid" that results in replacing the current standing bid. We call such orders market orders in our experimental markets. Event 3 and Event 4 are "new bid = standing bid" and "new bid < standing bid," respectively. These orders will be placed in the limit order book according to price priority over time priority rule, and they are called limit orders. Event 5 is "cancellation" that removes a previously entered market or limit order. The event types on the sellers' side are defined analogously.

Table 5.6 reports the unconditional frequencies of ten event types over the data pooled across 15 periods for each of the four data sets. Among the buyers' side activities, "take ask" is the most frequent event type and "new bid > standing bid" is the second most frequent event type in two of the four data sets, while the latter is the most frequent event type in the other two. In all of the four data sets, the orders away from the standing quotes,

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[20]This differentiation of event types follows the method used by Biais, Hillion, and Spatt (1993), except that they have 15 categories instead of 10. This is because in our experimental market the subjects are instructed not to enter multiple–unit orders, whereas in the Paris Bourse it is allowed as a matter of course. Therefore, Biais *et al.*'s differentiation includes the notion of the size of an order such as "large buy" and "small buy." We also do not have a category that they call "applications," which are prearranged trades put through the market at or within the best quotes.

i.e., "new bid < standing bid," occupy low frequencies. Analogously, on the sellers' side the events such as "take bid," "new ask < standing ask," and "new ask = standing ask" receive high frequencies compared to "new ask > standing ask" and "cancel ask." In general, most of the activities is within the bid–ask spread and at the standing quotes, which agrees with Biais *et al.*'s findings from the Paris Bourse data.[21] The subjects are anxious to participate actively in the trading processes instead of waiting in line to be hit by the other side of the market, reflecting the competition for price priority. The uninformed subjects are also hesitant to place orders away from the standing quote with the risk of being hit in case of unexpected informational events, whereas the informed subjects are reluctant to take the risk of revealing their privately held information too soon by letting the book carry more informaiton. Hence, the subjects seem to recognize the higher adverse selection cost associated with the quotes away from the standing quotes.

Tables 5.7a through 5.7d provide the unconditional frequencies of ten event types computed for each period of the four data sets. The data included in these tables seem to conform to the statement above. It has to be noted, however, that the event frequencies are found to be dependent on the market environment as it was discussed in the previous section. This finding can be considered as a piece of evidence that the order flow reflects the information dissemination process, and that it presumably reflects different information differently. Hence, the more of the order flow one has an access to observe, the more information one has a chance to extract. Consequently, this point confirms the contention made by Bollerslev *et al.* that the longer book seems to provide more information.

[21] Biais *et al.* also cite Harris and Hasbrouck's (1992) result on the NYSE data that is also consistent with our findings.

We depict the intraperiod frequencies of events in Figures 5.5a1 through 5.5d2. We divide each period into 10 intervals of 30 seconds, and compute the frequencies of events in one interval relative to the other intervals. In the Paris Bourse data Biais *et al.* found a prominent U-shaped pattern in the frequencies of orders and trades, in which market activities are relatively frequent in the morning and near the end of the trading day. In our data sets this pattern is not observed. In fact, in the market 1 environment it is difficult to identify if there is any pattern among the event frequencies. The markets seem to remain active throughout the trading period. This may be due to the short time limit we have set for each period. In the market 2 environment it appears that the market is relatively active right after the opening and that events gradually become less frequent as the competitive equilibrium is achieved. These figures show this contrast of markets with information asymmetry and without one clearly.

5.4.1 Frequencies of Event, Conditional on the Previous Event Type

Tables 5.9a through 5.9d document the frequencies of ten event types conditional on the previous event type that is also identified as one of the ten event types.[22] The price improvement tends to occur right after a trade; that is, "new bid > standing bid" and "new ask < standing ask" have high frequencies relative to the other events after "take ask" or "take bid" has just occurred. This observation reflects the subjects' competition for the supply of liquidity after the liquidity is consumed by the trade. The high probability of "new ask < standing ask" after "take bid" and of "new bid > standing bid" after "take ask" could also be an evidence of information effects in the order flow. After one seller observes that

[22]Hence, each row in the tables sums to 100.

another seller has accepted a standing bid, he may react to this event by entering an ask that can replace the current standing ask as a part of information adjustment process. Biais *et al.* (1993) interpret the shift in the order book due to large transactions as information effects. But they do not observe such shifts due to small transactions. Our finding, however, is an indication that traders are capable of extracting some information even from a unit transaction. The frequencies of these overbidding and undercutting event types conditional on the last event being a trade also tend to be higher than their unconditional frequencies. They also have high probabilities of occurrence after the placement of new orders at or within the standing quotes on the same side of the market, although they are not most frequent under this condition unlike Biais *et al.*'s finding in their data set.[23] This is also a reflection of the competitive behavior of the subjects for the supply of liquidity.[24]

The frequency of "take bid" by a seller is high right after an overbidding action by a buyer, and the frequency of "take ask" by a buyer is high right after an undercutting action by a seller. This shows the existence of traders waiting to pick up a more favorable offer and competing for trade execution.

The diagonal effect that was observed in Biais *et al.*'s analysis of the Paris Bourse data is also present in our data sets. The frequencies on the diagonal of each table are generally large compared to the frequencies in the other rows of the same column; that is, the same type of event tends to occur in succession.[25] In addition, they tend to be larger than the unconditional counterpart of the frequencies, meaning that the probability of a

[23]Note that the probabilities of overbidding and undercutting behavior are also high after the price improvement on the opposite side of the market.

[24]Biais *et al.* (1993) cite Ho and Stoll (1983) and Kyle (1985) for the models of competition for the supply of liquidity.

[25]Hasbrouck and Ho (1987) find strongly positive autocorrelations in the buy–sell indicator series for NYSE data.

particular event's occurrence is higher after the same type of event has just occurred than the probability expected unconditionally. This observation of positive serial correlation in event occurrence can be provided with several interpretations. The effect may reflect the subjects' behavioral pattern that they tend to interpret and react to the available information in a similar fashion, resulting in the succession of the same event type. A sequence of consecutive overbidding behavior, for example, may initiate an upward shift in the bid–ask spread leading to a permanent information adjustment. It may also reflect another behavioral pattern that some subjects learn to imitate the action of other subjects who are trusted for their accurate interpretation of information. Hence, some subjects extract information from the flow of other subjects' actions that works as a signal of what they are supposed to do. Or since the subjects are instructed to place a single–unit order at a time in our experiments, the same person may enter the same order consecutively with a short time interval in order to acquire an opportunity for trading multiple units. The diagonal effect of actions that result in an immediate trading such as "take ask" and "take bid" also reflects the positive relationship between the intensity of trade and the liquidity. If traders have discretion over the timing of their trades, they tend to bunch at times when they expect the others to be trading as well, for that is the time at which the liquidity is highest.[26]

The statistical significance of the differences between the conditional and unconditional frequencies noted in the discussion above is shown by the following argument. We calculate

[26]This is the central finding by Admati and Pfleiderer (1988). Kyle (1985) shows that this effect is enhanced with the presence of informed traders.

the multinomial chi-square sum in order to test the hypotheses,

$$H_0 \ : \ F^i_{j,k} = UF^i_k \ \forall \ k$$

$$H_a \ : \ F^i_{j,k} \neq UF^i_k \text{ for some } k,$$

where $F^i_{j,k}$ is the frequency of event k in the data set i, conditional on the previous event being j, and UF^i_k is the unconditional frequency of event k in the data set i. The χ^2 statistic is calculated for ten different last event types in each of the four data sets. For example, the χ^2 statistic for event type j in data i is

$$\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(F^i_{j,k} - UF^i_k)^2}{UF^i_k},$$

where N_j is the number of the observations with the previous event type being j. This follows the χ^2 distribution with the degrees of freedom equal to 9, which is the number of event types minus 1.[27] The computed χ^2 values are included in Table 5.10. The null hypothesis is clearly rejected for any last event type in all of the four data sets, indicating that the difference among the unconditional and conditional frequencies are statistically significant. Therefore, we can conclude that a current event type is not independent of the last event type in our experimental financial markets.

[27]The critical values for the significance levels 0.05 and 0.01 are $\chi^2_{0.05}(9df) \approx 16.919$ and $\chi^2_{0.01}(9df) \approx 21.666$, respectively.

5.4.2 Frequencies of Event, Conditional on the State of the Order Book

In this section we look at the discrepancy between the unconditional frequencies and the frequencies of event conditional on the previous state of the limit order book. The limit order book is characterized by nine different states, which depend on the relative size of bid–side and ask–side spreads and of bid–side and ask–side depths of the book. The bid–side spread is defined as the difference in francs between the standing bid quote and the lowest bid quote listed last in the limit order book.[28] The bid–side depth is defined as the number of orders listed in the limit order book plus 1 unit of the standing bid.[29] The ask–side spread and depth are defined similarly. State 1, for example, is "(bid–side spread > ask–side spread) ∧ (bid–side depth > ask–side depth)," state 2 is "(bid–side spread > ask–side spread) ∧ (bid–side depth = ask–side depth)," state 3 is "(bid–side spread > ask–side spread) ∧ (bid–side depth < ask–side depth)," and so on. The state of the book is updated whenever there is a new event. Tables 5.11a through 5.11d report the conditional frequencies, in which each row corresponds to each state.[30]

The frequencies of "take bid" tend to be high when the ask–side depth is larger than or equal to the bid–side depth, whereas those of "take ask" tend to be high when the ask–side depth is smaller than the bid–side depth. The former reflects the selling pressure with more people waiting to sell, and the latter reflects the buying pressure with more people waiting to buy. A similar pattern is observed in the undercutting and the overbidding behavior. The high probabilities of the placement of new asks within the bid–ask spread

[28] If there is no bid or only a standing bid, the spread is defined to be 0.

[29] Since only a sigle–unit order is allowed, this is equivalent of the total number of units listed in the book including 1 unit of the standing bid. For instance, the depth is 0 if there is no order, and the depth is 1 if there is only a standing bid and no order in the book.

[30] Hence, each row in the tables sums to 100.

or at the standing quotes when the ask–side depth is larger than or equal to the bid–side depth indicate that a number of the participants are anxious to gain time priority to sell, while the high probabilities of new bids within or at the market standing quotes reflect the opposite.

We conduct the χ^2 test to confirm the discrepancy between the unconditional and the conditional frequencies. The hypotheses tested are

$$
\begin{aligned}
H_0 &: BF^i_{j,k} = UF^i_k \ \forall\ k \\
H_a &: BF^i_{j,k} \neq UF^i_k \text{ for some } k,
\end{aligned}
$$

where $BF^i_{j,k}$ is the frequency of event k in the data set i, conditional on the state of the book being j, and UF^i_k is the unconditional frequency of event k in the data set i. For example, the χ^2 statistic for the book state j in data i is

$$
\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(BF^i_{j,k} - UF^i_k)^2}{UF^i_k},
$$

where N_j is the number of observations with the state of the book being j. Again, this chi-square statistic has 9 degrees of freedom, which is the number of event types minus 1. Table 5.12 contains the computed test statistics. The null hypothesis is rejected for any reasonable level of significance. Therefore, we have confirmed the presence of dependency between the order flow and the state of the order book in our experimental financial markets.

5.4.3 Frequencies of Event, Conditional on the Size of Bid–Ask Spread

In Tables 5.13a through 5.16 we define the size of the bid–ask spread to be large if it is at least as large as the time–series mean of bid–ask spreads through 15 periods, and to be small otherwise.[31] The frequencies of "take ask" and "take bid" are relatively high when a bid–ask spread is smaller than the mean bid–ask spread; that is, trades tend to occur when a bid–ask spread is tight. The frequencies of "new bid > standing bid" and "new ask < standing ask" are relatively high when a bid–ask spread is larger than or equal to the mean bid–ask spread; that is, overbidding and undercutting behavior tend to occur when a bid–ask spread is large. Other events such as new orders away from the standing quote do not appear to be affected by the size of bid–ask spread. These findings are consistent with Biais *et al.*'s findings on the Paris Bourse data.

We conduct a χ^2 test again to see the statistical significance of the discrepancy between the conditional and unconditional frequencies of the events. The hypotheses tested are

$$H_0 \ : \ SF^i_{j,k} = UF^i_k \ \forall \ k$$

$$H_a \ : \ SF^i_{j,k} \neq UF^i_k \text{ for some } k,$$

where $SF^i_{j,k}$ is the frequency of event k in period i, conditional on the size of the bid–ask spread right before the event being j, and UF^i_k is the unconditional frequency of event k in period i. For example, the χ^2 statistic for the bid–ask spread size j in period i is

$$\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(SF^i_{j,k} - UF^i_k)^2}{UF^i_k},$$

[31] Note that in Tables 5.13a' and 5.13b' the mean is computed for X–, Y–, and Z–state periods.

where N_j is the number of observations with the bid–ask spread being j. This chi-square has nine degrees of freedom, which is the number of event types minus one. As it is clear by observing the computed χ^2 statistics reported in Table 5.14, we can reject the null hypothesis at any reasonable level of significance. Hence, we can conclude that the size of bid–ask spreads affects the frequencies of different events in our experimental financial markets.

5.4.4 Time Intervals between Events

The distributions of time intervals between two consecutive events are graphed in Figure 5.8a through Figure 5.8d. Over 50 percent of all events occur in 2 seconds after another event has occurred in the market 1 environment, and about 50 percent in 3 seconds in the market 2 environment. Part of the short time intervals may be due to the stringent time limit we set for each period, or may be reflecting quick responses of subjects to observed events in competition for time priority. The frequency of events almost monotonically decreases as the time interval increases. Hausman *et al.* (1992) studied 1988 transactions data for selected U.S. stocks, and found similar patterns in the depiction of the time–between–trades for the stocks with relatively large market capitalization.[32] Biais *et al.*'s empirical distribution of the time intervals of the Paris Bourse data also showed the similar pattern.

Tables 5.15a and 5.15b show mean time intervals between two events conditional on the last event type, the state of the order book, the size of bid–ask spread, and the size of the last time interval. The size of the last time interval is defined to be large if it is larger than

[32]In their sample derived from the Institute for the Study of Security Markets database, International Business Machines Corporation has the largest market capitalization with a market value of $69.8 billion, and Handy and Harman Company has the smallest. The smaller, less liquid stocks showed a very different pattern.

or equal to the time series mean of time intervals computed from the data pooled across 15 periods, and to be small otherwise. The mean time interval after a small time interval is smaller than that after a large time interval in all of the four data sets. This suggests the existence of an alternating pattern of intense and sparse activities during a trading period. This pattern was also observed by Biais *et al.* The differences between the means conditional on the last time interval are shown to be statistically significant below. The mean time interval after the large bid–ask spread is larger than that after the small bid–ask spread in three of the four data sets. This contradicts the findings by Biais *et al.* on the Paris Bourse data. But in two out of the three cases the difference is not significant.

Mean time intervals after order placements at standing quotes are short, reflecting that some subjects wishing to trade multiple units at the same price enter multiple orders for a single unit of the same price within a short time.[33] An event after a cancellation tends to occur quickly, too. In fact, another cancellation often follows one cancellation with a short time interval, reflecting a quick response of other observant subjects or a sequence of multiple cancellations by the same subject to incorporate newly acquired information into their decisions. On the other hand, mean time intervals after a transaction tend to be larger than their unconditional counterpart. The intervals after overbidding or undercutting events are smaller than the intervals after a transaction in 042393c data, but they are larger in 042393b data though not by much.[34]

The intervals conditional on the state of the book indicate that an event tends to occur quickly when the state of the book is strongly asymmetric between the bid–side and the

[33] Note again that in our experiments the subjects are not allowed to enter multiple–unit orders.

[34] In Biais *et al.*'s analysis of the Paris Bourse data, they found that the mean time interval is shortest after market sell orders, and relatively short after market buy orders.

ask–side. In other words, if a difference between the bid–side and the ask–side spread is large at the same time when a difference between the bid–side and the ask–side depth is large, then the next event tends to occur in short time. If those differences are small, the time interval before the next event tends to be large. This is the indication that the subjects interpret that a shape of a book may become very asymmetric in a process of information assimilation, reflecting a significant amount of private information and emitting a signal that the current price needs to incorporate the information quickly. Hence, it results in the subjects' reacting to their observation quickly.

In order to support the above observations formally, we perform an analysis of variance to test each of the following hypotheses.

$$(A)\ H_0 \ : \ \mu_{event1} = \mu_{event2} = \cdots = \mu_{event10}$$

$$(B)\ H_0 \ : \ \mu_{state1} = \mu_{state2} = \cdots = \mu_{state10}$$

$$(C)\ H_0 \ : \ \mu_{spreadlg} = \mu_{spreadsm}$$

$$(D)\ H_0 \ : \ \mu_{intervallg} = \mu_{intervalsm},$$

where μ_{event1}, μ_{state1}, $\mu_{spreadlg}$, and $\mu_{intervallg}$ are the mean time intervals conditional on the previous event type being 1, the state of the book being 1, the size of bid–ask spread being large, and the previous time interval being large, respectively, and the others are defined similarly. Computed F ratios for the treatments are reported in Table 5.16. According to these statistics, it is difficult to conclude that the size of the previous bid–ask spread has a significant influence over the time interval, for the null hypothesis (C) cannot be rejected in three of the four data sets. On the other hand, in three out of the four data sets the null hypothesis (B) is rejected, indicating the time interval may be influenced by the state of the book. In addition, the time intervals are clearly dependent on the previous event type and the last time interval. Hence, we may conclude that the time interval between two trades or events carries information that is valuable to the market participants' decisions.

It is also interesting to inspect Tables 5.17a and 5.17b, which contain mean time intervals jointly conditional on the previous event type, the state of the book, and the size of the last time interval.

5.5 Concluding Remarks

We conclude this chapter by describing potential research interests on our data sets.

Prices provide an important source of information along with other variables. In fact, it is a common practice for securities' traders to study price changes in forming their investment decisions. Our empirical investigation on the determinants of the order flow indicates that the complex nature of order placements is closely related to price dynamics. It brings us to study how this interdependence between the order flow and the price dymanics functions. We need to analyze the movement of intra–period price changes conditional on the history of order flow, whereas many previous works have focused on unconditional distribution of price changes.[35] In so doing discreteness of price changes should not be ignored especially for intraperiod price movements, since such finely–sampled price changes may take on only several distinct values.

As it was indicated in the previous section, orders arrive in varied time intervals. Since

[35] Easley and O'Hara (1987) find that the order flow affects the conditional distribution of the next price change.

the order flow is not independent of the time intervals, the information contained in the length of time between, for example, two transactions may have a significant influence over price dynamics. In the process of the model specification, we need to incorporate a variable that reflects the information in the intervals.

In addition, we may include independent variables that describe the state of the order book on the ask side and on the bid side. For example, first we can examine several simple regression models without applying a time–series model to the residual series and analyze each model for cases in which independent variables are at $t+1$, $t+2$, and so on. Then choose the model with the best fit and use it to construct a combined regression–time-series model. In other words, give an ARIMA specification of the residual series. One of the goals is to conclude whether or not the state of the book is a good indicator of future transaction prices. We conjecture that the order book carries a significant amount of information, so that the presence of the book enhances the better forecasting by traders. We can repeat the similar procedure with other independent variables such as bid–ask spreads.

