ESSAYS ON SPECULATION AND FUTURES MARKETS

Thesis by

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Abstract

The thesis consists basically of two parts. The first part deals with speculators in commodity markets. In particular, we are interested in the role of speculators in stabilizing or destabilizing market price. The second part takes up hedgers in commodity futures markets. Here, we are concerned with the asymmetries between short and long hedgers. Specifically, we study whether or not the asymmetries discussed in the literature will lead to a backwardation equilibrium in futures markets.

The two approaches differ in the way speculators are treated in the framework as market participants. In the literature dealing with speculators and stabilization, the non-speculators are inactive; their only role is to provide an (exogenous) non-speculative excess demand function based on which speculators choose their transactions to maximize their objective functions. Conversely, in the futures market literature, under rational expectations and common beliefs on the part of all traders, speculators are only the supporting actors while hedgers play the leading roles; speculators act only to reduce the imbalance between short and long hedging. The difference between these two approaches is, however, not as clear-cut as it seems to be. The reason is simply that hedgers often take some speculative positions in their decision-making process. Consequently, it can be argued that both speculators and non-speculators are active participants in the futures markets. This specific characteristic

thus generates the ambiguities about the role of speculators in stabilizing or destabilizing market price in the futures market framework.

The main results of the thesis are as follows. From an ex post viewpoint, Chapter 1 indicates that profitable speculation will necessarily stabilize market price if and only if the non-speculative excess demand function is linear, with no lag structure and with the law of demand being satisfied. This conclusion falsifies the famous Friedman conjecture (i.e., profitable speculation necessarily stabilizes market price). We then study the case of linear non-speculative excess demand function using an ex ante approach. At a rational expectations equilibirum, it is shown that Friedman's conjecture holds when speculators' expected utility function can be expressed in terms of mean-variance consideration. Whether or not there are nonlinear non-speculative excess demand functions that verify the Friedman conjecture in ex ante framework is a matter for future research.

In Chapters 3 through 5, we deal with two well-known asymmetries between short and long hedging, namely, asymmetric arbitrage opportunities and the so-called Houthakker effect. First, we show that the asymmetric arbitrage argument has no standing in the way of establishing the existence of a backwardation equilibrium in forward markets, whereas some highly restrictive assumptions must be imposed for the asymmetric arbitrage argument to lead to a backwardation equilibrium in a true futures market. Thus the theoretical argument for a link between asymmetric arbitrage opportunities and a backwardation equilibrium is weak. Yet the question remains as to whether or not asymmetric arbitrage opportunities prevail in functioning futures markets. This is studied in Chapter 4 with respect to wheat and corn futures contracts traded on the Chicago Board of Trade (CBOT). The results indicate that asymmetric arbitrage opportunities have impacts upon CBOT wheat futures markets, but not upon CBOT corn futures markets. Consequently, the asymmetric arbitrage argument may apply only to some specific commodities.

Finally, in Chapter 5, we apply the same sample to test the existence of the Houthakker effect. Again, the hypothesis is rejected. Therefore, the two well-known asymmetries between short and long hedging do not have impacts upon CBOT wheat and corn futures markets, notwithstanding their roles in the way of a backwardation equilibrium.

The thesis is concerned with developing an understanding of the way in which futures markets function, and the role of speculators and hedgers in the markets. The results presented here indicate that it is only under rather restrictive conditions that definite results concerning these issues can be derived, particularly in the context of the true futures markets, that is, markets in which several delivery options exist under a futures contract.

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Introduction

In arguing the case for flexible versus fixed exchange rates, it was maintained by Friedman (1953) that profitable speculation necessarily stabilizes prices. The reason is that speculators can make profits only when they buy at low prices and sell at high prices. Both activities act to reduce the variability of the market price and hence are stabilizing. This intuitive argument involves several difficulties. First, we need a definition for speculators. In fact, Friedman defined a speculator to be a person who buys for future sales, not for his own consumption. As recognized by Friedman, this definition works well for ex post analyses; but it does not apply as well to decision making under uncertainty, where in the absence of a complete set of contingent claim markets, essentially all market participants are (or plan to be) speculators in this sense (see Feiger (1976)).

To see this, we first adopt a more precise definition of speculators based on Friedman (1953) and Kaldor (1939). That is, speculators are agents who trade to a bundle of goods from which they hope to trade away profitably after some price movements (see Feiger (1976)). Now, consider an exchange economy in which agents initially have an endowment contingent upon states of nature. Assume there are two time periods: a first period, denoted by $t = 0$, in which there is no uncertainty and a second period, denoted by $t = 1$, in which any one of S states of nature may prevail. There are G goods but no contingent claim markets exist.

An agent, a, has a state-dependent intertemporal utility function,

$$U_s^a = U_s^a(c_o^a, c_s^a), s = 1,\ldots,S, \; a = 1,\ldots,A;$$

Also, he has subjective probabilities of the occurrence of the states:

$$\pi_s^a \geq 0, \sum_{s=1}^{S} \pi_s^a = 1, \; a = 1,\ldots,A.$$

Define h_o^a to be the vector of commodities bought in the first period to be hoarded by agent a.(Essentially, this, implies the agent is an ex ante speculator; for an ex ante nonspeculator, b, h_o^b is constrained to be zero.) Agent a consumes the vector c_o^a in the first period and c_s^a, should state s eventuate in the second. And he possesses first-period (w_o^a) and contingent (w_s^a) endowments, traded in the first period and the second at prices P_o and P_s, respectively. Agents thus face a recursive decision problem in deciding their consumption bundles and hoardings. If state s occurs in the next period, for given c_o^a the agents would face the decision problem:

$$\max_{c_s^a} U_s^a(c_o^a, c_s^a)$$

$$\text{subject to } P_s \cdot (w_s^a + h_o^a - c_s^a) \geq 0$$

The solution to this problem defines an indirect utility function:

$$F_s^a = F_s^a(c_o^a, h_o^a, P_s).$$

Since markets are incomplete, some prices are uncertain; therefore, the agent may take an expectation over them to leave

$$E\{F_s^a\} = E\{F_s^a(c_o^a, h_o^a, P_s)\}$$

Thus, in the first period agents face the decision problem:

$$\max_{c_o^a, h_o^a} \sum_{s-1}^{S} \pi_s^a \cdot E\{F_s^a\}$$

$$\text{subject to } P_o \cdot (c_o^a + h_o^a - w_o^a) \leq 0.$$

Following Feiger (1976, Proposition 2), it was shown that the optimal solution to the second period problem (after c_o^a and h_o^a have been determined from solving the first-period problem), c_s^a, will equal to $w_s^a + h_o^a$ only coincidentally. Hence, every agent will choose $w_o^a + h_o^a$ first; then, upon the realization of the state, a trade to c_s^a will occur. As such, every agent is a speculator in the sense of "buying for resales."

From an ex post point of view with only one commodity being considered, it may as well be the case that upon the realization of the state, some agents do not engage in further transactions on that commodity. Consequently, they will be classified as nonspeculators (although they are speculators from an ex ante point of view); all others who engage in further transactions are thus speculators.

The above analysis justified the premise that everyone who might engage in hoarding is a speculator from an ex ante point of view. In cases where some agents, b = A + 1, A + 2,...,A + B, are excluded from hoarding due to lack of storage capacity, they face a constrained decision problem with $h_o^b = 0$. Consequently, they will be nonspeculators from both ex post and ex ante points of view because they cannot buy for resales. The solution to their decision problems then give rise to the nonspeculative excess demand function in an ex ante framework. To this point, it is clear that speculators are whose who might engage in hoarding from an ex ante point of view, while they consist of two groups, those lack of storage capacity and those who have storage capacity but happen to choose not to hoard, from an ex post point of view.

We now turn to the second problem associated with Friedman's argument. Mainly, speculators operating under uncertainty cannot know when prices are "low" or "high", so that any ex ante analysis has to be supplemented by some assumption such as rational expectations, if ex ante expected profitability is to be used as a measure of "profitable" speculation(see Telser (1959)).

Finally, we need a measure for market price stability. In the literature, the basic approach has been to study the effect on the variance of the market price of profitable speculative activity. That is, variance was adopted as the measure of price stability[1]. It is not at all clear that this is the appropriate measure from the point of view of evaluating the effects of speculation on social welfare (see Johnson (1976)). In this chapter, we will study the Friedman conjecture from an ex post point of view. Therefore, Friedman's definition of speculator and the forecasting ability of speculators will not cause any difficulty. Moreover, we also use the variance to measure price stability.

The problem of concern is whether or not Friedman's conjecture is correct in general, and, If not, to what extent it is correct. Concerning this, Baumol constructed a theoretical counterexample to show that the Friedman conjecture is not always true[2]. Nonetheless, the counterexample requires a non-speculative excess demand function that depends on previous price levels. Friedman then argued that such a functional form does not qualify as a non-speculative excess demand function because all persons whose demand or supply depends on previous prices must be classified as speculators. Since the issue is still unresolved, the two possible functional forms are studied in this chapter.

Later, Stein (1961) also provided a real-life counterexample to invalidate Friedman's conjecture. In his paper, the example involved only two dates; therefore, it is unclear what kind of functional form the non-speculative excess demand actually took. More importantly, whether or not the two data points lie on the same non-speculative excess demand remains unclear. In fact, it is easy to show that, if the non-speculative excess demand function changes over time, the Friedman's conjecture is generally false. In the literature, however, it is generally accepted that we are dealing with a non-speculative excess demand function that remains unchanged over time.

The above two papers looked at the negative aspects of the

Friedman conjecture. On the other hand, both Kemp (1963) and Telser (1959) showed that when the non-speculative excess demand function is linear with no lag structure and satisfies the law of demand, Friedman's conjecture is always true. At this point in the debate, it was clear there were only certain classes of non-speculative excess demand functions which can validate Friedman's conjecture. The problem is what classes and the the question of whether or not these classes can generally describe market behavior.

Farrell (1966) tackled these problems, and showed that, (i) for a two-period model, any continuous negatively sloped non-speculative excess demand function would validate Friedman's argument if there is no lag structure; and (ii) given the independence assumption and the law of demand, for a T-period model with $T \geq 3$, a negatively sloped linear non-speculative excess demand function is necessary and sufficient for Friedman's conjecture to be true if there is no lag structure. Schimmler (1973) generalized Farrell's results to the case of lag-responsive non-speculative excess demand, showing again that linearity coupled with the law of demand is necessary and sufficient to validate Friedman's argument.

However, there are some problems in Farrell's and Schimmler's approaches that invalidate their proofs. In this paper we will show that, after correcting these slips, Farrell's two results are in fact correct, and we will redo Schimmler's problem for time-independent non-speculative excess demand functions. The conclusions derived for Schimmler's problem are (i) for two-period models, any continuously differentiable non-speculative excess demand $f(P_t, P_{t-1})$ with $f_1(P_t,P_{t-1}) < 0$, $f_2(P_t,P_{t-1}) \leq 0$ (where $f_1(P_t,P_{t-1}) = \frac{\partial f(P_t,P_{t-1})}{\partial P_t}$, $f_2(P_t,P_{t-1}) = \frac{\partial f(P_t,P_{t-1})}{\partial P_{t-1}}$) will validate Friedman's conjecture; (ii) for T-period models ($T \geq 3$), within the class of twice continuously differentiable functions, linear non-speculative excess demand functions $f(P_t,P_{t-1},\cdots,P_{t-T+1})$ satisfying $f_1 < 0$, $f_2 = f_3 = \cdots = f_{t-T+1} = 0$ represent necessary and sufficient conditions for Friedman's conjecture to be true.

On the other hand, while Friedman's conjecture does not hold with nonlinear non-speculative excess demand functions, this still leaves open the question as to whether the conjecture is valid at a rational expectations equilibrium (where profits are maximized). Given a three-period model, we show that examples can be constructed of a rational expectations equilibrium such that prices are destabilized, given that the market clearing prices without speculation differ in all three periods. Thus, while the analysis of this paper takes an ex post approach, still there are certain ex ante implications of the analysis as well.

The appendix contains proofs of the most general results of the paper. We also prove a basic result on linear demand functions. Proofs of most other results are omitted, but the basic strategy underlying such proofs follows that of the three proofs presented.

Farrell's Framework: A Re-examination

Farrell considered a discrete time abstract market model, where the associated commodity is storable. Let $t = 1, 2, \cdots, T$ denote T periods. Within any period all transactions are assumed to take place at the same price. Also, let P^W_t, $t = 1, 2, \cdots, T$ denote the price in period t when there is no speculation, and let P^S_t, $t = 1,2,\ldots,T$ denote the price in period t given the speculation sequence $\{s_1, s_2, \cdots, s_T\}$, where s_t, $t = 1, 2, \cdots, T$, is the speculative sales in period t. To make the effects of speculation sequences welldefined, we need a clear-cut terminal date. Therefore, following Farrell, we define a complete speculation sequence as a speculation sequence $\{s_1, s_2, \cdots, s_T\}$ such that

$$\sum_{t=1}^{T} s_t = 0. \qquad (1)$$

By sales and purchases in the market, speculators' profits are

$$\pi = \sum_{t=1}^{T} P^S_t \cdot s_t. \qquad (2)$$

The introduction of speculation changes the variance of prices according to

$$C' = \{ [\sum_{t=1}^{T} (P^S_t)^2 - \frac{1}{T}(\sum_{t=1}^{T} P^S_t)^2] - [(\sum_{t=1}^{T} (P^W_t)^2 - \frac{1}{T}(\sum_{t=1}^{T} P^W_t)^2] \} \cdot \frac{1}{T} \qquad (3)$$

where

$$C = TC' = [\sum_{t=1}^{T} (P_t^s)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^s)^2] - [\sum_{t=1}^{T} (P_t^w)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^w)^2] \quad (3)'$$

is taken to be the measure of the stabilizing effect of speculation. That is, if $C > 0$, we say the speculation sequence destabilizes prices; if $C < 0$, we say the speculation sequence stabilizes prices.

Since Farrell considered only complete speculation sequences, Friedman's conjecture can be formalized as follows:

$$\text{When } \sum_{t=1}^{T} s_t = 0, \text{ if } \pi > 0, \text{ then } C < 0. \quad (4)$$

To derive his two results about (4), Farrell employed an independence assumption; i.e., he assumed that the non-speculative excess demand function has the following property:

$$P_t^s - P_t^w = h(s_t), \ \forall t, \text{ for some function } h(\cdot) \text{ such that } h(0) = 0 \text{ and } h'(.) < 0. \quad (5)$$

In other words, suppose we have a non-speculative excess demand function $f(\cdot)$ such that $Q_t^w = f(P_t^w)$. When there are speculative sales s_t, P_t^w must be adjusted to P_t^s in order to clear the market, i.e., we must have $Q_t^w + s_t = f(P_t^s)$ which implies $s_t = f(P_t^s) - f(P_t^w)$. Therefore, we can rewrite (5) as

$$P_t^s - P_t^w = h(f(P_t^s) - f(P_t^w)). \quad (5)'$$

Under Eq. (5) (or equivalently Eq. (5)'), Farrell derived the results: (i) for a two-period model, any continuous negatively sloped non-speculative excess demand function will satisfy (4); (ii) for a T-period model ($T \geq 3$), a negatively sloped linear non-speculative excess demand function is necessary and sufficient for (4) to be true.

The problem with Farrell's proofs is that there is a tautology involved. To see this, we ask when we can write Eq. (5). Equivalently, what functional form for non-speculative excess demand is consistent with Eq. (5)'?

Theorem 1

Let $Q^S = f(P^S)$, $Q^W = f(P^W)$. Then, within the class of continuous, differentiable functions, the only functional form $h(\cdot)$ which can satisfy $P^S - P^W = h(Q^S - Q^W)$ for all $P^S \geq 0$, $P^W \geq 0$ is linear. Also, $f(\cdot)$ must be linear.

[Proof]

$$P^S - P^W = h(Q^S - Q^W) = h(f(P^S) - f(P^W)), \quad \forall P^S, P^W$$

Taking the partial derivative with respect to P^S, we have

$$1 = h'(f(P^S) - f(P^W))f'(P^S), \quad \forall P^S, P^W.$$

Similarly, taking the partial derivative with respect to P^W,

$$-1 = h'(f(P^S) - f(P^W))(-f'(P^W)), \quad \forall P^S, P^W.$$

Hence, $(f'(P^S) - f'(P^W))h'(f(P^S) - f(P^W)) = 0, \quad \forall P^S, P^W$

$\Rightarrow \quad f'(P^S) = f'(P^W), \quad \forall P^S, P^W$; i.e., $f(\cdot)$ is linear. Therefore, $h'(Q^S - Q^W)$ is also a constant which implies that $h(\cdot)$ is linear.

Q.E.D.

Theorem 1 shows that only linear non-speculative excess demand functions are consistent with (5)′. Therefore, Farrell's proofs involve writing down a functional form (5) which can be satisfied only by a linear non-speculative excess demand function. Farrell then proved (4) is true only when we have linear non-speculative excess demand. Obviously, this involves a tautology.

Nonetheless, for every non-linear non-speculative excess demand function f(.), if we fix P^W and allow s to vary, we can still find a correspondence h between $P^S - P^W$ and s. But that correspondence depends as well on P^W; i.e., $P^S - P^W = h(s,P^W) = h(Q^S - Q^W,P^W)$. Farrell's proof depends on the assumption that h is independent of P^W, since the proof proceeds by exploiting the properties of a single curve h in (P^W,s) space. Once it is recognized that h must be parameterized by P^W, the geometric approach requires a family of curves, each member corresponding to some price P^W, if non-linear non-speculative excess demands are to be included in the analysis.

While Farrell's proofs are incorrect, the remaining question is whether his two results are still true. The following three theorems show that both his claims are, in fact, correct.

Theorem 2

For a two-period model, any differentiable, negatively sloped non-speculative excess demand function will satisfy Friedman's conjecture (i.e., Eq. (4)).

Theorem 3

For a three-period model, the only (nontrivial[3]) continuously differentiable functional form for the non-speculative excess demand function which can satisfy Friedman's conjecture is linear with negative slope.

Theorem 4

For a T-period model with $T \geq 3$, the only nontrivial continuously differentiable non-speculative excess demand functional form which can satisfy Friedman's conjecture is linear with negative slope.

Actually, Theorems 2 and 3 are special cases of Theorems 6 and 7; hence, we won't provide proofs (interested readers may check Lien (1984)). Theorem 4 is a generalization of Theorem 3 which can be easily proved by arbitrarily taking three consecutive periods and then applying Theorem 3.

Lag-Responsive Non-speculative Excess Demand

Friedman made the argument that the presence of a lagged response in non-speculative excess demand functions converts non-speculators into speculators. Nonetheless, it is still of interest to see to what extent the above results generalize to this case.

Schimmler investigated this problem by generalizing Farrell's approach to consider interdependent demand situations. First, let $P^S = [P_1^S, P_2^S, \ldots, P_T^S]'$, $P^W = [P_1^W, P_2^W, \ldots, P_T^W]'$, $Q^S = [Q_1^S, Q_2^S, \ldots, Q_T^S]'$, $Q^W = [Q_1^W, Q_2^W, \ldots, Q_T^W]'$, $S = Q^S - Q^W$, all $(T \times 1)$ vectors. Schimmler assumed that the non-speculative excess demand function has the following property:

$$P^S - P^W = H(S), \tag{6}$$

where H is a mapping from $\mathbb{R}^T$ to $\mathbb{R}^T$. Under (6), Schimmler showed that a necessary and sufficient condition for Friedman's conjecture to be true is that $H^*(S) = H(S) - \frac{H(S)'U}{T}U = b(S)'S$, where b(S) is a real-valued function and $U = [1,1,1,\ldots,1]'$ is a $(T \times 1)$ vector.

Since Schimmler's assumption (Eq. (6)) is only a generalized version of Farrell's assumption (Eq. (5)), then, by employing similar procedures, we can prove once again that only linear mappings are consistent with Eq. (6).

Theorem 5

Let P^S, P^W, Q^S, Q^W be $(T \times 1)$ vectors satisfying $Q^S = f(P^S)$, $Q^W = f(P^W)$. Within the class of continuously differentiable mappings, the only mapping $H(\cdot)$ which can satisfy $P^S - P^W = H(Q^S - Q^W)$ for all P^S, P^W is linear. Also, $f(\cdot)$ is linear.

[Proof] See Lien (1984, p.14).

Again, Theorem 5 shows that Schimmler's approach involved the same problems as Farrell's. Therefore, we reconsider interdependent excess demand situations and provide the following theorems:[4]

<u>Theorem 6</u>

Any continuously differentiable non-speculative excess demand function $f(P_t, P_{t-1})$ with $f_1(P_t, P_{t-1}) < 0$, $f_2(P_t, P_{t-1}) \leq 0$ will satisfy Friedman's conjecture in the two-period model.

<u>Theorem 7</u>

For a three-period model, within the class of twice continuously differentiable functions, the only (nontrivial) non-speculative excess demand functional form, $f(P_t, P_{t-1}, P_{t-2})$, that can satisfy Friedman's conjecture is linear with $f_1 < 0$, $f_2 = f_3 = 0$.

Theorem 6 shows that, as in the independent excess demand case, if we consider only two-period models, a large class of continuously differentiable non-speculative excess demand functions will satisfy Friedman's conjecture, even though they involve past prices. However, Theorem 7 states that, for Friedman's conjecture to be true in a three-period model, we can never have a lag structure, if we assume a non-speculative excess demand functional form that is independent of time. (The proofs of Theorems 6 and 7 are described in the Appendix.) This implies that Friedman's classification of speculators cannot be relaxed; otherwise, his conjecture will in general be invalidated. In fact, Theorem 7 can be easily generalized to a T-period model with $T \geq 3$ as follows:

Theorem 8

For a T-period model with $T \geq 3$, within the class of twice continuously differentiable functions, the only (nontrivial) non-speculative excess demand functional form $Q_t = f(P_t, P_{t-1}, \ldots, P_{t-(T-1)})$ which can satisfy Friedman's conjecture is linear with $f_1 < 0$, $f_2 = f_3 = \cdots = f_{t-(T-1)} = 0$.

Now, we really come to a dead end. That is, if non-speculative excess demand involves a non-degenerate lag structure, then Friedman's conjecture is always false. On the other hand, Friedman has already rejected this class of excess demand functions, because of his claim that such excess demand functions can only represent speculators, and not non-speculators. These conclusions apply only to T-period models, when $T \geq 3$. When T=2, there is a large class of functions which can satisfy Friedman's conjecture even if a lag structure exists.

Maximum Profit vs. Price Stabilization

While nonlinear non-speculative excess demand functions will invalidate Friedman's conjecture, (i.e., there exist profitable speculation sequences which destabilize prices), a further question is whether this can be true for a profit maximizing speculation sequence. In particular, given that a speculator has perfect information about future prices, at a rational expectations equilibrium, the speculator will choose a profit-maximizing speculation sequence. Can such a speculation sequence destabilize prices?

To tackle this problem, consider the following maximization problem:

$$\max_{\{P_t^s\}} \sum_{t=1}^{T} P_t^s\{f(P_t^s) - f(P_t^w)\}$$

$$\text{Subject To:} \quad \sum_{t=1}^{T}\{f(P_t^s) - f(P_t^w)\} = 0$$

$$\text{and } \mathrm{Var}\{P_t^s\} \geq \mathrm{Var}\{P_t^w\}.$$

Forming the Lagrangian, we have

$$\mathbb{L} \equiv \sum_{t=1}^{T} P_t^s\{f(P_t^s) - f(P_t^w)\} + \lambda\sum_{t=1}^{T}\{f(P_t^s) - f(P_t^w)\} + \mu(\mathrm{Var}\{P_t^s\} - \mathrm{Var}\{P_t^w\}).$$

Therefore, if there is a profit-maximizing speculation sequence which destabilizes prices, we must have $\mu = 0$. In this case, assuming $T = 3$, the first-order conditions are

$$(P_t^s + \lambda)f'(P_t^s) + f(P_t^s) = f(P_t^w), \ \forall\ t = 1, 2, 3 \tag{7}$$

$$\sum_{t=1}^{3}\{f(P_t^s) - f(P_t^w)\} = 0 \tag{8}$$

$$\mathrm{Var}\ \{P_t^s\} > \mathrm{Var}\ \{P_t^w\}. \tag{9}$$

Now, summing up Eq. (7) over t, we have $\lambda = \dfrac{\sum_{t=1}^{3} P_t^s f'(P_t^s)}{-\sum_{t=1}^{3} f'(P_t^s)} < 0.$

Also, from (7), $\sum_{t=1}^{3} P_t^s\{f(P_t^s) - f(P_t^w)\} = -\sum_{t=1}^{3} P_t^s(P_t^s + \lambda)f'(P_t^s) =$

$\dfrac{-\sum_{i=1}^{3}\sum_{j=1,j>i}^{3}(P_i^s - P_j^s)^2 f'(P_i^s)f'(P_j^s)}{\sum_{t=1}^{3} f'(P_t^s)} \geq 0$, and equality holds only when

$P_i^S = P_j^S$, $\forall$ i, j = 1, 2, 3.

