DECAY OF TURBULENCE

BEHIND THREE SIMILAR GRIDS

Thesis by

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NOTATION

$x =$ distance downstream along the tunnel, generally measured from the grid.

$y, z =$ horizontal and vertical coordinates respectively, measured in any cross section.

$U =$ mean air velocity in the $x$-direction.

$u', v', w' =$ instantaneous values of the fluctuating components of velocity in the $x, y$ and $z$ directions respectively.

$\frac{\sqrt{\left\langle u'^2 \right\rangle}}{\overline{U}}, \frac{\sqrt{\left\langle v'^2 \right\rangle}}{\overline{U}}, \frac{\sqrt{\left\langle w'^2 \right\rangle}}{\overline{U}} =$ turbulence levels in the $x, y$ and $z$ directions.

$R(y) = \frac{\left\langle u'u'_x \right\rangle}{\left\langle uu'_x \right\rangle} = \frac{\left\langle uu'_x \right\rangle}{u'_x} =$ correlation coefficient.

$L = \int_{0}^{\infty} R(y) dy =$ scale of turbulence.

$\nu = \frac{\nu}{\rho} =$ kinematic viscosity.

$M =$ mesh size of grid.

$t =$ time.

$K =$ constant used in determining compensation resistance (see Appendix I).

Note: since we are never directly concerned with the instantaneous values of the turbulent velocity components, $u'$ and $v'$ will often be used as a brief form of $\sqrt{\left\langle u'^2 \right\rangle}$ and $\sqrt{\left\langle v'^2 \right\rangle}$.

$u'_x, u'_y =$ simultaneous values of $u'$ at two points located at the same $x$ and separated by a distance $y$. 
\[ -a = \frac{-5}{5 + AB} \] is the exponent of \( \frac{x}{M} \) in the simplified form of the Karman equation for the decay of isotropic turbulence.

\( A, B = \) numerical constants.

\( C_1 = \) the coefficient of \( \left( \frac{x}{M} \right)^{-a} \) in the same equation, which is

\[
\frac{\sqrt{u^2}}{U} = C_1 \left( \frac{x}{M} \right)^{-a}
\]
SUMMARY

u' and v' decay measurements were made behind three similar grids. The u' readings were corrected for wire length and the v' readings were corrected for the sensitivity of the v'-meter to u'. These corrected values were plotted against $X/\lambda$ on logarithmic cross-section paper. The slope of these lines gave the exponent in the theoretical law for the decay of isotropic turbulence.

A consistent deviation was found between the average u' and v' values. A suitable length correction applied to the v' values would increase them somewhat, lessening the discrepancy, and giving a true comparison between the two components of the turbulent energy.

From a comparison of the present u' values behind the 1" grid with the u' values measured with a free stream turbulence level 10 times as great as the present one, it is concluded that the change in initial turbulence has no appreciable effect on the u' behind a grid.
INTRODUCTION

It is known that the flow behind a regular grid in a uniform air stream retains a definite pattern for some distance downstream from the grid. The turbulent mixing diffuses the wind shadows until, after a certain (unspecified) region, the turbulence is uniform across any section. This uniform turbulence is generally assumed to be isotropic, and in fact the isotropy has been checked by some experimenters. D. C. MacPhail (5) has checked the isotropy by means of correlation measurements, and H. L. Dryden and C. Thiele made some (unpublished) $u'$ and $v'$ decay measurements here at the GALCIT in 1940, and these agreed within the experimental scatter.

The only systematic decay measurements previously made at the GALCIT are those of S. Atsumi (3) whose data consists only of $u'$ values behind a 1" mesh grid of \( \frac{1}{4} " \) rods and a 1\( \frac{3}{8} " \) mesh grid of \( \frac{1}{2} " \) rods.

The purpose of this investigation was to determine experimental values of the exponent in the theoretical relation derived by von Karman (1) for the rate of decay of isotropic turbulence in a uniform air stream. Since the technique of measurement of the $v'$ component of turbulence has had considerable development in the last two or three years, the previous experiments could be extended considerably.