Another interesting extension is to investigate how traders' private information regarding a liquidation value of risky assets becomes conveyed into market variables such as transaction prices in laboratory asset markets, where asymmetrically informed traders can submit both market and limit orders. We are particularly interested in analyzing how the presence of the limit order book contributes to the process of disseminating the traders' private information. Previous experimental investigations regarding information dissemination focuses on the analysis of efficiency measures. They don't look at second–by–second movement of activities and variables. We feel the need to look for the sign of information revelation in the order flow and its determinant variables. By giving a closer look at the order flow and investigating the intraperiod pattern of the variables that are empirically proven to be the determinants of the order flow in the previous sections, we should be able to show the sign of information dissemination in the experimental asset markets that are not accommodated with the sufficient conditions given by Forsythe and Lundholm (1990).[36] In other words, Rational Expectation equilibria do not have to be achieved to be able to conclude that the market has provided information dissemination institution. Instead, look at the movement of, for example, a bid–ask spread, and look for the sign of information dissemination based on the previous theoretical findings and empirical findings on the real data. Each period may have different insiders and a different subject may have a different way of revealing one's private information, indicating that it may be harder for others to extract information when a certain person is an insider. Hence, every period doesn't necessarily converge to a Rational Expectation equilibrium price within the limited length of time. But can we reject Rational Expectation hypothesis just because transaction prices have not converged to what it predicts? We argue that the answer is no. We can find the information dissemination process among the order flow and other variables that affect the order flow. Hence, it's possible that information is revealed right before a given period ends and thus the fact that information has been successfully revealed is not reflected in the equilibrium price yet. We need to look at other variables to find the sign of revelation.

Though Rational Expectation equilibrium is not really achieved, we may still be able to show the evidence of information dissemination by looking at the shifts in the order book. If the shift of bid–ask spread is not transient, reflecting the permanent adjustment

[36]Forsythe *et al.* found, through the analysis of experimental data, that participants' trading experience and common knowledge of dividends are jointly sufficient to achieve a Rational Expectation equilibrium.

to the newly revealed information, then we may claim that it is information effects and is the evidence of the information dissemination process. We may be able to observe from figures the movement of intraperiod bid–ask spread for a few periods of X, Y, and Z that reflects the permanent adjustment. We provided four examples related to this topic in figures 5.7a1, 5.7a2, 5.7b1, and 5.7b2. The different sizes of the spreads may indicate that bid-ask spreads are larger for assets where information asymmetries are more pervasive. It would be worthwhile to verify empirically that information asymmetries affect the ability of executing trades. We can also compare standard deviations of price changes (or transaction prices) between early part and late part within each period. We hope to see $\sigma_{early} > \sigma_{late}$. We should also comment on observed information mirage or bubbles as a consequence of speculative behavior.

Yet another interesting extension is to investigate the concept of dynamic equilibrium in laboratory experiments.[37] This will require us to make a slight modification of the experimental design. Suppose that there are two types of traders in equilibrium in the market that form two different trends. For example, we may design the experiment in which one type is strongly influenced by a short–term view and the other's behavior is affected by a long–term prospect. We may be able to observe two different types of fair valuation of assets, one mostly reflecting the short–term information, and the other reflecting the state after the market has fully adjusted for the information. In such a case the equilibrium may shift between the two fair valuations with a certain length of time interval that is required for the

[37] We computed the Hurst exponent with our data sets to see the evidence of relaxation processes. Indeed the computed exponent indicated 1/f noise in our data. But it is still inconclusive, since we had to aggregate the data for all the 15 periods. We included the step–by–step computation process of the Hurst exponent as a reference in Appendix 5C.

market to absorb the information shocks. In other words, we should design the experiment in such a way that the same information may affect different subjects differently depending on each subject's investment horizon. We believe that analyzing the price dynamics and other details of the order flow in such trading environments may help us understand a part of the stock price movements that are sometimes hard to explain within the framework of the Efficient Market Hypothesis.

Appendix 5A

Tables for Chapter 5

Table 5.1a

Summary of the Experimental Design (Market 1)

Experiment Number	Book	Short Sales	Initial Endowment Certificates	Initial Endowment Francs	Dollars per Franc	Number of Traders	Number of Insiders	Trader Types	Dividends X	Dividends Y	Dividends Z	Minutes per Period	Subjects' Experience	Dividends Common Knowledge	Uncertainty of Insiders
042393b	Yes	No	8	10,000	0.001	8	3	I	120	330	40	5	None	No	No
								II	205	90	125				
042393c	Yes	No	8	10,000	0.001	8	3	I	120	330	40	5	None	No	No
								II	205	90	125				

Table 5.1b

States of Trading Periods (Market 1)

Trading Periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
States	Y	X	Z	Z	Y	X	Y	X	X	Z	X	Y	Y	Z	X

Table 5.1c

Summary of the Experimental Design (Market 2)

Experiment Number	Book	Short Sales	Initial Units	Endowment Francs	Dollars per Franc	Number of Traders	Trader Types	Minutes per Period	Subjects' Experience
042393b	Yes	No	10	10,000	0.003	8	4 types of buyers and 4 types of sellers as indicated in Tables 5.2b and 5.2c	5	None
042393c	Yes	No	10	10,000	0.003	8	4 types of buyers and 4 types of sellers as indicated in Tables 5.2b and 5.2c	5	None

Table 5.1d
Equilibrium Predictions in Market 1

		With Insiders							Without Insiders	
Market		*X*	*Y*	*Z*	*X*	*Y*	*Z*		Any State	Any State
042393b,c	RE	205	330	125	II	I	II	RE	163.33	I
	PI	205	330	163	II	I	II	PI	163.33	I
	MM	205	330	125	II	I	II	MM	90	II

Table 5.2a

State Contingent Dividend Parameters in Market 1

	State X	State Y	State Z
Type I	120	330	40
Type II	205	90	125

Table 5.2b

Buyers' Redemption Values in Market 2

Unit	Type 1	Type 2	Type 3	Type 4
1	230	224	234	238
2	196	190	204	212
3	162	156	170	178
4	128	138	138	146
5	94	128	108	116
6	60	100	74	82
7	26	66	40	48
8	10	32	10	10
9	10	10	10	10
10	10	10	10	10

Table 5.2c

Sellers' Inventory Use Costs in Market 2

Unit	Type 1	Type 2	Type 3	Type 4
1	32	28	28	36
2	62	54	48	70
3	96	88	82	104
4	128	120	116	138
5	158	150	144	172
6	192	184	178	206
7	226	218	212	240
8	245	250	250	245
9	250	255	255	250
10	255	260	260	255

Table 5.2d

Types of Each Subject in Market 1 and Market 2

Subject ID #	Market 1	Market 2
0	Type I	Buyer Type 1
1	Type I	Seller Type 1
2	Type I	Buyer Type 2
3	Type I	Seller Type 2
4	Type II	Buyer Type 3
5	Type II	Seller Type 3
6	Type II	Buyer Type 4
7	Type II	Seller Type 4

Table 5.3a

Selective Descriptive Statistics on Market Activities (Data 042393b : Market 1)

	Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Trading Volume	18	14	20	20	36	25	32	22	38	26	30	39	28	17	25
Number of Events	41	58	61	57	92	69	71	71	94	95	98	152	86	94	100
Transaction Price (Francs)															
High	410	170	150	200	300	200	250	219	225	140	225	300	100	150	150
Low	100	100	30	100	120	150	130	150	100	5	136	150	80	100	80
Mean	221.11	130.00	74.25	143.25	172.14	186.48	194.03	170.36	184.08	81.65	189.93	214.92	91.96	116.47	111.92
Std. dev.	71.69	20.00	30.79	23.41	36.07	16.47	32.93	19.59	28.47	22.13	18.53	33.84	4.70	13.20	17.21
Price Change (Francs)															
Mean	9.41	1.54	1.05	1.58	4.00	-0.83	1.45	-2.10	1.49	-1.60	2.72	3.68	0.37	1.88	1.67
Std. dev.	94.30	23.04	39.67	30.46	14.26	10.63	45.83	14.33	24.94	31.36	11.97	37.17	4.89	19.05	22.25
Bid-Ask Spread (Francs)															
Mean	104.02	40.86	32.54	104.26	65.77	92.58	107.17	78.45	70.80	53.11	40.33	56.11	36.77	47.16	56.20
Std. dev.	115.80	26.18	40.12	67.62	40.62	48.30	44.38	29.73	40.05	36.89	22.73	42.02	33.65	34.21	40.81
Time between Events (Seconds)															
Mean	7.39	6.29	4.89	5.33	3.15	4.32	4.25	4.23	3.19	3.26	3.00	2.01	3.40	3.26	3.06
Std. dev.	13.33	7.96	10.09	5.95	3.93	6.88	4.20	4.01	6.09	3.22	5.73	3.59	5.48	5.75	6.16

Table 5.3b

Selective Descriptive Statistics on Market Activities (Data 042393c : Market 1)

	Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Trading Volume															
	18	22	20	14	24	13	31	15	23	19	27	32	25	12	21
Number of Events															
	37	59	49	42	61	80	79	66	64	73	116	78	60	34	56
Transaction Price															
(Francs)															
High	165	170	120	150	230	199	250	180	180	145	200	250	200	149	170
Low	100	89	95	115	90	110	145	140	125	50	145	141	150	70	125
Mean	119.39	137 91	109.20	122.50	188.13	148.39	164.68	161.00	161.96	110.00	162.11	184.03	165.60	97.58	150.48
Std. dev.	23.72	18.81	6.13	8.49	35.44	27.64	18.62	12.98	15.86	19.29	15.29	31.21	12.94	24.73	8.93
Price Change															
(Francs)															
Mean	3.82	3.86	1.05	1.92	6.09	3.33	1.17	2.86	1.14	-1.94	1.62	2.26	0.83	-1.82	0.00
Std. dev.	22.11	15.15	7.89	7.78	20.22	31.33	21.32	7.26	14.30	24.56	11.02	35.00	12.99	38.24	9.03
Bid-Ask Spread															
(Francs)															
Mean	42.24	24.07	22.47	8.48	48.85	93.50	21.41	78.02	39.44	45.60	16.70	48.42	30.37	46.35	32.86
Std. dev.	24.78	23.81	29.53	7.58	42.36	58.72	14.29	95.01	19.24	38.14	17.31	44.22	25.20	21.76	15.92
Time between Events															
(Seconds)															
Mean	8.11	5.10	5.76	7.05	4.97	3.74	3.78	4.48	4.70	4.08	2.55	3.87	5.05	8.91	5.39
Std. dev.	9.27	8.57	6.61	7.90	5.55	5.22	4.12	6.08	6.34	4.93	4.56	4.27	5.19	10.70	7.17

Table 5.3c

Selective Descriptive Statistics on Market Activities (Data 042393b : Market 2)

	Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Trading Volume	9	12	13	16	16	15	12	13	14	14	14	14	13	12	13
Number of Events	43	53	51	57	53	49	47	59	56	42	41	33	48	44	46
Transaction Price (Francs)															
High	165	170	160	158	143	150	139	144	140	137	136	135	133	149	130
Low	140	130	130	125	110	122	120	130	125	120	120	120	20	121	120
Mean	156.56	149.33	147.85	138.50	133.00	135.13	129.33	137.54	134.36	129.64	128.86	129.14	120.00	135.83	127.46
Std. dev.	8.17	10.91	11.17	9.32	9.13	10.23	8.52	4.37	5.67	5.58	5.17	5.27	30.43	11.38	7.41
Price Change (Francs)															
Mean	0.50	0.18	-2.50	-0.33	-2.00	-0.21	0.00	0.75	-0.31	-0.77	-0.62	0.46	9.42	2.18	1.17
Std. dev.	12.33	14.74	17.65	9.07	10.39	7.66	12.47	3.02	5.91	3.81	6.10	6.75	30.52	5.56	8.64
Bid-Ask Spread (Francs)															
Mean	30.77	23.17	24.10	16.91	16.21	17.73	16.74	12.64	14.63	9.29	16.73	14.03	11.65	39.84	16.61
Std. dev.	19.94	14.51	8.44	10.82	12.81	7.92	15.68	9.31	7.01	5.85	10.93	18.71	7.42	1.88	4.35
Time between Events (Seconds)															
Mean	7.23	5.63	5.80	5.30	5.77	6.47	6.43	5.11	5.36	7.24	7.40	9.18	6.38	6.95	7.33
Std. dev.	13.52	9.04	9.02	6.59	4.60	7.98	6.66	6.40	8.00	11.23	11.09	10.38	8.05	13.26	14.56

Table 5.3d

Selective Descriptive Statistics on Market Activities (Data 042393c : Market 2)

	Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Trading Volume	11	11	13	12	13	11	12	14	13	14	13	13	12	14	15
Number of Events	42	46	41	41	37	32	34	58	35	50	45	51	47	52	54
Transaction Price (Francs)															
High	200	200	175	175	175	170	174	165	160	160	170	155	155	155	155
Low	140	150	145	140	145	145	142	140	142	120	140	120	140	120	130
Mean	164.55	166.82	157.92	157.00	159.08	157.18	155.92	153.64	152.54	148.36	152.77	146.31	147.92	145.14	147.20
Std. dev.	21.15	19.40	11.94	12.02	10.92	9.02	8.76	7.15	5.64	9.83	9.20	9.65	4.32	9.94	6.56
Price Change (Francs)															
Mean	-6.00	-5.00	-2.50	-3.18	0.00	-1.50	-1.18	-1.92	-1.50	-3.08	-2.50	-2.92	0.00	-2.31	-1.71
Std. dev.	14.30	10.27	9.29	10.98	14.62	9.28	12.19	5.16	3.68	7.45	7.18	8.81	5.85	6.16	4.16
Bid-Ask Spread (Francs)															
Mean	98.79	66.67	7.34	19.93	5.27	11.25	12.50	12.33	8.80	8.88	9.44	11.29	15.04	7.85	6.85
Std. dev.	32.46	28.12	9.05	14.22	7.07	10.58	10.42	8.53	5.89	7.44	5.86	8.15	9.66	7.82	5.80
Time between Events (Seconds)															
Mean	7.38	6.59	7.40	7.22	8.22	7.69	8.24	5.21	8.54	6.26	6.43	5.88	6.13	6.02	5.61
Std. dev.	7.73	10.64	9.63	6.56	11.37	8.02	19.71	6.63	12.14	9.78	8.56	9.26	6.85	8.32	9.98

Table 5.4

Statistics for Tests Concerning Equality of Selected Variables between Market 1 and Market 2

(A) $H_0 : \sigma^2_{tr1} = \sigma^2_{tr2}$; $H_a : \sigma^2_{tr1} > \sigma^2_{tr2}$

$$F = \frac{\sigma^2_{tr1}}{\sigma^2_{tr2}} \approx 8.348 > 1.130 \approx F_{0.05}(705, 390)$$

(B) $H_0 : \sigma^2_{pc1} = \sigma^2_{pc2}$; $H_a : \sigma^2_{pc1} > \sigma^2_{pc2}$

$$F = \frac{\sigma^2_{pc1}}{\sigma^2_{pc2}} \approx 6.832 > 1.140 \approx F_{0.05}(675, 360)$$

(C) $H_0 : \sigma^2_{ba1} = \sigma^2_{ba2}$; $H_a : \sigma^2_{ba1} > \sigma^2_{ba2}$

$$F = \frac{\sigma^2_{ba1}}{\sigma^2_{ba2}} \approx 4.779 > 1.000 \approx F_{0.05}(2192, 1386)$$

(D) $H_0 : \mu_{pc1} = \mu_{pc2}$; $H_a : \mu_{pc1} > \mu_{pc2}$

$$Z = \frac{\mu_{pc1} - \mu_{pc2}}{\sqrt{\frac{\sigma^2_{pc1}}{N_{pc1}} + \frac{\sigma^2_{pc2}}{N_{pc2}}}} \approx 2.244 > 1.645 \approx Z_{0.05}$$

(E) $H_0 : \mu_{ba1} = \mu_{ba2}$; $H_a : \mu_{ba1} > \mu_{ba2}$

$$Z = \frac{\mu_{ba1} - \mu_{ba2}}{\sqrt{\frac{\sigma^2_{ba1}}{N_{ba1}} + \frac{\sigma^2_{ba2}}{N_{ba2}}}} \approx 27.910 > 1.645 \approx Z_{0.05}$$

Table 5.5a

ANOVA Tables for Tests Concerning Difference among X, Y, and Z States
(Data 042393b : Market 1)

(A) $H_0 : \mu_{trX} = \mu_{trY} = \mu_{trZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2,\ 387)$
States	2	3.39882e+005	1.69941e+005	80.25	3.02
Residuals	387	8.19558e+005	2.11772e+003		
Total	389	1.15944e+006			

(B) $H_0 : \mu_{pcX} = \mu_{pcY} = \mu_{pcZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2,\ 372)$
States	2	6.10750e+002	3.05375e+002	0.29	3.02
Residuals	372	3.90453e+005	1.04961e+003		
Total	374	3.91064e+005			

(C) $H_0 : \mu_{baX} = \mu_{baY} = \mu_{baZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2,\ 1236)$
States	2	1.92890e+004	9.64450e+003	3.96	2.99
Residuals	1236	3.00852e+006	2.43408e+003		
Total	1238	3.02781e+006			

Table 5.5b

ANOVA Tables for Tests Concerning Difference among X, Y, and Z States
(Data 042393c : Market 1)

(A) $H_0 : \mu_{trX} = \mu_{trY} = \mu_{trZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2, 313)$
States	2	1.45682e+005	7.28410e+004	110.87	3.03
Residuals	313	2.05636e+005	6.56984e+002		
Total	315	3.51318e+005			

(B) $H_0 : \mu_{pcX} = \mu_{pcY} = \mu_{pcZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2, 298)$
States	2	3.27186e+002	1.63593e+002	0.40	3.03
Residuals	298	1.23145e+005	4.13238e+002		
Total	300	1.23472e+005			

(C) $H_0 : \mu_{baX} = \mu_{baY} = \mu_{baZ}$

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean square*	F	$F_{0.05}(2, 951)$
States	2	3.06200e+004	1.53100e+004	7.50	3.00
Residuals	951	1.94038e+006	2.04036e+003		
Total	953	1.97100e+006			

Table 5.6

Unconditional Frequencies of Events

Data → Event Type ↓	042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Take ask	22.14	17.68	13.30	16.15
New bid > standing bid	12.58	21.73	18.67	13.08
New bid = standing bid	10.70	20.02	3.11	4.15
New bid < standing bid	2.61	0.64	3.68	1.23
Cancel bid	1.31	1.06	6.79	1.85
Take bid	9.72	15.97	14.99	13.23
New ask < standing ask	21.32	15.76	23.20	24.15
New ask = standing ask	11.76	4.69	10.33	11.38
New ask > standing ask	5.64	1.06	3.96	6.31
Cancel ask	2.21	1.38	1.98	8.46

The number of each event's occurrence is added across 15 periods for each data set, and divided by the total number of observations to obtain the percentage frequencies.

