THEORY AND PRACTICE IN THE ANALYSIS OF COMMODITY FUTURES PRICE DISTRIBUTIONS

By

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DEDICATION

For my wife, Leslie

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Introduction

This thesis is concerned with commodity futures markets. More specifically, it addresses itself to two issues— the behavior of commodity futures prices and the effect of these price distributions on hedgers in commodity markets. On the former issue, the distribution of futures prices, the aim is to bring heretofore neglected theoretical implications to an empirical investigation into distributional form. Concerning the latter issue, price distributions and hedging activity, the arguments behind possible trends in futures prices due to short hedging dominance (short hedging in excess of offsetting long hedging, across the entire market) are highlighted, formalized, and tested empirically.

The theory of futures trading is reviewed in Chapter 1. This theory is diverse in approach and ranges from formal expected utility maximization models to less rigorous approaches. In an effort to bring cohesion to this diverse literature, a general framework regarding the participants in futures trading (long and short hedgers, and speculators) is used to derive propositions about the assorted "pieces" and how they fit together. The tie that binds this literature is the effect of stock levels on participant activity. The roots of the long-standing controversy over whether or not there are trends in the futures price are shown to be related to seasonality in commodity stocks. A variety of views on price trends exist, from arguments against any trend, to arguments of a rising futures price throughout the duration of the contract, to a seasonal trend in prices argument. However, each is tied crucially to the level of commodity stocks.

On the issue of distributional form, the theory reviewed in Chapter 1 also gives sound reasons for why the distribution of futures prices is not constant over the harvest cycle. The reasons are directly related to the behavior of participants based on stock levels. That distributions may not be constant over time has been studied by many writers concerned primarily with identifying the distributional characteristics of price series. A survey of such work comprises Chapter 2. By focusing on exceptions to the rule, it is argued there that past work ignores theoretical implications for empirical studies of commodity futures price distributions.

Chapter 3 formally states the theoretical implications for empirical testing. First, while the relative changes in futures prices perform a random walk, the parameters of the random walk may change. The factor behind such a change is the predictable behavior of commodity stock levels and futures market participants during the harvest cycle. Second, also based upon the level of stocks, Houthakker's notion that changes in the correlation between cash and futures prices at low versus high stock levels causes short hedgers to dominate in futures trading is discussed and a related argument based on cumulative density functions of the futures price is set out for empirical analysis. The empirical approach, taken from the implications of the theory, is to construct time series samples for the periods before the peak in commercial stocks and after. Then, the two theoretical implications are examined, empirically. Chapter 4 contains the results and the thesis concludes in Chapter 5, with a summary and suggestions for future research. Another interesting aspect of futures prices is addressed in Chapter 4, the imposition of limits on futures price movements by the exchange. Designed to minimize default in the event of drastic price changes, such limits open the question of "censoring" problems in the data.

Before proceeding, it is worth stating some of the terms and concepts connected with commodities futures. Commodity futures markets serve the need to allocate stocks of stored commodities over time. In the case of agricultural commodities, stocks exist because it is more economical to produce and store output than it is to produce continually. Incorrect consumption and production plans that lead to unintended excess amounts of a commodity contribute further to stock levels in the form of carryover from one harvest period to the next.

A variety of markets pertain to this allocative need. Spot markets provide for immediate delivery of a given commodity grade, or quality specification, at a currently quoted price. Forward markets concern later delivery of a particular commodity grade at a specific delivery location and date. Forward contracts are often so detailed as to specify the amounts and types of pesticides which can be used in the production of the commodity under contract. No payment on the forward contract is due until delivery is made. Given the attention to detail exhibited in forward contracts, they are virtually impossible to resell; forward contracts are intended for the transfer of physical units of the commodity. Since money changes hands immediately in spot transactions and at termination of the forward contract, between the parties to the original agreement, these two markets are often referred to as the cash market.

The essential difference between the cash market and the futures market lies in the flexibility and liquidity of the futures contract. Like the forward contract, the futures contract concerns delivery at a later date but the contracts are quite distinct on all other counts. Forward markets are heterogeneous in commodity quality, delivery location, and time of delivery. Futures contracts, on the other hand, specify a particular delivery grade but allow for premium and penalty payments for failure to deliver the specified grade. Further, rather than a specific delivery location and date, the futures contract allows for a variety of delivery locations and specifies only the delivery month. Since the seller of a futures contract has discretion over so many aspects of delivery, futures contracts are poorly suited to physical delivery needs; futures contract buyers do not have more than a vague notion of the quality and location of the commodity they would have to accept by taking delivery on the futures contract. However, the high degree of contract standardization in futures makes them an extremely viable means of exchange.

Cash markets are characterized by heterogeneity and high transactions costs; the cost of bringing buyers and sellers together is high. On the other hand, since futures contracts are nothing more than a claim to title over resources at some future date, they are well-suited to traders who may not wish to bind themselves irreversibly to an agreement to deliver. Speculators are among those who wish flexibility but to an important extent so are hedgers.

In addition to flexibility in commitments, futures contracts provide an important link between the cash and futures markets. Delivery on futures contracts is largely a fiction, but the threat of delivery or acceptance of delivery has an important implication: the price of a futures contract at the delivery date must equal the cash price of at least one deliverable grade at that point in time. Indeed, since the grade to deliver is at the seller's discretion, it is the lowest priced among cash grades which is most likely to be delivered. The establishment of this arbitrage imposed relationship between cash and futures prices provides traders with a means to earn profits. In this respect, the difference between current quotes of futures and cash prices provide an approximate index of the returns to be earned from storing a commodity over time. This difference is commonly referred to as the basis, and the uncertainty associated with its behavior is called basis risk. More generally, the differece between current quotes of futures prices for different delivery dates provides a basis for longer storage horizons and indexes the returns from trading only in futures contracts, the activity of pure speculators.