The possibility was felt that the grid mesh size employed in ref. (3) may have been large relative to the cross-sectional
dimensions of the wind tunnel working section, with a resulting wall influence on the rates of decay and growth of scale of the turbulence. Since basically the same wind tunnel was to be used in this research as was employed in ref. (3), two finer grids of \( \frac{1}{2}'' \) and \( \frac{3}{4}'' \) mesh were tested, in addition to the \( 1'' \) grid previously used. The nature of the turbulence must vary with varying mesh size relative to rod diameter, and this independent variable was eliminated by making the \( \frac{1}{2}'' \) and \( \frac{3}{4}'' \) grids proportional to the \( 1'' \) grid, that is, with \( \frac{1}{8}'' \) and \( \frac{1}{16}'' \) rods respectively. This would make it easier to see Reynolds Number effects, if any exist.

Furthermore, the free stream turbulence level of the wind tunnel has been lowered considerably by the construction of a new entrance and contraction section, and by the insertion of a fine-mesh precision screen in the large section upstream from the beginning of the contraction. Thus the effect of lowering the initial turbulence level could be deduced from a comparison of the present measurements on the \( u' \) decay behind the \( 1'' \) grid with the results of ref. (3).
DESCRIPTION OF EQUIPMENT

A. The Wind Tunnel

The wind tunnel used for this investigation was originally constructed especially for decay and correlation measurements in 1938 (2). It is of the Eiffel type with a 16 to 1 contraction ratio. The working section is 20" square and 12 feet long, and is provided with adjustable side walls for the attainment of a constant velocity along its entire length. The air enters through a cheesecloth screen which is backed by a honey-comb having \( \frac{1}{2} '' \) cells with a depth of \( 4\frac{1}{2} '' \). The principal dimensions of the flow passage are given on figure 13.

A longitudinal slot running the full length of the working section permits movement of the measuring instrument support and is kept sealed by a continuous shim-stock belt attached to the support. The power unit is a 5 h.p. D.C. motor driving a two-bladed wooden fan 30" in diameter.

By adjustment of the walls, the mean velocity at the center of the working section was made constant to within 1 percent along its entire length. The tunnel has a very steady mean velocity in the range employed for these experiments.

The \( u' \) and \( v' \) components of the free stream turbulence level are both on the order of 0.05 percent at the beginning of the working section, with a gradual increase to about 0.07 percent at the extreme downstream end of the working section, just before the
There are appreciable variations in the level from day to day and with varying temperature distribution in the laboratory.

B. The Grids

All three grids are of the bi-plane type with the horizontal and vertical rods merely in contact. The 1" grid is the same one tested by Atsumi and later by Thiele and also by Dryden when he brought some of his hot wire equipment along on a visit to the GALCIT laboratory. It is made up of ¼" wooden dowel, very accurately spaced. The ½" grid is made up of 1/8" diameter welding rod and is not quite as accurately spaced as the former, but it is still quite good. On the other hand, the ¼" grid made of 1/16" welding rod has noticeable deviations from regular spacing. However, it was felt that decay measurements would be much less sensitive to these irregularities in the ¼" grid than would scale measurements. An idea of the accuracy of the grids can be gotten from the photographs of fig. 16. All grids are fitted with frames which form a fairly smooth connection between the entrance and the working section of the tunnel.

It might be pointed out that the wooden grid probably has greater surface roughness relative to its other dimensions than do the other two.

C. Measuring Equipment

All measurements were made with platinum hot wires
0.00024" in diameter. The wire is Wollaston, and the silver was etched off before soldering. The u' wires are about 1.2 mm in length. The v' meters are of the bi-plane X type with wires about 4.5 mm long and a vertex angle of about 30°.

Both the u' and the v' meters were calibrated for sensitivity only, since a statistical survey of all the time constant calibrations here at the GARCIT for the past two years showed that for a given diameter of wire the probable error from the mean value was small enough to have no appreciable effect on the output reading. Consequently, the mean value of K was always used, as outlined in Appendix I. Then, the compensation resistance to be used depends merely upon the ratio $\frac{\Delta R}{R}$ at which the hot wire is operating.