The Appendix provides a proof of the following.

Theorem 9

For a three-period model with $P_i^W = P_j^W$ for some $i \neq j$, i, j = 1, 2, 3, any profit-maximizing speculation sequence will stabilize prices, given a continuous, differentiable non-speculative excess demand function with negative slope everywhere.

We next take up the case where P_1^W, P_2^W and P_3^W are all distinct. Consider the following numerical example[5]: $P_1^W = 7$, $P_2^W = 12$, $P_3^W = 2$; $\lambda = -10.95$; $P_1^S = 10.9$, $P_2^S = 11$, $P_3^S = 2.1$; $f(2) = 85$, $f(2.1) = 83$, $f(7) = 72$, $f(10.9) = 70$, $f(11) = 64$, $f(12) = 60$; $f'(2.1) = -\frac{2}{8.85}$, $f'(10.9) = -40$, $f'(11) = -80$ (see Fig. 1). These values will satisfy Eqs. (7) - (9). Specifically, Var $\{P_t^W\} = \frac{50}{3} < \frac{52.2}{3} =$ Var $\{P_t^S\}$. Similar examples can be constructed whenever P_1^W, P_2^W, and P_3^W are distinct, as is easily verified by examining the conditions (7) and (8) in this case. Therefore, we have the following theorem:

Theorem 10

Let T = 3. Given any sequence $\{P_1^W, P_2^W, P_3^W\}$ of market clearing prices in the absence of speculation where P_1^W, P_2^W and P_3^W are distinct, there exists a non-linear excess demand function f(P) with $f'(P) < 0$ such that (i) $f(P_t^W) = Q_t^W$, $\forall$ t = 1, 2, 3; (ii) at a rational expectations equilibrium with a complete speculation sequence $\{s_t\}$, $f(P_t^S) = Q_t^W + s_t$, $\forall$ t = 1, 2, 3 and Var $\{P_t^S\} >$ Var $\{P_t^W\}$.

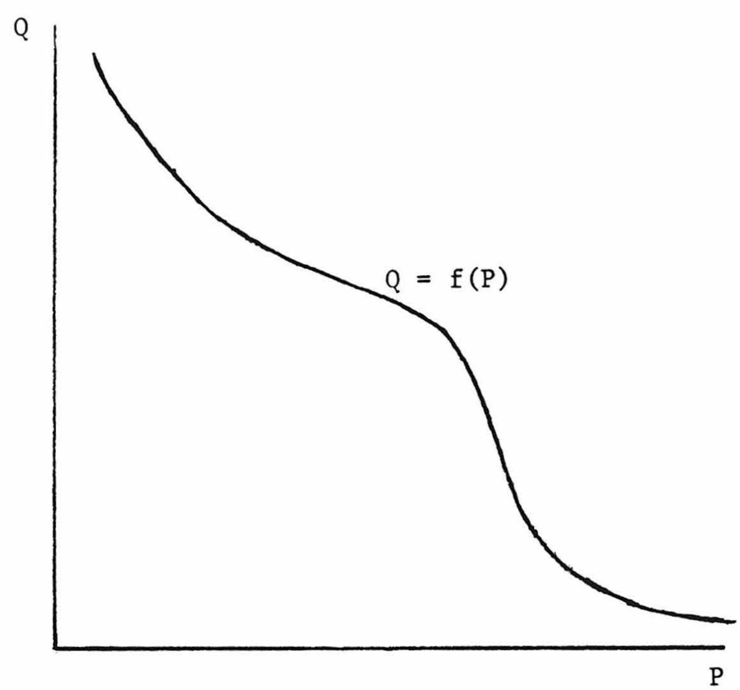

FIGURE 1.1 THE CASE OF RATIONAL DESTABILIZING SPECULATION

Theorem 10 establishes that the Friedman conjecture does not hold at a rational expectations equilibrium. Thus Farrell's original findings, while phrased in ex post terms, actually have some applications in ex ante analysis as well.

Appendix To Chapter 1

1. Proof of Theorem 6

Let P_0 be the exogeneously determined price at 0 period, and let P_1, P_2 be the prices in periods 1 and 2 under the speculation sequence $\{S_1, S_2\}$. Also, let q_1, q_2 be the prices in periods 1 and 2 when there is no speculation. Therefore, $S_1 = Q_1^S - Q_1^W = f(P_1,P_0) - f(q_1,P_0)$ and $S_2 = Q_2^S - Q_2^W = f(P_2,P_1) - f(q_2,q_1)$.

Now, consider the following minimization problem:

$$\min_{\{q_1,q_2\}} V(q_1,q_2) = (q_1^2 + q_2^2) - \tfrac{1}{2}(q_1 + q_2)^2.$$

$$\text{Subject To: } f(P_1,P_0) - f(q_1,P_0) + f(P_2,P_1) - f(q_2,q_1) = 0 \quad \text{(A1)}$$

$$P_1[f(P_1,P_0) - f(q_1,P_0)] + P_2[f(P_2,P_1) - f(q_2,q_1)] \geq 0. \quad \text{(A2)}$$

To verify Friedman's conjecture, we must require (P_1,P_2) to achieve the minimum; i.e., (P_1,P_2) must be a minimum point.

Case A: If both (A1) and (A2) are binding, then

$[f(P_1,P_0) - f(q_1^*,P_0)](P_1 - P_2) = 0 \Rightarrow q_1^* = P_1$, if $P_1 \neq P_2$
$\Rightarrow q_2^* = P_2$ (by (A1)).

If $P_1 = P_2$, then it can easily be shown that $q_1^* = q_2^* = P_1 = P_2$.

Case B: If (A2) is not binding, then we have $q_1^* = q_2^* = q^*$ (by solving first-order conditions), and $[f(P_1,P_0) - f(q^*,P_0)](P_1 - P_2) \geq 0$.

(i) If $P_1 > P_2$, then $f(P_1,P_0) \geq f(q^*,P_0) \Rightarrow f(P_2,P_1) - f(q^*,q^*) \leq$

0 and $P_1 \leq \bar{q}^*$.

Since $f_1 < 0$, $f_2 \leq 0$, we must have $P_2 \geq q^* \Rightarrow P_2 \geq P_1$, contradicting $P_1 > P_2$.

(ii) Similarly, if $P_1 < P_2$, then we have $P_1 \geq P_2$, also a contradiction.

Thus far, we have shown that the optimum point must be (P_1,P_2) for any given (P_1,P_2). To make sure V achieves a minimum, we'll check the local properties of V at (P_1,P_2), while still satisfying (A1) and (A2) as follows:

(i) If $P_1 > P_2$, then to satisfy (A1) and (A2), we must have

$$(f(P_1,P_0) - f(q_1,P_0))(P_1 - P_2) \geq 0,$$

which implies $f(P_1,P_0) \geq f(q_1,P_0) \Rightarrow P_1 \leq q_1$. Now by (A1), we have $f(P_2,P_1) \leq f(q_2,q_1) \Rightarrow P_2 \geq q_2$. Hence, $V(P_1,P_2) \leq V(q_1,q_2)$.

(ii) Similarly, if $P_1 < P_2$, we also have $V(P_1,P_2) \leq V(q_1,q_2)$.

(iii) If $P_1 = P_2$, then (P_1,P_2) is the only feasible point satisfying (A1) and (A2).

Therefore, (P_1,P_2) achieves a minimum of $V(q_1,q_2)$ subject to constraints (A1) and (A2), implying that Friedman's conjecture is satisfied.

2. <u>Proof of Theorem 7</u>

Let P_{-1} and P_0 be exogeneously determined prices in periods -1 and 0, respectively, and let P_1, P_2 and P_3 be prices in periods 1, 2 and 3 associated with speculative sequence $\{S_1,S_2,S_3\}$. Also, let q_1,

q_2, and q_3 be prices in periods 1, 2 and 3 when there is no speculation. Hence, we have:

$$S_1 = Q_1^S - Q_1^W = f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1})$$
$$S_2 = Q_2^S - Q_2^W = f(P_2,P_1,P_0) - f(q_2,q_1,P_0)$$
$$S_3 = Q_3^S - Q_3^W = f(P_3,P_2,P_1) - f(q_3,q_2,q_1).$$

Now, the minimization problem under consideration is

$$\underset{\{q_1,q_2,q_3\}}{\text{Min}} \sum_{i=1}^{3} q_i^2 - \frac{1}{3}(\sum_{i=1}^{3} q_i)^2,$$

subject to:

$$f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1}) + f(P_2,P_1,P_0) - f(q_2,q_1,P_0) + f(P_3,P_2,P_1) - f(q_3,q_2,q_1) = 0 \tag{A3}$$

and

$$P_1[f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1})] + P_2[f(P_2,P_1,P_0) - f(q_2,q_1,P_0)] + P_3[f(P_3,P_2,P_1) - f(q_3,q_2,q_1)] \geq 0. \tag{A4}$$

Forming the Lagrangian, we can derive the first-order conditions as following:

$$2P_1 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda[f_1^{(1)} + f_2^{(2)} + f_3^{(3)}] - \mu[f_1^{(1)}P_1 + f_2^{(2)}P_2 + f_3^{(3)}P_3] = 0 \tag{A5}$$

$$2P_2 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda[f_1^{(2)} + f_2^{(3)}] - \mu[f_1^{(2)}P_2 + f_2^{(3)}P_3] = 0 \tag{A6}$$

$$2P_3 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda f_1^{(3)} - \mu f_1^{(3)}P_3 = 0, \tag{A7}$$

where $f_i^{(1)} = f_i(P_1,P_0,P_{-1})$, $f_i^{(2)} = f_i(P_2,P_1,P_0)$, $f_i^{(3)} = f_i(P_3,P_2,P_1)$,

$i = 1, 2, 3.$

Now, consider only the binding case, i.e., $\lambda \neq 0$, $\mu \neq 0$. Then to satisfy Friedman's conjecture, we require (P_1, P_2, P_3) to satisfy Eqs. (A5) to (A7). By adding up (A5) to (A7),

$$\lambda = \frac{f_1^{(1)} P_1 + (f_1^{(2)} + f_2^{(2)}) P_2 + (f_1^{(3)} + f_2^{(3)} + f_3^{(3)}) P_3}{f_1^{(1)} + f_1^{(2)} + f_2^{(2)} + f_1^{(3)} + f_2^{(3)} + f_3^{(3)}} \cdot \mu \tag{A8}$$

Since for any given P_1, P_2 and P_3, Eqs. (A5) - (A8) must always hold, we can choose P_1, P_2, P_3 such that $P_1 + P_2 = 2P_3$ and $P_1 \neq P_3$, $P_2 \neq P_3$. Thus, from (A7),

$$\lambda f_1^{(3)} - \mu f_1^{(3)} P_3 = 0 \Rightarrow f_1^{(3)}(\lambda - \mu P_3) = 0 \Rightarrow \lambda = \mu P_3,$$

since f is nontrivial. Substituting this into (A8), and using the fact that $\mu \neq 0$, $P_1 = 2P_3 - P_2$, $P_2 \neq P_3$, we have $f_1(P_2, P_1, P_0) - f_1(P_1, P_0, P_{-1}) = -f_2(P_2, P_1, P_0)$, when $P_1 + P_2 = 2P_3$, and $P_1 \neq P_2$. Now, since we can arbitrarily change P_3 also by twice continuous differentiability of f, we have

$$f_1(P_2, P_1, P_0) - f_1(P_1, P_0, P_{-1}) = -f_2(P_2, P_1, P_0), \ \forall P_2, P_1, P_0, P_{-1}. \tag{A9}$$

Note that P_{-1} is arbitrarily given, and only $f_1(P_1, P_0, P_{-1})$ involves this term, implies that $f_{13}(P_1, P_0, P_{-1}) = 0$, $\forall\ P_1, P_0, P_{-1}$ $\Rightarrow f_{13}(P_t, P_{t-1}, P_{t-2}) = 0$, $\forall\ P_t, P_{t-1}, P_{t-2}$.

Similarly, if we fix P_1, P_0, P_{-1} and change P_2, we'll have

$$f_{11}(P_2,P_1,P_0) + f_{21}(P_2,P_1,P_0) = 0, \quad \forall\, P_2,P_1,P_0$$

$$\Rightarrow \quad f_{11}(P_t,P_{t-1},P_{t-2}) + f_{21}(P_t,P_{t-1},P_{t-2}) = 0, \quad \forall\, P_t,P_{t-1},P_{t-2}, \qquad \text{(A10)}$$

which is a partial differential equation. The solution of (A10) is $f_1(P_t,P_{t-1},P_{t-2}) = a + bP_t - bP_{t-1}$ for some constants a and b (Note: $f_{13} = 0$.) Substituting into (A9),

$$f_2(P_t,P_{t-1},P_{t-2}) = -bP_t + 2bP_{t-1} - bP_{t-2}, \quad \forall\, P_t,\ P_{t-1},\ P_{t-2}.$$

Now, choose P_1,P_2,P_3 such that $P_1 + P_3 = 2P_2$, $P_1 \neq P_2$, $P_3 \neq P_2$; then (A6) reduces to

$$\lambda(f_1^{(2)} + f_2^{(3)}) - \mu(f_1^{(2)}P_2 + f_2^{(3)}P_3) = 0.$$

Substituting into (A8), after algebraic operations, we have

$$f_1^{(1)} - f_1^{(3)} = f_3^{(3)}, \text{ since } f_1^{(2)} \neq 0,\ P_2 \neq P_3,\ f_2^{(3)} = 0$$

$$\Rightarrow f_3(P_3,P_2,P_1) = (a + bP_1 - bP_0) - (a + bP_3 - bP_2)$$

$$= -bP_3 + bP_2 + bP_1 - bP_0, \text{ when } P_1 + P_3 = 2P_2,\ P_1 \neq P_3.$$

Hence, by arbitrarily changing P_0, we must have

$$\partial f_3(P_3,P_2,P_1)/\partial P_0 = -b = 0,$$

which implies $f_1(P_t,P_{t-1},P_{t-2}) \equiv a$ and $f_2(P_t,P_{t-1},P_{t-2}) \equiv 0$

$\Rightarrow\ f_3^{(3)} = 0$ when $2P_2 = P_1 + P_3$ and $P_1 \neq P_3$, i.e.,

$$f_3(P_3,P_2,2P_2 - P_3) = 0, \quad \forall\, P_2,P_3.$$

However, $f_{31}(P_t,P_{t-1},P_{t-2}) = f_{13}(P_t,P_{t-1},P_{t-2}) \equiv 0$, $f_{32}(P_t,P_{t-1},P_{t-2}) = f_{23}(P_t,P_{t-1},P_{t-2}) \equiv 0$; hence, $f_3(P_t,P_{t-1},P_{t-2}) =$

$g(P_{t-2})$ for some function $g(\cdot)$, and $f_3(P_3,P_2,2P_2 - P_3) = g(2P_2 - P_3)$. Since we can arbitrarily choose P_2, P_3; therefore, $g(2P_2 - P_3) = 0$, $\forall\ P_2,P_3$ implies that $g(P_{t-2}) \equiv 0$, and $f_3(P_t,P_{t-1},P_{t-2}) \equiv 0$.

Combining the above results, we have $f(P_t,P_{t-1},P_{t-2}) = K + aP_t$ for some constants K and a.

Next, consider the nonbinding case (i.e., $\lambda = \mu = 0$); then we must require that (A4) not be satisfied. This will lead us to $a < 0$, which completes the proof.

3. Proof of Theorem 9

Let $x_t = P_t^s$, $y_t = P_t^w$, $\forall\ t = 1, 2, 3$. Now, assume $y_1 = y_2$; we have three cases:

Case A: $y_3 > y_1$

(i) If x_1, $x_2 > y_1$, then $f(x_1)$, $f(x_2) < f(y_1)$.
$\Rightarrow (x_1 + \lambda)f'(x_1) > 0$, $(x_2 + \lambda)f'(x_2) > 0 \Rightarrow x_1, x_2 < -\lambda$.
By (8), $x_3 < y_3$, since x_1, $x_2 > y_1 = y_2$ and $f'(\cdot) < 0$. This implies $f(x_3) > f(y_3) \Rightarrow (x_3 + \lambda)f'(x_3) < 0 \Rightarrow x_3 > -\lambda$.
Hence, $y_1 = y_2 < x_1, x_2 < -\lambda < x_3 < y_3 \Rightarrow \text{Var}\,\{P_t^w\} > \text{Var}\,\{P_t^s\}$, contradicting (9).

(ii) If x_1, $x_2 < y_1$, then x_1, $x_2 > -\lambda$. Also, $x_3 > y_3 \Rightarrow x_3 < -\lambda$.
Therefore, $y_1 > x_1, x_2 > -\lambda > x_3 > y_3$, contradicting $y_3 > y_1$.

(iii) If $x_1 > y_1$, $x_2 < y_1$, then $x_1 < -\lambda$, $x_2 > -\lambda \Rightarrow x_2 > x_1$. But $x_1 > y_1 > x_2$; hence we also have a contradiction.

(iv) If $x_1 = y_1$, then $x_1 = -\lambda$.

(a) If $x_2 > y_1$, then $x_2 < -\lambda \Rightarrow x_2 < x_1 = y_1$, contradiction.

(b) If $x_2 < y_1$, then $x_2 > -\lambda \Rightarrow x_2 > x_1 = y_1$, contradiction.

(c) If $x_2 = y_1$, then by (8), $x_3 = y_3 \Rightarrow \text{Var}\ \{P_t^W\} = \text{Var}\ \{P_t^S\}$, contradicting (9).

<u>Case</u> <u>B</u>: $y_3 < y_1$

Clearly, the proof of case A is applicable here as well, producing a contradiction in each instance.

<u>Case</u> <u>C</u>: $y_3 = y_1$

(i) Assume $x_1 = x_2 = x_3$ is not true; then, without loss of generality, let $x_1 > y_1$ and $x_2 < y_1$. Then $x_2 < y_1$ $\Rightarrow f(x_2) > f(y_1) \Rightarrow (x_2 + \lambda)f'(x_2) < 0 \Rightarrow x_1 > y_1 > x_2 > -\lambda$. But $x_1 > -\lambda \Rightarrow (x_1 + \lambda)f'(x_1) + f(x_1) < f(x_1) < f(y_1)$, a contradiction.

(ii) Assume $x_1 = x_2 = x_3$; then $\text{Var}\ \{P_t^S\} = \text{Var}\ \{P_t^W\}$, contradicting (9).

Combining these three cases, it follows that $\mu = 0$ doesn't provide a solution for the associated maximization problem, meaning that the profit maximizing speculation sequence must stabilize price. This completes the proof.

Footnotes For Chapter 1

1. Some other measures of price stability are summary statistics of price peaks and price troughs as provided by Baumol(1957) and market equilibrium stability provided by Kemp(1963). Although there are problems associated with using the variance as a measure of price stability, yet it seems to be widely accepted(see Telser (1959, p.296)).

2. Actually, Baumol provided two counterexamples. The other, as pointed out by Telser, is a case in which speculators' profits are unrealized. Consequently, Baumol withdrew this counterexample. Any valid counterexample to Friedman's conjecture must involve a complete speculation sequence, that is, one in which speculative profits are actually realized.

3. By "nontrivial," we mean that the non-speculative excess demand $Q_t = f(P_t, P_{t-1}, \ldots)$ has the property:

$$\frac{\partial f(P_t, P_{t-1}, \ldots)}{\partial P_t} \neq 0, \ \forall \ P_t.$$

4. There are two important points worth noting about Theorem 6 and 7:
(a) Though we assume $Q_t = f(P_t, P_{t-1})$ in two-period models, $Q_t = f(P_t, P_{t-1}, P_{t-2})$ in a three-period model, actually we can add more lags since they're irrelevant in the proofs.
(b) In this paper, we consider only time-independent lag-responsive speculative excess demand. Therefore, Theorems 6 and 7

apply only to this case.

5. Second-order conditions for this numerical example can be established by requiring f''(.) to satisfy some necessary conditions at P_t^s, t = 1, 2, 3.

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Chapter 2. Speculative Holdings Under Linear Expectation Processes: A Mean-Variance Approach

Introduction

Friedman's conjecture raises the general question as to whether speculators operating in an uncertain environment, acting in a manner consistent with a rational expectations equilibrium, will trade in such a way as to stabilize prices. The basic approach is as follows. We first assume the speculators have some price forecasting function and some objective functions. Based on their forecasting outcome, rational speculation sequences (that is, the speculation sequences maximizing the objective function) will be chosen, which in turn affect the realized market price in the next period. The rational expectations equilibrium then requires the equality between the forecasting outcome and the realized price. The main thrust of this approach then rests on the characterization of speculators' behavior under the expectation processes they use. Thereafter, the investigation of the Friedman conjecture is an immediate consequence.

In the literature, there have been several studies that attempt to characterize speculators' behavior under a linear expectation process (Kohn (1978), Rogerson (1979)). However, these papers assumed that speculators are expected profit-maximizers, regardless of the riskiness of their market operations.[1] In this paper, we assume that speculators employ a mean-variance approach[2], and then characterize their impacts on the market again, assuming a linear expectation process. Within this framework, and assuming a linear non-speculative excess demand function, Friedman's conjecture holds (i.e., profitable speculation necessarily stabilizes prices) from an ex ante point of view.

The plan of this paper is as follows: We first describe the market structure and the speculator's problem. Dynamic programming is then applied to solve the problem and some general properties of the solution are exploited. As expected, the results depend crucially on the specific expectation process assumed. To proceed further, linear expectation rules are introduced. Thereafter, we consider first the special case when inventory cost is a fixed constant. Then we take up the general quadratic inventory cost cases.

In modelling the rational expectations equilibrium, a non-speculative excess demand function is introduced. By taking into account rational speculation sequences, we can derive the probability distribution of the market price over time. Given a rational expectations equilibrium, we examine the Friedman conjecture. The summary of the main results then conclude the paper.

Market Structure

Consider a discrete time spot market, where the associated commodity is storable. There is no forward nor futures market in this commodity, and short-selling in the spot market is prohibited. The market opens at time $t = 0, 1, 2, \ldots$ and transactions take place immediately thereafter.

There are three different types of agents in this market: producers, speculators and consumers. Producers and consumers as a group are called non-speculators. The type of each agent is exogeneously determined. We also assume that the decisions of producers and consumers are made without considering the effects of speculators. Hence, we can treat non-speculative excess demand as exogeneously given. Random effects that enter the model come from either the production side or the non-speculative demand side but are assumed to be independent of speculators' behavior.

Each speculator takes prices as given (i.e., the case of competitive speculation), and he employs a mean-variance approach to solve his decision problem, using all information available to him. Let S_t denote the stock level at time t for a specified speculator (later, we'll assume all speculators are identical). Now, at time t, the speculator observes the market price P_t and his carry-over from the previous period S_{t-1}. He then constructs a probability density function to summarize his expectations about next period's market price P_{t+1} using all available information. From this p.d.f., he determines his stock level S_t. Any inventory holding cost $h(S_t)$ is assumed to be incurred at time t.

Let β be the discount factor employed by this speculator and let $\lambda/2$ be the weighting factor of market risk (variance) in his objective function. Then, the speculator entering the market at time t solves the following problem:

$$\text{(A)} \qquad \underset{\{S_i\}_{i=t}^{\infty}}{\text{Max}} \sum_{i=t}^{\infty} \beta^{i-t+1}\{E[P_{i+1}(S_i - S_{i+1}) - h(S_{i+1})]$$

$$- \frac{\lambda}{2} \text{ Var } [P_{i+1}(S_i - S_{i+1}) - h(S_{i+1})]\} + P_t(S_{t-1} - S_t) - h(S_t),$$

where the expectations are taken, conditional on his available information; hence, they are different operators at different points in time. Speculators are assumed to be risk-averse; hence $\lambda > 0$.

Before trying to solve (A), we make two further assumptions:[3]

(1) $\text{Var } (P_t | P_{t-1}) = \sigma^2, \; \forall \; P_t, P_{t-1}, \; t = 0, 1, 2, \ldots;$

(2) $h(S) = \frac{c(S-b)^2}{2} + d, \; \forall \; S \geq 0$. Assumption (2) incorporates a convenience-yield effect; i.e., at stock level b, we achieve minimum inventory cost. If there is no convenience-yield effect, then minimum inventory cost should be achieved at $S = 0$, which implies $b = 0$.[4] (See Rogerson (1979)).

Optimal Speculative Carry-over

To solve problem (A), assume that $\lim_{t \to \infty} S_t = b$ (equivalently, this says that in the limit, the speculator will choose a minimum-cost inventory stock level), and consider a decision beginning at $t = 0$. Under some regularity assumptions, we can utilize dynamic programming to solve the speculator's problem. Specifically, assume at time T that the speculator's problem is over and his stock decision is $S_t^* = b; \; \forall \; t \geq T$. Therefore, at time (T-1), his problem is:

(A1) $$\max_{S_{T-1}} \; \beta[E\{P_T(S_{T-1} - b)\} - \frac{\lambda}{2} \text{ Var } \{P_T(S_{T-1} - b)\}].$$

$$+ P_{T-1}(S_{T-2} - S_{T-1}) - \{\frac{c(S_{T-1} - b)^2}{2} + d\}$$

(Note that, at (T-1), P_{T-1} and S_{T-2} are both known.)