Table 5.7a

Unconditional Frequencies of Events by Period (Data 042393b : Market 1)

	Event Type	Unconditional Frequency (%)														
		Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Buy Side	Take ask	34.146	8.621	6.557	29.825	36.957	23.188	36.620	30.986	35.106	7.368	26.531	23.684	8.140	12.766	12.000
	New bid > standing bid	12.195	20.690	31.148	8.772	10.870	21.739	15.493	4.225	11.702	20.000	12.245	7.895	8.140	3.191	14.000
	New bid = standing bid	0.000	15.517	0.000	0.000	10.870	1.449	0.000	1.408	6.383	0.000	18.367	37.500	20.930	7.447	4.000
	New bid < standing bid	0.000	0.000	0.000	10.526	0.000	2.899	1.408	12.676	0.000	2.105	0.000	0.658	9.302	1.064	2.000
	Cancel bid	0.000	17.241	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.053	0.000	3.289	0.000	0.000	0.000
Sell Side	Take bid	9.756	15.517	26.230	5.263	2.174	13.043	8.451	0.000	5.319	20.000	4.082	1.974	24.419	5.319	13.000
	New ask < standing ask	36.585	17.241	16.393	40.351	29.348	26.087	23.944	32.394	19.149	18.947	18.367	19.737	19.767	17.021	12.000
	New ask = standing ask	2.439	0.000	19.672	3.509	2.174	7.246	8.451	12.676	17.021	2.105	15.306	1.316	0.000	34.043	40.000
	New ask > standing ask	4.878	5.172	0.000	1.754	6.522	4.348	5.634	2.817	5.319	26.316	1.020	3.289	9.302	2.128	2.000
	Cancel ask	0.000	0.000	0.000	0.000	1.087	0.000	0.000	2.817	0.000	2.105	4.082	0.658	0.000	17.021	1.000

Table 5.7b

Unconditional Frequencies of Events by Period (Data 042393c : Market 1)

	Event Type	Unconditional Frequency (%)														
		Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Buy Side	Take ask	18.919	25.424	30.612	11.905	1.639	3.750	16.456	7.576	29.688	21.918	17.241	15.385	20.000	11.765	33.929
	New bid > standing bid	27.027	15.254	8.163	28.571	42.623	17.500	32.911	18.182	15.625	10.959	16.379	41.026	26.667	14.706	16.071
	New bid = standing bid	5.405	18.644	14.286	26.190	9.836	48.750	1.266	37.879	17.188	16.438	43.103	0.000	1.667	20.588	8.929
	New bid < standing bid	0.000	0.000	0.000	0.000	1.639	0.000	2.532	0.000	0.000	0.000	0.000	1.282	0.000	5.882	0.000
	Cancel bid	0.000	0.000	0.000	0.000	0.000	3.750	0.000	0.000	0.000	8.219	0.000	0.000	0.000	0.000	1.786
Sell Side	Take bid	29.730	11.864	10.204	21.429	37.705	12.500	22.785	15.152	6.250	4.110	6.034	25.641	21.667	23.529	3.571
	New ask < standing ask	16.216	23.729	30.612	7.143	4.918	11.250	10.127	12.121	21.875	27.397	8.621	10.256	23.333	23.529	26.786
	New ask = standing ask	0.000	5.085	6.122	4.762	1.639	0.000	6.329	3.030	7.813	4.110	8.621	6.410	0.000	0.000	8.929
	New ask > standing ask	2.703	0.000	0.000	0.000	0.000	0.000	3.797	1.515	1.563	4.110	0.000	0.000	1.667	0.000	0.000
	Cancel ask	0.000	0.000	0.000	0.000	0.000	2.500	3.797	4.545	0.000	2.740	0.000	0.000	5.000	0.000	0.000

Table 5.7c

Unconditional Frequencies of Events by Period (Data 042393b : Market 2)

	Event Type	Unconditional Frequency (%)														
		Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Buy Side	Take ask	13.953	11.321	17.647	12.281	16.981	8.163	10.638	11.864	10.714	14.286	14.634	21.212	14.583	13.636	6.522
	New bid > standing bid	18.605	28.302	11.765	19.298	16.981	28.571	14.894	20.339	14.286	16.667	26.829	21.212	16.667	22.727	17.391
	New bid = standing bid	0.000	0.000	0.000	0.000	0.000	0.000	4.255	1.695	7.143	0.000	4.878	0.000	8.333	4.545	15.217
	New bid < standing bid	0.000	1.887	0.000	5.263	7.547	2.041	0.000	0.000	5.357	2.381	9.756	6.061	6.250	2.273	6.522
	Cancel bid	0.000	11.321	1.961	7.018	7.547	6.122	4.255	8.475	8.929	0.000	7.317	0.000	12.500	6.818	13.043
Sell Side	Take bid	6.977	11.321	7.843	15.789	13.208	22.449	14.894	10.169	14.286	19.048	19.512	21.212	12.500	13.636	21.739
	New ask < standing ask	13.953	11.321	35.294	24.561	30.189	24.490	31.915	22.034	28.571	30.952	17.073	24.242	16.667	29.545	10.870
	New ask = standing ask	25.581	24.528	21.569	12.281	3.774	4.082	17.021	11.864	5.357	7.143	0.000	6.061	2.083	0.000	6.522
	New ask > standing ask	11.628	0.000	3.922	3.509	3.774	4.082	2.218	8.475	0.000	9.524	0.000	0.000	8.333	0.000	2.174
	Cancel ask	9.302	0.000	0.000	0.000	0.000	0.000	0.000	5.085	5.357	0.000	0.000	0.000	2.083	6.818	0.000

Table 5.7d

Unconditional Frequencies of Events by Period (Data 042393c : Market 2)

	Event Type	Unconditional Frequency (%)														
		Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7	Per 8	Per 9	Per 10	Per 11	Per 12	Per 13	Per 14	Per 15
Buy Side	Take ask	26.190	23.913	17.073	17.073	27.027	21.875	20.588	10.345	22.857	10.000	11.111	13.725	4.255	13.462	9.259
	New bid > standing bid	9.524	4.348	12.195	9.756	5.405	12.500	8.824	10.345	8.571	18.000	13.333	11.765	39.298	17.308	14.815
	New bid = standing bid	0.000	4.348	2.439	4.878	5.405	6.250	5.882	6.987	5.714	0.000	4.444	0.000	2.128	0.000	12.963
	New bid < standing bid	0.000	0.000	0.000	0.000	0.000	0.000	2.941	1.724	2.857	0.000	0.000	1.961	6.383	0.000	1.852
	Cancel bid	0.000	0.000	0.000	0.000	0.000	3.125	0.000	5.172	0.000	0.000	0.000	1.961	0.000	3.846	9.259
Sell Side	Take bid	0.000	0.000	14.634	12.195	8.108	12.500	14.706	13.793	14.286	18.000	17.778	11.765	21.277	13.462	18.519
	New ask < standing ask	47.619	41.304	36.585	34.146	35.135	28.125	20.588	20.690	25.714	18.000	15.556	19.608	10.638	19.231	16.667
	New ask = standing ask	7.143	15.217	12.195	9.756	8.108	9.375	5.882	13.793	11.429	18.000	20.000	5.882	6.383	15.385	5.556
	New ask > standing ask	4.762	8.696	2.439	7.317	2.703	6.250	20.588	12.069	5.714	0.000	0.000	15.686	0.000	1.923	5.556
	Cancel ask	4.762	2.174	2.439	4.878	8.108	0.000	0.000	5.172	2.857	18.000	17.778	17.647	10.638	15.385	5.556

Table 5.8a : Cross-tabulation of Event Types by State

Data 042393b : Market 1

State → Event Type ↓	X	Y	Z
Take ask	114 23.3	117 26.5	40 13.0
New bid > standing bid	67 13.7	45 10.2	46 15.0
New bid = standing bid	39 8.0	85 19.2	7 2.3
New bid < standing bid	13 2.7	10 2.3	9 2.9
Cancel bid	10 2.0	5 1.1	1 0.3
Take bid	40 8.2	36 8.1	43 14.0
New ask < standing ask	99 20.2	106 24.0	67 21.8
New ask = standing ask	85 17.3	11 2.5	48 15.6
New ask > standing ask	16 3.3	25 5.7	28 9.1
Cancel ask	7 1.4	2 0.5	18 5.9

Data 042393c : Market 1

State → Event Type ↓	X	Y	Z
Take ask	81 18.4	45 14.3	40 20.2
New bid > standing bid	73 16.6	110 34.9	29 14.6
New bid = standing bid	141 32.0	10 3.2	37 18.7
New bid < standing bid	0 0.0	4 1.3	2 1.0
Cancel bid	4 0.9	0 0.0	6 3.0
Take bid	40 9.1	85 27.0	25 12.6
New ask < standing ask	70 15.9	39 12.4	46 23.2
New ask = standing ask	25 5.7	11 3.5	8 4.0
New ask > standing ask	2 0.5	5 1.6	3 1.5
Cancel ask	5 1.1	6 1.9	2 1.0

(Each cell contains the number of observations in integer form and the column percentage of observations.)

Table 5.8b : Statistics for Tests Concerning Equality of Unconditional Event Frequencies among Three Different States
($H_0 \,:\, P_{iX} = P_{iY} = P_{iZ}, \;\; for\; i = 1, \ldots, 10$)

Data 042393b : Market 1

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{3} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} \approx 176.344 > 34.805 \approx \chi^2_{0.01}(18)$$

Data 042393c : Market 1

$$\chi^2 = \sum_{i=1}^{10} \sum_{j=1}^{3} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} \approx 192.687 > 34.805 \approx \chi^2_{0.01}(18)$$

Table 5.9a

Frequencies of Events, Conditional on the Type of the Previous Event (Data 042393b : Market 1)

Event Type t / $t-1$	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	38.491	11.321	0.000	3.019	0.000	4.528	38.113	1.132	1.887	1.509
New bid > standing bid	8.280	14.013	21.019	2.548	0.637	26.752	21.019	1.274	3.822	0.637
New bid = standing bid	6.202	1.550	74.419	1.550	0.000	2.326	12.403	0.000	0.775	0.775
New bid < standing bid	19.355	16.129	0.000	25.806	3.226	6.452	25.806	0.000	3.226	0.000
Cancel bid	0.000	12.500	0.000	0.000	75.000	0.000	12.500	0.000	0.000	0.000
Take bid	18.644	22.034	0.000	4.237	0.000	27.119	21.186	1.695	4.237	0.847
New ask < standing ask	31.227	16.729	0.743	1.487	0.372	5.204	18.587	15.242	9.665	0.743
New ask = standing ask	13.986	3.497	0.000	0.000	0.000	6.294	7.692	67.133	1.399	0.000
New ask > standing ask	18.841	20.290	0.000	1.449	0.000	5.797	17.391	0.000	33.333	2.899
Cancel ask	11.111	11.111	0.000	0.000	3.704	3.704	11.111	0.000	0.000	59.259

The total number of each event conditional on the last event type j in 15 periods is divided by the total number of all events conditional on the last event type j in 15 periods to give a percentage frequency, where j corresponds to the conditioning variable in each row. Therefore, each row sums to 100.

Table 5.9a' : Frequencies of Events by State, Conditional on the Type of the Previous Event (Data 042393b : Market 1)

Event Type $t-1$	t	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	X	40.179	10.714	0.000	5.357	0.000	4.464	33.036	2.679	1.786	1.786
	Y	39.474	9.649	0.000	0.877	0.000	4.386	42.982	0.000	1.754	0.877
	Z	30.769	17.949	0.000	2.564	0.000	5.128	38.462	0.000	2.564	2.564
New bid > standing bid	X	7.463	20.896	11.940	4.478	0.000	25.373	22.388	2.985	2.985	1.493
	Y	8.889	6.667	48.889	0.000	2.222	4.444	24.444	0.000	4.444	0.000
	Z	8.889	11.111	6.667	2.222	0.000	51.111	15.556	0.000	4.444	0.000
New bid = standing bid	X	5.263	2.632	78.947	0.000	0.000	0.000	13.158	0.000	0.000	0.000
	Y	7.143	1.190	73.809	1.190	0.000	3.571	10.714	0.000	1.190	1.190
	Z	0.000	0.000	57.143	14.286	0.000	0.000	28.571	0.000	0.000	0.000
New bid < standing bid	X	25.000	25.000	0.000	16.667	0.000	0.000	33.333	0.000	0.000	0.000
	Y	20.000	10.000	0.000	30.000	10.000	20.000	10.000	0.000	0.000	0.000
	Z	11.111	11.111	0.000	33.333	0.000	0.000	33.333	0.000	11.111	0.000
Cancel bid	X	0.000	10.000	0.000	0.000	90.000	0.000	0.000	0.000	0.000	0.000
	Y	0.000	20.000	0.000	0.000	60.000	0.000	20.000	0.000	0.000	0.000
	Z	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000	0.000	0.000
Take bid	X	23.077	25.641	0.000	2.564	0.000	28.205	12.821	2.564	5.128	0.000
	Y	22.222	8.333	0.000	8.333	0.000	38.889	16.667	0.000	5.556	0.000
	Z	11.628	30.233	0.000	2.326	0.000	16.279	32.558	2.326	2.326	2.326
New ask < standing ask	X	29.592	13.265	1.020	1.020	1.020	0.000	21.429	23.469	8.163	1.020
	Y	40.000	15.238	0.952	1.905	0.000	8.571	18.095	3.810	11.429	0.000
	Z	19.697	24.242	0.000	1.515	0.000	7.576	15.152	21.212	9.091	1.515
New ask = standing ask	X	17.647	3.529	0.000	0.000	0.000	7.059	4.706	65.882	1.176	0.000
	Y	18.182	9.091	0.000	0.000	0.000	0.000	9.091	63.636	0.000	0.000
	Z	6.383	2.128	0.000	0.000	0.000	6.383	12.766	70.213	2.128	0.000
New ask > standing ask	X	25.000	43.750	0.000	0.000	0.000	6.250	18.750	0.000	6.250	0.000
	Y	32.000	24.000	0.000	0.000	0.000	4.000	16.000	0.000	24.000	0.000
	Z	3.571	3.571	0.000	3.571	0.000	7.143	17.857	0.000	57.143	7.143
Cancel ask	X	28.571	14.286	0.000	0.000	0.000	0.000	14.286	0.000	0.000	42.857
	Y	0.000	50.000	0.000	0.000	0.000	0.000	50.000	0.000	0.000	0.000
	Z	5.556	5.556	0.000	0.000	5.556	5.556	5.556	0.000	0.000	72.222

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.

Table 5.9b

Frequencies of Events, Conditional on the Type of the Previous Event (Data 042393c : Market 1)

Event Type t / t − 1	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	35.583	22.699	0.000	0.000	1.227	9.816	30.061	0.000	0.000	0.613
New bid > standing bid	9.524	17.143	30.476	1.429	0.000	27.143	12.381	0.000	0.952	0.952
New bid = standing bid	0.538	9.677	65.591	0.000	0.538	10.753	11.828	0.000	0.000	1.075
New bid < standing bid	16.667	50.000	0.000	16.667	0.000	0.000	16.667	0.000	0.000	0.000
Cancel bid	10.000	0.000	0.000	0.000	60.000	10.000	10.000	0.000	0.000	10.000
Take bid	15.172	39.310	1.379	1.379	0.000	24.828	13.793	0.690	0.690	2.759
New ask < standing ask	33.766	24.026	0.000	0.000	0.649	9.091	14.935	15.584	1.948	0.000
New ask = standing ask	18.605	25.581	0.000	0.000	0.000	2.326	6.977	44.186	0.000	2.326
New ask > standing ask	10.000	20.000	0.000	0.000	0.000	20.000	20.000	0.000	30.000	0.000
Cancel ask	16.667	25.000	0.000	0.000	0.000	25.000	8.333	0.000	8.333	16.667

Table 5.9b' : Frequencies of Events by State, Conditional on the Type of the Previous Event (Data 042393c : Market 1)

Event Type $t-1$	t	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	X	40.741	16.049	0.000	0.000	1.235	4.938	37.037	0.000	0.000	0.000
	Y	32.558	34.884	0.000	0.000	0.000	16.279	13.953	0.000	0.000	2.326
	Z	28.205	23.077	0.000	0.000	2.564	12.821	33.333	0.000	0.000	0.000
New bid > standing bid	X	9.589	12.329	61.644	0.000	0.000	8.219	8.219	0.000	0.000	0.000
	Y	11.111	22.222	6.481	2.778	0.000	39.815	13.889	0.000	1.852	1.852
	Z	3.448	10.345	41.379	0.000	0.000	27.586	17.241	0.000	0.000	0.000
New bid = standing bid	X	0.714	8.571	68.571	0.000	0.714	8.571	11.429	0.000	0.000	1.429
	Y	0.000	40.000	10.000	0.000	0.000	50.000	0.000	0.000	0.000	0.000
	Z	0.000	5.556	69.444	0.000	0.000	8.333	16.667	0.000	0.000	0.000
New bid < standing bid	X	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Y	25.000	50.000	0.000	0.000	0.000	0.000	25.000	0.000	0.000	0.000
	Z	0.000	50.000	0.000	50.000	0.000	0.000	0.000	0.000	0.000	0.000
Cancel bid	X	0.000	0.000	0.000	0.000	50.000	25.000	25.000	0.000	0.000	0.000
	Y	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Z	16.667	0.000	0.000	0.000	66.667	0.000	0.000	0.000	0.000	16.667
Take bid	X	16.216	35.135	0.000	0.000	0.000	27.027	16.216	0.000	0.000	5.405
	Y	8.333	45.238	2.381	1.190	0.000	25.000	13.095	1.190	1.190	2.381
	Z	37.500	25.000	0.000	4.167	0.000	20.833	12.500	0.000	0.000	0.000
New ask < standing ask	X	36.232	26.087	0.000	0.000	0.000	7.246	8.696	20.290	1.449	0.000
	Y	25.641	35.897	0.000	0.000	0.000	12.821	7.692	15.385	2.564	0.000
	Z	36.957	10.870	0.000	0.000	2.174	8.696	30.435	8.696	2.174	0.000
New ask = standing ask	X	32.000	12.000	0.000	0.000	0.000	4.000	8.000	44.000	0.000	0.000
	Y	0.000	63.636	0.000	0.000	0.000	0.000	0.000	36.364	0.000	0.000
	Z	0.000	14.286	0.000	0.000	0.000	0.000	14.286	57.143	0.000	14.286
New ask > standing ask	X	50.000	50.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Y	0.000	20.000	0.000	0.000	0.000	40.000	20.000	0.000	20.000	0.000
	Z	0.000	0.000	0.000	0.000	0.000	0.000	33.333	0.000	66.667	0.000
Cancel ask	X	0.000	0.000	0.000	0.000	0.000	25.000	25.000	0.000	25.000	25.000
	Y	16.667	33.333	0.000	0.000	0.000	33.333	0.000	0.000	0.000	16.667
	Z	50.000	50.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.