Detailed descriptions of hot wire equipment and the principles upon which it works can be found in several places in the technical literature, for instance, ref. (6). The heating circuits and amplifier used in the present investigation are essentially the same as those used in ref. (3), with the important modification that large chokes have been put into the wire heating circuits so that the current fluctuation is negligible. This amplifier has correct characteristics to within ±5 percent over a frequency range from 2 to 8000 c.p.s. Output readings are taken on a wall galvanometer having a period of about 20 seconds and a full scale deflection of 50 cm. The noise level is negligible at the amplification used in measuring the turbulence behind a grid, but may amount to 10 percent of the total reading at the high amplification necessary to measure the free stream turbulence.
RESULTS OF VON KARMAN THEORY

The following few equations give the pertinent results of ref. (1) on the rate of decay and growth of scale in isotropic turbulence. In order to adapt the derived equations to the case of steady flow in a wind tunnel, the variable \( t \) (time) is replaced everywhere by its equivalent, \( \frac{x}{U} \) or \( \frac{RM}{MU} \), and the grid is taken as the origin instead of the point \( x_0 \).

The first relation derived by von Kármán is

\[
\frac{d\overline{u'^2}}{d \frac{x}{M}} = -\frac{10}{\lambda^2} \frac{\nu M}{U} \cdot \frac{\overline{u'^2}}{U} \tag{1}
\]

where \( \lambda \) is a quantity related to the correlation function, \( R(y) \).

By assuming similarity for the shape of the correlation function and by making an approximation valid for large Reynolds Numbers, he obtained a set of relations between the rate of dissipation of turbulent energy and the correlation function.

We define the scale of turbulence as \( L = \int_0^\infty R(y)dy \), and then

\[
\frac{\overline{u'^2}}{L \cdot \nu} \frac{\lambda^2}{R} = A \tag{2}
\]

\[
\frac{dL}{d \frac{x}{M}} = MB \frac{\overline{u'^2}}{U} \tag{3}
\]

where \( A \) and \( B \) are numerical constants.

Then, from (1), (2) and (3),

\[
\lambda^2 = \lambda_0^2 + (5+AB) \frac{\nu M}{U} \cdot \frac{x-x_0}{M} \tag{4}
\]
\[
\frac{\sqrt{u'^2}}{U} = \frac{\sqrt{u'^2}}{U} \left\{ 1 + \frac{(S+AB)U/M}{U \chi^2 - \lambda_0^2} \cdot \frac{\chi - x_o}{M} \right\}^{-\frac{5}{S+AB}}
\]

(5)

\[
\frac{L}{M} = L_o \left\{ 1 + \frac{(S+AB)U/M}{U \chi^2 - \lambda_0^2} \cdot \frac{\chi - x_o}{M} \right\}^{\frac{AB}{S+AB}}
\]

(6)

where the subscript \(o\) denotes the value of a quantity at an arbitrary distance \(x_o\) behind the grid.

From equation (4), we see that \(\lambda^2\) is linear in \((x-x_o)\) so that if we could take our origin for \(x\) at the point where \(\lambda^2\) goes to zero, we could write \(\lambda^2\) simply as proportional to \(x\):

\[
\lambda^2 = (S + AB) \frac{U/M}{U} \cdot \frac{x}{M}
\]

(7)

In fact, Atsumi (3) did find that his lines of \(\lambda^2\) versus \(x\) measured from the grid went consistently very near the grid location for \(\lambda^2 = 0\). In the present measurements, the curves of \(\lambda^2\) vs. \(x\) often deviated appreciably from a straight line, but nearly always a mean straight line drawn through the points would pass fairly close to the origin; see, for instance fig. 10.

This means that to a reasonably good approximation we can use equation (7) with the grid location taken as the origin of \(x\). Furthermore, with \(\lambda^2\) proportional to \(x\), equations (5) and (6) take the following simplified forms:

\[
\frac{\sqrt{u'^2}}{U} = C_1 \left( \frac{x}{M} \right)^{-\frac{5}{S+AB}}
\]

(8)

\[
\frac{L}{M} = C_2 \left( \frac{x}{M} \right)^{\frac{AB}{S+AB}}
\]

(9)
For brevity we shall write, \( \frac{5}{5 + AB} = a \). If equation (8) holds reasonably well, we can now evaluate \( a \) by merely measuring the slope of the decay curve plotted to a double logarithmic scale. Figure II is a typical example of such a plot. All the values of \( a \) in the following table were gotten in this fashion. We can also get the value of the exponent \( \lambda \) from equation (7) if we have \( \lambda^2 \).

