ANALYTICAL MODELS FOR THE DYNAMICS OF BUILDINGS

Thesis by

Sudhir Kumar Jain

In Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

California Institute of Technology
Pasadena, California

1983

(Submitted May 25, 1983)

ABSTRACT

This thesis investigates the significance of in-plane floor flexibility on the dynamics of buildings, and develops analytical models for structures that have flexible floor diaphragms. Experience with past earthquakes demonstrates that this feature is particularly important for long, narrow buildings and buildings with stiff end walls. In the method developed in this study, the equations of motion and appropriate boundary conditions for various elements of the structure are written in a single coordinate system and then are solved exactly.

One- and two-story buildings with end walls are analyzed by treating their floors and walls as bending and shear beams, respectively. The resulting equations of motion and the boundary conditions are solved to obtain the dynamic properties of the structure. The expected low torsional stiffness of the end walls or frames is confirmed by analysis of a single-story example structure. Study of a similar two-story building showed that the first two modes, dominated by the floor and the roof vibrations, make the largest contributions to the total base shear in the structure.

Floors of multistory buildings with end walls (or frames) are idealized as equivalent, distributed beams while the walls or frames are treated as bending or shear beams. Analysis of a nine-story building showed that the structure possesses several lower modes in which floors vibrate essentially as pinned-pinned beams.

Buildings with large numbers of uniform stories and frames (or walls) are treated as vertically-oriented anisotropic plates. It is concluded that the floors in such buildings can be assumed rigid for seismic analysis, since the modes involving floor deformations are not excited by uniform ground motion.

The approach can be generalized further to study more complex structures. An example is the Imperial County Services Building, which has two end walls in the upper stories and several walls in the ground story. The analytical model of this building predicts several important features of the complex dynamic behavior of the structure.

ACKNOWLEDGMENTS

I wish to express my sincere thanks and gratitude to my advisor Professor P. C. Jennings for the encouragement and guidance he has provided during the course of this study and in the preparation of this thesis. I am also thankful to Professor G. W. Housner for many useful discussions.

This investigation was supported by the National Science Foundation, Earthquake Hazard Mitigation Program under Grant No. CEE-81-19962. The financial support through the Robert A. Millikan Fellowship, as well as the teaching and research assistantships and the tuition scholarships provided by the California Institute of Technology are gratefully acknowledged.

Special thanks are due to Mrs. Sharon Beckenbach and Mrs. Gloria Jackson for their skillful and speedy typing of the manuscript, and to Miss C. Lin for helping me with the illustrations.

Finally, I wish to dedicate this thesis to my parents, Sri Narendra Kumar Jain and Smt. Magan Mala Jain who have done so much for me, and to my eighth grade teacher Sri Mueed Ahmed Azmi without whose inspiration and encouragement this thesis may never have been written.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

1.1 SCOPE

It is important in seismically active areas to provide safe and economical protection for life and limb by making adequate provisions for earthquake resistance in buildings. For most ordinary buildings, it is sufficient to provide earthquake resistance in the buildings by means of a suitable building code. This usually involves static analysis of the building for prescribed lateral forces, which take into account in an approximate manner the effects of building characteristics, soil characteristics, seismic risk in the area, importance of the building, etc. However, there are buildings that have some special characteristics which make it difficult to model their dynamic behavior satisfactorily by a code-type, static analysis. Such buildings warrant detailed dynamic analyses for satisfactory answers to questions concerning their behavior during earthquakes. Included in this category are high-rise buildings, buildings with extreme plan dimensions (e.g., long and narrow buildings), buildings with eccentric centers of mass or stiffness (this leads to coupled torsional and translational motion), buildings with vertical set-backs, soft first-story buildings or buildings with other unusual characteristics (Arnold, 1980b; Arnold and Elsesser, 1980).

Several computer codes are commercially available for dynamic analysis of buildings. These codes treat a building as a reduced system, with finite number of degrees of freedom, while the actual building is a continuum with an infinite number of degrees of freedom. To keep the computer costs down, it is important to reduce the number of degrees of freedom involved in the calculations to a relatively small number, and to achieve this, assumptions have to be made about the behavior of the building. One such assumption, that is included as a requirement in almost all of the popular computer programs available for the dynamic analysis of buildings, is that the floors are rigid in their own planes. This implies rigid body motion in these planes, and thus the degrees of freedom for lateral earthquake analysis reduce to three per story: two translational and one rotational degree of freedom for every floor. The most common alternative to the assumption of rigid floors would be to use finite element methods to model the girders, beams, etc., of the floor system. This approach allows for flexibility of the floors, but involves many more degrees of freedom.

The rigid floor assumption is a valid assumption for many buildings. However, there are situations where the floor diaphragms cannot be considered as rigid. In fact, there are buildings which have exhibited significant in-plane floor flexibility during earthquakes; some of these buildings are described in Chapter II.

The flexibility of floor diaphragms can alter the dynamic behavior of a building, from that obtained assuming rigid diaphragms, in many ways. For instance, in analysis of rigid diaphragms, the various lateral load resisting elements, e.g., walls or frames, are assumed to share the total lateral load in proportion to their stiffnesses. This is due to the condition that at each floor level the lateral displacements in all the frames or walls have to be the same (for buildings with no torsional coupling). However, a flexible floor diaphragm may distribute the loads in a different manner. This may result in certain frames receiving much higher lateral loads than expected from an analysis using the rigid-floor assumption. As another example, the deformation in the diaphragm may induce torsional moments in frames or walls in addition to the expected shear. Thus, if the joints of the structure are not adequately designed for these moments, or if the frame (or wall) is not ductile enough, torsional damage may occur during earthquakes.

This study treats buildings for which floor diaphragms should be considered as flexible. The emphasis of the work is upon developing and presenting continuum models for some important classes of buildings with flexible floor diaphragms. From these results, it is possible to make some general conclusions regarding the nature and importance of the effects of in-plane floor flexibility on the earthquake response of the structures.

1.2 PAST WORK AND CURRENT STATUS

Blume, Sharpe and Elsesser (1961) seem to have been the first to report "long natural periods of roof or floor diaphragms" in some one-, two- and three-story buildings. Blume (1962) calculates the "diaphragm period" by considering the roof diaphragm as beams with simply supported or fixed-fixed boundary conditions. Nielsen (1964, 1966) reported one "free-free beam mode" with a frequency of 4.9 Hz in his dynamic tests on a 9-story steel frame building at the Jet Propulsion Laboratory, in Pasadena. Udwadia and Trifunac (1974) give mode shapes, some of them involving significant floor-diaphragm deformations, obtained from ambient vibration tests carried out on the same building.

To obtain the natural periods and mode shapes of multistory buildings with flexible floors, Goldberg and Herness (1965), and Goldberg (1966) have suggested use of the slope-deflection equations, while lumping the mass at the intersections of floors and frames (or walls). In an another study, Maybee, Goldberg and Herness (1966) developed a "separable model" for buildings with identical floors and identical frames. They showed that for such buildings one could obtain the frequencies and the mode shapes for the entire building by solving one typical floor problem and one frame problem. Recently, it was shown by the writer (1983) that for such "separable buildings" the modal participation factors for uniform earthquake ground motion are zero for modes involving floor diaphragm deformations.

Two "typical" two-story buildings were studied using lumped-mass model by Shepherd and Donald (1967), and they have concluded that neglecting the floor flexibility "does not significantly change the dynamic properties" of buildings. However, in a study on a single-story building by lumped-mass approach, Blume and Jhaveri (1969) have shown that the floor flexibility could indeed be very significant, especially for the type of buildings they have analyzed.

Other analytical studies on such buildings include one by Ostrom (1974) where he has modelled the floors by beams and columns or walls by springs. Irwin (1975) has presented a "stiffness matrix method" to analyze such "multistory shear wall buildings." Karadogan (1980) has suggested a "simplified force method" for the analysis of "slab type" structures. A method for one type of structures has been presented by Rutenberg (1980) which allows examination of the flexibility of floor slabs using plane frame procedures. Unemory (1978), and Unemory, Roesset and Becker (1980) have carried out a parametric study on crosswall building systems including floor flexibility using finite element models.

Karadogan, et al. (1980) and Nakashima, Huang and Lu (1981) have reported the results of in-plane shear tests on reinforced concrete flat plates; and Kolston and Buchanan (1980) have discussed the design requirements for reinforced concrete diaphragms.

1.3 OUTLINE OF PRESENT WORK

This study develops some continuum techniques for the analyses of buildings which have the possibility of significant in-plane floor flexibility. For simple single- or two-story buildings, the floors and the walls (or columns) have been treated as beams, and the resulting beam equations and boundary conditions have been combined to obtain the characteristic equation of the combined system. This equation can easily be solved on a small computer or programmable calculator to obtain the natural periods. Thus, the dynamic properties of the building can be obtained in an "exact" manner. In addition, to simplify the numerical work even further in some instances, perturbation techniques have been used to obtain the first order correction terms, to be added to the results of simple standard cases, for example, a pinned-pinned beam.

Multistory buildings with lateral load resistance systems consisting of only two end walls or frames are treated next. For such buildings, the floors are approximated by a continuous distribution of thin floors along the height of the building. These thin, distributed floors have no contact with the adjacent floors, and have been treated as beams. The end walls (or frames) are treated as uniform beams (bending or shear). The resulting system has been solved exactly to obtain the characteristic equation. From the roots of this equation, it is possible to obtain the natural frequencies, mode shapes and the participation factors for the entire building.

Multistory buildings that have uniformly distributed frames or walls are treated as vertically-oriented plates. It has been a common practice in earthquake engineering to model some features of multistory buildings by shear beams (e.g., Jennings, 1969; Hoerner, 1971). This plate concept, introduced in Chapter VI, is a generalization of that concept, and should give results that are comparable in applicability to those of the shear beam. This plate model allows one to obtain closed form solutions for frequencies, mode shapes and participation factors. These results, though approximate, are sufficiently accurate to allow various qualitative conclusions about the behavior of such buildings.

The above concepts have been generalized further to study buildings with some unusual features, such as a soft first story. By adding extra elements, such as beams, a distributed column system, etc., it is possible to include the influence of end walls, or a different story height in the first or the top story in an otherwise uniform building. The Imperial County Services Building, a six-story building with a soft first story, that sustained severe damage during the October 15, 1979 earthquake is studied using the concepts developed in this part of the study.

1.4 ORGANIZATION

This thesis has been divided into eight (8) chapters. Chapter I is an introduction, while Chapter II describes the evidence of significant floor flexibility as seen in past earthquakes. Chapters III and IV present treatments on single- and two-story buildings, respectively. Chapter V treats multistory buildings with lateral load resistance system (walls or frames) only at the two ends. Chapter VI describes the "plate concept" developed for multistory buildings with uniformly distributed frames. Multistory buildings that could be a combination of the earlier types, are addressed in Chapter VII, and Chapter VIII presents a summary and conclusions.

Chapter III also contains summaries of beam theories and relevant boundary conditions, a discussion of the distributed floor concept, and a note on matching the boundary conditions at junctions of elements. The concepts are extensively used in subsequent chapters. Thus, after reading this background material in Chapter III, it should be possible to read the following chapters independently of each other.

Mathematical notations have been defined where they first appear, and are also listed in the "Notation" section.

CHAPTER II

EVIDENCE OF FLOOR FLEXIBILITY IN PAST EARTHQUAKES

2.1 INTRODUCTION

Past earthquakes have been a great source of information for structural engineers about the dynamic behavior of structures. During an earthquake, when a structure sustains damage of any kind, it tells something about the structure. An investigation of the extent and pattern of the damage may uncover the weaknesses that led to the damage, thereby enabling one to avoid the same mistakes in new buildings. In fact, the situation can be compared to an actual full-scale destructive test of a structure, under field conditions. Hence, it is important to analyze past failures carefully, and to learn relevant lessons from them.

In recent years, because of increased interest in the earthquake safety of structures, there has been an increased number of installations of instruments in buildings in order to measure the motion of various parts of the structure during an earthquake. This provides data which can be used to interpret the cause of damage in the building, should a building suffer damage. Also, even if the building is undamaged after the earthquake, these records provide valuable insight into structural modelling and data about the dynamic properties of buildings, for example, the amount of equivalent damping.

In this chapter, five buildings are described which have exhibited significant floor-diaphragm deformations during past earthquakes. The first four sustained severe damage due to strong shaking, while the fifth one was undamaged after the earthquake. The later two buildings were instrumented, and the records obtained from them indicate the significance of in-plane floor flexibility in the dynamics of actual buildings. The five buildings represent a wide variety of building types; this indicates that in-plane floor flexibility may be more significant than it has been acknowledged to be in the past.

2.2 ARVIN HIGH SCHOOL BUILDING

In 1952, Arvin High School consisted of a large group of buildings, constructed during 1949-51. Because they were new, they met the requirements of California's Field Act. During the magnitude 7.7 Kern County (southern California) earthquake of July 21, 1952, most of these buildings performed extremely well. The only exception was the two-story Administrative Building. This long, narrow building had a roof 197 ft long and 46 ft wide. In the transverse direction, the lateral load resistance was provided by the end walls while the more flexible intermediate columns took only vertical loads. The building was a "reinforced concrete building with brick veneer on walls except that the second story wall at the west end was 8-1/2 inches thick reinforced grouted brick masonry without openings" (Steinbrugge and Moran, 1954). Figure (2.1) gives some details of the building.

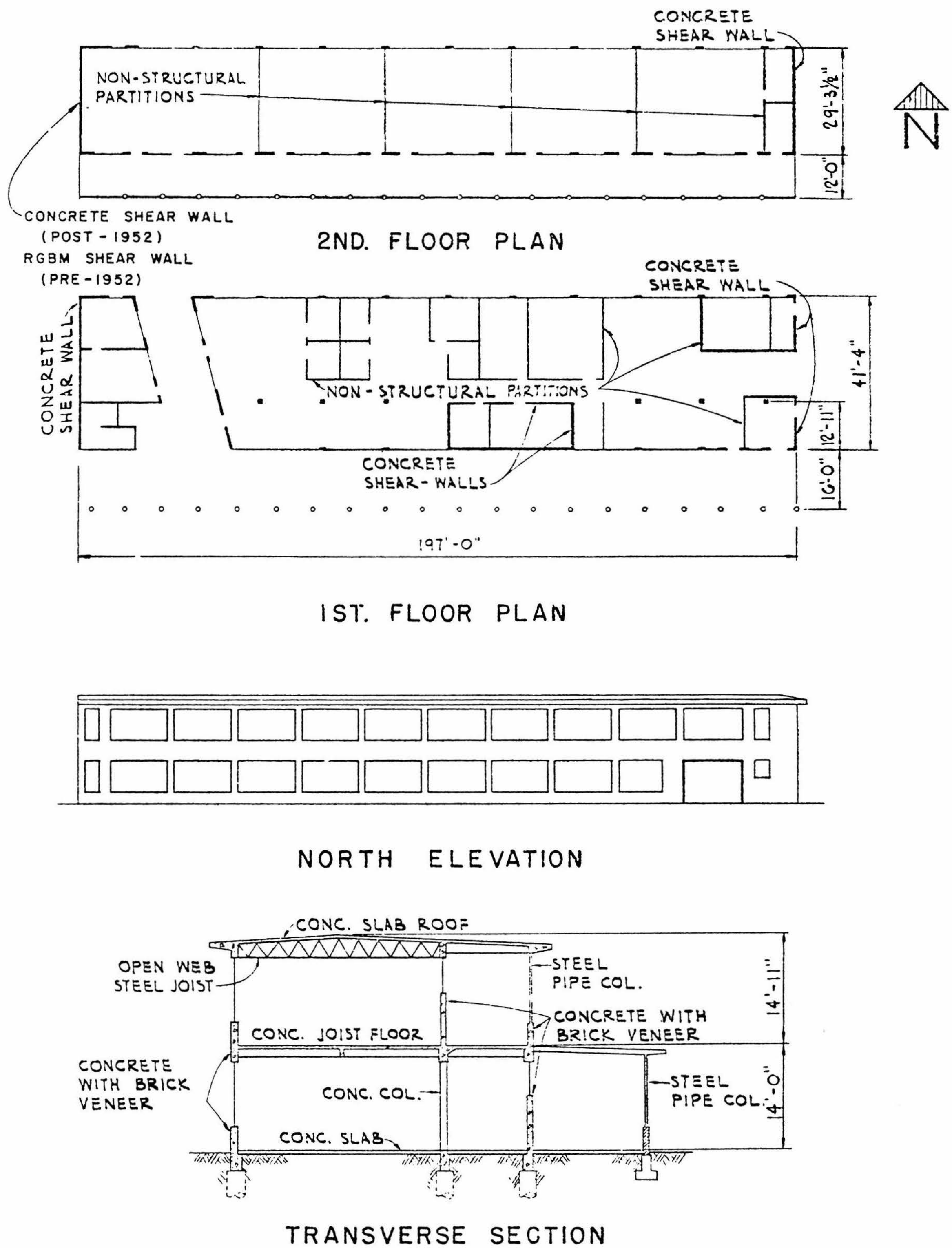


Figure 2.1. DETAILS OF THE ARVIN HIGH SCHOOL, ADMINISTRATIVE BUILDING (from Blume, et al., 1961).

The second story wall at the west end was extensively damaged as a result of the main shock. The effect of the earthquake upon this wall has been described by Steinbrugge (1970):

> The damage to this second-story wall consisted of x-cracks from diagonal tension forces, plus separation at the building corners due to diaphragm deflections causing torsional stresses in the damaged wall.

Figure (2.2) shows some of the details of damage to the building. Besides the flexible diaphragm causing damage by forcing the wall to twist, poor workmanship in the wall was noted. One- and two-story models of this building, studied in subsequent chapters of this thesis, reveal that in the fundamental mode of vibration the flexibility of the diaphragm was much more significant than the flexibility of the end walls.

2.3 WEST ANCHORAGE HIGH SCHOOL

During the Alaskan earthquake (magnitude 8.4) of March 27, 1964, the classroom wing of the West Anchorage High School suffered severe damage. The building was built in 1952-53, with flat-slab construction of reinforced, cast-in-place concrete. The building was designed for zone 2 requirements of 1949 Uniform Building Code. The framing plans of this two-story building are shown in Figure (2.3). Such buildings, consisting of two wings joined at an angle (e.g., L- or V-shape plans) are very susceptible to damage induced by floor flexibility, because the

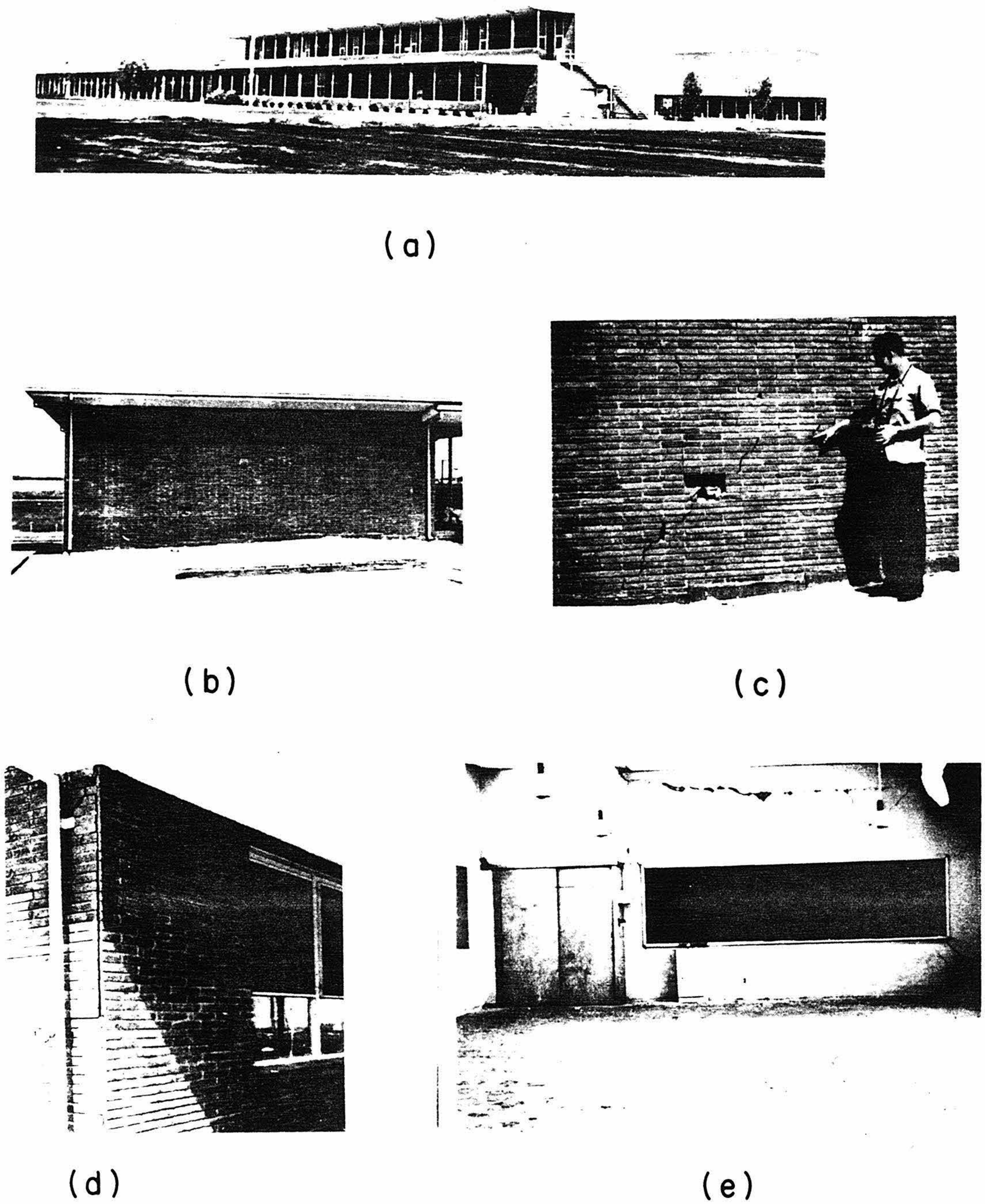
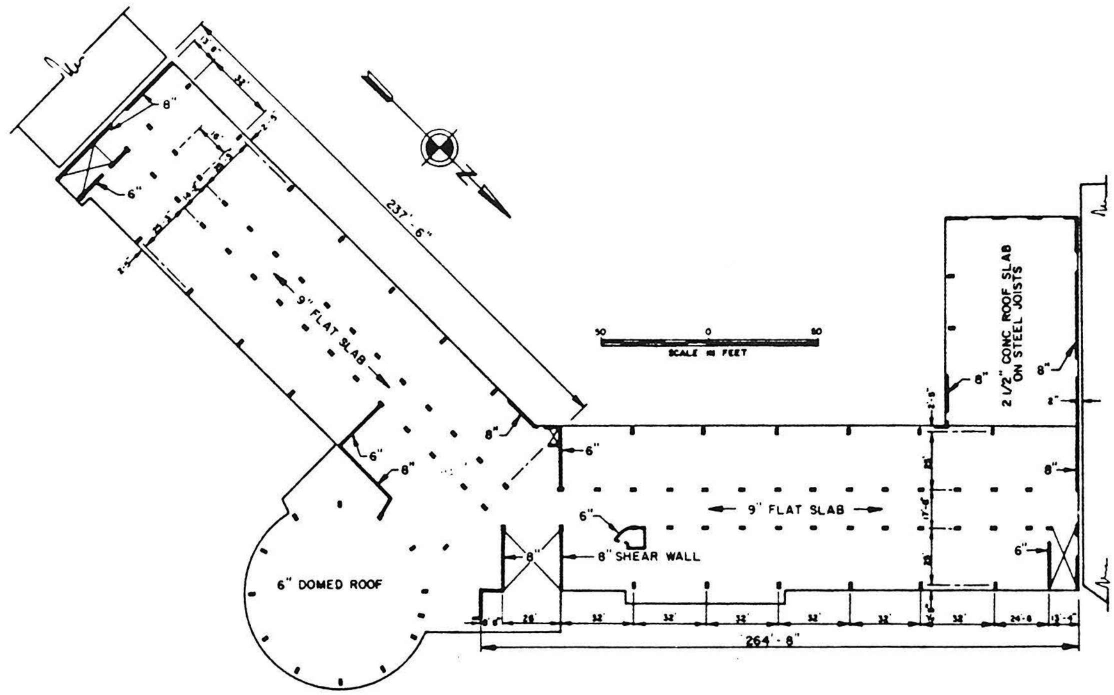
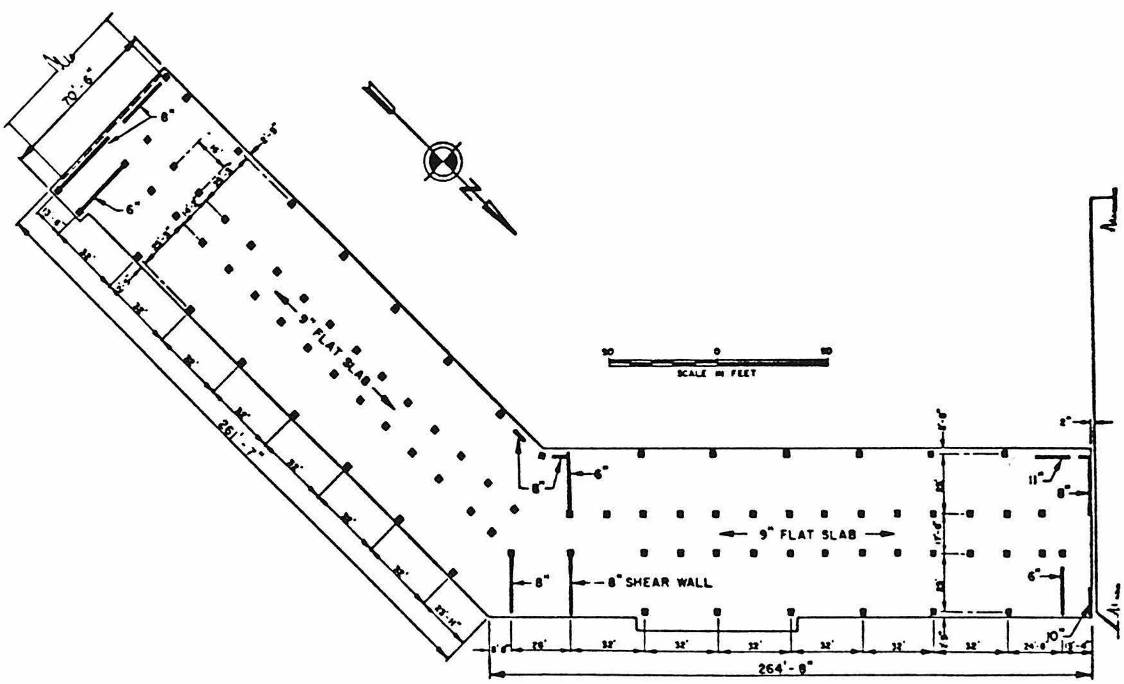


Figure 2.2. DETAILS OF DAMAGE TO ARVIN HIGH SCHOOL, ADMINISTRATIVE BUILDING (from Steinbrugge and Moran, 1954). (a) GENERAL VIEW. (b) SECOND STORY BRICK WALL. (c) CLOSE-UP OF THE BRICK WALL. (d) CRACK AT MITERED CORNER. (e) INTERIOR OF THE WALL SHOWN IN LAST THREE FIGURES.



(a)



(b)

Figure 2.3. FRAMING PLANS OF WEST ANCHORAGE HIGH SCHOOL BUILDING (from George, et al., 1973). (a) FIRST FLOOR PLAN. (b) SECOND FLOOR PLAN.

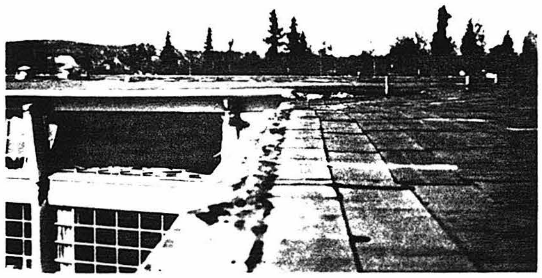
fan-like deformations in the two wings of diaphragm lead to a stress-singularity at the junction of the two wings. This building provides a spectacular example of such damage.

Figure (2.4) shows the damage in the building. The damage below the second floor was less than that above; this was attributed to "a different arrangement of shear walls and the fact that the floor diaphragm had a large stair opening at the intersection of the two wings", and "it is believed that this floor opening permitted a partial hinge to form in the remaining portion of the floor" (Meehan, 1967).

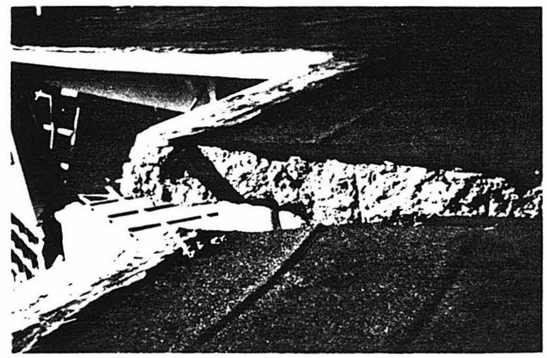
The cause and sequence of the damage in the building has been described by Meehan (1967) as follows:

> One cannot be certain of the sequence or path of distress; however, it is believed that the initial damage occurred in the roof diaphragm at the vertex of the angle formed by the two portions of the classroom wing due to torsional moment developed in this diaphragm. It is also believed that, after the roof diaphragm separated at this point, each portion of the classroom wing essentially formed individual buildings, thus necessitating a redistribution of load in the shear walls. The shear walls were not capable of resisting this redistribution of load and were apparently damaged next. The exterior second-floor columns were then unable to resist the total load alone, and damage developed in these.

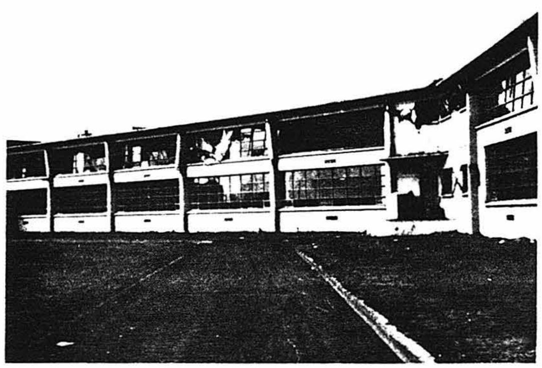
The above clearly indicates the importance of in-plane flexibility of floors in this type of building.



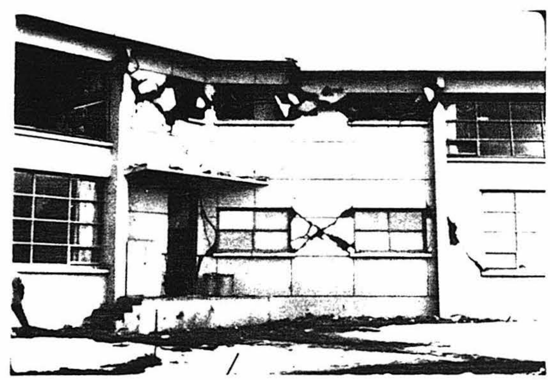
(a)



(b)



(c)



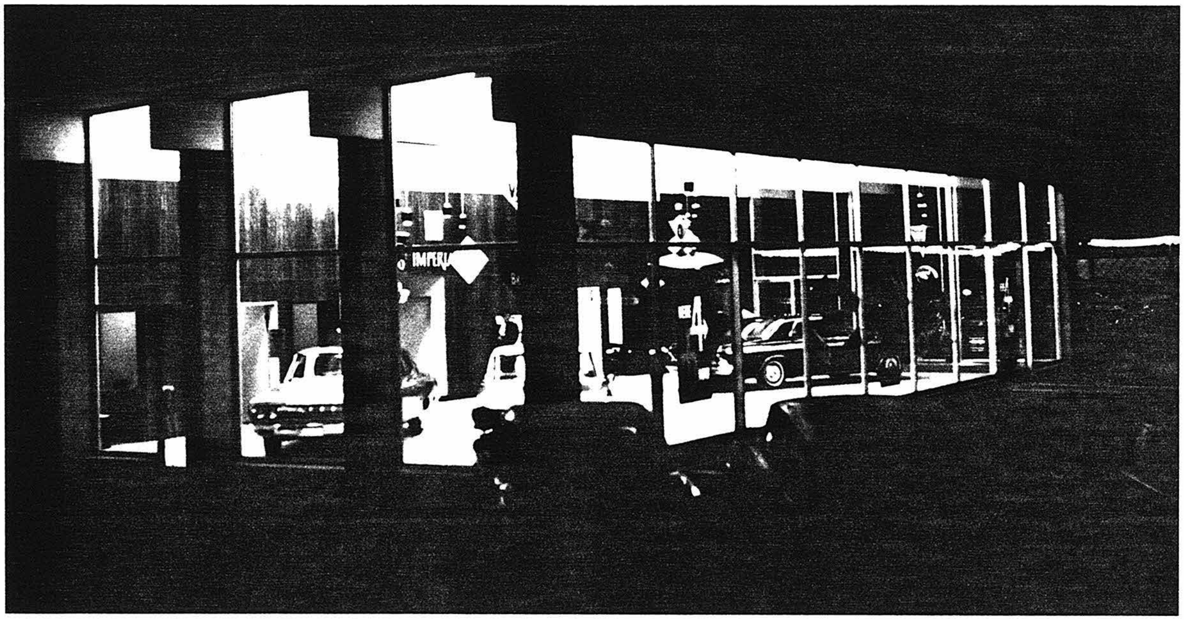
(d)

Figure 2.4. DETAILS OF DAMAGE TO WEST ANCHORAGE HIGH SCHOOL BUILDING (from Meehan, 1967). (a) ROOF DIAPHRAGM DAMAGE. (b) CLOSE-UP OF DAMAGE TO ROOF DIAPHRAGM. (c) WEST SIDE OF NORTHWEST WING. (d) DAMAGE IN THE SHEAR WALL AT INTERSECTION OF WINGS.

2.4 FIFTH AVENUE CHRYSLER CENTER

The Fifth Avenue Chrysler Center in Anchorage (Alaska) was a one-story rectangular building (about 157 ft long and 70 ft wide), that suffered extensively in the Alaska earthquake of March 27, 1964 (Steinbrugge, et al., 1967; Berg, 1973). This building provides another good example of significant in-plane floor flexibility in buildings. The front end of the building, facing south, was a showroom. The lateral load resistance in the longitudinal (north-south) direction was provided by two 8 inch concrete block walls at the sides along the length of the building, except in the showroom portion. In the transverse direction (east-west), there were 8 inch concrete block walls, one at the north end of the building, another wall at the center of the building and two stub walls extending from the sides just to the rear of the showroom. The roof of the building consisted of 20 prestressed precast reinforced concrete tees, 8 ft wide, that were placed side by side, spanning the whole width of the building. Sixteen of these were supported by the side walls, while 4 tees in the showroom portion were supported by 12" x 24" concrete block columns (Figure 2.5a). The flanges of the adjacent tees were connected together by welding the bar anchors which were embedded in the flanges. Figure (2.6) gives the first floor and roof plans of the building.

As a result of the earthquake, the showroom part of the building was extensively damaged, and the roof tees in this portion fell to the south of the building (Figure 2.5b). There was also significant damage



(a)

(b)

Figure 2.5. FIFTH AVENUE CHRYSLER CENTER (from Steinbrugge, et al., 1967). (a) PRE-EARTHQUAKE VIEW. (b) COLLAPSE AT THE SHOW-ROOM END.

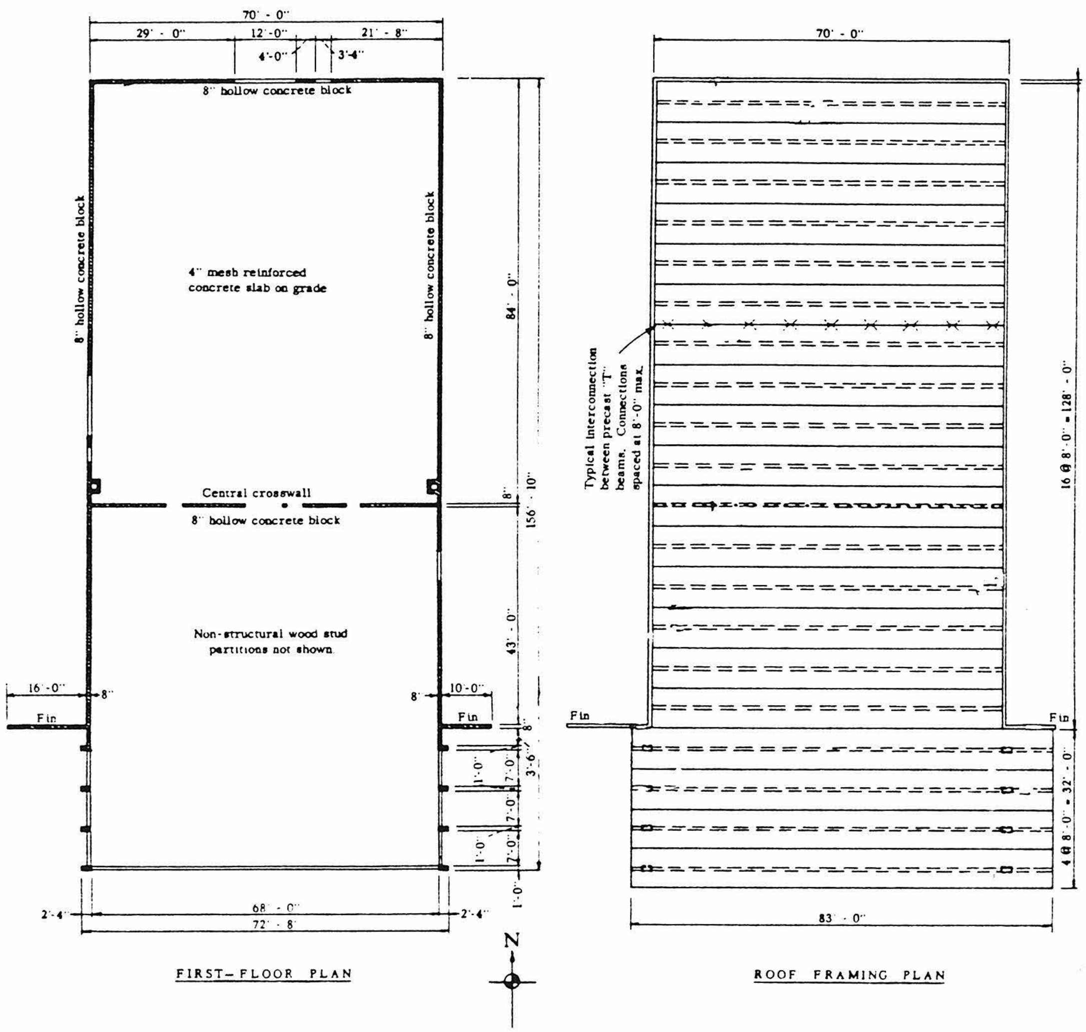


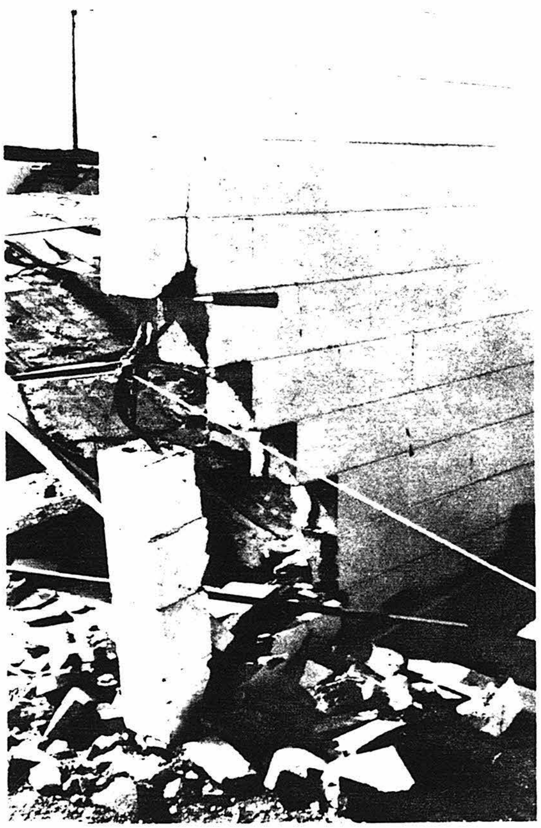
Figure 2.6. FRAMING PLANS OF THE FIFTH AVENUE CHRYSLER CENTER (from Steinbrugge, et al., 1967)

in the roof between the showroom and the central wall. The top half of the west stub wall collapsed (Figure 2.5b) while the east stub wall had damage at the base (Figure 2.7a). There was also some damage at the north ends of the side walls (Figure 2.7b).

