TRANSIENT RESPONSE OF A MULTIFREQUENCY REED GAGE

Thesis by

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ABSTRACT

The purpose of this thesis is to extend the usefulness of a multifrequency reed gage, a simple mechanical instrument, which records essential features of transient motions. This instrument, which can be considered a dynamic model of a multiple degree of freedom structure, consists of a set of one degree of freedom cantilever beams having various frequencies. The information from the instrument consists of the maximum positive and negative displacement of each beam. It is desired to obtain as many features as possible about the exciting motion from this information alone.

The instrument response is obtained analytically for various exciting pulses. Both symmetrical pulses and non-symmetrical pulses with rise time to decay time ratios from zero to infinity are considered. The impulsive response is considered separately.

From a study of these responses a correlation is made between the response and the excitation. Table #5, page 32, summarizes the information about these studies and indicates the methods for obtaining maximum information from the reed gage.

TABLE OF CONTENTS

NOTATION

a	maximum value of acceleration pulse, $\ddot{y}(t)$.
B	viscous damping coefficient
C	capacitance
$D=B/2\sqrt{km}$	percent of critical damping
$D_e=R\sqrt{C/4L}$	percent of critical damping for electrical circuit
$E(t)$	transient voltage
$f=p/2\pi$	frequency of reed
F	force applied to main body
g	gravitational constant (ft/sec^2)
i	current
I	Impulse
K	spring constant
L	inductance
m	effective mass of reed
M	mass of main body
$p=\sqrt{k/m}$	natural undamped frequency of reed in radians
q	electrical charge
r	ratio of decay time/rise time for acceleration pulse
R	resistance
t	time
t'	time to maximum value of acceleration pulse
t'_e	time to maximum value of electrical pulse
$u=x_{max}/\delta_{st}=x_{max}p^2/a$	dynamic amplification factor
u_n	ordinate of maximum negative tangent to the dynamic amplification curve
u_o	the value of u_n at $p=0$

$w=1/\sqrt{LC}$	electrical frequency in radians
$x(t)$	relative displacement of m with respect to the base
$\dot{x}$	relative velocity of m with respect to the base
x_{max}	maximum value of x
$x_p(\ddot{y})$	particular solution of the differential equation of motion as a function of $\ddot{y}$
$\ddot{y}(t)$	acceleration of main body and reed gage base
$\delta_{st}=am/k$	static deflection of m if a were applied gradually

Chapter I

INTRODUCTION

A new type of instrument for indicating some dynamic characteristics of transient accelerations has provided the incentive for this thesis. This instrument, which has been developed over the past few years, is called a reed gage. It is the purpose of the present work to extend the usefulness of this device by suggesting how additional information can be derived from it.

When measuring a transient acceleration two methods are generally used. The first is to use a seismic type accelerometer with associated amplifying and recording equipment. This method usually necessitates considerable space and is prone to failure. The second is to measure peak acceleration by a simple mechanical device. This method needs little or no associated equipment and is reliable in its operation. Its shortcoming is in the lack of information as to time variation of the acceleration pulse being investigated.

The reed gage as considered in this report is in an intermediate position. It requires no external equipment and does give some of the features of acceleration time record indirectly.

A reed gage consists of a set of one degree of freedom systems, usually in the form of cantilever beams, each of a

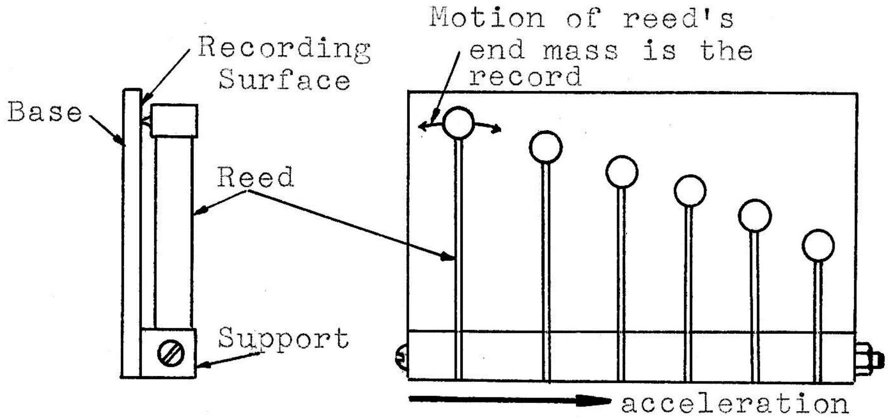


fig. 1
Reed Gage

different natural frequency, p (radians per second or f cps) fig. 1. For a structure which has natural frequencies of p_1, p_2,......p_n, a reed gage having single degree of freedom reeds at p_1.....p_n can be considered the dynamic model of the structure. For a transient acceleration the response of each natural mode of vibration of the structure will correspond to the response of the corresponding frequency reed in the reed gage.

The primary information obtained from a reed gage record is a dynamic amplification curve for a givên transient acceleration pulse. The coordinates of the dynamic amplification curve are u, the dynamic amplification factor, vs. $2ft'=pt'/\pi$, the frequency parameter. The dynamic amplification factor is defined as the ratio of the maximum response of an oscillator due to an acceleration pulse compared to the static deflection of the same system if the maximum acceleration were applied gradually. The variable in the frequency parameter is the natural undamped frequency of the oscillator; the factor, t', is the time from the initiation of the pulse to its maximum value.

The dynamic amplification curve has been generally obtained for various pulses using the maximum positive response for each frequency, p. Little has been done in the past in obtaining the negative dynamic amplification curve by using the maximum negative response. The negative dynamic response will be considered here for all transient acceleration pulses.

The dynamic amplification curve can be used in two ways. The stresses in a structure due to a transient acceleration can be found directly from the dynamic amplification curve (ref. 1). Secondly, some information about the acceleration pulse producing a given dynamic amplification curve can be deduced. It is the latter information with which this thesis is concerned.

The David Taylor Model Basin has designed and constructed several reed gages for studying the motions of a ship due to transient loads such as torpedo, mine, or bomb blasts as well as excitation from the ship's own guns. Dynamic amplification curves were computed for some typical excitations and the reed gage results from tests were compared to these dynamic response curves (ref. 2, 3 and 4).

Other examples of transient forces where reed gages might be applied are:

1. Packaging problems
    a. shipping
    b. shock mounting

2. Impact machine parts

    a. punch presses

    b. air hammer operations

3. Military applications

    a. ships excited by various explosions

    b. other military machines and equipment

4. Earthquake measurements

In the light of the preceding background the object of the present thesis can now be stated. The object of this study is to determine how to get the maximum information as to the acceleration time curve from a reed gage record. The method used is to obtain dynamic response curves for some representative symmetrical and non-symmetrical acceleration pulses: Then a correlation is made between the curves and the pulses.

Generally, the reeds in a reed gage have practically zero damping, therefore the main interest is centered on the zero damping case. For many of the above acceleration pulses the dynamic amplification curves are obtained for several values of damping so that the effect of damping can be considered.

The negative dynamic amplification curves are obtained for all pulses.

All the pulses considered start with a positive slope and proceed such that the slope varies without reversal to a final negative value. It may be possible to deduce the effect of some minor reversals in slope on the dynamic amplification curve. Some examples of pulses with reversals are a ripple superimposed on an acceleration pulse, or a small negative overshoot of acceleration at the end of a pulse which returns to zero slowly. These cases are not to be considered further in the present thesis.

Chapter II

DYNAMIC AMPLIFICATION CURVES

At the onset of this study the dynamic amplification curves were obtained for symmetrical acceleration pulses. The results did not seem to be sufficient for many generalizations, so the study progressed to investigation of non-symmetrical pulses which are identical up to their maximum value and have varying decay times. The latter study proved successful in obtaining useful correlation between pulse shapes and dynamic amplification curves.

This section will concern itself with obtaining the dynamic amplification curves for all the transient acceleration pulses. The following section will deal with the analysis of these curves.

The system to be employed is a single degree of freedom oscillator with damping as illustrated in fig. 2a. This system

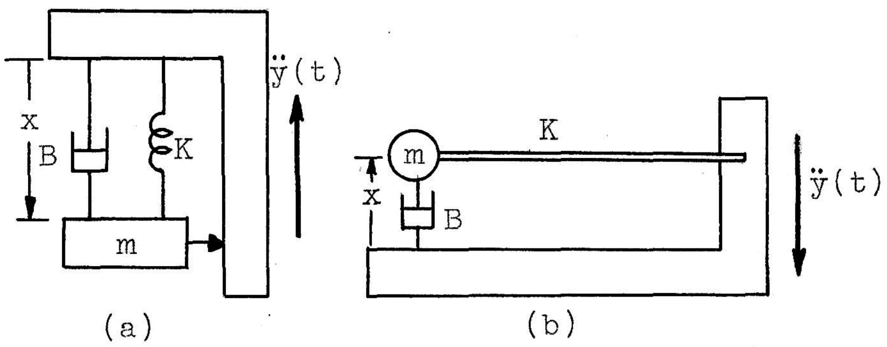


fig. 2

is analogous to the single degree of freedom reed shown in fig. 2b where the spring constant is supplied by the stiffness of the cantilevered reed. The transient acceleration $\ddot{y}(t)$

is applied to the base of the instrument.

The equation governing the motion of the mass is,

$$m\ddot{x} + B\dot{x} + Kx = m\ddot{y}(t)$$

(1) or $$\ddot{x} + 2pD\dot{x} + p^2x = \ddot{y}(t)$$

The initial conditions for all cases are,

(2) $$x = \dot{x} = 0 \text{ at } t = 0$$

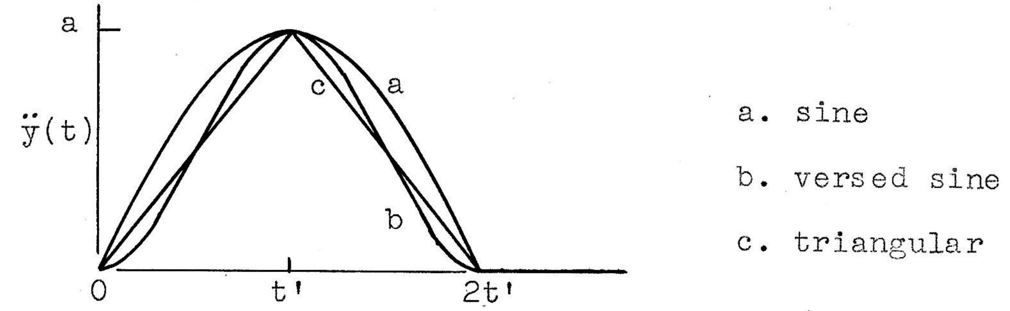
The solution for x is,

$$x = e^{-pDt}(A \sin p\sqrt{1-D^2}\, t + B \cos p\sqrt{1-D^2}\, t) + x_p(\ddot{y})$$

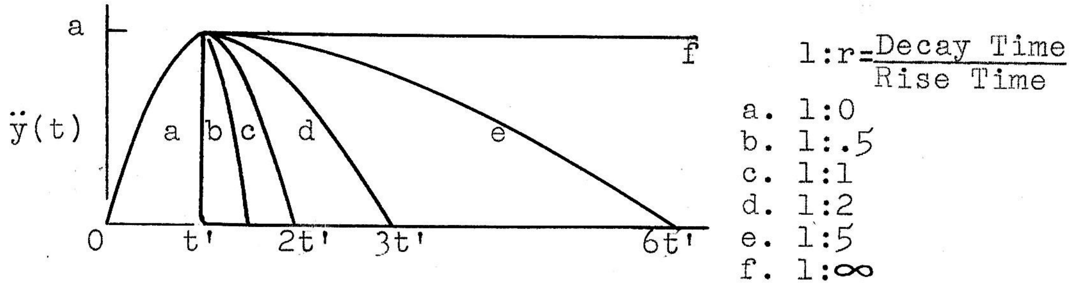
When the damping is zero the solution reduces to,

$$x = A \sin pt + B \cos pt + x_p(\ddot{y})$$

The transient acceleration pulses $\ddot{y}(t)$ to be treated are shown in fig. 3. The time to the maximum value of each pulse is t'.



