STUDIES OF THE COSMIC RAY LATITUDE EFFECTS

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Dedicated to

My Wife

ABSTRACT

A cosmic ray experiment in a B-29 airplane conducted through a wide range of latitudes at high altitudes is described. The chief experimental result is that the various cosmic ray components behave in quite similar fashion and hence are probably produced by the same primaries in a type of interaction roughly independent of energy of the primary.

A discussion is given of cosmic ray fluctuations. An unsuccessful attempt to correlate fluctuations at high altitude and at sea level is described. A study of electroscope data taken over a period of years shows a hitherto unsuspected regularity in the nature of the fluctuations. Attempts to explain this type of fluctuation are given but the conclusions are uncertain. Allowing properly for fluctuations, the electroscope and counter data give no evidence for a heliomagnetic cut-off south of 60° geomagnetic latitude.

The curves of minimum momentum for primary particles are corrected for use in the range of the airplane and electroscope experiments. The airplane experiment is consistent with these theoretical curves except south of 20° geomagnetic latitude. Analysis of the airplane experiments shows that there can be almost no negative particles in the primary radiation. The effect of fluctuations and other effects on the determination of the primary momentum distribution is considered and a new distribution curve drawn. The relation of the reading of a Geiger counter telescope to the primary

flux is studied and the "multiplicity" function evaluated.

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PART I COSMIC RAY MEASUREMENTS IN A B-29 AIRPLANE AT HIGH ALTITUDES

A. Introduction

During the summer of 1947 a cosmic ray group of the California Institute of Technology made a study of the cosmic ray latitude effect employing Geiger counter telescopes carried to very high altitudes in balloons[1]. The results indicated that during the period in which the measurements were made, significant fluctuations in cosmic ray intensity took place. To measure the latitude effect in a short time so as to avoid such fluctuations it was decided to put the telescopes in an airplane and fly from high latitudes to the geomagnetic equator at a constant atmospheric pressure. The greater carrying capacity of the airplane made many other sorts of experiments possible at the same time.

The experiment was performed at an atmospheric pressure of 310 gm cm^{-2} along the 80^{o}W geographic meridian over the range of geomagnetic latitude from 64^{o} N to 0^{o}. The trip was made in a two week period in May and June of 1948.

A list of the experiments performed on this trip as well as on a high altitude flight over the geomagnetic equator follows:

I. Atmospheric pressure of 310 gm cm^{-2}

A. Latitude effect, continuous measurements both coming and going:

1. Vertical radiation: two independent telescopes with no lead absorber and one each with 10 cm and 20 cm.
2. East-west effect at 45^{o} zenith angle: two independ-

ent telescopes with no lead absorber and one each with 10 cm and 20 cm.

3. Extensive showers: four trays of counters with maximum extension of 4 meters.

4. Total radiation: a continuously recording ionization chamber of the type used by Millikan and Neher in their sea level and airplane surveys[2].

B. Other experiments at atmospheric pressure 310 gm cm^{-2}

1. East-west effect at the geomagnetic equator: measurements with 0, 10, and 20 cm of lead absorber at zenith angles of 0°, $22\frac{1}{2}^\circ$, 45°, $67\frac{1}{2}^\circ$, and 90° in the east-west plane.

2. Zenith measurements: made in north-south plane at 48° geomagnetic north on flight from Provo, Utah to Springfield, Illinois, along a constant geomagnetic latitude. No absorber, 10 and 20 cm of lead absorber; zenith angles: 0°, $22\frac{1}{2}^\circ$, 45°, $67\frac{1}{2}^\circ$, 90°. These same measurements were repeated on the east-west flight along the geomagnetic equator from longitude 80° W to Lima, Peru.

3. Side showers: To provide data on the corrections due to showers from the sides to be made to the counting rate of the counter telescopes the center tray of counters on two telescopes was displaced sideways outside the telescope aperture. This was done both at northern latitudes and at the equator.

II. Atmospheric pressure of 235 gm cm^{-2}

A. Azimuth experiments: Two hexagonal courses were flown, one hexagon rotated 30° with respect to the other, thus giving data every 30° in azimuth. Measurements with 0, 10, and 20 cm of lead absorber; zenith angles: 0°, $22\frac{1}{2}^\circ$, 45°, $67\frac{1}{2}^\circ$.

B. Extensive showers were also measured.

III. Miscellaneous experiments

A. Counting rate versus atmospheric pressure: at nearly all times when the plane was ascending, all telescopes were turned vertically and records kept of the outside air pressure versus time.

B. Calibration checks: On several occasions on the flight down and return all telescopes were turned vertically and long runs made for intercomparison.

In a short time there will appear in the Physical Review an exhaustive description and analysis of these experiments; consequently, the present report will be limited to a brief survey sufficient for understanding the attempts made below to correlate different parts of this experiment and the present data with data of other experiments.

B. Apparatus

The cosmic ray telescopes used were only slightly modified from those used in the balloon experiments[3]. Three trays containing eight Geiger counters apiece were connected in triple coincidence. The trays were 23.1 cm by 28.5 cm, but due to the cracks between counters had an effective area of only about 600 cm^2; the outer trays were separated by 80.0 cm. Two telescopes of three trays were used with no absorber to measure the vertical radiation. Two telescopes of four trays were mounted so they could be rotated about the axis of the airplane. The top three and the bottom three trays were in coincidence separately; between the bottom two trays were placed 10 and 20 cm of lead respectively. These telescopes were used in measuring the intensity at zenith angles other than the vertical. A similar double-telescope had 20 cm of lead between the bottom two trays and 10 cm between the next two; it was used to measure the intensity of the hard vertical radiation. The Geiger tubes were from the same batch prepared for the balloon experiments.

The amplifier and coincidence circuits were the same as before. New apparatus[4] was constructed for scaling the counting rates down by a factor four.

Counts were recorded by mechanical counters which were photographed once every minute during operation and read visually once every ten minutes.

Responsibility for remaining at a constant atmospheric pressure was left with the pilot. He performed this tedious task very well and stayed within 100 feet of 30,000 feet (310 gm cm^{-2}) on all but very exceptional occasions.

Position of the plane was determined periodically by an experienced navigator.

C. Treatment of the Data

The data were read from the film records and averaged for 30 minute intervals with the following exceptions: 1) zenith data were averaged from all available data, 2) azimuth data were only available for 8 or 10 minute intervals, 3) altitude data were averaged for 8 minute intervals, 4) extensive showers were averaged within ranges of latitude, e.g. 0° to 10°, etc., and 5) side shower data were averaged from all available data. The electroscope film was read for 30 minute intervals at 0°, 10°,...50°, 60°, on both the north and south trips.

All telescope data were adjusted by suitable correction factors to read what set #1 would have read in the same circumstances. Set #1 was one of the vertical telescopes with no absorber; it was one which was thought to have shown no inconsistency throughout the experiment. This reduction was made possible for the sets with no absorber by the frequent intercomparison mentioned above. For those telescopes containing lead absorber only limited data taken on the ground with the lead removed were available. The iron boxes which contained the lead were still in place but absorption by them could be corrected for by determining the decrease in counting rate of set #1 when iron sheets of the same thickness were placed between its trays.

All of the data were reduced to absolute units, i.e. to counts cm^{-2} sec^{-1} ster adian^{-1}. In the fall of 1947, Montgomery[5] determined the factor for reducing the readings of the telescopes used in the balloon experiment to absolute units by making a determination of the absolute intensity of cosmic rays at sea level. If one assumes that this absolute intensity did not change in the interim, one can find the reduction factor for the present experiments by measuring their counting rate at sea level. Another way of determining this factor is to compute it from the change in geometry of the telescopes (see Appendix A). Both methods were tried and agreed to within a percent. It is evident that this reduction is a little more uncertain than the relative adjustment of the readings of the telescopes.

Corrections for accidentals were made using general formulae given by Schiff[6]; allowance was made for the different counting rates of the trays when under lead or when tipped. Allowance was made for the fact that the center tray in the telescopes was not in the exact center; this affects the coincidence rate of pairs of trays.

The deadtime of the counters was corrected for using general formulae which permitted taking into account the shielding of some of the trays by lead.

Several sources of error were considered to be of negligible importance. The correction for variations in atmospheric pressure was about 0.6% per 100 feet; since, however, averages were taken over such long periods of time, it was felt that any such small variations would average out. The pressure efficiency of the counters arranged in a telescope is about 99%[5]; this small correction is included in the absolute calibration described above. Since the number of counts was scaled by four, the pulses fed to the mechanical recorder were reduced in number and randomness sufficiently that the efficiency of the recorder was essentially 100%. It was estimated that the roll of the plane caused a statistical fluctuation in tilt of the telescopes of about 0.25° and that a 1° tilt would cause 3% error; no correction was made because it would be small and tend to average itself out.

No correction was made for the fact that although the atmospheric pressure was maintained constant the actual altitude, or the extension of the atmosphere, did change. This might cause systematic errors in the measurement of any component connected with a decay process. Whenever possible, readings were taken with a radio altimeter to give true altitude; these indicate that the atmosphere was more extended over the tropics which is what one might expect.

No correction was made for the amount of absorbing material in the walls of the airplane and the Geiger counters. This will affect the measurement of the soft component only. The counter walls amount to a minimum of 1.6 gm cm^{-2} of iron for the soft component, and the walls of the plane add about 1 gm cm^{-2} more, chiefly of aluminum.

The largest single correction that it was found necessary to make was for side showers. This amounted to a maximum of -13%. The correction was estimated from the experiment in which the center tray of the telescope was displaced and from the counting rates for a zenith angle of 90°.

The fact that the side shower experiment shows a latitude effect indicates that one is dealing with small showers set off in nearby material rather than with extensive showers. In this case it is reasonable to suppose that the event setting off the telescope is the passage of one particle through two trays and another through the third[5]. Hence we may consider the three trays in the side shower experiment as an equivalent system of three telescopes with associated side trays as indicated in Fig. 1. From Auger's data[7] on shower rate as a function of counter separation we can estimate the relative probabilities of the associated side trays being set off. From either Kraybill's[8] or our own results on the variation of showers with altitude we can estimate the relative amounts of shower producing radiation for the telescopes tipped at different angles; this assumes that this radiation follows a mass absorption law and behaves in a manner similar to the extensive showers. The soft component and extensive showers, however, behave in essentially the same way after they have reached a maximum, so we can safely use results for the latter in estimating intensities of the former, which is probably responsible for our side showers. Using these relative rates it was computed that the side shower experiment rate should decrease by about 40% if the arrangement were tipped over at 45^{o}; this is precisely what was found experimentally after subtracting out the coincidences due to extensive showers. Hence considerable confidence was felt in the computation by this same method of the factors by which the rates observed in the side shower experiment should be increased to give the side shower rate when the center tray is in its proper place. In the case of a vertical telescope this gave a factor of 2.7 which can be compared to a factor of 2.5 deduced from work of Montgomery[5] at sea level on telescopes of slightly different geometry.

When a telescope is tipped over to 90° with the vertical, it should be responding to radiation essentially the same as that encountered at sea level. Furthermore, if there is a block of lead between two of its trays it will be shielded against any of the side showers set off in the immediate neighborhood. Experimentally, in such a case the counting rate is 21 per minute; however, about 7 of these are due to extensive showers. Consequently, 7 hard particles pass through the telescope in each direction per minute; if these are 70% of the total, then the telescope without lead between its trays, but protected at one end by such a block of lead, should be counting 26 per minute. It is actually counting 102, so the other 76 are attributed to side showers.

The arguments above give us the side shower correction at zenith angles of 0°, 45°, and 90°. Reasonable interpolations were used to get the corrections for angles of $22\frac{1}{2}°$ and $67\frac{1}{2}°$. The correction was assumed to have the same latitude effect as the total radiation (the side shower experiment indicates that this is true). No asymmetry was assumed for this correction at the equator. It was assumed that the lead blocks shielded the telescopes measuring the hard components sufficiently that no shower correction was necessary.

The effect of the finite aperture of the telescope is different at different zenith angles. This is illustrated in Appendix A where the effect of the aperture is calculated for any zenith angle under rather simple assumptions concerning the zenith angle dependence. This formula indicates that the aperture correction differs by 2.0%, or less, from that for the vertical telescopes for zenith angles up to 45°. At 67½° a correction of -5% is found in this way. However, it is felt that this is not an accurate method of determining this correction in this case. The formula assumes that the intensity follows a simple cos z power law which implies, for mass absorption, a power law absorption with depth. Now, in the range of equivalent depths covered by the telescope at 67½°, the power law of absorption is found to undergo a significant change; this is not true at the other zenith angles. Since the assumptions are not fulfilled for using the formula, we must resort to numerical integration using the empirical depth law converted into the corresponding zenith law. This method yields a correction of -23% at northern latitudes.

D. Results

In Fig. 2 are plotted the results of the latitude experiment at 310 gm cm^{-2} using no lead absorber. Results for the vertical and 45° zenith angle east and west are given. The experimental points and the adopted curves are shown after all corrections and reductions have been made.

Fig. 3 shows the same things for the telescopes which had 10 and 20 cm of lead absorber respectively.

When the data for the azimuth experiment at 235 gm cm^{-2} at the equator were plotted as a function ofazimuth angle, it was found they varied sinusoidally; hence, in Fig. 4 the results are for no lead absorber plotted against the sine of the azimuth angle and a least squares fit of the points drawn in. The empirical equations for these curves are as follows:

TABLE I

Empirical Expressions for Azimuth Experiment.

Z	J'
$22\frac{1}{2}°$	$0.174 - 0.0137 \sin\alpha$
$45°$	$0.132 - 0.0192 \sin\alpha$
$67\frac{1}{2}°$	$0.0518 - 0.0099 \sin\alpha$

For details of the other experiments the reader is referred to the article which is to appear shortly.

In Table II appears a list of corrections made to the unshielded data with maximum values and estimates of the uncertainties in them and in the reduction factors.

TABLE II

Corrections and Reductions of Unshielded Data.

	Maximum Correction	Uncertainty
1. Accidentals	-1.5%	15%
2. Dead time	4.0%	20%
3. Side showers	-13.0%	10%
4. Finite aperture (except $67\frac{1}{2}^{\circ}$)	2.0%	10%
5. Internal calibration		0.75%
6. Reduction to absolute		2%

The corrections and uncertainties are larger for data taken under lead but are not larger than two times those for the unshielded case.

In this experiment we seem to have avoided any major fluctuations during the course of the experiment. There were no serious magnetic changes or solar phenomena during the period of measurement[9]. Similar data taken at different times agree very well in general. Comparable data for this experiment is a few percent higher than for our balloon experiments of the previous summer. Some of this may be due to the presence of the airplane but data provided by Dr. Forbush indicate that there was a 2% increase in sea level intensity between these times. We have some meager information which indicates that sea level changes may be accompanied by larger changes at high altitude (see below), so the difference may be due to an actual change in the intensity of radiation. Preliminary results from another similar airplane experiment indicate that the intensity may have changed since the present experiment[4].

Examination of the experimental results shows no marked plateaus of intensity such as previously found[10]. The latitude and azimuth experiments together cover a momentum range for the primary particles of 1 to 33 Bev/c. It seems safe to say that in this range there are no gaps in the primary momentum distribution.

In Table III are listed the usual coefficients by which experimental results are given. By latitude effect is meant the percent decrease in intensity in going from very high latitudes to the equator. The east-west effect is the difference in intensities in these directions divided by their average value and expressed in percent.

TABLE III

Latitude and East-West Effects.

Absorber (cm lead)	Latitude Effect (310 gm/cm^2)	East-West Effect (310 gm/cm^2)	East-West Effect (235 gm/cm^2)
0	37	23	29
10	40	28	33
20	33	30	35

The fact that we find nearly equal east-west effects for the hard as for the soft component is in contradiction to the result of Johnson[11]. He found a negligible east-west effect for the soft component though there was one for the hard. He concluded that the hard and soft particles were produced by different sorts of primary particles and that those producing the hard were predominantly positive. The present results indicate that such a conclusion is probably not true, and that it is sufficient to think of one sort of primary for both components. Furthermore, the existence of a west excess indicates this primary is predominantly positive, and in Part III evidence will be given that the negative component is completely negligible.

Another way of stating the rough equality of the coefficients of Table III is to say that the percent of the total intensity in the hard and soft components is nearly constant. This will be assumed in Part III so that only the total counting rate will be analyzed; this is convenient because it is known much more accurately than the others. A constant ratio of the hard to soft component has already been found in sea level latitude measurements, but here it is not surprising because most of the soft component at these depths has originated in the decay of the hard component. The constancy of this ratio at the higher altitudes of this experiment indicates that the reactions entered into by the primary particles are independent of the energy of the particle. This is the sort of thing predicted by Heitler[12]; he finds that at cosmic ray energies cross sections for the interaction of nucleons are roughly constant and that nucleons probably lose their energy in a number of successive interactions of the same type.

PART II. FLUCTUATIONS OF COSMIC RAY INTENSITY

A. Introduction

Several types of fluctuations have been recognized in the cosmic ray intensity at sea level[13]. Diurnal and seasonal effects related to solar time are generally accepted to exist while a variation with sidereal time is in doubt. A recurrence phenomenon of rather peculiar character has been found; it has a twenty-seven day period, but its correlation with any sort of solar activity is uncertain. Forbush has found that small disturbances take place in the intensity at stations all over the earth; these variations are random in time but possibly associated with terrestrial magnetic activity. Sudden large changes taking place at the time of magnetic storms have been noted by many observers; oddly enough, however, some storms occur with no detectable change in the radiation. Forbush[14] has discovered that on occasion solar flares are followed within a short time by sudden very large increases in intensity; this observation has been confirmed by many observers.

Some work has been done on variations at mountain top altitudes, but practically nothing at very high altitudes. One would expect that all the fluctuations mentioned above could be detected at high altitudes. The periodic and quasi-periodic changes may very possibly be associated with changes in atmospheric conditions related to the solar cycles and activity. In this case the changes in intensity throughout the atmosphere would probably be rather small; the total energy incident on the atmosphere would not be changed, and electroscope measurements could be made in such a way that these changes would have no effect. (See below.) It is hard to see how the aperiodic changes can be due to anything but actual changes in intensity of the primary cosmic ray particles. It is to be expected then that these changes would be of about the same relative magnitude at high altitudes as at sea level; such has been observed to be the case for the magnetic storm effect[15].

In the extensive work of Millikan and his collaborators using electroscopes, several types of fluctuations have been observed. First, during a flight there may be sudden very large variations much larger than the statistical errors. It is possible that these changes are instrumental, although their source is not understood. Second, flights made a few days apart at a given station may show maximum intensities which differ by as much as 5%[16]. It is felt that these fluctuations are real and not due to instruments. The differences between the curves made on successive days show up only at quite high altitudes, and may, therefore, be judged to be due to primaries of quite low energy, or, possibly, those incident at very large zenith angles. Finally, if the flights made at a given station during a year (they are generally made in a period of a week or so) are averaged and these results compared with those obtained in other years at the same station, large differences of the order of 25% are found to occur[17]. These changes will be discussed at greater length below.

In the summer of 1947 the cosmic ray group with which the author is associated made a series of measurements of intensity versus latitude using Geiger counter telescopes. Large discrepancies from any continuous latitude effect were found[1] to occur and were attributed to variations in intensity while the observers were moving from one station to the next. The variations found using telescopes in balloons and airplanes have been made the subject of extensive study by Mr. A. T. Biehl[4].