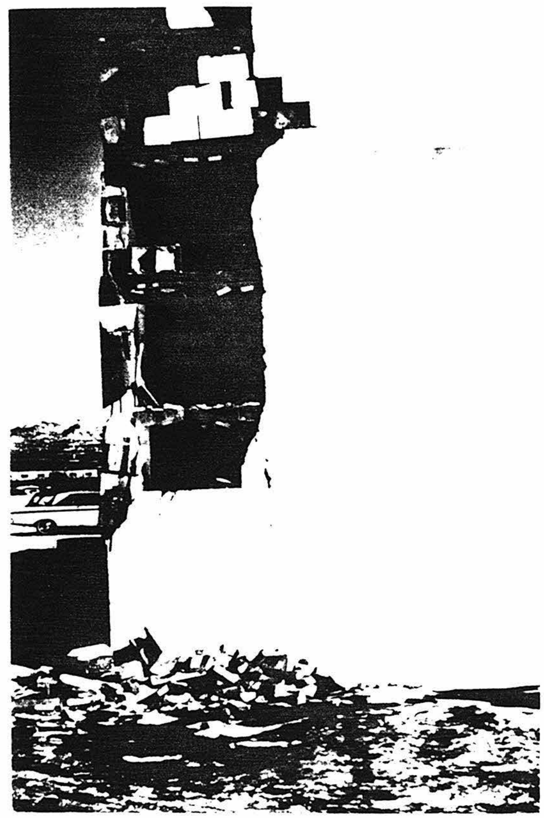
The cause of damage in the building has been attributed to vibrations in transverse direction. A possible explanation for the damage has been provided by Berg (1973) as follows:

> If we consider the roof as a beam lying on its side and oscillating in the lateral direction, it would act -- in its gross behavior -- as a beam on three resilient supports, and its fundamental mode of oscillation would be approximately as shown in Figure (2.8). The left support (the stub walls) is less rigid than the other two supports (the full transverse walls). At the left support, both shear and bending moment in the beam would be high. Shear would tend to shear the connections between adjacent tee flanges, and bending moment would tend to pull apart the same connections. At the right support (rear walls) the shear would also be high, tending to shear the connections between the flange of the end roof tee and the rear wall, the tee flange connections did indeed fail at these points. Once the connections between the fourth and fifth roof tees failed, there would be only nominal resistance to the southward collapse of the front part of the building. Because the left support was more flexible than the other two supports, the distortion in the left part of the beam would be greater than in the right part. The corresponding behavior in the building is greater lateral movement to the south of the middle wall than north of the middle wall, and it was the part south of the middle wall that collapsed.

A simple calculation will indicate that an assumption of a rigid floor diaphragm (i.e., a rigid beam on three similar springs in Figure 2.8) will lead to high shear and bending moments in the beam at the central support rather than at the left support. Had this been true, one would have observed more damage in the connections of roof



(a)



(b)

Figure 2.7. DAMAGE IN THE FIFTH AVENUE CHRYSLER CENTER (from Steinbrugge, et al., 1967). (a) DAMAGE AT BASE OF THE EAST FIN. (b) NORTH END OF THE WEST ELEVATION.

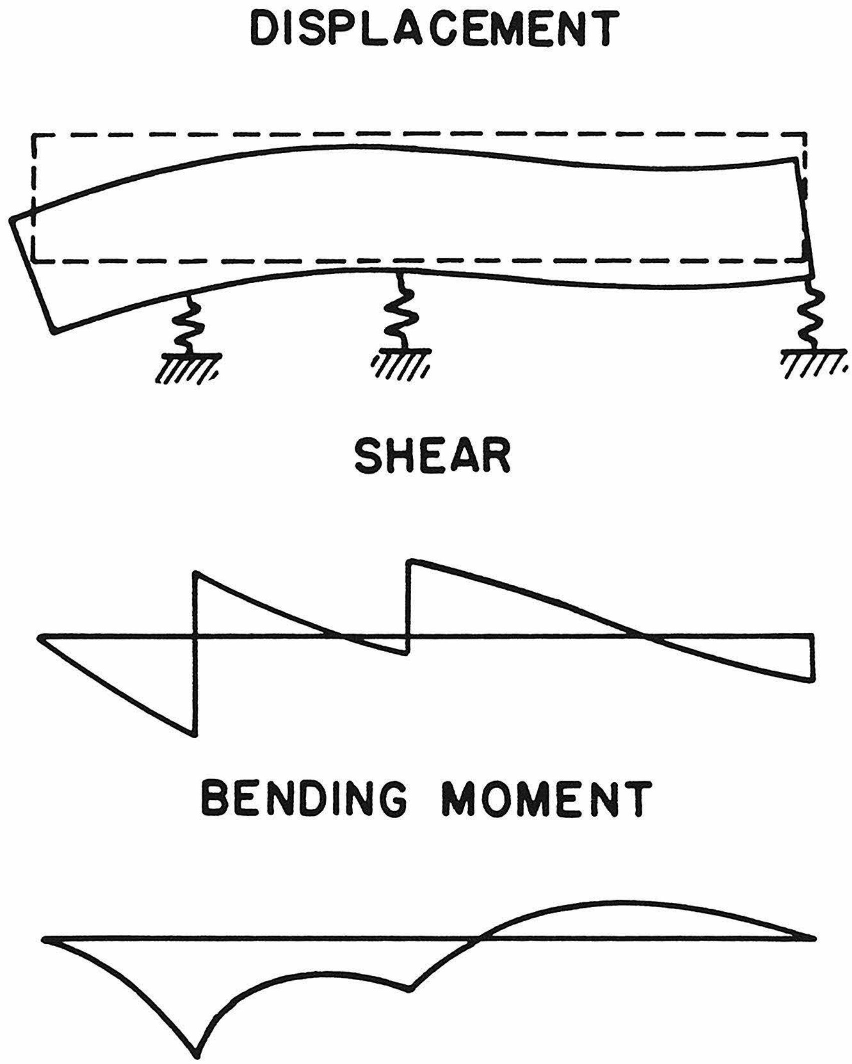


Figure 2.8. FUNDAMENTAL MODE OF ROOF DIAPHRAGM, FIFTH AVENUE CHRYSLER CENTER (from Berg, 1973).

tees near the central wall. Hence, it is evident that roof-diaphragm flexibility contributed to the damage in this building.

2.5 IMPERIAL COUNTY SERVICES BUILDING

During the Imperial County earthquake (magnitude 6.6) of October 15, 1979, the Imperial County Services Building, a six-story reinforced concrete structure, was the only modern building to have sustained severe damage (Jain and Housner, 1983a). The building was extensively instrumented under the program of the California State Office of Strong Motion Studies to record the motion at various locations should a large earthquake occur in the area. In the Imperial Valley earthquake, these instruments provided records which are very valuable to structural engineering, as they give information concerning the possible causes of the damage (e.g., Jennings, 1983; Pauschke, et al., 1981).

Figure (2.9) shows a schematic plan of the building. Note that in the upper stories of the building, the lateral load resistance was provided only by the end walls. Even though the aspect ratio of the building is not large (length = 136'-10", width = 85'-4"), a study of the records obtained from the roof, by Pauschke, et al., (1981) reveals that there was significant floor-diaphragm deformation (Figure 2.10). This in-plane floor flexibility is not considered to have been responsible for the initiation of the damage in the building. However, the fact

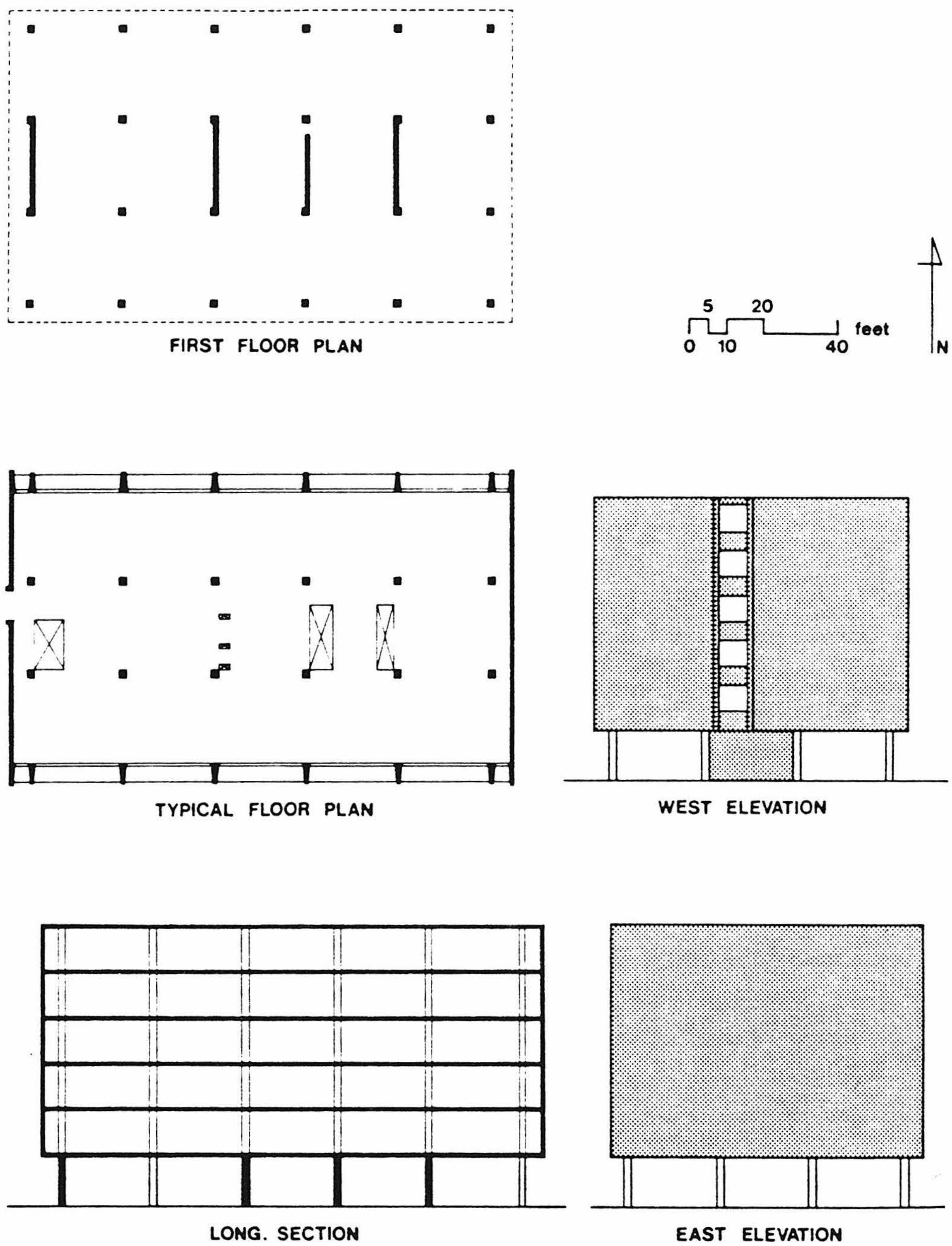


Figure 2.9. SCHEMATIC PLAN OF THE IMPERIAL COUNTY SERVICES BUILDING (from Arnold, 1980a).

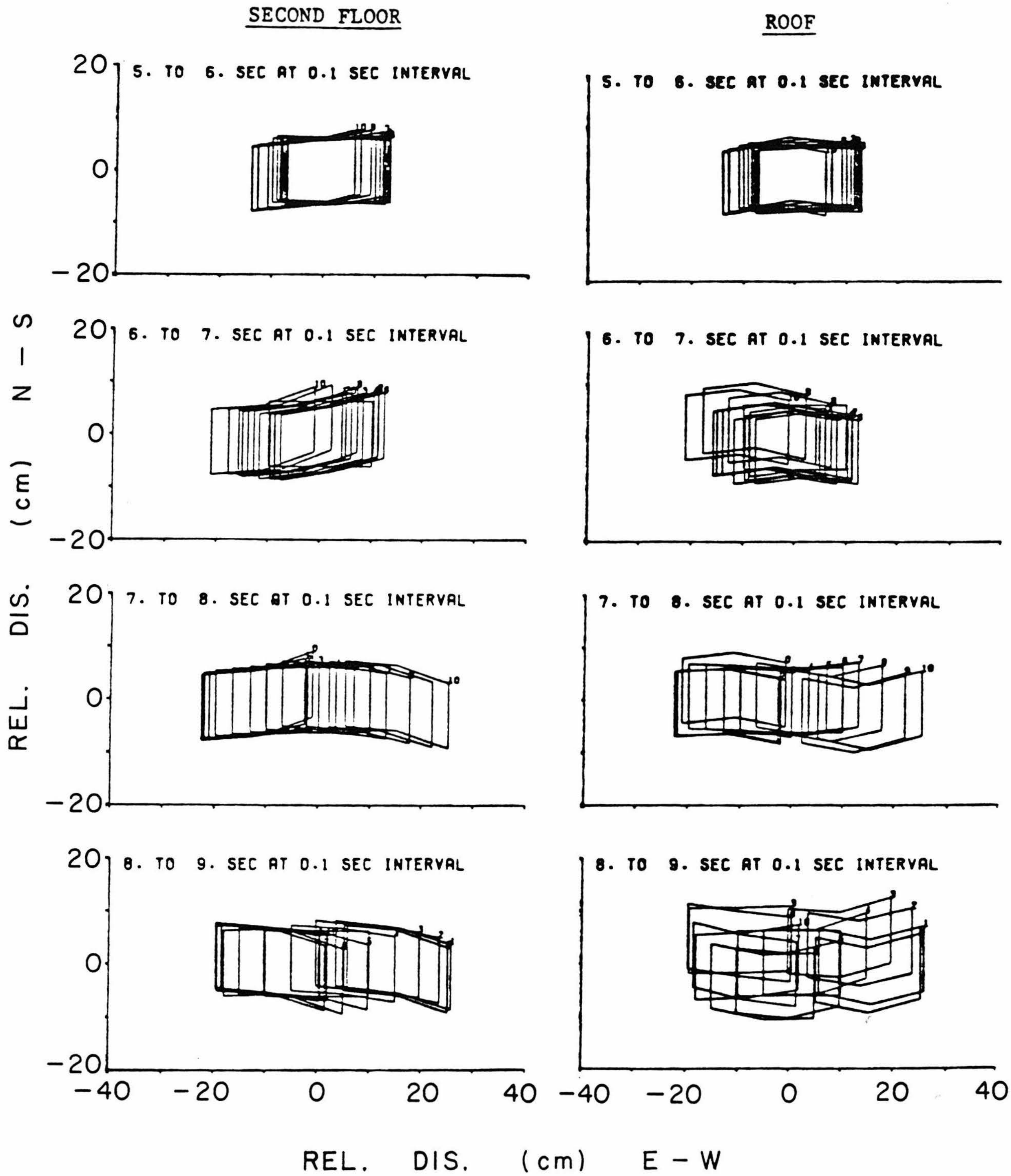


Figure 2.10. PLAN VIEW OF SECOND FLOOR AND ROOF MOTION BETWEEN 5 AND 9 SECONDS, IMPERIAL COUNTY SERVICES BUILDING (from Pauschke, *et al.*, 1981).

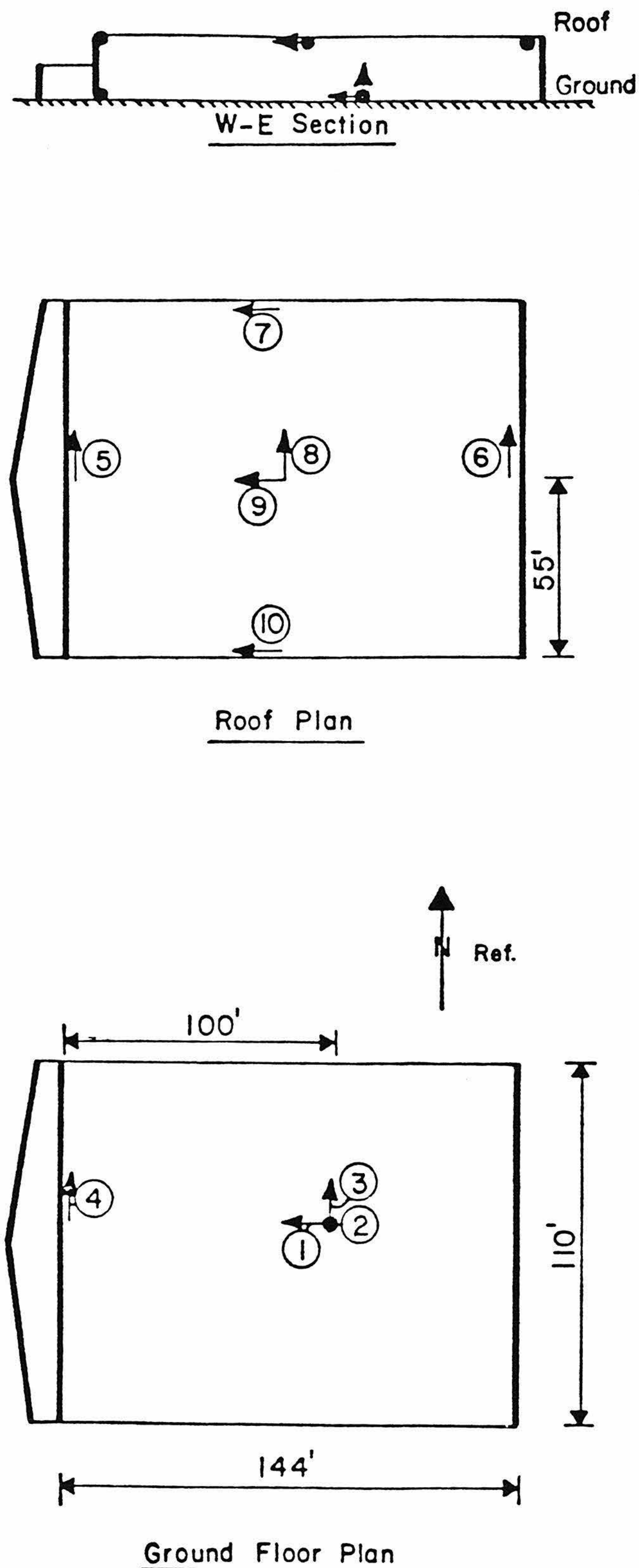
that it was significant, even in a building with a low aspect ratio, points out the importance of floor flexibility.

Figure (2.10) shows the displacements at the roof and the second floor in the transverse direction, recorded by instruments at the two ends and at mid-span. In this figure, the roof and floor dimensions have been "disproportionately reduced in order to magnify the relative displacements." It is clear that the roof and the second floor indeed had significant in-plane deformations. Also, one notices that the two floors show opposite curvatures. A model of this building studied in a later chapter also reveals this feature.

2.6 MAMMOTH HIGH SCHOOL GYMNASIUM

Strong motion accelerograms obtained from the single-story Mammoth High School Gymnasium building, during the May, 1980, earthquakes provide another good example of significant floor flexibility in low aspect-ratio buildings. This building, 144 ft long and 110 ft wide, has reinforced concrete exterior walls. The roof is supported by slightly inclined Warren trusses, spanning the width of the building. These trusses are braced vertically to prevent excessive lateral deflections. Also, horizontal steel bracing has been provided in the plane of the lower chord of the roof trusses.

Again under the State of California's program, the building was instrumented with 10 accelerometers located at various locations in the structure (Figure 2.11). During the earthquake swarm of May, 1980, they



Installation Notes;

Accelerometers 1,2,3 and 4 are installed on the ground floor slab.

Accelerometers 5,8 and 9 are attached to the roof trusses at the bottom chord level.

Accelerometers 6,7 and 10 are attached to the roof trusses at the top chord level.

Recorder trace order:

Accelerometer	1
Fixed trace	-
Accelerometer	2
" "	3
Fixed trace	-
Accelerometer	4
" "	5
Fixed trace	-
Accelerometer	6
" "	7
Fixed trace	-
Accelerometer	8
" "	9
Fixed trace	-
Accelerometer	10

Figure 2.11. STRONG MOTION INSTRUMENTATION SCHEME, MAMMOTH HIGH SCHOOL GYMNASIUM (from Turpen, 1980).

provided very useful sets of records. Three of these sets are shown in Figures (2.12) through (2.14). Of particular interest to this study are the records obtained from the roof of the building.

In each figure, a comparison of traces (5), (6) and (8), which recorded the motion of the roof in the transverse direction, indicates that the motion of the center of the roof was much more vigorous than that at the two ends of the roof span. This clearly indicates that the flexibility of the roof diaphragm cannot be neglected for this building in the transverse direction.

In the longitudinal direction, the motion of roof was recorded by instruments (7), (9) and (10). Despite the fact that the aspect ratio for this direction is less than one, one notices the same phenomenon, i.e., the instrument at mid-span registered a much larger response than registered by the instruments at the two ends. The high frequency content in trace (9) is due to the lateral vibrations of the lower chord segment of the roof truss, at whose mid-span this instrument was located. However, one can still see, in this trace, a significant motion at a lower frequency of about 4.5 Hz. This is thought to be the fundamental natural frequency of the roof and wall system along the longitudinal direction.

The above observations, which are applicable to the three different sets of records, prove that the floor flexibility in this building is not negligible in either direction despite the low aspect ratios.

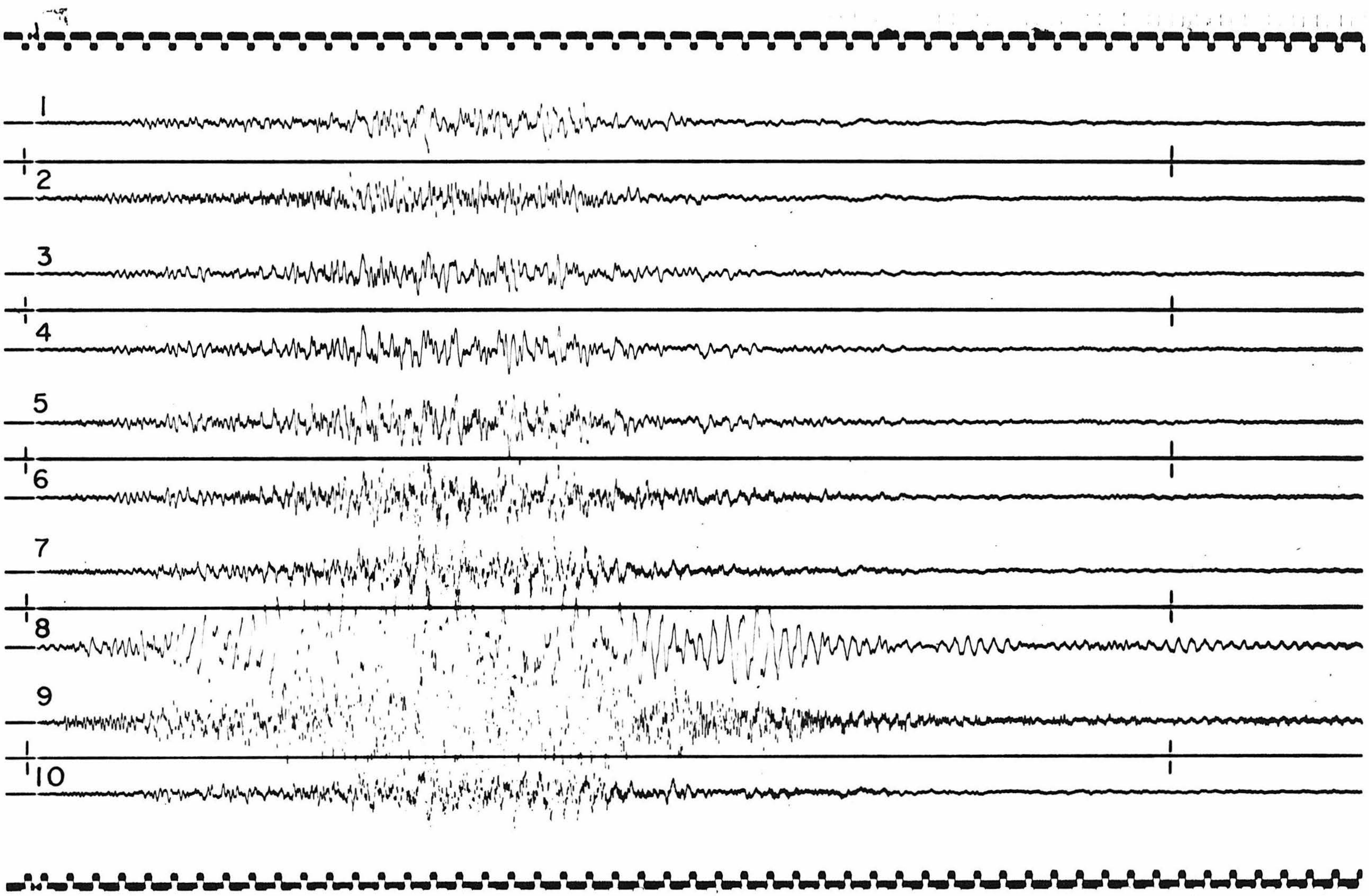


Figure 2.12. STRONG MOTION ACCELEROGRAM RECORDED MAY 25, 1980, IN MAMMOTH HIGH SCHOOL GYMNASIUM (Max. Ground Acceleration 0.32g)(California Division of Mines and Geology).

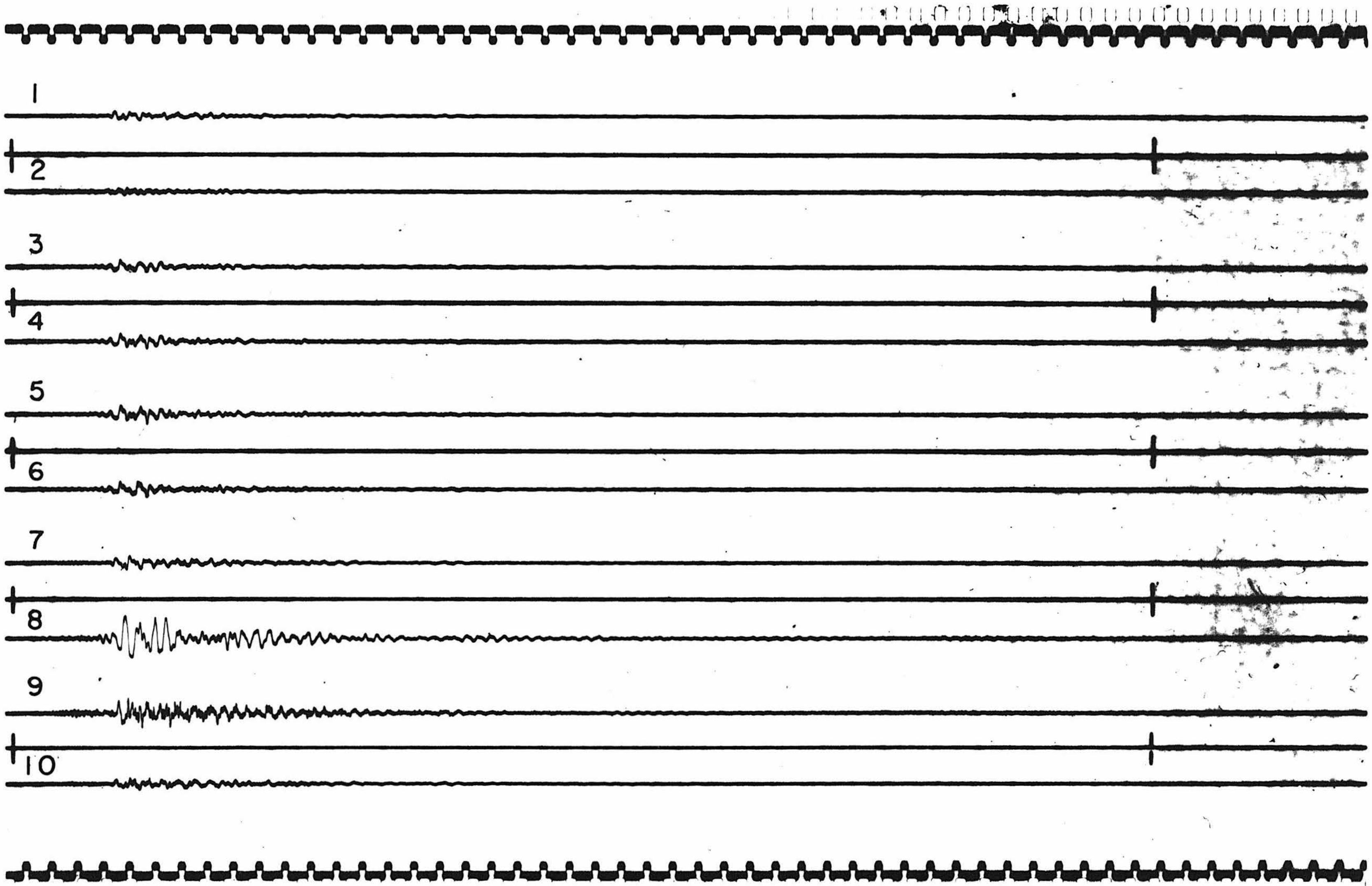


Figure 2.13. STRONG MOTION ACCELEROGRAM RECORDED MAY 25, 1980 IN MAMMOTH HIGH SCHOOL GYMNASIUM (Max. Ground Acceleration 0.07g)(California Division of Mines and Geology).

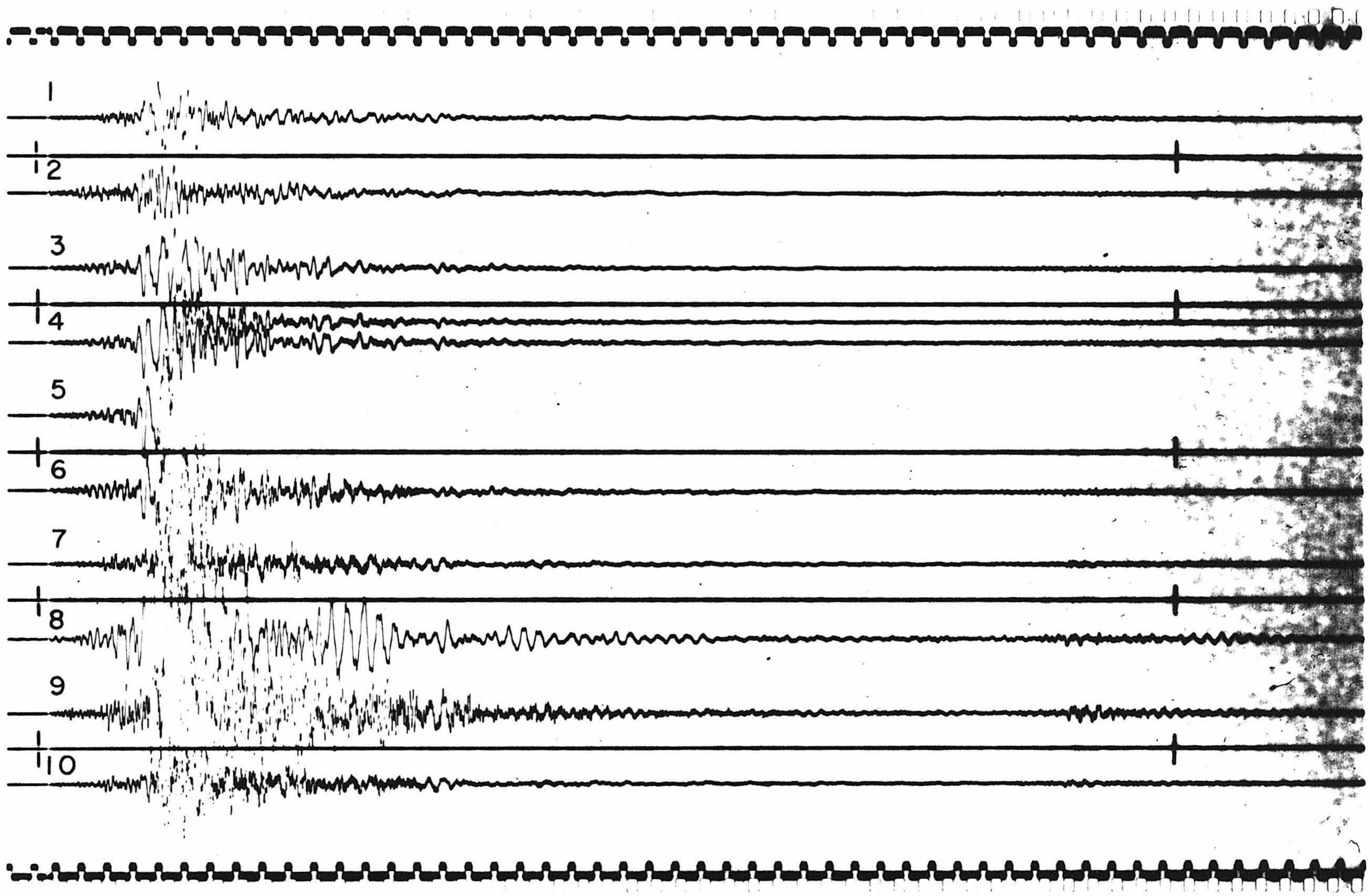


Figure 2.14. STRONG MOTION ACCELEROGRAM RECORDED MAY 25, 1980 IN MAMMOTH HIGH SCHOOL GYMNASIUM (Max. Ground Acceleration 0.43g)(California Division of Mines and Geology).

2.7 DISCUSSION

The five buildings discussed in this chapter provide ample evidence to indicate that in-plane floor flexibility can be very important in the earthquake response of some buildings. Three of the examples were school buildings, which suggests that the architectural layout of school buildings may make them more susceptible than other structures to problems caused by flexible floor diaphragms.

Also, three of these buildings had lateral load resistance systems consisting of only end walls. One of these three had a span to width ratio of only 0.76, but still exhibited significant diaphragm flexibility. The reason for this is the relative flexibility of the diaphragm with respect to the end walls. Even though floors with low aspect ratio may not seem very flexible in their plane, their flexibility may still be quite significant and may indeed dominate the dynamic response, when compared to the flexibility of very rigid walls.

The classroom wing of the West Anchorage High School had a plan consisting of two wings joined at an angle, thus forming a 'V'. Buildings such as this, forming an L, V, T, H, etc., warrant special considerations. The San Marcos Building, a four-story, reinforced concrete L-shape building is another example in this category (Dewell and Willis, 1925). This building was extensively damaged while the corner section was "totally destroyed," during the 1925 Santa Barbara earthquake. Its damage has been attributed to the shape of the ground plan, in addition to poor workmanship in the concrete. The Mene Grande Building, a sixteen-story H-shaped reinforced concrete structure, was heavily damaged during the Venezuela earthquake of 1967 (Hanson and Degenkolb, 1969; Sozen, et al., 1968). Besides other damage, this building experienced some floor-cracks "especially where the wings connected with the core" (Hanson and Degenkolb, 1969). Again this observation indicates a stress concentration at the corner.

This brief summary of cases of earthquake damage in buildings that can be attributed to the response of the floors as flexible diaphragms shows the importance of this phenomenon in the earthquake response of structures. The strong motion records obtained from the two buildings mentioned in this chapter, and some others (Porcella, et al., 1979), also lead to similar observations. Hence, the experience in past earthquakes indicates clearly that floor flexibility can be a potential problem unless considered in the design.

CHAPTER III

SINGLE-STORY BUILDINGS WITH FLEXIBLE FLOORS

3.1 INTRODUCTION

There has been considerable interest in the past in the dynamic analyses of long, narrow one-story buildings because this type of building is commonly adopted for schools, hospitals and offices (Blume, *et al.*, 1961, Blume and Jhaveri, 1969). Many such buildings have only two end walls in the transverse direction to provide lateral support, while any intermediate columns share only the vertical loads. This is largely due to the needs of functional flexibility, which requires movable partition walls. Hence, one has a roof mounted on two end walls that acts like a beam in the transverse direction due to its large span to width ratio. In such a situation, it is important to consider the flexibility of the roof diaphragm in the dynamic analysis of the building.

Moreover, there are other situations where even though the aspect ratio (length to width ratio) of the building is not large, the floor flexibility cannot be neglected. Mammoth High School Gymnasium, discussed in the previous chapter is one such example. In this type of building the roof flexibility, though small in absolute terms, is significant when compared to that of the stiff end walls.

It is shown in this chapter that such buildings can easily be treated analytically. Furthermore, the technique is general enough to allow the solutions of more complicated structures, e.g., buildings with different end walls. The method described in the following sections consists of treating the roof as a bending beam and the end walls as shear beams. The dynamic equations of motion for the roof and the walls can be written, and these equations can be solved for appropriate boundary conditions. This gives a transcendental characteristic equation, the solution of which provides the natural frequencies of the system. With these frequencies known, the mode shapes and the participation factors for earthquake excitation can be obtained, thus enabling one to calculate the dynamic response of the building.

The method described herein can also be extended to take into account the flexibility of the foundation. For example, one could model the effects of the supporting soil by Winkler's representation, thereby replacing the foundation by appropriate springs. This only affects the boundary condition to be satisfied at the bottom ends of the vertical beams representing the walls.

In the following parts of this chapter, bending and shear beam theories are discussed first. Next, the concept of "laminae" or "equivalent distributed beams" is presented. A note on how to match the boundary conditions between members meeting at a point is also included. A simple case of a one-story building with two, identical end walls is then solved. A section has been included on the use of perturbation theory to obtain an approximate value of the fundamental natural frequency of the structure. Also, solutions are given for some interesting but more complex structures. The chapter ends with a numerical example derived from the top story of the two-story Administrative Building at Arvin High School.

3.2 BEAM THEORIES

In this and subsequent chapters, extensive use will be made of the equations and properties of both Euler-Bernaulli and shear beams. For convenience, this section presents the well known theories for these two types of beams. Also, the concept of "laminae" or "equivalent-distributed beams" as applied to the present problem is introduced. This concept proves to be useful in modelling the floors of multistory buildings.

3.2.1 Bending Beam (Euler-Bernaulli)

For beams whose length to depth ratio is large, the bending deformations are large compared to those caused by shear, and, therefore, it is a common practice to neglect the shearing deformations for static analysis and for the analysis of the lower modes of vibration. Also, for the lower modes of vibration, the effect of rotatory inertia is small for such beams, and can be neglected. The resulting mathematical model for a beam is termed a bending beam or Euler-Bernaulli beam.