Table 5.9c

Frequencies of Events, Conditional on the Type of the Previous Event (Data 042393b : Market 2)

Event Type t / $t-1$	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	29.348	16.304	0.000	3.261	8.696	11.957	27.174	0.000	2.174	1.087
New bid > standing bid	7.194	14.388	6.475	7.194	4.317	30.216	25.899	0.719	2.158	1.439
New bid = standing bid	0.000	18.182	50.000	9.091	0.000	9.091	13.636	0.000	0.000	0.000
New bid < standing bid	16.000	28.000	4.000	8.000	4.000	8.000	28.000	4.000	0.000	0.000
Cancel bid	17.391	17.391	0.000	4.348	36.957	2.174	17.391	0.000	2.174	2.174
Take bid	12.871	27.723	0.990	0.990	6.931	14.851	33.663	0.990	0.990	0.000
New ask < standing ask	11.905	16.667	0.000	3.571	4.762	16.667	22.024	20.238	2.976	1.190
New ask = standing ask	9.589	21.918	0.000	0.000	1.370	1.370	12.329	49.315	4.110	0.000
New ask > standing ask	14.286	14.286	0.000	0.000	0.000	10.714	14.286	0.000	46.429	0.000
Cancel ask	7.692	15.385	0.000	0.000	0.000	7.692	7.692	0.000	0.000	61.538

Table 5.9d

Frequencies of Events, Conditional on the Type of the Previous Event (Data 042393c : Market 2)

Event Type t / $t-1$	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Take ask	17.476	16.505	0.000	0.000	1.942	11.650	41.748	1.942	3.883	4.854
New bid > standing bid	9.091	19.318	14.773	3.409	0.000	15.909	28.409	2.273	2.273	4.545
New bid = standing bid	3.704	7.407	44.444	3.704	0.000	18.519	22.222	0.000	0.000	0.000
New bid < standing bid	12.500	25.000	0.000	0.000	0.000	50.000	0.000	0.000	0.000	12.500
Cancel bid	25.000	0.000	0.000	0.000	33.333	16.667	16.667	8.333	0.000	0.000
Take bid	6.410	20.513	0.000	2.564	3.846	26.923	24.359	1.282	3.846	10.256
New ask < standing ask	25.150	10.778	0.599	1.198	1.796	9.581	18.563	17.964	8.383	5.988
New ask = standing ask	20.270	8.108	0.000	0.000	0.000	5.405	13.514	50.000	2.703	0.000
New ask > standing ask	19.512	7.317	2.439	0.000	0.000	7.317	24.390	2.439	36.585	0.000
Cancel ask	7.692	7.692	0.000	0.000	0.000	9.615	21.154	0.000	1.923	51.923

Table 5.10

χ^2 Statistics for the Discrepancy between Unconditional Frequencies and Frequencies Conditional on the Last Event Type

Data Set → / Event Type $(t-1)$ ↓	042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Take ask	271 × 52.601	166 × 60.383	94 × 36.189	105 × 29.700
New bid > standing bid	154 × 60.043	204 × 25.599	132 × 37.372	85 × 51.939
New bid = standing bid	131 × 428.577	188 × 136.438	22 × 757.562	27 × 438.527
New bid < standing bid	32 × 238.051	6 × 482.420	26 × 25.755	8 × 164.882
Cancel bid	16 × 4213.632	10 × 3386.943	48 × 162.171	12 × 577.880
Take bid	119 × 61.630	150 × 43.926	106 × 25.190	86 × 42.326
New ask < standing ask	261 × 22.844	148 × 66.133	164 × 14.390	157 × 15.973
New ask = standing ask	144 × 300.228	44 × 373.319	73 × 179.300	74 × 161.069
New ask > standing ask	69 × 168.001	10 × 823.538	28 × 487.096	41 × 170.412
Cancel ask	27 × 1522.027	13 × 254.811	14 × 1836.224	55 × 252.958

$$\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(F_{j,k} - UF_k)^2}{UF_k}$$

where N_j is the number of observations with the previous event type being j, $F_{j,k}$ is the frequency of event k conditional on the previous event being j, and UF_k is the unconditional frequency of event k.

Table 5.11a

Frequencies of Events, Conditional on the State of the Order Book (Data 042393b : Market 1)

State of the Book ↓	Event Type →	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	26.786	7.143	31.250	0.893	2.232	3.125	21.875	3.571	2.679	0.446
	Bid-side depth = Ask-side depth	0.000	5.263	5.263	10.526	0.000	10.526	57.895	5.263	0.000	5.263
	Bid-side depth < Ask-side depth	36.709	6.329	1.266	6.329	0.000	3.797	17.722	21.519	1.266	5.063
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	50.000	50.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	7.143	32.143	0.000	0.000	0.000	0.000	42.857	7.143	10.714	0.000
	Bid-side depth < Ask-side depth	5.556	11.111	0.000	0.000	0.000	11.111	11.111	50.000	11.111	0.000
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	28.319	6.195	13.274	3.540	4.425	3.540	30.088	5.310	3.540	1.770
	Bid-side depth = Ask-side depth	14.754	4.918	13.115	3.279	1.639	8.197	45.902	0.000	8.197	0.000
	Bid-side depth < Ask-side depth	20.147	16.176	5.294	2.500	0.735	14.118	16.324	14.853	7.059	2.794

The total number of each event given the state of the book is divided by the total number of all the events given the state of the book in order to give a percentage frequency. Therefore, each row sums to 100.

Table 5.11a'

Frequencies of Events by State, Conditional on the State of the Order Book (Data 042393b : Market 1)

Event Type →			Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
State of the Book ↓												
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	X	29.730	8.108	21.622	1.351	0.000	1.351	24.324	8.108	4.054	1.351
		Y	25.503	6.711	36.242	0.671	3.356	4.027	20.134	1.342	2.013	0.000
		Z	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	X	0.000	7.143	7.143	7.143	0.000	7.143	57.143	7.143	0.000	7.143
		Y	0.000	0.000	0.000	25.000	0.000	25.000	50.000	0.000	0.000	0.000
		Z	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	X	37.179	6.410	1.282	6.410	0.000	3.846	16.667	21.795	1.282	5.128
		Y	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Z	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000	0.000	0.000
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	X	0.000	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Y	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Z	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	X	12.500	25.000	0.000	0.000	0.000	0.000	50.000	0.000	12.500	0.000
		Y	7.143	42.857	0.000	0.000	0.000	0.000	28.571	7.143	14.286	0.000
		Z	0.000	16.667	0.000	0.000	0.000	0.000	66.667	16.667	0.000	0.000
	Bid-side depth < Ask-side depth	X	0.000	28.571	0.000	0.000	0.000	14.286	14.286	42.857	0.000	0.000
		Y	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Z	0.000	0.000	0.000	0.000	0.000	10.000	10.000	60.000	20.000	0.000
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	X	23.077	2.564	23.077	0.000	12.821	5.128	20.513	7.692	5.128	0.000
		Y	37.931	10.345	6.897	1.724	0.000	3.448	32.759	0.000	3.448	3.448
		Z	6.250	0.000	12.500	18.750	0.000	0.000	43.750	18.750	0.000	0.000
	Bid-side depth = Ask-side depth	X	16.667	8.333	16.667	0.000	8.333	16.667	25.000	0.000	8.333	0.000
		Y	12.121	6.061	15.152	3.030	0.000	6.061	48.485	0.000	9.091	0.000
		Z	18.750	0.000	6.250	6.250	0.000	6.250	56.250	0.000	6.250	0.000
	Bid-side depth < Ask-side depth	X	20.319	18.327	3.984	2.390	1.594	11.952	15.936	21.912	3.187	0.398
		Y	28.249	11.299	12.429	3.390	0.000	14.124	17.514	4.520	8.475	0.000
		Z	14.286	17.460	1.587	1.984	0.397	16.270	15.873	15.079	9.921	7.143

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.

Table 5.11b

Frequencies of Events, Conditional on the State of the Order Book (Data 042393c : Market 1)

State of the Book ↓	Event Type →	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	17.958	21.303	18.310	0.528	0.880	18.486	15.669	5.458	0.704	0.704
	Bid-side depth = Ask-side depth	22.222	29.630	7.407	0.000	3.704	7.407	18.519	3.704	3.704	3.704
	Bid-side depth < Ask-side depth	26.667	20.000	3.333	0.000	3.333	6.667	20.000	0.000	10.000	10.000
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	5.882	20.588	52.941	0.000	0.000	5.882	8.824	5.882	0.000	0.000
	Bid-side depth = Ask-side depth	8.889	42.222	17.778	2.222	0.000	8.889	20.000	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	0.000	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	10.714	20.000	32.857	0.000	2.143	15.000	9.286	5.000	1.429	3.571
	Bid-side depth = Ask-side depth	16.667	0.000	25.000	0.000	0.000	25.000	33.333	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	32.927	17.073	7.317	2.439	0.000	13.415	23.171	3.659	0.000	0.000

Table 5.11b'

Frequencies of Events by State, Conditional on the State of the Order Book (Data 042393c : Market 1)

Event Type → / State of the Book ↓			Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	X	24.382	13.074	29.329	0.000	0.353	7.420	18.728	6.714	0.000	0.000
		Y	10.648	33.333	2.315	1.389	0.000	33.796	10.648	4.167	1.852	1.852
		Z	14.493	17.391	23.188	0.000	5.797	15.942	18.841	4.348	0.000	0.000
	Bid-side depth = Ask-side depth	X	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Y	17.647	47.059	0.000	0.000	0.000	11.765	17.647	5.882	0.000	0.000
		Z	30.000	0.000	20.000	0.000	10.000	0.000	20.000	0.000	10.000	10.000
	Bid-side depth < Ask-side depth	X	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Y	26.316	26.316	0.000	0.000	0.000	10.526	21.053	0.000	5.263	10.526
		Z	27.273	9.091	9.091	0.000	9.091	0.000	18.182	0.000	18.182	9.091
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	X	0.000	20.000	70.000	0.000	0.000	0.000	10.000	0.000	0.000	0.000
		Y	28.571	28.571	14.286	0.000	0.000	28.571	0.000	0.000	0.000	0.000
		Z	0.000	14.286	42.857	0.000	0.000	0.000	14.286	28.571	0.000	0.000
	Bid-side depth = Ask-side depth	X	0.000	56.250	18.750	0.000	0.000	12.500	12.500	0.000	0.000	0.000
		Y	15.000	40.000	20.000	5.000	0.000	10.000	10.000	0.000	0.000	0.000
		Z	11.111	22.222	11.111	0.000	0.000	0.000	55.556	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	X	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Y	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		Z	0.000	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	X	9.091	15.455	37.273	0.000	2.727	15.455	8.182	5.455	1.818	4.545
		Y	20.000	55.000	0.000	0.000	0.000	10.000	10.000	5.000	0.000	0.000
		Z	10.000	0.000	50.000	0.000	0.000	20.000	20.000	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	X	50.000	0.000	0.000	0.000	0.000	0.000	50.000	0.000	0.000	0.000
		Y	0.000	0.000	0.000	0.000	0.000	50.000	50.000	0.000	0.000	0.000
		Z	12.500	0.000	37.500	0.000	0.000	25.000	25.000	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	X	25.000	50.000	0.000	0.000	0.000	0.000	25.000	0.000	0.000	0.000
		Y	55.556	11.111	0.000	0.000	0.000	11.111	22.222	0.000	0.000	0.000
		Z	30.435	15.942	8.696	2.899	0.000	14.493	23.188	4.348	0.000	0.000

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.

Table 5.11c

Frequencies of Events, Conditional on the State of the Order Book (Data 042393b : Market 2)

State of the Book ↓	Event Type →	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	12.048	15.663	3.614	4.819	4.819	21.687	31.325	4.819	1.205	0.000
	Bid-side depth = Ask-side depth	12.500	30.000	2.500	2.500	2.500	20.000	20.000	10.000	0.000	0.000
	Bid-side depth < Ask-side depth	15.205	16.374	2.924	4.678	10.526	12.281	19.298	11.696	2.339	4.678
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	25.000	25.000	25.000	0.000	0.000	0.000	25.000	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	0.000	19.231	3.864	7.692	0.000	3.846	34.615	26.923	3.846	0.000
	Bid-side depth < Ask-side depth	11.111	27.778	0.000	2.778	2.778	8.333	8.333	27.778	11.111	0.000
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	0.000	14.286	0.000	0.000	0.000	42.857	42.857	0.000	0.000	0.000
	Bid-side depth = Ask-side depth	10.000	30.000	0.000	0.000	0.000	40.000	20.000	0.000	0.000	0.000
	Bid-side depth < Ask-side depth	14.242	17.879	3.333	3.030	7.273	14.545	23.939	8.485	5.455	1.818

Table 5.11d

Frequencies of Events, Conditional on the State of the Order Book (Data 042393c : Market 2)

State of the Book ↓	Event Type →	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	9.677	19.355	6.452	3.226	0.000	35.484	19.355	0.000	0.000	6.452
	Bid-side depth = Ask-side depth	0.000	16.667	16.667	0.000	8.333	25.000	25.000	0.000	0.000	8.333
	Bid-side depth < Ask-side depth	11.905	9.524	4.762	0.000	4.762	16.667	19.048	11.905	7.143	14.286
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	0.000	12.500	18.750	0.000	12.500	0.000	31.250	25.000	0.000	0.000
	Bid-side depth = Ask-side depth	7.895	15.789	7.895	0.000	2.632	2.632	28.947	26.316	7.895	0.000
	Bid-side depth < Ask-side depth	15.789	17.544	3.509	1.754	0.000	5.263	24.561	28.070	0.000	3.509
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	10.000	30.000	10.000	10.000	0.000	20.000	0.000	0.000	20.000	0.000
	Bid-side depth = Ask-side depth	0.000	28.571	0.000	14.286	14.286	14.286	14.286	0.000	14.286	0.000
	Bid-side depth < Ask-side depth	19.222	11.442	2.746	0.915	1.144	13.272	24.943	8.924	7.323	10.069

Table 5.12

χ^2 Statistics for the Discrepancy between Unconditional Frequencies and Frequencies Conditional on the State of the Book

State of the Book $(t-1)$ ↓	Data Set →	042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	224 × 57.725	568 × 1.182	83 × 14.283	31 × 68.523
	Bid-side depth = Ask-side depth	19 × 130.736	27 × 35.011	40 × 18.199	12 × 107.028
	Bid-side depth < Ask-side depth	79 × 47.010	30 × 164.622	171 × 8.559	42 × 14.103
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	7 × 211.636	50 × 75.936	13 × 208.383	21 × 176.467
	Bid-side depth = Ask-side depth	12 × 95.264	18 × 48.608	10 × 67.201	20 × 47.624
	Bid-side depth < Ask-side depth	29 × 164.164	12 × 360.183	43 × 67.353	70 × 42.194
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	113 × 25.265	140 × 19.206	7 × 112.640	10 × 174.012
	Bid-side depth = Ask-side depth	61 × 51.641	12 × 56.557	10 × 79.714	7 × 294.863
	Bid-side depth < Ask-side depth	680 × 8.681	82 × 34.884	330 × 1.209	437 × 2.639

$$\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(BF_{j,k} - UF_k)^2}{UF_k}$$

where N_j is the number of observations with the state of the book being j, $BF_{j,k}$ is the frequency of event k conditional on the state of the book being j, and UF_k is the unconditional frequency of event k.

Table 5.13a

Frequencies of Events, Conditional on the Size of Bid–Ask Spread (Data 042393b : Market 1)

Event Type → / Bid–Ask Spread ↓	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥ Mean bid–ask spread	20.8	12.0	9.7	3.4	0.2	7.7	28.5	10.1	7.2	0.4
Bid–ask spread < Mean bid–ask spread	23.2	13.0	11.5	1.9	2.2	11.4	15.3	13.2	4.3	3.7

Mean spread is the time–series mean of bid–ask spreads through 15 periods.

Table 5.13a'

Frequencies of Events by State, Conditional on the Size of Bid–Ask Spread (Data 042393b : Market 1)

Event Type → / Bid–Ask Spread ↓		Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥	X	0.0	16.7	0.0	6.7	0.0	16.7	13.3	46.7	0.0	0.0
Mean bid–ask spread	Y	0.0	8.3	25.0	16.7	0.0	0.0	41.7	0.0	8.3	0.0
	Z	0.0	0.0	14.8	3.7	0.0	11.1	18.5	48.1	3.7	0.0
Bid–ask spread <	X	25.1	13.2	8.6	2.4	2.2	7.7	20.0	15.6	3.5	1.5
Mean bid–ask spread	Y	27.5	10.1	19.3	1.9	1.2	8.5	22.8	2.6	5.6	0.5
	Z	14.5	16.3	1.1	2.9	0.4	14.5	21.4	12.7	9.8	6.5

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.
Mean spread is the time–series mean of bid–ask spreads through X-state periods for row Xs, through Y-state periods for row Ys, and through Z-state periods for row Zs.

Table 5.13b

Frequencies of Events, Conditional on the Size of Bid–Ask Spread (Data 042393c : Market 1)

Event Type → / Bid–Ask Spread ↓	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥ Mean bid–ask spread	10.4	25.7	23.6	0.9	0.9	15.8	17.6	2.7	1.5	0.9
Bid–ask spread < Mean bid–ask spread	21.7	19.5	18.0	0.5	1.2	16.1	14.7	5.8	0.8	1.7

Mean spread is the time–series mean of bid–ask spreads through 15 periods.

Table 5.13b'

Frequencies of Events by State, Conditional on the Size of Bid–Ask Spread (Data 042393c : Market 1)

Event Type → / Bid–Ask Spread ↓		Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥	X	0.0	33.3	22.2	0.0	0.0	11.1	33.3	0.0	0.0	0.0
Mean bid–ask spread	Y	10.5	52.6	5.3	0.0	0.0	5.3	26.3	0.0	0.0	0.0
	Z	11.1	11.1	25.9	7.4	0.0	22.2	22.2	0.0	0.0	0.0
Bid–ask spread <	X	19.0	15.5	32.6	0.0	0.9	9.2	15.3	5.9	0.5	1.2
Mean bid–ask spread	Y	14.8	33.3	3.1	1.4	0.0	28.9	11.0	3.8	1.7	2.1
	Z	22.2	15.0	18.0	0.0	3.6	11.4	22.2	4.8	1.8	1.2

X-state periods are 2, 6, 8, 9, 11, and 15, Y-state periods are 1, 5, 7, 12, and 13, and Z-state periods are 3, 4, 10, and 14.
Mean spread is the time–series mean of bid–ask spreads through X-state periods for row Xs, through Y-state periods for row Ys, and through Z-state periods for row Zs.

Table 5.13c

Frequencies of Events, Conditional on the Size of Bid–Ask Spread (Data 042393b : Market 2)

Event Type → / Bid–Ask Spread ↓	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥ Mean bid–ask spread	10.4	22.6	5.1	4.0	5.1	10.8	24.2	13.5	3.0	1.3
Bid–ask spread < Mean bid–ask spread	15.4	15.9	1.7	3.4	8.0	18.0	22.4	8.0	4.6	2.4

Table 5.13d

Frequencies of Events, Conditional on the Size of Bid–Ask Spread (Data 042393c : Market 2)

Event Type → / Bid–Ask Spread ↓	Take ask	New bid > standing bid	New bid = standing bid	New bid < standing bid	Cancel bid	Take bid	New ask < standing ask	New ask = standing ask	New ask > standing ask	Cancel ask
Bid–ask spread ≥ Mean bid–ask spread	17.6	12.3	3.2	1.1	0.0	5.3	36.4	10.7	7.5	5.9
Bid–ask spread < Mean bid–ask spread	15.6	13.4	4.5	1.3	2.6	16.4	19.2	11.7	5.8	9.5

Table 5.14

χ^2 Statistics for the Discrepancy between Unconditional Frequencies and Frequencies Conditional on Bid–Ask Spread

Data Set → Bid–Ask Spread $(t-1)$ ↓	042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Bid–ask spread ≥ Mean bid–ask spread	557 × 6.367	335 × 5.904	297 × 5.835	187 × 14.265
Bid–ask spread < Mean bid–ask spread	667 × 4.412	604 × 1.869	410 × 2.969	463 × 2.316

$$\chi^2 = N_j \cdot \sum_{k=1}^{10} \frac{(SF_{j,k} - UF_k)^2}{UF_k}$$

where N_j is the number of observations with the bid–ask spread being j, $SF_{j,k}$ is the frequency of event k conditional on the bid–ask spread being j, and UF_k is the unconditional frequency of event k.