(a) Symmetrical Acceleration Pulses



(b) Non-Symmetrical Sine Segment Acceleration Pulses

fig. 3

The dynamic amplification factor as previously defined on page 2 can be expressed as,

$$u = \frac{x \max}{\delta_{st}} = \frac{x \max p^2}{a}$$

Symmetrical acceleration pulses.

The transient accelerations, $\ddot{y}(t)$, for the three symmetrical cases can be expressed as follows,

a. sine

$t < 0$	$\ddot{y}=0$
$0 < t < 2t'$	$\ddot{y}=a \sin\frac{\pi}{2}\frac{t}{t'}$
$2t' < t$	$\ddot{y}=0$

b. versed sine

$t < 0$	$\ddot{y}=0$
$0 < t < 2t'$	$\ddot{y}=\frac{a}{2}(1-\cos\pi\frac{t}{t'})$
$2t' < t$	$\ddot{y}=0$

c. triangular

$t < 0$	$\ddot{y}=0$
$0 < t < t'$	$\ddot{y}=a\frac{t}{t'}$
$t' < t < 2t'$	$\ddot{y}=a(2-\frac{t}{t'})$
$2t' < t$	$\ddot{y}=0$

The responses for the above cases can be expressed in the dimensionless form $x(t)p^2/a$. This form is chosen so that when $x(t)$ is evaluated at its maximum value then the expression becomes u, the dynamic amplification factor.

a. sine

(3a) $$x(t)\frac{p^2}{a} = \frac{1}{k^2-1}(\sin\frac{\pi}{2}\frac{t}{t'} - k\sin pt) \qquad 0 < t < 2t'$$

(3b) $$x(t)\,\frac{p^2}{a} = \frac{2k}{k^2-1}\cos pt' \sin p(t'-t) \qquad 2t' < t$$

$$k = \frac{\pi}{2pt'}$$

b. versed sine

(4a) $$x(t)\,\frac{p^2}{a} = \frac{1}{2}\;\frac{1}{1-4k^2}\left[(1-\cos pt)4k^2-(1-\cos\pi\frac{t}{t'})\right] \qquad 0 < t < 2t'$$

(4b) $$x(t)\,\frac{p^2}{a} = \frac{4k^2}{4k^2-1}\sin pt' \sin p(t'-t) \qquad 2t' < t$$

c. triangular

(5a) $$x(t)\,\frac{p^2}{a} = \frac{\sin pt}{pt'} - \frac{t}{t'} \qquad 0 < t < t'$$

(5b) $$x(t)\,\frac{p^2}{a} = \frac{t}{t'} + \frac{\sin pt}{pt'} - 2 - \frac{2\sin p(t-t')}{pt'} \qquad t' < t < 2t'$$

(5c) $$x(t)\,\frac{p^2}{a} = \frac{4}{pt'}\sin^2\frac{pt'}{2}\,\sin p(t'-t) \qquad 2t' < t$$

The dynamic amplification factor is obtained for each of the above when expressed at the maximum value in time, i.e.,

$$u = x_{max}p^2/a$$

The dynamic amplification curve can be plotted as a function of pt'/π . Figure 4 shows these three curves plotted. The negative dynamic amplification curves are also plotted.

Non-symmetrical acceleration pulses.

Fig. 3b defines the six non-symmetrical acceleration pulses.

The forcing function $\ddot{y}(t)$ for the six cases can be expressed,

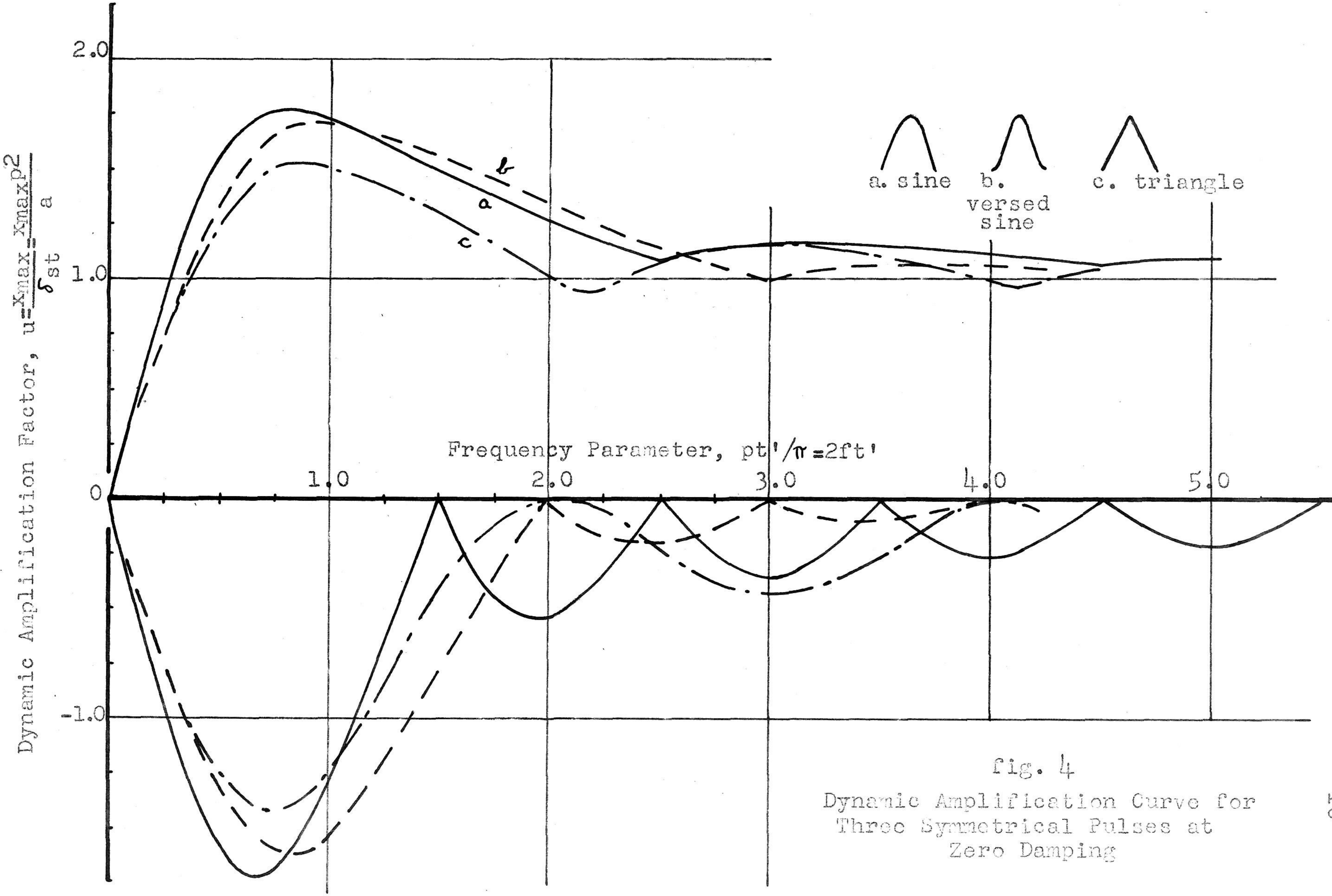


fig. 4
Dynamic Amplification Curve for Three Symmetrical Pulses at Zero Damping

$t < 0$	$\ddot{y}(t) = 0$
$0 < t < t'$	$\ddot{y}(t) = a \sin \frac{\pi}{2} \frac{t}{t'}$
$t' < t < (r+1)t'$	$\ddot{y}(t) = a \sin \frac{\pi}{2r}\left(\frac{t}{t'} + r - 1\right)$
$(r+1)t' < t$	$\ddot{y}(t) = 0$

The undamped response for the above forcing function is,

(6a) $$x(t)\frac{p^2}{a} = \frac{1}{k^2-1}\left(\sin\frac{\pi}{2}\frac{t}{t'} - R\sin\frac{\pi}{2k}\frac{t}{t'}\right) \qquad 0 < t < t'$$

(6b) $$x(t)\frac{p^2}{a} = -\sqrt{(D-E)^2 - 2(D-E)kD\sin\frac{\pi}{2k} + k^2D^2}\,\cos\left(\frac{\pi}{2k}\frac{t}{t'} - \varphi\right)$$

$$-E \sin \frac{\pi}{2r}\left(\frac{t}{t'} + r - 1\right) \qquad t' < t < (r\ 1)t'$$

$$k = \frac{\pi}{2pt'}$$

$$D = \frac{1}{k^2-1} \qquad E = \frac{r^2}{k^2-r^2}$$

$$\varphi = \tan^{-1}\left(\tan\frac{\pi}{2k} \quad \frac{r^2}{k(r^2-1)} \cos\frac{\pi}{2k}\right)$$

The solution for $t > 2t'$ is not presented since the computational method for obtaining the above responses was not used. The electric analog computer at the California Institute of Technology was used for obtaining the response to the first five acceleration pulses described in this section. The solution as presented on the analog computer gave the complete response of a single degree of freedom system at various values of natural frequency (p). The maximum and minimum values of the response x(t) were measured directly on an oscilloscope screen (ref. appendix). It was desired to obtain the dynamic amplification curves for several values of damping. The analog method lent itself easily to the damping case since damping is easily introduced into the electric analog of a single degree of freedom system by the addition of resistance. The analog computer was therefore used to obtain the positive and negative dynamic responses of an oscillator at 12 different natural frequencies, and six values of damping for each of acceleration shapes. The case of $r \rightarrow \infty$ was not obtained on the analog computer. The 6 values of damping were D=0, .05, .10, .30, .60 and 1.00.

The dynamic amplification curves obtained are shown in figures 5-9. Each figure is for one acceleration pulse and shows the curves for all values of damping. Figure 10 shows all the D=0 cases for the sine segment pulses including $r \rightarrow \infty$.

In the case when r=5, a few of the points did not seem to form a smooth curve (see fig. 9). It was decided to compute the positive response for these points. All the points fell in the range $t' < t < (m+1)t'$ hence the solution is given by equation 6b. The computed points are marked on fig. 9 and do form a satisfactory curve.

Impulsive response.