Consider the free vibrations of a bending beam. Let $v_b(x,t)$ be the lateral displacement at time t of a point at a distance x from the origin (Figure 3.1a). From the free body diagram of an element of width dx, one obtains

$$dQ = \rho A \frac{\partial^2 v_b}{\partial t^2} dx$$

$$dM = -Q dx$$

Hence, the equation governing the free vibrations of the beam may be written as

$$\frac{\partial^2}{\partial x^2}\left(EI(x) \frac{\partial^2 v_b}{\partial x^2}\right) + \rho A(x) \frac{\partial^2 v_b}{\partial t^2} = 0 \qquad (3.1a)$$

For beams with uniform cross-section, this becomes

$$EI \frac{\partial^4 v_b}{\partial x^2} + \rho A \frac{\partial^2 v_b}{\partial t^2} = 0 \qquad (3.1b)$$

For beams vibrating due to uniform earthquake excitation, $\ddot{u}_g(t)$ in the plane of the beam (Figure 3.1b), the equation of motion is

$$\frac{\partial^2}{\partial x^2}\left(EI(x) \frac{\partial^2 v_b(x,t)}{\partial x^2}\right) + \rho A(x) \frac{\partial^2 v_b(x,t)}{\partial t^2} = -\rho A(x)\ddot{u}_g(t) \qquad (3.2a)$$

If the cross-section is uniform, this reduces to

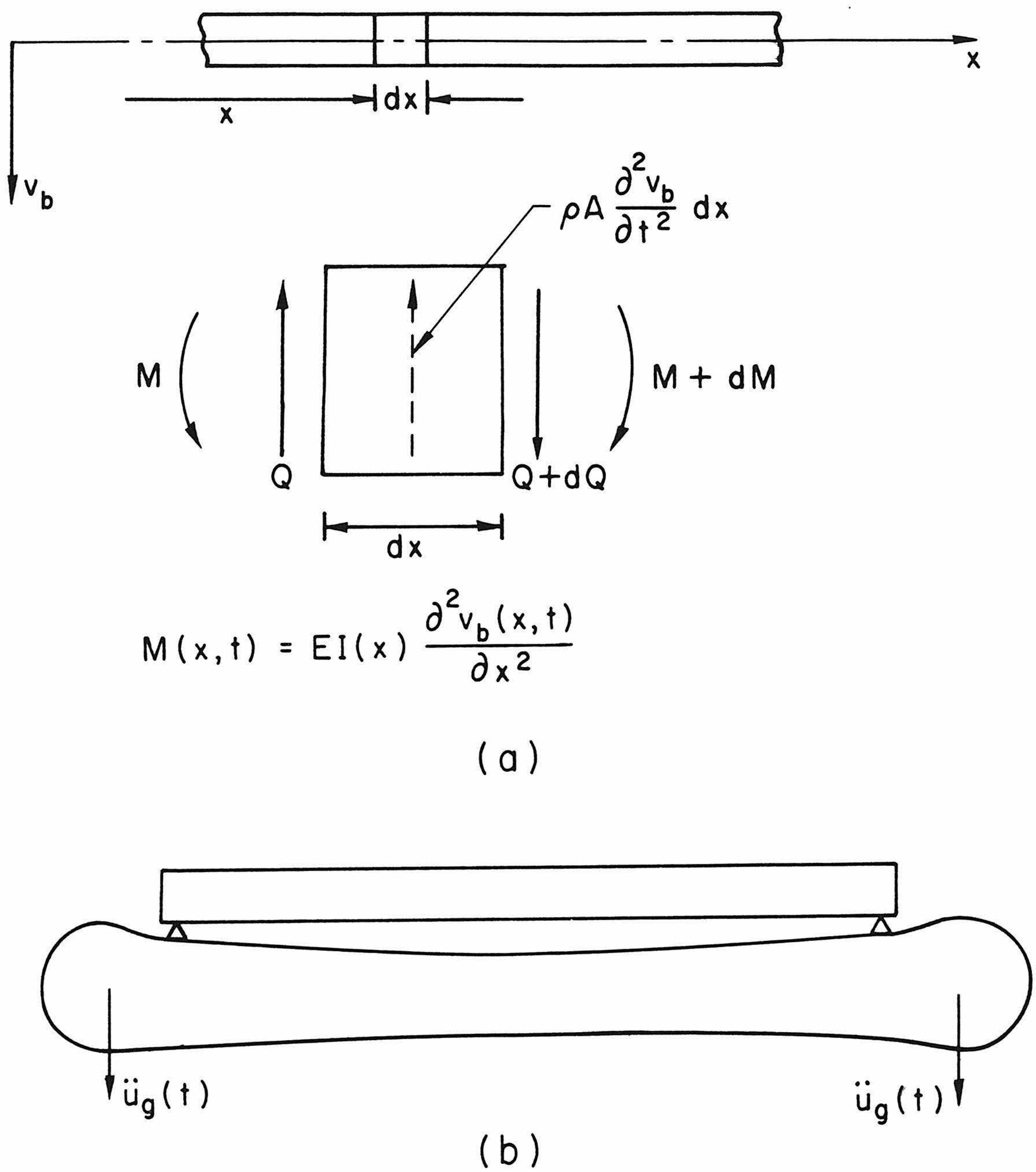
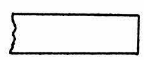
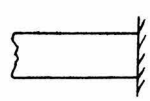
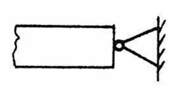
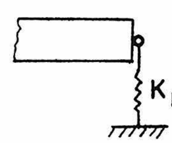
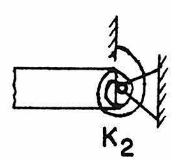


Figure 3.1. VIBRATION OF A BENDING BEAM. (a) FREE VIBRATION. (b) EARTHQUAKE MOTION.

$$EI \frac{\partial^4 v_b(x,t)}{\partial x^4} + \frac{\partial^2 v_b(x,t)}{\partial t^2} = -\rho A \ddot{u}_g(t) \qquad (3.2b)$$

In addition to the governing differential equation, the specification of boundary conditions is required to solve the vibration problems of beams. Table (3.1) gives some of the common boundary conditions for a bending beam.

TABLE 3.1. VARIOUS BOUNDARY CONDITIONS FOR A BENDING BEAM

Type of Boundary	Sketch of Boundary	Boundary Conditions
Free End		$[EI(x)v''_b]_{end} = 0$ (moment=0) $[(EI(x)v''_b)']_{end} = 0$ (shear=0)
Fixed End		$[v_b]_{end} = 0$ (displacement=0) $[v'_b]_{end} = 0$ (slope=0)
Pinned End		$[v_b]_{end} = 0$ (displacement=0) $[EI(x)v''_b]_{end} = 0$ (moment=0)
Spring Supported End		$[EI(x)v''_b]_{end} = 0$ (moment=0) $[(EI(x)v''_b)'] = K_1[v_b]$ (for right end) $[(EI(x)v''_b)'] = -K_1[v_b]$ (for left end)
Pinned End with Torsional Spring		$[v_b]_{end} = 0$ (displacement=0) $[EI(x)v''_b] = -K_2[v'_b]$ (for right end) $[EI(x)v''_b] = K_2[v'_b]$ (for left end)

3.2.2 Shear Beam

This beam theory is applicable for beams that exhibit bending deformations that are small compared to shear deformations. In the theory, the bending deformations are neglected reducing the governing equation to one of second order. Experience has shown that this beam models some important features of the dynamic behavior of buildings of moderate height (e.g., Jennings, 1969; Hoerner, 1971).

Consider the free vibrations of a shear beam as shown in Figure (3.2a). Let A(x) be the cross-sectional area at x, ρ be the density of beam material, and k(x) [=k'A(x)G] be the shear rigidity of the beam. Let $v_s(x,t)$ be the lateral displacement of a point at a distance x from the origin, at time t. From the free body diagram of an element dx, one obtains

$$dQ = \rho A \frac{\partial^2 v_s(x,t)}{\partial t^2} dx$$

Thus, the equation of motion for the free vibrations of a shear beam may be written as:

$$\frac{\partial}{\partial x}\left(k(x)\ \frac{\partial v_s(x,t)}{\partial x}\right) - \rho A(x)\ \frac{\partial^2 v_s(x,t)}{\partial t^2} = 0 \tag{3.3a}$$

For a beam with uniform cross-section, the shear rigidity k(x) is not a function of x, and the equation of motion becomes

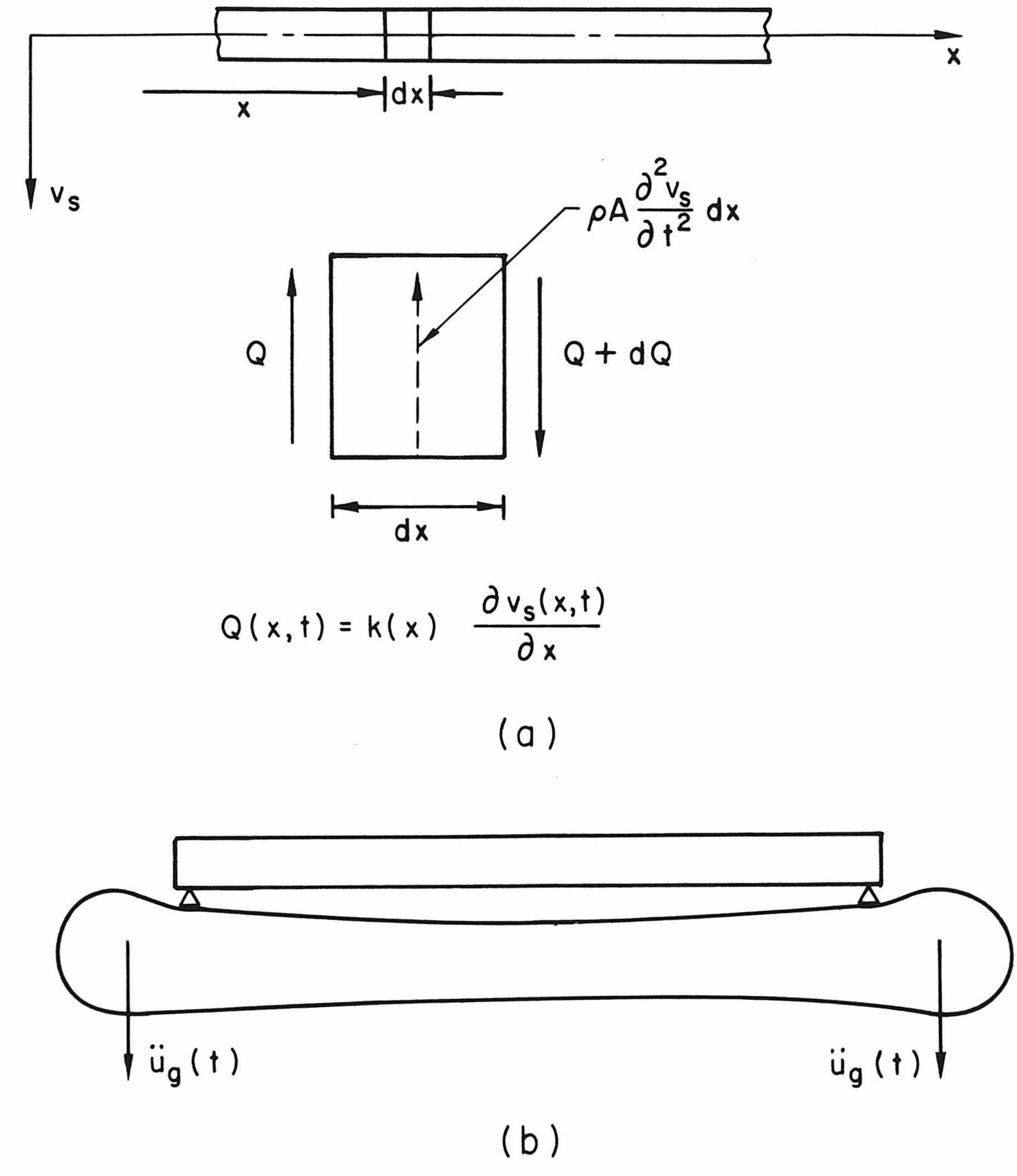


Figure 3.2. VIBRATION OF A SHEAR BEAM. (a) FREE VIBRATION. (b) EARTHQUAKE MOTION.

$$k\frac{\partial^2 v_s(x,t)}{\partial x^2} - \rho A\frac{\partial^2 v_s(x,t)}{\partial t^2} = 0 \qquad (3.3b)$$

For beams vibrating under ground excitation (Figure 3.2b), these equations become

$$\frac{\partial}{\partial x}\left(k(x)\frac{\partial v_s(x,t)}{\partial x}\right) - \rho A(x)\,\frac{\partial^2 v_s(x,t)}{\partial t^2} = \rho A(x)\ddot{u}_g(t) \qquad (3.4a)$$

and,

$$k\,\frac{\partial^2 v_s}{\partial x^2}(x,t) - \rho A\,\frac{\partial^2 v_s(x,t)}{\partial t^2} = \rho A\ddot{u}_g(t) \qquad (3.4b)$$

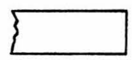
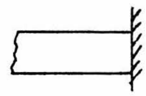
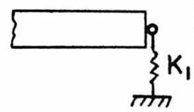
for beams with variable and uniform cross-sections, respectively.

Some of the common boundary conditions for a shear beam are tabulated in Table (3.2).

3.2.3 Equivalent Distributed Beam System (Laminae)

In the present work, floors in buildings have been treated as bending beams in order to include the effects of in-plane floor flexibility. For buildings with a large number of stories and uniform floor properties along the height of the building, it is sometimes convenient to replace the floor-beams by an "equivalent distributed beam system." This beam system consists of a continuum of independently-acting beams with infinitesimal thicknesses. The independent action of the infinitesimally thin beams means that an individual beam does not have any contact with adjacent beams. The stiffness and mass distributions

TABLE 3.2. VARIOUS BOUNDARY CONDITIONS FOR A SHEAR BEAM

Type of Boundary	Sketch of Boundary	Boundary Conditions
Free End		$[k(x)v'_s]_{end} = 0$ (shear=0)
Fixed End		$[v_s]_{end} = 0$ (displacement=0)
End Supported on a Spring		$[k(x)v'_s] = -K_1[v_s]$ (for right end) $[k(x)v'_s] = K_1[v_s]$ (for left end)

Note: For a shear beam, there is no counterpart to an end supported on a torsional spring, since sections perpendicular to the axis before deformation do not rotate as the beam-axis deforms.

of the system are obtained from the stiffnesses and masses of the actual floors by distributing the total stiffness and the total mass of all the floors evenly along the height of the building. These equivalent beams or laminae have been used in other problems in structural mechanics. First developed to analyze the deformations of aircraft wheels, this concept has been used extensively in civil engineering in the study of coupled shear walls (e.g., Chitty, 1947; Beck, 1962).

The equivalent system can be defined with the help of Figure (3.3). Let E be the modulus of elasticity of the floor material. Let I^* be the moment of inertia per unit height of the equivalent, distributed floor system, and m^* be the mass per unit length and per unit height of the system. Let Δy be width (along the height of building) of a thin beam, where Δy is very small. With this notation, the equation of motion for

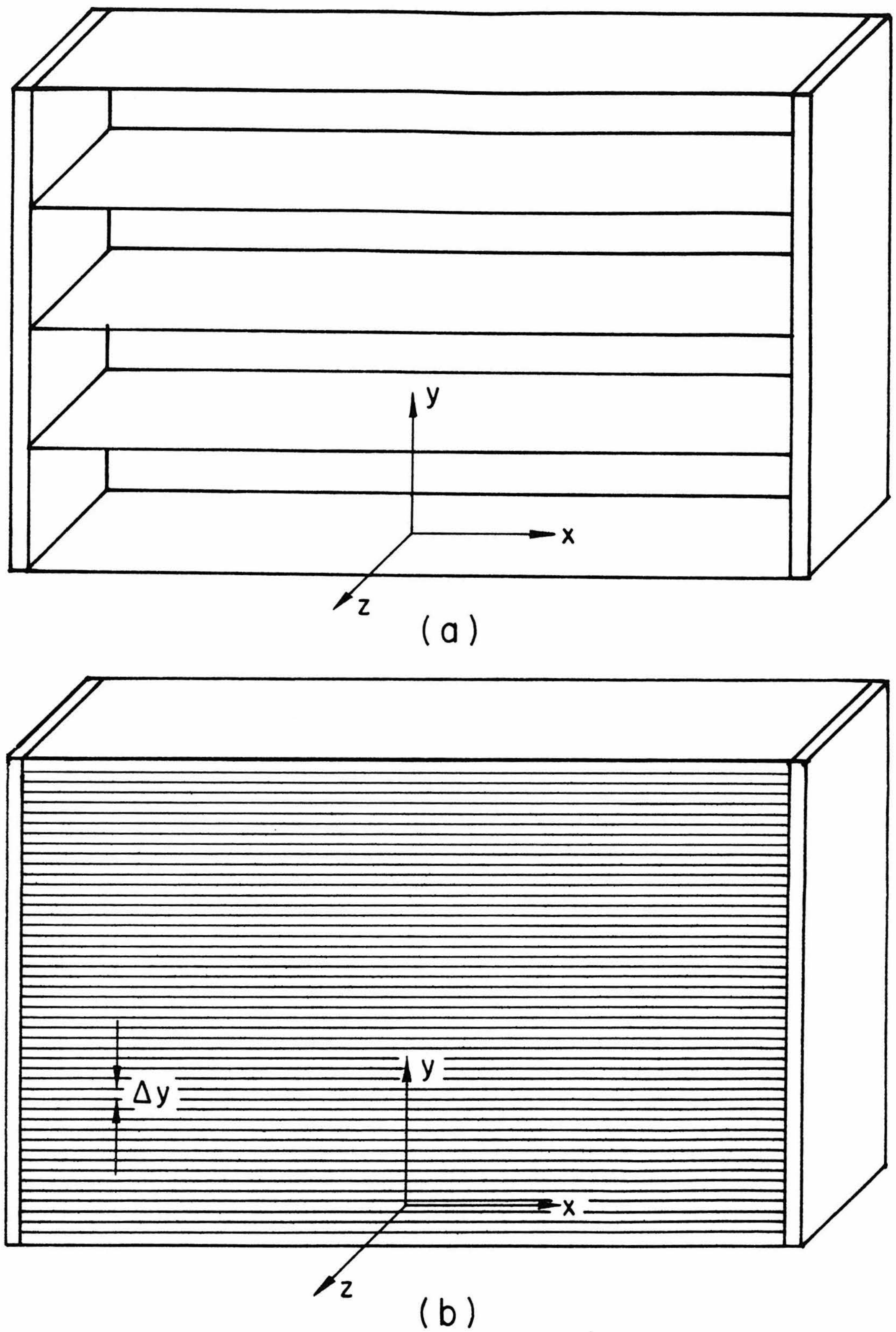


Figure 3.3. (a) N-STORY STRUCTURE WITH IDENTICAL FLOOR SYSTEM.
(b) EQUIVALENT DISTRIBUTED BEAM SYSTEM OR LAMINAE.

free vibrations of the system may be written as

$$\frac{\partial^2}{\partial x^2}\left(EI^* \Delta y \, \frac{\partial^2 u(x,y,t)}{\partial x^2}\right) + m^* \Delta y \, \frac{\partial^2 u(x,y,t)}{\partial t^2} = 0 \qquad (3.5a)$$

or,

$$\frac{\partial^2}{\partial x^2}\left(EI^* \, \frac{\partial^2 u(x,y,t)}{\partial x^2}\right) + m^* \, \frac{\partial^2 u(x,y,t)}{\partial t^2} = 0 \qquad (3.5b)$$

Here $u(x,y,t)$ is the displacement at point x, at time t in an infinitesimal thin beam located at height y. The common boundary conditions for this system are the same as those of bending beams (Table 3.1).

3.3 NOTE ON MATCHING BOUNDARY CONDITIONS

In the rest of this report, extensive use will be made of matching the boundary conditions at the junctions of two or more perpendicularly-intersecting beams. Hence, this section reviews some of the concepts involved. In general, the following situations will be encountered in this study: two beams joined at a right angle at their ends; three beams joined at their ends (arising from one beam joining the interior of another beam at a right angle); and an equivalent distributed beam system joining an ordinary bending or shear beam.

3.3.1 Two Beams Joined Perpendicularly at Their Ends

Consider two beams that are joined together at their ends such that they are perpendicular. Figure (3.4) shows the coordinate system (x,y,z), that is common for both beams. Let the two beams be joined at the point $x = L$ and $y = h$, and let $u(x,t)$ and $v(y,t)$ be the displacements in the z-direction, in beams (1) and (2), respectively. The following boundary conditions apply at the corner:

(i) The displacements in the two beams are the same at the junction, i.e.,

$$u(L,t) = v(h,t) \tag{3.6}$$

(ii) The end shears in the two beams are equal in magnitude and opposite in direction.

(a) Both beams are bending beams

$$\left[\frac{\partial}{\partial x}\left(E_1 I_1 \frac{\partial^2 u}{\partial x^2}\right)\right]_{x=L} + \left[\frac{\partial}{\partial y}\left(E_2 I_2 \frac{\partial^2 v}{\partial y^2}\right)\right]_{y=h} = 0 \tag{3.7a}$$

where $E_1 I_1$ and $E_2 I_2$ are the flexural stiffnesses of beams (1) and (2), respectively.

(b) Beam (1) is bending beam and beam (2) is shear beam:

$$\left[\frac{\partial}{\partial x}\left(E_1 I_1 \frac{\partial^2 u}{\partial x^2}\right)\right]_{x=L} - \left[k_2 \frac{\partial v}{\partial y}\right]_{y=h} = 0 \tag{3.7b}$$

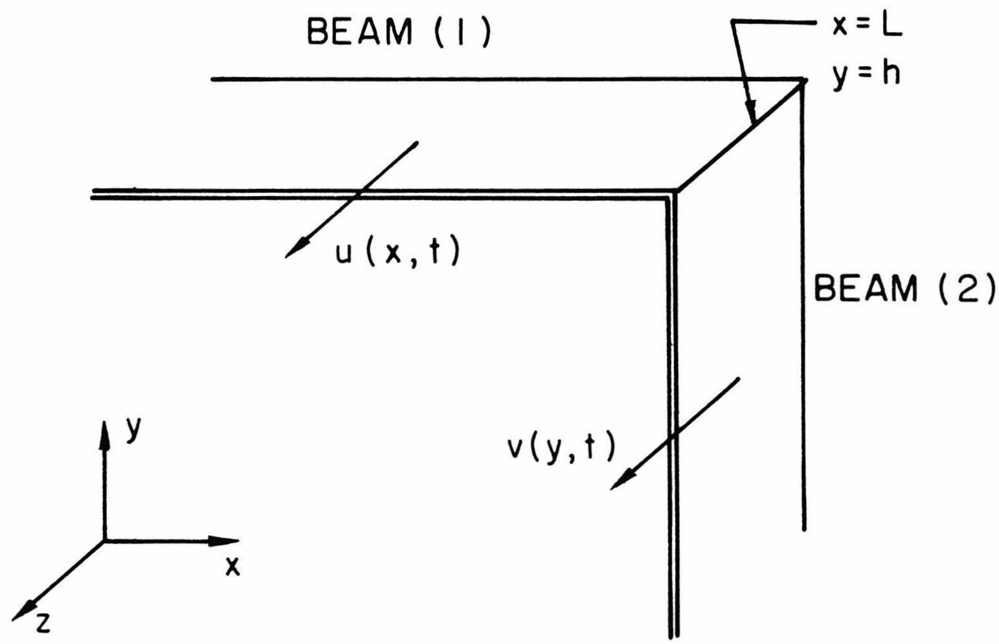


Figure 3.4. TWO BEAMS JOINED AT THEIR ENDS.

where E_1I_1 and k_2 are the flexural stiffness for beam (1) and the shear stiffness for beam (2), respectively.

(c) Both beams are shear beams:

$$\left[k_1 \frac{\partial u}{\partial x}\right]_{x=L} + \left[k_2 \frac{\partial v}{\partial y}\right]_{y=h} = 0 \qquad (3.7c)$$

where k_1 and k_2 are the shear stiffnesses for beams (1) and (2), respectively.

The algebraic signs in these equations will be different when beam (2) is joined at the left end of beam (1), rather than the right end, as considered here.

(iii) The end moment in beam (1) is equal in magnitude and opposite in direction to the torsional moment (torque) in beam (2). Assuming that the other end of beam (2) allows no rigid-body rotation of beam (2), this condition will be

$$\left[E_1I_1 \frac{\partial^2 u}{\partial x^2}\right]_{x=L} = -C_2 \left[\frac{\partial u}{\partial x}\right]_{x=L} \qquad (3.8)$$

where C_2 is the torsional stiffness of beam (2), given by the torque at y = h required to produce a unit rotation at that point. $\left[\frac{\partial u}{\partial x}\right]_{x=L}$ is the end rotation (slope) in beam (1), which is equal to the torsional rotation in beam (2).

For most applications, the walls can be treated as thin rectangular sections to obtain their torsional stiffness. For such sections, the torsional stiffness C_2 is given by (e.g., Timoshenko and Goodier, 1969)

$$C_2 = \frac{bc^3}{3h} G \tag{3.9}$$

where

b = Length of wall.

c = Thickness of wall.

h = Height of wall.

G = Shear modulus.

(iv) The end moment in beam (2) is equal in magnitude and opposite in direction to the torsional moment at the end of beam (1). This case is similar to case (iii) above.

3.3.2 Three beams Joined at Their Ends

Consider three beams meeting at a point as shown in Figure (3.5). The coordinate system (x,y,z) is common for all three beams, and each beam of the system is vibrating in the z-direction. Let u(x,t), v(y,t) and w(y,t) be the displacements in the z-direction, in beams (1), (2) and (3), respectively. At the point x = L, y = 0, the following boundary conditions must be satisfied:

(i) The end displacements in the three beams are the same:

$$u(x = L,t) = v(y = 0,t) = w(y = 0,t) \tag{3.10}$$

(ii) The resultant of the end-shears in the three beams is zero:

(a) The three beams are bending beams, and E_1I_1, E_2I_2, E_3I_3 are

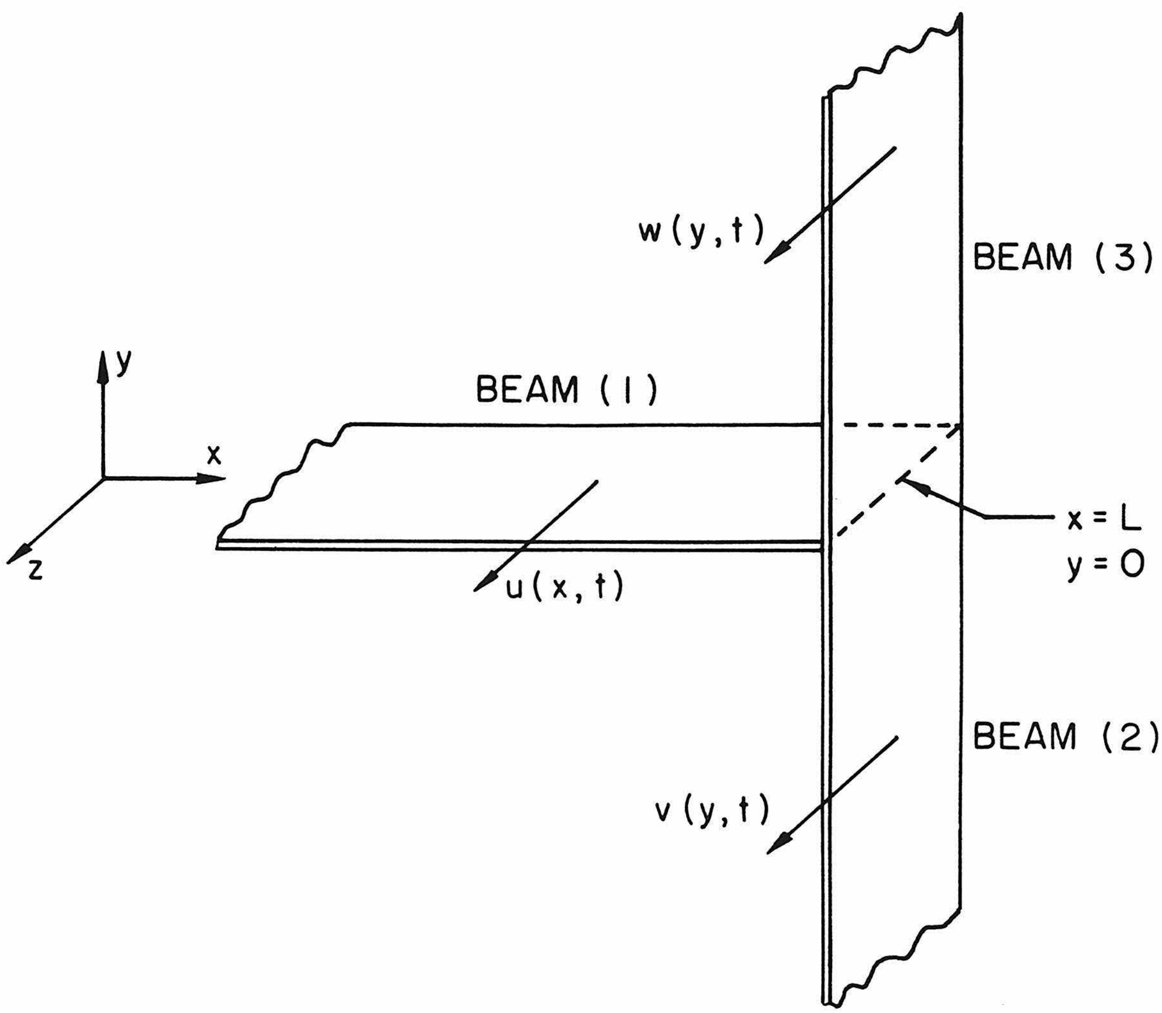


Figure 3.5. THREE BEAMS JOINED AT THEIR ENDS.

their flexural stiffnesses, respectively. The condition can be written as:

$$\left[\frac{\partial}{\partial x}\left(E_1 I_1 \frac{\partial^2 u}{\partial x^2}\right)\right]_{x=L} + \left[\frac{\partial}{\partial y}\left(E_2 I_2 \frac{\partial^2 v}{\partial y^2}\right)\right]_{y=0} - \left[\frac{\partial}{\partial y}\left(E_3 I_3 \frac{\partial^2 w}{\partial y^2}\right)\right]_{y=0} = 0 \qquad (3.11a)$$

(b) Beam (1) is a bending beam with $E_1 I_1$ as its flexural stiffness. Beams (2) and (3) are shear beams, and their shear stiffness are given by k_2 and k_3, respectively. The condition is

$$\left[\frac{\partial}{\partial x}\left(E_1 I_1 \frac{\partial^2 u}{\partial x^2}\right)\right]_{x=L} - \left[k_2 \frac{\partial v}{\partial y}\right]_{y=0} + \left[k_3 \frac{\partial w}{\partial y}\right]_{y=0} = 0 \qquad (3.11b)$$

(iii) The resultant of end moment in beam (1), torsional moment in beam (2) and torsional moment in beam (3) is zero at the junction point.

Define positive torsional moment for beams (2) and (3) as one that produces clockwise torsional rotation in the beam with respect to the bottom end of the beam. Let T_2 $(y = 0,t)$ and T_3 $(y = 0,t)$ be the twisting moments in beams (2) and (3), respectively, at the junction. The boundary condition can be written as

$$M_1(x=L,t) - T_2(y=0,t) + T_3(y=0,t) = 0 \qquad (3.12)$$

where

$M_1(x=L,t)$ = End moment in beam (1)

$= E_1 I_1 \frac{\partial^2 u}{\partial x^2}(x=L,t)$

$T_2(y=0,t) = C_2[\theta_2(y=0,t) - \theta_2(y=-h_1,t)]$

$T_3(y=0,t) = C_3[\theta_3(y=h_2,t) - \theta_3(y=0,t)]$

C_2, C_3 = Torsional stiffnesses of beams (2) and (3), respectively.

h_1, h_2 = Height of beams (2) and (3) , respectively.

θ_2, θ_3 = Torsional rotation in beams (2) and (3), respectively.

However, $\theta_2(y=0,t)$ and $\theta_3(y=0,t)$ are each equal to the end rotation in beam (1) $\left[= \frac{\partial u}{\partial x}(x=L,t)\right]$.

It will be shown in a subsequent section that for the purposes of the present work, one can neglect the torsional stiffnesses of beams (2) and (3). This results in the much simplified boundary condition that there is zero bending moment in beam (1) at x = L, i.e.,

$$E_1 I_1 \frac{\partial^2 u}{\partial x^2}(x=L,t) = 0 \qquad (3.13)$$

(iv) The torsional moment in beam (1) at x = L, and the bending moments in beams (2) and (3) at y = 0 have a zero resultant.

Neglecting the torsional stiffness of beam (1) and assuming beams (2) and (3) as bending beams, this condition is

$$E_2 I_2 \frac{\partial^2 v}{\partial y^2}(y=0,t) - E_3 I_3 \frac{\partial^2 w}{\partial y^2}(y=0,t) = 0 \qquad (3.14)$$

3.3.3 An Equivalent Distributed Beam System Joined to a Perpendicular Beam

Consider a distribution of laminae that is joined at one end to a perpendicular beam (Figure 3.6). The coordinate system is shown in the figure. Let u(x,y,t) and v(y,t) be the displacements in the z-direction in the distributed beam system and the perpendicular beam (beam 2), respectively. In this situation, the shears in the distributed beams are applying a distributed force on the perpendicular beam, and the equation of motion for the beam (2) is modified accordingly.

Let E_1 be the modulus of elasticity, I^*_1 be the moment of inertia per unit height and m^*_1 be the mass per unit length per unit height of the distributed beam system. The equation of motion for free vibrations of the distributed beams has been given in subsection (3.2.3). The equation of motion for beam (2) is as follows:

(a) For the case when beam (2) is a bending beam with $E_2 I_2$ its flexural stiffness and m_2 its mass per unit height, the equation of motion for free vibration is:

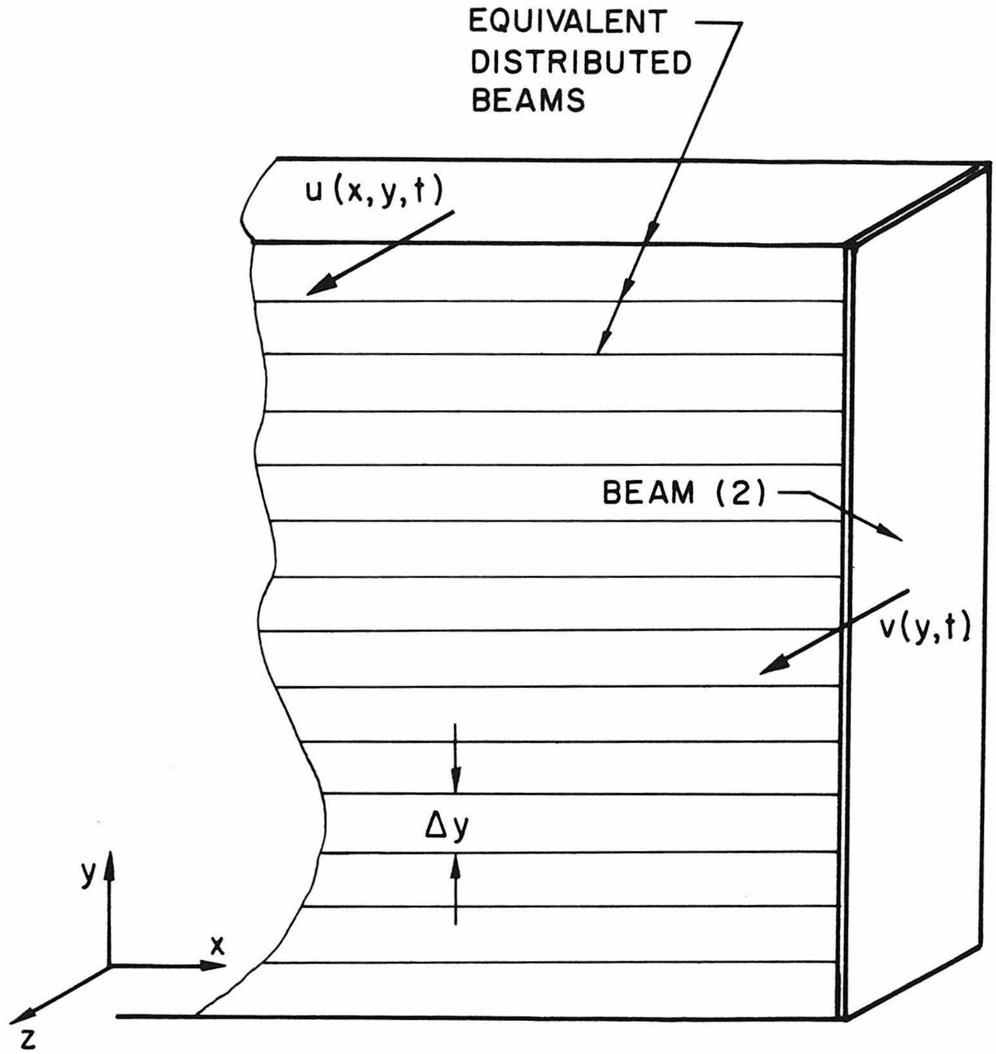


Figure 3.6. EQUIVALENT DISTRIBUTED BEAMS JOINED TO A PERPENDICULAR BEAM.

$$\frac{\partial^2}{\partial y^2}\left(E_2 I_2 \frac{\partial^2 v(y,t)}{\partial y^2}\right) = -m_2 \frac{\partial^2 v(y,t)}{\partial t^2} + \left[\frac{\partial}{\partial x}\left(E_1 I_1^* \frac{\partial^2 u(x,y,t)}{\partial x^2}\right)\right]_{x=L} \quad (3.15a)$$

(b) When beam (2) is a shear beam with stiffness k_2 and mass m_2 per unit height, the equation of motion for beam (2) is

$$\frac{\partial}{\partial y}\left(k_2 \frac{\partial v(y,t)}{\partial y}\right) = m_2 \frac{\partial^2 v(y,t)}{\partial t^2} - \left[\frac{\partial}{\partial x}\left(E_1 I_1^* \frac{\partial^2 u(x,y,t)}{\partial x^2}\right)\right]_{x=L} \quad (3.15b)$$

The algebraic sign of the last term in the above two equations will be different if beam (2) is joined at the left end of the distributed beam system.

The following boundary conditions must be satisfied for the system at the junction of the laminae and beam (2):

(i) The end displacements in the distributed beams are the same as the displacements in beam (2), i.e.,

$$u(x{=}L,y,t) = v(y,t) \quad (3.16)$$

(ii) The bending moments in the distributed beams at their ends must be in equilibrium with the torsional moment in the perpendicular beam.

This condition can be derived by taking the angle of twist in beam (2) equal to the end rotation in a distributed beam, and applying equilibrium. In mathematical form, the condition is

$$E_1 I_1^* \frac{\partial^2 u}{\partial x^2}(x=L,y,t) = \frac{bc^3 G_2}{3} \frac{\partial^2}{\partial y^2} \left[\frac{\partial u}{\partial x} (x=L,y,t)\right] \qquad (3.17)$$

where b, c, and G_2 are the length, thickness and shear modulus, respectively, for beam (2).

It will be acceptable for many applications to neglect the torsional rigidity of beam (2). This leads to a simpler condition, i.e.,

$$E_1 I_1 \frac{\partial^2 u}{\partial x^2} (x=L,y,t) = 0 \qquad (3.18)$$

3.4 ONE-STORY BUILDING WITH TWO IDENTICAL END WALLS

In this section, the solution for free vibrations of a one-story building with two end walls is presented. The walls are assumed to be identical. As mentioned earlier, this is a frequently used structural system for schools, hospitals, offices, etc. Typically, in these situations the end walls are quite short and wide, and can be treated as shear walls. The roof, usually being long and narrow, is modelled as a bending beam.

Consider a one-story building with two identical end walls (shear beams) of height h, and a long, narrow roof (bending beam) of length 2L (Figure 3.7). The building is being analyzed for motion in the z'-direction. Let the following be the roof and wall properties, assumed to be uniform:

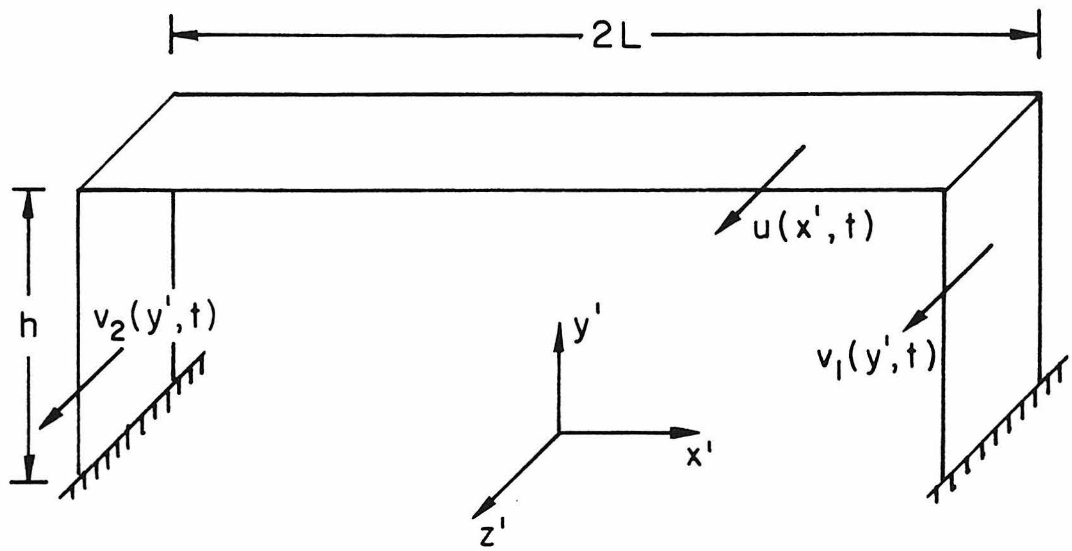


Figure 3.7. MODEL OF A ONE-STORY BUILDING WITH TWO END WALLS.

E_1 = Young's modulus for the roof.

I_1 = Moment of inertia about the center-line (parallel to the y'-axis) of roof cross-section.

k_2 = Shear rigidity of wall cross-section ($k_2 = k'A_2G_2$).

k' = Shape factor.

A_2 = Area of cross-section of wall.

G_2 = Shear modulus of wall.

m_1 = Mass per unit length of roof.

m_2 = Mass per unit height of wall.