Table 5.15a

Mean Time Interval between Two Events (seconds)

Data Set → Conditioning Variables $(t-1)$ ↓		042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Take ask		4.585	6.417	7.380	7.262
New bid > standing bid		4.656	4.433	7.158	7.261
New bid = standing bid		1.310	2.565	1.273	1.703
New bid < standing bid		4.484	4.333	6.680	7.250
Cancel bid		1.313	0.600	4.870	8.167
Take bid		3.763	6.276	7.040	10.269
New ask < standing ask		3.888	4.701	7.196	7.491
New ask = standing ask		1.720	2.558	2.603	2.919
New ask > standing ask		4.710	2.300	3.250	6.756
Cancel ask		2.074	2.667	5.308	3.404
Bid-side spread > Ask-side spread	Bid-side depth > Ask-side depth	2.228	4.136	7.831	7.000
	Bid-side depth = Ask-side depth	7.000	6.074	4.150	2.833
	Bid-side depth < Ask-side depth	4.089	4.200	6.363	8.000
Bid-side spread = Ask-side spread	Bid-side depth > Ask-side depth	2.286	4.620	4.154	1.905
	Bid-side depth = Ask-side depth	5.833	8.222	5.500	4.800
	Bid-side depth < Ask-side depth	3.793	6.333	2.744	3.214
Bid-side spread < Ask-side spread	Bid-side depth > Ask-side depth	4.584	3.879	6.571	2.200
	Bid-side depth = Ask-side depth	5.574	8.750	7.300	3.714
	Bid-side depth < Ask-side depth	3.504	6.622	6.403	7.586
Unconditional		3.588	4.563	6.171	6.632

Table 5.15b

Mean Time Interval between Two Events (seconds)

Data Set → / Conditioning Variables $(t-1)$ ↓	042393b : Market 1	042393c : Market 1	042393b : Market 2	042393c : Market 2
Bid–ask spread ≥ Mean bid–ask spread	4.294	4.866	5.882	6.711
Bid–ask spread < Mean bid–ask spread	2.999	4.396	6.380	6.600
Time interval ≥ Mean time interval	4.900	5.186	8.899	8.541
Time interval < Mean time interval	2.974	4.120	5.156	5.891
Unconditional	3.588	4.563	6.171	6.632

Table 5.16

Statistics for Tests Concerning Equality of Mean Time Intervals among Different Conditioning Variables

	(A) $H_0 : \mu_{event1} = \mu_{event2} = \cdots = \mu_{event10}$
042393b : Mkt1	$F \approx 26.88 > 1.88 \approx F_{0.05}(9, 1214)$
042393c : Mkt1	$F \approx 6.64 > 1.89 \approx F_{0.05}(9, 929)$
042393b : Mkt2	$F \approx 3.15 > 1.90 \approx F_{0.05}(9, 697)$
042393c : Mkt2	$F \approx 4.22 > 1.90 \approx F_{0.05}(9, 640)$

	(B) $H_0 : \mu_{state1} = \mu_{state2} = \cdots = \mu_{state9}$
042393b : Mkt1	$F \approx 7.40 > 1.94 \approx F_{0.05}(8, 1215)$
042393c : Mkt1	$F \approx 3.59 > 1.95 \approx F_{0.05}(8, 930)$
042393b : Mkt2	$F \approx 1.49 < 1.95 \approx F_{0.05}(8, 698)$
042393c : Mkt2	$F \approx 3.03 > 1.95 \approx F_{0.05}(8, 641)$

	(C) $H_0 : \mu_{spreadlg} = \mu_{spreadsm}$
042393b : Mkt1	$F \approx 14.03 > 3.84 \approx F_{0.05}(1, 1222)$
042393c : Mkt1	$F \approx 1.25 < 3.85 \approx F_{0.05}(1, 937)$
042393b : Mkt2	$F \approx 0.51 < 3.85 \approx F_{0.05}(1, 705)$
042393c : Mkt2	$F \approx 0.02 < 3.85 \approx F_{0.05}(1, 648)$

	(D) $H_0 : \mu_{intervallg} = \mu_{intervalsm}$
042393b : Mkt1	$F \approx 27.07 > 3.84 \approx F_{0.05}(1, 1207)$
042393c : Mkt1	$F \approx 6.47 > 3.84 \approx F_{0.05}(1, 922)$
042393b : Mkt2	$F \approx 24.04 > 3.84 \approx F_{0.05}(1, 690)$
042393c : Mkt2	$F \approx 10.19 > 3.84 \approx F_{0.05}(1, 633)$

$$F = \frac{MS(Tr)}{MSE}$$

where $MS(Tr)$ is the treatment mean square and MSE is the error mean square.

Table 5.17a

Mean Time Intervals between Two Events, Jointly Conditional on the Previous Event Type, the State of the Order Book, and the Size of the Last Time Interval
(Data 042393b : Market 1 in roman type ; 042393c : Market 1 in **bold** type)

Time interval →	Time interval ≥ Mean time interval									Time interval < Mean time interval								
State of the book → / Last event type ↓	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9
Event 1	5.11	7.50	3.40	3.00	7.00		0.78	0.75	0.33	2.65	5.50	5.39	2.00	2.00		6.04	8.28	3.30
	7.06	**8.00**	**1.00**	**11.00**	**8.00**		**8.83**	**17.00**	**8.63**	**5.82**	**6.50**	**5.20**	**6.00**	**7.00**		**6.00**	**10.50**	**5.00**
Event 2	4.40	2.00	6.75		2.50	11.00	3.00	3.38	6.24	7.50	1.00	15.00		4.07	3.00	2.00	2.50	2.44
	3.77	**3.33**	**3.00**	**2.33**	**0.75**		**2.64**	**9.00**	**2.33**	**5.35**	**0.50**	**2.00**	**4.00**	**17.40**	**3.00**	**2.23**		**5.00**
Event 3										0.50	0.00					3.13	1.17	1.90
				7.67						**2.36**	**0.00**		**3.30**			**2.54**	**2.00**	**0.00**
Event 4	4.00		2.00				5.67		5.83	3.67		4.00				4.50	6.00	3.03
	4.67								**0.00**	**7.00**								**5.00**
Event 5	1.00						0.00			2.25						0.00	0.00	1.83
	1.00						**0.00**			**0.00**	**0.00**	**0.00**				**2.50**		
Event 6		0.00	2.50		20.00	0.00	4.00	2.50	5.10	3.67		3.00				2.00	0.00	3.26
	5.04	**24.00**	**18.00**	**3.00**	**6.07**	**9.50**	**7.29**		**8.30**	**5.63**	**7.00**	**0.50**	**5.00**			**3.50**		**11.00**
Event 7	1.67	37.50	5.08	5.00	2.00	0.00	3.48	0.88	3.91	1.68	1.00	4.71		6.67	6.00	2.29	6.00	3.54
	2.41	**0.00**	**5.00**	**0.50**			**5.69**	**19.50**	**7.91**	**3.03**	**7.67**	**7.20**		**5.00**	**2.50**	**4.00**	**2.00**	**7.13**
Event 8	1.00		1.00						1.00	0.67	0.00	0.50			4.09	4.00	9.00	1.63
	3.00									**1.32**	**7.50**	**6.00**	**2.50**			**3.86**		**2.00**
Event 9			18.00				7.00		4.05	2.40	4.00				2.50	10.67		3.85
	1.00		**2.00**				**14.00**			**0.67**		**1.33**				**0.00**		
Event 10	9.00		0.00				10.00		1.80	2.00	0.00	0.00				12.00		1.00
	6.00						**0.00**			**2.75**	**1.50**	**3.00**				**4.50**		

Blank entries indicate no valid observations for the cell.

Table 5.17b

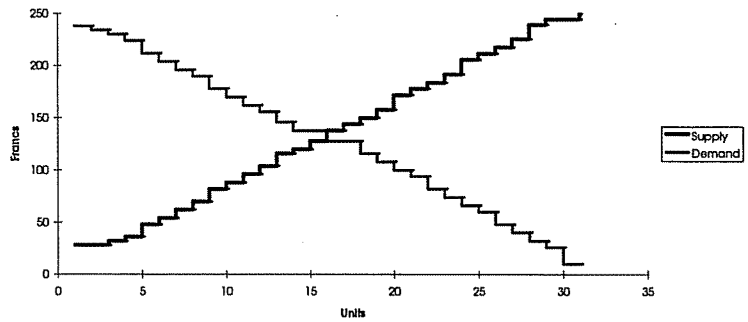
Mean Time Intervals between Two Events, Jointly Conditional on the Previous Event Type, the State of the Order Book, and the Size of the Last Time Interval
(Data 042393b : Market 2 in roman type ; 042393c : Market 2 in **bold** type)

Time interval →	Time interval ≥ Mean time interval									Time interval < Mean time interval								
State of the book → / Last event type ↓	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9
Event 1	1.07	3.50	8.44		1.00	[illegible]	1.00	3.00	11.46	10.31	5.50	8.93			3.00			5.19
	7.33		**7.50**			**32.00**			**11.06**	**4.00**	**4.00**	**6.00**	**2.50**	**5.60**	**4.18**			**5.30**
Event 2	10.17	5.00	9.00		20.00		18.00	7.50	10.20	9.00	4.33	7.20		1.00	0.00	1.00	10.00	3.61
	12.25		**4.00**		**4.00**	**0.00**		**5.00**	**8.50**	**8.22**	**2.50**	**6.67**	**4.00**	**3.00**	**3.29**	**1.00**	**6.00**	**11.11**
Event 3				0.00					0.00	2.00	0.00	0.00	0.00				2.00	2.00
										2.50	**0.00**		**2.00**		**2.00**	**6.00**	**2.00**	**1.09**
Event 4	5.50		10.00						4.00	5.80	5.33	5.83						8.67
									4.00	**11.00**		**4.00**			**3.00**			**7.00**
Event 5	14.50		3.00						8.00	6.00	0.00	3.50			4.00			4.74
			0.00						**7.50**			**1.50**	**0.00**	**0.00**	**4.00**			**30.50**
Event 6	10.50	3.00	8.83			2.00			11.31	5.75	4.43	6.47	0.00	2.00	6.67	5.00	4.50	0.68
			12.00		**4.00**	**8.50**			**8.31**	**5.25**	**1.50**	**11.00**	**1.50**		**11.00**	**5.50**	**6.00**	**15.14**
Event 7	7.44	14.33	14.55		2.67	7.00		6.00	8.49	3.17	2.29	5.33	2.00	7.67	4.50	2.00	11.00	7.43
	2.00		**10.80**				**0.00**	**2.00**	**8.12**		**7.00**	**7.00**	**4.00**	**12.33**	**3.50**	**0.00**	**2.00**	**8.19**
Event 8						2.00				5.00	0.00	2.29			0.75			4.14
									3.00			**4.80**	**0.00**	**0.00**	**1.38**			**4.03**
Event 9									5.00	6.00		10.50						2.23
			28.00						**12.00**			**10.00**				**1.00**	**3.00**	**3.67**
Event 10									15.00			2.43						2.33
			2.50			**1.00**			**3.00**	**1.00**	**0.00**	**10.00**		**3.33**				**3.07**

Appendix 5B

Figures for Chapter 5

Figure 5.1 : Demand and Supply Schedules in Market 2



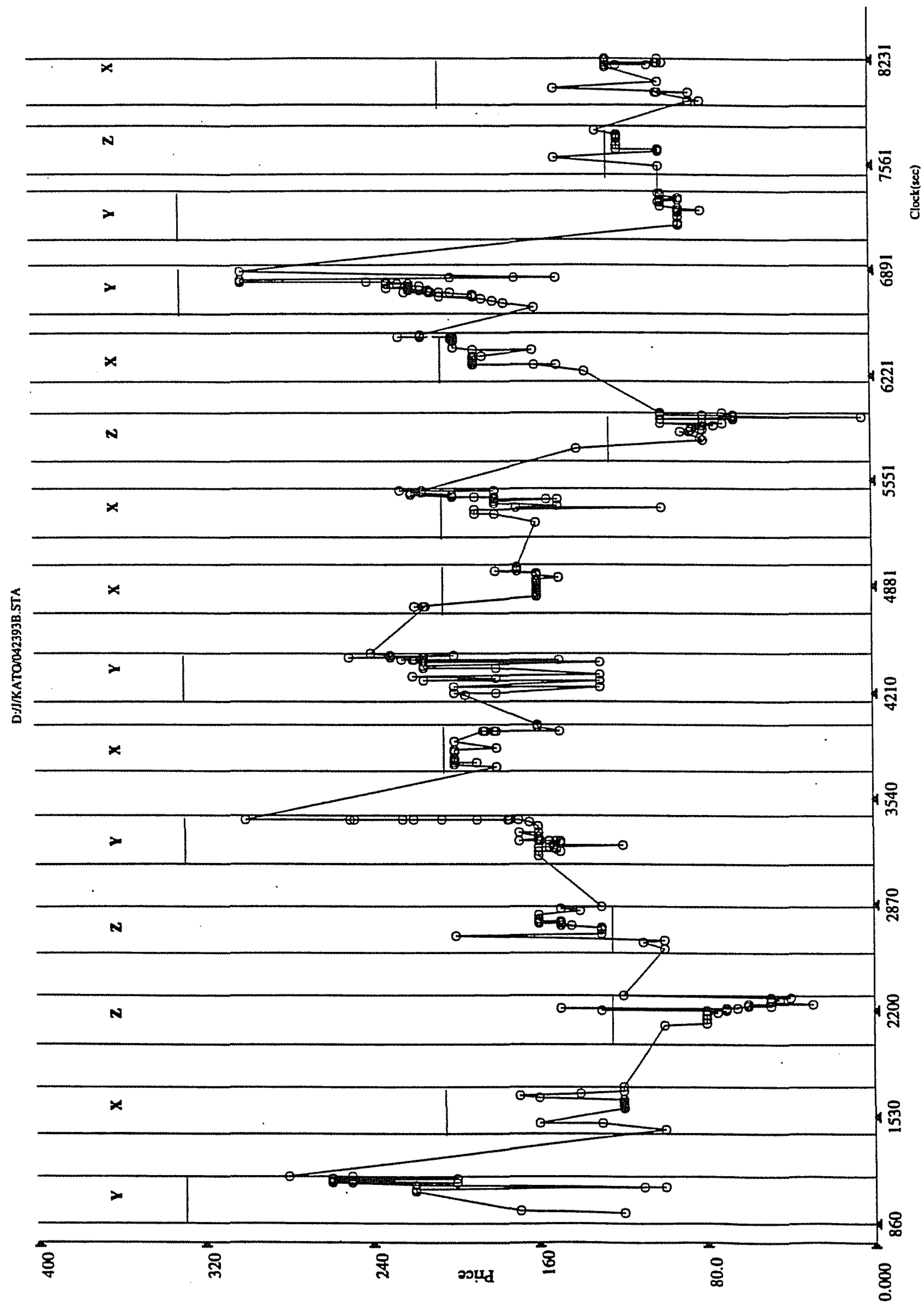


Figure 5.2a : Time Series of Transaction Prices (Data 042393b : Market 1)

Figure 5.2b : Time Series of Transaction Prices (Data 042393c : Market 1)

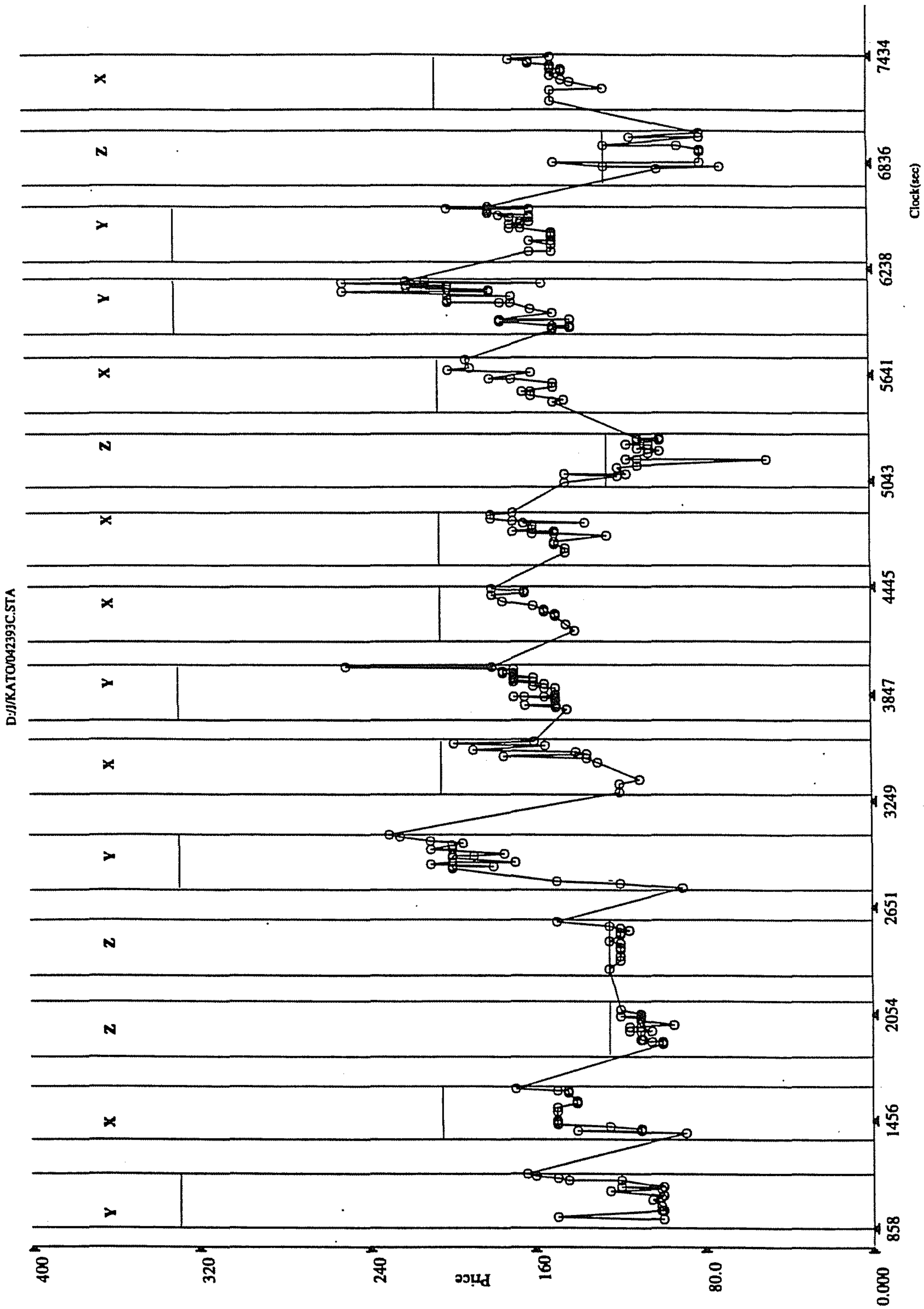
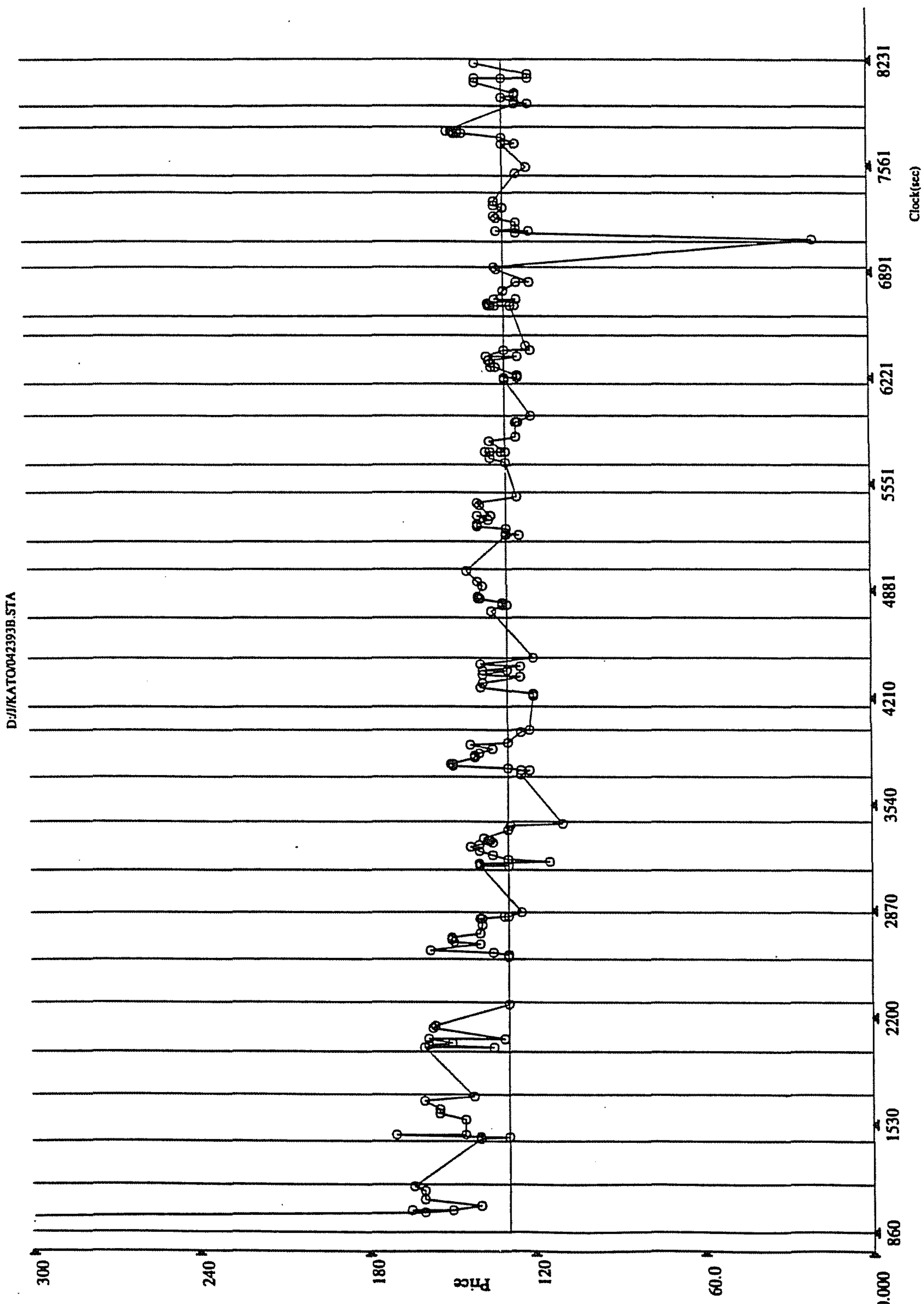


