SIGNALING GAMES: THEORY AND APPLICATIONS

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For my wife, Shannon.

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ABSTRACT

This thesis concerns the interactions between asymmetrically informed agents where information can potentially be transmitted through the actions of the agents. Refinements of the sequential equilibrium concept are derived and applied to (i) a model of pretrial bargaining between litigants to a civil suit, where both parties possess private information, and (ii) a model of electoral competition where the voters attempt to deduce the private information held by the candidates.

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INTRODUCTION

The majority of economic models through the 1960s assumed that agents possessed complete information in regards to decision parameters for themselves as well as all other agents. This in turn facilitated development of analytic-based predictions of the performance of agents in a wide array of economic activity. Recently, however, analysts have been viewing the presence of complete information as a degenerate case of the seemingly more realistic environment where some if not all decision parameters of one agent are unknown to the remaining agents. This has led on the one hand to theoreticians establishing methodologies to guide analysis of such situations and on the other to applied economics considering which of the parameters lend themselves to relaxing the assumption of complete information and thereby lead to realistic, tractable, or interesting results.

The work presented below attempts to discuss and influence both of these ongoing areas of research. The state of the art on the theory side has generated predictions from equilibrium analysis in which the agents' beliefs over unknown parameters are explicitly recognized in the equilibrium concept. Many of these predictions are a function of the latitude the agents are grants in their beliefs when faced with events inconsistent with the equilibrium path; i.e., events which the equilibrium predicts would occur with probability zero. Although any objection to this may at first glance appear innocuous, the fact that the probability of any event occurring is determined endogenously by the behavior of the agents and the ability to influence others' actions by different threats of response implies the critical nature of beliefs out of equilibrium. Chapter I below deals with reasonable types of restrictions to place on out-of-equilibrium beliefs so as to limit the set of equilibria in any particular application. The general form of these restrictions concerns the informativeness of the equilibrium payoffs in establishing the willingness and ability of informed agents with certain information to deviate from the equilibrium path. Hence, in a restricted concept of equilibrium uninformed agents should deduce and incorporate this information into their beliefs, thereby requiring out-of-equilibrium beliefs to be consistent with some type of information about the unknown parameters.

The goal of the subsequent chapters is to apply these refinement techniques to two models of strategic interaction. In Chapter II a model of pretrial negotiations between litigants to a civil suit is analyzed, where both plaintiff and defendant know whether or not they were negligent in actions prior to the accident, but where they don't know whether the other was negligent. Information about the defendant's negligence is potentially transmitted through a settlement offer to the plaintiff. If the plaintiff rejects the offer, they proceed to trial where the court determines the negligence of both parties and determines the appropriate allocation of resources. This allocation is a function of the liability rule in place, where a liability rule assigns to each possible state of parameters one of the parties as responsible for covering the damages. Four liability rules are considered, and their effects on the equilibrium outcomes are contrasted.

In Chapter III a model of electoral competition is developed where the two candidates possess private information regarding the position on a unidimensional policy space they will enact if elected. The candidates simultaneously announce positions on the policy space as a function of their "true" position, after which the voters attempt to infer these true positions and subsequently vote for one or the other candidate. This model generalizes earlier models of electoral competition in two fundamental respects: (i) it allows for a candidate's announced position and true position to differ, and (ii) it assumes voters cannot deduce ex ante what the candidates' positions if elected will be. Although the utility functions of the candidates are left unspecified except for signing the derivatives, the ability to restrict the voters' out-of-equilibrium beliefs inherent in the equilibrium concepts of Chapter I facilitates the predictive power of the model to a degree sufficient to state potentially interesting and insightful results.

CHAPTER I. EQUILIBRIUM SELECTION IN SIGNALING GAMES

1. INTRODUCTION

This chapter investigates the relationship between Kreps and Wilson's (1982) concept of sequential equilibria and Kohlberg and Mertens's (1984) concept of stability. It introduces a restriction on off-the-equilibrium-path beliefs that refines the set of sequential equilibria in signaling games. We call any sequential equilibria that satisfy our restriction on beliefs divine. For generic signaling games, every equilibrium contained in a stable component is divine. Moreover, the solution concept is restrictive enough to rule out all of the equilibria that Kreps (1985) and others dismiss on intuitive grounds. Thus, divinity provides an independent theoretical foundation for discarding non-intuitive equilibria in signaling games. In addition, a subsequent refinement of divinity, called universal divinity, is introduced. It is shown that, as with divinity, every equilibrium contained in a stable component is universally divine, while an example implies that the set of universally divine equilibria may be strictly contained in the set of divine equilibria.

We provide a generic example to show that universally divine, hence divine equilibria may not be contained in any stable component. However, the chapter presents an explicit characterization of stability in terms of off-the-equilibrium-path beliefs. That is, an equilibrium of a generic signaling game is in a stable component if and only if it can be supported as a sequential equilibrium with restricted off-the-equilibrium-path beliefs. Just as Kreps and Wilson (1982) characterize perfect equilibria for generic extensive-form games in terms of sequential equilibrium strategies and beliefs, our result characterizes stable outcomes for generic signaling games in terms of sequential equilibrium strategies and restrictions on beliefs. The characterization may be a useful way to compute stable equilibrium outcomes and to evaluate the consequences of using stability to select equilibria in extensive-form games.

Independent of our work, Cho and Kreps (1986) analyze the power of stability to select equilibria in signaling games. Their results closely parallel our own. They identify restrictions on equilibria similar to those embodied by divinity. In addition, they also state our characterization result (Theorem 3). Cho (1985) extends a restriction identified in Cho and Kreps to obtain a solution concept that refines the set of sequential equilibria in general extensive-form games.

Our debt to the existing literature on solution concepts for noncooperative games is obvious. Recent work on this topic includes papers by Kreps and Wilson (1982), Selten (1975), and McLennan (1985), who present refinement concepts for extensive-form games; and Myerson (1978), Kalai and Samet (1984), and Kohlberg and Mertens (1984), who present refinement concepts for normal-form games.

2. THE MODEL

In this chapter we analyze the equilibria of signaling games with finite action sets. There are two players, a Sender (S) and a

Receiver (R). The Sender has private information, summarized by his type, t, an element of a finite set T. There is a strictly positive probability distribution p(t) on T; p(t), which is common knowledge, is the ex ante probability that S's type is t. After S learns his type he sends a message m to R; m is an element of a finite set M. In response to m, R selects an action a from a finite set A(m); k(m) is the cardinality of A(m). S and R have von Neumann-Morgenstern utility functions u(t,m,a) and v(t,m,a), respectively.

For fixed T, M, and A(m) for m $\in$ M, the utility functions u(t,m,a) and v(t,m,a) completely determine the game. Therefore, if $L = [\bar{T} \times \sum_{i=1}^{\bar{M}} k(i)]^2$, where $\bar{T}$ is the cardinality of T and $\bar{M}$ is the cardinality of M, then every element of $\mathbb{R}^L$ determines a signaling game. We call a property of a signaling game <u>generic</u> if there exists $D \subset \mathbb{R}^L$ such that the property holds for all signaling games determined by d $\in$ D and a closed set of Lebesgue measure zero contains $\mathbb{R}^L \setminus D$. If a property of a signaling game is generic, then we say it holds for generic signaling games.

For any positive integer k, let $\Delta_k = \{\delta = (\delta(1),\dots,\delta(k))$: $\delta(i) \geq 0 \quad \forall\, i$ and $\sum_{i=1}^{k} \delta(i) = 1\}$ be the (k - 1)-dimensional simplex. We refer to the $(\bar{T} - 1)$-dimensional simplex most often; to simplify notation, we write Δ instead of $\Delta_{\bar{T}}$. A <u>signaling</u> <u>rule</u> for S is a function

$q: T \to \Delta_{\bar{M}}$;

q(m|t) is the probability that S sends the message m, given that his type is t. An <u>action rule</u> for R is an element of $\prod_{m\varepsilon M} \Delta_{k(m)}$; $r(a|m)$ is the probability that R uses the pure strategy a when he receives the message m.

We extend the utility functions u and v to the strategy spaces $\Delta_{k(m)}$ by taking expected values; for all $t \varepsilon T$, let

$$u(t,m,r(\cdot)) = \sum_{a\varepsilon A(m)} u(t,m,a)\ r(a|m)$$

$$v(t,m,r(\cdot)) = \sum_{a\varepsilon A(m)} v(t,m,a)\ r(a|m).$$

Also, for each $\lambda \varepsilon \Delta$ and $m \varepsilon M$ let

$$BR(\lambda,m) \equiv \underset{r(m)\varepsilon\Delta_{k(m)}}{\arg\max} \sum_{t\varepsilon T} v(t,m,r(m))\lambda(t)$$

be the best-response correspondence for R and for $\Lambda \subset \Delta_{k(m)}$, let $BR(\Lambda,m) \equiv \bigcup_{\lambda\varepsilon\Lambda} BR(\lambda,m)$.

<u>Definition</u>. A sequential equilibrium for a signaling game consists of signaling rules $q(t)$ for S, action rules $r(m)$ for R, and beliefs $\mu(\cdot|m) \varepsilon \Delta$ for R, such that

1) $\forall\ t \varepsilon T$, $q(m^*|t) > 0$ only if

$$u(t,m^*,r(m^*)) = \max_{m\varepsilon M} u(t,m,r(m));$$

2) $\forall\ m \varepsilon M$, $r(a^*|m) > 0$ only if

$$\sum_{t\varepsilon T} v(t,m,a^*)\mu(t|m) = \max_{a\varepsilon A(m)} \sum_{t\varepsilon T} v(t,m,a)\mu(t|m);$$

3) if $\sum_{t\varepsilon T} q(m|t)p(t) > 0$, then

$$\mu(t^*|m) = \frac{q(m|t^*)p(t^*)}{\sum_{t\varepsilon T} q(m|t)p(t)} .$$

In words, (1) states that q(·) maximizes S's expected utility, given R's strategy; (2) states that r(·) maximizes R's expected utility, given beliefs μ(·); and (3) states that R's beliefs given S's strategy are rational in the sense that Bayes' Rule determines $\mu(t|m)$ whenever the probability that S sends m in equilibrium is positive. If $q(m|t) = 0$, for all $t \varepsilon T$, then sequential rationality does not determine $\mu(t|m)$. However, the refinement concept introduced in Section 3 restricts the values that these beliefs may take.

Next, we describe stable equilibria. Our introduction follows Cho and Kreps (1986). Fix a signaling game; let $\tilde{\rho} = (\tilde{\rho}_R,\tilde{\rho}_S)$ satisfy $0 < \tilde{\rho}_i < 1$, $i = R,S$, and let $\tilde{q}$ and $\tilde{r}$ be strategies for S and R respectively that satisfy $\tilde{q}(m|t) > 0$, $\forall\ m \varepsilon M$, $\forall\ t \varepsilon T$ and $\tilde{r}(a|m) > 0$, $\forall\ a \varepsilon A(m)$, $\forall\ m \varepsilon M$. A $(\tilde{\rho},\tilde{q},\tilde{r})$-<u>perturbation</u> of the original game is the signaling game in which, if the players choose strategies q and r from the original game, then the outcome is the outcome of the original game if the strategy chosen by S is $(1 - \tilde{\rho}_S)q + \tilde{\rho}_S\tilde{q}$ and the strategy chosen by R is $(1 - \tilde{\rho}_R)r + \tilde{\rho}_R\tilde{r}$. We refer to $(\tilde{\rho},\tilde{q},\tilde{r})$ as trembles. Let (q,r) be Nash equilibrium strategies for a perturbed game. If $q(m|t) > 0$, we say that a type t Sender voluntarily sends m and we say that R voluntarily uses the mixed strategy r(m).

For a given signaling game, we call a subset C of the set of Nash equilibria stable if, for every $\varepsilon > 0$ there exists $\delta > 0$ such that every $(\tilde{\rho},\tilde{q},\tilde{r})$-perturbation of the original game with $0 < \tilde{\rho}_i < \delta$, i = R,S has an equilibrium no more than ε from the set C.

Definition. A stable component is a minimal (by set inclusion) stable set of equilibria.

Our analysis depends on several properties.[1]

Proposition 1. For generic extensive-form games, the set of equilibrium probability distributions on endpoints[2] is finite and all equilibria within a given connected component induce the same probability distributions on endpoints.

Proposition 2. Every game has at least one stable component.

Proposition 3. A stable set of equilibria remains so when one deletes a strategy that is not a best reply against any equilibrium in the set.

Therefore, in generic signaling games, there exists a stable set of equilibria with the property that every equilibrium in the set agrees along the equilibrium path; the equilibrium may vary off the equilibrium path. A variety of off-the-equilibrium-path responses may be needed to guarantee that any perturbation of the game has an equilibrium path close to a particular equilibrium path. Therefore, a single equilibrium need not be a stable set. However, we use Proposition 1 to justify an abuse of terminology. We call an equilibrium stable if it agrees with an element of a stable component along the equilibrium path. In particular, in generic signaling games, if an equilibrium is stable, then every perturbation has an equilibrium with payoffs close to the original equilibrium payoffs.

3. DIVINE EQUILIBRIA

Previous refinements of the Nash equilibrium concept place rationality restrictions on zero-probability events. In particular, sequential rationality requires that players respond optimally to some consistent assessment of how the game has been played. These equilibrium concepts do not require a player to draw any conclusion when a zero-probability event takes place. That is, although the refinement concepts embodied in sequential rationality and perfectness require that equilibria of games induce equilibria on any continuation of the game, these concepts do not require that a player systematically draw an inference from an opponent's unexpected move. Nevertheless, in order to decide how to respond to an unexpected signal, R should evaluate the willingness of S-types to deviate from equilibrium, and then incorporate into his beliefs the information that deviations from equilibrium might reveal.

This section presents an equilibrium concept that refines the set of sequential equilibria in signaling games by placing restrictions on off-the-equilibrium-path beliefs. We begin by describing two restrictions on beliefs along with the intuition behind them, and then proceed to define an equilibrium concept that incorporates these restrictions.

The first intuitive restriction on beliefs that we discuss requires R's off-the-equilibrium-path beliefs to place positive probability only on those Sender types who might not lose from a defection. Formally, this condition requires that if a type t sender receives utility $u^*(t)$ in equilibrium and

$J = \{t: u^*(t) > u(t,m,r(m))$ for all $r(m) \in BR(\Delta,m)\}$,

then $r^*(m) \in BR(\Delta_{T\setminus J},m)$.[3] Cho and Kreps (1986) also identify this condition and show that if an equilibrium is stable, then the condition must hold.[4]. Our refinement notion includes this type of restriction on beliefs.

Figure 1[5] describes a special case of a sequential settlement game (see Salant (1984) or Sobel (1985)). There are two types of S (the "defendant"): type t_2 defendants are negligent; type t_1 defendants are not negligent. S offers a low settlement, m_1, or a high settlement, m_2. R (the "plaintiff") either accepts (a_1) or rejects (a_2) the offer. If R accepts S's offer, S pays R an amount that depends only on the offer. If R rejects the offer, S must pay court costs and a transfer depending only on his type (e.g. the court finds out with certainty whether or not S was negligent). If $p(t_1) = p(t_2) = \frac{1}{2}$, then the game depicted in Figure 1 has two types of equilibria.

m_1	a_1	a_2
t_1	-3,3	-6,0
t_2	-3,3	-11,5

m_2	a_1	a_2
t_1	-5,5	-6,0
t_2	-5,5	-11,5

Figure 1

In one type of equilibrium, both types of S offer m_1, and R accepts any offer; $q(m_1|t_i) = 1$, $i = 1,2$, $r(a_1|m_j) = 1$, $j = 1,2$. In the other type of equilibrium, both types of S offer m_2 and R accepts m_2 and rejects m_1; $q(m_1|t_i) = 0$, $i = 1,2$, $r(a_1|m_1) = 0$, $r(a_1|m_2) = 1$. In order to support this behavior, we need $\mu(t_1|m_1) \leq \frac{2}{5}$. We claim that the second equilibrium is not plausible because, in order to support it, R must believe that t_2 is more likely than t_1 to offer m_1. However, t_1 prefers to defect whenever t_2 does (and not conversely: consider an equal mixture of a_1 and a_2 given m_1). Thus, a reasonable restriction on beliefs would require that the relative probability of t_1 should increase if R observes m_1. Our refinement notion captures this argument as well.

Fix an equilibrium in which a Sender of type t obtains utility $u^*(t)$, and, for all $t \in T$, the probability that t sends m is zero. We intend to restrict the beliefs that R can have given the message m. Since we deal with only one unsent message at a time, for notational convenience we drop the argument m from R's response function.

Recall that $\Delta_{k(m)}$ consists of all actions, r, available to R given m. Let

$$A_G = \{r \in \Delta_{k(m)}: u(t,m,r) \geq u^*(t), \text{ for some } t \in T\}$$

be the set of actions that some S-type weakly prefers to equilibrium actions, conditional on sending m. Our initial restriction is that R should believe that any type who sends m instead of the equilibrium signal does not expect to lose by doing so.[6] Thus, if R receives the signal m (as a defection from equilibrium), he should believe that S expects him to take an action in A_G.

For all $r \in \Delta_{k(m)}$, let

$$\bar{\mu}(t,r) = \begin{cases} 1 & \text{if } u(t,m,r) > u^*(t) \\ [0,1] & \text{if } u(t,m,r) = u^*(t) \\ 0 & \text{if } u(t,m,r) < u^*(t) \end{cases}$$

be the frequency that $t \in T$ would send m if he believed that m would induce the action r and t had a choice between sending m or obtaining $u^*(t)$. Next, let

$$\Gamma(r) = \{\gamma \in \Delta: \exists\ \mu(t) \in \bar{\mu}(t,r) \text{ and } c > 0 \quad \text{such that}$$
$$\gamma(t) = c\mu(t)p(t), \quad \forall\ t \in T\}.$$

Notice that $\Gamma(r)$ is nonempty if and only if $r \in A_G$. If it is common knowledge that m induces r, then the posterior probability distribution over T must be an element of $\Gamma(r)$. Thus, $\Gamma(r)$ is the set of beliefs consistent with R taking the action r in response to m (and t earning $u^*(t)$ otherwise).

Finally, let

$$\bar{\Gamma}(A) = \text{convex hull}\left[\bigcup_{r \in A} \Gamma(r)\right].$$

Thus, if A is closed, then $\bar{\Gamma}(A)$ is a closed, convex subset of the simplex Δ, and is empty if and only if $A_G \cap A$ is empty. Since $\bar{\Gamma}(\Delta_{k(m)})$ is empty only if $u^*(t) > u(t,m,r)$, $\forall\, t \in T$, $\forall\, r \in \Delta_{k(m)}$, R truly would be surprised by a defection from equilibrium, and there seems to be no reason to select one inference over another in response to m. Indeed, in this case, <u>any</u> conjecture supports the equilibrium. When $A_G \neq \emptyset$, and hence $\bar{\Gamma}(\Delta_{k(m)}) \neq \emptyset$, we think that it is not plausible for R to hold beliefs outside of $\bar{\Gamma}(\Delta_{k(m)})$ given the signal m. If R observes a defection from the equilibrium path, then he must form a conjecture over T <u>based on that defection</u>.

Notice that any equilibrium in which beliefs lie in $\bar{\Gamma}(\Delta_{k(m)})$ satisfies the intuitive restrictions that we described earlier. All conjectures in $\bar{\Gamma}(\Delta_{k(m)})$ assign zero probability to any $t \in T$ with $u(t,m,r) < u^*(t)$, $\forall\, r \in \Delta_{k(m)}$. Furthermore, if there exists $t,t' \in T$ such that $\bar{\mu}(t,r) = 1$ implies $\bar{\mu}(t',r) = 1$, $\forall\, r \in \Delta_{k(m)}$, then for all beliefs in $\bar{\Gamma}(\Delta_{k(m)})$, the ratio of the probability of t' given m to the probability of t given m is at least as great as $\frac{p(t')}{p(t)}$. That is, R believes that t' is at least as likely to defect as t.

Beliefs must lie in $\bar{\Gamma}(\Delta_{k(m)})$ provided two conditions hold. First, R believes that no type t would use m if t expected R to take an action that resulted in utility less than $u^*(t)$. This means that S expects R to take actions in A_G given the signal m. Second, S-types have a common conjecture over the distribution of actions that R would take as a response to a defection. This second condition may seem odd, since there is only one Sender. However, a "type" is a specification of the information S has concerning decision parameters that are not common knowledge. Thus, it is possible for two S-types to have different conjectures over R's actions in equilibrium. If it is common knowledge that R holds beliefs in $\bar{\Gamma}(\Delta_{k(m)})$, then S should expect m to induce an action in $BR(\bar{\Gamma}(\Delta_{k(m)}),m)$. This observation suggests the following iterative procedure. Let

$$\Gamma_0 = \Delta, \quad A_0 = \Delta_{k(m)}, \quad \text{and for } n > 0,$$

$$\Gamma_n \equiv \begin{cases} \bar{\Gamma}(A_{n-1}) & \text{if } \bar{\Gamma}(A_{n-1}) \neq \emptyset \\ \Gamma_{n-1} & \text{if } \bar{\Gamma}(A_{n-1}) = \emptyset \end{cases}$$

$$A_n \equiv BR(\Gamma_n,m), \quad \Gamma^* = \bigcap_n \Gamma_n, \quad A^* = \bigcap_n A_n .$$

Others use iterative procedures in the definition of equilibrium concepts. Specifically, given the assumptions that S expects R to take actions in A_G given an unexpected signal m and that S-types have a common conjecture over the actions that R would take in response to m, our iterative procedure coincides with that used by Bernheim (1984) and Pearce (1984) to define the set of rationalizable equilibria.

Theorem 1. In generic signaling games, if an equilibrium in which $q(m|t) = 0 \ \forall\, t \in T$ is stable, then there exists $r^* \in A^*$ such that $u(t,m,r^*) \leq u^*(t), \quad \forall\, t \in T$.

Theorem 1 is a direct consequence of Proposition 3. It states that if an equilibrium is stable, then there exist beliefs in Γ^* that support it. We discuss the proof later in this section.

Definition. A sequential equilibrium in a signaling game is divine if it is supported by beliefs in Γ^*.

Thus, by Theorem 1, every stable component contains a divine equilibrium. Therefore, Proposition 2 implies our next result.[7]

Theorem 2. Every signaling game has a divine equilibrium.

We believe that divinity captures a minimal restriction on off-the-equilibrium path beliefs. Stability implies much more, but we are not convinced that these restrictions are plausible.

The set of beliefs in Γ^* depend on the prior distribution of Sender types. To check this property, one need only note that in the game that Figure 1 describes,

$$\Gamma^* = \{\lambda \in \Delta: \lambda(t_1) \geq p(t_1)\}$$

for the equilibrium in which both t_1 and t_2 send m_2 with probability one. Let Γ^{**} be the intersection of the Γ^* taken over all nondegenerate priors on Sender types. We can show that in generic signaling games, if an equilibrium is stable, then it can be supported by beliefs in Γ^{**}. Call an equilibrium supported by beliefs in Γ^{**} universally divine. To see that universal divinity is more restrictive than divinity alone, note that in Figure 1, the sequential equilibrium in which S sends m_2 with probability one is divine provided that $p(t_1) \leq \frac{2}{5}$, but it is never universally divine since, regardless of the prior probability that S is t_1, R must believe that the unexpected signal m_1 comes from t_1.[8]

Cho and Kreps use Proposition 3 to further refine the equilibrium set. For a fixed equilibrium outcome and unsent signal m, call a type t <u>bad</u> for m if, for every equilibrium giving rise to this outcome, a t-Sender strictly prefers the equilibrium outcome to sending m.[9] Proposition 3 implies that a stable equilibrium can be supported by beliefs that give no weight to any type that is bad for m (if all types are bad for m, then the equilibrum payoffs strictly dominate any payoff S can obtain from a best response to m). To see that this condition is more restrictive than universal divinity, note that for generic signaling games, if t is not bad for m, then e(t), the element of Δ with t-th component equal to one, is an element of Γ^{**}.[10] Thus, Proposition 3 also implies that in generic signaling games, if an equilibrium is stable, then there exist beliefs in Γ^{**} that support it. Since $\Gamma^{**} \subset \Gamma^{*}$, Theorem 1 follows from Proposition 3.