Let $u(x',t)$ be the displacement in the z'-direction at time t of point x' of the roof. Similarly, let $v_1(y',t)$ and $v_2(y',t)$ be the displacements in the z'-direction at time t of points y' in the right and left end walls, respectively. The equations of motion for free vibrations for the system consisting of roof and end walls can be written as:

$$E_1 I_1 \frac{\partial^4 u(x',t)}{\partial x'^4} = -m_1 \frac{\partial^2 u(x',t)}{\partial t^2} \tag{3.19a}$$

$$k_2 \frac{\partial^2 v_1(y',t)}{\partial y'^2} = m_2 \frac{\partial^2 v_1(y',t)}{\partial t^2} \tag{3.19b}$$

$$k_2 \frac{\partial^2 v_2(y',t)}{\partial y'^2} = m_2 \frac{\partial^2 v_2(y',t)}{\partial t^2} \tag{3.19c}$$

It will be useful to carry out the further development in terms of nondimensional coordinates. Letting

$$x = \frac{x'}{L} \quad , \quad y = \frac{y'}{h} \tag{3.20}$$

and substituting these into the equations of motion gives

$$\frac{\partial^4 u(x,t)}{\partial x^4} = -\frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} \tag{3.21a}$$

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} = \frac{m_2 h^2}{k_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} \tag{3.21b}$$

and

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} = \frac{m_2 h^2}{k_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} \tag{3.21c}$$

Separation of variables is used to solve the problem of free vibrations of the system. Let

$$u(x,t) = U(x)e^{i\omega t} \tag{3.22a}$$

$$v_1(y,t) = V_1(y)e^{i\omega t} \tag{3.22b}$$

$$v_2(y,t) = V_2(y)e^{i\omega t} \tag{3.22c}$$

where ω is the natural frequency of the motion. Substitution into equations (3.21a, b, c) gives

$$\frac{d^4 U}{dx^4} - \alpha^4 U = 0 \tag{3.23a}$$

$$\frac{d^2V_1}{dy^2} + \beta^2 V_1 = 0 \tag{3.23b}$$

$$\frac{d^2V_2}{dy^2} + \beta^2 V_2 = 0 \tag{3.23c}$$

where

$$\alpha^4 = \frac{m_1 L^4}{E_1 I_1}\,\omega^2 \quad , \quad \beta^2 = \frac{m_2 h^2}{k_2}\,\omega^2 \tag{3.24}$$

The solutions for the above equations can be written as

$$U(x) = A_1 \sin \alpha x + A_2 \cos \alpha x + A_3 \sinh \alpha x + A_4 \cosh \alpha x \tag{3.25a}$$

$$V_1(y) = B_1 \sin \beta y + B_2 \cos \beta y \tag{3.25b}$$

$$V_2(y) = B'_1 \sin \beta y + B'_2 \cos \beta y \tag{3.25c}$$

where the A's and B's are constants to be determined by the governing boundary conditions.

Since the structure is symmetric about the y'-axis, it possesses symmetric and antisymmetric modes of vibrations. It is convenient to solve for the two types of modes separately, by making use of symmetry and by analyzing only the right half of the structure.

(a) Symmetric Modes

For the symmetric modes (or translational modes), there will be zero slope and zero shear at mid-span of the roof. The following are the boundary conditions for the right half of the structure:

(i) Slope at mid-span of roof is zero:

$$\frac{dU}{dx}(x=0) = 0$$

(ii) Shear at mid-span of roof is zero:

$$\frac{d^3U}{dx^3}(x=0) = 0$$

(iii) Displacement at the bottom end of wall is zero:

$$V_1(y=0) = 0$$

(iv) Displacements at the corner match:

$$U(x=1) = V_1(y=1)$$

(v) The shears at the junction balance

$$\frac{d^3U}{dx^3}(x=1) = q_1 \frac{dV_1}{dy}(y=1)$$

where, $$q_1 = \frac{k_2 L^3}{E_1 I_1 h} \quad (3.26)$$

(vi) The moments at the junction balance:

$$\frac{d^2U}{dx^2}(x=1) = -q_2 \frac{dU}{dx}(x=1)$$

where, $$q_2 = \frac{C_2 L}{E_1 I_1} \qquad (3.27)$$

and, C_2 is the torsional rigidity of the end walls.

The boundary conditions (i) and (ii) require,

$$A_1 = A_3 = 0$$

Similarly from boundary condition (iii)

$$B_2 = 0$$

From (iv), (v) and (vi), one obtains,

$$A_2 \cos\alpha + A_4 \cosh\alpha = B_1 \sin\beta \qquad (3.28a)$$

$$\alpha^3\left[A_2 \sin\alpha + A_4 \sinh\alpha\right] = q_1\beta\, B_1 \cos\beta \qquad (3.28b)$$

and

$$\alpha^2\left[-A_2 \cos\alpha + A_4 \cosh\alpha\right] = -q_2\alpha\left[-A_2 \sin\alpha + A_4 \sinh\alpha\right] \qquad (3.28c)$$

The determinant of these three equations can be solved to obtain the following characteristic equation for the natural frequencies, and

and the equations can then be solved for the mode shapes:

$$(\alpha \cosh \alpha + q_2 \sinh \alpha)(\alpha^3 \sin \alpha \sin \beta - q_1\beta \cos \alpha \cos \beta) +$$

$$+ (\alpha \cos \alpha + q_2 \sin a)(\alpha^3 \sinh \alpha \sin \beta - q_1\beta \cosh \alpha \cos \beta) = 0 \quad (3.29)$$

$$U(x) = A\left[(\alpha \cosh \alpha + q_2 \sinh \alpha)\cos \alpha x + (\alpha \cos \alpha + q_2 \sin \alpha) \cosh \alpha x\right]$$

$$0 \leq x \leq 1 \quad (3.30a)$$

and

$$V_1(y) = A\left[\frac{\cos \alpha(\alpha \cosh \alpha + q_2 \sinh \alpha) + \cosh \alpha(\alpha \cos \alpha + q_2 \sin \alpha)}{\sin \beta}\right] \sin \beta y$$

$$0 \leq y \leq 1 \quad (3.30b)$$

where A is an arbitrary constant.

The mode shapes for the left half of the structure can be obtained by symmetry. Hence, for the whole building, the mode shapes are

$$U(x) = A\left[(\alpha \cosh \alpha + q_2 \sinh \alpha) \cos \alpha x + (\alpha \cos \alpha + q_2 \sin \alpha) \cosh \alpha x\right]$$

$$-1 \leq x \leq 1 \quad (3.31a)$$

$$V_1(y) = A\left[\frac{\cos \alpha(\alpha \cosh \alpha + q_2 \sinh \alpha) + \cosh \alpha(\alpha \cos \alpha + q_2 \sin \alpha)}{\sin \beta}\right] \cdot$$

$$\cdot \sin \beta y \qquad 0 \leq y \leq 1 \quad (3.31b)$$

and

$$V_2(y) = A\left[\frac{\cos\alpha(\alpha\cosh\alpha + q_2\sinh\alpha) + \cosh\alpha(\alpha\cos\alpha + q_2\sin\alpha)}{\sin\beta}\right] \cdot \sin\beta y \qquad 0 \le y \le 1 \tag{3.31c}$$

For a given structure, equations (3.24) provide α and β as functions of the frequency ω. Thus, one can solve the characteristic equation (3.29) and obtain natural frequencies of the system. For known frequencies, the corresponding mode shapes are given by equations (3.31a, b and c).

(b) Antisymmetric Modes

For antisymmetric modes (or torsional modes), there will be zero displacement and zero moment at the mid-span of the roof. Thus, the boundary conditions are

(i) $U(x=0) = 0$

(ii) $\frac{d^2U}{dx^2}(x=0) = 0$

(iii) $V_1(y=0) = 0$

(iv) $U(x=1) = V_1(y=1)$

(v) $\frac{d^3U}{dx^3}(x=1) = q_1 \frac{dV_1}{dy}(y=1)$

where, $q_1 = \frac{k_2 L^3}{E_1 I_1 h}$

(vi) $\frac{d^2U}{dx^2}(x=1) = -q_2 \frac{dU}{dx}(x=1)$

where, $q_2 = \frac{C_2 L}{E_1 I_1}$

These boundary conditions can be applied to equations (3.25a) and (3.25b) to obtain the following characteristic equation and mode shapes:

$$(\alpha \sin \alpha - q_2 \cos \alpha)(\alpha^3 \cosh \alpha \sin \beta - q_1 \beta \sinh \alpha \cos \beta) -$$

$$- (\alpha \sinh \alpha + q_2 \cosh \alpha)(\alpha^3 \cos \alpha \sin \beta + q_1\beta \sin \alpha \cos \beta) = 0 \quad (3.32)$$

$$U(x) = A\left[(\alpha \sinh \alpha + q_2 \cosh \alpha) \sin \alpha x + (\alpha \sin \alpha - q_2 \cos \alpha) \sinh \alpha x\right]$$

$$0 \leq x \leq 1 \quad (3.33a)$$

$$V_1(y) = A\left[\frac{(\alpha \sinh \alpha + q_2 \cosh \alpha)\sin \alpha + (\alpha \sin\alpha - q_2 \cos \alpha) \sinh \alpha}{\sin \beta}\right] \cdot$$

$$\cdot \sin \beta y \qquad 0 \leq y \leq 1 \quad (3.33b)$$

where A is an arbitrary constant.

The mode shapes for the whole building can be obtained from the antisymmetry condition, as:

$$U(x) = A\left[(\alpha \sinh \alpha + q_2 \cosh \alpha) \sin \alpha x + (\alpha \sin \alpha - q_2 \cos \alpha) \sinh \alpha x\right]$$

$$-1 \leq x \leq 1 \quad (3.34a)$$

$$V_1(y) = A\left[\frac{(\alpha \sinh \alpha + q_2 \cosh \alpha) \sin \alpha + (\alpha \sin \alpha - q_2 \cos \alpha) \sinh \alpha}{\sin \beta}\right] \cdot$$

$$\cdot \sin \beta y \qquad 0 \leq y \leq 1 \quad (3.34b)$$

$$V_2(y) = -A\left[\frac{(\alpha \sinh \alpha + q_2 \cosh \alpha) \sin \alpha + (\alpha \sin \alpha - q_2 \cos \alpha) \sinh \alpha}{\sin \beta}\right] \cdot$$

$$\cdot \sin \beta y \qquad 0 \leq y \leq 1 \qquad (3.34c)$$

Here again, equation (3.32) can be solved to obtain natural frequencies, and equations (3.34) give corresponding mode shapes. It is important to note here that the way this problem has been formulated, the contributions of any longitudinal walls to the torsional stiffness of the system are neglected. Also, the polar moment of inertia of the roof is underestimated. The two effects are small for long and narrow buildings, and they have opposite influences on the values of the torsional frequencies. Typically, the first effect is more important. There are instances where these factors are important enough to be included in the analysis for torsional modes. To do so within the present framework, one can simply increase the wall stiffness and the floor mass used in the calculation to the values that are appropriate for the actual building. The new stiffness and mass can then be used for the analysis of the antisymmetric modes.

Limiting Case

In most applications, it is thought acceptable to neglect the torsional rigidity (C_2) of the end walls. The resulting, simpler solutions (characteristic equations and mode shapes) can be obtained by taking the limit, as q_2 approaches zero, in the expressions derived above. Hence, for symmetric modes, the characteristic equation and the mode shapes reduce to:

$$\alpha^3(\tan\alpha + \tanh\alpha) - 2q_1\beta\cot\beta = 0 \qquad (3.35)$$

and

$$U(x) = A\left[\cosh\alpha\cos\alpha x + \cos\alpha\cosh\alpha x\right] \qquad (3.36a)$$

$$V_1(y) = 2A\cos\alpha\cosh\alpha\,\mathrm{cosec}\,\beta\sin\beta y \qquad (3.36b)$$

and

$$V_2(y) = 2A\cos\alpha\cosh\alpha\,\mathrm{cosec}\,\beta\sin\beta y \qquad (3.36c)$$

Similarly, for antisymmetric modes, the characteristic equation and the mode shapes are:

$$\alpha^3(\coth\alpha - \cot\alpha) - 2q_1\beta\cot\beta = 0 \qquad (3.37)$$

$$U(x) = A(\sinh\alpha\sin\alpha x + \sin\alpha\sinh\alpha x) \qquad (3.38a)$$

$$V_1(y) = 2A\sin\alpha\sinh\alpha\,\mathrm{cosec}\,\beta\sin\beta y \qquad (3.38b)$$

$$V_2(y) = -2A\sin\alpha\sinh\alpha\,\mathrm{cosec}\,\beta\sin\beta y \qquad (3.38c)$$

Orthogonality of Modes

The following analysis demonstrates the orthogonality of the modes of vibration. Let ω_i and $U_i(x)$, $V_{1i}(y)$, $V_{2i}(y)$ be the natural frequency and mode shape for the i^{th} mode. Similarly, ω_k, $U_k(x)$, $V_{1k}(y)$ and $V_{2k}(y)$ correspond to the k^{th} mode. Substitution into equations (3.23a, b and c) gives:

$$\frac{d^4U_i(x)}{dx^4} - \frac{m_1L^4}{E_1I_1}\omega_i^2U_i(x) = 0 \qquad (3.39a)$$

$$\frac{d^2V_{1i}(y)}{dy^2} + \frac{m_2h^2}{k_2}\omega_i^2V_{1i}(y) = 0 \qquad (3.39b)$$

$$\frac{d^2V_{2i}(y)}{dy^2} + \frac{m_2h^2}{k_2}\omega_i^2V_{2i}(y) = 0 \qquad (3.39c)$$

and,

$$\frac{d^4U_k(x)}{dx^4} - \frac{m_1L^4}{E_1I_1}\omega_k^2U_k(x) = 0 \qquad (3.40a)$$

$$\frac{d^2V_{1k}(y)}{dy^2} + \frac{m_2h^2}{k_2}\omega_k^2V_{1k}(y) = 0 \qquad (3.40b)$$

$$\frac{d^2V_{2k}(y)}{dy^2} + \frac{m_2h^2}{k_2}\omega_k^2V_{2k}(y) = 0 \qquad (3.40c)$$

From these equations, one obtains

$$\int_{-1}^{1}\left[\frac{d^4U_i}{dx^4}U_k - \frac{d^4U_k}{dx^4}U_i\right]dx - \frac{m_1L^4}{E_1I_1}(\omega_i^2 - \omega_k^2)\int_{-1}^{1}U_iU_kdx = 0 \qquad (3.41a)$$

$$\int_{0}^{1}\left[\frac{d^2V_{1i}}{dy^2}V_{1k} - \frac{d^2V_{1k}}{dy^2}V_{1i}\right]dy + \frac{m_2h^2}{k_2}(\omega_i^2 - \omega_k^2)\int_{0}^{1}V_{1i}V_{1k}dy = 0 \qquad (3.41b)$$

$$\int_0^1 \left[\frac{d^2V_{2i}}{dy^2} V_{2k} - \frac{d^2V_{2k}}{dy^2} V_{2i} \right] dy + \frac{m_2 h^2}{k_2} (\omega_i^2 - \omega_k^2) \int_0^1 V_{2i} V_{2k} dy = 0 \qquad (3.41c)$$

Next, integrate the first integrals in these equations by parts, and apply the appropriate boundary conditions, to obtain

$$\left[\frac{d^3U_i}{dx^3} U_k - \frac{d^3U_k}{dx^3} U_i \right]_{x=-1}^{x=1} - \frac{m_1 L^4}{E_1 I_1} (\omega_i^2 - \omega_k^2) \int_{-1}^1 U_i U_k dx = 0 \qquad (3.42a)$$

$$\left[\frac{dV_{1i}(y=1)}{dy} V_{1k}(y=1) - \frac{dV_{1k}(y=1)}{dy} V_{1i}(y=1) \right] +$$

$$+ \frac{m_2 h^2}{E_2 I_2} (\omega_i^2 - \omega_k^2) \int_0^1 V_{1i} V_{1k} dy = 0 \qquad (3.42b)$$

$$\left[\frac{dV_{2i}(y=1)}{dy} V_{2k}(y=1) - \frac{dV_{2k}(y=1)}{dy} V_{2i}(y=1) \right] +$$

$$+ \frac{m_2 h^2}{E_2 I_2} (\omega_i^2 - \omega_k^2) \int_0^1 V_{2i} V_{2k} dy = 0 \qquad (3.42c)$$

However,

$$U(x=1) = V_1(y=1)$$

$$U(x=-1) = V_2(y=1)$$

$$\frac{E_1 I_1}{L^3} \frac{d^3U}{dx^3} (x=1) = \frac{k_2}{h} \frac{dV_1}{dy} (y=1)$$

$$-\frac{E_1 I_1}{L^3} \frac{d^3U}{dx^3} (x=-1) = \frac{k_2}{h} \frac{dV_2}{dy} (y=1)$$

Hence, equations (3.42a,b,c) can be combined to obtain

$$(\omega_i^2 - \omega_k^2)\left[m_1 L \int_{-1}^{1} U_i U_k dx + m_2 h \int_0^1 V_{1i} V_{1k} dy + m_2 h \int_0^1 V_{2i} V_{2k} dy\right] = 0 \quad (3.43)$$

For $i \neq k$, $\omega_i \neq \omega_k$,

$$m_1 L \int_{-1}^{1} U_i U_k dx + m_2 h \int_0^1 V_{1i} V_{1k} dy + m_2 h \int_0^1 V_{2i} V_{2k} dy = 0 \quad (3.44)$$

Thus, the modes are orthogonal, with the orthogonality condition given by equation (3.44).

Participation Factors for Earthquake Ground Motion

The equations of motion, including earthquake ground motion, for the roof and the walls can be written as:

$$\frac{\partial^4 u(x,t)}{\partial x^4} + \frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} = - \frac{m_1 L^4}{E_1 I_1} \ddot{u}_g(t) \quad (3.45a)$$

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} = \frac{m_2 h^2}{k_2} \ddot{u}_g(t) \quad (3.45b)$$

and,

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} = \frac{m_2 h^2}{k_2} \ddot{u}_g(t) \quad (3.45c)$$

where $\ddot{u}_g(t)$ is the earthquake ground acceleration in the z'-direction.

Expanding the response in terms of the normal modes of the system

$$u(x,t) = \sum_{i=1}^{\infty} U_i(x)T_i(t) \qquad (3.46a)$$

$$v_1(y,t) = \sum_{i=1}^{\infty} V_{1i}(y)T_i(t) \qquad (3.46b)$$

and

$$v_2(y,t) = \sum_{i=1}^{\infty} V_{2i}(y)T_i(t) \qquad (3.46c)$$

Substituting these into equations (3.45a,b,c) yields

$$\sum_i \frac{d^4U_i}{dx^4} T_i + \frac{m_1L^4}{E_1I_1} \sum_i U_i\ddot{T}_i = -\frac{m_1L^4}{E_1I_1} \ddot{u}_g(t) \qquad (3.47a)$$

$$\sum_i \frac{d^2V_{1i}}{dy^2} T_i - \frac{m_2h^2}{k_2} \sum_i V_{1i}\ddot{T}_i = \frac{m_2h^2}{k_2} \ddot{u}_g(t) \qquad (3.47b)$$

and,

$$\sum_i \frac{d^2V_{2i}}{dy^2} T_i - \frac{m_2h^2}{k_2} \sum_i V_{2i}\ddot{T}_i = \frac{m_2h^2}{k_2} \ddot{u}_g(t) \qquad (3.47c)$$

Substitution from equations (3.39a,b,c) gives

$$\sum_i \left[U_i(x)\ddot{T}_i(t) + \omega_i^2 U_i(x)T_i(t)\right] = -\ddot{u}_g(t) \qquad (3.48a)$$

$$\sum_i \left[V_{1i}(y)\ddot{T}_i(t) + \omega_i^2 V_{1i}(y)T_i(t)\right] = -\ddot{u}_g(t) \tag{3.48b}$$

and

$$\sum_i \left[V_{2i}(y)\ddot{T}_i(t) + \omega_i^2 V_{2i}(y)T_i(t)\right] = -\ddot{u}_g(t) \tag{3.48c}$$

Next, multiply equation (3.48a) by $m_1L\ U_k(x)$, equations (3.48b,c) by $m_2hV_{1k}(y)$ and $m_2hV_{2k}(y)$ respectively, integrate and add together to get

$$\sum_i \left[\left(m_1L\int_{-1}^{1} U_i(x)U_k(x)dx + m_2h\int_0^1 V_{1i}(y)V_{1k}(y)dy + m_2h\int_0^1 V_{2i}(y)V_{2k}(y)dy\right) \cdot \left(\ddot{T}_i\ (t) + \omega_i^2\ T_i(t)\right)\right]$$

$$= -\left[m_1L\int_{-1}^{1} U_k(x)dx + m_2h\int_0^1 V_{1k}(y)dy + m_2h\int_0^1 V_{2k}(y)dy\right]\ddot{u}_g(t) \tag{3.49}$$

Finally, one applies the orthogonality condition to obtain

$$\left[m_1L\int_{-1}^{1} (U_k(x))^2dx + m_2h\int_0^1 (V_{1k}(y))^2dy + m_2h\int_0^1 (V_{2k}(y))^2dy\right] \cdot (\ddot{T}_k + \omega_k^2 T_k)$$

$$= -\left[m_1L\int_{-1}^{1} U_k(x)dx + m_2h\int_0^1 V_{1k}(y)dy + m_2h\int_0^1 V_{2k}(y)dy\right]\ddot{u}_g(t) \tag{3.50}$$

or

$$\ddot{T}_k + \omega_k^2 T_k = -P_k \ddot{u}_g(t) \tag{3.51}$$

where P_k is the participation factor corresponding to the k^{th} mode, given by

$$P_k = \frac{m_1 L \int_{-1}^{1} U_k(x)\,dx + m_2 h \int_0^1 V_{1k}(y)\,dy + m_2 h \int_0^1 V_{2k}(y)\,dy}{m_1 L \int_{-1}^{1} (U_k(x))^2 dx + m_2 h \int_0^1 (V_{1k}(y))^2 dy + m_2 h \int_0^1 (V_{2k}(y))^2 dy} \tag{3.52}$$

Mode shapes from equations (3.31a,b,c) and (3.34a,b,c) can be substituted to obtain modal participation factors corresponding to a particular mode. For antisymmetric modes, substitution from equations (3.34a,b,c) into equation (3.52) gives zero modal participation factors. Hence, as could be anticipated, the assumed uniform ground motion does not excite antisymmetric modes in this symmetric structure.

3.5 PERTURBATION METHOD FOR FUNDAMENTAL NATURAL FREQUENCY

In the previous section, the characteristic equations obtained for the natural frequencies of the building are transcendental in nature, and have to be solved numerically. Although it is not difficult to solve these equations on a programmable calculator, it is of interest to have a simpler way of solving them, even if the solution is approximate. In this section, it is shown that one can use perturbation theory to obtain the fundamental natural frequency of the system without having to solve the equation numerically. In many applications, the fundamental frequency may be the only frequency of real concern, as most one-story structures are rather stiff and have high frequencies of vibration.

For a long, narrow one-story building with only two end walls in the transverse direction, the fundamental natural frequency of the building is close to the fundamental natural frequency of the roof when treated as a pinned-pinned beam. Thus, one can take the simple solution for the simply-supported beam as the unperturbed solution for such buildings, and seek the first order correction term in order to obtain an approximate solution for the first natural frequency of the whole system.

Let ω_0 be the fundamental natural frequency of the roof when treated as a pinned-pinned beam (e.g., Meirovitch, 1975). Thus,

$$\alpha_0^4 = \frac{m_1 L^4}{E_1 I_1} \omega_0^2$$

$$= \frac{m_1 L^4}{E_1 I_1} \frac{\pi^4 E_1 I_1}{16 m_1 L^4}$$

which gives,

$$\alpha_0 = \frac{\pi}{2} \tag{3.53}$$

Let the correct solution for equation (3.29) be α. Then,

$$\alpha = \alpha_0 + \alpha_1 + O(\alpha_1^2) \tag{3.54}$$

where α_1 is the first order correction term and is small compared to α. The analysis then proceeds on the basis that terms containing higher powers of α_1 can be neglected. From equation (3.24)

$$\beta^2 = \frac{m_2 h^2}{k_2} \frac{E_1 I_1}{m_1 L^4} \alpha^4$$

or,

$$\beta = p(\alpha_0 + \alpha_1)^2 = p(\frac{\pi^2}{4} + \pi\alpha_1) \tag{3.55}$$

where,

$$p = \left(\frac{m_2 E_1 I_1 h^2}{m_1 k_2 L^4}\right)^{1/2} \tag{3.56}$$

Since α_1 is small,

$$\sin\alpha = \sin(\frac{\pi}{2} + \alpha_1) = \cos\alpha_1 \cong 1$$

$$\cos\alpha = \cos(\frac{\pi}{2} + \alpha_1) = -\sin\alpha_1 \cong -\alpha_1$$

$$\begin{aligned}\sinh\alpha &= \sinh(\frac{\pi}{2} + \alpha_1) \\ &= \sinh\frac{\pi}{2}\cosh\alpha_1 + \cosh\frac{\pi}{2}\sinh\alpha_1 \\ &\simeq 2.3013 + 2.5092\alpha_1\end{aligned}$$

and,

$$\begin{aligned}\cosh\alpha &= \cosh(\frac{\pi}{2} + \alpha_1) \\ &\simeq 2.5092 + 2.3013\alpha_1\end{aligned}$$

Substitution of these into equation (3.29) gives a linear equation in α_1:

$$\alpha_1 = \frac{(15.28 + 17.84q_2)\sin(\frac{\pi^2}{4}p) + 6.19pq_1q_2\cos(\frac{\pi^2}{4}p)}{\left[(29.18 + 36.48q_2 - 19.45p^2q_1q_2)\sin(\frac{\pi^2}{4}p) + (47.99 - 19.45q_1 + 56.04q_2 + 7.88q_1q_2)p\cos(\frac{\pi^2}{4}p)\right]} \quad (3.57)$$

Here p, q_1, q_2 are functions of the structural properties and have been defined earlier. Substitution of these into this equation gives the correction to be applied to α_0, i.e.,

$$\alpha = \frac{\pi}{2} + \alpha_1$$

The frequency of the first mode of vibration is found from α via equation (3.24).

Neglecting the torsional rigidity of end walls ($q_2 \to 0$), the equation (3.57) can be simplified further to

$$\alpha_1 = \frac{15.28\sin(\frac{\pi^2}{4}p)}{29.18\sin(\frac{\pi^2}{4}p) + (47.99 - 19.45q_1)p\cos(\frac{\pi^2}{4}p)} \quad (3.58)$$

Similar expressions can be obtained for antisymmetric modes of vibration. For this, the unperturbed solution (α_0) can be obtained from the natural frequency for the first antisymmetric mode of the roof when treated as a pinned-pinned beam.

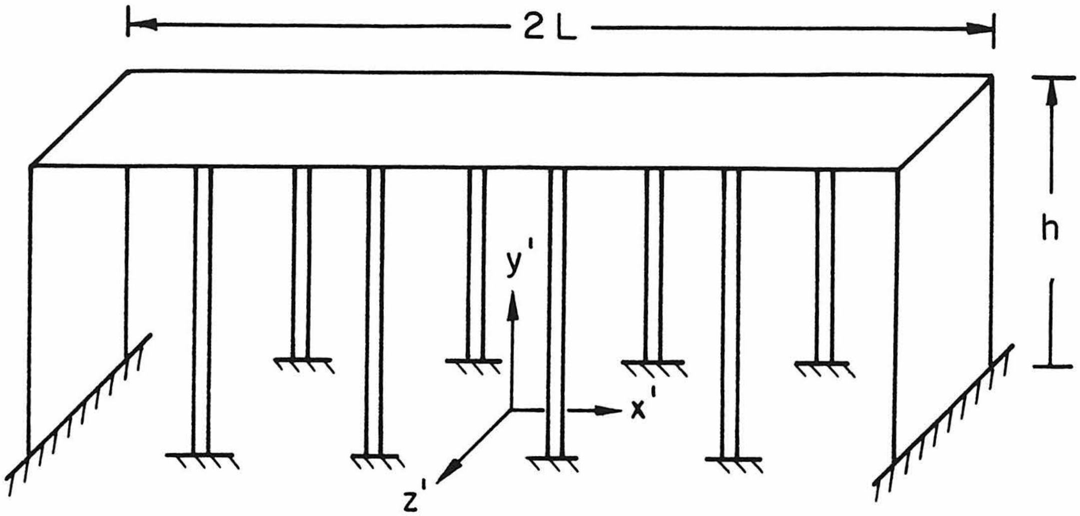
3.6 OTHER CASES OF SINGLE-STORY BUILDINGS

A particularly simple structure was chosen in section (3.4) to demonstrate the method. However, the technique is more general and can be applied to other single-story buildings. This section explains how some of the more general problems can be formulated and solved.

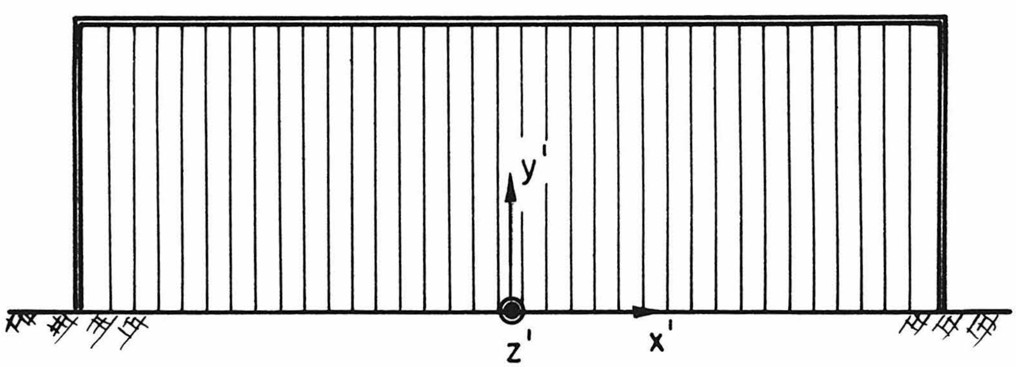
3.6.1. Symmetric Buildings with End Shear Walls and Distributed Columns

In the previous section, any columns between the end walls were assumed to take only vertical loads. However, one can easily include the lateral stiffness of these columns in the analysis. In this case, the roof and the end walls can be treated as bending and shear beams, respectively, as was done in section (3.4). The columns between the end walls can be modelled by uniformly distributed, thin columns with only bending flexibility, provided the spacing between adjacent columns along the x'-axis is not large. This leads to boundary conditions at the top end of the columns similar to those discussed in section (3.3.3).

Consider one such building (Figure 3.8). Let $w(x',y',t)$ be the displacement at time t, in the z'-direction, of a point (x',y') in the continuum modelling the columns. Let E_3 be the modulus of elasticity, I^*_3 be the moment of inertia per unit width and m^*_3 be the mass per unit area (in elevation) of the column continuum. Properties for the walls and the roof are the same as defined in section (3.4). The governing equations for free vibrations in the nondimensional coordinates are:



(a)



(b)

Figure 3.8. (a) ONE-STORY BUILDING WITH END SHEAR WALLS AND DISTRIBUTED COLUMNS. (b) DISTRIBUTED IDEALIZATION.

$$\frac{\partial^4 u(x,t)}{\partial x^4} + \frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{E_3 I_3^* L^4}{E_1 I_1 h^3} \frac{\partial^3 w(x,y=1,t)}{\partial y^3} \tag{3.59a}$$

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} = 0 \tag{3.59b}$$

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} = 0 \tag{3.59c}$$

and

$$\frac{\partial^4 w(x,y,t)}{\partial y^4} + \frac{m_3^* h^4}{E_3 I_3^*} \frac{\partial^2 w(x,y,t)}{\partial t^2} = 0 \tag{3.59d}$$

Considering only the right half of the structure, the following are the boundary conditions:

(i) $v_1(y=0,t) = 0$

(ii) $w(x,y=0,t) = 0$

(iii) $\frac{\partial w}{\partial y}(x,y=0,t) = 0$

(iv) $\frac{\partial^2 w}{\partial y^2}(x,y=1,t) = 0$

(v) $u(x=1,t) = v_1(y=1,t)$

(vi) $u''(x=1,t) = 0$

(vii) $u'''(x=1,t) = q_1 v_1'(y=1,t)$

(viii) $u(x,t) = w(x,y=1,t)$

(ix) $u'(x=0,t) = 0$ (for symmetric modes)

$u(x=0,t) = 0$ (for antisymmetric mode)

(x) $u'''(x=0,t) = 0$ (for symmetric modes)

$$u''(x=0,t) \quad = \quad 0 \quad \text{(for antisymmetric modes)}$$

Here condition (iv) is valid if the columns are free to rotate at their top ends. However, when there are rigid beams connecting the columns along the z'-axis, this cannot be applied; such a case is discussed in the next sub-section. Condition (vi) corresponds to zero torsional stiffness of the end walls.

The above differential equations and the boundary conditions can be combined together and solved. The resulting characteristic equations and mode shapes are given below:

(a) Symmetric Modes

The characteristic equation is given by

(i) For $\alpha^4 > 0$:

$$\alpha^3(\tan\alpha + \tanh\alpha) - 2q_1\beta\cot\beta = 0 \tag{3.60a}$$

(ii) For $\alpha^4 < 0$

$$4\xi^3 \frac{(\mathrm{cosec}\,2\xi + \mathrm{cosech}\,2\xi)}{(\tan\xi\tanh\xi + \cot\xi\coth\xi)} + q_1\beta\cot\beta = 0 \tag{3.60b}$$

where

$$\alpha^4 = \frac{m_1 L^4}{E_1 I_1}\omega^2 - \frac{E_3 I^*{}_3 L^4}{E_1 I_1 h^3}\frac{\gamma^3(1 + \cos\gamma\cosh\gamma)}{(\sin\gamma\cosh\gamma - \sinh\gamma\cos\gamma)} \tag{3.61a}$$

$$\beta^2 = \frac{m_2 h^2}{k_2}\omega^2 \tag{3.61b}$$

$$\gamma^4 = \frac{m_3^* h^4}{E_3 I^*_3} \omega^2 \tag{3.61c}$$

$$\xi^4 = -\frac{\alpha^4}{4} \tag{3.61d}$$

The mode shapes are

$$U(x) = A\left[\frac{\cos \alpha x}{\cos \alpha} + \frac{\cosh \alpha x}{\cosh \alpha}\right] \quad \text{for } \alpha^4 > 0 \tag{3.62a}$$

$$U(x) = A[\sin \xi x \sinh \xi x + \cot \xi \coth \xi \cos \xi x \cosh \xi x]$$

$$\text{for } \alpha^4 < 0 \tag{3.62b}$$

$$V_1(y) = V_2(y) = 2A \operatorname{cosec} \beta \sin \beta y \quad \text{for } \alpha^4 > 0 \tag{3.62c}$$

$$V_1(y) = V_2(y) = A \operatorname{cosec} \beta \cos \xi \cosh \xi (\tan \xi \tanh \xi$$

$$+ \cot \xi \coth \xi) \sin \beta y \quad \text{for } \alpha^4 < 0 \tag{3.62d}$$

$$W(x,y) = C(x)\left[\frac{\sin \gamma y - \sinh \gamma y}{\sin \gamma + \sinh \gamma} - \frac{\cos \gamma y - \cosh \gamma y}{\cos \gamma + \cosh \gamma}\right] \tag{3.62e}$$

where

$$C(x) = U(x) \frac{(\sin \gamma + \sinh \gamma)(\cos \gamma + \cosh \gamma)}{2(\sin \gamma \cosh \gamma - \cos \gamma \sinh \gamma)} \tag{3.63}$$

and A is an arbitrary constant.

(b) Antisymmetric Modes

For the antisymmetric modes, the characteristic equation is given by

(i) For $\alpha^4 > 0$:

$$\alpha^3(\coth \alpha - \cot \alpha) - 2q_1\beta \cot \beta = 0 \qquad (3.64a)$$

(ii) For $\alpha^4 < 0$:

$$4\xi^3 \frac{(\operatorname{cosec} 2\xi - \operatorname{cosech} 2\xi)}{(\tan \xi \coth \xi + \cot \xi \tanh \xi)} + q_1\beta \cot \beta = 0 \qquad (3.64b)$$

where α, β and ξ are given by equations (3.61a,b,c,d).

The mode shapes are given as follows:

$$U(x) = A\left[\frac{\sin \alpha x}{\sin \alpha} + \frac{\sinh \alpha x}{\sinh \alpha}\right] \quad \text{for } \alpha^4>0 \qquad (3.65a)$$

$$U(x) = A[\sin \xi x \cosh \xi x + \cot \xi \tanh \xi \cos \xi x \sinh \xi x)$$

$$\text{for } \alpha^4<0 \qquad (3.65b)$$

$$V_1(y) = -V_2(y) = 2A \operatorname{cosec} \beta \sin \beta y \quad \text{for } \alpha^4>0 \qquad (3.65c)$$

$$V_1(y) = -V_2(y) = A \operatorname{cosec} \beta \cos \xi \sinh \xi (\tan \xi \coth \xi$$

$$+ \cot \xi \tanh \xi) \sin \beta y \quad \text{for } \alpha^4<0 \qquad (3.65d)$$

$$W(x,y) = C(x)\left[\frac{\sin \gamma y - \sinh \gamma y}{\sin \gamma + \sinh \gamma} - \frac{\cos \gamma y - \cosh \gamma y}{\cos \gamma + \cosh \gamma}\right] \qquad (3.65e)$$

where,

$$C(x) = U(x) \frac{(\sin \gamma + \sinh \gamma)(\cos \gamma + \cosh \gamma)}{2(\sin \gamma \cosh \gamma - \cos \gamma \sinh \gamma)} \qquad (3.66)$$

and A is an arbitrary constant.

The orthogonality condition and the modal participation factors can be obtained following a procedure similar to the one used in section

(3.4). Again it will be found that all antisymmetric modes have zero participation factors for uniform ground motion.

3.6.2 Symmetric Buildings with End Shear Walls and Distributed Portal Frames

This situation is similar to the previous case, except that now there are transverse beams (along the z'-direction) that connect the columns at the roof level. Thus, the columns aligned with the z'-axis, along with the corresponding beam on top, act as a portal frame and the top end of the column is not free to rotate. Therefore, the boundary condition (iv) of section (3.6.1) is no longer applicable. Instead, another boundary condition can be found from the analysis of portal frames that gives a relation between the moment and the angle of rotation in the column at y = 1. For instance, if the beam is much more rigid than the columns, no end rotation will be allowed at the top end. In that case, condition (iv) in the previous section will be replaced by

$$\frac{\partial w}{\partial y}(x,y=1,t) = 0$$

The rest of the boundary conditions of the previous case are the same, and hence, the problem can be solved following a similar procedure.

However, in this type of building, another complication may arise if there are longitudinal roof beams as well, and the junctions between the longitudinal and transverse beams are designed to resist moment. In that case, the beam grid at the roof level acts as a shear beam, and hence, it becomes necessary to treat the roof as a bending-shear beam.

On the other hand, such a beam grid tends to make the roof diaphragm quite stiff, and it may be possible to treat the roof as a rigid diaphragm. This makes the problem considerably simpler.

3.6.3 Buildings with Two Similar End Walls and One Wall in the Center

One-story buildings with only end walls may have very large lateral displacements at the mid-span of the roof during an earthquake. One effective and convenient way to control this is to add another wall in the center. Consider one such structure (Figure 3.9). Let k_3 $(=k'A_3G_3)$ and m_3 be the shear rigidity and the mass per unit height, of the wall in the center. The properties for the roof and the end walls are the same as in section (3.4). The dynamic equations of motion for the free vibrations, in terms of dimensionless coordinates x and y, are:

$$\frac{\partial^4 u(x,t)}{\partial x^4} + \frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} = 0 \qquad (3.67a)$$

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} = 0 \qquad (3.67b)$$

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} - \frac{m_2 h^2}{k_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} = 0 \qquad (3.67c)$$

and

$$\frac{\partial^2 w(y,t)}{\partial y^2} - \frac{m_3 h^2}{k_3} \frac{\partial^2 w(y,t)}{\partial t^2} = 0 \qquad (3.67d)$$

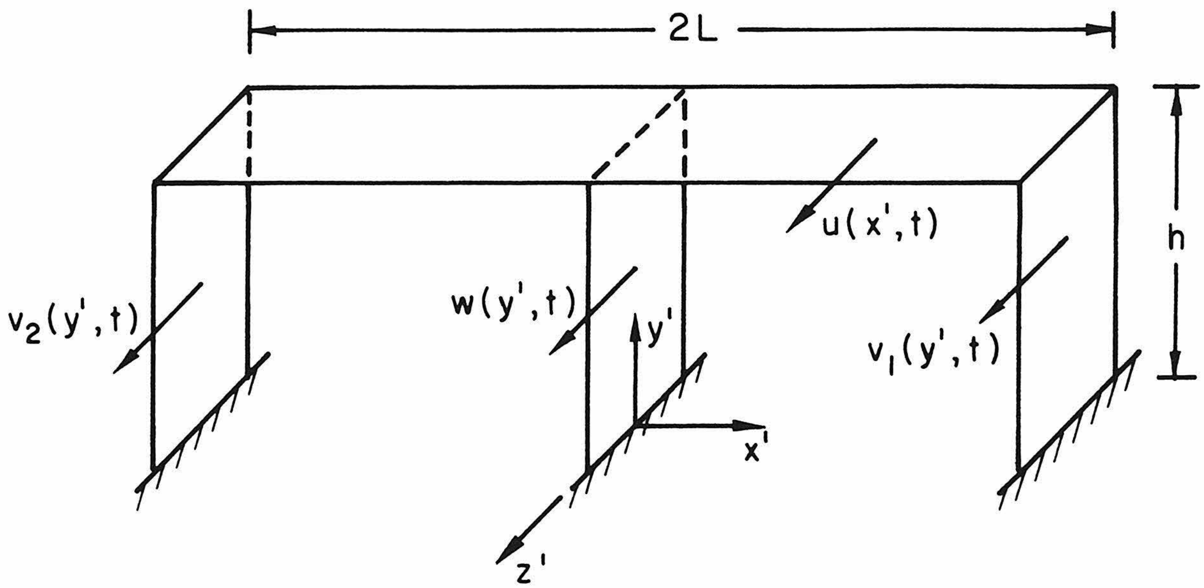


Figure 3.9. MODEL OF A ONE-STORY BUILDING WITH TWO END WALLS AND ONE WALL IN CENTER.

where w(y,t) is the displacement in the center wall.