\( \lambda^2 \) is computed from a modified form of equation (1). The derivation is worked out in ref. (5), and the convenient form is

\[
\frac{1}{\lambda^2} = \frac{U}{5V\eta} \frac{d \log \frac{U}{\eta}}{d\left(\frac{\chi}{\eta} \right)}
\]  

(10)
EXPERIMENTAL RESULTS

The following values of $C_1$ and $a$ were obtained from logarithmic plots of corrected decay curves. The $u'$ curves have been corrected for the finite hot wire length (4), while the $v'$ curves have been corrected for sensitivity of the $v'$ meter to $u'$ fluctuations (due to lack of perfect symmetry). So far, no rational method of applying length corrections to $v'$ readings has been published. A brief qualitative discussion will follow in the next section.

<table>
<thead>
<tr>
<th>Grid</th>
<th>$M^*$</th>
<th>$u$</th>
<th>$u'$</th>
<th>$v'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/s</td>
<td>a</td>
<td>$C_1$</td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td>15.0</td>
<td>0.65</td>
<td>0.24</td>
<td>0.59</td>
</tr>
<tr>
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<td>15.0</td>
<td>0.66</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
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<td>0.65</td>
<td>0.25</td>
<td>0.59</td>
</tr>
<tr>
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<td>0.67</td>
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<tr>
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<td>0.55</td>
<td></td>
</tr>
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<td>0.66</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
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<td>0.61</td>
<td>0.19</td>
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<td></td>
</tr>
<tr>
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<td>0.19</td>
<td>0.58</td>
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<td>0.28</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Figures 1, 2 and 3 give a complete picture of the uncorrected measurements for the three grids. However, the absolute levels of turbulence for different grids cannot be directly compared until the length corrections have been applied. Since we have reliable length correction formulae only for \( u' \) wires, the \( v' \) values for the three grids cannot be compared with each other. Figures 4, 5 and 6 are plots of corrected \( u' \) decay runs at a given speed, for any of the three grids tested at the particular speed.

It must be remembered that the uncorrected \( v' \) values cannot be directly compared even for the same grid. This is because practically every \( v' \) meter has some unsymmetry which makes it more or less sensitive to \( u' \) fluctuations as well. Since this \( u' \)-sensitivity varies with the wind speed, the various \( v' \) curves in figures 1, 2 and 3 are not in their correct relation to each other, so the \( v' \) decay runs for each grid have been properly adjusted according to the calibration of the \( v' \) meter, and they are replotted in figures 7, 8 and 9.

Figure 10 is an example of the nature of the \( \lambda^2 \) curves obtained in the present investigation. In fact, these two are rather closer to straight lines than most. The great difficulty in the calculation of \( \lambda^2 \) lies in having to measure graphically the slopes along the curve of \( \log \frac{U}{u'} \) vs. \( \frac{X}{M} \).

Figure 11 is a logarithmic plot of the measurements that also correspond to fig. 10, and illustrates how the values of the
exponent $a$ are measured. In general, only the section of the curve for $\frac{X}{M}$ greater than 40 is considered in the determination of the average slope.

It was originally hoped that there would be sufficient time to carry out complete correlation measurements in addition to the decay, but such was not the case, and figure 12 shows the only correlation curve run up to the present time in this investigation. With it is plotted the one of Atsumi's (3) correlation curves that comes closest to the same conditions. It is interesting to note that they cross twice. The unsimilarity of the curves may be a Reynolds' Number effect, or it may be due to the lower initial turbulence in the wind tunnel. The matter will be investigated further shortly.