4. A CHARACTERIZATION OF STABLE EQUILIBRIA

This section gives necessary and sufficient conditions for a sequential equilibrium in a generic signaling game to be stable. First, we present an example of a signaling game that has an unstable, divine equilibrium. The example motivates the notion of stable beliefs that we need to prove our equivalence theorem.

Consider the signaling game in Figure 3.

m_1	a
t_1	0,0
t_2	0,0

m_2	a_1	a_2	a_3	a_4
t_1	-1,3	-1,2	1,0	-1,-2
t_2	-1,-2	1,0	1,2	-2,3

Figure 3

Let $p(t_1) = \frac{1}{2}$. There exists a sequential equilibrium to this game in which $q(m_1|t_i) = 1$, $i = 1,2$, $r(a_1|m_2) = 1$ supported by beliefs $\mu(t_1|m_2) \geq \frac{2}{3}$. This equilibrium is universally divine since

$$\Gamma^* = \Gamma^{**} = \Delta \text{ and}$$

$a_1 \in BR(\Gamma^*, m_2)$; also, neither t_1 nor t_2 is bad for m_2 so that the Proposition 3 does not restrict beliefs. However, this equilibrium is not stable.

The stable equilibrium for this example involves both t_1 and t_2 sending m_2 with probability one and R responding to m_2 with actions a_2 and a_3 with probability $\frac{1}{2}$ each.

Now we argue that the equilibrium in which S does not use m_2 is not stable. Notice that if S voluntarily sends m_2 an equilibrium to the perturbed game in which S types expect to receive 0, then R must either use an equal mixture of a_1 and a_2 or an equal mixture of a_3 and a_4 in response to m_2. Hence, R must believe that the probability of t_1 given m_2 is equal to either $\frac{2}{3}$ or $\frac{1}{3}$. Any other strategy for R leads to positive payoffs for at least one S type or negative payoffs to both. Moreover, when R mixes equally between a_1 and a_2, t_1 does not voluntarily send m_2 and when R mixes equally between a_3 and a_4, t_2 does not voluntarily send m_2. This argument establishes that if $\mu(t_1|m_2)$, the probability of t_1 given m_2 if S does not voluntarily send m_2, is an element of $(\frac{1}{3},\frac{2}{3})$, then there is an equilibrium to the perturbed game close to the original equilibrium only if the tremble induces R to take an action given m_2 that does not attract either type of S. Moreover, if $\mu(t_1|m_2) \notin (\frac{1}{3},\frac{2}{3})$, then the perturbed game has an equilibrium that is close to the original game and in which either t_1 or t_2 voluntarily sends m_2. Therefore, the equilibrium in the example is stable if and only if, given m_2, every best response to the set of beliefs in which the probability of t_1 given m_2 is an element of $(\frac{1}{3},\frac{2}{3})$ leads to nonpositive expected payoffs to both S types. Since $a_3 \in BR((\frac{1}{3},\frac{2}{3}),m_2)$ yields positive payoffs to both S types, the equilibrium is not stable. We apply an analogous argument in general signaling games. First, we identify the set of trembles that cannot induce voluntary action in any equilibrium to the perturbed game that is close to the original equilibrium. Second, we prove that an equilibrium is stable precisely when no best response to this set of trembles induces S to voluntarily send m.

As in the previous section, fix an equilibrium that leads to utility levels $u^*(t)$, $\forall\ t \in T$, and in which $q(m|t) = 0$, $\forall\ t \in T$. For each $J \subset T$, define

$$I(J) \equiv \{r \in \Delta_{k(m)}: u^*(t) \geq u(t,m,r) \ \forall\, t \in T, \text{ and}$$
$$u^*(t) = u(t,m,r) \text{ if and only if } t \in J\},$$

and, for $r \in I(J)$, define

$$\hat{\Lambda}(J,r) \equiv \{\lambda \in \text{int}\,\Delta: \nexists\, \lambda^* \in \Delta \text{ with } r \in BR(\lambda^*,m)$$

$$\text{such that } \lambda^* = \sum_{t \in J} \alpha(t)e(t) + \beta\lambda, \text{ for}$$

$$\alpha(t) \geq 0, \ \ 1 - \sum_{t \in J} \alpha(t) = \beta > 0\},$$

where $e(t) \in \Delta$ is the vector with t-th component equal to one and all other components equal to zero. Finally, let

$$\Lambda(J) \equiv \begin{cases} \bigcap_{r \in I(J)} \hat{\Lambda}(J,r) & \text{if } I(J) \neq \emptyset \\ \Delta & \text{if } I(J) = \emptyset \end{cases}$$

$$\text{and } \Lambda^* = \bigcap_{J \subset T} \Lambda(J).$$

Consider a perturbed game in which trembles induce a belief λ given m unless some type voluntarily uses m. For sufficiently small trembles, there exists an equilibrium to the perturbed game, with payoffs close to $u^*(t)$, in which R takes action r given m if and only if $\lambda \notin \hat{\Lambda}(J,r)$ for some J; the action r is not a best response to any beliefs obtained by "adding" combinations of $t \in J$ to λ if and only if $\lambda \in \hat{\Lambda}(J,r)$. As only S-types in J voluntarily use m in an equilibrium in which they could obtain $u^*(t)$ by not sending m, $\hat{\Lambda}(J,r)$ contains exactly the beliefs that may cause instability if R takes action r given m. Thus, $\bigcap_{J \neq \emptyset \subset T} \Lambda(J)$ is the set of trembles that cannot induce voluntary action in any equilibrium. However, $\Lambda(\emptyset)$ are those beliefs which give rise to actions attractive to some S types. This argument leads to our characterization theorem.

Theorem 3. In generic signaling games, an equilibrium is stable if and only if, for all unused signals m, $\Lambda^* = \emptyset$.

5. EXTENSIONS

While we confine our discussion in this chapter to signaling games, Propositions 1-3 hold for generic extensive-form games. Since these results combine to imply Theorems 1 and 2, we can use our techniques to rule out implausible sequential equilibria in more general extensive-form games. We suspect that divinity is easier to verify than stability and may be simpler to generalize to games with infinite strategy spaces. On the other hand, Theorem 3 and possible generalizations appear to be valuable only as a characterization of stable equilibria.

We conclude by noting that our techniques do not refine the set of sequential equilibria in signaling games in which signals are costless. Specifically, let A(m), u(t,m,a), and v(t,m,a) be independent of m. These games are not generic, so we cannot apply our results directly. However, it is easy to verify that $\Gamma^* = \Delta$ for any unused signal. This is because if t induces the action $a \in \bar{A}$ with signal m', then there exist beliefs for which a is a best response to the (unused) signal m. When signaling is costless, t is indifferent between sending m and m′ and no other agent strictly prefers m to his equilibrium payoff. In addition, straightforward arguments show that stability does not restrict the set of equilibria, although this kind of game always has an equilibrium in which all types of S send the same signal and typically has other, more appealing, equilibria. Farrell[11] (1984) and Myerson (1983) present ideas that apply to costless signaling games. Myerson presents an axiomatic solution that limits the outcomes in a mechanism-design problem that usually has a large number of sequential equilibria, but it is not clear that his ideas extend in a sensible way to a noncooperative framework. Farrell argues that an equilibrium outcome is not plausible if there exists an unused signal m, a nonempty set J, and an action $r \in BR(\lambda,m)$ such that

$$J = \{t: u^*(t) < u(t,m,r)\}, \text{ where}$$

$$\lambda(t) = \begin{cases} p(t)/\sum_{t' \in J} p(t') & \text{if } t \in J \\ 0 & \text{if } t \notin J \end{cases}$$

is the conditional probability of t given $t \in J$. That is, Farrell argues that R should interpret a defection that benefits exactly the set J as evidence that exactly those t in J use m. Farrell calls an equilibrium in which this type of defection does not exist neologism proof. Neologism-proof equilibria do not exist in general, and, in games with costly signaling, need not be divine.

NOTES

1. Kreps and Wilson (1982) prove Proposition 1. Kohlberg and Mertens (1984) prove Propositions 1-3.

2. An equilibrium induces a probability distribution on the endpoints of the tree. An equilibrium probability distribution on endpoints is a probability distribution on endpoints induced by some equilibrium.

3. If J = T, then no action R can take in response to the signal m induces S to send m. In this case, any beliefs are permissible.

4. Kreps (1985) suggests a less restrictive version of this condition. Kreps discards an equilibrium in which there exists a sender type who would like to defect for every action in $BR(\Delta_{T\setminus J},m)$.

5. We represent examples with a bi-matrix B(m) for each $m \in M$. There is one column in B(m) for each strategy in A(m) and one row for each type. The entry in the t-th row and the a-th column is (u(t,m,a), v(t,m,a)). In each of these examples, the qualitative properties that we discuss in the text remain valid if we perturb the entries in B(m).

6. It does not change our results to require that R believes that any type who sends m instead of the equilibrium signal expects to benefit strictly by doing so. Thus, we can use a strong inequality in the definition of A_G.

7. Strictly speaking, Theorem 1 and Proposition 3 imply the existence of divine equilibria in generic signaling games. A limiting argument,

based on the upper hemi-continuity of divine equilibrium paths, establishes Theorem 2. Cho (1985) gives the details of a related argument.

8. Harris and Raviv (1983) study a game in which there is a divine equilibrium that is not universally divine, hence not stable. Their comparative-statics analysis concentrates on the stable path.

9. McLennan (1985) defines a refinement concept that is similar in spirit to this requirement. Specifically, call an action useless if it has a suboptimal payoff in every sequential equilibrium of a game (not just those equilibria in a stable component). McLennan shows that there exist sequential equilibria with beliefs restricted so that, at each information set, they assign positive probability only to nodes reached by the fewest useless actions. From this, McLennan recursively defines higher-order uselessness and arrives at a set of justifiable equilibria. In generic signaling games, only strongly dominated actions are useless, thus any divine equilibrium is justifiable.

10. This condition is strictly more restrictive than universal divinity. In the game described in Figure 2, there is a sequential equilibrium in which both S types send m_1 with probability one and R takes a_3 given m_2. It is straightforward to check that $\Gamma^{**} = \Delta$. However, the message m_2 is bad for t_2. When R believes only t_1 would send m_2, R's best response given m_2 is a_1. Therefore, the equilibrium is not stable.

m_1	a
t_1	0,0
t_2	0,0

m_2	a_1	a_2	a_3	a_4
t_1	-1,3	1,2	-1,0	1,-2
t_2	1,-2	1,0	-2,2	-1,3

Figure 2

11. Grossman and Perry's (1984) concept of perfect sequential equilibria is similar to Farrell's concept. However, Grossman and Perry analyze a particular game with costly signaling.

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CHAPTER II. LIABILITY RULES AND PRETRIAL SETTLEMENT

1. INTRODUCTION

The literature concerning the interaction of parties prior to and following the occurrence of an accident in which legal recourse to resolve financial liability exists currently lacks a fair degree of cohesion. Authors such as Brown (1973), Green (1976), Diamond (1974a,b), and Shavell (1983) have studied the effect on caretaking of various liability, or cost distribution, rules under the hypothesis that the goal of liability law is to create incentives for the efficient use of resources in the prevention of accidents [Posner (1972)].[1] This work has typically ignored the bargaining opportunities available to the injurer and victim in a civil suit prior to a court decision, assuming instead that the liability rule is enforced without alternative. Conversely, the work of Bebchuk (1984), Samuelson (1983), P'ng (1983, 1984), and Salant (1984) has focused on the proper modeling of the bargaining problem inherent in the legal process subsequent to an accident in the study of the strategic aspects of legal settlements, while avoiding the comparative analysis undertaken by Brown, Green, etc. This is quite understandable given the embryonic nature of bargaining theory and the analysis of strategic interaction of parties holding private and valuable information. However, explicitly incorporating the ability of injurer and victim to come to terms prior to trial identifies an area of generalization in regards to research into caretaking prior to an accident. Papers by Reinganum and Wilde (1985) and Sobel (1985) have focused on the effect of alternative court cost allocation schemes and discovery rules in analyzing pretrial bargaining models with asymmetric information. Similar work in terms of liability rules seems justified.

This paper is an initial step in such a direction. A model is developed which promotes the comparison of liability rules in regards to their influence on settlement decisions of injurers and victims in a civil suit. Though the model itself is somewhat simplistic, it seems to capture the leverage one or another party is granted in terms of pretrial bargaining by the liability rules as well as the differential behavior of negligent or nonnegligent parties.

The paper is organized as follows: the following section presents a review of the aforementioned models of pretrial bargaining. Section 3 describes the model, the equilibrium concept to be employed, and characterizations of the four liability rules to be analyzed: negligence, strict liability with contributory negligence, negligence with contributory negligence, and strict liability with dual contributory negligence.[2] Section 4 describes the divine equilibria under the four liability rules and compares the conditions and the outcomes of these equilibria. Section 5 characterizes similar results for universally divine equilibria, and Section 6 concludes with some areas of further research.

2. REVIEW: PRETRIAL BARGAINING IN A CIVIL SUIT

Early attempts to model decisions by plaintiffs and defendants over settlement vs. litigation typically revolved around single-person decision theory [Gould (1973), Posner (1973), Shavell (1982)]. As time progressed, analysts began viewing these decisions as part of a bargaining process between two possibly asymmetrically informed rational actors, and substituted game theory as the vehicle of analysis. Salant and Rest (1982), Salant (1984), and Reinganum and Wilde (1985) present models where the plaintiff has private information as to the level of damages incurred in the accident. The plaintiff makes some settlement demand, which the defendant either accepts or rejects, with rejection implying the litigants proceed to trial. In Salant and Rest (1982), however, the settlement demand is exogenously fixed; hence the ability of the plaintiff to signal his information is restricted, as is any analysis of equilibrium settlement demands. This restriction is lifted in Salant (1984), where the settlement demand is a function of the plaintiff's information. This information is assumed to take on two values (low and high), whereas Reinganum and Wilde (1985) generalize the model to allow for a continuum of possible damage levels, implying a continuum of plaintiff "types." They further examine the effects of different cost allocation systems on the equilibrium behavior of the litigants.

Bebchuk (1984) develops a model where it is the defendant who possesses the private information, its nature being the probability of the plaintiff prevailing if they were to end up in court. The plaintiff makes a settlement demand, after which the defendant either accepts or rejects the demand. Bebchuk solves for the unique sequential equilibrium; however, since the uninformed litigant moves first and only once, there is no possibility of information transmission.

In P'ng's (1983) model, it is again the defendant who has the private information: either the defendant was negligent or not negligent in regard to the accident. The defendant makes the first-and-final settlement offer, which the plaintiff can either accept or reject. However, as in Salant and Rest (1982), the settlement amount is fixed, implying the same shortcomings as in their analysis. Another shortcoming is that P'ng (1983) employs the Nash (as opposed to sequential) equilibrium concept, which allows nuisance suits to arise in equilibrium. Both of these problems are alleviated in P'ng (1984). This model is equivalent to the model below when the negligence liability rule is in force, although P'ng (1984) arbitrarily restricts attention to a subset of the sequential equilibria.

Cooter, Marks, and Mnookin (1982) and Samuelson (1983) develop models where both plaintiff and defendant possess private information, but where the litigants make their settlement offers/demands simultaneously. If the plaintiff's demand is less than or equal to the defendant's offer, then they settle; if not, they go to trial. Unfortunately, the assumption of simultaneous moves disallows the ability to transmit and learn of private information.

Sobel (1985) describes a model where again both litigants have private information and where the focus of analysis is on the effect of disclosure rule on pretrial behavior. The sequence of moves is as follows: the defendant makes a settlement offer, which the plaintiff either accepts or rejects. If the latter, the defendant either discloses his information or not, according to the disclosure rule in place. The plaintiff subsequently makes a counteroffer, which the defendant either accepts or rejects. Sobel (1985) uses the universally divine equilibrium concept described in Chapter I above to obtain a unique equilibrium outcome under either disclosure rule.

Finally, Spulber (1985) abandons the formal game-theoretic approach and instead analyzes a direct revelation game, the goal being "to avoid a priori restrictions on the information structure or on the strategy space of the negotiation game." The Revelation Principle allows Spulber to characterize the set of interim incentive efficient solutions to the game where both litigants have private information. It is unclear whether usage of the Revelation Principle for the underlying multi-stage process of offers and counteroffers is appropriate, and it is inconclusive whether all of the interim incentive efficient solutions can be generated as equilibria to any of the games described previously.

3. THE MODEL

Analysis of the settlement and liability issues is based on the following sequence of actions and events: an accident occurs involving two parties, one of which incurs monetary damages $m' > 0$, which is assumed to be common knowledge. This party, called the plaintiff, costlessly initiates a legal suit against the other party, now called the defendant, to recover the damages. At issue in the case is the negligence or nonnegligence of both parties in terms of actions directly related to the occurrence of the accident. It is assumed that the negligence standard in use is common knowledge, but each party's negligence or nonnegligence is known only to that party. Given the state of his negligence the defendant makes a monetary offer $m \in \mathbb{R}_+$ to the plaintiff to drop the suit. If the plaintiff accepts the offer, the amount m is transferred from the defendant to the plaintiff and the case is terminated. If the plaintiff rejects the offer, the parties proceed to court, where it is assumed that the court determines without error the negligence or nonnegligence of each party, and resolves the financial dispute. The monetary payoffs for the parties from the court decision are functions both of the negligence of each party as well as the liability rule in force, where it is assumed that both parties possess a priori knowledge of the liability rule.

We model this interaction as a game of incomplete information where the plaintiff, p, can be one of two types, p_1 (not negligent), or p_2 (negligent). Let $P = \{p_1, p_2\}$. Similarly, the defendant, d, can be either d_1 (not negligent) or d_2 (negligent), where $D = \{d_1, d_2\}$. It is assumed that p_1 occurs with probability γ and d_1 occurs with probability λ, where the random variables p_i and d_i are uncorrelated.

The set of pure strategies for d is the nonnegative real line $\mathbb{R}_+$; a <u>strategy</u> for d is a function

$$q : D \rightarrow \Delta_{\mathbb{R}_+},$$

where $\Delta_{\mathbb{R}_+}$ is the set of probability distributions on $\mathbb{R}_+$. Thus $q(m|d_i)$ is the probability that d offers m, given that his type is d_i. A pure strategy for p assigns an element of the set $A = \{a_1, a_2\}$ for each possible offer, where

a_1 = accept d's offer, and

a_2 = reject d's offer.

A <u>strategy</u> for p is a function

$$r : \mathbb{R}_+ \times P \rightarrow \Delta_A,$$

where Δ_A is the 1-dimensional simplex describing probability distributions over (in this case) A. Thus $r(a_i|m,p_j)$ is the probability that p takes action a_i, given that d has offered m, and p's type is p_j. In general, we can describe the utility functions for d and p as $u(d_i,p_j,m,a_k)$ and $v(d_i,p_j,m,a_k)$, respectively. We extend these functions to the strategy space Δ_A by taking expected values; let

$$u(d_i,p_j,m,r(\cdot,p_j)) = \sum_{a_k \in A} u(d_i,p_j,m,a_k)r(a_k|m,p_j)$$

$$v(d_i,p_j,m,r(\cdot,p_j)) = \sum_{a_k \in A} v(d_i,p_j,m,a_k)r(a_k|m,p_j).$$

Since d has no opportunity to gain information about p's type, we can suppress the p_i term in d's utility function by redefining the function as:

$$u(d_i,m,r(\cdot,\cdot)) = \gamma u(d_i,p_1,m,r(\cdot,p_1)) + (1 - \gamma)u(d_i,p_2,m,r(\cdot,p_2)).$$

Also, for each $\rho \in \Delta_D$ (i.e., probability distributions over D), $m \in \mathbb{R}_+$, and $p_j \in P$, let

$$BR(\rho,m,p_j) = \underset{r(\cdot,p_j)\in\Delta_A}{\text{argmax}} \sum_{d_i \in D} v(d_i,p_j,m,r(\cdot,p_j))\rho(d_i)$$

be the best response correspondence for p, given his type.

The utility payoffs for d and p are as follows: if p accepts an offer of m from d, then the payoffs for d and p are $(-m,m - m')$, respectively, regardless of p or d's type. If p rejects d's offer, both parties incur court costs ($c_p,c_d > 0$, resp.) and the payoffs are determined by p and d's types and the liability rule, but not by d's offer. Each liability rule we analyze can be described by a 2 X 2 matrix, constituting the four underlying states with entries of either 0 or 1, where 0 implies that p is liable for the damages and 1 implies that d is liable. The payoffs for d and p, respectively, are:

$$0 : (-c_d,-c_p - m')$$

$$1 : (-c_d - m',-c_p).$$

Thus, if p is held liable, he receives no compensation from d, while still incurring the court costs, as does d. [We assume that the American system of allocating court costs is in force, where each party pays his own costs irrespective of the court's decision.] Similarly, if d is held liable, he transfers m' to p, as well as paying his court costs (thus, we assume no punitive damages). The four liability rules we analyze are:

1. Negligence[3]

$$\begin{array}{c} \\ d_1 \\ d_2 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

Under the negligence rule, the court's decision is contingent only on d's type: i.e., whether or not p was negligent is not at issue.

2. Strict liability with contributory negligence

$$\begin{array}{c} \\ d_1 \\ d_2 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \end{array}$$

Under this rule, d's type is not at issue; d is assumed a priori (strictly) liable, but can use as a defense p's (contributory) negligence.

3. Negligence with contributory negligence

$$\begin{array}{c} \\ d_1 \\ d_2 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \end{array}$$

If d is negligent and p is not, then d is liable for damages;

otherwise p is liable.

4. Strict liability with dual contributory negligence

$$\begin{array}{c|cc} & p_1 & p_2 \\ d_1 & 1 & 0 \\ d_2 & 1 & 1 \end{array}$$

If p is negligent and d is not, then p is liable: otherwise d is liable.

These rules constitute four of the six "noncomparative" liability rules studied by Brown (1973), noncomparative implying that the negligence of either party is not a function of the other party's actions. The two remaining rules, no liability and strict liability, can be analyzed as degenerate cases of the strict liability with contributory negligence rule, with prior probabilities $\gamma = 0$ and $\gamma = 1$, respectively.

The generalization of the sequential equilibrium concept from Chapter I is readily apparent.

Definition: A sequential equilibrium to any of the above games consists of strategies $\{q^*(\cdot), r^*(\cdot,\cdot)\}$ for d and p, and beliefs $\mu(\cdot|m) \in \Delta_D$ for p such that

1. $\forall d_i \in D$, $q^*(m'|d_i) > 0$ only if

$$u(d_i, m', r^*(m', \cdot)) = \max_{m \in \mathbb{R}_+} u(d_i, m, r^*(m, \cdot))$$

2. $\forall m \in \mathbb{R}_+$, $\forall p_j \in P$, $r^*(a_k, m, p_j) > 0$ only if

$$\sum_{d_i \varepsilon D} v(d_i,p_j,m,a_k)\mu^*(d_i|m) = \max_{a_k \varepsilon A} \sum_{d_i \varepsilon D} v(d_i,p_j,m,a)\mu(d_i|m)$$

3. If $\sum_{d_i \varepsilon D} q(m|d_i)pr(d_i) > 0$, then

$$\mu^*(d_i|m) = \frac{q^*(m|d_i)pr(d_i)}{\sum_{d_i \varepsilon D} q^*(m|d_i)pr(d_i)},$$

where $pr(d_1) = \lambda$, $pr(d_2) = 1 - \lambda$.