Since the structure is symmetric, the symmetric and antisymmetric modes can be analyzed separately, and only the right half of structure need be considered. Following are the boundary conditions applicable to the right half of the structure:

(i) $v_1(y=0,t) = 0$

(ii) $w(y=0,t) = 0$

(iii) $u'(x=0,t) = 0$ (for symmetric modes)

$u(x=0,t) = 0$ (for antisymmetric modes)

(iv) $u'''(x=0,t) = -\frac{1}{2} q_3 w'(y=1,t)$ (for symmetric modes)

$u''(x=0,t) = \frac{1}{2} q_4 u'(x=0,t)$ (for antisymmetric modes)

where, $q_3 = \dfrac{k_3 L^3}{E_1 I_1 h}$

$q_4 = \dfrac{C_3 L}{E_1 I_1}$

and, C_3 is the torsional rigidity of the center wall.

(v) $u(x=0,t) = w(y=1,t)$

(vi) $u(x=1,t) = v_1(y=1,t)$

(vii) $u'''(x=1,t) = q_1 v'(y=1,t)$

(viii) $u''(x=1,t) = -q_2 u'(x=1,t)$

Equations (3.67) and the boundary conditions, as above, can be combined and solved to obtain the governing characteristic equations for the symmetric and antisymmetric modes as was done in section (3.4). However, this involves a considerable amount of algebra.

The method developed in this chapter can be applied to even more complex structures than those analyzed herein. The principal limitation is that as the structures get more complex, the algebra gets considerably more involved; and as a result the method loses the advantage of being simple. Even under these circumstances, there may be instances when the method is preferable to the finite element or lumped-mass methods.

3.7 NUMERICAL EXAMPLE

In order to illustrate the method described in this chapter, a single-story building with two identical end walls has been analyzed numerically to determine natural frequencies and mode shapes. The solution also allows comparison of natural periods obtained using several assumptions, e.g., neglecting the torsional stiffness of the end walls, etc.

For convenience, the properties for the example structure have been derived from the top story of the two-story Administrative Building at Arvin High School, discussed in the previous chapter. The appropriate data have been obtained from Steinbrugge and Moran (1954), Blume, et al. (1961), and Blume and Jhaveri (1969). The following are the building properties taken for the analysis:

Roof: span (2L) = 197.0 ft

weight ($m_1 g$) = 3770 lbs per ft

modulus of elasticity (E_1) = 2.0×10^6 psi

	moment of inertia (I_1)	= 66.2×10^6 (in)^4
End Wall:	height (h)	= 14'-11"
	weight (m_2g)	= 3300 lbs per ft
	shear modulus (G_2)	= 0.855×10^6 psi
	area of cross section (A_2)	= 3160 sq in
	shape factor (k')	= 0.833
	width of wall (b)	= 352 in
	thickness of wall (c)	= 9.0 in

The characteristic equations (3.29) and (3.32) for symmetric and antisymmetric modes, respectively, were solved for these properties. The roots give the natural frequencies for the example structure. In addition, characteristic equations (3.35) and (3.37), which correspond to zero torsional stiffness of the end walls, have been solved. Finally, equation (3.57) was used to approximate the fundamental frequency using the perturbation method. The natural periods obtained as indicated above and from modelling the roof by a pinned-pinned beam are given in Table (3.3). A comparison of these periods indicates that neglecting the torsional stiffness of the end walls does not introduce any significant error in the calculation of the lower frequencies. Also, in this example the perturbation method gives a very good estimate for the fundamental period, while avoiding the need to solve the transcendental characteristic equation.

TABLE 3.1. NATURAL PERIODS FOR THE SINGLE-STORY BUILDING

	Torsional Stiffness of Wall Included	Torsional Stiffness of Wall Neglected	Roof Modelled as Pinned-Pinned Beam	Perturbation Method
	(Sec)	(Sec)	(Sec)	(Sec)
First symmetric mode	0.283	0.283	0.279	0.280
First antisymmetric mode	0.0743	0.0744	0.0697	-
Second symmetric mode	0.0367	0.0367	0.0310	-
Second antisymmetric mode	0.0246	0.0246	0.0174	-
Third symmetric mode	0.0185	0.0185	0.0111	-
Third antisymmetric mode	0.0135	0.0135	0.0077	-

The mode shapes have been obtained using equations (3.31) and (3.34) and the first six are given in Table (3.4), while the first four are plotted in Figure (3.10). It is obvious from these mode shapes that the floor flexibility dominates the dynamic response of this example. For instance, in the fundamental mode, the center of the roof moves 45 times as much as the ends of the roof.

The modal participation factors for the symmetric modes were obtained from equation (3.52), and are given in Table (3.4). They are normalized by the displacement of the top of the end walls. The low numerical value for the participation factor for the first mode does not imply a relatively small contribution from that mode, because the participation factors depend on the way the mode shapes are normalized.

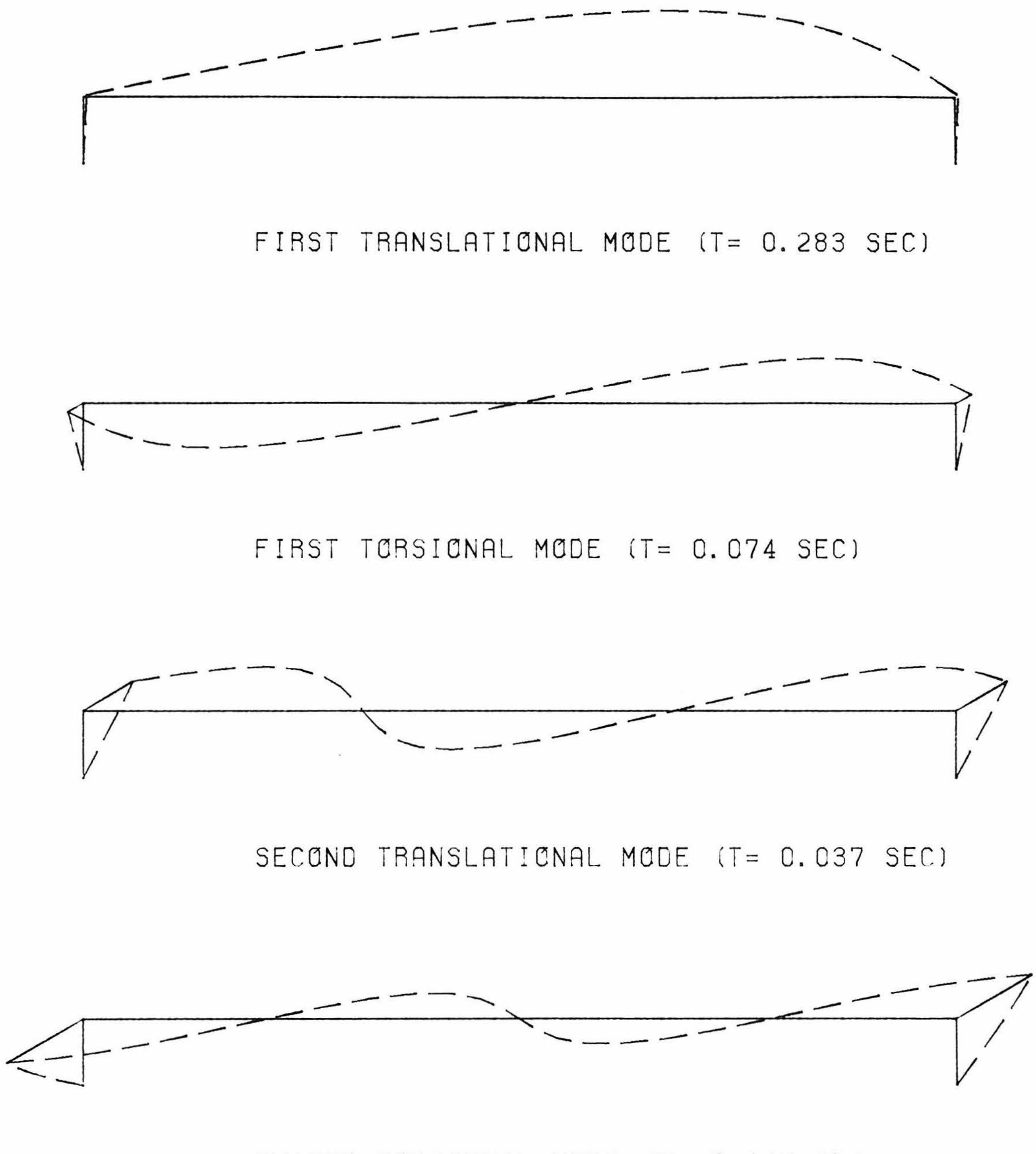


Figure 3.10. MODE SHAPES FOR THE SINGLE-STORY BUILDING.

TABLE 3.4. MODE SHAPES AND PARTICIPATION FACTORS FOR THE ONE-STORY BUILDING

	First Symmetric Mode	First Torsional Mode	Second Symmetric Mode	Second Torsional Mode	Third Symmetric Mode	Third Torsional Mode
U(x=0)	45.36	0.00	-1.32	0.00	0.51	0.00
U(x=0.1)	44.81	1.51	-1.20	-0.30	0.42	0.44
U(x=0.2)	43.19	2.88	-0.85	-0.51	0.18	0.66
U(x=0.3)	40.53	3.99	-0.33	-0.58	-0.12	0.56
U(x=0.4)	36.89	4.76	0.25	-0.49	-0.37	0.20
U(x=0.5)	32.37	5.09	0.80	-0.25	-0.48	-0.26
U(x=0.6)	27.08	4.98	1.23	0.08	-0.40	-0.58
U(x=0.7)	21.15	4.43	1.47	0.42	-0.14	-0.58
U(x=0.8)	14.71	3.52	1.48	0.70	0.23	-0.24
U(x=0.9)	7.94	2.33	1.30	0.89	0.63	0.34
U(x=1.0)	1.00	1.00	1.00	1.00	1.00	1.00
V_1(y=1.0)	1.00	1.00	1.00	1.00	1.00	1.00
V_1(y=0.9)	0.90	0.90	0.91	0.91	0.94	0.98
V_1(y=0.8)	0.80	0.80	0.81	0.83	0.86	0.93
V_1(y=0.7)	0.70	0.70	0.72	0.74	0.78	0.86
V_1(y=0.6)	0.60	0.60	0.62	0.64	0.68	0.78
V_1(y=0.5)	0.50	0.50	0.52	0.54	0.58	0.68
V_1(y=0.4)	0.40	0.40	0.42	0.44	0.47	0.56
V_1(y=0.3)	0.30	0.30	0.31	0.33	0.36	0.43
V_1(y=0.2)	0.20	0.20	0.21	0.22	0.24	0.29
V_1(y=0.1)	0.10	0.10	0.11	0.11	0.12	0.15
V_1(y=0)	0.00	0.00	0.00	0.00	0.00	0.00
Participation Factor	0.0283	0.00	0.412	0.00	0.650	0.00

The antisymmetric modes, as expected, have zero modal participation factors for uniform ground motion.

CHAPTER IV

TWO-STORY BUILDINGS WITH FLEXIBLE FLOORS

4.1 INTRODUCTION

The dynamic behavior of two-story, long, narrow buildings, like similar single-story buildings, is of significant interest to structural engineers due to their frequent use as office, school or hospital buildings. One such two-story school building was damaged during the 1952 Kern County earthquake, and has been discussed in Chapter II. The discussion in Chapter III about the dynamic behavior of single-story buildings is also valid for similar two-, three-, or more-story buildings and is not repeated here.

It will be shown in this chapter that the techniques developed for the single-story buildings can be applied to these buildings as well. The problem that arises in treating the multistory structures with the previously discussed methods is that the algebra tends to get very complicated with the increased number of stories. Hence, beyond a certain number of stories, this method loses the advantage of simplicity. For such situations, less accurate but simple and economic, methods are presented in the following chapters.

A two-story building, with identical end walls and no other lateral load resistance element, is the structure treated in this chapter. A characteristic equation for the natural frequencies, and expressions for the mode shapes and the participation factors are given in general form.

A numerical example, based on the Arvin High School Building, described in Chapter II, is used to illustrate the use of the method.

4.2 TWO-STORY BUILDINGS WITH TWO IDENTICAL END WALLS

Consider a two-story building that is long, narrow and has two identical end walls (Figure 4.1). Intermediate columns, if any, are assumed to take only vertical loads and provide no lateral resistance. The roof and the floor, having large span to width ratios, are treated as bending beams. The end walls are presumed to have small height to width ratios and are modelled as shear beams.

Although it is possible to consider different story heights and different wall properties in the first and the second story, for simplicity of analysis it has been assumed that the two story heights are the same, and that the walls are uniform throughout the building height. Let 2L be length of the roof and the floor, and h be the story height. Let the following be the roof, floor and wall properties, assumed to be uniform.

E_1, E_2 = Young's modulus for the floor and the roof, respectively.

I_1, I_2 = Moment of inertia of floor and roof cross sections, respectively.

$k_3 = k'A_3G_3$ = Shear rigidity of wall cross-section.

k' = Shape factor

A_3 = Area of cross-section of the wall.

G_3 = Shear modulus of the wall.

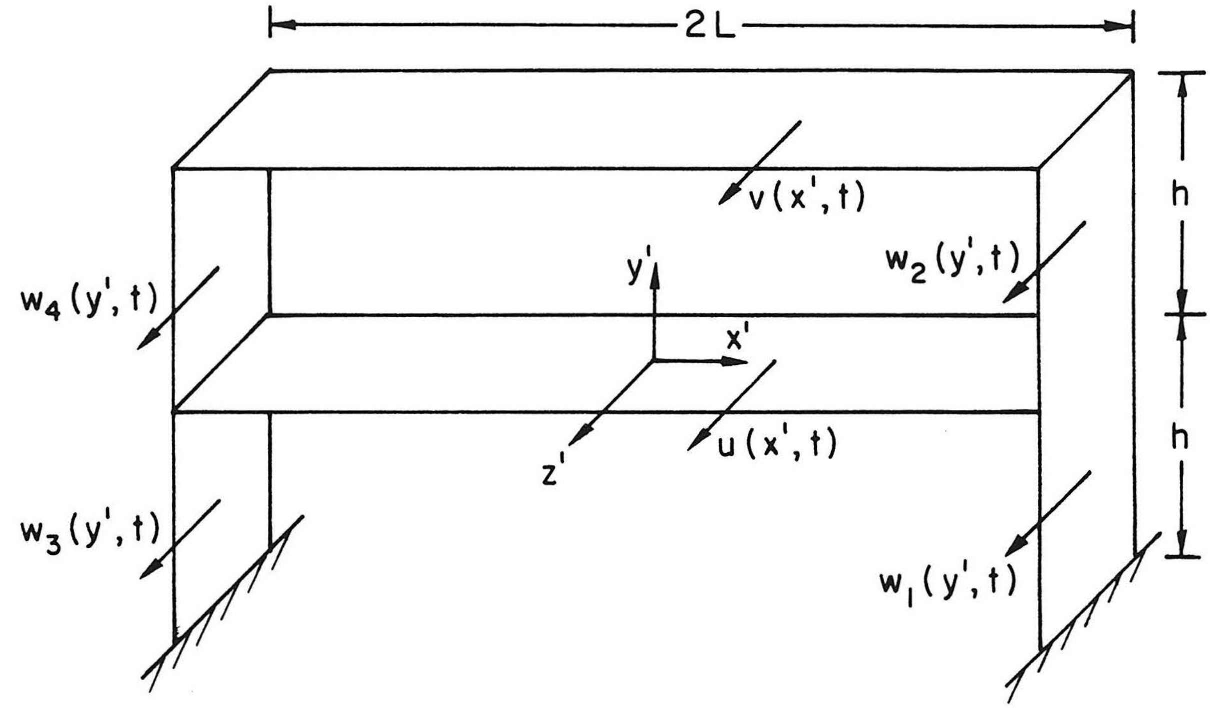


Figure 4.1. MODEL OF A TWO-STORY BUILDING WITH TWO END WALLS.

m_1, m_2, m_3 = Mass per unit length of floor, roof and wall, respectively.

Since the structure is symmetric about the y'-axis, it is possible to separate the vibrational modes into symmetric and antisymmetric modes. It is convenient to consider only the right half of the structure, and to treat the symmetric and antisymmetric modes separately. Let $u(x',t)$, $v(x',t)$, $w_1(y',t)$ and $w_2(y',t)$ be the displacements in the z'-direction in the floor, roof, and the first and the second story of the right end wall, respectively. The equations of motion for free vibrations of the right half of the structure are:

$$E_1 I_1 \frac{\partial^4 u(x',t)}{\partial x'^4} = -m_1 \frac{\partial^2 u(x',t)}{\partial t^2} \tag{4.1a}$$

$$E_2 I_2 \frac{\partial^4 v(x',t)}{\partial x'^4} = -m_2 \frac{\partial^2 v(x',t)}{\partial t^2} \tag{4.1b}$$

$$k_3 \frac{\partial^2 w_1(y',t)}{\partial y'^2} = m_3 \frac{\partial^2 w_1(y',t)}{\partial t^2} \tag{4.1c}$$

and

$$k_3 \frac{\partial^2 w_2(y',t)}{\partial y'^2} = m_3 \frac{\partial^2 w_2(y',t)}{\partial t^2} \tag{4.1d}$$

It is useful to perform the further analysis in terms of dimensionless coordinates, x and y, defined as

$$x = \frac{x'}{L} \quad , \quad y = \frac{y'}{h} \qquad (4.2)$$

Using these coordinates, the equations of motion can be rewritten as

$$\frac{\partial^4 u(x,t)}{\partial x^4} = - \frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} \qquad (4.3a)$$

$$\frac{\partial^4 v(x,t)}{\partial x^4} = - \frac{m_2 L^4}{E_2 I_2} \frac{\partial^2 v(x,t)}{\partial t^2} \qquad (4.3b)$$

$$\frac{\partial^2 w_1(y,t)}{\partial y^2} = \frac{m_3 h^2}{k_3} \frac{\partial^2 w_1(y,t)}{\partial t^2} \qquad (4.3c)$$

and,

$$\frac{\partial^2 w_2(y,t)}{\partial y^2} = \frac{m_3 h^2}{k_3} \frac{\partial^2 w_2(y,t)}{\partial t^2} \qquad (4.3d)$$

The analysis uses the method of separation of variables to solve the problem of free vibrations of the system. Let

$$u(x,t) = U(x)\ e^{i\omega t} \qquad (4.4a)$$

$$v(x,t) = V(x)\ e^{i\omega t} \qquad (4.4b)$$

$$w_1(y,t) = W_1(y)\ e^{i\omega t} \qquad (4.4c)$$

$$w_2(y,t) = W_2(y)\ e^{i\omega t} \qquad (4.4d)$$

where ω is the natural frequency of the motion. Substitution into equations (4.3a,b,c,d) gives

$$\frac{d^4U}{dx^4} - \alpha^4 U = 0 \tag{4.5a}$$

$$\frac{d^4V}{dx^4} - \beta^4 V = 0 \tag{4.5b}$$

$$\frac{d^2W_1}{dy^2} + \gamma^2 W_1 = 0 \tag{4.5c}$$

$$\frac{d^2W_2}{dy^2} + \gamma^2 W_2 = 0 \tag{4.5d}$$

where,

$$\alpha^4 = \frac{m_1 L^4}{E_1 I_1}\,\omega^2 \;, \quad \beta^4 = \frac{m_2 L^4}{E_2 I_2}\,\omega^2 \;, \quad \text{and} \quad \gamma^2 = \frac{m_3 h^2}{k_3}\,\omega^2 \tag{4.6}$$

The solutions for the above equations are:

$$U(x) = A_1 \sin \alpha x + A_2 \cos \alpha x + A_3 \sinh \alpha x + A_4 \cosh \alpha x \tag{4.7a}$$

$$V(x) = B_1 \sin \beta x + B_2 \cos \beta x + B_3 \sinh \beta x + B_4 \cosh \beta x \tag{4.7b}$$

$$W_1(y) = C_1 \sin \gamma y + C_2 \cos \gamma y \tag{4.7c}$$

$$W_2(y) = D_1 \sin \gamma y + D_2 \cos \gamma y \tag{4.7d}$$

Here, the A's, B's, C's and D's are constants to be determined from the boundary conditions of the problem. The appropriate boundary conditions and solutions for the symmetric and the antisymmetric modes of the structure are listed below.

(a) Symmetric Modes

For the symmetric (i.e., translational) modes, the boundary conditions are:

(i) $\frac{dU}{dx}(x=0) = 0$

(ii) $\frac{d^3U}{dx^3}(x=0) = 0$

(iii) $\frac{dV}{dx}(x=0) = 0$

(iv) $\frac{d^3V}{dx^3}(x=0) = 0$

(v) $W_1(y=-1) = 0$

(vi) $W_1(y=0) = U(x=1)$

(vii) $W_2(y=0) = U(x=1)$

(viii) $W_2(y=1) = V(x=1)$

(ix) $\frac{d^3U(x=1)}{dx^3} + q_1 \frac{dW_2(y=0)}{dy} - q_1 \frac{dW_1(y=0)}{dy} = 0$

where,

$$q_1 = \frac{k_3L^3}{E_1I_1h} \qquad (4.8)$$

(x) $\frac{d^3V(x=1)}{dx^3} = q_2 \frac{dW_2(y=1)}{dy}$

where,

$$q_2 = \frac{k_3 L^3}{E_2 I_2 h} \tag{4.9}$$

(xi) $\frac{d^2U}{dx^2}(x=1) = 0$

(xii) $\frac{d^2V}{dx^2}(x=1) = 0$

The last two conditions correspond to zero torsional stiffness of the end walls. One can write the boundary conditions for finite torsional stiffness of the end walls, as shown in section (3.3.2), but the example problem in the previous chapter suggests that this complexity is not required.

The boundary conditions (i) and (ii) require:

$$A_1 = A_3 = 0 \tag{4.10}$$

Similarly, from (iii) and (iv),

$$B_1 = B_3 = 0 \tag{4.11}$$

From (xi) and (xii), respectively,

$$A_2 \cos \alpha = A_4 \cosh \alpha \tag{4.12}$$

and,

$$B_2 \cos \beta = B_4 \cosh \beta \tag{4.13}$$

Thus,

$$U(x) = A_2 \left(\cos \alpha x + \frac{\cos \alpha}{\cosh \alpha} \cosh \alpha x \right) \tag{4.14}$$

$$V(x) = B_2 \left(\cos \beta x + \frac{\cos \beta}{\cosh \beta} \cosh \beta x \right) \tag{4.15}$$

From (vi) and (vii), respectively

$$C_2 = 2A_2 \cos \alpha \tag{4.16}$$

and,

$$D_2 = 2A_2 \cos \alpha \tag{4.17}$$

From (v) and (viii), respectively

$$-C_1 \sin \gamma + C_2 \cos \gamma = 0 \tag{4.18}$$

and,

$$D_1 \sin \gamma + D_2 \cos \gamma = 2B_2 \cos \beta \tag{4.19}$$

Conditions (ix) and (x) give,

$$A_2\alpha^3[\sin \alpha + \frac{\cos \alpha}{\cosh \alpha} \sinh \alpha] + q_1\gamma D_1 - q_1\gamma C_1 = 0 \tag{4.20}$$

and,

$$B_2\beta^3[\sin \beta + \frac{\cos \beta}{\cosh \beta} \sinh \beta] = q_2\gamma(D_1 \cos \gamma - D_2 \sin \gamma) \tag{4.21}$$

These equations can be combined to obtain the following characteristic equation:

$$\frac{\beta^3}{2q_2\gamma}\left[2 - \frac{\alpha^3}{2q_1\gamma}(\tan\alpha + \tanh\alpha)\tan\gamma\right](\tan\beta + \tanh\beta)\sin\gamma$$

$$+ \frac{\alpha^3}{2q_1\gamma}(\tan\alpha + \tanh\alpha)\sin\gamma + \tan\gamma\sin\gamma - \cos\gamma = 0 \qquad (4.22)$$

Here, α, β and γ are known functions of ω (equations 4.6), and q_1 and q_2 are known parameters for a given structure (equations 4.7, 4.8). Thus, equation (4.22) can be solved to obtain the natural frequencies of the symmetric modes of vibration of the structure. The mode shapes are given by

$$U(x) = A_2(\cos\alpha x + \frac{\cos\alpha}{\cosh\alpha}\cosh\alpha x) \qquad 0\leq x\leq 1 \qquad (4.23a)$$

$$V(x) = B_2(\cos\beta x + \frac{\cos\beta}{\cosh\beta}\cosh\beta x) \qquad 0\leq x\leq 1 \qquad (4.23b)$$

$$W_1(y) = C_1\sin\gamma y + C_2\cos\gamma y \qquad -1\leq y\leq 0 \qquad (4.23c)$$

$$W_2(y) = D_1\sin\gamma y + D_2\cos\gamma y \qquad 0\leq y\leq 1 \qquad (4.23d)$$

where,

$$A_2 = \tfrac{1}{2}C_1\sec\alpha\tan\gamma \qquad (4.24a)$$

$$B_2 = \tfrac{1}{2}\sec\beta\sin\gamma\left[2 - \frac{\alpha^3}{2q_1\gamma}(\tan\alpha + \tanh\alpha)\tan\gamma\right]C_1 \qquad (4.24b)$$

$$C_2 = C_1\tan\gamma \qquad (4.24c)$$

$$D_1 = \left[1 - \frac{\alpha^3}{2q_1\gamma}(\tan\alpha + \tanh\alpha)\tan\gamma\right]C_1 \qquad (4.24d)$$

$$D_2 = C_1 \tan \gamma \tag{4.24e}$$

The mode shapes for the left half of the structure can be obtained from the conditions of symmetry.

$$U(x) = A_2(\cos \alpha x + \frac{\cos \alpha}{\cosh \alpha} \cosh \alpha x) \qquad -1 \leq x \leq 1 \tag{4.25a}$$

$$V(x) = B_2(\cos \beta x + \frac{\cos \beta}{\cosh \beta} \cosh \beta x) \qquad -1 \leq x \leq 1 \tag{4.25b}$$

$$W_1(y) = W_3(y) = C_1 \sin \gamma y + C_2 \cos \gamma y \qquad -1 \leq y \leq 0 \tag{4.25c}$$

$$W_2(y) = W_4(y) = D_1 \sin \gamma y + D_2 \cos \gamma y \qquad 0 \leq y \leq 1 \tag{4.25d}$$

where $W_3(y)$ and $W_4(y)$ correspond to the displacements in the left end wall in the first and the second story, respectively.

Thus, for a given natural frequency (obtained from equation 4.22), the symmetric mode shapes can be obtained from equations (4.24) and (4.25).

(b) Antisymmetric Modes

For antisymmetric (i.e., torsional) modes of vibration, the boundary conditions are:

(i) $U(x=0) = 0$

(ii) $\frac{d^2U}{dx^2}(x=0) = 0$

(iii) $V(x=0) = 0$

(iv) $$\frac{d^2V}{dx^2}(x=0) = 0$$

(v) $$W_1(y=-1) = 0$$

(vi) $$W_1(y=0) = U(x=1)$$

(vii) $$W_2(y=0) = U(x=1)$$

(viii) $$W_2(y=1) = V(x=1)$$

(ix) $$\frac{d^3U(x=1)}{dx^3} + q_1 \frac{dW_2(y=0)}{dy} - q_1 \frac{dW_1(y=0)}{dy} = 0$$

(x) $$\frac{d^3V(x=1)}{dx^3} = q_2 \frac{dW_2(y=1)}{dy}$$

(xi) $$\frac{d^2U(x=1)}{dx^2} = 0$$

(xii) $$\frac{d^2V(x=1)}{dx^2} = 0$$

where q_1 and q_2 are given by equations (4.8) and (4.9), respectively. The last two conditions correspond to zero torsional rigidity of the end walls.

These boundary conditions can be applied to the solutions of the differential equations (equations 4.7), to obtain the following characteristic equation and mode shapes:

$$\frac{\beta^3}{2q_2\gamma}\left[2 - \frac{\alpha^3}{2q_1\gamma}(-\cot\alpha + \coth\alpha)\tan\gamma\right](-\cot\beta + \coth\beta)\sin\gamma$$

$$+ \frac{\alpha^3}{2q_1\gamma}(-\cot\alpha + \coth\alpha)\sin\gamma + \tan\gamma\sin\gamma - \cos\gamma = 0 \qquad (4.26)$$

$$U(x) = A_1\left(\sin\alpha x + \frac{\sin\alpha}{\sinh\alpha}\sinh\alpha x\right) \qquad -1\leq x\leq 1 \qquad (4.27a)$$

$$V(x) = B_1\left(\sin\beta x + \frac{\sin\beta}{\sinh\beta}\sinh\beta x\right) \qquad -1\leq x\leq 1 \qquad (4.27b)$$

$$W_1(y) = C_1\sin\gamma y + C_2\cos\gamma y \qquad -1\leq y\leq 0 \qquad (4.27c)$$

$$W_2(y) = D_1\sin\gamma y + D_2\cos\gamma y \qquad 0\leq y\leq 1 \qquad (4.27d)$$

$$W_3(y) = -C_1\sin\gamma y - C_2\cos\gamma y \qquad -1\leq y\leq 0 \qquad (4.27e)$$

$$W_4(y) = -D_1\sin\gamma y - D_2\cos\gamma y \qquad 0\leq y\leq 1 \qquad (4.27f)$$

where,

$$A_1 = \tfrac{1}{2}C_1\,\mathrm{cosec}\,\alpha\tan\gamma \qquad (4.28a)$$

$$B_1 = \tfrac{1}{2}\,\mathrm{cosec}\,\beta\sin\gamma\left[2 - \frac{\alpha^3}{2q_1\gamma}(-\cot\alpha + \coth\alpha)\tan\gamma\right]C_1 \qquad (4.28b)$$

$$C_2 = C_1\tan\gamma \qquad (4.28c)$$

$$D_1 = \left[1 - \frac{\alpha^3}{2q_1\gamma}(-\cot\alpha + \coth\alpha)\tan\gamma\right]C_1 \qquad (4.28d)$$

$$D_2 = C_1 \tan \gamma \qquad (4.28e)$$

The roots of equation (4.26) produce the natural frequencies for the antisymmetric modes of the structure. For a particular natural frequency, equations (4.27) and (4.28) give the corresponding mode shape of the structure.

<u>Orthogonality of Modes</u>

The following analysis demonstrates the expected orthogonality of the modes of vibration. Let ω_i and $U_i(x)$, $V_i(x)$, $W_{1i}(y)$, $W_{2i}(y)$, $W_{3i}(y)$, $W_{4i}(y)$ be the frequency and the mode shape for the i^{th} mode. Similarly, ω_k, $U_k(x)$, $V_k(x)$, $W_{1k}(y)$, $W_{2k}(y)$, $W_{3k}(y)$ and $W_{4k}(y)$ correspond to the k^{th} mode. Substitution into equations (4.5) gives,

$$\frac{d^4U_i(x)}{dx^4} - \frac{m_1L^4}{E_1I_1}\,\omega_i^2U_i(x) = 0 \qquad (4.29a)$$

$$\frac{d^4V_i(x)}{dx^4} - \frac{m_2L^4}{E_2I_2}\,\omega_i^2V_i(x) = 0 \qquad (4.29b)$$

$$\frac{d^2W_{1i}(y)}{dy^2} + \frac{m_3h^2}{k_3}\,\omega_i^2W_{1i}(y) = 0 \qquad (4.29c)$$

$$\frac{d^2W_{2i}(y)}{dy^2} + \frac{m_3h^2}{k_3}\,\omega_i^2W_{2i}(y) = 0 \qquad (4.29d)$$

$$\frac{d^2W_{3i}(y)}{dy^2} + \frac{m_3h^2}{k_3}\,\omega_i^2W_{3i}(y) = 0 \qquad (4.29e)$$

$$\frac{d^2W_{4i}(y)}{dy^2} + \frac{m_3h^2}{k_3}\,\omega_i^2 W_{4i}(y) = 0 \qquad (4.29f)$$

Similar equations can be written for vibrations in the k^{th} mode. The two sets of equations can be combined to obtain

$$\int_{-1}^{1}\left[\frac{d^4U_i}{dx^4}U_k - \frac{d^4U_k}{dx^4}U_i\right]dx - \frac{m_1L^4}{E_1I_1}(\omega_i^2-\omega_k^2)\int_{-1}^{1}U_iU_k dx = 0 \qquad (4.30a)$$

$$\int_{-1}^{1}\left[\frac{d^4V_i}{dx^4}V_k - \frac{d^4V_k}{dx^4}V_i\right]dx - \frac{m_2L^4}{E_2I_2}(\omega_i^2-\omega_k^2)\int_{-1}^{1}V_iV_k dx = 0 \qquad (4.30b)$$

$$\int_{-1}^{0}\left[\frac{d^2W_{1i}}{dy^2}W_{1k} - \frac{d^2W_{1k}}{dy^2}W_{1i}\right]dy + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{-1}^{0}W_{1i}W_{1k}dy = 0 \qquad (4.30c)$$

$$\int_{0}^{1}\left[\frac{d^2W_{2i}}{dy^2}W_{2k} - \frac{d^2W_{2k}}{dy^2}W_{2i}\right]dy + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{0}^{1}W_{2i}W_{2k}dy = 0 \qquad (4.30d)$$

$$\int_{-1}^{0}\left[\frac{d^2W_{3i}}{dy^2}W_{3k} - \frac{d^2W_{3k}}{dy^2}W_{3i}\right]dy + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{-1}^{0}W_{3i}W_{3k}dy = 0 \qquad (4.30e)$$

and,

$$\int_{0}^{1}\left[\frac{d^2W_{4i}}{dy^2}W_{4k} - \frac{d^2W_{4k}}{dy^2}W_{4i}\right]dy + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{-1}^{0}W_{4i}W_{4k}dy = 0 \qquad (4.30f)$$

Integrate the first integrals of these equations by parts, and apply the appropriate boundary conditions. This gives,

$$\left[\frac{d^3U_i}{dx^3}U_k - \frac{d^3U_k}{dx^3}U_i\right]_{x=-1}^{x=1} - \frac{m_1L^4}{E_1I_1}(\omega_i^2-\omega_k^2)\int_{-1}^{1}U_iU_k dx = 0 \qquad (4.31a)$$

$$\left[\frac{d^3V_i}{dx^3}V_k - \frac{d^3V_k}{dx^3}V_i\right]_{x=-1}^{x=1} - \frac{m_2L^4}{E_2I_2}(\omega_i^2-\omega_k^2)\int_{-1}^{1}V_iV_k dx = 0 \qquad (4.31b)$$

$$\left[\frac{dW_{1i}}{dy}(y=0)W_{1k}(y=0) - \frac{dW_{1k}}{dy}(y=0)W_{1i}(y=0)\right]$$

$$+ \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{-1}^{0}W_{1i}W_{1k}dy = 0 \qquad (4.31c)$$

$$\left[\frac{dW_{2i}}{dy}W_{2k} - \frac{dW_{2k}}{dy}W_{2i}\right]_{y=0}^{y=1} + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{0}^{1}W_{2i}W_{2k}dy = 0 \qquad (4.31d)$$

$$\left[\frac{dW_{3i}}{dy}(y=0)W_{3k}(y=0) - \frac{dW_{3k}}{dy}(y=0)W_{3i}(y=0)\right]$$

$$+ \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{-1}^{0}W_{3i}W_{3k}dy = 0 \qquad (4.31e)$$

$$\left[\frac{dW_{4i}}{dy}W_{4k} - \frac{dW_{4k}}{dy}W_{4i}\right]_{y=0}^{y=1} + \frac{m_3h^2}{k_3}(\omega_i^2-\omega_k^2)\int_{0}^{1}W_{4i}W_{4k}dy = 0 \qquad (4.31f)$$

However, from the boundary conditions,

$$W_1(y=0) = U(x=1), \qquad W_3(y=0) = U(x=-1)$$

$$W_2(y=0) = U(x=1), \qquad W_4(y=0) = U(x=-1)$$

$$W_2(y=1) = V(x=1), \qquad W_4(y=1) = V(x=-1)$$

$$\frac{d^3U}{dx^3}(x=1) + \frac{k_3L^3}{E_1I_1h}\frac{dW_2}{dy}(y=0) - \frac{k_3L^3}{E_1I_1h}\frac{dW_1}{dy}(y=0) = 0$$

$$\frac{d^3V}{dx^3}(x=1) = \frac{k_3L^3}{E_2I_2h}\frac{dW_2}{dy}(y=1)$$

$$\frac{d^3U}{dx^3}(x=-1) - \frac{k_3L^3}{E_1I_1h}\frac{dW_4}{dy}(y=0) + \frac{k_3L^3}{E_1I_1h}\frac{dW_3}{dy}(y=0) = 0$$

and,

$$\frac{d^3V}{dx^3}(x=-1) = -\frac{k_3L^3}{E_2I_2h}\frac{dW_4}{dy}(y=1)$$

Equations (4.31) can be combined such that the boundary terms cancel, and the following is obtained,

$$(\omega_i^2-\omega_k^2)\left[m_1L\int_{-1}^{1} U_iU_k dx + m_2L\int_{-1}^{1} V_iV_k dx + m_3h\int_{-1}^{0} W_{1i}W_{1k}dy\right.$$

$$\left.+ m_3h\int_{0}^{1} W_{2i}W_{2k}dy + m_3h\int_{-1}^{0} W_{3i}W_{3k}dy + m_3h\int_{0}^{1} W_{4i}W_{4k}dy\right] = 0 \qquad (4.32)$$

for $i \neq k$, $\omega_i \neq \omega_k$. From equation (4.32), it follows that

$$m_1 L \int_{-1}^{1} U_i U_k dx + m_2 L \int_{-1}^{1} V_i V_k dx + m_3 h \int_{-1}^{0} W_{1i} W_{1k} dy$$

$$+ m_3 h \int_{0}^{1} W_{2i} W_{2k} dy + m_3 h \int_{-1}^{0} W_{3i} W_{3k} dy + m_3 h \int_{0}^{1} W_{4i} W_{4k} dy = 0 \qquad (4.33)$$

Hence, the modes of vibration are orthogonal and equation (4.33) gives the orthogonality condition.

Participation Factors for Earthquake Ground Motion

The equations of motion due to uniform, transverse earthquake excitation can be written as:

$$\frac{\partial^4 u(x,t)}{\partial x^4} + \frac{m_1 L^4}{E_1 I_1} \frac{\partial^2 u(x,t)}{\partial t^2} = - \frac{m_1 L^4}{E_1 I_1} \ddot{u}_g(t) \qquad (4.34a)$$

$$\frac{\partial^4 v(x,t)}{\partial x^4} + \frac{m_2 L^4}{E_2 I_2} \frac{\partial^2 v(x,t)}{\partial t^2} = - \frac{m_2 L^4}{E_2 I_2} \ddot{u}_g(t) \qquad (4.34b)$$

$$\frac{\partial^2 w_1(y,t)}{\partial y^2} - \frac{m_3 h^2}{k_3} \frac{\partial^2 w_1(y,t)}{\partial t^2} = \frac{m_3 h^2}{k_3} \ddot{u}_g(t) \qquad (4.34c)$$

$$\frac{\partial^2 w_2(y,t)}{\partial y^2} - \frac{m_3 h^2}{k_3} \frac{\partial^2 w_2(y,t)}{\partial t^2} = \frac{m_3 h^2}{k_3} \ddot{u}_g(t) \qquad (4.34d)$$

$$\frac{\partial^2 w_3(y,t)}{\partial y^2} - \frac{m_3 h^2}{k_3} \frac{\partial^2 w_3(y,t)}{\partial t^2} = \frac{m_3 h^2}{k_3} \ddot{u}_g(t) \qquad (4.34e)$$

and

$$\frac{\partial^2 w_4(y,t)}{\partial y^2} - \frac{m_3 h^2}{k_3}\frac{\partial^2 w_4(y,t)}{\partial t^2} = \frac{m_3 h^2}{k_3}\ddot{u}_g(t) \qquad (4.34f)$$

where $\ddot{u}_g(t)$ is the earthquake acceleration in the z'-direction, $w_3(y,t)$ and $w_4(y,t)$ are the displacements in the first and the second story of the left end wall. All other terms have been defined earlier.