When the time duration of a transient pulse is less than about one-fifth the period of the responding system, then the loading can be considered impulsive within a maximum error of five percent. The response of a system to an impulse is equivalent to that caused by an instantaneous velocity change. The base of a reed gage mounted on a main body (M) receiving an impulse will experience the velocity change,

$$V = I/M$$

where the impulse

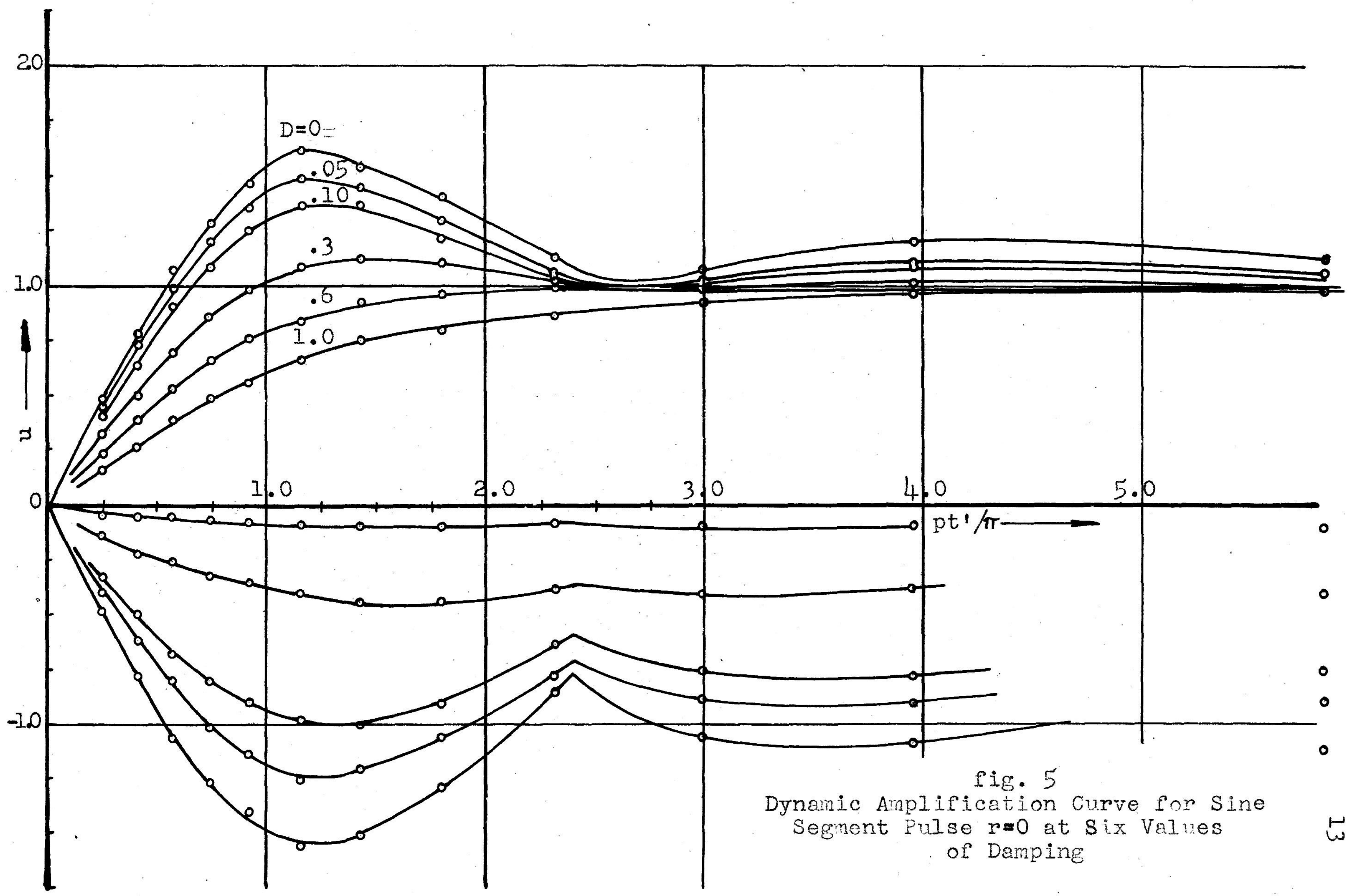


fig. 5
Dynamic Amplification Curve for Sine Segment Pulse r=0 at Six Values of Damping

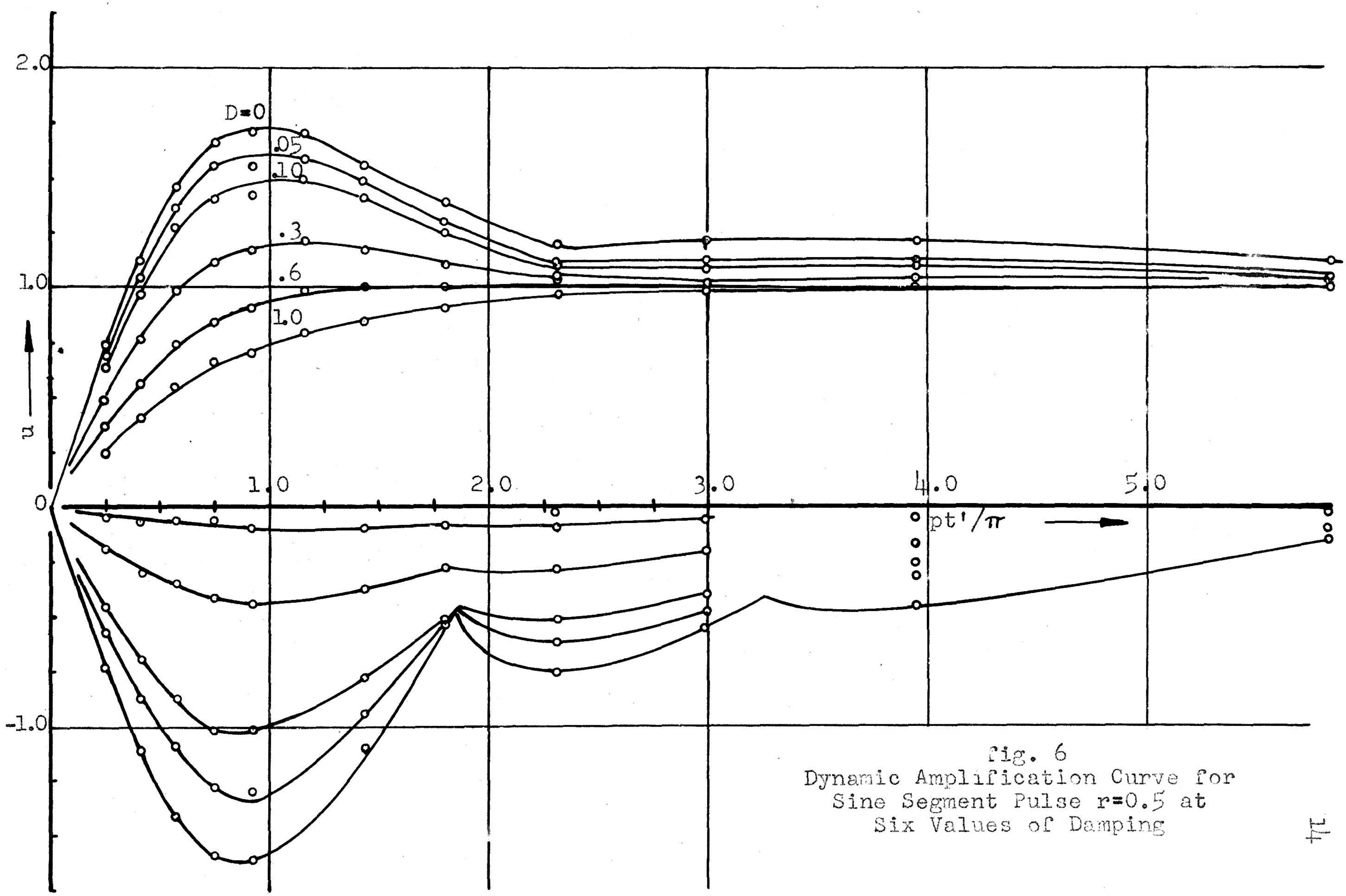


fig. 6
Dynamic Amplification Curve for Sine Segment Pulse r=0.5 at Six Values of Damping

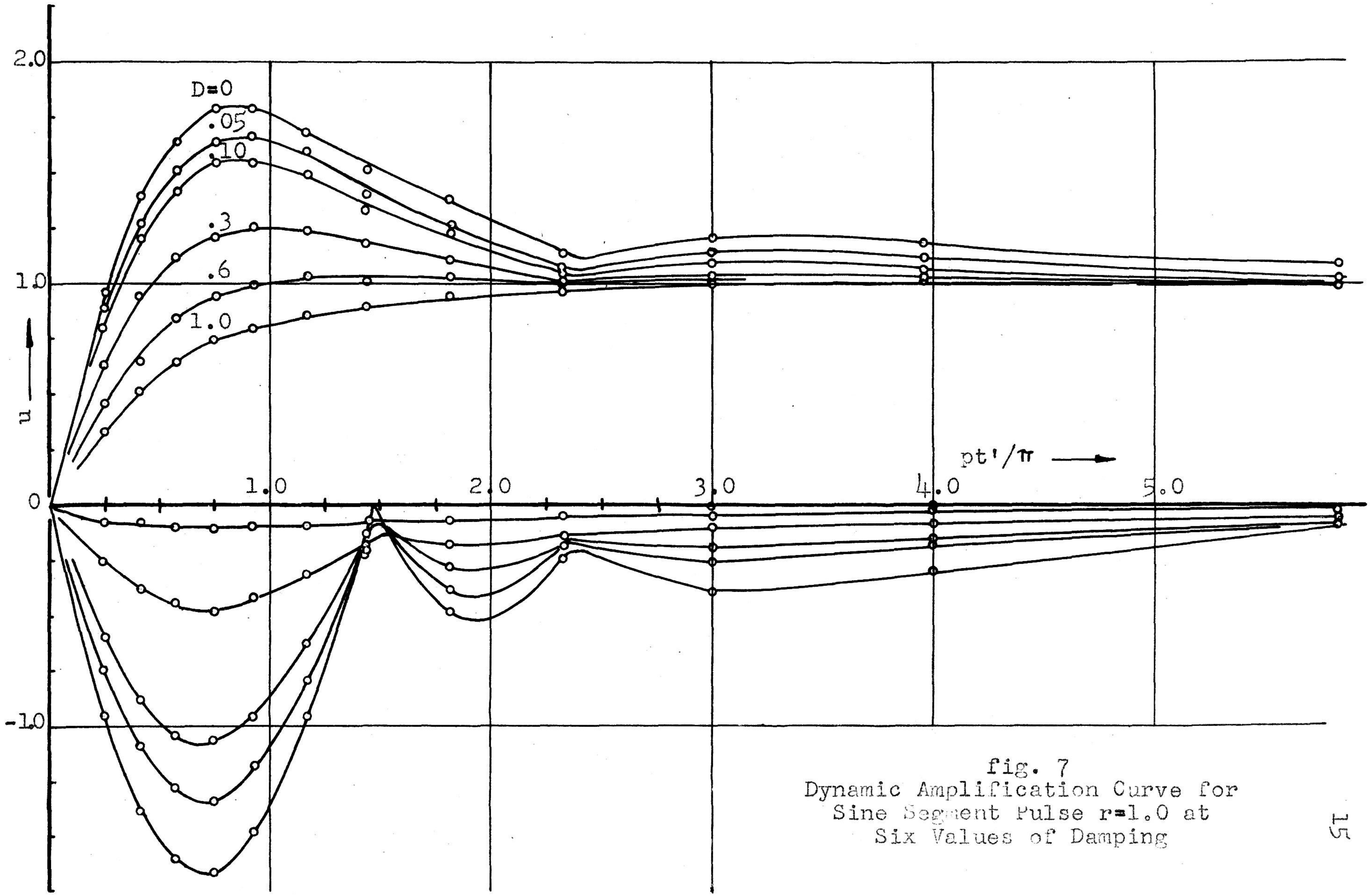


fig. 7
Dynamic Amplification Curve for Sine Segment Pulse r=1.0 at Six Values of Damping