The first-order condition for (A1) is

$$\beta E_T - \beta\lambda\sigma^2(S^*_{T-1} - b) - P_{T-1} - c(S^*_{T-1} - b) = 0$$

$$\Rightarrow S^*_{T-1} = \frac{\beta E_T - P_{T-1} + bc}{\beta\lambda\sigma^2 + c} + \frac{b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c},$$

where $E_T = EP_T$, the conditional expectation of P_T using information about P_{T-1}, and S^*_t is the optimal choice of S_t, t = 0, 1, ...

Next, at time t = T - 2, his problem becomes

$$\text{(A2)} \quad \underset{S_{T-2}}{\text{Max}} \; \beta[E\{P_{T-1}(S_{T-2} - S^*_{T-1})\} - \frac{\lambda}{2}\text{Var}\{P_{T-1}(S_{T-2} - S^*_{T-1})\}]$$

$$+ P_{T-2}(S_{T-3} - S_{T-2}) - \{\frac{c(S_{T-2} - b)^2}{2} + d\} + K_0,$$

where K_0 is a constant term independent of S_{T-2}.

Since

$$\text{Var}\{P_{T-1}(S_{T-2} - S^*_{T-1})\} =$$

$$\text{Var}\{P_{T-1}(S_{T-2} - \frac{\beta E_T - P_{T-1} + bc + b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\} =$$

$$E\{S_{T-2}(P_{T-1} - E_{T-1}) - \frac{bc + b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}(P_{T-1} - E_{T-1}) -$$

$$(P_{T-1}\frac{\beta E_T - P_{T-1}}{\beta\lambda\sigma^2 + c} - E[P_{T-1}\frac{\beta E_T - P_{T-1}}{\beta\lambda\sigma^2 + c}])\}^2 =$$

$$\sigma^2 S^2_{T-2} - \frac{2(bc + b\beta\lambda\sigma^2)}{\beta\lambda\sigma^2 + c}\sigma^2 S_{T-2} - \frac{2S_{T-2}}{\beta\lambda\sigma^2 + c}.$$

$$\text{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1})) + K_1,$$

where K_1 is a constant term independent of S_{T-2} and $E_{T-1} = EP_{T-1}$; therefore, the first-order condition for (A2) is

$$\beta\{E_{T-1} - \lambda\sigma^2 S^*_{T-2} + \frac{\lambda(bc + b\beta\lambda\sigma^2)\sigma^2}{\beta\lambda\sigma^2 + c} +$$

$$\frac{\lambda}{\beta\lambda\sigma^2 + c} \text{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1}))\} - P_{T-2} -$$

$$c(S^*_{T-2} - b) = 0$$

$$\Rightarrow (\beta\lambda\sigma^2 + c)S^*_{T-2} = \beta E_{T-1} - P_{T-2} + bc + \frac{bc\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} +$$

$$\frac{b(\beta\lambda\sigma^2)^2}{\beta\lambda\sigma^2 + c} + \frac{\lambda\beta}{\beta\lambda\sigma^2 + c} \text{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1}))$$

$$\Rightarrow S^*_{T-2} = \frac{\beta E_{T-1} - P_{T-2}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\{1 + \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\} +$$

$$b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^2 + \frac{\lambda\beta}{(\beta\lambda\sigma^2 + c)^2}\text{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1})).$$

In general, define $f_0(P_t) = \text{Cov}(P_t, P_t(\beta E_{t+1} - P_t))$, $\forall$ t and $f_k(P_t) = \text{Cov}(P_t, P_t f_{k-1}(P_{t+1}))$, $\forall$ k = 1, 2, . . ., $\forall$ t. Then we can state the following theorem:

<u>Theorem 1</u>

The optimal speculative stock level that solves (A) under the assumption that T is the terminal date is:

$$S^*_t = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^i + b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t}$$
$$+ \frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-2}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i f_i(P_{t+1}), \quad (1)$$

$\forall$ t = 0, 1, 2, . . ., T-1.

[Proof]

The cases t = T-1, T-2 can be checked easily. Now, assume at time t, S^*_t satisfies Eq. (1). Then, at time (t-1), the speculator's problem is

$$\text{(A3)} \quad \max_{S_{t-1}} \; \beta[E\{P_t(S_{t-1} - S_t^*)\} - \frac{\lambda}{2} Var\{P_t(S_{t-1} - S_t^*)\}] +$$

$$P_{t-1}(S_{t-2} - S_{t-1}) - \{\frac{c(S_{t-1} - b)^2}{2} + d\} + K_2,$$

where K_2 is a constant term independent of S_{t-1}. From Eq. (1), we have

$$Var\{P_t(S_{t-1} - S_t^*)\}$$

$$= E\{(P_t - E_t)S_{t-1} - \frac{bc}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} (\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^i (P_t - E_t) -$$

$$b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t}(P_t - E_t) - \frac{1}{\beta\lambda\sigma^2 + c}(P_t(\beta E_{t+1} - P_t) -$$

$$E[P_t(\beta E_{t+1} - P_t)]) - \frac{\beta\lambda}{\beta\lambda\sigma^2 + c)^2} \sum_{\lambda=0}^{T-t-2} (\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i (P_t f_i(P_{t+1}) -$$

$$E[P_t f_i(P_{t+1})])\}^2$$

$$= \sigma^2 S_{t-1}^2 - \frac{2bc\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=1}^{T-t-1} \phi^i S_{t-1} - 2b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t}\sigma^2 -$$

$$\frac{2}{\beta\lambda\sigma^2 + c} Cov(P_t, P_t(\beta E_{t+1} - P_t)) -$$

$$\frac{2\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} (\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i \; Cov(P_t, P_t f_i(P_{t+1}))$$

$$= \sigma^2 S_{t-1}^2 - \frac{2bc\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} \phi^i S_{t-1} - 2b\phi^{T-t}\sigma^2 - \frac{2}{\beta\lambda\sigma^2 + c} f_0(P_t) -$$

$$\frac{2\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} (\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i f_{i+1}(P_t),$$

where $\phi = \beta\lambda\sigma^2/(\beta\lambda\sigma^2 + c)$. Therefore, the first-order condition for (A3) is:

$$\beta\{E_t - \lambda\sigma^2 S_{t-1}^* + \frac{bc\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} \phi^i + b\lambda\sigma^2\phi^{T-t} + \frac{\lambda}{\beta\lambda\sigma^2 + c} f_0(P_t) +$$

$$\frac{\beta\lambda^2}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} (\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i f_{i+1}(P_t)\} - P_{t-1} - c(S_{t-1}^* - b) = 0$$

$$\Rightarrow S^*_{t-1} = \frac{\beta E_t - P_{t-1}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c} + bc\frac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-1}\phi^i + b\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\phi^{T-t} +$$

$$\frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\{f_0(P_t) + \sum_{i=0}^{T-t-2}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^{i+1}f_{i+1}(P_t)\}$$

$$\Rightarrow S^*_{t-1} = \frac{\beta E_t - P_{t-1}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{k=0}^{T-t}\phi^k +$$

$$b\phi^{T-t+1} + \frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\sum_{k=0}^{T-t-1}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^k f_k(P_t),$$

by letting $K = i + 1$. Therefore, the proof is completed.

Q.E.D.

Eq. (1) expresses the optimal speculative stock level as the summation of four terms: (1) the current expected profit effect $\frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c}$; (2) the terminal convenience yield effect $b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t}$; (3) the cost-factor-and-convenience-yield interaction effect

$$\frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^i \quad ;$$

and (4) the covariance risk effect

$$\frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-2}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i f_i(P_{t+1}).$$

Among these four effects, (2) vanishes as T approaches infinity, while (3) and (4) are special features rising because the mean-variance approach is used to describe the speculator's preferences.(That is, we consider a risk averse speculator instead of a risk neutral one.) These will be discussed further in the following section.

Properties of the Optimal Stock Level

First, note that in the derivations leading to Eq. (1), we implicitly assumed $S_t^* \geq 0$. However, since short selling is not allowed, the optimal speculative stock level should be written as $\hat{S}_t = \max(S_t^*, 0)$ for every $t > 0$; and if we have t', such that $S_{t'}^* < 0$, then all formulas for S_t^*, $t \leq t'$ are now invalid. This introduces a complex discontinuity into the problem. In the general case, we will simply assume $S_t^* \geq 0$. [There are some special cases, however, in which $S_t^* \geq 0$ can be proved (i.e., the case where $c \equiv 0$).]

Second, assume $\lambda = 0$ and let T approach infinity. Then Eq. (1) reduces to the case considered in Rogerson (1979); i.e., all competitive speculators are expected-profit maximizing agents. Hence, Eq. (1) becomes $S_t^* = \frac{\beta E_{t+1} - P_t}{c} + b$, since the terminal convenience yield effect (2) and covariance risk effect (4) both vanish when $\lambda = 0$, $T \rightarrow \infty$, and the interaction effect becomes b. There is one period time-lag difference between our model and that used in Rogerson as to when the inventory cost occurs. Adjusting for this, we obtain the optimal stock level derived in Rogerson (1979), which is therefore a special case of our model.

Third, note that Eq. (1) holds when T is the terminal date. But to solve (A), we must let T approach infinity, creating convergence problems. Note that if $c > 0$, then $0 < \delta = \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1$, and convergence problems arise only from the covariance risk effect. However, if $c = 0$ (i.e., there are no variable inventory costs), then all the terms except (2) require further consideration.

The last points we want to make are about the interaction effect and the covariance risk effect. Each of these is a discounted sum of a sequence but uses apparently different discount rates, $\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}$ and $\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}$ respectively. This is somewhat misleading. When we introduce $f_K(\cdot)$ into Eq. (1), it turns out that both expressions involve the same discount rate $\phi = \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}$. If we let b = 0 or c = 0, then the interaction effect vanishes (but the covariance risk effect remains). As for $f_K(\cdot)$, these functions all take the covariance operator form. For example, $f_0(P_t) = \text{Cov}(P_t, P_t(\beta E_{t+1} - P_t))$ is the covariance between price and expected profit for the next period;

$$f_K(P_t) = \text{Cov}(P_t, P_t f_{K-1}(P_{t+1}))$$
$$= \text{Cov}(P_t, P_t \text{Cov}(P_{t+1}, P_{t+1} f_{K-2}(P_{t+2})) = \ldots$$
$$= \text{Cov}(P_t, P_t \text{Cov}(P_{t+1}, P_{t+1} \text{Cov}(P_{t+2}, \ldots,$$
$$\text{Cov}(P_{t+K}, P_{t+K}(\beta E_{t+K+1} - P_{t+K}))$$

measures the covariance between price and expected profit K periods later (by updating information at each subsequential future period). Therefore, we named (4) as the covariance risk effect. Note that this effect comes across time, rather than across alternatives at a point in time (leading to a covariance risk effect in the Capital Asset Pricing Models).

<u>Linear Expectation Rule</u>

Now, assume every speculator is identical with price expectation formation equation given by:

$$P_t^e = \delta + \alpha P_{t-1} + \varepsilon_t, \quad \forall\ t, \tag{2}$$

where α is the price expectation adjustment coefficient; $\delta/(1 - \alpha)$ is the long-run rational expectations equilibrium price; $\{\varepsilon_t\}$ is a sequence of identically independently distributed random variables with $E(\varepsilon_t | P_{t-1}) = 0$; $Var(\varepsilon_t | P_{t-1}) = \sigma^2$ and $E(\varepsilon_t^3 | P_{t-1}) = 0$ (i.e., the probability density function of ε_t is symmetric with respect to zero), $\forall$ t. When $\alpha > 1$, we say the speculator is responsive; when $\alpha < 1$, we say he is unresponsive.

Using (2), we can determine $f_K(P_t)$ for every $K \geq 0$. For example,

$$\begin{aligned} f_0(P_t) &= Cov(P_t, P_t(\beta E_{t+1} - P_t)) = Cov(P_t, P_t(\beta\delta + (\beta\alpha - 1)P_t)) \\ &= \beta\delta\sigma^2 + (\beta\alpha - 1)\ Cov(P_t, P_t^2) \\ &= \beta\delta\sigma^2 + (\beta\alpha - 1)E\{\varepsilon_t \cdot [2(\delta + \alpha P_{t-1})\varepsilon_t + \varepsilon_t^2 - \sigma^2]\} \\ &= \{\beta\delta + 2(\beta\alpha - 1)(\delta + \alpha P_{t-1})\}\sigma^2; \end{aligned}$$

also,

$$\begin{aligned} f_1(P_t) &= Cov(P_t, P_t f_0(P_{t+1})) = Cov(P_t, P_t(\beta\delta + 2(\beta\alpha - 1)(\delta + \alpha P_t))\sigma^2) \\ &= \sigma^4\{\beta\delta + 2(\beta\alpha - 1)\delta\} + 2\sigma^2(\beta\alpha - 1)\alpha\ Cov(P_t, P_t^2) \\ &= \{\beta\delta + 2(\beta\alpha - 1)\delta + 2\alpha(\beta\alpha - 1)(\delta + \alpha P_{t-1})\}\sigma^4. \end{aligned}$$

In general, we can prove the following theorem:

Theorem 2

Under the linear expectation rule (Eq. (2)),

$$f_k(P_t) = \{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{K+1}\alpha^{j-1} + 2\alpha^{K+1}(\beta\alpha - 1)P_{t-1}\}\sigma^{2K+2}, \quad \forall K, \ \forall P_t. \tag{3}$$

[Proof]

The cases where K = 0 and K = 1 can be easily checked. Now, assume for K = i, $f_k(P_t)$ satisfies Eq. (3); hence,

$$\begin{aligned} f_{i+1}(P_t) &= \mathrm{Cov}(P_t, P_t f_i(P_{t+1})) \\ &= \mathrm{Cov}(P_t, P_t(\beta\delta + \delta\sum_{j=1}^{i+1} 2(\beta\alpha - 1)\alpha^{j-1} + 2\alpha^{i+1}(\beta\alpha - 1)P_t)\sigma^{2i+2}) \\ &= \{\beta\delta + \delta\sum_{j=1}^{i+1} 2(\beta\alpha - 1)\alpha^{j-1}\}\sigma^{2i+4} + 2\alpha^{i+1}(\beta\alpha - 1)\sigma^{2i+2}\mathrm{Cov}(P_t, P_t^2) \\ &= \{\beta\delta + \delta\sum_{j=1}^{i+1} 2(\beta\alpha - 1)\alpha^{j-1}\}\sigma^{2i+4} + 2(\beta\alpha - 1)\alpha^{i+1}\sigma^{2i+4}(\delta + \alpha P_{t-1}) \\ &= \{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{i+2}\alpha^{j-1} + 2(\beta\alpha - 1)\alpha^{i+2}P_{t-1}\}\sigma^{2i+4}, \end{aligned}$$

which completes the proof.

Q.E.D.

Substituting Eq. (3) into Eq. (1), we have

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^i + b\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^{T-t} +$$

$$\frac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-2}\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^i.$$

$$\{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{i+1}\alpha^{j-1} + 2\alpha^{i+1}(\beta\alpha - 1)P_t\} \tag{4}$$

Next we will consider the problem of the condition under which S_t^* will converge as $T \to \infty$.

<u>Zero Variable Inventory Cost</u>

When c = 0, Eq. (4) becomes

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \sum_{i=0}^{T-t-2}\{\frac{\beta\delta}{\beta\lambda\sigma^2} + \frac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2}\sum_{j=1}^{i+1}\alpha^{j-1} + \frac{2\alpha^{i+1}(\beta\alpha -)P_t}{\beta\lambda\sigma^2}\}$$

$$= \begin{cases} \dfrac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \displaystyle\sum_{i=0}^{T-t-2}\{\frac{\beta\delta}{\beta\lambda\sigma^2} + \frac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2}\cdot\frac{1 - \alpha^{i+1}}{1 - \alpha} + \frac{2\alpha^{i+1}(\beta\alpha - 1)P_t}{\beta\lambda\sigma^2}\}, & \text{if } \alpha \neq 1. \\ \dfrac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \displaystyle\sum_{i=0}^{T-t-2}\{\frac{\beta\delta}{\beta\lambda\sigma^2} + \frac{2\delta(\beta - 1)(i + 1)}{\beta\lambda\sigma^2} + \frac{2(\beta - 1)P_t}{\beta\lambda\sigma^2}\}, & \text{if } \alpha = 1 \end{cases} \tag{4'}$$

<u>Theorem 3</u>

Given c = 0, assume $\delta \neq 0$. Then, if $T \to \infty$, S_t^* is unbounded for every t.

[Proof]

Obviously, when $\alpha = 1$, $\delta \neq 0$, $T \to \infty$, then $S_t^* \to -\infty$. On the other hand, if $\alpha \neq 1$, then for S_t^* to be bounded, we must require that: (1) $\alpha < 1$ and (2) $\lim_{i \to \infty} \frac{\beta\delta}{\beta\lambda\sigma^2} + \frac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2} \cdot \frac{1 - \alpha^{i+1}}{1 - \alpha} = 0$. Now, (2) implies $\beta(1 - \alpha) + 2(\beta\alpha - 1) = 0 \Rightarrow \beta(\alpha + 1) = 2$, contradicting $\alpha, \beta < 1$. Hence, S_t^* is unbounded when $\delta \neq 0$, $T \to \infty$.

Q.E.D.

Although S_t^* is unbounded from below, yet since short selling is prohibited, S_t must be non-negative. Hence, the optimal stock $\hat{S}_t = \max(S_t^*, 0)$ is either ∞ or 0, when $\delta \neq 0$ and $T \to \infty$. This implies

Corollary 1

Given $c = 0$, $0 \leq \hat{S}_t < \infty$, $\forall\, t$ and $\hat{S}_t > 0$ for some t implies one of the following conditions:

(i) $\delta = 0$, $\alpha < 1$

(ii) T is finite.

Corollary 2

Given $c = 0$, $\delta = 0$, $\alpha < 1$, and $T \to \infty$ implies $\hat{S}_t = 0$, $\forall\, t$.

Corollaries 1 and 2 show that, with zero variable inventory holding costs, when $T \to \infty$, and short selling is prohibited, then the speculator accumulates either unbounded stocks or no stocks at all;, i.e., speculators are either highly active or totally inactive. For example, when $\delta = 0$, $\alpha < 1$, they always hold zero stock. In the other cases, when $\delta \neq 0$, they might switch from an unbounded stock to zero at some points and then remain for few periods; finally, they switch from zero to an unbounded level of stocks. This implies that they are highly active.

Theorem 4

Given $c = 0$, when $T < \infty$, any time-independent linear expectation of speculators won't be fulfilled.

The proof of Theorem 4 involves the structure of non-speculative excess demand; therefore, we put it into the Appendix after the introduction of a market demand structure. Nonetheless, the reason we state Theorem 4 here is to claim that "$T < \infty$" is also not a useful assumption to avoid the "unboundedness" problems that arise when $c = 0$. In the following sections, $c \neq 0$ is assumed.

Nonzero Variable Inventory Cost

When $c \neq 0$, Eq. (4) can be written as:

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + M_{1t} + M_{2t} + M_{3t} + M_{4t} + M_{5t},$$

where

$$M_{1t} = \frac{bc}{\beta\lambda\sigma^2 + c} \cdot \frac{1 - \phi^{T-t}}{1 - \phi},$$

$$M_{2t} = b\phi^{T-t}$$

$$M_{3t} = \frac{\beta^2\delta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \cdot \frac{1 - \phi^{T-t-1}}{1 - \phi}.$$

$$M_{4t} = \begin{cases} \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2} \displaystyle\sum_{i=0}^{T-t-2} \phi^i \cdot \frac{1 - \alpha^i}{1 - \alpha}, \text{ where } \alpha \neq 1 \\[2ex] \dfrac{2\delta(\beta - 1)\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2} \displaystyle\sum_{i=0}^{T-t-2} \phi^i (i + 1), \text{ when } \alpha = 1 \end{cases}$$

$$M_{5t} = \frac{2\beta\lambda\sigma^2(\beta\alpha - 1)P_t}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \alpha^{i+1}\phi^i$$

and $$\phi = \frac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)}$$

Hence, as $T \to \infty$, $M_{1t} \to \dfrac{bc}{\beta\lambda\sigma^2 + c} \cdot \dfrac{1}{1 - \phi} = b$; $M_{2t} \to 0$;

$$M_{3t} \to \frac{\beta^2\delta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \cdot \frac{1}{1 - \phi} = \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)}$$

and

$$M_{5t} \to \begin{cases} \dfrac{2\alpha\beta\lambda\sigma^2(\beta\alpha - 1)P_t}{(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]}, \text{ when } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1 \\[2ex] \infty, \text{ when } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \geq 1 \end{cases}$$

$$M_{4t} \to \begin{cases} \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)} - \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{(1 - \alpha)(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]}, \\ \qquad \text{when } \alpha \neq 1 \text{ and } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1 \\[2ex] \pm\infty \text{ when } \alpha \neq 1 \text{ and } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \geq 1 \\[2ex] \dfrac{2\delta(\beta - 1)\beta\lambda\sigma^2}{c^2} \text{ when } \alpha = 1. \end{cases}$$

Note that when $\alpha = 1$, $\dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1$ is satisfied. Therefore, we have the following theorem:

Theorem 5

Assume $T \to \infty$. If $\alpha\beta\lambda\sigma^2 \geq \beta\lambda\sigma^2 + c$, then S_t^* is unbounded. Furthermore, whether $S_t^* = \infty$ or $-\infty$ depends on M_{4t} and M_{5t}. On the other hand, if $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, then as $T \to \infty$, S_t^* will converge to

$$\bar{S}_t = M + \frac{(\beta\alpha - 1)P_t}{\beta\lambda\sigma^2 + c}\left[\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\right]$$

where

$$M = \frac{\beta\delta}{\beta\lambda\sigma^2 + c} + \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)} + b + \frac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)} - \frac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]},$$

when $\alpha \neq 1$;

and

$$M = \frac{\beta\delta}{\beta\lambda\sigma^2 + c} + \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)} + b + \frac{2\delta(\beta - 1)\beta\lambda\sigma^2}{c^2},$$

when $\alpha = 1$.

Corollary 3

Given $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, $\bar{S}_t > 0$ implies

(i) $\partial\bar{S}_t/\partial b = 1, \ \forall\, t.$

(ii) $\mathrm{sgn}(\partial\bar{S}_t/\partial P_t) = \mathrm{sgn}(\beta\alpha - 1), \ \forall\, t.$

[Proof]

(i) is obvious. For (ii), if $\alpha < 1$, then $\text{sgn}(\partial\bar{S}_t/\partial P_t) = \text{sgn}(\beta\alpha - 1) < 0$, since $1 - \alpha > 0$ and $\beta, \alpha, \sigma^2, c > 0$. If $\alpha > 1$, then since $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c \Rightarrow c + (1 - \alpha)\beta\lambda\sigma^2 > 0$, hence $\text{sgn}(\partial\bar{S}_t/\partial P_t) = \text{sgn}(\beta\alpha - 1)$, $\forall$ t.

Q.E.D.

Theorem 5 shows that $\{\bar{S}_t\}$ is the solution for problem (A) when $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$ and $c > 0$ (note that, by hypothesis, $\bar{S}_t \geq 0$, $\forall$ t); therefore, the optimal speculative stock level is fully characterized. Otherwise, we always have $\bar{S}_t = \infty$ or 0. Corollary 3 shows that, as the minimum-cost stock level b changes by one unit, the optimal stock level $\bar{S}_t$ also changes by one unit in the same direction for every t. Furthermore, when the current price P_t changes, which direction $\bar{S}_t$ will change to is determined by the sign of $(\beta\alpha - 1)$.

Market Price Behavior

Now, since we take the behavior of non-speculators as given, we can summarize their impacts on the market by a non- speculative excess demand function. Following the literature, we postulate a linear non-speculative excess demand function of the form:

$$D_t = -aP_t + \gamma_t, \quad a > 0, \tag{5}$$

where $\{\gamma_t\}$ is a sequence of identically independently distributed random variables with $E(\gamma_t) = \mu$, $\text{Var}(\gamma_t) = V$.