Note that, although divinity and universal divinity were originally defined for signaling games; i.e., where p can be only one type, generalization to this model is saved from some difficulties by the fact that, under the negligence rule, payoffs are not a function of p's type (so we can without loss of generality assume only one type of plaintiff) whereas, in the other three liability rules at least one type of plaintiff has a dominant strategy, implying that such a type's best response correspondence is (subject to indifference) not a function of his beliefs. Thus, in using divinity to refine the set of sequential equilibria we will typically need to inspect the beliefs of only one type of plaintiff.

4. DIVINE EQUILIBRIA

4.1 Negligence

Without loss of generality let $r(\cdot|\cdot,p_1) = r(\cdot|\cdot,p_2) = r(\cdot|\cdot)$.[5] For any offer m, the payoffs for d and p can be characterized by the following bi-matrix:

m	a_1	a_2
d_1	$-m, m - m'$	$-c_d, -c_p - m'$
d_2	$-m, m - m'$	$-c_d - m', -c_p$

Define $\alpha(m) = \frac{m' - m - c_p}{m'}$; $\alpha(m)$ is the probability of d_1 such that p is indifferent between accepting and rejecting the offer m. Define $m_\lambda = (1 - \lambda)m' - c_p$; given beliefs λ, p is indifferent between accepting and rejecting m_λ. Note that $m_\lambda \geq 0 \iff \lambda \leq \frac{m' - c_p}{m}$. If $m_\lambda > 0$, Fig. 1 describes p's decision problem. Suppose that $m_\lambda \leq c_d$; then both d_1 and d_2 would prefer to offer $m \in [m_\lambda, c_d]$ and have it accepted, than to make any other offer and have it rejected. By Fig. 1 if both d_1 and d_2 make an offer $m \geq m_\lambda$, p can (in equilibrium) accept. Thus, there exist sequential pooling equilibria $m^* \in [\max\{0, m_\lambda\}, c_d]$ of the form:[6]

$$q^*(m^*|d_1) = q^*(m^*|d_2) = 1, \tag{1.1}$$

$$r^*(a_1|\hat{m}) = 1, \quad \forall \hat{m} \geq m^*, \tag{1.2}$$

$$r^*(a_1|\hat{\hat{m}}) = 0, \quad \forall \hat{\hat{m}} < m^*. \tag{1.3}$$

To check whether any of these pooling equilibria are divine, we use the following: given equilibrium payoffs $u^*(d_1)$ at m^* define

$$\theta_1(\bar{m}|m^*) = r(a_1|\bar{m}) \text{ s.t.}$$
$$u^*(d_1) = r(a_1|\bar{m})(-\bar{m}) + (1 - r(a_1|\bar{m}))(-c_d);$$

similarly, define

$$\theta_2(\bar{m}|m^*) = r(a_1|\bar{m}) \text{ s.t.}$$
$$u^*(d_2) = r(a_1|\bar{m})(-\bar{m}) + (1 - r(a_1|\bar{m}))(-c_d - m').$$

Since the payoffs of d_1 and d_2 are increasing in $r(\cdot|\cdot)$, d_1 would prefer to deviate if $r(\cdot|\cdot) > \theta_1$, and d_2 would prefer to deviate if $r(\cdot|\cdot) > \theta_2$. Recalling the conditions for divinity, $\theta_1 < \theta_2 \Rightarrow \mu(d_1|m) \geq \lambda$, and vice versa. From Fig. 1 we see that, for equilibrium offers $m^* > m_\lambda$ and unsent offer $\bar{m} \in (m_\lambda, m^*)$, p's beliefs must be such that $\mu(d_1|\bar{m}) < \lambda$, in order to reject the offer $\bar{m}$. Calculating $\theta_1(\bar{m}|m^*)$ and $\theta_2(\bar{m}|m^*)$ we get

$$\theta_1 = \frac{c_d - m^*}{c_d - \bar{m}} \tag{2}$$

$$\theta_2 = \frac{m' - m^* + c_d}{m' - \bar{m} + c_d}\,. \tag{3}$$

Thus, $\theta_1, \theta_2 \leq 1 \Rightarrow \bar{m} \leq m^*$, and $\frac{\partial\theta_i}{\partial\bar{m}} > 0$, $i = 1,2$. Cancelling terms we find that, for $\bar{m} \leq m^*$, $\theta_1 \leq \theta_2$, as in Fig. 2.

Thus, divinity implies $\mu(d_1|\bar{m}) \geq \lambda$; but $\mu(d_1|\bar{m}) \geq \lambda$ and $\bar{m} > m_\lambda$ imply p should accept $\bar{m}$ with probability one. Thus, the only divine pooling equilibrium offer is at

$$m^* = \max\{0, m_\lambda\}.$$

However, an offer of m_λ leaves p indifferent between acceptance and rejection, allowing p to mix between these two actions. Thus, a complete characterization of the equilibria is:

$$q^*(m_\lambda | d_i) = 1, \quad i = 1,2 \tag{4.1}$$

$$r^*(a_1 | m_\lambda) = \sigma \geq \frac{c_p + c_d}{c_p + \lambda m' + c_d} \tag{4.2}$$

$$r^*(a_1 | m < m_\lambda) \leq \frac{\sigma(c_d - m_\lambda)}{c_d - m} \tag{4.3}$$

$$r^*(a_1 | m' - c_p > m > m_\lambda) \leq \frac{\sigma(m' - m_\lambda + c_d)}{m' - m + c_d} \tag{4.4}$$

$$r^*(a_1 | m \geq m' - c_p) = 1. \tag{4.5}$$

Since p will accept <u>any</u> offer from d_1 if p knew it was from d_1, there does not exist a sequential separating equilibrium under the negligence rule. There does, however, exist sequential semi-pooling equilibria under certain conditions. It is easily shown that it is not possible to make both d_1 and d_2 indifferent between making two offers; hence the semi-pooling equilibria will consist of d_2 mixing between two offers, d_1 sending one of the offers (the "common" offer) with probability one, and p mixing between acceptance and rejection at the common offer. From Fig. 1 we see that p will accept with probability one an offer of $m \geq m' - c_p$ even if he knows its from d_2; furthermore it must be that $m_\lambda \geq 0$ for p to be indifferent between acceptance and rejection. Thus, if $m_\lambda \geq 0$, there exist sequential semi-pooling equilibria with common offer $m^* \in [0, \min\{m_\lambda, c_d\}]$. At m^*, d_2 is indifferent between m^* and $m' - c_p$ if $r(a_1 | m^*)$ solves

$$c_p - m' = r(a_1|m^*)(-m^*) + (1 - r(a_1|m^*))(-c_d - m').$$

Calculating through, we get

$$r^*(a_1|m^*) = \frac{c_p + c_d}{m' - m^* + c_d} . \tag{5}$$

$r^*(a_1|m^*)$ is always positive, and

$$r^*(a_1|m^*) \leq 1 \iff m^* \leq m' - c_p.$$

Since $q^*(m^*|d_1) = 1$, to get $\alpha(m^*) = \dfrac{m' - m^* - c_p}{m'}$, $q^*(m^*|d_2)$ must solve

$$\frac{m' - m^* - c_p}{m'} = \frac{\lambda}{\lambda + (1 - \lambda)q^*(m^*|d_2)} ,$$

which implies

$$q^*(m^*|d_2) = \frac{\lambda(m^* + c_p)}{(1 - \lambda)(m' - m^* - c_p)} . \tag{6}$$

Thus, the full description of the sequential semi-pooling equilibria is: for $m^* \varepsilon [0,\min\{m_\lambda, c_d\}]$,

$$q^*(m^*|d_1) = 1 \tag{7.1}$$

$$q^*(m^*|d_2) = \frac{\lambda(m^* + c_p)}{(1 - \lambda)(m' - m^* - c_p)} \tag{7.2}$$

$$q^*(m' - c_p|d_2) = \frac{(1 - \lambda)m' - m^* - c_p}{(1 - \lambda)(m' - m^* - c_p)} \tag{7.3}$$

$$r^*(a_1|m^*) = \frac{c_p + c_d}{m' - m + c_d} \tag{7.4}$$

$$r^*(a_1|\tilde{m} < m' - c_p, \tilde{m} \neq m^*) = 0 \tag{7.5}$$

$$r^*(a_1|\hat{m} \geq m' - c_p) = 1. \tag{7.6}$$

To check divinity, we can redefine $\theta_i(\bar{m}|m^*)$ in terms of the common offer m^*. Thus, after solving for d_1's equilibrium utility, $\theta_1(\bar{m}|m^*)$ solves

$$\frac{1}{m' - m^* + c_d}\{c_p c_d - m^* c_p - m' c_d\} = \theta_1(\bar{m}|m^*)(-\bar{m}) + (1 - \theta_1)(\bar{m}|m^*))(-c_d),$$

calculating, we get

$$\theta_1(\bar{m}|m^*) = \frac{(c_d + c_p)(c_d - m^*)}{(c_d - \bar{m})(m' - m^* + c_d)}. \tag{8}$$

Since d_2's utility is the same in <u>all</u> the semi-pooling equilibria, $c_p - m'$, $\theta_2(\bar{m}|m^*)$ is simply the equilibrium mix at $\bar{m}$:

$$\theta_2(\bar{m}|m^*) = \frac{c_p + c_d}{m' - \bar{m} + c_d}. \tag{9}$$

Note that, at $m^* = \bar{m}$, $\theta_1 = \theta_2$, and

$$\frac{\partial\theta_1}{\partial\bar{m}} = \frac{(c_d + c_p)(c_d - m^*)}{(c_d - \bar{m})^2(m' - m^* + c_d)} > 0, \tag{10}$$

$$\frac{\partial\theta_2}{\partial\bar{m}} = \frac{(c_p + c_d)}{(m' - \bar{m} + c_d)^2} > 0. \tag{11}$$

Solving for the ordering of θ_1 and θ_2, we get:

$$\theta_1 \leq \theta_2 \iff \bar{m} \lesssim m^*.$$

Fig. 3 describes the situation. Thus, divinity requires that $\mu(d_1) \geq \lambda$ for $\bar{m} \lesssim m^*$, and $\mu(d_1) \lesssim \lambda$ for $\bar{m} \gtrsim m^*$. However, for $m^* \lesssim m_\lambda$, $\lambda \lesssim \alpha(m^*)$, so that divinity allows p to reject offers below the common offer. Thus, all the sequential semi-pooling equilibria are divine.

To summarize the results under the negligence rule:

(i) if $m_\lambda \lesssim c_d$, there exists a divine pooling equilibrium offer at $m^* = \max\{0, m_\lambda\}$, which p accepts with positive probability.

(ii) if $m_\lambda \gtrsim 0$, there exist divine semi-pooling equilibria with common offer $m^* \in [0, \min\{m_\lambda, c_d\}]$, and where the probability of trial is

$$\left[\lambda + \frac{(1-\lambda)\lambda(m^* + c_p)}{(1-\lambda)(m' - m^* - c_p)}\right] \cdot \left[\frac{m' - m^* - c_p}{m' - m^* + c_d}\right] = \frac{\lambda m'}{m' - m^* + c_d} .$$

3.2 Strict Liability with Contributory Negligence

Without loss of generality let $q(\cdot|d_1) = q(\cdot|d_2) = q(\cdot)$. Given an offer m, the payoffs to d and p are:

m	a_1	a_2
p_1	$-m, m - m'$	$-c_d - m', -c_p$
p_2	$-m, m - m'$	$-c_d, -c_p - m'$

We see that both p_1 and p_2 have (weakly) dominant strategies: p_1 should reject all offers less than $m' - c_p$, and accept all offers greater than or equal to $m' - c_p$ and p_2 should accept any offer. Since the sequential equilibrium concept limits players to undominated strategies off the equilibrium path, p_1 and p_2 cannot threaten to take any other action (e.g., it is not a sequential equilibrium if $r(a_1|m,p_2) < 1$, for any $m \in \mathbb{R}_+$). Thus in a sequential equilibrium,

$$r^*(a_1|m,p_2) = 1, \quad \forall m, \tag{12.1}$$

$$r^*(a_1|m,p_1) = \begin{cases} 0 & \text{if } m < m' - c_p \\ 1 & \text{if } m \geq m' - c_p . \end{cases} \tag{12.2}$$

For d, given $\gamma \in (0,1)$, any offer $m \in (0,m' - c_p)$ is dominated by offering $m = 0$, given p's equilibrium strategy; similarly $m \in (m' - c_p,\infty)$ is dominated by offering $m = m' - c_p$. Thus, in a sequential equilibrium,

$$q^*(m) > 0 \Rightarrow m \in \{0,m' - c_p\}.$$

Now,

$$u(d,m = 0,r^*(\cdot)) = -\gamma c_d - \gamma m';$$

$$u(d,m = m' - c_p,r^*(\cdot)) = c_p - m'.$$

Let $m_\gamma = (1 - \gamma)m' - c_p$. Thus, we get:

$$\text{(i)} \quad \text{if } \gamma c_d < m_\gamma, \text{ then } q^*(m = 0) = 1 \tag{12.3}$$

$$\text{(ii)} \quad \text{if } \gamma c_d > m_\gamma, \text{ then } q^*(m = m' - c_p) = 1 \tag{12.4}$$

In words, if $\gamma c_d < m_\gamma$, then the unique sequential (hence divine) equilibrium is for d to offer $m = 0$, for p_1 to reject and go to court, and for p_2 to accept and drop the case. If $\gamma c_d > m_\gamma$, then the unique divine equilibrium involves d offering $m = m' - c_p$, and both p_1 and p_2 accepting. Note that if $\gamma = 0$, (i) always holds; if p is always liable, then d should give p nothing (as in the case of "no liability"). If $\gamma = 1$, (ii) always holds, and d should offer $m' - c_p$ (as in the case of "strict liability").

4.3 Negligence with Contributory Negligence

For an offer m from d, the payoffs to d and p are:

$p = p_1$

m	a_1	a_2
d_1	$-m, m - m'$	$-c_d, -c_p - m'$
d_2	$-m, m - m'$	$-c_p - m', -c_p$

$p = p_2$

m	a_1	a_2
d_1	$-m, m - m'$	$-c_d, -c_p - m'$
d_2	$-m, m - m'$	$-c_p, -c_p - m'$

Note that the decision problem of p_1 is similar to that of p under the negligence rule, while the decision problem of p_2 is similar to that of p_2 under the strict liability with contributory negligence rule. Thus, Fig. 1 characterized p_1's problem, while p_2 has a dominant strategy to accept any offer.

As under the negligence rule, there exists a continuum of pooling sequential equilibria under certain conditions. Here the condition is that $m_\lambda \le \gamma c_d$, for d_1 can guarantee himself (in expected value terms) γc_d by sending m = 0 and having p_1 reject and p_2 accept. Formally, the equilibria are:[7]

$$m^* \varepsilon [m_\lambda, \gamma c_d], \tag{13.1}$$

$$q^*(m^*|d_1) = q^*(m^*|d_2) = 1, \tag{13.2}$$

$$r^*(a_1|\hat{m},p_1) = 1, \quad \forall\hat{m} \geq m^* \tag{13.3}$$

$$r^*(a_1|\hat{\hat{m}},p_1) = 0 \quad \forall\hat{\hat{m}} < m^*, \tag{13.4}$$

$$r^*(a_1|m,p_2) = 1, \quad \forall m. \tag{13.5}$$

To check for divinity, we calculate $\theta_1(\bar{m}|m^*)$ and $\theta_2(\bar{m}|m^*)$ as under the negligence rule where, since p_2 has a dominant strategy to accept any offer, $\theta_i(\bar{m}|m^*)$ is the probability that p_1 accepts $\bar{m}$ such that d_i is indifferent between the equilibrium payoffs at m* and deviating to $\bar{m}$. Thus $\theta_1(\bar{m}|m^*)$ solves

$$-m^* = \gamma(\theta_1(\bar{m}|m^*)(-\bar{m}) + (1 - \theta_1(\bar{m}|m^*))(-c_d)) + (1 - \gamma)(-\bar{m})$$

which gives

$$\theta_1(\bar{m}|m^*) = \frac{(1 - \gamma)\bar{m} - m^* + \gamma c_d}{\gamma(c_d - \bar{m})} . \tag{14}$$

Similarly, $\theta_2(\bar{m}|m^*)$ solves

$$-m^* = \gamma(\theta_2(\bar{m}|m^*)(-\bar{m}) + (1 - \theta_2(\bar{m}|m^*)(-c_d - m')) + (1 - \gamma)(-\bar{m})$$

which gives

$$\theta_2(\bar{m}|m^*) = \frac{(1 - \gamma)\bar{m} - m^* + \gamma(c_d + m')}{\gamma(m' - \bar{m} + c_d)} . \tag{15}$$

Ordering $\theta_1(\cdot)$ and $\theta_2(\cdot)$, we get, as under the negligence rule,

$$\theta_1(\cdot) \leq \theta_2(\cdot) \iff \bar{m} \leq m^*,$$

as in Fig. 2. Thus, the only divine pooling equilibrium offer which both p_1 and p_2 accept with positive probability is at $m^* = m_\lambda$. There are, however, conditions under which another divine pooling equilibrium exists. Suppose that both d_1 and d_2 offer $m = 0$. If $m_\lambda > 0$, then p_1 will reject the offer (see Fig. 1), and p_2 will accept, giving d_1 a utility of $-\gamma c_d$ and d_2 a utility of $-\gamma(c_d + m')$. If $r(a_1|m,p_1) = 0$, $\forall m < m' - c_p$, then the only deviation viable to d_2 is $m = m' - c_p$, which both p_1 and p_2 will accept. Thus, the condition for $m^* = 0$ to be a sequential pooling equilibrium is that

$$-\gamma(c_d + m') \geq c_p - m', \text{ or}$$

$$m_\gamma \geq \gamma c_d.$$

Checking divinity, we get that

$$\theta_1(\bar{m}|m^*) = \frac{(1 - \gamma)\bar{m}}{\gamma(c_d - \bar{m})} , \text{ and} \tag{16}$$

$$\theta_2(\bar{m}|m^*) = \frac{(1-\gamma)\bar{m}}{\gamma(m' - m^* + c_d)} . \qquad (17)$$

Hence, $\theta_1(0|0) = \theta_2(0|0) = 0$, $\frac{\partial\theta_i}{\partial\bar{m}} > 0$, $i = 1,2$, and $\theta_1(\cdot) > \theta_2(\cdot)$ implying Fig. 4. Divinity implies that $\mu(d_1|\bar{m}) \leq \lambda$, $\forall\bar{m} \in (0, \gamma(m' + c_d))$, so that p_1 can reject all offers less than $m' - c_p$ in a divine pooling equilibrium at $m^* = 0$.

As in the negligence case there exists sequential semi-pooling equilibria in which d_2 is indifferent between offers $m^* \leq m_\lambda$ and $m' - c_p$ and mixes between them, and d_1 sends m^* with probability one. Recall that p_2 has a dominant strategy: $r^*(a_1|m,p_2) = 1$, $\forall m$, so that only p_1-type plaintiffs mix between acceptance and rejection. For p_1 to be indifferent he must believe that d_1 occurs with probability $\alpha(m^*) = \frac{m' - m^* - c_p}{m'}$. For d_2 to be indifferent between m^* and $m' - c_p$ it must be that $r(a_1|m^*,p_1)$ solves

$$c_p - m' = \gamma(r(a_1|m^*,p_1)(-m^*) + (1 - r(a_1|m^*,p_1))(-c_d - m')) + (1-\gamma)(-m^*),$$

so that

$$r^*(a_1|m^*,p_1) = \frac{(1-\gamma)(m^* - m') + \gamma c_d + c_p}{\gamma(m' - m^* + c_d)} . \qquad (18)$$

Now $r^*(a_1|m^*,p_1) \leq 1$ implies $m^* \leq m' - c_p$, while $r^*(a_1|m^*,p_1) \geq 0$ implies

$$m^* \geq \frac{(1-\gamma)m' - \gamma c_d - c_p}{(1-\gamma)} \equiv \tilde{m}. \qquad (19)$$

Since (as in the negligence case) d_2 can only make p_1 indifferent for offers less than m_λ, a condition for the existence of sequential semi-pooling equilibria is that

$$\tilde{m} \lesssim m_\lambda.$$

Also, it must be that $\tilde{m} \lesssim \gamma c_d$; otherwise d_1 would be better off offering $m = 0$ and having p_1 reject and p_2 accept.

Since p_1 is indifferent at $m*$ in the semi-pooling equilibrium and $q*(m*|d_1) = 1$, $q*(m*|d_2)$ must solve

$$\alpha(m*) = \frac{m' - m* - c_p}{m'} = \frac{\lambda}{\lambda + (1 - \lambda)q*(\cdot)} \text{, or}$$

$$q*(m*|d_2) = \frac{\lambda(m* + c_p)}{(1 - \lambda)(m' - m* - c_p)} , \qquad (20)$$

which is the same as under the negligence rule.

