Asymmetric Information and Cooperation

Thesis by

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Abstract

This thesis investigates the theory of cooperative behavior in the presence of asymmetric information.

Traditionally, the core has been a powerful and much used solution concept to describe cooperative outcomes. In settings where agents have some private information, it may be appropriate to include the opportunity for communication in the development of the core. I study the relationship of various core solution concepts with prevalent noncooperative solution concepts for environments with asymmetric information. The core definitions examined vary by the level of communication assumed. In Chapter 2, I investigate the welfare properties of market equilibria. I demonstrate that appropriate communication restrictions can be placed on the core (and efficiency) in order to obtain first and second welfare theorems. In Chapter 3, I discuss the Bayesian implementation of core solutions. If full communication is assumed, Palfrey and Srivastava (1987) have shown that the core is not Bayesian implementable: a game cannot be constructed that has *only* core allocations as its equilibria. I demonstrate that communication restrictions on the core are sufficient to obtain positive Bayesian implementation results in the environment studied by Palfrey and Srivastava. In other words, a game can be constructed that entices noncooperative players to choose strategies that are cooperative under limited communication.

In Chapter 4, I examine cooperation between bidders in private value, sealed bid auctions. I assume that bidders can overcome their one period temptation to break any collusive agreement, and that they attempt to formulate a *collusive mechanism.* However, each bidder's valuations are still his own, private information. If he is not given the proper incentives, he may lie about his values in order to increase his profits. Therefore, any collusive mechanism must be incentive compatible and is likely to be, at a minimum, interim efficient. I demonstrate that the theory provides some predictions about the set of collusive mechanisms chosen by bidders and that, when

moving to a setting where multiple objects are for sale, the set of feasible collusive mechanisms grows. When multiple objects are for sale, there exist incentive compatible mechanisms that are preferred by all bidders to the only incentive compatible mechanisms in the single object case. Laboratory experiments indicate that these predictions are often consistent with actual behavior. However, deviations by some bidders suggest some weaknesses in this approach.

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Chapter 1 Introduction

What decisions will a group of privately informed agents arrive at? This question obviously has many possible answers depending on the specific environments examined and the assumptions on behavior. In this thesis, two distinct approaches are taken. First, in Chapter 2 and 3 the core is examined in economies with asymmetric information. The core has been a popular description of cooperative behavior. The core is compared with two prominent noncooperative solution concepts for asymmetric information economies: rational expectations equilibrium (REE), and Bayes Nash equilibrium. I study the conditions under which cooperative behavior, described by the core, is supported by noncooperative behavior. Second, a particular setting where cooperation is of considerable interest is in auctions. Possible cooperative (collusive) strategies for a private informed bidders are suggested and studied in experiments.

A first step in approaching this topic is to understand exactly what is meant by the two terms in the title. First, asymmetric information is a description of the distribution of information in the economy. At least one agent possesses some information about preferences, endowments, or other relevant information that other agents do not know. It is also assumed that this information is truly private: it is impossible for an agent to credibly prove that he has some particular information. Second, cooperation is defined as coordinated behavior between agents. When agents are acting noncooperatively, they are assumed to act unilaterally in their own self-interest. However, this unilateral action by all agents can lead to decisions that are inferior to some other feasible choices for all agents. Therefore, cooperative behavior may allow them to obtain preferred outcomes through coordinated action.

1.1 A Brief History of the Core

While the concept of the core was originally introduced by Edgeworth (1881), and von Neumann and Morgenstern (1944) studied similar notions of coalitional stability, the formal definition of, and use of the term, the core was first provided by Gillies (1959) and Shubik (1959). In its broadest sense, the core is defined as *any outcome that is not blocked (or dominated)*[1] *by any coalition.*

The core is meant to represent a kind of coalitional (or cooperative) stability: if at a core point, then no individual, or group, would want to move away from that point. However, at any point not in the core, at least one group would have an incentive to coordinate their activities in order to reap greater utility.

> At any [allocation] in the core all players obtain at least as much as they are able to enforce in any coalition. By means of n-person game analysis and an examination of the core, we can give meaning to such vague ideas as "a world of monopolies." In any economy, any [allocation] of wealth which does not lie in the core implies that some group of individuals is profiting at the expense of another group. (Shubik 1959)

As Aumann (1961) describes, in order to transform this intuitive definition of the core into a formal mathematical notion, it is necessary to answer three questions:

1. What is the setting? (i.e., Is this an exchange economy, noncooperative game, cooperative game, etc.?)
2. What are the possible 'outcomes'?
3. What does it mean for a coalition to 'block'?

Different answers to these three question can lead to widely varying results.

For example, Gillies (1959) originally defined the core in the same setting as von Neumann and Morgenstern's (1944) seminal analysis of cooperative games. The ability of agents to completely transfer utility amongst themselves is assumed. Therefore,

[1] Shapley (1972) has criticized the use of the term *blocked* in favor other terms such as *dominated.*

the *value*, $V(S)$, of any coalition can be represented by a single number and any allocation such that $\sum_S x_i < V(S)$ is blocked by coalition S. In this transferable utility (TU) setting, the core is a subset of the von Neumann and Morgenstern stable set. The von Neumann and Morgenstern stable set requires that any allocation not in the set be blocked by some allocation in the set, and that any allocation in the set not be blocked by any other allocation in the set. Aumann (1961) studied the core in a more restrictive class of situations known as non-transferable utility (NTU) games. The set of feasible allocations for a coalition can no longer be characterized by a single number. Instead, $V(S)$ is the set of possible allocations for coalition S.

One of the primary reasons for the historical success of the core solution is the role it has played in describing competitive equilibrium behavior in large economies.[2] Edgeworth was the first to describe this feature. In a two agent exchange economy, the set of mutually beneficial trades (the contract curve) is much larger than the set of competitive equilibria. However, as the economy grows (under some reasonable assumptions) and bargaining between more than two players is allowed, this "range of indeterminacy" or contract curve shrinks. In the limit, the contract curve is identical to the competitive allocation. His conjecture was formalized and proven first by Shubik (1959) in a limited setting, and proven in a more general setting by Debreu and Scarf (1963).[3] The core limit theorem provides a strong justification for competitive equilibrium: for extremely large economies the difference between competitive equilibria and cooperative equilibria is negligible. In other words, in the limit, the price taking assumption of competitive equilibrium is justified. No agent or group of agents can benefit by choosing some other set of feasible trades.

Most of the utilization of the core described up to this point is in games with transferable utility and pure exchange economies. In these settings, it is fairly obvious how to define the set of outcomes for each coalition. For example, in an exchange economy, a coalition can obtain any allocation that is equal to their endowment ($\sum_S x_i \leq \sum_S e_i$). However, in more complex environments defining the set of fea-

[2]Schotter (1973) provides an in depth survey of the core's relation to competitive equilibria.
[3]Numerous other studies have extended limit results in even less restrictive settings.

sible outcomes becomes more difficult. In economies with production, production is typically introduced by the inclusion of a set Y that describes how inputs may be turned into output for the whole economy. What can a coalition produce? In order to define feasible outcomes for a coalition, a production possibilities set, Y_S, for each coalition must be defined. The addition of externalities can cause similar difficulties. If a coalition breaks off, do they enjoy the externality provided by the whole economy? Or, are they to be treated as a completely separate economy? Foley (1970) discusses versions of the core that assume different possibilities for coalitions. Even the setting of noncooperative games complicates the discussion of outcomes. Suppose that a coalition decides on a particular strategy in the game. Then their payoff will vary by the choices of strategies by the other $N - S$ players. The α-core and β-core are the prominent methods for rectifying this difficulty (Aumann 1961). In the α-core the payoff to a coalition for any strategy is assumed to be the minimum feasible payoff: the $N - S$ coalition chooses a strategy which punishes the deviating coalition to the greatest extent possible. The β-core makes the opposite assumption. Of course, a problem with both these core versions is that they may not make sense for the $N - S$ coalition to actually follow such strategies.

The setting that is of primary interest in this thesis is where agents have private information. In all the settings discussed previously, it was assumed that all the agents knew everything there is to know about the other agents. However, in most realistic economic situations, there is significant asymmetric information. For example, I know things about my preferences that you do not know, and it is impossible, without the proper incentives, to force me to truthfully reveal this information. A classic and simple example of such a setting is known as a *lemons market* originally due to Akerlof (1970). In this setting a seller has some private information about the quality of the product he is selling. The potential buyer has to consider the fact that the product may be of a high or low quality when deciding to make a purchase at a particular price. The existence of private information can lead to new difficulties in defining the core. First, it is not necessarily reasonable to assume full communication. Agents may not want to fully communicate all of their private information. For example, a seller with a high quality product would want to announce the quality of his product. However, holders of low quality products would not be so willing to communicate. Second, what is a feasible outcome at any given time can vary according to the private information of the agents. Wilson (1978) tackled the first issue in his seminal discussion of the application of the core to asymmetric information exchange economies. He defined the *fine* core, which assumes full communication between agents, and the *coarse* core, which assumes no communication between agents. Yannelis (1991) and Allen (1991) represented these communication restrictions by measurability restrictions on the set of attainable outcomes for coalitions. They also described a new core concept known as the *private* core. While numerous other variants of the core have been developed for asymmetric information economies (see (Hahn and Yannelis 1997b, Lee 1997, Lee and Volij 1997, Volij 1997, Vohra 1997)), much of the focus has been placed on formulating new variations of the core rather than demonstrating the efficacy, or usefulness, of a particular version. My intention here is to begin to unlock the conundrum of the core with asymmetric information by identifying the properties the core must satisfy in order to obtain results that the core (in perfect information settings) naturally satisfies.

In Chapters 2 and 3, I examine the core in pure exchange, differential information economies. I take a similar approach to Yannelis and Allen in utilizing measurability restrictions to define different levels of assumed communication. Given a particular communication structure for each coalition, outcomes are required to be consistent with this level of communication: if, after communication, an agent still cannot distinguish two states, then his allocation should not vary over those states. I also examine some features of the definition of blocking for these economies. In Chapter 2, I look at the difference between strong and weak blocking, and in Chapter 3 I discuss the use of a *one state deviation* principle instead of the typical requirement that an allocation must be preferred in all states.

1.2 Would I Lie to You?

An assumption of most of the literature on the core with asymmetric information is that, if agents communicate, then they communicate truthfully. This is a strong assumption about cooperation between agents. However, as the lemons market example indicates, some agents may be more than willing to lie about their information: the low quality seller may be willing to communicate but he would undoubtedly never acknowledge that he had a low quality product. A more realistic assumption might be that agents need to be given the proper incentives in order to truthfully reveal their information. There are two justifications for the imposition of an incentive compatibility constraint. First, if agents are making a group decision and they do not know that the information revealed by all agents is truthful then they have to discount what they say (or not use their information). Second, the Revelation Principle guarantees that any outcome which can be attained as a Bayes Nash equilibrium of some game can also be attained through a direct revelation mechanism (Gibbard 1973, Dasgupta, Hammond, and Maskin 1979). In other words, any noncooperative equilibrium can be replicated by a mechanism that is incentive compatible.

In Chapter 4, I focus on how incentive compatibility limits the choice of possible strategies by bidders in an auction. Bidders would ideally like to arrive at an allocation which maximizes their joint surplus, but these incentive constraints limit their effectiveness. While in single object auctions these restrictions can be quite severe (at least when side payments are not allowed), in multiple object auctions a larger variety of uniformly more profitable collusive strategies are possible.

1.3 Notational and Mathematical Preliminaries

Every effort has been made to keep notation consistent across chapters.[4] Each chapter contains a section that describes the notation used, assumptions made, and other particular features. A few preliminary notational conventions are worthy of mention.

[4]However, Chapter 4 contains notation that is significantly different than that of the preceding two chapters.

Let x and y be two vectors with x_i and y_i specific elements within the vector. The following operators are defined as follows:

$$x > y \quad \text{if and only if} \quad x_i > y_i \text{ for all } i$$
$$x \geq y \quad \text{if and only if} \quad x_i \geq y_i \text{ for all } i$$
$$x = y \quad \text{if and only if} \quad x_i = y_i \text{ for all } i$$

The vector containing a zero in all of its elements is given by $\mathbf{0}$. Let A be a finite set. Then $\#A$ indicates the cardinality of the set.

I define terms that are used without definition but may be unfamiliar to the reader. Aliprantis and Border (1994) is a common reference for most of these definitions and provides a more thorough investigation into their properties.

Let F and G be two cumulative distribution functions on the same space X. The distribution F first-order stochastic dominates G if it is always more likely to take on higher values.

1.3.1 Definition *F first-order stochastic dominates G if*

$$F(x) \leq G(x) \text{ for all } x \in X.$$

Let (X, Σ_1) and (Y, Σ_2) be two measurable spaces where Σ_1 and Σ_2 are both σ-fields of X and Y respectively. Let f be a function mapping (X, Σ_1) to (Y, Σ_2).

1.3.2 Definition *A function f is (Σ_1, Σ_2) measurable if for all $S \in \Sigma_2$, $f^{-1}(S) \in \Sigma_1$.*

1.3.3 Definition *A vector space X and an order relation $\geq$ is a Riesz space if:*

1. *$x \geq y$ implies $x + z \geq y + z$ for all $z \in X$,*

2. *$x \geq y$ implies $\alpha x \geq \alpha y$ for all $\alpha \geq 0$, and*

3. *If each pair of elements $x, y \in X$ has a supremum and an infimum.*

1.3.4 Definition *A space X is a Banach Lattice if it is a norm complete Riesz space.*

1.3.5 Definition *A real function, $\| \cdot \|$, on a vector space X is a norm if:*

1. $\| x+y \| \leq \| x \| + \| y \|$,

2. $\| \alpha x \| = |\alpha| \| x \|$ *for all* $\alpha \geq 0$, *and*

3. $\| x \| = 0$ *if and only if* $x = 0$.

Let x_α be a net. Then we can define an order continuous norm.

1.3.6 Definition *A norm $\| \cdot \|$ is order continuous if $x_\alpha \downarrow 0$ implies $\| x_\alpha \| \downarrow 0$.*

Let (Y, Σ, μ) be a measure space and X be a Banach lattice. A function, $g : Y \to X$ is *simple* if there exists (finite) $x_1, x_2, \ldots, x_n \in X$ and $S_1, S_2, \ldots, S_n \in \Sigma$ such that $g = \sum_{i=1}^{n} x_i \chi_{S_i}$ where χ_{S_i} is an indicator function ($\chi_{S_i} = 1$ if $y \in S_i$).

1.3.7 Definition *A measurable function $f : Y \to X$ is Bochner integrable if there exists a sequence of simple functions $(f_n)_{n=1}^{\infty}$ such that*

$$\lim_{n \to \infty} \int_Y \| f_n(y) - f(y) \| \, d\mu(y) = 0$$

Chapter 2 The Welfare Properties of Rational Expectations Equilibria: The Core

Abstract

By utilizing restrictions on the measurability of allocations, the welfare properties of REE are clarified. Measurability restrictions may be thought of as exogenous constraints on the level of communication between differently informed agents. I present first and second welfare theorems for REE. The measurability restrictions necessary to obtain these results highlight both the sources of inefficiency for REE and the information processing embodied in REE. In simple replica economies, core equivalence cannot be obtained. Finally, I present a new market equilibrium concept, pseudo rational expectations equilibrium (PREE), which yields positive welfare theorems and core equivalence.

2.1 Introduction

Rational expectations equilibria (REE) may not be ex post, interim, or ex ante efficient (Laffont 1985). On the other hand, fully revealing REE are generally ex post efficient but may not be interim efficient (Allen 1981, Grossman 1981). The existence of partially revealing REE may eliminate insurance opportunities and Pareto improving trades. Jordan (1983) has demonstrated that only for limited parametric classes of utility functions are all REE efficient. What more positive can be said about the welfare properties of REE? Can some allocations be ruled out as possible REE? What properties must an allocation have in order for it to be a REE? In perfect information economies, these questions are answered by the first and second welfare theorems which say that all Walrasian equilibria are efficient (and in the core) and that any efficient allocation can become a Walrasian equilibrium.

In this chapter, I study one possible technique for obtaining similar results in asymmetric information exchange economies: the use of measurability restrictions. In these economies, where there are multiple possible states of nature, how an allocation varies over these states may be related to the information agents possess. For example, if agents are unable to distinguish between two states, then they should not contract different allocations in each state (just as a player in an extensive form game cannot choose different actions at different nodes in the same information set). Recently, such an approach has been used to provide *constrained* definitions of efficiency and the core in differential information economies (Hahn and Yannelis 1997b, Yannelis 1991, Koutsougeras and Yannelis 1993, Allen 1993). Using measurability restrictions can be thought of as exogenously imposing communication restrictions on agents and coalitions (Wilson 1978). *Classical* versions of ex ante and interim efficiency necessarily imply full communication between all agents by allowing allocations to vary over every state. Measurability constrained versions enlarge the set of 'efficient' or 'core' allocations by shrinking the set of blocking allocations.

In addition to measurability restrictions, in asymmetric information economies the informational stage becomes a relevant concern. There are three primary stages of interest: ex ante, interim, and ex post. At the ex ante stage agents do not yet possess their private information, yet are aware of what their preferences, endowments, etc. would be *if* they did know the state of the world. At the interim stage, each agent has observed some private information which allows him to rule out certain events. Finally, at the ex post stage all relevant uncertainty is revealed and each agent knows the state of the world. Much of the analysis of measurability constrained core and efficiency solutions has focused on evaluations made at the ex ante stage (Hahn and Yannelis 1997b, Koutsougeras and Yannelis 1993, Page 1997, Srivastava 1984). Analysis at the ex post stage becomes equivalent to a discussion of perfect information economies (since all private information is revealed). When discussing the welfare properties of REE, the interim informational stage seems most appropriate. REE is, at least implicitly, an interim concept: agents make choices of quantities to demand having learned their private information and additional information revealed by prices. Therefore, interim welfare concepts which consider what agents prefer given their private information are examined.

I present measurability restrictions which are sufficient to obtain first and second welfare theorems in the interim stage. All REE fall within the set of allocations which are not blocked by coalitions of agents where each agent uses only his private information (the fine private core). However, in order to construct an allocation which is itself a REE, one must also know that it is not blocked by the grand coalition using its pooled information (fine efficient). The measurability restrictions required for the second welfare theorem are more restrictive than those required for the first welfare theorem. This suggests that, while it is only possible to make minimal inferences about the level of communication necessary to yield REE, strong communication assumptions must be made in order to find sufficient conditions for an allocation to be a REE. The inconsistency between these two results suggests a new critique of the REE concept, since it calls into question how a market process (or auctioneer) might arrive at the allocations and prices described by REE.

I relax the measurability assumptions of REE and describe a new market equilibrium concept called pseudo rational expectations equilibrium (PREE). While agents still only use the information revealed by prices in order to update their preferences, there are no information restrictions on the set of feasible allocations. The measurability restrictions needed for welfare theorems for PREE do not share the previous inadequacy. They also demonstrate that a weakening of fully revealing REE is sufficient to obtain ex post efficient allocations.

Replica economies are generalized to differential information economies. After demonstrating that the fine private core does not converge to the set of REE, I show that core equivalence for PREE can be obtained.

In Section 2.2 I present the basic model of a Radner differential information economy. In Section 2.3, I describe how measurability restrictions are used to provide constrained efficiency and core definitions. I also note an important difference between weak and strong blocking in these environments. Section 2.4 presents first and second welfare theorems for REE and PREE. Finally, in Section 2.5 I discuss core convergence in replica economies. General sufficient conditions are provided for core definitions to satisfy equal treatment.

2.2 The Model

The model of an exchange economy presented here is the same as that used by Page (1997) and Yannelis (1991). Let $N = \{1, 2, \ldots, n\}$ be the number of agents in an exchange economy. The commodity space is given by $Y = \mathbb{R}^\ell$ with positive orthant $Y_+ = \mathbb{R}^\ell_+$.[1] Let $(\Omega, \mathcal{F}, \mu)$ be the probability space describing uncertainty in the model where μ is a probability measure representing ex ante probabilities and $\mathcal{F}$ is a σ-field over Ω. Let the set of all possible state contingent allocations be given by $L_1(\Omega, \mathcal{F}, \mu; Y)$ or the space of equivalence classes of $\mathcal{F}$-measurable, (Bochner) integrable functions $x : \Omega \to Y$. An agent, i, is a five-tuple $(Y_i, \mathcal{F}_i, u_i, e_i, \mu_i)$ where:

$Y_i : \Omega \to 2^{Y_+}$ is the state dependent consumption set correspondence of agent i,

[1] All the results of this chapter remain true if Y is a Banach lattice with order continuous norm.

$\mathcal{F}_i$ is a sub-σ-field of $\mathcal{F}$ that represents agent i's private information,

$u_i : Y \times \Omega \to \mathbb{R}$ is agent i's state contingent utility function,

$e_i : \Omega \to Y_+$ is a function denoting agent i's state contingent initial endowment of commodities, and

μ_i is a probability measure on $(\Omega, \mathcal{F})$ that represents agent i's prior beliefs.

An economy with asymmetric information $\mathcal{E} = \langle (Y_i, \mathcal{F}_i, u_i, e_i, \mu_i)_{i \in N} \rangle$ is a finite collection of agents. The following assumptions are imposed on the model.

2.2.1 Assumption *For each $\omega \in \Omega$, $Y(\omega) \subset Y_+$ is a nonempty, convex, and sequentially closed set.*

2.2.2 Assumption *For all $i \in N$*

i. u_i is $(\mathcal{F}, \mathcal{B}_Y)$ measurable where $\mathcal{B}_Y$ is the Borel σ-field of Y,

ii. For all $\omega \in \Omega$ $u_i(\omega, \cdot)$ is concave, sequentially weakly upper semicontinuous on Y_+, and

iii. $u_i(\cdot, \cdot)$ is integrably bounded.

Measurability of the utility function assures that $\mathcal{F}$ captures all relevant information in the economy. Utility is also assumed to be integrably bounded in order to ensure existence of an expected utility representation.

2.2.3 Assumption *For all $i \in N$*

i. e_i is $\mathcal{F}_i$ measurable,

ii. e_i is (Bochner) integrable, and

iii. $e_i(\omega) \in Y_i(\omega)$ a.e.$[\mu]$.

Assuming that endowments are measurable with respect to private information is standard (Koutsougeras and Yannelis 1993, Page 1997). If the endowment were not $\mathcal{F}_i$ measurable, then an agent could use the fact that his endowment varies over states which he previously could not distinguish to refine his information.

I also make a variety of assumptions on the structure of information ($\mathcal{F}_i$) and beliefs (μ_i) for each agent. A class of subsets, $\mathcal{G}$, of Ω are said to *generate* $\mathcal{F}$ if $\mathcal{F}$ is the smallest σ-field containing $\mathcal{G}$.

2.2.4 Assumption *$\mathcal{F}$ is finitely generated.*

Finitely generated σ-fields can be generated by finite partitions.[2] Therefore, finitely generated σ-fields represent finite information structures where each atom, denoted by $F(\omega)$, represents the *smallest* discernible event.

2.2.5 Assumption $\mathcal{F} = \sigma(\bigcup_{i \in N} \mathcal{F}_i)$.

This is the same assumption used by Allen (1981) to eliminate the possibility that prices may depend on more information than all the agents jointly possess. In other words, *sun spot* equilibria, i.e., equilibria which depend on information which *none* of the agents have are ruled out.

Each agent's prior is assumed to be absolutely continuous with respect to μ.

2.2.6 Assumption *For each $i \in N$, and for $E \in \mathcal{F}$, $\mu(E) = 0$ if and only if $\mu_i(E) = 0$.*

In order to ensure that interim expected utilities are well defined, measures are assumed to be purely atomic.

2.2.7 Assumption *$\mu(F(\omega)) > 0$ for all $\omega \in \Omega$.*

There are no events to which any agent assigns zero probability. Under Assumption 2.2.6, this assumption is without loss of generality: any economy for which there are states to which all agents assign zero prior probability, a modified economy

[2] The partition formed by the σ-field's atoms will generate the σ-field.

which excludes these states and satisfies Assumption 2.2.7 can be constructed. Since these excluded states were assigned zero probability, they could not have effected any agent's preferences.

A bundle for agent i is a function $x_i : \Omega \to Y_+$ that assigns to agent i a commodity vector in each state of the world. The set of bundles for agent i is denoted by $\mathbb{B}_i$, where $\mathbb{B}_i = \{x_i : x_i(\omega) \in Y_i(\omega) \text{ a.e.}[\mu]\}$. An allocation for a coalition $S \subseteq N$ is a vector $x \in (\mathbb{B}_i)_{i\in S}$ denoted by x_S. I denote the set of all feasible allocations for this coalition by $\mathbb{B}(S)$ where

$$\mathbb{B}(S) = \left\{ x_S | \sum_{i\in S} x_i(\omega) = \sum_{i\in S} e_i(\omega) \text{ a.e.}[\mu] \right\}.$$

Let $\mathbb{G}$ be the set of all possible sub-σ-fields of $\mathcal{F}$ satisfying the assumptions given above. The *interim expected utility* function $V_i : \mathbb{B}_i \times \Omega \times \mathbb{G} \to \mathbb{R}$ of agent i is defined by

$$V_i(x_i, \omega, \mathcal{G}) = \frac{1}{\mu_i(G(\omega))} \int_{\omega' \in G(\omega)} u_i(x_i(\omega'), \omega') d\mu_i(\omega').$$

When not explicitly stated, interim expected utility will be assumed to be taken with respect to agent i's private information, $\mathcal{F}_i$, and will be denoted $V_i(x_i, \omega)$.

2.2.8 Lemma *For all $i \in N$, if $u_i(\omega, \cdot)$ is upper semicontinuous and concave for every $\omega \in \Omega$, then $V_i(\cdot, \omega, \mathcal{F})$ is weakly upper semicontinuous and concave for all $\omega \in \Omega$.*

Proof: See Theorem 2.8 in Balder and Yannelis (1993). ∎

2.2.9 Lemma *For all $i \in N$, if $u_i(\omega, \cdot)$ is strictly concave for every $\omega \in \Omega$, then $V_i(\omega, \cdot)$ is strictly concave for all $\omega \in \Omega$.*

Proof: See Allen (1993). ∎

2.3 Information Restrictions, Efficiency, and the Core

If an agent or set of agents cannot distinguish two states of the world, then it is reasonable to expect that their actions should not vary between those states. At the interim stage, agents possess their private information $\mathcal{F}_i$. In other words, one would expect their choices to remain constant for $F_i(\omega)$. However, in the process of coming to a group decision (which implicitly the core is) agents may *communicate* some of this private information. Since agents are asymmetrically informed, each agent may be able to improve his information (distinguish between more states). Unconstrained versions of interim efficiency and the core allow allocations to vary over any state (see Holmström and Myerson (1983)). Thus, it is as if the agents in N (the grand coalition) fully communicate their information to each other. Constrained versions of efficiency and the core can be developed by placing measurability restrictions upon the definitions. Under the assumptions made on the probability space, the extent of communication between agents is representable by measurability restrictions on the set of allocations. A function $f : \Omega \to X$ is measurable with respect to $\mathcal{G}$ if:

$$f(\omega') = f(\omega), \text{ for all } \omega' \in G(\omega), \text{ for all } \omega \in \Omega$$

where $G(\omega)$ is an atom. For example, unconstrained core and efficiency concepts allow each agent's allocation, x_i, to be $\mathcal{F}$ measurable. This approach has been used extensively in the developing literature on the core in economies with differential information (Yannelis 1991, Allen 1993, Srivastava 1984) and has also recently been applied to definitions of efficiency by Morris (1994) and Hahn and Yannelis (1997b).

There are two types of restrictions that may be imposed: restrictions on the final allocation, and restrictions on the allocations which coalitions use to block. Let $\mathcal{G}_i^*$ be the σ-field with respect to which the final allocation, x_i, is assumed to be measurable for each i. Let $\mathcal{G}_i(S)$ be the σ-field with respect to which blocking allocations, y_i, for $i \in S$ can be measurable. By restricting $\mathcal{G}_i^*$ to a sub-σ-field of $\mathcal{F}$, the number of available allocations shrinks. However, restricting $\mathcal{G}_i(S)$ to a sub-σ-field of $\mathcal{F}$ decreases the number of blocking allocations, thus potentially increasing the set of solutions.

An allocation is *blocked* if there is a coalition, S, and a state of the world such that all agents in the coalition strictly prefer some other feasible allocation at that state.

2.3.1 Definition *An allocation, x, is $\mathcal{G}_i(S)$ blocked by S if there exists an $\omega \in \Omega$ and a $y \in \mathbb{B}(S)$ such that y_i is $\mathcal{G}_i(S)$ measurable for all $i \in S$ and $V_i(y_i, \omega) > V_i(x_i, \omega)$ for all $i \in S$.*

2.3.2 Definition *x is $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ efficient if:*

i. x_i is $\mathcal{G}_i^*$ measurable for each $i \in N$,

ii. $x \in \mathbb{B}(N)$, and

iii. x is not $\mathcal{G}_i(N)$ blocked by N.

A general definition for the core with measurability restrictions is as follows.

2.3.3 Definition *x is in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core if:*

i. x_i is $\mathcal{G}_i^*$ measurable for each $i \in N$,

ii. $x \in \mathbb{B}(N)$, and

iii. For all $S \subseteq N$, x is not $\mathcal{G}_i(S)$ blocked by S.

A variety of combinations of measurability restrictions on both the allocation, $\mathcal{G}_i^*$, and the blocking allocations, $\mathcal{G}_i(S)$, may be used to formulate alternative versions of efficiency and the core. Represent coarse information sharing by $\overline{\mathcal{F}}_S = \sigma(\bigcap_{i \in S} \mathcal{F}_i)$: the largest σ-field common to all σ-fields $\mathcal{F}_i$. All events $E \in \sigma(\bigcap_{i \in S} \mathcal{F}_i)$ are said to be common knowledge for coalition S. Also, let $\underline{\mathcal{F}}_S = \sigma(\bigcup_{i \in S} \mathcal{F}_i)$ be the coarsest σ-field containing all the σ-fields $\mathcal{F}_i$ representing distributed (or pooled) information of coalition S. Hahn and Yannelis (1997b) have extensively discussed various constrained versions of efficiency. One such definition is that of *fine efficiency* which requires that

$\mathcal{G}_i^* = \mathcal{F}_i$ and $\mathcal{G}_i(N) = \underline{\mathcal{F}}_N$ for all $i \in N$. While the final allocation for each agent may only be measurable with respect to his private information, blocking allocations may be measurable with respect to the agents' pooled information implying full communication. Versions of the coarse and fine core originally defined by Wilson (1978) and presented in terms of measurability restrictions by Yannelis (1991) can also be captured using this approach. The coarse core encompasses the restriction that coalitions can only make decisions over those events which are commonly known to the coalition. Therefore, $\mathcal{G}_i(S)$ equals $\overline{\mathcal{F}}_S$ for each coalition. In keeping with Yannelis' presentation, the final allocation is allowed to be measurable with respect to each agent's private information (or $\mathcal{G}_i^* = \mathcal{F}_i$). The fine core describes the case where coalitions are able to fully communicate. Therefore, it is assumed that $\mathcal{G}_i(S) = \underline{\mathcal{F}}_S$ for each coalition. The most common version of the core is one where agents may only choose allocations which vary with respect to their private information ($\mathcal{G}_i(S) = \mathcal{F}_i$). This private or publicly predictable information core ($\mathcal{PC}(\mathcal{E})$) has been used extensively by Yannelis (1991), Koutsougeras and Yannelis (1993), and Allen (1992b, 1993). While the coarse and private cores are non-empty under the assumptions made here, it can be easily shown that the fine core is empty in many reasonable situations.

2.3.1 Weak vs. Strong Blocking

The definition of blocking given in Definition 2.3.1 makes the strong requirement that all agents in a coalition strictly prefer the blocking allocation. A weaker definition may be more reasonable: a coalition will block an allocation if it can find an alternative which all agents weakly prefer and at least one agent strictly prefers.

2.3.4 Definition (Weak Blocking) *An allocation, x, is weakly $\mathcal{G}_i(S)$ blocked by S if there exists an $\omega \in \Omega$ and a $y \in \mathbb{B}(S)$ such that y_i is $\mathcal{G}_i(S)$ measurable for all $i \in S$ and $V_i(y_i, \omega) \geq V_i(x_i, \omega)$ for all $i \in S$, with strict inequality for some $j \in S$.*

A general definition of efficiency using weak blocking can then be given.

2.3.5 Definition *x is $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ efficient with weak blocking if:*

i. x_i *is* $\mathcal{G}_i^*$ *measurable for each* $i \in N$,

ii. $x \in \mathbb{B}(N)$, *and*

iii. x *is not weakly* $\mathcal{G}_i(N)$ *blocked by* N.

Then a general definition for the core with weak blocking is as follows.

2.3.6 Definition x *is in the* $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ *core with weak blocking if:*

i. x_i *is* $\mathcal{G}_i^*$ *measurable for each* $i \in N$,

ii. $x \in \mathbb{B}(N)$, *and*

iii. For all $S \subseteq N$, x *is not weakly* $\mathcal{G}_i(S)$ *blocked by* S.

In perfect information exchange economies, weak blocking and the earlier *strong* notion of blocking are identical under minimal assumptions: strong monotonicity and continuity of preferences are sufficient for strong and weak blocking to be equivalent. But, in differential information economies, weak blocking may allow more blocking allocations thus shrinking the size of the efficient allocations or the core. The following example illustrates this with state-independent preferences that are monotonic and continuous.

2.3.7 Example Consider an economy with three types of agents (denoted by 1, 2, and 3) and two agents of each type (labeled A and B respectively). Let there be three states of nature denoted by a, b, c. There are two goods in each state, and each agent has a state-independent utility function given by $u_i = x_{i1}^{1/2} x_{i2}^{1/2}$. All agents assign equal prior probability to each state of nature. The agents' endowments and information are described by Table 2.1. The allocation described in Table 2.2 is a feasible allocation. However, if blocking allocations are allowed to be measurable with respect to private information, i.e., $\mathcal{G}_i(S) = \mathcal{F}_i$, the allocation is weakly blocked by the coalition $S = \{1B, 2B, 3B\}$. Table 2.3 gives the weakly blocking allocation. However, S cannot strongly block the original allocation, since that would require that agent 2B transfer a small amount of positive allocation to agent 1B in states

a and b. Since the original allocation gives 2B zero of both goods in state b, this is impossible. △

			Endowments		
Agent i	$\mathcal{F}_i$	State:	a	b	c
1	$\{a, b\}, \{c\}$		(2,0)	(2,0)	(1,3)
2	$\{a\}, \{b, c\}$		(0,2)	(0,0)	(0,0)
3	$\{a\}, \{b\}, \{c\}$		(0,0)	(0,2)	(3,1)

Table 2.1: Endowments and Information for Example 2.3.7

The problem rests in the requirement that each agent's blocking allocation be measurable with respect to $\mathcal{G}_i(S)$. An agent who is made strictly better by an allocation which is weakly blocking must be able to redistribute the allocation in a manner that is $\mathcal{G}_i(S)$ measurable for each agent. A sufficient technique is for the agent to give each other agent an increase in his allocation over an event which is measurable with respect to $\sigma(\bigcap_{i \in S} \mathcal{G}_i(S))$, the finest σ-field for which the allocation is measurable with respect to $\mathcal{G}_i(S)$ for all i in S. As the example illustrates, agents may have a zero allocation at some state in this event, prohibiting this redistribution. However, as long as allocations are strictly positive, weak and strong blocking are equivalent.

2.3.8 Proposition *Let preferences be continuous, strongly monotonic and x^* and x be feasible allocations. If x $\mathcal{G}_i(S)$ weakly blocks x^* for coalition S and $x_i(\omega) > 0$ for all $\omega \in \Omega$, then x^* is strongly blocked by S.*

Proof: Let x weakly block x^* for some coalition S. Then, for some ω^*, $V_i(x_i, \omega^*) \geq V_i(x_i^*, \omega^*)$ for all $i \in S$, with strict inequality somewhere. Let j be an agent such that $V_j(x_i, \omega^*) > V_j(x_i^*, \omega^*)$. Let $\overline{F}_S(\omega^*)$ be the atom generated by the coarsening σ-field $\sigma(\bigcap_{i \in S} \mathcal{G}_i(S))$. Define a strong blocking allocation, x', as follows.

For all $i \in S \backslash \{j\}$

$$x_i'(\omega) = \begin{cases} x_i(\omega) + \epsilon & \omega \in \overline{F}_S(\omega^*) \\ x_i(\omega) & \text{otherwise} \end{cases}$$

		Allocation		
Agent i	State:	a	b	c
1A		$(1,1)$	$(1,1)$	$(2,2)$
1B		$(1,1)$	$(1,1)$	$(2,2)$
2A		$(2,2)$	$(0,0)$	$(0,0)$
2B		$(0,0)$	$(0,0)$	$(0,0)$
3A		$(0,0)$	$(1,1)$	$(2,2)$
3B		$(0,0)$	$(1,1)$	$(2,2)$

Table 2.2: A Weakly Blocked Allocation for Example 2.3.7

		Allocation		
Agent i	State:	a	b	c
1B		$(1,1)$	$(1,1)$	$(2,2)$
2B		$(1,1)$	$(0,0)$	$(0,0)$
3B		$(0,0)$	$(1,1)$	$(2,2)$

Table 2.3: A Weakly Blocking Allocation for Example 2.3.7

and for j

$$x'_j(\omega) = \begin{cases} x_j(\omega) - |S|\epsilon & \omega \in \overline{F}_S(\omega^*) \\ x_j(\omega) & \text{otherwise} \end{cases}$$

where $\epsilon > 0$ is such that

i. $x_j(\omega) - |S|\epsilon > 0$ for all $\omega \in \overline{F}_S(\omega^*)$,

ii. $V_j(x_j - |S|\epsilon, \omega^*) > V_j(x_j^*, \omega^*)$,

iii. $V_i(x_i + \epsilon, \omega^*) > V_i(x_i^*, \omega^*)$.

Since $x_j(\omega) > 0$ for all $\omega \in \Omega$ and preferences are continuous, an $\epsilon > 0$ exists which satisfies the first two conditions. Strong monotonicity ensures that the third condition holds. Clearly, x' is still $\mathcal{G}_i(S)$ measurable for all $i \in S$. Thus, x' strongly blocks x^*. ∎

As the proof of Proposition 2.3.8 makes clear, positivity of blocking allocations is necessary to ensure that an agent who strictly prefers x to x^* can offer a new blocking allocation which is measurable for each member of the coalition. On the other hand, there are natural measurability restrictions under which every weakly blocked allocation is also strongly blocked. One such condition is that each agent in S be able to obtain blocking allocations which are measurable with respect to the same σ-field.

2.3.9 Proposition *Let x^* and x be feasible allocations. Assume preferences are continuous, strongly monotonic and $\mathcal{G}_i(S) = \mathcal{G}_j(S)$ for all $i, j \in S$. If x weakly blocks x^* for coalition S, then x^* is strongly blocked by S.*

Proof: Notice that by strong monotonicity, in order for $V_j(x_j, \omega^*) > V_j(x_j^*, \omega^*)$ it must be that $x_j(\omega) > 0$ for all $\omega \in F_i(S)(\omega^*)$, and that $\sigma(\bigcap_{i \in S} \mathcal{G}_i(S)) = \mathcal{G}_i(S)$ for all $i \in S$. Then, apply the proof for Proposition 2.3.8. ∎

In standard general equilibrium theory with perfect information, the difference between strong and weak blocking can be mitigated with a few simple assumptions.