B. Cosmic Ray Effects from Solar Flares and Magnetic Storms

In the summer of 1946 the author and Dr. Neher were fortunate in having a recording electroscope in operation at the Mount Wilson Observatory during the occurrence of a large solar flare and the subsequent magnetic storm and cosmic ray disturbance. During the magnetic storm, data were obtained by a balloon flight with an electroscope at Fort Worth, Texas. All of this was made the subject of a report[15] which has been put in Appendix C. In addition, Fig. 23 has been prepared which shows just how close the correlation was between the cosmic ray change and the magnetic storm.

One of the important discoveries made from our data was that the cosmic ray change correlated with the flare was exponential in build-up and decay. Since the publication of the report, Dr. Forbush of the Carnegie Institution of

Washington, who first noticed the result of the flare, has supplied the author with his data for the time of the flare. His electroscope was read only every two hours, and as a consequence cannot be used to establish this exponential character as well as ours, which could be read every fifteen minutes, though hourly averages were used. Nevertheless, his data are consistent with an exponential variation; it certainly does not contradict it.

Forbush, Gill, and Vallarta have recently published[18] a study of a mechanism for the production of charged particles of high energy during solar flares and for getting them through the magnetic field of the sun to the earth. It should be noted that the presence of the sun's magnetic field is actually essential to their theory since it is called upon to explain why all flares are not accompanied by cosmic ray fluctuations. The theory predicts correctly the lapse of time between the flare and the start of the cosmic ray increase, but as yet no explanation of the exponential variation has been put forward.

C. Relation of Fluctuations at Sea Level to Those at High Altitudes

Dr. Neher has found a possible relation between fluctuations at sea level and fluctuations at high altitudes. Sea level ionization data furnished by Dr. Forbush were found to show small variations during the period of the 1947 latitude survey mentioned above. If the fluctuations at sea level were increased in magnitude by a factor of about 10, they were found to account pretty well for the odd character of the results plotted versus latitude.

As was pointed out above, it is quite reasonable that the fluctuations found at sea level should have their counterpart higher up. Since the author was engaged in a study of fluctuations found in previous experiments, he undertook an attempt to see if this correlation could explain the variations in other results. All of the electroscope, single counter, and telescope data accumulated by the group associated with Millikan, Neher, and Pickering, were compared, in so far as was possible, to see if high altitude changes were related to those obtained by Dr. Forbush at sea level.

First, the average result for the maximum value from the radiation versus depth curves obtained at each station in a given series of measurements was compared with the average sea level intensity. The results shown in Table IV for Bismarck, North Dakota, are typical.

TABLE IV

Comparison of Maximum Ionization at Bismarck, N.D., with Sea Level Intensities.

Year	Maximum Ionization	Sea Level Intensity* Huancayo	Cheltenham
1938	370	4.7	6.5
1940	478	5.0	6.2
1946	381	4.6	4.8

(* Measured as % deviation from a standard value for the particular station.)

It is quite evident that there is no possibility of correlating these two sets of numbers.

Next, it was felt that the smaller fluctuations around the average values might correlate with the sea level data. In Fig. 5 are shown plotted the percentage change at the maximum from the average value of that series of measurements versus the deviations of the sea level intensities at the time of the flights. Again there appears to be no correlation.

Similar procedures were carried out for the single counter and telescope data, but here there is little that can really be compared since it is seldom that there are very many flights at any one station. Again the result was negative.

The fact that this attempt at correlation failed is not completely conclusive. The sea level fluctuations are quite small and their "true" value may have been masked by other effects. The sea level data were obtained at stations (Huancayo, Peru, and Cheltenham, Maryland, U.S.A.) quite far removed from the points at which the upper altitude measurements were made. Of course, one would expect changes in the primary flux to be world-wide, but a certain amount of irregularity cannot be ruled out. Furthermore, the major changes at high altitude seem to be due to low energy rays whose effects might not be felt at sea level.

Other attempts to explain the irregularities in the 1947 latitude experiment have been made. Sunspot activity was high but there were no great flares; the earth's magnetic field was quiet[9]. J. F. Dennise[19], using an analysis just like that of Dr. Neher, except that he used the radio frequency radiation emitted by the sun for comparison, has succeeded in smoothing out the latitude curve.

D. Variations in Intensity Over Long Periods of Time

In the summer of 1946 the author made a series of balloon flights with electroscopes at Bismarck, N. D. The techniques used and results obtained were quite similar to those of previous observers[10] so no detailed description will be given here though the results will be used below. These flights were extended over a period of about two months and as a consequence the deviations from the average value were larger than those obtained previously. It was decided to make a comprehensive study of all electroscope data available to establish as many facts as possible concerning these fluctuations. Through the kindness of Dr. Neher, the author was given access to the original records of all the electroscope flights of the Millikan, Neher, Pickering group.

It was mentioned above that periodic and quasi-periodic fluctuations due to atmospheric changes can be eliminated from electroscope data. This is possible because the area under the ionization versus depth curve gives a measure of the total energy incident at the top of the atmosphere. Since this will not be changed by variations in atmospheric conditions, expression of the results of electroscope flights in terms of the areas under the curves should be free from all variations except true changes in primary flux. The areas under the curves of all the old data were remeasured.

The electroscope areas were converted into energy units and then plotted in Fig. 6 versus geomagnetic latitude. In general the average value for a series of flights at each station was plotted. This figure reveals a previously unsuspected regularity in these long period fluctuations. Measurements made in the year 1940 fall on one smooth latitude curve; measurements made in 1937, 1938, and 1946 fall on another smooth latitude curve which is much lower than the other at high latitudes. The two curves appear to join somewhere around geomagnetic latitude 40°. Flights made in 1936 at stations south of 40° agree quite closely with those made in 1940.

Since only one series of points disagrees with all of the rest, one at first suspects that there may have been some difficulties of instrumentation or calibration causing the difference. It is very hard to see how this might have happened however. The instruments used were quite sensitive but very rugged quartz fiber electroscopes developed by Dr. Neher[20]. Only about twenty of these electroscopes were ever made so the large number of flights made represent repeated use of the same instruments. Before each series of flights the electroscopes were subjected to intercomparison and calibration tests and only very rarely was the calibration found to have changed. After the high flights of 1940, the electroscopes were checked before the 1946 flights and no significant changes in calibration found. Most of the flights of 1946 are in agreement with those of 1937 and 1938. What has been said of the electroscopes is also true of the barometers they contain. The zero point of these barometers changes from flight to flight but this is a known effect and is corrected for; the other calibration constant was not found to change in the testing prior to the flights for the different years. Since ionization and pressure are the only two measurements which enter into these curves, it seems quite possible to assume we are dealing with a real phenomenon and not one due to instrumental variations.

Several things can be said about this type of fluctuation. Since it shows a latitude effect, it must be due to charged particles. Investigation shows no pronounced terrestrial magnetic activity[21] during the periods in which the flights studied were made; hence we may conclude that the particles causing the difference were affected by the same terrestrial magnetic field and consequently have the same ratio of momentum to charge as the particles ordinarily incident at these latitudes. This means that for singly charged particles their momenta lie in the range 0 to 5 Bev/c. The actual factor by which the momentum spectrum of the primaries changes can be deduced and is given in Table V below; it is identical with the "change in transmission factor". More detailed examination of the ionization curves is difficult but seems to indicate that the particles causing the difference are absorbed in the same way as the particles ordinarily incident in these latitudes and hence are probably not much different in nature. We have already seen that there is no correlation between these fluctuations and sea level fluctuations. Most of the flights were made during the summer months so that one cannot explain the change as a seasonal one in the primary flux; in fact, the Omaha flights of 1938 were made in the middle of winter and show no discrepancy with the lower curve so we may take this as an indication that there is no such seasonal variation.

Because of the averaging process used in obtaining Fig. 6, it has seemed appropriate to call the fluctuations dealt with here long period ones. In the two month period of the author's measurements at Bismarck in 1946, however, although most of the flights agreed pretty well with those made in 1938 there was one (plotted separately in Fig. 6) that reached about one third of the way up to the 1940 curve. This might be classed with the short period variations considered earlier, but since it is so large it might be taken as indicating that the short period changes are just smaller instances of the type of fluctuation considered here. There exist no data taken simultaneously at different stations which would enable a test of this point.

Since there seems to have been no extraordinary terrestrial magnetic activity during the period of these measurements, it is natural to next turn to the sun for a hypothesis as to the cause of this fluctuation. The sunspot activity was not any different during 1940[21] than it was during several of the other periods of measurement and hence need not be considered. Another possibility is that the magnetic field of the sun changes. The solar field is generally credited with a cutting off action on the low energy particles incident on the earth[22]; a change in the solar field would change the amount cut out and this change would be observed at high latitudes which is just what is found to happen. Furthermore, measurements of the strength of the solar field made by Babcock[23] over a period of about ten years indicate that the field may be changing in strength.

To make a quantitative test of this hypothesis an approximate theory of the action of the solar field on cosmic ray particles developed by Epstein[22] may be used. He considered a solar field which was a dipole in nature with a polar field of 25 gauss. His formulae and curves are easily extended to other values of the field. In Fig. 7 are shown curves of the solar "transmission factor" versus momentum for various values of of the solar field expressed in gauss at the pole; by "transmission factor" is meant the fraction of the cosmic ray intensity at great distances from the solar system which will reach the earth in the presence of the given solar field. From our experimental results we can find the change in this transmission factor. Below we will plot the incident energy versus the minimum momentum for incidence rather than against geomagnetic latitude (Fig. 14). By taking the negative slopes of these curves we get the incident energy for a small range of momenta; by taking the ratio of the slopes we get the fractional change in this incident energy, or, since the average energy will be the same, the change in numbers of particles incident. This will also be the factor by which the transmission factor changes; these are given in Table V. In Fig. 7 the dashed curves represent what the transmission factor would have to have been in 1937, 1938, and 1946 to give the transmission coefficient corresponding to 0, 20, and 30 gauss fields in 1940.

TABLE V

Change in Transmission Factor or Change in Primary Momentum Spectrum

Momentum	Changed by a factor
1.5	3.4
2.5	2.4
3.5	1.6
4.5	1.5
5.5	1.2

It is evident that none of these curves will coincide with any computed transmission curve. This might be all right because the theory is only approximate; it considers the earth as a point in the magnetic field of the sun but with no magnetic field of its own. It is possible that a more accurate theory would give differently shaped curves which would give a better fit for the experimental points. A more serious objection is that apparently a very strong solar field would be required during the years of low intensity if its removal is supposed to cause increases such as that observed in 1940. Such strong solar fields would, as mentioned above, cause a cut off in the cosmic ray intensity below a certain energy. As discussed below, there is very little reason to suppose that such a cut off exists.

Some thought might be given to the possibility that these fluctuations are associated with the recently discovered heavy nuclei component of the primary flux[24]. The changes being discussed are of the order of 30% of the total incident energy or a factor of better than two in the energy due to particles of momentum-charge ratio less than five. The question of heavy nuclei will be discussed again below where it will be shown that they may have enough energy to be considered as a source of these variations. It has been pointed out above that the particles causing the difference in the two curves behave in pretty much the same way as those ordinarily incident at these latitudes; this does not rule out the heavy nuclei however because they are either absorbed or produce nuclear reactions at heights greater than that reached by the electroscopes and it would be their reaction products which would penetrate to greater depths. These reaction products would probably be no different than those produced by lighter particles.

The possibility of heavy nuclei is interesting because considerable difficulty is found in explaining how they might get their high energies in processes which would be adequate for lighter particles[25]. It is possible that the heavy and light particles are accelerated by different processes and that the one for the former is subject to some sort of variation.

E. The Heliomagnetic Cut-off

As mentioned above it has often been supposed that the solar magnetic field prevents particles of low energy from reaching the earth. The principal evidence for this has been the existence of a plateau of intensity above 50° geomagnetic latitude[26]. To establish that such a plateau existed it was necessary to compare the results of three different groups of observers and to normalize them suitably. Now, it is very unsafe to draw any conclusions in this manner; first, because the process of normalization is always a very uncertain one, and second, because of the existence of fluctuations of the sort found above. It was formerly supposed that the electroscope data used above, which extends up to 60°, was in agreement with the hypothesis of such a plateau[16]. This was based on the comparison of data without proper allowance for fluctuations. Fig. 6 shows absolutely no indication of any sort of plateau south of 60°.

The latitude data taken with telescopes in 1947 showed a very uneven latitude effect. We can, however, use the data taken in the airplane experiment described in Part I to normalize the 1947 curves and so construct what the latitude effect should have been. This particular normalization is a fairly safe one; it is done for nearly identical instruments at a common altitude, and after the airplane results have been corrected for the showers which were probably largely due to the structure of the airplane. This is done in Fig. 8. The old values of the maximum counting rates are shown with their probable errors; the normalized values are shown with the same probable errors although the errors should now undoubtedly be made much larger. The result is not perfect, but most of the irregularities have been eliminated. The source of the considerable discrepancy at 56° (Bismarck) is not known; it may very well be that at the pressure of 310 gm cm^{-2} where the normalization was made, the 1947 curve may have been quite inaccurate resulting in the low value shown. It is to be noted that this curve too now shows no plateau of intensity south of 60°.

Judging from these two results it seems fairly certain that the sun is not cutting out any particles which can arrive at 60°; such particles have momenta of about 1 Bev/c. From the curves in Fig. 7 this implies that if the sun has a magnetic field it is probably less than 10 gauss at the pole. This conclusion is in agreement with that reached by Oppenheimer, et.al.[27], from study of the heavy nuclei.

A heliomagnetic cut-off has been assumed necessary to account for the maximum observed for the soft component at high altitudes[28]. Janossy gives the following expression for the intensity y of the soft component at atmospheric pressure x when it is started by a continuous distribution of electron primaries including the very low energies:

$$y = A\ e^{-ax} + B\ e^{-bx}$$

A, B, a, b are constants. It is evident that this shows no maximum in the atmosphere. In Part III we will add to the accumulation of evidence that the primary particles are all positively charged and are therefore presumably not electrons. In that case the cascade component must start from electrons or photons formed in the atmosphere. In the particular example above one would have to find the intensity of the soft component from

$$y'(x) = \int_0^x y(x-t)\ \left|dN(t)\right|$$

where N(t) is the number of primary particles at depth t capable of giving rise to cascade producing radiation.

If $N = N_o e^{-kx}$, this becomes

$$y' = kN_o \left\{ \frac{A}{a-k} (e^{-kx} - e^{-ax}) - \frac{B}{b-k} (e^{-kx} - e^{-bx}) \right\}$$

an expression which does show a maximum in the atmosphere. Although this was presented merely as an example to show that a heliomagnetic or other form of cut-off is not a necessary assumption it is interesting to know that the maximum so predicted comes at about 50 to 100 gm cm^{-2} atmospheric pressure if k is taken as 1/125 gm cm^{-2} as is done by Rossi[29]. Actually the expression for y must be different from the one used.

III. GEOMAGNETIC ANALYSIS OF THE LATITUDE EFFECTS

A. The Theory of the Geomagnetic Analysis

The fundamental result of the analysis of the motion of charged particles in the magnetic field of the earth[30] is that if it were not for the atmosphere the intensity at any point of observation could be written to a good approximation as (see Appendix E for notation),

$$J = \int_{p}^{\infty} j(p')\, dp'$$

where j(p') is the momentum distribution of the particles at a large distance from the earth, and p is a minimum momentum which is a function of the sign of the charge, the geomagnetic latitude, the distance to the dipole, and the direction of observation. The function p for negative particles is the mirror image in the north-south plane of the function for positive particles. Possession of curves giving the dependence of this minimum momentum on these parameters is fundamental to any discussion of the results of a latitude effect experiment.

Vallarta and his colleagues[31] have calculated the values of the minimum momentum for the vertical direction in the latitude range 0° to 45°. For higher latitudes it is known that the values will approach those which can be calculated on the earlier theory of Störmer so it is possible to determine fairly accurately just how the curve of minimum momentum versus latitude should be drawn.

This curve is shown in Fig. 9.

Detailed calculations have also been made for all zenith and azimuth angles[31] for latitudes of 0°, 20°, and 30°. The author has scaled from the figures given by Vallarta the azimuth dependence of the minimum momentum at zenith angles of $22\frac{1}{2}^\circ$, 45°, and $67\frac{1}{2}^\circ$ for the equator. The curves for positive particles are given in Fig. 10. Instead of presenting the results for the minimum momentum directly, the dependence of the Störmer variable r is given; this variable is related to the minimum momentum by

$$p = 59.6\, r^2 \quad (Bev/c)$$

The latitude dependence of the minimum momentum for a zenith angle of 45° in either the east or the west is not known very well. Vallarta has published[32] pairs of curves, one of which (p_1) gives a lower limit to the minimum momentum, and the other of which (p_2) gives an upper limit. It is known that the desired curve should follow the p_2 curve near the equator and should then cross over in the intermediate latitudes till it meets the p_1 curve. Little is known theoretically of where it actually crosses over. This situation will be discussed more later. Fig. 11 shows the p_1 and p_2 curves for positive particles after they have been corrected as described below.

Before proceeding to use these curves, it is desirable to correct them for certain things, all of which are due to the fact that the dipole which best approximates the earth's field does not coincide with the center of the earth[33], but is located 342 km from it in latitude (geog.) 6.5°N and longitude 161.8°E. Also the axes are not parallel; the dipole axis is parallel to a line with coordinates: latitude 78.5°N and longitude 69°W. This eccentricity makes necessary corrections for the varying distance to the dipole and for the difference between the geographic and geomagnetic verticals. Also it is customary to compute geomagnetic latitudes ignoring the eccentricity; hence, a correction for variation in latitude should be made.

Some, at least, of these corrections were evidently computed once by Johnson[34]; however, the method of computation was not given and the results were presented in such a way as to be of but little use. General formulae will be given below for these corrections and the curves of Figs. 9, 10, and 11 will be corrected as for stations along the 80°W meridian; they will then apply with negligible correction to the stations in the United States where electroscope flights have been made.

The corrections will be assumed small so that they can be computed using differentials. For variations of distance from the dipole we use the relation

$$p = 59.6\ r^2$$

and the fact that r is directly proportional to the distance from the dipole. For the zenith and latitude corrections we can use the Störmer formula

$$\sin\beta = \frac{2}{r\cos\Lambda} - \frac{\cos\Lambda}{r^2}$$

Let α = distance from the center of the earth to the dipole divided by the radius of the earth (= 0.0536). Define the following unit vectors:

a from center of earth to observation point

b along dipole axis

c from center of earth to center of dipole

d = b x ρ, geomagnetic east at observation point

e direction of observation

x geographic east at observation point

y " north " " "

z " vertical " "

ρ from center of dipole to observation point.

Then the following formulae for the fractional correction dp/p to the minimum momentum are obtained: for the radius correction

$$\frac{dp}{p} = 2\alpha\,(a\cdot c)$$

for the latitude correction

$$\frac{dp}{p} = \alpha\,\frac{\sin\Lambda}{\cos^2\Lambda}\;\frac{2r - \cos^2\Lambda}{r - \cos^2\Lambda}\left\{(a\cdot b)(a\cdot c) - (b\cdot c)\right\}$$

which for the vertical becomes

$$\frac{dp}{p} = -4\,\alpha\,\frac{\sin\Lambda}{\cos^2\Lambda}\left\{(a\cdot b)(a\cdot c) - (b\cdot c)\right\}$$

for the zenith correction

$$\frac{dp}{p} = \frac{r^2\cos\Lambda}{r - \cos^2\Lambda}\left\{(x-d)\cdot e\right\}$$

which for the vertical becomes

$$= -\,\alpha\,\frac{\cos^3\Lambda}{2}\,(a\cdot b\times c)$$

Expressions for the different unit vectors will be found in Appendix D.