To check for divinity, we calculate $\theta_1(\bar{m}|m*)$ and $\theta_2(\bar{m}|m*)$. As under the negligence rule,

$$\theta_2(\bar{m}|m*) = r*(a_1|\bar{m},p_1) = \frac{(1 - \gamma)(\bar{m} - m') + \gamma c_d + c_p}{\gamma(m' - \bar{m} + c_d)} , \qquad (21)$$

while $\theta_1(\bar{m}|m*)$ solves

$$\frac{c_p c_d - m' c_d - m* c_p}{m' - m* + c_d} = \gamma[\theta_1(\bar{m}|m*)(-\bar{m}) + (1 - \theta_1(\bar{m}|m*))(-c_d)] + (1 - \gamma)(-\bar{m}),$$

which gives

$$\theta_1(\bar{m}|m*) = \frac{(1 - \gamma)[\bar{m}(c_d - m*) - m'(c_d - \bar{m})] + (c_d - m*)(\gamma c_d + c_p)}{\gamma(c_d - \bar{m})(m' - m* + c_d)} .$$

(22)

As a check, we see that, at $m^* = \bar{m}$,

$$\theta_1(\cdot) = \theta_2(\cdot); \text{ also} \tag{23}$$

$$\frac{\partial \theta_1}{\partial \bar{m}} = \frac{(c_d - m^*)(c_d + c_p)}{\gamma(c_d - \bar{m})^2(m' - m^* + c_d)} > 0, \tag{24}$$

$$\frac{\partial \theta_2}{\partial \bar{m}} = \frac{(c_d + c_p)}{\gamma(m' - \bar{m} + c_d)^2} > 0.$$

Note that these partial derivatives are the same as in the semi-pooling equilibria under the negligence rule multiplied by $1/\gamma$. Thus, all the sequential semi-pooling equilibria under the rule of negligence with contributory negligence are divine. To summarize:

(i) if $m_\lambda \leq \gamma c_d$, there exists a divine pooling equilibria at $m^* = \max\{0, m_\lambda\}$, where both p_1 and p_2 accept with positive probability;

(ii) if $m_\lambda > 0$ and $\gamma c_d \leq m_\gamma$, there exists a divine pooling equilibria at $m^* = 0$ where p_1 rejects and p_2 accepts;

(iii) if $m_\lambda > 0$, $\tilde{m} \leq m_\lambda$, $\tilde{m} \leq \gamma c_d$, there exists semi-pooling divine equilibria with common offer $m^* \varepsilon [\tilde{m}, \min\{m_\lambda, \gamma c_d\}]$ and where the probability of trial is

$$\left[\lambda + \frac{(1 - \lambda)\lambda(m^* + c_p)}{(1 - \lambda)(m' - m^* \_ c_p)}\right] \cdot \left[\frac{\gamma(m' - m^* - c_p)}{\gamma(m' - m^* + c_d}\right] = \frac{\lambda m'}{m' - m^* + c_d} .$$

3.4 Strict Liability with Dual Contributory Negligence

Given an offer m, the payoffs to d and p are:

$p = p_1$

m	a_1	a_2
d_1	$-m, m - m'$	$-c_d - m', -c_p$
d_2	$-m, m - m'$	$-c_d - m', -c_p$

$p = p_2$

m	a_1	a_2
d_1	$-m, m - m'$	$-c_d, -c_p - m'$
d_2	$-m, m - m'$	$-c_d - m', -c_p$

Thus the undominated strategies for p_1 can be characterized as

$$r(a_1|m,p_1) = \begin{cases} 0 & \text{if } m < m' - c_p \\ 1 & \text{if } m \geq m' - c_p \end{cases}, \tag{25}$$

while p_2 faces a decision problem similar to that of p in the negligence case (see Fig. 1). Again there exist sequential pooling equilibria $m^* \geq m_\lambda$ with the following constraints:

(i) $m^* \leq c_d + \gamma m'$; since p_1 will reject any offer less than $m' - c_p$, and in a pooling equilibrium p_2 will typically reject all offers lower than the equilibrium offer; and

(ii) $m^* \leq \tilde{m}$; since both d_1 and d_2 obtain $\gamma(-c_d - m') + (1 - \gamma)(-m^*)$ in a pooling equilibrium at m^*, it must be that d_1 and d_2 prefer this payoff to that which they would receive by offering $m = m' - c_p$ and having it accepted with probability one. Thus,

$\gamma(-c_d - m') + (1 - \gamma)(-m^*) \geq c_p - m'$, which implies

$$m^* \leq \frac{(1 - \gamma)m' - \gamma c_c - c_p}{(1 - \gamma)} \equiv \tilde{m}. \tag{26}$$

In terms of divinity,

$$\theta_1(\bar{m}|m^*) = \frac{c_d - m^*}{c_d - \bar{m}} \text{ , while} \tag{27}$$

$$\theta_2(\bar{m}|m^*) = \frac{m' - m^* + c_d}{m' - \bar{m} + c_d} \text{ , so that,} \tag{28}$$

for $m^* < \bar{m}$, $\theta_1 < \theta_2$ and the only divine pooling equilibrium offer of this type is at $m^* = m_\lambda$.

Suppose now that $\tilde{m} < m_\lambda$, so that an equilibrium with the above conditions fails to exist. Hence both d_1 and d_2 prefer to offer $m = m' - c_p$ and have it accepted by p_1 and p_2 than to offer $m = m_\lambda$ and have it accepted only by p_2. Furthermore, if d_1 (and hence d_2) prefer to offer $m = m' - c_p$ than $m = 0$ and having the offer rejected by both p_1 and p_2, it must be that

$$\gamma(-c_d - m') + (1 - \gamma)(-c_d) < c_p - m',$$

which implies

$$m_\gamma < c_d.$$

Under these conditions there exists a divine pooling equilibrium at $m^* = m' - c_p$ which p_1 accepts and where p_2 adopts the strategy

$$r^*(a_1|m,p_2) = \begin{cases} 0 & \text{if } m < m_\lambda \\ 1 & \text{if } m \geq m_\lambda \end{cases} . \qquad (29)$$

To check for divinity, we see that $\theta_1(\bar{m}|m' - c_p)$ solves

$$c_p - m' = \gamma(-c_d - m') + (1 - \gamma)[\theta_1(\bar{m}|m' - c_p)(-\bar{m}) + (1 - \theta_1(\bar{m}|m' - c_p))(-c_d)]$$

which implies

$$\theta_1(\bar{m}|m' - c_p) = \frac{c_d + c_p - (1 - \gamma)m'}{(1 - \gamma)(c_d - \bar{m})} . \qquad (30)$$

$\theta_2(\bar{m}|m' - c_p)$ solves

$$c_p - m' = \gamma(-c_d - m') + (1 - \gamma)[\theta_2(\bar{m}|m' - c_p)(-\bar{m}) + (1 - \theta_2(\bar{m}|m' - c_p))(-c_d - m')]$$

which gives

$$\theta_2(\bar{m}|m' - c_p) = \frac{c_p + c_d}{(1 - \gamma)(m' - \bar{m} + c_d)} . \qquad (31)$$

If $\theta_1(\cdot)$, $\theta_2(\cdot) > 1$, $\forall \bar{m}$, then divinity places no restrictions on beliefs. From the above equations we see that

$$\theta_1(\bar{m}|m' - c_p) \leq 1 \iff \bar{m} \leq \tilde{m},$$
$$\theta_2(\bar{m}|m' - c_p) \leq 1 \iff \bar{m} \leq \tilde{m},$$

so there does not exist an offer $\hat{m}$ such that d_i would prefer to send $\hat{m}$ under some mixed strategy by p_2 while d_j would never prefer to deviate to $\hat{m}$. Since divinity allows p_2 to use the prior probability over D when the issue is only the <u>ordering</u> of θ_1 and θ_2, and since the prior supports p_2's equilibrium strategy (see Fig. 1), the equilibrium is divine.

The sequential semi-pooling equilibria in this case will involve p_1 rejecting the common offer and accepting $m = m' - c_p$, p_2 indifferent between accepting and rejecting the common offer, d_1 sending the common offer with probability one, and d_2 indifferent between the common offer and $m = m' - c_p$. Thus, d_2 is indifferent at m^* if $r(a_1|m^*,p_2)$ solves

$$c_p - m' = \gamma(-c_d - m') + (1 - \gamma)[r(a_1|m^*,p_2)(-m^*) + (1 - r(a_1|m^*,p_2))(-c_d - m')]$$

or,

$$r^*(a_1|m^*,p_2) = \frac{c_p + c_d}{(1 - \gamma)(m' - m^* + c_d)} . \tag{32.1}$$

Now $r^*(\cdot) > 0$, $\forall m^*$, while $r^*(\cdot) \leq 1 \Leftrightarrow m^* \leq \tilde{m}$, so that $\tilde{m} < 0$ implies there does not exist any sequential semi-pooling equilibria. [Note: $\tilde{m} \leq 0 \Leftrightarrow \gamma c_d \geq m_\gamma$.]

As above, p_2 is indifferent at m^* if

$\mu(d_1|m^*) = \alpha(m^*) \equiv \dfrac{m' - m^* - c_p}{m'}$, so that

$$q^*(m^*|d_2) = \frac{\lambda(m^* + c_p)}{(1 - \lambda)(m' - m^* - c_p)} \text{ , while} \tag{32.2}$$

$$q^*(m^*|d_1) = 1. \tag{32.3}$$

Completing the equilibrium strategies,

$$r^*(a_1|m,p_1) = \begin{cases} 0 & \text{if } m < m' - c_p \\ 1 & \text{if } m \geq m' - c_p \end{cases}, \tag{32.4}$$

$$r^*(a_1|m,p_2) = \begin{cases} 0 & \text{if } m \neq m^*, m < m' - c_p \\ 1 & \text{if } m \neq m^*, m \geq m' - c_p \end{cases}. \tag{32.5}$$

Checking divinity, $\theta_2(\bar{m}|m^*)$ is simply p_2's equilibrium mix at $\bar{m}$:

$$\theta_2(\bar{m}|m^*) = \frac{c_p + c_d}{(1 - \gamma)(m' - \bar{m} + c_d)}, \tag{33}$$

while (omitting the algebra)

$$\theta_1(\bar{m}|m^*) = \frac{(c_d + c_p)(c_d - m^*)}{(1 - \gamma)(c_d - \bar{m})(m' - m^* + c_d)}. \tag{34}$$

Note that for $m^* = \bar{m}$, $\theta_1(\cdot) = \theta_2(\cdot)$, and

$$\frac{\partial\theta_1}{\partial\bar{m}} = \frac{(c_d + c_p)(c_d - m^*)}{(1 - \gamma)(c_d - \bar{m})^2(m' - m^* + c_d)} > 0, \tag{35}$$

$$\frac{\partial\theta_2}{\partial\bar{m}} = \frac{(c_p + c_d)}{(1 - \gamma)(m' - \bar{m} + c_d)^2} > 0, \tag{36}$$

which are the partial derivatives of $\theta_1(\cdot)$ and $\theta_2(\cdot)$ under the negligence rule multiplied by $1/(1 - \gamma)$. Hence the semi-pooling sequential equilibria, with common offer $m^* \varepsilon [0,\min\{\tilde{m},m_\lambda,c_d + \gamma m']$ are divine. In summary:

(i) if $m_\lambda \leq \tilde{m}$, $m_\lambda \leq c_d + \gamma m'$, and $\tilde{m} > 0$, there exists a divine pooling equilibria at $m = m_\lambda$ which p_1 rejects and p_2 accepts;

(ii) if $m_\gamma < c_d$, there exists a divine pooling equilibria at $m = m' - c_p$, which both p_1 and p_2 accept;

(iii) if $m_\lambda \geq 0$ and $\tilde{m} \geq 0$ there exist divine semi-pooling equilibria with common offer $m^* \in [0, \min\{\tilde{m}, m_\lambda, c_d + \gamma m'\}]$. The probability of trial is

$$\left[\lambda + \frac{(1-\lambda)\lambda(m^* + c_p)}{(1-\lambda)(m' - m^* - c_p)}\right] \cdot \left[\gamma + \frac{(1-\gamma)(m' - m^*) - \gamma c_d - c_p}{m' - m^* + c_d}\right]$$

$$= \left[\frac{\lambda m'}{m' - m^* - c_p}\right] \cdot \left[\frac{m' - m^* - c_p}{m' - m^* + c_d}\right] = \frac{\lambda m'}{m' - m^* + c_d} .$$

4.5 Summary

The divine equilibrium paths under the four liability rules are:

1. negligence

(i) if $m_\lambda \leq c_d$, there exists a pooling equilibrium offer at $m^* = \max\{0, m_\lambda\}$, which p accepts with positive probability; the maximum probability of rejection is $\frac{\lambda m'}{c_p + \lambda m' + c_d}$;

(ii) if $m_\lambda \geq 0$, there exist semi-pooling equilibria with common offer $m^* \in [0, \min\{m_\lambda, c_d\}]$, where p mixes between acceptance and rejection; the probability of rejection (hence a trial decision) at m^* is $\frac{\lambda m'}{m' - m^* + c_d}$.

2. strict liability with contributory negligence

(i) if $\gamma c_d < m_\gamma$, the equilibrium offer by d is at $m^* = 0$, which p_1 rejects and p_2 accepts; thus the probability of trial is $Pr(p_1) = \gamma$;

(ii) if $\gamma c_d > m_\gamma$, the equilibrium offer is at $m^* = m' - c_p$, which both p_1 and p_2 accept.

3. negligence with contributory negligence

(i) if $m_\lambda \leq \gamma c_d$, there exists a pooling equilibrium offer at $m^* = \max\{0, m_\lambda\}$, which p_2 accepts with probability one and p_1 accepts with positive probability. The maximum probability of rejection by p_1 is $\frac{\lambda m'}{c_p + \lambda m' + c_d}$;

(ii) if $m_\lambda > 0$ and $\gamma c_c < m_\gamma$, there exists a pooling equilibrium offer at $m^* = 0$, which p_1 rejects and p_2 accepts; thus the probability of trial is $Pr(p_1) = \gamma$;

(iii) if $m_\lambda \geq 0$, $\tilde{m} \leq m_\lambda$, $\tilde{m} \leq \gamma c_d$, there exist semi-pooling equilibria with common offer $m^* \varepsilon [\tilde{m}, \min\{m_\lambda, \gamma c_d\}]$ which p_2 accepts and p_1 mixes between acceptance and rejection; the probability of trial at m^* is $\frac{\lambda m'}{m' - m^* + c_d}$.

4. strict liability with dual contributory negligence

(i) if $m_\lambda \leq \tilde{m}$ and $m_\lambda \leq c_d + \gamma m'$, there exists a pooling equilibrium offer at $m^* = \max\{0, m_\lambda\}$, which p_1 rejects and p_2 accepts with positive probability; the probability of trial is at least $Pr(p_1) = \gamma$;

(ii) if $\tilde{m} \leq m_\lambda$ and $m_\gamma \leq c_d$, there exists a pooling equilibrium offer at $m^* = m' - c_p$, which both p_1 and p_2 accept;

(iii) if $\tilde{m} \geq 0$, $m_\lambda \geq 0$, there exist semi-pooling equilibria with common offer $m^* \in [0, \min\{\tilde{m}, m_\lambda, c_d + \gamma m'\}]$, where the probability of trial at m^* is $\frac{\lambda m'}{m' - m^* + c_d}$.

Since $\gamma c_d \geq m_\gamma \iff \tilde{m} \leq 0$, there exist comparisons between the divine equilibria of different liability rules in terms of the set of parameters for which the equilibria exist. The most interesting comparison seems to be negligence v. negligence with contributory negligence, and strict liability with contributory negligence v. strict liability with dual contributory negligence, which for notational simplicity we label n, ncn, slcn, sldcn, respectively. We begin by partitioning the space of parameters into two sets.

A. $m_\lambda < 0$ (i.e., $\lambda > \frac{m' - c_p}{m'}$).

(i) There exist no semi-pooling equilibria.

(ii) If there exists a pooling equilibrium at $m^* = 0$ under slcn, then there exists a pooling equilibrium at $m^* = 0$ under sldcn; thus, the equilibrium $m^* = 0$ exists "more often" (in terms of a probability distribution over parameter values) under sldcn than under slcn.

(iii) If there exists a pooling equilibrium at $m^* = m'$ c_p under sldcn, then there exists a pooling equilibrium at $m^* = m' - c_p$ under slcn.

(iv) The only equilibria under n and ncn is at $m^* = 0$.

B. $m_\lambda \geq 0$.

(i) The pooling equilibrium offer $m^* = m' - c_p$ exists more often under sldcn than under slcn.

(ii) The pooling equilibrium offer $m^* = 0$ exists more often under slcn than under sldcn.

(iii) The pooling equilibrium offer $m^* = m_\lambda$ exists more often under n than under ncn.

(iv) The pooling equilibrium offer $m^* = 0$ exists more often under ncn than under n.

(v) The semi-pooling equilibria exist more often under n than under ncn; more over, the set of common offers is smaller under ncn than under n.

Thus we see that, given $m_\lambda \geq 0$, the pooling offers tend to be smaller going from n to ncn and slcn to sldcn, while with $m_\lambda < 0$ there is no difference between n and ncn, and the pooling offers are on average larger under sldcn than under slcn.

5. UNIVERSALLY DIVINE EQUILIBRIA

Given the characterization of divine beliefs in Section 4, the further restriction to universally divine beliefs is easily stated. For all out-of-equilibrium messages $\bar{m}$, if

$$\theta_i(\bar{m}|m^*) < \theta_i(\bar{m}|m^*) \text{ and}$$
$$\theta_i(\bar{m}|m^*) \leq 1,$$

then universal divinity implies $\mu(d_i|\bar{m}) = 1$, where m* is the equilibrium offer.

5.1 Negligence

Let $\lambda^* = \frac{m' - c_p}{m'}$. Then

$$m_\lambda \gtrless 0 \quad \text{as} \quad \lambda^* \gtrless \lambda.$$

If $\lambda < \lambda^*$, then there exists a divine equilibrium offer at $m^* = m_\lambda$. To support this equilibrium, p_1 and p_2 must reject lower offers with a higher probability than in equilibrium. By Fig. 2, however, $\theta_1(\bar{m}|m_\lambda) < \theta_2(\bar{m}|m_\lambda)$, $\forall\bar{m} < m_\lambda$. Thus, universal divinity implies $\mu(d_1|\bar{m}) = 1$, which implies acceptance, by Fig. 1. This then upsets the equilibrium. If, on the other hand, $\lambda > \lambda^*$, then $m^* = 0$; by Fig. 2, this equilibrium will be universally divine.

For the semi-pooling divine equilibria, which exist if $\lambda < \lambda^*$, by Fig. 3 we see that, given a common offer $m^* \varepsilon [0,m_\lambda]$, universal divinity again requires that $\mu(d_1|\bar{m}) = 1$, $\forall\bar{m} < m^*$. Thus the only semi-pooling universally divine equilibrium is where the common offer is at $m^* = 0$.

Fig. 5 summarizes these results.

5.2 Strict Liability with Contributory Negligence

Let $\gamma^* = \frac{m' - c_p}{m' + c_d}$. Then

$$m_\gamma \gtrless \gamma c_d \quad \text{as} \quad \gamma^* \gtrless \gamma; \text{ also}$$

$$\tilde{m} \gtrless 0 \quad \text{as} \quad \gamma^* \gtrless \gamma.$$

Since both p_1 and p_2 have weakly dominant strategies, the restriction of universally divine beliefs will not alter the set of equilibria. Thus, if $\gamma < \gamma^*$, the unique universally divine equilibrium has d_1 and d_2 offering $m^* = 0$, which p_1 rejects and p_2 accepts. If $\gamma > \gamma^*$, then the unique universally divine equilibrium has d_1 and d_2 offering $m^* = m' - c_p$, which both p_1 and p_2 accept. See Fig. 6.

5.3 Negligence with Contributory Negligence

Recall that there exist two types of pooling equilibria in this case: (i) if $m_\lambda < \gamma c_d$, the pooled offer is at $m^* = \max\{0, m_\lambda\}$; (ii) if $\lambda < \lambda^*$ and $\gamma < \gamma^*$, the pooled offer is at $m^* = 0$. As in the negligence case above, if $\lambda < \lambda^*$, then the type (i) equilibrium cannot be supported by universally divine beliefs; see Figs. 1 and 2. Thus, a type (i) equilibrium is universally divine only if $\lambda > \lambda^*$, implying $m^* = 0$, which both p_1 and p_2 accept. For a type (ii) equilibrium, we see that, by Fig. 4, universal divinity implies $\mu(d_2|\bar{m}) = 1$, $\forall \bar{m} < m' - c_p$, thereby allowing p_1 to reject all offers less than $m' - c_p$. Thus, if $\lambda < \lambda^*$ and $\gamma < \gamma^*$, there exists a universally divine equilibrium where d_1 and d_2 offer $m^* = 0$, which p_1 rejects and p_2 accepts.

The conditions for a semi-pooling divine equilibria are that $\lambda < \lambda^*$, $\tilde{m} \leq m_\lambda$, and $\tilde{m} \leq \gamma c_d$. By the arguments for the semi-pooling equilibria in the negligence case, we know that the lowest common offer is the only potentially universally divine equilibrium offer.

Thus $m^* = \tilde{m}$ would be the common offer. However, if $\tilde{m} > 0$, then by Fig. 3 we see that universal divinity implies $\mu(d_1|\bar{m}) = 1$, $\forall\bar{m} < m^*$. Thus, by Fig. 1, both p_1 and p_2 would accept offers lower than m^*, upsetting the equilibrium. The only instance where a semi-pooling universally divine equilibrium exists is when $\tilde{m} \lesssim 0$; i.e. $\gamma > \gamma^*$, implying $m^* = 0$. Thus, the conditions for such an equilibrium are that $\lambda < \lambda^*$ and $\gamma > \gamma^*$. Fig. 7 summarizes these results.

5.4 Strict Liability with Dual Contributory Negligence

Again there exists two types of pooling divine equilibria: (i) if $m_\lambda \leq \tilde{m}$, $\tilde{m} > 0$, and $m_\lambda \leq c_d + \gamma m'$, then $m^* = \max\{0, m_\lambda\}$; (ii) if $m_\gamma < c_d$, then $m^* = m' - c_p$. For type (i) equilibria, since $\theta_1(\bar{m}|m^*) < \theta_2(\bar{m}|m^*)$, $\forall\bar{m} < m^*$, universal divinity implies $\mu(d_1|\bar{m}) = 1$. Hence, p_1 should accept these offers, upsetting the equilibrium. Thus, as above the only type (i) equilibrium which is universally divine exists when $\lambda > \lambda^*$ and $\gamma < \gamma^*$, where $m^* = 0$ and p_1 rejects and p_2 accepts. For type (ii) equilibria, we have

$$\bar{m} < \tilde{m} \Rightarrow \theta_1(\bar{m}|m^*) < \theta_2(\bar{m}|m^*), \text{ and}$$
$$\bar{m} > \tilde{m} \Rightarrow \theta_1(\bar{m}|m^*) > \theta_2(\bar{m}|m^*) > 1.$$

When $\theta_1(\cdot)$, $\theta_2(\cdot) > 1$, then universal divinity places no restrictions on beliefs. However, $\bar{m} < \tilde{m}$ implies $\mu(d_1|\bar{m}) = 1$; thus p_2 should accept these offers. If $\tilde{m} > 0$, then an offer of $m = 0$ would give d_1 a utility of $\gamma(-c_d - m')$. To maintain the equilibrium, then, it must be that $\gamma(-c_d - m') < c_p - m'$, or equivalently $m_\gamma \lesssim \gamma c_d$. But this is the same as $\tilde{m} \lesssim 0$. Thus, the only type (ii) equilibrium which is universally divine exists when $\gamma > \gamma^*$.

The conditions for a semi-pooling divine equilibrium are that $\lambda < \lambda^*$ and $\gamma < \gamma^*$. Using again Figs. 1 and 3, we see that the only semi-pooling universally divine equilibrium is at $m^* = 0$. Fig. 8 summarizes these results.

5.5 Summary

As in Section 4 we compare the equilibrium predictions of negligence vs. negligence with contributory negligence, and strict liability with contributory negligence vs. strict liability with dual contributory negligence, where we again use the shorthand of n, ncn, slcn, and sldcn, respectively. Under universal divinity these comparisons are facilitated by the fact of unique of equilibrium predictions. Thus, we can compare the liability rules in terms of the preferences of the litigants.

1. n vs. ncn.

For $\lambda > \lambda^*$, we see by Figs. 5 and 7 that the equilibrium predictions are equivalent. For $\lambda < \lambda^*$ and $\gamma > \gamma^*$, it is easily seen that p_1 and p_2 achieve the same utility under both rules. Since d_2 is mixing in these equilibria, it must be that d_2's utility is equal to $c_p - m'$ under both rules. Under n, d_1 receives utility $\frac{m' - c_p}{m' + c_d}(-c_d)$; i.e., court costs times the probability of trial. Under ncn, p_2 accepts d_1's offer of $m^* = 0$, while p_1 rejects $m^* = 0$ with probability

$\frac{m' - c_p}{\gamma(m' + c_d)}$. Since d_1 faces p_1 with probability γ, d_1's utility under ncn is also $\frac{m' - c_p}{m' + c_d}(-c_d)$.

Thus, the only time utilities differ between n and ncn is when $\lambda < \lambda^*$ and $\gamma < \gamma^*$. Under n, p_1 receives utility

$$\frac{\lambda m'}{m' - c_p} (-m') + (1 - \frac{\lambda m'}{m' - c_p})(-c_p),$$

while under ncn p_1 receives utility $-\lambda m' - c_p$. Cancelling terms, we see that these two expressions are equal. Also, p_2's utility under n is the same as p_1's. Under ncn, p_2 receives utility $-m'$. Since $c_p < m'$, p_2 prefers a convex combination of $-m'$ and $-c_p$ to $-m'$ with certainty. Hence, p_1 receives the same utility under n and ncn, and p_2 prefers n to ncn.