By the market clearing condition,[5] we have

$$\bar{S}_{t-1} - \bar{S}_t = -aP_t + \gamma_t, \ \forall\ t$$

$$\Rightarrow zP_{t-1} - zP_t = -aP_t + \gamma_t, \ \forall\ t$$

$$\Rightarrow P_t = \frac{z}{z - a}P_{t-1} - \frac{\gamma_t}{z - a}, \ \forall\ t \tag{6}$$

$$\Rightarrow P_t = (\frac{z}{z - a})^t P_0 - \sum_{j=0}^{t-1} (\frac{z}{z - a})^j \frac{\gamma_{t-j}}{z - a}, \ \forall\ t = 0, 1, 2, \ldots, \tag{7}$$

where $z = \frac{(\beta\alpha - 1)}{\beta\lambda\sigma^2 + c}\{\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\}$. Note that this result is derived when $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$ and $0 \leq \bar{S}_t < \infty, \ \forall\ t$.

From (7), we have

$$EP_t = (\frac{z}{z - a})^t P_0 - \sum_{j=0}^{t-1} (\frac{z}{z - a})^j \frac{\mu}{z - a}$$

$$= (\frac{z}{z - a})^t P_0 - \frac{\mu}{z - a} \cdot \frac{1 - (\frac{z}{z - a})^t}{1 - \frac{z}{z - a}}$$

$$= w^t P_0 + \frac{\mu}{a}(1 - w^t), \text{ where } w = \frac{z}{z - a};$$

$$\text{Var } P_t = \text{Var}\{(\frac{z}{z - a})^t P_0 - \sum_{j=0}^{t-1} (\frac{z}{z - a})^j \frac{\gamma_{t-j}}{z - a}\}$$

$$= \frac{V}{(z - a)^2} \sum_{j=0}^{t-1} (\frac{z}{z - a})^{2j} = \frac{(1 - w)^2 V}{a^2} \cdot \frac{1 - w^{2t}}{1 - w^2}.$$

Since $w = \frac{z}{z - a} \Rightarrow za - aw = z \Rightarrow z = \frac{aw}{w - 1} \Rightarrow z - a = \frac{a}{w - 1}$,

$$\text{Cov}(P_t, P_{t-h}) = \text{Cov}(-\sum_{j=0}^{t-1} w^j \frac{\gamma_{t-j}}{z - a}, -\sum_{k=0}^{t-h-1} w^k \frac{\gamma_{t-h-k}}{z - a})$$

$$= \sum_{k=0}^{t-h-1} w^{2k+h} \cdot \frac{V}{(z - a)^2} = \frac{(1 - w)^2 V}{a^2} \cdot w^h \cdot \frac{1 - w^{2t-2h}}{1 - w^2},$$

when $0 \leq h < t$.

This proves:

<u>Theorem 6</u>

Given $c \neq 0$, $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, $0 \leq \bar{S}_t < \infty \ \forall t$,

$$\text{(i)} \quad \lim_{t \to \infty} EP_t = \begin{cases} \frac{\mu}{a}, & \text{if } |w| < 1 \\ \pm\infty, & \text{if } |w| > 1 \end{cases}$$

$$\text{(ii)} \quad \lim_{t \to \infty} \text{Var}P_t = \begin{cases} \frac{(1 - w)V}{a^2(1 + w)}, & \text{if } |w| < 1 \\ \infty, & \text{if } |w| > 1 \end{cases}$$

$$\text{(iii)} \quad \lim_{t \to \infty} \text{Cov}(P_t, P_{t-h}) = \begin{cases} \frac{w^h(1 - w)V}{a^2(1 + w)}, & \text{if } |w| < 1 \\ \pm\infty, & \text{if } |w| > 1. \end{cases}$$

Since we want price to be non-negative, we make the following assumptions: (i) $w > 0$ and (ii) $P_0 > \frac{\mu}{a}$. Therefore, for $|w| < 1$, we need $0 < \frac{z}{z - a} < 1 \Rightarrow z < 0 \Rightarrow \beta\alpha < 1$, since $(1 - \alpha)\beta\lambda\sigma^2 + c > 0$. Now, if $c > (1 - \beta)\lambda\sigma^2$, then $\beta\lambda\sigma^2 + c > \lambda\sigma^2 \Rightarrow 1 + \frac{c}{\beta\lambda\sigma^2} > \frac{1}{\beta}$; hence, $\alpha < \frac{\beta\lambda\sigma^2 + c}{\beta\lambda\sigma^2} \not\Rightarrow \alpha < \frac{1}{\beta}$, which establishes the following:

<u>Theorem 7</u>

Assume $\alpha > 1$. If $c > (1 - \beta)\lambda\sigma^2$, then S_t^* bounded and $\lim_{t \to \infty} EP_t$, $\lim_{t \to \infty} \text{Var}P_t$ unbounded do not violate market clearing. Under this configuration, the action of competitive speculators will destabilize prices.

As to whether the speculator's expectations will be fulfilled, we can compare Eq. (6) and Eq. (2) to derive the following theorem:

Theorem 8

Fulfilling of speculator's expectation implies:

(i) $\frac{z}{z - a} = \alpha$ and (ii) $\frac{\gamma_t}{a - z} = \delta + \varepsilon_t$, where

$$z = \frac{\beta\alpha - 1}{\beta\lambda\sigma^2 + c}\{\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\},$$

and (ii) holds for every $t = 1, 2, \ldots.$

Corollary 4

If speculators' expectations are fulfilled, then

(i) $\alpha < 1$, (ii) $\mu = \delta(a - z)$, (iii) $V^2 = (a - z)^2\sigma^2$.

[Proof]

Assume $\alpha = 1$; then, fulfilling expectation implied $\frac{z}{z-a} = 1$ $\Rightarrow$ $z = z - a$ $\Rightarrow$ $a = 0$, a contradiction. On the other hand, if $\alpha > 1$, then $\frac{z}{(z - a)} = \alpha$ $\Rightarrow$ $z > a > 0$ and $\frac{\partial \bar{S}_t}{\partial P_t} = z > a$. Therefore, as $P_t \rightarrow \infty$, $\bar{S}_t \rightarrow \infty$ which is unbounded. Since we dealt only with bounded $\bar{S}_t$, hence $\alpha < 1$ is required. (ii) and (iii) are derived from $E(\frac{\gamma_t}{a - z}) = E(\delta + \varepsilon_t)$ and $Var(\frac{\gamma_t}{a - z}) = Var(\delta + \varepsilon_t)$, respectively (where expectations are conditional on available information).

Q.E.D.

Therefore, when speculators' expectations are fulfilled, EP_t, $VarP_t$ and $Cov(P_t, P_{t-h})$ are all bounded. Also, S_t^* is bounded.

Profitable Speculation

In this section, we turn to Friedman's conjecture; i.e., profitable speculation necessarily stabilizes prices. Recall that, in problem (A), $S_t = b, \ \forall \ t$ is a feasible strategy; therefore, any strategy $\{S_t\}$ with $S_t \neq b$, for some t certainly incurs positive profits (actually, the profits must be high enough to cover the losses in expected utility due to nonzero variance). From Theorem 5, this implies $M + zP_t \neq b$ for some t which is easily satisfied.

Now, from Theorem 7, when $\alpha > 1$, there would be destabilizing profitable speculation. However, in this case, the speculators' expectations won't be fulfilled. On the other hand, when their expectations are fulfilled, $\alpha < 1$ and

$$VarP_t = \frac{(1 - w)V}{a^2(1 + w)} \cdot (1 - w^{2t}) < \frac{V}{a^2},$$

(since $\alpha < 1 \Rightarrow z < 0 \Rightarrow 0 < w = \frac{z}{z - a} < 1$), where $\frac{V}{a^2} = VarP_t$ when there are no speculators. Therefore,

Theorem 9

At a rational expectations equilibrium (i.e., speculators' expectations are fulfilled), and given a linear non-speculative excess demand, profitable speculation always stabilizes prices.

Theorem 9 leaves it open whether, at a rational expectations equilibrium with non-linear non-speculative excess demand, profitable speculation always stabilizes prices. Because of earlier results (see Farrell (1966), Lien (1984), Schimmler (1973)), it seems unlikely that Friedman's conjecture will hold with non-linear excess demands, however.

Conclusion

In this paper, speculators are taken to be risk-averse, and a mean-variance approach is employed. Under this approach, the optimal stock level for speculators has been derived. Nonetheless, this stock level might be unbounded. To carry the analysis further, we found that, when the marginal inventory cost is zero, speculators are either highly active ($S_t = \infty$) or inactive ($S_t = 0$). To resolve the problem of unboundedness of S_t when $c = 0$ requires either the assumption that the long-run equilibrium price equals zero (which leads to $S_t = 0$, $\forall$ t) or the assumption of a finite horizon (in which case speculators' expectations won't be fulfilled[6]).

On the other hand, when the inventory carrying cost function is of a non-degenerate quadratic form, one possible equilibrium configuration involves bounded stock levels and unbounded prices, with the expectation adjustment coefficient greater than 1. However, this does not constitute a rational expectations equilibrium.[7] When a rational expectations equilibrium exists, given linear non-speculative excess demand, the stock level is bounded, price is also bounded, and

Friedman's conjecture is verified; *i.e.*, profitable speculation necessarily stabilizes prices.

Appendix To Chapter 2

Proof of Theorem 4

By inspecting Eq. (2) and Eq. (4′), we know that unless $\beta\alpha - 1 = 0$, the speculators' expectations won't be fulfilled, since, in (4′), price terms involve multiplicative time factors when $\beta\alpha \neq 1$. On the other hand, when $\beta\alpha - 1 = 0$, then

$$S_t^* = \frac{\beta\delta}{\beta\lambda\sigma^2} + b + \frac{\beta\delta}{\beta\lambda\sigma^2}(T - t - 1) = \frac{\beta\delta}{\beta\lambda\sigma^2}(T - t) + b \geq 0, \quad \forall\ t \leq T$$

$$\Rightarrow S_{t-1}^* - S_t^* = \frac{\delta}{\lambda\sigma^2}, \quad \forall\ t \leq T.$$

Therefore, the market clearing condition becomes

$$- aP_t + \gamma_t = \frac{\delta}{\lambda\sigma^2}, \quad \forall\ t \leq T$$

$$\Rightarrow P_t = \frac{\gamma_t}{a} - \frac{\delta}{a\lambda\sigma^2}, \quad \forall\ t \leq T.$$

Now, for expectations to be fulfilled, we need $\alpha = 0$, which contradicts $\beta\alpha - 1 = 0$. Hence, the proof is completed.

Footnotes For Chapter 2

1. On the other hand, both Sarris (1984) and Turnovsky (1983) employed mean-variance approach to determine one-period optimal stock level, without taking account of the dynamic effects.

2. When dealing with risk neutral speculators, it is only the expectation of market price that matters. Consequently, the rational expectations equilibrium in this case requires only the equality with respect to the first moment of forecasted and realized prices. On the other hand, if risk-averse speculators are considered, the equality of second moments is also required.

3. The assumption $Var(P_t | P_{t-1}) = \sigma^2$, $\forall$ t can be relaxed to $Var(P_t | P_{t-1}) = \sigma_t^2$, which is a constant term independent of P_t, P_{t-1}, but might change over time. Under this assumption, the results can be easily adjusted to characterize the optimal stock level. Nonetheless, the market price process will be highly complex and difficult to analyze.

4. If the inventory cost function were chosen to be linear rather than quadratic, then, when b = 0 (i.e., no convenience yield), the inventory cost curve is a straight line over $[0,\infty)$. However, if $b \neq 0$, then we have to introduce a kinked point in the inventory cost curve.

5. Strictly speaking, the left-hand side of the market clearing

equation should be multiplied by the number of representative speculators, but, without loss of generality, we set this factor to be one.

6. In the one-period framework, considering all the agents in the market, Turnovsky showed that constant marginal inventory cost may lead to the nonexistence of rational expectations equilibrium in futures market as well.

7. Without considering the "unboundedness" problem of the optimal stock level, when $\alpha > 1$ and the speculator's expectation is fulfilled, the optimal stock level will increase monotonically over time. Therefore, there will never be realized profits.

References For Chapter 2

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Chapter 3. Asymmetric Arbitrage and the Pattern of Futures Prices under Rational Expectations

Introduction

It is now over sixty years since Keynes (1930) first argued that the "normal" state of affairs on futures markets was one of backwardation, here interpreted as a situation in which the current price of a futures contract is less than its expected price at maturity of the futures contract. Keynes argued that short hedgers (long in the cash market, short in the futures market) would pay a risk premium to speculators, this premium representing the degree of backwardation in the market. Keynes did not explain why it was that only short hedgers, and not both short and long hedgers, would have to pay such a premium. (Long hedgers are long in the futures market and short in the cash market.) Later, Hicks (1965) argued that a preponderance of short over long hedgers was to be expected because purchasers of inputs have more possibilities of substitution available to them than do the producers of a commodity. Kaldor (1939) admitted the possibility of an excess of long over short hedging on the market, in part because of the quantity risks that a producer exposes himself to, if he engages in a hedge to avoid price risks. In the more recent literature, backwardation and the preponderance of short over long hedging has been attributed to information asymmetries (Danthine (1978)), to a highly elastic demand for the final good (Macminn, Morgan and Smith (1984)) or to the fact that futures contracts provide poor consumption hedges (Richard and Sundaresan (1981)), and sometimes backwardation is simply imposed ad hoc as a condition of the model describing the futures market (Baesel and Grant (1982)).

In this paper, we explore an explanation for backwardation advanced by Houthakker (1959), namely, the idea that arbitrage on the futures market is asymmetric in such a way as to favor short hedgers over long hedgers. The idea here is that, at any point in time, the futures price cannot exceed the cash price plus carrying costs to the maturity date of the futures contract, since otherwise there is a riskless profit to be earned by selling a futures contract, buying cash and storing to deliver on the futures. Arbitrage thus provides an upper limit on the amount by which the futures price can exceed the cash price, but there is no corresponding arbitrage operation available to limit the amount by which the cash price can exceed the futures price.

Actually, Houthakker suggested two explanations for backwardation in his seminal work of the 1950s and 1960s, the second being the tendency for the delivery alternatives admissible under a futures contract to be better substitutes for one another at low rather than at high cash prices. In a recent paper, Fort and Quirk (1984) show that under an appropriate specification of such a "Houthakker effect," a backwardation equilibrium can be constructed, even when there is an equal number of short and long hedgers on the market, with identical utility functions and densities over cash and futures prices.

The existence of a "Houthakker effect," however, is closely tied to seasonality in production patterns and the existence of multiple delivery alternatives under futures contracts. These features are present in agricultural futures contracts but are not as common in financial futures or in futures covering metals and other industrial commodities. For this reason among others, it is important to determine whether the presence of asymmetric arbitrage in and of itself is sufficient to generate a pattern of backwardation on a futures market.

Briefly, our results in the present paper are the following. In a world with an equal number of short and long hedgers, with identical utility functions and densities over cash and futures prices, asymmetric arbitrage has no effect on the pattern of cash and futures prices when the futures market is in fact a forward market, that is, a market in which the cash and futures prices are identically equal at maturity of the futures contract. In such a world, under rational expectations, the resulting equilibrium is a martingale equilibrium in the futures market (current price of the futures contract equals its expected price next period), with the current futures price equal to the current cash price plus carrying costs to maturity of the futures contract.

The situation is different in a true futures market, that is, a market in which there are two or more delivery alternatives admissible under the futures contract, these being less than perfect substitutes for one another.

In a true futures market, the effect of asymmetric arbitrage on a previous martingale equilibrium is indeterminate in the general case; it might be to produce a backwardation equilibrium, or a contango, or no change at all. Given an arbitrary symmetric joint density over the cash and futures prices and given an arbitrary concave utility function for traders, the introduction of asymmetric arbitrage does not even necessarily encourage short hedging and discourage long hedging, despite the intuitive appeal of Houthakker's argument. But even when, under highly specialized conditions, it can be shown that the Houthakker conjecture holds in the sense that short hedging is encouraged and long hedging is discouraged by asymmetric arbitrage, additional restrictions need to be imposed to guarantee a backwardation equilibrium. Moreover, imposing a rational expectations framework on the model of the futures market implies that, given a T-period futures contract, the effects of asymmetric arbitrage show up only in the futures markets for periods T - 1 and T, while, in earlier periods, the futures market behaves like a forward market. In effect, rational expectations, by precluding the possibility of capital gains by traders in earlier periods, rule out speculation as a market force during those periods.

The upshot of all this is that, despite its intuitive appeal, Houthakker's argument that asymmetry of arbitrage works to produce a backwardation equilibrium has no standing when the market is a forward market and is at best highly conjectural when applied to a true futures market.

The Model

We consider a world in which there is a futures market as well as cash markets in the grade-location alternatives deliverable under the futures contract. This is a T period ($t = 0,1,2,\ldots,T$) world. There is one futures contract available, maturing at time T. Traders on the futures market are long (L) hedgers, short (S) hedgers, and speculators. All traders are assumed to have the same strictly concave utility function over income and the same probability beliefs concerning futures and cash prices for periods in the future.

Let p_t^c denote the cash price at time t of a grade-location alternative deliverable under the futures contract. Let p_t^f denote the price of the futures contract at time t.

We take short hedgers to be elevator operators, while long hedgers are millers. For short hedgers, let W_t^S denote the cash commitment (no. of bushels stored) for a typical short hedger at time t, and let V_t^S denote the number of futures contracts sold at time t. Profits for the short hedger during period t, beginning at t - 1 and ending at t, are denoted by π_t^S, which is given by

$$\pi_t^S = (p_t^c - p_{t-1}^c - k_t)W_{t-1}^S + (p_{t-1}^f - p_t^f)V_{t-1}^S + R_t(X_t^S). \qquad (1)$$

In (1), k_t is the carrying cost per bushel of wheat over the t^{th} period, which is taken to be a known constant independent of prices and of storage levels. X_t^S denotes the flow of wheat through the elevator during the t^{th} period, so that $X_t^S = W_{t-1}^S - W_t^S$. $R_t(\cdot)$ is a strictly concave function, giving the net revenue earned by the elevator operator from those activities (grading, handling, etc.) for which per-bushel profits are unrelated to changes in cash prices. In the formulation adopted here, it is assumed that the short hedger in effect liquidates both his cash and futures positions at the end of each period.

For a typical long hedging miller, let W_t^L denote the cash (wheat) commitment of the miller at time t. We interpret this as follows. At t = 0, the miller signs forward contracts to deliver flour to its customers between t = 0 and t = T. These contracts provide for a delivered price (in wheat equivalent terms) equal to $p_0^c + \sum_{\tau=1}^{t} k_\tau$ plus a milling fee, if delivery occurs at time t. Option as to the delivery date is with the miller.

Let V_t^L denote the number of futures contracts bought at time t and let $X_t^L = W_{t-1}^L - W_t^L$. Then it is convenient to write the expression for profits, π_t^L, in the form

$$\pi_t^L = (p_{t-1}^c + k_t - p_t^c)W_{t-1}^L + (p_t^f - p_{t-1}^f)V_{t-1}^L + R_t(X_t^L). \qquad (2)$$

Equation (2) expresses profits in the t^{th} period under the assumption that the cash and futures positions are liquidated at time t. Thus, if the cash position were liquidated at time t, wheat sufficient to deliver on outstanding forward contracts could be purchased at p_t^c per bushel (for instantaneous processing). If $X_t^L < W_{t-1}^L$, this means that, in effect, the miller "repurchases" his remaining forward contracts at the beginning of the t + 1st period, increasing the selling price by the carrying charge k_{t+1}. Expressing profits in this way makes the long hedger's profit function directly comparable to that of the short hedger. The $R_t(\cdot)$ function gives (net) revenues to the miller from activities where per bushel earnings are not related to changes in the cash prices, for example, milling operations. To complete the picture, we assume an equal number of elevator operators and millers, operating in competitive cash and futures markets. For symmetry, we take the $R_t(\cdot)$ functions for both the short and long hedgers to be identical.

It is assumed that the commodity in question is a seasonal good, but the argument can be extended in a natural fashion to cover financial futures or futures in non-agricultural commodities. Time $t = 0$ can be thought of as the harvest time, with no harvest occurring again until after time $t = T$. Thus all of the commodity available for use at time $t = 1$ to $t = T$ is represented by the cash commitments of short hedgers (elevator operators) at time $t = 0$. Similarly, it is assumed that all of the commitments for consumption at $t = 1$ to $t = T$ are represented by the cash commitments of long hedgers (millers) at time $t = 0$. Assuming an equal number of identical short and long hedgers, we have the following market clearing conditions.

<u>Cash Markets</u>:

$$X_t^S = X_t^L \text{ and } W_t^S = W_t^L, \; t = 0, 1, \ldots, T \tag{3}$$

Futures Markets:

$$V_t^S = V_t^L + V_t^{spec}, \quad t = 0, 1, \ldots, T - 1, \tag{4}$$

where V_t^{spec}, $t = 0,1,2,\ldots,T - 1$ is the number of purchases of futures contracts by pure speculators. Speculators buy futures whenever expected profits from purchases are positive ($Ep_t^f > p_{t-1}^f$) and sell futures whenever expected profits from sales are positive ($Ep_t^f < p_{t-1}^f$). We assume that the aggregate (excess) demand functions for futures by speculators are of less than infinite elasticity.

In describing the pattern of prices on the futures market, we use the following terminology. The futures market attains a <u>martingale equilibrium</u> at time t - 1 if the market clearing prices p_{t-1}^f, p_t^f satisfy the condition:

$$E(p_t^f | p_{t-1}^f) = p_{t-1}^f. \tag{5}$$

The futures market is said to exhibit <u>backwardation</u> at time t - 1, if

$$E(p_t^f | p_{t-1}^f) > p_{t-1}^f. \tag{6}$$

Similarly, the futures market exhibits a <u>contango</u> at time t - 1 if

$$E(p_t^f | p_{t-1}^f) < p_{t-1}^f. \tag{7}$$

In analyzing the effect of asymmetric arbitrage on the pattern of prices on the futures market, it is helpful to distinguish between two cases, the case of a forward market, and the case of a "true" futures market. A forward market is one where there is only one grade-location alternative deliverable under the futures contract, so that $p_T^c = p_T^f$ is known to be the relationship that will hold at time T between the market clearing prices on the cash and futures markets. This is the case where "perfect hedges" occur and is the case typically studied in the theoretical literature dealing with futures markets (e.g., see Anderson and Danthine (1983)).

In contrast, if two or more grade-location alternatives are deliverable under a futures contract, with these being less than perfect substitutes for one another, then we have the case of a "true" futures market.

Because choice of the grade-location alternative to deliver under the futures contract is up to the seller, buyers and sellers in a true futures market know that what will be delivered under the futures contract will be that delivery alternative with the lowest cash price at time T. Hence arbitrage ensures that the relationship between equilibrium prices of the futures and any delivery alternative at time t = T in a true futures market takes the less restrictive form $p_T^f \leq p_T^c$; hedges now become "imperfect" and there is a nondegenerate joint pdf over p_T^f, p_T^c that must be analyzed in examining the time pattern of cash and futures prices.

Moreover, at any time $t < T$, arbitrage imposes additional constraints on the futures price through the relationship $p_t^f \leq p_t^c + \sum_{\tau=t+1}^{T} k_\tau$, where k_τ is the cost of carrying a unit of the commodity over the τ^{th} period. If this constraint were violated, then there would be a riskless profit that could be earned by selling a futures, buying a deliverable grade on the cash market, and then storing to deliver at time T under the futures contract. Because arbitrage acts only to impose an upper (but not a lower) bound on p_t^f, arbitrage is asymmetric. We first investigate the effect of asymmetric arbitrage on a forward market.

Price Patterns in a Forward Market

Since $p_T^c = p_T^f$ is the equilibrium condition at time T in a forward market, thus at t = T - 1, cash and futures commitments of short and long hedgers are chosen under the degenerate joint density $f(p_T^c)(\equiv f(p_T^f))$, held in common by all traders. Further, we have $W_T^S = W_T^L = 0$ so that $X_T^S = W_{T-1}^S$ and $X_T^L = W_{T-1}^L$. Hence, first-order conditions for a short hedger are given by

$$\int_0^\infty u'(\pi_T^S)[p_T^c - p_{T-1}^c - k_T + R_T']f(p_T^c)dp_T^c = 0$$

$$\int_0^\infty u'(\pi_T^S)[p_{T-1}^f - p_T^c]f(p_T^c)dp_T^c = 0. \tag{8}$$

Similarly, first-order conditions for the long hedger are given by

$$\int_0^\infty u'(\pi_T^L)[p_{T-1}^c + k_T - p_T^c + R_T']f(p_T^c)dp_T^c = 0$$

$$\int_0^\infty u'(\pi_T^L)[p_T^c - p_{T-1}^f]f(p_T^c)dp_T^c = 0. \tag{9}$$

Consider as a possible candidate for equilibrium in the t = T - 1 cash and futures market the following price and commitment pattern:

$$p^f_{T-1} = p^c_{T-1} + k_T, \text{ with } W^S_{T-1} = W^L_{T-1} \equiv W_{T-1}, \text{ and } V^S_{T-1} = V^L_{T-1} = W_{T-1},$$

where $R_T'(W_{T-1}) = 0$; and $E(p^f_T | p^f_{T-1}) = p^f_{T-1}$, $E(p^c_T | p^c_{T-1}) = p^c_{T-1} + k_T$.