The corrections found from these formulae agree as closely as can be checked with those found by Johnson. Curves computed by Vallarta for the radius correction at certain latitudes are of the wrong sign[32].

It is possible to represent the minimum momentum curves for the azimuth experiments at the equator by rather simple analytical expressions. After corrections have been made, the formulae for the Störmer variable are,

at $z = 22\tfrac{1}{2}^\circ$ $\qquad r = \dfrac{0.493}{1 - 0.1033\sin\alpha}$

at $z = 45^\circ$ $\qquad r = \dfrac{0.518}{1 - 0.199\sin\alpha}$

at $z = 67\tfrac{1}{2}^\circ$ $\qquad r = \dfrac{0.574}{1 - 0.330\sin\alpha}$

where α is the azimuth angle measured from the north, positive in the east.

B. Experimental Results as Functions of Momentum

In Fig. 12 the counting rates obtained in the 1948 airplane experiment with vertical telescopes are plotted as a function of the minimum momentum for incidence in the vertical direction.

Lack of knowledge of the momentum curve prevents preparation of a similar curve for the intensity at the 45° zenith angles in the east and west. It is possible however to use the experimental data to fix the momentum curve. In a certain range of latitudes it is possible to choose two latitudes such that the intensity in the east at one is equal to the intensity in the west at the other; these should then be latitudes of equal minimum momentum for the respective directions. If vertical lines are drawn at these latitudes on the momentum versus latitude diagram, there should be a horizontal line, a value of momentum, which will intersect these vertical lines at points between the p_1 and p_2 curves furnished by Vallarta. When this process is carried out using the corrected curves, it turns out that there is no arbitrariness whatever. In fact, for the low latitudes the only choice is to take a line which cuts the p_2 curve of the west and the p_1 curve of the east and experimentally it is possible to do so. For the rest of the points it is possible to make use of the fact known from theory that the minimum momentum curve will follow the p_1 curve at high latitudes; since the momentum curve in the east has already reached the p_1 curve, it can be assumed to follow it for all higher latitudes and thus to fix the position of the momentum curve for the west. Points plotted in this manner and the curve so determined are shown in Fig. 11. Using this curve and the eastern p_1 curve one can also plot the zenith results as functions of momentum as was done for the vertical intensity; this is done in Fig. 13. Points taken from the curves are plotted to show just how well this process makes the two sets of measurements agree; this agreement, of course, has been forced.

The experimental fact that the results can be fitted into the geomagnetic theory in the manner described using the curves for positive primary particles is a good argument that if there are any negative primaries they are relatively quite small in number.

To compare the zenith data and the vertical data as functions of momentum, the former have been normalized to the latter at $p = 7$ Bev/c and plotted as the dashed curve in Fig. 12. Evidently the agreement is excellent.

Here is another even better proof that the primaries with which we are dealing are predominantly positive. Because of the mirror property of the minimum momentum functions for oppositely charged particles, the ratio of the intensity at 45° zenith angle to the intensity at the vertical will be constant only if one sign of charge is present.

The results from the azimuth experiment at the geomagnetic equator have been corrected to the lower altitude of the latitude measurements by means of the observed altitude dependence at the 45° zenith angle and also by means of the altitude dependence of the vertical radiation at, however, a depth corresponding to the greater zenith angle. The corrected result is shown in Fig. 13 for comparison with the latitude data at the same zenith angle.

Examination of Figs. 12 and 13 shows that there are some regions in which the intensity shows an odd behavior. For p greater than 12 Bev/c for the vertical and greater than 14 Bev/c for the eastern zenith data and 9 Bev/c for the western, the intensity seems to flatten off in a peculiar manner. Reference back to the momentum curves shows that these are all characterized by being data taken at geomagnetic latitudes less than 20°. The azimuth data indicate the same sort of effect; the curve is much flatter than for comparable latitude data.

A tendency toward a similar effect has been noticed previously in the sea level data taken off Peru[35]. At about 20° the intensity seems to flatten off at times, and gives no further change on going down to the equator. The sea level curve taken on the other side of the earth shows a steady decrease all the way to the equator. A smaller decrease is to be expected near Peru due to the longitude effect, but an absolute lack of change in intensity is unexpected.

Related to this same discrepancy is the fact that several authors[36] have estimated the percentage east-west effect to be expected at the equator and have obtained answers ranging from 40 to 60%. If our latitude data are extrapolated in a way that seems quite reasonable, the estimate is nearly 70%. The experimental results are 23% at 310 gm cm^{-2} and 29% at 235 gm cm^{-2}.

These facts might seem to indicate that somehow radiation is being kept out of this particular region of the earth. Evidence to be presented later will indicate that all intensities are down by a factor of about two.

One hypothesis that might lead to an explanation of this effect is that there may be a large magnetic anomaly off the coast of South America which is sufficient to affect the motion of cosmic ray particles. Maps of the earth's magnetic field showing lines of equal horizontal force show rather queer behavior off South America; there exists a very large region in which there is almost no change in the horizontal component; along the coast these lines run almost north and south. On the other hand Bauer[37] finds that, after subtracting the dipole field from the earth's observed field, the residual field off the South American coast is as small or smaller than the residual field over the United States. In the opinion of the author, an equally good hypothesis is that the calculations of minimum momentum are at fault. In the equatorial regions these depend on a very complex theory of forbidden and allowed regions for entry; if these regions are sensitive to the presence of non-dipole components of the earth's field, it may very well be that they are inaccurate to such an extent as to introduce the discrepancies above.

As remarked earlier, the corrected momentum curves apply with negligible error to the stations in the United States from which electroscope flights were made. The stations in India need to be calculated separately; the momenta for vertical incidence are found to be as follows: Peshawar, 12.0; Agra, 13.8; and Bangalore-Madras, 15.6 Bev/c. With this additional data the electroscope data used in Part II can be plotted as a function of the momentum for vertical incidence; this is done in Fig. 14.

C. The Momentum Distribution of the Primary Particles

It is known that the area under the ionization versus depth curve can be changed into the energy incident at the top of the atmosphere[38]. To be precise, it is approximately (See Appendix E for notation)

$$\int_{p}^{\infty} \overline{W}\, j_{\square}(p)\, dp$$

One approximation consists in setting a unique lower limit to the integral. Actually there will be a range of momenta which will not get in with full intensity but should be weighted by a factor increasing from zero to one. At low momenta, however, this range in momentum is not great; at high momenta the value of p for which the weight function would be $\frac{1}{2}$ is generally used. This should be fairly accurate at low momenta, but may be in error by an uncertain amount at high momenta where the cones of entry of particles are very distorted and where the spectrum of particles is changing more rapidly. The other approximations in this expression which are due to experimental faults and possible loss of energy by effects other than ionization have recently been discussed by Montgomery[38].

The quantity which one would like to deduce from the electroscope measurements is j(p). We can evidently find $\overline{W}\, j_{\square}(p)$ approximately by taking the negative derivative of the curve giving total energy versus minimum momentum for the vertical as given in Fig. 14. Then, if we assume that the radiation is uniform from above, there is a factor π between j and $j_{\square}$[39]. Finally we must consider $\overline{W}$. It is the average energy of the particles with momenta between p and p + dp; that it is not necessarily unique is due to the fact that we may be dealing with particles of different rest mass.

So far as the geomagnetic analysis is concerned, a particle with charge Z and momentum $\frac{p}{Z}$ is equivalent to a singly charged particle of momentum p. For nuclei of atoms which are completely ionized, the rest mass is about 2Z times that of a proton. Hence, if M is the rest mass of a proton, the energy of such a particle would be larger than that of the corresponding proton by a factor,

$$\left\{ \frac{\frac{1}{Z^2}\left(\frac{p}{Mc^2}\right)^2 + 4\ Z^2}{1 + \left(\frac{p}{Mc^2}\right)^2} \right\}^{\frac{1}{2}}$$

Such fully ionized heavy nuclei have been observed by Oppenheimer and his colleagues[24]. They find values of p/Mc^2 from 1 to 2, for which the above factor is slightly more than Z. For the heavy nuclei they find an intensity of 1/500 that of the protons and an average Z of about 20. This means they carry about 18% as much energy as the protons of the same momentum. The ratio of the number of alpha particles to protons is much larger, $\frac{1}{4}$; consequently they may carry about as much energy as the protons.

Such figures as these might mean serious difficulty in determining $\overline{W}$ if it were not for other considerations. The range of the heavy nuclei themselves is very short; alpha particles of the momenta observed would penetrate only about 25 gm cm^{-2} of air. The electroscope data used in Part II did not go to such pressures. The reaction products of these nuclei might travel farther, but the mean range for absorption is apparently somewhat smaller than that for nuclear interactions; so probably less than one half of these nuclei will have reactions.

Finally the p/Z ratios observed are of the order unity, so these heavy fragments could have an effect only at the highest latitudes. In general then, these nuclei will be ignored and it will be assumed that W is the energy of a proton of momentum p.

On the basis of this theory Millikan, Neher, and Pickering, have prepared a curve for $W\ j_{\square}\ (p)$ from which j (p) can be derived[40]. This gives an exponential curve which cannot be valid over too great a range because integrated to infinity it does not give the right energy for the electroscope curve at the equator. The process of differentiation mentioned above was replaced by taking finite differences between different electroscope curves; this process is, of course, subject to considerable error in the case of small differences.

In Fig. 14 we have total energy versus momentum. From this curve, using the data of 1936, 1937, 1938, 1946, and the above theory, the following expressions for j (p) are found:

$$2 \leqslant p \leqslant 11 \text{ Bev/c} \qquad j\ (p) = \frac{(3.18 \pm 0.1) \times 10^{-2}}{p^{1.00 \pm 0.05}}$$

$$12.5 \leqslant p < \infty \text{ Bev/c} \qquad j\ (p) = \frac{4.75 \pm 0.3}{p^{3.0 \pm 0.2}}$$

These are plotted in Fig. 15. The errors indicated were estimated from the accuracy with which the curves could be drawn. Numerically the results are not much different from that of Millikan, Neher, and Pickering, which is plotted as the dashed curve in the same figure; but they are somewhat simpler and give the right energy for the equator when integrated to infinity. The p^{-3} dependence at high energies agrees roughly with that of other authors who base their estimates on different sorts of experiments[41]. The analysis of the next section does not depend critically on which curve is taken for j (p), but the one here deduced will be used.

D. Relation of Telescope Readings to Primary Intensity

It is now desired to relate the reading of a telescope to what was found above for the primary intensity. To do this, the concept of the intensity j' (p) at the point of observation due to primaries with energies between p and p + dp is introduced. Then, ignoring the effects of scattering, the telescope reading is given by,

$$J' = \int_p^\infty j'(p')\,dp'$$

where p is the minimum momentum as before. Here there is no approximation for the lower limit as there was for the electroscope case. Evidently j' is found by taking the negative derivative of the counting rate curve versus momentum.

Before evaluating j' for the airplane data, we should determine how the effects of the fluctuations discussed in Part II will enter. In Fig. 16 are plotted the readings of the ionization chamber carried in the airplane (solid curve) and readings of balloon electroscopes scaled from the depth-ionization curves for different years (dashed curves). The curves cannot be expected to be comparable at low latitudes because of the anomaly discussed above and the longitude effect. At high latitudes they cannot be compared because of fluctuations. In intermediate latitudes they might be compared. The instrument in the airplane will be expected to read much higher becasue of the showers from the airplane, the presence of radioactive dials, and the greater burst production in the walls which were thicker than for the balloon electroscopes. If we normalize the curves at intermediate latitudes, it is seen that at high latitudes the airplane data will coincide with that taken in balloons in 1937, 1938, 1946; hence, we may properly take the j (p) curve deduced above in the analysis of the airplane data.

If the average of the vertical and the normalized 45° zenith angle data (Fig. 12) is plotted on log-log paper as in Fig. 17, a region at high momenta follows a $p^{-0.635}$ power law from which we deduce,

$$10 \leqslant p \text{ Bev/c} \qquad\qquad j'(p) = \frac{0.546 \pm 0.015}{p^{1.64 \pm 0.04}}$$

If the intensity at high latitudes minus the intensity at a given momentum is plotted on log-log paper versus that momentum, as in Fig. 18, it is found to follow a p^2 law from which we deduce,

$$2 \leqslant p \leqslant 10 \text{ Bev/c} \qquad j'(p) = (9.40 \pm 0.45) \times 10^{-4} p^{1.00 \pm 0.06}$$

The ratio $j'(p)/j(p) = m(p)$ has been named the multiplicity. This is somewhat unfortunate because there are many multiple processes being studied in cosmic ray investigations and there are many different "multiplicities". From the results above we find,

$$2 \leqslant p \leqslant 10 \text{ Bev/c} \qquad m(p) = (2.95 \pm 0.2) \times 10^{-2}\, p^{2.00 \pm 0.08}$$

$$10 \leqslant p \text{ Bev/c} \qquad = (0.115 \pm 0.006)\, p^{1.36 \pm 0.2}$$

These curves are given in Fig. 19.

The multiplicity can also be determined from the azimuth experiment although the accuracy of the momentum dependence is not great. It has been shown above that J' is of the form,

$$J' = A - B \sin \alpha$$

and

$$r = \frac{c}{1 - D \sin \alpha}$$

now,

$$j'(p) = -\frac{\partial J'}{\partial p}$$

$$= - \frac{\partial J'}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} \right)^{-1} \frac{\partial r}{\partial p}$$

Using this expression, the data at 45° zenith angle after correction for altitude gives,

$$10 \leqslant p \leqslant 23 \text{ Bev/c} \qquad m(p) = (0.055 \pm 0.001)\, p^{1.5 \pm 0.2}$$

This is also plotted in Fig. 19. The momentum dependence is only slightly different, but the magnitude is off by a factor of about 2. This is further evidence for the nature of the anomaly south of 20° geomagnetic latitude.

Some theoretical estimates have been made of late by Oppenheimer, Chew, and others[42] of the "multiplicity" of meson production by primary particles. The momentum dependence predicted is usually p^{α} with $1/3 \leqslant \alpha \leqslant \frac{1}{2}$. The fact that this is not the momentum dependence obtained above should not be surprising. Consider a very simple case; suppose radiation were incident only from the vertical. Then for each particle there will be a region (Fig. 20) in which it or its reaction products might be detected. At any cross section AA' there will be a probability function P(r) for the particle being detected. This P(r) will depend both on the number of particles formed in the initial acts and on subsequent scattering processes. For a uniform radiation, the response of the instrument will be proportional to

$$2\pi \int_0^{\infty} r\, P(r)\, dr$$

which will further complicate matters. In the actual case, allowance would have to be made for rays incident at different zenith angles and for the directions of the secondary particles. This could only be done if a very detailed knowledge of the multiplication and scattering processes were available.

The rapid decrease of the multiplicity function, shown in Fig. 19, makes it easy to understand why the intensity might appear to level off at high latitudes and cause a plateau which might be interpreted as due to a heliomagnetic cut-off. Since $j'(p) = m(p)\, j(p)$, it seems much more natural to assume the approach of j' to zero to be due to the vanishing of m rather than the vanishing of j.

APPENDIX A CALCULATION FOR TELESCOPE APERTURE

The calculation for a vertical telescope in $j_o \cos^2 z$ radiation field has been made by Montgomery[5]. This is here generalized to a telescope with axis at zenith angle θ in a $j_o \cos^\alpha z$ field. Referring to Fig. 21 the counting rate k of a telescope is given by

$$k = \iiiint j_o \cos^\alpha z \quad \frac{\cos^2\Delta \, dx \, dy \, d\xi \, d\eta}{r^2}$$

We have the relations

$$\cos z = \cos\Delta \cos\theta \left[1 - \tan\theta \left(\frac{\xi - x}{L} \right) \right]$$

$$r^2 = L^2 \left[1 + \left(\frac{\xi - x}{L} \right)^2 + \left(\frac{\eta - y}{L} \right)^2 \right]$$

which can be expanded in series and the integration performed term by term. The result, accurate to better than ½% for A/L about 1/3, is

$$k = j_o \cos^\alpha\theta \, \frac{A^2 B^2}{L^2} \left[1 - \left(\frac{\alpha + 4}{12} \right) \left(\frac{A^2 + B^2}{L^2} \right) + \frac{\alpha(\alpha - 1)}{12} \tan^2\theta \left(\frac{A}{L} \right)^2 + \left(\frac{\alpha^2 + 10\alpha + 24}{120} \right) \left(\frac{A^4 + 5/6 A^2 B^2 + B^4}{L^4} \right) - \frac{(\alpha + 4)\alpha(\alpha - 1)}{60} \tan^2\theta \left(\frac{A^4 + 5/12 A^2 B^2}{L^4} \right) \right].$$

At the 90° zenith angle a similar integration can be carried out making j = 0 for z less than 90°. The result is

$$k = j_o \frac{A^2B^2}{L^2}\left(\frac{A}{L}\right)^{\alpha}\Bigg[\frac{1}{(\alpha+1)(\alpha+2)}$$

$$-\left(\frac{\alpha+4}{2}\right)\left(\frac{A^2}{(\alpha+3)(\alpha+4)L^2} + \frac{B^2}{6(\alpha+1)(\alpha+2)L^2}\right)$$

$$+\left(\frac{\alpha^2+10\alpha+24}{8}\right)\left(\frac{A^4}{(\alpha+5)(\alpha+6)L^4} + \frac{A^2B^2}{3(\alpha+3)(\alpha+4)L^4} + \frac{B^4}{15(\alpha+1)(\alpha+2)L^4}\right)\Bigg]$$

APPENDIX B COUNTING RATES OF SINGLE COUNTERS

This calculation was not mentioned in the above report but is included here because of its general interest.

Assume a $j_0 \cos^\alpha z$ radiation field and compute the counting rate of a flat plate of unit area including rays going in both directions. Referring to Fig. 22a the rate K' is

$$K' = \int j_0 \cos^\alpha z \, d\omega$$

$$= j_0 \int_{-\infty}^{\infty}\int_{-\operatorname{ctn}\theta}^{\infty} \frac{\xi \sin\theta + \cos\theta}{(1+\xi^2+\eta^2)^{\frac{\alpha+4}{2}}} \, d\xi \, d\eta$$

$$- j_0 \int_{-\infty}^{\infty}\int_{-\infty}^{-\operatorname{ctn}\theta} \frac{\xi \sin\theta + \cos\theta}{(1+\xi^2+\eta^2)^{\frac{\alpha+4}{2}}} \, d\xi \, d\eta$$

When α has integral values, these are elementary integrals and the results are:

$\alpha = 0$ $\qquad K' = \pi j_0$

$= 1$ $\qquad = \frac{2}{3} j_0 \left\{ 2\sin\theta + (\pi - 2\theta)\cos\theta \right\}$

$= 2$ $\qquad = \frac{\pi j_0}{4} \left\{ 1 + \cos^2\theta \right\}$

$= 3$ $\qquad = \frac{8 j_0}{15} \left\{ \sin^5\theta + \sin^3\theta\cos^2\theta + \frac{3}{2}\sin\theta\cos^2\theta + \frac{3}{2}\left(\frac{\pi}{2} - \theta\right)\cos\theta \right\}$

Note that the cases when the plate is horizontal or vertical can always be integrated by methods used by Van Allen[43]:

Horizontal: $$K' = \frac{2\pi}{\alpha+2}\, j_0$$

Vertical: $$K' = \frac{4\,\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{\alpha+1}{2}\right)}{\alpha(\alpha+2)\,\Gamma\left(\frac{\alpha}{2}\right)}$$

Knowing the rates for flat plates, the rate K for a cylindrical counter is easily computed. Referring to Fig. 22b; take $\alpha = 2$ as an example:

Ends: $$\frac{\pi j_0}{4}\left(1+\cos^2\theta\right)\pi r^2$$

Cylinder: $$\frac{\pi j_0}{4}\int_{-\pi/2}^{\pi/2} r\, l\left(1+\cos^2\alpha\,\sin^2\theta\right)d\alpha$$

$$= \frac{\pi^2 j_0 r l}{4}\left(1+\tfrac{\pi}{2}\sin^2\theta\right)$$

So

$$K = \frac{\pi^2 j_0 r l}{4}\left\{\left(1+\frac{r}{l}\right)+\left(\frac{\sin^2\theta}{2}+\frac{r\cos^2\theta}{l}\right)\right\}$$

Similarly, for $\alpha = 1$:

$$K = \frac{2 j_0 r l}{3}\left\{8E(\sin\theta) - 4\cos^2\theta\, K(\sin\theta) + \frac{\pi r}{l}\left[2\sin\theta + (\pi-2\theta)\cos\theta\right]\right\}$$

For r = 1.65 cm and l = 23.1 cm, turning a counter from vertical to horizontal will drop the counting rate by 27% for $\alpha = 2$, 17% for $\alpha = 1$, and 0% for $\alpha = 0$.