First, expand the response in terms of the normal modes of the system. Let

$$u(x,t) = \sum_{i=1}^{\infty} U_i(x)T_i(t) \qquad (4.35a)$$

$$v(x,t) = \sum_{i=1}^{\infty} V_i(x)T_i(t) \qquad (4.35b)$$

$$w_1(y,t) = \sum_{i=1}^{\infty} W_{1i}(y)T_i(t) \qquad (4.35c)$$

$$w_2(y,t) = \sum_{i=1}^{\infty} W_{2i}(y)T_i(t) \qquad (4.35d)$$

$$w_3(y,t) = \sum_{i=1}^{\infty} W_{3i}(y)T_i(t) \qquad (4.35e)$$

$$w_4(y,t) = \sum_{i=1}^{\infty} W_{4i}(y)T_i(t) \qquad (4.35f)$$

Next, substitute these and equations (4.29) into equations (4.34) to obtain

$$\sum_i [U_i(x)\ddot{T}_i(t) + \omega_i^2 U_i(x)T_i(t)] = -\ddot{u}_g(t) \tag{4.36a}$$

$$\sum_i [V_i(x)\ddot{T}_i(t) + \omega_i^2 V_i(x)T_i(t)] = -\ddot{u}_g(t) \tag{4.36b}$$

$$\sum_i [W_{1i}(y)\ddot{T}_i(t) + \omega_i^2 W_{1i}(y)T_i(t)] = -\ddot{u}_g(t) \tag{4.36c}$$

$$\sum_i [W_{2i}(y)\ddot{T}_i(t) + \omega_i^2 W_{2i}(y)T_i(t)] = -\ddot{u}_g(t) \tag{4.36d}$$

$$\sum_i [W_{3i}(y)\ddot{T}_i(t) + \omega_i^2 W_{3i}(y)T_i(t)] = -\ddot{u}_g(t) \tag{4.36e}$$

$$\sum_i [W_{4i}(y)\ddot{T}_i(t) + \omega_i^2 W_{4i}(y)T_i(t)] = -\ddot{u}_g(t) \tag{4.36f}$$

These equations can be combined to yield

$$\begin{aligned}
&\sum_i \Bigg[\Bigg(m_1 L \int_{-1}^{1} U_i U_k dx + m_2 L \int_{-1}^{1} V_i V_k dx + m_3 h \int_{-1}^{0} W_{1i} W_{1k} dy \\
&+ m_3 h \int_{0}^{1} W_{2i} W_{2k} dy + m_3 h \int_{-1}^{0} W_{3i} W_{3k} dy + m_3 h \int_{0}^{1} W_{4i} W_{4k} dy\Bigg) \\
&\cdot \left(\ddot{T}_i(t) + \omega_i^2 T_i(t)\right)\Bigg] \\
&= -\Bigg[m_1 L \int_{-1}^{1} U_k dx + m_2 L \int_{-1}^{1} V_k dx + m_3 h \int_{-1}^{0} W_{1k} dy + m_3 h \int_{0}^{1} W_{2k} dy \\
&+ m_3 h \int_{-1}^{0} W_{3k} dy + m_3 h \int_{0}^{1} W_{4k} dy\Bigg]\ddot{u}_g(t)
\end{aligned} \tag{4.37}$$

Finally, apply orthogonality condition (equation 4.33) to obtain

$$\ddot{T}_k(t) + \omega_k^2 T_k(t) = -P_k \ddot{u}_g(t) \tag{4.38}$$

where P_k, the participation factor for the k^{th} mode, is given by

$$P_k = \frac{\left[m_1 L\int_{-1}^{1} U_k dx + m_2 L\int_{-1}^{1} V_k dx + m_3 h\int_{-1}^{0} W_{1k} dy + m_3 h\int_{0}^{1} W_{2k} dy + m_3 h\int_{-1}^{0} W_{3k} dy + m_3 h\int_{0}^{1} W_{4k} dy\right]}{\left[m_1 L\int_{-1}^{1} (U_k)^2 dx + m_2 L\int_{-1}^{1} (V_k)^2 dx + m_3 h\int_{-1}^{0} (W_{1k})^2 dy + m_3 h\int_{0}^{1} (W_{2k})^2 dy + m_3 h\int_{-1}^{0} (W_{3k})^2 dy + m_3 h\int_{0}^{1} (W_{4k})^2 dy\right]} \tag{4.39}$$

For a particular mode, substitution of the expressions for the mode shape into equation (4.39) gives the corresponding modal participation factor. When antisymmetric mode shapes (equations 4.27) are used in this expression, they yield zero modal participation factors. Hence, as anticipated in this symmetric structure, a uniform ground motion does not excite torsional modes.

4.3 NUMERICAL EXAMPLE

As an illustration of the method described in this chapter, the Administrative Building of the Arvin High School has been modelled and analyzed in this section. The results of this approximate analysis include the natural frequencies, the mode shapes and the modal participation factors for uniform ground motion.

The properties for the structure have been obtained from Steinbrugge and Moran (1954), Blume, et al. (1961), and Blume and Jhaveri (1969). The shear wall near the center in the first story has been neglected because of its small size and the complexities it causes in the analysis. The properties of the end walls in the first and the second story have been assumed to be the same, and the right and the left end walls are taken to be identical. The story height for the first and second story has been taken to be the same.

The building properties used for this example problem are as follows:

Floor:	span (2L)	=	197.0 ft
	weight ($m_1 g$)	=	7330.0 lb per ft
	modulus of elasticity (E_1)	=	2.0×10^6 psi
	moment of inertia (I_1)	=	41.0×10^6 $(in)^4$
Roof:	weight ($m_2 g$)	=	3770.0 lb per ft
	modulus of elasticity (E_2)	=	2.0×10^6 psi
	moment of inertia (I_2)	=	66.2×10^6 $(in)^4$
Walls:	story height (h)	=	14.0 ft
	weight ($m_3 g$)	=	3710.0 lb per ft
	shape factor (k')	=	0.833
	shear modulus (G_3)	=	0.855×10^6 psi
	area of cross-section (A_3)	=	3560.0 sq in

The characteristic equations (4.22) and (4.26) for the symmetric and the antisymmetric modes, respectively, were solved to obtain the natural frequencies of the structure. For these frequencies, the mode shapes were obtained using equations (4.23, 4.24) and (4.27, 4.28), and equation (4.39) gave the corresponding modal participation factors for uniform earthquake ground motion. These results are tabulated in Table (4.1), and the first four mode shapes are plotted in Figure (4.2).

A comparison of the natural frequencies indicates that the first natural frequency is close to the fundamental frequency of the second floor vibrating as a pinned-pinned beam. Similarly, the second natural frequency is approximately equal to the fundamental frequency of the roof when treated as a pinned-pinned beam. This second mode period (0.29 sec) is in good agreement with the 0.25 sec "horizontal roof-diaphragm period" reported by Blume, et al. (1961) during their forced-vibration tests on the same building. The third symmetric mode period for the structure is 0.061 sec and probably corresponds to the mode reported by Blume, et al. (1961) as "fundamental translation mode" with a measured period of 0.10 sec.

Table (4.2) gives the base shear in the structure in various modes of vibration under earthquake motion characterized by a constant acceleration spectrum value of 0.20g. As expected, the antisymmetric modes do not get excited by this type of ground motion and thus contribute nothing towards the base shear. It is obvious from Table (4.2) that the first two modes, dominated by floor or roof vibrations,

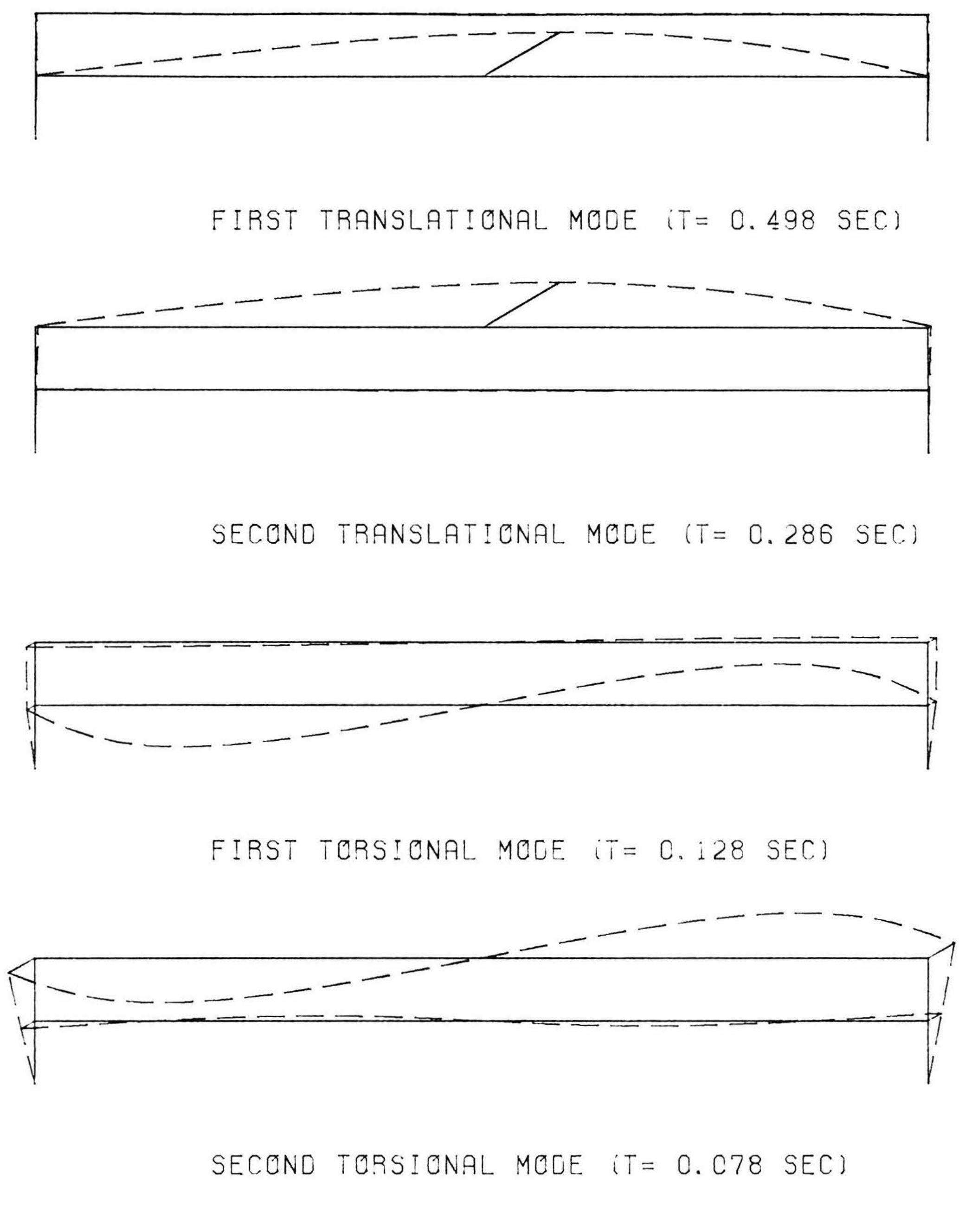


Figure 4.2. MODE SHAPES FOR THE TWO-STORY BUILDING.

TABLE 4.1. PERIODS, MODE SHAPES AND PARTICIPATION FACTORS FOR THE TWO-STORY BUILDING

	First Transl. Mode	Second Transl. Mode	First Tors. Mode	Second Tors. Mode	Third Transl. Mode	Third Tors. Mode
Period	0.498 sec	0.286 sec	0.128 sec	0.078 sec	0.061 sec	0.040 sec
U(x=0.0)	76.17	-0.46	0.00	0.00	-1.68	0.00
U(x=0.2)	72.49	-0.41	4.75	-0.25	-1.05	-0.54
U(x=0.4)	61.81	-0.27	7.77	-0.33	0.39	-0.47
U(x=0.6)	45.18	-0.06	7.98	-0.20	1.58	0.15
U(x=0.8)	24.22	0.21	5.34	0.12	1.71	0.72
U(x=1.0)	0.98	0.50	0.92	0.52	0.83	0.82
V(x=0.0)	1.58	24.93	0.00	0.00	-0.48	0.00
V(x=0.2)	1.55	23.76	0.35	1.63	-0.36	-0.42
V(x=0.4)	1.47	20.37	0.65	2.73	-0.05	-0.54
V(x=0.6)	1.34	15.08	0.86	2.95	0.35	-0.26
V(x=0.8)	1.18	8.40	0.96	2.26	0.71	0.33
V(x=1.0)	1.00	1.00	1.00	1.00	1.00	1.00
W_1(y=-1.0)	0.00	0.00	0.00	0.00	0.00	0.00
W_1(y=-0.8)	0.20	0.10	0.19	0.10	0.17	0.17
W_1(y=-0.6)	0.39	0.20	0.37	0.21	0.34	0.34
W_1(y=-0.4)	0.59	0.30	0.56	0.31	0.51	0.50
W_1(y=-0.2)	0.79	0.40	0.74	0.41	0.67	0.66
W_1(y=0)	0.98	0.50	0.92	0.52	0.83	0.82
W_2(y=0.2)	0.99	0.60	0.94	0.62	0.87	0.87
W_2(y=0.4)	0.99	0.70	0.96	0.71	0.91	0.92
W_2(y=0.6)	0.99	0.80	0.97	0.81	0.94	0.95
W_2(y=0.8)	1.00	0.90	0.99	0.91	0.97	0.98
W_2(y=1.0)	1.00	1.00	1.00	1.00	1.00	1.00
Participation Factor	0.0178	0.0518	0.00	0.00	0.474	0.00

make the largest contributions to the total base shear for the structure. The third symmetric mode, with less pronounced floor and roof motions, gives a base shear only about 1/3 that of the second mode.

The numerical results suggest another interesting feature that may occur in multistory buildings that are relatively uniform and have flexible floors. In the example, the first two natural frequencies are

TABLE 4.2. MAXIMUM BASE SHEARS FROM SYMMETRIC MODES IN THE TWO-STORY BUILDING (S_A = 0.20g)

Symmetric Mode	Period (sec)	Base Shear* (lb)	Base Shear (percentage of total weight)
1	0.50	13.6×10^4	6%
2	0.29	24.3×10^4	11%
3	0.061	7.0×10^4	3%
4	0.042	6.4×10^4	3%

*The total weight of the structure is 230×10^4 lb.

close to the natural frequencies of the floor and the roof when treated as independent, pinned-pinned beams. Therefore, for multistory buildings that have nearly identical floors and stiff end walls, some of the lower frequencies may correspond to floor motions. It can be expected that such frequencies may be very nearly equal, leading to additional complications in the analysis and response of the buildings.

CHAPTER V

MULTISTORY BUILDINGS WITH END WALLS

5.1 INTRODUCTION

This chapter deals with multistory buildings whose lateral load resistance system consists only of two walls or frames at the two ends of the building. The one- and two-story counterparts of such buildings have been studied in Chapters III and IV, respectively. It was noted in Chapter IV that the approach developed in Chapter III could be applied to buildings with more stories but that the algebra was increasingly complex as more stories are considered. Although the approach allows one to analyze the system "exactly," it loses its simplicity for multistory buildings. In this chapter, a simpler approach has been developed that allows the analysis of such multistory buildings. However, the approach requires some additional assumptions about the building.

In this new approach, the end walls or frames are represented by an appropriate beam (bending beam or shear beam). The floors, treated as separate beams in the previous chapter, are now modelled as an equivalent distributed beam system, discussed in section (3.2.3). The distributed system is attached uniformly along the height of the vertical end beams. Thus, the floors are no longer assumed to be attached to the end walls (or frames) at discrete points, and a single differential equation applies to the whole wall. Similarly, only one differential equation is needed to represent the floors. However, for practical purposes, one has to assume the distribution of mass and stiffness in the floors and in the end walls to be uniform (or some simple variation) along the height of the building.

5.2 BUILDINGS WITH END WALLS MODELLED AS BENDING BEAMS

As the height to width ratio of an end wall increases, the wall tends to behave more like a bending beam and less like a shear beam. In this section, it has been assumed that the building has two identical end walls, that have a height to width ratio large enough that the bending flexibility is much larger than the shear flexibility. Consider one such building as shown in Figure (5.1). The building height is h and the length of the floors is 2L. The following list gives the properties of the end walls and the distributed floor system. They are assumed to be uniform along the height of the building. In addition, the floor-system properties are uniform along the length of the building.

E_1, E_2 = Young's modulus of elasticity for the floor system and the wall, respectively.

I^*_1 = Moment of inertia of floor-system cross-section per unit height.

I_2 = Moment of inertia of the end wall cross-section.

m^*_1 = Mass per unit area (in x'-y' plane) of the distributed floor system.

m_2 = Mass per unit height of the end wall.

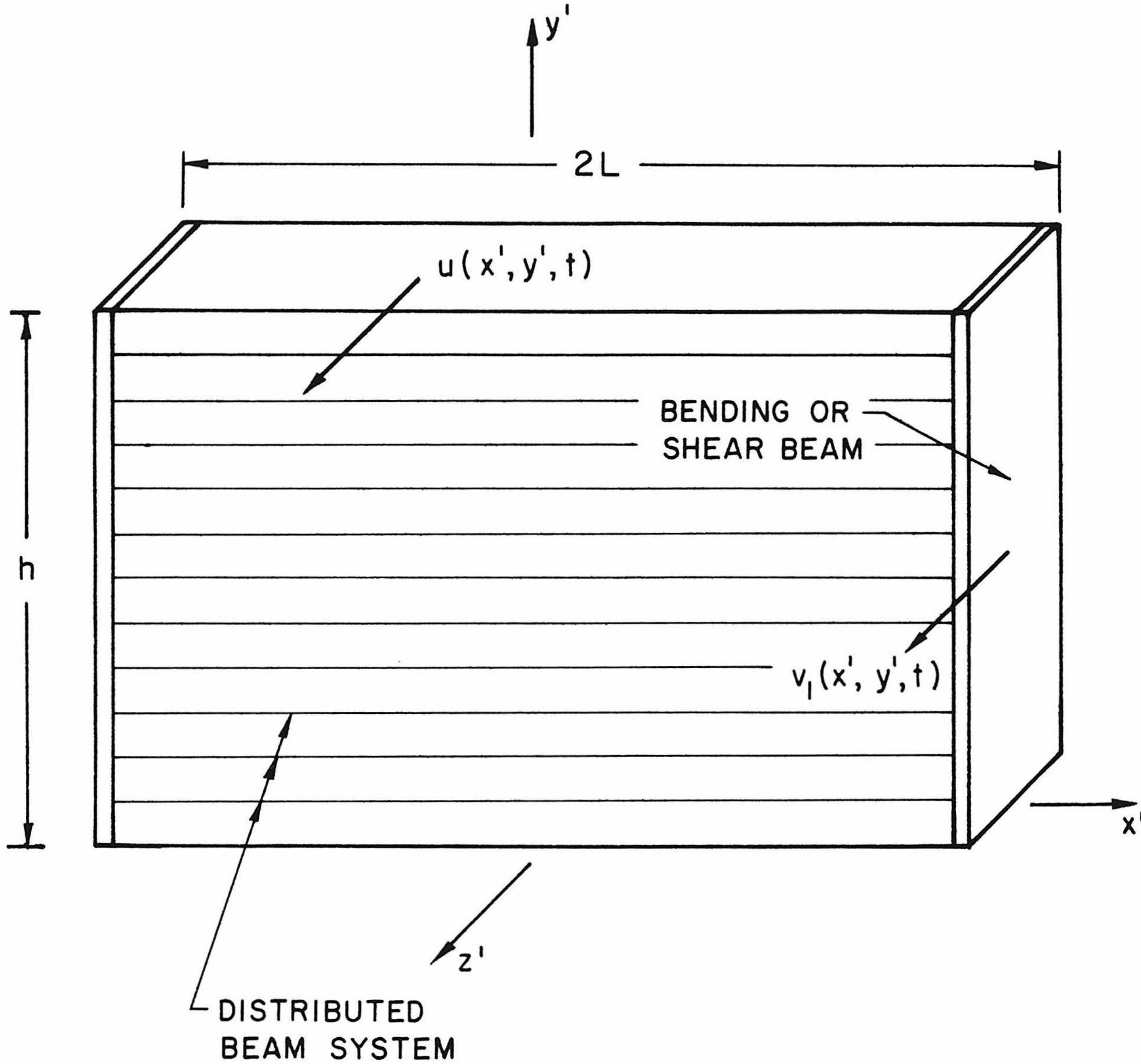


Figure 5.1. MODEL OF A MULTISTORY BUILDING WITH END WALLS OR FRAMES.

Let $u(x',y',t)$ be the displacement in the z'-direction, at point (x',y') of the floor system at an instant t. Similarly, $v_1(y',t)$ and $v_2(y',t)$ are the displacements in the z'-direction, in the right and the left end walls.

The equations of motion for free vibrations of the system can be written as,

$$E_1 I^*_1 \frac{\partial^4 u(x',y',t)}{\partial x'^4} = -m^*_1 \frac{\partial^2 u(x',y',t)}{\partial t^2} \tag{5.1a}$$

$$E_2 I_2 \frac{\partial^4 v_1(y',t)}{\partial y'^4} = -m_2 \frac{\partial^2 v_1(y',t)}{\partial t^2} + \left[E_1 I^*_1 \frac{\partial^3 u(x',y',t)}{\partial x'^3}\right]_{x'=L} \tag{5.1b}$$

and,

$$E_2 I_2 \frac{\partial^4 v_2(y',t)}{\partial y'^4} = -m_2 \frac{\partial^2 v_2(y',t)}{\partial t^2} - \left[E_1 I^*_1 \frac{\partial^3 u(x',y',t)}{\partial x'^3}\right]_{x'=-L} \tag{5.1c}$$

Let x and y be the nondimensional coordinates defined by:

$$x = \frac{x'}{L} \quad , \quad y = \frac{y'}{h} \tag{5.2}$$

Equations (5.1) can be written in the new coordinate system as

$$\frac{\partial^4 u(x,y,t)}{\partial x^4} = -\frac{m^*_1 L^4}{E_1 I^*_1} \frac{\partial^2 u(x,y,t)}{\partial t^2} \tag{5.3a}$$

$$\frac{\partial^4 v_1(y,t)}{\partial y^4} = -\frac{m_2 h^4}{E_2 I_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} + \frac{E_1 I^*_1 h^4}{E_2 I_2 L^3} \frac{\partial^3 u(x=1,y,t)}{\partial x^3} \tag{5.3b}$$

$$\frac{\partial^4 v_2(y,t)}{\partial y^4} = -\frac{m_2 h^4}{E_2 I_2}\frac{\partial^2 v_2(y,t)}{\partial t^2} - \frac{E_1 I^*_1 h^4}{E_2 I_2 L^3}\frac{\partial^3 u(x=-1,y,t)}{\partial x^3} \qquad (5.3c)$$

Separation of variables is used to analyze the free vibration problem of the system. Let,

$$u(x,y,t) = U(x,y)e^{i\omega t} \qquad (5.4a)$$

$$v_1(y,t) = V_1(y)e^{i\omega t} \qquad (5.4b)$$

$$v_2(y,t) = V_2(y)e^{i\omega t} \qquad (5.4c)$$

where ω is the natural frequency of the motion. Substitution into equations (5.3) produces

$$\frac{\partial^4 U(x,y)}{\partial x^4} - \frac{m^*_1 L^4}{E_1 I^*_1}\omega^2 U(x,y) = 0 \qquad (5.5a)$$

$$\frac{d^4 V_1(y)}{dy^4} - \frac{m_2 h^4}{E_2 I_2}\omega^2 V_1(y) = \frac{E_1 I^*_1 h^4}{E_2 I_2 L^3}\frac{\partial^3 U}{\partial x^3}(x=1,y) \qquad (5.5b)$$

$$\frac{d^4 V_2(y)}{dy^4} - \frac{m_2 h^4}{E_2 I_2}\omega^2 V_2(y) = -\frac{E_1 I^*_1 h^4}{E_2 I_2 L^3}\frac{\partial^3 U}{\partial x^3}(x=-1,y) \qquad (5.5c)$$

Because of the symmetry in the structure about the y'-axis, it is convenient to treat the symmetric and the antisymmetric modes of vibration of the structure separately.

Symmetric Modes

The following are the boundary conditions applicable to the symmetric modes of vibration:

(i) $\frac{\partial U}{\partial x}(x=0,y) = 0$

(ii) $\frac{\partial^3 U}{\partial x^3}(x=0,y) = 0$

(iii) $\frac{\partial^2 U}{\partial x^2}(x=1,y) = 0$

(iv) $V_1(y=0) = 0$

(v) $\frac{dV_1}{dy}(y=0) = 0$

(vi) $\frac{d^2V_1}{dy^2}(y=1) = 0$

(vii) $\frac{d^3V_1}{dy^3}(y=1) = 0$

(viii) $U(x=1,y) = V_1(y)$

Here, (iii) assumes that the end walls have zero torsional stiffness. The solution of equation (5.5a) can be written as:

$$U(x,y) = A_1(y) \sin \alpha x + A_2(y) \cos \alpha x + A_3(y) \sinh \alpha x + A_4(y) \cosh \alpha x \tag{5.6}$$

where,

$$\alpha^4 = \frac{m^*{}_1 L^4}{E_1 I^*{}_1} \omega^2 \tag{5.7}$$

and $A_1(y)$, $A_2(y)$, $A_3(y)$ and $A_4(y)$ are some functions of y.

From boundary conditions (i) and (ii),

$$A_1(y) = 0 \quad , \quad A_3(y) = 0 \tag{5.8}$$

Boundary condition (iii) gives:

$$A_4(y) = \frac{\cos \alpha}{\cosh \alpha} A_2(y) \tag{5.9}$$

Thus,

$$U(x,y) = A_2(y) \left[\cos \alpha x + \frac{\cos \alpha}{\cosh \alpha} \cosh \alpha x\right] \tag{5.10}$$

From boundary condition (viii)

$$A_2(y) = \frac{V_1(y)}{2 \cos \alpha} \tag{5.11}$$

Next, substitute (5.10) and (5.11) into equation (5.5b) to obtain

$$\frac{d^4 V_1(y)}{dy^4} - \beta^4 V_1(y) = 0 \tag{5.12}$$

where,

$$\beta^4 = \frac{m_2 h^4}{E_2 I_2} \omega^2 + \frac{E_1 I^*_1 h^4}{E_2 I_2 L^3} \frac{\alpha^3}{2} (\tan \alpha + \tanh \alpha) \tag{5.13}$$

The solution for equation (5.12) is

$$V_1(y) = B_1 \sin \beta y + B_2 \cos \beta y + B_3 \sinh \beta y + B_4 \cosh \beta y \tag{5.14}$$

where the B's are constants to be determined from the boundary conditions of the system. From (iv) and (v),

$$B_4 = -B_2 \quad , \quad B_3 = -B_1 \tag{5.15}$$

Therefore,

$$V_1(y) = B_1(\sin \beta y - \sinh \beta y) + B_2(\cos \beta y - \cosh \beta y) \tag{5.16}$$

From (vi) and (vii),

$$(-\sin \beta - \sinh \beta)B_1 + (-\cos \beta - \cosh \beta)B_2 = 0 \tag{5.17}$$

and,

$$(-\cos \beta - \cosh \beta)B_1 + (\sin \beta - \sinh \beta)B_2 = 0 \tag{5.18}$$

From these equations, the condition for a nontrivial solution is obtained as

$$\cos \beta \cosh \beta + 1 = 0 \tag{5.19}$$

and, for each β satisfying equation (5.19)

$$B_2 = - \frac{\sin\beta + \sinh\beta}{\cos\beta + \cosh\beta} B_1 \qquad (5.20)$$

Hence, equation (5.19) is the characteristic equation for the problem,* where β is related to the natural frequency ω through equations (5.7) and (5.13). These equations can be solved to obtain the natural frequencies of the symmetric modes of the structure.

The translational mode shapes for the building are given by,

$$V_1(y) = V_2(y) = B\left[\frac{\sin\beta y - \sinh\beta y}{\sin\beta + \sinh\beta} - \frac{\cos\beta y - \cosh\beta y}{\cos\beta + \cosh\beta}\right] \quad 0 \leq y \leq 1 \qquad (5.21a)$$

and

$$U(x,y) = \frac{B}{2}\left[\frac{\cos\alpha x}{\cos\alpha} + \frac{\cosh\alpha x}{\cosh\alpha}\right] \cdot$$

$$\cdot \left[\frac{\sin\beta y - \sinh\beta y}{\sin\beta + \sinh\beta} - \frac{\cos\beta y - \cosh\beta y}{\cos\beta + \cosh\beta}\right] \quad \begin{matrix} -1 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix} \qquad (5.21b)$$

Here B is an arbitrary constant.

Antisymmetric Modes

The following boundary conditions apply to the antisymmetric modes of vibrations:

(i) $U(x=0,y) = 0$

(ii) $\frac{\partial^2 U}{\partial x^2}(x=0,y) = 0$

* This is the same characteristic equation with different definition of β that governs the vibrations of a cantilever bending beam.

(iii) $\frac{\partial^2 U}{\partial x^2}(x=1,y) = 0$

(iv) $V_1(y=0) = 0$

(v) $\frac{dV_1}{dy}(y=0) = 0$

(vi) $\frac{d^2V_1}{dy^2}(y=1) = 0$

(vii) $\frac{d^3V_1}{dy^3}(y=1) = 0$

(viii) $U(x=1,y) = V_1(y)$

Equations (5.5) can be solved for these boundary conditions in a manner similar to that for the symmetric modes. The characteristic equation for these modes is again

$$\cos\beta \cosh\beta + 1 = 0 \tag{5.22a}$$

with β now defined by

$$\beta^4 = \frac{m_2 h^4}{E_2 I_2}\omega^2 + \frac{E_1 I^*{}_1 h^4}{E_2 I_2 L^3}\frac{\alpha^3}{2}(-\cot\alpha + \coth\alpha) \tag{5.22b}$$

$$\alpha^4 = \frac{m^*{}_1 L^4}{E_1 I^*{}_1}\omega^2 \tag{5.22c}$$

Equations (5.22) can be solved to obtain the natural frequencies for the antisymmetric (torsional) modes. The mode shapes are given by

$$V_1(y) = -V_2(y) = B\left[\frac{\sin\beta y - \sinh\beta y}{\sin\beta + \sinh\beta} - \frac{\cos\beta y - \cosh\beta y}{\cos\beta + \cosh\beta}\right] \quad 0\leq y\leq 1 \qquad (5.23a)$$

and,

$$U(x,y) = \frac{B}{2}\left[\frac{\sin\alpha x}{\sin\alpha} + \frac{\sinh\alpha x}{\sinh\alpha}\right]\cdot$$

$$\cdot\left[\frac{\sin\beta y - \sinh\beta y}{\sin\beta + \sinh\beta} - \frac{\cos\beta y - \cosh\beta y}{\cos\beta + \cosh\beta}\right] \quad \begin{matrix}-1\leq x\leq 1\\ 0\leq y\leq 1\end{matrix} \qquad (5.23b)$$

where B is an arbitrary constant.

Orthogonality of Modes

Beginning with the differential equations of the two walls and the distributed beam system modelling the floors, integration by parts and use of the boundary conditions produce the expected orthogonality condition. For $\omega_i \neq \omega_j$, where i and j denote two modes of vibration, the condition is

$$m^*_1 L \int_0^1 \int_{-1}^1 U_i(x,y)U_k(x,y)\,dxdy + m_2 \int_0^1 V_{1i}(y)V_{1k}(y)\,dy$$

$$+ m_2 \int_0^1 V_{2i}(y)V_{2k}(y)\,dy = 0 \qquad (5.24)$$

Participation Factors for Earthquake Ground Motion

The equations of motion due to uniform ground motion may be written as:

$$\frac{\partial^4 u(x,y,t)}{\partial x^4} + \frac{m^*{}_1 L^4}{E_1 I^*{}_1} \frac{\partial^2 u(x,y,t)}{\partial t^2} = - \frac{m^*{}_1 L^4}{E_1 I^*{}_1} \ddot{u}_g(t) \tag{5.25a}$$

$$\frac{\partial^4 v_1(y,t)}{\partial y^4} + \frac{m_2 h^4}{E_2 I_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} - \frac{E_1 I^*{}_1 h^4}{E_2 I_2 L^3} \frac{\partial^3 u(x=1,y,t)}{\partial x^3} = - \frac{m_2 h^4}{E_2 I_2} \ddot{u}_g(t) \tag{5.25b}$$

and,

$$\frac{\partial^4 v_2(y,t)}{\partial y^4} + \frac{m_2 h^4}{E_2 I_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} + \frac{E_1 I^*{}_1 h^4}{E_2 I_2 L^3} \frac{\partial^3 u(x=-1,y,t)}{\partial x^3} = - \frac{m_2 h^4}{E_2 I_2} \ddot{u}_g(t) \tag{5.25c}$$

Here $\ddot{u}_g(t)$ is the earthquake acceleration in the z'-direction.

The normal modes of vibration are used to expand the response of the structure. Let

$$u(x,y,t) = \sum_{i=1}^{\infty} U_i(x,y) T_i(t) \tag{5.26a}$$

$$v_1(y,t) = \sum_{i=1}^{\infty} V_{1i}(y) T_i(t) \tag{5.26b}$$

$$v_2(y,t) = \sum_{i=1}^{\infty} V_{2i}(y) T_i(t) \tag{5.26c}$$

Next, substitute equations (5.26) into equations (5.25) to obtain

$$\sum_i \left[U_i(x,y)\ddot{T}_i(t) + \omega_i^2 U_i(x,y)T_i(t)\right] = -\ddot{u}_g(t) \qquad (5.27a)$$

$$\sum_i \left[V_{1i}(y)\ddot{T}_i(t) + \omega_i^2 V_{1i}(y)T_i(t)\right] = -\ddot{u}_g(t) \qquad (5.27b)$$

$$\sum_i \left[V_{2i}(y)\ddot{T}_i(t) + \omega_i^2 V_{2i}(y)T_i(t)\right] = -\ddot{u}_g(t) \qquad (5.27c)$$

These equations can be combined to yield,

$$\sum_{i=1}^{\infty} \left[\left(m^*_1 L \int_0^1\int_{-1}^1 U_i U_k dxdy + m_2 \int_0^1 V_{1i}V_{1k}dy + m_2 \int_0^1 V_{2i}V_{2k}dy\right) \cdot \right.$$

$$\left. \cdot(\ddot{T}_i + \omega_i^2 T_i)\right] = -\left[m^*_1 L \int_0^1\int_{-1}^1 U_k dxdy + m_2 \int_0^1 V_{1k}dy \right.$$

$$\left. + m_2 \int_0^1 V_{2k}dy\right]\ddot{u}_g(t) \qquad (5.28)$$

Applying the orthogonality condition (equation 5.24), equation (5.28) becomes

$$\ddot{T}_k + \omega_k^2 T_k = -P_k \ddot{u}_g(t) \qquad (5.29)$$

where P_k, the participation factor for the k^{th} mode, is given by

$$P_k = \frac{m^*_1 L \int_0^1 \int_{-1}^1 U_k dxdy + m_2 \int_0^1 V_{1k}dy + m_2 \int_0^1 V_{2k}dy}{m^*_1 L \int_0^1 \int_{-1}^1 (U_k)^2 dxdy + m_2 \int_0^1 (V_{1k})^2 dy + m_2 \int_0^1 (V_{2k})^2 dy} \qquad (5.30)$$

Thus, the mode shapes obtained earlier can be substituted into equation (5.30) to obtain the corresponding modal participation factors. As expected, the participation factors are zero for the antisymmetric modes of vibration.

5.3 BUILDINGS WITH END WALLS (OR FRAMES) MODELLED AS SHEAR BEAMS

Walls with low height to width ratios and moment-resisting frames of low to moderate height, can be modelled as shear beams. Hence, buildings whose lateral load resistance system consists of only two such end walls or end frames can be treated in a manner similar to the previous section. The only difference is that the walls (or frames) now have to be modelled as shear beams rather than as bending beams. As shown in Figure (5.1), let the building height be h and the plan length be 2L.

The two end walls are assumed to be identical and uniform throughout the height of the building. The floors are taken as uniform along the length and are identical along the height of the building. Let the following be the mass and the stiffness properties of the structure:

E_1 = Young's modulus of elasticity for the floor system.

I^*_1 = Moment of inertia of the floor system cross-section per unit height.

$k_2 = k'A_2G_2$ = Shear rigidity of the shear beam that models the end frame or wall.

k' = Shape factor

A_2 = Area of cross-section of the end wall.

G_2 = Shear modulus of elasticity for the end wall.

m^*_1 = Mass per unit area (in x'-y' plane) of the distributed floor system.

m_2 = Mass per unit height of the end beam.

Let $u(x',y',t)$ be the displacement in the z'-direction, at point (x',y') of the floor system at an instant t. Similarly, $v_1(y',t)$ and $v_2(y',t)$ are the displacements in the z'-direction, in the right and the left end beams. Let x and y be the nondimensional coordinates defined as:

$$x = \frac{x'}{L} \quad , \quad y = \frac{y'}{L} \tag{5.31}$$

The equations of motion for free vibrations of the structure can be written in terms of these nondimensional coordinates as

$$\frac{\partial^4 u(x,y,t)}{\partial x^4} = - \frac{m^*_1 L^4}{E_1 I^*_1} \frac{\partial^2 u(x,y,t)}{\partial t^2} \tag{5.32a}$$

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} = \frac{m_2 h^2}{k_2} \frac{\partial^2 v_1(y,t)}{\partial t^2} - \frac{E_1 I^*_1 h^2}{k_2 L^3} \frac{\partial^3 u}{\partial x^3} (x=1,y,t) \tag{5.32b}$$

and,

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} = \frac{m_2 h^2}{k_2} \frac{\partial^2 v_2(y,t)}{\partial t^2} + \frac{E_1 I^*_1 h^2}{k_2 L^3} \frac{\partial^3 u}{\partial x^3}(x=-1,y,t) \qquad (5.32c)$$

As in the previous case, let

$$u(x,y,t) = U(x,y)e^{i\omega t} \qquad (5.33a)$$

$$v_1(y,t) = V_1(y)e^{i\omega t} \qquad (5.33b)$$

$$v_2(y,t) = V_2(y)e^{i\omega t} \qquad (5.33c)$$

where ω is the natural frequency of the motion. Equations (5.33) can be substituted into equations (5,32) to obtain

$$\frac{\partial^4 U(x,y)}{\partial x^4} - \frac{m^*_1 L^4}{E_1 I^*_1} \omega^2 U(x,y) = 0 \qquad (5.34a)$$

$$\frac{d^2 V_1(y)}{dy^2} + \frac{m_2 h^2}{k_2} \omega^2 V_1(y) = -\frac{E_1 I^*_1 h^2}{k_2 L^3} \frac{\partial^3 U}{\partial x^3}(x=1,y) \qquad (5.34b)$$

$$\frac{d^2 V_2(y)}{dy^2} + \frac{m_2 h^2}{k_2} \omega^2 V_2(y) = \frac{E_1 I^*_1 h^2}{k_2 L^3} \frac{\partial^3 U}{\partial x^3}(x=-1,y) \qquad (5.34c)$$

These equations can be solved separately for the symmetric (translational) and the antisymmetric (torsional) modes of vibration by considering only the right half of the structure.

Symmetric Modes

The boundary conditions applicable to the symmetric modes of vibration are as follows:

(i) $\frac{\partial U}{\partial x}(x=0,y) = 0$

(ii) $\frac{\partial^3 U}{\partial x^3}(x=0,y) = 0$

(iii) $\frac{\partial^2 U}{\partial x^2}(x=1,y) = 0$

(iv) $V_1 \ (y=0) = 0$

(v) $\frac{dV_1}{dy}(y=1) = 0$

(vi) $U(x=1,y) = V(y)$

As in the previous section, (iii) assumes zero torsional stiffness of the end walls. Equations (5.34) can be solved for these boundary conditions, in a manner similar to the previous section. This enables one to obtain the following characteristic equation:

$$\frac{m_2 h^2}{k_2}\omega^2 + \frac{E_1 I^*{}_1 h^2}{k_2 L^3}\frac{\alpha^3}{2}(\tan\alpha + \tanh\alpha) = \beta^2 \qquad (5.35a)$$

where

$$\alpha^4 = \frac{m^*{}_1 L^4}{E_1 I^*{}_1} \omega^2 \qquad (5.35b)$$

$$\beta = \frac{(2i-1)\pi}{2} \qquad i=1,2,3,... \qquad (5.35c)$$

From these equations, the natural frequency ω_i for the i^{th} mode can be obtained. The corresponding mode shape is given by,

$$U(x,y) = \frac{B}{2}\left[\frac{\cos \alpha x}{\cos \alpha} + \frac{\cosh \alpha x}{\cosh \alpha}\right] \sin \beta y \qquad \begin{matrix} -1 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix} \qquad (5.36a)$$

and,

$$V_1(y) = V_2(y) = B \sin \beta y \qquad 0 \leq y \leq 1 \qquad (5.36b)$$

where B is an arbitrary constant.