In Chapter 1, the theory of futures markets is discussed in three sections, which can be summarized under two theory headings. The high transactions costs and associated riskiness in the cash market led to the theory of the risk premium. Quite simply, traders holding the cash commodity face risks that other traders may be willing to bear more cheaply. It is the price of risk bearing that was coined the risk premium. Payment to inventory holders for providing the service of moving resources through time led to the theory of the price of storage.

According to this theory, inventory is held in the expectation that profits will result and it is hedgers, rather than speculators, who must be paid for the services they provide. The aim of the theory review is to outline the richness of the theoretical implications for an analysis of futures price distributions. The conclusion of the review is that such implications are fertile areas for cultivating a broader understanding of the behavior of futures prices. Contributions in that direction constitute the remainder of the thesis.

CHAPTER 1

Review of the Theory of Futures Markets

Introduction

Students of commodity futures markets are both blessed and damned by the volume of past theoretical works. On one hand, the traditional depth and evolution of the theory make for fascinating and rewarding study. On the other, the theory provides little in the way of uniform study material; given the diversity in analytical technique, identifying precisely the assumptions and connecting threads of past lines of argument is a challenging task. But the richness of theoretical implications for an analysis of futures price distributions makes any attempt at providing a cohesive overview well worth the effort. The attempt here is to fit as much of the literature as possible into a general explanatory framework by identifying where, within the framework, different portions of the literature lie. In some cases, the results are quite successful and many of the reasons for past conflicts are unveiled. In others, the violations of the framework are particularly revealing. In still others, the framework offers little more than the original works themselves. The goals of the chapter are to bring cohesion to a diverse literature and highlight the theoretical implications for analyzing futures price distributions.

The general framework adopted is based on expected utility analysis. To see that the framework is indeed general, the following overview is offered. In the case of an underlying normal distribution, the expected utility model reduces to mean-variance portfolio analysis. within both models, subject to further restrictions, lie two special cases which dominate the literature: futures markets where no basis risk exists and the case of pure forward, as opposed to futures, trading. Hence, the framework allows the various threads of the literature to be tied into a cohesive unit. Further, the special cases provide important insight into the distribution of futures prices. Most importantly, the framework reveals the evolution of ideas culminating in Houthakker's observations about the interrelationships between three factors: hedging behavior, the correlation between cash and futures prices, and the level of commodity stocks.

This review will not cover the theory concerned with information aggregation problems in commodity futures markets (Grossman and Stiglitz [15], Danthine [9], Bray [5] or general equilibrium exchange economy applications (Hirschleifer [18], Feiger [12], Salant [31], Richard and Sundaresan [28]). Futures price patterns in the former are based on information asymmetries and establishing a rationale for such occurrences is difficult. Futher, fundamental characteristics of futures markets are ignored, most notably a full variety of participants and true futures, as opposed to forward, trading. The point of describing a model that is generally descriptive of futures trading is to encompass the elements of functioning markets. For example, general equilibrium exchange approaches deal with consumers rather than firms and are unwieldy for an analysis of operational markets where specialized assumptions on firm behavior prove most revealing.

The General Model

The approach here is adopted from Anderson and Danthine [2] but the assumptions governing revenue functions confronting the variety of futures market participants are quite distinct and the analysis extends beyond their focus upon pure forward, as opposed to futures, trading. A two-period model is assumed throughout (times 0 and 1). Participants can be involved in productive transformation of the spot commodity and have access to a single futures contract defined on that commodity.

A more descriptive model of futures trading would include a multiplicity of contracts and an extended time period. Anderson and Danthine [1] allow for a multiplicity of contracts but under the special case of mean-variance analysis. None of the literature treated here is time-general.[1] The aim is to bring a variety of theories together and the level of generality chosen suffices for that purpose. The following model is neither time-general nor designed to allow trading in more than one contract but neither is the literature it treats. With the exception noted above, the literature covers single contract, two-period models.

The participants can be described as we turn to the specification of production interests. Identified by subscript p, producers are assumed to choose a nonstochastic output level y_p at time 0, incurring costs associated with that level of $b(y_p)$. The assumption of nonstochastic production is clearly unrealistic and will be examined later. Net revenues from production of the commodity are

$$(1)\ \Pi_p(y_p) = C_1 y_p - b(y_p) + R(y_p),$$

where C_1 is the time 1 cash price and $R(y_p)$ represents returns strictly related to activities concerning the spot commodity, independent of changes in cash or futures prices, e.g., commodity "grading" by elevator operators. The function $R(\bullet)$ will be assumed the same for all participants involved with transformations of the commodity and strictly concave, i.e., $R(0) = 0$, $R' > 0$, and $R'' < 0$. This can be justified by the equating of returns across industries through free entry. The subscript e identifies elevator operators who commit themselves to carry an amount of the commodity y_e between times 0 and 1. Assuming the commodity is perfectly nonperishable, storage revenues are

$$(2)\ \Pi_e(y_e) = (C_1 - C_0)y_e - k(y_e) + R(y_e),$$

where $k(\bullet)$ represents known costs of storage, C_0 is the time 0 cash price, and $R(\bullet)$ is as specified in (1). At this point, the relation between storage costs and the level of the spot commitment is left unspecified for reasons that will be made clear later. The subscript m refers to millers who require the amount y_m as an input at time 1 and are assumed to make competitive bids on the sale of their output based upon the known costs of storage in (2). Including their entrepreneurial return results in a net spot revenue function exactly symmetric to storage operation revenues:

$$(3)\ \Pi_m(y_m) = (C_0 - C_1)y_m + k(y_m) + R(y_m).$$

All profit functions are assumed strictly concave, [$\Pi(0) = 0$,

$\Pi' < 0$, $\Pi'' < 0$]. Speculators indicated by subscript s are defined as having no production-oriented interest in the commodity other than profiting from changes in its price; $y_s = \Pi_s = 0$.

