AN AERODYNAMIC STUDY OF THE "ELECTRIC WIND"

Thesis by

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ABSTRACT

The basic physical process considered is the appearance of a pressure gradient in a slightly ionized gas under the influence of an electric field. On a molecular scale this process involves the acceleration of ions in an electric field and the transfer of momentum to the main body of the fluid through collisions.

A particular phenomenon involving this basic process is the pressure gradient in a point corona discharge and the resulting "electric wind". The experimental arrangement studied is a point to a downstream coaxial cylinder discharge in air.

The difficulity of applying boundary conditions to the mathematical description of the process requires some simplifying assumptions. A simple one-dimensional model of the physical situation is considered, the electrical field equations being approached from the consideration of a space charge limited current. The functional relations derived appear consistent with the experimental results for the limiting cases implied by the assumptions.

Velocity variations encountered in the induced flow of air present questions regarding the bulk parameter which describes the molecular interchange of momentum, the ion mobility. While this experiment was not for the purpose of measuring ion mobilities, it is believed that in any applications involving the basic process in the working fluid of the aerodynamicist the time, density, temperature and humidity dependence of the ion mobility requires data which is not now available.

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SYMBOLS

a	$= 2/\epsilon$
A	Cross sectional area of tube.
E	Electric field strength.
i	Current in amperes
j	Current density - current/area.
k	Ion mobility.
ℓ	Axial distance between electrodes.
p	Hydrodynamic pressure.
u, U	Airstream velocity.
u_r	Ratio of airstream velocity to the maximum ion drift velocity.
V	Electric potential.
x	Axial distance measured from point.
ϵ	Dielectric constant for air.
η	Efficiency in %.
ρ	Air density.
σ	Charge density.

UNITS

A rationalized length-mass-time-charge system is employed where charge is in coulombs, and, compatible with the usual aerodynamic system, length, mass, and time have units feet, slugs, and seconds. It is found convenient to retain the volt as the unit of electric potential with the result that force is given in volt-coulombs per foot. In this system the dielectric constant has the value

$$\epsilon = 2.70 \times 10^{-12} \frac{\text{coulombs}}{\text{volt-feet}}$$

The hydrodynamic pressure in electrical units is in $\frac{\text{volt-coulombs}}{\text{ft}^3}$.

For p in lb./ft^2,

$$1 \frac{\text{volt-coulombs}}{\text{ft}^3} = .7376 \frac{\text{lb.}}{\text{ft}^2}$$

I. INTRODUCTION

With a view toward possible aerodynamic applications of electrical field forces in an ionized gas there is considered here an effect due to such forces which, although small by comparison to the effects of dynamic forces generally considered is, nevertheless, real and measurable. This effect, which is well known but of only passing interest to the physicist and electrical engineer, is the "electric wind" or the flow of air induced by a point corona discharge.

The complete problem involves an interaction between the electrical field and the flow field of the air. The electric field induces an air flow, and the air flow, established according to additional boundary conditions, in turn affects the electric field.

To prevent the analysis of the problem from becoming hopelessly complex so that the "how much" as well as the "why" may be answered, a simplified one-dimensional model of the system is considered. For this model approximations are required only for the boundary conditions since the governing field equations are quite simple.

Initally considered is the pressure resulting from a given electric field and charge distribution - the charge distribution being determined by the properties of the ions, the properties of the air (mainly velocity and density), and by the electric field. Secondly, the effect of the airstream on the electric field is estimated from the measurable characteristics of the discharge varying the air velocity. Then, for given boundary conditions on the air flow the induced airstream velocity may be determined as a function of the electric current.

II. PRESSURE DUE TO ION MOTION

Momentum Exchange and Ion Mobility

Of interest in this study is the behavior of a slightly ionized gas under the influence of an applied electric field. "Slightly ionized" refers to the proportion of molecules ionized and does not imply that the charge density or the electrical forces on the charges need be small. The condition that the gas is slightly ionized holds generally for a point corona discharge except for a small localized region of breakdown near the point. In addition, outside of this localized region, the ions are unipolar, and we may consider the motion of ions of only one sign.

On a molecular scale the ions are accelerated by the electric field and interchange momentum with the neutral gas molecules by collision.

On a macroscopic scale the bulk property which characterises this molecular interchange is the ion mobility. If u_i is the average ion drift velocity and E the magnitude of the electric field, then the ion mobility, k, is defined by

$$u_i = kE$$

To show that k is actually a parameter which relates the electric current to the momentum exchange with the gas, consider ion motion through a stationary gas. Multiply both sides of the defining equation for k by the charge density, σ ,

$$\sigma u_i = kE\sigma$$

$$\text{or} \quad \sigma E = \frac{\sigma u_i}{k} = \frac{j}{k}$$

Now, σu_i is just the current density, j, while σE is a force per unit volume or better a pressure gradient exerted on the fluid or conductor.

The ion mobility is readily related to the conductivity of a continuous medium. If the conductivity, c, is defined by the relation

$j = cE$ where j is the current density

then $c = \sigma k$

The conductivity is more familiar as describing the macrosopic properties of electron motion. In this case the momentum transfer to a continuous medium is extremely small, k being of the order of 10^4 times the value for that of negative ions in air.

The functional relation of k with the molecular properties of the ions and neutral molecules may be derived using the kinetic theory of gases after some suitable choice of a molecular model. Different molecular models result in differences in the calculated temperature dependence of k. Phillips[(6)] measured the mobility of positive and negative ions in air at constant pressure for temperatures between 94° and 411° K. These results reduced to constant density (using the perfect gas equation) show the mobility independent of temperature above 250° K. In agreement both experimentally and as derived for all molecular models is the density dependence of k. The mobility is inversely proportional to the density of the gas.

The experimental values of the ion mobility are not too well defined except for well controlled experiments with pure gases where there is some certainty about the nature of the ions. There is especially some uncertainty about the value of k for the following experiments which were performed using room air.

In air the nature of the ion is difficult to define. This is especially true for the positive ion for which there is a characteristic time at which ions of a different mobility appear. Experiments indicate that for small time the mobility of the positive ion is approximately the same as that for the stable negative ion - the mobility decreasing with time. This "aging" of the positive ion is generally accepted to result from combination of the ions with other neutral molecules. To give an order of magnitude of the characteristic time of the positive ion Loeb (2) states that "no more than two mobilities and usually only one mobility appear for ions in a pure gas at any age below 10^{-2} seconds."

The primary negative ion in air in a corona discharge is formed by electron attachment to oxygen molecules. Thus, in a negative point corona discharge there is also a characteristic time for the formation of the negative ions. This time dependence of the ion mobility will later be considered as a possible explanation of variations in the induced velocity of the air in the corona discharge.

For certain limiting conditions the ion mobility may be considered constant. As an example, if the electrode geometry and/or the airstream velocity are such that the time of passage of a positive ion is less than the characteristic time for which ion-molecule combinations are probable then the mobility may reasonably be assumed constant.

With regard to the density dependence of k, in the experiments of this study the airstream velocities range only to the order of 200ft/sec. allowing the approximation of an incompressible flow of air. It will further be seen that the electric field forces are an order of magnitude smaller than the maximum dynamic forces of the airstream so that density changes due to pressure may be neglected. Thus, throughout the region of the discharge the ion mobility will be assumed independent of density.

Approximate values for the "aged" positive ion, k^+, and for "new" positive ions and negative ions, k^-, in dry air at standard sea level density are

$$k^+ = 1.5 \times 10^{-3} \frac{\text{ft/sec}}{\text{volts/ft.}} \qquad (1)$$

Ref: Thomson p.123

$$k^- = 2.3 \times 10^{-3} \frac{\text{ft/sec}}{\text{volts/ft.}}$$

Pressure Gradients in an Ionized Gas

Consider the motion of a uniform airstream with velocity, u, in the direction of a high voltage electrode point. Outside of the small region of break-down near a positive point there is a density of positive charge, σ, which moves under the action of the airstream and the electric field, E,.

For this one-dimensional incompressible flow of air the momentum equation of continuum mechanics reduces to

$$\nabla p = \sigma \vec{E}$$

i.e., the pressure gradient is proportional to the charge density and the resultant electric field.