Under n, d_1 receives $\frac{m' - c_p}{m' + c_d}(-c_d)$; under ncn, d_1 receives $\gamma(-c_d)$. Working through the algebra, we see that, since $\gamma < \gamma^*$, d_1 prefers ncn to n. Similarly for d_2, since $\gamma < \gamma^*$, $\gamma(-c_d - m') > c_p - m'$; thus, d_2 prefers ncn to n.

2. slcn vs. sldcn.

For $\gamma > \gamma^*$, from Figs. 6 and 8 the equilibrium predictions are equivalent. Furthermore, for $\lambda > \lambda^*$ and $\gamma < \gamma^*$ the predictions are also equivalent. Hence we again focus on $\lambda < \lambda^*$ and $\gamma < \gamma^*$. Under slcn, p_1 receives utility $-c_p$, while under sldcn p_1 receives $-c_p$ if the offer is $m^* = 0$ and he rejects, and $-c_p$ if the offer is $m^* = m' - c_p$ and he accepts. Thus, p_1 is indifferent between slcn and sldcn. Under slcn, p_2 receives $-m'$, while under sldcn, p_2 receives a convex combination of $-m'$ and $-c_p$. Thus, p_2 prefers sldcn to slcn.

Under slcn, d_1 receives $\gamma(-c_d - m')$, while under sldcn d_1 receives $\gamma(-c_d - m') + \frac{m' - c_p}{m' + c_d}(-c_d)$. Thus, d_1 prefers slcn to sldcn. Under slcn, d_2 also receives $\gamma(-c_d - m')$, while under sldcn d_2 receives $c_p - m'$, since d_2 is indifferent and his mixing between $m^* = 0$ and $m^* = m' - c_p$. Now $\gamma(-c_d - m') > c_p - m'$, since as above $\gamma < \gamma^*$. Thus d_2 also prefers slcn to sldcn.

6. CONCLUSION

We have seen how the liability rule in force can influence the behavior of plaintiff and defendant in the pretrial bargaining of a civil suit. A generalization of the model would be to allow the plaintiff the ability to make the first offer, which the defendant can either accept or make a counteroffer, and the plaintiff either accepting this or rejecting and going to court. This would allow the defendant the opportunity to gain insight into the plaintiff's type prior to making his offer, an opportunity which does not exist in the model above. Note that, if the defendant rejected a pooled offer from the plaintiff, the subsequent behavior would fall directly under the model of this paper; given a pooled offer by the plaintiff, the defendant gains no information; given that he's rejected the offer, he proceeds to make his own offer.

In terms of analyzing behavior prior to an accident, notice that defendants prefer outcomes when $\lambda > \lambda^*$, so that for a fixed damage size m′, there is an incentive as a group to maintain a high prior probability of nonnegligence in the eyes of potential plaintiffs. Similarly, plaintiffs prefer outcomes when $\gamma > \gamma^*$, so that there are incentives for (potential) plaintiffs to maintain a high probability of nonnegligence as a group. Analysis such as this is fairly ad hoc, however; a more complete development will be the topic of subsequent papers.

NOTES

1. Epstein (1973) posits an alternative goal of liability law, that of "corrective justice."

2. Two other rules, no liability and strict liability, will be seen to be degenerate cases of strict liability with contributory negligence.

3. P'ng's (1984) analysis basically deals with this rule.

4. Of course, one could have initially defined equilibrium and subsequently added divinity; however, divinity grew out of the methodology of the sequential equilibrium concept and as such is easier to characterize as a refinement of sequential equilibrium.

5. Since the payoffs are not a function of the plaintiff's type, the plaintiff's strategy can be a nontrivial function of type only if he is indifferent between a_1 and a_2. In this case, the (mixed) strategies of the plaintiff below can be interpreted as those which arise after taking expectations over p_1 and p_2.

6. Some mixing between a_1 and a_2 is allowed out of equilibrium, as shown below; this does not alter the set of nondivine sequential equilibria.

7. See note 6 above.

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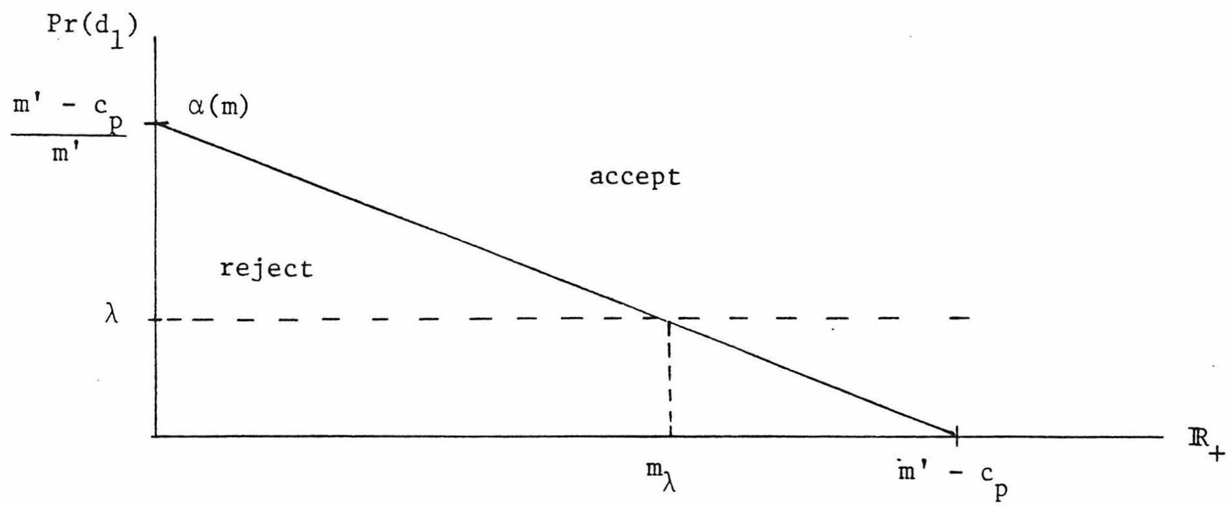

Figure 1

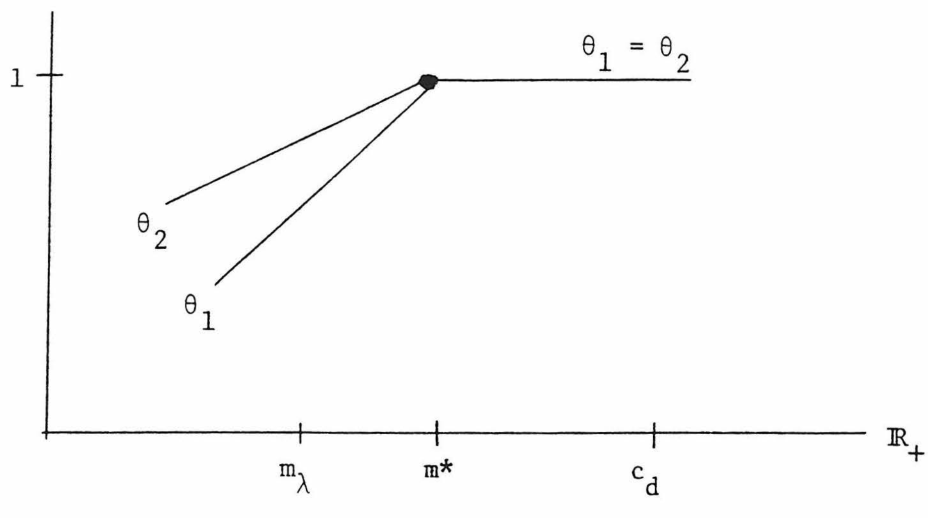

Figure 2

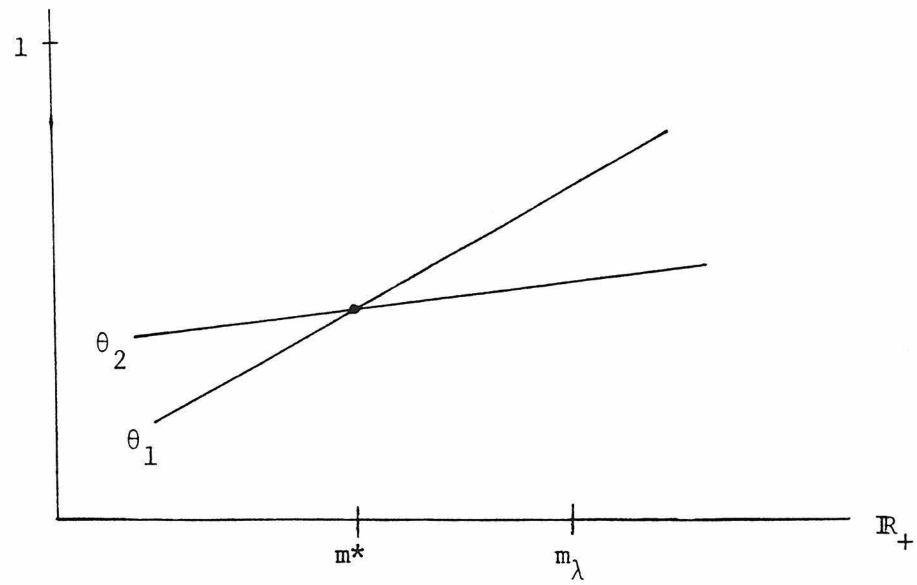

Figure 3

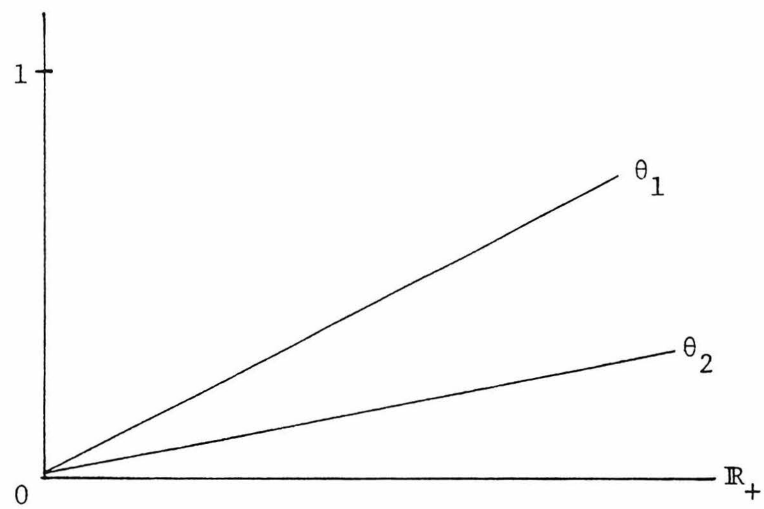

Figure 4

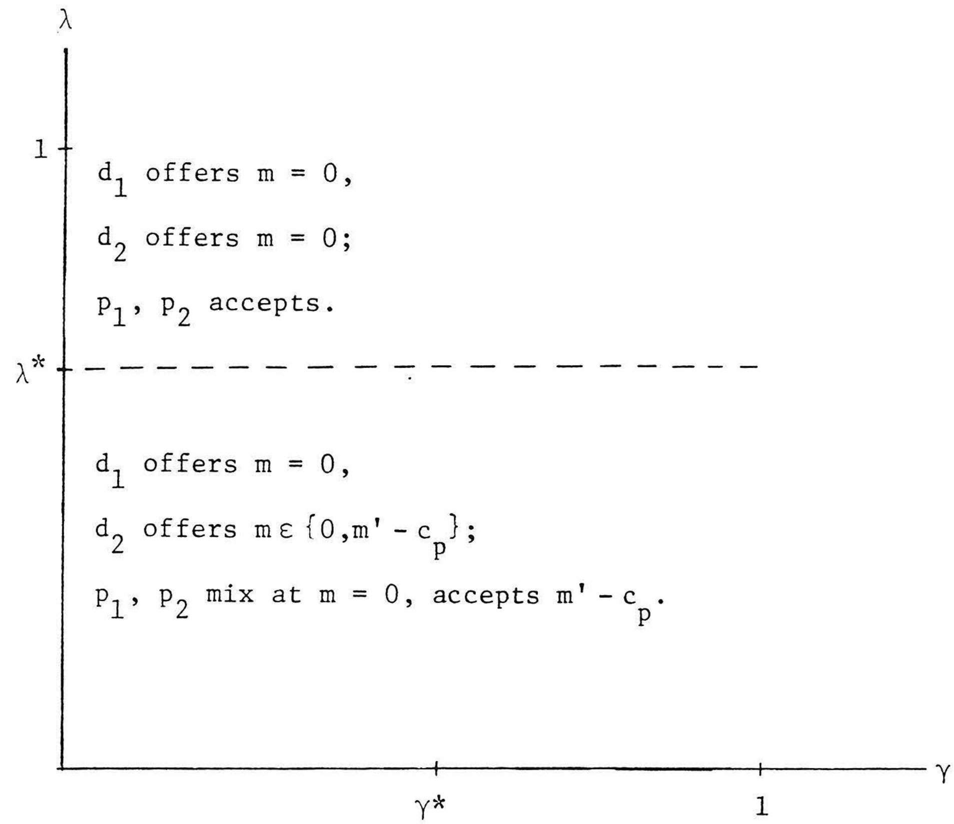

Figure 5: Negligence

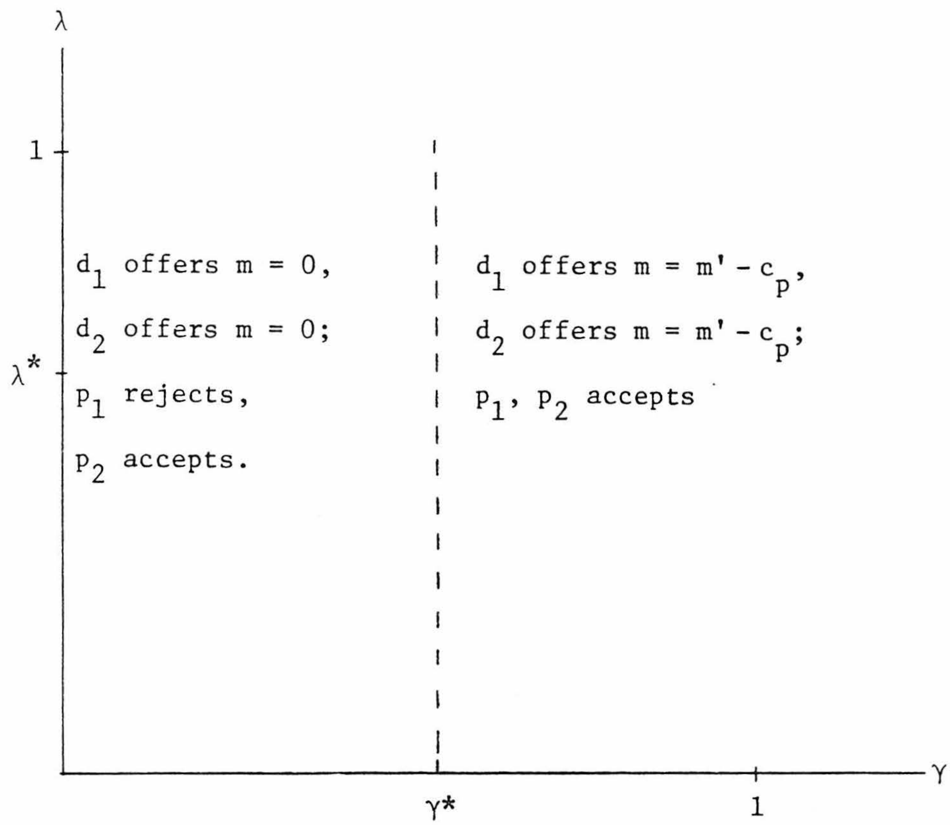

Figure 6: Strict Liability with Contributory Negligence

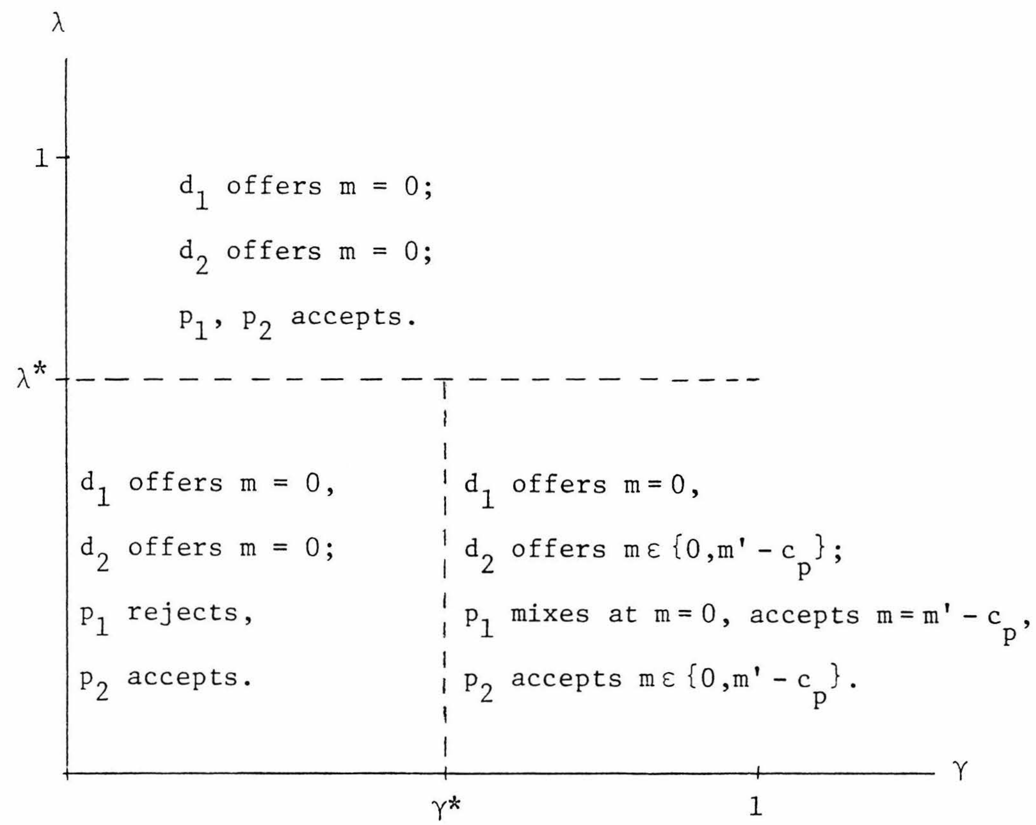

Figure 7: Negligence with Contributory Negligence

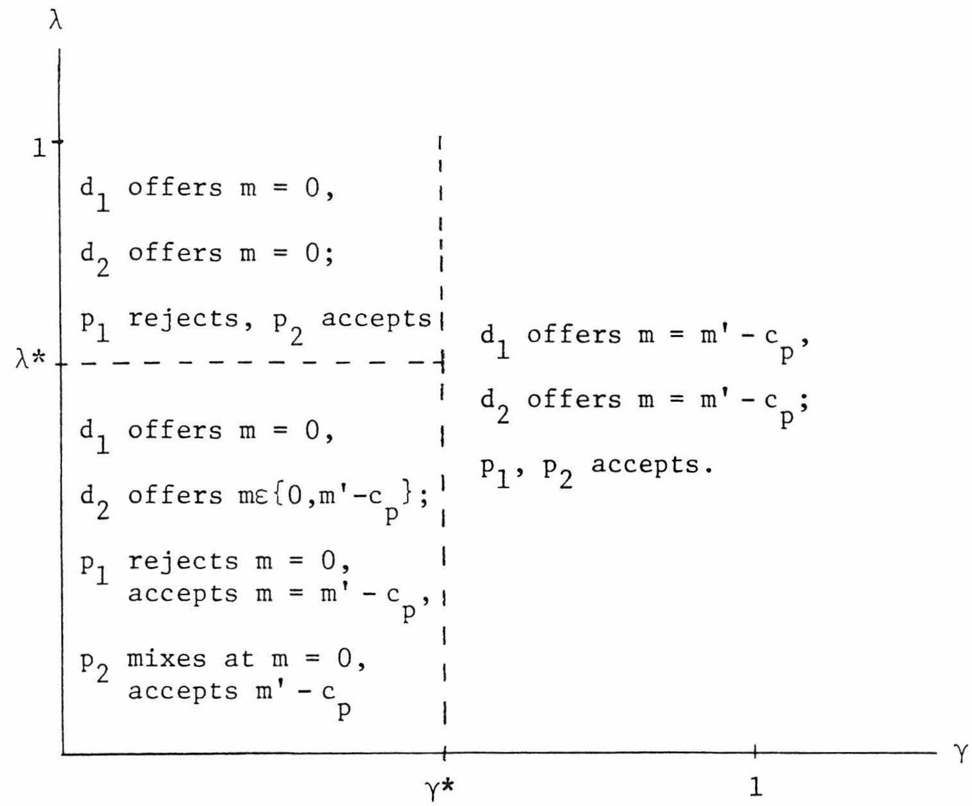

Figure 8: Strict Liability with Dual Contributory Negligence

CHAPTER III. A MODEL OF ELECTORAL COMPETITION WITH INCOMPLETE INFORMATION

1. INTRODUCTION

Modern attempts to model the competition between two candidates in an electoral setting have typically involved eliminating the restrictive assumptions inherent in the Downsian model. For instance, the Downsian model assumes that candidates and voters possess complete information concerning candidate positions and voter preferences. McKelvey and Ordeshook (1984, 1985) relax the former assumption by modeling a subset of the voters as uninformed about candidate positions, while Ledyard (1986) relaxes the latter by assuming that neither candidate knows precisely what the distribution of ideal points is. Another strong assumption implicit in the model of Downs is that the positions the candidates announce prior to an election will be the positions they subsequently enact once in office. Since the model assumes that voters have utility over the positions the candidates enact, not the position they announce, but their only information consists of these announcements, the tractability of the model may lead one to simply assume that what a candidate says he will do is in fact what he will do.

This paper describes a model which attempts to remove this restriction. The model assumes that prior to an election each candidate knows what position he will adopt once in office, but the other candidate as well as the voters do not possess this information. The candidates simultaneously choose positions to announce, after which voters attempt to infer from these announcements what they would actually do if elected. There are costs involved in announcing a position different from the actual position, these costs only accruing to the winning candidate. Thus, the winning candidate's true position is revealed after an election, since he implements his policy position, while the losing candidate's position is not revealed. The game is modelled as a game of incomplete information (Harsanyi (1967-68)) and the sequential equilibrium concept (Kreps and Wilson (1982)) is used to describe equilibrium behavior. Given their beliefs over the candidates' true positions, the voters vote for the candidate giving them their highest expected utility. Similarly, given the strategies of the voters and the strategy of the other candidate along with the (common knowledge) prior over the other candidate's true position, the candidates choose to announce positions which maximize their expected utility. If furthermore the beliefs of the voters satisfy certain consistency criteria related to the strategies of the candidates, then an equilibrium is achieved.