Note that combining the two first-order conditions in (8) and (9) we have

$$[p^f_{T-1} - p^c_{T-1} - k_T + R_T'(W^S_{T-1})] = 0.$$

$$[p^f_{T-1} - p^c_{T-1} - k_T + R_T'(W^L_{T-1})] = 0.$$

Setting W_{T-1} such that $R_T'(W_{T-1}) = 0$, then $p^f_{T-1} = p^c_{T-1} + k_T$, and we also satisfy the cash market equilibrium condition in (3).

Further, integrate the first integral in (8) by parts to obtain

$$u'(\pi^S_T)\int_0^{p^c_T}[x-p^c_{T-1}-k_T + R_T'(W^S_{T-1})]f(x)dx \Big|_0^\infty$$

$$-(W^S_{T-1}-V^S_{T-1})\int_0^\infty u''(\pi^S_T)\int_0^{p^c_T}[x-p^c_{T-1}-k_T + R_T'(W^S_{T-1})]f(x)dxdp^c_T = 0. \tag{10}$$

Given that $E(p^c_T | p^c_{T-1}) = p^c_{T-1} + k_T$ with $R_T'(W^S_{T-1}) = 0$, strict concavity of u implies that $W^S_{T-1} = V^S_{T-1}$. A similar development establishes that $W^L_{T-1} = V^L_{T-1}$. Hence, we satisfy the market clearing condition $V^S_{T-1} = V^L_{T-1}$ for the futures market.

Consider, next, equilibrium in the time t = T - 2 market.

Suppose it ~~is~~ common knowledge at t = T - 2 that all traders have identical utility functions and identical probability beliefs about time t = T - 1 and time t = T cash and futures prices. Then we claim that a rational expectations equilibrium at time t = T - 2 is one such that $p^f_{T-2} = p^c_{T-2} + k_{T-1} + k_T$, with $E(p^f_{T-1} | p^f_{T-2}) = p^f_{T-2}$ and $E(p^c_{T-1} | p^c_{T-2}) = p^c_{T-2} + k_{T-1}$. Further, $X^S_{T-1} = X^L_{T-1} = X_{T-1}$, where $X_{T-1} = W_{T-2} - W_{T-1}$ satisfies $R'_{T-1}(X_{T-1}) = 0$. Here $W_{T-2} = W^S_{T-2} = W^L_{T-2}$ with $V^S_{T-2} = V^L_{T-2} = W_{T-2}$.

The argument is much like the one above establishing the martingale property in the time t = T - 1 markets. Given the common knowledge assumption, each trader knows that the equilibrium price pattern in the time t = T - 1 markets is one such that $p^f_{T-1} = p^c_{T-1} + k_T$. Since $p^f_{T-1} = p^c_{T-1} + k_T$, again we can describe the probability beliefs of traders in terms of a degenerate density over p^c_{T-1} only, say $g(p^c_{T-1})$. Further, W^S_{T-1} and W^L_{T-1} are known to equal W_{T-1}, so that $X^S_{T-1} = W^S_{T-2} - W_{T-1}$ and $X^L_{T-1} = W^L_{T-2} - W_{T-1}$.

First-order conditions for the short hedger are then given by

$$\int_0^\infty u'(\pi^S_{T-1})[p^c_{T-1} - p^c_{T-2} - k_{T-1} + R'(X^S_{T-1})]g(p^c_{T-1})dp^c_{T-1} = 0$$

$$\int_0^\infty u'(\pi^S_{T-1})[p^f_{T-2} - p^c_{T-1} - k_T]g(p^c_{T-1})dp^c_{T-1} = 0. \tag{11}$$

The long hedger's first-order conditions are

$$\int_0^\infty u'(\pi^L_{T-1})[p^c_{T-2} + k_{T-1} + R'(X^L_{T-1}) - p^c_{T-1}]g(p^c_{T-1})dp^c_{T-1} = 0$$

$$\int_0^\infty u'(\pi^L_{T-1})[p^c_{T-1} + k_T - p^f_{T-2}]g(p^c_{T-1})dp^c_{T-1} = 0. \tag{12}$$

Using the earlier approach, it immediately follows from (11) and (12) that, if $W^S_{T-2} = W^L_{T-2} = W_{T-2}$ such that $R'_{T-1}(X^S_{T-1}) = R'_{T-1}(X^L_{T-1}) = 0$, then market clearing prices in the $t = T - 2$ markets satisfy

$$p^f_{T-2} = p^c_{T-2} + k_{T-1} + k_T \text{ with } E(p^f_{T-1} | p^f_{T-2}) = p^f_{T-2},$$

$$E(p^c_{T-1} | p^c_{T-2}) = p^c_{T-2} + k_{T-1}.$$

Similarly, the above arguments apply to $t = T - 3$, $T - 4, \ldots, 0$. Thus, we have established the following.

Proposition 1. Given a forward market with an equal number of short and long hedgers, each with identical utility functions and densities over cash and futures prices, there exists a rational expectations equilibrium which is also a martingale equilibrium, satisfying

$$p^f_t = p^c_t + \sum_{\tau=t+1}^{T} k_\tau, \quad t = 0,1,\ldots,T - 1,$$

$$p^f_T = p^c_T \text{ with}$$

$$E(p^f_t | p^f_{t-1}) = p^f_{t-1}, \quad t = 1,\ldots,T$$

$$E(p^c_t | p^c_{t-1}) = p^c_{t-1} + k_t, \quad t = 1,2,\ldots,T$$

$$W^S_t = W^L_t = W_t, \; V^S_t = V^L_t = W_t, \text{ with } R'_t(W_{t-1} - W_t) = 0, \; t = 0,1,\ldots,T - 1.$$

One thing to note about this rational expectations martingale equilibrium is that there is no role for Houthakker's "asymmetric arbitrage" to play in influencing the configuration of equilibrium prices, or the decisions taken by short or long hedgers. In fact, with a forward market, the futures prices at all times $t = 0,1,2,\ldots,T$ are set at the maximum levels permitted by arbitrage (futures price equals the cash price plus carrying cost to maturity of the futures contract).

Price Patterns on a True Futures Market

The situation is quite different once we move to a true futures market, with two or more delivery options available under the futures contract. In a true futures market, asymmetric arbitrage can impose a binding constraint on the joint pdf over the cash and futures prices and hence can have an impact on the decisions of hedgers concerning their cash and futures commitments, which in turn has an effect on the pattern of the market clearing prices in the cash and futures markets.

Recall that in a true futures market, arbitrage ensures that $p_T^f \leq p_T^c$, and $p_t^f \leq p_t^c + \sum_{\tau=t+1}^{T} k_\tau$, $t = 0,1,\ldots,T-1$, but there are no corresponding constraints limiting the amount by which the cash price can exceed the futures price at any point in time.

Consider now a futures market in which arbitrage is not permitted to occur. Let $h(p_t^c, p_t^f)$ denote the joint density over the cash and futures prices at time t in such a situation, held by all traders at time t - 1.[1] Our approach is first to construct an equilibrium for the case where arbitrage is not permitted to occur and then to contrast the resulting pattern of market clearing prices with that which obtains under arbitrage.

Because we wish to explore the effects of asymmetric arbitrage under as simple conditions as possible, it is convenient to begin with a set of assumptions under which the equilibrium (without arbitrage) is a martingale equilibrium. In particular, assume that the density held by traders at $t = T - 1$ is symmetric about Ep_T^c, Ep_T^f, and consider as a candidate for equilibrium in the $T - 1$ markets the price and commitment relationships:

$$p_{T-1}^f = p_{T-1}^c + k_T.$$

$$E(p_T^f | p_{T-1}^f) = p_{T-1}^f, \quad E(p_T^c | p_{T-1}^c) = p_{T-1}^c + k_T$$

with $W_{T-1}^S = W_{T-1}^L = W_{T-1}$ satisfying $R_T'(W_{T-1}) = 0$ and
with $V_{T-1}^S = V_{T-1}^L$.

Once again we have $W_T^S = W_T^L = 0$, so that $X_T^S = W_{T-1}^S$ and $X_T^L = W_{T-1}^L$. At $t = T - 1$ the first-order conditions for the short hedger are

$$\frac{\partial E_h u^S}{\partial W_{T-1}^S} = \int_0^\infty\int_0^\infty u'(\pi_T^S)[p_T^c - p_{T-1}^c - k_T + R_T'(X_T^S)]h(p_T^c, p_T^f)dp_T^c dp_T^f = 0$$

$$\frac{\partial E_h u^S}{\partial V_{T-1}^S} = \int_0^\infty\int_0^\infty u'(\pi_T^S)[p_{T-1}^f - p_T^f]h(p_T^c, p_T^f)dp_T^c dp_T^f = 0. \tag{13}$$

Similarly, the first-order conditions for the long hedger are

$$\frac{\partial E_h u^L}{\partial W_{T-1}^L} = \int_0^\infty\int_0^\infty u'(\pi_T^L)[p_{T-1}^c + k_T + R_T'(X_T^L) - p_T^c]h(p_T^c,p_T^f)dp_T^f dp_T^c = 0$$

(14)

$$\frac{\partial E_h u^L}{\partial V_{T-1}^L} = \int_0^\infty\int_0^\infty u'(\pi_T^L)[p_T^f - p_{T-1}^f]h(p_T^c,p_T^f)dp_T^f dp_T^c = 0.$$

Suppose that $W_{T-1}^S = W_{T-1}^L = W_{T-1}$ satisfies $R_T'(W_{T-1}) = 0$, and assume that $V_{T-1}^S = V_{T-1}^L = V_{T-1}$. Let $x = p_T^c - Ep_T^c$, $y = p_T^f - Ep_T^f$. Then by symmetry, $h(Ep_T^c + x, Ep_T^f + y) = h(Ep_T^c - x, Ep_T^f - y)$ for all x,y. Also, given that $Ep_T^c = p_{T-1}^c + k_T$, $Ep_T^f = p_{T-1}^f$, we have $\pi_T^S(x,y) = W_{T-1}x - V_{T-1}y + R_T(W_{T-1}) = \pi_T^L(-x,-y)$.

Rewriting the first-order conditions (13) and (14), we have

$$\frac{\partial E_h u^S}{\partial W_{T-1}^S} = \int_{-Ep_T^c}^\infty\int_{-Ep_T^f}^\infty u'(\pi_T^S(x,y))xh(Ep_T^c + x, Ep_T^f + y)dydx = 0$$

$$\frac{\partial E_h u^L}{\partial W_{T-1}^L} = -\int_{-Ep_T^c}^\infty\int_{-Ep_T^f}^\infty u'(\pi_T^L(x,y))xh(Ep_T^c + x, Ep_T^f + y)dydx = 0 \qquad (15)$$

$$\frac{\partial E_h u^S}{\partial V_{T-1}^S} = \int_{-Ep_T^c}^\infty\int_{-Ep_T^f}^\infty u'(\pi_T^S(x,y))yh(.,.)dydx = 0$$

$$\frac{\partial E_h u^L}{\partial V_{T-1}^L} = -\int_{-Ep_T^c}^\infty\int_{-Ep_T^f}^\infty u'(\pi_T^L(x,y))yh(.,.)dydx = 0.$$

Clearly, by substituting (-x,-y) for (x,y) in the second and fourth equations, these reduce to the first and third. Hence, market clearing in both the cash and futures markets is consistent with the first order conditions in the $t = T - 1$ markets.

We might note that, in contrast to the $t = T - 1$ equilibrium in the case of a forward market, here there is no guarantee that all cash commitments will be hedged; all we know is that $V_{T-1}^S = V_{T-1}^L$.

Consider now the $t = T - 2$ markets. Again invoking a common knowledge assumption, all traders know that the equilibrium pattern of prices on the $t = T - 1$ markets will satisfy $p_{T-1}^f = p_{T-1}^c + k_T$, with $W_{T-1}^S = W_{T-1}^L = W_{T-1}$ such that $R_T'(W_{T-1}) = 0$. Using the line of reasoning employed earlier, we can show that a rational expectations equilibrium exists on the $t = T - 2$ markets such that $p_{T-2}^f = p_{T-2}^c + k_{T-1} + k_T$ with $E(p_{T-1}^f | p_{T-2}^f) = p_{T-2}^f$, and $E(p_{T-1}^c | p_{T-2}^c) = p_{T-2}^c + k_{T-1}$, with $W_{T-2}^S = W_{T-2}^L = W_{T-2}$ satisfying $R_{T-1}'(W_{T-2} - W_{T-1}) = 0$, and with $V_{T-2}^S = V_{T-2}^L = W_{T-2}$. Note that we do not require symmetry of the density over time $t = T - 1$ prices, since the rational expectations assumption reduces the $t = T - 1$ market to a forward market. Similarly, the same argument applies to $t = T - 3, T - 4, \ldots, 0$. We formalize this as follows.

<u>Proposition 2</u>. Given an equal number of short and long hedgers, each with identical utility functions and density functions over futures and cash prices, and with the density over $t = T$ prices symmetric about the mean cash and futures prices, there exists a rational

expectations-equilibrium which is also a martingale equilibrium, satisfying

$$p_t^f = p_t^c + \sum_{\tau=t+1}^{T} k_\tau, \; t = 0,1,2,\ldots,T-1.$$

$$E(p_t^f | p_{t-1}^f) = p_{t-1}^f, \; t = 1,2,\ldots,T$$

$$E(p_t^c | p_{t-1}^c) = p_{t-1}^c + k_t, \; t = 1,2,\ldots,T.$$

We next examine the effects of asymmetric arbitrage on the cash and futures commitments of traders. A natural way to incorporate asymmetric arbitrage into the picture is to assume that, if $h(p_t^c, p_t^f)$ is the density when arbitrage is not permitted, and $f(p_t^c, p_t^f)$ is the density when arbitrage can occur, then

$$f(p_t^c, p_t^f) = \begin{cases} h(p_t^c, p_t^f) & \text{for } p_t^f < p_t^c + \phi(t) \\ \int_{p_t^c+\phi(t)}^{\infty} h(p_t^c, p_t^f)\, dp_t^f & \text{for } p_t^f = p_t^c + \phi(t) \\ 0 & \text{for } p_t^f > p_t^c + \phi(t) \end{cases} \quad (16)$$

where $\phi(t) = \sum_{\tau=t+1}^{T} k_\tau$.

Thus, the effect of arbitrage is to concentrate at $(p_t^c, p_t^c + \phi(t))$ all the probability weight assigned under h to (p_t^c, p_t^f) for higher values of p_t^f. Given this specification of f, it immediately follows that h stochastically dominates f in the sense of first-degree stochastic dominance (see Quirk and Saposnik (1963)), since, for any p_t^c, we have

$$\int_0^{p_t^f} h(p_t^c, v)dv \leq \int_0^{p_t^f} f(p_t^c, v)dv$$

for all p_t^f, with strict inequality for some values of p_t^f. By the well-known property of dominating distributions, $E_h u > E_f u$ if u is monotone increasing in p_t^f, and $E_h u < E_f u$ if u is monotone decreasing in p_t^f. Hence we have the following.

<u>Proposition 3</u>. Arbitrage acts to increase expected utility for short hedgers, and to decrease expected utility for long hedgers.

[Proof]

For every W,V, $E_h u(\pi^S(W,V)) < E_f u(\pi^S(W,V))$ since π^S is monotone decreasing in p_t^f while u is monotone increasing in π^S. Let W^*, V^* maximize $E_h u(\pi^S)$ and let W^{**}, V^{**} maximize $E_f u(\pi^S)$. Then $E_h u(\pi^S(W^*, V^*)) < E_f u(\pi^S(W^*, V^*)) \leq E_f u(\pi^S(W^{**}, V^{**}))$. A similar argument establishes the proposition for long hedgers.

When arbitrage is permitted, the first-order conditions for short hedgers at T - 1 are given by

$$\frac{\partial E_f u^S}{\partial W_{T-1}^S} = \int_0^{\infty}\{\int_0^{p_T^c} u'(\pi_T^S(p_T^c, p_T^f))[p_T^c - p_{T-1}^c - k_T + R_T'(W_{T-1}^S)]h(p_T^c, p_T^f)dp_T^f$$

$$+ u'(\pi_T^S(p_T^c, p_T^c))[p_T^c - p_{T-1}^c - k_T + R_T'(W_{T-1}^S)]\int_{p_T^c}^{\infty} h(p_T^c, p_T^f)dp_T^f\}dp_T^c = 0 \qquad (17)$$

$$\frac{\partial E_f u^S}{\partial V^S_{T-1}} = \int_0^\infty \{\int_0^{p^c_T} u'(\pi^S_T(p^c_T, p^f_T))[p^f_{T-1} - p^f_T]h(p^c_T, p^f_T)dp^f_T$$

$$+ u'(\pi^S_T(p^c_T, p^c_T))[p^f_{T-1} - p^f_T]\int_{p^c_T}^\infty h(p^c_T, p^f_T)dp^f_T\}dp^c_T = 0.$$

Let $\tilde{W}^S_{T-1}, \tilde{V}^S_{T-1}$ denote the optimal choices of the short hedger under arbitrage, satisfying (17), and let $\bar{W}^S_{T-1}, \bar{V}^S_{T-1}$ denote the choices of the short hedger when arbitrage is not permitted, satisfying (13). Evaluate the first order conditions in (17) at $\bar{W}^S_{T-1}, \bar{V}^S_{T-1}$, and consider

$\frac{\partial E_h u^S}{\partial W^S_{T-1}} - \frac{\partial E_f u^S}{\partial W^S_{T-1}}$, $\frac{\partial E_h u^S}{\partial V^S_{T-1}} - \frac{\partial E_f u^S}{\partial V^S_{T-1}}$, evaluated at $\bar{W}^S_{T-1}, \bar{V}^S_{T-1}$.

Then we have

$$\frac{\partial E_h u^S}{\partial W^S_{T-1}} - \frac{\partial E_f u^S}{\partial W^S_{T-1}} \tag{18}$$

$$= \int_0^\infty [p^c_T - p^c_{T-1} - k_T]\{\int_{p^c_T}^\infty [u'(\pi^S_T) - u'(\pi^0)]h(p^c_T, p^f_T)dp^f_T\}dp^c_T$$

$$\frac{\partial E_h u^S}{\partial V^S_{T-1}} - \frac{\partial E_f u^S}{\partial V^S_{T-1}}$$

$$= \int_0^\infty \int_{p^c_T}^\infty [p^f_{T-1} - p^f_T][u'(\pi^S_T) - u'(\pi^0)]h(p^c_T, p^f_T)dp^f_T dp^c_T, \tag{19}$$

where $\pi^0 = \pi^S_T$ evaluated at $p^f_T = p^c_T$, with $W^S_{T-1} = \bar{W}^S_{T-1}, V^S_{T-1} = \bar{V}^S_{T-1}$.

In order to show that arbitrage encourages short hedging, in effect we need to solve a comparative statics problem where the exogenous shift involves the change from the density h to the density f.

Assuming a regular maximum, the local solution to the comparative statics problem can be written as

$$\begin{bmatrix} \Delta W^S \\ \Delta V^S \end{bmatrix} = \frac{1}{J} \begin{bmatrix} \dfrac{\partial^2 Eu^S}{\partial V^2} & \dfrac{\partial^2 Eu^S}{\partial W \partial V} \\ \dfrac{\partial^2 Eu^S}{\partial W \partial V} & \dfrac{\partial^2 Eu^S}{\partial W^2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial E_h u^S}{\partial W^S_{T-1}} - \dfrac{\partial E_f u^S}{\partial W^S_{T-1}} \\ \dfrac{\partial E_h u^S}{\partial V^S_{T-1}} - \dfrac{\partial E_f u^S}{\partial V^S_{T-1}} \end{bmatrix} \tag{20}$$

where ΔW^S is the change in W^S_{T-1} induced by the change from h to f, and ΔV^S is the corresponding change in V^S_{T-1}. J is the determinant of the Hessian matrix, with $J > 0$ at a regular maximum.

To solve the above comparative statics problem from qualitative information, for an arbitrary density h and an arbitrary concave utility function, the signs of (18) and (19) should be determinate. Using integration by parts, it is straightforward to establish that if the utility function satisfies constant or decreasing absolute risk aversion, then (19) is negative for an arbitrary symmetric density h.

Thus, we can write (19) as

$$\frac{\partial E_h u^S}{\partial V^S_{T-1}} - \frac{\partial E_f u^S}{\partial V^S_{T-1}} =$$

$$\int_0^{\infty}\{[u'(\pi_T^S) - u'(\pi^0)]\int_0^{p_T^f}(p_{T-1}^f - x)h(p_T^c,x)dx\Big|_{p_T^c}^{\infty}$$

$$- \int_{p_T^c}^{\infty}(\int_0^{p_T^f}(p_{T-1}^f - x)h(p_T^c,x)dx)[u''(\pi_T^S) - u''(\pi^0)](- V_{T-1}^S)dp_T^f\}dp_T^c.$$

Since $\pi_T^S = \pi^0$ when $p_T^f = p_T^c$, the first term under the integral is negative. Given a martingale equilibrium in (13), and with the utility function exhibiting constant or decreasing absolute risk aversion, $u''' > 0$, and the second term is negative, so that (19) is negative for an arbitrary symmetric h.

However, the sign of (18) depends on obscure properties of the utility function and the density even when there is constant or decreasing absolute risk aversion, as is easy to verify. Hence, despite the intuitive appeal of the asymmetric arbitrage argument, it turns out, that in the general case, we cannot even show that the presence of arbitrage encourages short hedging (and discourages long hedging), much less that arbitrage leads to a backwardation equilibrium.

Thus, integrating by parts, the term inside the curved brackets in (18) is positive, but in order to sign (18) we need to know whether this term increases or decreases with p_T^c. Even assuming constant or decreasing absolute risk aversion does not resolve the ambiguity, which requires, instead, restrictive conditions on the derivative of h with respect to p_T^c.

To indicate the difficulty, differentiate the term in curved brackets in (18) with respect to p_T^c to obtain

$$\frac{\partial}{\partial p_T^c}\{\int_{p_T^c}^{\infty}[u'(\pi_T^S) - u'(\pi^0)]h(p_T^c,p_T^f)dp_T^f\} =$$

$$\int_{p_T^c}^{\infty}[u''(\pi_T^S)\overline{W}_{T-1}^S - u''(\pi^0)(\overline{W}_{T-1}^S - \overline{V}_{T-1}^S)]h(\cdot)dp_T^f$$

$$+ \int_{p_T^c}^{\infty}[u'(\pi_T^S) - u'(\pi^0)]\frac{\partial h(\cdot)}{\partial p_T^c}dp_T^f \ . \qquad (21)$$

The assumption that u is characterized by constant or decreasing absolute risk aversion signs the first term on the RHS of (21) as negative, but the second term is generally of ambiguous sign. It is true that if h is nonincreasing with respect to p_T^c, then the second term also is signed as negative. However, since h is taken to be symmetric about Ep_T^f, this means that only in the uniform case would this argument be relevant. (In the uniform case, with a constant or decreasing absolute risk aversion utility function, W and V are cooperative inputs ($\frac{\partial^2 Eu}{\partial W \partial V} > 0$), and it can be seen from (20) that Houthakker's conjecture holds in that asymmetric arbitrage leads to a larger cash and hedging commitment by short hedgers, while the opposite is true for long hedgers.)

Thus, excluding the uniform case, (18) can be signed only is restrictive conditions are imposed on the higher derivatives of h, so that, despite its intuitive appeal, it is clear that to establish the

Houthakker conjecture concerning the effect of asymmetric arbitrage on hedging behavior will require highly restrictive conditions on the underlying joint density over cash and futures prices.

The basic reason that Houthakker's intuitive argument does not hold in general is that, when arbitrage is introduced, this produces effects on both the cash and futures commitments of traders. If we ignored the effect of arbitrage on the cash commitment (so that the right-hand side of (18) were identically zero), then the Houthakker conjecture--that arbitrage encourages short hedging and discourages long hedging--is immediate, since the right-hand side of (19) is always negative, implying that $\Delta V^S > 0$ at a stable equilibrium, from (20). But the indeterminacy of sign in (18) in the general case means that it is possible that introducing arbitrage can actually decrease the (expected) marginal utility of cash commitments. When this happens, it raises the further possibility that the increase in expected utility for short hedgers engendered by arbitrage occurs through a decrease, rather than an increase, in the amount hedged.

But this doesn't end things. Note that in the uniform case there is no guarantee that the equilibrium when arbitrage is present in a backwardation equilibrium. The reason is that the introduction of arbitrage makes short hedging more attractive in part because it lowers the expected value of the futures price at time T, since the upper tail of the density h is lopped off by arbitrage. What is required for arbitrage to lead to backwardation is not simply that short hedging be encouraged and long hedging be discouraged; net short hedging must-be encouraged enough so that the fall in the futures price at t = T - 1 more than compensates for the fall in the expected value of the futures price at t = T. This requires further restrictive quantitative conditions on the utility function and on the density, even beyond such highly specialized conditions as that h be uniform or the condition that the right hand side of (18) be zero. It is clear that the presence of asymmetric arbitrage is at best a highly tenuous argument for a backwardation equilibrium.