Figure 5.2c : Time Series of Transaction Prices (Data 042393b : Market 2)



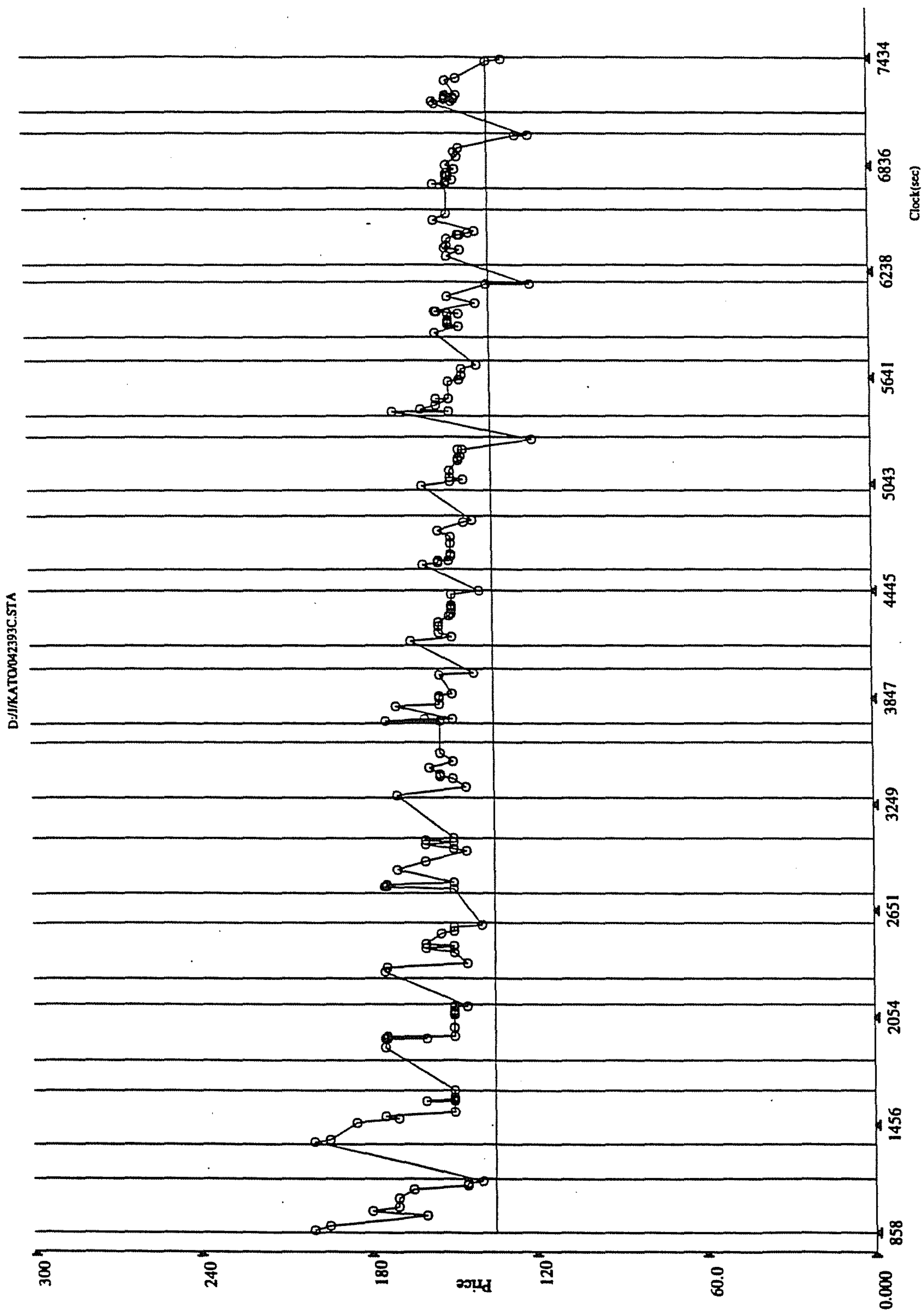


Figure 5.2d : Time Series of Transaction Prices (Data 042393c : Market 2)

Figure 5.3a : Average and Total Number of Bids and Asks (Data 042393b : Market 1)

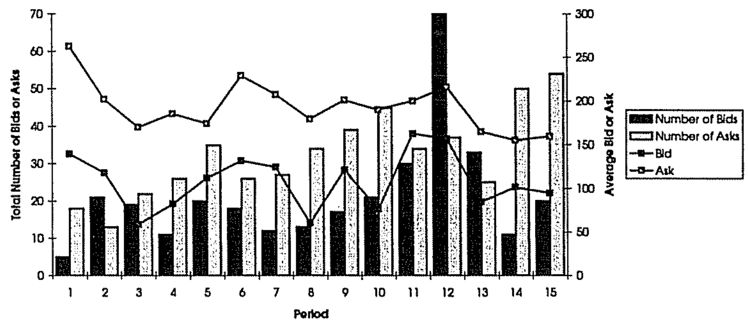


Figure 5.3b : Average and Total Number of Bids and Asks (Data 042393c : Market 1)

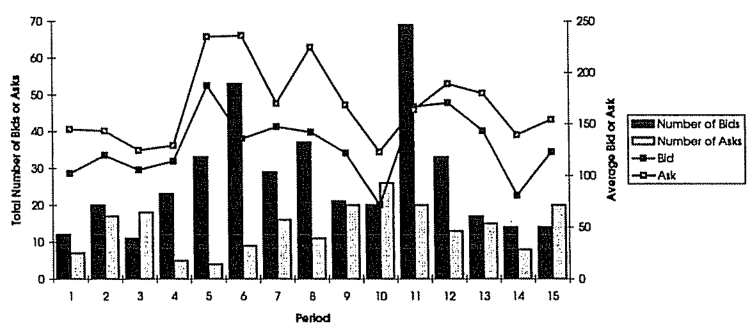


Figure 5.3c : Average and Total Number of Bids and Asks (Data 042393b : Market 2)

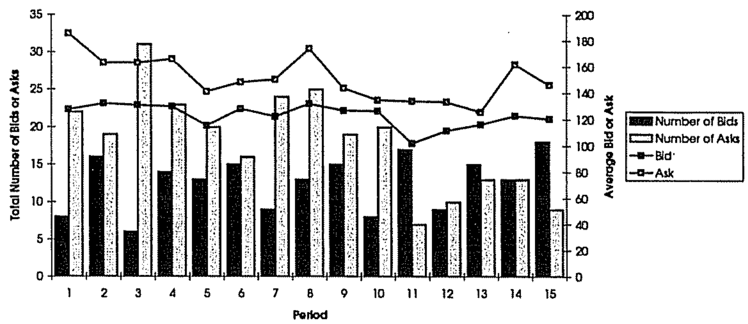


Figure 5.3d : Average and Total Number of Bids and Asks (Data 042393c : Market 2)

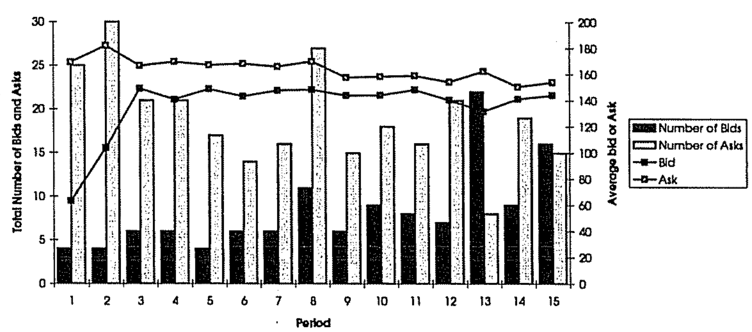


Figure 5.4a : State of the Order Book (Data 042393b : Market 1)

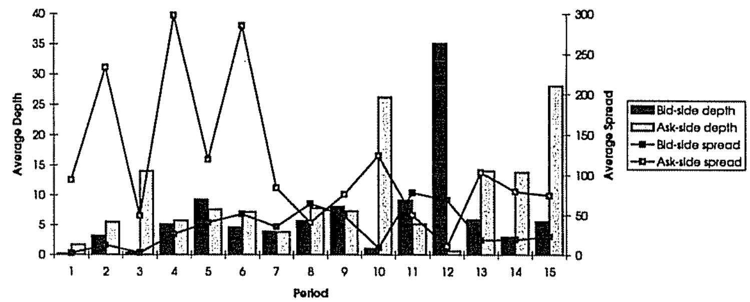


Figure 5.4b : State of the Order Book (Data 042393c : Market 1)

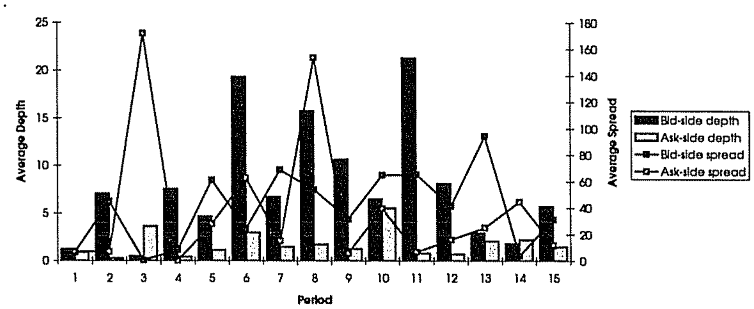


Figure 5.4c : State of the Order Book (Data 042393b : Market 2)

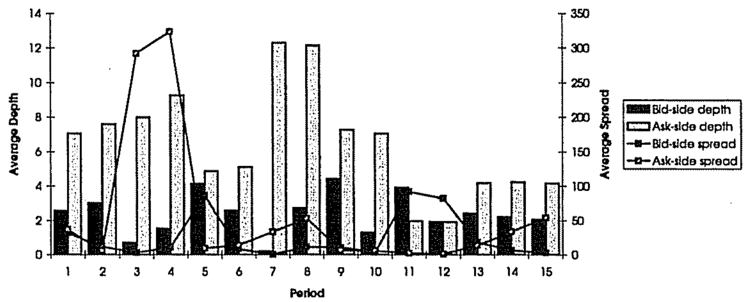


Figure 5.4d : State of the Order Book (Data 042393c : Market 2)

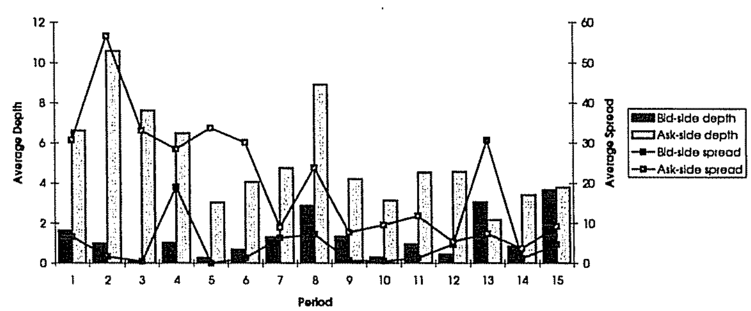


Figure 5.5a1 : Intraperiod Patterns of Market Activities (Data 042393b : Market 1)

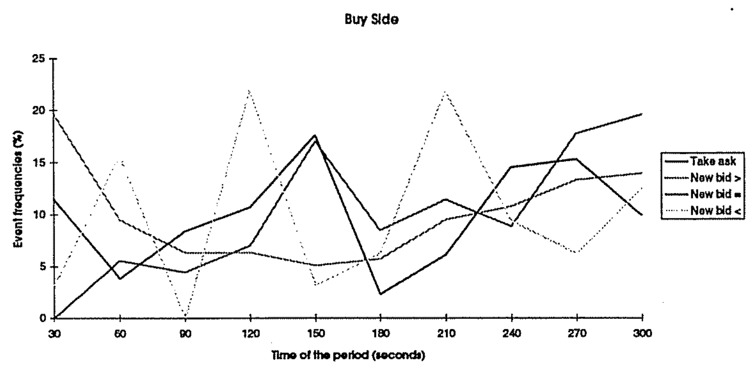


Figure 5.5a2 : Intraperiod Patterns of Market Activities (Data 042393b : Market 1)

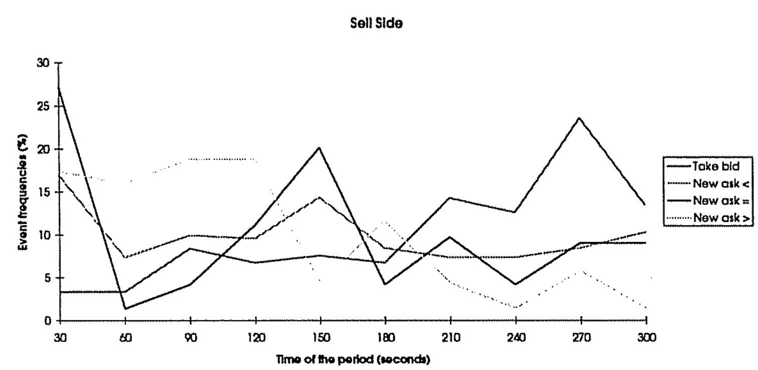


Figure 5.5b1 : Intraperiod Patterns of Market Activities (Data 042393c : Market 1)

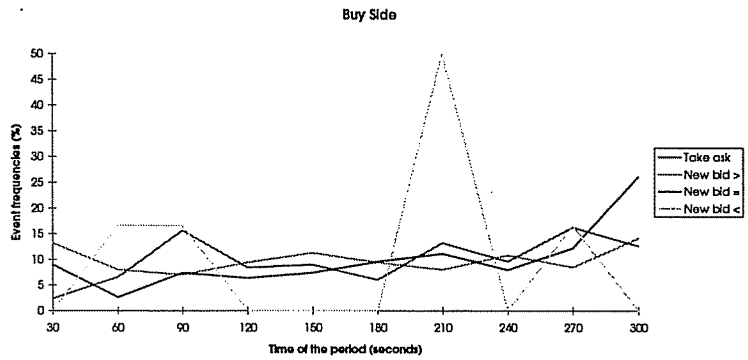


Figure 5.5b2 : Intraperiod Patterns of Market Activities (Data 042393c : Market 1)

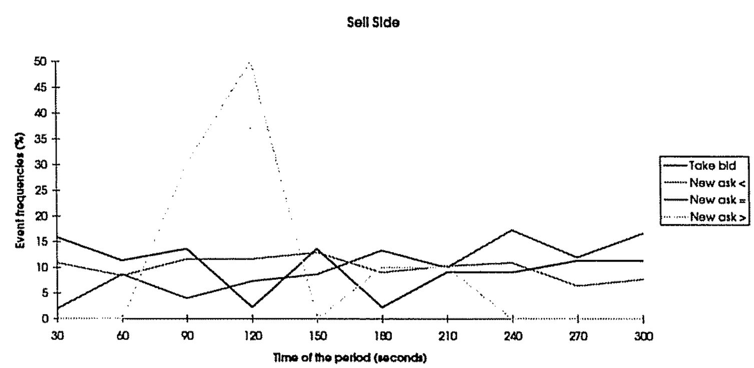


Figure 5.5c1 : Intraperiod Patterns of Market Activities (Data 042393b : Market 2)

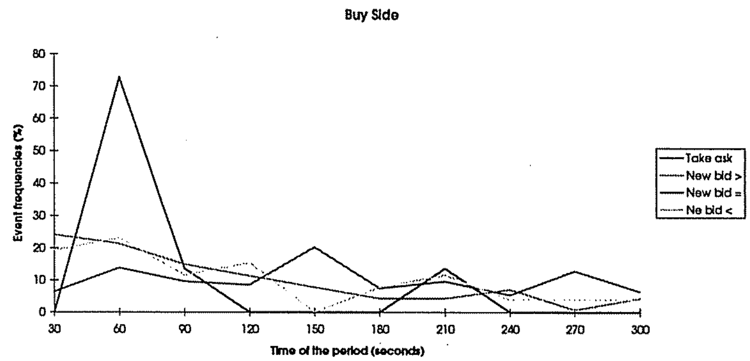


Figure 5.5c2 : Intraperiod Patterns of Market Activities (Data 042393b : Market 2)

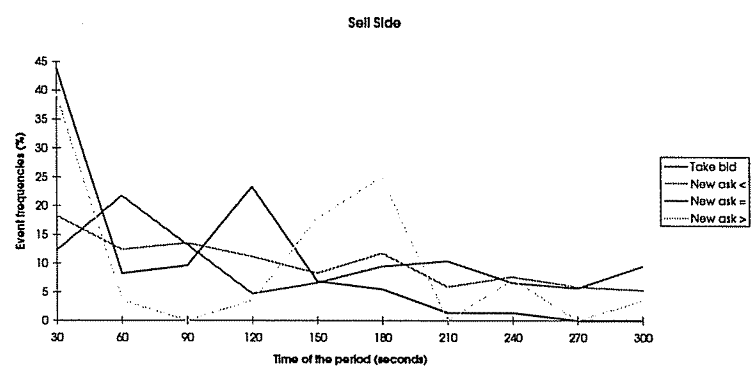


Figure 5.5d1 : Intraperiod Patterns of Market Activities (Data 042393c : Market 2)

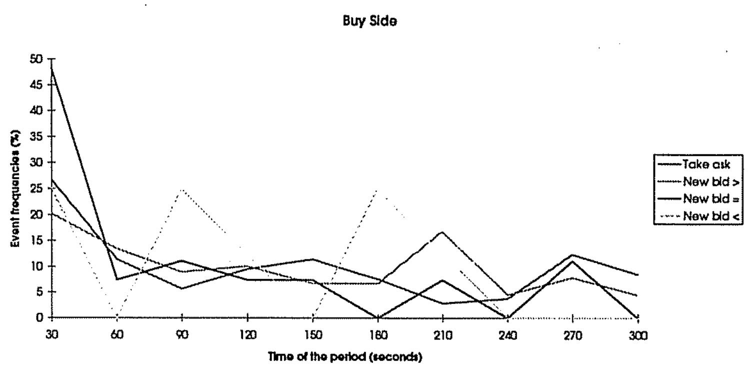


Figure 5.5d2 : Intraperiod Patterns of Market Activities (Data 042393c : Market 2)

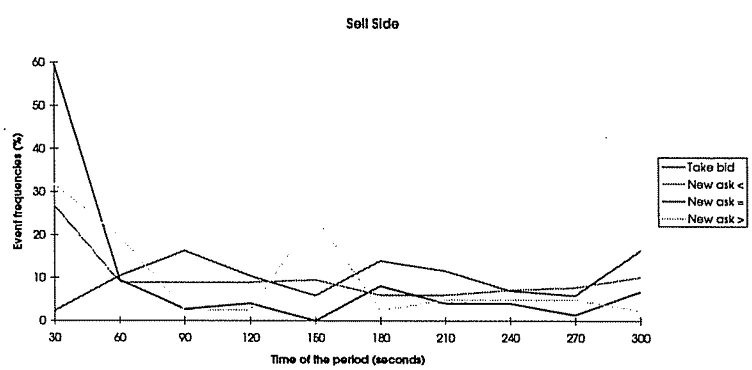


Figure 5.6a : Histograms of Intraperiod Price Changes (Data 042393b : Market 1)

(The horizontal axis ranges from –100 to 100 with ticksize 5.)