In addition to the formal decay runs, there are some interesting miscellaneous experimental results. An attempt was made to measure a correlation curve for the free tunnel turbulence. The points showed considerable scatter, but the general nature of the correlation is as follows: from $y = 1$ mm to $y = 10$ cm, $R(y)$ gradually decreases from 0.94 to 0.70, and is even concave upward, which indicates that there is about 60% correlation across the half width of the working section, since the curve is practically horizontal at 10 cm. The drop off from 1.0 to 0.94 apparently takes place extremely suddenly. This seems to indicate that a small part of the tunnel "turbulence" is perhaps actually decayed turbulence from the screen in the entrance section. By far the larger part of it seems to be either large pressure fluctuations, or perhaps just plain noise. The curve is not plotted here because it contains a
large region of scatter which, judging from oscilloscope observations, seemed to be due to drumming of the tunnel wall upon which the correlation arm was mounted. At any rate, the question of the free stream correlation should obviously be investigated further. At such low levels of turbulence it is very likely that the exact nature of the turbulent fluctuations becomes increasingly important.

At the lowest test velocity, 5 m/s, both \( u' \) and \( v' \) readings showed considerable fluctuation at the higher values of \( \frac{X}{M} \). The oscilloscope showed that in this region, the frequency of the turbulent fluctuations is on the order of that in an ordinary laminar boundary layer. As would be expected, it was found that at any given tunnel speed, the predominating frequencies in the flow behind a grid decrease very much with increasing \( \frac{X}{M} \). This is, of course, an indication of the growth of scale. Naturally, for any given grid and \( \frac{X}{M} \), the predominating frequencies increase with increasing wind velocity.

During the measurement of one \( v' \) decay curve, a couple of points were checked for \( w' \) simply by rotating the \( X \)-wires 90° about the \( x \)-axis. Both points gave \( w' \) about 5% higher than \( v' \), based on the \( v' \) calibration of the \( X \)-wires. Probably the \( w' \) sensitivity calibration would give a slightly different result, so we cannot conclude that \( w' > v' \). The matter was tabled pending the calibration of some \( X \)-wires in the vertical plane.

In the course of a systematic variation of experimental conditions for the purpose of discovering the reason for the anisotropic
character of the turbulence as measured behind all three grids, the free stream turbulence level was raised from about .05 % to about 0.6 % by means of tape hung all over the honeycomb and the precision screen. Since the absolute level of the u' turbulence behind the 1" grid is quite close to that measured by Atsumi and later by Thiele, it is assumed that the "difficulty" lies with the lateral component of the turbulence. Therefore, v' was measured behind a grid for both values of the free stream level. The result was that with the .6% initial turbulence, the v' behind the grid increased only about 2 % of its value behind the grid in the low turbulence flow.

DISCUSSION

It is quite evident from the first three figures that the discrepancy between the mean values of u' and v' is much greater than the scatter of either one. In view of the fact that the isotropy of the turbulence behind a grid has been explicitly checked by Dryden at the Bureau of Standards (4) and by Dryden and Thiele here at the GARCIT, and has been implicitly checked by MacPhail (5), it is evident that this result must be regarded very cautiously.

As mentioned before, a special effort has been made to vary the experimental conditions as much as possible, but although an occasional v' curve may approach the u' level, the average still remains low. Also, it should be noted on the page 12 tabulation experimental results that, with but two exceptions, the value of
exponent \( a \) is less for \( v' \) than for \( u' \). In other words, the rate of
decay of \( v' \) is less than that of \( u' \). This at least means that if
\( v' \) is actually lower than \( u' \), the condition of isotropy is never-
theless being approached. The very fact that the rates of decay are
different would seem to keep open the possibility that the two
turbulence levels are also unequal.

However, the final decision must await further and more
conclusive experimental evidence.

In connection with the general possibilities for experimental
error, Dryden \(^{(2)}\) stresses three points in particular: errors due to
wire length, errors due to cut-off frequencies of amplifier, and
errors due to current variation. None of these points causes any trouble
in the \( u' \) decay runs. The correction for wire length is applied
as given by Skramstad \(^{(4)}\), the amplifier has a more than adequate
frequency range as mentioned before, and large chokes in the wire
heating circuits prevent any appreciable current variation.