From the discussion above it is easily seen that the model involves two basic presumptions. The first is that the candidates' positions are known to them prior to the election; i.e., there is no room for a posteriori decisions on policy. This can possibly be justified by either assuming that candidates have policy as well as vote-maximizing preferences and, conditional on winning, would faithfully carry out these preferences, or by appealing to the characteristics of the party nomination process and the existence of party control of its members. Suppose the political parties themselves have preferences over the policy space and through the nomination process choose the candidate most likely to carry out the party's preferred position given a favorable election outcome. Furthermore, the party allows the candidate to "win any way he can," that is, announce any position that will get him elected. If we treat the parties' preferences as exogenous and unknown, the model follows. The second presumption is that the candidate's utility, conditional on winning, is a decreasing function of the distance between his announced position and his actual position. Implicit in this formulation is a dynamic structure in which the winning candidate proceeds to enter another election, where his "reputation" for deceit is harmful to his chances (this interpretation of reputations in elections is slightly different than that found in Ingberman (1985)). Suppose that in this second election the candidate faces a new challenger on a new policy space, and where the voters discount the utility gained from the (previously victorious) candidate winning by some factor, an argument of which is such a distance. If this second election, and any subsequent elections, were explicitly in the model, the subgames they induce could be solved recursively for the utility payoffs of this first election. This is not done in the present paper; a functional form for the utilities is assumed incorporating what might be thought as this type of reputation effect. However, if the model developed below proves to be of value in describing behavior in elections, it may provide the groundwork for a more general model of electoral dynamics and reputation formation.

2. THE MODEL

Consider a policy space $P \subseteq \mathbb{R}$, a closed, convex interval, with $|P| = D$, where without loss generality we assume that the midpoint of P is at zero. Thus, $P = [\frac{-D}{2}, \frac{D}{2}]$. There are two candidates, A and B, whose "types," or true policy positions, are i.i.d. random variables with cumulative distribution $F(\cdot)$ and density $f(\cdot)$, where $f(\cdot) > 0$ for all points in P and $f(\cdot)$ is symmetric about zero. (Thus, $f(\cdot)$ is the common knowledge prior over both candidates' types.) A <u>strategy</u> for candidate A is a function

$$s_A : P \rightarrow P,$$

where $s_A(a)$ is the announced position of candidate A whose type is $a \in P$; $s_B(\cdot)$ is similarly defined. There exists a finite set $N = \{1,\ldots,n\}$ of voters, n odd, where each voter $i \in N$ has a single-peaked utility function $u_i(\cdot)$ over P; let p_i be the ideal point of voter i. Assume that the median voter $v \in N$ has an ideal point equal to the midpoint of the policy space, i.e., $p_v = 0$. A <u>strategy</u> for voter i is a function

$$r_i : P \times P \rightarrow \{0, \tfrac{1}{2}, 1\}$$

where $r_i(p_A, p_B)$ is the probability that voter i votes for candidate A, given that i sees announced positions p_A and p_B. Thus, voters either vote for A or B with probability one, or vote for A and B with probability one-half each.

A1. Utility of voters. $\forall i \in N$, the utility of policy position p is $u_i(p) = -(p_i - p)^2$. Thus, given beliefs $g_A(\cdot)$ over candidate A, i's expected utility from A winning is given by

$$E_g U_i = -(\bar{a} - p_i)^2 - \sigma_a^2,$$

where $\bar{a}$ is the mean and σ_a^2 the variance associated with the density $g_A(\cdot)$. Let

$$v_A(p_A, p_B) = |\{i \in N: r_i(p_A, p_B) = 1\}|,$$

$$v_B(p_A, p_B) = |\{i \in N: r_i(p_A, p_B) = 0\}|$$

A2. Utility of candidates. The utility of candidate A with type a, given positions (p_A, p_B), is zero if $v_B(p_A, p_B) \geq \frac{n+1}{2}$, $U(a, p_A)$ if $v_A(p_A, p_B) \geq \frac{n+1}{2}$, and is

$$U(a, p_A) \left(\tfrac{1}{2}\right)^{(n-v_A-v_B)} \sum_{j=\frac{n+1}{2}-v_A}^{n-v_A-v_B} \binom{n - v_A - v_B}{j}$$

otherwise. The utility of candidate B is similarly defined, using $U(b, p_B)$. The function $U(\cdot,\cdot)$ is a continuous concave decreasing function of the distance between the arguments, so that

$$\forall (x,y) \in P \times P, \text{ where } x \neq y,$$

$$\frac{\partial U(x,y)}{\partial x} \gtrless 0 \text{ as } y \gtrless x,$$

$$\frac{\partial^2 U(x,y)}{\partial x^2} \leq 0,$$

$$\frac{\partial U(x,y)}{\partial y} \gtrless 0 \text{ as } x \gtrless y,$$

$$\frac{\partial^2 U(x,y)}{\partial y^2} \leq 0,$$

$$U(y, y + \varepsilon) = U(y, y - \varepsilon), \text{ and}$$

$$U(x + \varepsilon, x) = U(x - \varepsilon, x).$$

Also, for all $x \in P$, we assume that there exists a (non-empty) region $P(x) \subseteq P$ such that $U(x,p) \geq 0$ *iff* $p \in P(x)$. [We allow $U(\cdot,\cdot)$ to be non-differentiable at $x = y$ so as to permit utility functions which are linear in $d(x,y) = |x - y|$.]

Considering again the utility functions of the voters, note that for any beliefs $g_A(\cdot)$ and $g_B(\cdot)$ over candidate types, where $\bar{a} \neq \bar{b}$, there exists a unique position $\bar{p} \in P$ defined by

$$\bar{p} = \frac{\bar{a} - \bar{p}}{2} + \frac{(\sigma_a^2 - \sigma_b^2)}{2(\bar{a} - \bar{b})}$$

such that all voters with $p_i < \bar{p}$, should vote for one candidate, and all voters with $p_i > \bar{p}$ should vote for the other candidate, if all voters have the same beliefs and behave in the optimizing manner described below. If $\bar{a} = \bar{b}$ and $\sigma_a^2 = \sigma_b^2$, then all voters are indifferent between voting for A or B, while if $\bar{a} = \bar{b}$ and $\sigma_a^2 \neq \sigma_b^2$, all voters vote for the candidate with the lower variance. Now an implication of the consistency criterion inherent in the sequential equilibrium concept employed below is that all voters *do* hold the same beliefs, in *and* out of equilibrium. This is so because beliefs in a sequential equilibrium are the limit of a sequence of beliefs derived using Bayes' rule on candidate strategies which take every position with some positive probability. Since this completely determines beliefs for every possible position, voters' beliefs are the same all along the sequence, and hence at its limit. Thus, given beliefs $g_A(\cdot)$ and $g_B(\cdot)$, if the median voter v is not indifferent between voting for A or B, then whoever v votes for would win the election. In what follows we will restrict indifferent voters in that we assume they vote for each candidate with probability one-half. Hence, if v is indifferent, and all other voters are not, then $v_A(\cdot,\cdot) = v_B(\cdot,\cdot) = \frac{n-1}{2}$, and by assumption each candidate wins with probability one-half. Finally if all voter are indifferent, the probability of being elected is again equal to one-half for each candidate, or equivalently is equal to the probability that v votes for them. Thus, the expected utility for candidate A, given positions (p_A,p_B) is equal to

$$r_v(p_A,p_B)U(a,p_A),$$

while given a strategy $s_B(\cdot)$ and taking expectations over B-types A's utility is

$$U(a,p_A)\int_P r_v(p_A,s_B(b))f(b)db;$$

the expected utility for B is similarly defined.

<u>Def</u>. A (perfect) sequential equilibrium to the above model consists of strategies $s_A^*(\cdot)$, $s_B^*(\cdot)$, $r_i^*(\cdot,\cdot)$, $\forall i \in N$, and beliefs

$\mu_A^*(\cdot|\cdot)$, $\mu_B^*(\cdot|\cdot)$, such that

i) $\forall a \in P$, $s_A^*(a)$ maximizes

$$U(a,s_A(a))\int_P r_v^*(s_A(a),s_B^*(b))f(b)db$$

ii) $\forall b \in P_B$, $s_B^*(b)$ maximizes

$$U(b,s_B(b))\int_P [1 - r_v^*(s_A^*(a),s_B(b))]f(b)db$$

iii) $\forall i \in N$, $\forall(p_A,p_B) \in P \times P$,

$$r_i^*(p_A,p_B) = \begin{cases} 1 \\ \frac{1}{2} & \text{as} \\ 0 \end{cases}$$

$$\int_P u_i(a)\mu_A^*(a|p_A)da \gtrless \int_P u_i(b)\mu_B^*(b|p_B)db;$$

iv) if $s_A^{*-1}(p_A) \neq \emptyset$, then $\mu_A^*(t_A|p_A)$ is the conditional probability (relative to the prior $f(\cdot)$) that $a \in t_A \cap s_A^{*-1}(p_A)$ given $a \in s_A^{*-1}(p_A)$, where $t_A \subseteq P$;

v) if $s_B^{*-1}(p_B) \neq \emptyset$, then $\mu_B^*(t_B|p_B)$ is the conditional probability (relative to the prior $f(\cdot)$) that $b \in t_B \cap s_B^{*-1}(p_B)$ given $b \in s_B^{*-1}(p_B)$, where $t_B \subseteq P$.

Parts i) and ii) of the definition are self-explanatory. Part iii) says that, although voters have preferences over the <u>winning</u> candidate's position and they may be indifferent between voting for A or B in that they are not pivotal in the election, they always vote for the candidate giving them the highest expected utility, which is their weakly dominant strategy. This is in the spirit of the perfectness criteria of Selten (1975), but is not captured in the sequential equilibrium concept. An equivalent way of getting this condition is to assume that the voters get some amount of utility from voting for their most preferred candidate, irregardless of the electoral outcome; this is used in McKelvey and Ordeshook (1985). Part iii) also implies that, for every pair of possible announcements (p_A, p_B) by the candidates, the voters form some beliefs over candidate types and maximize their utility according to these beliefs; thus, voters cannot threaten to take an action which is strictly dominated by some other action, since a dominated action is a best response to no beliefs. Also, all voters hold the same beliefs about the candidates in and out of equilibrium, as discussed above. Finally, parts iv) and v) imply that voters use Bayes' rule to update their beliefs on candidate types by their knowledge of the equilibrium strategies of the candidates. Thus, if only one type of candidate A makes a particular announcement, then if the voters see that announcement they must believe it is that type of candidate with probability one; if a subset of types make the same announcement the voters assign positive probability only to those types in the subset, and use the prior to deduce the posterior probability distribution.

3. SYMMETRIC EQUILIBRIA

In what follows we will look only at equilibria which are symmetric with respect to the candidates (i.e., $a = b \Rightarrow s_A(a) = s_B(b)$), thus allowing us to drop the subscript on candidate strategies and w.l.o.g. focus on candidate A. Furthermore, for the non-(pure)pooling equilibria we will examine equilibria which are symmetric with respect to the origin (i.e., $s(a) = -s(-a)$), implying we can concentrate on the half-policy space $[0,\frac{D}{2}]$.

One feature of all symmetric equilibria in the model is that the equilibrium strategy $s(\cdot)$ is monotone increasing; i.e. $\forall a,a' \in [0,\frac{D}{2}], a < a'$ implies $s(a) \leq s(a')$. To see this, fix an equilibrium and let $\lambda(a)$ be the probability that an a-type candidate wins the election, where we suppress the other arguments of this function. In equilibrium it must be that no type is better off emulating another type; thus $\forall a$,

$$\lambda(a)\ U(a,s(a)) \geq \lambda(a')\ U(a,s(a')),\ \forall a'. \tag{1}$$

Suppose that $s(\cdot)$ is not monotone increasing, so that $a < a'$ but $s(a) > s(a')$, and assume $a > s(a)$. Rewriting (1) as

$$\frac{\lambda(a)}{\lambda(a')} \geq \frac{U(a,s(a'))}{U(a,s(a))} \tag{2}$$

and differentiating the right hand side of (2) with respect to a, holding $s(a)$ and $s(a')$ fixed, gives

$$\frac{\partial[\cdot]}{\partial a} = \frac{\frac{\partial U(a,s(a'))}{\partial a}\cdot U(a,s(a)) - \frac{\partial U(a,s(a))}{\partial a}\cdot U(a,s(a'))}{[U(a,s(a))]^2}. \tag{3}$$

The denominator is positive, while the numerator is negative since $U(a,s(a)) > U(a,s(a'))$ and $\frac{\partial U(a,s(a'))}{\partial a} < \frac{\partial U(a,s(a))}{\partial a} < 0$. Thus, the RHS of (2) decreases as a increases. But this implies that, at a',

$\lambda(a)\ U(a',s(a)) > \lambda(a')\ U(a', s(a'))$. Thus, a' would prefer to send $s(a)$ than $s(a')$, thereby contradicting the assumption that $s(\cdot)$ is an equilibrium strategy. The arguments for the cases $s(a) > s(a') > a'$ and $s(a) > a$, $s(a') < a'$ are analogous.

Since all equilibrium strategies are monotonic, any discontinuities will be jump discontinuities. Furthermore, it is easily seen that an equilibrium strategy must have a jump discontinuity when going from a "pooled" position; i.e. a position taken by more than one type, to a separating position, where each type takes a unique position. Suppose that there exists a type $a \in [0,\frac{D}{2}]$ and numbers ε_1, $\varepsilon_2 > 0$ such that all $a' \in [a-\varepsilon_1,a)$ take the same position and all $a' \in [a,a + \varepsilon_2]$ take a unique position. Letting $\lambda(\cdot)$ again denote the equilibrium probability of election, there must be a jump discontinuity at $\lambda(a)$. By the continuity of $U(\cdot,\cdot)$, it also must be the case that a must be indifferent between pooling with $[a - \varepsilon_1,a)$ and separating. But to be indifferent with a jump discontinuity in $\lambda(\cdot)$ it must be that $s(\cdot)$ has a jump discontinuity at a as well, thus proving the claim. In what follows we will examine four kinds of equilibria: 1) pooling equilibria, where all types take the same position, 2) semi-pooling equilibria, where subsets of types take the same position, 3) separating equilibria, where each type takes a unique position, and 4) hybrid equilibria, where some types separate, while other types pool. Since the hybrid equilibria allow for any combination of the first three types, this categorization will exhaust the possibilities for the sequential equilibrium of the model. In each case, we will look at conditions under which these equilibria exist, and whether or not they satisfy the restrictions on off-the-equilibrium-path beliefs set forth in Chapter I above. In particular, we will attempt to characterize the universally divine equilibria for each case.

3.1 POOLING EQUILIBRIA

It is easy to see that a pooling equilibria will exist only if there exists a position $p \in P$ such that $U(a,p) \geq 0$, $\forall a$. Furthermore, if such a position exists, then beliefs of the form

$$\mu(\tfrac{D}{2}|m) = 1 \text{ if } m > p$$
$$\mu(-\tfrac{D}{2}|m) = 1 \text{ if } m < p$$

will support the equilibrium. This is so because, given a pooled position, voter v simply uses the prior $f(\cdot)$ as his beliefs which, given the symmetry of $f(\cdot)$, gives a mean of zero and a variance σ_f^2. The out-of-equilibrium beliefs above give a mean of $\frac{D}{2}$ (or $-\frac{D}{2}$) and a variance of zero. Hence, given the voters' quadratic utility functions and the fact that $\sigma_f < \frac{D}{2}$, for any out-of-equilibrium message by a voter v would respond by voting for B with probability one. Thus, the equilibrium is maintained.

To see whether or not these beliefs are reasonable, we calculate (as in Chapter II) the probability of voting for A which makes a given type indifferent between sending the equilibrium position or defecting. Since equilibrium utility is $\frac{1}{2}U(a,p)$, define

$$\theta(a,m|p) = \frac{1U(a,p)}{2U(a,m)} \tag{4}$$

as the value making an a-type indifferent between sending p and m; any value higher than this would lead to defection. For each m, we want to calculate the type who is most likely to defect, i.e., the type which minimizes $\theta(a,m|p)$. Note that we need only look at types for which $U(a,m) > 0$, since otherwise the type would never defect and send m. Fix $m > p$. Differentiating (1) with respect to a gives

$$\frac{\partial\theta(a,m|p)}{\partial a} = \frac{1}{2}\,\frac{\left[\frac{\partial U(a,p)}{\partial a}\cdot U(a,m) - \frac{\partial U(a,m)}{\partial a}\cdot U(a,p)\right]}{[U(a,m)]^2}\,. \tag{5}$$

The denominator is always positive, by assumption. We examine the sign of (2) in 3 regions:

i) $a > m > p$. Then

$$\frac{\partial U(a,p)}{\partial a} < \frac{\partial U(a,m)}{\partial a} < 0, \text{ and}$$

$$0 < U(a,p) < U(a,m);$$

thus, $\frac{\partial\theta(a,m|p)}{\partial a} < 0.$

ii) $a \varepsilon (p,m)$. Then

$$\frac{\partial U(a,p)}{\partial a} < 0 < \frac{\partial U(a,m)}{\partial a}\ ;$$

thus, $\frac{\partial\theta(a,m|p)}{\partial a} < 0.$

iii) $a < p < m$. Then

$$0 < \frac{\partial U(a,p)}{\partial a} < \frac{\partial U(a,m)}{\partial a}, \text{ and}$$

$$0 < U(a,m) < U(a,p);$$

thus, $\frac{\partial\theta(a,m|p)}{\partial a} < 0.$

(Although $\theta(a,m|p)$ is continuous in a, it may not be differentiable at $a = p$ or $a = m$.) Thus, for $m > p$, $\frac{D}{2} = \underset{a}{\text{argmin}}\ \theta(a,m|p)$. Similarly, for $m < p$, $-\frac{D}{2} = \underset{a}{\text{argmin}}\ \theta(a,m|p)$. Hence, all pooling equilibria are universally divine.

3.2 SEMI-POOLING EQUILIBRIA

The semi-pooling equilibria we examine are characterized as follows:

$$\forall a \in (-a_1,a_1),\ s(a) = 0 \equiv a_0;$$
$$\forall a \in [a_1,a_2),\ s(a) = a_1;$$
$$\forall a \in (-a_2,-a_1],\ s(a) = -a_1;$$
$$\forall a \in [a_2,a_3),\ s(a) = a_2, \text{ etc.}$$

See Figure 1. Those types sending a_0 will win the election with probability $2(1 - F(a_1)) + F(a_1) - F(a_0)$, or equivalently $2 - F(a_0) - F(a_1)$. In general, if $a \in (a_i,a_{i+1})$, then a wins with probability $2 - F(a_i) - F(a_{i+1})$, giving an equilibrium expected utility of

$$[2 - F(a_i) - F(a_{i+1})]U(a,a_i)\ . \tag{6}$$

It is assumed that a_1 is indifferent between sending a_0 and a_1; thus, a_1 solves

$$\frac{U(a_1,a_0)}{U(a_1,a_1)} = \frac{2 - F(a_1) - F(a_2)}{2 - F(a_0) - F(a_1)}\ ; \tag{7}$$

thus a_1 is a function of a_2. In general, a_i solves

$$\frac{U(a_i,a_{i-1})}{U(a_i,a_i)} = \frac{2 - F(a_i) - F(a_{i+1})}{2 - F(a_i) - F(a_{i-1})} \quad . \tag{8}$$

If there are $\ell - 1$ such points on $[0,\frac{D}{2}]$ in equilibrium, then (5) gives $\ell - 1$ equations in ℓ unknowns. The final equation is given by $a_\ell = \frac{D}{2}$. It is easy to see that, if $a = a_i$ is indifferent between sending a_{i-1} and a_i, then all types $a \in (a_{i-1},a_i)$ prefer to send a_{i-1}, and all types $a \in (a_i,a_{i+1})$ prefer to send a_i, thus assuring an equilibrium considering only those positions sent with positive probability. For the out-of-equilibrium positions, again beliefs such as

$$\mu(\tfrac{D}{2}|m) = 1 \text{ if } m > 0$$
$$\mu(-\tfrac{D}{2}|m) = 1 \text{ if } m < 0$$

will support the equilibrium.

To examine out-of-equilibrium beliefs, we look at $\theta(a,m|s(a))$ as in 3.1, although the probability of winning now differs across subsets of types sending different positions. However, given the equalities in (5), $\theta(a,m|s(a))$ will still be continuous, although non-differentiable at $a_0,a_1,a_2,\ldots a_\ell$. Furthermore, since the probability of winning is not a function of type except at these points, we can ignore this term in signing $\frac{\partial\theta(a,m|s(a))}{\partial a}$, thus giving the form found in (2), replacing p with s(a).

Suppose $m \in (a_{i-1},a_i)$, $m > 0$. We examine at four regions:

i) $a < a_{i-1}$. Then

$$0 < \frac{\partial U(a,s)}{\partial a} < \frac{\partial U(a,m)}{\partial a}, \text{ and}$$

$$U(a,m) < U(a,s(a));$$

$$\text{thus, } \frac{\partial \theta(a,m|s(a))}{\partial a} < 0.$$

ii) $a \varepsilon (a_{i-1},m)$. Then

$$\frac{\partial U(a,s)}{\partial a} < 0 < \frac{\partial U(a,m)}{\partial a};$$

$$\text{thus, } \frac{\partial \theta(a,m|s(a))}{\partial a} < 0.$$

iii) $a \varepsilon (m,a_i)$. Then

$$\frac{\partial U(a,s)}{\partial a} < \frac{\partial U(a,m)}{\partial a} < 0, \text{ and}$$

$$U(a,s(a)) < U(a,m);$$

$$\text{thus, } \frac{\partial \theta(a,m|s(a))}{\partial a} < 0.$$

iv) $a > a_i$. Then

$$0 > \frac{\partial U(a,s)}{\partial a} > \frac{\partial U(a,m)}{\partial a}, \text{ and}$$

$$U(a,s(a)) > U(a,m);$$

$$\text{thus, } \frac{\partial \theta(a,m|s(a))}{\partial a} > 0.$$

Figure 2 describes the situation. Thus, universal divinity implies that, for all $m \varepsilon (a_{i-1},a_i)$, $\mu(a_i|m) = 1$. With this belief, A defeats B with probability $2(1 - F(a_i))$. As $m \Rightarrow a_i$, $\theta(a,m|a_i) \Rightarrow 2 - F(a_i) - F(a_{i+1})$. But $2(1 - F(a_i)) > 2 - F(a_i) - F(a_{i+1})$. Thus, for m sufficiently close to a_i, a_i would want to send m as opposed to a_i, thereby upsetting the equilibrium. Hence, the semi-pooling equilibria described above are not universally divine.

3.3 SEPARATING EQUILIBRIA

The strategy $s(\cdot)$ is part of a separating equilibrium only if $s(\cdot)$ is one-one; we will also assume that it is continuous. In a separating equilibrium, the payoffs to each type are $2(1 - F(a))U(a,s(a))$. An equilibrium condition is that no type is better off emulating another type. Thus, $\forall a$,

$$2(1 - F(a))U(a,s(a)) \geq 2(1 - F(a'))U(a,s(a')), \quad \forall a'. \tag{9}$$

Three conditions of a separating equilibrium are immediately apparent:

1) $s(a) \leq a, \ \forall a.$

If not, let $a' = s(a) > a$, for some a; then a' is better off sending $s(a)$ than $s(a')$, since a' receives a higher utility upon winning and a higher probability of winning, thus contradicting the assumption of an equilibrium.

2) $\frac{\partial s}{\partial a} > 0, \ \forall a.$

If not, then there exists $a,a' > 0$ such that $a > a'$ and $s(a) < s(a')$; by 1) then, $s(a) < s(a') \leq a' < a$; thus a is better off sending $s(a')$: contradiction.

3) $s(\frac{D}{2})$ is such that $U(\frac{D}{2},s(\frac{D}{2})) = 0.$

Note that, if $\frac{D}{2}$ separates, then he wins with probability zero. If $U(\frac{D}{2},s(\frac{D}{2})) > 0$, then $\exists \ \varepsilon_1 > 0$ such that $U(\frac{D}{2},s(\frac{D}{2}) - \varepsilon_1) > 0$ by the continuity of $U(\cdot,\cdot)$; but the probability that $s^{-1}(s(\frac{D}{2}) - \varepsilon_1)$ wins is strictly positive. Thus, $\frac{D}{2}$ is better off sending $s(\frac{D}{2}) - \varepsilon_1$. If $U(\frac{D}{2},s(\frac{D}{2})) < 0$, then $\exists \ \varepsilon_2 > 0$ such that the probability that $\frac{D}{2} - \varepsilon_2$

wins is strictly positive, but $U(\frac{D}{2} - \varepsilon_2, s(\frac{D}{2} - \varepsilon_2)) < 0$, by the continuity of $s(\cdot)$ and $U(\cdot,\cdot)$. Thus, $\frac{D}{2} - \varepsilon_2$ is better off sending $s(\frac{D}{2})$. Contradiction.