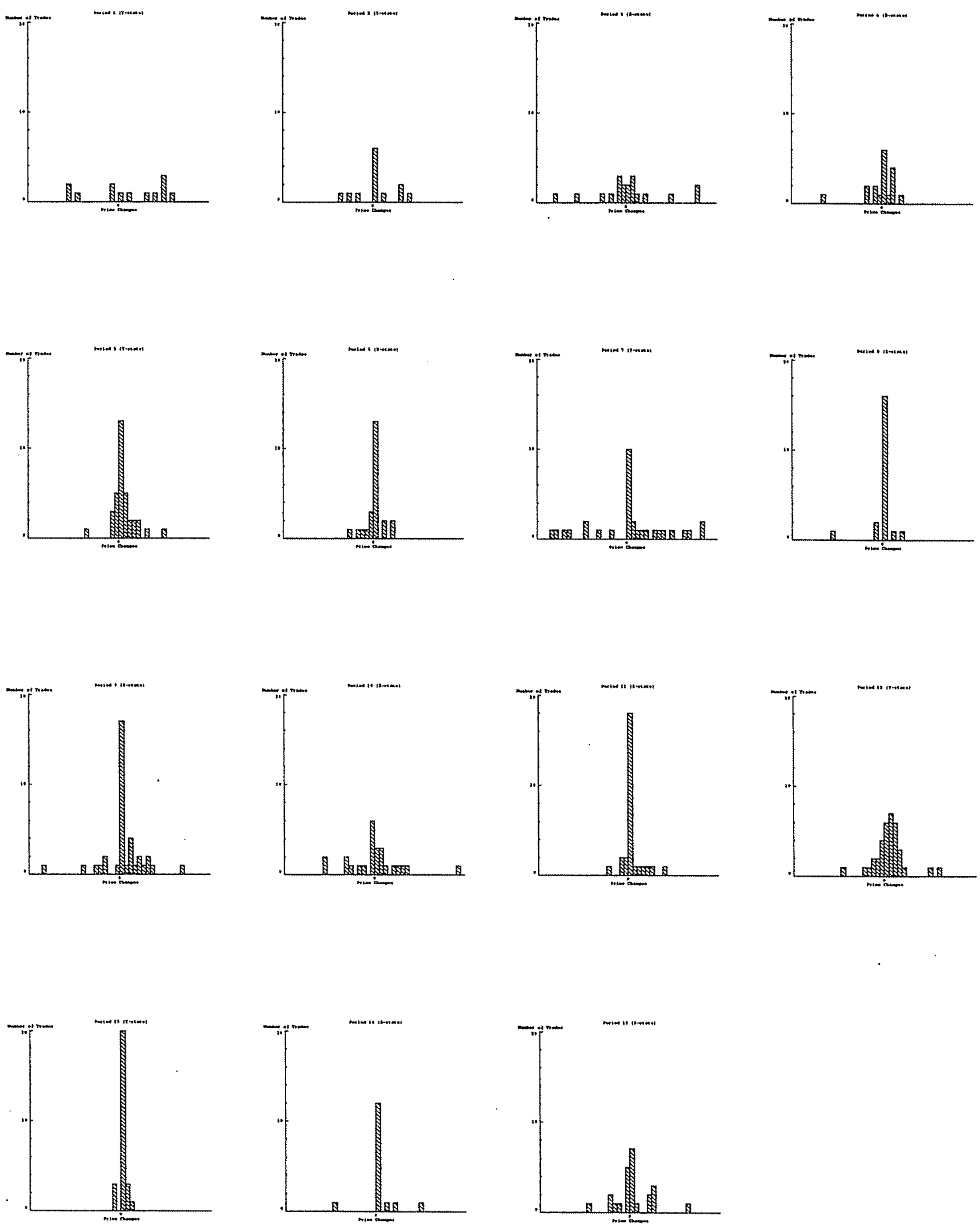


Figure 5.6b : Histograms of Intraperiod Price Changes (Data 042393c : Market 1)

(The horizontal axis ranges from –100 to 100 with ticksize 5.)

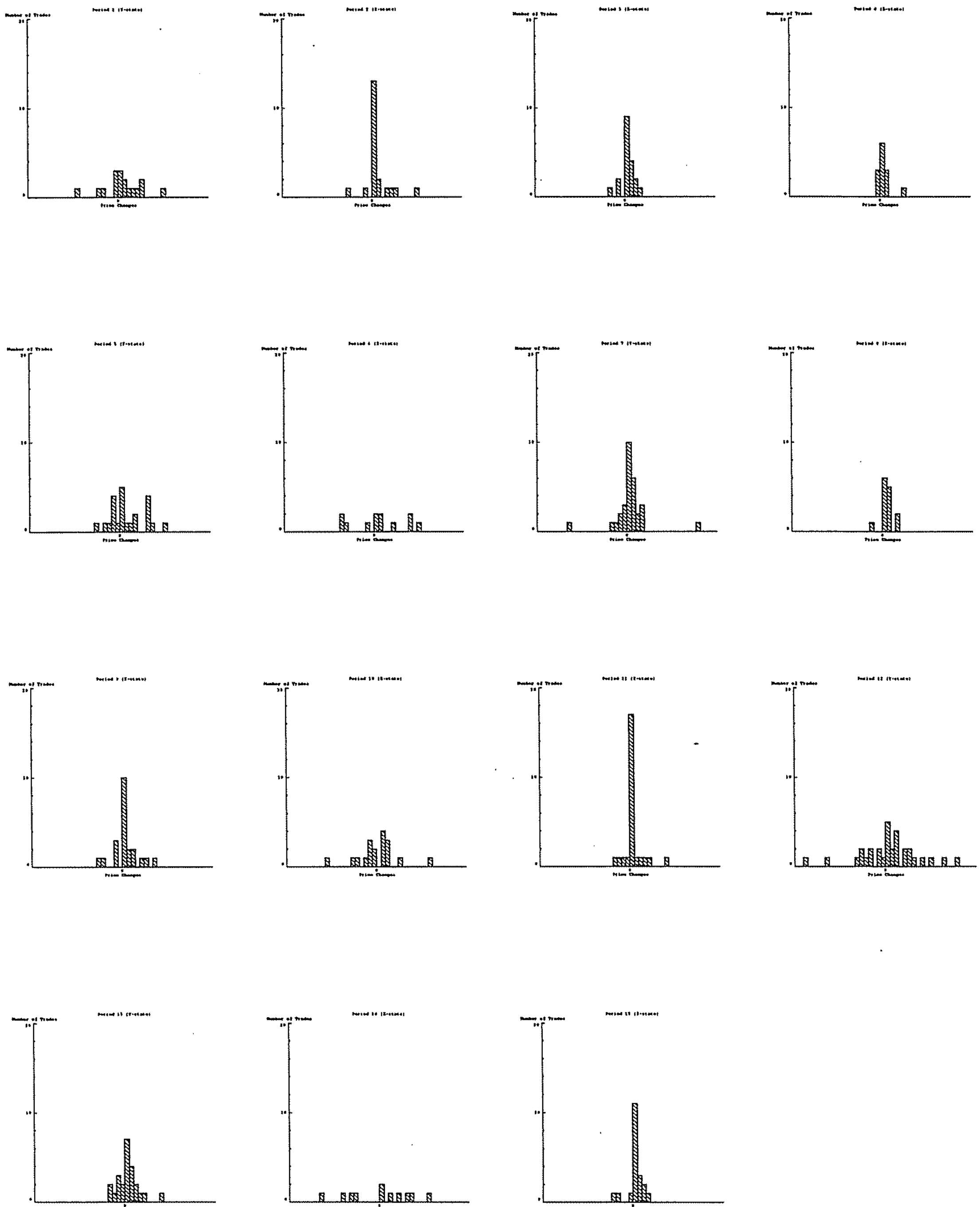


Figure 5.6c : Histograms of Intraperiod Price Changes (Data 042393b : Market 2)

(The horizontal axis ranges from –40 to 40 with ticksize 2.)

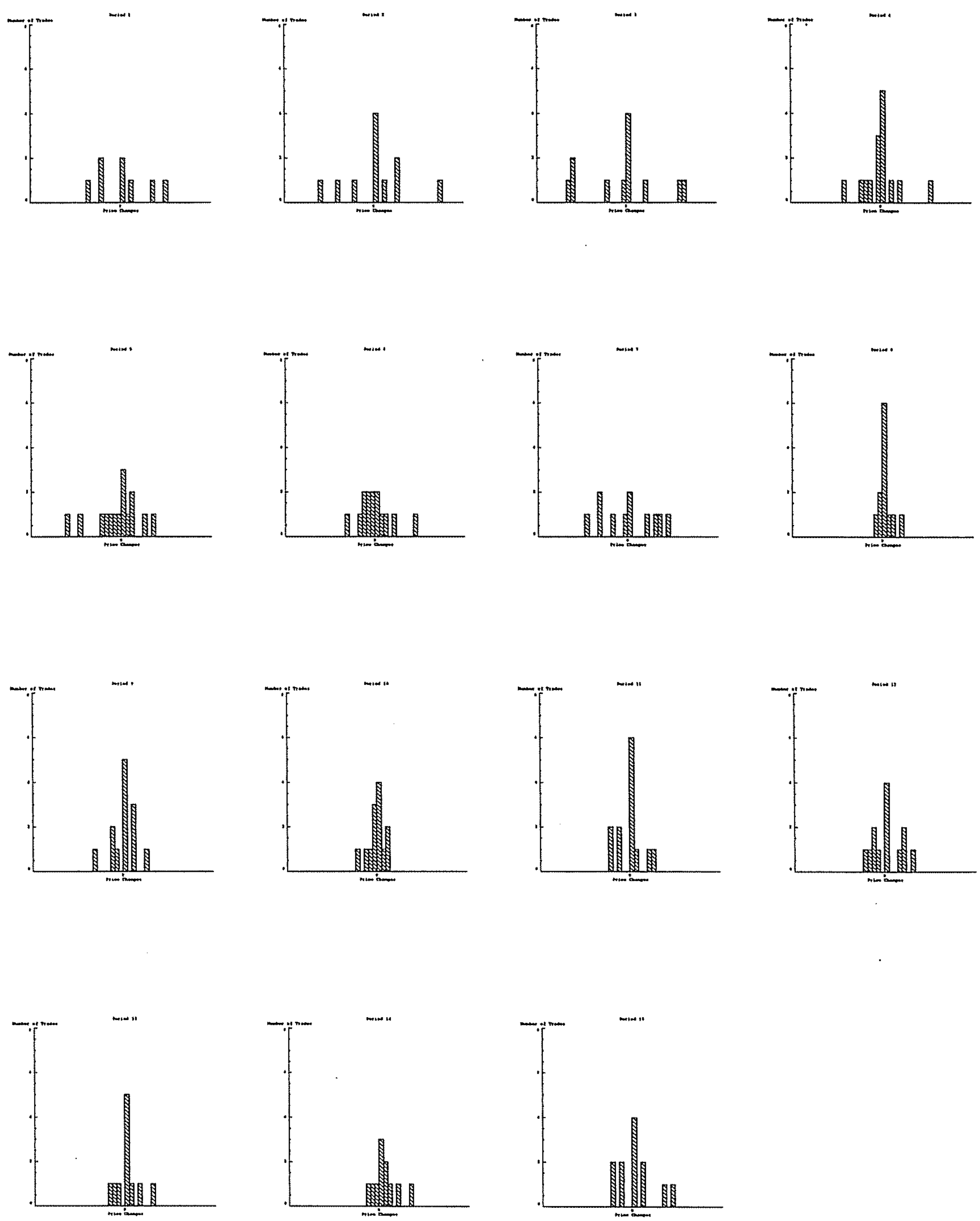


Figure 5.6d : Histograms of Intraperiod Price Changes (Data 042393c : Market 2)

(The horizontal axis ranges from –40 to 40 with ticksize 2.)

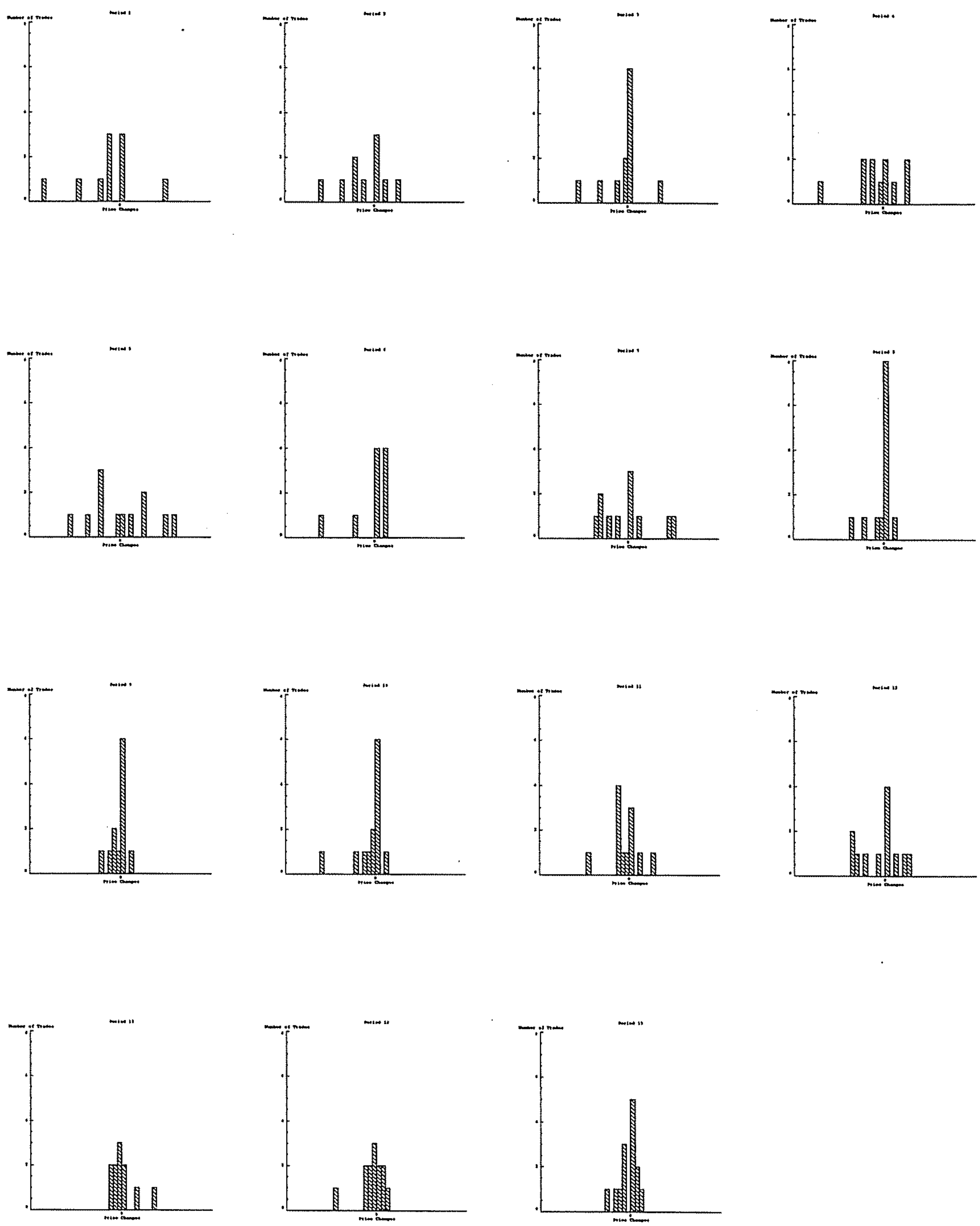


Figure 5.7a1 : Intraperiod Pattern of Bid–Ask Spreads (Data 042393b : Market 1)

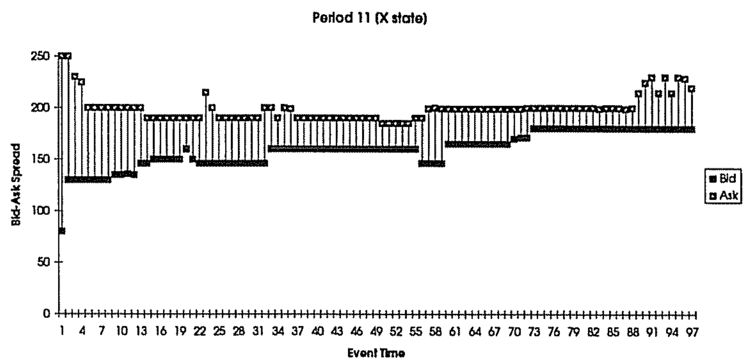


Figure 5.7a2 : Intraperiod Pattern of Bid–Ask Spreads (Data 042393b : Market 1)

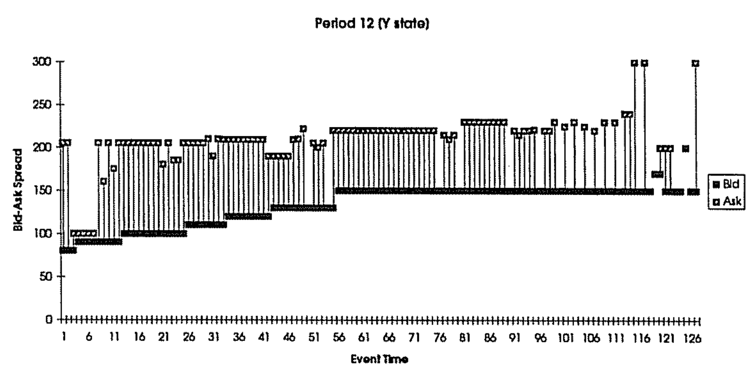


Figure 5.7b1 : Intraperiod Pattern of Bid–Ask Spreads (Data 042393c : Market 1)