For the \( v' \) wires, however, there is as yet no sound method published
for the length correction. As a matter of fact, for a \( v' \) meter of the
conventional X type, two corrections must be computed: one for the
lack of 100% correlation over the length of the wires themselves as
in the case of the \( u' \) wire; and a second for the lack of 100% 
correlation between the two wires, which cross each other with a
finite gap, and are even farther apart at all other points.

Perhaps a suitable correction could be arrived at by the method
Skramstad uses in N.A.C.A. T. R. 581, but it would no doubt be
extremely complex. At any rate, it would seem that the correction
to \( v' \) meters must be greater than that to the \( u' \) wires.
Although a small damping device was added to the cantilever hot wire supporting arm, some noticeable vibration was still encountered when either this tunnel or the 10' GALCIT tunnel ran at certain critical speeds. However, the effect of this vibration is not of the order of magnitude of the gap between the u' and v' decay curves, since this vibration occasionally started during a run, and merely increased the scatter a bit.

Returning for a moment to the tabulation of data in the section on experimental results, it must be remembered that the u' curves from which the data were gotten were corrected for wire length, while the v' curves were not. The wire length corrections will in general not appreciably affect the slope of the logarithmic plot, but they will change the level of the curve, and consequently the value of C1.

A very interesting and rather surprising result is that at any given value of \( \frac{X}{M} \), the absolute level of turbulence is very nearly the same for the three similar grids. Obviously this approximate equality must break down when the grid becomes either so small or so large that a critical Reynolds Number is reached for the cylindrical rods.

In ref. (2) Dryden has a plot that seems to indicate a considerable lowering of the turbulence level behind a grid when the free stream level is very much lowered. However, he points out elsewhere in the paper that no two tunnels seem to get the same results anyway. Therefore, we might possibly conclude that the difference in level is a tunnel characteristic. In the present investigation, the free tunnel turbulence is about .05% as compared with about .2% for the
measurements of Atsumi and Thiele. However, the level of \( u' \) turbulence behind the 1" grid checks very closely with that measured with the higher initial turbulence.

CONCLUSIONS

The following conclusions are rather tentative; it is hoped that more certain results, and in particular the reasons for them, will come out of further investigation.

1) The isotropy of turbulence behind a regular grid is not verified, but there is insufficient evidence to say that it is disproved, considering the strong experimental group who have checked the isotropy.

2) Lowering the initial turbulence level from 0.2 % to 0.05 % has no appreciable effect on the level of \( u' \) behind a grid.

3) The values of the exponent \( a \), indicating the rate of decay of the turbulence behind the grids, is as tabulated on page 12. The values for the \( \frac{1}{8} \)" grid are somewhat in doubt due to irregularities in the rod spacing.
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APPENDIX I — Time constant calibration to determine proper amplifier compensation resistance.

Because of the thermal capacity of the wire and the limited rate of supply of energy, it is obvious that the hot wire will lag rapid fluctuations in speed and that the lag will increase with increasing frequency. Since the theory of this lag for small speed fluctuations gives results which are the same as the expression giving the magnitude and phase of the current relative to the impressed voltage in a circuit containing inductive reactance and resistance, it is possible to compensate for the frequency characteristics of the hot wire by introducing an electrical circuit with opposite frequency characteristics into the amplifier. A complete discussion is given in reference (8). In general there is a constant inductive reactance in the compensating circuit, and an adjustable resistance which is set according to the frequency characteristics of the given hot wire at a given current and wind speed, as determined by the time constant calibration.

By proper manipulation of King's equation for the cooling of a small heated cylinder in an air stream (8) an expression for the time constant is derived:

\[
M = \frac{4.2 \text{ ms} (R-R_a)}{i^2 R_a R_o \alpha}
\]