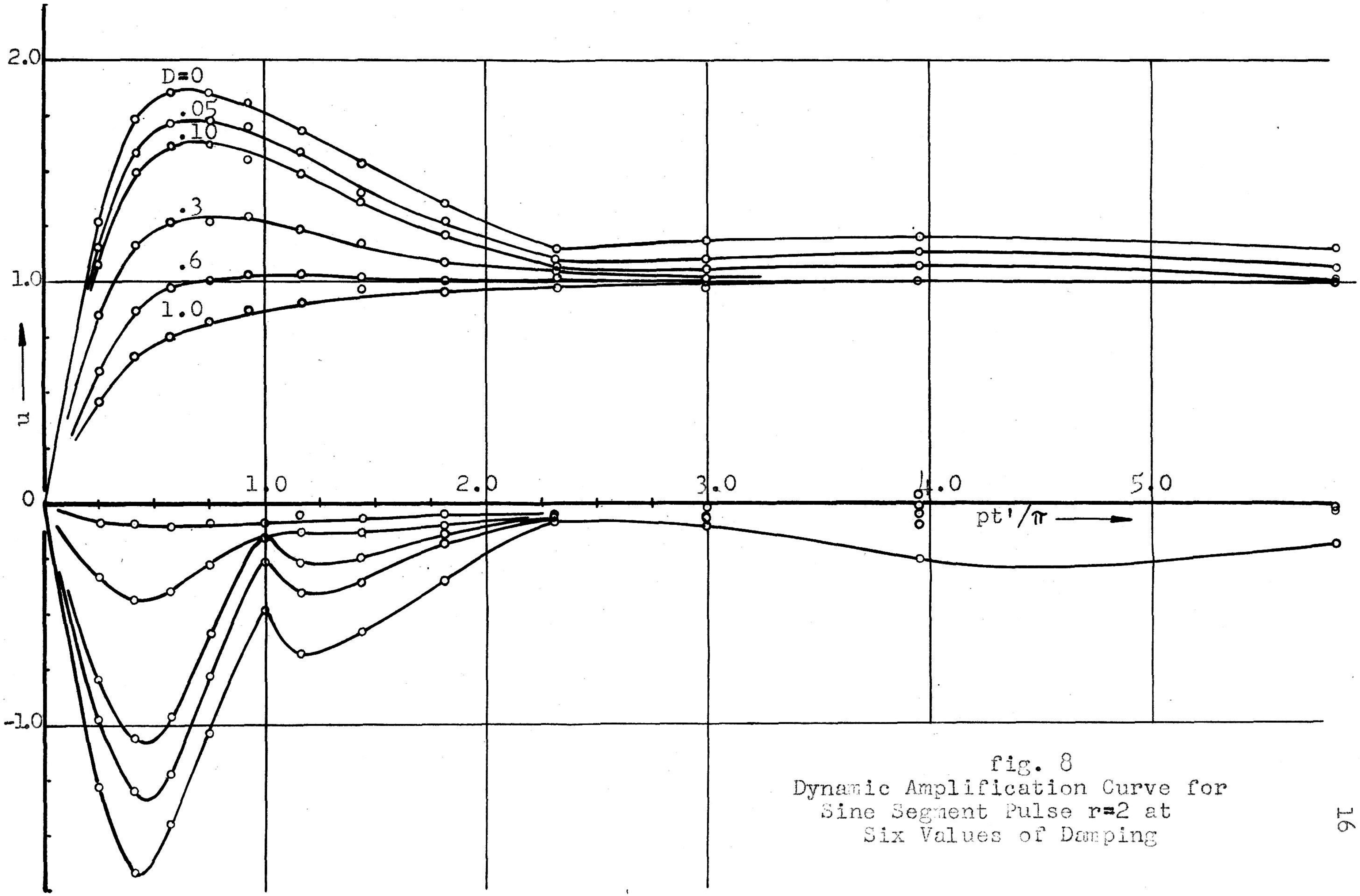


fig. 8
Dynamic Amplification Curve for
Sine Segment Pulse r=2 at
Six Values of Damping

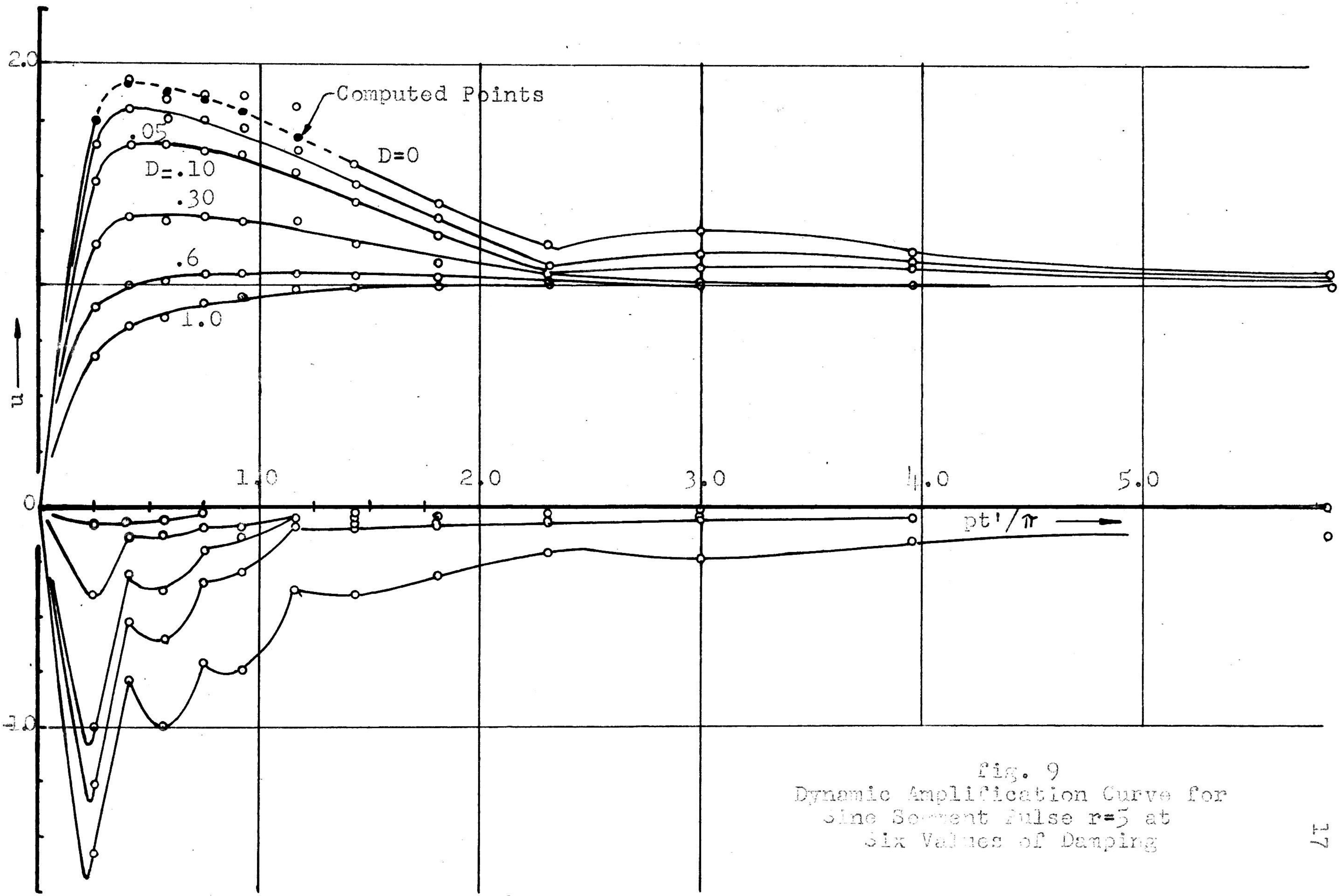


fig. 9
Dynamic Amplification Curve for
Sine Segment Pulse r=5 at
Six Values of Damping

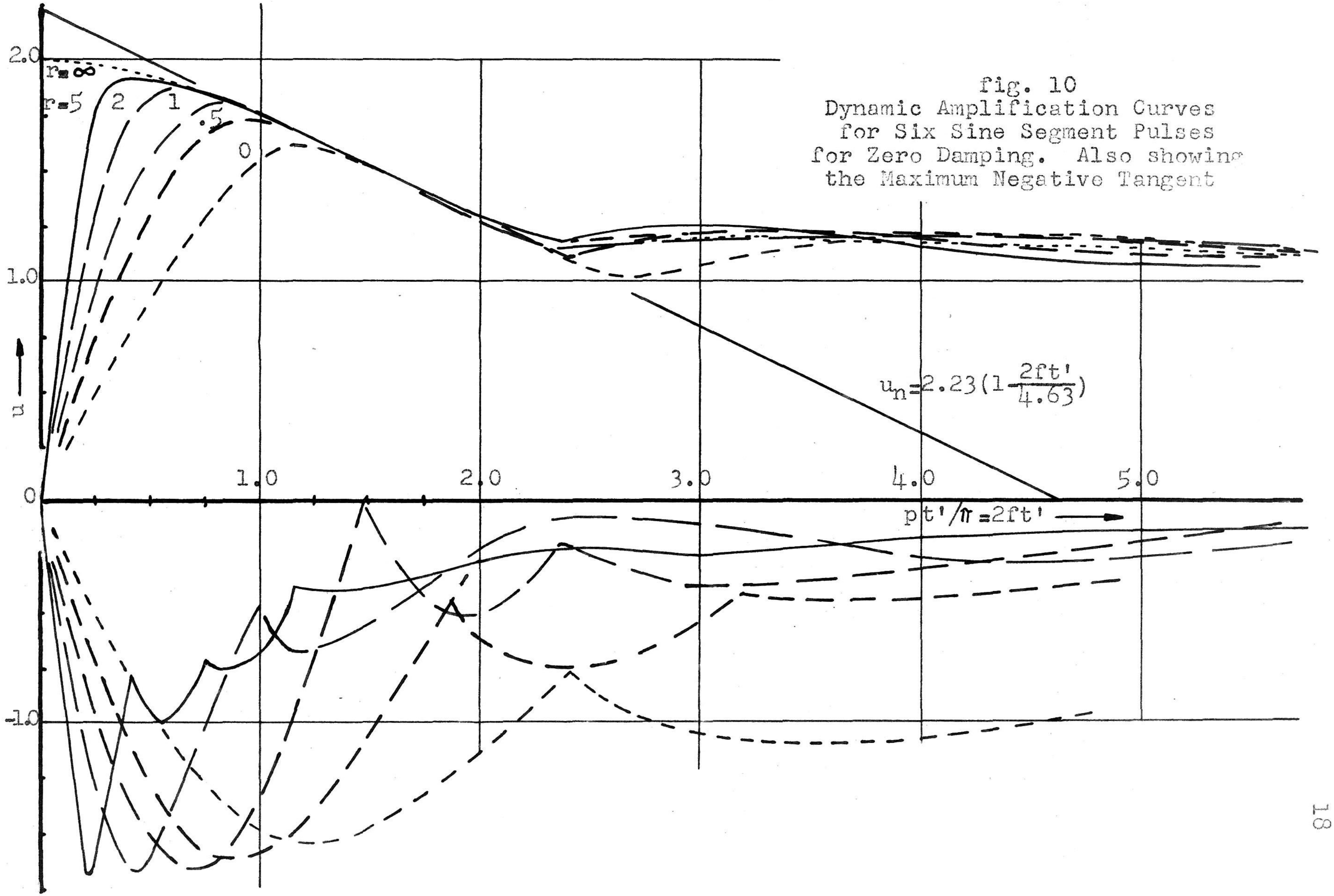


fig. 10
Dynamic Amplification Curves for Six Sine Segment Pulses for Zero Damping. Also showing the Maximum Negative Tangent

$$I = \int F dt$$

The response to a step velocity change is,

$$x = -\frac{v_o}{p} \sin pt$$

or $$\ddot{x} = v_o p \sin pt$$

hence

(7) $$\ddot{x}_{max} = x_{max} p^2 = v_o p$$

This result is plotted in fig. 11.