In differential information economies, without making the additional assumption of strictly positive endowments, the choice of blocking technique can change the set of core allocations when $\mathcal{G}_i(S) \neq \mathcal{G}_j(S)$ for some i, j. In Section 2.5, this difference will become relevant when discussing equal treatment. Except when noted, strong blocking (as in Definition 2.3.1) is assumed.

2.4 Welfare Theorems

Rational expectations equilibrium (REE) is meant to capture the fact that prices may convey information. For example, if an agent observes a high price for a particular good, he may be able to rule out certain states of the world where such a high price would not be consistent with utility maximization by the other agents. Therefore, a REE is a fixed point not only over the space of feasible allocations, but also over the information that prices (and possibly the allocation) transmit.

A *price vector* is a measurable function $p : \Omega \to Y'_+$ where Y'_+ is the dual space of positive linear functionals. The information conveyed by the prices is the partition of Ω given by $\mathcal{P}_p$ such that $\mathcal{P}_p(\omega) = \{\omega' \in \Omega | p(\omega) = p(\omega')\}$. Let $\mathcal{F}_p$ be the σ-field generated by this partition. Given a price vector, each agent may be able to improve his private information. Thus $\mathcal{F}_{i\cup p} = \sigma(\mathcal{F}_i \cup \mathcal{F}_p)$ is agent i's information refined by prices.

The definition of rational expectations equilibrium I use is equivalent to that used by Allen (1981) and Radner (1979).

2.4.1 Definition *A rational expectations equilibrium (REE) for an economy, $\mathcal{E}$, is a (p, x) such that for all $i \in N$,*

i. x_i is $\mathcal{F}_{i\cup p}$ measurable,

ii. For all $\omega \in \Omega$, there does not exist a y_i such that y_i is $\mathcal{F}_{i\cup p}$ measurable, $p(\omega)y_i(\omega) \leq p(\omega)e_i(\omega)$, and $V_i(y_i, \omega, \mathcal{F}_{i\cup p}) > V_i(x_i, \omega, \mathcal{F}_{i\cup p})$, and

iii. $\sum_{i\in N} x_i = \sum_{i\in N} e_i$ a.e.$[\mu]$.

The requirement that the allocations only be measurable with respect to each agent's private information and the prices ensures that the allocation does not provide agents with additional information. If the allocation were not $\mathcal{F}_{i \cup p}$ measurable for each agent, there would be states of the world which agents could distinguish by the fact that they received different allocations. It would then be reasonable to assume that agents may refine their information with respect to the allocation which may change their preferences and lead to the allocation no longer being an equilibrium. Therefore, this definition captures the notion that the allocation as well as prices may reveal information.

One of the classical results in general equilibrium theory is the first welfare theorem: under weak assumptions, the set of Walrasian equilibria is contained in the core. Since the first welfare theorem implies that a decentralized price process leads to core allocations, this result provides a compelling argument in favor of Walrasian equilibria. Laffont (1985) has shown that REE are not always unconstrained interim efficient. However, it is possible to use exogenously imposed communication restrictions in order to find a constrained version of the core containing all REE. In order to obtain a first welfare theorem, the private core must be weakened to the fine private core $\mathcal{FPC}(\mathcal{E})$ where the final allocations are measurable with respect to the grand coalition's pooled information ($\mathcal{G}_i^* = \underline{\mathcal{F}}_N$).

2.4.2 Definition (Fine Private Core) *An allocation x is in the fine private core for $\mathcal{E}$ if the followings conditions hold:*

i. each x_i is $\underline{\mathcal{F}}_N$ measurable,

ii. $x \in \mathbb{B}(N)$, and

iii. For all $\omega \in \Omega$, there does not exist a coalition S and an allocation, y_S, such that $y_S \in \mathbb{B}(S)$, y_i is $\mathcal{F}_i$ measurable for all $i \in S$, and $V_i(y_i, \omega) > V_i(x_i, \omega)$ for all $i \in S$.

Since more allocations are allowed, it is clear that $\mathcal{PC}(\mathcal{E}) \subseteq \mathcal{FPC}(\mathcal{E})$. The fine private core is nonempty since it satisfies the no-insider condition in Page (1997) which is sufficient for existence.

Using the fine private core, I provide a constrained first welfare theorem for REE.

2.4.3 Theorem *If (x^*, p) is a REE then x^* is in the fine private core.*

Proof: Suppose not. Two possible cases must be checked.

Case 1:

x_i^* is not measurable with respect to $\sigma(\bigcup_{i\in N} \mathcal{F}_i)$. Since (x^*, p) is a REE, it must be that x_i^* is $\mathcal{F}_{i\cup p}$ measurable. Since $\sigma(\bigcup_{i\in N} \mathcal{F}_i) = \mathcal{F}$, x_i^* must be measurable with respect to $\underline{\mathcal{F}}_N$. A contradiction.

Case 2:

There exists a coalition S and some feasible allocation x', with x_i' measurable with respect to $\mathcal{F}_i$, for all $i \in S$, such that for all $i \in S$ and for some $\omega \in \Omega$,

$$V_i(x_i', \omega) > V_i(x_i^*, \omega) \tag{2.1}$$

and

$$\sum_{i\in S} x_i'(\omega) = \sum_{i\in S} e_i(\omega) \; a.e.[\mu]. \tag{2.2}$$

(2.1) implies that there exists some $\omega'' \in F_i(\omega)$ such that $V_i(x_i', \omega'', \mathcal{F}_i \cup \mathcal{F}_p) > V_i(x_i^*, \omega'', \mathcal{F}_i \cup \mathcal{F}_p)$, but by the definition of REE it must be that $p(\omega')x_i'(\omega') > p(\omega')e_i(\omega')$ for all $i \in S$ and for ω' since x_i' measurable with respect to $\mathcal{F}_i$ implies that it is also measurable with respect to $\sigma(\mathcal{F}_i \cup \mathcal{F}_p)$ (See Lemma A.0.1). However, this implies that for some $\omega' \in \Omega$

$$p(\omega') \sum_{i\in S} x_i'(\omega') > p(\omega') \sum_{i\in S} e_i(\omega'). \tag{2.3}$$

Since both p and x' are $\mathcal{F}$ measurable it must be that (2.3) holds for all $\omega \in F(\omega')$. Since the probability measures are assumed to be purely atomic, it must be that for all $i \in S$, $\mu_i(F(\omega')) > 0$ which contradicts (2.2). ∎

The proof is similar to the standard proof of the first welfare theorem for Walrasian equilibria except that the measurability of blocking allocations becomes important.

Since the blocking allocation is measurable with respect to $\mathcal{F}_i$, it must also be measurable with respect to $\mathcal{F}_{i\cup p}$ which tells us, by the definition of REE, that the blocking allocation cannot be affordable. On the other hand, any measurability restriction which is not a coarsening of $\mathcal{F}_i$ may not be measurable with respect to $\mathcal{F}_{i\cup p}$.

An obvious corollary to Theorem 2.4.3 is that the set of REE also fall within the set of fine private efficient ($\mathcal{G}_i^* = \mathcal{F}$, $\mathcal{G}_i(N) = \mathcal{F}_i$) allocations. A more traditional first welfare theorem is then:

2.4.4 Corollary *If (x^*, p) is a REE then x^* is fine private efficient.*

Is the fine private core the smallest core concept which contains the set of REE? The following two examples demonstrate that for two common refinements of the fine private core there are REE which fall outside them.

2.4.5 Example There exist economies such that REE $\not\subseteq \mathcal{PC}(\mathcal{E})$. Consider an economy with two agents (denoted by 1,2) and three states of nature (denoted by a, b, c). There are two goods in each state, and each agent has a state-independent utility function given by $u_i = x_{i1}^{1/2} x_{i2}^{1/2}$. All agents assign equal prior probability to each state of nature. The agents' endowment and information are described by Table 2.4. The allocation and prices in Table 2.5 are a REE. The prices fully reveal the state of nature and, therefore, the allocation varies over every state. Thus, the allocation is not measurable with respect to $\mathcal{F}_i$ for some agents, implying that it cannot be a private core allocation. △

			Endowments		
Agent i	$\mathcal{F}_i$	State:	a	b	c
1	$\{a\}, \{b, c\}$		(4,0)	(6,3)	(6,3)
2	$\{a, b\}, \{c\}$		(0,4)	(0,4)	(3,6)

Table 2.4: Endowments and Information for Example 2.4.5

Another core concept, the weak fine core (Koutsougeras and Yannelis 1993), is a weakening of the fine core to allow the actual allocation to be $\underline{\mathcal{F}}_N$ measurable.

		Allocation		
Agent i	State:	a	b	c
1		$(2,2)$	$(4\frac{2}{7},5)$	$(4\frac{1}{2},4\frac{1}{2})$
2		$(2,2)$	$(1\frac{5}{7},2)$	$(4\frac{1}{2},4\frac{1}{2})$
Prices		$(\frac{1}{2},\frac{1}{2})$	$(\frac{7}{13},\frac{6}{13})$	$(\frac{1}{2},\frac{1}{2})$

Table 2.5: REE for Example 2.4.5

2.4.6 Definition (Weak Fine Core) *An allocation x is in the weak fine core for $\mathcal{E}$ if the followings conditions hold:*

i. each x_i is $\mathfrak{F}_N$ measurable,

ii. $x \in \mathbb{B}(N)$, and

iii. For all $\omega \in \Omega$, there does not exist a coalition S and an allocation, $y_S \in \mathbb{B}(S)$, such that y_i is $\mathfrak{F}_S$ measurable for all $i \in S$, and $V_i(y_i, \omega) > V_i(x_i, \omega)$ for all $i \in S$.

The set of allocations in the fine core for economy $\mathcal{E}$ is denoted by $\mathcal{WFC}(\mathcal{E})$. Unlike the fine core, the weak fine core exists under the assumptions made here. Also, since more allocations can block under this definition, $\mathcal{WFC}(\mathcal{E}) \subseteq \mathcal{FPC}(\mathcal{E})$.

2.4.7 Example There exist economies such that REE $\not\subseteq \mathcal{WFC}(\mathcal{E})$. Consider an economy with two agents (denoted by 1,2) and two states of nature (denoted by a, b). There are two goods in each state. Agent 1 has a state-independent utility function given by $u_1 = x_{11} + x_{12}$. Agent 2's state-dependent utility function is given by:

$$u_2(x,\omega) = \begin{cases} \frac{2}{3}\log x_{21} + \frac{1}{3}\log x_{22} & \omega = a \\ \frac{1}{3}\log x_{21} + \frac{2}{3}\log x_{22} & \omega = b \end{cases}$$

The agents' endowments and information are described by Table 2.6. The allocation and prices described in Table 2.7 constitute a REE. In fact agent 2's expected utility

			Endowments	
Agent i	$\mathcal{F}_i$	State:	a	b
1	$\{a\},\{b\}$		(0,5)	(5,5)
2	$\{a,b\}$		(5,0)	(5,0)

Table 2.6: Endowments and Information for Example 2.4.7

		Allocation	
Agent i	State:	a	b
1		$(2\frac{1}{2}, 2\frac{1}{2})$	$(7\frac{1}{2}, 2\frac{1}{2})$
2		$(2\frac{1}{2}, 2\frac{1}{2})$	$(2\frac{1}{2}, 2\frac{1}{2})$
Prices		$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$

Table 2.7: REE for Example 2.4.7

for an allocation that does not vary within his information is given by

$$u_2 = \frac{1}{2}\log x_{21} + \frac{1}{2}\log x_{22}.$$

However, this allocation is not in the weak fine core. The allocation given in Table 2.8 is measurable with respect to the agents' distributed information, and is strictly preferred in all states by both agents. △

In fact, the two examples show something more: for any strict refinement of the fine private core there are always economies such that REE are not contained in the core. Example 2.4.5 is an example of a fully revealing REE (FRREE). Since FRREE will always be measurable with respect to $\mathcal{F}$, regardless of the restrictions on $\mathcal{G}_i(S)$

		Allocation	
Agent i	State:	a	b
1		$(2, 3\frac{1}{10})$	$(8\frac{1}{10}, 2)$
2		$(3, 1\frac{9}{10})$	$(1\frac{9}{10}, 3)$

Table 2.8: Blocking Allocation for Example 2.4.7

the core allocations, $\mathcal{G}_i^*$, must be likewise measurable (see Section 2.4.1 for a more complete discussion of FRREE). In the economy constructed for Example 2.4.7, the only possible refinement of the fine private core via changes in $\mathcal{G}_i(S)$ comes from refining agent 2's information. However, as the example illustrates, there exist partially revealing REE that would then be blocked with this improved information.

2.4.8 Theorem *For any* $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ *core such that* $\mathcal{G}_i^*$ *is coarser than* $\mathcal{F}$*and/or* $\mathcal{G}_i(S)$ *is finer than* $\mathcal{F}_i$ *for all* i *in* S*, there exist economies such that the set of REE is not contained in that solution.*

Whereas previous results have only shown that REE, in general, fall outside *classical* (no measurability restrictions) efficiency and core solutions, I have demonstrated that there exists a welfare concept, the fine private core, in which all rational expectations equilibria are contained. Due to the existence of partially revealing REE, the constrained welfare concept used cannot imply full communication. Instead, agents can be presumed to use only their own private information in blocking allocations. Furthermore, I have shown (Theorem 2.4.8) that there is no refinement, at least in terms of measurability conditions, of the fine private core which yields a similar inclusion for all economies.

A second classical result in general equilibrium theory is the second welfare theorem: for any Pareto optimal allocation, prices can be constructed such that it is also a Walrasian equilibrium for a modified economy. While the first welfare theorem establishes that all Walrasian equilibria are Pareto optimal, the second welfare theorem allows us to conclude that there are endowments (given a fixed set of agents) that lead to *all* Pareto optimal allocations being Walrasian equilibria. Thus, Walrasian equilibria are *unbiased*: all Pareto optimal allocations can be decentralized by prices. I ask whether some notion of interim efficiency yields a similar result for rational expectations equilibria. If so, then any allocation in this set can be *decentralized* via market prices described by REE. This turns out to be the case for the definition of fine efficiency introduced earlier.

2.4.9 Definition x *is fine efficient if* $x \in \mathbb{B}(N)$*,* x_i *is* $\mathcal{F}_i$ *measurable for all* $i \in N$*,* *there does not exist another allocation $x' \in \mathbb{B}(N)$ that is $\underline{\mathcal{F}}_N$ measurable, and for some ω and for all $i \in N$, $V_i(x_i', \omega) > V_i(x_i, \omega)$.*

2.4.10 Theorem *Let preferences be strictly monotonic. If x^* is a fine efficient allocation such that $x_i^*(\omega) > 0$ for all $i \in N$ and for all $\omega \in \Omega$, then there exists a price vector, p such that (x^*, p) is a REE for the initial endowment $e = x^*$.*

Proof: Let x^* be a fine efficient allocation. Define a correspondence $\beta_i : \Omega \twoheadrightarrow L_1(\Omega, \mathcal{F}, \mu; Y)$ as follows:

$$\beta_i(\omega) = \{x_i \in L_1(\Omega, \mathcal{F}, \mu; Y) \mid V_i(x_i, \omega) > V_i(x_i^*, \omega)\}$$

which indicates the set of allocations which agent i strictly prefers to x_i^*. Note that $\beta_i(\omega)$ is convex as long as u_i is concave and integrably bounded. Let $\beta : \Omega \twoheadrightarrow L_1(\Omega, \mathcal{F}, \mu; Y)$ be defined as the sum of individual allocations in β_i or,

$$\beta(\omega) = \{z \in L_1(\Omega, \mathcal{F}, \mu; Y) \mid z = \sum_{i \in N} x_i \text{ and } x_i \in \beta_i(\omega)\ \forall i \in N\}.$$

This is the set of all allocations which at ω can be redistributed to the agents so they will strictly prefer it to x^*. Being the sum of convex sets, $\beta(\omega)$ is convex. Let $e = \sum_{i \in N} x_i^*$. Since x^* is fine efficient, it is clear that for all $\omega \in \Omega$ there does not exist an allocation x' such that $\sum_{i \in N} x_i' = e$, x_i' is $\mathcal{F}$ measurable, and $V_i(x_i', \omega) > V_i(x_i^*, \omega)$ for all $i \in N$. Thus, $e \notin \beta(\omega)$ for all $\omega \in \Omega$.

Define the correspondence $\varphi : \Omega \twoheadrightarrow Y'$ where Y' is the dual space of linear functionals on Y, by

$$\varphi(\omega) = \{p \in Y' \mid p \cdot z(\omega) \geq p \cdot e(\omega),\ \forall z \in \beta(\omega)\}.$$

Note that if $\beta(\omega)$ is convex and nonempty then the set $B(\omega) = \{x \in Y \mid x = z(\omega) \text{ for some } z \in \beta(\omega)\}$ is also convex and nonempty. Let x and x' be two points in $B(\omega)$. Then there exist z and z' such that $x = z(\omega)$ and $x' = z'(\omega)$. Since $\beta(\omega)$ is convex, it is obvious that $z''(\omega) = \alpha z(\omega) + (1 - \alpha) z'(\omega) \in \beta(\omega)$, which implies that $B(\omega)$ is convex. Therefore, by the Separating Hyperplane Theorem, $\varphi(\omega)$ is nonempty. $\varphi(\omega)$ is closed-valued by Lemma A.0.2, and it is weakly measurable by Lemma A.0.3. Therefore, we can apply the Kuratowski-Ryll-Nardzewski Selection Theorem (see Aliprantis and Border (1994) p. 505) to find a measurable selector $p : \Omega \to Y'$ such that for all $\omega \in \Omega$ and for all $z \in \beta(\omega)$

$$p(\omega) \cdot z(\omega) \geq p(\omega) \cdot e(\omega)$$

or

$$p(\omega) \cdot (z(\omega) - \sum_{i \in N} x_i^*(\omega)) \geq 0.$$

The proof proceeds in five steps:

1: I first show that $p(\omega)$ is a positive linear functional for all $\omega \in \Omega$.

Let $\delta > 0$. Then, by monotonicity, $e(\omega) + \delta \in \beta(\omega)$, which implies that

$$p(\omega) \cdot (e(\omega) + \delta - e(\omega)) \geq 0$$

and

$$p(\omega) \cdot (\delta) \geq 0$$

which implies that $p(\omega)$ is positive.

2: Show that for all $\omega \in \Omega$, $x_i \in \beta_i(\omega)$ implies that $p(\omega) \cdot x_i(\omega) \geq p(\omega) \cdot x_i^*(\omega)$ for all $i \in N$.

By strong monotonicity, there exist a $\delta > 0$ such that

$$z_i(\omega) = (1 - \delta) x_i(\omega)$$

and for all $j \neq i$,

$$z_j(\omega) = x_j^*(\omega) + \frac{\delta x_i(\omega)}{N - 1}$$

and z is $\mathcal{F}$ measurable, integrably bounded, and $z_i \in \beta_i(\omega)$ for all $i \in N$. Thus,

$z = \sum_{i \in N} z_i$ is in $\beta(\omega)$. Therefore,

$$\begin{aligned} p(\omega) \cdot z(\omega) &\geq p(\omega) \cdot x^*(\omega), \\ p(\omega) \cdot ((1-\delta)x_i(\omega) + \sum_{j \neq i} x_i^*(\omega) + \delta x_i(\omega)) &\geq p(\omega) \cdot \left(\sum_{i \in N} x_i^*(\omega) \right), \text{ and} \\ p(\omega) \cdot x_i(\omega) &\geq p(\omega) \cdot x_i^*(\omega). \end{aligned}$$

3: Show that for all $\omega \in \Omega$, $x_i \in \beta_i(\omega)$ implies that $p(\omega) \cdot x_i(\omega) > p(\omega) \cdot x_i^*(\omega)$.

From step 2 I know that $p(\omega) \cdot x_i(\omega) \geq p(\omega) \cdot x_i^*(\omega)$. Since preferences are upper semicontinuous, there exists some $0 < \theta < 1$ such that $V_i(\theta x_i, \omega, \mathfrak{F}_i) > V(x_i^*, \omega, \mathfrak{F}_i)$. Thus it must be that

$$p(\omega) \cdot (\theta x_i(\omega)) \geq p(\omega) \cdot x_i^*(\omega)$$

or

$$\theta(p(\omega) \cdot x_i(\omega)) \geq p(\omega) \cdot x_i^*(\omega). \tag{2.4}$$

Since $x_i^*(\omega) > 0$ and $p(\omega)$ is a positive linear functional, $p(\omega) \cdot x_i^*(\omega) > 0$. Thus, if $p(\omega) \cdot x_i(\omega) = p(\omega) \cdot x_i^*(\omega)$, it must be that $\theta(p(\omega) \cdot x_i(\omega)) < p(\omega) \cdot x_i^*(\omega)$, which contradicts (2.4).

4: Show that the set of allocations which are preferred under agents' updated information, $\mathfrak{F}_{i \cup p}$, is still not affordable.

Let $\beta_i^*(\omega)$ be the better than correspondence using the σ-field $\mathfrak{F}_{i \cup p}$, or

$$\beta_i^*(\omega) = \{x_i \in L_1(\Omega, \mathfrak{F}, \mu; Y) \mid V_i(x_i, \omega, \mathfrak{F}_{i \cup p}) > V_i(x_i^*, \omega, \mathfrak{F}_{i \cup p})\}.$$

If $x_i \in \beta_i^*(\omega)$, then there exists an allocation x_i' such that $x_i' \in \beta_i(\omega)$ and $x_i(\omega') = x_i'(\omega')$ for all $\omega' \in F_{i \cup p}(\omega)$. Let $x_i \in \beta_i^*(\omega)$. Then $V_i(x_i, \omega, \mathfrak{F}_{i \cup p}) > V_i(x_i^*, \omega, \mathfrak{F}_{i \cup p})$, or

$$\int_{\omega' \in F_{i \cup p}(\omega)} u_i(x_i(\omega'), \omega') d\mu_i(\omega') > \int_{\omega' \in F_{i \cup p}(\omega)} u_i(x_i^*(\omega'), \omega') d\mu_i(\omega'). \tag{2.5}$$

Since $\mathcal{F}_i$ is a sub σ-field of $\mathcal{F}_{i\cup p}$, $F_{i\cup p}(\omega) \subseteq F_i(\omega)$. Define a new allocation, x_i', as

$$x_i'(\omega') = \begin{cases} x_i(\omega') & \omega' \in F_{i\cup p}(\omega) \\ x_i^*(\omega') & \text{otherwise} \end{cases}$$

which by (2.5) is strictly greater than $V_i(x_i^*, \omega, \mathcal{F}_i)$, implying $x_i' \in \beta_i(\omega)$. This result obviously implies that $B^*(\omega) \subseteq B(\omega)$. Thus, for all $x_i \in \beta_i^*(\omega)$, $p(\omega) \cdot x_i(\omega) > p(\omega) \cdot x_i^*(\omega)$.

5:

By the definition of a REE I am only interested in allocations, x_i, that are $\mathcal{F}_{i\cup p}$ measurable as opposed to those that are $\mathcal{F}$ measurable. Therefore, these allocations are a subset of $\beta_i^*(\omega)$ which implies that the previous steps apply for this restriction. Finally, since x_i^* is assumed to be $\mathcal{F}_i$ measurable for all $i \in N$, it must be that it is $\mathcal{F}_{i\cup p}$ measurable for all $i \in N$. Thus, I have constructed a REE. ∎

While the proof is similar in style to the standard proof of the second welfare theorem for Walrasian equilibria, there are some added complications due to measurability requirements. First, it is necessary for the price function to be measurable. However, since the correspondence of possible prices at each state is non-empty, closed-valued and weakly measurable, there exists a measurable price function. Second, the measurability restrictions of fine efficiency ($\mathcal{G}_i(N) = \mathcal{F}$, and $\mathcal{G}_i^* = \mathcal{F}_i$) are necessary. If $\mathcal{G}_i(N) \neq \mathcal{F}$, one could not be certain that the set of allocations which are preferred to x^* under agents' private information contains the set of allocations preferred to x^* when information is refined by prices. In Example 2.4.7, if $\mathcal{G}_i(N) = \mathcal{F}_i$, the allocation $(3, 1\frac{9}{10})$ in both states $\{a, b\}$ would not be preferred to $(2\frac{1}{2}, 2\frac{1}{2})$ in both states for agent 2. However, if $p(a) \neq p(b)$, then $(3, 1\frac{9}{10})$ would be preferred to $(2\frac{1}{2}, 2\frac{1}{2})$ in state a. The restriction of $\mathcal{G}_i^* = \mathcal{F}_i$ ensures that x^* satisfies the measurability requirement for endowments (Assumption 2.2.3) as well as the requirement that REE be $\mathcal{F}_{i\cup p}$ measurable.

Constrained versions of efficiency and the core can be developed in order to provide first and second welfare theorems for REE. Unfortunately, the informational constraints needed in order to obtain these two results are different. The fine private core allows the allocations to be $\underline{\mathfrak{F}}_N$ measurable whereas fine efficiency requires the allocation to be privately measurable ($\mathfrak{F}_i$). Under fine efficiency the grand coalition can block with its pooled information $\underline{\mathfrak{F}}_N$. The fine private core only allows privately measurable blocking for each coalition including the grand coalition. Therefore, fine efficient allocations are a subset of fine private efficient allocations. Putting the two welfare theorems together suggests a large degree of indeterminacy. The set of allocations which can be rationalized as a REE for some endowment (given fixed preferences and private information) will always include fine efficient allocations, but may vary between fine efficient and fine private efficient allocations. Therefore, REE do not, in general, satisfy the same unbiasedness property that Walrasian equilibria naturally satisfy, i.e., some fine private efficient allocations may not be REE. Why is this the case? In the case of the first welfare theorem, without specific knowledge of the prices, it is impossible to tell the amount of information revealed by a REE. The only allocations which can be ruled out *for certain* are those that use each individual's private information ($\mathfrak{F}_i$). In terms of the second welfare theorem, lack of specific knowledge of prices works in the opposite direction. In order to be certain that prices and the allocation will be a fixed point in both preferences and information, one must know that the allocation would be efficient for any possible refinement of private information which the prices might cause. Therefore, without the use of parameters that vary between economies (such as particular classes of preferences), it is not possible to say much more about the welfare properties of REE.

This inconsistency between the two welfare theorems for REE is another critique of REE as a solution concept. As with Walrasian equilibria, one would like to imagine some decentralized mechanism (the auctioneer) leading to equilibrium prices. However, as Theorem 2.4.10 indicates, this decentralized price setter would need to possess the grand coalition's pooled information in order to come up with equilibrium prices. In reality, the most appealing REE may be those which do not fully reveal private information (precisely those which drive the result in Theorem 2.4.3). Unfortunately, it is not yet clear how those solutions might be arrived at.

2.4.1 PREE and FRREE

I can provide an alternative equilibrium notion which does not suffer from the same information inconsistencies between the first and second welfare theorems as REE. It turns out that in order to do this one must allow allocations to be measurable only with respect to distributed information. This is the distinguishing feature of the pseudo rational expectations equilibrium.

2.4.11 Definition *A pseudo rational expectations equilibrium (PREE) for an economy $\mathcal{E}$ is a (p, x) such that for all $i \in N$,*

i. *x_i is $\underline{\mathcal{F}}_N$ measurable,*

ii. *For all $\omega \in \Omega$, there does not exist a y_i such that y_i is $\underline{\mathcal{F}}_N$ measurable, $p(\omega)y_i(\omega) \leq p(\omega)e_i(\omega)$, and $V_i(y_i, \omega, \mathcal{F}_{i\cup p}) > V_i(x_i, \omega, \mathcal{F}_{i\cup p})$, and*

iii. $\sum_{i \in N} x_i = \sum_{i \in N} e_i$ *a.e.*$[\mu]$.

A PREE may be thought of as a competitive equilibrium of a market, in the interim stage, for futures contracts. While agents can use only their private information (refined by prices) to make market decisions, the actual trades will occur in the ex post stage. Since all private information will be revealed at that stage, the actual allocation may vary across any state.[3]

PREE are included in the weak fine core discussed earlier ($\mathcal{G}_i^* = \mathcal{F}$ and $\mathcal{G}_i(S) = \underline{\mathcal{F}}_S$). This core solution is smaller than the fine private core used for Theorem 2.4.3 since coalitions are now assumed to pool their information.

2.4.12 Theorem *If (x^*, p) is a PREE then x^* is in the weak fine core.*

[3]It is interesting to note that each agent's maximal allocation (in the interim stage) will be $\mathcal{F}_{i\cup p}$ measurable, but that the easing of the measurability restrictions allows agents to propose trades that, while not maximal, may be improving and feasible for all agents. In this manner, more information is also captured by the prices.

Proof: Simply apply the proof of Theorem 2.4.3 with the necessary changes in measurability restrictions. ∎

As a corollary to Theorem 2.4.12, PREE are weak fine efficient. This version of efficiency permits more final allocations than fine efficiency ($\mathcal{G}_i^* = \mathcal{F}$ as opposed to $\mathcal{G}_i^* = \mathcal{F}_i$). Weak fine core allocations are a subset of the weak fine efficient allocations.

2.4.13 Definition *x is weak fine efficient if $x \in \mathbb{B}(N)$, x_i is $\underline{\mathcal{F}}_N$ measurable for all $i \in N$, there does not exist another allocation $x' \in \mathbb{B}(N)$ that is $\underline{\mathcal{F}}_N$ measurable, and $V_i(x_i', \omega) > V_i(x_i, \omega)$ for some ω for all $i \in N$.*

2.4.14 Corollary *If (x^*, p) is a PREE then x^* is weak fine efficient.*

A second welfare theorem can be developed for PREE by utilizing weak fine efficiency as opposed to the fine efficiency of Theorem 2.4.10.

2.4.15 Theorem *Let x^* be a weak fine efficient allocation such that $x_i^*(\omega) > 0$ for all $i \in N$ and for all $\omega \in \Omega$ and preferences are strictly monotonic. Then there exists a price function p such that (x^*, p) is a PREE for the initial endowment $e = x^*$.*

Proof: The proof follows as in Theorem 2.4.10 except that step 5 may be omitted. ∎

These two concepts, the weak fine core and weak fine efficiency, are congruent. For both solutions $\mathcal{G}_i^* = \mathcal{F}$ and $\mathcal{G}_i(N) = \underline{\mathcal{F}}_N$. Therefore, at least for PREE, I obtain the desired result: the set of pseudo rational expectations equilibria and weak fine efficient allocations are essentially equivalent.

Fully revealing rational expectations equilibria have received substantial attention. Particularly, Radner (1979) and Allen (1981) have demonstrated that FRREE generically exist under the assumptions made here. Likewise, since all *knowable* private information is assumed to be the private information of some agent (see Assumption 2.2.5), FRREE are ex post Pareto optimal. Prices are said to fully reveal $\mathcal{F}$ if

$$\mathcal{P}_p(\omega) = F(\omega) \text{ a.e.}[\mu].$$

FRREE is the result of refining REE by asking that prices fully reveal $\mathcal{F}$.

2.4.16 Definition *A Fully Revealing Rational Expectations Equilibrium (FRREE) for an economy, $\mathcal{E}$, is a REE such that p fully reveals $\mathfrak{F}$.*

Since FRREE are a subset of PREE, a version of the first welfare theorem follows as a Corollary to Theorem 2.4.12.

2.4.17 Corollary *If (x^*, p) is a FRREE then x^* is in the weak fine core.*

Unfortunately, the development of a second welfare theorem is not as easy. While there exists a measurable price function which separates preferred allocations from the endowment, it is impossible to say anything about its specific measurability properties. In fact, prices may not vary over some atoms of $\mathfrak{F}$ (i.e., $p(\omega) = p(\omega')$ for some $F(\omega) \neq F(\omega')$). While prices would still be $\mathfrak{F}$ measurable, they would no longer necessarily be fully revealing.

Since fine information sharing can be thought to lead to the ex post efficient allocations, the results for PREE suggest that full revelation need not be necessary for ex post efficiency.

2.5 Replica Economies

A classical result about Walrasian equilibria is that as the economy grows, the set of core allocations which are not Walrasian equilibria shrinks (Debreu and Scarf 1963, Hildenbrand and Kirman 1988). This is typically interpreted as: in large economies, the difference between cooperative *bartering*, the core, and decentralized (noncooperative) markets, Walrasian equilibria, is negligible.

I begin by extending the standard definition of replica economies to economies with differential information. Let $\mathcal{E} = \langle N, (Y_i^*, \mathfrak{F}_i^*, u_i^*, e_i^*, \mu_i^*)_{i \in N} \rangle$ be the *original* economy. Then an r replica economy, $\mathcal{E}^r = \langle N^r, (Y_i, \mathfrak{F}_i, u_i, e_i, \mu_i)_{i \in N^r} \rangle$, is constructed as follows:

i. $N^r = \bigcup_{n \in N} N_n^r$ where for all n, n', $N_n^r \cap N_{n'}^r = \emptyset$.

ii. $|N_n^r| = r$ for all $n \in N$.

iii. For all $i \in N_n^r$, $(Y_i, \mathfrak{F}_i, u_i, e_i, \mu_i) = (Y_n^*, \mathfrak{F}_n^*, u_n^*, e_n^*, \mu_n^*)$.

This replication procedure is the simplest process one could imagine (and that originally used by Debreu and Scarf (1963)). If this simple procedure does not lead to positive convergence results, then there is little reason to suspect that more complicated techniques will yield positive results. Notice that each agent's private information is also replicated. While simple replication of information is certainly the most straight-forward application of Debreu-Scarf type replica economies to situations with differential information, it eliminates each agent's informational advantage. As soon as the economy is replicated once, someone in the economy knows everything that each agent already knows.[4]

The first step in examining core convergence is determining what it means for two solutions to converge. To measure the difference between two allocations, the uniform metric is used. Let x_i, y_i be two individual allocations then

$$d(x_i, y_i) = \sup_{\omega \in \Omega} \parallel x_i(\omega) - y_i(\omega) \parallel .$$

Then, as in Hildenbrand and Kirman (1988), the difference between two sets (the core and REE) for a given economy is defined as $\delta(\mathcal{E})$: the smallest δ such that for every $x \in \mathcal{FPC}(\mathcal{E})$[5] there exists an allocation $y \in REE(\mathcal{E})$ such that $d(x_i, y_i) \leq \delta$ for all $i \in N$, or

$$\delta(\mathcal{E}) = \sup_{x \in \mathcal{FPC}(\mathcal{E})} \inf_{y \in REE(\mathcal{E})} \sup_{i \in N} d(x_i, y_i).$$

In order to demonstrate convergence, I would like to show that for the sequence of economies defined by the replication technology, $\delta(\mathcal{E}^r) \xrightarrow{r} 0$, or the maximal difference between any allocation in the core and any allocation that is a REE is becoming arbitrarily small.

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[4]Palfrey and Srivastava (1986) suggest a stochastic replication procedure that may be more reasonable here. In each replication, while each agent's preferences and endowments are replicated, his private information is drawn randomly. Therefore, each agent retains some informational advantage. However, such a procedure leads to the possibility that the set of REE may also change with each replication. I leave the discussion of convergence under stochastic replication to future work.

[5]Any alternative core definition could be used.

2.5.1 Equal Treatment

Equal treatment is the cornerstone of core convergence. Price equilibria, such as REE, PREE, and FRREE, obviously satisfy equal treatment when preferences are strictly concave.[6] Hence, if a core definition does not satisfy equal treatment, there will always be core allocations which cannot be the same as those of the price equilibrium.

2.5.1 Definition (Equal Treatment) *An allocation, x, satisfies equal treatment if, for all $n \in N$ and for all $i, j \in N_n^r$, $x_i(\omega) = x_j(\omega)$ a.e.$[\mu]$.*

Srivastava (1984) showed that a version of the ex ante core with differential information satisfies equal treatment. Koutsougeras and Yannelis (1993) have suggested that the ex ante private core does not satisfy equal treatment. In the interim stage, equal treatment cannot be obtained for the fine private core. The problem lies in the measurability requirements of the definition. If a fine private core allocation treats two identical agents differently in some state, the agents will not necessarily be able to find a privately measurable allocation for some coalition that will be blocking. If the fine private core consists of more than one allocation for $r = 1$, then unequal treatment allocations can be constructed by assigning each complete set of N agents a different allocation from the $r = 1$ fine private core. The following example demonstrates the failure of equal treatment.

2.5.2 Example Consider an economy with two types of agents (denoted by 1 and 2) and two agents of each type (labeled A and B respectively). Let there be three states of nature (denoted by a, b, c). There are two goods in each state, and each agent has a state-independent utility function given by $u_i = x_{i1}^{1/2} x_{i2}^{1/2}$. All agents assign equal prior probability to each state of nature. The agents' endowments and information are described by Table 2.9. The allocation described in Table 2.10 is a fine private core allocation. The allocation is obviously $\mathcal{F}$ measurable. However, agents of both types are treated unequally. There are no privately measurable allocations for the

[6] Since each agent of identical type is maximizing his utility with respect to the same prices, strict concavity implies the existence of a unique maximizer for each type.

agents which improve them in all states of the world. Therefore, any $\mathcal{F}$ measurable allocation which is weakly improving over the endowment cannot be blocked. $\triangle$

			Endowments		
Agent i	$\mathcal{F}_i$	State:	a	b	c
1	$\{a,b\},\{c\}$		(5,3)	(5,3)	(3,5)
2	$\{a\},\{b,c\}$		(3,5)	(5,3)	(5,3)

Table 2.9: Endowments and Information for Example 2.5.2

The fact that coalitions can only use their private information to block leads to asymmetries. Is it that, in the limit, these differences are small? Unfortunately, the answer is no. In Appendix A I construct an economy similar to the one described in Example 2.5.2.

2.5.3 Proposition *There exist economies $\mathcal{E}$ such that the difference between the fine private core and the set of REE does not tend to zero, i.e.,*

$$\lim_{r\to\infty} \delta(\mathcal{E}^r) \neq 0.$$

Proof: Let $(\Omega, \mathcal{F}, \mu)$ be an arbitrary probability space satisfying the original assumptions. Define the original economy as follows. Let $N = \{1,2\}$ and $Y_i(\omega) = Y_+$ for all $\omega \in \Omega$ and $i \in N$. Let $(F^k)_{k=1}^m$ be the finite collection of m atoms for $\mathcal{F}$. Then there are two cases: m is even, and m is odd. First, let m be even. Let $\mathcal{F}_1 = \mathcal{F}$ and let $\mathcal{F}_2$

		Allocation		
Agent i	State:	a	b	c
1A		$(5,3)$	$(5,3)$	$(3,5)$
1B		$(4,4)$	$(5,3)$	$(4,4)$
2A		$(4,4)$	$(5,3)$	$(4,4)$
2B		$(3,5)$	$(5,3)$	$(5,3)$

Table 2.10: An Unequal Fine Private Core Allocation for Example 2.5.2

be generated by the collection of $m/2$ atoms given by:

$$F_2^k = F^k \cup F^{m-k+1}.$$

Let both agents have state independent utility functions given by

$$u_i(x, \omega) = x_{i1}^{1/2}(\omega) x_{i2}^{1/2}(\omega) \text{ for all } \omega \in \Omega.$$

Endowments are given as follows:

$$e_1(\omega) = \begin{cases} (3,5) & \omega \in \bigcup_{k=1}^{m/2} F_1^k \\ (5,3) & \text{otherwise} \end{cases}$$

and

$$e_2(\omega) = (5,3) \text{ for all } \omega \in \Omega.$$

Let μ_i be any absolutely continuous measure.