Equation (7) holds with equality at $a' = a$; thus a (first-order) necessary condition for an equilibrium is that

$$\frac{\partial}{\partial a'}[2(1 - F(a'))U(a,s(a'))]\Big|_{a'=a} = 0, \text{ or}$$

$$-f(a)U(a,s(a)) + \frac{\partial U(a,s(a))}{\partial s}\frac{\partial s(a)}{\partial a}(1 - F(a)) = 0. \qquad (10)$$

The strategy $s(\cdot)$ must also satisfy the second-order condition

$$-f'(a)U(a,s(a)) - 2f(a)\frac{\partial U(a,s)}{\partial s}\frac{\partial s(a)}{\partial a} + \frac{\partial^2 U(a,s)}{\partial s^2}\frac{\partial s(a)}{\partial a}(1 - F(a))$$
$$+ \frac{\partial U(A,s)}{\partial s}\frac{\partial^2 s(a)}{\partial a^2}(1 - F(a)) < 0. \qquad (11)$$

Equation (10) gives a first-order differential equation, while condition 3) above gives a value restriction. However, condition 1) implies that we must also have $s(0) = 0$. Thus, we have two value restrictions on a first-order differential equation, implying the generic non-existence of a solution. However, we incorporate the above mathematics in the following case.

3.4 HYBRID EQUILIBRIA

The hybrid equilibria we examine are of two configurations: C_I, where types toward the midpoint of the policy space separate, and those towards the extremes pool, and C_{II}, where the extreme types separate and the others pool. Figures 3 and 4 give examples of what the equilibria might look like. For both C_I and C_{II}, we assume that there exists a type, a_I and a_{II}, respectively, who is indifferent between separating and pooling. By the results in 3.2 above, we need only consider one pooling position per half-policy space in attempting to characterize the universally divine equilibrium, since more than one would imply (as above) that some type would want to defect from the equilibrium when we restrict beliefs.

C_I

Given a separating position $s(a_I)$, a_I is indifferent between pooling with types $a \varepsilon (a_I, \frac{D}{2})$ and separating if

$$2(1 - F(a_I))U(a_I,s(a_I)) = (1 - F(a_I))U(a_I,a_I), \text{ or}$$

$$\frac{U(a_I,s(a_I))}{U(a_I,a_I)} = \frac{1}{2} . \tag{12}$$

To assure an equilibrium it must be that differentiating the LHS of (12) with respect to the first argument in $U(\cdot,\cdot)$ is nonpositive:

$$\frac{\partial U(a,s(a_I))}{\partial a} \cdot U(a,a_I) - \frac{\partial U(a,a_I)}{\partial a} \cdot U(a,s(a_I)) \leq 0. \tag{13}$$

But

$$\frac{\partial U(a,s(a_I))}{\partial a} < \frac{\partial U(a,a_I)}{\partial a} < 0, \text{ and}$$

$$U(a,s(a_I)) < U(a,a_I),$$

so that (13) holds. Thus, if a_I is indifferent between sending $s(a_I)$

and a_I, all types $a > a_I$ strictly prefer to send a_I.

For $m > a_I$, it is obvious that universal divinity implies $\mu(\frac{D}{2}|m) = 1$, thus supporting the equilibrium. For $m \varepsilon (s(a_I),a_I)$, we have the following:

for $a \lesssim a_I$,

$$\theta(a,m|s(a)) = 2(1 - F(a)) \frac{U(a,s(a))}{U(a,m)} . \tag{14}$$

Differentiating (14) and ignoring the denominator, we get

$$\frac{\partial\theta(a,m|s(a))}{\partial a} = 2\left[-f(a)U(a,s(a)) + \left[\frac{\partial U(a,s)}{\partial a} + \frac{\partial U(a,s)}{\partial a} \cdot \frac{\partial s(a)}{\partial a}\right](1 - F(a))\right] \times U(a,m) - 2\frac{\partial U(a,m)}{\partial a}(1 - F(a))U(a,s(a)) . \tag{15}$$

By equation (10) however, this simplifies to

$$\frac{\partial\theta(a,m|s(a))}{\partial a} = 2(1 - F(a))\left[\frac{\partial U(a,s)}{\partial a} \cdot U(a,m) - \frac{\partial U(a,m)}{\partial a} \cdot U(a,s(a))\right] \tag{16}$$

Two regions are of interest:

i) $a > m > s(a)$. Then

$$\frac{\partial U(a,s)}{\partial a} < \frac{\partial U(a,m)}{\partial a} < 0, \text{ and}$$

$$U(a,s(a)) < U(a,m);$$

thus, $\frac{\partial\theta(a,m|s(a))}{\partial a} < 0.$

ii) $m > a > s(a)$. Then

$$\frac{\partial U(a,s)}{\partial a} < 0 < \frac{\partial U(a,m)}{\partial a};$$

thus, $\frac{\partial\theta(a,m|s(a))}{\partial a} < 0.$

for $a > a_I$, we have

$$\theta(a,m|a_I) = \frac{(1 - F(a_I))U(a,a_I)}{U(a,m)} . \tag{17}$$

Differentiating with respect to a gives

$$\frac{\partial\theta(a,m|a_I)}{\partial a} = (1 - F(a_I))\left[\frac{\partial U(a,a_I)}{\partial a} \cdot U(a,m) - \frac{\partial U(a,m)}{\partial a} \cdot U(a,a_I)\right] \tag{18}$$

Then,

$$0 > \frac{\partial U(a,a_I)}{\partial a} > \frac{\partial U(a,m)}{\partial a}, \text{ and}$$

$$U(a,a_I) > U(a,m);$$

thus, $\frac{\partial\theta}{\partial a} > 0$. See Figure 5. Hence, universal divinity implies that, $\forall\ m \in (s(a_I),a_I)\ \mu(a_I|m) = 1$. Thus, as in the semi-pooling equilibria above, if a sends m, then a wins with probability $2(1 - F(a_I))$, while as $m \to a_I$, $\theta(a,m|a_I) \to 1 - F(a_I)$, implying in particular that a_I would defect from the equilibrium. Since $a_I \neq s(a_I)$ by (12), there exist such out-of-equilibrium positions. Hence, no C_I equilibria are universally divine.

C_{II}

Given a separating position $s(a_{II})$, a_I is indifferent between pooling with $[-a_{II},a_{II})$ at $a_0 \equiv 0$ and separating if

$$2(1 - F(a_{II})U(a_{II},s(a_{II})) = [F(a_0) + 1 - F(a_{II})]U(a_{II},a_0), \tag{19}$$

or,

$$\frac{U(a_{II},s(a_{II}))}{U(a_{II},a_0)} = \frac{\frac{3}{2} - F(a_{II})}{2 - 2F(a_{II})} . \tag{20}$$

To assure an equilibrium differentiating the LHS of (20) with respect to the first argument in $U(\cdot,\cdot)$ must be positive (again ignoring the denominator):

$$\frac{\partial U(a,s(a_{II}))}{\partial a} \cdot U(a,m) - \frac{\partial U(a,m)}{\partial a} \cdot U(a,s(a_{II})) > 0. \qquad (21)$$

i) $0 < a < m < s(a_{II})$. Then

$$\frac{\partial U(a,s(a_{II}))}{\partial a} > \frac{\partial U(a,m)}{\partial a} > 0, \text{ and}$$

$$U(a,m) > U(a,s(a_{II})).$$

ii) $m < a < s(a_{II})$. Then

$$\frac{\partial U(a,s|a_{II})}{\partial a} > 0 > \frac{\partial U(a,m)}{\partial a}.$$

Thus, (21) holds. If a_{II} is indifferent between sending a_0 and $s(a_{II})$, then all types $a \varepsilon (a_0,a_{II})$ prefer to send a_0. For $m > s(\frac{D}{2})$, it is easy to see that universal divinity implies $\mu(\frac{D}{2}|m) = 1$, thus supporting the equilibrium. For $m \varepsilon (a_0,s(a_{II}))$ we have the following:

Let $a > a_{II}$; then

$$\theta(a,m|s(a)) = \frac{2(1 - F(a))U(a,s(a))}{U(a,m)}. \qquad (22)$$

Differentiating with respect to a, substituting in (10) and ignoring the denominator gives

$$\frac{\partial\theta(a,m|s(a))}{\partial a} = 2(1 - F(a))\left[\frac{\partial U(a,s)}{\partial a} \cdot U(a,m) - \frac{\partial U(a,m)}{\partial a} \cdot U(a,s)\right] \qquad (23)$$

Since $m < s(a) < a$,

$$0 > \frac{\partial U(a,s)}{\partial a} > \frac{\partial U(a,m)}{\partial a}, \text{ and}$$

$$U(a,s) > U(a,m);$$

$$\text{thus, } \frac{\partial \theta(a,m|s(a))}{\partial a} > 0.$$

Let $a \varepsilon [a_0, a_{II})$; then

$$\theta(a,m|a_0) = [\tfrac{3}{2} - F(a_{II})] \frac{U(a,a_0)}{U(a,m)} . \tag{24}$$

Thus,

$$\frac{\partial \theta(a,m|a_0)}{\partial a} = \left[\frac{3}{2} - F(a_{II})\right]\left[\frac{\partial U(a,a_0)}{\partial a} \cdot U(a,m) - \frac{\partial U(a,m)}{\partial a} \cdot U(a,a_0)\right] \tag{25}$$

i) $a > m > a_0$. Then

$$\frac{\partial U(a,a_0)}{\partial a} < \frac{\partial U(a,m)}{\partial a} < 0, \text{ and}$$

$$U(a,a_0) < U(a,m);$$

$$\text{thus, } \frac{\partial \theta(a,m|a_0)}{\partial a} < 0.$$

ii) $m > a > a_0$. Then

$$\frac{\partial U(a,a_0)}{\partial a} < 0 < \frac{\partial U(a,m)}{\partial a};$$

$$\text{thus, } \frac{\partial \theta(a,m|a_0)}{\partial a} < 0.$$

Thus, replacing a_I with a_{II} in Figure 5 we get the equivalent sketch of $\theta(a,m|\cdot)$ for C_{II}. Universal divinity thus implies that, $\forall\, m \varepsilon (a_0, s(a_{II}))$, $\mu(a_{II}|m) = 1$. Any type sending m would win with probability $2(1 - F(a_{II}))$. For the C_{II} equilibria to be universally divine, it must be that

$$2(1 - F(a_{II})) \lesssim \theta(a,m|\cdot), \ \forall a, \ \forall m \varepsilon (a_0, s(a_{II})), \tag{26}$$

or

$$2(1 - F(a_{II})) \lesssim \inf_{m\varepsilon(a_0, s(a_{II}))} \left[\min_a \theta(a,m|\cdot) \right] . \tag{27}$$

By the arguments above, a_{II} minimizes the term in brackets. Since (22) and (24) are equal for $a = a_{II}$ for all m (by the definition of a_{II}), we can substitute in either into (27). Thus, using (22),

$$2(- F(a_{II})) \lesssim \inf_{m\varepsilon(a_0, s(a_{II}))} \frac{2(1 - F(a_{II}))U(a_{II}, s(a_{II}))}{U(a,m)} . \tag{28}$$

Since $s(a_{II}) \lesssim a_{II}$, $U(a_{II}, s(a_{II})) > U(a,m)$, $\forall m < s(a_{II})$; thus (28) holds with strict inequality, since (dividing through by $2(1 - F(a_{II}))$, the LHS of (28) equals one, while the RHS is greater than or equal to one. Hence, all C_{II} equilibria are universally divine.

A C_{II} equilibria exists if there exists a type $a > 0$ such that equation (20) holds, where $s(\cdot)$ satisfies (10) as well as the value restriction $U(\frac{D}{2}, s(\frac{D}{2})) = 0$, and the constraint $0 \lesssim s(a) \lesssim a$. Note that if the value restriction cannot be met, then there exist pooling equilibria but no C_{II} equilibria; if the restriction can be met, then there exist C_{II} equilibria but no pooling equilibria.

To see whether a solution to (20) exists, let

$$h(a,s) = \frac{U(a,s(a))}{U(a,0)},$$

$$g(a) = \frac{\frac{3}{2} - F(a)}{2 - 2F(a)}.$$

Then,

$$\frac{\partial h(a,s)}{\partial a} = \frac{\left[\frac{\partial U(a,s)}{\partial a} + \frac{\partial U(a,s)}{\partial s}\frac{\partial s}{\partial a}\right] U(a,0) - \frac{\partial U(a,0)}{\partial a} \cdot U(a,s)}{[U(a,0)]^2}, \quad (29)$$

$$\frac{\partial g(a)}{\partial a} = \frac{-f(a)[2 - F(a)] + 2f(a)[\frac{3}{2} - F(a)]}{[2 - 2F(a)]^2}. \quad (30)$$

Working through the algebra, we see that $\frac{\partial g(a)}{\partial a} > 0$, $g(a)$, $\frac{\partial g(a)}{\partial a} \to \infty$ as $a \to \frac{D}{2}$, and $\frac{\partial h(a,s)}{\partial a} > 0$ for $a < a'$, $h(a,s)$, $\frac{\partial h(a,s)}{\partial a} \to \infty$ as $a \to a'$, where a' solves $U(a,0) = 0$. Furthermore, $g(0) = 1$, and $h(s^{-1}(0),s) = 1$. Thus, if $s^{-1}(0) \geq 0$ and $h(a,s)$ is convex, then $h(a,s)$ and $g(a)$ cross at exactly one point. See Figure 6.

To get some idea of how this C_{II} equilibrium changes as one varies the utility functions of the candidates, let the constant k measure the degree of concavity of the functions; that is, as k increases, the utility functions become narrower, shrinking the sets P(a) defined above. One can think of k as a measure of the costs involved in deviating (after elected) from an announced position. Furthermore, assume that the separating strategy is a continuously differentiable function of k.

Rewrite (20) as

$$2(1 - F(a))U(a,s(a;k);k) - (\frac{3}{2} - F(a))U(a,0;k) = 0. \quad (31)$$

Thus, a_{II} which solves (31) will be a function of k. To calculate

$\frac{\partial a_{II}}{\partial k}$, note first that by the envelope theorem, the derivative of (31) with respect to k when a is replaced by $a_{II}(k)$ is zero. Applying the chain rule then,

$$\frac{\partial a_{II}}{\partial k} = -\frac{\partial[\cdot]/\partial k}{\partial[\cdot]/\partial a}, \tag{32}$$

where $[\cdot]$ is the LHS of (31). The denominator of (32) is by (10),

$$\frac{\partial[\cdot]}{\partial a} = 2(1 - F(a))\frac{\partial U(a,s)}{\partial a} + f(a)U(a,0) - (\tfrac{3}{2} - F(a))\frac{\partial U(a,0)}{\partial a}, \tag{33}$$

which is positive since $0 > \frac{\partial U(a,s)}{\partial a} > \frac{\partial U(a,0)}{\partial a}$ and $(\frac{3}{2} - F(a)) > 2(1 - F(a)) > 0$. The numerator of (32) is

$$\frac{\partial[\cdot]}{\partial k} = 2(1 - F(a))\left[\frac{\partial U(a,s)}{\partial s}\frac{\partial s}{\partial k} + \frac{\partial U(a,s)}{\partial a}\right] - (\tfrac{3}{2} - F(a))\frac{\partial U(a,0)}{\partial k}. \tag{34}$$

Since $0 > \frac{\partial U(a,s)}{\partial k} > \frac{\partial U(a,0)}{\partial k}$ by assumption, $2(1 - F(a))\frac{\partial U(a,s)}{\partial k} - (\frac{3}{2} - F(a))\frac{\partial U(a,0)}{\partial k} > 0$. Thus, if $\frac{\partial s}{\partial k} > 0$, then the numerator of (32) will be positive, thus giving $\frac{\partial a_{II}}{\partial k} < 0$. Hence, as the costs increase, the set of separating types increases, while the set of pooling types decreases. This continues until $s^{-1}(0) = 0$; i.e., $a_{II}(k) = 0$.

Note that if $s^{-1}(0) = (0)$, so that we have a separating equilibrium, and $U(\cdot,\cdot)$ is strictly concave with $\frac{\partial U(a,s)}{\partial a} = 0$ at $a = s$, then equation (10) is going to hold at values arbitrarily close to $a = 0$ only if $k = \infty$. Thus, in this case, the limit of the C_{II} equilibria $k \to \infty$ is the separating equilibrium.

To get an idea about the sign of $\frac{\partial s}{\partial k}$, differentiating (10) with respect to k, where s(a) is replaced with s(a;k), and applying the envelope theorem implies

$$\frac{\partial s}{\partial k} = \frac{f(a)\frac{\partial U}{\partial k} - (1 - F(a))\left[\frac{\partial^2 U}{\partial k \partial s}\frac{\partial s}{\partial a} + \frac{\partial U}{\partial s}\frac{\partial^2 s}{\partial k \partial a}\right]}{-f(a)\frac{\partial U}{\partial s} + (1 - F(a))\frac{\partial^2 U}{\partial s^2}\frac{\partial s}{\partial a}} . \quad (35)$$

The denominator of (35) is negative, as is all but the last term in the numerator. Hence, a sufficient condition for $\frac{\partial s}{\partial k} > 0$ is that $\frac{\partial^2 s}{\partial k \partial a} > 0$. Assuming that s(a;k) is continuously differentiable, we can rewrite (35) as

$$\frac{d}{da}\left[\frac{\partial s}{\partial k}\right] + \frac{\partial s}{\partial k}\alpha(a) = \beta(a), \quad (36)$$

where

$$\alpha(a) = \frac{(1 - F(a))\frac{\partial^2 U}{\partial s^2}\frac{\partial s}{\partial a} - f(a)\frac{\partial U}{\partial s}}{(1 - F(a))\frac{\partial U}{\partial s}}, \quad \text{and}$$

$$\beta(a) = \frac{f(a)\frac{\partial U}{\partial k} - (1 - F(a))\frac{\partial^2 U}{\partial k \partial s}\frac{\partial s}{\partial a}}{(1 - F(a))\frac{\partial U}{\partial s}} .$$

Equation (36) is a first-order linear differential equation in $\frac{\partial s}{\partial k}$, which has as a solution

$$\frac{\partial s}{\partial k} = e^{-\int \alpha(a)da}\left[\int (e^{\int \alpha(a)da} - \beta(a))da + q\right], \quad (37)$$

where q is a constant. Thus, $\frac{\partial s}{\partial k}$ is positive if and only if the bracketed term in (37) is positive. Given the complicated expressions $\alpha(a)$ and $\beta(a)$, this is yet to be computed.

In summary, universal divinity implies the following: suppose $a = 0$ pools with other types. Increasing from $a = 0$, universal divinity restricts the equilibrium strategy to pooling out to $a = \frac{D}{2}$, or jumping to a separating segment, but not jumping to another pooling segment. Once separating, universal divinity restricts the equilibrium strategy to separating out to $a = \frac{D}{2}$. Hence, if $k \leq k^*$, where k^* solves $U(\frac{D}{2},0;k) = 0$, then the only universally divine equilibria are pooling equilibria, restricted to some interval about the median. If $k > k^*$, then the only equilibria are C_{II} hybrid equilibria, which will be unique for each value k. If $\frac{\partial U(a,s)}{\partial a} = 0$ at $a = s$, then as k approaches ∞ the hybrid equilibria approach the separating equilibrium. If, as is assumed in the following section, $U(\cdot,\cdot)$ is linear in distance, then there will exist a value $\bar{k} < \infty$ which supports the separating equilibrium characterized by equation (10). For values above $\bar{k}$, it is easily seen that the strategy defined by (10) would have $s(a') = a'$, for some $a' > 0$ and $s(a) > a$ for all $a < a'$, which won't be an equilibrium. However, there __is__ a separating equilibrium defined as $s(a) = a$, $\forall\, a \leq a'$, and $s(a) = s^*(a)$, $\forall\, a > a'$, where $s^*(a)$ solves (10). Thus, in the linear case, types continue to separate for parameter values above $\bar{k}$, where the separating strategy is now "kinked" and follows the 45° line for values less than a'.

4. EXAMPLES OF SYMMETRIC EQUILIBRIUM BEHAVIOR

To the assumptions in section 2 we add the following:

A3. $F(\cdot)$ is a uniform distribution over P.

A4. $U(x,y) = r - k(|x - y|)$.

Thus, if a candidate wins the election, he receives an amount (r) for winning minus a constant (k) times the distance between his true position, or type, and his announced position.

By A4, we see that if $\frac{D}{2} \leq \frac{r}{k}$, then there will exist pooling equilibria at all positions satisfying $r - k(\frac{D}{2} + p) \geq 0$; there will exist no C_{II} hybrid equilibria. If $\frac{D}{2} \in (\frac{r}{k}, \frac{2r}{k})$, then there will exist C_{II} hybrid equilibria, unique for each parameter specification $(D, \frac{r}{k})$, while no pooling equilibria will exist.

To calculate the C_{II} hybrid equilibria, we begin by calculating the separating segment of the equilibrium strategy. From section 3 we know that

$$\frac{\partial}{\partial a'}[(1 - \frac{2a'}{D})(r - k(a - s(a')))]\Big|_{a'=a} = 0, \tag{38}$$

which implies

$$s' - \frac{1}{(\frac{D}{2} - a)}s = (\frac{r}{k} - a)(\frac{1}{\frac{D}{2} - a}), \tag{39}$$

which is a first-order linear differential equation. The solution to (39) is

$$s(a) = \frac{1}{(\frac{D}{2} - a)}(\frac{r}{k}a - \frac{a^2}{2} + h), \tag{40}$$

where h is a constant determined by the value restriction $U(\frac{D}{2}, s(\frac{D}{2})) = 0$. This restriction implies the condition $h = \frac{D}{2}(\frac{D}{4} - \frac{r}{k})$; thus after canceling terms,

$$s(a) = \frac{1}{2}a + \frac{D}{4} - \frac{r}{k}. \tag{41}$$

$s(\cdot)$ is of a particularly simple form: $\frac{\partial s}{\partial a} = \frac{1}{2}$, $\frac{\partial^2 s}{\partial a^2} = 0$.

Furthermore, $\frac{\partial s}{\partial k} > 0$, so that $\frac{\partial a_{II}}{\partial k} < 0$, where $a_{II}(k,r)$ solves

$$\frac{k(\frac{1}{2}a - \frac{D}{4})}{r - ka} = \frac{1 - \frac{a}{D}}{1 - \frac{2a}{D}}. \tag{42}$$

In the following four examples, assume D = 4.

Example 1. Let $\frac{r}{k} = 3$. Then there exists pooling equilibria at positions $p \in [-1,1]$.

Example 2. Let $\frac{r}{k} = 2$. Then there exists a pooling equilibrium at $p = 0$.

Example 3. Let $\frac{r}{k} = 1.5$. Then there exists a hybrid equilibrium of the form

$$s(a) = \begin{cases} 0 & \text{if} \quad a \in [0,\frac{4}{3}) \\ \frac{1}{2}a + \frac{1}{2} & \text{if} \quad a \in (\frac{4}{3},2] \end{cases}.$$

See Figure 7.