Let F_0 and F_1 be the time 0 and 1 futures prices, respectively, and Q the number of futures contracts bought or sold. Buyers earn $(F_1 - F_0)Q$ and sellers earn $(F_0 - F_1)Q$ on their transactions. Holders of the spot commodity, here the producers and elevator operators, attempt to reduce the risk of changes in the value of their holdings by selling futures. Millers face the opposite problem and buy futures while speculators can either sell or buy since they have no spot commitment. The following expressions define the sum of production and futures trading revenues:

(4) $V_p = \Pi_p(y_p) + (F_0 - F_1)Q_p$

(5) $V_e = \Pi_e(y_e) + (F_0 - F_1)Q_e$

(6) $V_m = \Pi_m(y_m) + (F_1 - F_0)Q_m$

(7) $V_s = (F_0 - F_1)Q_s$.

It is further assumed that at time 0 all participants 1) maximize the same strictly concave Von Neuman-Morgenstern utility function, 2) hold identical marginal pdfs, $h_F(F_1)$ and $h_C(C_1)$, over the random time 1 futures and cash price, respectively, and 3) hold the same joint pdf, $h(F_1,C_1)$ over these prices. Speculators maximize $\int\int U(V_s)h(F_1, C_1)dF_1dC_1$ with respect to Q_s. ($Q_s < 0$ represents purchases and $Q_s > 0$ speculative sales of futures contracts). The first-order condition is (sufficient as well, for strictly concave utility):

(8) $F_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 - \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1 = 0.$

Producers, elevator operators, and millers maximize the following (appropriately subscripted) with respect to y_i and Q_i, $i = p, e$, and m: $\iint U(V_i)h(F_1, C_1)dF_1dC_1$. The first-order conditions for producers with respect to y_p and Q_p are (again, also sufficient):

$$(9)\quad [R'(y_p) - b'(y_p)] \iint U'(V_p)h(F_1,C_1)dF_1dC_1 + \iint C_1U'(V_p)h(F_1,C_1)dF_1dC_1 \le 0$$

$$(10)\quad F_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 - \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1 = 0,$$

and expression (9) will be strictly equal to zero for $y_p > 0$. For elevator operators, the first-order conditions with respect to y_e and Q_e are

$$(11)\quad [R'(y_e) - k'(y_e) - C_0] \iint U'(V_e)h(F_1,C_1)dF_1dC_1 + \iint C_1U'(V_e)h(F_1,C_1)dF_1dC_1 \le 0$$

$$(12)\quad F_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 - \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1 = 0,$$

and expression (11) is strictly equal to zero for $y_e > 0$. Finally, the solution to the miller's problem must satisfy (with respect to y_m and Q_m)

$$(13)\quad [C_0 + k'(y_m) + R'(y_m)] \iint U'(V_m)h(F_1,C_1)dF_1dC_1 - \iint C_1U'(V_m)h(F_1,C_1)dF_1dC_1 \le 0$$

$$(14)\quad \iint F_1U'(V_m)h(F_1,C_1)dF_1dC_1 - F_0 \iint U'(V_m)h(F_1,C_1)dF_1dC_1 = 0,$$

and expression (13) is strictly equal to zero for $y_m > 0$. In what follows, the analysis is restricted to the case of interior maxima. Expressions (8) - (14), for the general case, provide some insight into the behavior of the various participants. For participants with a production interest in the spot commodity, we are interested in circumstances under which the spot position is or is not completely covered with futures contract commitments. About this relationship, the preceding development offers little but under later restrictions proves enlightening. For speculators, however, the following is seen to hold in general.

Proposition 1:

Speculators, whose problem is represented by expression (8), will sell futures contracts if and only if $F_0 > EF_1$, buy if and only if $F_0 < EF_1$, and assume no futures position if and only if $F_0 = EF_1$.

Proof: The second derivative of the speculator's expected utility problem, with respect to Q_s is

$$\iint (F_0 - F_1)^2 U''(V_s) h(F_1, C_1) dF_1 dC_1 < 0,$$

strictly negative since $U(V_s)$ is strictly concave, assuming a nondegenerate pdf h. Hence, expected utility is strictly concave in Q_s, implying for any two distinct futures choices, Q_s^0 and Q_s^1, that $Q_s^0 \geq Q_s^1$ is equivalent to

$$\partial EU(Q_s^0)/\partial Q_s \leq \partial EU(Q_s^1)/\partial Q_s.$$

Let Q_s^* solve (8), so that $\partial EU/\partial Q_s = 0$.

Now, $Q_s^* \geq Q_s$ <u>iff</u> $\partial EU(Q_s)/\partial Q_s \geq 0$

$(Q_s^* > 0)$. But $\partial EU/\partial Q_s \geq 0$ is equivalent

to

$$F_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 \geq \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1,$$

by (8). In particular, let $Q_s = 0$, so that

$U'[(F_0 - F_1)0] = U'(0) > 0$ can be cancelled. The

remaining result is $Q_s^* \geq 0$ <u>iff</u> $F_0 \geq EF_1$, and

$Q_s^* > 0$ <u>iff</u> $F_0 > EF_1$. On the other hand, for

$Q_s^* < 0$, $Q_s^* \leq Q_s$ <u>iff</u>

$\partial EU(Q_s)/\partial Q_s \leq 0$. But $\partial EU(Q_s)/\partial Q_s \leq 0$ is

equivalent to

$$F_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 \leq \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1.$$

Again, letting $Q_s = 0$, one finds $Q_s^* \leq 0$ <u>iff</u>

$F_0 \leq EF_1$. Hence, $Q_s^* \gtrless 0$ <u>iff</u>

$F_0 \gtrless EF_1$.