Given this economy, the unique REE is characterized by the following fully revealing prices:

$$p(\omega) = \begin{cases} (\frac{1}{2}, \frac{1}{2}) & \omega \in \bigcup_{k=1}^{m/2} F_1^k \\ (\frac{1}{19}, \frac{18}{19}) & \text{otherwise} \end{cases}$$

and the following allocation:

$$x_i(\omega) = \begin{cases} (4,4) & \omega \in \bigcup_{k=1}^{m/2} F_1^k \\ (5,3) & \text{otherwise} \end{cases}$$

for $i = 1, 2$. Notice that there are no partially revealing REE for this example due to the symmetry of the problem: any prices that do not vary on one of agent 2's information sets will necessarily lead to excess demand by agent 1 in one of the states.

However, for each replication r the allocation given by the endowment is in the fine private core since there is no privately measurable allocation which improves a coalition. Then, $d(x_i, y_i) = \sqrt{2}$ for all i, and for all r. Thus, it must be that $\delta(\mathcal{E}^r) \geq \sqrt{2}$ for all r, or $\lim_{r\to\infty} \delta(\mathcal{E}^r) \neq 0$.

If m is odd, the same construction will work except each agent has an additional partition element. For 2, $F_2^{(m/2)+1} = F^{(m/2)+1}$, and endowments can be assumed to be $(5, 3)$ for both agents. ∎

The measurability of blocking allocations plays a prominent role in obtaining core definitions which satisfy equal treatment. I proceed by abstracting from the specific core concepts presented earlier and provide general sufficient conditions for equal treatment of interim core concepts. Along with the standard requirement of strict concavity, blocking allocations being at least as fine as the final allocation is sufficient to obtain equal treatment in the interim stage with weak blocking.

2.5.4 Lemma *If preferences are strictly concave and for all $i \in N$, $\mathcal{G}_i(N) \supset \mathcal{G}_i^*$, then the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core with weak blocking exhibits equal treatment.*

Proof: Suppose not. Then there exists a measurable event, G, such that for some $n \in N$ and $i, j \in N_n^r$, $x_i(\omega) \neq x_j(\omega)$ for all $\omega \in G$. Fix $\omega^* \in G$. By the measurability assumptions of the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core and the information structure, there exists an atom $G_i^*(\omega^*) \subseteq G$ and for all $\omega' \in G_i^*(\omega^*)$, $x_i(\omega') = x_i(\omega^*)$, for all $i \in N_n^r$, for all $n \in N$. Likewise, $u_k(\cdot, \omega') = u_k(\cdot, \omega)$.

Form a blocking coalition, S, as follows. Let S be composed of one agent from each type (N_n^r) such that for each $n \in N$,

$$i \in S \text{ implies } i \in \arg\min_{i \in N_n^r} \{V_i(x_i, \omega^*)\}.$$

Since x is feasible for a.e.$[\mu]$,

$$\frac{1}{r} \sum_{n \in N} \sum_{i \in N_n^r} x_i(\omega) = \frac{1}{r} \sum_{n \in N} \sum_{i \in N_n^r} e_i(\omega) \text{ a.e. } [\mu], \text{ and}$$

$$\sum_{n\in N}\overline{x}_n(\omega) = \sum_{n\in N} e_n(\omega) \text{ a.e.}[\mu].$$

Thus, $\overline{x}_S \in \mathbb{B}(S)$. Since for all $i \in S$, $V_i(x_i, \omega^*) \leq V_i(x_j, \omega^*)$ for all $j \in N_n^r$. By strict concavity of u, $V_i(\overline{x}_i, \omega^*) \geq V_i(x_i, \omega^*)$ for all $i \in S$, with strict inequality for some agent. As constructed, $\overline{x}$ is $\mathcal{G}_i(S)$ measurable. Therefore, $\overline{x}$ blocks x, contradicting the assumption that $x \in (\mathcal{G}_i^*, \mathcal{G}_i(S))$ core. ∎

Using Propositions 2.3.8 and 2.3.9, I can use Lemma 2.5.4 in order to obtain sufficient conditions for equal treatment under strong blocking.

2.5.5 Theorem *Let preferences be strictly concave, strongly monotonic, and continuous. If for all $i \in N$, $\mathcal{G}_i(N) \supset \mathcal{G}_i^*$, and either*

i. $e_i(\omega) > 0$ for all $i \in N$ and for all $\omega \in \Omega$, or

ii. $\mathcal{G}_i(N) = \mathcal{G}_j(N)$ for all $i, j \in N$,

then the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core exhibits equal treatment.

Proof: *Case 1:*
If $e_i(\omega) \gg 0$ for all $i \in N$ then $\overline{x}_i(\omega)$ is obviously strictly positive. Thus, Proposition 2.3.8 obviously allows us to apply Lemma 2.5.4.
Case 2:
If $\mathcal{G}_i(N) = \mathcal{F}_j(N)$ for all $i, j \in N$ then Proposition 2.3.9 and Theorem 2.5.4 can be applied. ∎

As a consequence of Theorem 2.5.5, equal treatment can be easily obtained for the weak fine core.

2.5.6 Corollary *Let preferences be strictly concave, strongly monotonic, and continuous. Then the weak fine core satisfies the equal treatment property.*

Since PREE are contained in the weak fine core, PREE obviously satisfy equal treatment as well. The standard arguments of Debreu and Scarf (1963) can then be applied to obtain a core convergence result.

2.5.7 Theorem *The difference between the weak fine core and the set of PREE tends to zero, i.e.,*

$$\lim_{r \to \infty} \delta(\mathcal{E}^r) = 0.$$

Under the assumptions made here, the weak fine core is non-empty. Unfortunately, the conditions sufficient to obtain equal treatment, and, thus, core convergence are only rarely consistent with the sufficient conditions given by Page (1997) for existence of the core. His *no-insider* condition requires that for all $S \subset N$, and for all $i \in S$, $\mathcal{G}_i(S) \subset \mathcal{G}_i^*$. Therefore, non-emptiness and equal treatment are simultaneously satisfied only when for all $i \in N$, $\mathcal{G}_i(N) = \mathcal{G}_i^*$. In order to provide sufficient conditions for existence, Page finds conditions which make the implied game balanced. In order for balancedness to be obtained, feasible blocking allocations for coalitions must be translatable to final allocations. If the information of the coalition is finer (or better) than that of the final allocation, it is possible that balancedness will not be obtained. However, in order to obtain equal treatment, it must be that any allocation which can be a final allocation is no finer than the information the grand coalition can use to block.

2.6 Conclusion

I have demonstrated that there are measurability restricted definitions of efficiency and the core which yield first and second welfare theorems for REE and PREE. Two observations come out of this exercise. First, the restrictions on core allocations necessary to obtain a first welfare theorem are incompatible with those needed for a second welfare theorem for generic rational expectations equilibria. In environments without private information, the standard definition of the core is an obvious subset of the set of efficient allocations. Walrasian equilibria are always in the core and any Pareto optimal point can be turned into a Walrasian equilibrium. But, when allocations are *observed* to be REE, one can only conclude that they have not been blocked by each agent's private information; one cannot presume that prices have managed to completely reveal individual information. However, any allocation that cannot be improved upon by any allocation which is measurable with respect to agents' pooled information ($\mathcal{F}_N$) can be rationalized as a REE. The stringent informational requirements of fine efficiency make constructing a REE simple.

Second, a new market equilibrium concept, PREE, can be developed which does not share these difficulties. Ironically, removing the restriction that allocations be measurable with respect to prices (implying that agents do not use observations of the allocation to refine their information), I obtain first and second welfare theorems which necessarily imply full communication. Core equivalence can also be demonstrated for PREE.

I have imposed measurability restrictions as an exogenous constraint on the set of feasible allocations. Therefore, the difference in informational requirements for the first and second welfare theorems should not be that surprising: there are economies where agents may want to share their information and others where they may not. The information revealed by REE is at least partially endogenous: individuals change their preferences based on information revealed by prices which, in turn, must change to reflect these changes until a fixed point in terms of both information and preferences is reached. Can a more endogenous, yet well defined, welfare concept be developed which more consistently captures these variations? One possible approach is to incorporate incentive compatibility as a restriction on allocations as opposed to measurability restrictions. Allen (1992a) has examined incentive compatible versions of the core, and Koutsougeras and Yannelis (1993) have suggested that in exchange economies private measurability implies incentive compatibility. Such a restriction *might* allow different information to be revealed when it is consistent with individual incentives.

Chapter 3 Bayesian Implementable Core Allocations

Abstract

I examine the implementation of core allocations when agents are differently informed. A one state deviation principle (an allocation cannot be improved at any state) and measurability restrictions (blocking allocations may only be measurable with respect to each agent's private information) are sufficient to yield interim core solutions that are Bayesian implementable. Private measurability of blocking allocations is necessary for implementation. Similar results hold for interim efficiency. However, the results cannot be extended to exclusive information environments.

3.1 Introduction

In complete information environments, core allocations are Nash implementable (Maskin 1998, Repullo 1987). In other words, a game can be constructed such that the set of Nash equilibria exactly coincide with the core. The power of such a result is that it reveals that the difference between cooperative behavior (ostensibly described by the core) and noncooperative behavior (described by Nash equilibria) in complete information settings is nonexistent. However, when agents have some private information, the results are not as satisfying. Palfrey and Srivastava (1987) demonstrated that even in a very limited class of asymmetric information environments similar results do not hold. For *any* game there will be Bayes Nash equilibria that are not ex ante, interim, or ex post core allocations. The addition of even limited private information creates enough problems that there will always be outcomes such that there is room for cooperative agreements to be made. Myerson and Satterthwaite (1983) even describe a simple environment where *all* possible Nash equilibria are inefficient.[1]

While the proper definition of the core under complete information (and no externalities) is not in dispute, defining the core under incomplete information is more contentious. Should full communication be assumed? When would a coalition decide to block an allocation? These are all relevant questions in settings with private information. The definition of the core which Palfrey and Srivastava use is consistent with Holmström and Myerson's (1983) original definition for efficient allocations. The object of this chapter is to investigate alternative candidate definitions of the core in this setting. I will demonstrate that some particular changes from Holmström and Myerson's (1983) original version are necessary and sufficient to obtain positive Bayesian implementation results. The fact that such cores are Bayesian implementable should not be justification on its own for them to be used as a description of cooperative behavior, but the results suggest which assumptions must be made if one wants an equivalence between cooperative and noncooperative behavior.

Palfrey and Srivastava (1987) demonstrate that another solution concept, rational

[1]Their example, however, does not satisfy the assumption I make in this paper.

expectations equilibrium (REE), is implementable under conditions of Non-Exclusive Information (NEI) and state-independent endowments. What leads to REE being implementable but core allocations not? Two salient features appear to separate REE from the standard definition of the core. First, in a REE, the allocation each agent demands does not vary between states he cannot distinguish. No restrictions are placed on how allocations in the core may vary. Second, in order to demonstrate that an allocation is not a REE it is only necessary to demonstrate that, at *one* state, prices do not separate the strictly preferred allocations for some agents. On the other hand, an allocation is not in the core only if there exists an allocation that at least weakly improves all agents for *all* states.

I develop a class of alternative core definitions by incorporating features similar to REE. First, blocking allocations are required to satisfy certain measurability restrictions. Specifically, only allocations that do not vary across each agent's private information are allowed to block. Despite the fact that, even in the interim stage, two states of the world may be indistinguishable for an agent, the classical definition of the core permits the agent to choose different allocations in each state. Second, a *one state deviation* principle is imposed. A core allocation satisfies the one state deviation principle if it cannot be improved upon at *any* state. This principle is similar to the notion of durability originally presented by Holmström and Myerson (1983). While core allocations that are not durable describe the set of allocations that *may not* be blocked, durable allocations are those that will *definitely not* be blocked.

While neither of these changes are sufficient on their own to produce implementable allocations, when combined they lead to positive results. Under the same environment restrictions used by Palfrey and Srivastava (1987), I demonstrate that core definitions that require blocking allocations to be privately measurable and final allocations to satisfy the one state deviation principle are Bayesian implementable. Private measurability is also necessary for an implementable core definition. Similar results also follow for efficient allocations.

Since implementation investigates when a social choice set can be rationalized as the equilibrium of a game, it is logical that making a social choice set more like a noncooperative equilibrium will improve its chances of being implemented. The one state deviation principle requires that there is no feasible *deviation* at one point (state) that improves some agents. In games, a strategy is not an equilibrium if there is a single beneficial deviation. Second, measurability restrictions are analogous to the obvious requirement that, in a game, players cannot choose different actions in the same information set.

There are three informational stages at which welfare can be evaluated: ex ante, interim, and ex post. At the ex ante stage agents do not yet possess their private information but they are aware of what their preferences, endowments, etc., would be *if* they did know the state of the world. At the interim stage, each agent has observed some private information which allows him to rule out certain events. Finally, at the ex post stage all relevant uncertainty is revealed and each agent knows the state of the world. Palfrey and Srivastava show that core allocations are not implementable with respect to preferences in any of these informational stages. I focus on the interim stage for two reasons. First, Bayes Nash equilibrium is a solution concept in the interim stage: players choose strategies having observed their private information. Second, the one state deviation principle is only meaningful in the interim stage.

Hahn and Yannelis (1997a) also study the implementation of measurability constrained core allocations in differential information economies. However, they define a new solution concept, coalitional Bayes Nash equilibrium, and find core allocations that are implementable using that solution. Their solution concept is explicitly cooperative: no coalition can unilaterally move to a new strategy. While their results describe allocations that can be obtained as the cooperative outcome of a game, it is of interest to examine whether explicitly cooperative social choice sets (the core) can be implemented by noncooperative solution concepts (Bayes Nash equilibrium). Also, Hahn and Yannelis only demonstrate that one core solution, the private core, is implementable in coalitional Bayes Nash equilibria. The implementation results presented here describe a large class of implementable social choice sets.

The chapter is organized as follows. Section 3.2 describes the basic model and definitions. Section 3.3 briefly describes and provides intuition for the use of measurability restrictions and the one state deviation principle. The implementability of welfare allocations with durability and measurability restrictions is then proven in Section 3.4.

3.2 The Model and Definitions

The model of a differential information exchange economy used here is similar to that used by Palfrey and Srivastava (1987). Let $N = \{1, 2, \ldots, n\}$ be the number of agents in the exchange economy. The commodity space is given by $Y = \mathbb{R}^m$ with positive orthant $Y_+ = \mathbb{R}^m_+$. Let $(\Omega, \mathcal{F}, \mu)$ be the probability space describing uncertainty in the model where μ is a probability measure representing ex ante (prior) probabilities and $\mathcal{F}$ is a σ-field. Let the set of all possible state contingent allocations be given by $L_1(\Omega, \mathcal{F}, \mu; \mathbb{R}^m)$ or the space of equivalence classes of $\mathcal{F}$-measurable, integrable functions $x : \Omega \to Y$. An agent, i, is a fivetuple $(Y_i, \mathcal{F}_i, u_i, e_i, \mu_i)$ where:

$Y_i : \Omega \to 2^{Y_+}$ is the state dependent consumption set correspondence of agent i,

$\mathcal{F}_i$ is a sub-σ-field of $\mathcal{F}$ that represents agent i's private information,

$u_i : Y \times \Omega \to \mathbb{R}$ is agent i's state contingent utility function,

$e_i : \Omega \to Y_+$ is a function denoting agent i's state contingent initial endowment of commodities, and

μ_i is a probability measure on $(\Omega, \mathcal{F})$ that represents agent i's prior beliefs.

An economy with asymmetric information $\mathcal{E} = \langle (Y_i, \mathcal{F}_i, u_i, e_i, \mu_i)_{i \in N} \rangle$ is a finite collection of agents.

Impose the following assumptions on the model.

3.2.1 Assumption *For each $i \in N$ and $\omega \in \Omega$, $Y(\omega) = \mathbb{R}^m_+$.*

There are no allocations that are automatically ruled out by consumption requirements including the $\mathbf{0}$ allocation which assigns an agent 0 in each state.

3.2.2 Assumption *For all $i \in N$*

i. u_i *is* $(\mathcal{F}, \mathcal{B}_Y)$ *measurable where* $\mathcal{B}_Y$ *is the Borel* σ*-field of* Y.

ii. For all $\omega \in \Omega$, $u_i(\cdot, \omega)$ *is concave, strictly increasing, and bounded below.*

iii. For all $\omega \in \Omega$, $u_i(0, \omega) = 0$.

Measurability of the utility function assures that $\mathcal{F}$ captures all relevant information in the economy. Agents' utility functions are normalized to equal 0 when they receive the 0 allocation. Given that u_i is assumed to be strictly increasing and bounded below, this assumption is without loss of generality.

3.2.3 Assumption (State Independent Endowments) *For all* $i \in N$ *and for all* $\omega \in \Omega$, $e_i(\omega) = e_i \gg 0$.

Endowments are assumed to be state independent and strictly positive. This requirement ensures that a feasible allocation in one state is also feasible in any other state.

I make a variety of assumptions on the structure of information $(\mathcal{F}_i)$ and beliefs (μ_i) for each agent.

3.2.4 Assumption Ω *is finite.*

Since Ω is assumed to be finite, $\mathcal{F}$ must be generated by a finite collection of atoms. Let $F(\omega)$ represent the *smallest* discernible event at ω, and $F = (F(\omega))_{\omega\in\Omega}$ is the finite collection of distinct atoms that form a partition of Ω. For notational simplicity, assume, without loss of generality, that $F(\omega) = \{\omega\}$. For each agent, $F_i(\omega)$ is the collection of states viewed as possible at ω, and $F_i = (F_i(\omega))_{\omega\in\Omega}$ is a finite partition of Ω.[2]

3.2.5 Assumption (No Redundant States) $\mathcal{F} = \sigma(\bigcup_{i\in N} \mathcal{F}_i)$.

[2]As in Palfrey and Srivastava (1987), agents' private information is a function of Ω. Alternatively, agents' private information could be treated as the primitive, and states determined as a collection of private information. Harsanyi (1967) shows these approaches to be equivalent.

This is identical to the requirement of no redundant states used by Palfrey and Srivastava (1987). In other words, $\{\omega\} = \cap_{i\in N} F_i(\omega)$. This assumption insures that all relevant private information is held by some agent in the economy. Each agent's prior is assumed to be absolutely continuous with respect to μ.

3.2.6 Assumption *For each $i \in N$, $\mu(E) = 0$ if and only if $\mu_i(E) = 0$.*

In order to ensure that interim expected utilities are well defined, measures are assumed to have full support.

3.2.7 Assumption *For all $\omega \in \Omega$, $\mu(\{\omega\}) > 0$.*

There are no events to which any agent assigns zero probability. Under Assumption 3.2.6, this assumption is without loss of generality: in any economy for which there are states to which all agents assign zero prior probability, one can simply construct a modified economy which excludes these states and satisfies Assumption 3.2.7. Since these states were originally assigned zero probability, they could not have effected individuals' expected utility calculations.

The set of feasible allocations for coalition S is denoted by

$$\mathbf{A}(S) = \left\{ z \in \mathbb{R}_+^{sm} \mid \sum_{i\in S} z_i \leq \sum_{i \in S} e_i \right\}$$

where $s = \#S$. An allocation for agent i is a function $x_i : \Omega \to \mathbb{R}_+^m$ that assigns to agent i a commodity vector in each state of the world. The set feasible (state-contingent) allocations are then given by

$$\mathbb{B}(S) = \{x : \Omega \to \mathbf{A}(S)\}.$$

Let $\mathbb{G}$ be the set of all possible sub-σ-fields of $\mathcal{F}$ satisfying the assumptions given above. The *interim expected utility* function $V_i : \mathbb{B}_i \times \Omega \times \mathbb{G} \to \mathbb{R}$ of agent i is defined by

$$V_i(x_i, \omega, \mathcal{G}) = \frac{1}{\mu_i(G(\omega))} \int_{\omega' \in G(\omega)} u_i(x_i(\omega'), \omega') d\mu_i(\omega').$$

When not explicitly stated, interim expected utility will be assumed to be taken with respect to agent i's private information, $\mathcal{F}_i$, and will be denoted $V_i(x_i, \omega)$.

Denote the set of economies that satisfy Assumptions 3.2.1-3.2.7 as $\mathbf{E}$ where $\mathcal{E}$ is a particular economy.

3.2.1 Implementation

I use the same terminology as Palfrey and Srivastava (1987) to describe a collection of implicitly *desirable* allocations. The structure of the economy is assumed to be common knowledge among all agents. The only uncertainty is contained in the private information of agents. Given a particular economy $\mathcal{E} = \langle (Y_i, \mathcal{F}_i, u_i, e_i, \mu_i)_{i \in N} \rangle$, a *social choice set* C is a subset of $\mathbb{B}(S)$: a collection of feasible state-contingent allocations. A social choice set describes a set of "desirable" allocations given a particular economy. In this paper, the social choice sets of interest are different variants of interim efficient and core allocations.

A social choice set is Bayesian implementable if the full set can be obtained as the set of Bayes Nash equilibrium outcomes of some noncooperative game (or mechanism). In this chapter, I use the necessary and sufficient conditions for Bayesian implementation provided in Palfrey and Srivastava (1989).

Define a mechanism for a given economy as a pair (M, g) where $M = M_1 \times M_2 \times \cdots \times M_n$ is a list of messages for each agent, and $g : M \to \mathbf{A}(N)$ is a function which maps lists of messages to outcomes.

Since the only unknown information in the economy is each agent's private information, a strategy for agent i is a function that maps from information partitions F_i to a message $\sigma_i : F_i \to M_i$.[3] Given a state of the world ω,

$$\sigma(\omega) = (\sigma_1(F_1(\omega)), \ldots, \sigma_n(F_n(\omega)))$$

is the collection of messages by each agent. A strategy is a *Bayes Nash equilibrium* if for all ω no agent would want to unilaterally deviate from his strategy.

[3] As is common in the implementation literature, only pure strategies are allowed.

3.2.8 Definition (Bayes Nash Equilibrium) *σ is a Bayes Nash equilibrium to (M, g) if for all $i \in N$, and for all $\omega \in \Omega$*

$$V_i(g_i(\sigma_i, \sigma_{-i}), \omega) \geq V_i(g_i(\hat{\sigma}_i, \sigma_{-i}), \omega) \text{ for all } \hat{\sigma}_i.$$

Since σ is a function of ω, $g(\sigma)$ determines a feasible state-contingent allocation. Given an economy $\mathcal{E}$, say that a mechanism (fully) Bayesian implements a social choice set C if an allocation is in C if and only if it is the outcome of some (pure strategy) Bayes Nash equilibrium of the mechanism.

3.2.9 Definition *Given an economy $\mathcal{E}$, a mechanism (M, g) Bayesian implements C if*

i. *For all $x \in C$, there exists a Bayes Nash equilibrium σ to (M, g) such that $g(\sigma) = x$.*

ii. *If σ is a Bayes Nash equilibrium to (M, g) then $g(\sigma) \in C$.*

A social choice set C is then said to be *Bayesian implementable* if there exists a mechanism which Bayesian implements C.

Let $\alpha = \{\alpha_1, \ldots, \alpha_n\}$ be a list of functions where $\alpha_i : F_i \to F_i$. A deception, α_i, is the private information each agent reports at each of his *true* private information sets. Deceptions describe all possible equilibrium strategies. Discussion can be restricted to deceptions that are compatible with the underlying information structure.

3.2.10 Definition *α is compatible with F if for all $(E_1, \ldots, E_n)$ with $E_i \in F_i$ for all i, $\cap_{i \in N} E_i \neq \emptyset$ implies $\cap_{i \in N} \alpha_i(E_i) \neq \emptyset$.*

For α that are not compatible with F (i.e., $\cap_{i \in N} E_i \neq \emptyset$ but $\cap_{i \in N} \alpha_i(E_i) = \emptyset$), it is easy to construct payoffs that make incompatible strategies dominated. For example, give $\mathbf{0}$ to the agents who make incompatible reports. Given Assumption 3.2.5 of no redundant states, it is clear that $\cap_{i \in N} \alpha_i(E_i) \neq \emptyset$ if and only if it is an atom of $\mathcal{F}$ (an element of F). For any α compatible with F define the following terms:

$$\alpha(\omega) = \bigcap_{i \in N} \alpha_i(F_i(\omega))$$

is the atom of $\mathcal{F}$ that the compatible α leads to at ω. If agents truthfully report at ω, then $\alpha(\omega) = F(\omega) = \{\omega\}$. For compatible α define

$$x_\alpha(\omega) = x(\alpha(\omega))$$

and

$$x_\alpha = (x_\alpha(\omega))_{\omega \in \Omega}.$$

Maskin (1998) demonstrated that monotonicity is a necessary condition for implementation in Nash equilibria. Roughly speaking, monotonicity requires that if an allocation is in a social choice set at one state, and at another state it is preferred to more allocations, then it should be in the social choice set at that state as well. Monotonicity can be generalized to differential information environments.

3.2.11 Definition (Bayesian Monotonicity) *A social choice set C satisfies Bayesian Monotonicity if for all α compatible with F, if*

i. $x \in C$

ii. For all i, for all ω, and for all $y \in \mathbb{B}(N)$,

$$V_i(x_i, \alpha(\omega)) \geq V_i(y_i, \alpha(\omega)) \Rightarrow V_i(x_{i\alpha}, \omega) \geq V_i(y_{i\alpha}, \omega)$$

then $x_\alpha \in C$.

A key assumption needed to obtain implementable allocations is Non-Exclusivity of Information (NEI). Information is non-exclusive if any agent's private information is known by the other $N-1$ agents' pooled information. In NEI environments, incentive compatibility becomes unimportant. If one agent lies about his private information and the $N-1$ other agents tell the truth, it will be obvious that the agent is lying (his report will be inconsistent with the report of the other agents). As in implementation in Nash strategies, any deviation by one agent can be detected (and punished) by the other agents. Therefore, it is never in the interest of agents to unilaterally deviate.

3.2.12 Assumption (Non-Exclusivity of Information) *$F_i(\omega) \supset \bigcap_{j \neq i} F_j(\omega)$ for all $i \in N$ and $\omega \in \Omega$*

Necessary and sufficient conditions for implementability are then given as follows.

3.2.13 Theorem (Palfrey and Srivastava (1989)) *If C is Bayesian implementable, then C satisfies Bayesian Monotonicity. If $N \geq 3$, C satisfies Bayesian Monotonicity, NEI is satisfied, and $C \neq \mathbf{0}$, then C is Bayesian implementable.*

$C \neq \mathbf{0}$ if for all $x \in C$, $x_i(\omega) \neq 0$ for all i and ω.

If $\mathbf{D}$ is a collection of economies, then a social choice correspondence (SCC) on $\mathbf{D}$, c, is a set-valued function assigning for every $\mathcal{E} \in \mathbf{D}$ a social choice set $c(\mathcal{E}) \subset \mathbb{B}(N)$.

3.2.14 Definition *A SCC c is Bayesian implementable on $\mathbf{D}$ if for all $\mathcal{E} \in \mathbf{D}$, $c(\mathcal{E})$ is Bayesian implementable.*

In other words, there exists a mechanism for each $\mathcal{E} \in \mathbf{D}$ such that the set of pure strategy Bayes Nash equilibria of that mechanism exactly coincide with the socially desirable outcomes for that economy as described by $c(\mathcal{E})$. The mechanism used to implement each $c(\mathcal{E})$ will obviously depend upon the particular commonly known features of $\mathcal{E}$.

In this chapter, the collection of economies that I will be concerned with Bayesian implementation on are those which satisfy the Non-Exclusivity of Information assumption. Let $\mathbf{E}^1$ be the subset of $\mathbf{E}$ such that Assumption 3.2.12 is satisfied and $N \geq 3$.

3.3 Welfare, Deviations, and Measurability

I begin by presenting the notion of interim efficiency originally used by Holmström and Myerson (1983). It is also the welfare concept discussed by Palfrey and Srivastava (1987) and has become standard.

An allocation is dominated if there exists an alternative allocation that is preferred by all agents (at least weakly) for all possible states.

3.3.1 Definition *x is interim dominated for S if there exists a $y \in \mathbb{B}(S)$ such that $V_i(y_i, \omega) \geq V_i(x_i, \omega)$ for all $i \in S$, for all $\omega \in \Omega$ with strict for at least one i and ω.*

Using this notion of domination, define interim efficiency. The HM is added to delineate this version from others that will be discussed later.

3.3.2 Definition *x is HM interim efficient if $x \in \mathbb{B}(N)$ and x is interim undominated for N.*

Although HM did not specifically discuss core allocations, their interim efficiency concept can logically be extended to coalitional deviations.

3.3.3 Definition *x is in the HM interim core if $x \in \mathbb{B}(N)$, and for all $S \subseteq N$, x is interim undominated for S.*

Palfrey and Srivastava demonstrate, using an example, that both HM interim efficient and HM interim core allocation cannot be globally implemented. I replicate the example here for completeness. It will be used later to motivate changes in the definition of interim efficiency and the core.

3.3.4 Example Consider an economy with four agents (denoted by 1, 2, 3, and 4). Let there be three states of nature denoted by a, b, c. There is one good in each state. Agents 1 and 2 are completely informed and have strictly increasing preferences over the good in each state. Neither agents 3 nor 4 can distinguish the states b and c $(\{a\}, \{b, c\})$. State dependent preferences are given as follows:

$$u_i(x_i(\omega), \omega) = \beta_i(\omega) \log(x_i(\omega))$$

		Allocation		
Agent i	State:	a	b	c
1		1.0	1.0	1.0
2		1.0	1.0	1.0
3		1.0	0.5	1.5
4		1.0	1.5	0.5

Table 3.1: A HM Interim Efficient Allocation for Example 3.3.4

with $\beta_3(a) = 0.5, \beta_3(b) = 0.25, \beta_3(c) = 0.75$, and $\beta_4(a) = 0.5, \beta_4(b) = 0.75, \beta_4(c) = 0.25$. The allocation described in Table 3.1 is HM interim efficient. The following α is compatible with F: $\alpha_i(F_i(\omega)) = F_i(a)$ for all i, ω and $\alpha(\omega) = \{a\}$ for all ω. Consider the Bayesian Monotonicity condition for agent 3 is as follows:

$$0.5\log(1) \geq 0.5\log(y(a))$$

implies

$$0.5[0.25\log(1)] + 0.5[0.75\log(1)] \geq 0.5[0.25\log(y(a))] + 0.5[0.75\log(y(a))].$$

Both expressions are identical. The same holds true for agent 4. Therefore, Bayesian Monotonicity would require the allocation $(1, 1, 1, 1)$ for all agents be HM interim efficient. However, the allocation given in Table 3.1 interim dominates this allocation. Thus, HM interim efficiency is not Bayesian Monotonic, and, therefore, it cannot be globally implementable. $\triangle$

3.3.1 One State Deviations

The definition of domination presented in the previous section is a stringent requirement on improving allocations. In order for agents to pick one allocation over another it must be common knowledge that they prefer that allocation. However, there may be states of the world where all agents know that they prefer another allocation yet it is not common knowledge. The one state deviation principle incorporates these allocations: if it is ever the case that all agents prefer one allocation to another then that allocation will not be a feasible outcome. I incorporate this principle into the definition of blocking.

3.3.5 Definition *x is interim blocked by S if there exists an $\omega \in \Omega$, and a $y \in \mathbb{B}(S)$, such that $V_i(y_i, \omega) > V_i(x_i, \omega)$ for all $i \in S$.*

Replace domination (Definition 3.3.1) with blocking (Definition 3.3.5) to obtain an alternative version of the interim core.

3.3.6 Definition *x is in the interim core if $x \in \mathbb{B}(N)$, and for all $S \subseteq N$, x is not interim blocked by S.*

Define the set of allocations that are not blocked for the grand coalition N as being interim efficient.

3.3.7 Definition *x is interim efficient if $x \in \mathbb{B}(N)$, and x is not interim blocked by N.*

The one state deviation principle can greatly reduce the set of feasible allocations. In abstract environments, such a stringent requirement may lead to the social choice set being empty. In economic environments, however, sufficient conditions for existence of core allocations are satisfied. Page (1997) shows that as long as any coalition cannot block with better (inside) information than is assumed to be available for the grand coalition, a balanced characteristic form game can be constructed, satisfying the sufficient conditions for a non-empty core. The core makes no restrictions on information sharing for any coalition.

The one state deviation principle used here is not equivalent to the notion of durability defined by Holmström and Myerson (1983). In their construction, an allocation is durable if it is never an equilibrium, in a voting game, for agents to unanimously approve another allocation over it. Their definition allows for information leakage: each agent bases his approval decision on the fact that the other $N-1$ agents also approved the change. Blocking (Definition 3.3.5) does not imply such sophisticated contingent logic. The following example demonstrates that the two concepts are not the same.

3.3.8 Example Consider an economy with two agents (denoted 1, and 2). Let there be three states of nature denoted by a, b, c. Both agents view each state as being equally likely. There are two goods in each state. Agent 1 has the following state dependent preferences:

$$u_1(x,\omega) = \begin{cases} .9 \log x_{11} + .1 \log x_{12} & \omega = a \\ .25 \log x_{11} + .75 \log x_{12} & \omega \in \{b, c\}. \end{cases}$$

Preferences for agent 2 are as follows:

$$u_2(x,\omega) = \begin{cases} .9 \log x_{21} + .1 \log x_{22} & \omega = a \\ .5 \log x_{21} + .5 \log x_{22} & \omega \in \{b, c\}. \end{cases}$$

			Allocation		
Agent i	$\mathcal{F}_i$	State:	a	b	c
1	$\{a, b\}, \{c\}$		(3,5)	(3,5)	(3,5)
2	$\{a\}, \{b, c\}$		(5,3)	(5,3)	(5,3)

Table 3.2: A Blocked Allocation in Example 3.3.8

Consider the information structure and allocation given in Table 3.2. The allocation is interim blocked (as in Definition 3.3.5) by coalition N. If $\omega = b$ the allocation $(4,4)$ for both agents in all states is strictly preferred by both 1 and 2. However, this blocking allocation would not block under the HM version of durability. The allocation $(5,3)$ is strictly preferred by agent 2 to $(4,4)$ in state a. Therefore, agent 1 can infer that unanimous approval of $(4,4)$ in all states must mean that state b has occurred (agent 1 has learned). Given that 1 now knows that b has occurred, he strictly prefers the allocation $(3,5)$ in b to $(4,4)$ in b. Both agents would never unanimously approve $(4,4)$ in all states. $\triangle$

The one state deviation concept used here is more correctly thought of as *naive durability*. It would be ideal to include a more sophisticated definition of blocking similar to Holmström and Myerson's (1983) durability. However, these blocking allocations are extremely difficult to characterize since it requires accounting for agents refining their information based upon the approval of the other agents: an allocation would block if all agents in a coalition were improved at state ω given their information refined by the approval of the $S-1$ other agents. Volij (1997) presents a core definition that allows for this advanced logic: preference for one allocation over the other "become[s] common knowledge after a long handshake."

If one selects an allocation that satisfies the one state deviation principle, he can be certain that the agents will never move away from it: there will be no information such that another allocation is thought to be preferred. Undominated allocations provide an upper bound on the set of allocations one can expect: there is no allocation that is always preferred. Naively durable allocations are closer to the outcome of equilibrium play of a noncooperative game. If, at any state, there is a strategy, $\hat{\sigma}_i$, for some individual such that $V_i(g_i(\hat{\sigma}_i, \sigma_{-i}), \omega) > V_i(g_i(\sigma), \omega)$, then the original strategy cannot be an equilibrium.

Prohibiting one state deviations by itself is not enough. In Example 3.3.4, the HM interim efficient allocation is also unblocked, demonstrating that interim core and efficient allocations are also not globally implementable.

The definition of blocking presented here uses a *strong* form of blocking: all agents in a coalition must strictly prefer an alternative allocation for it to block. Alternatively, one may wish to use a weaker form of blocking in which all agents in a coalition must weakly prefer an alternative allocation (with at least one agent strictly preferring it) to be willing to switch. In Chapter 2, I show that strictly positive endowments is sufficient to obtain equivalence of these blocking notions.

3.3.2 Measurability

Another change to the classical versions of interim welfare allocations can come in the imposition of measurability restrictions. A function $f : \Omega \to X$ is measurable with respect to $\mathcal{G}$ if:

$$f(\omega') = f(\omega), \text{ for all } \omega' \in G(\omega), \text{ for all } \omega \in \Omega$$

where $G(\omega)$ is an atom. Thus, measurability restrictions constrain allocations to not vary over certain states. Both versions of the interim core and efficiency presented earlier allow any $\mathcal{F}$ measurable allocation. A more thorough justification for the use of measurability restrictions is given in Section 2.3.

It is this lack of information restrictions that makes the HM interim efficient allocation in Example 3.3.4 troubling. Agents 3 and 4 are asked to contract different allocations in states b and c. However, at the interim stage (when the decision is assumed to be made) neither agent can distinguish between these two states. This is equivalent to asking players in an extensive form game to choose different strategies in the same information set. The following example demonstrates this.

3.3.9 Example Consider the extensive form game described in Figure 3.1 in which nature plays T with probability p and B with probability $1 - p$ in the first stage. Then, player 1, who cannot distinguish between the two states of the world, then chooses U or D. Player 2, who knows either T or B and U or D, then picks either u or d in each node. If $p \geq 1/3$, then a Bayes Nash equilibrium for this game is 1 plays U and 2 plays u, d, u, d (from the top node to the bottom). However, if Player 1 could choose strategies contingent on T and B, he would choose U in T and D in B, increasing both players' payoff in state B from $(5, 10)$ to $(6, 11)$. While this new outcome would clearly block the previous outcome, Bayes Nash equilibrium requires that each agent pick a strategy that is consistent with his private information: σ_i maps from F_i to messages, not from Ω to messages. △

There are two types of measurability restrictions that may be imposed: on the

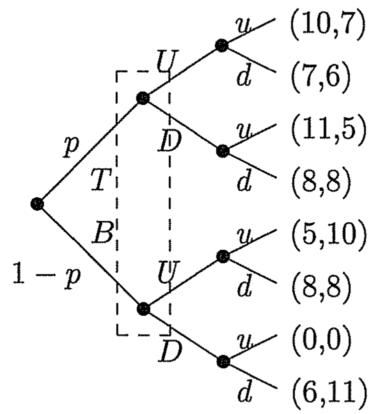

Figure 3.1: A Game with Incomplete Information

final allocation, and on the allocations that coalitions use to block. Let $\mathcal{G}_i^*$ be the σ-field with respect to which the final allocation, x_i, is assumed to be measurable for each i. Let $\mathcal{G}_i(S)$ be the σ-field with respect to which blocking allocations, y_i, for $i \in S$ can be measurable. By restricting $\mathcal{G}_i^*$ to a sub-σ-field of $\mathcal{F}$, the number of available allocations shrinks. However, restricting $\mathcal{G}_i(S)$ to a sub-σ-field of $\mathcal{F}$ decreases the number of blocking allocations thus potentially increasing the set of solutions.