Example 4. Let $\frac{r}{k} = 1$. Then there exists a hybrid equilibrium of the form $s(a) = \frac{1}{2}, \forall a$. See Figure 8.

Thus, the limit of the hybrid equilibria as $\frac{r}{k}$ decreases to $\frac{D}{4}$ is a separating equilibrium.

5. CONCLUSION

We have developed a model which explicitly incorporates the ability of candidates to misrepresent themselves to voters in regard to their true positions on a policy space. Using equilibrium refinement techniques described in Chapter I, we were able to eliminate numerous types of symmetric equilibria from consideration and were left with equilibrium predictions which, examining the comparative statics, have an intuitively realistic feature; namely, as the costs of misrepresentation increase, candidate types will be more likely to reveal their true position and hence less likely to imitate the median candidate.

Further research will concentrate on, i) asymmetric equilibria in the above model, ii) asymmetric priors over candidate types, and iii) modeling explicitly a multi-election game in which the costs of misrepresentation are endogenously generated.

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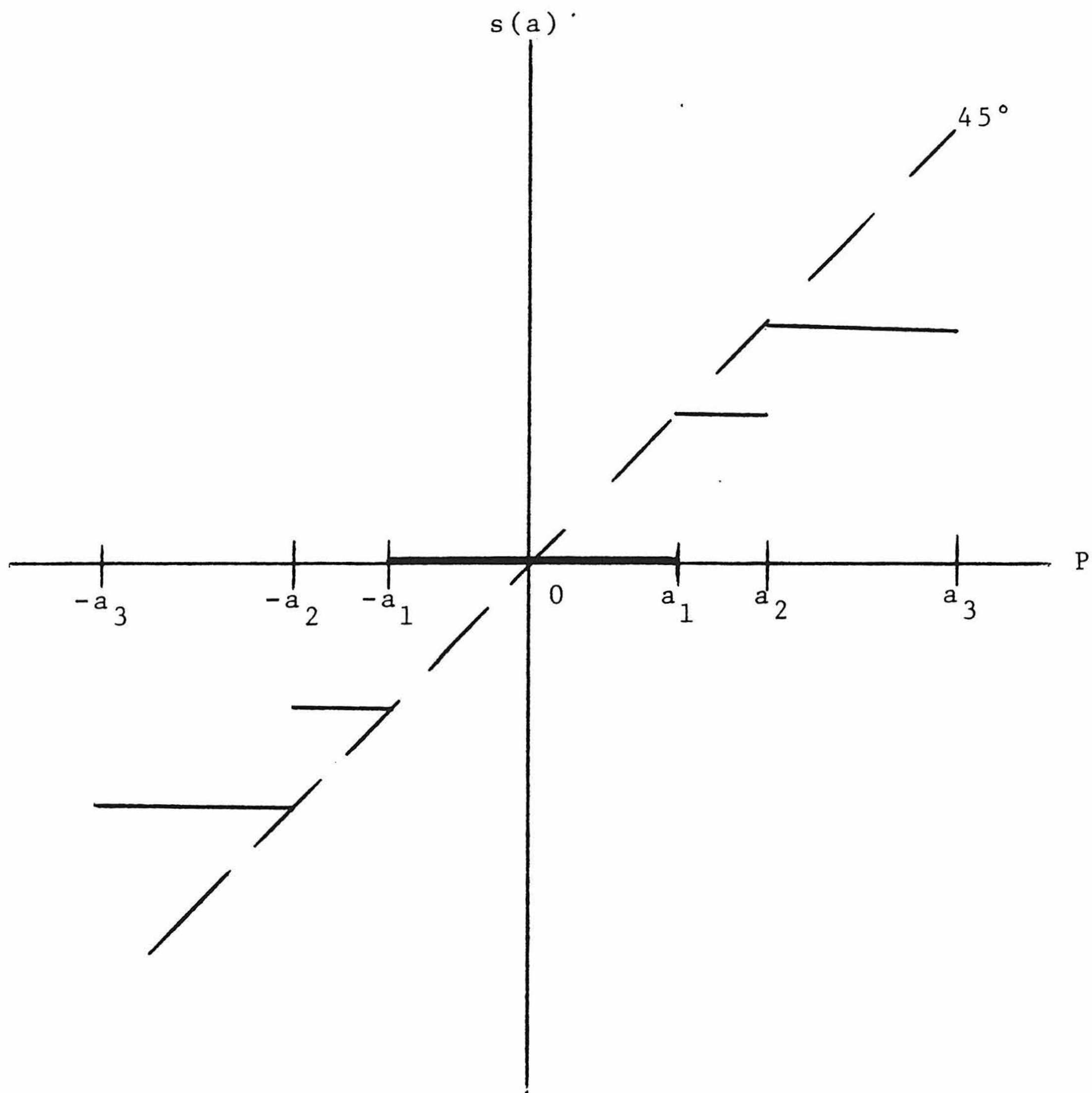


Figure 1

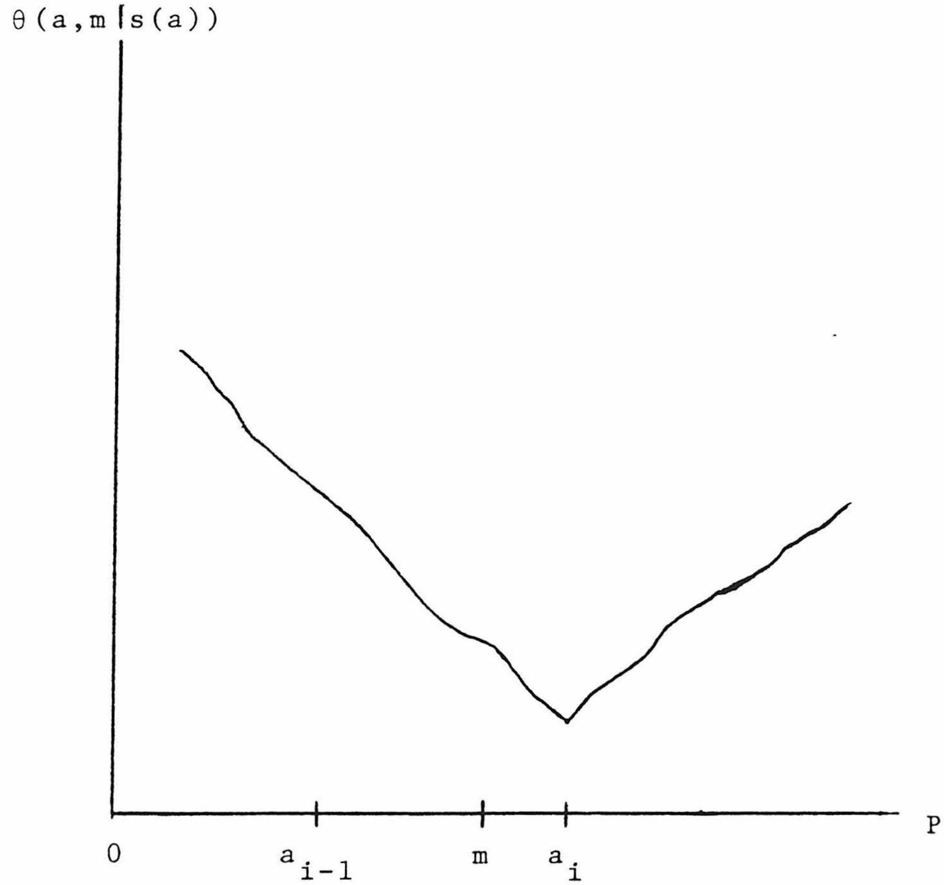
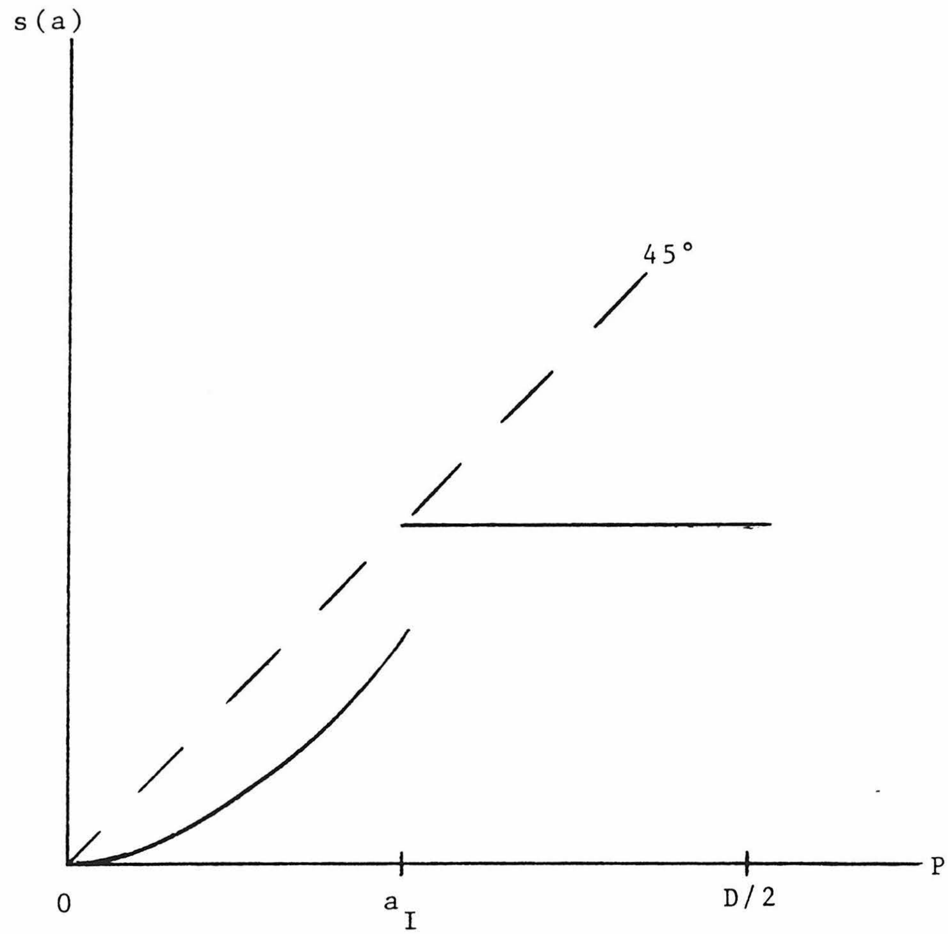


Figure 2



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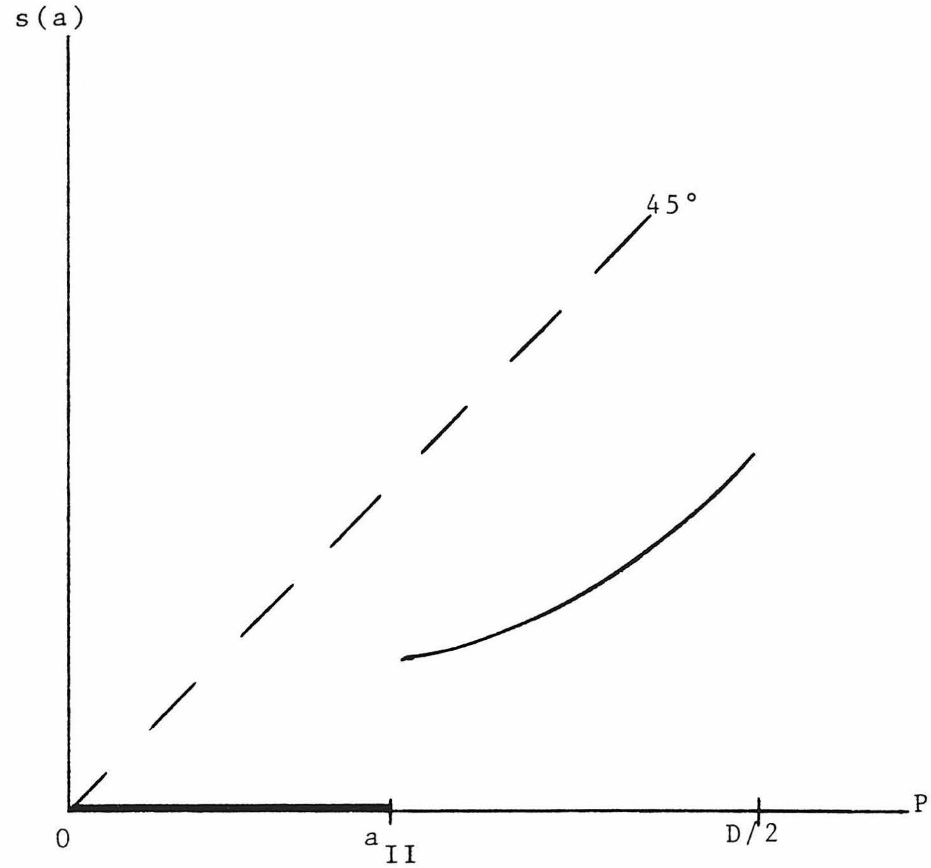

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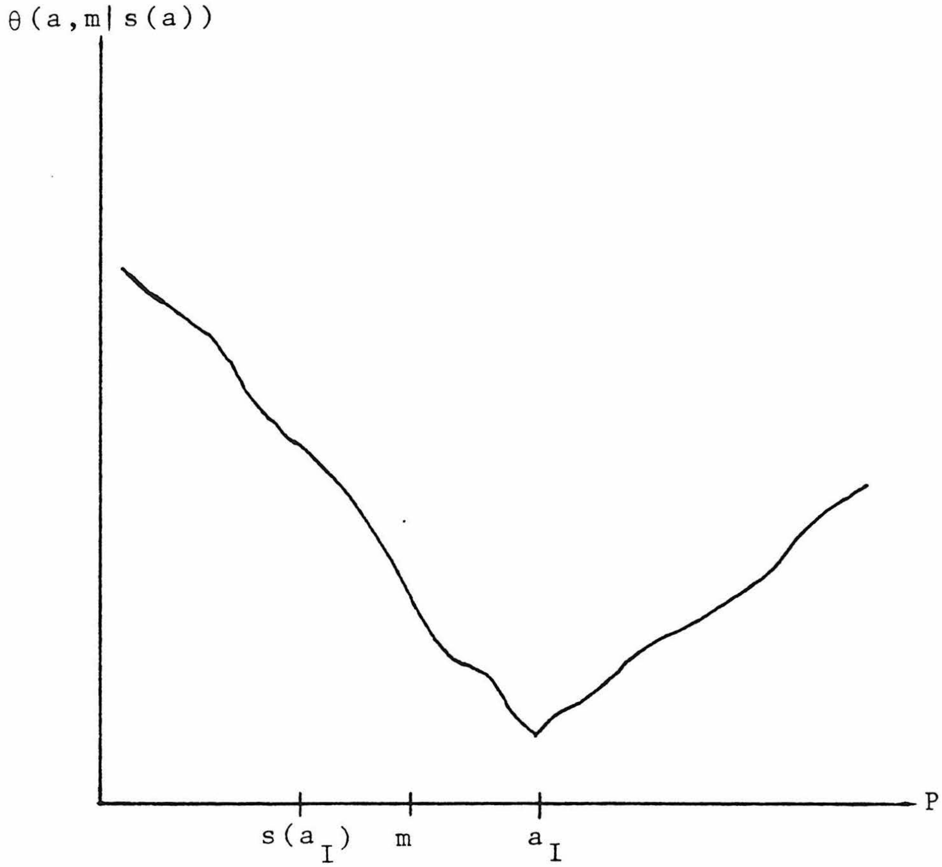

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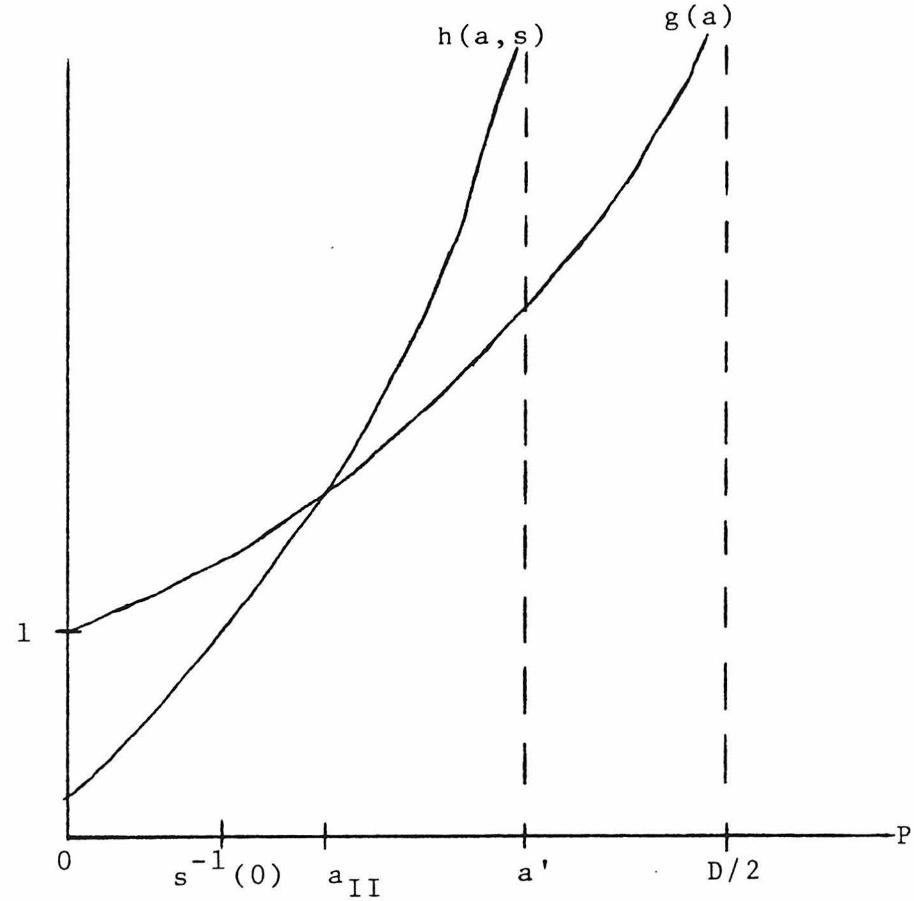

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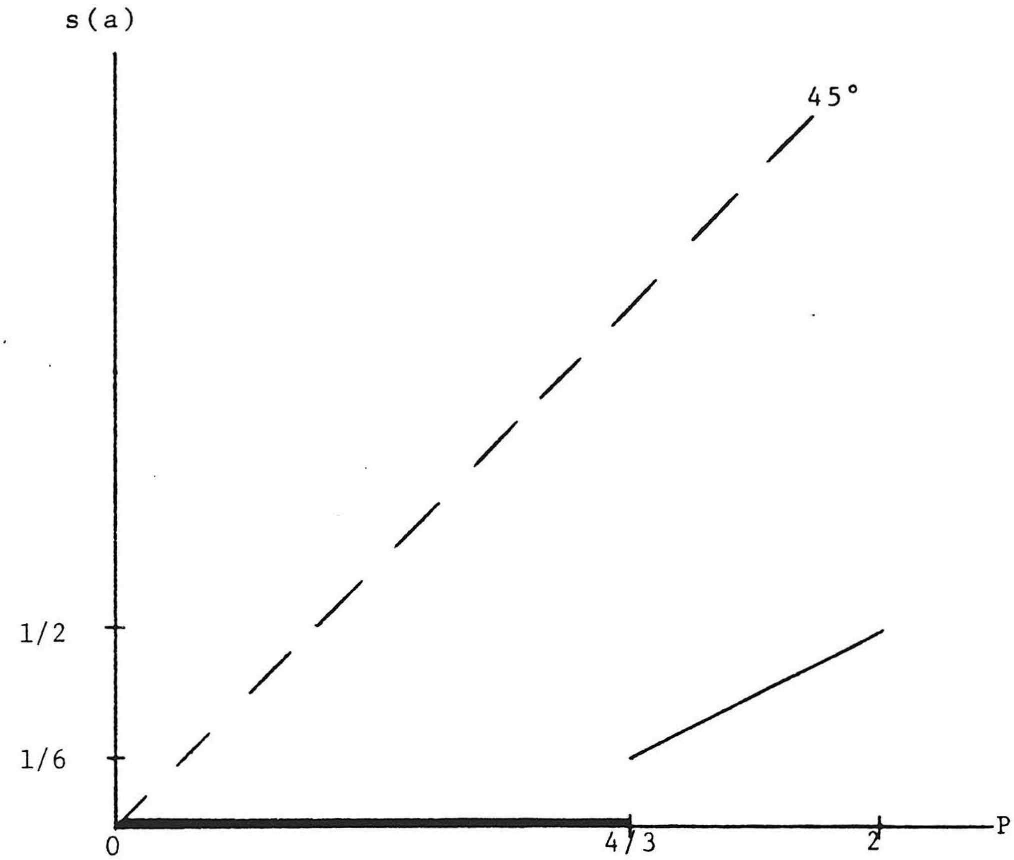

Figure 7

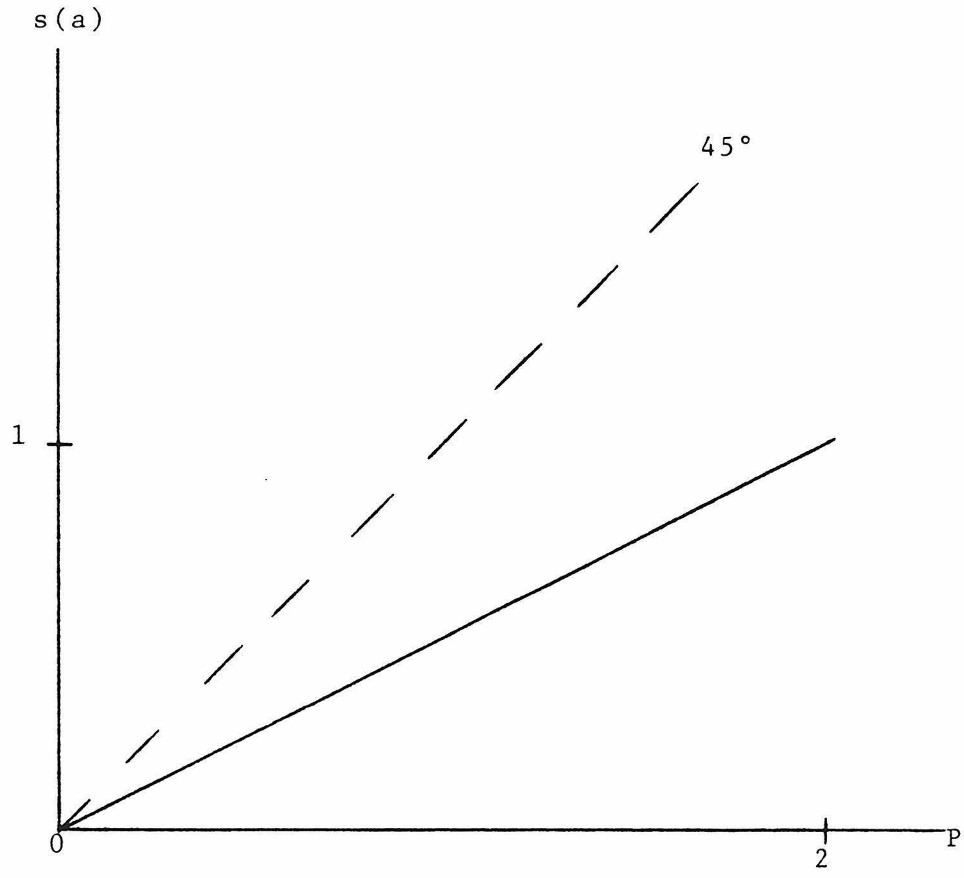

Figure 8