The proposition is an extension of that given in Anderson and Danthine [2] to the general case of an arbitrary strictly concave utility function in a true futures market. Others have proven more restrictive versions of Proposition 1 and their contributions will be noted in the cases examined shortly.

Clearance in the market for futures contracts is defined by

(15) $n_pQ_p + n_eQ_e = n_mQ_m + n_sQ_s,$

where n denotes the number of participants of each subscripted type. As noted before, little can be said regarding participants with a production interest at this point.[2] Whether the market exhibits excess sales or purchases, in the absence of speculators, is of crucial interest in the cases to which we now turn our attention; analysis of those with spot commitments is most revealing there.

Mean-Variance Analysis

The distinguishing characteristic of mean-variance analysis (MVA) is its assumption that participants choose spot and futures positions on the basis of resulting means and variances of final profits. The primary theoretical endeavors in the MVA tradition seek to determine conditions under which hedges will violate the "routine" hedge (futures position exactly offsetting the spot position), how hedging opportunities affect real output decisions, and most recently the circumstances concerning hedging availability when the particular spot commodity has no organized futures market. The mean-variance assumption is satisfied when final profits are normally distributed. Further assuming constant absolute risk aversion results in well-behaved (linear) utility functions.

The unknown time 1 cash and futures prices are assumed to have a subjectively viewed joint probability density; means EC_1 and EF_1, positive finite variances $\sigma^2_{C_1}$ and $\sigma^2_{F_1}$, and covariance $\sigma_{C_1F_1}$ are all known. The problem is to maximize expected utility and, under the assumptions of normally distributed final profits and constant absolute risk aversion, we write

$$(16)\quad \max EV - (1/2)\,\chi\sigma^2_V,$$

where V is as defined for the various participants in expressions (4) - (7) and χ is the individual risk aversion parameter which will be assumed positive (risk aversion). The variance of returns confronting participants concerned with a production interest in the spot commodity is

$$(17)\quad \sigma^2_V = y^2\sigma^2_{C_1} - 2Qy\sigma_{C_1F_1} + Q^2\sigma^2_{F_1},$$

while for pure speculators the variance is $Q^2\sigma^2_{F_1}$. The first-order condition from (16), using expression (7) for speculators, implies

$$(18)\quad Q^*_s \gtrless (F_0 - EF_1)/\ \chi\sigma^2_{F_1} \ \text{ as } F_0 \gtrless EF_1,$$

where Q^*_s is the optimal futures position. It is clear in (18) that speculative sales occur if and only if $F_0 > EF_1$ and speculative purchases when $EF_1 > F_0$. Hence, as in the general case, Proposition 1 holds for MVA.

In order to give the flavor of the solution to hedgers' problems under MVA, analysis of the producer suffices. The first-order conditions for (16), assuming an interior solution and using (4) for producers, with respect to y and Q, respectively, are

$$(19)\quad EC_1 - b'(y_p) + R'(y_p) + \chi(Q_p\sigma_{C_1F_1} - y_p\sigma^2_{C_1}) = 0$$

$$(20)\quad F_0 - EF_1 - \chi(Q_p\sigma^2_{F_1} - y_p\sigma_{C_1F_1}) = 0.$$

In this case of only one futures contract, the optimal spot and futures positions, y^*_p and Q^*_p, respectively, are

$$(21)\quad y_p^* = [EC_1 + R'(y_p) - b'(y_p) + \beta_1(F_0 - EF_1)]/\chi\sigma^2{}_{C_1}(1 - \rho^2)$$

$$(22)\quad Q_p^* = [F_0 - EF_1 + \beta_2(EC_1 - b'(y_p) + R'(y_p))]/\ \chi\sigma^2{}_{F_1}(1 - \rho^2).$$

The terms β_1 and β_2 are coefficients from simple regressions of cash on futures prices and futures on cash prices, respectively. For example, $\beta_1 = \sigma_{C_1F_1}/\sigma^2{}_{F_1}$. ρ^2 is the simple correlation coefficient squared. Anderson and Danthine [1] show that the following line of investigation is more profitable. By (20),

$$(23)\quad Q_p^* = (F_0 - EF_1)/\ \chi\sigma^2{}_{F_1} + y_p^*\beta_1;$$

there is a speculative component, equivalent to a pure speculative decision as in (18), and a pure risk-reducing component, dependent upon the explanatory power of the cash price regarding the futures price (β_1), in the optimal hedge.

Additional insight into the optimal spot position can be gained by rearranging (21):

$$(24)\quad EC_1 + R'(y_p^*) + (F_0 - EF_1)\beta_1 = \chi y_p^*\sigma^2{}_{C_1}(1 - \rho^2) + b'(y_p^*).$$

This is simply the necessary condition for y, given that Q adjusts optimally to variations in y. The L.H.S. is marginal revenue and the R.H.S. is marginal cost. The latter is the sum of marginal production costs and what can be termed a risk premium; the combined influences of individual risk aversion, cash price variance, and the portion of the total variation in the futures price that remains unexplained by the cash price. The risk premium has the property that it decreases with increases in ρ^2.[3] The difference between futures and cash prices, or what is commonly referred to as the "basis," is often of theoretical interest. Adding and subtracting $\beta_1(EC_1 - C_0)$ on the L.H.S. of (24), marginal revenue becomes

$$(25) \quad EC_1 - \beta_1(EC_1 - C_0) + \beta_1(B_0 - EB_1) + R'(y_p^*),$$

where $B_t = F_t - C_t$ is the basis at time t.