It will be shown later that the coordinates of the reed gage data are those plotted in fig. 11.

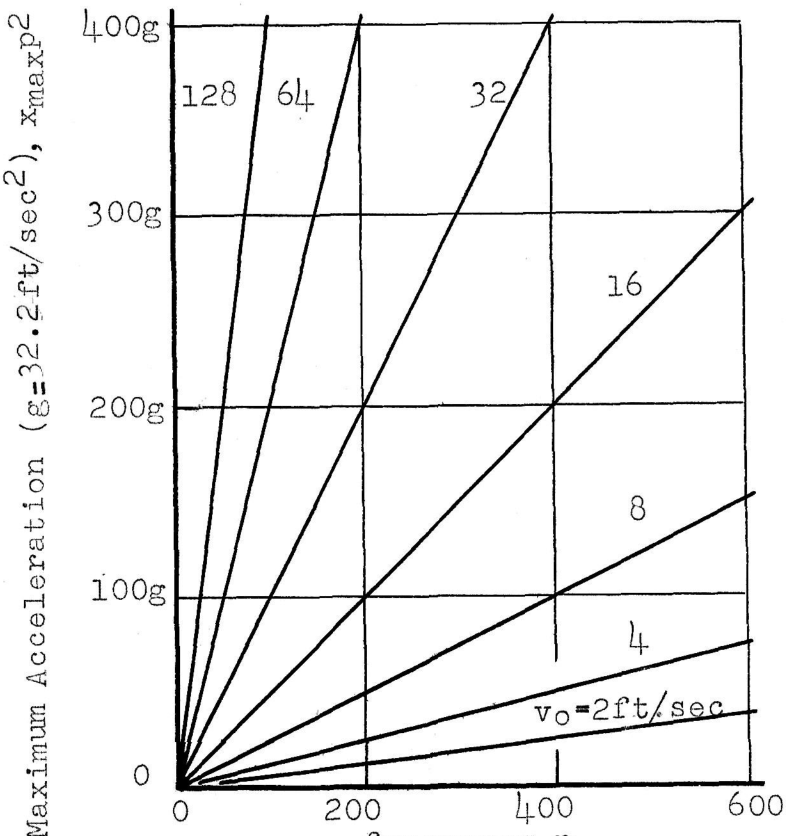


fig. 11
Acceleration of an Oscillator of Frequency p for Various Values of Step Velocity

The initial slopes of a dynamic amplification curve (figs. 4-10) thus correspond to impulsive loading. Hence we may expect the initial slope of any dynamic amplification curve to indicate the velocity change associated with a particular pulse.

The following extension of the above momentum equation is made for sine segment pulses.

$$\int M\ddot{y}(t)\,dt = M\Delta v = Mv_o$$

The total area of a pulse is therefore

$$\int_0^{t'} y\,dt + \int_{t'}^{t'(1+r)} y\,dt = v_o$$

(8) $$v_o = a\frac{2t'}{\pi}(1+r) = (x_{max}p)$$

Solving for r

(9) $$r = \frac{1}{2}\left(\frac{x_{max}p^2}{a}\right)\cdot\left(\frac{\pi}{pt'}\right)-1$$

Equations (7) and (9) are to be used in the following section in order to obtain information about the pulse from a reed gage record.

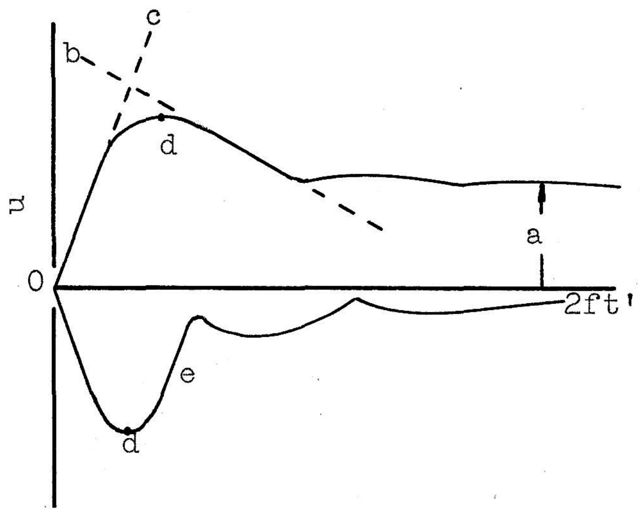
Chapter III

ANALYSIS OF RESPONSE CURVES

The purpose of this section is to determine how much information about the transient acceleration pulse can be determined from its dynamic amplification curve alone. The following properties are important in defining the pulse,

1. Peak acceleration (a)
2. Time duration (t')
3. Area (v_o)
4. Shape (r)

The following portions of the dynamic amplification curves are to be considered from the standpoint of their usefulness in giving the above desired information: (fig. 12)



a. Steady value u at large values of 2 ft'

b. The maximum negative tangent

c. The maximum leading edge tangent

d. Extreme values

e. Negative portion of the dynamic amplification curve

fig. 12
Typical Dynamic Amplification Curve

In addition to these factors, the effect of reed damping is discussed, and finally, a possible extension of the information obtained for sine segment pulses to other shape pulses is considered.

Steady value of u at large values of 2ft'.

In the case of zero damping it can be seen on fig. 9 that the dynamic response approaches unity for all pulses after 2ft'(the frequency parameter)$\doteq$ 2.5. The manner in which the various curves approach unity depends upon the exact pulse shape.

At a value of 2ft'=3, $1 < u < 1.25$ at 2ft'=6, u is restricted to smaller range namely, $1 < u < 1.13$. At values of 2ft'$>$ 6 the value of u diminishes in an envelope which approaches u=1 as 1/f. Hence for 2ft'=12 we would expect $1 < u < 1.06$.

From the above one might expect that in order to find the maximum value of the pulse (a), it is only necessary to use a reed of sufficiently high frequency and it will give the exact value for a, where

$$x_{max}\, p^2/a = u - 1$$

or $$a = x_{max} p^2$$

However, for a given value of a, x_{max} (deflection of the reed) is diminished as $1/p^2$ and therefore the error in the ability to read the reed gage record goes up. It is pointed out in ref. 4 that reed gage records cannot be measured much closer than 0.01 inches, which amounts to recorded value errors as shown in Table #1.

It is, therefore, not always possible to use the reed gage information to find a in the range where $u \rightarrow 1$, due to the error associated with reading the reed amplitude.

TABLE #1

f(cps)	Error (in g's)
20	.4
40	1.39
100	8.32
210	37.7
345	120
430	185
570	316

It will now be shown that there are other independent means of determining a either as a check of the above method or when the above method does not yield accurate results.

Maximum negative tangent.

Figure 10 shows quite clearly that the maximum negative tangent for the dynamic amplification curves of all the sine segment pulses are approximately equal. Since all these pulses are identical from time zero to their maximum, the maximum negative tangent can be expected to define this similarity of all the pulses.

The maximum negative slope intersects the u ordinate at 2.23 ± .03 for the dynamic amplification curves of all the sine segment pulses (fig. 10). The equation of this tangent line is,

(10) $$u_n = 2.23\left(1 - \frac{2ft'}{4.63}\right)$$

where the value 4.63 (± .10) is where the tangent line intersects the frequency (2ft') axis.

This line can be located with a fair amount of accuracy if the reed gage has two or three reeds in the range where the maximum negative tangent and the dynamic amplification curve coincide.

To illustrate clearly two pieces of information to be obtained from this line, assume fig. 13 is a curve obtained from reed gage data and plotted as $x_{max}p^2$ vs. p.

Then $$\frac{x_{max}p^2|_A}{a} = 2.23$$

(11) $$a = \frac{0A}{2.23}\left[1 \pm (.015 + e_s)\right]$$

and $$\frac{p_B t'}{\pi} = 2f_B t' = 4.63(1 \pm .022)$$

(12) $$t' = \frac{4.63\,\pi}{0B}\left[1 \pm (.022 + e_s)\right]$$

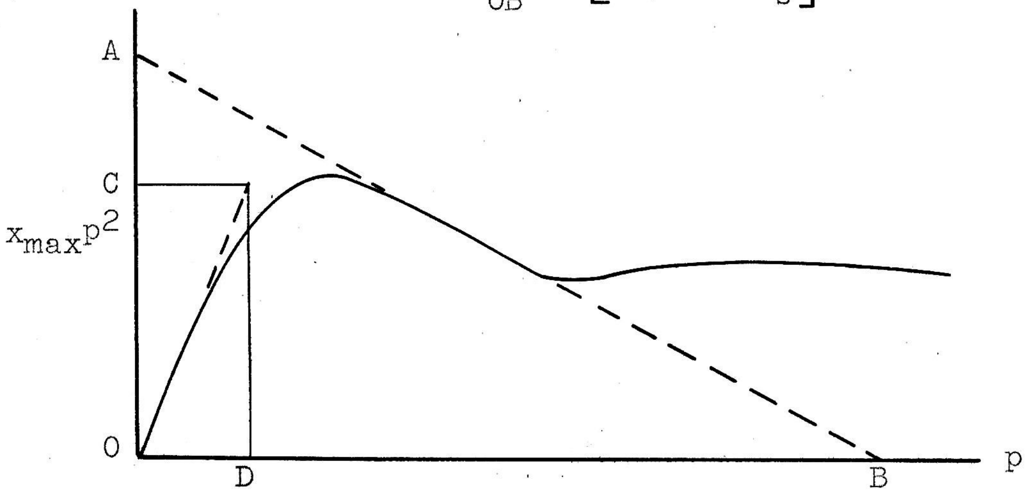


fig. 13
Dynamic Amplification Curve for Example in Text

These two relations are quite easy to use and give desired information about the applied acceleration. The first error term in each expression is due to the inaccuracy or variations in slope found in the dynamic amplification curves for the sine segment acceleration pulses. The second error term is the error in obtaining the points A and B in fig. 13 as determined by evaluating the reed gage data.

Maximum leading edge tangent.

The maximum leading edge tangent of a dynamic amplification curve is determined by the change of velocity of the reed gage base as shown on pages 19 and 20. The relation obtained for the maximum acceleration of a reed is,

$$x_{max}p^2 = v_o p$$

or

$$\frac{(x_{max}p^2)}{(p)} = v_o$$

As an example consider the curve of fig. 13. Then,

(13) $$v_o = \frac{OC}{OD}$$

The values of v_o vs. r for the sine segment pulses are plotted on fig. 14 based on equation (8).

If the value of a and t' are known, then we have from equations (9), (11), and (12), and fig. 13,

(9) $$r = \frac{1}{2}\,\frac{(x_{max}p^2)}{(p)} \cdot \frac{\pi}{at'} - 1$$

(14) or $$r = 0.241\frac{OC}{OD} \cdot \frac{OB}{OA} - 1$$

From the consideration in this section, two more pieces of information about the pulse have been obtained.

Equation (13) gives the relation for the change of velocity applied to the base of the reed gage and equation (14) gives the shape of the sine segment by the relation for r.