The conservation of charge is expressed by the equation

$$\vec{j} = \sigma (\vec{u} + k\vec{E})$$

where $\vec{j}$ is the current density (current/area) and k is the ion mobility. The sum $(\vec{u} + k\vec{E})$ represents the resultant ion velocity — the ion drift velocity, $k\vec{E}$, now being measured relative to the moving airstream.

For the stationary case, u=0, the problem is much simplified since

$$\sigma = \frac{|\vec{j}|}{k|\vec{E}|}$$

the current now having the same direction as E.

Thus $$\nabla p = \frac{|\vec{j}|}{k} \frac{\vec{E}}{|\vec{E}|} = \frac{\vec{j}}{k}$$

since as before $\vec{E}$ and $\vec{j}$ are collinear.

This rather simple result shows that for this stationary case the pressure gradient may be determined without any explicit knowledge of the field strength or charge density which may be difficult to determine for all but the simplest of geometries. Chattock[4] used this result to measure the mobilities of ions produced in a point-to-ring corona discharge. Enclosing the electrode pair in a glass cylinder and making the distance between the electrodes large compared to the diameter of the ring anode, the current was assumed one-dimensional giving

$$\frac{dp}{dx} = \frac{j}{k}$$

and $$\Delta p = \frac{i\ell}{Ak}$$

where i is the total measured current, Δp the pressure difference along the tube, A the cross-sectional area of the tube, and ℓ the axial distance between the electrodes. With this arrangement k was determined by measuring the pressure difference and the current, giving the "right answer" after some manipulation with the geometrical parameters.

For the case of an air flow superimposed on the discharge the problem is much more complicated even with the one-dimensional assumption which will be maintained. The one dimensional momentum equation is

$$\frac{dp}{dx} = \sigma E$$

where now

$$\sigma = \frac{j}{u+kE}.$$

giving $$\frac{dp}{dx} = j \frac{E}{u+kE}$$

In an integration over x the current density might legitimately be assumed constant if the electrode spacing is large compared to the radius of the outside electrode. However, the field strength now appears explicitly. An attempt to apply the simplifying assumption of a one-dimensional field leads to some difficulty with boundary conditions especially at $x = 0$, i.e., in the neighborhood of the point. Here the field is highly non-uniform, a condition which is necessary for the occurrence of the corona discharge.

The problem of determining the pressure difference across the corona discharge from the above equation resolves itself to finding an approximate one-dimensional expression for the electric field. This requires a consideration of the characteristics of the point corona discharge and how these characteristics are affected by an airstream.

III. CHARACTERISTICS OF THE CORONA DISCHARGE IN AN AIRSTREAM

The Point Corona Discharge

The electrical discharge at a point may be described in various phases. For a positive point the phases may be enumerated as follows:

1.) Conduction below ionization by collision.

For low applied voltage the current to the point depends upon ions produced by an external agent. In normal room air the magnitude of this current is minute and discontinuous.

2.) Conduction with Ionization by collision.

For higher applied voltages the field strength increases to the point where electrons resulting from external ioniztion gain sufficient energy in the field to cause further ionization by collision. For sufficiently large field strength the "Townsend avalanche" effect results in a current pulse, which greatly magnifies the effect of the original electron.

3.) The Geiger Counter regime.

For still higher field strength a single externally produced ion results in an "avalanche" in which sufficient radiation is produced to cause further photoelectric ionization in the gas. A large current pulse builds up to a point where the space charge distortion of the field chokes the process of ionization by collision and the discharge ceases.

4.) Continuous corona regime.

For yet higher applied voltages the space charge distortion cannot reduce the field strength to the point of choking and there results a self sustained discharge. The visible corona glow now appears about the point. This discharge can be maintained in the absence of another electrode by photoelectric ionization in the gas and at lower potentials by additional photoelectric ionization at another electrode.

5.) Breakdown stage.

The field strength may be increased to the point where the breakdown of the gas extends across the electrode gap. This may initially be in the form of "pre-breakdown streamers" or"brushes". Finally, the corona discharge culminates in an arc discharge.

The only phase of the discharge which is considered here is the continuous or self sustaining discharge. Further, it is considered only from the viewpoint of continuum mechanics without reference to the "mechanistic" descriptions of electron avalanches, etc. From this point of view there are no distinctions to be made in the description of the electric field in a positive or negative discharge. The only difference considered is that of the bulk parameter k, the ion mobility.

Point to Cylinder Discharge

The geometrical electrode arrangement to be considered is that of a point to coaxial cylinder with the cylinder displaced downstream of the point as illustrated in figure 5. This arrangement presents itself as aerodynamically "clean" with respect to the flow of air. This is important since in the measurements made the electrical pressure forces are of the same order of magnitude as the frictional forces.

The description of the electric field in such a discharge presents a difficult task if an exact solution is desired. Even for the simpler case of no space charge, exact solutions are readily obtained only for very simple geometrical electrode arrangements, i.e. where the boundary conditions are simply given in an appropriate coordinate system. With appreciable space charge, which is the case in the region near the point electrode, the boundary conditions to be applied in the solutions of the field equation are not only difficult to express mathematically but are difficult to determine.

Reasonably accurate solutions for the infinitly long, thin, wire to coaxial cylinder corona discharge have been developed and are presented by Loeb.[(2)] A solution of Laplace's equation is derived neglecting the space charge outside of a cylindrical surface at which the boundary condition is applied. This procedure of neglecting the space charge and using Laplace's equation as an approximation to the field seems to be a generally accepted method of approximating the field for general electrode arrangements.

If this approximation is accepted for the point corona discharge, then it would be expected that pressures due to momentum exchange would dominantly occur in the vicinity of the point electrode where the field forces as given by Laplace's equation are extremely high, falling to negligibly small values over a distance small compared with that of the electrode spacing. Evidence that this is not the case is presented by Chattock (4) in his preliminary investigations of the pressures on a flat plate in a point to plane discharge. Pressure distributions were obtained at constant current and electrode spacing for two cases. The first case was for a simple point to plane geometry. The second case involved an additional electrode surrounding the point which collimated the field except in the neighborhood of the point. With the field collimated the pressure distribution was sharply peaked just opposite the point and fell off rapidly with the radial distance from the centerline. Without the collimator the pressure distribution was relatively flat with appreciable pressure being maintained on the plate to the limit of the traverse mechanism. Chattock's conclusions were that

"The pressure cannot be due either wholly or in considerable part to the imparting of momentum to the gas at the point itself; otherwise the pressure curves a and b would be alike, since the field at a discharging point is constant for a given current".

This evidence and the apparent success obtained by Chattock in using a one-dimensional analysis in the point to ring discharge discouraged the use of an electric field calculated by neglecting space charge and applying this result to determining the pressures. Such an analysis would be productive if a suitable boundary condition could be calculated and applied at some surface outside of the immediate neighborhood of the point i.e., at some surface outside of which the space charge might legitimately be neglected in describing the field. Such an analysis, while complex for a stationary fluid, becomes further complicated for the present case in which the air stream and the discharge are interdependent. The airstream influences the boundary conditions of the electric field and the electric field in turn influences the induced velocities due to the pressure gradient.

The simplified model to be considered is that of a one-dimensional space charge limited current in a one-dimensional incompressible flow of air. The one-dimensional assumption is maintained throughout, including the ion source. The ions are assumed to emanate from a planar source at a potential equal to that of the starting potential of the continuous corona discharge. The potential drop or anode-fall is taken as the difference between the applied potential and the starting potential of the discharge. This energy loss may be considered as that required for creation of the ions including losses by radiation, excited molecular states, and chemical reactions as well as the energy manifesting itself in useful ionization.