Expression (23) can be used to summarize MVA findings regarding the routine hedge. Again, continuing the analysis of the producer suffices to make the point. Producers would usually be viewed as selling futures against their production that will be available at time 1. Let $z = (F_0 - EF_1)/\ \chi\sigma^2_{F_1}$ in (23) and suppose the producer subjectively views the covariance as positive. If he further believes that $EF_1 < F_0$ (expected fall in the futures price), since $y_p^* > 0$, the traditional view holds and the producer short hedges. However, if $EF_1 < F_0$ (expected rise in the futures price), we may observe the producer buying futures, or, as it has come to be called, reverse hedging. If $EF_1 > F_0$ and $z < y_p^*\beta_1$, again, the producer short hedges. But if $z > y_p^*\beta_1$, the producer buys futures even though he is committed to future purchases of the spot commodity, provided he views the covariance as positive. Now, one can think of the $\chi\sigma^2_{F_1}$ term as the total risk weight associated with the futures position and $y_p^*\beta_1$ as the predictable change in the value of the cash position, associated with a change in the futures price (by definition,

$\beta_1 = \partial C_1 / \partial F_1$). If we rewrite

$z < y_p^* \beta_1$ as

$$(26)\quad F_0 - EF_1 < \chi\sigma^2_{F_1} y_p^* \beta_1,$$

then with $EF_1 > F_0$ and positive covariance, a traditionally held short futures trader such as a producer will actually buy futures when the expected rise in the futures price exceeds the risk weighted change in the value of the spot commitment associated with the futures price change. The story is exactly symmetric when the covariance is believed to negative, as shown in Table 1, where for a short time Z stands in place of $y_p^* \beta_1$.

Table 1. The Variety of Hedging Possibilities Revealed by MVA.

		Covariance			
		Positive		Negative	
		$EF_1 < F_0$	$EF_1 > F_0$	$EF_1 < F_0$	$EF_1 > F_0$
Futures	Sell	never	$z > Z$	$z < Z$	always
	Buy	always	$z < Z$	$z > Z$	never

The MVA assumption of normally distributed final profits allows for substantial insight into the behavior of participants with production interests in the spot commodity. However, the normality assumption is quite restrictive when one looks at true futures (as contrasted with forward) markets. In the succeeding sections, other restrictions on the general model in the form of assumptions regarding the relation between cash and futures prices round out the requirements for a fairly in-depth study of the literature.

<u>The Absence of Basis Risk</u>

One class of restrictions imposed upon the general model involves price relationships. The least restrictive, of interest in analyzing the literature, is the case of equal absolute changes in the difference between the futures and the spot price. This difference is known as the basis and equiproportionate changes in the basis mean that there is no risk from price movements, even when the later prices are unknown:

$$(27)\quad B_1 = F_1 - C_1 = F_0 - C_0 = B_0.$$

Technically, there is no basis risk when changes in the spot price equal changes in the futures price, plus or minus a constant term. Hence, the definition in (27) is adopted only for simplicity's sake. There are two important virtues of this special case. First, the market for later delivery remains a true futures market, as opposed to a pure forward trading market covered later. To see this, note only that the cash and futures market do not converge at time one, i.e., the time one result is not $F_1 \equiv C_1$. Second, it allows for heretofore unexplored unification of past work.

Optimality under the case of no basis risk is first examined for speculators. Given (27), we may rewrite (8) as

$$(28)\quad C_0 \iint U'(V_s)h(F_1,C_1)dF_1dC_1 - \iint F_1U'(V_s)h(F_1,C_1)dF_1dC_1 = 0.$$

This leads immediately to the following corollary to Proposition 1.

Corollary 1.1:

In the absence of basis risk, as defined in (27), speculators whose problem is represented by expressions (8) will sell futures contracts if and only if $F_0 > EF_1$ $(C_0 > EC_1)$, buy futures contracts if and only if $F_0 < EF_1$ $(C_0 < EC_1)$, and assume no position if and only if $F_0 = EF_1$ $(C_0 = EC_1)$.

Proof: Identical to the proof of Proposition 1 where, by virtue of (27), C_0 and C_1 can be substituted for F_0 and F_1, respectively.

Hence, the assumption of no basis risk provides one simplification unavailable to speculators under the preceding general model; namely, that the relation between current and expected spot prices can be substituted at will for the futures price relation of Proposition 1.

For producers, elevator operators, and millers, expression (27) gives

$$(29) \quad C_0 \iint U'(V_i)h(F_1,C_1)dF_1dC_1 = \iint C_1U'(V_i)h(F_1,C_1)dF_1dC_1,$$

$i = p, e, m$, when substituted into their respective first-order conditions with respect to Q. Subsequent substitution into their respective first-order conditions with respect to y for $\iint C_1U'(V_i)h(F_1,C_1)dF_1dC_1$, $i = p, e, m$, gives:

$$(30) \quad R'(y_p) = b'(y_p) - C_0 \qquad \text{(for producers)}$$

$$(31) \quad R'(y_e) = k'(y_e) \qquad \text{(for elevator operators)}$$

$$(32) \quad R'(y_m) = k'(y_m) \qquad \text{(for millers).}$$

Expression (30) shows that producers choose their output level so as to equate marginal returns to the excess of marginal production costs over the current cash price. This choice is independent of their expectations about the later spot price or the form of their utility functions (i.e., risk attitudes). The ability to separate spot from futures choices is strictly the result of assuming no basis risk and nonrandom production. Expressions (31) and (32) show that this "separation result" also holds for the remaining participants, and emphasizes the assumed symmetry between elevator operators and millers. As noted at the outset, and again for Proposition 1, the separation result here is an extension of the outcome found by Anderson and Danthine [2] in the case of pure forward trading.