3.3.10 Definition *An allocation, x, is $\mathcal{G}_i(S)$ interim blocked by S if there exists an $\omega \in \Omega$ and a $y \in \mathcal{B}(S)$ such that y_i is $\mathcal{G}_i(S)$ measurable for all $i \in S$ and $V_i(y_i, \omega) > V_i(x_i, \omega)$ for all $i \in S$.*

In addition to the one state deviation principle, blocking is restricted to a subset of all feasible allocations. This form of blocking defines a class of social choice correspondences that depend upon the measurability restrictions imposed. A general definition for the class of core allocations with measurability restrictions is as follows.

3.3.11 Definition *x is in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ interim core if:*

i. *x_i is $\mathcal{G}_i^*$ measurable for each $i \in N$.*

ii. $x \in \mathbb{B}(N)$

iii. For all $S \subseteq N$, x is not $\mathcal{G}_i(S)$ interim blocked by S.

3.3.12 Definition *x is $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ interim efficient if:*

i. x_i is $\mathcal{G}_i^*$ measurable for each $i \in N$.

ii. $x \in \mathbb{B}(N)$

iii. x is not $\mathcal{G}_i(N)$ interim blocked by N.

Interim efficient and core allocations as defined in Section 3.3.1 are then just one element of this class of social choice sets given by the restriction that $\mathcal{G}_i^* = \mathcal{G}_i(S) = \mathcal{F}$ for all S, and for all $i \in S$.

Sub-σ-fields of $\mathcal{F}$ can be partially ordered by the *at least as fine as* relation. We say that

$$\mathcal{G} \text{ is at least as fine as } \mathcal{F} \Leftrightarrow \mathcal{F} \subseteq \mathcal{G}$$

Therefore, if $\mathcal{G}$ is at least as fine as $\mathcal{F}$ and f is $\mathcal{F}$ measurable then f is also $\mathcal{G}$ measurable.

As long as $\mathcal{G}_i^*$ is at least as fine as $\mathcal{G}_i(S)$ for all i and S, the core will be non-empty (Page 1997). Since $e_i \gg 0$ for all i, $\mathbf{0}$ will not be in the set of $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ efficient or core allocations (See Chapter 2).

3.4 Implementation

These changes to interim efficiency and the core lead to positive results. By altering the social choice set to exhibit more *game theoretic-like* properties, Bayesian Monotonicity is satisfied.

3.4.1 Theorem *If for all S, $\mathcal{F}_i$ is at least as fine as $\mathcal{G}_i(S)$ for all $i \in S$ then the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ interim core is Bayesian implementable on $\mathbf{E}^1$.*

Proof: Let $x \in C$. Let α be compatible with F and suppose that the Bayesian Monotonicity conditions are satisfied. Suppose that x_α is not in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core.

Therefore, there exists a coalition S, a state ω and an allocation $z \in \mathbb{B}(S)$ with z_i $\mathcal{G}_i(S)$ measurable for each $i \in S$ such that

$$V_i(z, \omega) > V_i(x_\alpha, \omega) \text{ for all } i \in S. \tag{3.1}$$

Let $\omega' = \alpha(\omega)$. Then $x_\alpha(\omega) = x(\omega')$. Let $\sigma(\bigcap_{i \in S} \mathcal{G}_i(S))$ be the smallest common coarsening of $\mathcal{G}_i(S)$ where $\overline{G_i(S)}(\omega)$ is an atom. Define a new allocation for coalition S as follows:

$$y_i(\omega'') = \begin{cases} z_i(\omega) & \omega'' \in \overline{G_i(S)}(\omega') \\ e_i & \text{otherwise} \end{cases}$$

which is feasible by the assumption of state independent endowments. Also, notice that for all $i \in S$ for all $\hat{\omega} \in F_i(\omega)$ implies that $\alpha(\hat{\omega}) \in F_i(\omega')$. Therefore, for all $\hat{\omega} \in F_i(\omega)$, $y_\alpha(\hat{\omega}) = z_i(\omega)$ since $\mathcal{F}_i$ is at least as fine as $\mathcal{G}_i(S)$. Then, by (3.1)

$$V_i(y_\alpha, \omega) > V_i(x_\alpha, \omega) \text{ for all } i \in S$$

which by Bayesian Monotonicity implies that

$$V_i(y, \omega') > V_i(x, \omega') \text{ for all } i \in S.$$

However, since y_i is $\mathcal{G}_i(S)$ measurable, this contradicts the assumption that x is in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core. ∎

The logic behind the result is similar to the reason that REE are implementable (Palfrey and Srivastava 1987). If there is an allocation that blocks the allocation attained via some deception, then a new allocation can be constructed that blocks the original allocation. Private measurability and state independent endowments ensure that this new allocation can be feasibly constructed.

A similar result can be stated for efficient allocations.

3.4.2 Corollary *If $\mathcal{F}_i$ is at least as fine as $\mathcal{G}_i(N)$ for all $i \in N$, then $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ interim efficient allocations are Bayesian implementable on* $\mathbf{E}^1$.

Measurability restrictions must only be placed on blocking allocations. Consider Example 3.3.4. Forcing blocking allocations to be privately measurable ensures that $(1,1)$ in all states will not be blocked. However, there is no need in terms of implementation to eliminate the original, non-$\mathcal{F}_i$ measurable, allocation. When $\mathcal{F}_i = \mathcal{F}$ for all i, these results are equivalent to the results for Nash implementation of efficient and core allocations (Maskin 1998).

By restricting the set of blocking allocations to be at least privately measurable, the size of the social choice set has increased. Therefore, more allocations are *optimal* under this version of interim efficiency. Is private measurability the finest possible measurability restriction?

3.4.3 Proposition *If the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ interim core is Bayesian implementable on $\mathbf{E}^1$, then $\mathcal{F}_i$ is at least as fine as $\mathcal{G}_i(S)$ for all S, and for all $i \in S$.*

Proof: Let the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core be implementable and suppose that for some S and some $i \in S$, $\mathcal{G}_i(S) \not\subseteq \mathcal{F}_i$. Therefore, there exists an $\omega \in \Omega$ such that $F_i(\omega) \not\subseteq G_i(S)(\omega)$. Construct an economy for which Bayesian Monotonicity fails for the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core. Assume that $F_i(\omega) \neq \Omega$, and assume that for all $j \neq i$, $\mathcal{F}_j = \mathcal{F}$, or all agents other than i are perfectly informed. Assume that for all $i \in N$, $e_i = (1,1)$. Let the $j \neq i$ agents have state independent preferences given by:

$$u_j(x,\omega) = x_{j1}(\omega) + x_{j2}(\omega).$$

Define the following two events:

$$A = F_i(\omega) \cap G_i(S)(\omega) \text{ and } B = F_i(\omega) \setminus G_i(S)(\omega)$$

where $G_i(S)(\omega)$ is an atom of the σ-field $\mathcal{G}_i(S)$. Let i have state-dependent preferences as follows:

$$u_i(x,\omega) = \begin{cases} .75 \log x_{i1}(\omega) + .25 \log x_{i2}(\omega) & \omega \in A \\ .25 \log x_{i1}(\omega) + .75 \log x_{i2}(\omega) & \omega \in B \\ .5 \log x_{i1}(\omega) + .5 \log x_{i2}(\omega) & \text{otherwise} \end{cases}$$

Also, let i's prior be such that

$$\frac{\mu_i(A)}{\mu_i(F_i(\omega))} = .5 \text{ and } \frac{\mu_i(B)}{\mu_i(F_i(\omega))} = .5$$

Consider the compatible α such that $\alpha(\omega) = \omega'$ for all $\omega \in \Omega$, where $\omega' \notin F_i(\omega)$. If x is in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core, then it must be that $x_i(\omega') = x_j(\omega') = (1,1)$. Bayesian Monotonicity then implies that $x_i(\omega) = x_j(\omega) = (1,1)$ for all $\omega \in \Omega$, is in the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core. However, this allocation is blocked by coalition S at ω by the following allocation:

$$y_i(\omega) = \begin{cases} (1.2, .7) & \omega \in G_i(S)(\omega) \\ (.7, 1.2) & \omega \in \bigcup_{\omega'' \in B} G_i(S)(\omega'') \\ (1,1) & \text{otherwise} \end{cases}$$

and for $j \neq i$ in S

$$y_j(\omega) = \begin{cases} (1 - .2/\#S, 1 + .3/\#S) & \omega \in G_i(S)(\omega) \\ (1 + .3/\#S, 1 - .2/\#S) & \omega \in \bigcup_{\omega'' \in B} G_i(S)(\omega'') \\ (1,1) & \text{otherwise.} \end{cases}$$

This implies that the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core cannot be implementable. ∎

If coalitions can propose blocking allocations which are not consistent with their private information, there will be economies for which Bayesian Monotonicity fails.

If a social choice set in the class of $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ interim cores, or $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ interim efficient allocations is either not $\mathcal{F}_i$ measurable or finer than $\mathcal{F}_i$, then there are example economies such that the social choice set cannot be implemented. However, Palfrey and Srivastava (1987) and Blume and Easley (1990) show that, in this class of economies, expectations equilibria will be implementable. Any expectations equilibria uses some statistic (prices in REE or any arbitrary statistic in Blume and Easley's formulation) that agents can refine their information with respect to. Let γ be a random variable (such as prices). The partition formed by variation in γ generates a σ-field $\mathcal{F}_\gamma$, and agents refine their private information by this statistic. Therefore, blocking allocations that are finer than private information are permitted. Theorem 3.4.3 can be altered to allow for the discussion of expectations equilibria: take $\mathcal{F}_i' = \sigma(\mathcal{F}_i \cup \mathcal{F}_\gamma)$ to be each agent's private information refined by the statistic. Given this new information structure, private measurability will be necessary for implementation. Without knowledge of the exact features of γ, one cannot infer any blocking besides private information. If $\mathcal{F}_i$ is at least as fine as $\mathcal{F}_\gamma$, then the statistic will be uninformative. In Chapter 2, I demonstrated that expectations equilibria will be contained in the fine private core ($\mathcal{G}_i^* = \mathcal{F}$, $\mathcal{G}_i(S) = \mathcal{F}_i$). Therefore, the implementable allocations outlined here contain all expectations equilibria. If one is interested in examining situations in which prices are unknown or irrelevant, then this result demonstrates what efficiency properties can be induced via a noncooperative game (mechanism).

Blume and Easley (1990) show that the Non-Exclusivity of Information is necessary for expectations equilibria to be implementable. Given the similarity between expectations equilibria and measurability constrained welfare allocations discussed here, it would be surprising if the same result did not hold here. At least for a subset of social choice sets identified earlier, NEI is necessary.

3.4.4 Proposition *Let $\mathcal{F}_i$ be at least as fine as $\mathcal{G}_i(S)$, and $\mathcal{G}_i^*$ be finer than $\mathcal{F}_i$. If the $(\mathcal{G}_i^*, \mathcal{G}_i(S))$ core is Bayesian implementable on $\mathbf{D}$, then all $\mathcal{E} \in \mathbf{D}$ satisfy NEI.*

In order for a social choice set to be implementable in exclusive information environments, an incentive compatibility condition must be satisfied (Palfrey and Srivastava

1989). However, as long as the final allocation is allowed to be finer than each agent's private information, non-incentive compatible allocations will be in the social choice set. If $\mathcal{G}_i^* = \mathcal{F}$, then all rational expectations equilibria will be in the solution including fully revealing REE. However, fully revealing REE are not generally incentive compatible (Palfrey and Srivastava 1986, Blume and Easley 1990).

3.5 Conclusion

The results presented here are mixed. There are changes to the definition of optimal allocations in the interim stage that yield a class of implementable social choice sets. Before, even in the most limited economic environments, the same could not be said of HM interim efficiency or the core. If we imagine that decisions on allocations are actually made at the interim stage, then these changes seem reasonable. First, the one state deviation principle recognizes that, having observed their private information, agents will not care about the allocation at states they can rule out ($\omega' \notin F_i(\omega)$). Second, measurability restrictions impose the requirement that agents do not demand different allocations at states they cannot distinguish. However, the results cannot be extended to a larger set of environments in which the NEI condition is not satisfied. Most differential information settings include at least some exclusive information. When NEI holds, no agent has truly private information: there is some subset of the other agents who know at least what he knows.

The analysis here has focused only on changes to the set of blocking allocations. These appear to be the changes that are important for Bayesian Monotonicity to be satisfied since monotonicity is related to the set of blocking allocations across possible deceptions. However, in order to obtain positive implementation results in exchange economies, incentive compatibility must also be satisfied which appears to be related to the set of admissible final allocations. For example, Koutsougeras and Yannelis (1993) show that the private core ($\mathcal{G}_i^* = \mathcal{F}_i$ and $\mathcal{G}_i(S) = \mathcal{F}_i$) is incentive compatible. Therefore, checking the more stringent Bayesian Monotonicity requirement for exclusive information environments is the only difficulty in order to demonstrate implementability in exclusive information environments.

The requirement that endowments be state independent (Assumption 3.2.3) greatly simplifies the proofs. In order to prove global implementability of $(\mathcal{G}_i^*, \mathcal{G}_i(N))$ interim efficient allocations, it is only necessary to assume that the sum of the endowments be constant across states. It is worthwhile to examine whether similar results can be obtained when the endowments vary across states.

Chapter 4 Collusion in Multiple Object Simultaneous Auctions: Theory and Experiments

Abstract

The choice of strategies by bidders who are allowed to communicate in auctions is studied. Using the tools of mechanism design, the possible outcomes of communication between bidders participating in a series of simultaneous first-price auctions are investigated. A variety of mechanisms are incentive compatible when side payments are not allowed. When attention is restricted to mechanisms that rely only on bidders' ordinal ranking of markets, incentive compatibility is characterized and the ranking mechanism of Pesendorfer (1996) is interim incentive efficient. Laboratory experiments were completed to investigate the existence, stability, and effect on bidder and seller surplus of cooperative agreements in multiple object simultaneous first-price auctions. Collusive agreements are stable in the laboratory. The choices of the experimental subjects often closely match the choices predicted by the ranking and serial dictator mechanisms. However, a few notable exceptions raise interesting prospects for the theoretical development of models of cooperative behavior.

4.1 Introduction

Collusion by bidders is thought to be a prominent feature of auctions for antiques, fish, wool, timber, school milk, and oil drainage leases (Cassady 1967, Pesendorfer 1996, Hendricks and Porter 1988). In fact, from 1979 to 1988, 81% all of Sherman Act cases filed by the U.S. Department of Justice involved auctions (Froeb 1988). Bidders have incentives to coordinate their behavior to increase their surplus by eliminating competition amongst each other. If they can find an equitable technique for dividing the spoils from such collusive behavior, bidder *rings* can be quite successful.

In auctions, bidders are asymmetrically informed; they know their own values for the objects but not those of the other agents. In order to limit the amount of surplus that the auctioneer accumulates, the bidders would like to reach a preauction bidding agreement. However, any agreement may reveal the bidders' private information, causing their decisions to change. All bidders face a temptation to increase their one period profits by defecting from the collusive bidding agreement. Three primary questions which need to be addressed in the auction setting are:

1. Do bidders form cooperative agreements in simultaneous first-price auctions?

2. If they do, what sort of strategies do they utilize?

3. How do these strategies affect market efficiency, bidder surplus, and seller surplus?

The objective of this chapter is to begin grappling with these questions by providing a theoretical and experimental examination of cooperative agreements in first-price sealed bid auctions. Collusion in single object auctions has been extensively discussed (Graham and Marshall 1987, McAfee and McMillan 1992, Güth and Peleg 1996). Pesendorfer (1996) suggests some collusive mechanisms for multi-object sealed bid auctions, and, recently, Brusco and Lopomo (1999) have examined how multiple objects can increase the number of 'collusive' Bayes Nash equilibria in English auctions. Multiple object simultaneous sealed bid auctions are not completely unfamiliar. For example, auctions for school milk contracts are held under this procedure

(Pesendorfer 1996). Milgrom (1996) has recently suggested that simultaneous sealed bid auctions be used for determining the Carrier of Last Resort (COLR) privileges by the FCC.

Collusion is modeled as the choice of a *collusive mechanism* by the bidders. Given that they face a game defined by a series of simultaneous first-price auctions, bidders select a mechanism that maps from their valuations for each object to a set of bids in the auction. While noncooperative (Bayes Nash equilibrium) bidding is one possible mechanism, there are potentially many other, more profitable, mechanisms. When side payments are allowed between bidders, an interim incentive efficient mechanism that dominates the noncooperative outcome for the bidders is identified. I then examine collusive mechanisms under the restriction that no side payments may be made between bidders. In the multiple object setting, the number of potential incentive compatible mechanisms increases significantly. Three mechanisms that, in general, are preferred by all bidders to Bayes Nash bidding are presented. On a restricted domain, the ranking mechanism of Pesendorfer (1996) is shown to be interim incentive efficient. These findings suggest that, if given the opportunity, bidders should be able to find a mechanism that they prefer to noncooperative behavior (cooperative agreements will be formed) and that there are some intuitively simple mechanisms that can be predicted as possible stable outcomes. Laboratory experiments are then conducted that often support these theoretical predictions. However, in a few experiments, bidders appear to deviate from theoretical predictions. They choose mechanisms that are not consistent with individual incentives yet lead to higher profits. These deviations suggest an avenue for future research.

In Section 4.2, the general framework of this institution is developed. The tools of mechanism design are used to develop a model of cooperative behavior in simultaneous first-price sealed bid auctions in Section 4.3. The experimental design is presented in Section 4.4. Section 4.5 is a general discussion of the findings of these experiments. Proofs of relevant lemmas are provided in Appendix A.

4.2 The Model

There are n bidders bidding on m objects in separate, simultaneous first-price auctions. Bidders are assumed to have independent private values for each of the m objects. Bidder i's valuation for object j is drawn from a continuous distribution F_{ij} that is assumed to be independent of each bidder's distribution for the other $m-1$ other objects. It is assumed that for all i and for all j, F_{ij} has a common support given by $[\underline{v}, \overline{v}]$ with $\underline{v} \geq 0$. The density, f_{ij}, is defined and strictly positive. Assume that bidders' valuations in each market are symmetric, or $F_{ij} = F_{kj}$ for all i, j, k.[1] Let $v = (v_1, v_2, \ldots, v_n)$ be the vector of individual valuations where $v_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ is the vector of valuations in each market for individual i. Let b be a vector of bids similarly defined.

The simultaneous first-price auctions determine an allocation $x \in \{0,1\}^{m \cdot n}$ and prices based on the bids placed, where $x_{ij} = 1$ indicates that bidder i has been allocated object j. Feasibility requires that $\sum_{i=1}^{n} x_{ij} = 1$ for all j. The function $g : [\underline{v}, \overline{v}]^{m \cdot n} \rightarrow [0,1]^{m \cdot n}$ determines the probability that each bidder is allocated each object:[2]

$$g_{ij}(b) = \begin{cases} \frac{1}{k} & b_{ij} \geq b_{\ell j} \text{ for all } \ell \\ 0 & \text{otherwise} \end{cases}$$

where $k = \#\{b_{ij} | b_{ij} \geq b_{\ell j} \ \forall \ell\}$ is the number of high bidders. Thus, each object is allocated to the highest bidder with ties broken randomly. The price paid by each bidder is given by $p : X \rightarrow [\underline{v}, \overline{v}]^{m \cdot n}$ which is defined as

$$p_{ij}(x) = \begin{cases} b_{ij} & x_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

That is, if a bidder wins an item, then he pays his bid. Let $G_{ij}(b_{ij})$ be the expected

[1] Many of the results presented here are also true when values are drawn from different distributions, but symmetry is maintained for simplicity.

[2] It can be easily verified that no bidder would ever want to place a bid outside of $[\underline{v}, \overline{v}]$.

probability that a bid of b_{ij} by bidder i is highest in market j. If the bid functions for all the bidders are symmetric and monotonic, then $G_{ij}(b_{ij})$ will be the probability that v_{ij} is greater than $n-1$ draws from the distribution F_j, or $G_{ij}(b_{ij}) = F_j(v_{ij})^{n-1}$. Let $P_{ij}(b_{ij})$ be the expected price paid by bidder i for object j when he has placed a bid of b_{ij}. Since first-price auctions are being modeled, the expected price can be simplified to $P_{ij}(b_{ij}) = b_{ij}G_{ij}(b_{ij})$. Assume that bidders are risk neutral. The expected utility for individual i is given by

$$U_i(b_i, v_i) = \sum_{j=1}^{m} G_{ij}(b_{ij})(v_{ij} - b_{ij}).$$

The auctioneer may want to set a reserve price $c > \underline{v}$ to maximize revenue. For simplicity, assume that the auctioneer is passive and sets $c = \underline{v}$. Also assume that the bidders cannot resell the objects; the allocation decision of the auctioneer is binding.

The outcome of noncooperative behavior in this environment has been extensively studied. The optimal bidding strategy of each player is given by the Bayes Nash equilibrium of a game with asymmetric information. Maskin and Riley (1996) provide the most general sufficient conditions for the existence and uniqueness of a Bayes Nash equilibrium bid function. Given the assumption that F_{ij} and f_{ij} are strictly positive and bidders are risk neutral, a unique, monotonic Bayes Nash equilibrium exists. In the case of symmetric distributions, the symmetric bid function for each bidder is given by the simple bid function

$$b_{ij}(v_{ij}) = v_{ij} - \int_{\underline{v}}^{v_{ij}} \left(\frac{F_j(y)}{F_j(v_{ij})} \right)^{n-1} dy \text{ for all } i, j \tag{4.1}$$

where $F_j(v) = F_{ij}(v)$ for all i.

4.3 Cooperative Equilibria

If all bidders act noncooperatively, their attempts to outbid each other will give most of the surplus to the auctioneer. When all bidders' values are drawn from the uniform distribution, bidders will obtain only $1/n$ of the surplus. If the bidders can find an agreement in which they place very low bids in the auction, they can expropriate most of the surplus from the seller. However, finding such an agreement is not necessarily an easy task. In single unit first-price auctions, collusion is considered to be difficult to sustain. Robinson (1985) shows that, with commonly known values, collusive agreements are not stable. However, in an independent private values framework, McAfee and McMillan (1992) show that collusion is possible. However, Güth and Peleg (1996) note that, by using repeated play to support their collusive equilibrium, McAfee and McMillan (1992) diminish the problem of enforcement in their analysis. Güth and Peleg (1996) show that no collusive mechanism satisfies both the no-envy property and their weaker form of incentive compatibility when the item is being sold at the first-price. However, under more general conditions, Güth and Peleg (1996) describe equilibrium strategies. They find that when the object is being sold in a first-price sealed bid auction a ring leads to the same profits for both the buyer and seller as in the competitive case. In their view, the inability of collusion in first-price auctions to lead to profitable agreements may explain the general predominance of first-price sealed bid auctions. In the multiple object setting, however, the opportunities for collusive equilibria increase.

In order to collude in this auction environment, bidders must come to a voluntary agreement about what bids are to be placed at the auction (which, in turn, determines who will be the winner of each item) as well as what sort of side payments are to be made between members. Assume that bidders can communicate, and that they coordinate their bidding in each market in some sort of group decision process.

How is this group decision process modeled? Assume that bidders formulate a *collusive mechanism.* A collusive mechanism is a game played by the bidders, the outcome of which is a set of bids in the auction. As in Laffont and Martimort (1998), assume that the objective of the mechanism is to maximize the expected utility of each bidder.[3]

[3]Laffont and Martimort (1998) examine collusive mechanisms in public goods environments. They propose that the collusive mechanism is designed by a benevolent planner (or centre). The perspective taken here is similar, but I aspire to allow the bidders to select the mechanism themselves.

Attempting to characterize the collusive mechanisms that may arise as the outcome of all possible cooperative games between bidders is a daunting task. Fortunately, by assuming that any collusive agreement must be compatible with individual incentives, that search can be drastically limited. The Revelation Principle guarantees that any outcome which can be attained as the Bayes Nash equilibrium of some mechanism can also be attained as the Bayes Nash equilibrium of a direct revelation mechanism (Gibbard 1973, Dasgupta, Hammond, and Maskin 1979). A direct revelation mechanism is a direct mechanism which satisfies individual incentive compatibility (IC). A mechanism is direct if the strategy space is equivalent to the type space. In this case, agents report a vector of valuations r_i. Thus, the outcome of communication between bidders can be thought of as a mechanism, (β, s), which determines the bids to be placed and the payments to be made between members. In other words, $\beta : [\underline{v}, \overline{v}]^{m \cdot n} \to [\underline{v}, \overline{v}]^{m \cdot n}$ is a function such that $\beta_{ij}(r)$ specifies a bid by i in market j. The function $s : [\underline{v}, \overline{v}]^{m \cdot n} \to \mathbb{R}^{m \cdot n}$ specifies the payment (possibly negative) that each bidder pays in addition to his bid price if he is the winning bidder. Hence, $s_{ij}(r)$ is the payment bidder i pays in market j.

If the private information of all the agents were known, then agents could evaluate each mechanism with respect to their *ex post* utility given by

$$u_i((\beta, s)|v) = \sum_{j=1}^{m} g_{ij}(\beta(v))(v_{ij} - \beta_{ij}(v)) - s_{ij}(v). \tag{4.2}$$

Throughout this chapter, I assume that bidders decide upon a mechanism in the *interim* stage: after they have seen their own values in each market but they remain uncertain as to the actual valuations of the other bidders. No bidder has the ability to coerce another bidder to reveal his valuation. Thus, all information about individual preferences for markets must come from the mechanism itself. At the interim stage, each bidder's interim expected utility is given by

$$U_i((\beta, s)|v_i) = \sum_{j=1}^{m} G_{ij}(B_{ij}(v_i))(v_{ij} - B_{ij}(v_i)) - S_{ij}(v_i) \tag{4.3}$$

where

$$B_{ij}(r_i) = \int_{r_{-i}\in V_{-i}} \beta_{ij}(r_i, r_{-i}) dF_{-i}(r_{-i})$$

and

$$S_{ij}(r_i) = \int_{r_{-i}\in V_{-i}} s_{ij}(r_i, r_{-i}) dF_{-i}(r_{-i})$$

are the *reduced form* equations representing the bidder's expected bid and expected payment. G_{ij} is given by the rules of the auction; it is the probability that i's bid is greater than the $n-1$ other bids placed.

A mechanism satisfies (interim) incentive compatibility (IC) if it is in the best interest of every individual to report his true valuation for the objects for all possible values that the other bidders might have.

4.3.1 Definition (Incentive Compatible) *A mechanism (β, s) is (interim) incentive compatible if, for all r_i and v_i*

$$\sum_{j=1}^{m} G_{ij}(B_{ij}(v_i))(v_{ij} - B_{ij}(v_i)) - S_{ij}(v_i) \geq \sum_{j=1}^{m} G_{ij}(B_{ij}(r_i))(v_{ij} - B_{ij}(r_i)) - S_{ij}(r_i).$$

Finally, if bidders were to decide on a mechanism at the *ex ante* stage, they would evaluate each mechanism with respect to their ex ante expected utility:

$$V_i(\beta, s) = \int_{v_i\in V_i} U_i((\beta, s)|v_i) dF_i(v_i).$$

A restriction that makes analysis of the various mechanisms substantially easier is *anonymity*, which requires that bidders with the same valuations are treated the same under the mechanism.

4.3.2 Definition (Anonymity) *A mechanism (β, s) satisfies anonymity if for all permutations $\sigma : N \to N$, $B_i(v_i) = B_{\sigma(i)}(v_{\sigma(i)})$ and $S_i(v_i) = S_{\sigma(i)}(v_{\sigma(i)})$ for all v_i, and for all i.*

As in Ledyard and Palfrey (1994), when examining situations in which agents' valuations are drawn from identical distributions, it is assumed that mechanisms are anonymous (or symmetric).[4] Lemmas A.0.4 and A.0.5 in the appendix demonstrate that when bidders' values are drawn from identical distributions, there are no non-anonymous mechanisms that are preferred by all bidders to anonymous mechanisms.

An IC collusive mechanism that is always feasible is the *noncooperative mechanism*: bids are placed that are consistent with the symmetric Bayes Nash equilibrium.

4.3.3 Example (Noncooperative Mechanism)

$$\begin{aligned} \beta^*_{ij}(r) &= v_{ij} - \int_{\underline{v}}^{v_{ij}} \left(\frac{F_j(y)}{F_j(v_{ij})} \right)^{n-1} dy \\ s^*_{ij}(r) &= 0 \end{aligned} \tag{4.4}$$

△

Let (β^*, s^*) denote the noncooperative mechanism. If bidders cannot find a collusive mechanism that is preferred to this strategy, there is little hope for successful collusion. Laffont and Martimort (1998) examine, in a public goods setting, whether some mechanism dominates the noncooperative mechanism. The objective here is to go a step further by describing the possible mechanisms.

Assume that bidders do not deviate from the collusive mechanism. While bidders are able to misrepresent their values *within* the mechanism, once bids are determined by the mechanism the bids are perfectly enforced in the auction. This approach may be justified by repeated play. If bids are placed that are inconsistent with the mechanism, bidders will use a trigger strategy to punish deviant bidders.

4.3.4 Proposition *If $V_i(\beta, s) > V_i(\beta^*, s^*)$ and (β, s) is IC, then (β, s) can be supported as a stationary equilibrium of an infinitely repeated game.*

Begin by defining the structure of the repeated game. Let $\delta \in (0, 1]$ be a common discount factor. At each time t there are two stages. At the beginning of each date, all players observe their valuations (or type) for that period. Stage t_1 is the *negotiation* phase. All players submit a report of their valuations ($r^t_i \in [\underline{v}, \overline{v}]^m$). A suggested bid

[4] In situations in which values are drawn from different distributions for the same market, the mechanism should be allowed to vary with different distributions as well as with different values.

and side payment $(\beta(r), s(r))$ is announced. Then, at stage t_2, bidders simultaneously, and independently select a bid and side payments (b_i^t, σ_i^t). A mechanism (β, s) is said to be supported as a stationary equilibrium of this game if there exists a $\delta > 0$ such that for all possible type draws the equilibrium outcome for all t is $v_i^t = r_i^t$, $b_i^t = \beta_i(v^t)$, and $\sigma_i^t = s_i(v^t)$.

Proof: The equilibrium outcome can be supported by the following strategy. At time t,

1. If for all $t' < t, \beta_i(r^{t'}) = b_i^{t'}$ and $s_i(r^{t'}) = \sigma_i^{t'}$ for all i, then $\beta_i(r^t) = b_i^t$ and $s_i(r^t) = \sigma_i^t$.

2. Otherwise all play (β^*, s^*).

First, notice that by the assumption that (β, s) is IC, if a bidder chooses to play $\beta_i(r^t) = b_i^t$ and $s_i(r^t) = \sigma_i^t$ it is optimal for him to choose $r_i^t = v_i^t$. Second, demonstrate that deviating from the collusive mechanism (β, s) is not optimal. For any collusive mechanism, each player's one period gain from deviating from the collusive mechanism is bounded by $(\overline{v} - \underline{v})^m$.[5] By assumption, $V_i(\beta, s) > V_i(\beta^*, s^*)$. Define

$$c = V_i(\beta, s) - V_i(\beta^*, s^*).$$

Then, an agent will not deviate at t if

$$\sum_{T=0}^{\infty} \delta^T c > (\overline{v} - \underline{v})^m.$$

Thus as long as

$$\delta > \frac{(\overline{v} - \underline{v})^m - c}{(\overline{v} - \underline{v})^m}$$

agents will prefer to always play the collusive mechanism. Since c is also bounded by $(\overline{v} - \underline{v})^m$, it follows that there exists $\delta < 1$ satisfying this condition. ∎

[5] This is the utility of one agent if he has the highest value in all the markets and is able to win all of them at a price equal to $\underline{v}$.

McAfee and McMillan (1992) and Pesendorfer (1996) use this approach to find profitable collusive mechanisms. Otherwise, bidders' incentives to increase their bids cannot be avoided. The negative results of Güth and Peleg (1996) are largely due to the fact that they assume that bidders may place any bid in the auction. The repeated game approach appears to be consistent with previous experimental evidence on cooperative agreements (see Section 4.4). Also, assume that bidders' values are not ex post observable. After an auction, bidders cannot observe values in order to determine whether bids were truthful. Therefore, collusive mechanisms must be independent of actions in previous auctions.

A first step in determining what mechanisms might be expected is to propose a reasonable mechanism and investigate its characteristics. A *reduced bidding* mechanism is one possibility. Under this mechanism, each bidder agrees to bid some fraction (α_j) of his value in each market.

4.3.5 Example (Reduced Bidding Mechanism)

$$\begin{aligned} \beta_{ij}(r) &= \alpha_j r_{ij} \\ s_{ij}(r) &= 0 \end{aligned}$$

$\triangle$

The reduced bidding mechanism represents limited competition between bidders. By choosing such a mechanism, if the bidders truthfully report their valuations, the objects will be won by the bidders with the highest valuations, and, if the α's are small, the cartel will capture most of the surplus. The following lemma characterizes IC reduced bidding mechanisms.

4.3.6 Lemma *Let $\Gamma_{ij}(r_{ij}) = \frac{G_{ij}(r_{ij})}{g_{ij}(r_{ij})}$. For each market, j, there exists an $\alpha_j \in (0, 1]$ such that the Reduced Bidding mechanism is Bayesian Incentive Compatible if and only if $\frac{d\Gamma_{ij}(r_{ij})}{dr_{ij}} = c_j$ for all i, where $c_j \in \mathbb{R}_+$. Furthermore, α_j is given by $\frac{d\Gamma_{ij}(r_{ij})}{dr_{ij}} = \frac{1-\alpha_j}{\alpha_j}$.*

The implication of Lemma 4.3.6 is that the only IC reduced bidding mechanisms are those that yield bidder profits identical to noncooperative bidding.

4.3.7 Theorem *If an IC α_j exists for all markets then the resulting bid function is equivalent to the noncooperative mechanism.*

Proof: Let (β, s) be a reduced bidding mechanism satisfying incentive compatibility (and thus the conditions of Lemma 4.3.6). Suppose that $b_{ij}(v_{ij}) = \alpha_j v_{ij}$ is not a Bayes Nash equilibrium. In the noncooperative setting the first order conditions for maximization are given by

$$\frac{1}{\phi_{ij}(b) - b} = \sum_{k \neq i} H_{kj}(\phi_{kj}(b))\phi'_{kj}(b) \text{ for all } i$$

where

$$H_{kj}(v) = \frac{f_{kj}(v)}{F_{kj}(v)}$$

is the hazard rate for each individual, and $\phi_{ij}(b)$ is the inverse bid function. If $b_{ij}(v_{ij}) = \alpha_j v_{ij}$ is not an equilibrium then it must be that

$$\frac{1}{v_{ij} - \alpha_j v_{ij}} \neq \sum_{k \neq i} H_{kj}(v_{ij}) \left(\frac{1}{\alpha_j}\right)$$

for some i and j which implies that

$$\alpha_j \neq v_{ij}(1 - \alpha_j) \sum_{k \neq i} H_{kj}(v_{ij}).$$

Note that

$$g_{ij}(v_{ij}) = G_{ij}(v_{ij}) \sum_{k \neq i} H_{kj}(v_{ij}).$$

Thus, multiplying by $G_{ij}(v_{ij}) > 0$ leads to

$$G_{ij}(v_{ij})\alpha_j \neq v_{ij}(1 - \alpha_j) g_{ij}(v_{ij})$$

which is a contradiction with the first order conditions for IC given by Equation A.6.

Thus, if the reduced bidding mechanism is IC it is also the outcome of competitive behavior. ∎

If $\alpha_j v_{ij}$ is not equal to the noncooperative bid strategy, bidders have an incentive to increase their reported values to increase their probability of winning in the auction. Since the cartel members cannot directly observe each other's values, all agents will partake in this destructive behavior as long as they are bidding below the Bayes Nash equilibrium. Therefore, the only IC reduced bidding mechanism is the Bayes Nash equilibrium. This result is similar to Güth and Peleg's (1996): any mechanism which allows for positive bidding must be equivalent to the Bayes Nash equilibrium.

Given that mechanisms of this sort will be no better than the noncooperative mechanism, what types of mechanisms might one expect to see bidders select? Holmström and Myerson (1983) suggest that a reasonable class of mechanisms to eliminate are those that are *interim dominated* by another mechanism.

4.3.8 Definition (Interim Dominated) *A mechanism (β, s) is interim dominated by (β', s') if $U_i((\beta', s')|v_i) \geq U_i((\beta, s)|v_i)$ for all i and for all v_i with at least one strict inequality.*

If the bidders select a mechanism that is interim dominated then, even before they learn their values, bidders would unanimously prefer to switch to a mechanism that dominates it. Interim incentive efficient (Holmström and Myerson 1983) mechanisms are those which are not dominated.

4.3.9 Definition (Interim Incentive Efficient) *A mechanism is interim incentive efficient if there does not exist another IC feasible mechanism that interim dominates it.*

Holmström and Myerson (1983) show that if a mechanism is interim incentive efficient, then it can *never* be common knowledge that another IC mechanism interim dominates it. Thus, interim efficiency is a minimal standard for what is expected as the outcome of a cooperative process.[6]

[6]It is likely that the cooperative process would lead bidders to actually select a subset of the

In order to limit the incentives to misrepresent their valuations that arise in the reduced bidding mechanism, bidders might select one bidder as the sole bidder in each market. Define β° as follows.

$$\beta^{\circ}_{ij}(v) = \begin{cases} \underline{v} & \text{with probability } q_{ij}(v) \\ \emptyset & \text{with probability } (1 - q_{ij}(v)) \end{cases}$$

where $\emptyset$ indicates that the bidder does not participate in the auction,[7] and

$$\sum_{i=1}^{n} q_{ij} = 1 \text{ for all } j.$$

The function that determines the side payments as a function of bidders' valuations is now given by t. Let the class of mechanisms of this form be indicated by $\mathcal{B}^{\circ}$. Thus, (q, t) is now a new direct revelation mechanism that defines a bidder's ex post expected utility as

$$U_i(r, v) = \sum_{j=1}^{m} q_{ij}(r)(v_{ij} - \underline{v}) - t_{ij}(r),$$

and interim expected utility as

$$U_i(r_i, v_i) = \sum_{j=1}^{m} Q_{ij}(r_i)(v_{ij} - \underline{v}) - T_{ij}(r_i)$$

where the reduced form probability of being selected as the sole bidder Q_{ij} and side payment T_{ij} is:

$$\begin{aligned} Q_{ij}(r_i) &= \int q_{ij}(r_i, r_{-i}) dF_{-i} \\ T_{ij}(r_i) &= \int t_{ij}(r_i, r_{-i}) dF_{-i}. \end{aligned}$$

interim incentive efficient mechanisms. For example, Holmström and Myerson (1983) argue that the appropriate restriction in the face of communication between agents is the concept of *durability* (the bidders would never unanimously approve a change from one mechanism to another). For the duration of this chapter, I take the set of interim incentive efficient mechanisms to be a good first approximation.