Antisymmetric Modes

The boundary conditions for these modes are:

(i) $U(x=0,y) = 0$

(ii) $\frac{\partial^2 U}{\partial x^2} (x=0,y) = 0$

(iii) $\frac{\partial^2 U}{\partial x^2} (x=1,y) = 0$

(iv) $V_1 \ (y=0) = 0$

(v) $\frac{dV_1}{dy} (y=1) = 0$

(vi) $U(x{=}1,y) = V(y)$

These boundary conditions and the equations (5.34) can be combined to obtain the following characteristic equation for antisymmetric modes of vibration:

$$\frac{m_2 h^2}{k_2}\,\omega^2 + \frac{E_1 I^*_1 h^2}{k_2 L^3}\,\frac{\alpha^3}{2}\,(-\cot\alpha + \coth\alpha) = \beta^2 \tag{5.37a}$$

where

$$\alpha^4 = \frac{m^*_1 L^4}{E_1 I^*_1}\,\omega^2 \tag{5.37b}$$

$$\beta = \frac{(2i-1)\pi}{2} \qquad i=1,2,3,\ldots \tag{5.37c}$$

The corresponding mode shapes are:

$$U(x,y) = \frac{B}{2}\left[\frac{\sin\alpha x}{\sin\alpha} + \frac{\sinh\alpha x}{\sinh\alpha}\right]\sin\beta y \qquad \begin{matrix}-1\le x\le 1\\ 0\le y\le 1\end{matrix} \tag{5.38a}$$

and,

$$V_1(y) = B\sin\beta y \qquad 0\le y\le 1 \tag{5.38b}$$

$$V_2(y) = -B\sin\beta y \qquad 0\le y\le 1 \tag{5.38c}$$

where B is an arbitrary constant.

Orthogonality Condition

As in the previous case, the following orthogonality condition can be obtained:

$$m_1 L \int_0^1 \int_0^1 U_i(x,y) U_k(x,y) dxdy + m_2 \int_0^1 V_{1i}(y) V_{1k}(y) dy$$

$$+ m_2 \int_0^1 V_{2i}(y) V_{2k}(y) dy = 0 \qquad \text{for } i \neq k \tag{5.39}$$

Participation Factors for Ground Motion

Similarly, it can be shown that the modal participation factor for the k^{th} mode for earthquake ground motion is given by,

$$P_k = \frac{m_1 L \int_0^1 \int_{-1}^1 U_k dxdy + m_2 \int_0^1 V_{1k} dy + m_2 \int_0^1 V_{2k} dy}{m_1 L \int_0^1 \int_{-1}^1 (U_k)^2 dxdy + m_2 \int_0^1 (V_{1k})^2 dy + m_2 \int_0^1 (V_{2k})^2 dy} \tag{5.40}$$

5.4 NUMERICAL EXAMPLE

In this section, a multistory building with two end walls (modelled as bending beams) has been analyzed. Using the method described earlier in this chapter, the natural periods, the mode shapes and the modal participation factors have been obtained.

The example structure has been derived from Building 180 at the Jet Propulsion Laboratory in Pasadena. This long and narrow building (220' X 40' in plan) has 12 uniformly distributed moment-resisting frames to resist the lateral loads in the transverse direction. However, in the example structure, the frames are assumed to be capable of providing only the vertical support, while two 12" thick reinforced concrete walls have been added at the two ends of the building to provide all the lateral load resistance in the transverse direction.

The actual building is ten stories high with basement walls. The story heights are 14 ft except in the top story and in the basement and first stories where they are 16 ft. The lumped weight of the roof is 1517 kips while that of the typical floor is 1270 kips (Wood, 1972). In the example structure, the building is assumed to be rigidly held at ground level, thus neglecting the basement story. The nonuniformities in the story height and in the lumped masses have been neglected, and an average story height and average lumped weights have been taken. The floors are 5 in thick and made of light-weight concrete. Since the end walls are 40 ft wide and 130 ft high, it is reasonable to neglect their shear flexibility and treat them as bending beams. Similarly, the floors, which are 220 ft long and 40 ft wide, behave like bending beams and were treated as such. The following building properties were used for the analysis of the example structure:

Uniformly distributed floor system:

weight ($m^*_1 g$) = 409 lbs/ft^2

moment of inertia (I^*_1) = 154 $(ft)^4$/ft height

modulus of elasticity (E_1) = 2.0×10^6 psi

length of the floors (2L) = 220 ft

Walls:

weight ($m_2 g$) = 6000 lbs/ft

moment of inertia (I_2) = 5330 $(ft)^4$

modulus of elasticity (E_2) = 2.9×10^6 psi

building height (h) = 130 ft

As noted, an examination of equation (5.19) reveals that it is the same as the characteristic equation for the free vibration of a cantilever bending beam. The roots of this equation (i.e., values of β) are 1.875, 4.694, 7.855, 10.996, 14.137, 17.279, etc. (e.g., Timoshenko, et al., 1974). Here, β equal to 1.875 corresponds to the end wall deforming as the first mode of a cantilever beam. Similarly, the higher values of β correspond to the end wall deforming in higher cantilever modes.

For various values of β, equations (5.7 and 5.13) were solved to obtain the natural frequency (ω) for the symmetric modes of vibration. Also, equations (5.22b,c) were solved to obtain the frequencies for the antisymmetric modes.

For the appropriate natural frequencies, equations (5.21) and (5.23) gave the symmetric and the antisymmetric mode shapes. A few of the more important mode shapes are plotted in Figure (5.2). Equation (5.30) was used to calculate the modal participation factors for the various modes of vibration. The maximum base shear in various modes due to an earthquake motion assumed to have a constant acceleration spectrum value of 0.20g, was then calculated using these participation factors. Table (5.1) gives natural periods and maximum base shear for some of the lower modes.

TABLE 5.1. PERIODS AND MAXIMUM BASE SHEARS FOR SYMMETRIC MODES OF THE EXAMPLE STRUCTURE

Period (sec)	β	Maximum Base Shear (Kips)
0.929	1.875	1590
0.533	4.694	369
0.523	7.855	126
0.522	10.996	14
0.522	14.137	0.03
0.243	1.875	38.5
0.078	4.694	112
0.060	7.855	20
0.058	10.996	1.7
0.049	1.875	0.03
0.043	4.694	18.0

As expected, the antisymmetric modes have zero modal participation factors and do not contribute to the base shear. The natural periods for the lowest few antisymmetric modes of vibration were obtained as 0.524 sec (β = 1.875), 0.144 sec (β = 4.694), 0.132 sec (β = 7.855), 0.131 sec (β = 10.996), ... , 0.097 sec (β = 1.875), 0.059 sec (β = 4.694), and 0.035 sec (β = 7.855).

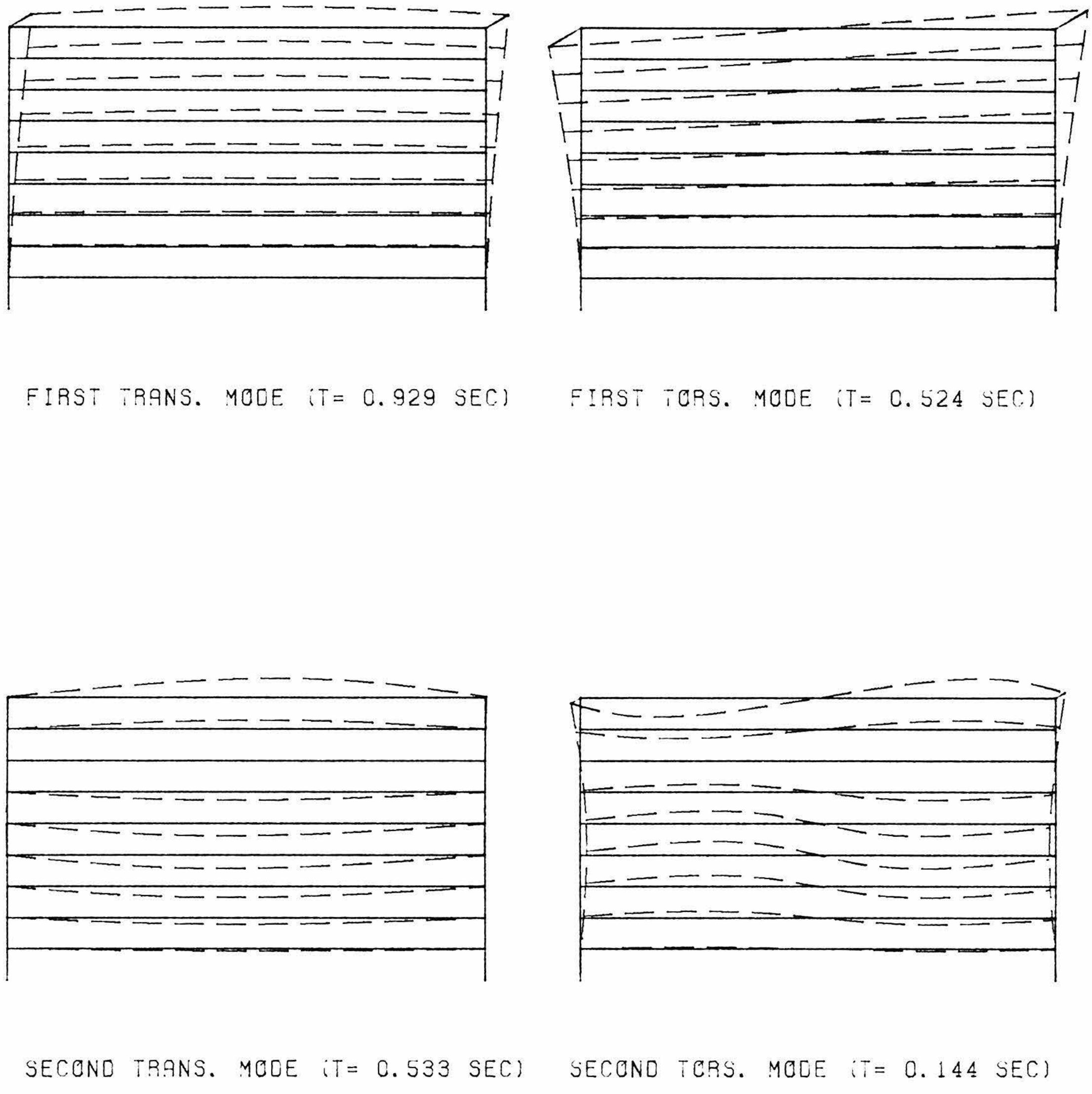


Figure 5.2. MODE SHAPES FOR THE 9-STORY BUILDING.

If one treats the floors as rigid and the end walls as bending beams, the fundamental period is obtained as 0.806 sec, while that for the model with floor flexibility is 0.929 sec. Thus, the floor flexibility makes the structure more flexible and increases its fundamental natural period. The fundamental period of the floors when treated as pinned-pinned beams is 0.522 sec. It is interesting to note that the use of Dunkerley's equation (Dunkerly, 1895; Thomson, 1965) gives a period of the combined system as 0.960 sec ($T^2 \simeq 0.806^2 + 0.522^2$) which is a reasonably good estimate of the actual period.

One notices from Table (5.1) that there are several modes with periods nearly equal to 0.522 sec, but with various values of β. Also, as noted above, the fundamental period of the floors when treated as pinned-pinned beams is 0.522 sec. Since the method of this chapter treats the floors as an infinite number of independently acting beams of infinitesimal thickness, it is reasonable to see many modes with periods close to 0.522 sec, for different values of β. Fortunately, one has to consider only the first few of these 0.522 sec modes since, as the value of β increases, the modes contribute less and less to the base shear.

The mode with period 0.243 sec is one in which the end walls deform in the first cantilever mode (since $\beta = 1.875$), while the floors deform in the second symmetric mode of a pinned-pinned beam. Some of the other periods can be interpreted in a similar manner.

CHAPTER VI

MULTISTORY BUILDINGS WITH UNIFORMLY DISTRIBUTED FRAMES OR WALLS

6.1 INTRODUCTION

Some of the earliest forced vibration tests on multistory buildings that indicated floor deformation modes were performed on Building 180 at the Jet Propulsion Laboratory in Pasadena (Nielsen, 1964,1966). The lateral load resistance in transverse direction in this long and narrow building (220′ X 40′ in plan) is provided by 12 moment-resisting frames that are uniformly placed along the length of the building. Thus, even though the span to width ratio of the floors is not large, the overall floor length to width ratio is quite large and, it is of interest to see how such floors affect the dynamic behavior of the structure. This chapter presents treatment on such buildings, i.e., multistory buildings with a large number of transverse frames or walls that are placed at equal intervals along the length of the building.

The floors, due to their large aspect ratio, can be treated as bending beams. The moment resisting frames are modelled as shear beams, due to the similarities in the mode shapes and in the spacing of frequencies. Thus, for transverse vibrations the building can be idealized as a grid consisting of vertical shear beams and horizontal bending beams. Similarly, a building that has a uniform distribution of walls with a large height to width ratio can be modelled as a grid consisting of bending beams in both directions. The vibration problem of such grids can be solved by the finite element model or by the finite difference method, using slope deflection equations (e.g., Wah, 1963; Goldberg and Herness, 1965; Goldberg, 1966).

Moreover, grids that have a large number of uniform, identical beams in both the vertical and the horizontal directions can also be modelled as vertically-oriented anisotropic plates by averaging the mass and the stiffness properties of the beams over the entire length and height of the grid (e.g., Timoshenko and Woinowsky-Krieger, 1959). It is proposed in this chapter to analyze buildings whose floors and frames are sufficiently uniform and numerous as vertically-oriented plates. For buildings with moment-resisting frames, the plate is such that a thin vertical strip cut from the plate has only shear flexibility, and thus behaves like a shear beam, while a thin horizontal strip cut from the plate behaves like a bending beam. Such plates will be referred to as "bending-shear" plates. Similarly, buildings with walls that behave like bending beams are treated as anisotropic plates, with only the bending deformations important along the two coordinate directions; such plates are referred to as "bending-bending" plates.

In the following parts of this chapter, equations of motion for these plates are discussed. Then, expressions for the natural frequencies, the mode shapes and the modal participation factors are obtained for buildings with moment-resisting frames. Buildings with slender walls, idealized as bending-bending plates, have similar solutions. Some conclusions based on these results are presented at the end of the chapter.

6.2 PLATE EQUATIONS

This section describes the equations of motion for the "bending-bending" and the "bending-shear" plates.

6.2.1 Bending-Bending Plate

The equation, for static loads, describing a plate that models a grid consisting of bending beams can be found in Timoshenko and Woinowsky-Krieger (1959):

$$\frac{E_1 I_1}{a_1} \frac{\partial^4 w(x,y)}{\partial x^4} + \left[\frac{C_1}{a_1} + \frac{C_2}{a_2}\right] \frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{E_2 I_2}{a_2} \frac{\partial^4 w(x,y)}{\partial y^4} = f(x,y) \tag{6.1}$$

where,

$E_1 I_1$ = Flexural rigidity of horizontal beams.

$E_2 I_2$ = Flexural rigidity of vertical beams.

C_1 = Torsional rigidity of horizontal beams.

C_2 = Torsional rigidity of vertical beams.

a_1 = Distance between two consecutive horizontal beams.

a_2 = Distance between two consecutive vertical beams.

$f(x,y)$ = Load at (x,y) acting perpendicular to the plane of the grid.

$w(x,y)$ = Displacement at (x,y) in z-direction.

The coordinate system (x,y,z) is shown in Figure (6.1). As discussed earlier, the torsional stiffness of the floors and the walls can be neglected in the present application. Thus, replacing $f(x,y)$ by the inertial force term and neglecting the torsional stiffness terms, the equation of motion for free vibrations of a "bending-bending" plate may be written as

$$\bar{D}_1 \frac{\partial^4 w(x,y,t)}{\partial x^4} + \bar{D}_2 \frac{\partial^4 w(x,y,t)}{\partial y^4} = -m \frac{\partial^2 w(x,y,t)}{\partial t^2} \qquad (6.2)$$

where

$\bar{D}_1$ = Flexural stiffness of a horizontal strip of the plate, of unit width $\left[= \frac{E_1 I_1}{a_1} \right]$.

$\bar{D}_2$ = Flexural stiffness of a vertical strip of the plate, of unit width $\left[= \frac{E_2 I_2}{a_2} \right]$.

m = Mass per unit area (in x-y plane) of the plate.

Equation (6.2) can easily be solved using the method of separation of variables.

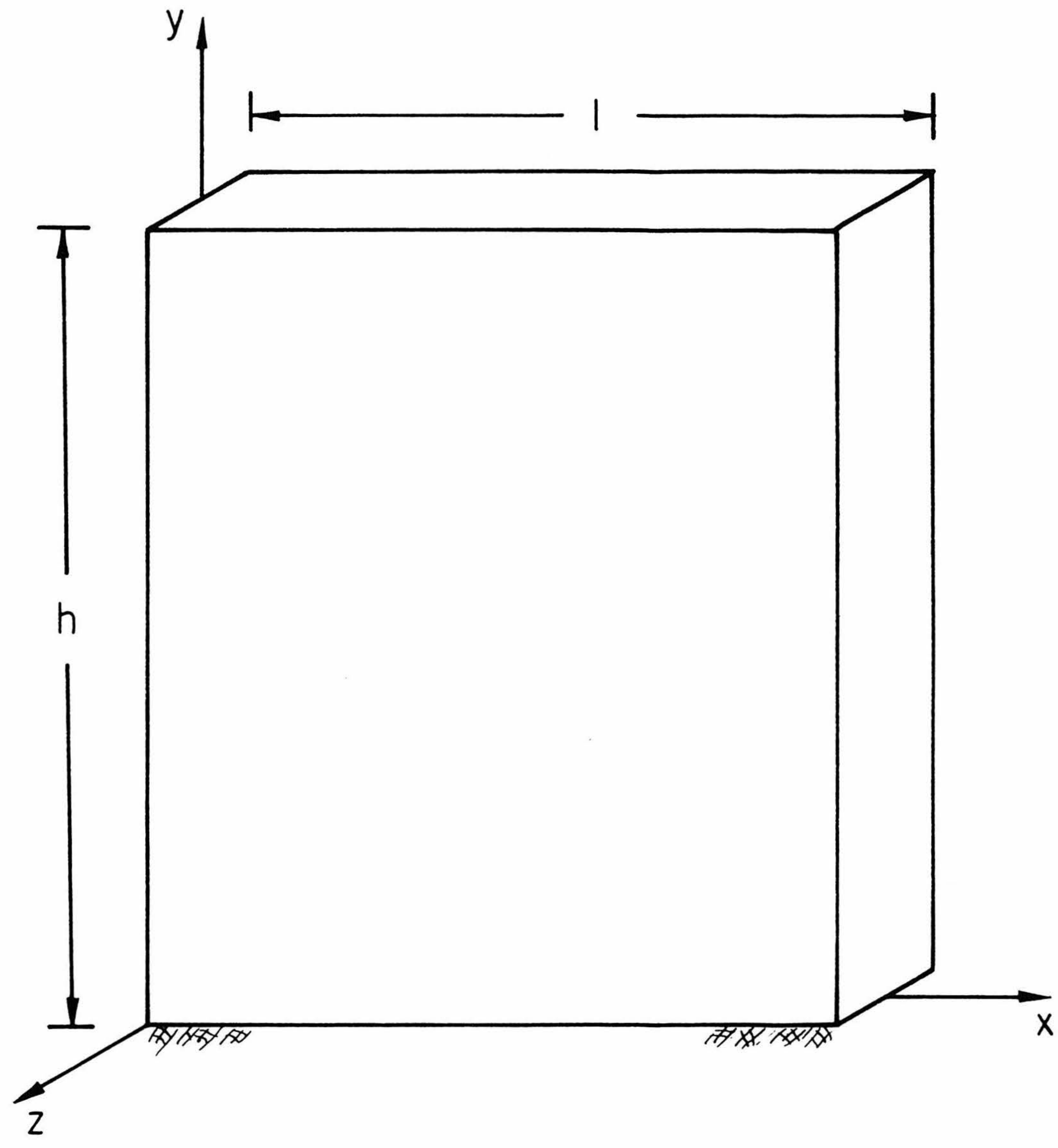


Figure 6.1. CONTINUOUS MODEL FOR BUILDINGS WITH UNIFORMLY DISTRIBUTED FRAMES OR WALLS.

6.2.2 Bending-Shear Plate

The equation of motion for free vibrations of a plate, modelling a grid that consists of bending beams in the x-direction and shear beams in the y-direction, can be derived from energy principles or obtained in analogy to equation (6.2) as

$$\bar{D}_1 \frac{\partial^4 w(x,y,t)}{\partial x^4} - \bar{K}_2 \frac{\partial^2 w(x,y,t)}{\partial y^2} = -m \frac{\partial^2 w(x,y,t)}{\partial t^2} \qquad (6.3)$$

where $\bar{K}_2$ is the shear stiffness of a vertical strip of the plate of unit width. All other terms have been defined earlier. Equation (6.3) can also be solved by the method of separation of variables.

6.3 MULTISTORY BUILDINGS WITH UNIFORMLY DISTRIBUTED FRAMES

Consider a multistory building with uniform and identical moment-resisting frames and uniform and identical floors. The spacing of the frames is uniform and all the story heights are the same. Such a structure can be modelled as a vertically-oriented "bending-shear" plate. Let l be the length and h be the height of the building. The coordinate system (x,y,z) is shown in Figure (6.1).

The equation of motion for free vibrations of the structure is given by equation (6.3). The assumed form of the solution is

$$w(x,y,t) = W(x,y)e^{i\omega t} = X(x)Y(y)e^{i\omega t} \qquad (6.4)$$

where ω is the natural frequency of the system. Substitution of equation (6.4) into equation (6.3) yields

$$\frac{d^4X(x)}{dx^4} - \alpha^4 X(x) = 0 \tag{6.5a}$$

$$\frac{d^2Y(y)}{dy^2} + \beta^2 Y(y) = 0 \tag{6.5b}$$

where α and β are constants to be determined from the boundary conditions of the problem, they satisfy the condition

$$\bar{D}_1\alpha^4 + \bar{K}_2\beta^2 = m\omega^2 \tag{6.6}$$

Solutions of equations (6.5) can be written as

$$X(x) = A_1 \sin \alpha x + A_2 \cos \alpha x + A_3 \sinh \alpha x + A_4 \cosh \alpha x \tag{6.7a}$$

$$Y(y) = B_1 \sin \beta y + B_2 \cos \beta y \tag{6.7b}$$

where the A's and B's are constants to be determined from the boundary conditions.

The boundary conditions for the plate in this case are fixed at (y=0) and free at the other three sides. In mathematical form, these can be expressed as

(i) $Y(y=0) = 0$

(ii) $\dfrac{dY(y=h)}{dy} = 0$

(iii) $$\frac{d^2X(x=0)}{dx^2} = 0$$

(iv) $$\frac{d^3X(x=0)}{dx^3} = 0$$

(v) $$\frac{d^2X(x=1)}{dx^2} = 0$$

(vi) $$\frac{d^3X(x=1)}{dx^3} = 0$$

Boundary conditions (i) and (ii) give

$$B_2 = 0 \tag{6.8}$$

and,

$$\beta B_1 \cos \beta h_1 = 0 \tag{6.9}$$

Thus, for a nontrivial solution

$$\beta_j = \frac{(2j-1)\pi}{2h} \qquad j=1,2,3,\ldots \tag{6.10}$$

From boundary conditions (iii) and (iv), one obtains

$$A_4 = A_2 \quad , \quad A_3 = A_1 \tag{6.11}$$

and, boundary conditions (v) and (vi) yield

$$A_1(-\sin \alpha l + \sinh \alpha l) + A_2(-\cos \alpha l + \cosh \alpha l) = 0 \tag{6.12}$$

$$A_1(-\cos \alpha l + \cosh \alpha l) + A_2(\sin \alpha l + \sinh \alpha l) = 0 \qquad (6.13)$$

This gives the condition for nontrivial solution as

$$\cos \alpha l \cosh \alpha l - 1 = 0 \qquad (6.14)$$

and

$$A_2 = -\frac{\sin \alpha l - \sinh \alpha l}{\cos \alpha l - \cosh \alpha l} A_1 \qquad (6.15)$$

The first few roots of equation (6.14) are given as (e.g., Timoshenko, et al., 1974)

$\alpha_1 l$	$\alpha_2 l$	$\alpha_3 l$	$\alpha_4 l$	$\alpha_5 l$	$\alpha_6 l$	$\alpha_7 l$
0	0	4.730	7.853	10.996	14.137	17.279

or,

$$\alpha_i = 0 \qquad i=1,2 \qquad (6.16a)$$

$$\alpha_i \simeq \left[i - \frac{3}{2}\right]\frac{\pi}{l} \qquad i=3,4,5,\ldots \qquad (6.16b)$$

The natural frequencies of the system can now be obtained using equations (6.6), (6.10) and (6.16) as

$$\omega_{ij} = \frac{(2j-1)\pi}{2h}\sqrt{\frac{\bar{K}_2}{m}} \qquad \begin{matrix} i=1,2 \\ j=1,2,3,\ldots \end{matrix} \qquad (6.17a)$$

$$\omega_{ij} \simeq \left[\frac{\left(i - \frac{3}{2}\right)^4 \pi^4 \bar{D}_1}{m l^4} + \frac{(2j-1)^2 \pi^2 \bar{K}_2}{4 m h^2}\right]^{1/2} \qquad \begin{matrix} i=3,4,5,\ldots \\ j=1,2,3,\ldots \end{matrix} \qquad (6.17b)$$

For i = 1 and 2, α vanishes and equation (6.5a) and the appropriate boundary conditions give

$$X(x) = A_1 + A_2 x \tag{6.18}$$

where A_1 and A_2 are arbitrary constants. This represents a rigid body translation and a rigid body rotation of floors. Thus, the mode shapes for i=1,2 are

$$W_{1j}(x,y) = A \sin \frac{(2j-1)\pi}{2h} y \qquad j=1,2,3,... \tag{6.19a}$$

$$W_{2j}(x,y) = A\left(x - \frac{1}{2}\right) \sin \frac{(2j-1)\pi}{2h} y \qquad j=1,2,3,... \tag{6.19b}$$

where A is an arbitrary constant. The modes represented by equation (6.19a) are translational modes that involve no floor deformations, with frequencies given by equation (6.17a). Similarly, equation (6.19b) gives the torsional mode shapes of the structure, again with no floor deformations. These mode shapes and the corresponding frequencies are the same as obtained from an analysis based on the assumption that the floors are rigid in their own plane.

The mode shapes that correspond to the higher values of α are

$$W_{ij}(x,y) = A\left[\frac{\sin \alpha_i x + \sinh \alpha_i x}{\sin \alpha_i l - \sinh \alpha_i l} - \frac{\cos \alpha_i x + \cosh \alpha_i x}{\cos \alpha_i l + \cosh \alpha_i l}\right] \cdot \sin \frac{(2j-1)\pi y}{2h} \qquad \begin{array}{l} i=3,4,5,... \\ j=1,2,3,... \end{array} \tag{6.20}$$

where A is an arbitrary constant. The modes represented by equation

(6.20) involve floor deformations. These mode shapes are simply the superposition of mode shapes of the floors when treated as free-free beams and those of the frames. Also, the corresponding frequencies, given by equation (6.17b), are the square root of the sum of the squares of the floor frequencies when treated as free-free beams and the frame frequencies. Thus, the dynamic analysis of such buildings can be carried out by separately analyzing a typical frame and a typical floor with free-free end conditions. This result supports similar observations made earlier by Maybee, et al. (1966), who treated the building as a discrete system, lumping the mass at the intersections of the floors and the frames.

Orthogonality of Modes

It can be shown that the modes of vibration of the structure are orthogonal and, that the orthogonality condition is given by

$$\int_0^h \int_0^1 W_{ij}(x,y) W_{rs}(x,y)\, dx dy = 0 \qquad (6.21a)$$

or

$$\int_0^h \int_0^1 X_i(x) X_r(x) Y_j(y) Y_s(y)\, dx dy = 0 \qquad (6.21b)$$

where $i \neq r$, and/or $j \neq s$.

Modal Participation Factors for Earthquake Ground Motion

The equation of motion for the bending-shear plate under uniform earthquake excitation can be written as

$$\bar{D}_1 \frac{\partial^4 w(x,y,t)}{\partial x^4} - \bar{K}_2 \frac{\partial^2 w(x,y,t)}{\partial y^2} = -m \frac{\partial^2 w(x,y,t)}{\partial t^2} - m\ddot{u}_g(t) \quad (6.22)$$

where $\ddot{u}_g(t)$ is the ground acceleration in the z-direction.

To solve, expand w(x,y,t) in terms of the normal modes of the system. Let

$$w(x,y,t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} X_i(x) Y_j(y) T_{ij}(t) \quad (6.23)$$

and substitute this into equation (6.22) to obtain

$$\bar{D}_1 \sum_i \sum_j \frac{d^4 X_i}{dx^4} Y_j T_{ij} - \bar{K}_2 \sum_i \sum_j X_i \frac{d^2 Y_j}{dy^2} T_{ij}$$

$$+ m \sum_i \sum_j X_i Y_j \frac{d^2 T_{ij}}{dt^2} = -m\ddot{u}_g(t) \quad (6.24)$$

This gives

$$\sum_i \sum_j (X_i Y_j \ddot{T}_{ij} + \omega_{ij}^2 X_i Y_j T_{ij}) = -\ddot{u}_g(t) \quad (6.25)$$

Next, multiply equation (6.25) by $X_r Y_s$, integrate and apply the orthogonality relationship (equation 6.21) to obtain

$$\ddot{T}_{rs} + \omega_{rs}^2 T_{rs} = -P_{rs}\ddot{u}_g(t) \tag{6.26}$$

where P_{rs}, the participation factor for the rs^{th} mode, is given by

$$P_{rs} = \frac{\int_0^h \int_0^1 X_r Y_s dxdy}{\int_0^h \int_0^1 X_r X_r Y_s Y_s dxdy} \tag{6.27}$$

For i=1, $X_i(x)$ = A (rigid body translation) and the orthogonality condition yields

$$\int_0^1 X_r(x)dx = 0 \qquad \text{for } r\neq 1 \tag{6.28}$$

Therefore, the numerator in equation (6.27) vanishes for all $r\neq1$, and the participation factors are zero for all the modes that involve floor deformations. This is a very useful result and shows that uniform ground motion excites only those translational modes that do not involve floor deformations. However, as noted earlier, such modes are the same as obtained by an analysis based on the rigid floor-diaphragm assumption. Therefore, in the dynamic analysis of such buildings for uniform earthquake ground motion, one need not take into account the in-plane flexibility of the floors.

However, this conclusion is valid only for the uniform ground motion and cannot be applied to other types of loading, e.g., spatially varying ground shaking. Also modes involving floor diaphragm deformations can be excited by forced vibration tests, as was done on Building

6.4 BUILDINGS WITH UNIFORMLY DISTRIBUTED WALLS

Multistory buildings that have a uniform distribution of identical walls with large height to width ratio, and satisfy other uniformity conditions discussed earlier in the chapter, can be modelled as "bending-bending" plates. The equation of motion for free vibrations of such structures is given by equation (6.2). This equation can be solved by the method of separation of variables in a manner similar to that followed in the preceding section. All the discussion in the previous section about the nature of the frequencies, the mode shapes and the modal participation factors is also valid for these buildings and is not repeated here.

6.5 DISCUSSION AND CONCLUSIONS

It has been shown that long and narrow buildings with a uniform distribution of identical frames (or walls) can be analyzed as vertically-oriented plates. This model is a two-dimensional analog of shear beam models for multistory buildings that have been extensively used in the past (e.g., Jennings, 1969; Hoerner, 1971). Based on this plate idealization, it has been shown that such buildings possess all the modes of vibration that one obtains by analysis based on the assumption of rigid floor diaphragms, plus additional modes that involve floor deformations similar to those of free-free beams. The mode shapes and the natural frequencies of the structure can be obtained by analyzing a typical floor and a typical frame. However, in doing so, care must be taken to include the floor masses in the frame analysis and vice-versa.

Also, it is seen that none of the modes that involve floor deformations are excited by uniform ground motion. Thus, there is no need to treat the floors of such buildings as flexible, when analyzing them for uniform seismic forces. This result has also been shown using discrete, lumped-mass models for such buildings (Jain, 1983). In addition, in a parametric study on a building with five cross walls, Unemori, et al. (1980) have found using finite element approach that the modes with floor deformations have very small modal participation factors.

It was assumed in this chapter that the number of frames (or walls) and the number of stories in the building are large. When this condition is not met, the proposed model may not be a good idealization of the structure. From a practical viewpoint, it seems that the building should have five or more floors and frames for the method to give reliable results.

CHAPTER VII

MULTISTORY BUILDINGS WITH MORE GENERAL FEATURES

7.1 INTRODUCTION

Using the tools developed in the previous chapters, it is possible to analyze multistory buildings with some nonuniformities as well. For instance, a multistory building that has a uniform distribution of frames along with two rather stiff end walls can be modelled as a plate with two end beams. The differential equations of motion for the plate and the end beams can be written and the system can be solved for the appropriate boundary conditions. Similarly, the problem of an otherwise uniform building with a more flexible, "soft " first story can be analyzed using these modelling techniques. For this situation, the columns or the walls in the first story can be modelled as a uniform distribution of infinitesimally thin vertical bending or shear beams, with the plate modelling the rest of the building joined to the top of these beams. Again the equations of motion and the boundary conditions can be solved to obtain a characteristic equation for the frequencies and expressions for the mode shapes.

In this chapter, an analysis is given for a multistory building that has two end walls in the upper stories to provide lateral resistance to the structure while in the first story the lateral support is provided by several uniformly distributed walls. Finally, the Imperial County Services Building which can be modelled approximately by this type of approach for transverse response is analyzed as an illustrative example.

7.2 MULTISTORY BUILDINGS WITH TWO END WALLS IN THE UPPER STORIES AND SEVERAL WALLS IN THE GROUND STORY

Consider a long and narrow multistory building whose lateral loads in the transverse direction are resisted by two end walls in the upper stories and by several uniformly placed walls in the first story. Thus, the upper floors transfer all the lateral loads to the end walls, which in turn transfer this load to the ground story walls through the second floor slab. It is of interest to see how this rather complex structural system can be analyzed by the methods developed in the previous chapters, while treating the floors as flexible.

A structure of this type can be modelled as shown in Figure (7.1). It is assumed that the floors, with their large aspect ratio, behave like bending beams. Hence, the floors above the second floor can be modelled as a uniformly distributed bending beam system, while the second floor is treated as a separate bending beam, due to its important role in transferring the loads from the end walls to the walls below. The upper story walls are assumed to have small height to width ratios and to behave as shear beams. The walls in the first story also have small height to width ratios and have been treated as a vertically-oriented uniform distribution of shear beams.

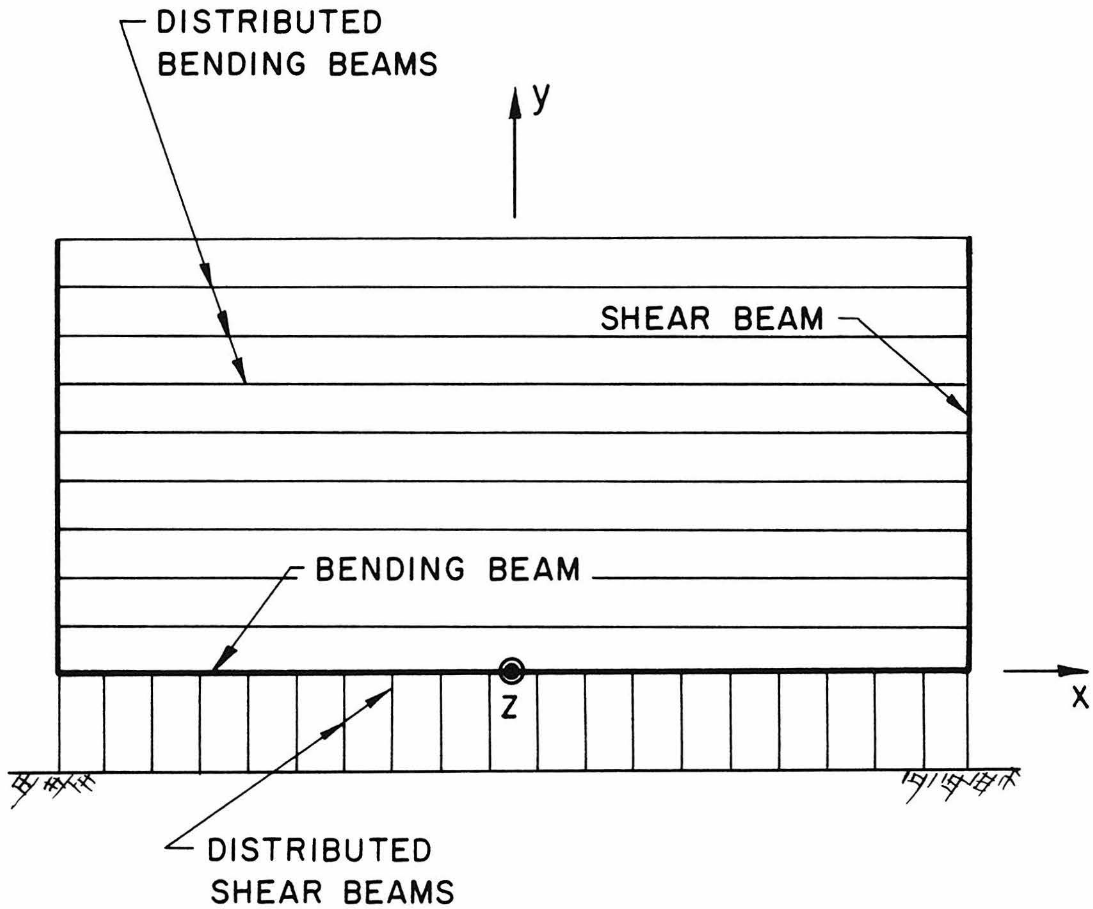


Figure 7.1. MODEL OF A MULTISTORY BUILDING WITH TWO END WALLS IN THE UPPER STORIES AND SEVERAL WALLS IN THE GROUND STORY.

Let the following be the mass and the stiffness properties of the various elements in the structure, assumed to be uniform:

E_1 = Young's modulus for the distributed floor system.

E_3 = Young's modulus for the second floor.

I^*_1 = Moment of inertia of the floor system cross-section per unit height.

I_3 = Moment of inertia of the second floor.

k^*_2 = Shear stiffness of the first-story wall system per unit length.

k_4 = Shear stiffness of the end walls.

m^*_1 = Mass per unit area (in x-y plane) of the distributed floor system.

m^*_2 = Mass per unit area (in x-y plane) of the ground wall system.

m_3 = Mass per unit length of the second floor.

m_4 = Mass per unit height of the end wall.