The importance of the result can be seen in the solution of the remaining participants' optimal futures positions. Let y^*, appropriately subscripted, solve (30) - (32), and rewrite (4) - (6), spot traders' profits, as

$$(33) \quad V_p = \Pi_p^* + (F_0 - F_1)D_p$$

$$\text{where } \Pi_p^* = C_0 y_p^* - b(y_p^*) + R(y_p^*)$$

$$D_p = Q_p - y_p^*$$

$$(34)\quad V_e = \Pi_e^* + (F_0 - F_1)D_e$$

$$\text{where } \Pi_e^* = R(y_e^*) - k(y_e^*)$$

$$D_e = Q_e - y_e^*$$

$$(35)\quad V_m = \Pi_m^* + (F_1 - F_0)D_m$$

$$\text{where } \Pi_m^* = R(y_m^*) + k(y_m^*)$$

$$D_m = Q_m - y_m^*.$$

Π^* is simply spot commodity transformation revenues at the optimal, and separable, spot choice. Hence, the optimal deviation from a futures position equal to the spot commitment can be had from maximizing $EU(V_i)$ with respect to Q_i, $i = p, e, m$ in (33) - (35), respectively. Thus, there is a speculative element in every maximization problem as can be easily seen by noting that $D = Q$ for speculators who have no production interest. Hence, along with Corollary 1.1, the case of no basis risk provides the following proposition for participants with production interests, by the same proof as for speculators (see Anderson and Danthine [2] for the case of pure forward trading).

Proposition 2:

In the absence of basis risk, as defined in (27), the deviation from a futures position just equal to the spot commitment occurs as follows.

(a) $D_p \gtrless 0$ iff $F_0 \gtrless EF_1$ ($C_0 \gtrless EC_1$)

(b) $D_e \gtrless 0$ iff $F_0 \gtrless EF_1$ ($C_0 \gtrless EC_1$)

(c) $D_m \gtrless 0$ iff $EF_1 \gtrless F_0$ ($EC_1 \gtrless C_0$)

where participants are as described in (9) - (14).

Participants involved in productive transformation of the spot commodity will cover their spot commitment, no more and no less, through futures contracting when, and only when, their expectation of the later futures (or spot) price equals the current futures (or spot) price. The routine hedge occurs when (and only when) $F_0 = EF_1$.

The separation result and use of the deviation notation allow the market clearance condition (15) to be written (substituting $Q = D + y$):

$$(36) \quad n_p y_p^* + n_e y_e^* - n_m y_m^* = n_m D_m + n_s Q_s - (n_p D_p + n_e D_e),$$

providing the following proposition which greatly facilitates reviewing past works.

Proposition 3:

$F_0 = EF_1$ ($C_0 = EC_1$) is an equilibrium which can be established without speculative participation under the following circumstances:

a) no basis risk (specifically, expression (27)),

b) nonstochastic productive transformation of the spot commodity,

c) identical strictly concave utility functions and identical subjective probability density functions for all participants,

d) $b'(y_p) = C_0$, $k'(y_e) = k'(y_m) = 0$, and, finally,

e) the number of participants who plan to sell spot at time 1 equals the number who plan to buy spot at time 1.

Proof: Let $F_0 = EF_1$ $(C_0 = EC_1)$. By Propositions 1 and 2, (36) implies that the futures market clears when

$$n_p y_p^* + n_e y_e^* = n_m y_m^*.$$

Let $b'(y_p) = C_0$, $k'(y_e) = k'(y_m) = 0$. Expressions (30) - (32) yield

$$y_p^* = y_e^* = y_m^* = y.$$

Now, the market clears when

$$(n_p + n_e - n_m)y = 0,$$

that is, when $n_p + n_e = n_m$. (But this is just the stipulation of part e.)

In summary, the assumption of no basis risk as in expression (27) provides two important results. The first, summarized in Corollary 1.1, is that expectations concerning the spot price may be interchanged with expectations about the futures price. The second, and more significant, result is that the spot and futures choices can be separated from each other; the futures choice remains dependent upon expectations of the later futures price while the optimal spot choice is independent of expectations and the form of the utility function. Imposing additional structure upon the relationship between cash and futures prices moves us into the final area of interest important to a review of the theoretical literature, pure forward trading.

Pure Forward Trading

The final restriction upon the relation between cash and futures prices common to the literature results in the futures market's actually becoming a pure forward trading market. The assumption is that the cash and futures market come together at time 1; $F_1 \equiv C_1$. Basically, all spot grades are perfect substitutes for one another and deliverable under the futures contract so that delivery on the futures contract becomes a reality rather than a fiction. The price for later delivery in this special case will be denoted f to distinguish it from the true futures price. In the case of forward trading, the analysis is on the same grounds chosen by Anderson and Danthine [2].

Again, turning first to the pure speculator's problem, $F_1 \equiv C_1$ allows us to rewrite (8) as

$$(37)\quad f_0 \int U'(V_s)g(f_1)df_1 - \int f_1 U'(V_s)g(f_1)df_1 = 0,$$

where $g(f_1)$ is the degenerate probability density function resulting from the identity $f_1 \equiv C_1$. The following, stated as a corollary, is the forward market analog to the futures market Proposition 1.

Corollary 1.2:

Speculators, whose problem in the case of pure forward trades is represented by (35), will sell forward if and only if $f_0 > EC_1$ ($\equiv Ef_1$), buy if and only if $f_0 < EC_1$ ($\equiv Ef_1$), and assume no forward position if and only if $f_0 = EC_1$ ($\equiv Ef_1$).