The Electric Field in One Dimension

If D is the electric excitation in units charge per unit area and σ is the charge density in units charge per unit volume, then

$$\nabla \cdot \vec{D} = \sigma$$

Using the electrostatic charge-force relation given by

$$\vec{D} = \varepsilon \vec{E}$$

where ε is the dielectric constant, the differential equation for the electric field vector, $\vec{E}$, is

$$\nabla \cdot \vec{E} = \frac{\sigma}{\varepsilon}$$

For the one-dimensional case

$$\frac{dE}{dx} = \frac{\sigma}{\varepsilon}$$

Using the equation expressing the conservation of charge, this becomes

$$\frac{dE}{dx} = \frac{1}{\varepsilon} \frac{j}{u + kE}$$

or

$$\frac{1}{2k} d(u + kE)^2 = \frac{j}{\varepsilon} dx$$

Integrating between the electrodes from $x = 0$ which is taken to be the edge of the small region of break-down about the point anode and letting E_0 be the field strength at $x = 0$

$$(u + kE)^2 - (u + kE_0)^2 = \frac{2}{\varepsilon} kjx$$

$$E = \frac{1}{k}\left[\frac{2}{\varepsilon} kjx + (u + kE_0)^2\right]^{\frac{1}{2}} - \frac{u}{k}$$

This expression may be further simplified if the one-dimensional approximation is maintained by assuming that the ion source is a planar ionized layer; then Thompson (1)# shows that the field strength at the edge of such a planar source may be neglected. With this assumption the expression for the electric field becomes

$$E = \frac{1}{k}\left[\frac{2}{\varepsilon}\,kjx + u^2\right]^{\frac{1}{2}} - \frac{u}{k}$$

Having an expression for the field strength, the equation for the pressure gradient in terms of j and u may be written down immediately. However, from the pressure gradient it would be difficult to experimentally check the validity of the one-dimensional assumption over any reasonable range of air stream velocities since electric field forces become very small in comparison to the dynamic pressure forces and the viscous forces at the boundary of the flow.

The range of velocities and currents for which this equation may be valid will be estimated by carrying the one-dimensional analysis further to obtain a relation for the current in terms of the velocity and applied potential. These relations are readily checked experimentally.

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One-Dimensional Voltage-Current Relation

As an electrostatic problem the field is irrotational, i.e.

$$E = -\frac{d\phi}{dx} \text{ where } \phi \text{ is the electric potential.}$$

Thus,

$$\phi = -\int_0^x E\,dx$$

Integrating the expression for E from $x = 0$ to $x = \ell$ and letting $\Delta\phi = \phi_0 - \phi_\ell$

$$\Delta\phi = \int_0^\ell E dx = \frac{1}{k}\int_0^\ell \left\{\left[ajkx + u^2\right]^{\frac{1}{2}} - u\right\} dx$$

$$= \frac{2}{3}\frac{1}{ajk^2}\left\{\left[ajk\ell + u^2\right]^{\frac{3}{2}} - u^3\right\} - \frac{\ell u}{k} \text{ where } a = \frac{2}{\varepsilon}$$

To get an approximate, explicit expression for the current the magnitudes of the various terms must be evaluated, especially the constants of integration ϕ_0. The current as a function of applied potential is zero up to a potential $V = V_s$ which is defined as the starting potential of the self sustaining corona discharge. The current is primarily a function of $(V - V_s)$, the difference between the applied potential difference and the starting potential.

In an attempt to avoid the details of the non-uniform field assume that $\Delta\phi = V - V_s$, i.e., that the anode fall of potential is $V - V_s$. Considering first the simpler case for $u = 0$

$$\Delta\phi = V - V_s = \frac{2}{3}\frac{\ell}{k}\left[ajk\ell\right]^{\frac{1}{2}}$$

Before checking the validity of this equation in the light of experimental results the order of magnitude of the quantity $ajk\ell$, which has the dimensions of a velocity squared, will be estimated. This velocity is in the one-dimensional case for $u = 0$ the maximum ion drift velocity attained at the downstream electrode. Corresponding to experimental values, let:

ℓ = axial distance between electrodes = 6.25×10^{-2} ft.

A = cross sectional area = 2.08×10^{-2} ft.

In addition, let

$$a = 7.41 \times 10^{11} \frac{\text{volt-ft}}{\text{coulomb}}$$ ~ dielectric constant in vacuo

$j = \frac{i}{A} \times 10^{-6}$ where i is the measured current in microamps.

and assume

$$k = 1.6 \times 10^{-3} \frac{\text{ft/sec}}{\text{volts/ft}}$$

then,

$$ajk\ell = \left(190 \frac{\text{ft}}{\text{sec}}\right)^2 / \mu\text{amp.}$$

and the current in microamps. is

$$i = 4.18 \times 10^{-8} (V-V_s)^2$$

The current - voltage relation of the point corona is usually described by an equation of the form

$$i = C V (V-V_s)$$

This agrees with experimental results over a large range of voltages. However, for large currents the data is better described by a relation of the form

$$i = C (V-V_s)^2$$

For large currents where the approximation

$ajk\ell \gg (kE_o)^2$ is valid the experimental results corresponding to the above equation gives

$$i = 3.3 \times 10^{-8} (V-V_s)^2$$

The agreement with the approximate equation is quite satisfactory since the concern here is primarily to establish some justification for approximate relations for the pressure difference across the discharge. Better agreement in the constant of proportionality might be obtained by some manipulation of the geometrical parameters. For instance the distance, ℓ, may be considered only as a characteristic length. Using the distance between the electrodes for this length is correct only as a orientating value.

Current as a Function of Voltage and Airstream Velocity

Under the influence of a moving airstream the current is given implicitly in the one-dimensional case by

$$V - V_s = \frac{2}{3}\frac{1}{ajk^2}\left\{\left[ajk\ell + u^2\right]^{\frac{3}{2}} - u^3\right\} - \frac{u\ell}{k}$$

For $aik\ell > u^2$,

$$V - V_s = \frac{2}{3}\frac{1}{ajk^2}\left\{\left[(ajk\ell)^{\frac{3}{2}} + \frac{3}{2}u^2(ajk\ell)^{\frac{1}{2}} + \cdots\right] - u^3\right\} - \frac{u\ell}{k}$$

$$\doteq \frac{2}{3}\frac{\ell}{k}(ajk\ell)^{\frac{1}{2}} + \frac{\ell}{k}\frac{u^2}{(ajk\ell)^{\frac{1}{2}}} - \frac{2}{3}\frac{\ell}{k}\frac{u^3}{aik\ell} - \frac{u\ell}{k}$$

Introducing the velocity ratio parameter $u_r = \frac{u}{(ajk\ell)^{\frac{1}{2}}}$

$$V - V_s = \frac{2}{3}\frac{\ell}{k}(ajk\ell)^{\frac{1}{2}} - \frac{u\ell}{k}\left[1 - u_r + \frac{2}{3}u_r^2\right]$$

Dividing by the current density at zero velocity, j_o, and rearranging

$$\left[\frac{j}{j_o}\right]^{\frac{1}{2}} = 1 + \frac{u\ell}{(V-V_s)k}\left(1 - u_r \quad \frac{2}{3}u_r^2\right) = \left[\frac{i}{i_o}\right]^{\frac{1}{2}}$$

This relation expresses the characteristics of the discharge in an airstream. The major physical argument leading to its derivation is that the ions enter the electric field with the velocity u and the electrostatic potential energy $(V-V_s)$.

The equation is still implicit in i since u_r is a function of i. A simple explicit expression is obtained for the case $u_r \ll 1$, viz.,

$$\left[\frac{i}{i_o}\right]^{\frac{1}{2}} = 1 + \frac{u\ell}{(V-V_s)k} \qquad (u_r \ll 1)$$

This relation expresses the experimental results for a limited range of voltage and velocity implied in the derivation. For high applied voltage, i.e., with increasing space charge, this relation appears to apply, as can be seen from figure 2.

The deviations from the linear relation of $\left(\frac{i}{i_o}\right)^{\frac{1}{2}}$ vs. $\left(\frac{u}{V-V_s}\right)$ occur with decreasing applied voltage.

In attempting to evaluate the geometrical factor, ℓ , using the supposedly more accurate expression involving the expansion in terms of the velocity ratio, u_r, it was found that the approximate expression for the current was a better functional relation in u. This would further limit the use of the assumed expression for the field strength to the condition that $u_r \ll 1$.

The data of figure 2 represents airstream velocities only as low as 75 ft/sec. Figure 3 indicates that the data may be linearly extrapolated to lower velocities.