APPENDIX C

Cosmic-Ray Effects from Solar Flares and Magnetic Storms

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Cosmic-ray data taken during the period of a solar flare and the magnetic storm that followed 26.5 hours later during July, 1946, are reported. The results following the flare agree with those of other investigators and, in addition, serve to establish the fact that the start of the cosmic-ray effect and the visual part of the solar flare were simultaneous. It is pointed out that increases of cosmic rays during solar flares suggest a mode of origin of the rays. The lack of effect of most solar flares on cosmic-ray intensity is noted and differences in intensity of the flares given as a possible reason. A high altitude balloon flight with an electroscope during the magnetic storm gave results in agreement with the current-sheet hypothesis of such storms but a serious objection to this hypothesis is given.

FORBUSH[1] has reported a sudden increase in cosmic-ray intensity following a solar flare on July 25, 1946. He had noted a similar effect in 1942. His previous observation had been confirmed by Duperier.[2] This last increase was also observed in England by Dolbear and Elliott.[3] During the period of this phenomenon we had an unshielded self-recording electroscope in operation at the Mount Wilson Observatory. The results from this instrument agree with those noted above and were briefly as follows: On July 25, 1946, after an intense flare over a spot on the sun, the cosmic-ray intensity increased about 18 percent. The prominent part of the increase was over in about ten hours. However, the intensity was still 1 or 2 percent above normal when a sudden large magnetic storm began 26.5 hours after the start of the flare. At this time there was a decrease in intensity of the type usually noted during magnetic storms.[4]

No increase in cosmic rays was reported by Forbush for the flares of either 1942 or 1946 at the equator. This suggests that, as Forbush has pointed out, the cosmic-ray increase was due to charged particles emitted by the sun and that their maximum energy was not sufficient to permit them to reach the earth at the magnetic equator but did permit them not only to come in at a magnetic latitude of 40° but also to penetrate the earth's atmosphere to sea level. The energies of at least some of these particles then seems to be less than 10 Bev but greater than 6 Bev.

One objection raised to the assumption that particles of these energies were actually emitted by the sun at this time is that the cosmic-ray "flare" did not occur simultaneously with the visual flare as one might expect for particles of these energies. The published data seem to indicate such a difference in times. This particular difficulty may possibly be removed by the following result. Our instrument gave readings every fifteen minutes; when hourly readings, corrected for barometric variations, are plotted against the time, it is seen that the cosmic-ray increase began at about the same time as the flare. Figure 1 shows the percentage increase in cosmic-ray intensity during this period. Plotted also is an optical observation of the flare, the relative change in the line width of the alpha-line in the Balmer series of hydrogen, as observed in the flare by M. A. Ellison[5] in Great Britain. These two results strongly suggest that the visual flare and the cosmic-ray "flare" began simultaneously. They reached their respective maximums, however, 2.2 hours apart. It is interesting to note that both the rise and fall of the cosmic-ray effect was exponential, as shown in Fig. 2.

[1] S. E. Forbush, Phys. Rev. **70**, 771 (1946).
[2] A. Duperier, Proc. Phys. Soc. **57**, 473 (1945).
[3] D. W. N. Dolbear and H. Elliott, Nature **159**, 58 (1947).
[4] T. H. Johnson, Rev. Mod. Phys. **10**, 193 (1938).
[5] M. A. Ellison, Nature **158**, 450 (1946).

According to the observations at Mount Wilson[6] solar flares occur at rates of from 2 to 10 per hundred hours. The majority of such flares would seem to be ineffective as regards cosmic rays or the effect would have been observed oftener. For example, a flare was observed July 21, 1946, four days before the one under discussion with no appreciable influence on cosmic rays. It is possible that the effective ones are characterized by unusually great intensity. Ellison[5] reports that this flare showed characteristics such as enhancement of the continuous spectrum which suggest unusually great intensity. Johnson and Korff[7] have reported a balloon flight with a Geiger counter during which a flare took place with no observable effect. This might be explained in two ways: (a) the flare may not have been an effective one, or (b) the flare may have occurred at such a time that its effect had not become noticeable during the time of the flight.

The fact that our sun appears to give off electrically charged particles at rare intervals having energies lying in the cosmic-ray region suggests a possible mode of origin of cosmic rays. It is at once apparent, however, that due to the infrequent occasions when our sun has given rise to such particles, that it cannot be a representative star if all cosmic rays have their origins in such flares. This is due to the fact that the total energy in visible star light is about the same as the total energy in cosmic rays[8] and our sun emits far more energy in visible light than it does energy in the cosmic-ray region.

For charged particles to leave the sun, particularly near its equator where a general magnetic field of the sun would make escape most difficult, some mechanism such as that suggested by Vallarta[9] would need to be invoked.

It should be pointed out that no decrease in the general magnetic fields either of the sun or the earth seem to explain this sudden increase in cosmic-ray intensity. Any decrease in the sun's field would affect only those particles having energies below about 2 Bev and these do not send their effects down to sea level. Any decrease in the earth's magnetic field would affect the equatorial regions at sea level but should not be felt at sea level farther north or south than about 40° geomagnetic latitude (see below). Any increase in the magnetic fields of either body could only decrease the cosmic-ray intensity. Magnetic records showed no unusual changes of the earth's magnetic field at the time of the flare.

The fact that no particular direction in space, in particular the direction of the Milky Way, appears to be a preferred direction for the incoming cosmic rays would not seem to be an objection to a theory of stellar origin because it has recently been discovered[10] that many stars possess large magnetic fields. It would be expected then that cosmic rays originating at a particular point would be rendered practically isotropic after passing through a number of stellar fields.

The magnetic storm following the flare by 26.5 hours and its effect on cosmic rays was of the kind usually noted. The ratio $H\Delta I/I\Delta H$ was about 10 which is of the order of magnitude observed in other storms. While the cosmic-ray intensity was at its lowest during this storm, a high altitude balloon flight with an electroscope was made near Fort Worth, Texas. The ionization was lower than that observed during previous and following flights. The peak of the curve was about 18 percent below that of the average for the flights made at Fort Worth during the

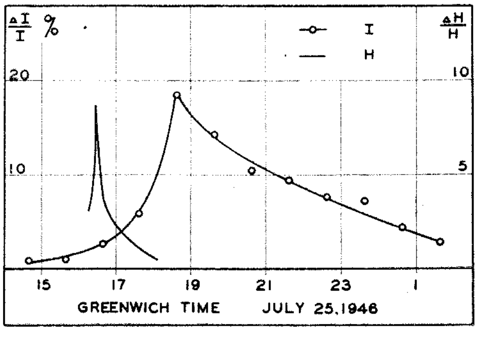

FIG. 1. Curves of (I) hourly readings of the percent increase of the cosmic-ray intensity above its pre-flare average; (H) the relative increase in the width of the H_α line above its normal value (Ellison[5]).

[6] Elizabeth Sternberg Mulders, Astro. Soc. Pac. **59**, 16 (1947).
[7] S. A. Korff, Rev. Mod. Phys. **11**, 211 (1939).
[8] I. S. Bowen, R. A. Millikan, and H. V. Neher, Phys. Rev. **44**, 246 (1933).
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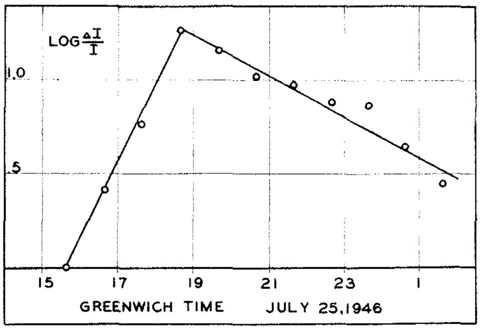

FIG. 2. Logarithm of the percent increase of cosmic-ray intensity above its pre-flare average.

summer of 1946. At the same time, the electroscope at Mount Wilson read 5 to 6 percent below normal. This should be comparable to the ground level reading at Fort Worth, as the two differ but slightly in magnetic latitude. The curve of ionization *vs.* depth below the top of the atmosphere was very similar to the one ordinarily obtained at San Antonio, 3.2° (magnetic) further south; it is not included here because the barometer data are not reliable. This is just the sort of thing predicted by any theory which treats magnetic storms as a weakening of the effective dipole moment of the earth.

Objections based on cosmic-ray phenomena have already been raised to theories such as the current-sheet theory which results in the weakening of the earth's dipole.[4] We should like to point out another objection. Changes in the dipole moment will shift the latitude of entry of a given energy particle either north or south. If one considers the sea level latitude effect, it is evident that the edge of the plateau usually found near 40° (magnetic) will shift either north or south, but unless this shifting is tremendous no change in sea level intensity should be felt in the high latitudes. However, there are reports of changes in cosmic rays as far north as 75° during storms[7] while the facts just reported (which ought to be quite typical) indicate that the latitude shifts would be only a few degrees.

We wish to thank Dr. S. B. Nicholson of the Mount Wilson Observatory for providing us with magnetic data and for looking after our electroscope while it was at the Observatory.

APPENDIX D CALCULATION OF CORRECTIONS

Expressions for vectors used in the theory:

$$a = \begin{pmatrix} \cos\lambda \cos\varphi \\ \cos\lambda \sin\varphi \\ \sin\lambda \end{pmatrix}$$

λ and φ geographic latitude and longitude

$$b = \begin{pmatrix} .0712 \\ -.186 \\ .980 \end{pmatrix}$$

$$c = \begin{pmatrix} -.944 \\ .310 \\ .113 \end{pmatrix}$$

$$b \times c = \begin{pmatrix} -.325 \\ -.933 \\ -.153 \end{pmatrix}$$

$$b \cdot c = -.0141$$

$$e = x \sin z \sin\alpha + y \sin z \cos\alpha + a \cos z$$

$$x = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \qquad y = \begin{pmatrix} -\sin\lambda \cos\varphi \\ -\sin\lambda \sin\varphi \\ \cos\lambda \end{pmatrix}$$

$$z = d.$$

APPENDIX E DEFINITION OF INTENSITIES

j dA dw dt dp = number of particles crossing area dA in time dt with directions in solid angle dw and momenta between p and p + dp.

j' is the number of particles as above but due to primaries with momenta between p and p + dp

In either case

$$J = \int_{p}^{\infty} j \, dp$$

The number of particles crossing a unit horizontal area is given by:

$$J_{\square} = \int J \cos z \, d\omega$$

also

$$j_{\square} = \int j \cos z \, d\omega$$

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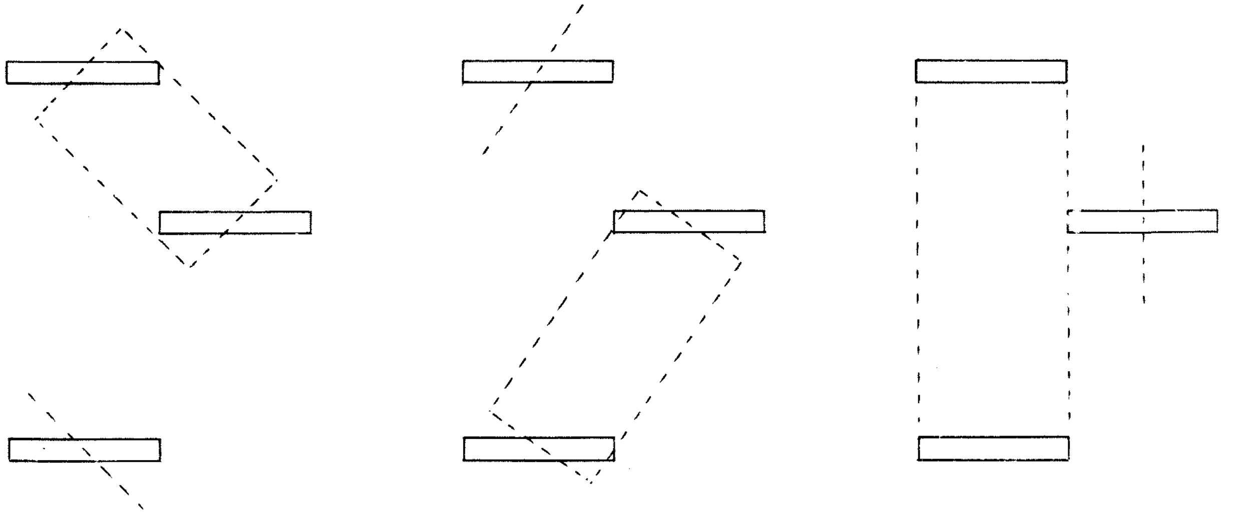


Fig. 1 System of equivalent telescopes and side trays for use in evaluating the side shower correction.

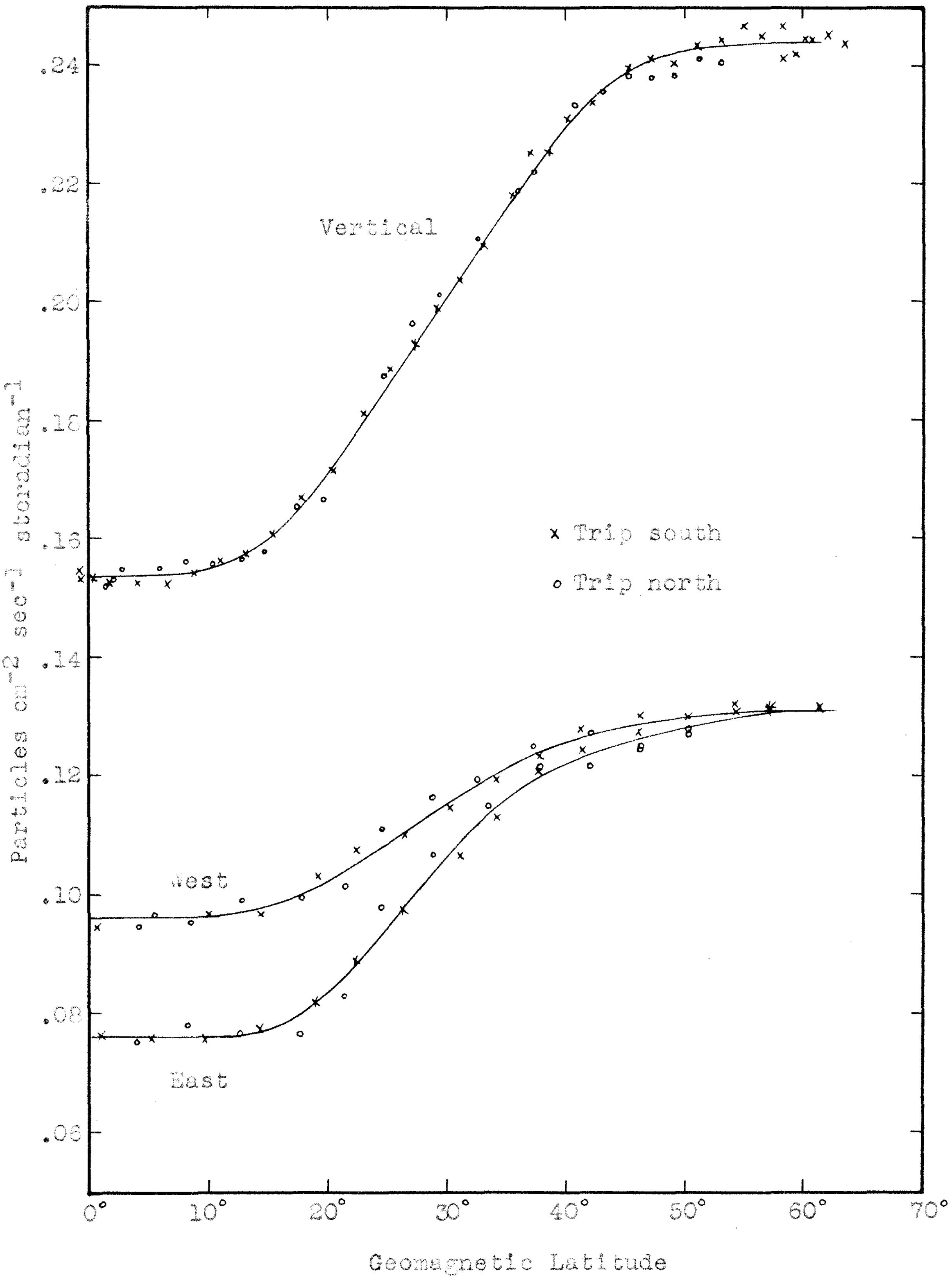


Fig. 2 Intensity with no lead absorber at the vertical and at 45^{0} zenith angle in the east and west versus geomagnetic latitude (pressure 310 gm cm^{-2}).