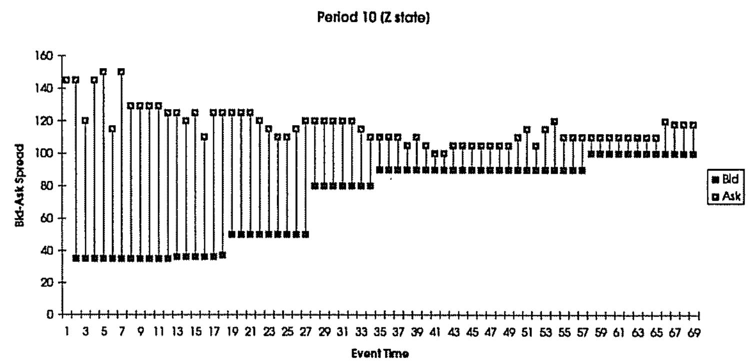


Figure 5.7b2 : Intraperiod Pattern of Bid–Ask Spreads (Data 042393c : Market 1)

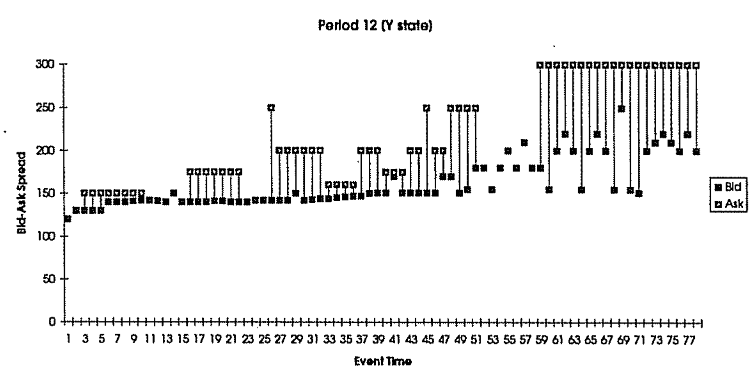


Figure 5.8a : Distribution of Time Interval between Orders (Data 042393b : Market 1)

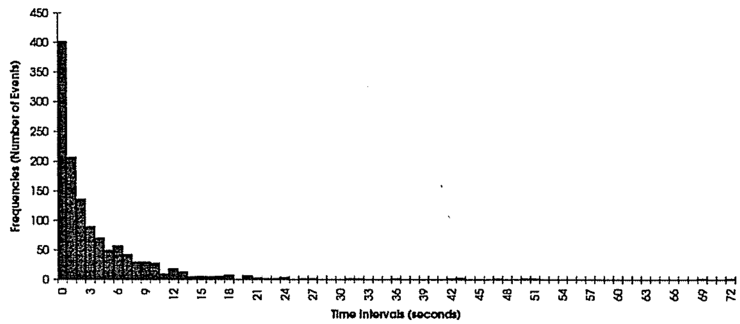


Figure 5.8b : Distribution of Time Interval between Orders (Data 042393c : Market 1)

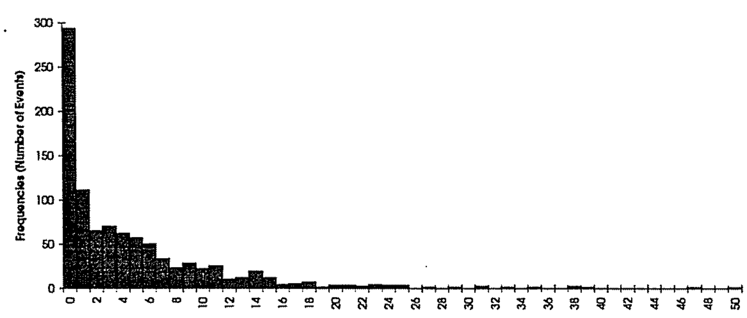


Figure 5.8c : Distribution of Time Interval between Orders (Data 042393b : Market 2)

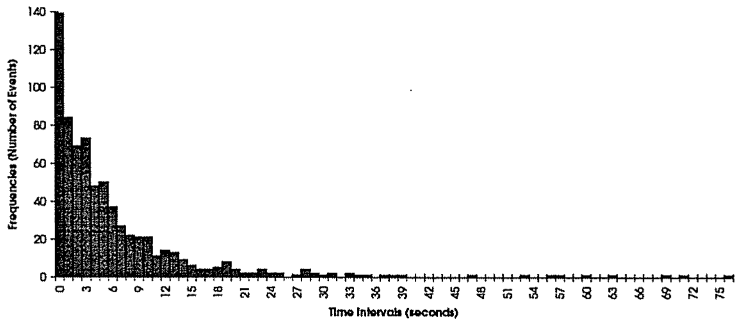
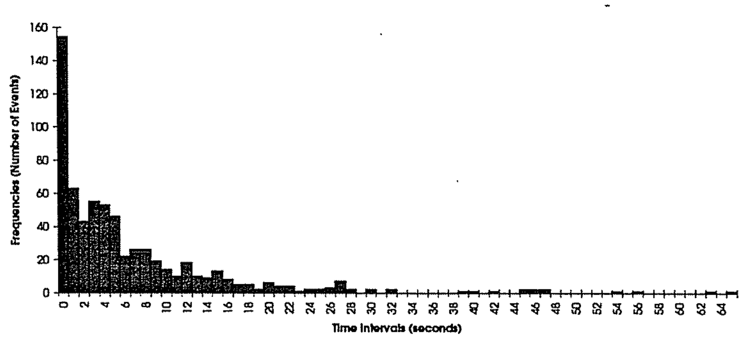


Figure 5.8d : Distribution of Time Interval between Orders (Data 042393c : Market 2)



Appendix 5C

Computation Steps of the Hurst Exponent

Suppose that a given time series has M observation points. First, we convert the series into a time series of length $N = M - 1$ of logarithmic ratios in the following manner.

$$N_i = \log \frac{M_{i+1}}{M_i}, i = 1, 2, \ldots, M - 1.$$

Then we divide the log–ratio time series into A consecutive subperiods of length n, i.e., $A \cdot n = N$. Each subperiod is labeled I_a, with $a = 1, 2, \ldots, A$, and each element in I_a is labeled $N_{j,a}$, with $j = 1, 2, \ldots, n$. For each I_a of length n, the average value is defined as

$$E_a(N) = \frac{1}{n} \sum j = 1nN_{j,a}$$

where $E_a(N)$ is an expected value of N_i in subperiod I_a. A series of cumulative deviations from the mean value for each subperiod I_a is defined as

$$X_{j,a} = \sum i = 1j(N_{i,a} - E_a(N)),\ j = 1, 2, \dots, n.$$

Hence, we have n values computed for each I_a. The range is defined as the difference between the maximum and the minimum value of $X_{j,a}$ in each I_a such that

$$R_a = \max(X_{j,a}) - \min(X_{j,a}),\ 1 \leq j \leq n.$$

The standard deviation calculated for each subperiod I_a is

$$\sigma_a = (\frac{1}{n} \sum j = 1n(N_{j,a} - E_a(N))^2)^{\frac{1}{2}}$$

We normalize each range R_a by dividing it by a corresponding standard deviation σ_a; that is, the rescaled range for each I_a is $\frac{R_a}{\sigma_a}$. The average R/S value for length n is defined as

$$(R/\sigma)_n = \frac{1}{A} \sum a = 1A \frac{R_a}{\sigma_a}.$$

The length n is increased to the next higher value, where $(M-1)/n$ is an integer. Then the procedure above is repeated for the new value of n, until $n = (M-1)/2$. Finally, we perform an ordinary least squares regression on $\log(n)$ as the independent variable and $\log(R/\sigma)_n$ as the dependent variable. The estimated slope of the equation is the estimate of the Hurst exponent. More detailed description of the rescaled range analysis can be found in Chapter 4 of Peters (1994).

Appendix 5D

An Instruction for the Experiments

INSTRUCTIONS

General:

This is an experiment in the economics of market decision making. Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money which will be paid to you in cash.

In this experiment there are two markets, called Market 1 and Market 2. In Market 1 you will buy and sell certificates in a sequence of market periods. Attached to the instructions you will find a sheet, labeled "Dividend Sheet," which helps determine the value to you of any decisions you might make in Market 1. In Market 2 you are either a buyer or a seller as indicated on your Market 2 Record Sheet. You will buy or sell units of goods in a sequence of market periods. On your Market 2 Record Sheet you will find *Redemption Value* or *Inventory Use Cost* of each unit, which helps determine the value to you of any decisions you might make in Market 2. You are not to reveal the information on your Dividend Sheet and the Record Sheet to anyone. It is your own private information.

The type of currency used in this market is francs. All trading and earnings will be in terms of francs. Each franc is worth ________ dollars to you in Market 1, and ________ dollars to you in Market 2. Do not reveal this number to anyone. At the end of the experiment your francs will be converted to dollars at this rate, and you will be paid in dollars. Notice that the more francs you earn, the more dollars you earn.

Market 1

Specific Instructions:

Your profits in Market 1 come from two sources — from collecting dividend earnings on all certificates you hold at the end of the period *and* from buying and selling certificates. During each market period you are free to purchase or sell as many certificates as you wish, provided you follow the rules below. The dividend per certificate depends on the *state* of the market period. Each market period will be in one of three states, X, Y, or Z. You can find the dividends associated with each of these three states on your "Dividend Sheet." You are assigned either Type I or Type II dividend sheet. Note that dividend values corresponding to a state may be different for different dividend types. For example, if the state is X, then the dividend you will receive might not be the same as the dividend received by someone else. The method by which the state is selected each period is explained later in these instructions.

Suppose that your dividend sheet was as follows. (The numbers are completely hypothetical.)

	State X	State Y	State Z
Dividend	100	70	50

At the end of each market period the state will be announced. You will compute your total dividend earnings for the period by multiplying the dividend per certificate, given the state, by the number of certificates held. That is, (number of certificates held) $\times$ (dividend per certificate) $=$ total dividend earnings. Suppose, for example, that you hold 5 certificates at the end of a period and that the state is X. If your dividend is 100 francs per certificate as in the example, then your total dividend earnings from the period would be $5 \times 100 = 500$ francs. Likewise, if the state is Y and if your dividend is 70 francs as in the example, then your total dividend earnings would be $5 \times 70 = 350$ francs. This number should be recorded in the box labeled **C** on your "Record Sheet" after each period.

Sales from your certificate holdings increase your francs on hand by the amount of the sale price. Similarly, purchases reduce your francs on hand by the amount of the purchase price. Thus you can gain or lose money on the purchase and resale of certificates. Your total gain or loss from buying and selling certificates should be recorded in the box labeled **B** on your "Record Sheet" after each period.

At the beginning of each period, each of you are provided with an initial holding of ____ certificates. You may sell these if you wish or you may hold them. If you hold a certificate, then you receive "dividend per certificate" at the end of the period. Notice therefore that for each certificate you hold initially, you can earn during the period *at least* the amount shown as "dividend per certificate." You earn this amount if you do not sell the certificate during the period.

In addition, at the beginning of each period you are provided with an initial amount of 10,000 francs on hand. You may keep this if you wish or you may use it to purchase certificates.

Thus at the beginning of each period you are endowed with holdings of ____ certificates and 10,000 francs on hand. You are free to buy and sell certificates as you wish according to the rules below. Your francs on hand at the end of each period are determined by your initial amount of francs on hand, dividends on certificate holdings at the end of the period, and gains and losses from purchases and sales of certificates. All francs on hand at the end of each period in excess of 10,000 francs are your total profits for the period and are yours to keep.

Determination of States:

The dividend you receive from the certificates you hold depends on the state of a market period. The state can be either X, Y, or Z. If the market period is in the state X, then your dividend per certificate is the one associated with the state X as given in your "Dividend Sheet." The state of a market period will be randomly determined before each period begins. But it will not be made public until the period ends. Each state is equally likely. A random number table was used and can be inspected by anyone after the experiment.

Information about States:

At the beginning of each market period, before trading starts, each of you will receive a clue card that may or may not carry some information regarding the state. If your clue card contains "X," then the state is X for sure. Similarly, "Y" and "Z" inform you that the state is Y and Z with certainty, respectively. If your clue card does not contain any information, then it means you have received no information for the period. In each period there will be exactly two people who receive the information, one person among Type I people and the other person among Type II people.

At the beginning of each period, each trader will draw a clue card out of a box that the experimenter had prepared. After you have drawn a clue card, write down the information you have received as it appears in the clue card in the box **A** of your "Record Sheet (Market 1)." If you have received no information, then write "No" in the box. The information given to you in a clue card is your private information, and you are not allowed to talk to each other regarding your private information.

Trading and Recording Rules:

1. All transactions are for one certificate at a time. After each of your sales or purchases you must record the TRANSACTION PRICE in the appropriate column on your

"Record Sheet" depending on the nature of the transaction.

2. You are free to sell or buy as many certificates as you wish. Notice that if you think that it would be profitable, then you can sell more certificates than you have on hand. In such a case, if you end up with a negative number of certificates on hand at the end of the period, then your dividend earnings would be negative. Suppose, for example, that you hold -2 certificates on hand at the end of period and that the dividend per certificate is 50 francs. Then your dividend earnings are $-2 \times 50 = -100$ francs. Of course if you sold the certificates for more than 50 francs each, then you have made a profit. But if you sold for less than 50 francs each, then you have made a loss.

3. At the end of each period, compute your total earnings from buying and selling from the period, and record it in the box labeled **B**.

4. At the end of each period, after the experimenter has announced the state of the period, compute your total dividend earnings from the period and record it in the box labeled **C**.

5. The price of the information given to you at the beginning of each period is zero in every market period. Therefore, 0 has been entered in the box labeled **D** in every market period.

6. At the end of each period, compute your total profits from the period by adding the numbers in boxes **B** and **C**, and record it in the box labeled **E**. Also record it on the appropriate row of your "Profit Sheet."

7. At the end of the experiment, add up your profit from all the periods, and record it on row 18 of your "Profit Sheet." Then, convert it into dollars by multiplying the profit by the conversion rate that is given on row 19 of your "Profit Sheet." Finally record your profit in dollars on row 20 of your "Profit Sheet." The experimenter will pay you this amount of money in cash.

Market Organization:

The market for these certificates is organized as follows. The market will be conducted in a series of market periods. Each market lasts for ______ minutes. The technology of trading will be explained to you.

Are there any questions?

Anyone wishing to purchase a certificate is free to do so by employing either or both of the following.

1. Enter the price you are willing to pay, let's call it 'buy order,' in the order box on your monitor and hit [F1] key, and wait until someone accepts your buy order. Note that if your buy order is *higher* than any other buy orders, then it will become a standing buy order and will appear in the buy order box on the monitor. If your buy order is *lower* than the current standing buy order, then it will be kept in a "Book," which you can always view by hitting [F5] key.

2. You can purchase a certificate by accepting a 'sell order' which appears in the sell order box on the monitor. In order to accept a sell order, hit [Ctrl] + [F1] keys simultaneously. Note that 'accepting a sell order' means that you are purchasing a certificate at a price that appears in the sell order box on your screen.

Similarly, anyone wishing to sell a certificate is free to do so by employing either or both of the following.

1. Enter the price you are willing to sell (a sell order) in the order box on your monitor and hit [F2] key, and wait until someone accepts your sell order. Note that if your sell order is *lower* than any other sell orders, then it will become a standing sell order and will appear in the sell order box on the monitor. If your sell order is *higher* than the current standing sell order, then it will be kept in a "Book" which you can always view by hitting [F5] key.

2. You can sell a certificate by accepting a 'buy order' which appears in the buy order box on the monitor. In order to accept a buy order, hit [Ctrl] + [F2] keys simultaneously. Note that 'accepting a buy order' means that you are selling a certificate at a price that appears in the buy order box on your screen.

Are there any questions?

FINANCIAL AGREEMENT

SHOULD MY EARNINGS FROM THE EXPERIMENT BE NEGATIVE, I AGREE TO WORK IN THE ECONOMIC SCIENCE LABORATORY AT A RATE OF 7 DOLLARS PER HOUR UNTIL THE LOSS IS REPAID.

NAME ______________________________________

DATE ______________________________________

Appendix 5E

Record Sheet and Dividend Sheet

Subject ID #____ Period #____

Record Sheet (Market 1 : Type I)

Information on a clue card is : [ ] **A**

Row	Sale Price	Purchase Price	
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			Revenue from
19			trading
20		−	=

B

Number of certificates on hand at the end		Dividend per certificate		Total dividend	
[ ]	×	[ ]	=	[ ]	**C**

Your profit in Market 1 from this period is :

B + **C** = [ ] **D**

Subject ID #____ Period #____

Record Sheet (Market 1 : Type II)

Information on a clue card is : [ ] **A**

Row	Sale Price	Purchase Price
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Revenue from trading

— = [ ] **B**

Number of certificates on hand at the end		Dividend per certificate		Total dividend	
[ ]	×	[ ]	=	[ ]	**C**

Your profit in Market 1 from this period is :

B + **C** = [ ] **D**

Subject ID #____ Period #____

Record Sheet (Market 2 : Buyer)

Row	Redemption Value (1)	Purchase Price (2)	Profit ((1) - (2))
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
		Your profit in Market 2 from this period is ⟶	

Subject ID #____ Period #____

Record Sheet (Market 2 : Seller)

Row	Sale Price (1)	Inventory Use Cost (2)	Profit ((1) - (2))
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
		Your profit in Market 2 from this period is ⟶	

Profit Sheet

Subject ID # ____

Row	Market Period	Profit (Market 1)	Profit (Market 2)
0	0 (Practice)		
1	1		
2	2		
3	3		
4	4		
5	5		
6	6		
7	7		
8	8		
9	9		
10	10		
11	11		
12	12		
13	13		
14	14		
15	15		
16	16		
17	17		
18	Total Profit in Francs		
19	Dollars per Franc		
20	Total Profit in Dollars		

NAME ______________________________________

DATE ______________________________________

Dividend Sheet (Market 1 : Type I)

	State X	State Y	State Z
Dividend	120	330	40

Dividend Sheet (Market 1 : Type II)

	State X	State Y	State Z
Dividend	205	90	125

Appendix 5F

Data Files

The raw data from the experiments reported here are saved at the Laboratory for Experimental Economics and Political Science at the California Institute of Technology. The file names follow the convention we use in the lab., 042393b.sta and 042393c.sta.

The raw data required several modifications due to obvious mistakes that were made by the experiment participants or problems associated with a computer software. These changes are documented below.

- (042393b, mkt1) Period 14 at 7499 seconds : The bid order by agent 5 has been modified into 5 units instead of 50 units.
- (042393c, mkt1) Period 7 at 3781 seconds : "dask" order was placed by agent 1 (1 unit of 250). However, the .col data set indicates that this ask order was never entered before the "dask" order. Hence, it was eliminated. MUDA was revised to correct such problems in the later version.

- (042393c, mkt1) Period 7 at 3933 seconds : "dask" order entered by agent 1 (1 unit of 200) had the same problem as above, and was eliminated.
- (042393c, mkt1) Period 10 at 5261 seconds : "dask" order entered by agent 3 (1 unit of 129) had the same problems as above, and was eliminated.
- (042393c, mkt1) Period 13 at 6524 seconds : "dask" order entered by agent 1 (1 unit of 250) had the same problem as above, and was eliminated.
- (042393c, mkt1) Period 15 at 7260 seconds : "dask" order entered by agent 5 (1 unit of 190) had the same problem as above, and was eliminated.
- (042393c, mkt2) Period 11 at 5714 seconds : "dask" order entered by agent 3 (1 unit of 160) had the same problem as above, and was eliminated.
- (042393c, mkt2) Period 14 at 6862 seconds : "dask" order entered by agent 1 (3 units of 155) had the same problem as above, and was eliminated.
- Multiple unit orders were modified into multiple orders of single unit.

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