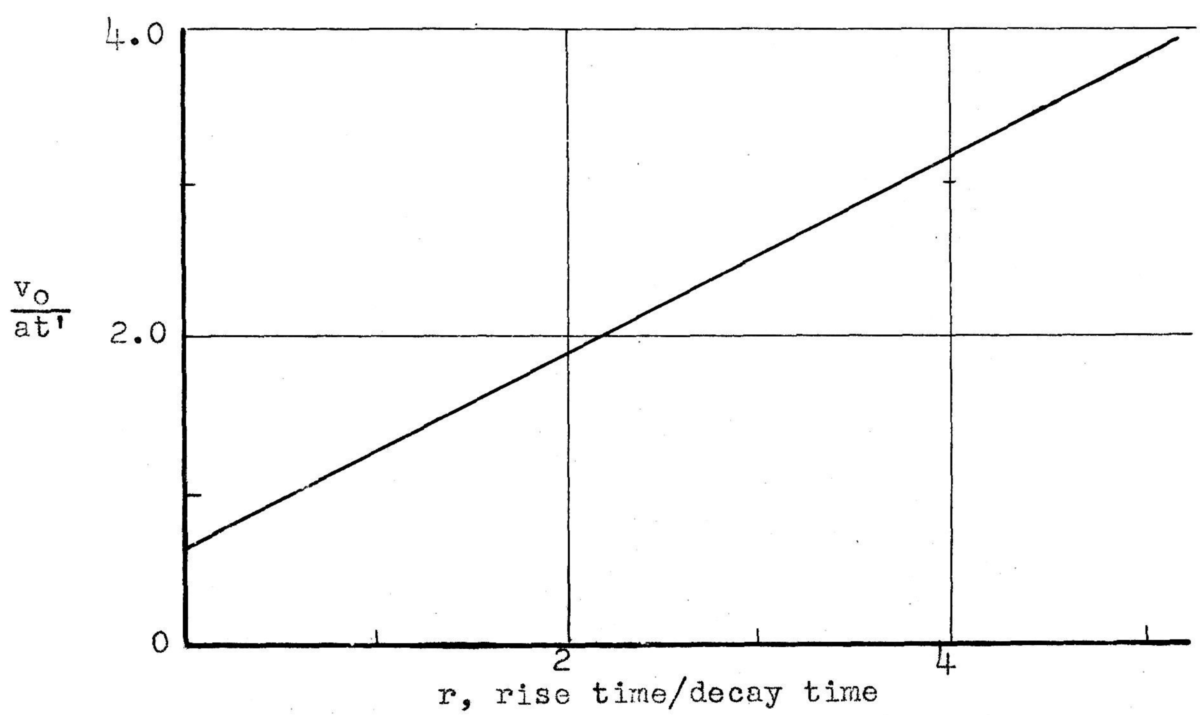


fig. 14
Velocity Change, v_o, vs. r

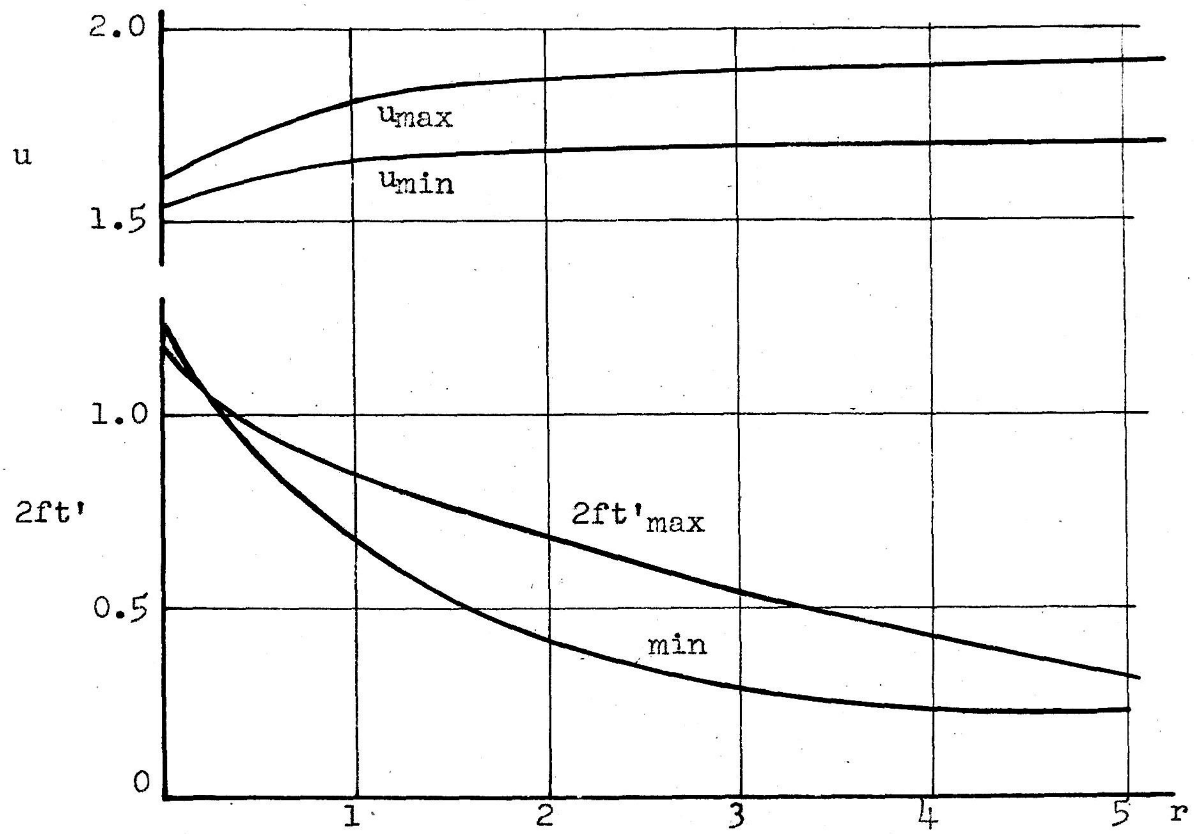


fig. 15
Extreme Values of the Dynamic Amplification Curves
vs. r for Zero Damping

Extreme values.

Figure 15 plots u and 2ft' vs. r as taken from figure 10 for the maximum and minimum of each dynamic acceleration curve. The extreme values of dynamic amplification factor, u, are not very sensitive to changes of r. However, the value of the frequency parameter, 2 ft', does show substantial variation with r for both the negative and positive portions of the dynamic response curves.

For a specific reed gage record fig. 15 will act as a guide to find the value of r, hence indicating the pulse shape.

Negative portion of the dynamic amplification curve.

In obtaining reed gage information the negative dynamic response is available with hardly any extra effort. In the past, little has been done with the extra available information. The negative response has been obtained for all the transient pulses considered in this report and are shown on figures 4-10.

The following observations are made about the character of the negative portion.

1. The shape of the first part looks like a negative sine wave (ode, fig. 12).
2. The maximum negative leading edge tangent is equal to the maximum positive leading edge tangent at zero damping only.
3. The minimum value of u is approximately constant at a given value of damping for all the sine segment pulses.

As can be seen in fig. 4 for the symmetrical pulses, the maximum negative value of u (undamped) does vary for different pulse shapes. A generality which seems evident is that the value of the minimum varies with the "bluntness" of the peak of the acceleration pulse. Bluntness shall be here defined as the area of an acceleration pulse that intersects the rectangle from 0 to t' and from a/2 to a. Table #2 gives the minimum value of u and the bluntness area; figure 16 shows a plot from the data in Table #2.

TABLE #2

Shape	u_{min}	Area of Peak From o-t' and a/2-a
triangle	1.43	.125 at'
versed sine	1.62	.160 at'
sine	1.72	.218 at'
rectangular	2.00	.500 at'

4. The frequency parameter varies approximately for both extreme values of the dynamic amplification factor as shown in fig. 15.
5. The negative response represents the steady unforced residual amplitude of the reed subsequent to the transient pulse. It may be possible to plot an energy spectrum from this data.

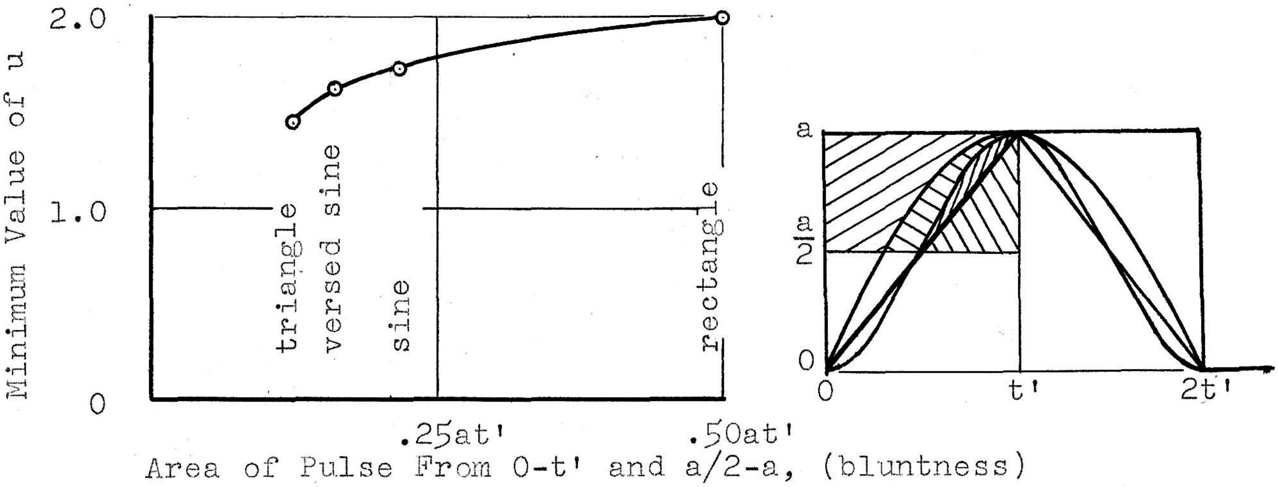


Fig. 16
Minimum Dynamic Amplification Factor Vs.
Area From a/2 to a and 0 to t'
For Symmetrical Acceleration Pulses

Damping.

The dynamic amplification curves for each sine segment pulse are shown at six values of damping in figs. 5-9.

Observation of these figures will show that the value of u approaches unity more rapidly for larger values of damping. Table #3 shows the range of u at two values of the frequency parameter and all the values of damping.

TABLE #3

	Range of u	
D	2ft'=3	2ft'=6
0	1-1.25	1.13
.05	1-1.15	1-1.06
.10	.98-1.09	1-1.04
.30	1-1.03	1-1.02
.60	.97-1.01	1.00
1.00	.87-1.00	.98-1.00

Hence it is seen that damping allows more accurate determination of a at lower values of 2ft' as compared to the undamped case. For an example, at D=.60 the value of u=1±.02 for all pulses at a value of 2ft' as low as 2.0.

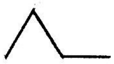
Figures 5-9 show that the essential features of the D=0 dynamic amplification curves have been preserved for D=.05 and .10. If a reed gage were operating with a known damping then most of the above derived relations could be altered to suit that value of damping.

In the curves for large amounts of damping ($D \gg .30$), the essential features of the curves become obscured and it is unlikely that a reed gage in this range of damping values would yield much information. The two features apparent for the $D \gg .30$ dynamic amplification curves are, 1. the steady values approach u=1 rapidly, and 2. the leading edge slope of the response curves vary according to pulse area, these slopes being smaller than for the corresponding zero damped cases.

Extension from sine segment pulses.

There is one important feature concerning the information derived from this study of sine segment pulses which applies to pulses of other shapes. The maximum negative tangent of the dynamic amplification curves for non-sine segment pulses intersect the ordinate axis of the response curve at similar values as the sine segment pulses. Table #4 was prepared using some information found in ref.6 for the value of u.

TABLE #4

Leading Edge	u_o	Returning to Zero	u_o	Sym. Pulse	u_o	Remaining Steady
Sine	2.24		2.23		2.24	
Triangle	2.00		2.15		2.30	
Versed Sine	*2.16		2.22		2.28	

*Estimated value assuming linear reduction from other values as found in the triangle case.