[7]A nearly equivalent version would allow one bidder to bid $\underline{v} + \epsilon$ and all others to bid $\underline{v}$.

The following theorem establishes that attention can be restricted to this particular class of mechanisms.

4.3.10 Theorem *If (β, s) is an incentive compatible direct mechanism such that $(\beta, s) \notin \mathcal{B}^\circ$, then there exists an incentive compatible, direct mechanism $(\beta', s') \in \mathcal{B}^\circ$ that interim dominates (β, s).*

Proof: Let (β, s) be an incentive compatible mechanism. Define

$$Q_{ij}(v_i) = G_{ij}(B_{ij}(v_i))$$

and

$$T_{ij}(v_i) = G_{ij}(B_{ij}(v_i))(B_{ij}(v_i) - \underline{v}) + S_{ij}(v_i).$$

Then,

$$\sum_{j=1}^{m} Q_{ij}(v_i)(v_{ij} - \underline{v}) - T_{ij}(v_i) \equiv \sum_{j=1}^{m} G_{ij}(B_{ij}(v_i))(v_{ij} - B_{ij}(v_i)) - S_{ij}(v_i) \; \forall v_i \; \forall i.$$

Since (β, s) satisfies that necessary and sufficient conditions for IC, then so to must (q, t).

To show the second part assume that $S(v) = \sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij}(v_i)$ and note that $T(v) = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{ij}(B_{ij}(v_i))(B_{ij}(v_i) - \underline{v}) + S_{ij}(v_i)$. Thus, since $G_{ij}(B_{ij}(v_i))$ is a probability and $(B_{ij}(v_i) - \underline{v}) \geq 0$ ($\underline{v}$ is the lower bound of the range of $B_{ij}(v_i)$), it must be that $T(v) \geq S(v)$ for all v. Thus, we can define a function $c_{ij}(v_{-i})$ such that $c_{ij}(v_{-i}) \geq 0$ and $\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}(v_{-i}) = T(v) - S(v)$. Let

$$c_{ij}(v_{-i}) = \frac{\sum_{k \neq i}^{m} G_{kj}(B_{kj}(v_k))(B_{kj}(v_k) - \underline{v})}{n - 1}.$$

Then, let

$$\hat{T}_{ij}(v_i) = T_{ij}(v_{ij}) - \int_{\underline{v}}^{\overline{v}} c_{ij}(v_{-i}) dF_{-i}$$

be the new expected tax then $(q, \hat{t})$ is a new mechanism that is still incentive compatible (since the new term is just a constant for any agent) but yields higher expected utility due to the lower expected taxes. ∎

The set of interim incentive efficient mechanisms lie within $\mathcal{B}^{\circ}$. Bidding leads to profits for the auctioneer which necessarily implies losses to the cartel. To achieve the greatest possible surplus bidders will allocate the object to the bidder who would have won the same object under the mechanism not in $\mathcal{B}^{\circ}$. Then, the bidders can divide up the gains from not bidding in a manner that does not affect incentives. This is the approach taken by Graham and Marshall (1987) when modeling collusion in second-price auctions. Noncooperative bidding specifies strictly positive bids for all bidders. The noncooperative mechanism is not in $\mathcal{B}^{\circ}$, implying that there exist collusive mechanisms which dominate it.

4.3.11 Corollary *The noncooperative mechanism is dominated when side payments are allowed.*

McAfee and McMillan (1992) provide an insight into possible mechanisms that might arise in this setting.

4.3.12 Definition (Ex post Efficient) *A mechanism (q, t) is ex post efficient if there does not exist another mechanism (q', t') such that $u_i((q', t')|v) \geq u_i((q, t)|v)$ for all i, and for all v with strict inequality somewhere.*

That is, a mechanism is said to be ex post efficient if it always assigns bidding rights in each market to the bidder with the highest valuation. Thus, the bidder with the highest valuation is chosen as the winning bidder with probability one.

4.3.13 Remark In order for a mechanism to be ex post efficient it must be that for all j, $q_{ij}(v) = 1$ if and only if $v_{ij} = \max\{v_{1j}, v_{2j}, \ldots, v_{nj}\}$.

Definition 4.3.12, however, did not impose incentive compatibility on the set of feasible mechanisms. It may be possible that the informational constraints prohibit mechanisms from always selecting the bidder with the highest value. In the single object setting, McAfee and McMillan (1992) show that there exists an ex post efficient and interim incentive compatible mechanism which can easily be extended to the multiple object environment developed here.

4.3.14 Example (Efficient Strong Cartel Mechanism)

$$q_{ij}(v) = \begin{cases} 1 & v_{ij} = \max\{v_{1j}, v_{2j}, \dots, v_{nj}\} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{ij}(v) = (F_j(v_{ij})^{-n}) \int_{\underline{v}}^{v_{ij}} (v_{ij} - \underline{v})(n-1)F_j(s)^{n-1} f_j(s)ds + \underline{v}$$

if $v_{ij} = \max\{v_{1j}, v_{2j}, \dots, v_{nj}\}$ and otherwise

$$t_{ij}(v) = -\frac{[t_{ik}(v) - \underline{v}]}{(n-1)} \tag{4.5}$$

△

Under this mechanism, the bidder with the highest valuation in each market is selected and splits between each of the $n-1$ other bidders the gain in surplus from limiting competition. Since this mechanism is dependent only upon valuation reports for each particular market it can be extended to the multiple object setting.

4.3.15 Theorem (McAfee and McMillan (1992)) *The efficient strong cartel mechanism is both incentive compatible and ex post efficient.*

When side payments are allowed, there exists a collusive mechanism that allows the bidders to capture all available surplus. Since ex post efficiency uniquely characterizes q (Remark 4.3.13), it must be that an interim efficient mechanism also satisfies that restriction.[8] Therefore, the strong cartel mechanism must be interim incentive efficient.

[8]The set of interim efficient mechanisms are a subset of the ex post efficient mechanisms.

4.3.16 Corollary *The efficient strong cartel mechanism is interim incentive efficient when side payments are allowed.*

4.3.1 Weak Cartels

McAfee and McMillan (1992) also examine collusive agreements in single object first-price auctions that prohibit side payments. A justification for this restriction is that antitrust laws and the threat of detection make actual side payments extremely risky, if not impossible. It is hard to imagine a large firm actually transferring funds to another firm. Thus, the only method for collusion is the division of bidding rights in various markets. Assume side payments are not possible. Under these *weak cartel* agreements, McAfee and McMillan (1992) show that the best mechanism for a ring of bidders is one in which they all place identical bids, or they commit to a rotation scheme that randomly chooses an exclusive bidder. Such rotation schemes are often called *phases of the moon* agreements (Bane 1973). In the mechanism design model just developed, such a restriction can be implemented by requiring that no transfers are made.

4.3.17 Assumption (No side payments) $t_{ij}(v) = 0$ *for all* i *and for all* j.

Let $\mathcal{B}^s$ be the subset of $\mathcal{B}^\circ$ satisfying Assumption 4.3.17. A mechanism with no side payments cannot be ex post efficient since the condition given by Remark 4.3.13 violates IC. The following result is a generalization of the result of Dudek, Kim, and Ledyard (1995).

4.3.18 Theorem *Let* $G_{ij}(r) = \prod_{k \neq i} F_{kj}(r)$. *If there is some* i *such that* $G_{ij}(\hat{v}_{ij}) \neq G_{ij}(v_{ij})$ *for some positive* $\hat{v}_{ij}, v_{ij} \in [\underline{v}, \overline{v}]$, *then there does not exist an IC* (β, s) *in* $\mathcal{B}^s$ *such that it is ex post efficient.*

Proof: Suppose q is an ex post efficient Bayesian Mechanism without transfers. Let $v_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ and $\hat{v}_i = (v_{ij}, \hat{v}_{i,-j})$ and agent i is as given above. Incentive compatibility requires that

$$\sum_{j=1}^{m} \int_{v_{-i} \in V_{-i}} \hat{v}_{ij} q_{ij}(\hat{v}_i, v_{-i}) dF_{-i}(v_{-i}) \geq \sum_{j=1}^{m} \int_{v_{-i} \in V_{-i}} \hat{v}_{ij} q_{ij}(v_i, v_{-i}) dF_{-i}(v_{-i})$$

and

$$\sum_{j=1}^{m} \int_{v_{-i} \in V_{-i}} v_{ij} q_{ij}(v_i, v_{-i}) dF_{-i}(v_{-i}) \geq \sum_{j=1}^{m} \int_{v_{-i} \in V_{-i}} v_{ij} q_{ij}(\hat{v}_i, v_{-i}) dF_{-i}(v_{-i}).$$

Since $q_{ij}(v) = 1$ only if $v_{ij} > v_{kj}$ for all $k \neq i$ and $q_{ij}(v) = 0$ otherwise, from ex post efficiency, I may simplify, so that $\int_{v_{-i} \in V_{-i}} q_{ij}(v_i, v_{-i}) dF_{-i}(v_{-i}) = G_{ij}(v_{ij})$. Thus,

$$\sum_{j=1}^{m} \hat{v}_{ij} G_{ij}(\hat{v}_{ij}) \geq \sum_{j=1}^{m} \hat{v}_{ij} G_{ij}(v_{ij})$$

and

$$\sum_{j=1}^{m} v_{ij} G_{ij}(v_{ij}) \geq \sum_{j=1}^{m} v_{ij} G_{ij}(\hat{v}_{ij})$$

But for all $k \neq j$ $v_{ik} = \hat{v}_{ik}$ implying that $G_{ik}(v_{ik}) = G_{ik}(\hat{v}_{ik})$. This allows me to simplify the expression to

$$\hat{v}_{ij} G_{ij}(\hat{v}_{ij}) \geq \hat{v}_{ij} G_{ij}(v_{ij})$$

and

$$v_{ij} G_{ij}(v_{ij}) \geq v_{ij} G_{ij}(\hat{v}_{ij}).$$

Rearranging terms yields

$$\hat{v}_{ij}[G_{ij}(\hat{v}_{ij}) - G_{ij}(v_{ij})] \geq 0$$

and

$$v_{ij}[G_{ij}(v_{ij}) - G_{ij}(\hat{v}_{ij})] \geq 0.$$

Since both $\hat{v}_{ij}, v_{ij} > 0$, it must be that $G_{ij}(\hat{v}_{ij}) = G_{ij}(v_{ij})$, which is a contradiction. ∎

Without the extra lever of side payments, the cartel cannot ensure that the bidder with the highest valuation is chosen. Every bidder (even the lowest types) must be given some positive probability of being chosen as the sole bidder. Therefore, there will always be mechanisms in $\mathcal{B}^{\circ}$ that (ex post) dominate mechanisms without side payments. If, instead of comparing mechanisms in $\mathcal{B}^{s}$ to *all* other mechanisms, attention is restricted to mechanisms *only* in $\mathcal{B}^{s}$, then any mechanism is ex post efficient. Raising any bidder's probability that he is the sole bidder in some market (given that all type information is revealed) necessarily requires lowering other bidders' probabilities of being the sole bidder. The decreased probability of winning cannot be offset by side payments as it is in the strong cartel situation.

There are limitations, however, to this result. $\mathcal{B}^{s}$ is assumed to be a subset of $\mathcal{B}^{\circ}$, mechanisms that select a single bidder for each market. There are incentive compatible mechanisms in $\mathcal{B}$ that do not involve side payments yet are ex post efficient. The noncooperative mechanism is one. When examining strong cartels, it was shown in Theorem 4.3.10 that any mechanism in $\mathcal{B}\backslash\mathcal{B}^{\circ}$ was dominated by a mechanism in $\mathcal{B}^{\circ}$. The result does not hold for mechanisms in $\mathcal{B}^{s}$. In fact, for any mechanism in $\mathcal{B}^{s}$ a set of value distributions can be constructed such that the noncooperative mechanism is not dominated. Therefore, assuming $\mathcal{B}^{s} \subset \mathcal{B}^{\circ}$ is a simplifying assumption that could potentially eliminate some *good* mechanisms.

Unfortunately, examining all possible mechanisms in $\mathcal{B}^{s}$ is still a very difficult task due to the dimensionality of each bidder's type (each bidder's type is an m-tuple of valuations). While Rochet (1987) provides necessary and sufficient conditions for a mechanism to be IC in very general multi-dimensional settings, finding the interim incentive efficient mechanism in this class is not trivial. I proceed by proposing a potential collusive mechanism in $\mathcal{B}^{s}$. In the single object setting, McAfee and McMillan (1992) show that random assignment of a winning bidder is the only IC collusive mechanism without side payments other than the noncooperative mechanism. The random assignment mechanism generalizes their result to the multiple object setting.

4.3.19 Example (Random Assignment Mechanism)

$$q_{ij}(v) = \frac{1}{n} \tag{4.6}$$

△

The random assignment mechanism is IC since it does not depend upon *any* individual information. Three procedures describe how bidders might arrive at the probabilities specified by this mechanism. First, in each market, the group could randomly select a sole bidder. Second, if the auctioneer randomizes amongst tie bids, all bidders could simply agree to place identical bids of $\underline{v}$ in each market. Finally, when $F_{ij} = F_{kj}$ for all i, j, and k and the auction is repeated many times, the assignment mechanism could also be approximated by each bidder bidding in only one market for all periods.

Is it the case that even this simple mechanism interim dominates noncooperative bidding? For a large class of distributions the random assignment mechanism will always be preferred to noncooperative bidding.

4.3.20 Proposition *If for all j, F_j first-order stochastically dominates the uniform distribution on $[\underline{v}, \overline{v}]$, then the random assignment mechanism dominates the noncooperative mechanism.*

Proof: In order for Proposition 4.3.20 to be true it must be that

$$U_i((\beta^*, s^*)|v_i) \leq \sum_{j=1}^{m} \frac{1}{n}(v_{ij} - \underline{v})$$

for all v_i, or

$$\sum_{j=1}^{m}(v_{ij} - \beta_{ij}^*(v_{ij}))F_j(v_{ij})^{n-1} \leq \sum_{j=1}^{m} \frac{1}{n}(v_{ij} - \underline{v}).$$

It is sufficient to demonstrate that for all j and v_{ij},

$$(v_{ij} - \beta_{ij}^*(v_{ij}))F_j(v_{ij})^{n-1} \leq \frac{1}{n}(v_{ij} - \underline{v}).$$

Substituting Equation 4.4 for β^*_{ij} and simplifying yields

$$\int_{\underline{v}}^{v_{ij}} F_j(y)^{n-1}dy \leq \frac{1}{n}(v_{ij} - \underline{v}).$$

F_j first order stochastic dominates the uniform distribution, or

$$\frac{y-\underline{v}}{\overline{v}-\underline{v}} \geq F_j(y)$$

for all $y \in [\underline{v}, \overline{v}]$ implying that

$$\begin{aligned}
\int_{\underline{v}}^{v_{ij}} F_j(y)^{n-1}dy &\leq \int_{\underline{v}}^{v_{ij}} \left(\frac{y-\underline{v}}{\overline{v}-\underline{v}}\right)^{n-1} dy \\
&= \frac{(\overline{v}-\underline{v})}{n}\left(\frac{v_{ij}-\underline{v}}{\overline{v}-\underline{v}}\right)^{n} \\
&\leq \frac{(\overline{v}-\underline{v})}{n}\left(\frac{v_{ij}-\underline{v}}{\overline{v}-\underline{v}}\right) \\
&= \frac{v_{ij}-\underline{v}}{n}.
\end{aligned}$$

∎

An implication of this result is that if F_j is convex for all markets, the random assignment mechanism will dominate the Bayes Nash equilibrium mechanism. A convex F_j implies that higher value draws are more likely, which encourages bidders to bid closer to their values in the Bayes Nash equilibrium.

However, for some distributions there will always be values for which bidders prefer the noncooperative mechanism to the random assignment mechanism. Consider an example where $v_{ij} \in [0, 1]$ and $F_{ij}(v) = v^{1/3}$ for all i. The Bayes Nash equilibrium bid function is given by

$$b_{ij}(v_{ij}) = \frac{n-1}{n+2}v_{ij}.$$

For any v_{ij}, a bidder's expected utility from the random assignment mechanism is

$\frac{1}{n}v_{ij}$. A bidder will prefer the noncooperative outcome in market j if

$$v_{ij} > \left(\frac{n+2}{3n}\right)^{\frac{3}{n-1}}. \tag{4.7}$$

For $n > 1$, there are feasible values satisfying this condition. As n increases, the right hand term of Equation 4.7 approaches 1. In general, as the set of bidders grows, the set of values under which the noncooperative mechanism is preferred to the random assignment mechanism shrinks.

4.3.21 Theorem *For any collection of distributions, $(F_j)_{j=1}^m$, there exists a number of bidders, n, such that the random assignment mechanism dominates the noncooperative mechanism.*

Proof: Lemma A.0.6 establishes that there exists a n such that

$$F_j(y)^{n-1} \leq \frac{y - \underline{v}}{\overline{v} - \underline{v}}$$

for all j and y. Then, apply Proposition 4.3.20. ∎

In general, only bidders with high valuations will prefer the noncooperative outcome. However, as n increases, it is more likely that there are other bidders with high values. This makes the Bayes Nash equilibrium strategy less profitable.

The fact that bidders are more likely to prefer the random assignment mechanism to noncooperative bidding when n is large is opposed to conventional wisdom on collusive behavior. Both experimentally and empirically, cartelization is thought to be much easier in small groups. However, as the group size increases, the benefits from noncooperative behavior shrink significantly.

For the single object case the random assignment mechanism is interim incentive efficient. For all distributions, there is a cutoff within $[\underline{v}, \overline{v}]$ between preference for the random assignment mechanism and the noncooperative mechanism. However, the noncooperative mechanism is the only other incentive compatible mechanism. Thus, random assignment cannot be dominated.[9]

4.3.22 Corollary *When the number of objects $m = 1$, the random assignment mechanism is interim incentive efficient.*

While random assignment mechanisms are the only IC collusive mechanisms in the single object environment, other IC mechanisms are available in the multiple object environment. Bidders may use more sophisticated *rotation schemes* utilizing the increased dimensionality of the type space to increase efficiency. Bidders are willing to trade-off probability of winning a lower valued object for increased probability of winning a higher valued object. These mechanisms are characterized by the strategic choice of sole bidders for each market based upon their reported values. The serial dictator mechanism (Satterthwaite and Sonnenschein 1981, Olson 1991) is an example of a rotation scheme.

4.3.23 Example (Serial Dictator Mechanism) For each random permutation of bidders: $(n_1, n_2, \ldots, n_m)$, where $n_k = i$ indicates that bidder i selects in spot k,

$$q_{(n_k)j} = \begin{cases} 1 & j = R(n_k) \\ 0 & \text{otherwise} \end{cases} \tag{4.8}$$

$$\tag{4.9}$$

where $R(n_k)$ is defined iteratively as follows. Let $R(n_0) = \emptyset$ and for $k \geq 1$

$$R(n_k) = \arg \max_{j \in \{1,\ldots,m\} \setminus \bigcup_{i=1}^{k-1} R(n_i)} \{v_{(n_k)j}\}.$$

△

The serial dictator mechanism selects the order in which each bidder is allowed to select the market in which he is the sole bidder. Each bidder is a *dictator* over the outcomes at a single point in time. Assume that the choice of dictator at any point

[9] Any randomization will be IC, but only the random assignment mechanism satisfies anonymity.

		Bidder				
		1	**2**	**3**	**4**	**5**
	A	1	3	3	5	5
	B	2	5	2	3	2
Market	**C**	3	2	1	1	1
	D	4	1	4	2	3
	E	5	4	5	4	4

Figure 4.1: An Example with 5 Bidders and 5 Markets

is random.[10] If there are m objects and n bidders, the probability that any bidder is selected to be the dictator for market i is $1/m$. In the example described by Figure 4.1, there are five bidders and five objects. The numbers indicate each bidder's relative ranking of his values. If the random draw of dictators yields the order $(1, 2, 3, 4, 5)$, then bidder 1 would select first and choose market A, 2 would select market D, 3 would select C, 4 would select B, and 5 would have no choice but to select market E. When the number of objects is less than or equal to the number of bidders, the serial dictator mechanism requires that each bidder be selected at most one time. The number of possible allocations predicted by the serial dictator mechanism can be large. In principle, each different permutation of the dictator order could lead to a different outcome.[11] Thus, the number of possible orderings, $n!$, acts as an upper bound on the number of possible outcomes.

The serial dictator mechanism is IC because stating one's true valuations maximizes the probability that higher valued objects are chosen first. The serial dictator mechanism highlights the increased richness of the set of possible mechanisms when examining multiple object auctions. More importantly, the multiple object environment makes the random assignment mechanism an inferior choice.

4.3.24 Proposition *The serial dictator mechanism interim dominates the random*

[10]This is necessary to maintain anonymity.

[11]Although that is not necessarily true. It is easy to imagine circumstances in which only one solution is possible. For example, suppose each bidder's maximal valuation is in a different market. Then, for any combination, each bidder will select the market he ranks highest.

assignment mechanism.

Proof: Let Q_{ij}^{SD} be the reduced form probabilities given by the serial dictator mechanism, and let $Q_{ij}^{A} = \frac{1}{n}$. Note that $\sum_{j=1}^{m} Q_{ij}^{SD} = \sum_{j=1}^{m} Q_{ij}^{A} = \frac{m}{n}$. Also, note that Q_{ij}^{SD} is a decreasing function of each markets ordinal ranking. Thus, w.l.o.g. let v_i be such that $v_{i1} \geq v_{i2} \cdots \geq v_{im}$. Note that $Q_{i1}^{SD} > Q_{i1}^{A} = \frac{1}{n}$ since with probability $\frac{1}{n}$, i gets to choose first; however, there is also a positive probability that i chooses at some other point but market 1 is still available. This is enough to apply Lemma A.0.7 (multiply the whole equation by $\frac{n}{m}$ in order to get a linear combination). Thus, it must be that for all v_i, $v_i \cdot Q_{ij}^{SD} \geq v_i \cdot Q_{ij}^{A}$. All that remains to be shown is that there exists a v_i such that $v_i \cdot Q_{ij}^{SD} > v_i \cdot Q_{ij}^{A}$ Let $v_i = (\overline{v}_{i1}, \ldots, \overline{v}_{ik}, \underline{v}_{i(k+1)}, \ldots, \underline{v}_{im})$. This yields our result. ∎

Ideally, I would continue examining generic weak cartel mechanisms. However, the serial dictator mechanism suggests a class of mechanisms that seem particularly reasonable as a first guess at the expected choice of mechanism in this setting and are easier to analyze. They are *ordinal mechanisms* which rely only on each individual's ranking of his markets.

4.3.25 Definition (Ordinal Mechanism) *Let $M = \{1, 2, \ldots, m\}$. Let $f : [\underline{v}, \overline{v}]^{m \cdot n} \to M^{m \cdot n}$ be a function defined as*

$$f(v_{ij}) = k \text{ if and only if } \#\{v_{il} : v_{il} \geq v_{ij}\} = k - 1.$$

q is an ordinal mechanism if for all v and v'

$$f(v) = f(v') \Rightarrow q(v) = q(v').$$

When a mechanism is ordinal, both q and the reduced form probabilities Q can be expressed as a function of each agent's ranks of the markets. Since any incentive compatible collusive mechanism will not be ex post efficient (Theorem 4.3.18), it must be that the mechanism makes limited use of the bidders' information. Also, when

$m = 1$, in order to satisfy IC, the mechanism must not depend on any private information. The class of ordinal mechanisms are one class of mechanisms that satisfy these constraints. There may be other incentive compatible mechanisms that use more information than ordinal mechanisms. However, as a first cut, the possible mechanisms given this restriction are examined. When considering ordinal mechanisms, IC is characterized by the following proposition.

4.3.26 Proposition *Any ordinal mechanism q is Bayesian Incentive Compatible if and only if for all i,*

i. *Q_{ij} is decreasing in the ranks (i.e., $Q_{ij}(m_i|m_{ij} = 1) \geq Q_{ij}(m_i|m_{ij} = 2) \ldots \geq Q_{ij}(m_i|m_{ij} = m)$), and*

ii. *$(Q_{ij}(m_i|m_{ij} = l) - Q_{ij}(m_i|m_{ij} = p)) = (Q_{ik}(m_i|m_{ik} = l) - Q_{ik}(m_i|m_{ik} = p))$ for all j, k and for all l, p.*

Proof: Necessity. Let an ordinal mechanism be IC and assume that 1 or 2 do not hold. Suppose there exists a $l < p$ and a j such that $Q_{ij}(m_{ij} = p) > Q_{ij}(m_{ij} = l)$. Let v_i be such that $v_{i1} > v_{i2} > \cdots > v_{im}$ such that $\#\{v_{il}|v_{il} > v_{ij}\} = l - 1$. IC implies that

$$\begin{aligned} Q_{ij}(m_{i1} = 1)v_{i1} + \cdots + Q_{ij}(m_{ij} = l)v_{ij} + \cdots + Q_{ik}(m_{ik} = p)v_{ik} + \\ + Q_{im}(m_{im} = m)v_{im} \geq Q_{ij}(m_{i1} = 1)v_{i1} + \\ + Q_{ij}(m_{ij} = p)v_{ij} + \cdots + Q_{ik}(m_{ik} = l)v_{ik} + \cdots + Q_{im}(m_{im} = m)v_{im} \end{aligned}$$

which implies that

$$\begin{aligned} Q_{ij}(m_{ij} = l)v_{ij} + Q_{ik}(m_{ik} = p)v_{ik} \geq Q_{ij}(m_{ij} = p)v_{ij} + Q_{ik}(m_{ik} = l)v_{ik} \\ v_{ij}(Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)) \geq v_{ik}(Q_{ik}(m_{ik} = l) - Q_{ik}(m_{ik} = l)) \end{aligned}$$

implying that $v_{ij} \leq v_{ik}$ which is a contradiction.

Suppose there exists $Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p) \neq Q_{ik}(m_{ik} = l) - Q_{ik}(m_{ik} = p)$. W.l.o.g. assume $Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p) > Q_{ik}(m_{ik} = l) - Q_{ik}(m_{ik} = p)$. Let

$c = \frac{Q_{ij}(m_{ij}=l)-Q_{ij}(m_{ij}=p)}{Q_{ik}(m_{ik}=l)-Q_{ik}(m_{ik}=p)} \geq 1$. Then choose v_i such that $\#\{v_{il}|v_{il} > v_{ij}\} = l-1$ and $\#\{v_{il}|v_{il} > v_{ik}\} = p-1$ and $v_{ik} > cv_{ij}$. Then, using the same argument as above, IC implies that

$$\begin{aligned} \frac{Q_{ij}(m_{ij}=l)-Q_{ij}(m_{ij}=p)}{Q_{ik}(m_{ik}=l)-Q_{ik}(m_{ik}=p)} &\geq \frac{v_{ik}}{v_{ij}} \\ &> \frac{cv_{ij}}{v_{ij}} \\ &= c \end{aligned}$$

which is a contradiction.

Sufficiency. Suppose 1 and 2 hold but the ordinal mechanism is not IC. Then there exists j, k such that $v_{ij} > v_{ik}$ and

$$Q_{ij}(m_{i1}=1)v_{i1} + \cdots + Q_{ij}(m_{ij}=l)v_{ij} + \cdots + Q_{ik}(m_{ik}=p)v_{ik} + \tag{4.10}$$

$$+Q_{im}(m_{im}=m)v_{im} < Q_{ij}(m_{i1}=1)v_{i1} + \tag{4.11}$$

$$+Q_{ij}(m_{ij}=p)v_{ij} + \cdots + Q_{ik}(m_{ik}=l)v_{ik} + \cdots + Q_{im}(m_{im}=m)v_{im} \tag{4.12}$$

which implies that

$$v_{ij}(Q_{ij}(m_{ij}=l) - Q_{ij}(m_{ij}=p)) < v_{ik}(Q_{ik}(m_{ik}=l) - Q_{ik}(m_{ik}=p))$$

given that 1 and 2 hold it must be that $v_{ij} < v_{ik}$ which is a contradiction. ∎

The first condition is a standard IC constraint that says a bidder will be willing to place each market in its proper rank only if doing so results in an increase in his probability of winning that object. The second condition constrains how the mechanism may vary across markets. The relative differences in Q between each rank must be the same in each market. Otherwise, there may be values for which the bidder would prefer to change his reported ranks.

The random assignment and serial dictator mechanisms are IC ordinal mechanisms. Pesendorfer (1996) suggests another ordinal mechanism that satisfies Bayesian incentive compatibility: the ranking mechanism. Each Bidder submits a report of his ranking of the markets.[12] Then, the bidder with the highest rank is selected in each market as the sole bidder in that market. If more than one bidder happens to report the same rank, then the sole bidder is chosen at random from those bidders. The example in Figure 4.1 is an illustration of such a mechanism. The ranking mechanism would select bidder 1 as the sole bidder in market A, either 1, 3, or 5 in market B, either 3, 4, or 5 in market C, 2 in market D, and 2, 4, or 5 in market E. Three features of the ranking mechanism are apparent. First, bidders can be selected as the sole bidder in more than one market. In this example, bidder 5 could potentially be selected as the bidder in three markets. It is possible that bidder 3 not be selected at all. Second, the sole bidder's rank can be very low. For example, in market E, the potential winning bidders' ranks are all 4, indicating that their valuations are likely to be quite low.

4.3.27 Example (The Ranking Mechanism)

$$q_{ij}(m) = \begin{cases} \frac{1}{k} & m_{ij} \leq m_{\ell j} \text{ for all } \ell \\ 0 & \text{otherwise} \end{cases}$$

where $k = \#\{\ell | m_{\ell j} = m_{ij}\}$ is the number of bidders who ranked market j the same as i. △

The reduced form probabilities for each bidder and each market are given by

$$Q_{ij}(m_{ij}) = \sum_{k=1}^{n} \left(\frac{1}{k}\right) \left(\frac{(n-1)!}{(k-1)!(n-k)!}\right) \left(\frac{1}{m}\right)^{k-1} \left(\frac{m - m_{ij}}{m}\right)^{n-k} \tag{4.13}$$

for $m_{ij} < m$, and for $m_{ij} = m$,

$$Q_{ij}(m) = \frac{1}{n}\left(\frac{1}{m}\right)^{n-1}.$$

[12]If one wishes to stick to the strict definition of a direct mechanism, imagine bidders submitting their valuations and some cartel centre ranking their values. Bidders are indifferent between reporting their true valuations and reporting any other order-preserving set of valuations.

Under the ranking mechanism, each agent's probability of being selected as the sole bidder in a particular market is independent of his ranks for the other markets. Obviously, the ranking mechanism satisfies incentive compatibility since the interim probability of being selected as the sole bidder is decreasing in the ranking. The probability that an individual is selected as the bidder in any particular market is simply the probability that no one ranked that market higher than he did, which is clearly decreasing in his ranking (for higher ranks (m_{ij}) each term in Equation 4.13 is smaller).

For a fixed number of bidders, Pesendorfer (1996) shows that expected efficiency converges to 100% as the number of markets increases.[13] The ranking mechanism will always select a bidder who ranked a particular market the highest as opposed to the serial dictator mechanism which may, due to the order of draws, select a bidder who does not have a high rank. Thus, in expectation, bidders' valuations should be higher. In fact, the ranking mechanism is an interim incentive efficient ordinal mechanism.

4.3.28 Theorem *If $\overline{v} > (n-1)\underline{v}$, the ranking mechanism is interim incentive efficient in the class of all anonymous ordinal mechanisms without side payments.*

Before showing the proof that the ranking mechanism is interim incentive efficient, some additional notation is in order. Since the ranking mechanism is assumed to be anonymous, it must be that $q_{ij}(m)$ is a function only of the number of individuals who have ranked each particular market in each spot. Thus, for simplicity let $E_j = \{(n_{1j}, n_{2j}, \ldots, n_{mj}) | \sum_{i=1}^{m} n_{ij} = n\}$ be the set of possible total ranks for a market where n_{ij} indicates that n_{ij} bidders ranked market j in their ith spot. Thus, $E = E_1 \times E_2 \times \cdots \times E_m$ is the set of possible events over which q_{ij} may vary. Let $\pi(e)$ be the probability that $e \in E$ occurs.

Proof: Suppose that there exists another ordinal mechanism such that $\sum_{j=1}^{m} Q'_{ij}(v_i)v_{ij} \geq \sum_{j=1}^{m} Q_{ij}(v_i)v_{ij}$ for all i and for all v_i Let v_i be such that $v_{i1} \geq v_{i2} \cdots \geq v_{im}$. Then it

[13]Bidder surplus as a percentage of the maximum possible surplus can be readily substituted for efficiency in these situations since bidders are essentially bidding zero which implies no seller's surplus.

must be that

$$Q'_{i1}(1)v_{i1} + Q'_{i2}(2)v_{i2} \ldots + Q'_{im}(m)v_{im} \geq Q(1)v_{i1} + Q(2)v_{i2} \ldots + Q(m)v_{im}$$

which implies that

$$(Q'_{i1}(1) - Q(1))v_{i1} + (Q'_{i2}(2) - Q(2))v_{i2} \ldots + (Q'_{im}(m) - Q(m))v_{im} \geq 0$$

This inequality implies that there exists a k such that $Q'_{ik}(k) \geq Q(k)$. If for all $k \leq m$, $Q'_{ik}(k) = Q(k)$ then for all i the outcome of Q' is identical to Q and they are equivalent mechanisms. On the other hand, let j be the first market such that $Q'_{ij}(j) > Q(j)$ Notice that,

$$Q'_{ij}(j) = \sum_{e \in E(j)} q'_{ij}(e)\pi(e) \tag{4.14}$$

$$Q(j) = \sum_{e \in E(j)} q_{ij}(e)\pi(e) \tag{4.15}$$

where $E(j) = \{e \in E | n_{jj} \geq 1\}$. Let $E(j)$ be partitioned into two sets: $E(j)_1 = \{e \in E(j) | q_{ij}(e) = 0\}$ and $E(j)_2 = \{e \in E(j) | q_{ij}(e) > 0\}$. In order for $Q'_{ij}(j) > Q(j)$ it must be that there exists a $e \in E$ such that $q'_{ij}(e) > q_{ij}$. Now, show, by cases, that an increase in q_{ij} for any event in either $E(j)_1$ or $E(j)_2$ will lead to a contradiction.

Case 1: Suppose there exists a $e \in E(j)_1$ such that $q'_{ij}(e) > q_{ij}(e)$.

Let $c = q'_{ij}(e)$ Then, since $q_{ij}(e) = 0$ it must be that there exists a $k < j$ such that $n_{kj} > 0$ or some other individual ranks the events lower than you. Let $k^* = \min\{k | n_{kj} > 0\}$. Under the ranking mechanism, it must be that $q_{ik^*}(e) = \frac{1}{n_{k^*}}$ and $q_{il}(e) = 0$ otherwise. Thus, since $q_{ij}(e) \geq 0$ for all j and $n_i q_{ik} = 1$ it must be that any change increase in $q'_{ij}(e)$ must come at a reduction in q_{ik}. Thus it must be that $q'_{ik} = \frac{1-n_j c}{n_k}$. Thus given the choice of j, it must be that

$$\left[\frac{1 - n_j c}{n_k} - \frac{1}{n_k}\right] \pi(e)v_{ik} + [c\pi(e)]v_{ij} \geq 0$$

which is true only if

$$v_{ij} \geq \frac{n_j}{n_k} v_{ik}$$

Notice that both n_j and n_k are greater than zero. This is only true for all $e \in E(j)_1$ if

$$v_{ij} \geq \frac{1}{n-1} v_{ik}$$

Let v_i be such that $v_{i1} = v_{i2} = \ldots = v_{i(j-1)} = \overline{v}$ and $v_{ij} = v_{i(j+1)} = \ldots = v_{im} = \underline{v}$. If q' is preferred to q it must be that $\underline{v} \geq \frac{1}{n-1}\overline{v}$. Since $\overline{v} > (n-1)\underline{v}$, it must be that $\underline{v} > \underline{v}$ which is a contradiction. Thus, Case 1 cannot hold.

Case 2: Suppose there exists a $e \in E(j)_2$ such that $q'_{ij}(e) > q_{ij}(e)$. If $e \in E(j)_2$, then it must be that $q_{ij}(e) = \frac{1}{n_j}$. Thus, it must be that $q'_{ij}(e) > \frac{1}{n_j}$ which violates feasibility of anonymous mechanisms (since this implies $q'_{kj}(e) \neq q'_{ij}(e)$ for some other individual who ranks the market in spot j). Thus, Case 2 cannot hold.

Thus, there cannot exist a q' such that it improves each agent's interim expected utility for all values. ∎

If $\underline{v} = 0$, then the condition on the support of the distribution is satisfied for all n. This suggests that, if a group of bidders are deciding on how to collude, they may very well want to pick the ranking mechanism since no other mechanism can do better for all possible values.[14]

There may exist other (non-ordinal) mechanisms which dominate the ranking mechanism. Since the noncooperative mechanism is not an ordinal mechanism, it is even possible that it may dominate the ranking mechanism. However, since the ranking mechanism dominates random assignment, Theorem 4.3.21 can be applied to the ranking mechanism: the ranking mechanism is not dominated by noncooperative bidding.

4.3.29 Corollary *For any collection of distributions, $(F_j)_{j=1}^m$, there exists a number*

[14]A similar result likely holds for asymmetric distributions and a slightly redefined ranking mechanism where the probability a bidder is assigned a market when there is a tie is dependent on his distribution of values in that market. However, the current version of the proof relies heavily upon the anonymity of the mechanism.

of bidders such that the ranking mechanism dominates the noncooperative mechanism.

4.3.2 Durability

Holmström and Myerson (1983) suggest a stronger standard that may be more logical when a decision rule is chosen in the interim stage. A mechanism is *durable* if the agents would never (for all possible draws of values) unanimously approve a change from that mechanism to another mechanism. Interim incentive efficiency guarantees that, from an ex ante perspective, the mechanism will not be blocked. However, once agents have observed their values, one agent (not *knowing* that the others will prefer a new mechanism) may propose a change which is unanimously accepted. Proposition 4.3.30 suggests that the ranking mechanism may not be durable. There are always distributions such that for some values all agents prefer the Bayes Nash equilibrium strategy to the ranking mechanism.