From the above it is expected that the equation for the assumed electric field may be applied to evaluating the pressure under the restrictions of high voltage and small velocities.

Empirical Current-Velocity Relation for Low Voltages

A strictly empirical reduction of the data gives the simple representation illustrated in figure 4. This relation is valid for lower applied voltages, i.e., below the region of validity of the equations of a one-dimensional, space charge limited current. The current is implicitly

$$\frac{i}{i_o} = \left[1 + c\,\frac{u^2}{i}\right]^2 \qquad \text{where } c = \text{constant.}$$

IV. PRESSURE DIFFERENCE ACROSS THE DISCHARGE

Calculated Pressure Difference

The one-dimensional pressure gradient is

$$\frac{dp}{dx} = \sigma E = i \frac{E}{u + kE}$$

Using the simplified expression for the field strength, viz.,

$$E = \frac{1}{k} \left[akjx + u^2 \right]^{\frac{1}{2}} - \frac{u}{k}$$

$$\frac{dp}{dx} = \frac{j}{k} \left[1 - \frac{u}{(akjx + u^2)^{\frac{1}{2}}} \right]$$

Letting $\Delta p = p_{x=\ell} - p_{x=o}$

$$\Delta p = \frac{j}{k} \int_o^{\ell} \left[1 - \frac{u}{(ajkx + u^2)^{\frac{1}{2}}} \right] dx$$

$$= \frac{j\ell}{k} \left[1 - \tfrac{1}{2} u_r \quad (1 + u_r{}^2)^{\frac{1}{2}} + u_r{}^2 \right] \text{ where } u_r = \frac{u}{\sqrt{ajk\ell}}$$

For $u_r \ll 1$

$$\Delta p \doteq \frac{j\ell}{k} \left[1 - \tfrac{1}{2} u_r \right]$$

This represents the pressure difference due to a space charge limited current for airstream velocities small in relation to the maximum ion drift velocity.

The above expression for the pressure involves only the current and the airstream velocity; the applied voltage does not appear explicitly. An expression for the pressure difference as it varies with airstream velocity, only, may be obtained by substituting the expression for the current as a function of velocity as previously derived.

The current as linearized for small velocities is

$$\frac{j}{j_o} = 1 + 3 \frac{u}{aj_o k\ell}$$

This gives to first order terms

$$\Delta p = \frac{j_0 \ell}{k}\left[1 + \left(3\sqrt{\frac{j}{j_0}} - \frac{1}{2}\right) u_r\right]$$

or approximately,

$$\Delta p = \frac{j_0 \ell}{k}\left[1 + \frac{5}{2}\,\frac{u}{\sqrt{a j_0 k \ell}}\right]$$

This rather encouraging expression indicates that the pressure difference increases if the airstream velocity is allowed to increase. The relation was not checked experimentally since measurements were obtained only for velocities determined by fixed boundary conditions. As will be seen the limits placed on the velocity by frictional losses are quite restrictive.

Measurements of Induced Velocity

Since the pressure difference across a point corona discharge is of the order of 10^{-2} inches of water at atmospheric pressure, it was decided that it would be much easier to measure the induced velocities. To do this a hot wire anemometer was constructed and calibrated on a whirling arm.

In order to take measurements with the hot wire equipment, which was at ground potential, it was necessary that the induced flow be transported to a region outside of the influence of the discharge. By extending the cylindrical electrode sufficiently downstream the influence of the hot wire probe became negligible. This was checked by noting changes in the corona current and by measuring the discharge current picked up by the probe. As a result of minimizing the effects of the probe on the discharge, the increased wall friction is by no means small in relation to the pressure forces in the discharge. For example, the pressure drop due to wall friction was of the order of two-thirds the pressure difference across the discharge.

In figure 5 two typical induced velocity profiles at the downstream end of the tube are illustrated. Included is a velocity profile without the discharge showing the magnitude of the effect of the wall friction. Figure 6 is a plot of the centerline velocities as a function of the current. While there is some variability in the plotted data the severity of actual variations is not shown since only time average values of the velocity are plotted. Velocity differences for curve (a) in figure 5 were of the order of ± .6 ft/sec. From the outset of the experiments which included a number of electrode arrangements these velocity fluctuations which could not be correlated with voltage or current fluctuations were encountered.

Discussion of Velocity Variations

The wide ranges of induced velocity which had been attained for a fixed applied potential and constant current suggested that the variation was due to changes occurring in the stream away from the immediate neighborhood of the point. For a fixed current, conditions at the point should be constant. It was then realized that the time dependency of the ion mobility due to ion-molecule combination might be of importance. One way of checking this was to reverse the polarity of the electrodes to see if the variations persisted for the negative ion. For the same geometrical arrangement the only factor which would be different would be the mobility.

To accomplish this check for the same geometry and to keep the hot-wire probe away from the "hot" electrode the arrangement shown at the top of figure 7 was used with the hot-wire upstream and a center electrode support added. This support further added to frictional losses and gives an increased amount of turbulence at the point as compared with the original arrangement. There, the center electrode extended as a thin rod from the stagnation region. However, these factors probably do not detract from the primary purpose of illustrating differences due to a difference in the ion mobility.

In figure 7 the induced velocity as a function of measured current shows a range of velocities for a given current for both the positive and negative point discharge. A most important feature is that the minimum induced velocity for the positive discharge is very nearly the same as the maximum for the negative discharge. An additional point to note was that one complete run was completed under approximately the same conditions during which the induced velocity was stable. This stable velocity corresponded to the minimum for the positive discharge.

An additional factor to be considered is the effect on the mobility of the moisture content of the air. This not only affects the value of the mobility but the time dependency as well. The relative humidity for the experiment in question was measured at 47% using a sling psychrometer.

It has been stated [1] that the mobility of positive ions is only slightly affected by moisture. This result actually refers to the so called "aged" positive ion. Tyndall and Grindley [5] found that

"The initial positive ion has a mobility which is indistinguishable from that of a negative ion, and which is affected by water vapor in the same way".

Thus, if the mobility that is of importance here is basically that of the initial positive ion, then appreciable changes due to moisture content can be expected to occur. However, they might be expected to be of importance only for the situation where the "life" of the ion is not very long or very short compared to the characteristic time.

Calculation of Pressure Difference from Velocity Measurements

In all the measurements obtained of the induced velocity the results are somewhat beclouded by the magnitude of the frictional losses and the variability noted in the velocity. For a best estimate of the pressure difference across the discharge use is made of the data of figure 7 for the steady positive discharge, i.e. the dividing line between negative and positive discharge. For this case the ion mobility appears to be "pinned down" at least to some constant value. The pressure drop due to frictional losses is estimated from velocity surveys at the exit of the tube in the absence of the discharge.

From these velocity surveys the mean velocity may be calculated. This is approximately the velocity at the entrance to the tube, where the velocity profile is nearly constant. The frictional pressure drop is then correlated with that for the induced flow at the same mean velocity. The pressure drop due to friction is readily obtained by applying Bernoulli's equation to the core of the flow which is practically unaffected by frictioh, i.e., unaffected by viscous shear, not unaffected in the sense that the core is accelerated. Thus, the pressure drop is obtained using only a velocity survey at the exit of the tube.

In the absence of frictional losses in the tube the pressure difference across the discharge would be just equal to the dynamic head measured at the inlet. Actually, the pressure difference due to the discharge is the sum of the inlet head plus the magnitude of the pressure drop due to friction. In figure 8 is shown the magnitude of the frictional loss at the measured mean velocity. In the same figure the total pressure difference due to the discharge is plotted as a function of the current. Approximately two-thirds of the pressure increase due to the discharge is lost as a result of friction.

Comparison With Calculated Pressure Difference

The data of figure 8 is best compared with the expression derived for the variation of the pressure difference as a function of the current and the velocity ratio, u_r, viz.

$$\Delta p = \frac{j\ell}{k}\left[1 - \tfrac{1}{2}\, u_r\right]$$

The limitations imposed on the velocity which are primarily due to frictional losses give a value of u_r of the order of 0.02. Thus, the effect of the airstream velocity gives corrections of the order of 1% which may be neglected.