Proof: Identical to the proof of Proposition 1, using the fact that $EC_1 \equiv Ef_1$ in pure forward markets.

The assumption of pure forward trading brings the focus of participants to bear upon the relation between prices for current delivery and prices for later delivery. However, it is only the simplifying assumption, and the resulting degenerate probability density function, which bring about the concern over this relationship. As seen in both Proposition 1 and Corollary 1.1, in more general instances the density function does not allow such a simplification.

Turning to producers, elevator operators, and millers, the identity between later cash and futures prices gives

$$(38) \quad f_0 \int U'(V_i)g(f_1)df_1 = \int C_1U'(V_i)g(f_1)df_1,$$

$i = p, e, m$, by their respective first-order conditions with respect to Q. Subsequent substitution into their respective first-order conditions with respect to y for $\int C_1U'(V_i)g(f_1)df_1$, $i = p, e, m$, gives

$$(39) \quad R'(y_p) - b'(y_p) + f_0 = 0$$

$$(40) \quad R'(y_e) - k'(y_e) - C_0 + f_0 = 0$$

$$(41) \quad R'(y_m) + k'(y_m) + C_0 - f_0 = 0.$$

As in the case of no basis risk, (39) - (41) show that the optimal spot position is independent of expectations about later prices and risk attitudes, in the case of pure forward trades. The spot and forward choices are separable from each other. In exactly the same manner as the proof of Proposition 2, the follwing result holds.

Corollary 2.1:

In the case of pure forward trades ($f_1 \equiv C_1$), the deviation from a forward position just equal to the spot commitment, with participants described by (9) - (14), occurs as follows.

(a) $D_p \gtrless 0$ <u>iff</u> $f_0 \gtrless EC_1 \quad (\equiv Ef_1)$

(b) $D_e \gtrless 0$ <u>iff</u> $f_0 \gtrless EC_1 \quad (\equiv Ef_1)$

(c) $D_m \gtrless 0$ <u>iff</u> $(Ef_1 \equiv)\ EC_1 \gtrless f_0$,

where $D_i = Q_i - y_i^*$, and $i = p, e, m$.

The corollary shows that the routine hedge occurs when (and only when) $f_0 = EC_1$.

Since the majority of early writers were interested in when, and why, there might be excess forward sales, or purchases, in the absence of speculative participation, the following corollary to Proposition 3 provides a useful benchmark.

Corollary 3.1:

For pure forward trading, $f_1 \equiv C_1$, $f_0 = EC_1$ ($\equiv Ef_1$) is an equilibrium which can be established without speculative participation under the following circumstances:

a) nonstochastic productive transformation of the spot commodity,

b) identical strictly concave utility functions and identical subjective probability density functions for all participants,

c) $b'(y_p) = f_0 = C_0 + k'(\bullet)$ and, finally,

d) the number of participants who plan to sell on the spot market at time 1 equals the number who plan to buy at time 1.

Proof: Let $F_0 = EC_1$ ($\equiv EF_1$). By Corollaries 1.2 and 2.1, (36) implies that the forward market clears when

$$n_p(y_p^* - y_m^*) + n_e(y_e^* - y_m^*) = 0,$$

since $n_p + n_e = n_m$, by part (d). Part (c) implies $y_p^* = y_e^* = y_m^*$ so that the market clears with no speculative participation.

Corollary 3.1 is not the most general description of forward market clearance in the absence of speculators, since all that is required is

$$(42) \quad (n_p/n_e) = (y_m^* - y_e^*)/(y_p^* - y_m^*).$$

But the corollary is quite useful, as will be seen as the analysis of the literature begins.

The preceding presentation, in its various degrees of generality, provides a framework for analyzing a wide variety of past theoretical work. For example, when it is agreed that particular relationships between spot and futures prices must hold at equilibrium, one is led to question how such claims compare to the benchmark Proposition 3, or its Corollary 3.1. That the "general" model has its shortcomings requires some emphasis. It is surely not the most general model of futures markets, since the behavior of some important agents is not accounted for, namely, the futures exchange and the government regulator of futures market activity. Taking the exchange as an example, it was noted in the introduction that movements in the futures price are limited to minimize default in times of rapid price change. In the most general case where propositions about hedgers are derived, the absence of basis risk, the imposition of such limits has no effect on participants. To see this, take the case of the elevator operator. Expressions (2) and(5) give the revenue function, first-order conditions (11) and (12) remain the same except for altering the integration with respect to F_1 (if A represents the imposed limit on price movements, $F_0 - A$ and $F_0 + A$ are the limits of integration), (29) is still the product of (27), again, with the limits of integration over F_1 altered, and (31) remains as the condition for optimal cash holdings. Hence, the separability result is unchanged and the optimal futures position still comes from (34). However, all this occurs in the special case of no basis risk. A more general model would be required to account for the interaction between the exchange authority, government regulators, and futures market participants.

Before proceeding to those works with implications for the distribution of futures prices, important works that provided building blocks deserve acknowledgement. The behavior of hedgers, outlined in the case of producers, has been examined by many MVA writers. McKinnon [27] and Rolfo [30] restrict their analysis to producers, assuming that both output quantity and price are variable. Rolfo's is the more general treatment. Johnson [23] examines storage operators and processors assuming an exogenous stock level carried by storage operators. The possibilities of Table 1, equally applicable to storage operators, are all found by Johnson. Ward and Fletcher [38] add a special case of processor, described by an ongoing production process at the time hedges are instituted (feedlot operators, whose cash inventory is in feeder cattle, hedging in live cattle). In addition to the previous acknowledgement of Anderson and Danthine [1], they also cover simultaneous trades in a multiplicity of contracts and cross hedging.