4.3.30 Proposition *For all n and m, there exist distributions such that for a set of values of positive measure the noncooperative is unanimously preferred to the ranking mechanism.*

Proof: Let $F_j(y) = (\frac{y-\underline{v}}{\overline{v}-\underline{v}})^{\frac{1}{2(n-1)}}$ for all j. Then the $n-1$ order statistic will always equal $F_j(y)^{n-1} = (\frac{y-\underline{v}}{\overline{v}-\underline{v}})^{\frac{1}{2}}$ for all j. This distribution is strictly concave thus there is always some large values where agents would prefer the noncooperative mechanism. Namely, whenever $v_{ij} > \frac{9}{16}\overline{v}$, for all j a bidder will prefer the noncooperative mechanism. ∎

This, however, does not mean that the ranking mechanism is not durable. Holmström and Myerson (1983) model durability by a specific voting game. It is necessary to consider what the bidders would learn if they unanimously approved a change to another mechanism. For example, if all bidders agreed to move from the ranking mechanism to the noncooperative mechanism, then each bidder could infer that everybody had high valuations. This updated information, however, would cause them to bid higher in the Bayes Nash equilibrium, making it a less attractive agreement. Consider an

Bidder

		1	2	3	4	5
	A	1	1	1	1	1
	B	2	2	2	2	2
Market	**C**	3	3	3	3	3
	D	4	4	4	4	4
	E	5	5	5	5	5

Figure 4.2: An Example that is Not Durable

example where $m = 1$ and $n = 2$. Let $v_i \in [0, 1]$ and $F_i(v) = v^{1/2}$. Both bidders will prefer the Bash Nash bidding only if $v_i > 9/16$. However, if the bidders condition their bids on the fact that their opponent has a value above 9/16, they will bid so high that they will no longer prefer noncooperative behavior.

Are these mechanisms durable in general? Holmström and Myerson show that if a mechanism is uniformly incentive compatible and interim incentive efficient, then it is durable.

4.3.31 Definition *A mechanism (β, s) is uniformly incentive compatible if for all v and for all $\hat{v}_i$*

$$u_i((\beta(v), s(v))|v) \geq u_i((\beta(\hat{v}_i, v_{-i}), s(\hat{v}_i, v_{-i}))|v).$$

Uniform IC is closer to strategy-proofness: an agent must not want to lie about his type even after he knows everyone else's types. Unfortunately, the ranking mechanism does not satisfy the sufficient conditions for a mechanism to be durable. Consider a truthful report of ranks as given in Figure 4.2. Having observed the other agents' reports, each agent's expected utility would be $\sum_{j=1}^{5} \frac{1}{5} v_{ij}$. However, as long as $v_{i2} > \frac{1}{4} v_{i1}$, agent i would prefer to 'flip' his reports for the first two markets. Doing so would allow him to capture v_{i2} for certain. The question of whether there are any durable collusive mechanisms without side payments remains open. However, since the serial dictator mechanism is strategy-proof, if it happens to be interim incentive efficient (in some environments), then the serial dictator mechanism will be durable.

4.3.3 Coalitional Deviations

Until this point, I have only examined the incentives for unilateral deviations and improvements for the *whole* group of bidders. Since the situation being modeled is assumed to be cooperative, it makes sense to allow for deviations by coalitions of agents. If one can find a collusive mechanism that is better than any coalition of agents can enforce, then the collusive agreement is considered to be stable and not taking advantage of one group of agents for the benefit of another.

In this setting the profitability of coalitional deviations is greatly affected by the assumption on what coalitions can do. In order to obtain some preliminary results, I make the following assumptions on coalitional behavior.

1. A coalition of any size can form and it can exclude new members from joining.

2. Coalition membership cannot be based upon private information.

3. The agents not in the coalition bid noncooperatively.

The first assumption gives power to coalitions. In some models of coalition formation with private information (such as the optimal taxation literature (Berliant 1992)) it is required that in addition to the coalition preferring its outcome, all members not in the coalition would not want to join the coalition. However, given that even simple mechanisms will often dominate noncooperative bidding, such a strong condition would almost surely mean death for any coalition not of size N. Assumption 2 requires that coalitions form at the ex ante stage. The coalition then picks a mechanism for itself in the interim stage. This assumption avoids the problem that for some draws of values a group of high valued bidders may want to split from low valued bidders. The justification is that, if a low valued bidder saw these bidders forming a coalition, he would want to claim to be a high valued bidder in order to join the group. Finally, 3 is similar to the α-core (Aumann 1961). If, instead, one was to assume that the $N \backslash S$ bidders formed their own collusive mechanism, the analysis would be greatly complicated (at least without specific knowledge of the optimal collusive mechanism). If a coalition of bidders use a mechanism that selects a single bidder for each market, then it will be optimal for that bidder to bid as if he were participating in an auction with $\#N\backslash S+1$ noncooperative bidders. Admittedly, many of these assumptions limit the power of coalitions, but they are a first step at understanding coalitional power in this setting.

Let $s = \#S$. Then, each bidder's interim expected utility given that he is in coalition S is given by

$$U_i(q|v_i, S) = \sum_{j=1}^{m} Q_{ij}(v_i|s)(v_{ij} - \beta^*_{ij}(v_{ij}|n-s+1))F_j(v_{ij})^{n-s} \tag{4.16}$$

where $\beta^*_{ij}(v_{ij}|n-s+1)$ is i's Bayes Nash equilibrium bid given that there are $n-s$ other *non-coalition* bidders. Equation 4.16 simplifies to

$$U_i(q|v_i, S) = \sum_{j=1}^{m} Q_{ij}(v_i|s) \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy.$$

Since coalitions are assumed to form in the ex ante stage, a coalition *blocks* a mechanism if it can find a feasible mechanism that always makes all members of the coalition better off.

4.3.32 Definition *A coalition S blocks a mechanism (β, s) if there exists a mechanism (β_S, s_S) such that for all i in S and all v_i,*

$$U_i((\beta_S, s_S)|v_i, S) \geq U_i((\beta, s)|v_i).$$

There are trade-offs involved with coalition formation. Small coalitions may want to form in order increase each bidder's chance of being selected as the sole bidder in a market. However, it is not optimal for any coalition of size smaller than n to bid zero in the auction: smaller coalitions mean that the coalition's bid in the auction must be higher. As long as F_j stochastically dominates the uniform distribution, coalitions will never find it in their interest to block using a random assignment mechanism.

4.3.33 Proposition *Let F_j first-order stochastically dominates the uniform distri-*

bution on $[\underline{v}, \overline{v}]$ for all j. Then there does not exist a coalition S such that the random assignment mechanism for S blocks either the random assignment, serial dictator, or ranking mechanisms.

Proof: We want to show that for all v_i

$$\sum_{j=1}^{m} \frac{1}{s} \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy \leq \sum_{j=1}^{m} \frac{1}{n}(v_{ij} - \underline{v}).$$

It is sufficient to show that for all j

$$\frac{1}{s} \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy \leq \frac{1}{n}(v_{ij} - \underline{v}).$$

Since F_j first-order stochastic dominates the uniform distribution, or

$$\frac{y - \underline{v}}{\overline{v} - \underline{v}} \geq F_j(y)$$

for all y in $[\underline{v}, \overline{v}]$ implying that

$$\begin{aligned}
\frac{1}{s} \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy &\leq \frac{1}{s} \int_{\underline{v}}^{v_{ij}} \left(\frac{y - \underline{v}}{\overline{v} - \underline{v}}\right)^{n-s} dy \\
&= \frac{(\overline{v} - \underline{v})}{s(n - s + 1)} \left(\frac{v_{ij} - \underline{v}}{\overline{v} - \underline{v}}\right)^{n-s+1} \\
&\leq \frac{(\overline{v} - \underline{v})}{n(n - s + 1)} \left(\frac{v_{ij} - \underline{v}}{\overline{v} - \underline{v}}\right) \\
&= \frac{v_{ij} - \underline{v}}{n(n - s + 1)} \\
&\leq \frac{v_{ij} - \underline{v}}{n}.
\end{aligned}$$

Since the serial dictator and ranking mechanisms dominate the random assignment mechanism, they too will not be blocked. ∎

The gains from increases in the probability of being selected as the bidder are more than offset by the increased bidding required to compete with the $n-s$ noncooperative bidders.

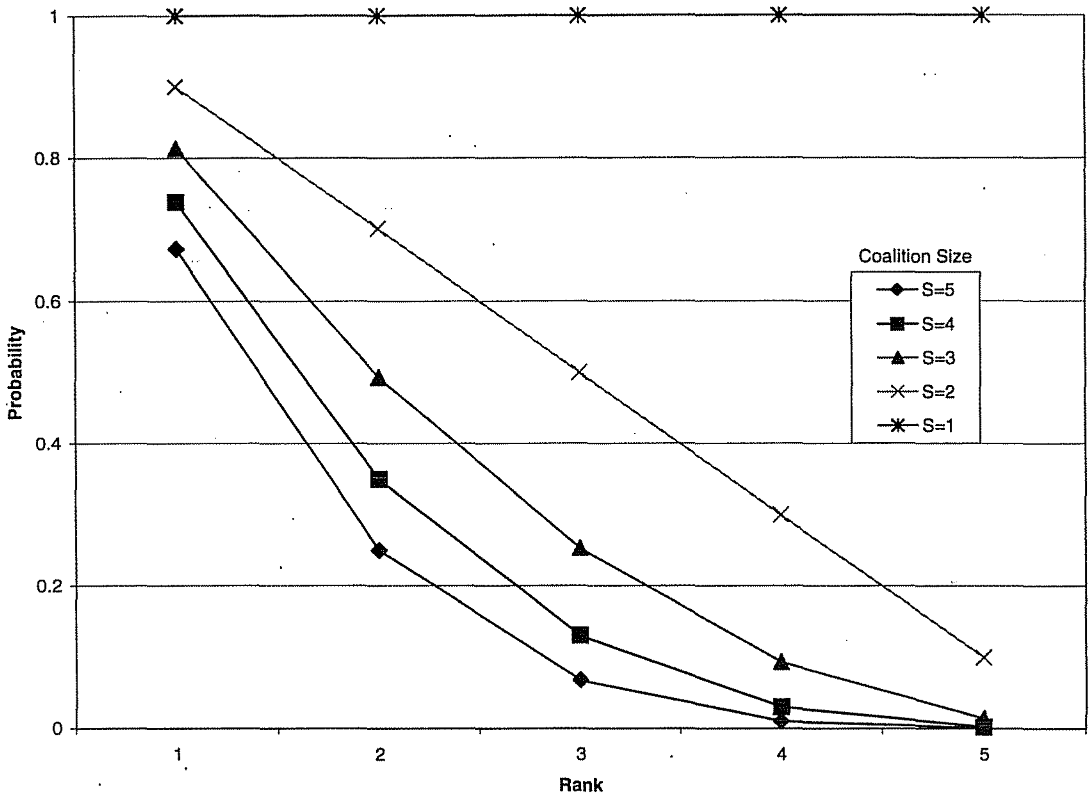

Figure 4.3: The Ranking Mechanism for $m = 5$ and $n = 5$

However, random assignment is not the best mechanism for a coalition. As in the case of the grand coalition (N), the ranking and serial dictator mechanisms will dominate random assignment for all coalitions. Likewise, the ranking mechanism is interim incentive efficient for each coalition. The reduced form probabilities under the ranking mechanism for 5 bidders and markets are illustrated in Figure 4.3. For all m_{ij}, the interim probability of being chosen as the sole bidder is higher ($Q_{ij}(m_{ij}|S) > Q_{ij}(m_{ij}|S+1)$). However, the increased probability is not enough to compensate for the increased bidding required. As long as $m \geq n$, coalitions will prefer to be larger.

4.3.34 Theorem *Let $m \geq n$ and F_j first-order stochastically dominate the uniform distribution on $[\underline{v}, \overline{v}]$ for all j. Then there does not exist an S that blocks the ranking mechanism.*

Proof: If we can establish that the ranking mechanism applied to any coalition S is dominated by the ranking mechanism for the grand coalition N, it will follow that no incentive compatible mechanism for coalition S will block. Let $Q_{ij}(m_{ij}|S)$ denote the reduced form probabilities of the ranking mechanism for S, and $Q_{ij}(m_{ij})$ the reduced form probabilities of the ranking mechanism for the grand coalition. We need to demonstrate that

$$\sum_{j=1}^{m} Q_{ij}(m_{ij}|S) \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy \leq \sum_{j=1}^{m} Q_{ij}(m_{ij})(v_{ij} - \underline{v}).$$

Using the same stochastic dominance arguments as in Proposition 4.3.33, it follows that for all j

$$Q_{ij}(m_{ij}|S) \int_{\underline{v}}^{v_{ij}} F_j(y)^{n-s} dy \leq \frac{Q_{ij}(m_{ij}|S)}{n-s+1}(v_{ij} - \underline{v}).$$

Therefore, it is sufficient to show that

$$\sum_{j=1}^{m} \frac{Q_{ij}(m_{ij}|S)}{n-s+1}(v_{ij} - \underline{v}) \leq \sum_{j=1}^{m} Q_{ij}(m_{ij})(v_{ij} - \underline{v}).$$

Note that by definition

$$\sum_{j=1}^{m} Q_{ij}(m_{ij}|S) = \frac{m}{s}$$

and

$$\sum_{j=1}^{m} Q_{ij}(m_{ij}) = \frac{m}{n}$$

implying that

$$\sum_{j=1}^{m} \frac{Q_{ij}(m_{ij}|S)}{n-s+1} \leq \sum_{j=1}^{m} Q_{ij}(m_{ij}).$$

Thus, the arguments of Lemma A.0.7 can be applied and it is sufficient to show that $\frac{Q_{ij}(1|S)}{n-s+1} \leq Q_{ij}(1)$. Show that $\frac{Q_{ij}(1|S)}{n-s+1}$ is increasing in s. Suppose not. Then for some s,

$$\frac{Q_{ij}(1|S)}{n-s+1} > \frac{Q_{ij}(1|S+1)}{n-s}.$$

Then,

$$\frac{(n-s)m(s+1)}{(n-s+1)s}\left[\sum_{k=1}^{s}\binom{s}{k}\left(\frac{(s-1)!}{(s-k)!(k-1)!)}\right)(m-1)^{s-k}\right]>$$
$$\left[\sum_{k=1}^{s+1}\binom{s+1}{k}\left(\frac{s!}{(s-k+1)!(k-1)!)}\right)(m-1)^{s-k+1}\right]$$

implying that

$$\left[\sum_{k=1}^{s}\left[\left(\frac{1}{k}\right)\left(\frac{(s-1)!}{(s-k)!(k-1)!)}\right)(m-1)^{s-k}\right]\right.$$
$$\left.\left(\left(\frac{n-s}{n-s+1}\right)m(s+1)-\frac{(s+1)m(m-1)}{s+1-k}\right)\right]-1>0$$

The only chance for this statement to be true is if for some $k=1,\ldots,s$

$$\left(\left(\frac{n-s}{n-s+1}\right)m(s+1)-\frac{(s+1)m(m-1)}{s+1-k}\right)>0.$$

Since this term is decreasing in k, let $k=1$ and simplify to obtain

$$(s+1)\left[\left(\frac{n-s}{n-s+1}\right)m-(m-1)\right]>0$$

which is only true if the bracketed term is positive. Noticing that this term is decreasing in s, set $s=1$ to obtain

$$\begin{aligned}\left(\frac{n-1}{n}\right)m-m+1 &> 0\\ \frac{-m}{n} &> -1\\ n &> m\end{aligned}$$

which is a contradiction with the assumption that $m\geq n$. ■

Figure 4.4 shows the reduced form probabilities divided by $n-s+1$ for an example with five bidders and five markets. Since the ranking mechanism for coalition S is not dominated by any other feasible, ordinal, incentive compatible mechanism for S,

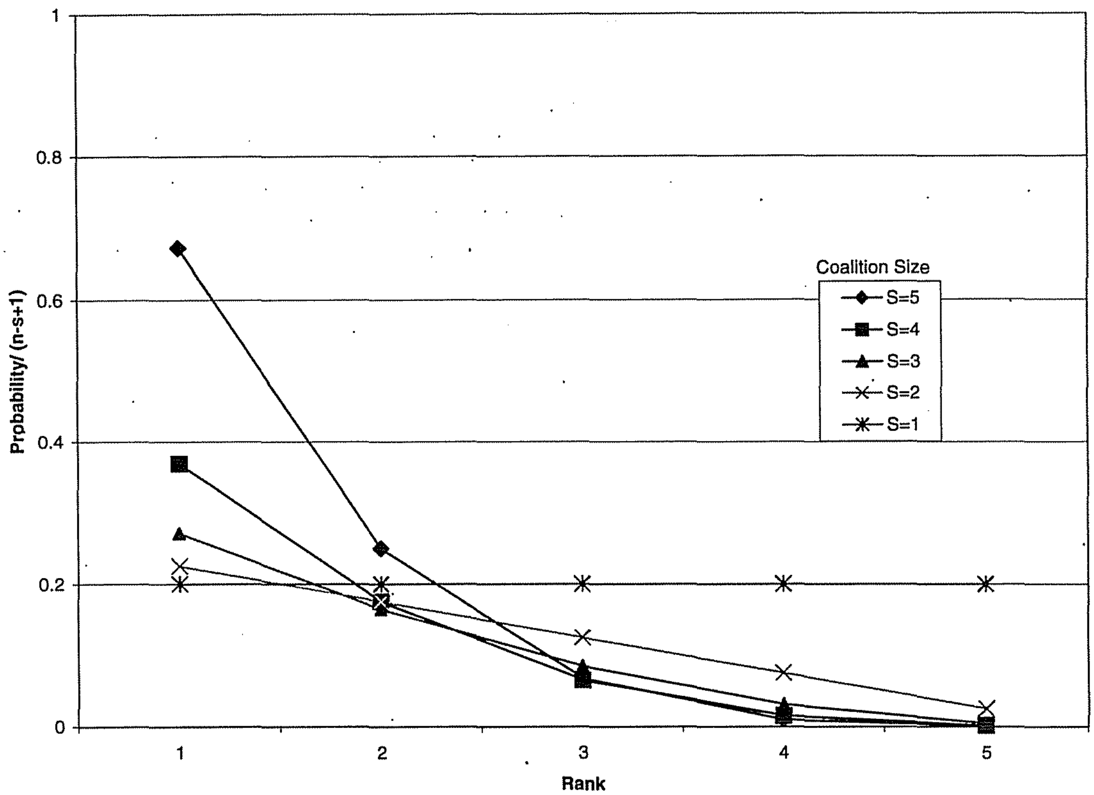

Figure 4.4: $Q_{ij}(m_{ij}|S)/(n-s+1)$ for $m=5$ and $n=5$

it must be the case that the ranking mechanism for N is not blocked by any other IC mechanism for S. It remains to be shown whether a similar result will hold for $m < n$. At least for a limited class of environments, there will be no profitable coalitional deviations.

4.3.4 Private Measurability

In Chapters 2 and 3 incentive problems were largely ignored in favor of examining the efficacy of communication restrictions upon the choices of agents. This chapter has focused on the incentive compatibility constraint and ignored communication restrictions. Is there a connection? In the previous chapters a private measurability restriction has played an important role in describing behavior. Koutsougeras and Yannelis (1993) have shown that private measurability implies incentive compatibility (but not vice versa). However, measurability restrictions have only been utilized in exchange economies. The first step is to define private measurability in an environment such as this. Let $f_i(v|v_i)$ be i's conditional density given v_i.

4.3.35 Definition *A mechanism is privately measurable (in outcomes) if for all* i *and for all* $v, \hat{v} \in V$ *such that* $f_i(v|v_i) > 0$ *and* $f_i(\hat{v}|v_i) > 0$*, then* $(\beta_i(v), s_i(v)) = (\beta_i(\hat{v}), s_i(\hat{v}))$.

Having observed v_i agent i cannot distinguish between v and $\hat{v}$. Thus, his actions should not vary across these states. Assuming that a mechanism must be privately measurable is similar to an assumption that the mechanism be *decentralized.* Mechanism design theory typically assumes a designer (or centre) who takes each agent's reported information and aggregates that information and proposes an action. Under that system, the players may appear to act as if they have more information than any one individual has. Private measurability assumes that agents cannot act through such a mediator. Therefore, their actions can only depend on the information available to themselves at the time that they make a decision (Vohra 1997).

Private measurability can greatly restrict the set of feasible mechanisms in many environments. In general, decision rules cannot be assumed to take on *private* elements (i.e., $(\beta_i(v), s_i(v)) = (\beta_j(v), s_j(v))$).[15] However, in private value auctions, private measurability simply implies that each bidder's specific element be only a function of their own values.

4.3.36 Proposition *Let* v *and* $\hat{v}$ *be such that* $v_i = \hat{v}_i$*. Then* $(\beta_i(v), s_i(v)) = (\beta_i(\hat{v}), s_i(\hat{v}))$.

Proof: By the independent private value assumption, $f_i(v|v_i) > 0$ and $f_i(\hat{v}|v_i) > 0$. Thus, $(\beta_i(v), s_i(v)) = (\beta_i(\hat{v}), s_i(\hat{v}))$. ∎

Many of the mechanisms discussed here can be implemented as privately measurable mechanisms. The random assignment mechanism is obviously privately measurable since it does not depend upon v. The noncooperative mechanism is also privately measurable: each bidder's bids in the auction are only a function of his values in the

[15] When this is true, and agents' beliefs have full support, the only privately measurable mechanisms will be constant mechanisms. Or, random assignment in the auction setting.

markets. Finally, the ranking mechanism can be operated in a privately measurable method. While the probabilities (q) that were used to describe the ranking mechanism depend upon all bidders' values, the ranking mechanism can be translated into a privately measurable bidding function in the auction. Let $\epsilon > 0$. The following bid function is privately measurable:

$$\beta_i j(v_i) = (m - m_{ij})\epsilon + \underline{v}.$$

If s bidders rank the market the same, then they will place the same bid in the auction. If they happen to have the highest rank, then they will have the highest bid, and they can rely on the auctioneer to perform the randomization. Thus, an interim incentive efficient mechanism can be devised without any explicit coordination from a centre.

4.3.5 Summary

Clearly, collusive agreements will most likely involve selecting a sole bidder to bid in each market. When side payments are allowed, an ex post efficient mechanism exists. However, with no side payments, ex post efficiency cannot be achieved. The fact that bidders are bidding on multiple objects allows them to choose a collusive agreement that yields higher expected surplus (and efficiency) than the best IC mechanism in the single object case (random assignment). The serial dictator and ranking mechanisms are two ordinal mechanisms which interim dominate the random assignment mechanism. This is only a partial analysis of the outcomes of collusive behavior in the multiple object setting. There remain many unanswered questions. For example, what is the full characterization of interim incentive efficient mechanisms? Also, what is the impact of communication and repeated play on the choice of strategies?

While the ranking mechanism is an interim incentive efficient mechanism (in the class of ordinal mechanisms), the serial dictator mechanism may have an advantage due to its simplicity. The structure of the serial dictator mechanism is similar to a typical description of a bidder *ring* in which each bidder takes a turn (in a ring) picking what he wants to bid on (Cassady 1967). However, this intuitive simplicity is at the cost of expected efficiency. Both these rotation schemes interim dominate the random assignment mechanism. On the other hand, the random assignment mechanism would be extremely simple for a group of bidders to utilize and monitor. An experimental examination of this mechanism design problem will give some initial insight into this trade-off between efficiency and simplicity.

4.4 Experimental Design

In the previous section, it was shown that different forms of collusive strategies could be used in multiple object simultaneous first-price auctions. A few strategies highlighted as possible choices by bidders are:

- Competitive bidding,
- Reduced bidding,
- Random assignment, and
- Rotation schemes (serial dictator or ranking).

The theory suggests that some of these mechanisms will most likely be preferred to others. For example, both the particular rotation schemes examined, the serial dictator and ranking mechanism, interim dominate the random assignment mechanism. Reduced bidding agreements are generally only IC if they yield the same profitability as competitive bidding.

In the analysis of Section 4.3, some possible collusive mechanisms are discussed given the assumption that bidders have agreed to cooperate. Will bidders actually decide to form cooperative agreements? In this vein, the experimental literature on cooperative behavior provides some initial insights. As is the case in prisoners' dilemma or public goods experiments, there are incentives for participants to coordinate their behavior to increase their overall payoffs. However, each participant also has an incentive to defect from any cooperative agreement. While only Isaac and

Walker (1985) examine collusive behavior in sealed bid auctions,[16] numerous other experimental studies have highlighted three factors that appear to affect the ability of groups to cooperate:

1. Communication,

2. Repeated play, and

3. Institutional structure.

In general, participants cannot form successful cooperative agreements unless they are given an opportunity to communicate and coordinate their strategies. Isaac, McCue, and Plott (1985) found that allowing communication in a public goods experiment led to a small but stable increase in the amount contributed to the public good. Daughety and Forsythe (1987) found that, with written communication, experimental subjects made choices closer to the collusive optimum. In addition, the method by which communication is allowed appears to be important. For example, Palfrey and Rosenthal (1991) found that in a public goods experiment, where binary signals were the only form of communication allowed, the resultant behavior was no more efficient, despite the fact that participants conditioned their behavior heavily on the signals. This suggests that the more extensive the communication that is allowed, the more likely it is that stable, cooperative outcomes will be observed. The psychology literature has focused on the ability of group discussion to change individual choices (Pruitt 1971). Numerous psychological factors can play important roles in the ability of a group discussion to lead to outcomes that are preferred by the group but may be contrary to individual incentives (i.e., providing a public good or participating in a cartel).

Repeated interaction appears to be a significant factor in the effectiveness of cooperation. If participants meet only one time, there is little incentive to choose a cooperative outcome. However, cooperative choices can be supported in repeated settings through the use of trigger strategies or Tit-for-Tat type behavior. Selten

[16]Kagel and Roth (1995) describe a series of in-class experiments that Kagel conducted with common values.

and Stoecker (1986) report a significant end-game effect in which participants tend to defect from cooperative agreements when they know the end of the experimental session is near. Palfrey and Rosenthal (1994) compare games in which participants in a public goods experiment are repeatedly matched with different individuals to games in which participants repeatedly interact with the same individual. They find that contributions increase slightly under the repeated treatment. Andreoni and Miller (1993) find a similar result in prisoner's dilemma experiments.

The institutional structure of the environment can drastically affect the level of cooperation observed. The best example of such a contrast is the difference in the effectiveness of collusion in double auction, posted-offer, and sealed bid auction institutions. Isaac, Ramey, and Williams (1984) and Clauser and Plott (1993) report that collusive agreements are more successful when sellers can place posted offers. In the double auction environment, in which each participant can change the current offer at any time, collusive efforts almost always break down. However, Isaac and Walker (1985) show that collusive agreements are relatively stable in sealed bid first-price auctions. In 7 out of 10 experiments, stable collusive agreements developed. One explanation for the contrast in the success of collusion under these various institutions is that in the double auction there is a continuous incentive to defect from the cooperative agreement. However, in both the posted-offer and sealed bid auctions, participants only make a single, binding decision; if they do not deviate when making that decision, it is impossible for them to deviate until the next period. Recent experimental evidence suggests, however, that this problem may not be present in one-sided auctions. Sherstyuk (1998, 1999) found that in ascending auctions bidders can often learn to coordinate their bids in order to collude. However, in her experiments, Sherstyuk increased the set of (Bayes) Nash equilibria to include these collusive outcomes by allowing for tie bids to be placed. In addition to communication and repeated play, the overall susceptibility of the underlying economic environment to cooperation should also be considered when determining the likelihood of cooperative results.

The multiple unit simultaneous sealed bid auction combines all of the above factors to create a situation that is conducive to cooperative outcomes. First, bidders are allowed to verbally communicate. Second, bidders repeatedly interact with the same individuals and, in most cases, do not know when the experiment will end.[17] Finally, the institution is an extension of the sealed bid auctions studied by Isaac and Walker (1985), which are susceptible to collusion.

With this previous experimental work in mind, stable and successful cooperative agreements are expected to form. However, participants can choose among many different cooperative strategies that vary significantly in their relative sophistication. Palfrey and Rosenthal (1994) note that, despite the increase in contributions from repeated play, participants fail to use sophisticated and more profitable strategies. However, when bidders are allowed to communicate, Isaac and Walker (1985) find that some groups attempt to use more sophisticated strategies where the bidder with the highest value is picked. The primary objective of this experimental study is to determine what types of strategies bidders are actually using in this environment. Also, as shown in Section 4.3, when side payments are not allowed these strategies are not expected to lead to ex post efficient auctions. The choice of collusive mechanism will affect the final efficiency of the auction as well as the surplus of both the bidders and the seller. In order to provide a better understanding of collusive agreements in first-price auctions, a series of laboratory experiments was designed that allowed for observation of bidders' choice of collusive mechanism.

In each experiment, five bidders participated in five simultaneous single unit first-price auctions, in which five objects were sold. An experimental design with the same number of bidders as objects was chosen for two reasons. First, I expect cooperative agreements to be more successful here (as opposed to a setting with fewer objects). Since my interest is primarily in the observation of cooperative strategies, such a design should maximize the number of observations. Second, when the number of bidders and objects is the same, bidders may utilize a relatively simple but

[17]Two experiments were conducted in which the final period was announced in order to test the end-game effect.

less profitable strategy of assigning (ex ante) one bidder to each market. In the first five periods of each session no communication was allowed. Then, in the next 12–17 periods subjects were allowed to communicate between each period. In general, subjects were undergraduate students at the California Institute of Technology. However, a few graduate students and staff members were participants. Each subject participated in only one experiment. Instructions can be found in Appendix B. The simultaneous first-price auctions were implemented using auction software designed by Wes Boudeville and Dave Porter.

Bidders were required to place a bid of at least one experimental dollar (franc) in each market.[18] This restriction ensured that subjects were unable to monitor adherence to collusive agreements via the sound of computer keys being hit indicating the submission of bids. Also, this allowed the experimenter to easily determine when all the bids had been placed. If ties occurred in the highest bids, the computer software randomized between the high bidders to determine the winner.

4.4.1 Symmetry

In the symmetric environment, valuations for all five markets and bidders were drawn from the same distribution. Integer values between 1 and 1000 were drawn using the discrete uniform distribution. Under the assumption that bidders are risk neutral, the unique, symmetric Bayes Nash equilibrium bid function is:

$$b_{ij}(v_{ij}) = .8v_{ij} \quad \text{for all } i, j.$$

Since the bid functions are symmetric and strictly monotonic, under competitive bidding the auction is expected to be ex post efficient.[19]

[18]The conversion rate of francs to dollars was either 250 or 500. Thus, a minimum bid of 1 franc was generally a trivial amount.

[19]An auction is ex post efficient if the winning bidder has the highest valuation for the object.

4.4.2 Asymmetry

In the asymmetric environment, valuations for four of the markets for each bidder were drawn from the same discrete uniform distribution with values between 1 and 1000. In the fifth market, valuations were drawn from a first-order stochastic dominant distribution, $F(v) = \frac{v^2}{1000^2}$, taking values between 1 and 1000 as well.[20] In each market, one bidder had a valuation drawn from this preferred distribution. The identity of that bidder was announced to all participants.

The asymmetric environment was used in order to give bidders a stronger incentive to use an inefficient but simple cooperative strategy: assign sole bidding rights to the bidder with the preferred distribution in each market. The expected efficiency of such a strategy is 80% (as opposed to 60% under symmetry) but there exist more sophisticated strategies which dominate it.[21]

When bidders are behaving noncooperatively, the Bayes Nash equilibrium bid function can be estimated numerically. Figure 4.5 is a plot of the estimated bid functions for each market when bidders have values drawn from the above distributions.[22] If bidder 1 has values drawn from the stochastically dominant distribution, $b_1(v) \leq b_i(v)$ for all other i and for all valuations. Thus, competitive bidding will not necessarily lead to full efficiency. However, in this case, the expected efficiency of competitive bidding is extremely close to 100% (at 99.983%).

4.4.3 Communication

After the fifth period it was announced that communication would be allowed between bidders. The following statement was handed out and read to subjects, who were then allowed to ask questions.

Communication with Other Participants

[20]The discrete analog to this distribution was actually used.

[21]We say one strategy dominates another if for all possible valuation draws all agents prefer that strategy.

[22]BIDCOMP2, a program developed by John Riley, was used to estimate these bid functions.

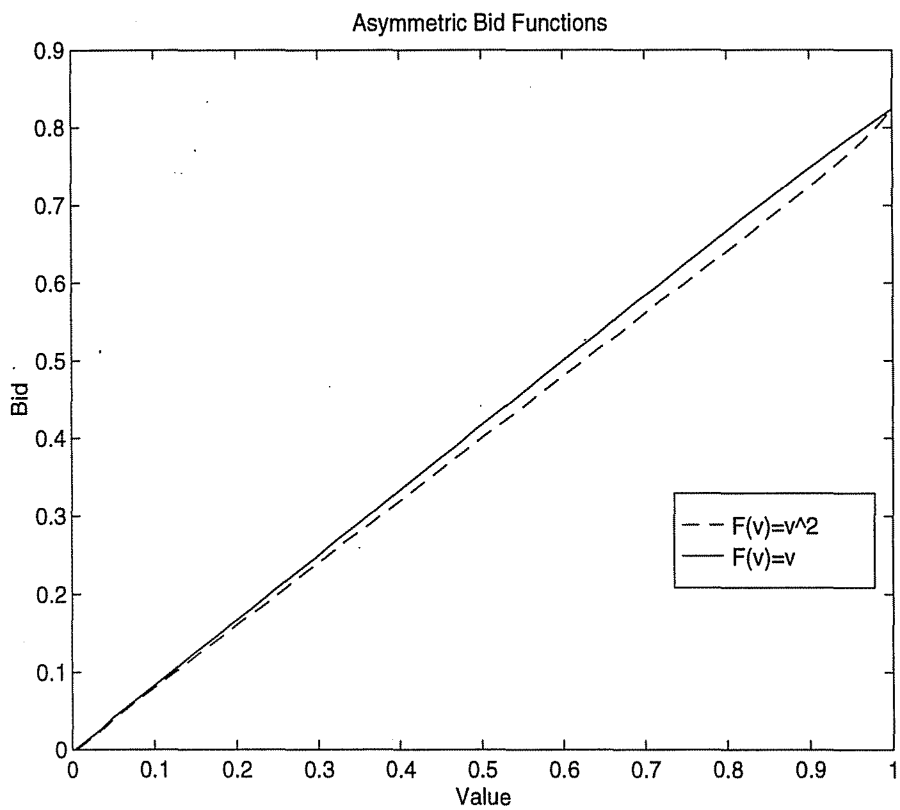

Figure 4.5: Asymmetric Bid Functions

Sometimes in previous experiments, participants have found it useful when the opportunity arose, to communicate with one another. You are going to be allowed this opportunity while the computers are reset between periods.

There will be some restrictions.

You are free to discuss any aspects of the experiment (or the market) that you wish, except that:

- You may not discuss any quantitative aspects of the private information on your value sheets.
- You are not allowed to discuss side payments or to use physical threats.

Since there are still some restrictions on your communications with one another, an experimenter will monitor your discussion between periods. To make this easier, all discussions will be at this site.

Remember, after the computers have been reset between periods (and the next period has begun) there will be no discussion until after the end of the next period.

We will allow a maximum of 4 minutes in any one discussion session.

In addition to these instructions, in every experiment except the first, subjects were also told that the number of rounds had been fixed. This announcement assured participants that lengthy communication would not reduce profits via a reduced number of periods. In most experiments, subjects had no problem understanding the limitations of their communication and only occasional reminders (or clarifications about the form of acceptable information) were required.

4.4.4 Information Conditions

The *limited information* environment was the most restrictive information condition utilized by Isaac and Walker (1985). The only information available to participants was the identity of the winning bidders and the prices they paid. A second, more limited, information condition not used by Isaac and Walker (1985) is the *zero information* condition which reports only the winning bids to the bidders. The identity of the winning bidder in each auction is unknown to everyone except the winner. Under the zero information condition, the participants can only determine who had placed winning bids through voluntary discussion. The increased difficulty in identifying and punishing deviant bidders was expected to make the zero information condition less conducive to cooperative behavior.

4.4.5 End of Experiment Changes

In order to determine whether communication or repeated play were important factors in the success of collusive agreements, two changes at the end of 5 of the 10 experiments were implemented. The first change was intended to determine the value of repeated play in this environment. Since it was not practical to conduct experiments in which cartel members did not repeatedly interact as in Palfrey and Rosenthal (1994), the end-game effect (EG) was studied (Selten and Stoecker 1986). At the end of experiments six and seven, it was announced that one more period would be conducted. In this final period, communication was allowed but otherwise complete anonymity was induced. Bidders drew their values randomly from a set of five envelopes. The identity of the winning bidders and their exact earnings were unknown to the experimenter and the other subjects.[23]

The second treatment was designed to demonstrate the importance of communication. In experiments 8 through 10, subjects were told at the beginning of their discussion for period 18 that it would be the last period of discussion (the experiment lasted for five periods beyond that).[24] Isaac and Walker (1988) and Daughety and Forsythe (1987) report that, while cooperation is greater with prior communication (PC) than with no communication, once communication ends the level of cooperation

[23]A third party not involved with the experiments paid the subjects for that period by placing their earnings in envelopes marked with an ID known only to the bidder.

[24]In experiment 10, discussion was ended after period 17 and 6 periods without communication were completed.

Exp.	Number of Periods	Environment	Information Condition
1	20	Symmetric	Limited
2	22	Symmetric	Limited
3	22	Symmetric	Limited
4	22	Symmetric	Limited
5	20	Asymmetric	Limited
6	20	Asymmetric	Limited
7	20	Asymmetric	Zero
8	18	Asymmetric	Zero
9	18	Symmetric	Zero
10	17	Symmetric	Zero

Table 4.1: Experimental Design

tends to gradually erode.[25]

Both of these changes were made near the end of the experimental session and subjects did not have any a priori knowledge of these treatments. Thus, observations of cooperative agreements in earlier periods should not be affected by either the EG or PC treatment.

4.5 Experiment Results

Ten experiments were completed with six experiments utilizing the symmetric environment and four using asymmetric valuation draws. Six experiments were conducted under the limited information setting; four experiments used the more limited zero information condition. A general summary of the experiments can be found in Table 4.1.

Subject earnings averaged $33.75 across all experiments. No experimental session

[25]Actually Isaac and Walker (1988) found that in 3 out of 4 experiments in their first experimental series no participants defected from the collusive agreement after communication was ended. However, in their second set of experiments, contributions declined in 11 of 17 experiments.

lasted longer than two hours, with the average length closer to one hour and thirty minutes. There was no significant variance of subject profits between and within periods.[26]

The behavior of the bidders in the first five periods of the auction, when communication was not allowed, was similar to previously observed results. Cox, Smith, and Walker (1988) found that bidders often place bids above the risk neutral Nash equilibrium prediction. However, for extremely low valuations, where bidders have little chance of winning, bidders typically place extremely low bids (often 0). The estimation of a simple linear regression on the bids placed in the auctions with symmetric valuations demonstrates these results. Estimating the linear regression of $b_i = \beta_1 + \beta_2 v_i + \epsilon$ for each bidder should lead to estimates of $\hat{\beta}_1 = 0$ and $\hat{\beta}_2 = .8$ if bidders are playing the risk neutral Nash equilibrium. (Cox, Smith, and Walker 1988) found that for many bidders $\hat{\beta}_1 < 0$ and $\hat{\beta}_2 > .8$. We found similar behavior in our experiments: 17 out of 30 (57%) subjects exhibited $\hat{\beta}_1 < 0$ and 16 out of 30 (53%) exhibited $\hat{\beta}_2 > .8$.