One other point should be made about the pattern of futures prices under asymmetric arbitrage, given the rational expectations framework. The common knowledge assumption that underlies rational expectations equilibria guarantees that the only effect that asymmetric arbitrage will have so far as backwardation (or a contango) is concerned is in the t = T - 1 market. The reason for this is that whatever the relationship between the market clearing cash and futures prices on the t = T - 1 markets, this relationship will be inferred by all traders at a rational expectations equilibrium at t = T - 2. Similar arguments apply to t = T - 3,T - 4,...,0. Backwardation (or a contango) can occur only in the t = T - 1 markets. This means, that at a rational expectations equilibrium, the upper limit on the futures price imposed by arbitrage does not constrain the equilibrium in any period prior to T - 1, and the futures market is reduced to a simple forward market in all such prior periods. The futures price in such periods simply equals the cash price plus carrying costs to maturity of the futures contract, and there is no role for speculation to play, since the futures market attains a martingale equilibrium. This might be viewed as a rationalization of sorts for the widespread use of two period models in the literature on futures markets or, alternatively, an indication of the limited usefulness of the rational expectations framework in analyzing futures markets.

Conclusion

In this paper we have explored the implications of asymmetric arbitrage on the pattern of prices on a futures market under rational expectations, and in particular we have looked into the question as to whether asymmetric arbitrage is a force making for backwardation. Our conclusions are generally negative. If the futures market is a forward market, then in a rational expectations framework, asymmetric arbitrage has no effect on the pattern of futures (or cash) prices. If we are dealing with a true futures market, then arbitrage will typically have some effect on the pattern of hedging and hence on the pattern of futures prices. However, there is no clear-cut conclusion that the introduction of arbitrage acts to encourage short hedging and to discourage long hedging; generally, this depends on the specific properties of the joint density over cash and futures prices and on the specific properties of the utility function. Furthermore, even when it is known that short hedging increases and long hedging decreases because of the introduction of arbitrage, this does not imply that a martingale equilibrium becomes a backwardation equilibrium; this requires even further quantitative restrictions.

Footnotes For Chapter 3

1. The pdf $h(p_t^c, p_t^f)$ reflects uncertainties on the part of traders as to the level of demand by consumers as to goods produced using the given grade-location alternative (say grade #1 winter wheat delivered at Chicago), as well as uncertainty as to the demands by consumers for goods produced using substitutes, including other grade-location alternatives deliverable under the futures contract, and of course there are also uncertainties as to general economic and political conditions which also have an impact on the cash and futures markets. In particular, since the terminal value of the futures price, p_T^f, equals the minimum of the cash prices at time T of deliverable grade-location alternatives, the state of the cash markets for these other alternatives plays a critical role in determining the form of h.

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Chapter 4. Asymmetric Arbitrage in Futures Markets: An Empirical Study

Introduction

Dating back to Keynes (1930), Hicks (1965), and Kaldor (1940), an extensive literature deals with the problem of backwardation in the futures markets. Herein, the term backwardation refers to a situation in which the current futures price is a downward biased estimator of the futures price in future periods. While not the notion of backwardation used by Keynes and the other earlier writers[1], it is the notion most often used in the recent futures market literature.

Under the assumption of common beliefs on the part of all traders, hedgers and speculators, a theoretical basis for backwardation requires establishing an argument for the dominance of short hedging over long hedging. By the statement, "short hedging dominates long hedging" we mean that if the current futures price is equal to its expected price at contract maturity[2], there is an excess of short hedging over long hedging. With common expectations and risk averse traders, speculative activity by itself cannot generate a backwardation (or contango) equilibrium but can act only to mitigate the extent of such an equilibrium. Several possible bases for the preponderance of short over long hedging have been identified, including information asymmetries (Danthine (1978)), highly elastic demand for the final good (Macminn, Morgan and Smith (1984)), and the fact that futures contracts provide a poor consumption hedge (Richard and Sundaresan (1981)), to name a few.

In this paper, we investigate another intuitive argument for backwardation proposed by Houthakker (1959, 1968), namely, asymmetric arbitrage opportunities[3]. This argument asserts that, at any point in time, the futures price cannot exceed the spot price plus carrying cost to the maturity date of the futures contract, since, otherwise, selling a futures contract, buying spot and storing to deliver would earn a riskless profit. Arbitrage thus provides an upper limit on the amount by which the futures price can exceed the spot price, but no corresponding arbitrage operation limits the amount by which the spot price can exceed the futures price. Thus, asymmetric arbitrage in and of itself benefits short hedgers by limiting their price risk, relative to long hedgers. As a result, Houthakker argued that asymmetric arbitrage acts to encourage short hedging and discourage long hedging.

Despite its intuitive appeal, the theoretical work dealing with the asymmetric arbitrage argument generally establishes negative results. Specifically, Lien and Quirk (1985) show that asymmetric arbitrage has no effect on the pattern of futures prices within a rational expectations framework if the futures market is a forward market. If we are dealing with a true futures market (one with several delivery alternatives), instances can be identified in which asymmetric arbitrage acts to encourage short hedging and discourage long hedging, but even this restrictively weak conclusion does not hold under gereral conditions. Moreover, it is shown that even when short hedging is encouraged and long hedging discouraged by asymmetric arbitrage, this is still not sufficient to establish a backwardation equilibrium. The Lien and Quirk paper assumes an efficient market operating under rational expectations and common beliefs, so that the theoretical conclusions derived in that paper concerning asymmetric arbitrage do not necessarily imply, of course, that it is not a factor present in functioning futures markets. Instead, empirical work is needed to determine the relevance of the Houthakker argument for functioning futures markets. That is the objective of the present paper.

We do not, however, attempt to resolve all of the issues relating to Houthakker's argument. In particular, we do not attempt to prove or disprove the contention that asymmetric arbitrage as it operates in practice leads to a backwardation equilibrium. The objective here is more modest. We wish to determine whether asymmetric arbitrage has an impact on the pattern of spot and futures prices, and if so, what that impact is. In this paper, we address these problems as they apply to Chicago wheat and corn futures markets[4].

As in all empirical studies, some limitations are imposed by the data. After adjusting for data deficiencies, the empirical test results generally support the notion that asymmetric arbitrage has an impact on the pattern of spot and futures prices when dealing with CBOT wheat futures; asymmetric arbitrage, however, is ineffective in the CBOT corn futures market.

The structure of the paper is as follows. In Section II, the theoretical aspects of asymmetric arbitrage are discussed. We then describe the statistical methods we employ and the nature of the data in Section III. The test results and their implications are presented in Section IV.

Theoretical Aspects of Asymmetric Arbitrage

As stated above, Houthakker maintained that asymmetric arbitrage favors short hedgers over long hedgers. Since short hedgers sell futures and long hedgers buy futures, Houthakker argued that asymmetric arbitrage tends to depress the current futures price and hence promotes backwardation. That is, at any point in time, "...the spot price cannot fall below the futures price by more than the cost of carrying inventories from now to the maturity of the futures contract; if it did, riskless profit could be made by buying spot, selling futures, and making deliveries. On the other hand, there is no limit to the amount by which the spot price can exceed the futures price, for the arbitrage just mentioned cannot be reversed.....As a result of this asymmetry short hedgers have a limited risk, while long hedgers have an unlimited risk, of adverse changes in the basis." (Houthakker (1968, p.196)). Thus, in a forward market with the deliverable grade and delivery location's being completely specified, if p^c_{1t} is the spot price of the deliverable grade at location 1 at time t and p^f_t is the futures price at time t, we have

$$p^f_t \leq p^c_{1t} + k_t, \tag{1}$$

where k_t is the carrying cost from t to the maturity date of the futures contract, including warehousing cost, insurance charges, transportation cost from the spot location l to the delivery location, and the interest cost arising from the cash commitment[5].

On the other hand, to avoid squeezes and corners, a true futures contract generally provides multiple grade-location alternatives with a specified premium/discount structure, the alternatives being less than perfect substitutes for one another. In the setting of a true futures contract, Equation (1) becomes

$$p_t^f \leq p_{ilt}^c + k_{idt} - m_i - n_d, \text{ where } i \in I,\ d \in D,\ l \in L, \qquad (2)$$

where I is the set of deliverable grades, D is the set of delivery locations and L is the set of all possible spot locations; p_{ilt}^c is the spot price of the i-th grade at location l at time t; k_{idt} is the carrying cost (including the transportation cost from l to d)for the i-th grade from t to the maturity date of the futures contract with delivery location being d, m_i is the premium/discount for delivering the i-th grade, and n_d is the premium/discount for delivering at location d. Thus, $p_{ilt}^c + k_{idt} - m_i - n_d$ is the actual cost of buying the i-th grade at spot location l and storing to deliver at location d at the maturity date of the futures contract. Equation (2) then insures that no riskless arbitrage opportunity exists in the true futures market.

Combining Equations (1)-(2), we can construct the upper bound U_t for p_t^f in a general futures market as follows:

$$p_t^f \lesssim U_t = \min \{ p_{ilt}^c + k_{idt} - m_i - n_d;\ (i,d,l) \in I \times D \times L \}. \quad (3)$$

If the futures market is actually a forward market, then I and D are both singletons (i.e., only one deliverable option exists), and $m_i = n_d = 0$. The equation for U_t, without a corresponding equation specifying the lower bound of p_t^f(other than zero), fully characterizes asymmetric arbitrage opportunities.

As noted earlier, Houthakker has argued that asymmetric arbitrage is a force working toward backwardation on the futures market. However, despite its intuitive appeal, asymmetric arbitrage alone is not sufficient to generate a backwardation equilibrium in a general framework[6]. Specifically, assume an equal number of short and long hedgers, with identical utility functions and identical densities over spot and futures prices, with speculators having the same densities, Lien and Quirk (1985) show that, under rational expectations, asymmetric arbitrage has no standing as an explanation for backwardation when the market is a forward market and is at best highly conjectural when applied to a true futures market[7].

The basic reason that Houthakker's intuitive argument does not hold in general is that asymmetric arbitrage affects both the cash and futures commitments of traders. If we ignored the effect of arbitrage on the cash commitment, then the Houthakker conjecture that asymmetric arbitrage encourages short hedging and discourages long hedging is immediate. But Lien and Quirk show that, once the cash commitment of a trader is taken as a decision variable, the introduction of asymmetric arbitrage no longer leads necessarily to an excess of short over long hedging at a martingale equilibrium. Then the indeterminacy of the effect of arbitrage on the cash commitment renders difficulties.

Note that the Lien and Quirk theoretical model excludes several other possible explanations for backwardation by the assumptions of identical utility functions and identical densities, etc. Of particular importance is the assumption that markets are characterized by rational expectations. Thus, whether functioning futures markets satisfy the Lien and Quirk assumptions is an open question, so that an empirical investigation of asymmetric arbitrage opportunities remains an open, interesting question.

Data Description and Statistical Methods

The market characteristics underlying Houthakker's asymmetric arbitrage argument are tested in this paper using the wheat and corn futures contracts traded on the Chicago Board of Trade (CBOT). As shown in Table 1, the wheat futures contract allows eleven different qualities of wheat to be delivered at Chicago or Toledo at any time in the maturity month with specified premiums/discounts. Previous studies indicate the following important characteristics of this futures market:

(1) Delivery rarely occurs under the CBOT wheat futures contract (Gray and Peck (1981)).

(2) Typically, the open interest (i.e., the number of contracts outstanding) in wheat futures contracts for a given delivery month begins building up slowly about a year before the delivery date, and subsequently rises more rapidly reaching a peak shortly after the

TABLE 4.1a STRCTURE OF PREMIUMS/DISCOUNTS FOR CBOT WHEAT FUTURES (MAY 1977+)

DELIVERABLE GRADES	PREMIUMS/DISCOUNTS(PER BUSHEL)
No. 2 Red Soft	par
No. 2 Red Hard	par
No. 2 Dark North Spring	par
No. 1 North Spring	par
No. 1 Red Soft	1.0 ¢
No. 1 Red Hard	1.0 ¢
No. 1 Dark North Spring	1.0 ¢
No. 3 Red Soft	-1.0 ¢
No. 3 Red Hard	-1.0 ¢
No. 3 Dark North Spring	-1.0 ¢
No. 2 North Spring	-1.0 ¢

DELIVERY LOCATIONS	PREMIUMS/DISCOUNTS(PER BUSHEL)
Chicago	par
Toledo	-2.0 ¢

TABLE 4.1b STRUCTURE OF PREMIUMS/DISCOUNTS FOR CBOT CORN FUTURES (MAY 1977+)

DELIVERABLE GRADES	PREMIUMS/DISCOUNTS(PER BUSHEL)
No. 1 Yellow	0.5 ¢
No. 2 Yellow	par
No. 3 Yellow	-0.5 ¢

DELIVERY LOCATIONS	PREMIUMS/DISCOUNTS(PER BUSHEL)
Chicago	par
Burn Harbor	par
Toledo	-4.0 ¢
St. Louis	-4.0 ¢

expiration of trading in prior month contracts from which point contracts are liquidated until the end of trading (Gray and Peck (1981)).

(3) In those cases where actual deliveries occur, they are almost universally No.2 Red Hard Winter wheat or No.2 Red Soft Winter wheat and deliveries occur at Chicago rather than Toledo[8] (Gay and Manaster (1984)).

The above observations lead us to choose two months prior to the maturity date as the sample period to test the impacts of asymmetric arbitrage. This time span enables us to cover the period during which the peak hedging activity in a contract takes place, close enough to contract maturity so that links between the futures and spot prices are closer than in earlier months when speculative forces may be of more importance in the futures market[9].

In the case of CBOT corn futures contracts, three different qualities of corn are deliverable at Chicago, Burn Harbor, Toledo or St. Louis at any time in the maturity month with specified premiums/discounts. For purpose of comparisons, we adopted the same approach as we did for wheat futures. That is, we also chose two months prior to the maturity date to test the impacts of asymmetric arbitrage.

Ideally, to test the presence of asymmetric arbitrage opportunities, we should consider all possible grade-location combinations (see Equation (3)). For any grade-location combination, a corresponding spot price series exists with all spot prices corresponding to different combinations being closely linked by the substitutabilities among grades and transportation costs. However, there are only a few spot price series that are readily available, reflecting the most heavily traded cash markets. In the case of wheat futures, most of the deliveries by traders occur at Chicago and No.2 Red Soft or No.2 Red Hard Winter wheat are the grades delivered. The spot price data available to us consist only of the spot price of No.2 Red Soft wheat at Chicago. While this situation is not ideal, nonetheless the price series applies to one of the most common delivery alternative. Accordingly, this spot price series is relevant to the asymmetric arbitrage conjecture. Similar arguments apply to CBOT corn futures in which the spot price series available are those of No.2 Yellow recorded at Chicago. The grade and delivery location both involve no premiums or discounts.

The futures price we use in this paper is the settlement price. It is essentially a weighted average price over the last few bid prices on any given day (Kolb (1984)). The settlement price is commonly used in empirical studies because it smooths out the noisy signals occurring at the end of each daily trading period. It provides a better representation of information flows in the futures market than does the closing price.

The basic data series were obtained from the Center for the Study of Futures Markets at Columbia University, and cover the period from 1966 to 1982 (for wheat futures) and from 1966 to 1985 (for corn futures). However, we dealt only with the most recent five-year period in which data are available for both markets. That is, we considered only the period from 1978 to 1982. Unfortunately, given these truncations, the wheat futures data (but not the corn futures data) still impose some limitations. Specifically, prior to May 1982, the spot price provided is that of No.2 Red Soft Winter wheat at Chicago; subsequent spot prices of No.2 Red Soft Winter wheat are recorded at St. Louis. Moreover, some futures contracts (for example, the contract with May 1980 being the delivery month) do not have futures price data in the maturity month. In this paper, only the contracts from 1978 to May 1982 with complete futures prices data are considered.

To test the impacts of asymmetric arbitrage, we also need estimates of the carrying cost for wheat and corn. The estimate used in this paper was derived from the "Uniform Storage Agreement Schedule of Rates" (USASR) published by the the U.S. Department of Agriculture (USDA). The USASR estimate includes warehousing cost, insurance charges, etc. It is essentially what USDA paid in total to public elevator operators on a daily basis for storage of goverment-owned grains listed by year. Another series of storage cost data appears in Gay and Manaster (1984), showing much higher storage cost levels than the USDA series. Two cost items have to be taken into account in adjusting the USDA data to reflect the total cost of private storage: the loading charge, which is about 5 cents per bushel during the 1978-1982 period and the interest cost. Our approach is to adjust the USASR estimates by adding those two costs, using the interest rate on

TABLE 4.2 SOME ITEMS OF CARRYING COST

	WHEAT		CORN*	
YEAR	USASR (CENTS PER BUSHEL PER DAY)	GAY-MANASTER (CENTS PER BUSHEL PER DAY)	USASR (CENTS PER BUSHEL PER DAY)	INTEREST RATE (% PER YEAR)
1978	0.073	0.10 before March 0.14 after March	0.072	7.94
1979	0.074	0.14	0.073	10.97
1980	0.079	0.14 before May 0.16 after May	0.078	12.66
1981	0.087	----	0.086	15.32
1982	0.091	----	0.089	11.89

* Corn data is based on the average over all grains.

three month commercial paper. Table 2 displays these cost items.

Now we turn to the statistical methods employed in this paper. Let p_t^c, p_t^f denote the spot price for No.2 Red Soft Winter wheat (or No.2 Yellow corn) at Chicago and the CBOT futures price (measured in cents per bushel) at time t, respectively; let T denote the maturity date of the futures contract with i denoting the interest rate and S the carrying cost from USASR. By Equation (2), a crude upper bound for p_t^f is

$$U_t = p_t^c \cdot [1 + i(T - t)] + S \cdot (T - t) + 5, \tag{4}$$

where U_t is measured in cents per bushel. Obviously, U_t as calculated in (4) is larger than the true upper bound calculated with all options being taken into account. Because arbitrage is asymmetric, no lower bound on p_t^f corresponds to U_t. However, were the arbitrage opportunities asymmetric, i.e., if arbitrage limited the risks of long hedgers as it did for short hedgers, a crude <u>lower</u> bound for p_t^f corresponding to U_t is given by:

$$L_t = p_t^c \cdot [1 - i(T - t)] - S \cdot (T - t) - 5, \tag{5}$$

where L_t is also measured in cents per bushel. This is, of course, not a true bound, since the futures price has no lower limit (other than 0).

To test whether the existence of an upper bound (but not a lower bound) has any potential effect on hedging decisions, one direct approach is to compute the number of cases where the futures price lies above the upper bound and the number of cases where the futures price falls below the (corresponding) lower bound. If the futures price seldom falls below the lower bound, then the upper bound should not have much impact on hedging, and hence on backwardation[10]. Moreover, when the futures price lies above the upper bound, a riskless profitable arbitrage opportunity occurs in the markets. In an efficient market, the transaction flows will quickly eliminate this opportunity; a long lasting arbitrage opportunity thus undoubtedly reflects measurement errors. Examples are September 1981 and December 1981 wheat futures contracts, where the futures price always lies above the upper bound as calculated in (4). Conversely, since no appropriate arbitrage operations are available in cases where the futures price falls below the lower bound, it may last for a long time. Figures 1 and 2 illustrate these two situations.

Figure 1 displays the futures price, the upper bound and the corresponding lower bound for December 1979 wheat futures contract. On several occasions the futures price lies above the upper bound, but in most of those cases the arbitrage opportunity lasts only one day. The longest period for consecutive arbitrage opportunities is four days. On the other hand, Figure 2 illustrates that almost all the time the futures price for May 1979 wheat futures contract falls below the lower bound, the longest period for which is nineteen days.

Empirical Test Results and Their Implications

The results from testing the impact of asymmetric arbitrage opportunities are summarized in Tables 3-7. We first discuss CBOT wheat futures market. The monthly average of $(p_t^f - U_t)$ [i.e., the

FIGURE 4.1 THE FUTURES PRICE PATTERN IN 79-DECEMBER WHEAT CONTRACT

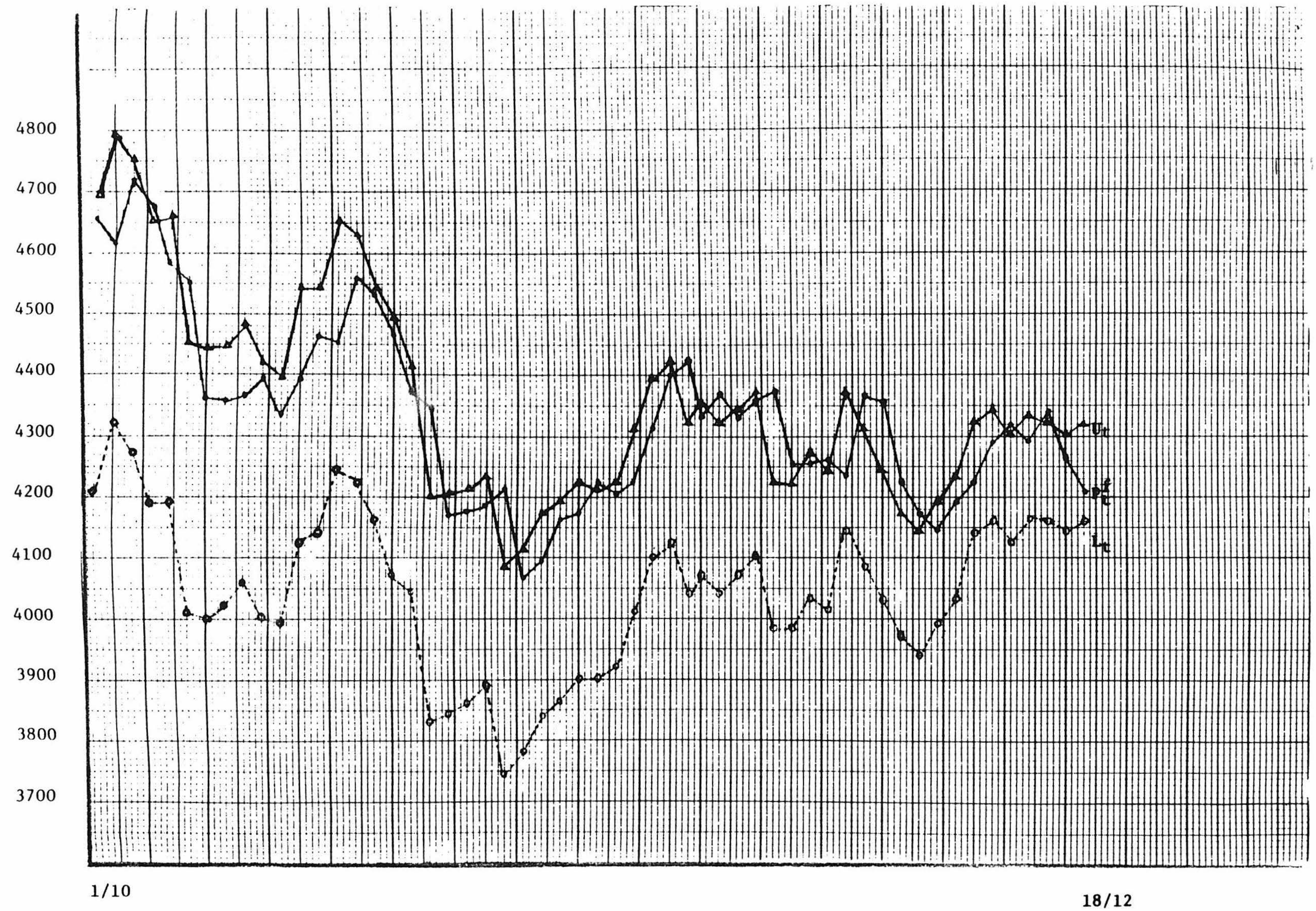

FIGURE 4.2 THE FUTURES PRICE PATTERN IN 79-MAY WHEAT CONTRACT

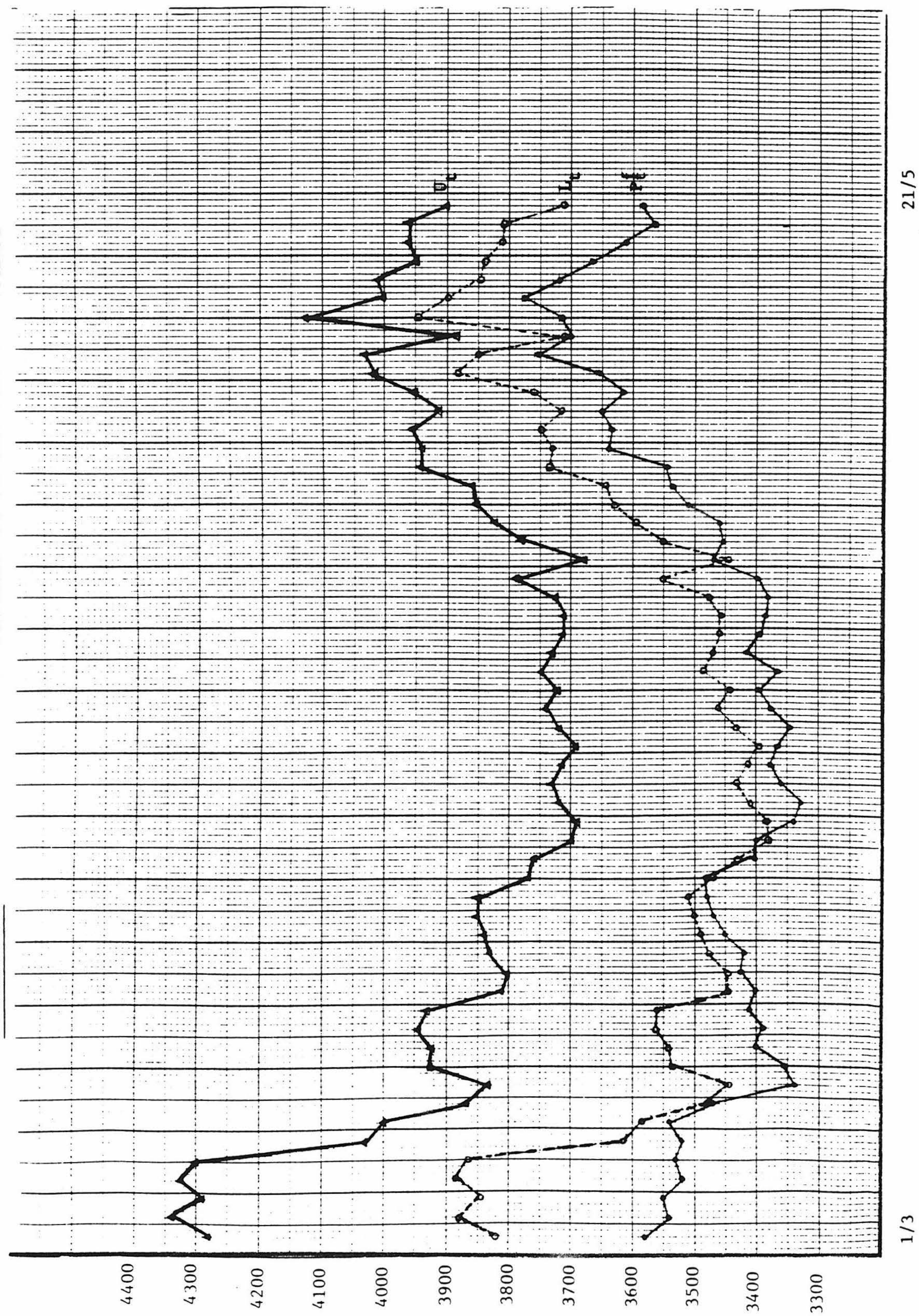

TABLE 4.3 TEST RESULTS (TWO MONTHS PRIOR TO THE DATE OF MATURITY)