If the axial distance between the electrodes is taken as ℓ and the current density computed using the cross-sectional area of the tube, then the value of k which matches the straight line of figure 8 is $k = 2.4 \times 10^{-3} \frac{\text{ft/sec}}{\text{volts/ft}}$. This is, to two significant figures, the value of k given on page 6 and corrected for the air density of the experiment. It is probably the correct value to use since the "life" of the ions is of the order of 10^{-3} sec., and, especially with moisture present, the mobility should correspond to the value of that for "new" positive ions.

V. CONCLUSIONS

1.) The restrictions placed on the induced velocity preclude confirmation of the equations for the pressure difference to anywhere near the limits for which the characteristics of the discharge in an airstream were obtained. However, the data indicates that the one-dimensional analysis gives essentially the correct result for low speeds for the pressure difference and, in addition, gives a reasonable description of the current-velocity relation for fairly high speeds. Thus, predictions based upon this model should not be too far from the truth.

2.) This study was intended to raise questions regarding applications of the basic process. The relation of the ion mobility with the properties of atmospheric air and the geometry of a particular problem is just such a question which presented itself and which was not answered.

3.) With respect to aeronautical engineering applications of electrical pressure forces, it was realized from the outset that this study might be classified in the realm of "bird - watching". Power inputs are small and power outputs in terms of directed energy are still much smaller.

The efficiency of the experimental device as a pump can be estimated from the expressions for the pressure difference and, for a given applied voltage, the expression for the current as a function of the airstream velocity. The power input is just

Vi and the power output is $uA\Delta p$. As an example, from the data of figure 1 for a fixed applied voltage of 11,600 volts, $i_o = 2.22\ \mu$amps, and considering airstream velocities small in comparison to the maximum ion drift velocity the efficiency in percent is

$$\eta = .22u\ \% \qquad \text{(u in ft/sec)}$$

For a mean induced velocity of the order of 5 ft/sec. the efficiency of the device as a pump is of the order of 1%.

The efficiency is limited to low values for two reasons:

a.) The most important restriction is that placed on the velocity of the flow. This is primarily the result of frictional losses.

b.) The corona discharge is an inefficient source of ionization, i.e., the useable power is $(V-V_s)i$ whereas the total power input is Vi. V_s is of the order of thousands of volts whereas the useful ionization requires an ionizing potential only of the order of 15 volts.

To increase the magnitude of the effects considered here, i.e., to introduce much higher charge and current densities other methods are certainly available. For example, the mechanism of ionization might employ alpha rays rather than the corona discharge. The magnitude of the effects due to a corona discharge is determined by the arcing limit. The high field intensity required for the sustenance of the discharge limits the current below the disruptive discharge to relatively low values.

References

1. Thomson, O.M. and Thomson, G.P., Conduction of Electricity Through Gases, Ed. 3, Vol. 1, Cambridge University Press, 1928
2. Loeb, L.B., Fundamental Processes of Electrical Discharge in Gases, John Wiley & Sons, New York, 1939
3. Loeb, L.B., Basic Processes of Gaseous Electronics, University of California Press, 1955
4. Chattock, A.P., Philisophical Magazine, Vol. 48, p. 401, 1899
5. Tyndall and Grindley, Proc. Roy. Soc., A, cx, p.341, 1926
6. Phillips, P., Proc. Roy. Soc. A78, 167, 1906

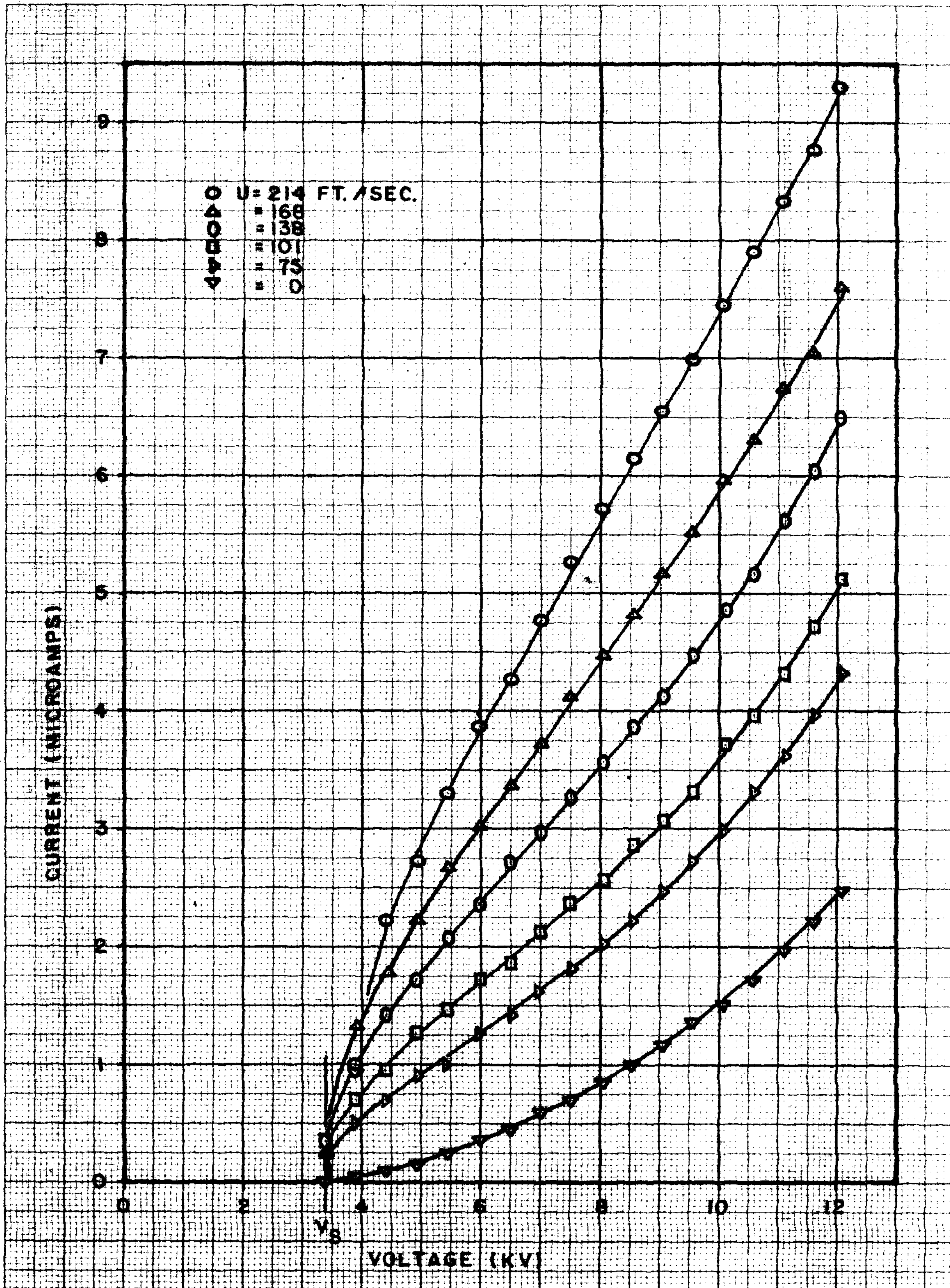

Fig. 1. Corona discharge current versus applied voltage with airstream velocity as a parameter.