With the exception of the first triangular pulse in table #4, the value of u_o that could apply to "almost" any pulse is $2.22 \pm .08$ or $2.22(1 \pm .036)$. From the value of u_o the value of maximum acceleration, a, can be determined for any pulse (except certain triangular pulses) to within four to six percent by the use of equation (11).

An important conclusion is obtained from the following. Using the value of u_o for any dynamic amplification curve, the value of a can be found. Then a value for t' can be obtained by the use of equation (12). Equation (12), however, is derived for a sine segment pulse. So the value of t' found, if the curve were another shape, would be for the equivalent sine segment pulse which gives the same dynamic response. In other words, given a reed gage record there can be found a sine segment pulse which produces a positive dynamic amplification curve similar to the actual pulse producing the reed gage record.

Summary.

Table #5 reviews all the acceleration pulse information which may be obtained from various parts of the dynamic response curve as found in Chapter III.

TABLE #5

Characteristics of Dynamic Amplification Curve (fig. 12, p21)	a	t'	v_0	m	Rising leading edge shape
1. value at large 2ft'	x				
2. max. negative tangent	x	x			
3. max. leading edge tangent			x		
4. (3) and (2)				x	
5. maximum alone					
6. (4) and (2)				x	
7. minimum alone					x
8. (6) and (2)				x	

Reed gage range.

There are five types of reed gage records that might be obtained depending upon the reed gage frequency range and the time to the maximum value of the pulse t'. The five types of records are shown in figs. 17a, b, c, d, e. If all the records of fig. 17 were taken with the same reed gage then (a) would represent a very short t', (e) a very large t'.

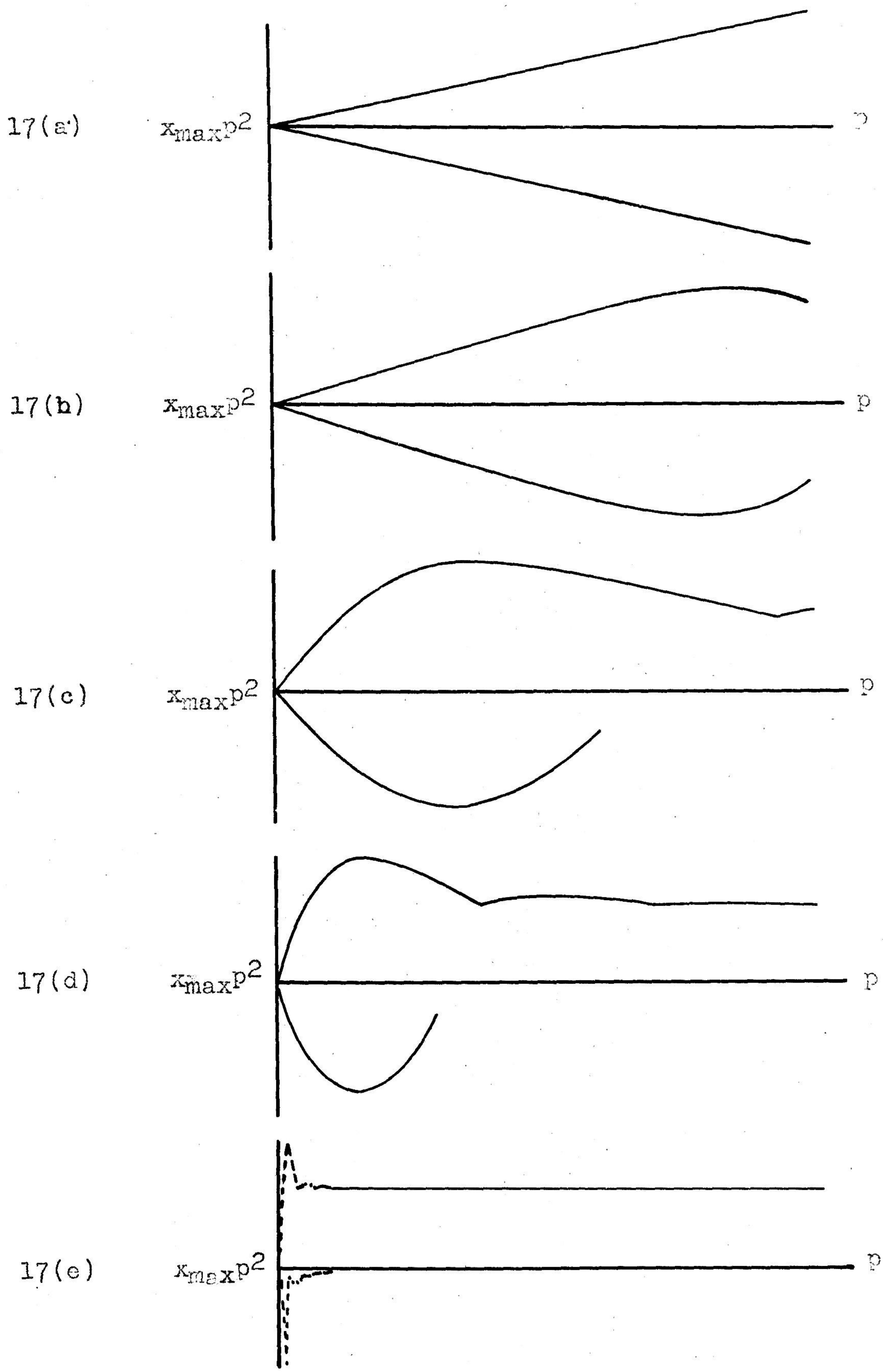


fig. 17
Five Possible Reed Gage Ranges for
a Given Transient Acceleration

Case 17(a). Here t' is smallest and the response is purely impulsive. Many of the reed gage records taken from the Navy Shock Machine described in ref. 5 are of this type. Information obtainable would be v_o.

Case 17(b). This is similar to 17(a) with additional information about the pulse shape. It is possible to guess or estimate the slope and position of the maximum negative slope and thus obtain approximate values for a and t'.

Case 17(c). This case yields practically all the information possible as listed in table #5.

Case 17(d). This case is practically like case 17(c) except it may be more accurate to obtain the value of a from large values of 2ft'. Then the maximum negative tangent can be located more accurately.

Case 17(e). This case is a static response. The value of a can be obtained but probably little else. Actually a is the only value desired in a static case.

APPENDIX

ANALOG COMPUTER METHOD

The analog computer at the California Institute of Technology was used for obtaining the dynamic response curves for various transient excitations, figs. 5-9.

The basic analogy between the mechanical and electrical systems is seen in the differential equations for the single degree of freedom system with damping (fig. 2) and a series loop of LRC and voltage E. They are,

$$m\ddot{x}+B\dot{x}+kx= -m\ddot{y}(t)$$

$$L\ddot{q}+R\dot{q}+\frac{1}{C}q= E(t) \qquad q=\int idt$$

These may be rewritten in the form,

$$\ddot{x}+2pD\dot{x}+p^2x= -\ddot{y}(t) \tag{1}$$

$$\ddot{q}+2wD_e\dot{q}+w^2q= \frac{E(t)}{L} \tag{15}$$

$$w^2=\frac{1}{LC} \qquad D_e= \frac{R}{2}\sqrt{\frac{C}{L}}$$

To make the motion similar for equations (1) and (15) the percent damping is equated and the same time ratio is used between the period of applied motion and the natural frequency of the system, i.e.,

$$D=D_e$$

and $\quad pt'=wt'_e$

E(t).

The transient voltage excitation is produced by first making a film record of each transient acceleration function on a circular track. This film is placed on a turntable and rotated at a constant speed of 10 RPS. A photo-electric cell picks up a light signal proportional to the track spacing which is amplified to become the input voltage E(t). The transient voltage is repeated every 0.1 second in which time the electrical system responds for 0.05 seconds and the entire system is allowed to come to equilibrium in the remaining 0.05 seconds before another voltage pulse starts a new cycle. The output is placed on an oscilloscope which has a sweep speed of 10 cps giving a stationary image.

Electrical circuit.

The circuit diagram used is shown in fig. 18(a). The main elements are LRC and E(t). A negative resistance element cancels all resistances when R is set to zero. Figure 18(b) shows the operation of the synchronous switch S-4 which permits a repetitious response to start with the system quiet. When S-4 is closed (shorting terminals N to O) the pulse E(t) starts the circuit in motion giving a voltage proportional to displacement in the mechanical system across the capacitor terminals P-H. Switch S-4 opens putting a resistance R-15 in series which makes the circuit approximately critically damped, hence quieting the circuit by the time S-4 closes and starts a new cycle.

When it is desired to set zero damping, in some cases it is necessary to use the special arrangment shown in fig. 18(a). Connection K-N is opened and L-O is closed. The synchronous switch S-5 puts a step voltage on the system; the output of which is a uniform amplitude oscillation. The control on the negative resistance system is set until the oscillating amplitude is exactly uniform.

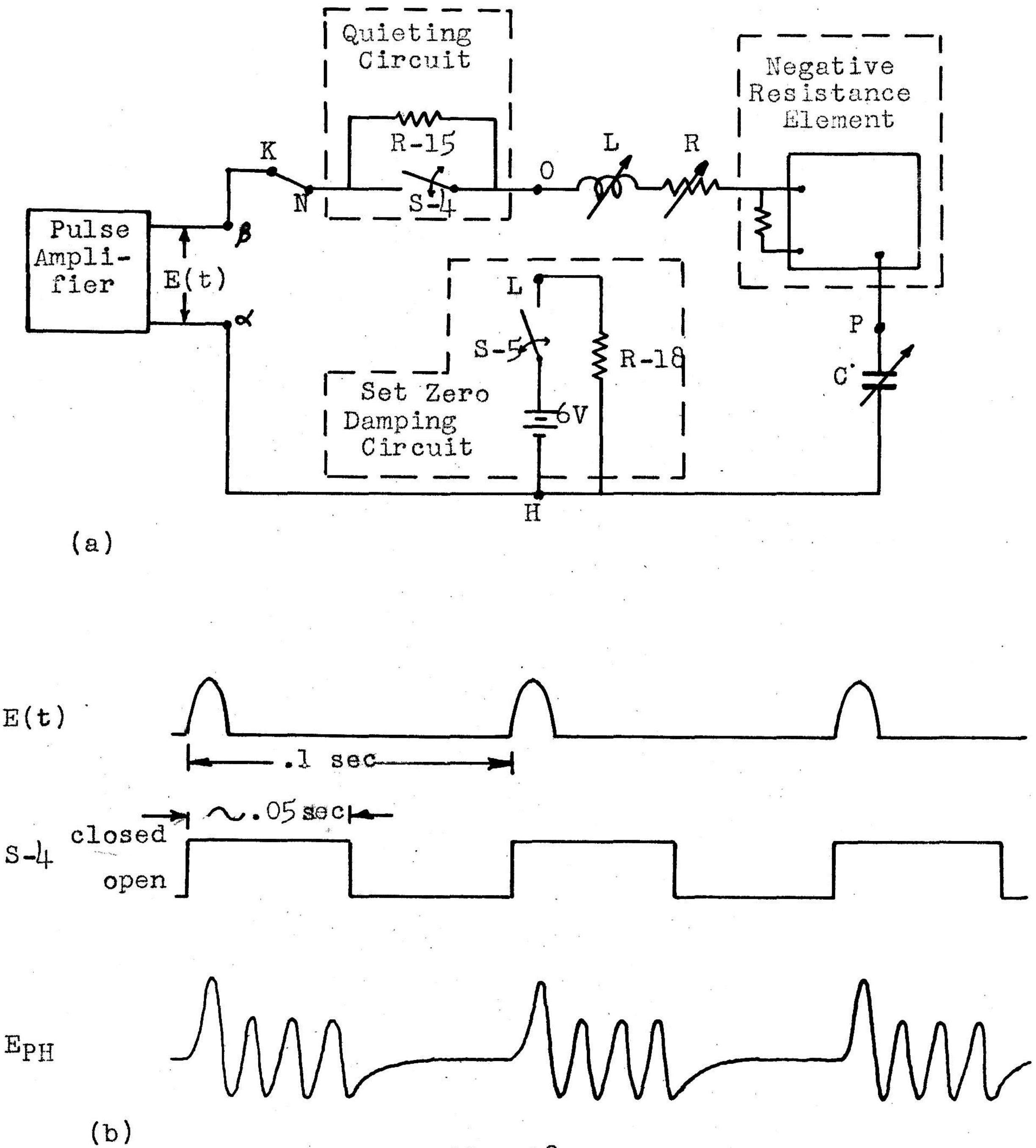

Quieting
Circuit
R-15
S-4
K
N
O
L
R
Negative
Resistance
Element
Pulse
Amplifier
E(t)
β
α
L
S-5
R-18
Set Zero
Damping
Circuit
6V
H
P
C
(a)
E(t)
.1 sec
~.05 sec
closed
S-4
open
E_{PH}
(b)

fig. 18

The response was measured for five pulses, 12 values of 2ft', and six values of damping. The maximum and minimum dynamic response was measured in each case. Hence 720 values were recorded.