	MONTHLY AVERAGE OF $(P_t^f - U_t)$		MONTHLY AVERAGE OF $(L_t - P_t^f)$	
CONTRACT TYPE#	WHEAT	CORN	WHEAT	CORN
7803	-11.24	-10.71	-18.36	- 7.41
7805	-16.27	- 9.39	-19.11	-18.74
7807	-19.79	-20.28	-16.48	- 9.89
7809	-20.03	- 5.62	-11.09	-21.91
7812	-16.80	- 8.58	-15.54	-19.78
7903	-45.14	-14.96	7.35	-16.22
7905	-43.94	-15.90	16.90	-16.82
7907	-43.65	-14.53	5.20	-18.70
7909	-13.58	-10.31	-27.00	-24.44
7912	- 4.11	-10.06	-36.19	-23.62
8003	-17.00	*	-26.42	*
8005	*	- 7.57	*	-27.52
8007	*	- 8.38	*	-28.02
8009	*	-13.03	*	-24.85
8103	*	- 5.91	*	-38.50
8105	*	-10.41	*	-34.78
8109	*	- 2.98	*	-40.25
8203	-10.29	- 8.72	-31.19	-27.00
8205	-18.20	*	-23.01	*

The contract with March 1978 being the delivery month is specified as 7803; similarly for other contracts.

TABLE 4.4 TEST RESULTS (ONE MONTH PRIOR TO THE DATE OF MATURITY)

	MONTHLY AVERAGE OF $(p_t^f - U_t)$		MONTHLY AVERAGE OF $(L_t - p_t^f)$	
CONTRACT TYPE	WHEAT	CORN	WHEAT	CORN
7803	-10.18	- 7.60	-16.57	- 3.76
7805	- 5.49	- 7.94	-17.76	-13.14
7807	- 9.39	- 6.57	-12.37	-15.09
7809	-20.33	- 9.05	- 2.11	-11.63
7812	-13.08	-12.50	-10.12	- 8.27
7903	-16.07	-14.25	- 9.96	- 9.20
7905	-31.09	-14.80	10.58	- 9.46
7907	-29.57	- 8.18	- 3.52	-13.71
7909	- 7.88	-12.87	-19.89	-11.91
7912	- 0.81	- 3.96	-26.48	-19.76
8003	- 2.56	*	-27.11	*
8005	*	- 6.48	*	-18.56
8007	*	- 5.39	*	-20.47
8009	*	- 8.56	*	-18.10
8103	*	- 3.16	*	-27.50
8105	*	- 4.16	*	-27.05
8109	*	- 7.63	*	-21.79
8203	- 6.43	- 7.68	-22.29	-18.23
8205	-13.83	*	-14.70	*

TABLE 4.5 TEST RESULTS (MATURITY MONTH)

CONTRACT TYPE	MONTHLY AVERAGE OF $(P_t^f - U_t)$ WHEAT	CORN	MONTHLY AVERAGE OF $(L_t - P_t^f)$ WHEAT	CORN
7803	- 9.41	- 6.66	- 7.10	1.71
7805	- 5.32	-10.69	- 7.89	- 4.68
7807	-15.97	- 3.53	1.35	-10.97
7809	- 9.17	- 9.47	- 6.25	- 4.95
7812	-11.52	-15.36	- 4.58	- 0.47
7903	-12.01	-13.53	- 5.21	- 2.49
7905	-30.77	-10.43	15.67	- 5.58
7907	- 5.56	- 1.81	-11.96	-14.16
7909	- 3.28	- 9.37	-14.21	- 6.77
7912	- 2.21	- 6.43	-16.22	-10.03
8003	- 4.68	*	-13.91	*
8005	*	- 7.92	*	- 9.12
8007	*	- 7.85	*	- 9.04
8009	*	- 0.90	*	-16.77
8103	*	- 5.39	*	-13.82
8105	*	- 9.51	*	-10.15
8109	*	- 1.98	*	-15.64
8203	-17.39	-16.51	- 6.22	- 0.61
8205	14.28	*	-32.92	*

TABLE 4.6 TEST RESULTS FOR CBOT WHEAT FUTURES (THE WHOLE SAMPLE PERIOD)

CONTRACT TYPE	NO. OF OBSERVATIONS	NO. OF POSITIVE $(P_t^f - U_t)$	NO. OF POSITIVE $(L_t - P_t^f)$	LONGEST DURATION* WITH CONSECUTIVELY POSITIVE $(P_t^f - U_t)$	LONGEST DURATION* WITH CONSECUTIVELY POSITIVE $(L_t - P_t^f)$
7803	52	1	3	1	2
7805	55	8	0	3	0
7807	57	0	7	0	4
7809	54	1	13	1	4
7812	53	3	9	2	3
7903	55	9	31	4	20
7905	56	0	53	0	19
7907	55	6	28	3	8
7909	54	11	1	2	1
7912	55	17	0	4	0
8003	52	8	0	3	0
8203	54	0	6	0	3
AVERAGE	54.3	5.3	12.6	2.0	5.3

* These numbers are expressed in terms of trading days.

TABLE 4.7 TEST RESULTS FOR CBOT CORN FUTURES (THE WHOLE SAMPLE PERIOD)

CONTRACT TYPE	NO. OF OBSERVATIONS	NO. OF POSITIVE $(p_t^f - U_t)$	NO. OF POSITIVE $(L_t - p_t^f)$	LONGEST DURATION* WITH CONSECUTIVELY POSITIVE $(p_t^f - U_t)$	LONGEST DURATION* WITH CONSECUTIVELY POSITIVE $(L_t - p_t^f)$
7803	54	1	11	1	5
7805	56	0	2	0	1
7807	55	3	0	2	0
7809	54	0	0	0	0
7812	53	0	8	0	6
7903	55	0	2	0	1
7905	56	0	0	0	0
7907	55	8	0	2	0
7909	54	0	1	0	1
7912	55	5	0	2	0
8005	53	0	0	0	0
8007	53	1	0	1	0
8009	54	5	0	2	0
8103	54	7	0	2	0
8105	55	3	1	3	1
8109	53	11	0	3	0
8203	54	0	7	0	5
AVERAGE	54.3	2.6	2.0	1.1	1.2

* These numbers are expressed in terms trading days.

futures price minus the upper bound] is always negative for the sample contracts, except for the May 1982 contract where a drastic drop of the spot price occurs because the measurement point moves from Chicago to St. Louis at the end of April, 1982. Thus, on average, the upper bound we calculated is effective. Conversely, for several cases the monthly average of $(L_t - p_t^f)$ [i.e., the corresponding lower bound minus the futures price] is positive; that is, the futures price falls below the lower bound on average. Based on the monthly average values of $(p_t^f - U_t)$ and $(L_t - p_t^f)$, the empirical results indicate that the hypothetical lower bound is a less effective constraint than the upper bound, suggesting that asymmetric arbitrage might indeed have an impact on hedgers' decisions.

As shown in Table 6, the number of cases where the futures price falls below the lower bound exceeds the number of cases where the futures price rises above the upper bound in eight out of twelve contracts. Averaging over all the sample contracts, we find the latter number is less than half of the former (i.e., 5.3:12.6). This result provides further evidence for the hypothesis that asymmetric arbitrage influences the distribution of spot and futures prices. Another comparison shown in the table indicates that the longest duration with consecutively positive $(p_t^f - U_t)$ is generally less than the duration of consecutively positive $(L_t - p_t^f)$; on average, the latter number is about 2.4 times of the former one. By the asymmetric arbitrage argument, this situation arises from the fact that a positive $(p_t^f - U_t)$ creates arbitrage opportunities[11], but no corresponding arbitrage operations are available working to eliminate a positive $(L_t - p_t^f)$. Combining the above three observations, we can conclude that asymmetric arbitrage does have an impact on the observed joint distribution of cash and futures prices and hence presumably on hedgers' decision making.

Thus, the range of the CBOT wheat futures price incorporates an effective upper bound while the (hypothetical) lower bound constructed from the symmetric assumption is less effective. Alternatively, this means the empirical probability density function of the basis (futures minus spot prices) is asymmetric with the highest positive value it can take being less than the absolute value of the lowest negative value. The risk of short hedgers is represented by positive values of the basis, while that of long hedgers is represented by the negative values of the basis. Thus, arbitrage acts to limit the risk of short hedgers relative to long hedgers. Consequently, the asymmetric arbitrage opportunities present in the CBOT wheat futures market will create asymmetries in the p.d.f. of the basis (or the joint p.d.f. of spot and futures prices) which are beneficial to short hedgers as compared to long hedgers.

We now turn to CBOT corn futures market. Here, as shown in Tables 3-5, the monthly average of $(p_t^f - U_t)$ and $(L_t - p_t^f)$ are almost always negative; the only exception is the March 1978 contract where the monthly average of $(L_t - p_t^f)$ is positive for the maturity month. Therefore, the hypothetical lower bound is actually an effective constraint. Moreover, as shown in Table 7, the number of cases where the futures price falls below the lower bound exceeds the number of cases where the futures price rises above the upper bound in seven contracts; however, the latter exceeds the former in eight contracts. Averaging over all the sample contracts, we find the latter is about 1.3 times the former (i.e., 2.6:2.0). Combining all these observations, we conclude that asymmetric arbitrage does not have impacts on the observed joint distribution of cash and futures prices. To this point, Houthakker's arguments in the way of a backwardation equilibrium cannot be applied since the precondition is already falsified.

The empirical results for corn are thus inconsistent with the Houthakker argument. That is, although we establish the preconditions of the Lien and Quirk results under rational expectations for CBOT wheat futures, yet whether asymmetric arbitrage's effects on the density of spot and futures prices is pronounced enough to lead to an excess of short over long hedging at a martingale equilibrium cannot be determined from the results reported here. On the other hand, the precondition is falsified in CBOT corn futures, and hence the Houthakker arguments will not hold. Consequently, at best we can only argue that Houthakker's arguments may hold for some commodity futures.

Footnotes For Chapter 4

1. Mathematically, the term "backwardation" employed in this paper can be written as:

$$p_t^f < E_t(p_{t+1}^f), \qquad (*)$$

where p_s^f is the futures price at time s, and $E_s(.)$ denotes the expectation operator at time s. On the other hand, the notion adopted by earlier writers is :

$$p_t^f < E_t(p_{t+1}^c), \qquad (**)$$

where p_s^c is the spot price at time s. When dealing with a forward contract, we have $p_T^f = p_T^c$ at the maturity date T. Consequently, the two versions are equivalent, once we set $T = t + 1$. However, if we deal with a true futures contract, there will be many "spot" commodities. For each specified spot commodity, we can only establish $p_T^f \leq p_T^c$. Thus, it appears that our version (i.e., equation (*)) is stronger than the earlier version (i.e., equation (**)); but the latter encounters a problem as to which spot commodity we refer to in the investigation of the existence of a backwardation equilibrium. Furthermore, if we require (**) be established for all spot commodities to verify the existence of a backwardation equilibrium, then the two versions will be equivalent, once we set $T = t + 1$.

2. This statement implies implicitly that we set $T = t + 1$ because we consider only the relationship between the current futures price

and the expected futures price at contract maturation.

3. Actually, Houthakker provided two arguments for a backwardation equilibrium in the futures market. The other argument, the so-called Houthakker effect, relates to the notion that the correlation between the spot and futures prices depends on the stocks of the commodity. Theoretical work dealing with the Houthakker effect validates Houthakker's conjecture under a quite restrictive specification of such an effect (see Fort and Quirk (1984)).

4. Originally, Houthakker tested his arguments on wheat, corn and cotton futures markets. For the purpose of comparisons, we also worked on these markets. Unfortunately, the daily changes in cotton futures prices and those in cotton spot prices are the same for most trading days in 1978-82, reflecting some sort of government intervention in the market. In fact, before USDA bids in the cotton spot market, the New York Cotton Exchange has already closed the daily transactions. Hence, USDA generally takes the daily change in cotton futures prices into account and adjusts the bid price by the very amount. Due to these market manipulaions, we exclude cotton futures from our empirical studies.

5. In fact, the carrying cost associated with a cash commitment is a random variable, since in particular the interest cost depends on the spot price, which is random. The simplifying assumption is

made here that carrying cost is known with certainty. This might possibly be rationalized in that we deal here only with relatively short-term (1 to 2.5 months) carrying periods, so that the uncertainties associated with the carrying cost are second-order effects relative to the uncertainties associated with the spot and futures prices.

6. Houthakker's arguments center on hedging activities. In some cases, speculative trading can dominate the role of hedging so far as the pattern of futures prices is concerned. Yamey (1971) made a similar point. In Lien and Quirk, the Houthakker arguments are examined under the assumptions of common beliefs and risk aversions on the part of all traders, so that speculative trading acts only to reduce the degree of backwardation or contango from an imbalace of hedging.

7. Another interesting result from Lien and Quirk is that, under rational expectations in a multiperiod framework, the futures market is always in martingale equilibrium except in the last period. Basically, the rational expectations assumptions rule out the possibilities of speculative profits (on average) in the market. On the one hand, this result justifies the studies of a two-period framework; on the other hand, it challenges the usefulness of the rational expectations concepts in futures markets.

8. This observation may be explained by Garbade and Silber's approach

(see Garbade and Silber (1983)). In fact, since no uniform characteristic is extractable from wheat, the discount/premium structure of CBOT wheat futures contract is a penalty system rather than an equivalence system. Thus, the deliveries at Chicago rather than Toledo are generally expected.

9. If we exclude maturity months from our consideration, the result for CBOT wheat futures indicates that the number of observations that fall below the lower bound is about 2.65 times the number of observations that exceed the upper bound (105:40). Again, the asymmetric arbitrage is effective in this case. Consequently, our conclusions remain the same whether or not maturity months are taken into account. Similar results can be established for CBOT corn futures; asymmetric arbitrage is ineffective whether or not maturity months are considered.

10. More specifically, the null hypotheses of our tests stated that the number of observations that exceed the upper bound equals the number of observations that fall below the lower bound. This property depends only on the symmetry conditions, which are independent of stationary patterns. Consequently, the nonstationary patterns of futures prices argued by Anderson (1985) would not cause any bias. Moreover, given the null hypotheses, we can apply a simple binomial test to investigate whether or not it is acceptable. Upon applying the normal approximation (for example, see Pollard (1977)) to the aggregate data, the null

hypothesis will be rejected at the 1% significant level for CBOT wheat futures markets. On the other hand, the null hypothesis cannot be rejected at the 10% significant level for CBOT corn futures markets.

11. Although a positive $(p_t^f - U_t)$ creates arbitrage opportunities which then generate a riskless profit, in our empirical studiess these profits are generally small. More specifically, in CBOT wheat futures, the average of these profits is 4.54 cents per bushel with the standard error being 3.86 cents per bushel; the maximum of these profits is 14.16 cents per bushel (which is about 4.88% of the corresponding futures price). In CBOT corn futures, the average is 2.55 cents per bushel with the standard error being 1.92 cents per bushel. Also, the maximum of these profits is 6.38 cents per bushel, about 2.41% of the corresponding futures price.

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Chapter 5. Testing the Houthakker Effect in Commodity Futures Markets

Introduction

In the previous chapter, we investigated the impacts of asymmetric arbitrage opportunities on the CBOT wheat and corn futures markets. Now we turn to the second argument provided by Houthakker (1968) in the way of a backwardation equilibrium, the so-called Houthakker effect. The Houthakker effect, as described by Fort and Quirk (1984), relates to the notion that the correlation between the spot and futures prices depends on the stocks of the commodity. Specifically, when inventories of a commodity are large, the spot and futures prices tend to be highly correlated. Since large inventories tend to be associated with low spot prices, and short hedgers (short in futures, long in spot) endeavor to avoid the risk associated with low spot prices, the correlation pattern thus renders the futures contract as an effective instrument for short hedgers. In contrast, long hedgers (long in futures, short in spot) try to avoid the risk associated with high spot prices, but the correlation pattern limits the effectiveness of futures contracts for such hedging purposes. The preponderance of short over long hedging is thus established.

Recent theoretical work dealing with the Houthakker effect does not provide unambiguous results concerning its relevance for backwardation. Specifically, Fort and Quirk show that it is possible to construct a backwardation equilibrium if the joint probability density function of spot and futures prices is characterized by a "Houthakker effect," but this requires a quite restrictive specification of such an effect. Consequently, the theoretical standing of the Houthakker effect is not all that convincing, suggesting in turn that empirical work is needed to determine the relevance of the Houthakker effect for functioning futures markets. That is the objective of the present paper.

We do not attempt to resolve all of the issues relating to the Houthakker effect in this paper. In particular, we do not attempt to prove or disprove the contention that the presence of a Houthakker effect leads to a backwardation equilibrium. The objective here is more modest. What we wish to do is to determine whether the pattern of spot and futures prices observed in the market exhibits a Houthakker effect. Again, we address these problems as they apply to the Chicago wheat and corn futures markets.

Upon comparing the two relevant conditional correlations, our empirical test results indicate that the Houthakker effect is absent in CBOT wheat and corn futures markets. This sheds some doubt about the Houthakker argument for normal backwardation.

The structure of the paper is as follows. First, we discuss the theoretical aspects of the Houthakker effect. We then describe the statistical methods we employ. Finally, the test results and their implications conclude the paper.

Theoretical Aspects of The Houthakker Effect

Basically, Houthakker's argument for backwardation is composed of two parts, namely, (i) the correlation pattern between spot and futures prices and (ii) the specific basis pattern in futures markets. Thus, Houthakker argued, "When inventories are large, there will in general be no excessive shortages or surpluses of particular grades and locations, so that all prices will tend to move closely together. On the other hand, when inventories are small, they are also likely to be unevenly distributed; the market will to some extent disintegrate into submarkets, and the correlation tends to be less close." (Houthakker (1968, p.198)).

Now, since hedgers' decisions about their participation in a futures market hinge on the correlation of spot and futures prices (at least, in a mean-variance framework), therefore, in a seasonal commodity market, the correlation pattern (i.e., the Houthakker effect) favors hedging (both short and long) in the middle of the crop year when inventories are large, but not before and immediately after harvest when commercial inventories are small. Note that this definition of the Houthakker effect is merely a seasonal phenomenon; that is, the specific correlation pattern is established across different months. Also, the correlation referred to is that of futures and spot-price movements, rather than the levels of futures and spot prices. Most importantly, this notion of a Houthakker effect does not establish the preponderance of short over long hedging; both are encouraged when inventories are large.

The second part of Houthakker's argument claims that, in the middle of the crop year, "...the algebraic change in the spot price exceeds the corresponding change in the futures price, but during the summer and autumn the spot price rises less, or falls more, than the futures price. This seasonal pattern means that short hedging is favored, and long hedging discouraged, during the period when stocks in commercial hands are large. Conversely, the seasonality of the basis is favorable to long hedging, and unfavorable to short hedging, when commercial stocks are small." (Houthakker (1968, p.200)) It is these two effects, the Houthakker effect and the basis pattern effect, which leads to a backwardation equilibrium in the futures market when (total) inventories are large.

The argument concerning the basis pattern is an empirical one, based on Houthakker's analysis of data in the corn and cotton markets. From a theoretical point of view, there is some circularity. To argue that short hedging is encouraged by a seasonal pattern of basis changes favorable to short hedging ignores the fact that basis movements reflect in turn the pattern of hedging that actually occurs. Thus, the basis pattern concept of Houthakker's argument is really an argument for a sort of reverse "bubble"-- short hedging increases because short hedgers expect short hedging to decline relative to long hedging during the middle of the crop year, thus producing a favorable pattern of basis change so far as short hedgers are concerned.

To see this, consider a simple two-period forward contract model and let p_t^f, p_t^c denote the futures and spot prices at time t, t = 0, 1, where t = 1 denotes the maturity date of the contract. The structure leads to a perfect hedge situation, i.e., the cash commitments of hedgers equal to the corresponding futures commitments. Let W^S, $W^L > 0$ denote the futures commitments of short hedgers and long hedgers, respectively; then the profits accrued to short hedgers (π^S) and long hedgers (π^L) are :

$$\pi^S = W^S[(p_0^f - p_1^f) + (p_1^c - p_0^c)] = W^S[\Delta p^c - \Delta p^f];$$
$$\pi^L = W^L[(p_1^f - p_0^f) + (p_o^c - p_1^c)] = W^L[\Delta p^f - \Delta p^c].$$

Thus, whenever $\Delta p^c > \Delta p^f$, π^S will increase and short hedgers will increase their futures commitments; also, π^L will decrease and long hedgers will decrease their futures commitments. On the other hand, at t = 1 the short hedgers buy back their futures contracts while the long hedgers sell futures to liquidate their futures commitments; hence $\Delta p^c > \Delta p^f$ implies that p_1^f is relatively low, which in turn implies the short hedging level is low since it represents the demand side so as to determine p_1^f the level. Consequently, the expectation of $\Delta p^c > \Delta p^f$ implies the expectation of declines in short hedging, which in turn encourages current short hedging so that a reverse bubble is thus produced.

Employing a different version of the Houthakker effect, Fort and Quirk actually prove the desirable result (linking the Houthakker effect to backwardation) within a world with an equal number of short and long hedgers, with identical utility functions and identical densities over spot and futures prices. Specifically, assume, at some time point before the maturity date of the futures contract, that hedgers hold—a common probability density function over spot and futures prices at the maturity date. Fort and Quirk interpret the Houthakker effect as stating that, when the (maturity date) spot price is low, the correlation between the spot and futures prices is high; and, conversely, when the spot price is high, the correlation tends to be low. The notion employed here extends the original Houthakker effect based on a seasonal pattern and total inventories to that based on a maturity date pattern and inventories of a specific grade, since low spot prices for a grade are associated with large inventories of the grade[1].

Intutively, since short hedgers endeavor to avoid the risk associated with low spot prices, hence given a Houthakker effect, the futures contract offers a desirable instrument for short hedgers. On the other hand, long hedgers try to avoid the risk of high spot prices, but the low correlation between spot and futures prices at high spot prices limits the effectiveness of the futures contract for long hedging purposes. Under an appropriate specification of the Houthakker effect, a preponderance of short over long hedging, and a backwardation equilibrium, can be established.