Let 2L be the length of the building, h_1 be the height of the building from the second floor level and h_2 be the story height of the first story. The coordinate system (x,y,z) is shown in Figure (7.1). Let $u(x,y,t)$, $v(x,t)$, $w_1(y,t)$, $w_2(x,y,t)$ and $w_3(y,t)$ be the displacements in the z-direction in the distributed floor system, the second floor, the right end wall, the ground story wall system and the left end wall, respectively. The equations of motion for free vibrations can be written for each element in the structure as:

$$E_1 I^*_1 \frac{\partial^4 u(x,y,t)}{\partial x^4} + m^*_1 \frac{\partial^2 u(x,y,t)}{\partial t^2} = 0 \tag{7.1a}$$

$$k^*_2 \frac{\partial^2 w_2(x,y,t)}{\partial y^2} - m^*_2 \frac{\partial^2 w_2(x,y,t)}{\partial t^2} = 0 \tag{7.1b}$$

$$E_3 I_3 \frac{\partial^4 v(x,t)}{\partial x^2} + m_3 \frac{\partial^2 v(x,t)}{\partial t^2} = -k^*_2 \frac{\partial w_2(x,y=0,t)}{\partial y} \tag{7.1c}$$

$$k_4 \frac{\partial^2 w_1(y,t)}{\partial y^2} - m_4 \frac{\partial^2 w_1(y,t)}{\partial t^2} = -E_1 I^*_1 \frac{\partial^3 u(x=L,y,t)}{\partial x^3} \tag{7.1d}$$

and

$$k_4 \frac{\partial^2 w_3(y,t)}{\partial y^2} - m_4 \frac{\partial^2 w_3(y,t)}{\partial t^2} = E_1 I^*_1 \frac{\partial^3 u(x=-L,y,t)}{\partial x^3} \tag{7.1e}$$

These equations can be solved using the method of separation of variables. Let

$$u(x,y,t) = U(x,y)e^{i\omega t} \tag{7.2a}$$

$$v(x,t) = V(x)e^{i\omega t} \tag{7.2b}$$

$$w_1(y,t) = W_1(y)e^{i\omega t} \tag{7.2c}$$

$$w_2(x,y,t) = W_2(x,y)e^{i\omega t} \tag{7.2d}$$

and

$$w_3(y,t) = W_3(y)e^{i\omega t} \tag{7.2e}$$

where ω is the natural frequency of the motion. Substitution into equations (7.1) gives

$$\frac{\partial^4 U(x,y)}{\partial x^4} - \frac{m^*_1}{E_1 I^*_1}\,\omega^2 U(x,y) = 0 \tag{7.3a}$$

$$\frac{\partial^2 W_2(x,y)}{\partial y^2} + \frac{m^*_2}{k^*_2}\,\omega^2 W_2(x,y) = 0 \tag{7.3b}$$

$$\frac{d^4 V(x)}{dx^4} - \frac{m_3}{E_3 I_3}\,\omega^2 V(x) = -\frac{k^*_2}{E_3 I_3}\,\frac{\partial W_2(x,y=0)}{\partial y} \tag{7.3c}$$

$$\frac{d^2 W_1(y)}{dy^2} + \frac{m_4}{k_4}\,\omega^2 W_1(y) = -\frac{E_1 I^*_1}{k_4}\,\frac{\partial^3 U(x=L,y)}{\partial x^3} \tag{7.3d}$$

and,

$$\frac{d^2 W_3(y)}{dy^2} + \frac{m_4}{k_4}\,\omega^2 W_3(y) = \frac{E_1 I^*_1}{k_4}\,\frac{\partial^3 U(x=-L,y)}{\partial x^3} \tag{7.3e}$$

Since the structure is symmetric, only the right half of it needs to be considered. The following are the boundary conditions that apply for the translational modes of the structure.

(i) $\frac{\partial U(x=0,y)}{\partial x} = 0$

(ii) $\frac{\partial^3 U(x=0,y)}{\partial x^3} = 0$

(iii) $\frac{\partial^2 U(x=L,y)}{\partial x^2} = 0$

(iv) $\frac{dV(x=0)}{dx} = 0$

(v) $\frac{d^3V(x=0)}{dx^3} = 0$

(vi) $\frac{d^2V(x=L)}{dx^2} = 0$

(vii) $\frac{dW_1(y=h_1)}{dy} = 0$

(viii) $W_2(x,y=-h_2) = 0$

(ix) $U(x=L,y) = W_1(y)$

(x) $V(x=L) = W_1(y=0)$

(xi) $W_2(x,y=0) = V(x)$

(xii) $E_3I_3 \frac{d^3V(x=L)}{dx^3} + k_4 \frac{dW_1(y=0)}{dy} = 0$

The solution of equation (7.3a) that satisfies the boundary conditions (i, ii, iii and ix) is:

$$U(x,y) = \frac{W_1(y)}{2} \left[\frac{\cos \alpha x}{\cos \alpha L} + \frac{\cosh \alpha x}{\cosh \alpha L}\right] \tag{7.4}$$

where,

$$\alpha^4 = \frac{m^*_1}{E_1 I^*_1} \omega^2 \tag{7.5}$$

Equation (7.3b) when solved for the boundary conditions (viii and xi) yields

$$W_2(x,y) = V(x)(\cot \beta h_2 \sin \beta y + \cos \beta y) \tag{7.6}$$

where,

$$\beta^2 = \frac{m^*_2}{k^*_2} \omega^2 \tag{7.7}$$

Substitution of equation (7.6) into equation (7.3c) gives

$$\frac{d^4V}{dx^4} - \left[\frac{m_3}{E_3I_3} \omega^2 - \frac{k^*_2}{E_3I_3} \beta \cot \beta h_2\right] V = 0 \tag{7.8}$$

The solution for this equation satisfying boundary conditions (iv, v and vi) is

$$V(x) = C\left[\frac{\cos \gamma x}{\cos \gamma L} + \frac{\cosh \gamma x}{\cosh \gamma L}\right] \tag{7.9}$$

for positive values of γ^4 where,

$$\gamma^4 = \frac{m_3}{E_3I_3} \omega^2 - \frac{k^*_2}{E_3I_3} \beta \cot \beta h_2 \tag{7.10}$$

For negative values of γ^4, equation (7.9) is replaced by

$$V(x) = C[\sin \xi x \sinh \xi x + \cot \xi L \coth \xi L \cos \xi x \cosh \xi x] \tag{7.11}$$

where,

$$\xi^4 = -\frac{\gamma^4}{4} \tag{7.12}$$

and C is an arbitrary constant.

Substitution of equation (7.4) into equation (7.3d) yields

$$\frac{d^2W_1}{dy^2} + \left[\frac{m_4}{k_4}\omega^2 + \frac{E_1I^*_1}{k_4}\frac{\alpha^3}{2}(\tan \alpha L + \tanh \alpha L)\right]W_1 = 0 \tag{7.13}$$

This equation can be solved for boundary condition (vii) as

$$W_1(y) = D(\sin \lambda y + \cot \lambda h_1 \cos \lambda y) \tag{7.14}$$

for positive values of λ^2, where λ^2 is given by

$$\lambda^2 = \frac{m_4}{k_4}\omega^2 + \frac{E_1I^*_1}{k_4}\frac{\alpha^3}{2}(\tan \alpha L + \tanh \alpha L) \tag{7.15}$$

For negative values of λ^2, the following equation replaces equation (7.14)

$$W_1(y) = D(\sinh \mu y - \coth \mu h_1 \cosh \mu y) \tag{7.16}$$

where D is an arbitrary constant and μ is given by

$$\mu^2 = -\lambda^2 \tag{7.17}$$

Boundary conditions (x) and (xii) can now be used to obtain the condition for a nontrivial solution, i.e., a characteristic equation for the natural frequencies given by

(i) For $\gamma^4 > 0$, $\lambda^2 > 0$:

$$E_3 I_3 \frac{\gamma^3}{2} (\tan \gamma L + \tanh \gamma L) + k_4 \lambda \tan \lambda h_1 = 0 \qquad (7.18a)$$

(ii) For $\gamma^4 < 0$, $\lambda^2 > 0$:

$$4E_3 I_3 \xi^3 \frac{(\text{cosec } 2\xi L + \text{cosech } 2\xi L)}{(\tan \xi L \tanh \xi L + \cot \xi L \coth \xi L)} - k_4 \lambda \tan \lambda h_1 = 0 \qquad (7.18b)$$

(iii) For $\gamma^4 > 0$, $\lambda^2 < 0$:

$$E_3 I_3 \frac{\gamma^3}{2} (\tan \gamma L + \tanh \gamma L) - k_4 \mu \tan \mu h_1 = 0 \qquad (7.18c)$$

(iv) For $\gamma^4 < 0$, $\lambda^2 < 0$:

$$4E_3 I_3 \xi^2 \frac{(\text{cosec } 2\xi L + \text{cosech } 2\xi L)}{(\tan \xi L \tanh \xi L + \cot \xi L \coth \xi L)} + k_4 \mu \tanh \mu h_1 = 0 \qquad (7.18d)$$

An approach like Holzer's method (Thomson, 1965) is used to obtain the natural frequencies of the translational modes of the system. That is, one chooses an initial value of ω and substitutes it into equations (7.5), (7.7), (7.10) and (7.15) to obtain α, β, γ^4 and λ^2. Next, one substitutes these into one of the equations (7.18), depending upon the signs of γ^4 and λ^2, to see if the equation is satisfied. If that is the case, that value of ω is the natural frequency of the structure. However, if it does not satisfy the equation, another value of ω is chosen and the process is repeated. This search for the roots can, of course, by systematized.

The mode shapes of the structure are given by:

$$U(x,y) = \frac{W_1(y)}{2}\left[\frac{\cos \alpha x}{\cos \alpha L} + \frac{\cosh \alpha x}{\cosh \alpha L}\right] \tag{7.19}$$

$$W_2(x,y) = V(x)(\cot \beta h_2 \sin \beta y + \cos \beta y) \tag{7.20}$$

$$V(x) = C\left[\frac{\cos \gamma x}{\cos \gamma L} + \frac{\cosh \gamma x}{\cosh \gamma L}\right] \qquad \text{for } \gamma^4>0 \tag{7.21a}$$

$$V(x) = C(\sin \xi x \sinh \xi x + \cot \xi L \coth \xi L \cos \xi x \cosh \xi x)$$

$$\text{for } \gamma^4<0 \tag{7.21b}$$

$$W_1(y) = W_3(y) = 2C(\tan \lambda h_1 \sin \lambda y + \cos \lambda y) \qquad \text{for } \gamma^4>0,\ \lambda^2>0 \tag{7.22a}$$

$$W_1 = W_3(y) = C\left[\frac{\sin^2 \xi L \sinh^2 \xi L + \cos^2 \xi L \cosh^2 \xi L}{\sin \xi L \sinh \xi L}\right]\cdot$$

$$\cdot(\tan \lambda h_1 \sin \lambda y + \cos \lambda y) \qquad \text{for } \gamma^4<0,\ \lambda^2>0 \tag{7.22b}$$

$$W_1(y) = W_3(y) = -2C(\tanh \mu h_1 \sinh \mu y - \cosh \mu y) \quad \text{for } \gamma^4>0,\ \lambda^2<0 \tag{7.22c}$$

$$W_1(y) = W_3(y) = -C\left[\frac{\sin^2 \xi L \sinh^2 \xi L + \cos^2 \xi L \cosh^2 \xi L}{\sin \xi L \sinh \xi L}\right]\cdot$$

$$\cdot(\tanh \mu h_1 \sinh \mu y - \cosh \mu y) \qquad \text{for } \gamma^4<0,\ \lambda^2<0 \tag{7.22d}$$

Similar expressions can also be obtained for the antisymmetric modes of vibration. It can be shown that all the modes satisfy the following orthogonality condition:

$$m^*_1 \int_0^{h_1} \int_{-L}^{L} U_i U_k dxdy + m^*_2 \int_{-h_2}^{0} \int_{-L}^{L} W_{2i} W_{2k} dxdy + m_3 \int_{-L}^{L} V_i V_k dx +$$

$$+ m_4 \int_0^{h_1} W_{1i} W_{1k} dy + m_4 \int_0^{h_1} W_{3i} W_{3k} dy = 0 \quad \text{for } i \neq k \tag{7.23}$$

Also, the following expression can be obtained for the modal participation factors for uniform ground motion:

$$P_k = \frac{\left(m^*_1 \int_0^{h_1} \int_{-L}^{L} U_k dxdy + m^*_2 \int_{-h_2}^{0} \int_{-L}^{L} W_{2k} dxdy + m_3 \int_{-L}^{L} V_k dx + m_4 \int_0^{h_1} W_{1k} dy + m_4 \int_0^{h_1} W_{3k} dy \right)}{\left(m^*_1 \int_0^{h_1} \int_{-L}^{L} U_k^2 dxdy + m^*_2 \int_{-h_2}^{0} \int_{-L}^{L} W_{2k}^2 dxdy + m_3 \int_{-L}^{L} V_k^2 dx + m_4 \int_0^{h_1} W_{1k}^2 dy + m_4 \int_0^{h_1} W_{3k}^2 dy \right)} \tag{7.24}$$

7.3 NUMERICAL EXAMPLE

The Imperial County Services Building, also discussed in Chapter II, was a six-story reinforced concrete structure. During the Imperial Valley earthquake of October 15, 1979, it was severely damaged and was eventually taken down. Some of the structural features and the lateral load transfer scheme of this building closely resemble those of the structure analyzed in the previous section. Hence, to illustrate the method developed in that section, this building has been modelled and solved for its dynamic properties.

Figure (2.9) shows the structural plan of the building. In the upper stories the lateral loads in the transverse direction were resisted by two end walls. However, the west wall was different from the east end wall since it had "smoke tower" openings in all the stories. At the second-floor level, the lateral shear was transferred to the four shear walls in the ground story through the second-floor slab, while the overturning moment was transferred to the four columns located just inside the end walls. The ground shear walls were not symmetrically placed. In order to use the model developed in the previous section, the "smoke tower" openings in the west end wall were neglected. Also, the asymmetry in the ground story walls was neglected; they were idealized as a uniformly distributed, equivalent shear beam system.

The aspect ratio of the floors was about 1.8 and, therefore, shear deformations in the floors cannot be neglected in comparison to bending deformations. To approximate the effects of the shear deformations and the rotatory inertia, the moment of inertia of the floors was multiplied by a factor of 0.47. This factor introduces enough bending in the floors to give the same fundamental period that would occur in the floor if the effects of shear deformation and the rotatory inertia were included (Timoshenko, et al., 1974). The adjusted stiffness of the roof and the upper floors, and the lumped mass at these levels, were uniformly distributed between the roof and the second floor level to represent the floors as equivalent, distributed bending beams. The following properties of the various elements were used in the analysis:

Equivalent distributed floor system:

weight ($m^*_1 g$) = 1020 lbs/ft^2

moment of inertia (I^*_1) = 1890 (ft)^4/ft height

modulus of elasticity (E_1) = 3.60×10^6 psi

length of the floors (2L) = 136'-3"

Equivalent distributed shear wall system:

weight ($m^*_2 g$) = 137 lb/ft^2

shear stiffness (k^*_2) = 1.69×10^8 lb/ft

height (h_2) = 16'-8"

Second floor:

weight ($m_3 g$) = 14100 lbs/ft

moment of inertia (I_3) = 33000 (ft)^4

modulus of elasticity (E_3) = 3.60×10^6 psi

Upper story end walls:

weight ($m_4 g$) = 8600 lb/ft

shear stiffness (k_4) = 1.00×10^{10} lb

height (h_1) = 67'-2"

The appropriate equations given in the previous section were solved to find the translational frequencies of the building. The corresponding mode shapes were obtained from equations (7.19) through (7.22); the first four are plotted in Figure (7.2).

The natural periods for the four lowest modes were found to be 0.23 sec, 0.11 sec, 0.10 sec and 0.099 sec. The first mode period is higher than that obtained by treating the floors as rigid, while modelling the walls as shear beams, which is 0.17 sec. Thus, the floor-flexibility does have a significant effect on the dynamic properties of the building. However, the 0.23 sec period is not in close agreement with the 0.38 sec period found from the strong-motion records obtained from the building during a small earthquake on March 28, 1978 (Jain and Housner, 1983b) or the 0.45 sec period observed during the ambient vibration tests performed on the building (Pardoen, _et al_., 1981). Foundation flexibility is thought to have contributed significantly to this discrepancy (Jain, _et al_., 1983). This effect can be included by representing the foundation by translational and rotational springs. Also, in the model the end walls transfer the overturning moment to the ground story walls, which are assumed to have negligible bending flexibility, while in the actual structure the overturning moment is transferred to the columns below. This makes the actual system more flexible than the model by allowing rotation of end walls as rigid bodies. The proposed model needs to be improved for application to this structure in order to incorporate this flexibility. In addition, the end walls could be treated as Timoshenko beams to include the bending flexibility, neglected in the analysis. Hence, in order to model this building more accurately, the above factors should be included in the model. However, this was considered to be beyond the scope of this thesis.

The fundamental mode shape, plotted in Figure (7.2), shows an interesting feature. It can be seen in the figure that the second floor bends in the opposite direction from the other floors. This is due to the role the second floor plays in transferring the loads. The lateral forces of all the upper floors are transferred to the second floor at its two ends through the upper story end walls. The floor, in turn, transfers them to the uniformly-distributed shear walls in the ground story. Thus, the second floor acts like a free-free beam on an elastic foundation, with two concentrated end loads. For such a system, the beam curvature will be as observed for the second floor. A beam loaded this way can actually experience uplift near the center (e.g., Hetenyi, 1946). This explains why in the fundamental mode, a portion of the second floor near the center is displaced in the opposite direction from the rest of the structure. This behavior is rather unusual. For most buildings, the fundamental mode has the property that the whole structure is displaced in the same direction.

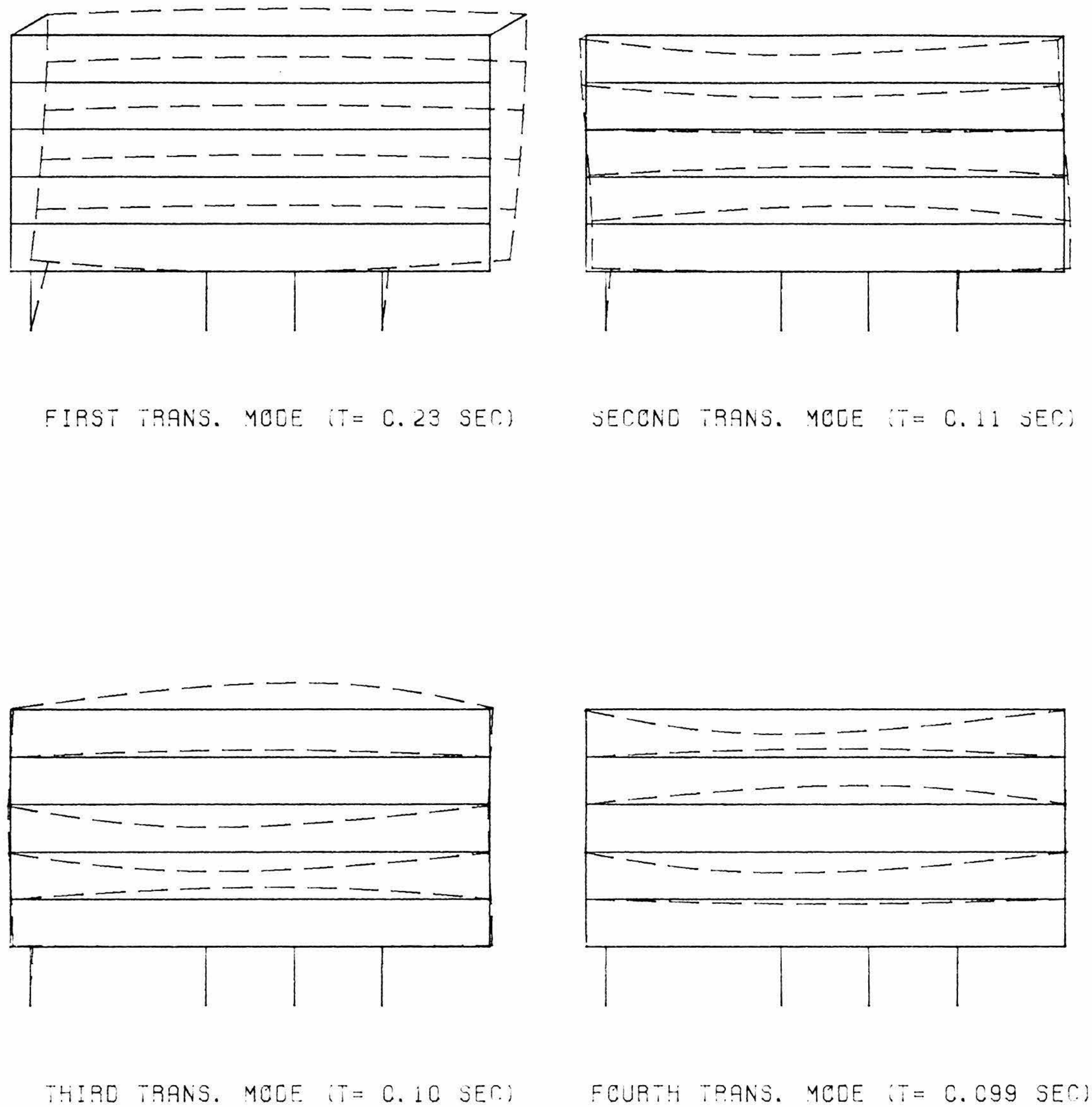


Figure 7.2. TRANSLATIONAL MODE SHAPES OF THE IMPERIAL COUNTY SERVICES BUILDING.

The opposite curvatures of the second floor and the upper floors are also confirmed from the strong-motion records obtained from the building. The relative displacement plots of the second floor and the roof, given by Pauschke, et al. (1981) and reproduced in Figure (2.10) of this thesis, clearly indicate this trend, although it is not clear in this figure as to whether or not the second floor actually has a negative displacement near the center. Gonzalez, et al. (1980) have subjected a finite element model of the building to the code-prescribed static lateral forces in the transverse direction. Their plots of the deformed shapes of the fourth floor and the second floor also show opposite curvature.

The second, third and fourth mode periods are nearly equal to the fundamental mode period (0.098 sec) of the upper floors when these are treated as pinned-pinned beams. This is similar to what has been observed in the example structure of Chapter V.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

This study investigates the significance of in-plane floor flexibility on the dynamic behavior of buildings and develops new analytical methods to analyze buildings with flexible floor diaphragms. A study of the literature on past earthquakes revealed that there have been several buildings that were damaged during strong ground shaking due to significant in-plane floor deformations. Also, strong-motion records obtained from some undamaged buildings have shown that floors can indeed be quite flexible in their plane. Some of the evidence that indicates the important role floors play in the dynamics of buildings is presented in Chapter II. It is observed there that long, narrow buildings are particularly susceptible to this phenomenon, although it can happen also in buildings with small aspect ratios, if stiff end walls are present. Buildings that consist of two or more wings joined at an angle (e.g., L- or V-shape plans) also warrant special attention to floor flexibility and the resulting stress concentration at the corners where the two wings meet. Three of the example structures discussed were school buildings, which suggests that the architectural layout of school buildings may make them more susceptible than other structures to problems caused by flexible floor diaphragms.

As preliminary information for later work, the mechanics of bending and shear beams are reviewed in Chapter III. In addition, the concept of equivalent, distributed beam systems, such as are used in the analysis of coupled shear walls, is presented. Also, a note is included on matching the boundary conditions at junctions of various elements. The first building studied is a one-story building whose lateral load resistance in the transverse direction is provided by two end walls. The structure is analyzed by treating the roof as a bending beam and the walls as shear beams. The equations of motion for these elements and the boundary conditions are combined to obtain the characteristic frequency equation, roots of which give the natural frequencies of the system. Also, expressions are obtained in general form for the mode shapes and the participation factors. Once the natural frequencies and the mode shapes are known for the structure, the complete dynamic response can be calculated. However, the characteristic equation is transcendental in nature, and must be solved numerically. For convenience, a perturbation technique is applied to obtain the fundamental natural frequency in an approximate but much simpler manner. Solutions are also discussed for some more complex single-story buildings.

As an illustration of the technique described in Chapter III, the top story of the Administrative Building in Arvin High School is modelled and its dynamic properties obtained. In this example, the perturbation method gives a very good estimate of the fundamental natural frequency. The low torsional stiffness expected for walls and frames was confirmed in this example and in all subsequent chapters the torsional stiffness of the walls or the frames is neglected.

The method developed in Chapter III is applied to two-story buildings with identical end walls in the subsequent chapter. Results include the characteristic frequency equation and expressions for the mode shapes and the participation factors. As an example application, the two-story Administrative Building at the Arvin High School has been modelled with this method and its dynamic properties are obtained for the first few modes. It is noted that the first two translational frequencies of the structure are close to the fundamental frequencies of the second floor and the roof, when treated as pinned-pinned beams. It is seen in this example that these first two modes, dominated by floor or roof vibrations, make the largest contributions to the total base shear for earthquake response of the structure. The third symmetrical mode, with less pronounced floor and roof motions, gives a base shear only about 1/3 that of the second mode.

The study of the two-story buildings also showed another interesting phenomenon. It was seen that some of the lower frequencies of multistory buildings that have nearly identical floors and stiff end walls may be very nearly equal. Besides finding the properties of such closely spaced modes, this technique can also be used to coalesce such modes into a single mode such that all the floors vibrate in phase.

Multistory buildings with two end walls or frames are treated in Chapter V by modelling the floors as an equivalent, distributed system of bending beams and the end walls or frames as bending or shear beams. As an illustration of the analysis, a long, narrow 9-story building with two slender end walls has been analyzed. As anticipated, the fundamental frequency of the structure was lower than that based on the assumption of stiff floors. It was noted that the fundamental period could be approximated closely by the use of Dunkerley's equation. The second mode frequency was found to be close to the fundamental frequency of a floor when treated as a pinned-pinned beam. Also, there were several other modes with nearly the same frequency, in which the floors essentially vibrate like the first mode of pinned-pinned beams. This is partly a consequence of the use of the equivalent, distributed system, which allows an infinite number of such modes, and the fact that at least n such modes are significant for the dynamics of an n-story building with flexible floors.

Chapter VI treats multistory buildings with uniformly spaced moment-resisting frames or walls. If the numbers of frames and stories are sufficiently large, such structures can be idealized as vertically-oriented anisotropic plates. A study of this type of model leads to the conclusion that the dynamic properties of such buildings can be obtained by separately analyzing one typical frame and one typical floor. The frequencies of the whole structure are simply the square root of the sum of the squares of the floor frequencies, when treated as free-free beams, and of the frame frequencies. Also, the mode shapes can be obtained by superposition of the floor modes and the frame modes. It is shown that such buildings possess all the modes of vibration that one obtains by an analysis based on the assumption of rigid floor diaphragm, plus extra modes that involve floor deformations similar to those of free-free beams. However, the modes that involve floor deformations have zero modal participation factors for uniform ground motion. Hence, it is concluded that the floors in such buildings can be treated as rigid in their own plane in earthquake analysis without introducing an additional approximation.

It is shown in Chapter VII that the concepts presented in the earlier chapters can be applied to study even more complex structures. As a specific case, the characteristic frequency equation and expressions for the mode shapes and the participation factors are obtained for a long, narrow building that has two end walls in the upper stories and several uniformly placed walls in the ground story. Thus, the lateral loads are resisted by the end walls in the upper stories, but are transferred to the ground story walls through the second-floor slab. The Imperial County Services Building, which has a similar structural system, is then analyzed using this model to obtain the first few frequencies and mode shapes. The fundamental mode shape displays some interesting features. First, the second floor deforms with opposite curvature from that of the upper floors. Also, a portion of the second floor near mid-span is displaced in the opposite direction from the rest of the structure. This is unusual, but is consistent with the mechanism of shear transfer from the upper walls to the ground shear walls.

All of the work has been carried out on symmetrical structures, although structures with asymmetry can also be analyzed using these methods. Even though the antisymmetric (torsional) modes of the symmetrical structures are not excited by uniform ground motion, results for these modes are presented. However, caution is needed in using these results, since the contribution of longitudinal frames or walls to the torsional stiffness of the structure is neglected in the way the problem has been formulated. Also, the polar moment of inertia of the floors is underestimated in this approach because the floors have been modelled as beams. The two effects are small in long, narrow buildings and have opposite, compensating effects on the dynamic properties of the structure. However, there may be situations where they cannot be neglected. To include these effects one can increase the end wall stiffness and the floor mass so as to obtain the same torsional stiffness and rotational inertia of the floor, that would actually occur in the building.

From this thesis, it is seen that the problem of significant in-plane floor deformations, important in the earthquake response of certain types of buildings, is amenable to analysis in many cases. It is hoped that the results of this study lead to better understanding of this phenomenon, and that the analytical methods presented will prove useful in the dynamic analyses of buildings.

As further areas of research, the present work can easily be extended to include both bending and shear deformations of the floors (or walls). The approach may also have application to buildings that consist of more than one wing joining at an angle (e.g., L-, V-, T-shape plans). Such structures need to be studied to learn the effects of floor-diaphragm deformations. Also, the method has potential for use in the study of buildings with vertical offsets.

REFERENCES

Arnold, C., 1980a, "Architectural Implications," in Reconnaissance Report, Imperial County, California Earthquake, October 15, 1979, Earthquake Engineering Research Institute, Berkeley, California, 94704, pp. 111-138.

Arnold, C., 1980b, "Building Configuration: Characteristics for Seismic Design," Proceedings of the Seventh World Conference on Earthquake Engineering, Istanbul, Turkey, Vol. 4, pp. 589-592.

Arnold, C. and E. Elsesser, 1980, "Building Configuration: Problems and Solutions," Proceedings of the Seventh World Conference on Earthquake Engineering, Istanbul, Turkey, Vol. 4, pp. 153-160.

Beck, H., 1962, "Contribution to the Analysis of Coupled Shear Walls," Journal of the American Concrete Institute, Vol. 59, pp. 1055-1069.

Berg, G.V., 1973, "Response of Buildings in Anchorage," in The Great Alaska Earthquake of 1964 (Engineering), National Academy of Sciences, Washington, D.C., pp. 247-282.

Blume, J.A., 1962, "The Earthquake Resistance of California School Buildings - Additional Analyses and Design Implications," Division of Architecture, California State Printing Division, Sacramento, California.

Blume, J.A. and D. Jhaveri, 1969, "Time-History Response of Buildings with Unusual Configurations," Proceedings of the Fourth World Conference on Earthquake Engineering, Santiago, Chile, Vol. 2, pp. 155-170.

Blume, J.A., R.L. Sharpe and E. Elsesser, 1961, "A Structural-Dynamic Investigation of Fifteen School Buildings Subjected to Simulated Earthquake Motion," Division of Architecture, California State Printing Division, Sacramento, California.

Chitty, L., 1947, "On the Cantilever Composed of a Series of Parallel Beams Interconnected by Cross Members," Philosophical Magazine (London), Vol. 38, pp. 685-699

Dewell, H.D., and B. Willis, 1925, "Earthquake Damage to Buildings," Bulletin of the Seismological Society of America, Vol. 15, No. 4, pp. 282-300.

Dunkerley, S., 1895, "On the Whirling and Vibration of Shafts," Philosophical Transactions, Royal Society, London Series A, Vol. 185, pp. 269-360.

George, W., P. Knowles, J.K. Allender, J.F. Sizemore, and D. E. Carson, 1973, "Structures in Anchorage," in The Great Alaska Earthquake of 1964 (Engineering), National Academy of Sciences, Washington, D.C., pp. 774-846.

Goldberg, J.E., 1966, "Analysis of Multistory Buildings Considering Shear Wall and Floor Deformations," Tall Buildings, Proceedings of the Symposium on Tall Buildings, held in University of Southampton, pp. 349-373.

Goldberg, J.E. and E.D. Herness, 1965, "Vibration of Multistory Buildings Considering Floor and Wall Deformations," Bulletin of the Seismological Society of America, Vol. 55, No. 1, pp. 181-200.

Gonzalez, R., L.W. Lu, and T. Huang, 1980, "Seismic Damage Analysis of the Imperial County Services Building," Fritz Engineering Laboratory Report No. 422.12, Lehigh University, Bethlehem, Pennsylvania.

Hanson, R.D. and H.J. Degenkolb, 1969, The Venezuela Earthquake, July 29, 1967, American Iron and Steel Institute, 150 East Forty-Second Street, New York, New York.

Hetenyi, M., 1946, Beams on Elastic Foundation, The University of Michigan Press, Ann Arbor.

Hoerner, J.B., 1971, "Modal Coupling and Earthquake Response of Tall Buildings," Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California.

Irwin, A.W., 1975, "Analysis of Tall Shear Wall Buildings Including In-Plane Floor Deformations," Build International, Vol. 8, pp. 43-55.

Jain, S.K., 1983, "Seismic Response of Buildings with Flexible Floors," to be published in the Journal of the Engineering Mechanics Division, American Society of Civil Engineers.

Jain, S.K., J.F. Hall and G.W. Housner, 1983, "Features of Structural Response - Computer Analysis (Imperial County Services Building)," in Engineering Features and Studies from the Imperial County Earthquake, October 15, 1979, J.F. Hall (Ed.), Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California (to be published).

Jain, S.K. and G.W. Housner, 1983a, "Description of Building and Damage (in the Imperial County Services Building)," in Engineering Features and Studies from the Imperial County Earthquake, October 15, 1979, J.F. Hall (Ed.), Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California (to be published).

Jain, S.K. and G.W. Housner, 1983b, "Features of Structural Response - Simple Analysis (Imperial County Services Building)," in Engineering Features and Studies from the Imperial County Earthquake, October 15, 1979, J.F. Hall (Ed.), Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California (to be published).

Jennings, P.C., 1983, "Visual Interpretation of Strong-Motion Records (Imperial County Services Building)," in Engineering Features and Studies from the Imperial County Earthquake, October 15, 1979, J.F. Hall (Ed.), Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California (to be published).

Jennings,P.C., 1969, "Spectrum Techniques for Tall Buildings," Proceedings of the Fourth World Conference on Earthquake Engineering, Santiago, Chile, Vol. 2, pp. A-3, 61-74.

Karadogan, H.F., 1980, "Earthquake Analysis of 3-D Structures with Flexible Roofs," Proceedings of the Seventh World Conference on Earthquake Engineering, Istanbul, Turkey, Vol. 5, pp. 162-268.

Karadogan, H.F., T. Huang, L. Lu and M. Nakashima, 1980, "Behavior of Flat Plate Floor Systems Under In-Plane Seismic Loading," Proceedings of the Seventh World Conference on Earthquake Engineering, Istanbul, Turkey, Vol. 5, pp. 9-16.

Kolston, D. and B.W. Buchanan, 1980, "Diaphragms in Seismic Resistant Buildings," Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 13, No. 2, pp. 162-170.

Maybee, J.S., J.E. Goldberg and E.D. Herness, 1966, "Dynamics of Separable Multistory Buildings," Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 92, No. EM2, pp. 153-176.

Meehan, J.F., 1967, "The Response of Several Public School Buildings in Anchorage, Alaska, to the March 27, 1964, Earthquake," in The Prince William Sound, Alaska, Earthquake of 1964 and Aftershocks, F.J. Wood (Editor-in-Chief), U.S. Department of Commerce, U.S. Government Printing Office, Washington, D.C., pp. 219-243.

Meirovitch, L., 1975, Elements of Vibration Analysis, McGraw-Hill Book Company, New York, New York.

Nakashima, M., T. Huang and L.W. Lu, 1981, "Seismic Resistance Characteristics of Reinforced Concrete Beam-Supported Floor Slabs in Building Structures," Fritz Engineering Laboratory Report No. 422.9, Lehigh University, Bethlehem, Pennsylvania.

Nielsen, N.N., 1964, "Dynamic Response of Multistory Buildings," Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California.

Nielsen, N.N., 1966, "Vibration Tests of a Nine-Story Steel Frame Building," Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 92, No. EM1, pp. 81-110.

Ostrom, D.K., 1974, "In-Plane Slab Deformation in Multistory Structures," thesis presented to the University of California, at Los Angeles, California in 1974, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering.

Pardoen, G.C., G. Hart and B.T. Bunce, 1981, "Earthquake and Ambient Response of El Centro County Services Building" in Dynamic Response of Structures: Experimentation, Observation, Prediction and Control, G. Hart (Ed.), American Society of Civil Engineers, New York, pp. 164-173.

Pauschke, J.M., C.S. Oliveira, H.C. Shah and T.C. Zsutty, 1981, "A Preliminary Investigation of the Dynamic Response of the Imperial County Services Building During the October 15, 1979 Imperial Valley Earthquake", Report No. 49, The John A. Blume Earthquake Engineering Center, Department of Civil Engineering, Stanford University, Stanford, California.

Porcella, R.L., R.B. Matthiesen, R.D. McJunkin and J.T. Ragsdale, 1979, "Compilation of Strong-Motion Records Recovered from the Coyote Lake Earthquake of 6 August 1979," Preliminary Report 25, Office of Mines and Geology, 2811 "O" Street, Sacramento, California.

Rutenberg, A., 1980, "Laterally Loaded Flexible Diaphragm Buildings," Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 106, No. ST9, pp. 1969-1973.

Shepherd, R. and R.A.H. Donald, 1967, "The Influence of In-Plane Floor Flexibility on the Normal Mode Properties of Buildings," Journal of Sound and Vibration, Vol. 5, No. 1, pp. 29-36.

Sozen, M.A., P.C. Jennings, R.B. Matthiesen, G.W. Housner and N.M. Newmark, 1968, Engineering Report on the Caracas Earthquake of 29 July 1967, National Academy of Sciences, Washington, D.C.

Steinbrugge, K.V., 1970, "Earthquake Damage and Structural Performance in the United States," in Earthquake Engineering, R.L. Wiegel (Ed.), Prentice-Hall, Inc., Englewood-Cliffs, New Jersey, pp. 167-226.

Steinbrugge, K.V., J.H. Manning and H.J. Degenkolb, 1967, "Building Damage in Anchorage," in The Prince William Sound, Alaska, Earthquake of 1964, and Aftershocks, F.J. Wood (Editor-in-Chief), U.S. Department of Commerce, U.S. Government Printing Office, Washington, D.C., pp. 7-217.

Steinbrugge, K.V. and D.F. Moran, 1954, "An Engineering Study of the Southern California Earthquake of July 21, 1952, and its Aftershocks," Bulletin of the Seismological Society of America, Volume 44, No. 2B, pp. 199-462.

Thomson, W.T., 1965, Vibration Theory and Applications, Prentice-Hall, Inc., Englewood Cliffs, N.J.

Timoshenko, S.P., and J.N. Goodier, 1970, Theory of Elasticity, McGraw-Hill Book Company, New York.

Timoshenko, S.P., and S. Woinowsky-Krieger, 1959, Theory of Plates and Shells, McGraw-Hill Book Company, New York.

Timoshenko, S.P., D.H. Young and W. Weaver, Jr., 1974, Vibration Problems in Engineering, John Wiley & Sons, New York.

Turpen, C.D., 1980, "Strong-Motion Records from the Mammoth Lakes, California, Earthquakes of May, 1980," Preliminary Report, Office of Strong-Motion Studies, California Division of Mines and Geology, 2811 'O' Street, Sacramento, California 95816.

Udwadia, F.E. and M.D. Trifunac, 1974, "Ambient Vibration Tests on Full-Scale Structures," Proceedings of the Fifth World Conference on Earthquake Engineering, Rome, Italy, Vol. 2, pp. 1430-1439.

Unemori, A.L., 1978, "Generalized Dynamic Behavior of Crosswall Building Systems," thesis presented to the Massachusetts Institute of Technology, Boston, in partial fulfillment of the requirements for the degree of Doctor of Science.

Unemori, A.L., J.M. Roesset, and J.M. Becker, 1980, "Effect of Inplane Floor Slab Flexibility on the Response of Crosswall Building Systems," in Reinforced Concrete Structures Subjected to Wind and Earthquake Forces, ACI Publication SP-63, pp. 113-134.

Wah, T., 1963, "Natural Frequencies of Univeform Grillages," Journal of Applied Mechanics, Transactions of the American Society of Mechanical Engineers, Vol. 30, pp. 571-578.

Wood, J.H., 1972, "Analysis of the Earthquake Response of a Nine-Story Steel Frame Building During the San Fernando Earthquake," Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, California.

NOTATION

Mathematical symbols have been defined where they first appear. They are summarized here in alphabetical order. Some symbols are given more than one meaning, when there is no question of confusion.

A = area of cross-section; constant of integration; arbitrary constant

A^* = area of cross-section per unit width

a_1 = spacing of floors

a_2 = spacing of frames

B = constant of integration

b = width of a wall

C = torsional stiffness; constant of integration; arbitrary constant

c = thickness of a wall

D = constant of integration

$\bar{D}$ = flexural rigidity of plate strip of unit width

E = Young's modulus

f = intensity of a continuously distributed static load

G = shear modulus

g = acceleration of gravity

h = height of building; story height

h_1 = height of roof from second floor

h_2 = first-story height

I = moment of inertia

I^*	=	moment of inertia per unit width or height
K	=	spring constant
$\bar{K}$	=	shear stiffness of plate strip of unit width
k	=	shear stiffness
k'	=	shape factor
L	=	half length of building
l	=	length of building
M	=	bending moment
m	=	mass per unit length
m^*	=	mass per unit area
P	=	modal participation factor
p	=	coefficient (dimensionless)
Q	=	shear force
q	=	coefficient (dimensionless)
S_A	=	acceleration spectrum value
T	=	twisting moment; period
T(t)	=	function of time
U	=	function of x; function of x and y
u	=	displacement
$\ddot{u}$	=	ground acceleration
V	=	function of x; function of y
v	=	displacement
W	=	function of y; function of x and y
w	=	displacement
X	=	function of x

Y	=	function of y
x',y',z'	=	cartesian coordinates
x,y,z	=	dimensionless cartesian coordinates; cartesian coordinates
α	=	coefficient (function of frequency)
β	=	coefficient (function of frequency)
γ	=	coefficient (function of frequency)
θ	=	angle of twist
λ	=	coefficient (function of frequency)
μ	=	coefficient (function of frequency)
ξ	=	coefficient (function of frequency)
ρ	=	density
ω	=	frequency

Subscripts

b	=	bending
i,j,k,r	=	integers
s	=	shear; integer