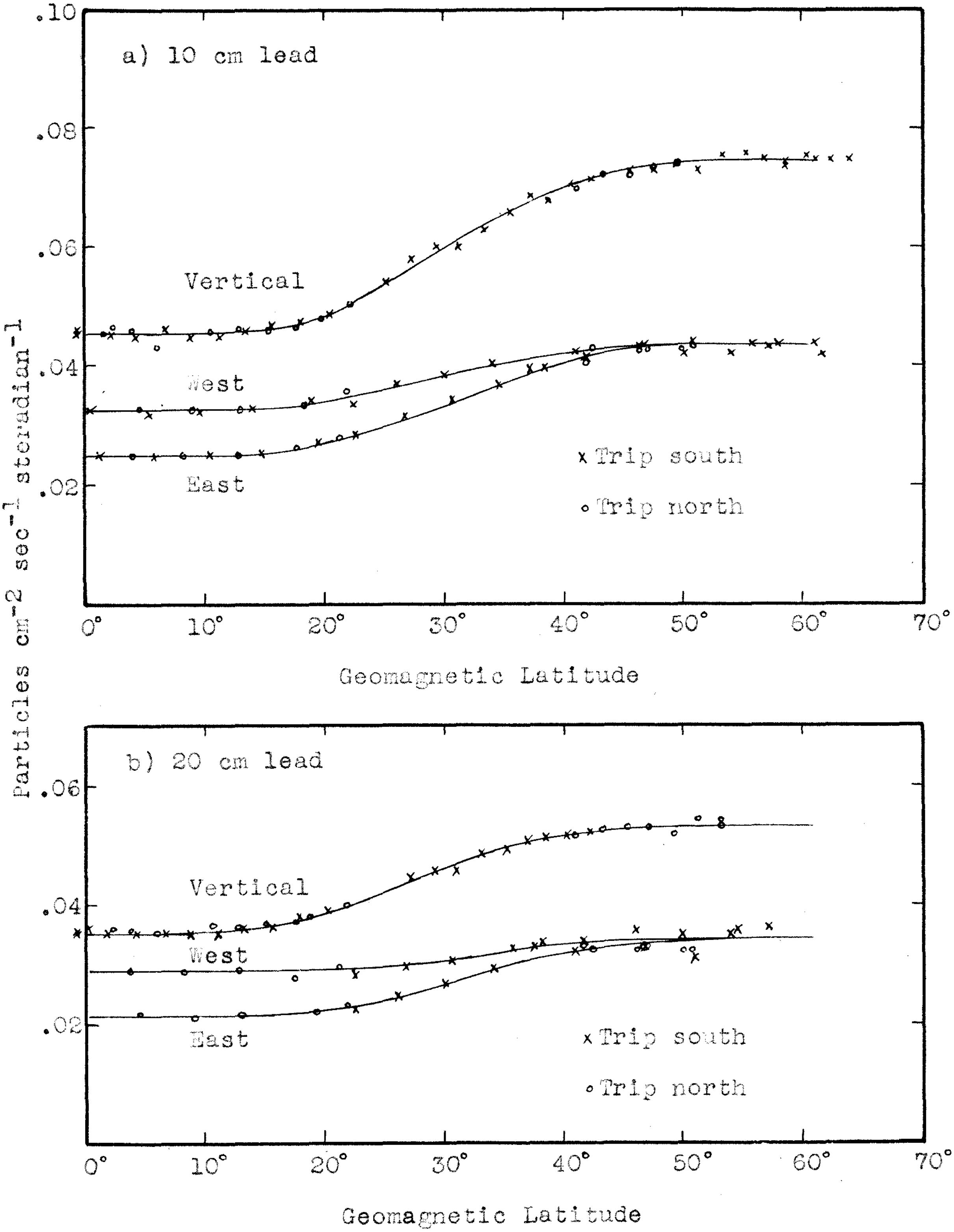


Fig. 3 Intensity with a) 10 cm and b) 20 cm of lead absorber at the vertical and at 45^{0} zenith angle in the east and west versus geomagnetic latitude (pressure 310 gm cm^{-2}).

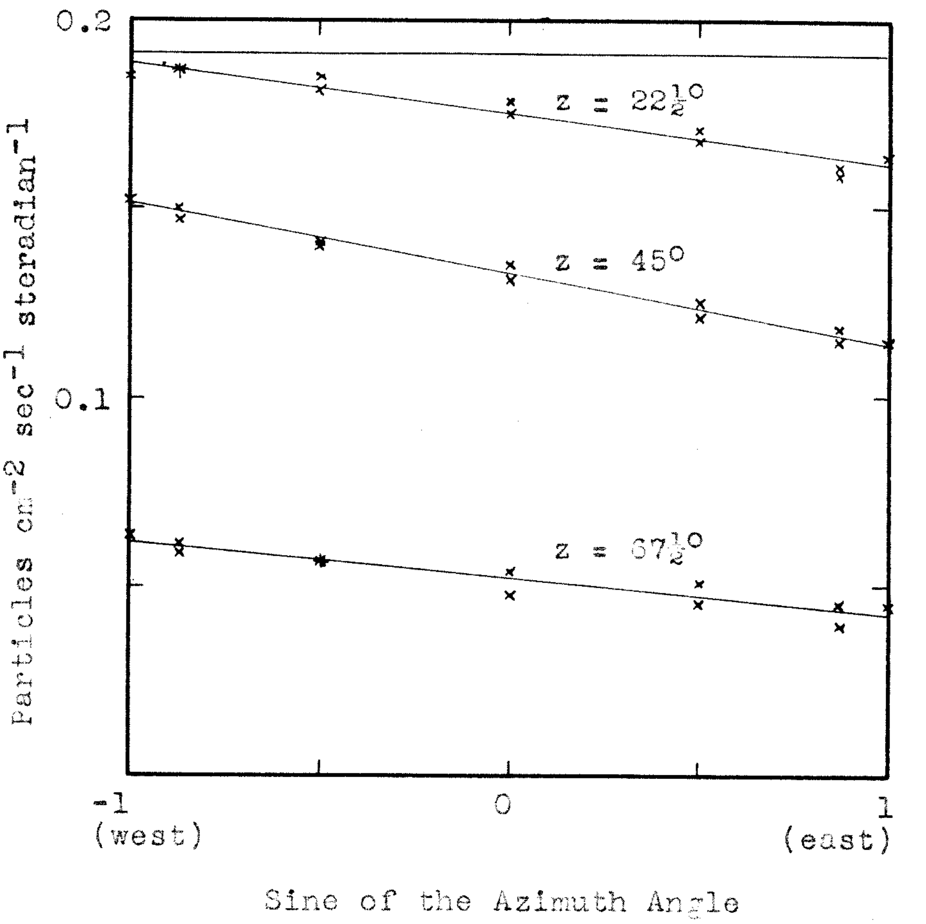


Fig. 4 Intensity with no lead absorber at the geomagnetic equator versus the sine of the azimuth angle for various zenith angles (pressure 235 gm cm^{-2}).

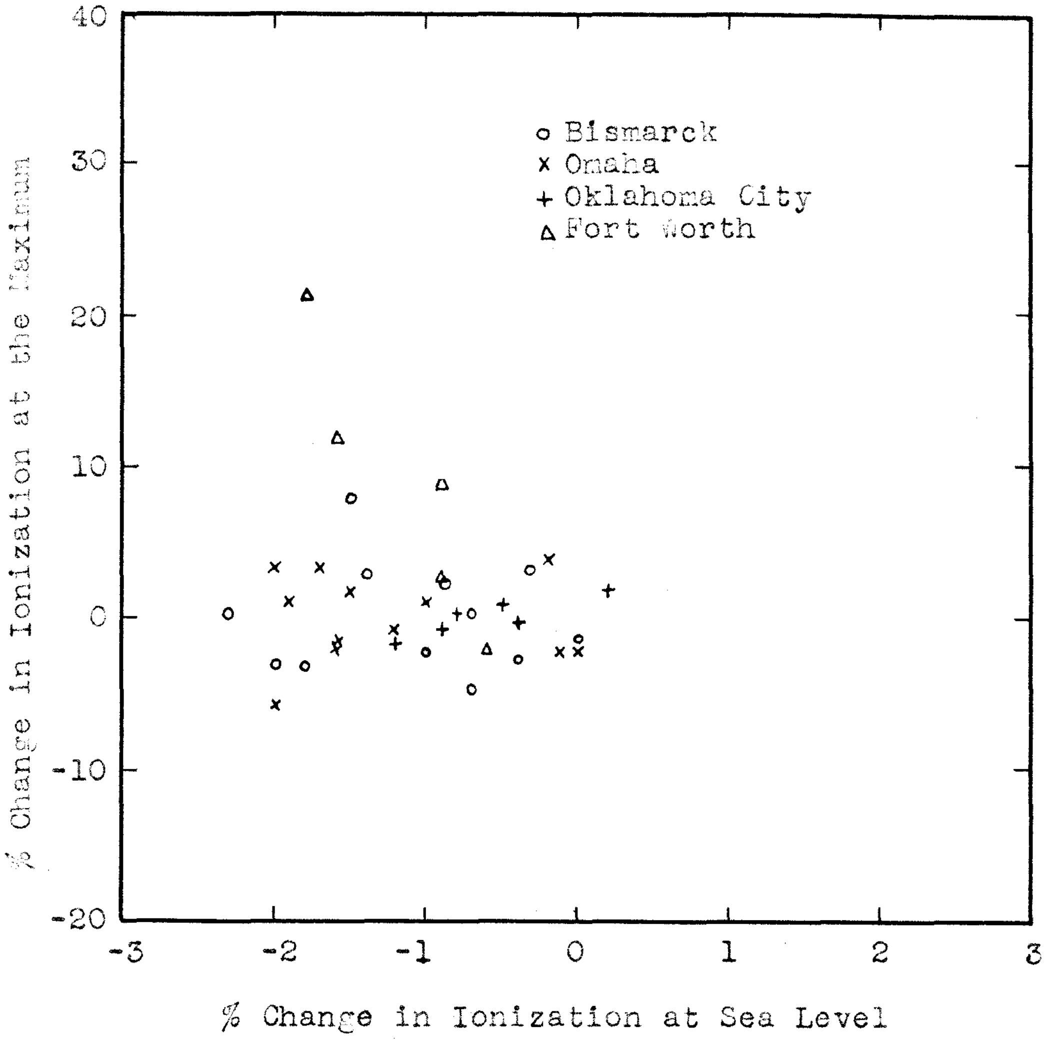


Fig. 5 Percentage change at the maximum of the intensity versus depth curve from the average value for a series of flights versus the sea level deviations at the time of the flights. (Electroscopes)

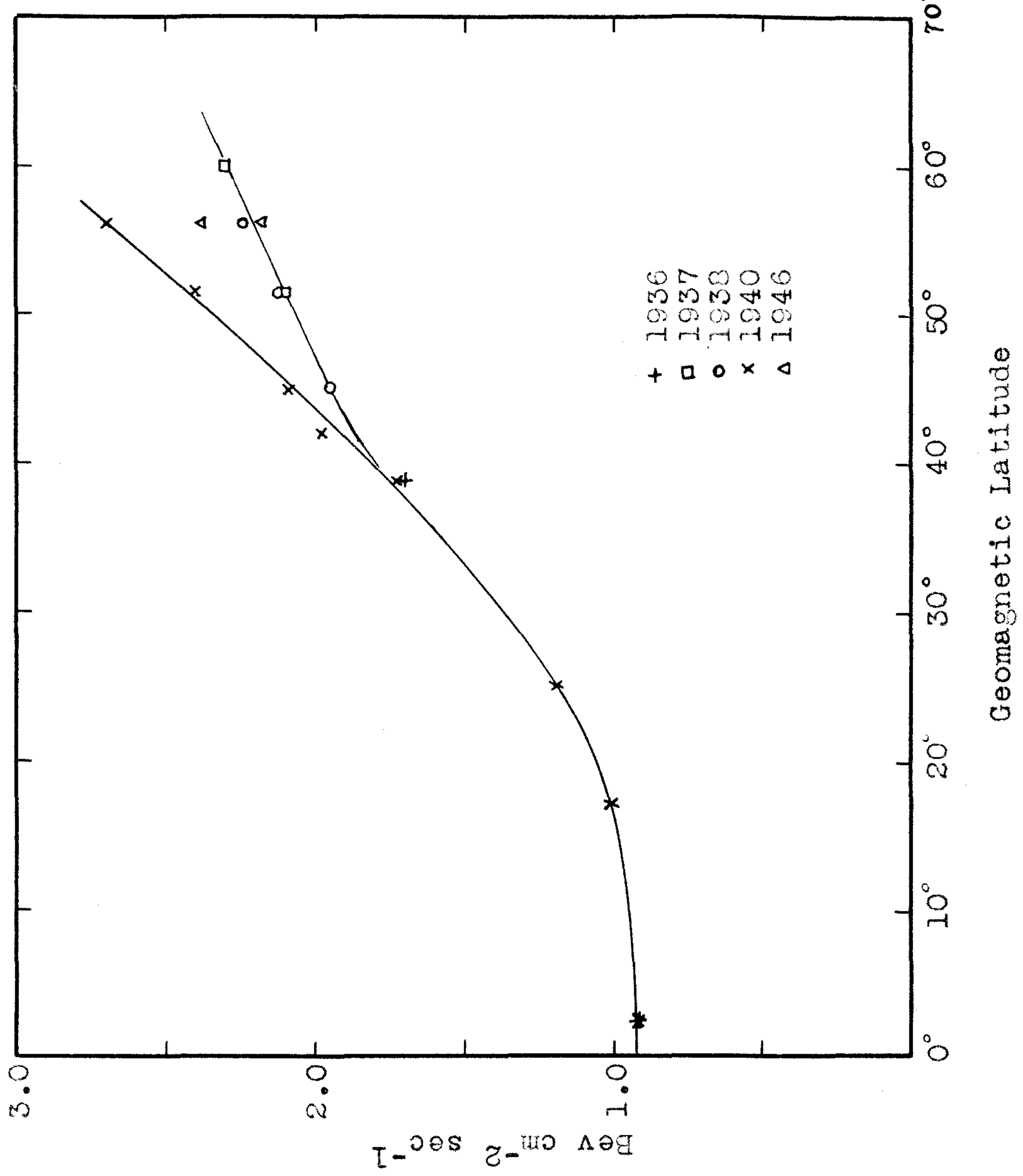


Fig. 6 The areas under the ionization curves versus geomagnetic latitude.

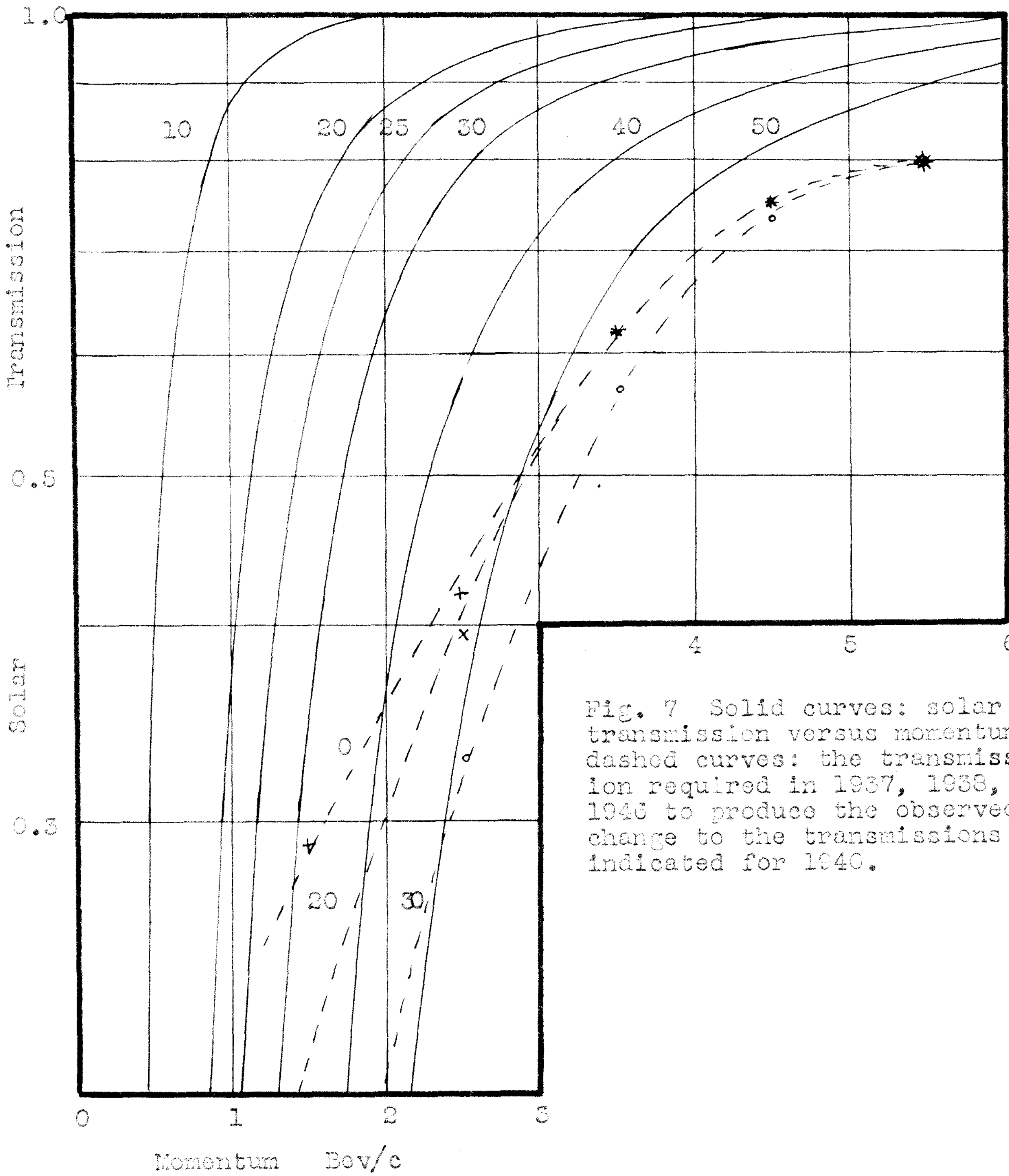


Fig. 7 Solid curves: solar transmission versus momentum; dashed curves: the transmission required in 1937, 1938, 1946 to produce the observed change to the transmissions indicated for 1940.