In the Fort-Quirk paper, the Houthakker effect is defined in terms of the "closeness" of the spot and futures prices at low versus high spot prices, rather than in terms of correlation levels. But again the conditions required to establish backwardation are highly specialized, and the question as to whether this version of the Houthakker effect is reasonable or not is a matter for empirical tests.

The Statistical Methods

The market characteristics underlying the Houthakker effect are tested using the wheat and corn futures contracts traded on the Chicago Board of Trade (CBOT). The nature and limitations of these data have been described in the previous chapter.

We now turn to the statistical methods employed in this paper. Let p_T^f, p_T^c denote the CBOT wheat (or corn) futures price and the spot price respectively for No.2 Red Soft wheat (or No.2 Yellow corn) at Chicago at the maturity date. The Houthakker effect can be characterized by the following mathematical statement:

$$\mathrm{Corr}(p_T^f,\ p_T^c | p_T^c \leq \underline{p}) > \mathrm{Corr}(p_T^f,\ p_T^c | p_T^c \geq \bar{p}), \qquad (1)$$

where $\underline{p}$ is an appropriately chosen "low" spot price, and $\bar{p}$ is some "high" spot price. For all nonnull events A, $\mathrm{Corr}(p_T^f, p_T^c | A)$ is the correlation of p_T^f, p_T^c conditional on A occurring; i.e.,

$$\mathrm{Corr}(p_T^f,\ p_T^c | A) = \frac{E[(p_T^f - E(p_T^f | A))(p_T^c - E(p_T^c | A)) | A]}{\{E[(p_T^f - E(p_T^f | A))^2 | A]\}^{1/2} \{E[(p_T^c - E(p_T^c | A))^2 | A]^2\}^{1/2}}. \qquad (2)$$

Also, $p_T^c \leq \underline{p}$ indicates that the spot price is low, while $p_T^c \geq \bar{p}$ is used in those cases when the spot price is high. Note that, if $\bar{p} \neq \underline{p}$, the specification of the Houthakker effect involves great arbitrariness, a reasonable choice will be $\bar{p} = \underline{p} = Ep_T^c$. That is what we employ in our empirical studies.

Given the above statements, the null hypothesis for testing the presence of the Houthakker effect is

$$H_0\!:\!\mathrm{Corr}(p_T^f, p_T^c | p_T^c \le Ep_T^c) > \mathrm{Corr}(p_T^c, p_T^c | p_T^c \ge Ep_T^c), \tag{3}$$

while the alternative hypothesis is

$$H_A\!:\!\mathrm{Corr}(p_T^f, p_T^c | p_T^c \le Ep_T^c) \le \mathrm{Corr}(p_T^f, p_T^c | p_T^c \ge Ep_T^c). \tag{4}$$

To simplify the notation, we use LCOR (Lower Conditional Correlation) to denote $\mathrm{Corr}(p_T^f, p_T^c | p_T^c \le Ep_T^c)$, UCOR (Upper Conditional Correlation) to denote $\mathrm{Corr}(p_T^f, p_T^c | p_T^c \ge E_T^c)$. Ideally, we would like to employ a nonparametric test in which no specific assumption on the joint distribution of (p_T^f, p_T^c) is required. This approach, however, is beyond the scope of this paper, simply because once truncations (or hazard rates) are encountered; there are no nonparametric tests available in the literature (see Lee (1984)). Consequently, a parametric test that requires the assumption of the underlying generating process of (p_t^c, p_t^f) must be proposed[2].

To tackle this problem, we assume that $(\log p_t^c, \log p_t^f)$ follows a vector AR(1) process. That is, we have

$$\begin{bmatrix} \log p_t^c \\ \log p_t^f \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \log p_{t-1}^c \\ \log p_{t-1}^f \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{5}$$

where $(\varepsilon_{1t}, \varepsilon_{2t})'$ is an identically, independently distributed bivariate normal random vector. The specification is more general than other specifications currently applied in the literature. For example, if we consider only the futures price equation and let $a_{21} = 0$, $a_{22} = 1$, then we derive a lognormal-normal process for futures prices (see Clark (1973)). Tests of the efficient market hypothesis, on the other hand, are generally based only on the spot price equation. There remain some doubts about the validity of Equation (5); but in general our empirical results indicate that the data fit well into the proposed models. To carry through our empirical studies, we asume the vector AR(1) process to be a corect specification and it is stationary (Obviously, these two assumptions may be invalid for some contracts as shown in Tables 1-4).

Given that $(\log p_t^f, \log p_t^c)$ follows a vector AR(1) process, the variance-covariance matrix of $(\log p_t^f, \log p_t^c)$ satisfies a first-order vector difference equation. Since we are concerned with the variance-covariance matrix at the maturity date, a natural estimate will be the stationary solution of the vector difference equation[3]. That is, if we let Q_t denote the variance-covariance matrix of $(\log p_t^c, \log p_t^f)$ and let $\sum$ denote the variance-covariance matrix of $(\varepsilon_{1t}, \varepsilon_{2t})$, then

$$Q_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} Q_{t-1} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} + \sum, \qquad (6)$$

which is a vector difference equation the stationary solution of which will be treated as the estimate of Q_T. Furthermore, since $(\log p_T^c, \log p_T^f)$ is a bivariate normal random vector, (p_T^c, p_T^f) is a bivariate lognormal random vector. The results derived in Lien (1985) can be applied to derive the estimates for terms included in equation (3).

More specifically, let (x, y) be a bivariate lognormal random vector such that

$$\begin{bmatrix} \log x \\ \log y \end{bmatrix} \sim N\left[\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \right]. \qquad (7)$$

Then, for any $A > 0$ with $B = \log A - \mu_1$, we have

(i) $E(x|x \geq A) = c\,\Phi(\sigma_1 - B/\sigma_1)/\,\Phi(-B/\sigma_1)$

(ii) $E(y|x \geq A) = d\,\Phi((\sigma_{12} - B)/\sigma_1)/\,\Phi(-B/\sigma_1)$

(iii) $E(x^2|x \geq A) = c^2\exp(\sigma_1^2)\Phi(2\sigma_1 - B/\sigma_1)/\,\Phi(-B/\sigma_1)$

(iv) $E(xy|x \geq A) = cd\,\exp(\sigma_{12})\Phi((\sigma_1^2 + \sigma_{12} - B)/\sigma_1)/\,\Phi(-B/\sigma_1)$

(v) $E(y^2|x \geq A) = d^2\exp(\sigma_2^2)\Phi((2\sigma_{12} - B)/\sigma_1)/\,\Phi(-B/\sigma_1)$,

where $c = \exp(\mu_1 + \sigma_1^2/2)$, $d = \exp(\mu_2 + \sigma_2^2/2)$ and $\Phi(.)$ is the cumulative density function of a standard univariate normal variable. Similarly, the conditional expectations, $E(x|x \leq A)$ and $E(x^2|x \leq A)$ etc., can be constructed. Consequently, LCOR and UCOR can be derived upon applying the above formulae once we replace (x, y) by (p_T^c, p_T^f) and set $A = \exp(\mu_1 + \sigma_1^2/2)$. More specifically, let W_{ij} denote the (i,j) element of Q_T. Then

$$\mathrm{Corr}(p_T^f, p_T^c | p_T^c \geq Ep_T^c) = \frac{\exp(W_{12})\Phi(\theta_4)\Phi(\gamma) - \Phi(\theta_1)\Phi(\theta_2)}{\{\exp(W_{11})\Phi(\theta_3)\Phi(\gamma) - \Phi^2(\theta_1)\}^{1/2}\{\exp(W_{22})\Phi(\theta_5)\Phi(\gamma) - \Phi^2(\theta_2)\}^{1/2}} \qquad (8)$$

$$\mathrm{Corr}(p_T^f, p_T^c | p_T^c \leq Ep_T^c) = \frac{\exp(W_{12})\Phi(\alpha_4)\Phi(\delta) - \Phi(\alpha_1)\Phi(\alpha_2)}{\{\exp(W_{11})\Phi(\alpha_3)\Phi(\delta) - \Phi^2(\alpha_1)\}^{1/2}\{\exp(W_{22})\Phi(\alpha_5)\Phi(\delta) - \Phi^2(\alpha_2)\}^{1/2}} \qquad (9)$$

where $\theta_1 = W_{11}^{1/2}/2$, $\theta_2 = W_{12}W_{11}^{-1/2} - (W_{11}^{1/2}/2)$, $\theta_3 = 3W_{11}^{1/2}/2$, $\theta_4 = W_{12}W_{11}^{-1/2} + (W_{11}^{1/2}/2)$, $\theta_5 = 2W_{12}W_{11}^{-1/2} - (W_{11}^{1/2}/2)$, $\gamma = -W_{11}^{1/2}/2 = -\delta$; also $\alpha_i = -\theta_i$, $\forall\, i = 1,\ldots,5$. In Lien (1985), it was also shown that the Houthakker effect may be accepted or rejected depending upon the values of the parameters given a bivariate lognormal specification of (p_T^f, p_T^c).

In fact, the appropriate test statistics for the presence of the Houthakker effect is certainly LUCOR = LCOR - UCOR. That is, if LUCOR > 0, then we accept the null hypothesis that the Houthakker effect prevails; otherwise, we reject the null hypothesis. The above formulae provide us with the estimate of LUCOR; yet the significance of a test result still requires the estimate of the variance of LUCOR. To this point, Rao's large sample theory can be applied once the sample size is sufficiently large[4]. Unfortunately, our empirical studies include only small samples because we have to consider each contract separately, which in turn is due to the nonstationary pattern of futures prices across contracts (see Anderson (1985))[5]. On the other hand, the exact distribution of LUCOR for finite sample cases is highly complicated. As a result, our conclusions will simply draw from the signs of LUCOR regardless of the significance level.

Test Results and Implications

The test results are presented in Tables 1-6. Tables 1 and 3 present the estimation of wheat and corn futures price equations, respectively. In general, the two coefficients associated with previous futures and spot prices (in logarithmic terms) are both highly significant different from zero. Moreover, $\bar{R}^2$ is also quite large while the corn futures price equation generally has a greater $\bar{R}^2$ than the wheat futures price equation. These results are expected, since the specific spot commodities we considered are typically the relevant delivery options in determining the futures prices at

TABLE 5.1 ESTIMATION OF FUTURES PRICE EQUATIONS (WHEAT)

CONTRACT TYPE	CONSTANT TERM	COEFFICIENT OF $\log(p^{f}_{t-1})$	COEFFICIENT OF $\log(p^{c}_{t-1})$	D.W. STATISTICS	$\bar{R}^2$
7803#	0.2756	0.2081**	0.7572**	1.3103	0.9466
7805	-0.9577**	0.2285**	0.8920**	0.9877	0.9651
7807	1.0602	0.7545**	0.1137	1.6113	0.6046
7809	-0.2750	0.7584**	0.2752**	2.0016	0.8034
7812	1.3874	0.5715**	0.2593**	1.6191	0.4551
7903	1.9679**	0.6290**	0.1317**	2.7273	0.8282
7905	0.3297	0.4662**	0.4900**	1.4086	0.9482
7907	-0.7518*	0.3523**	0.7367**	1.0233	0.9427
7912	1.2196*	0.2639**	0.5921**	0.6602	0.7865
8003	0.2915	0.7976**	0.1676*	1.8846	0.9188
8109	1.8329*	0.6598**	0.1201**	2.0206	0.6210
8112	-2.5904**	0.6672**	0.6456**	1.6954	0.9529
8203	-1.4028**	0.6146**	0.5572**	1.7828	0.9121
8205	1.7427**	0.5823**	0.2057**	1.2420	0.9274
8207	2.7716**	0.7480**	-0.0894	1.6478	0.5171
8209	1.7321	0.0410	0.7535**	0.4973	0.1695
8212	0.5564	1.1055**	-0.1743	1.4227	0.8432

* significant at the 90% level
** significant at the 95% level

\# The contract with March 1978 being the delivery month is specified as 7803; similarly for other contracts.

TABLE 5.2 ESTIMATION OF SPOT PRICE EQUATIONS (WHEAT)

CONTRACT TYPE	CONSTANT TERM	COEFFICIENT OF $\log(p^{f}_{t-1})$	COEFFICIENT OF $\log(p^{c}_{t-1})$	D.W. STATISTICS	$\bar{R}^2$
7803	0.3236	-0.0861	1.0454**	1.7995	0.8591
7805	1.1729	-0.0142	0.8684**	2.0083	0.6706
7807	2.5565**	-0.1367	0.8197**	1.7209	0.5506
7809	3.1498**	0.2745*	0.3381	1.8273	0.3575
7812	7.7938**	-0.3143	0.3642**	1.4694	0.1034
7903	1.2827	0.0424	0.8018**	2.1845	0.6582
7905	0.6432	0.6026**	0.3239	2.3525	0.8473
7907	1.6860**	0.0675	0.7320**	1.8808	0.7222
7909	1.5138	0.1549	0.6638**	1.9107	0.6381
7912	2.9259**	-0.1831	0.8328**	2.0681	0.5799
8003	1.9477**	0.6441**	0.1211	1.7751	0.7011
8109	-0.5931	0.8144**	0.2515	2.0165	0.1370
8112	0.1930	0.0985	0.8775**	1.4168	0.8817
8203	1.6721**	-0.2008**	0.9965**	1.6283	0.6843
8205	-1.8311	0.5377	0.6842**	1.9191	0.8469
8207	3.6760**	-0.2507	0.7975**	1.5296	0.5679
8209	2.8035**	-0.0231	0.6742**	1.6225	0.3710
8212	0.2710	0.2846*	0.6820**	1.4163	0.8306

TABLE 5.3 ESTIMATION OF FUTURES PRICE EQUATIONS (CORN)

CONTRACT TYPE	CONSTANT TERM	COEFFICIENT OF $\log(p^f_{t-1})$	COEFFICIENT OF $\log(p^c_{t-1})$	D.W. STATISTICS	$\bar{R}^2$
7803	0.4954	0.4465**	0.4904**	1.8139	0.9518
7805	-2.0608**	0.6700**	0.5928**	1.6075	0.8675
7807	-0.2700	0.5058**	0.5293**	2.0461	0.9867
7809	-0.0075	0.5316**	0.4691**	1.4802	0.7971
7812	-1.4965	0.8859**	0.3069**	1.4552	0.8667
7903	3.9123**	-0.1085	0.6038**	1.7514	0.4640
7905	-0.1976	0.2311**	0.7933**	1.7308	0.9595
7907	-0.1907	0.0091	1.0171**	1.0621	0.9770
7909	1.4781	0.0224	0.7909**	1.5052	0.6934
7912	2.0217**	0.5695**	0.1751**	1.4995	0.7565
8003	0.6532	0.9489**	-0.0319	2.5884	0.8893
8005	3.1592**	0.1705	0.4304**	1.1017	0.7451
8007	0.6889**	0.3254**	0.5901**	1.7963	0.9879
8009	-0.2090	0.1009*	0.9270**	1.2800	0.9678
8012	0.0167	0.8898**	0.1091	1.6147	0.7691
8103	-5.5203**	0.6014**	1.0768**	1.3843	0.9051
8105	-3.2578**	0.7758**	0.6236**	1.2978	0.9084
8107	1.4369	0.7165**	0.1070	1.5316	0.6340
8109	0.0874	0.4328**	0.5571**	1.4012	0.9794
8112	-0.5485	0.9123**	0.1574	1.7899	0.9548
8203	-1.3044*	0.9092**	0.2562**	2.3274	0.9564
8205	1.6161	0.9445**	-0.1498	2.4664	0.8694
8207	0.1132	0.6972**	0.2903**	2.1613	0.8563
8209	1.0708*	0.9015**	-0.0404	1.5653	0.8497
8212	2.4837**	0.6086**	0.0718	2.0521	0.7334

TABLE 5.4 ESTIMATION OF SPOT PRICE EQUATIONS (CORN)

CONTRACT TYPE	CONSTANT TERM	COEFFICIENT OF $\log(p^f_{t-1})$	COEFFICIENT OF $\log(p^c_{t-1})$	D.W. STATISTICS	$\bar{R}^2$
7803	-0.3185	0.2172	0.8241**	1.6463	0.8918
7805	2.0112*	0.0375	0.7050**	2.3844	0.4727
7807	-0.0457	-0.1415	1.1472**	1.4569	0.9627
7809	1.8982**	-0.1017	0.8546**	1.9336	0.6219
7812	2.0912*	-0.0383	0.7680**	1.6696	0.5228
7903	-0.3614	0.2383	0.8089**	2.0497	0.8332
7905	1.3115	0.1468	0.6868**	2.0729	0.6671
7907	0.7460	0.0573	0.8496**	1.6294	0.8870
7909	3.5678**	0.0156	0.5350**	1.8558	0.2523
7912	0.3281	0.1723	0.7858**	2.4470	0.8741
8003	5.7174**	0.2839*	-0.0103	1.4390	0.4694
8005	3.2113**	-0.3772**	0.9707**	2.5903	0.7390
8007	0.1457	-0.1403	1.1231**	1.8817	0.9575
8009	1.6184**	0.0202	0.7810**	1.9511	0.6950
8012	4.8979**	-0.1055	0.5058**	2.2097	0.2165
8103	4.5352**	0.0998	0.3435**	1.8868	0.1587
8105	2.5083**	0.0536	0.6390**	1.4169	0.4903
8107	3.1610	0.4780*	0.1311	1.9666	0.1292
8109	0.1536	0.3639**	0.6150**	2.0363	0.9217
8112	0.6772	0.0973	0.8157**	2.1428	0.8224
8203	1.4149**	-0.0963**	0.9167**	2.2910	0.8397
8205	2.6303**	-0.0244	0.6901**	2.1635	0.6602
8207	1.1549	-0.1026	0.9561**	1.6793	0.7362
8209	1.0524	0.0833	0.7795**	1.1122	0.7665
8212	2.0566**	0.0570	0.6773**	2.6284	0.8104

TABLE 5.5 TEST RESULTS FOR THE PRESENCE OF THE HOUTHAKKER EFFECT IN CBOT WHEAT FUTURES MARKET

CONTRACT TYPE	ESTIMATE OF LCOR	ESTIMATE OF UCOR	ESTIMATE# OF LUCOR
7803	0.1893	0.1924	-0.0031
7805	-0.0590	-0.0520	-0.0070
7807	-0.3074	-0.3053	-0.0021
7809	0.6923	0.6922	0.0001
7812	-0.3075	-0.2973	-0.0102
7903	0.3066	0.2821	0.0245
7905	0.6847	0.6865	-0.0018
7907	0.2199	0.2163	0.0036
7909	0.1279	0.1174	0.0105
7912	0.0153	0.0001	0.0152
8003	0.7867	0.7975	-0.0108
8109	0.5690	0.5714	-0.0024
8203	0.1013	0.1051	-0.0038
8205	0.7049	0.7047	0.0002
8207	-0.5065	-0.5031	-0.0034
8209	-0.0111	-0.0273	0.0162
8212	0.8117	0.8152	-0.0035

LUCOR = LCOR - UCOR

TABLE 5.6 TEST RESULTS FOR THE PRESENCE OF THE HOUTHAKKER EFFECT IN CBOT CORN FUTURES MARKET

CONTRACT TYPE	ESTIMATE OF LCOR	ESTIMATE OF UCOR	ESTIMATE OF LUCOR
7805	0.3645	0.5585	-0.1940
7807	0.8675	0.8699	-0.0024
7809	-0.0992	-0.1184	0.0192
7812	-0.4974	-0.4955	-0.0019
7903	0.1493	0.1502	-0.0008
7905	0.2174	0.2311	-0.0137
7907	0.0891	0.0897	-0.0006
7912	0.3282	0.3344	-0.0062
8003	0.8504	0.8584	-0.0079
8005	0.1235	0.1275	-0.0041
8012	-0.6799	-0.6683	-0.0117
8103	0.5129	0.5228	-0.0098
8105	0.6392	0.6190	0.0201
8107	0.5754	0.5767	-0.0013
8203	-0.2160	-0.2152	-0.0008
8207	-0.0487	-0.0514	0.0027
8209	0.5859	0.5896	-0.0038
8212	0.3388	0.2715	0.0673

termination of the futures contracts[6]. In other words, they are "basis grades" for CBOT wheat and corn futures contracts, respectively. Therefore, the close link between spot and futures prices is expected.

Moreover, since there are only three deliverable grades in corn futures while there are eleven deliverable grades in CBOT wheat futures, the uncertainties about which grade constitutes the basis grade are propably greater in the latter market. Consequently, the estimation of the corn futures price equation is expected to be better than that of the wheat futures price equation. In fact, the expectation is fulfilled in terms of $\bar{R}^2$ or Durbin-Watson statistics.

There are eight out of eighteen wheat futures price equations that correspond to unsatisfactory Durbin-Watson statistics. This implies that some sort of serial correlation in the disturbance terms may be present. For corn futures price equations, the Durin-Watson statistics are quite satisfactory in almost all cases[7].

Tables 2 and 4 present the estimation of the wheat and corn spot price equations. The results are not as good as those of the corresponding futures price equations. The reason is as follows : the specific spot price we utilized tends to be correlated with all the other spot prices; but the futures price reflects only the price level of the basis grade. Therefore, the current spot price is not closely related to the previous futures price when the basis grade changes frequently over time. The estimation results actually confirm the above statements : only five out of twenty five corn spot price equations and six out of eighteen wheat spot price equations contain significant coefficients associated with previous futures prices. The Durbin-Watson statistics for all spot price equations are quite satisfactory, either in terms of absolute levels or in comparison to corresponding futures price equations. This suggests that the AR(1) process may be a good approximation for wheat and corn spot prices.

Now we turn to the test results related to the Houthakker effect. From Table 5, ten out of seventeen CBOT wheat futures contracts have negative LUCOR, which is contradictory to the Houthakker effect conjecture. Although the three contracts with highest absolute values of LUCOR support the presence of the effect, the evidence is quite weak. On the other hand, fourteen out of eighteen CBOT corn futures contracts have negative LUCOR. While the other four contracts provide significantly positive LUCOR, this provides very weak support for the existence of the Houthakker effect. On the basis of the data summarized here, the Houthakker effect was shown not to be present in either the CBOT wheat nor the corn futures markets[8,9]. Consequently, the conjecture that the Houthakker effect is an empirically significant foundation for a backwardation equilibrium appears to be false. In particular, the empirical results indicate that the correlation between the futures and spot prices at the maturity date does not depend on the spot price level in the manner argued by Houthakker. In fact, the empirical results suggest that short and long hedgers are in symmetric positions in terms of correlation consideration. That is, short and long hedgers in CBOT corn or wheat futures markets actually face equivalent risk levels.

Footnotes For Chapter 5

1. For some other implications arising from the presence of the Houthakker effect, see Quirk(1985). Also, note that Quirk's expression for the Houthakker effect was in terms of correlation, which differs from that of Fort and Quirk (1984).

2. If the joint distribution of spot and futures prices is symmetric, then we have LCOR = UCOR (see Lien (1985)). Consequently, an asymmetric joint distribution is needed to effect test the presence of the Houthakker meanifully.

3. There are some cases in which the stationary solution does not exist; all these cases are excluded from our consideration when dealng with the tests on the Houthakker effect.

4. In fact, the variance of LUCOR cannot exceed 4. Nonetheless, applications of large sample theory to our models yield a much larger asymptotic variance. As a result, the significance level calculated with the asymptotic variance becomes meaningless. The difficulty is generally expected in the literature of futures markets (See Taylor (1985)).

5. The nonstationary patterns of futures prices across contracts eliminate the possibility of pooling all maturity date data over all contracts to investigate the joint distribution of spot and futures prices. On the other hand, if Houthakker's argument is

correct, the price level will certainly depend on the stock level. Thus, we cannot pool contracts together and then impose stationary assumptions.

6. Another interesting result is that four out of five insignificant estimates of coefficients associated with previous futures and spot prices in CBOT wheat futures markets appear after May 1982. Also, if we use May 1982 as the dividing line, we find that the $\bar{R}^2$ associated with the period after May 1982 is much lower than that associated with the period before May 1982. This indicates that Chicago is a better candidate for "basis location" than St. Louis, once we notice that, after May 1982, the spot price series is recorded at St. Louis instead of Chicago.

7. Since lag endogenous variables are invloved in the specification of Equation (5), the Durbin-Watson statistics will be biased toward the null hypotheses. Therefore, the statements associated with satisfactory Durbin-Watson statistics should be treated with caution.

8. We also tested the Houthakker effect and the basis pattern effect on CBOT wheat futures using Houthakker's approach. The result is again negative; but here the problem may arise because we consider only the five-years period, which is rather short compared to Houthakker's sample data.

9. If we investigate the possible seasonal patterns of our test

results (especially the one suggested by Houthakker), again there is no significant conclusion.

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