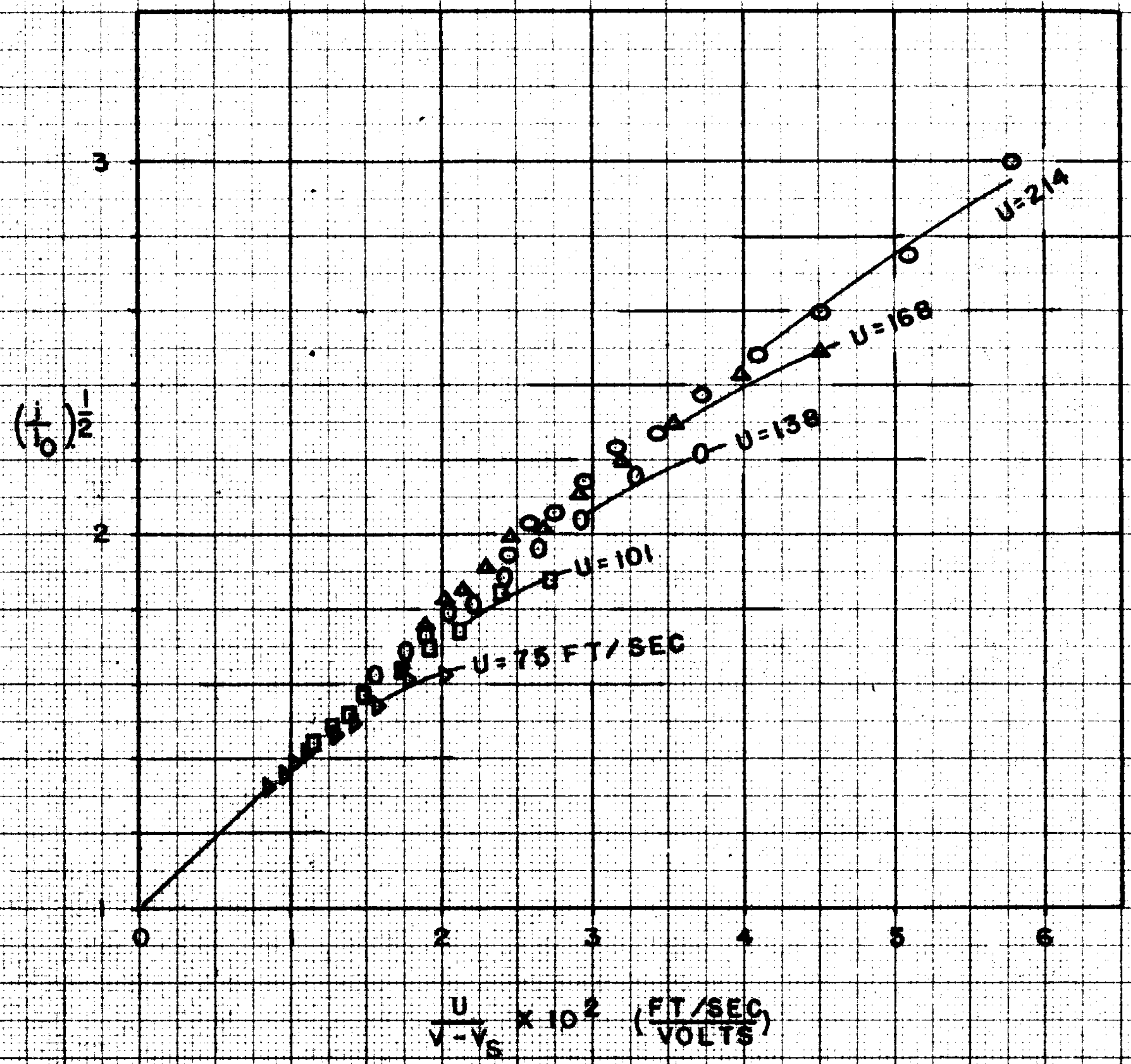

Fig. 2. Ratio of corona current to the stationary current as a function of the dimensional parameter $\frac{u}{V-V_s}$

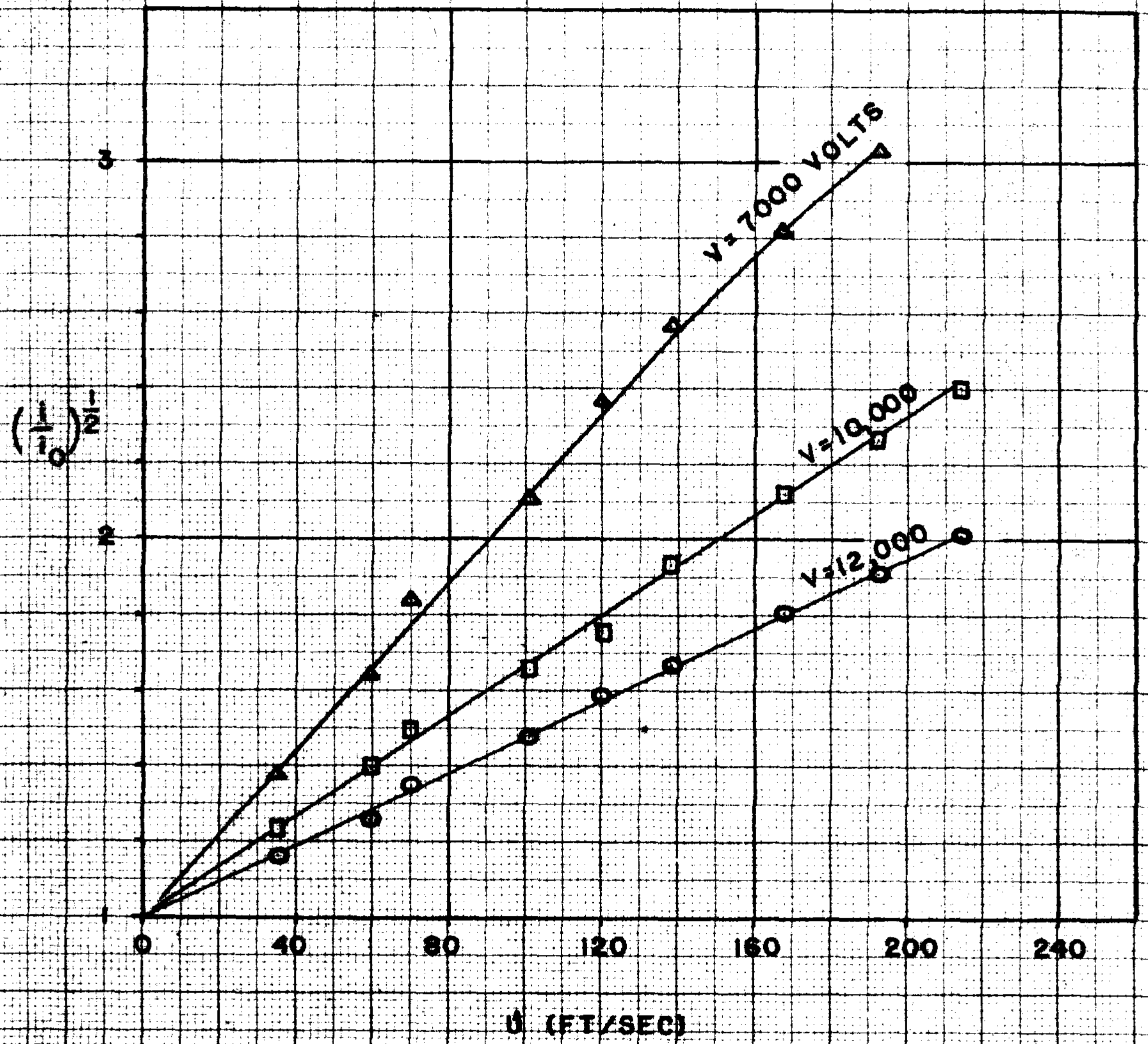

Fig. 5. Ratio of corona current to the stationary current at constant voltage as a function of airstream velocity.