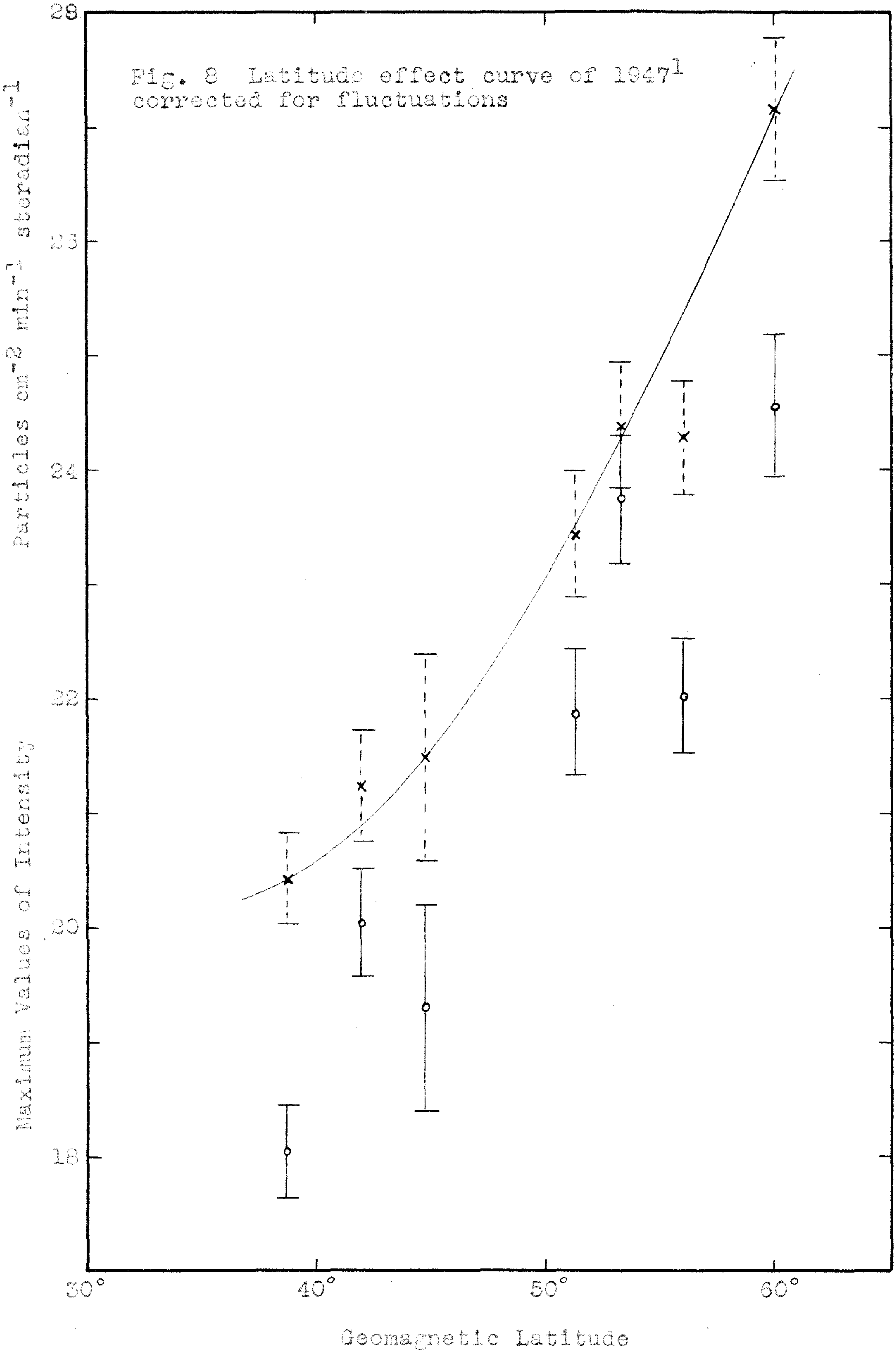


Fig. 8 Latitude effect curve of 1947[1]
corrected for fluctuations
28
26
24
22
20
18
Maximum Values of Intensity Particles cm⁻² min⁻¹ steradian⁻¹
30°
40°
50°
60°
Geomagnetic Latitude

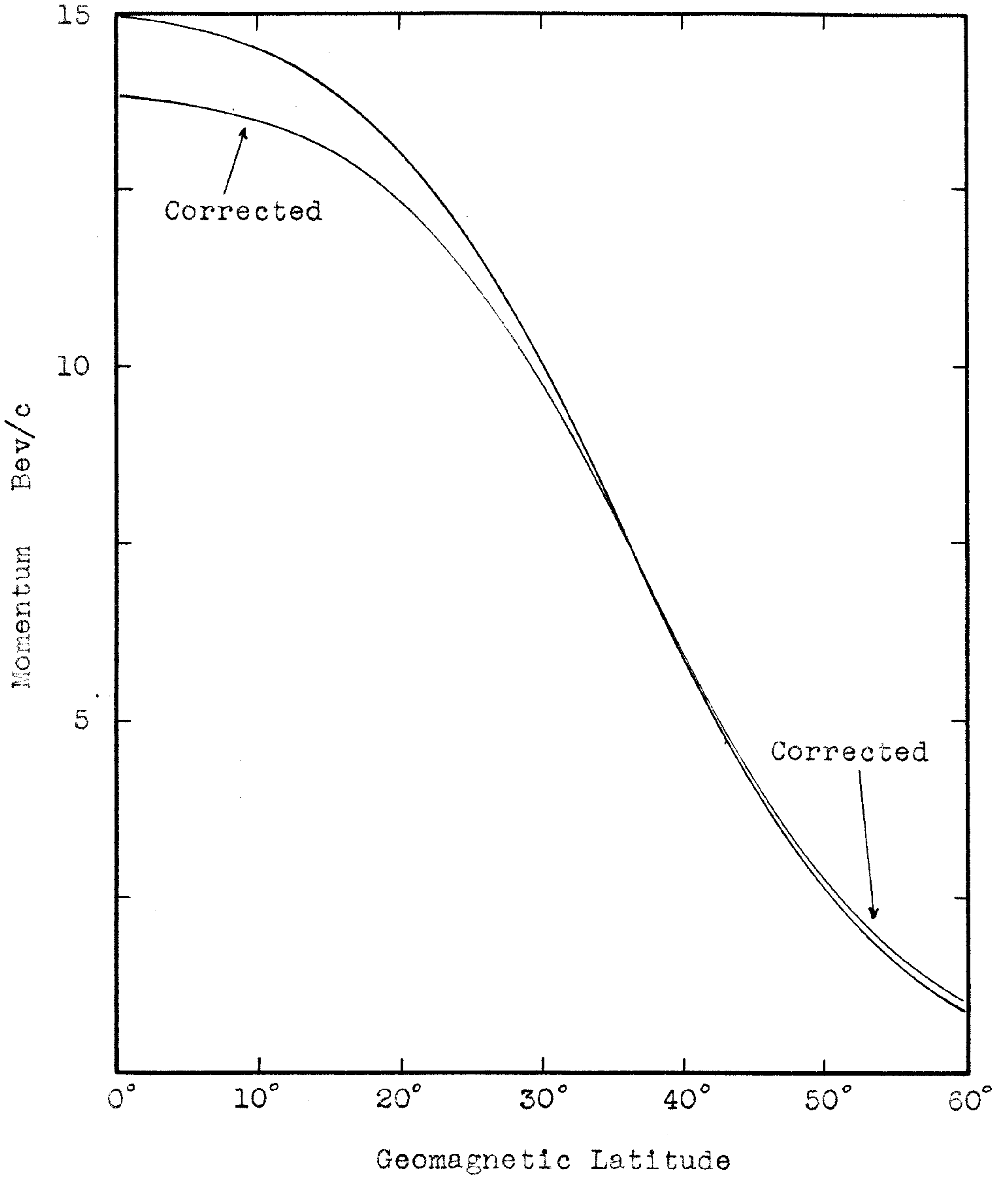


Fig. 9 Minimum momentum for entrance at the vertical versus geomagnetic latitude.

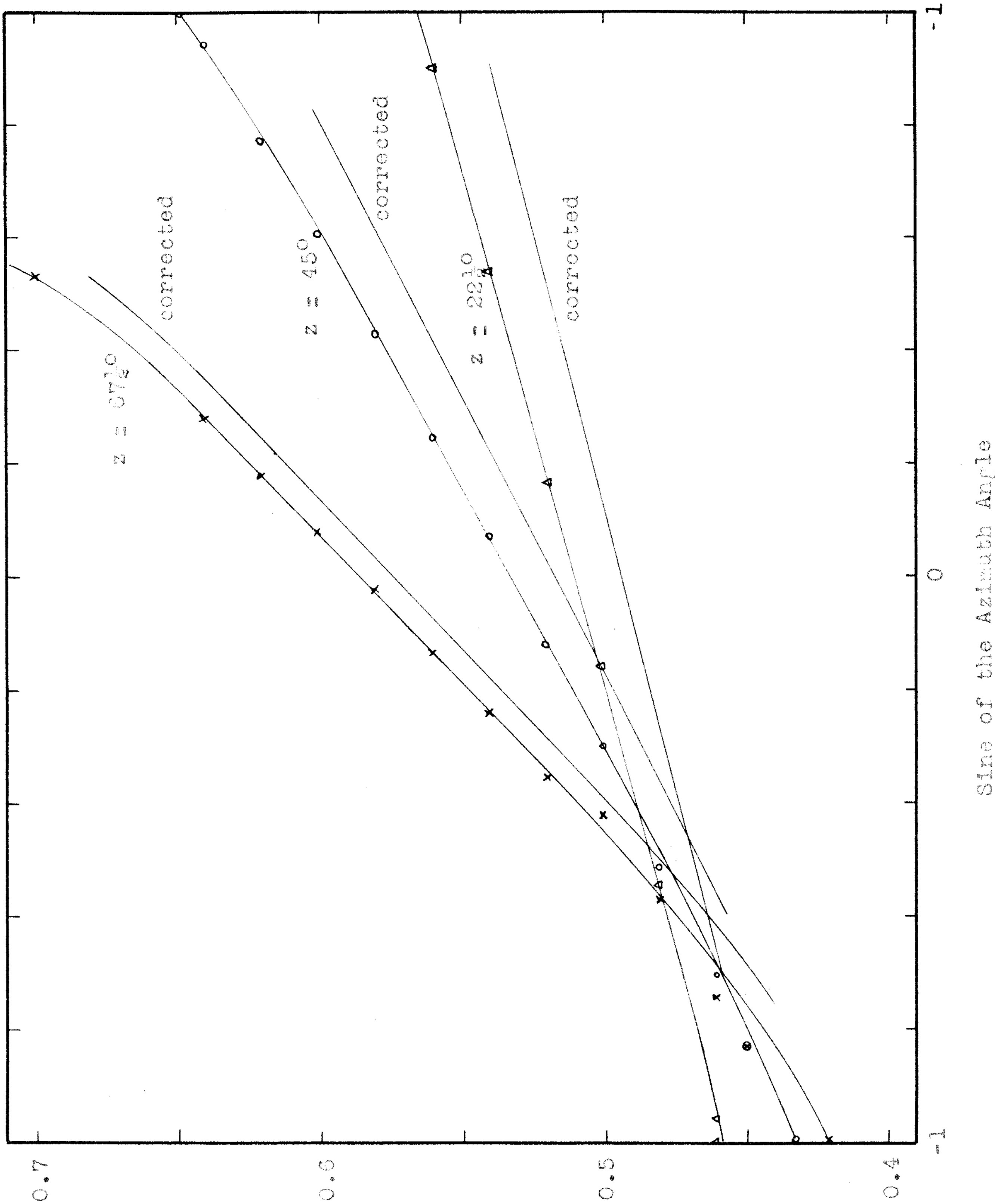


Fig. 10 Minimum Störmer variable for entrance versus sine of the azimuth angle for various zenith angles (positive particles).

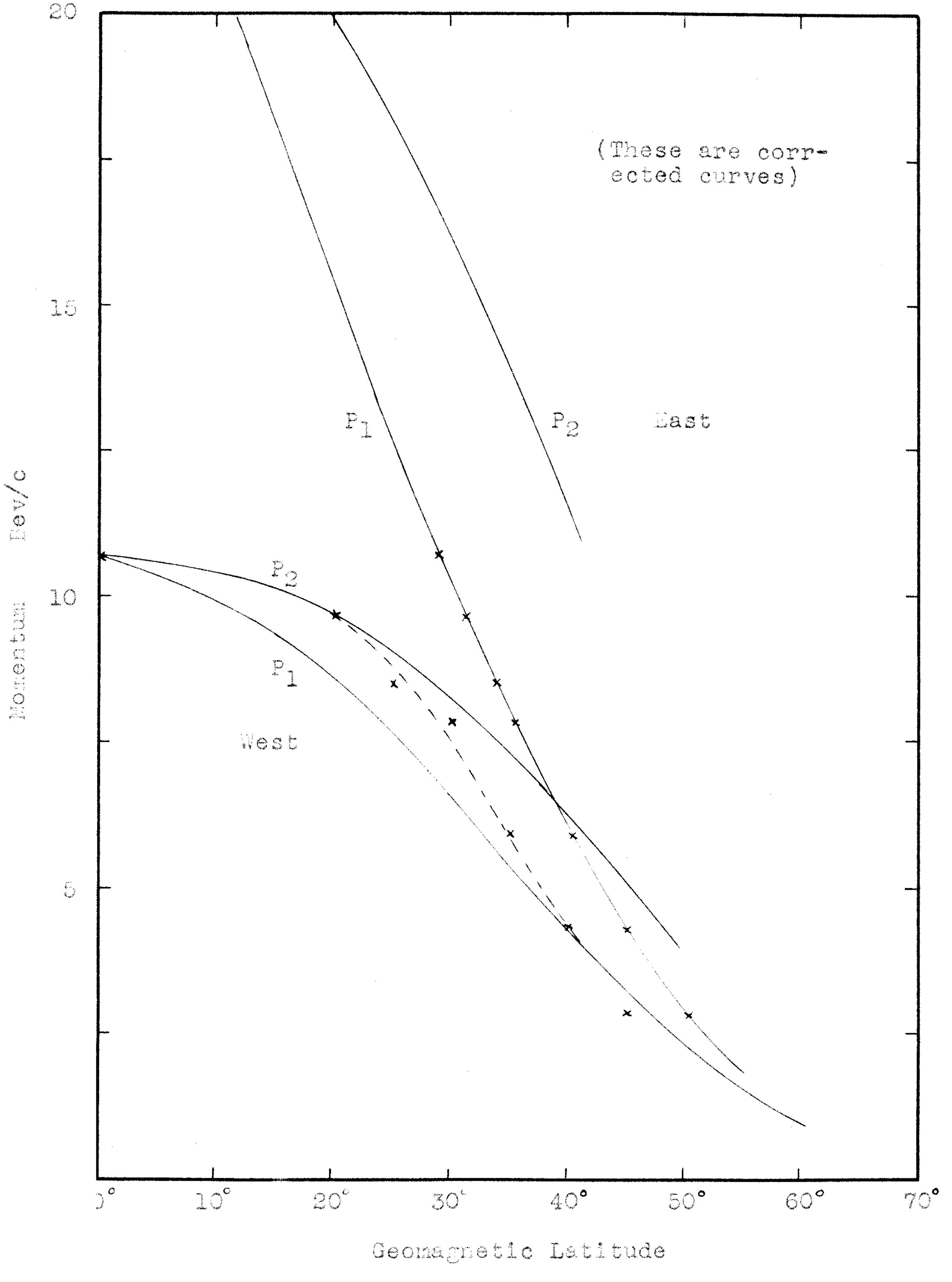


Fig. 11 Vallarta's p_1 and p_2 curves for entrance at 45° zenith angle in the east and west versus geomagnetic latitude (positive particles) and the empirical determination of the minimum momentum curve.

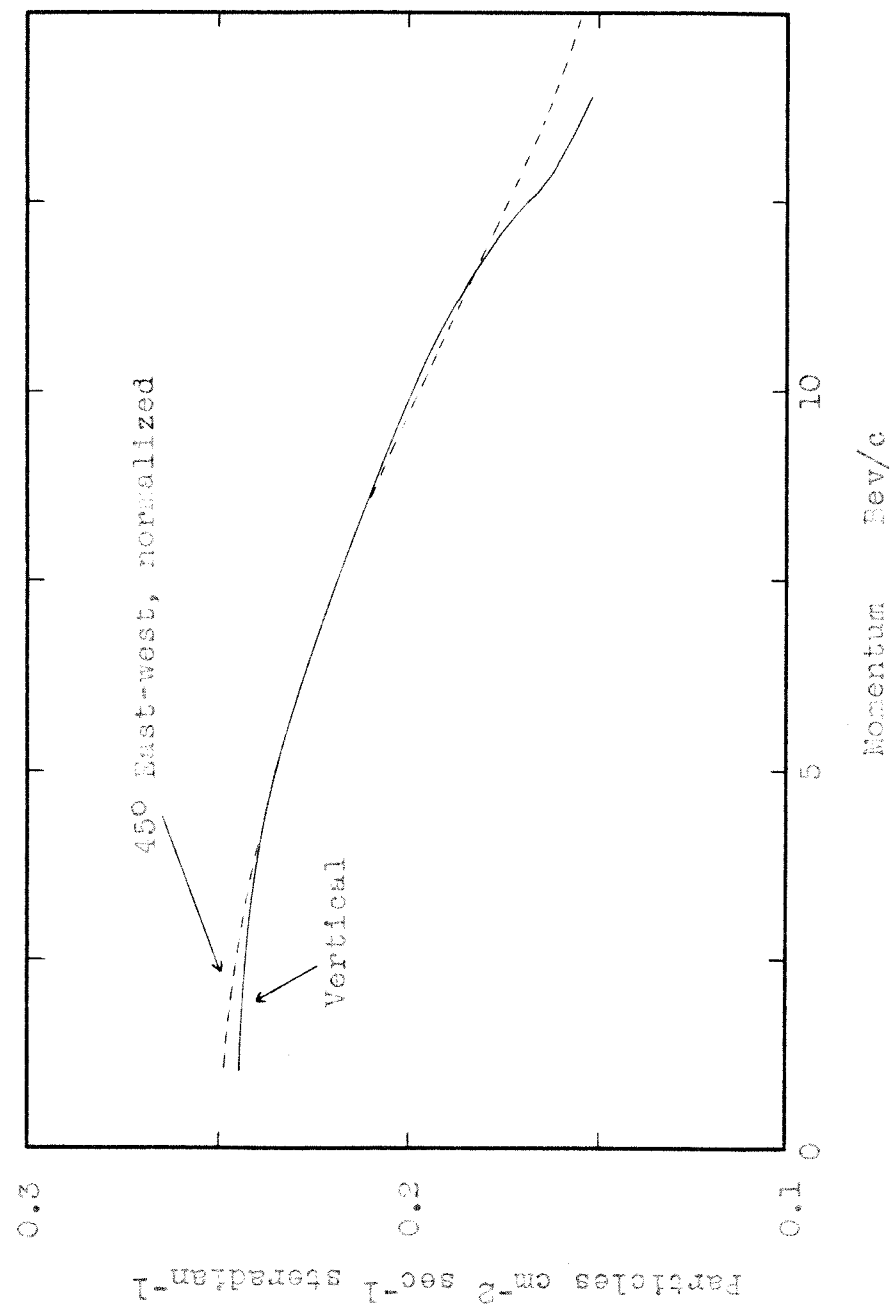


Fig. 12 Intensity at vertical versus minimum momentum for incidence of a single particle. Dashed curve is 45° zenith angle east-west result normalized at 7 Bev/c. (pressure 310 gm cm^{-2})

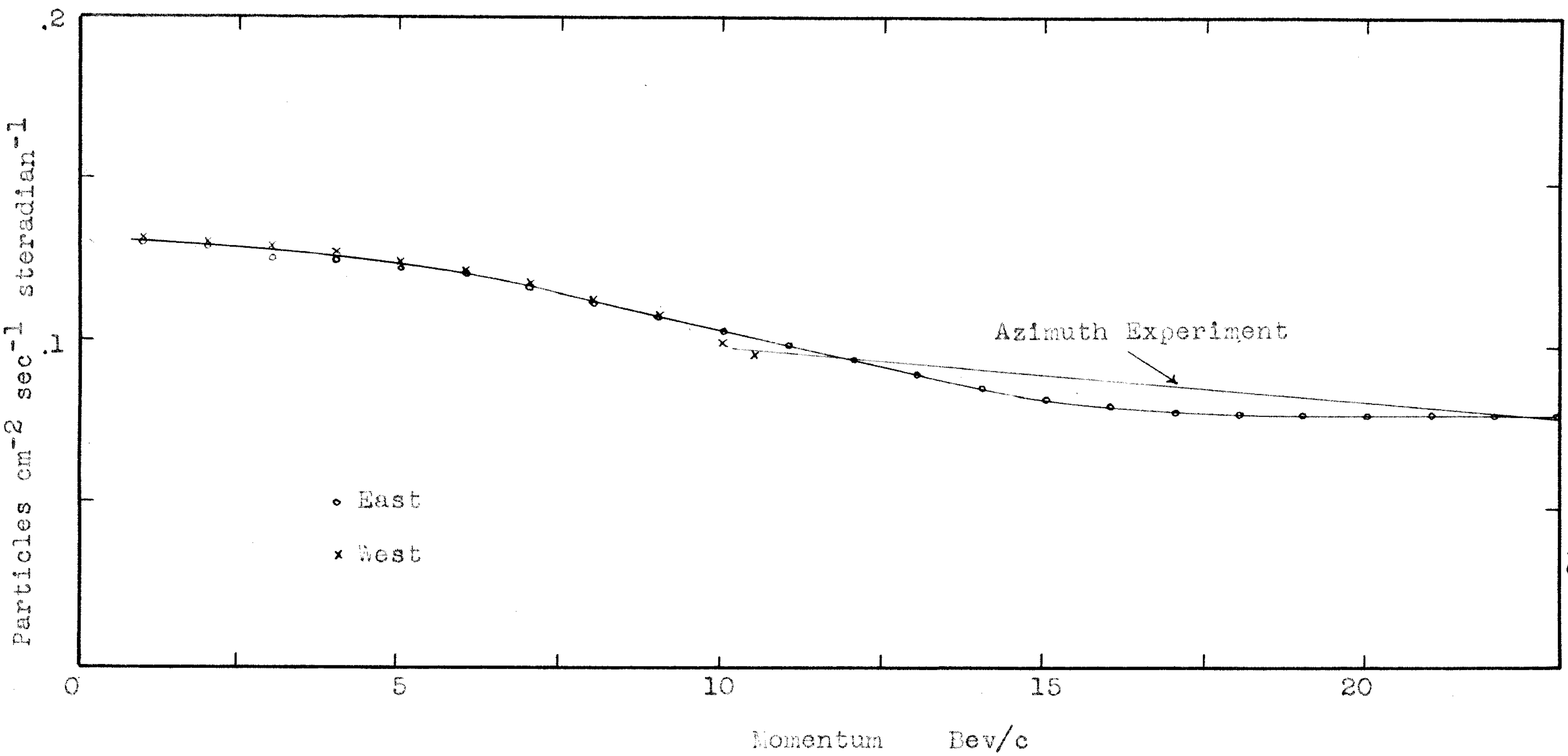


Fig. 13 Intensity at 45° zenith angle in the east and west versus minimum momentum for incidence for a single particle (pressure 310 gm cm^{-2}). Result for azimuth experiment at zenith angle 45° shown after correction for altitude