The five pulses used are shown in fig. 19.

The response for the pulse r=1 is shown for five of the twelve values of 2ft' in fig. 20. For 2ft'=3.96, the response at three values of damping, D=0, .05 and .60 are shown. In several cases for D=0 there is shown also the transient voltage producing the response.

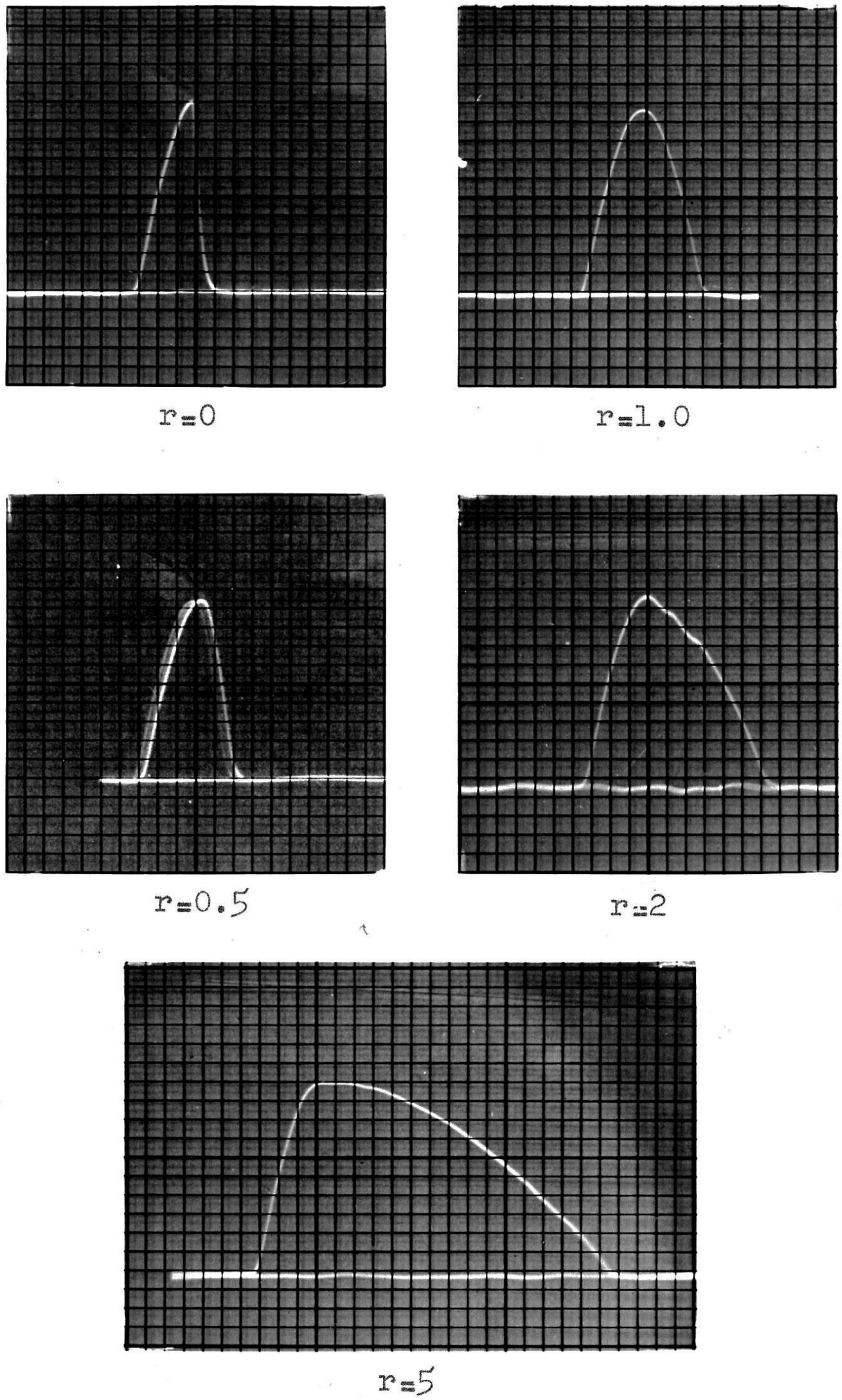
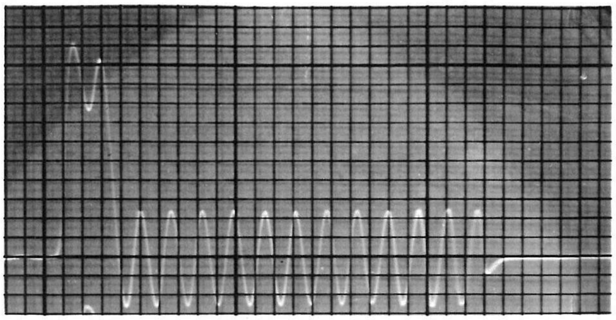
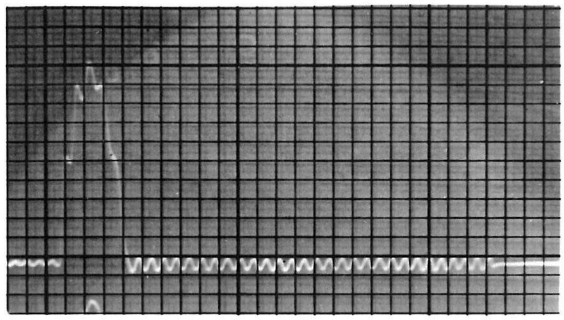


fig. 19

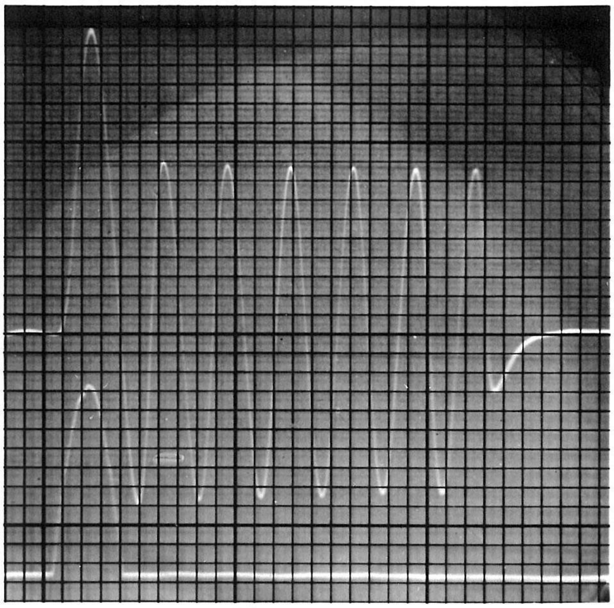
Transient Pulses



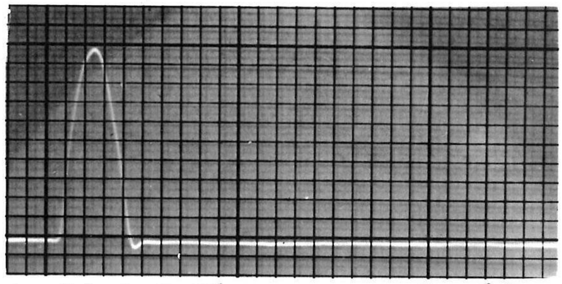
2f5'=2.32 D=0



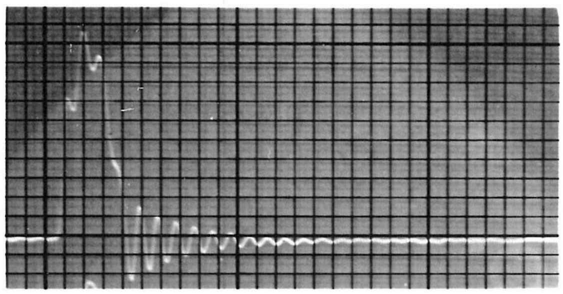
2ft'=5.84 D=0



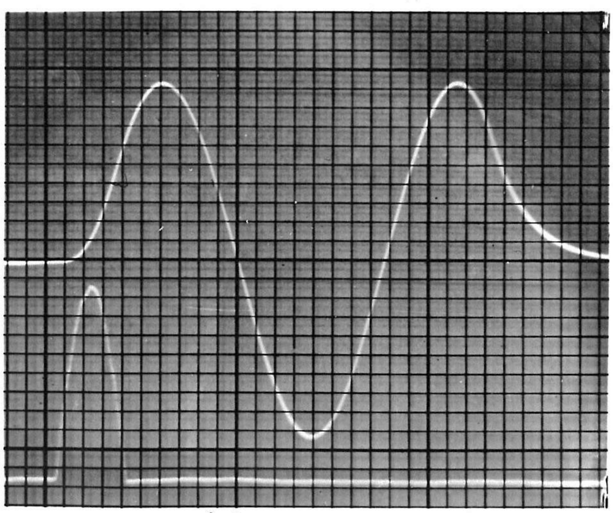
2ft'=1.166 D=0



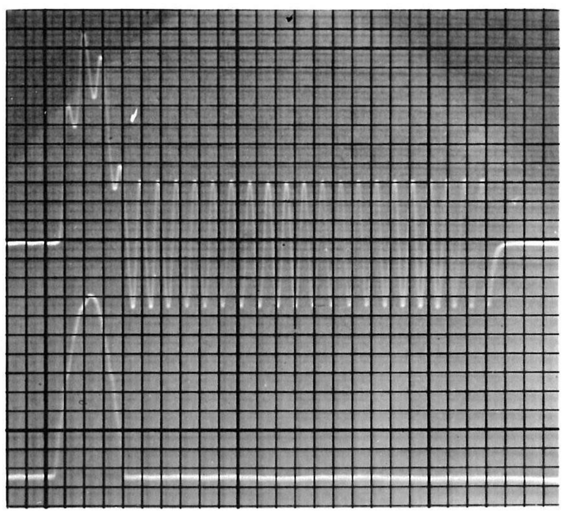
2ft'=3.96 D=.60



2ft'=3.96 D=.05



2ft'=.248 D=0



2ft'=3.96 D=0

fig. 20

Dynamic Response to Pulse r=1

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