(11)

where:  
\( M \) = time constant
\( R \) = wire resistance as operated
\( R_a \) = wire resistance at the air temperature
\[ R_0 = \text{wire resistance at } 0^\circ \]

\[ m = \text{mass of wire} \]

\[ s = \text{specific heat of wire material} \]

\[ i = \text{heating current} \]

\[ \alpha = \text{temperature coefficient of change of resistivity} \]

\[ M \text{ has the dimensions of time, and it can be shown (8) that it is the time necessary for a small temperature change in the wire to decrease to } \frac{1}{e} \text{ of its original value. It can also be shown that if } L \text{ and } Z \text{ are the inductance and resistance in the compensating circuit, the wire will be properly compensated if } \frac{L}{Z} = M. \text{ Then, from equation (11), the expression for the compensation resistance becomes} \]

\[ Z = \frac{L R_0 \alpha}{4.2 \text{ m} \cdot \text{s}} \cdot \frac{i^2 R_a}{R - R_a} \quad (12) \]

This is independent of wire length, and the factor \( \frac{L R_0 \alpha}{4.2 \text{ m} \cdot \text{s}} \) should be a constant for a given diameter platinum wire. We call this constant \( K \), and the calibration equation is written \[ Z = K \frac{i^2 R_a}{(R - R_a)} \].

Over the past two years a great many time constant calibrations have been made at the GALCIT, with both the dynamic vibrator (7) and the oscillator (8) methods. A statistical survey was made of the values of \( K \) resulting from these calibrations for both \( u' \) wires and \( v' \) meters (the latter can be calibrated by oscillator only, of course), with nominal wire diameters of both .00025 and .00024 inches. The difference in \( K \) for the two diameters was found to be small relative to the variation in the values for either diameter separately, so all the values were averaged together.
The results of the survey for \( u' \) wires gave \( K = 4.15 \times 10^5 \), with the mean deviation from the mean value being about \( 0.07 \times 10^5 \), or 1.7 \%. For the \( v' \) meters, \( K = 6.3 \times 10^5 \) with the mean deviation from the mean value equal to \( 0.09 \times 10^5 \) or about 1.4 \%.

Of course King's equation can be only a rough approximation for the wires in a \( v' \) meter, but the consistency of the values of \( K \) for wires which were certainly at assorted angles to the wind seems to indicate that this form is quite adequate.
Turbulence Decay: 1" Grid

- U = 15 m/s

\[ \sqrt{\frac{u^2}{u^2}} \]

- \( \times \) 15
- \( \triangle \) 10
- \( \triangledown \) 10
- \( \square \) 7.5

\( \sqrt{\frac{v^2}{v^2}} \)

- \( \times \) 15
- \( \triangle \) 10
- \( \triangledown \) 10

(uncorrected)
Turbulence Decay: $\frac{1}{2}$" Grid

$\sqrt{\bar{u}^2} \approx 14.6 \text{ m/s} = U$

$\sqrt{\bar{u}^2} \approx 10$

$\frac{\sqrt{\bar{u}^2}}{U} \approx 5$

(uncorrected)
Turbulence Decay
WITH LENGTH CORRECTION

- 1" Grid, \( U = 15 \text{ m/s} \)
- 1" Grid, \( U = 15 \)
- 0.75" Grid, \( U = 15 \)
- 0.5" Grid, \( U = 14.6 \)
$u'$ Turbulence Decay

With Length Correction

△ = 1" Grid; $U = 10$ m/s

△ = 1"

△ = $\frac{1}{2}$"

△ = $\frac{1}{4}$"
Turbulence Decay: 1" Grid

\[ u' \] corr. for \( u' \) sens.

\[ \langle u' \rangle = 15 \text{ m/s} \]

\[ \vartriangle \] 10

\[ \circ \] 10

Fig. 7
Turbulence Decay: 2" Grid

corr. for u' sens.

\[ U = 14.6 \text{ m/s} \]

\[ \nabla \quad 10 \]

\[ \circ \quad 5 \]
1" Grid @ \( U = 10 \% \)
Fig. 14 - The Wind Tunnel

Fig. 15 - Electrical Equipment and Manometer
Fig. 16a - 1" Grid

Fig. 16b - $\frac{1}{2}$" Grid
Fig. 16c - $\frac{3}{4}$" Grid

Fig. 17 - Hot Wire Mounted on Cantilever Arm; View into Working Section from Side of Top and Slightly Downstream; Longitudinal Slot Is Sealed with Endless Shimstock Belt below.