4.5.1 Do Bidders Form Cooperative Agreements?

The results of Isaac and Walker (1985) suggest that successful cooperation is expected here. A significant drop in bidding prices is one indicator of collusive behavior. The average bid in periods with communication drops to near zero. While the average bid in no communication periods was 428 francs, it was only 9.6 francs in communication periods. However, a reduction in bid levels is not necessarily an indicator of profitable collusive behavior; Isaac, Ramey, and Williams (1984) found that while prices increased when communication was allowed in posted-offer markets, profits did not necessarily increase. Isaac and Walker (1985) use an index of monopoly effectiveness (M), which is the proportion of maximum total possible surplus captured by the

[26]However, there was significant variation of profits across experiments due to the choice of cooperative strategy.

bidders:

$$M = \frac{\sum_{j=1}^{5} v_j^* - b_j^*}{\sum_{j=1}^{5} \max_i v_{ij}},$$

where v_j^* is the valuation of the winning bidder in market j and b_j^* is his bid. In these experiments, M increases from an average of .265 in no communication periods to .912 when communication is allowed. Bidders capture a significantly large proportion of the total surplus available. Perhaps the strongest evidence of successful cooperative behavior is that, despite a change in the conversion rate from 250 francs per dollar to 500 francs per dollar, average bidder per period profits rose from \$ .93 to \$ 1.51. In comparison, if the bidders were placing bids consistent with the risk neutral Nash equilibrium, they would have earned \$.33 on average in the communication periods.

1 Conclusion *When communication is allowed, under both environments and information conditions, collusive agreements are formed and are stable.*

Few deviations from collusive agreements were evident in the ten experiments. In early periods, bidders occasionally placed bids that were not in line with the collusive agreement. Excluding the first two periods of communication, there were only three out of 129 periods in which bidders made notable deviations from the cooperative agreements. In contrast to Isaac and Walker (1985), where collusion occasionally broke down, there is no evidence of sustained deviations in these experiments.[27]

Given the apparent strength of collusive agreements, the two changes mentioned in Section 4.4.5 were made to try to gain an insight into the source of the strength of these ties. Under the EG treatment, 9 out of 10 subjects did not deviate from the collusive agreement; only one bidder in experiment seven deviated.[28] This seems to indicate that even in one shot environments such collusive agreements are fairly stable. Thus, repeated play is not a particularly important factor in the success of cooperation in this setting.

[27] The graphs of bidders' surplus in Appendix C demonstrate the consistency of the cooperative agreements.

[28] That bidder placed a bid out of line with the collusive agreement in only one market.

However, a second change indicated weakness in collusive agreements. Three experiments were conducted with the PC treatment. As expected, in all three experiments, the bidders formulated an agreement on how to collude when discussion was not allowed. However, bidders were quick to deviate from their ex ante agreements. In the first period of no communication, one bidder deviated in every experiment (see Figure 4.6). The number of deviations typically increased and most bidders began to bid more aggressively. In one experiment, by the last period four of the five bidders placed bids roughly in line with competitive bidding. However, in the other two experiments, a few bidders were typically able to take advantage of the optimistic behavior of the other bidders. All in all, 12 of 15 bidders placed bids that were significantly different than the ex ante agreement reached by the group. Bidder surplus as a percentage of maximum total surplus dropped from 87.88% in the communication periods to 80.64% in the no communication periods.[29]

2 Conclusion *Communication is more important than repeated play in fostering successful collusive agreements.*

These results indicate that one of the most important features of such collusive agreements is the ability to discuss the outcomes and make after plans every period. A possible explanation is the need for the cartel to coordinate punishment strategies at the end of each period.

4.5.2 What Types of Strategies Do Bidders Utilize?

Closer examination of the periods in which communication was allowed reveals heterogeniety in the choice of cooperative strategies between some experimental sessions. Two distinct strategies can be discerned from the data and observation of preplay communication. The first, and most common strategy, was the utilization of bid rotation. Bid rotation strategies can be characterized by the selection of one bidder as the sole bidder in each auction. This bidder placed a low bid greater than 1 franc

[29] The null hypothesis that the mean surplus from the communication periods is less than or equal to the mean surplus with communication can be rejected at a 90% level of confidence by a rank sum test ($z = 1.317$).

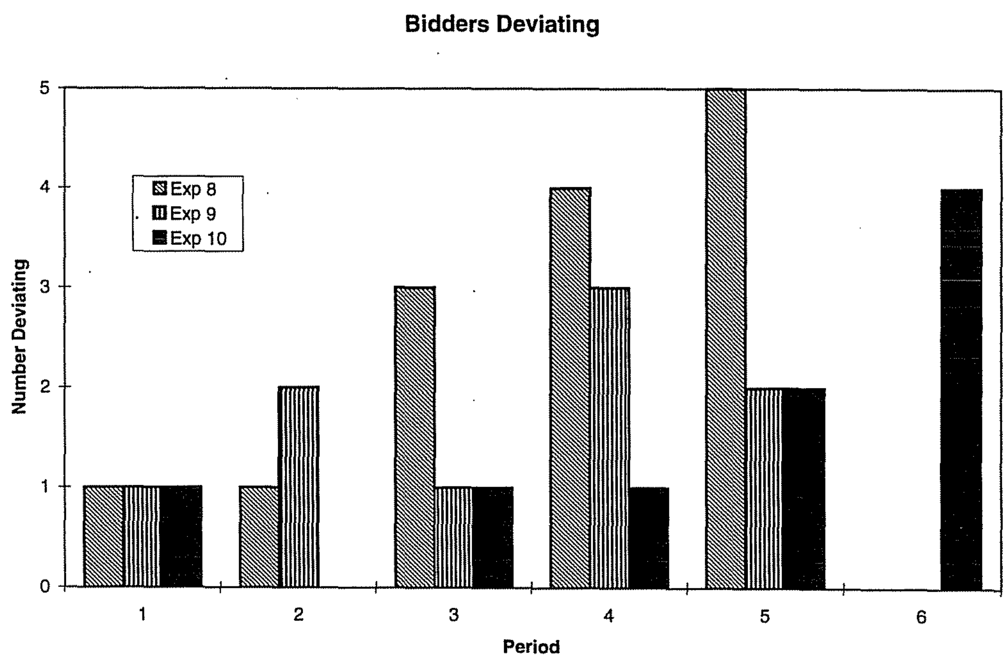

Figure 4.6: Bidders Deviating when No Communication was Allowed

(typically 2–5 francs) while all other bidders bid 1 franc in the auction. A second strategy observed in the data was a reduced bidding agreement. This strategy entails the agreement by all bidders to place bids which are linear transformations of their actual valuations. Since bidders are required to submit whole franc bids of at least 1 franc, reduced bidding will, in general, lead to higher average bids than bid rotation.[30]

3 Conclusion *In 7 out of 10 experiments, bidders used a bid rotation strategy. In experiments where bid rotation was not used, bidders used a reduced bidding strategy.*

The easiest method for discerning these two different strategies was observation of preplay communication. In the 7 experiments in which bid rotation was used, bidders attempted to reach some resolution of who would bid in each market. However, in the 3 bid reduction schemes, bidders determined a level of bidding. The difference between these experiments can also be seen in the level of bids placed. In the 7 rotation experiments, the average bid placed was 2.8 francs. In the reduced bidding experiments, the average bid was 23 francs.

Reduced Bidding

In two of the reduced bidding experiments, the cartel agreed to place bids that were 1% of redemption values.[31] In the other, bidders agreed to place bids that were 10% of valuations. Such agreements violate individual incentive compatibility (Section 4.3). Only an agreement to bid 80% of valuations is incentive compatible. Since their values are not ex post verifiable, bidders can increase their bids beyond either the 1% or 10% level without detection, and increase their probability of winning the object. Therefore, bidders would be expected to bid higher than their particular reduced bidding agreement dictates. Figure 4.7 shows the deviations from the agreed upon strategy. A deviation of zero indicates that the bidder placed his bid at the whole number nearest either 1% (for experiments 1 and 4) or 10% (for experiment 3) of his

[30] Bid rotation strategies lead to average bids that are close to 1 franc since all bidders except one bid 1 franc.

[31] Bidders in experiment 4 quickly switched from a 10% rule to a 1% rule after two periods.

	Experiment		
	1	3	4
Mean Deviation	-0.06481	-0.00774	0.167219
Std. Error	0.131955	0.076929	0.286164
Observations	375	425	425

Table 4.2: Mean Deviations from Reduced Bidding Agreement

value:

$$\text{Deviation} = \frac{\text{Actual Bid}}{\text{Predicted Bid}}.$$

A deviation of 100% indicates that the bidder placed a bid double that predicted by the particular linear bid reduction rule. In all three experiments the null hypothesis that the mean deviation is equal to zero is rejected at the 95% confidence level. Surprisingly, however, in two of the experiments, mean deviations are significantly below zero implying bidders were actually bidding below the agreement. Only in one experiment were deviations significantly above zero (See Table 4.2).

4 Conclusion *Bidders choose linear bid reduction strategies that are not incentive compatible. However, bidders rarely significantly misreport their redemption values.*

The fact that these reduced bidding agreements are replicated and appear to be relatively stable creates problems for the theory. Why did bidders not shade their bids up in two experiments? Bidders seem to ignore individual incentives, despite the fact that detection of placing higher bids is very difficult. Section 4.5.3 provides one possible explanation for the choice of this strategy.

Bid Rotation

The majority of the experiment sessions (7 out of 10) lead to bidding strategies that were classified as bid rotation agreements. There are many different mechanisms which are incentive compatible and look like bid rotation outcomes. The choice of mechanism by the group has significant implications for efficiency and thus the per-

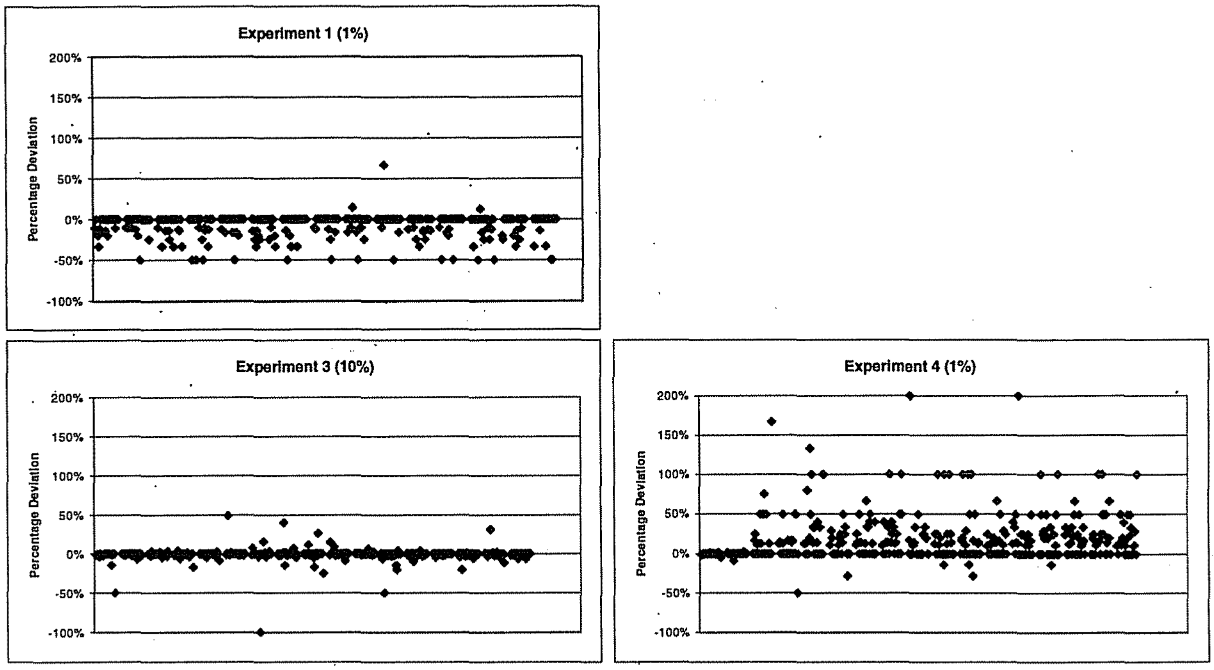

Figure 4.7: Deviations from Reduced Bidding Agreements

centage of maximum total surplus captured by the bidders. Four behavioral strategies which can lead to outcomes similar to those observed in these seven experiments are:

1. Ranking mechanism (R),
2. Serial dictator mechanism (SD),
3. Random assignment mechanism (A), and
4. Perfect information (P).

The ranking (R), serial dictator (SD), and random assignment (A) mechanisms were discussed in Section 4.3. The perfect information (P) strategy describes the possibility that bidders may perfectly collude by somehow determining the bidder with the highest valuation in each market.[32] The objective is to determine which of these

[32]This is a highly unexpected outcome given the limitations on bidder communications. However, it is still possible that this may be the *best* predictor of group behavior.

possible mechanisms was most likely utilized in each of these experiments. Three techniques that shed light on the choice of a strategy by bidders are:

1. Observation of preplay discussion,

2. Comparison of expected efficiencies with observed efficiencies, and

3. Comparison of predicted market division with observed choices.

Discussion

While observing bidder discussion is not a rigorous test for the predominance of one model over the other, simply listening to the conversations of the bidders can provide a great deal of insight into the intentions of the bidders. Bidder discussion was typically closer to the ranking mechanism than to the serial dictator mechanism. In most cases, bidders would begin their discussion by naming what they wanted first (their highest rank). If there was no conflict, discussion ended. If there was disagreement, those who had chosen conflicting markets would attempt to reach a compromise by naming their next best market. It is easy to see that such an iterative procedure leads to outcomes predicted by the ranking mechanism under the restriction that no bidder be chosen more than once. If the group discussion was consistent with the serial dictator mechanism, once a bidder had named a market in which he wished to bid, no other bidder could pick that market. Typically, conversation between bidders did not take this form.[33]

Efficiencies

An auction is efficient if the winner of each object is the bidder with the highest valuation. Efficiency is denoted by

$$\text{Efficiency} = \frac{\sum_{j=1}^{5} v_j^*}{\sum_{j=1}^{5} \max_i v_{ij}}.$$

[33]However, it is possible that there may have been some first-mover advantage; the bidder who made his announcement of preferred markets first got his favored market more often. Since the order of discussion was not recorded, this factor cannot be analyzed for these experiments.

Behavioral	**Predicted Efficiency**	
Strategy	Symmetry	Asymmetry
R	90.60%	92.12%
SD	85.20%	86.63%
A	60.00%	80.00%
P	100.0%	100.0%

Table 4.3: Predicted Efficiencies

	Experiment						
	2	5	6	7	8	9	10
Mean Efficiency	90.52%	92.70%	90.48%	92.77%	87.94%	89.94%	90.69%
Std. Error	0.0466	0.0545	0.0985	0.0755	0.1100	0.0729	0.0841
Observations	17	15	15	15	13	13	12

Table 4.4: Mean Efficiencies – Rotation

The predicted efficiencies for each of the behavioral strategies in this particular setting are given in Table 4.3. If a group is utilizing a particular mechanism, the average of the observed efficiencies should converge to the above efficiencies. The null hypothesis that the mean efficiency for each experiment was different than 90.60%, for the symmetric environment, and 92.12%, for the asymmetric environment, predicted by the ranking mechanism cannot be rejected at a 95% level of confidence in any of the seven experiments (See Table 4.4). However, in five of the seven experiments, the null hypothesis that the mean efficiency was equal to that predicted by the serial dictator mechanism (85.20% and 86.63%) can be rejected at a 95% level of confidence.[34] The observed efficiencies are also significantly different from the 60% and 80% predicted by an assignment mechanism. The perfect information model can also be rejected under this test in all seven experiments. A simple comparison of observed results seems to strongly favor the ranking mechanism as the best determinant of behavior in each of the seven experiments.

[34] Comparison of the mean bidder surplus yields similar results since bids placed are close to zero.

Comparing the Choices

An analysis of bidder discussion and efficiencies provides some support for the ranking mechanism. However, analysis of discussion is purely ad hoc and relies upon the judgment of the experimenter who observed the experimental session. Comparison of mean observations utilizes outcomes rather than choices.

A more rigorous test involves comparing the choices of the bidders to the choices predicted by each model. Initial examination of choices in each particular experiment indicates that the ranking mechanism is a good predictor of choices; 87% of all observed choices are consistent with the ranking mechanism. However, other mechanisms also correlate well with the observed choices. The likelihood-based classification procedure of El-Gamal and Grether (1995) provides a more rigorous statistical comparison of all the proposed models. Let $C_t = \{(c_1, c_2, c_3, c_4, c_5) | c_i \in \mathbb{Z}, 1 \leq c_i \leq 5,\ i = 1, 2, 3, 4, 5\}$ be the class of behavior rules for each period such that each bidder is selected as the sole bidder in a particular market. For example, $c_1 = 2$ indicates that bidder 2 was selected as the sole bidder in market A. Each model predicts a subset $B_t \subset C_t$ and $B = B_1 \times B_2 \times \cdots \times B_{p_s}$ where p_s is the number of periods completed in an experiment. Each experimental session is treated as a single subject, s, and it is assumed that each s chooses exactly one behavioral strategy. The error probability, ϵ, is assumed to be the same for all individuals, experimental sessions and choices. The choice by individual i in period t for a particular experimental session is denoted by a_{ti}. Then, for all B, let

$$x^s_{B,ti} = \begin{cases} 1 & a_{ti} \in B_t \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$X^s_B = \sum_{i=1}^{n} \sum_{t=1}^{p_s} x^s_{B,ti}$$

be the total number of choices predicted correctly for a particular session. The likelihood can be found to be

$$f^{B,\epsilon}(x^s) = \left(1 - \frac{\epsilon}{3}\right)^{X_B^s} \left(\frac{2\epsilon}{3}\right)^{p_s n - X_B^s}$$

for each behavioral strategy.[35] Under the assumption that participants in all S experiments are using the same mechanism, the maximum likelihood estimate is given by

$$(\hat{B}, \hat{\epsilon}) = \arg\max_{B,\epsilon} \prod_{s=1}^{S} f^{B,\epsilon}(x^s).$$

The algorithm suggested by El-Gamal and Grether (1995) is used to obtain the maximum likelihood estimate for any set of k behavioral strategies. Then, using a penalty function given by

$$g(k) = k\ln(4) + k\ln(3) + S\ln(k),$$

k is chosen to maximize the information criterion,

$$IC(k) = \ln\left(\prod_{s=1}^{S} \max_{h\in\{1,\ldots,k\}} f^{\hat{B},\hat{\epsilon}}(x^s)\right) - g(k).$$

Using this technique, I can test the ability of the four possible mechanisms to explain the observed choices by each experimental session. The choices of the ranking mechanism (R) are easily characterized by saying that an error was made in a particular market if the bidder chosen was not the individual with the highest rank in that market. Unfortunately, the serial dictator mechanism (SD) cannot be characterized as easily. Each possible permutation of the five bidders can potentially lead to a different choice of market assignment predicted by the mechanism. Almost any observed choice can be predicted by the mechanism. For any particular experiment the number of possible combinations of choices across periods is 120^{p_s} (which is 8.92×10^{24} in the experiment with the fewest periods). The choices predicted by the serial dic-

[35]It is assumed here that, if a bidder made an error, he chose the correct strategy with probability one-third and another strategy with probability two-thirds. In reality, a bidder could have a choice of between 5 (if he happens to be choosing first or if there is little conflict) to 1 (if he is choosing last or there is a great deal of conflict) markets. Since, on average, he will have a choice of three markets, $(\frac{1}{3}, \frac{2}{3})$ is selected as an approximation.

	Experiment						
	2	**5**	**6**	**7**	**8**	**9**	**10**
R	85.88%	90.67%	85.33%	93.33%	78.46%	89.23%	85.00%
SD	84.71%	65.33%	62.67%	64.00%	86.15%	86.15%	86.67%
A	29.41%	49.33%	40.00%	42.67%	33.85%	23.08%	30.00%
P	56.47%	58.67%	61.33%	60.00%	50.77%	53.85%	60.00%

Table 4.5: Percentage of Choices Explained by Models – Individual Experiments

tator mechanism are limited to a smaller set. It is assumed that each experimental group agrees to rotate the order of selection in each period. Thus, if the order of choosing was 1,2,3,4,5 in period t then it would be 2,3,4,5,1 in period $t + 1$. This limits the number of combinations predicted by the serial dictator mechanism to a more manageable 120 combinations. While limiting the serial dictator mechanism in this manner makes it less likely that it will be classified as the best fitting model, it is reasonable to assume that no individual bidder would approve of any combination that did not evenly spread out the right to pick early since early picking leads to higher individual surplus. The assignment mechanism (A) assumes that each bidder is selected as the sole bidder in his favored market when distributions are not symmetric. Thus, bidder 1 is assumed to always be the sole bidder in market A, bidder 2 in B, bidder 3 in C, bidder 4 in D, and bidder 5 in E. Finally, the perfect information model (P) represents the choices that would be made if the bidders were able to actually aggregate their information perfectly. The bidder with the highest value is picked in each market.

Table 4.5 presents the data for each experiment. In all experiments, the ranking and serial dictator mechanisms better explain the data than either random assignment or perfect information. Table 4.6 reports the results of the maximization of the information criterion to determine the optimal number of rules to choose. Using two rules best explains the choices observed in the seven experiments. In experiments 2, 5, 6, 7 and 9, the ranking mechanism is the behavioral strategy that best fits the experimental data. However, the serial dictator mechanism significantly adds to the

No. of Models	Rule(s) Chosen	No. Classified	$\hat{\epsilon}$	$g(k)$	IC
1	R	435	0.39	2.485	-203.101
2	**R,SD**	**333,108**	**0.354**	**9.822**	**-197.893**
3	R,SD,*	333,108,0	0.354	15.145	-203.216
4	R,SD,*,*	333,108,0,0	0.354	19.644	-207.715

Table 4.6: Estimated Models

		Information	
		Limited	**Zero**
Values	**Symmetric**	3 Reduced 1 Rotation	2 Rotation
	Asymmetric	2 Rotation	2 Rotation

Table 4.7: The Effect of Treatments

explanatory power of the model in experiments 8 and 10. Using this classification procedure, it is possible to rule out the random assignment model of collusive behavior. Also, bidders were apparently unable to perfectly aggregate information. However, the serial dictator mechanism cannot be eliminated.

5 Conclusion *The ranking mechanism is the best description of behavior in the rotation scheme experiments. However, the serial dictator mechanism cannot be ruled out in some experiments.*

The combination of these three methods of determining which bid rotation scheme was used gives strong evidence in favor of the ranking mechanism. The serial dictator mechanism, however, still appears to be a strategy which is used occasionally by groups in this setting, especially in experiment 8, in which both the observed efficiency and the choices of markets correlate well with the serial dictator mechanism.

6 Conclusion *Reduced bidding mechanisms are only observed under the limited information and symmetric environments.*

All three instances of utilization of reduced bidding strategies were in experiments in which bidders had uniform valuation draws in all five markets and were informed of the identity of the winning bidders (Table 4.7). While Isaac and Walker (1985) found no significant patterns between collusive agreements and their two information conditions of full information and limited information,[36] this result demonstrates that information matters. While it may not be significant in determining whether bidders collude, it does alter their choice of strategy. Bidders seem to be less willing to select a strategy which violates incentive constraints when they have less ex post information. Second, the switch to a less cooperative strategy in the asymmetric environment has some precedence. Isaac and Walker (1988) found that asymmetries in public goods experiments tended to decrease the level of voluntary contributions. While a complete breakdown of cooperation is never evident here, this result suggests that bidders' choices of strategies are affected by the environment.

4.5.3 What Effect Do Different Strategies Have on the Outcome of the Auction?

The choice of cooperative strategies can drastically affect the results of the auction. The differences between mechanisms can best be seen by examining the efficiency of the auction and the amount of surplus accruing to the bidders.

Efficiency

Despite the apparent problems with enforceability, reduced bidding agreements have advantages from a social welfare standpoint. In the three experiments which exhibited these collusive agreements, average efficiencies were 99.26%, 99.38%, and 98.36%. A rank sum test shows that the mean efficiency for these experiments is significantly different than the mean efficiency of experiments in which bidders used rotation schemes.[37]

7 Conclusion *Reduced bidding yields higher average efficiency than bid rotation.*

[36] The full information condition was a less restrictive environment which reported all the bids placed in the auction.

[37] The Wilcoxon-Mann-Whitney test with correction for ties yielded $z = 8.267$, which is greater than any reasonable critical value of the standard normal distribution.

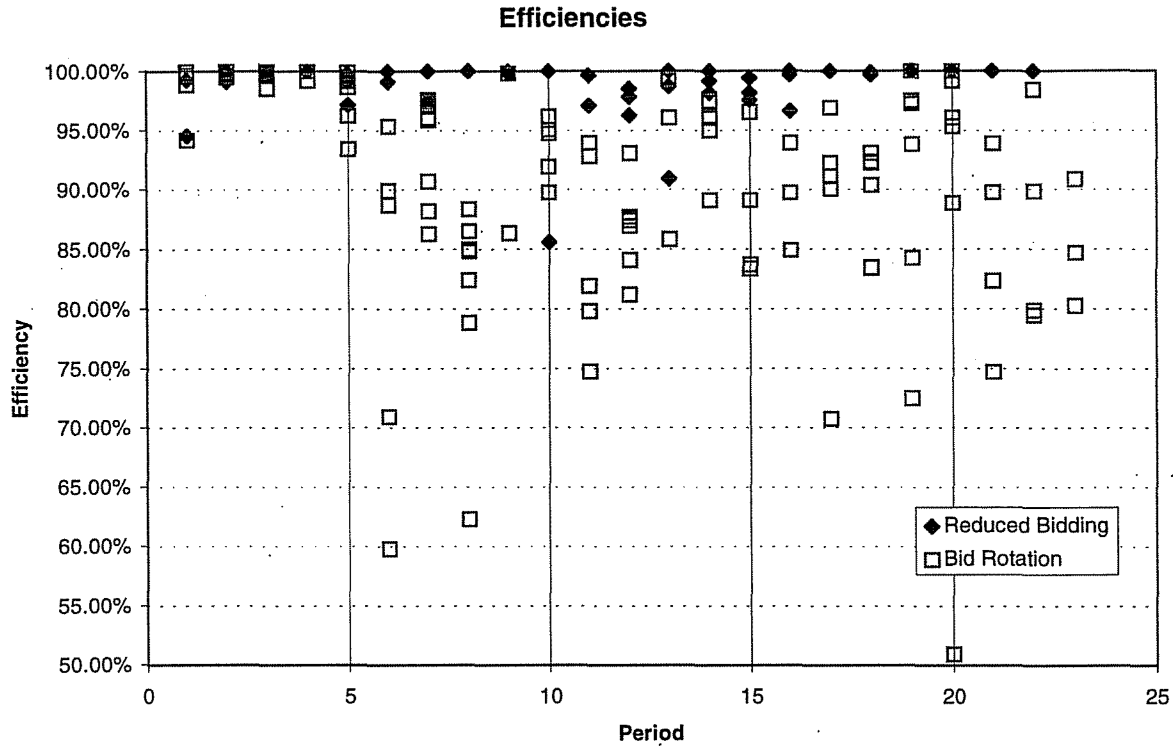

Figure 4.8: Reduced Bidding v. Rotation Efficiencies

This result is due to the ability of reduced bidding to select the highest bidder (assuming people do not deviate from the agreement). Figure 4.8 shows the efficiencies for the experiments in which reduced bidding was observed and the efficiencies for the experiments in which bid rotation was observed. Efficiency is also fairly stable under the reduced bidding agreements. When bidders are using rotation schemes, efficiency varies significantly due to the imprecision of the ranks. However, the reduced bidding agreement consistently yields efficiencies near 100%. The variance of the observed efficiencies for the seven non-reduced bidding experiments was always higher than the variance for the three reduced bidding experiments.

Bidder Surplus

The overall level of profitability for the bidders is best described by the index of monopoly effectiveness which reports the proportion of total possible surplus captured by the bidders. In experiments where bidders used rotation schemes, the average M was 0.898. The two 1% bid reduction experiments yielded an average effectiveness of

0.970, and the 10% bid reduction experiment averaged $M = 0.885$.

8 Conclusion *The index of monopoly effectiveness is highest for the bidders under the 1% reduced bidding rule.*

The 1% reduced bidding agreement was the most successful (profitable) collusive agreement. This result highlights the apparent trade-offs between these strategies. If bidders do not lie about their values, reduced bidding yields a much higher efficiency than rotation schemes. This increase in the size of the available surplus more than accounts for the increased level of bids required by a 1% agreement. The 10% agreement, on the other hand, entails too high a level of bidding to actually increase profitability over rotation schemes.

These two conclusions are the best argument in favor of a reduced bidding mechanism. Bidders select reduced bidding because it is more profitable than rotation mechanisms, despite the fact that it is not consistent with individual incentives. It appears that something in the nature of communication in the group decision making process allowed the bidders to ignore this problem.

4.6 Conclusion

The two primary contributions of this chapter are:

1. A description of possible collusive mechanisms when the number of objects is greater than 1,

2. An analysis of experimental data to identify the strategies chosen by bidders.

While incentive compatibility constraints severely limit the set of possible cooperative strategies in single unit auctions, there are many more sophisticated and profitable possible mechanisms when multiple objects are being auctioned simultaneously. Rotation schemes take advantage of bidders' willingness to trade-off probability of winning in lesser valued markets in return for an increased probability in higher valued markets. Of all rotation schemes that use only ordinal information, the ranking mechanism is interim incentive efficient and provides outcomes strictly preferred to those that are possible when only one item is being auctioned (random assignment).

Previous experimental investigations of cooperation in a wide variety of settings (auctions, markets, public goods, prisoners' dilemma) have almost solely focused on the formation of cooperative agreements. It has been well established that, in environments similar to the auction environment discussed here, experimental subjects will agree to cooperate. In this chapter, I examine the choice of cooperative strategies. Bidders can choose from a variety of strategies (including noncooperative behavior) that vary significantly in their complexity, profitability, and adherence to incentive constraints. Subjects exhibit behavior which is often consistent with strategies predicted by theory.

However, deviations in three of the experimental sessions from the choices predicted by theory suggest that a better theory of the cooperative choice of a decision rule needs to be formulated. In these experiments, bidders used a strategy that is not incentive compatible, but leads to higher profits when bidders do not lie about their values. The theory developed here assumes that bidders do not voluntarily communicate and that any information that is used must be consistent with their incentives. However, if, a priori, bidders could agree to credibly reveal their information, then reduced bidding agreements become possible. A complete theory of the choice of strategies when bidders are asymmetrically informed will treat the level of communication as an additional choice variable. Wilson (1978) proposes versions of interim efficiency that assume different levels of information sharing (coarse and fine). Interim efficiency is a very weak standard on the strategies chosen. Potentially, there are many interim efficient mechanisms. Since the behavior being modeled is explicitly cooperative, a more cooperative solution concept is in order. In many domains that concept is the core. However, finding core allocations in this setting is more difficult. For example, the feasible set of strategies for each coalition depends upon the actions of those outside the coalition, and the question of information sharing within coalitions becomes relevant. Myerson (1984) provides some initial insights by defining threat points as minimal levels of expected utility that each coalition must receive.

The inclusion of durability may also lead to a more satisfying theory. If a mechanism is not durable, then there will be instances in which bidders will reject it in favor of another mechanism. This behavior might be observable experimentally. Is there evidence of a move away from one mechanism based upon the values drawn? In terms of collusion, durability might even predict when collusion breaks down. In this experimental design, the random assignment, serial dictator, and ranking mechanisms dominate the noncooperative mechanism (due to the choice of distributions).

Finally, the auctioneer was assumed to be completely passive. In reality, the auctioneer can take steps to combat collusive behavior. Graham and Marshall (1987) highlight some techniques that the auctioneer may use in an English auction. In sealed bid auctions, the use of a reserve price becomes even more important for the auctioneer to earn revenue. Collusive strategies are also easily identifiable by a lack of bidding. If the auctioneer can punish collusive behavior, then bidders may need to formulate agreements that are less obvious. A full understanding of collusion in auctions requires an analysis of the steps an auctioneer can take to combat collusion.

Appendix A Lemmas

A.0.1 Lemma *Let $\mathcal{F}^*$ be a σ-field such that $\mathcal{F}^* = \sigma(\bigcup_{i \in N} \mathcal{F}_i)$. If a function $f : (X, \mathcal{F}_i) \to (Y, \mathcal{B}_Y)$ is $\mathcal{F}_i$ measurable for some $i \in N$, then it is also $\mathcal{F}^*$ measurable.*

A.0.2 Lemma *$\varphi(\omega)$ is closed.*

Proof: Let $\{p_n\}$ be a sequence of linear functionals in $\varphi(\omega)$ such that $p_n \to p$. Suppose $p \notin \varphi(\omega)$ or $p \cdot z(\omega) < p \cdot e(\omega)$ for some $z \in \beta(\omega)$. Then, $p \cdot (e(\omega) - z(\omega)) > 0$. Let $x = e(\omega) - z(\omega)$ and let $c \in \mathbb{R}_{++}$ such that $p \cdot x = c$. Let $\epsilon = \frac{c}{2 \cdot \|x\|} > 0$. Since $p_n \to p$ there exists a N such that for all $n' > N$,

$$\begin{aligned}
\| p_{n'} - p \| &< \epsilon \\
\| p_{n'} - p \| \cdot \| x \| &< \epsilon \cdot \| x \| \\
\| p_{n'} \cdot x - p \cdot x \| &< \epsilon \cdot \| x \| \\
c &< \epsilon \cdot \| x \| \\
c &< \frac{c}{2}
\end{aligned}$$

which is a contradiction. ∎

The *lower inverse* of a correspondence is defined by

$$\varphi^{\ell}(A) = \{\omega \in \Omega \mid \varphi(\omega) \cap A \neq \emptyset\}.$$

A correspondence is said to be *weakly measurable* if for all open subsets G of Y, $\varphi^{\ell}(G) \in \mathcal{F}$.

A.0.3 Lemma *φ is weakly measurable.*

Proof: Let G be an open set of Y'. If there does not exist an $\omega \in \Omega$ such that $\varphi(\omega) \cap G \neq \emptyset$ then $\varphi^{\ell}(G) = \emptyset \in \mathcal{F}$. Let ω be such that $\varphi(\omega) \cap G \neq \emptyset$. Then for all

$\omega' \in F(\omega)$, $\varphi(\omega') \cap G \neq \emptyset$, since $e(\omega) = e(\omega')$ and $\beta(\omega) = \beta(\omega')$. Thus, $F(\omega) \subseteq \varphi^{\ell}(G)$ and $\varphi^{\ell}(G) = \cup_{\omega' s.t. \varphi(\omega') \cap G \neq \emptyset} F(\omega') \in \mathfrak{F}$ since it is the union of at most countable many distinct atoms. ∎

A.0.4 Lemma *Let $(Q,T) = \{(Q_i, T_i)\}_{i=1}^{n}$ and $(Q',T') = \{(Q'_i, T'_i)\}_{i=1}^{n}$ be two generic, feasible, IC mechanisms. Then for all $\alpha \in [0,1]$, $(\alpha Q + (1-\alpha)Q', \alpha T + (1-\alpha)T')$ is also a feasible, IC mechanism.*

Proof: Since (Q,T) and (Q',T') are IC

$$\sum_{j=1}^{m} Q_{ij}(v_i)v_{ij} - T_{ij}(v_i) \geq \sum_{j=1}^{m} Q_{ij}(\overline{v}_i)v_{ij} - T_{ij}(\overline{v}_i) \ \forall \overline{v} \ \forall i \tag{A.1}$$

$$\sum_{j=1}^{m} Q'_{ij}(v_i)v_{ij} - T'_{ij}(v_i) \geq \sum_{j=1}^{m} Q'_{ij}(\overline{v}_i)v_{ij} - T'_{ij}(\overline{v}_i) \ \forall \overline{v} \ \forall i \tag{A.2}$$

which implies that

$$\begin{aligned} &\alpha \left(\sum_{j=1}^{m} Q_{ij}(v_i)v_{ij} - T_{ij}(v_i) \right) + (1-\alpha) \left(\sum_{j=1}^{m} Q'_{ij}(v_i)v_{ij} - T'_{ij}(v_i) \right) \geq \\ &\alpha \left(\sum_{j=1}^{m} Q_{ij}(\overline{v}_i)\overline{v}_{ij} - T_{ij}(\overline{v}_i) \right) + (1-\alpha) \left(\sum_{j=1}^{m} Q'_{ij}(\overline{v}_i)\overline{v}_{ij} - T'_{ij}(\overline{v}_i) \right) \end{aligned} \tag{A.3}$$

Rearranging and bringing the α inside the sum yields the desired result

$$\begin{aligned} &\sum_{j=1}^{m} (\alpha Q_{ij}(v_i) + (1-\alpha)Q'_{ij}(v_i))v_{ij} - (\alpha T_{ij}(v_i) + (1-\alpha)T'_{ij}(v_i)) \geq \\ &\sum_{j=1}^{m} (\alpha Q_{ij}(\overline{v}_i) + (1-\alpha)Q'_{ij}(\overline{v}_i))\overline{v}_{ij}) - (\alpha T_{ij}(\overline{v}_i) + (1-\alpha)T'_{ij}(\overline{v}_i)) \end{aligned} \tag{A.4}$$

Feasibility follows by simply allowing each agent to report their types v_i and using a public randomization device to choose (Q,T) with probability α and (Q',T') with probability $(1-\alpha)$. ∎

A.0.5 Lemma *If $F_{ij} = F_{kj}$ for all i, j, k and $(Q, T) = \{(Q_i, T_i)\}_{i=1}^n$ is feasible and*

$$\omega = \sum_{i=1}^{n} \frac{1}{n} \int (\lambda(v_i)(Q_i(v_i)v_i - T_i(v_i)))dF_i(v_i)$$

where λ is a social welfare weight on types, Q_i and T_i are $j \times 1$ vectors and F_i is the joint distribution of the j values of each agent. Then there exists $(\hat{Q}, \hat{T})$ such that $\hat{Q}_i = \hat{Q}_k$ and $\hat{T}_i = \hat{T}_k$ for all i, k and

$$\int (\lambda(v)\hat{Q}(v)v - \hat{T}(v))dF(v) = \omega$$

Also, if (Q, T) is IC then $(\hat{Q}, \hat{T})$ is IC as well.

Proof: Since values for all individuals are drawn from identical distributions and utilities are of an identical form, if (Q, T) is feasible and IC then for all $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ one-to-one (permutations) $\{Q_{\sigma(i)}, T_{\sigma(i)}\}_{i=1}^n$ is also feasible and IC. By Lemma A.0.4, every mechanism in the convex hull of all permutations of (Q, T) is feasible and IC. Let $(\hat{Q}, \hat{T})$ be the mechanism created by the convex combination of all $n!$ permutations of (Q, T) equally weighted by $\frac{1}{n!}$. Thus, since each (Q_i, T_i) appears exactly $(n-1)!$ times, $\hat{Q} = \frac{1}{n}\sum_{i=1}^n Q_i$ and $\hat{T} = \frac{1}{n}\sum_{i=1}^n T_i$. Thus, given λ, we have that

$$\begin{aligned}
\int (\lambda(v)(\hat{Q}(v)v - \hat{T}(v)))dF(v) &= \int (\lambda(v)(\frac{1}{n}\sum_{i=1}^{n} Q_i(v)v - \frac{1}{n}\sum_{i=1}^{n} T_i(v)))dF(v) \\
&= \sum_{i=1}^{n} \frac{1}{n} \int (\lambda(v)(Q_i(v)v - T_i(v)))dF_i(v) \\
&= \omega \qquad \text{(A.5)}
\end{aligned}$$

Thus, $(\hat{Q}, \hat{T})$ is symmetric, feasible and IC and leads to the same ex ante social value.