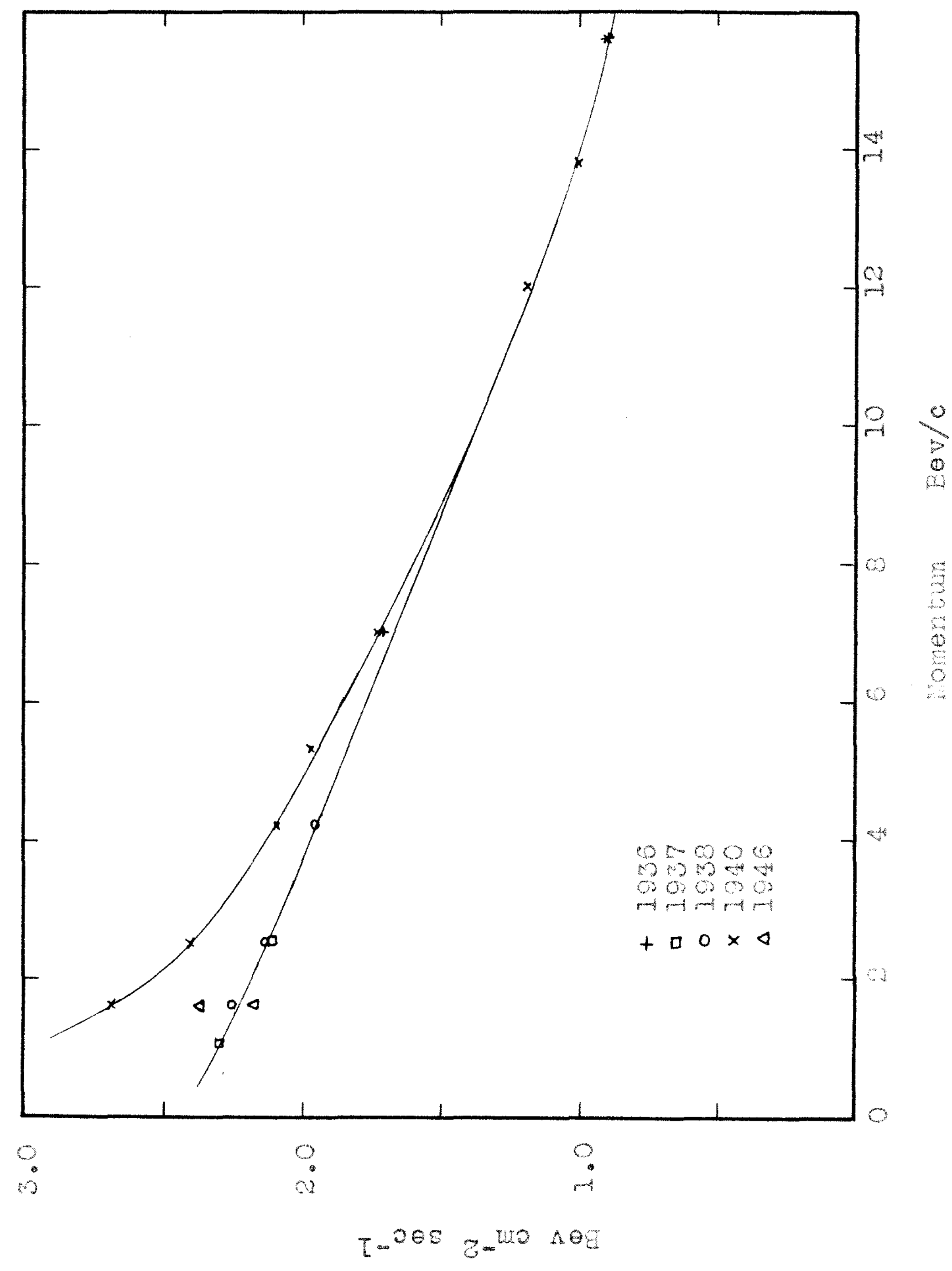


Fig. 14 The areas under the ionization curves versus the minimum momentum for incidence at the vertical.

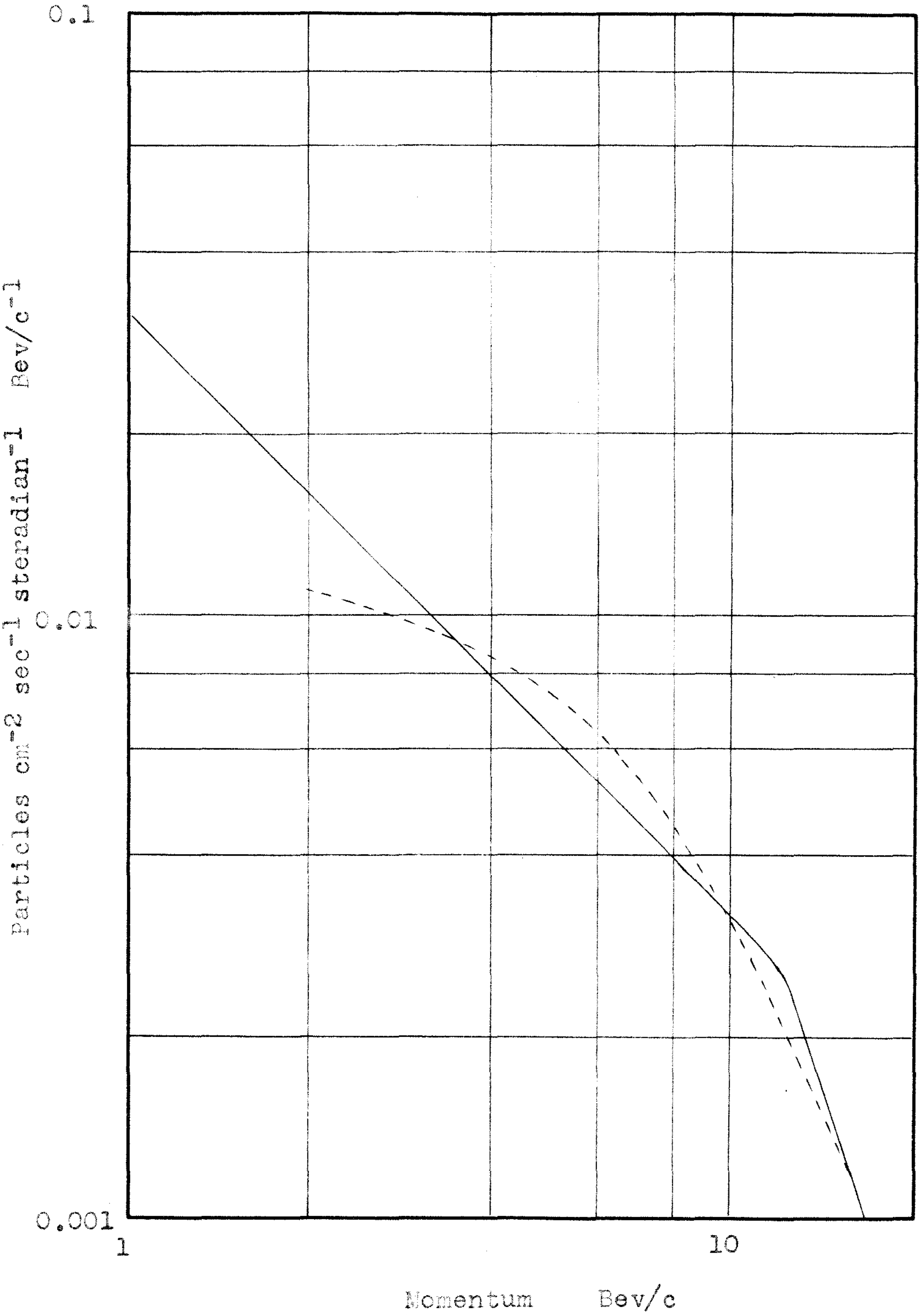


Fig. 15 j(p), the primary differential intensity distribution versus momentum. Dashed curve derived from results of Millikan, Neher, and Pickering[40].

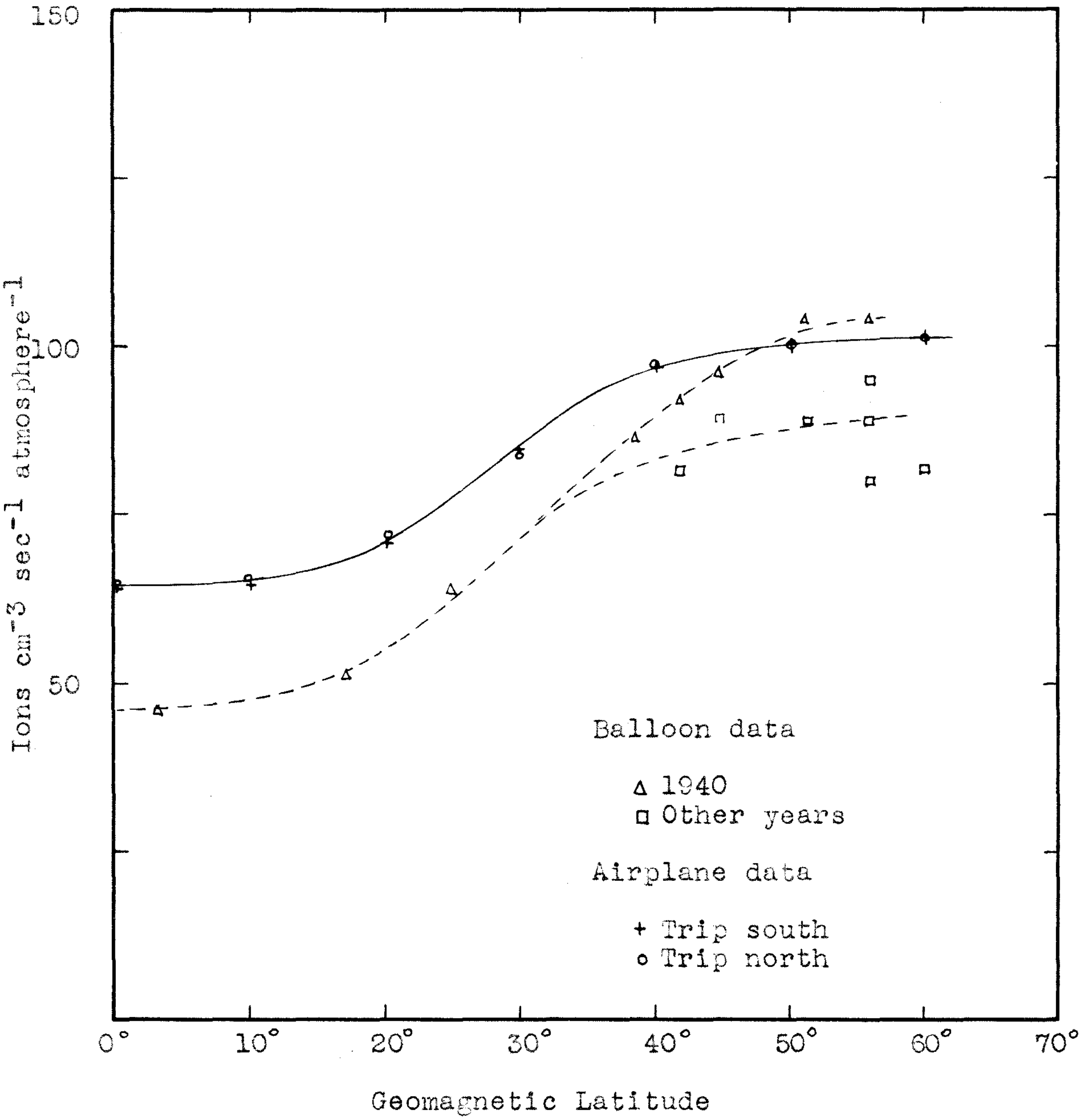


Fig. 16 Ionization as measured in the airplane experiment (1948), solid curve, as compared with similar results using balloons. (pressure 310 gm cm^{-2})

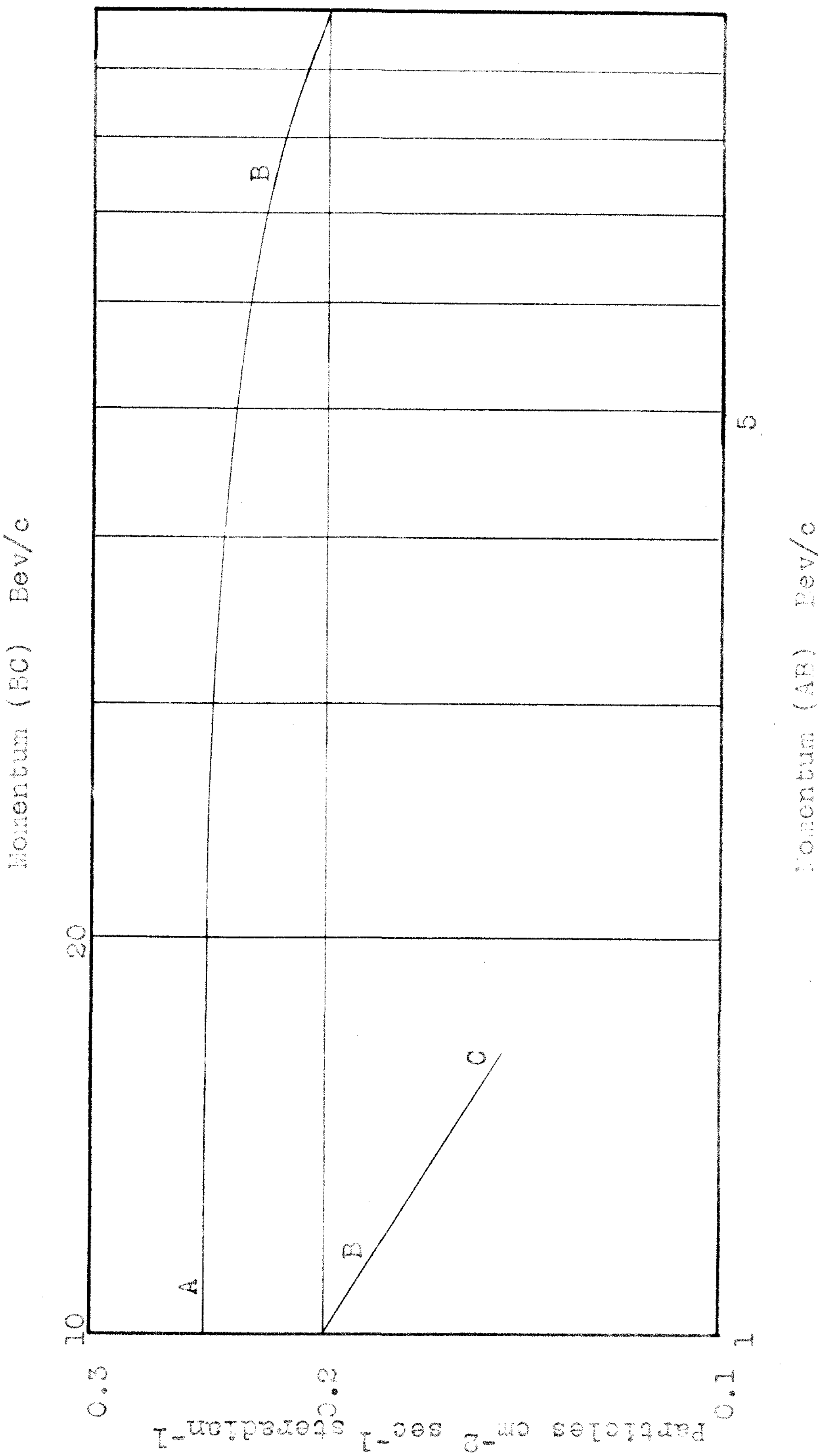


Fig. 17 Curve of Fig. 12 on a log-log plot.