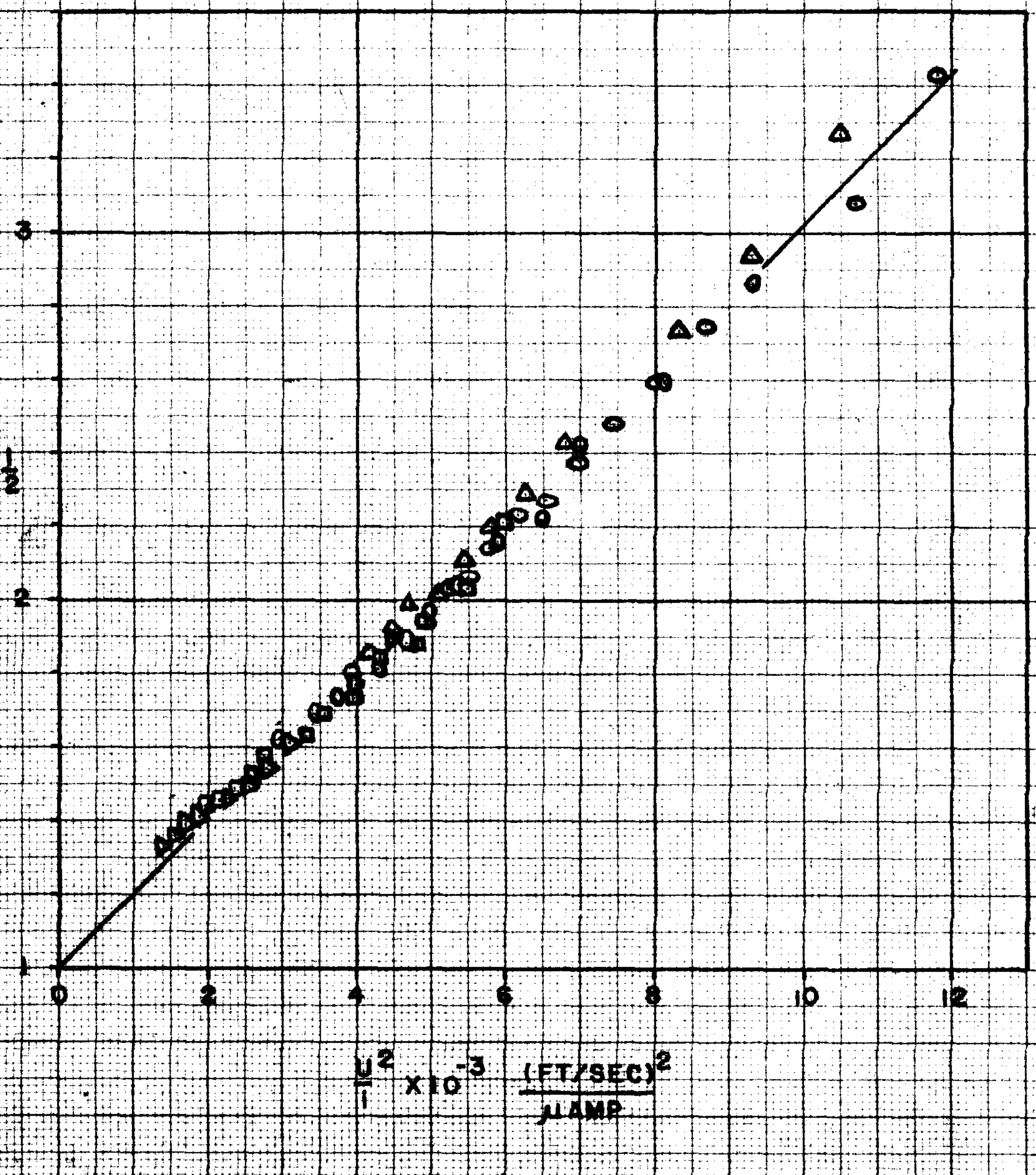

Fig. 4. Ratio of corona current to the stationary current as a function of the dimensional parameter $\frac{U^2}{I}$

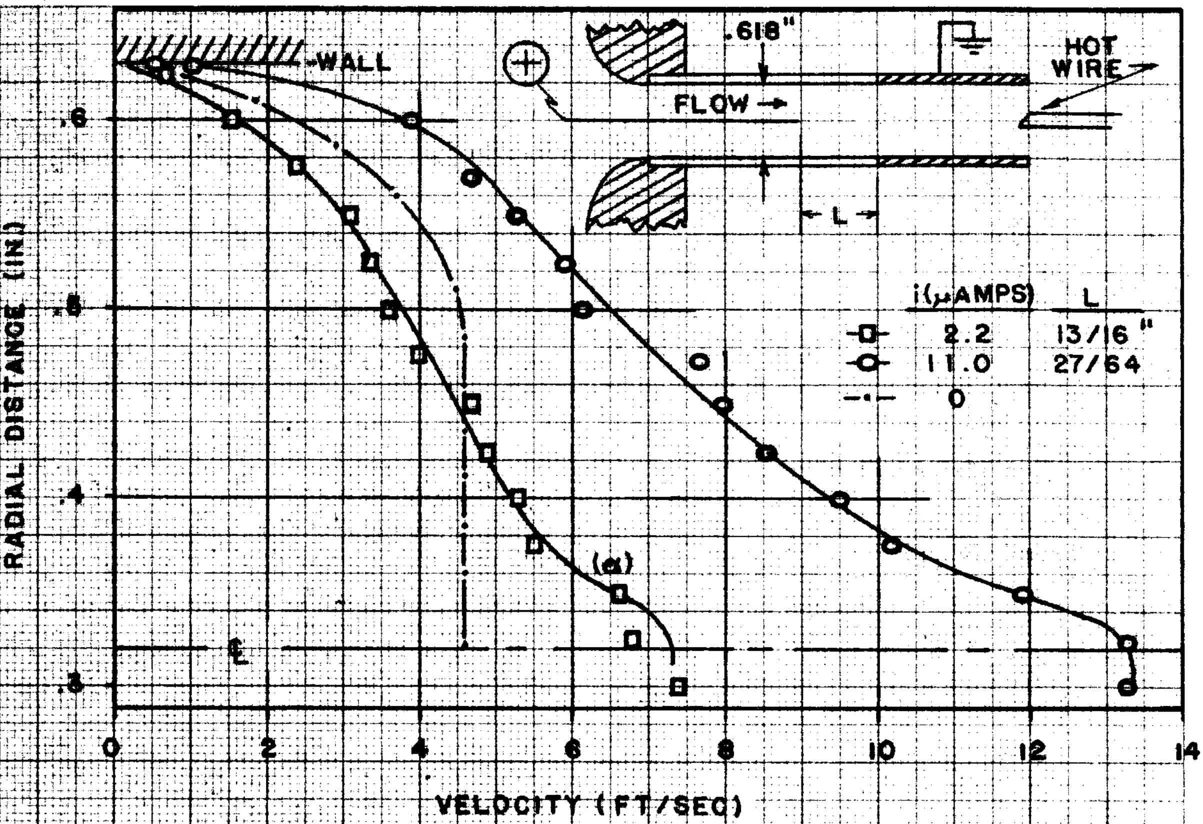

Fig. 5. Typical induced velocity profiles at the exit of the discharge tube.

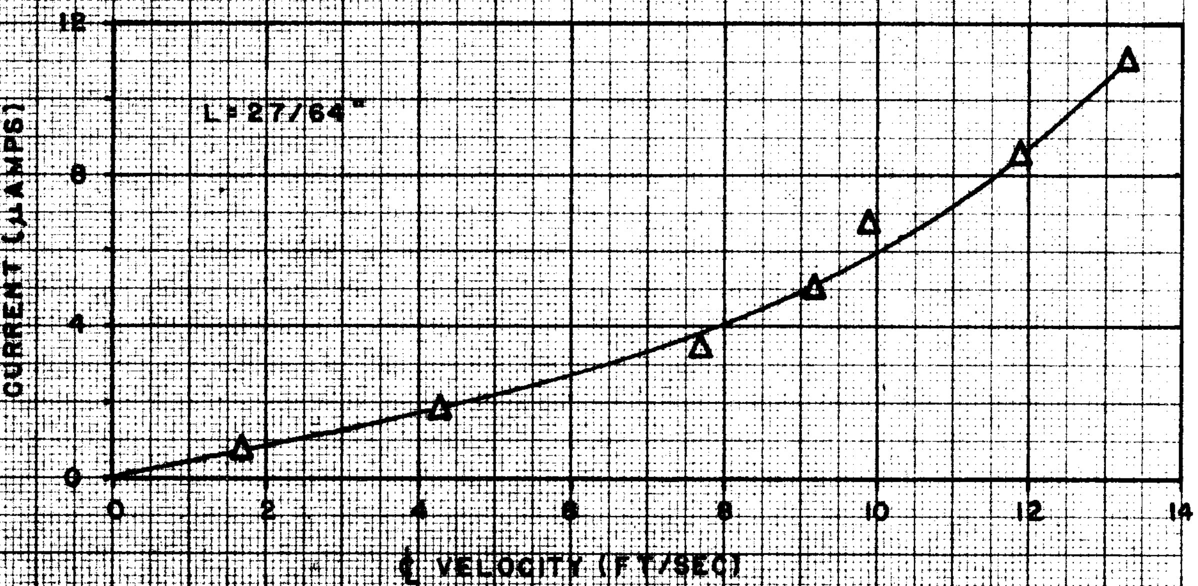

Fig. 6. Typical variations of induced centerline velocity with the current.