∎

Proof: Let the Reduced Bidding mechanism be IC. Then given the first order conditions for maximization of each agent's expected utility, it must be that

$$g_{ij}(v_{ij})v_{ij}(1-\alpha_j) - G_{ij}(v_{ij})\alpha_j = 0 \tag{A.6}$$

$$\Gamma_{ij}(v_{ij}) = v_{ij}\frac{(1-\alpha_j)}{\alpha_j} \tag{A.7}$$

Since this must be true for all $v_{ij} \in [\underline{v}, \overline{v}]$, differentiating with respect to v_{ij} yields

$$\frac{d\Gamma_{ij}(v_{ij})}{dv_{ij}} = \frac{1-\alpha_j}{\alpha_j} = c_j.$$

Obviously, the other direction can be trivially shown to hold by setting $\alpha_j = \frac{1}{\frac{d\Gamma_{ij}(v_{ij})}{dv_{ij}}+1}$. ∎

A.0.6 Lemma *Let $F(y)$ be any continuous distribution on $[\underline{v}, \overline{v}]$. There exists a N^* such that $F(y)^{N^*}$ first-order stochastic dominates the uniform distribution on $[\underline{v}, \overline{v}]$.*

Proof: It is sufficient to show that for some N^*, $F(y)^{N^*} \leq \frac{y-\underline{v}}{\overline{v}-\underline{v}}$ for all $y \in [\underline{v}, \overline{v}]$.

First, note that at $\underline{v}$ and $\overline{v}$, $F(y)^{N^*} = \frac{y-\underline{v}}{\overline{v}-\underline{v}}$. For all $y \in (\underline{v}, \overline{v})$, $F(y) < 1$ and $\frac{y-\underline{v}}{\overline{v}-\underline{v}} > 0$. Then, $\lim_{n\to\infty} F(y)^n = 0$, or there exists some N such that for all $n \geq N$,

$$F(y)^n \leq \frac{y-\underline{v}}{\overline{v}-\underline{v}}.$$

Let $N(y)$ be the function which maps from $y \in [\underline{v}, \overline{v}]$ to the natural numbers satisfying this condition. Let $N^* = \max N(y)$ which exists and is obtained since $[\underline{v}, \overline{v}]$ is compact. Then for all $y \in [\underline{v}, \overline{v}]$, $F(y)^{N^*} \leq \frac{y-\underline{v}}{\overline{v}-\underline{v}}$. ∎

In order to prove Proposition 4.3.24, we need the following lemma.

A.0.7 Lemma *Let α, $\alpha' \in \mathbb{R}^n_+$ and $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \alpha'_i = c$ where $c > 0$. If there exists a k such that for all $j \leq k$, $\alpha_j \geq \alpha'_j$ and for all $j > k$, $\alpha_j \leq \alpha'_j$, then for all $x \in \mathbb{R}^n$ such that $x_1 \geq x_2 \geq \cdots \geq x_n$, $\alpha \cdot x \geq \alpha' \cdot x$.*

Proof: Assume there is a x such that $\alpha \cdot x < \alpha' \cdot x$. Then it must be that

$$(\alpha_1 - \alpha_1')x_1 + (\alpha_2 - \alpha_2')x_2 + \cdots + (\alpha_n - \alpha_n')x_n < 0.$$

Since for all $j \leq k$, $(\alpha_j - \alpha_j') \geq 0$ and for all $j > k$, $(\alpha_j - \alpha_j') \leq 0$

$$\left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha_j'\right) x_k + \left(\sum_{j > k} \alpha_j - \sum_{j > k} \alpha_j'\right) x_{k+1} < 0.$$

Since α and α' both sum to c,

$$\left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha_j'\right) x_k < \left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha_j'\right) x_{k+1}$$

which implies the contradiction that $x_k < x_{k+1}$. ∎

Appendix B Experiment Instructions

Experiment Instructions

Introduction

You are about to participate in an experiment in the economics of market decision making in which you will earn money based on the decisions you make. All earnings you make are yours to keep and will be paid to you at the end of the experiment. In this experiment you are going to participate in a market in which you will be buying units in a sequence of independent market days or trading periods. You will each receive a sequence of numbers, five for each period, which describe the value to you of any decisions you might make. These numbers may differ among individuals. *You are not to reveal this information to anyone. It is your own private information.* From this point forward, you will be referred to by your bidder number. You are bidder number ___ in this experiment. In each trading period you will be able to place bids to purchase a single unit in all of five markets (labeled A-E).

Redemption Values and Earnings

During each market period you are free to purchase a unit in any of the five markets if you want. If you purchase a unit in that market, you will receive the redemption value indicated on your redemption value sheet for that period and that market. Your earnings from a unit purchase, which are yours to keep, are the difference between your redemption value for that unit and the price you paid for the unit. That is:

$$\text{Your earnings} = (\text{redemption value}) - (\text{purchase price})$$

Suppose for example that you buy a unit in market A and that your redemption value is 200 in market A. If you pay 150 for the unit then your earnings are

$$\text{Earnings from unit} = 200 - 150 = 50$$

You can calculate your earnings on your accounting sheet at the end of each period. The currency used in the markets is francs. The conversion rate of francs to dollars will be listed on your redemption value sheets. Your total earnings in any period are given by the sum of your earnings in each market. For example, if you purchased a unit in market A for earnings of 50 and a unit in market B for earnings of 80, then your total earnings that period would be 130 francs. Remember, if you purchase a unit in a particular market, you must use the redemption value from that market.

Market Organization

In each period five markets will be open. There will be 5 participants in each market. In the markets, buyers may submit bids by entering bids into the computer. The bids will be arranged from the highest bid to the lowest. The highest bid in each market will be announced by the computer as the buyer in that market. The identity of the highest bidder will not be announced. The buyer will pay a price equal to the bid and as a result will earn the difference between his/her redemption value for the unit and the highest bid placed. The bids of all other bidders are nullified. They receive no redemption value and pay nothing and so have earnings of zero for that market. If more than one bidder submits an identical high bid in a market, the buyer will be determined randomly (each tied bidder has an equal chance) and the price paid will be equal to their high bid in that market.

Submitting Bids

On your screen you will see a window titled, *Make A Bid.* In this window you select the market you want to bid in by clicking the square beneath an item's letter. When you click on a market the button will appear to be depressed in order to indicate that the market has been selected. Once you mark the desired market, you can enter the amount (in francs) you are willing to bid in the box with a dollar sign. Bids should be in whole francs only. After your order is specified, you can send it to the market by selecting *save.* Each bid you make must have only one market selected. You must place a bid of at least 1 franc in every market. However, you may bid as much as you choose in any period and any market. You will have approximately two minutes in order to submit your bids. The period will end when all bidders have placed a bid in each market. You may view your bids by clicking on the *Bids* button in your main window. Once all bidders have submitted their bids, the period will be closed and the results calculated. When the results are available, you may view the bids by clicking on the *Results* button in your main window. Selecting *Show* will display the results.

Determination of Redemption Values

For each buyer the redemption value for each market and each period will be between 1 and 1000. In four of the five markets, each number from 1 to 1000 has equal chance of appearing. It is as if each number between 1 and 1000 is stamped on a single ball and placed in an urn. A draw from the urn determines the redemption value for an individual. The ball is replaced and a second draw determines the redemption value for another player. The redemption values each period are determined the same way. The following is a table in which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Redemption value	Probability of a value in this range
1-100	10%
1-200	20%
1-300	30%
1-400	40%
1-500	50%
1-600	60%
1-700	70%
1-800	80%
1-900	90%
1-1000	100%

In the fifth market, redemption values are drawn in a different manner. Redemption values close to 1000 have a higher chance of appearing than do those close to 1. It is as if the number 1 is stamped on a single ball, 2 is stamped on 3 balls, 3 is stamped on 5 balls, and so on. For any value n between 1 and 1000, the number of balls equals 2n-1. All the balls are placed in an urn. A draw from the urn determines the redemption value for an individual. The ball is replaced and a second draw determines the redemption value for another player. The redemption values each period are determined the same way. The following is a table in which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Redemption value	Probability of a value in this range
1-100	1%
1-200	4%
1-300	9%
1-400	16%
1-500	25%
1-600	36%
1-700	49%
1-800	64%
1-900	81%
1-1000	100%

There will be one bidder whose values are drawn from this set of draws in each market. Bidder 1 will receive redemption values drawn in this manner in market A. Likewise, 2 in B, 3 in C, 4 in D, and 5 in E. For each bidder, the redemption values in the four other markets will be given by draws determined as previously described.

Your redemption value sheet may look something like this:

A 520

B 128

C 200

D 750

E 776

This indicates that you would receive a redemption value of 520 in market A if you place the highest bid in that market. Likewise, your value in market B would be 128 and so on. The first period will be practice. You will receive no earnings for this period. If you have a question, please raise your hand and a monitor will come by to answer your question.

To be read after round 5

Communication with Other Participants

Sometimes in previous experiments, participants have found it useful when the opportunity arose, to communicate with one another. You are going to be allowed this opportunity while the computers are reset between periods. There will be some restrictions. You are free to discuss any aspect of the experiment (or the market) that you wish, except that:

- You may not discuss any quantitative aspects of the private information on your value sheets.
- You are not allowed to discuss side payments or to use physical threats.

Since there are still some restrictions on your communications with one another, an experimenter will monitor your discussion between periods. To make this easier, all discussions will be at this site. Remember, after the computers have been reset between periods (and the next period has begun) there will be no discussion until after the end of the next period. We allow a maximum of 4 minutes in any one discussion session.

Appendix C Bidder and Seller Surplus

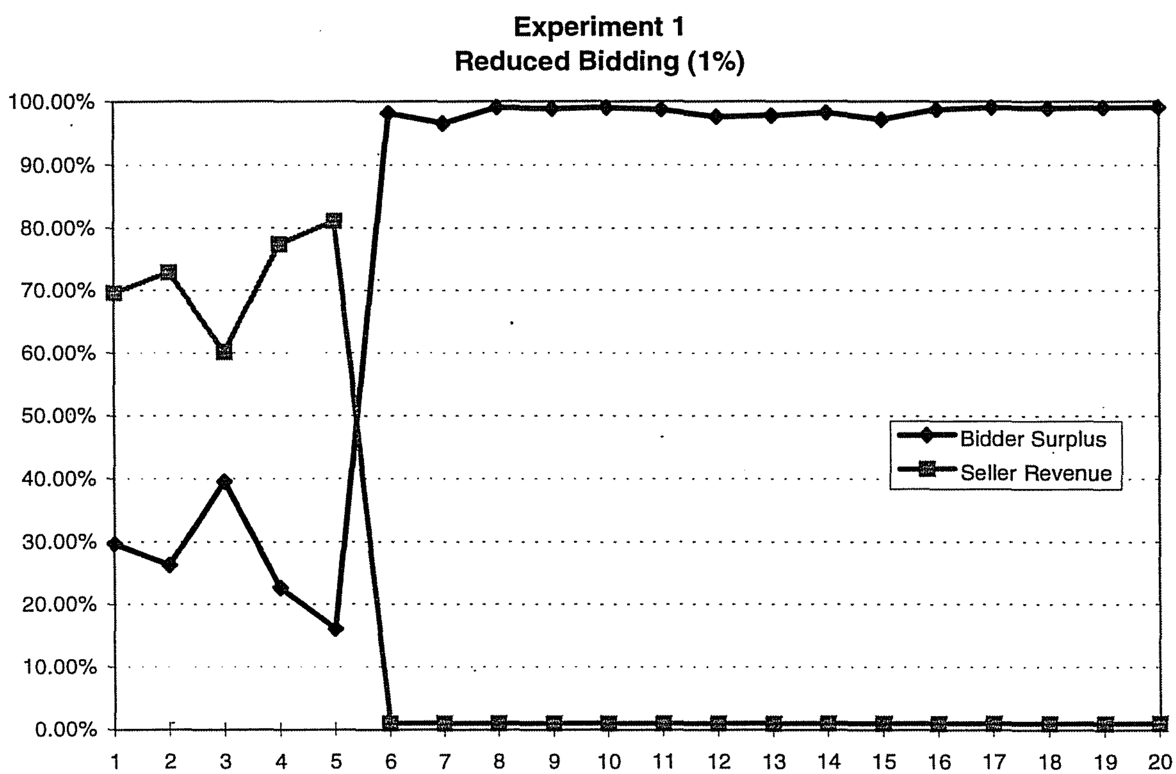

Figure C.1: Experiment 1

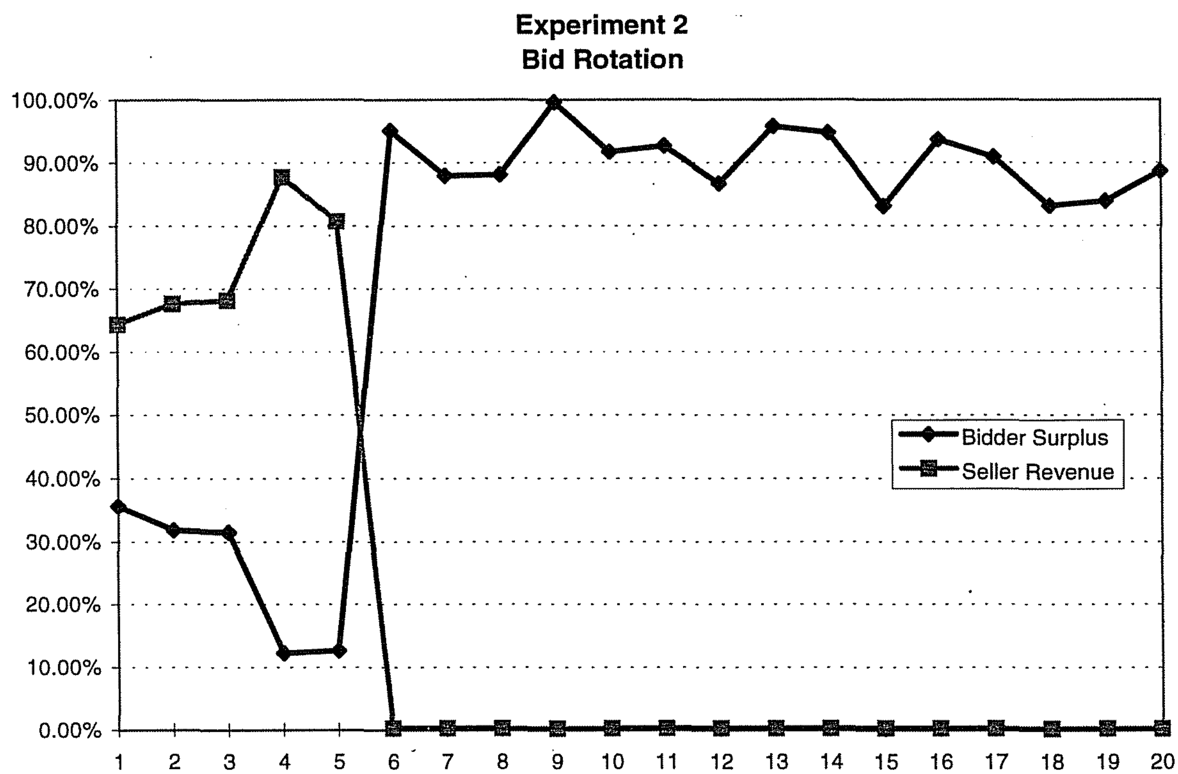

Figure C.2: Experiment 2

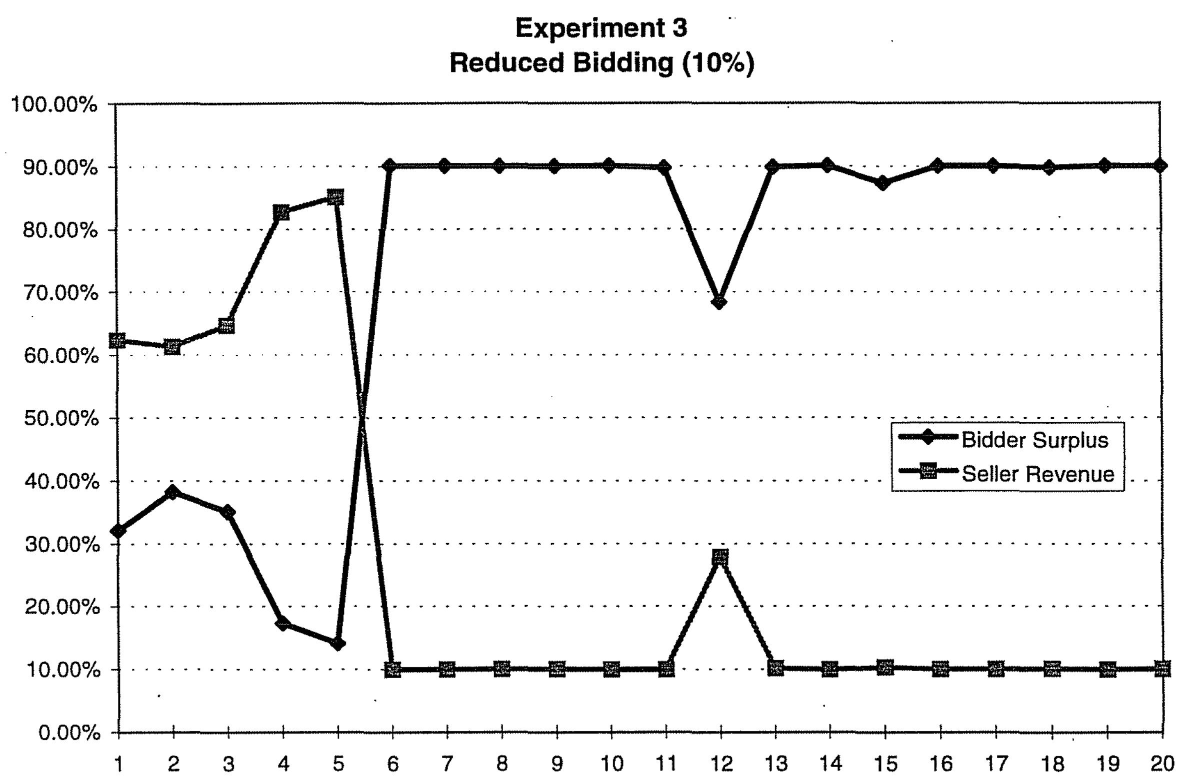

Figure C.3: Experiment 3

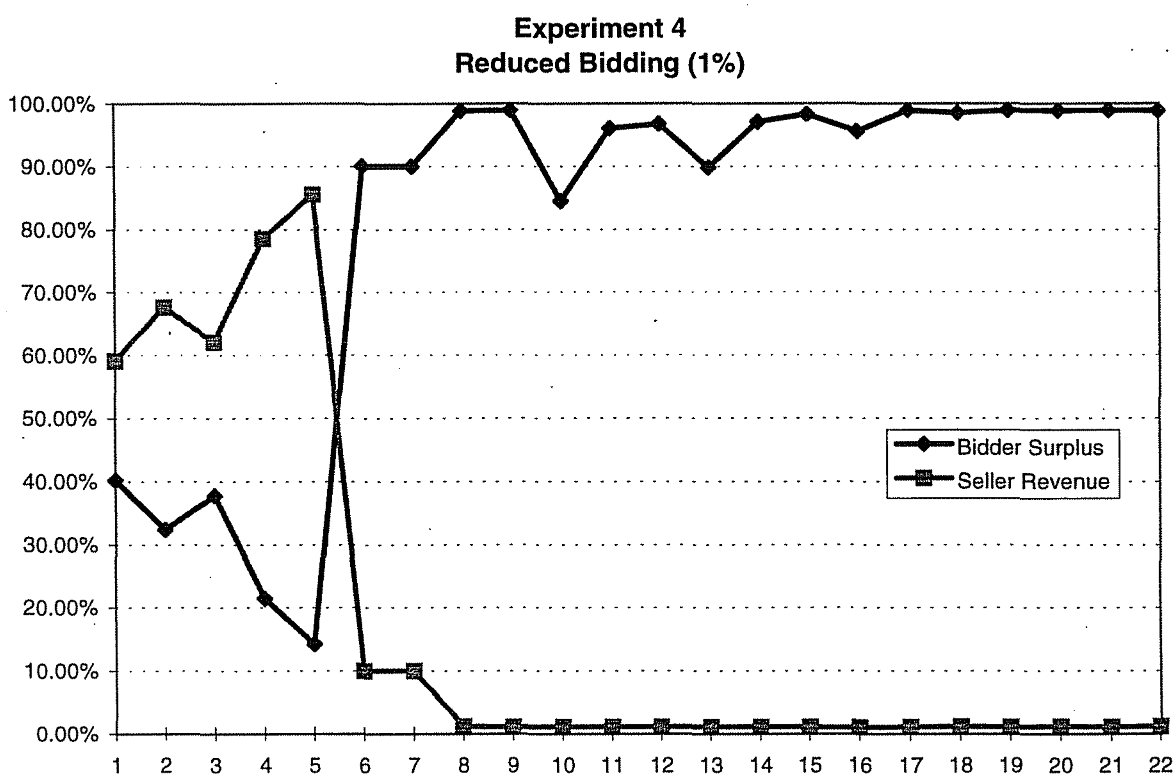

Figure C.4: Experiment 4

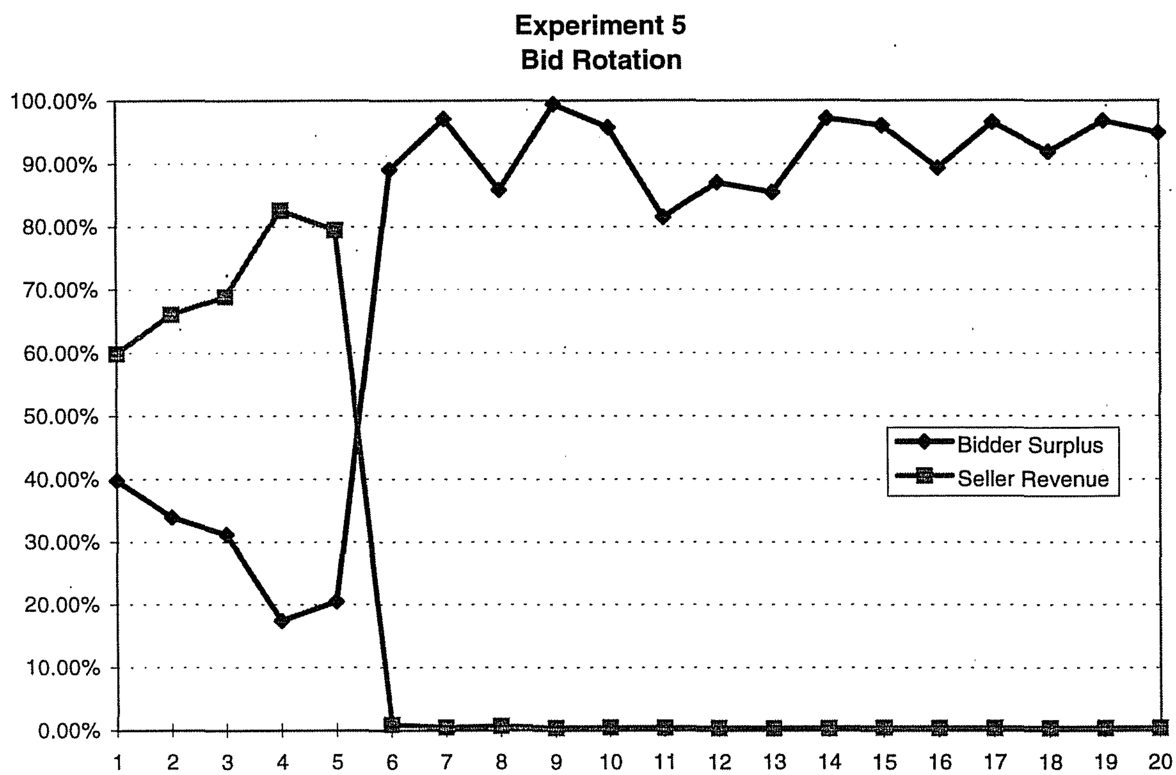

Figure C.5: Experiment 5

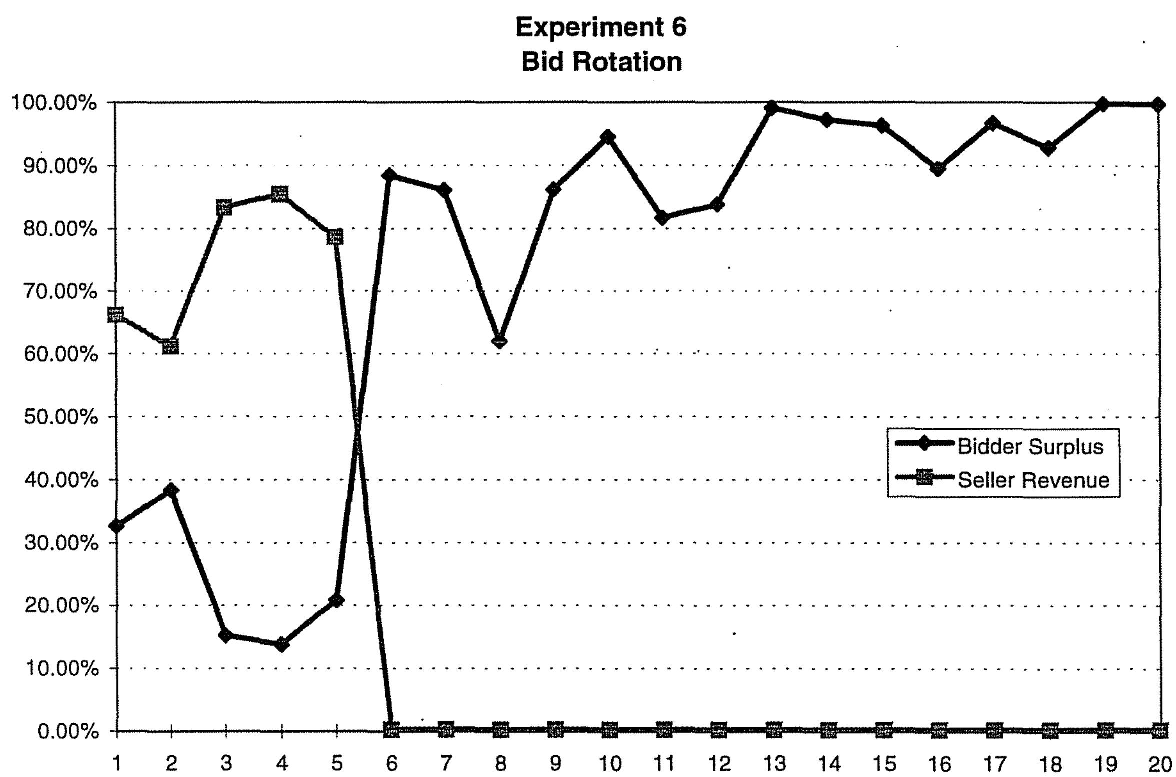

Figure C.6: Experiment 6

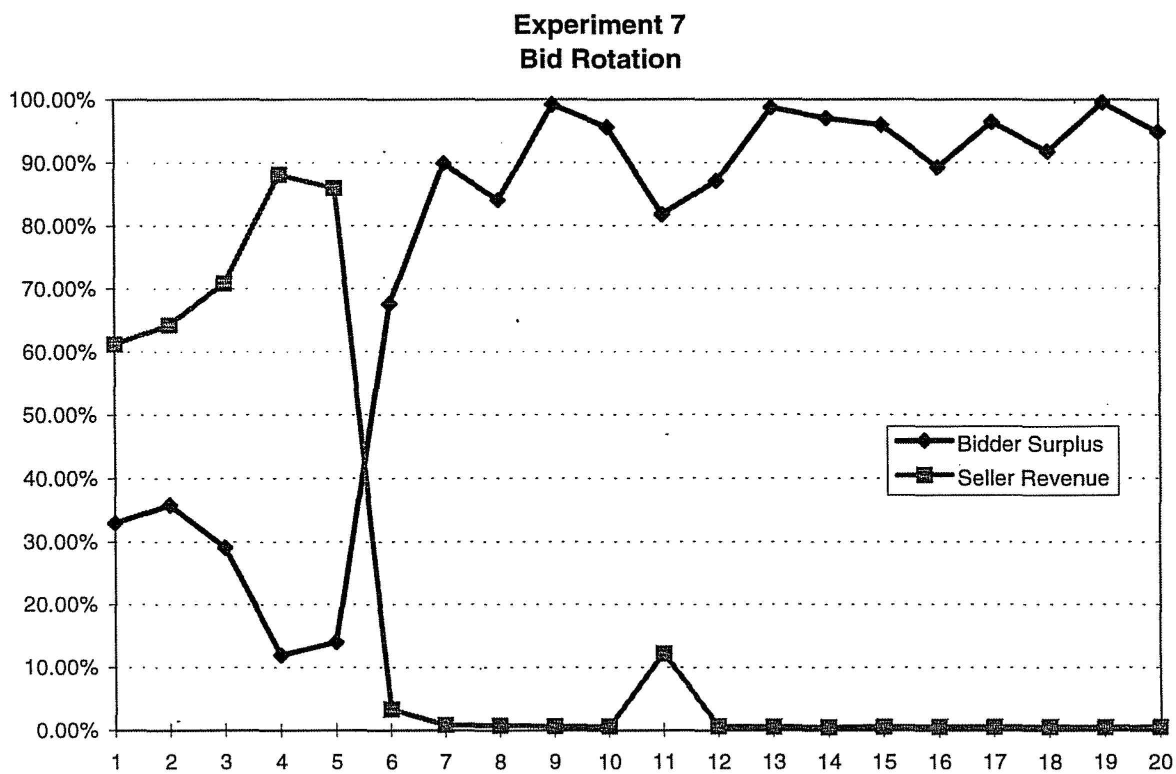

Figure C.7: Experiment 7

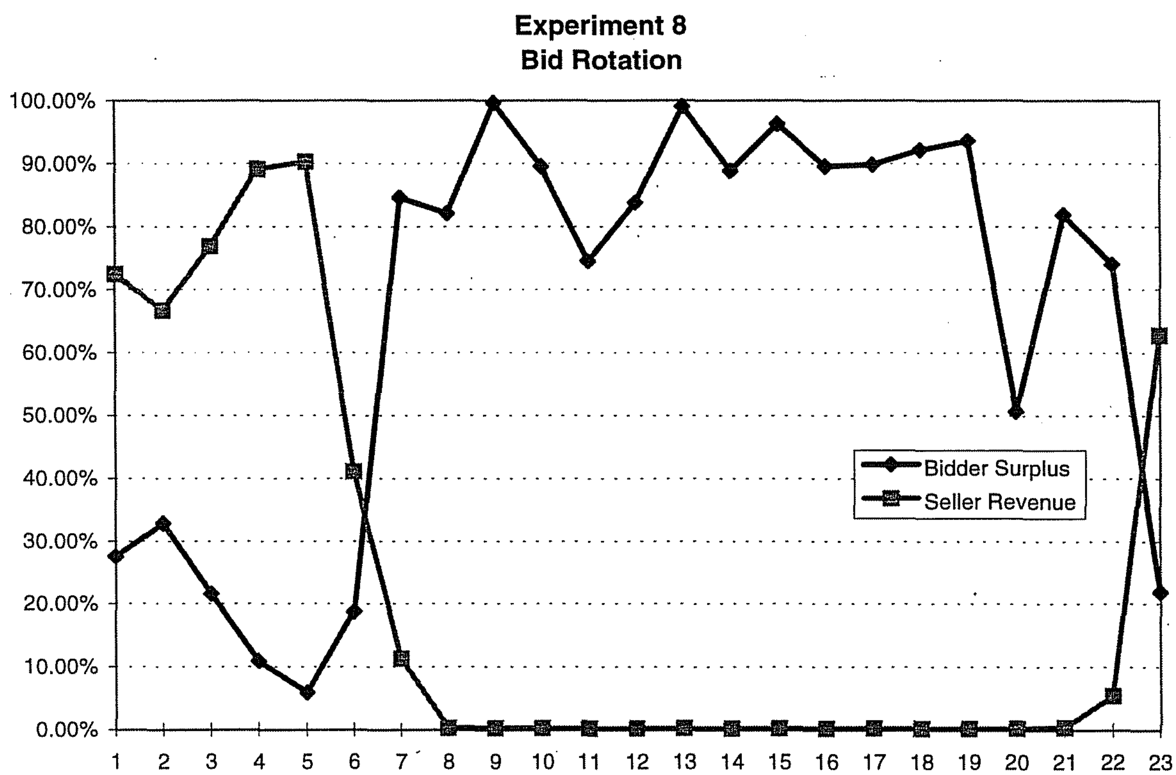

Figure C.8: Experiment 8

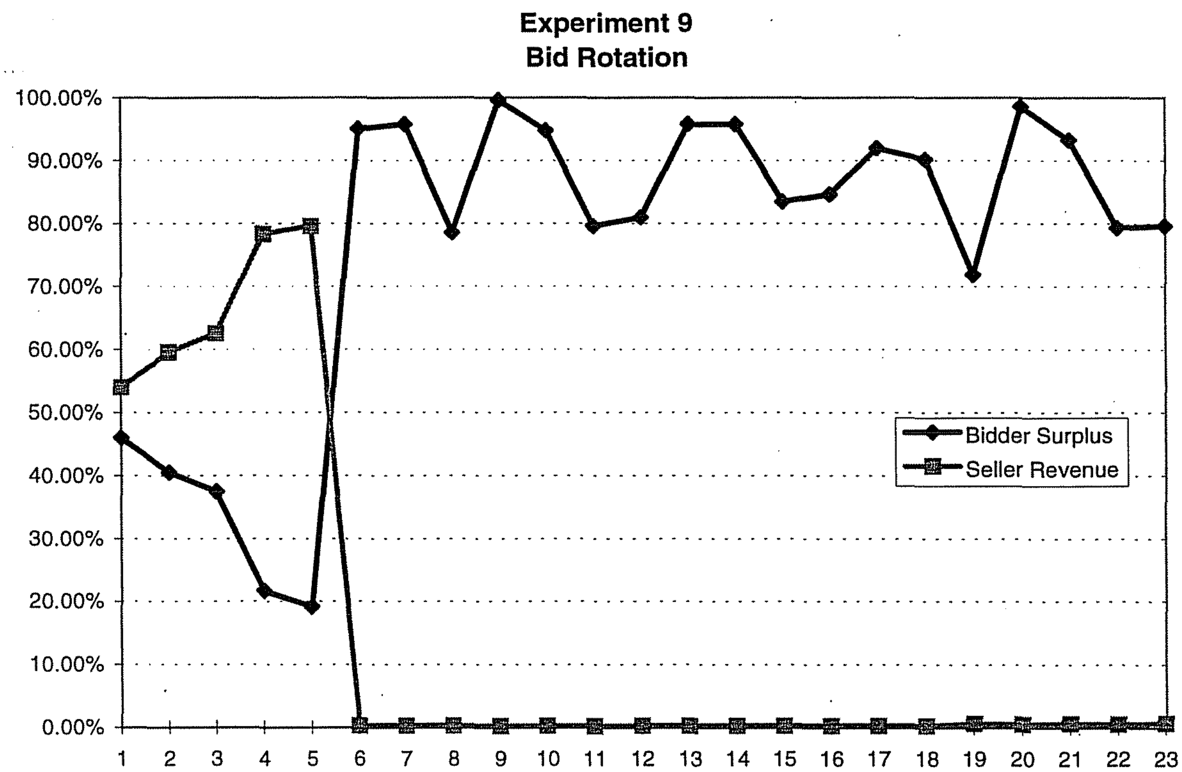

Figure C.9: Experiment 9

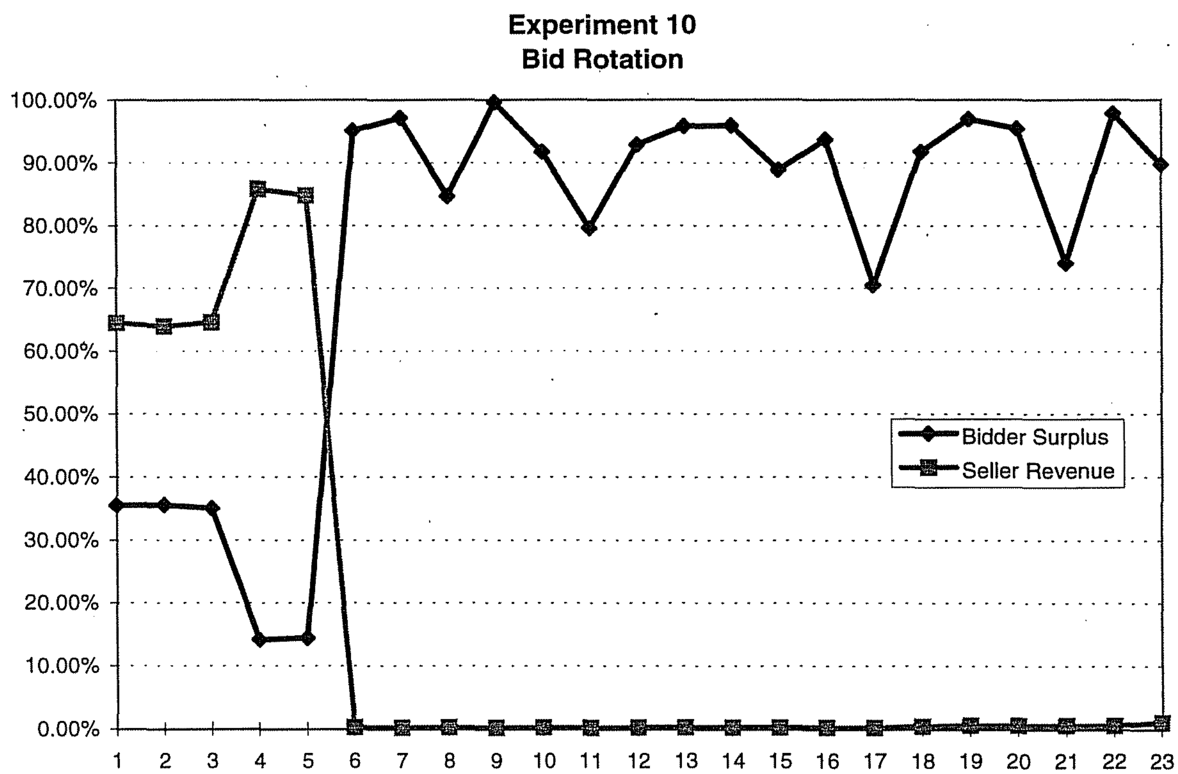

Figure C.10: Experiment 10

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