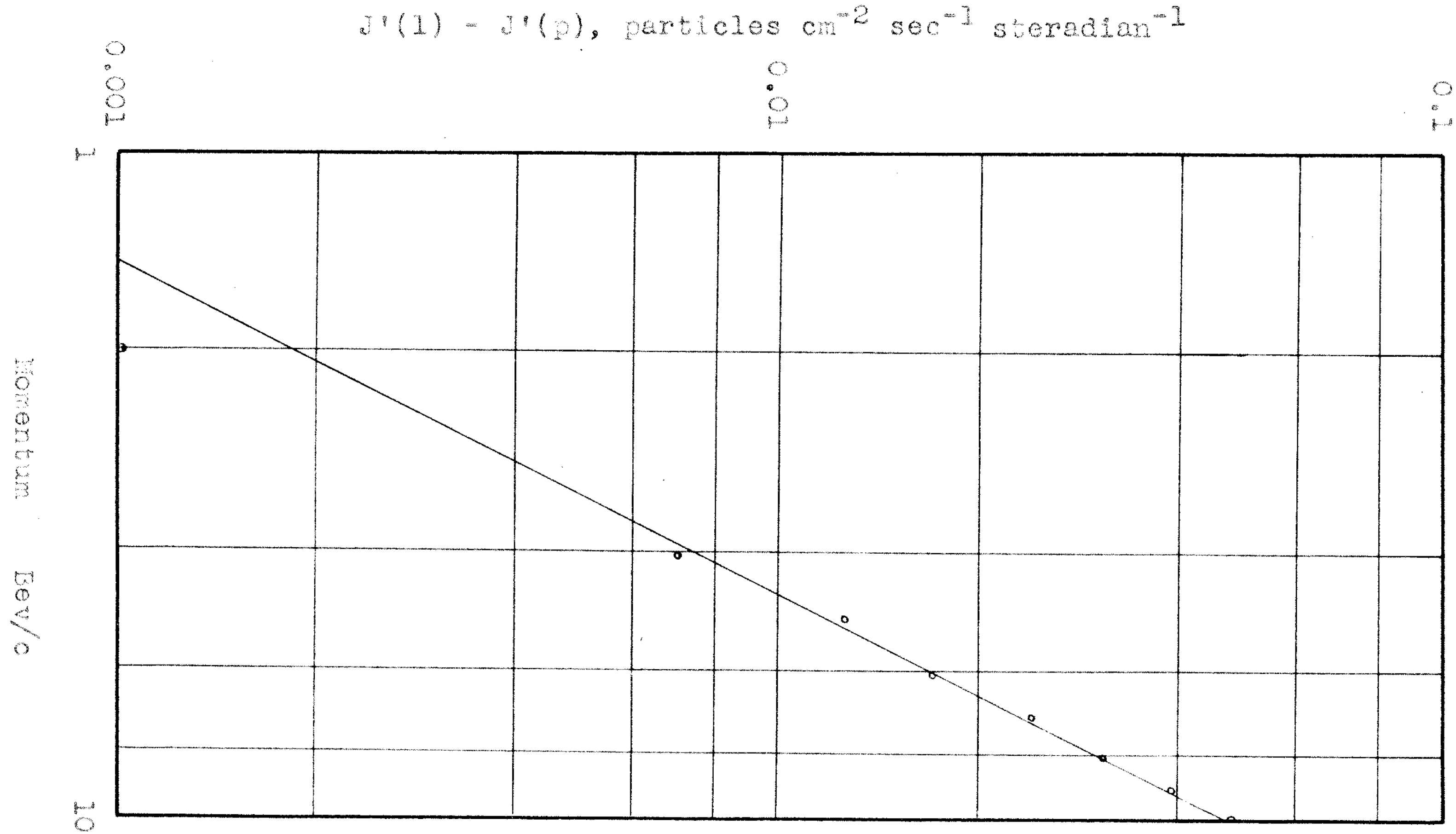


Fig. 18 Vertical intensity at high latitudes minus intensity for a given momentum versus that momentum. (pressure 310 gm cm^{-2})

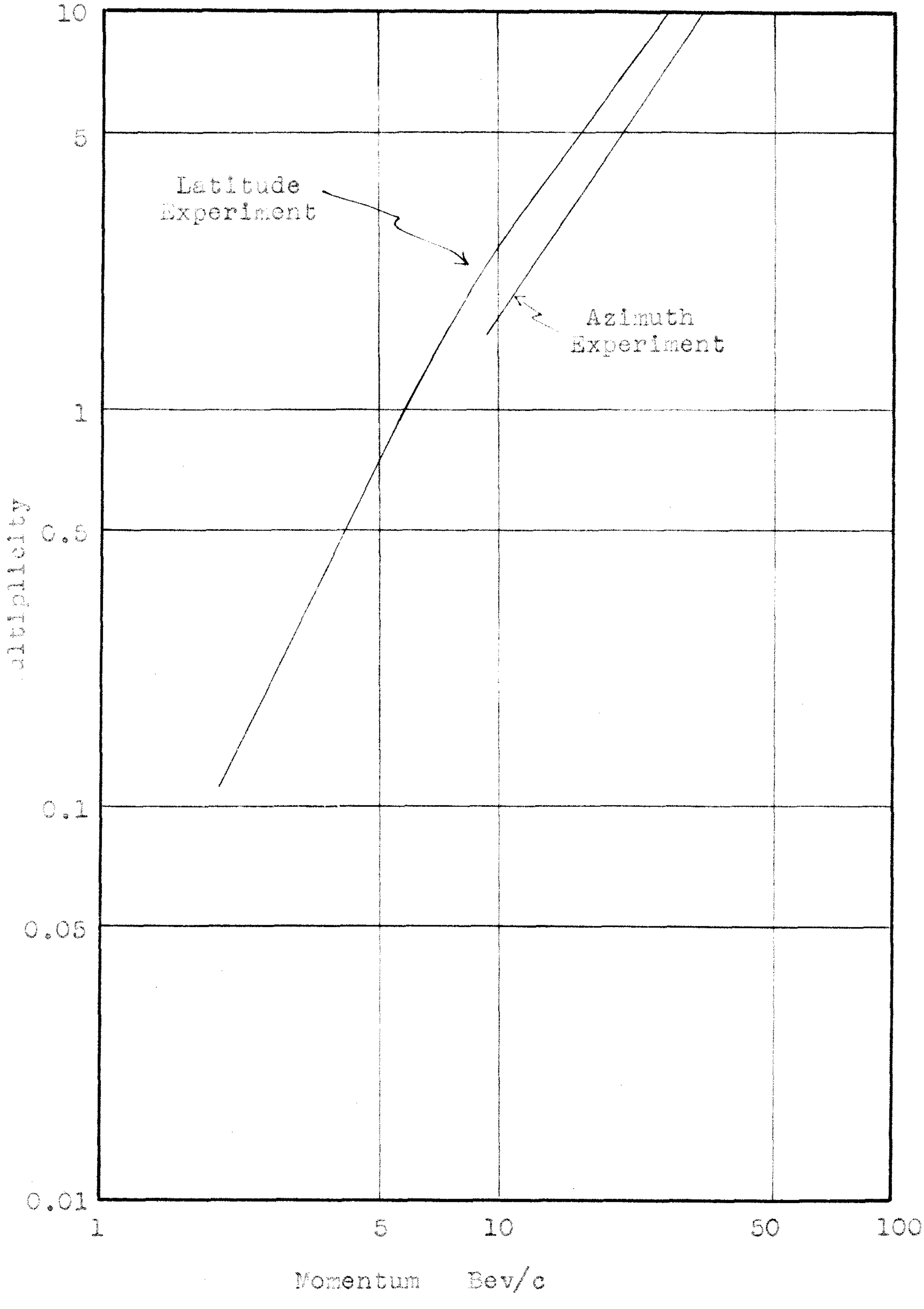


Fig. 19 m(p) = j'(p)/j(p), the multiplicity function at the vertical (pressure 310 gm cm^{-2}).

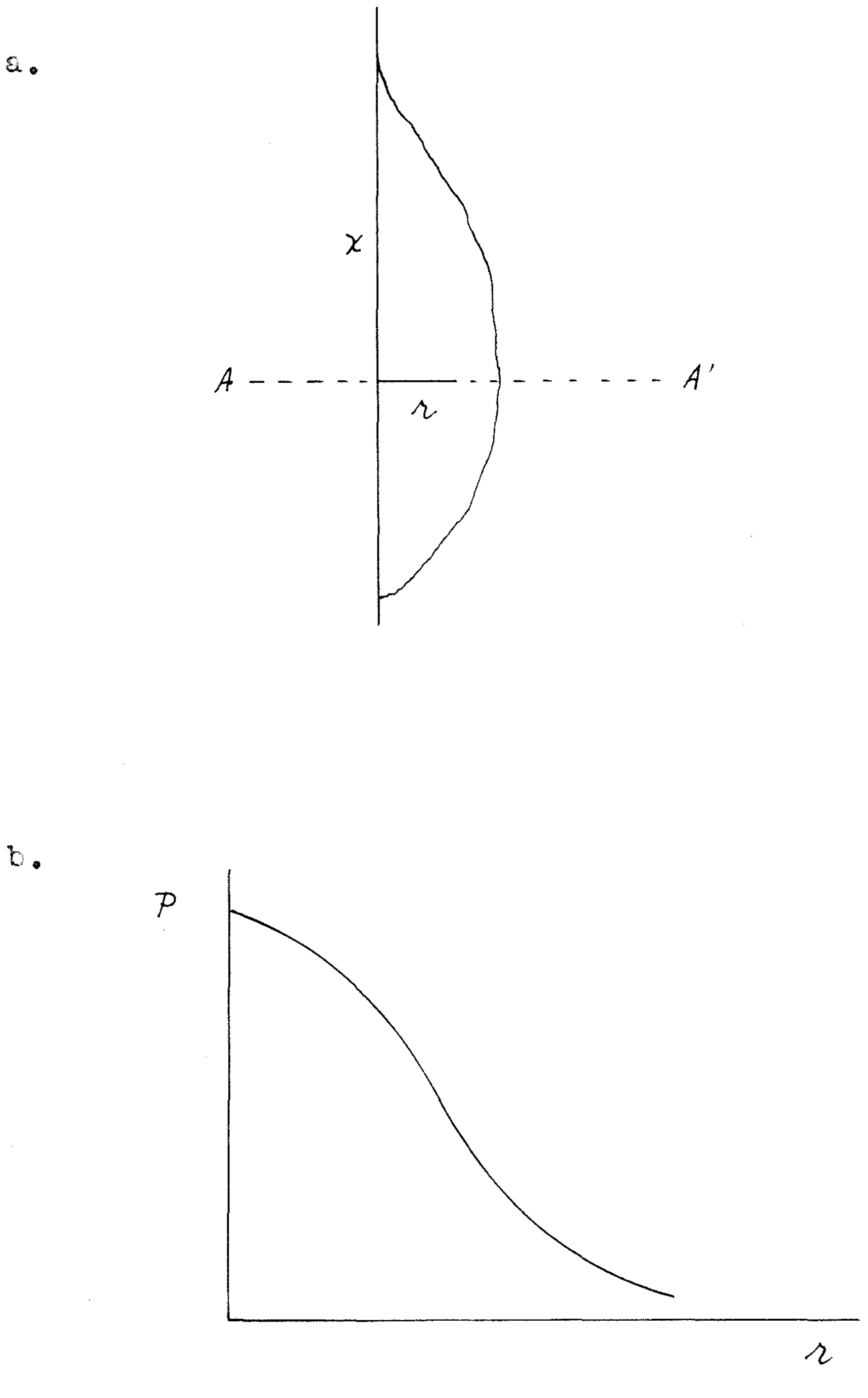


Fig. 20 Illustration of the multiplicity concept. a) Region in which effects of a particle may be detected, b) probability of detection versus distance from the axis at cross section AA'.

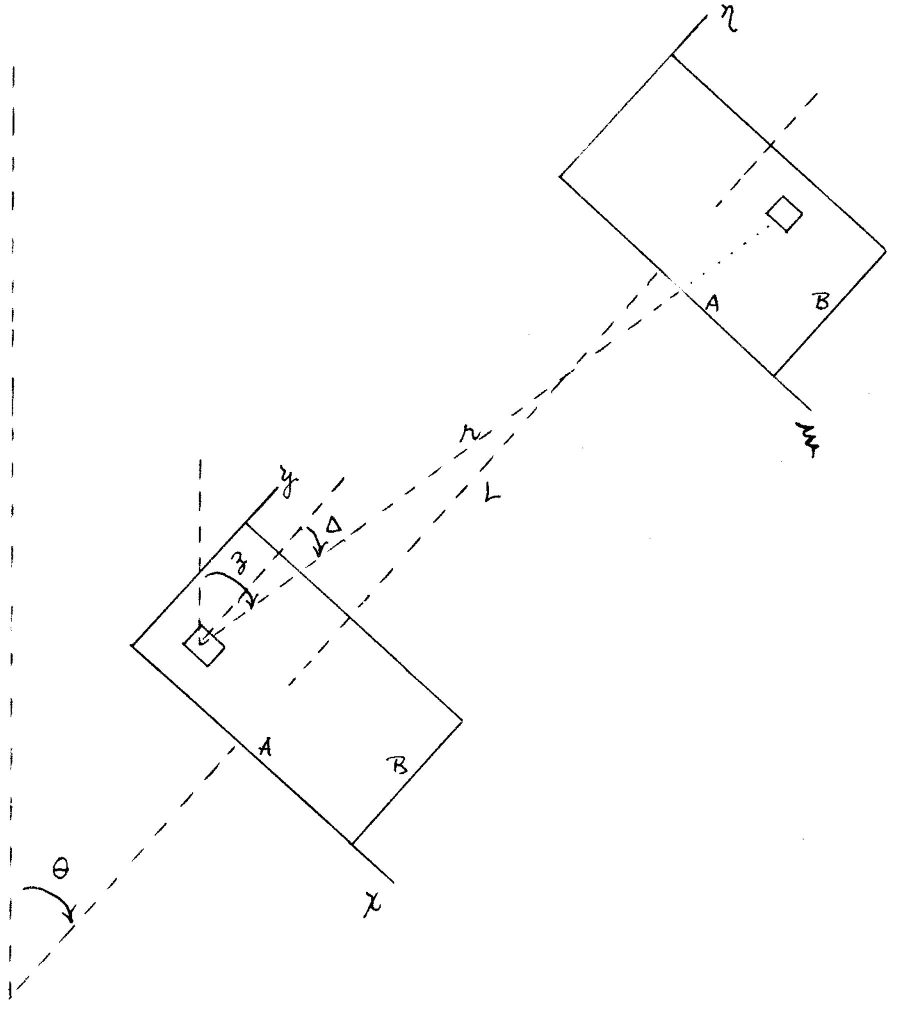
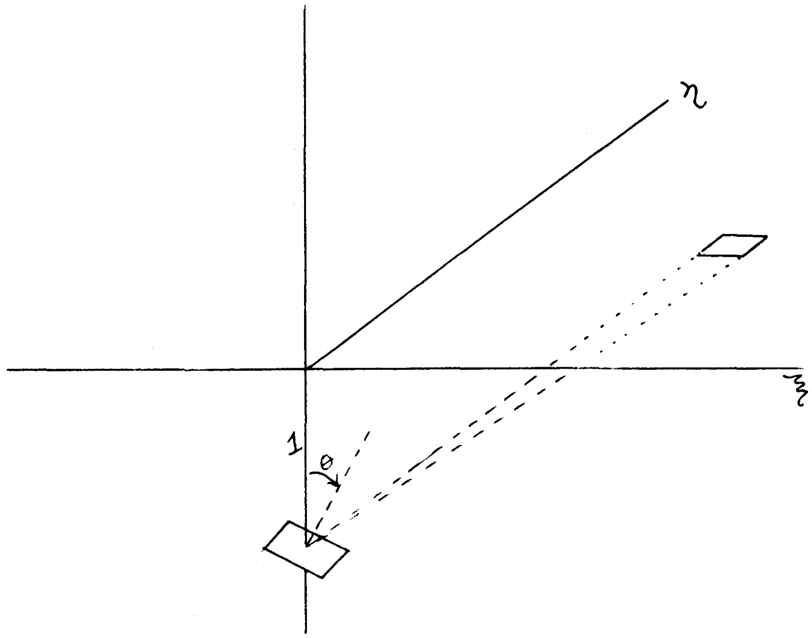


Fig. 21 Calculation of Telescope Apertures

a.



b.

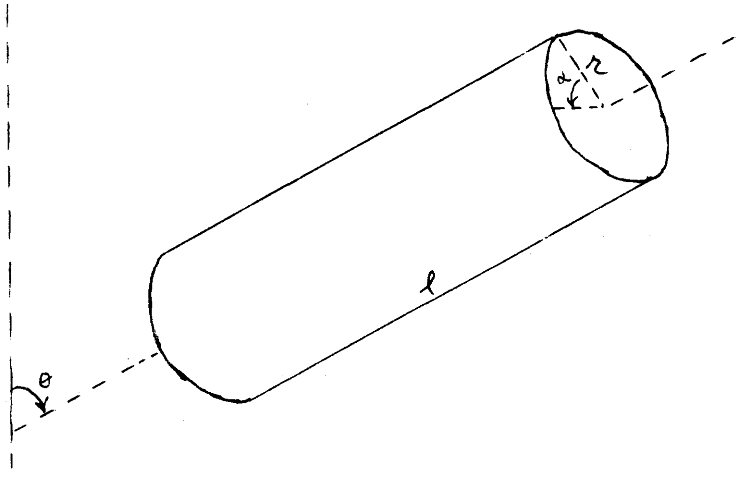


Fig. 22 Calculation of the counting rate of a single counter.

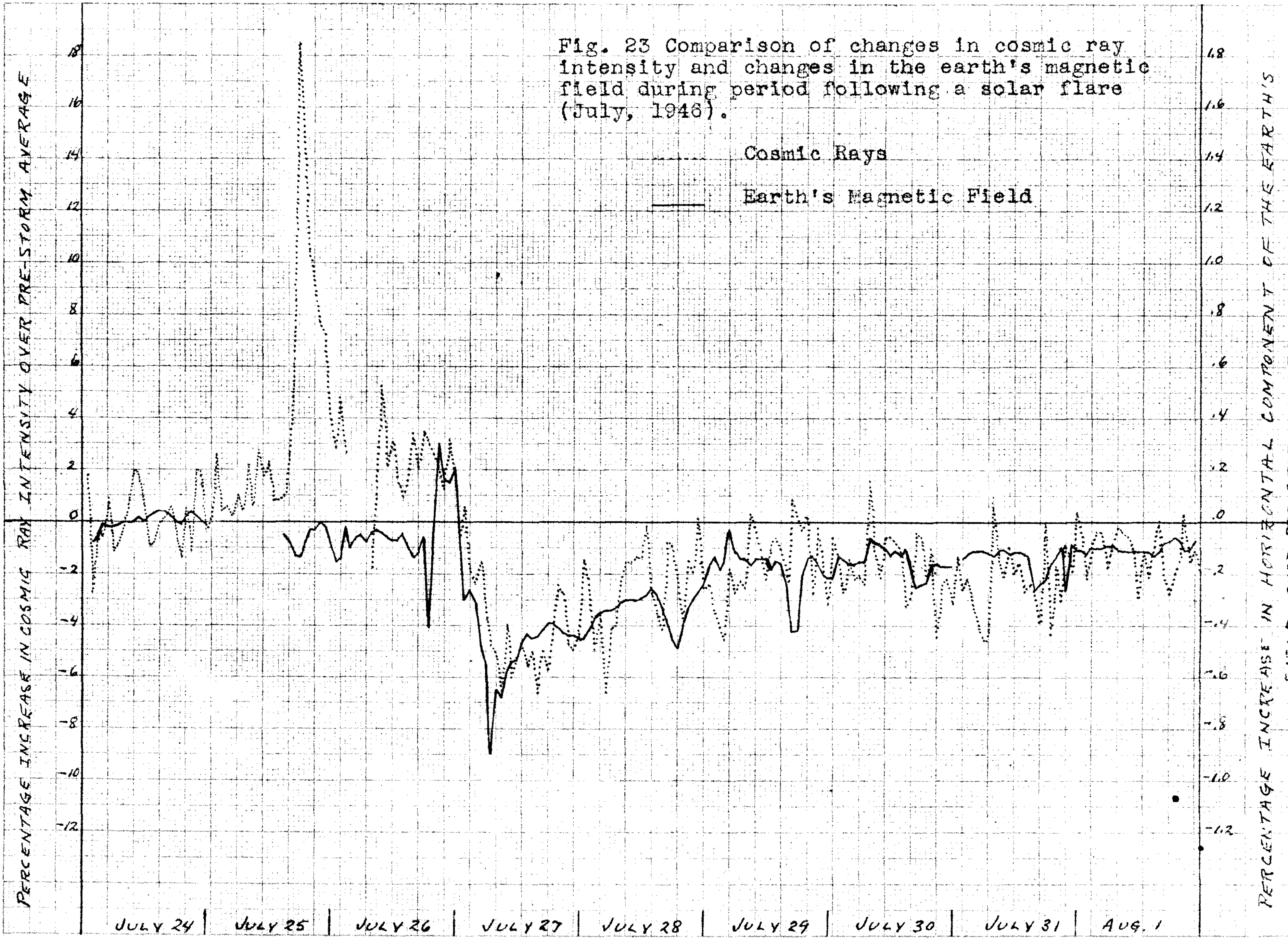


Fig. 23 Comparison of changes in cosmic ray intensity and changes in the earth's magnetic field during period following a solar flare (July, 1946).