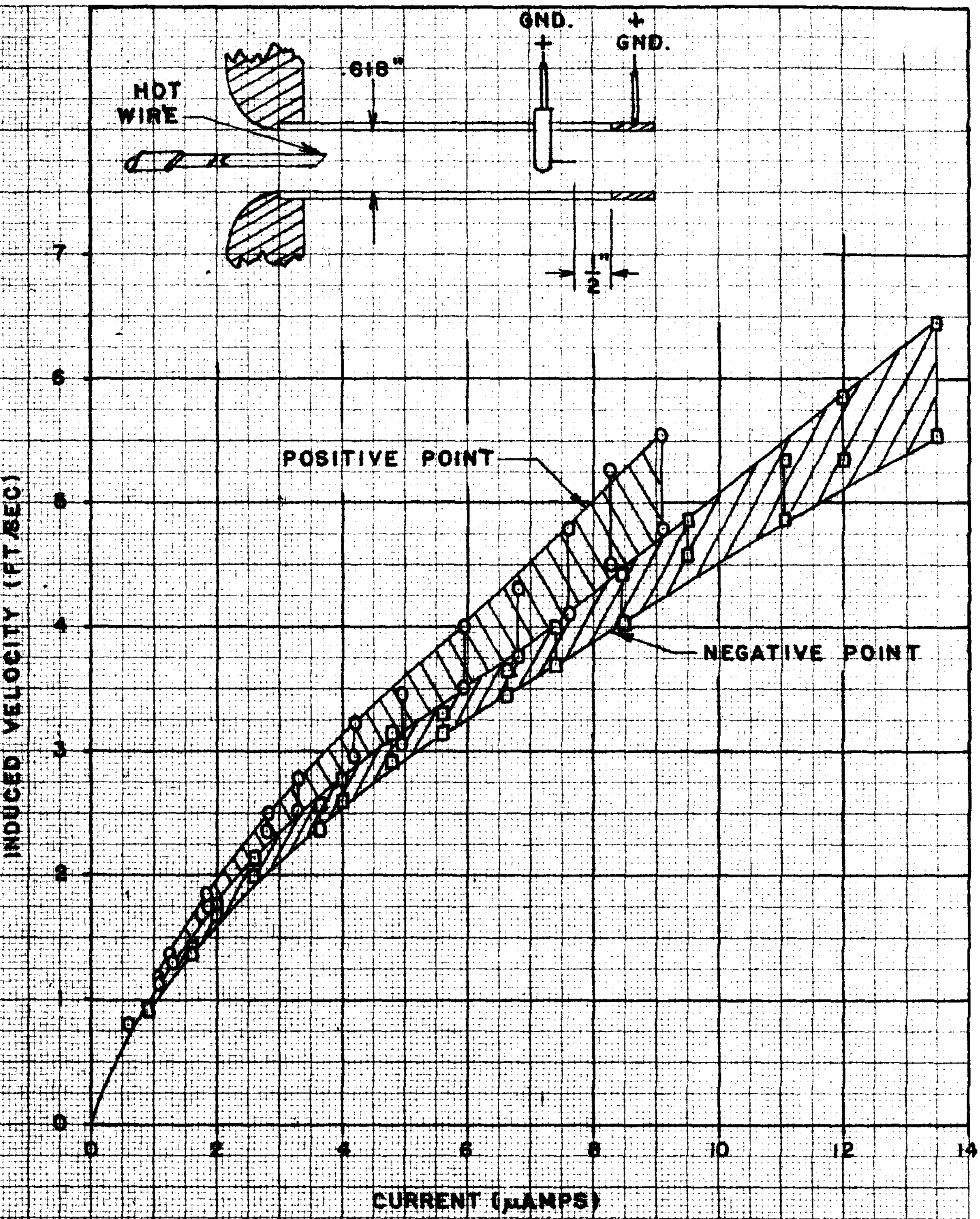

Fig. 7. Induced airstream velocity for positive and negative point discharges versus current

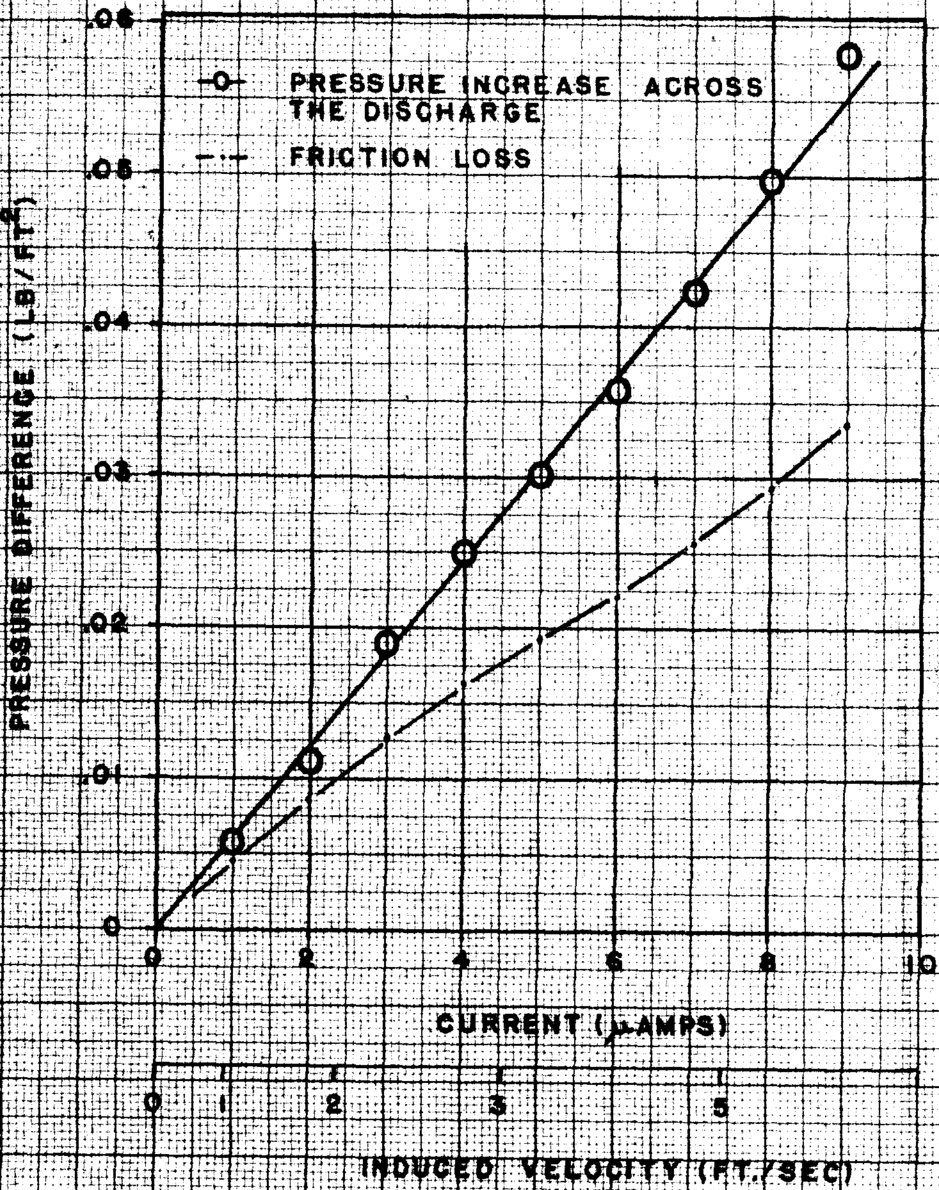

Fig. 2. Pressure difference across the corona discharge versus current. Magnitude of the pressure drop due to friction corresponding to the mean induced velocity.