The debt to Anderson and Danthine [2] was already mentioned. They also cover the case where productive transformations are subject to uncertainty and investigate the model equilibria under rational expectations. It should be pointed out that their analysis assumes pure forward trading. Holthausen [19] and Feder, Just, and Schmitz [11] also assume pure forward trading and nonstochastic production, deriving propositions regarding the separation result for the optimal spot commitment and the relation between the optimal forward and spot positions. These two works, however, restrict themselves only to the producer's decision and do not explicitly recognize that it is production certainty and the assumption of pure forward trading which drive these results. Another pure forward model is found in Baesel and Grant [3].

In most cases, the progression of the theoretical literature with a detectable interest in the pattern of futures prices over time began with a focus on pure forward trading and proceeded to the more general case of actual futures markets, and the review follows this progression. The overall conclusion is that price patterns in futures markets are the result of interrelationships between hedging behavior, the correlation between cash and futures prices, and the level of commodity stocks. En route, strong arguments are made in favor of futures prices that rise throughout the duration of a given contract, futures prices that fall throughout, and seasonality in the behavior of futures prices based upon stock levels. The headings which encompass these results are the theory of the risk premium, interim MVA results, and the theory of the price of storage.

Theory of the Risk Premium

Specification of speculative demands occupied nearly all of the early writers concerned with forward markets. Further, while primary producers do not as a rule participate in futures trading, their role is important in forward markets and, with one exception, the theory of the risk premium is a theory of forward trading. All of the early writers covered in this section consider the primary reason for the existence of markets for later delivery to be their efficacy in the allocation of risk bearing. None ever questioned the notion that participants with spot positions must pay in order to avoid fluctuations in the value of those positions. For these writers, the "risk premium" was an essential element in the relation between spot, forward, and futures prices.

In reviewing the theory of the risk premium, the following method is used. First, the writings of the major contributors are reviewed. In some cases, formal structure is provided for primarily verbal argument. Following this, the general framework of the preceding sections is brought to bear in an effort to identify the contentions of the early writers on the theory of the risk premium. Finally, the implications for distributions of futures market prices are highlighted.

Keynes [25] and Hicks [17] focus their attention on two states of the world. First, normalcy reigns when stocks of commodities are at a level to successfully maintain production of final output at a "normal" level (Keynes), or when "the conditions of supply and demand are stable, so that the spot price is expected to be about the same in a month's time as it is today" (Hicks, p. 138). Second, a period of "redundant" stocks in excess of normal levels can occur due to "miscalculation of supply and demand" (Keynes, p. 136). In the previous notation, normalcy implies $EC_1 = C_0$, while redundant stocks occur when expectations diverge from the current spot price. In the latter case, Keynes argues that redundant stocks imply $EC_1 > C_0$. To see this, let $C_0 - f_0 = RP_N$, the risk premium in normal times, and $EC_1 - f_0 = RP_R$, the risk premium during periods of redundant stocks. Keynes (p. 144) argues that $RP_R > RP_N$ due to "the additional element of uncertainty introduced by the existence of stocks and the additional supply of risk-bearing which they require." Hence, $EC_1 > C_0$.

The terms "normal" and "redundant" deserve further discussion in thier use concerning commodity stock levels. Normal stocks would be those at a level just satisfying all expectations about input requirements for a given period with nothing left over to be carried into the next period. Put another way, if $EC_1 = C_0$, there is no incentive to carry stocks that cannot be sold on a day-to-day basis. The question immediately arises as to how often one might expect "normalcy" to reign. Both Keynes and Hicks refer almost exclusively to stocks of manufactured goods (with some reference to tin and rubber by Keynes) whose production uncertainty is certainly lower than for other goods carried over time, such as agricultural commodities. For such manufactured goods, "normal" stocks would appear to be a much more applicable term than for agricultural commodities; the existence of normal stocks is a much more unlikely situation for agricultural commodities exhibiting a high degree of production uncertainty. Given the forward market context of Keynes and Hicks with their focus on stocks of manufactured goods, the normal versus redundant distinction is a meaningful one. However, the distinction cannot be meaningful in a general context since redundancy reigns in the agricultural commodities.

In the Keynes-Hicks formulation, equilibrium outcomes in these two states always see forward sellers paying a risk premium to forward buyers, including speculators. Hence, at all times $EC_1 > f_0$ but in normal periods $C_0 > f_0$ as well. A situation where the current spot price exceeds the current forward price is the original definition of backwardation. Over the period covered in this review, the term backwardation has come to be interpreted in a variety of ways, relative to expected prices and current forward, and futures, prices. No terminology wil be attached to such relationships here. See Gray and Rutledge [14] for an excellent treatment of this "confusion." For example, Keynes (p. 144) notes that, in periods of redundancy, arbitrage enforcements will result in $f_0 > C_0$, or "contango." Again, many subsequent writers have referred to $EC_1 > f_0$ during periods of redundancy as Keynesian backwardation, when Keynes quite clearly describes the excess of expected spot over current forward as the premium during redundant stock periods.

Regarding the supply of forward contracts in these equilibria, one finds that producers can sell forward without risk when the current forward price exceeds production costs while production cannot pay otherwise (Keynes, pp. 142-3) and that during periods of redundancy arbitrage will enforce the equality of the current forward price and the sum of the current spot price and costs of storage (Keynes, p. 144). Regarding the former view of the production decision, Keynes (pp. 142-3) states

> If this [forward] price shows a profit on his [the producer's] costs of production, then he can go full steam ahead, selling his product forward and running no risk. If, on the other hand, this price does not cover his costs (even after allowing for what he loses by temporarily laying up his plant), then it cannot pay him to produce at all.

After no small amount of pondering, this is interpreted as a statement that net revenue, in the notation here $f_0 y_p - b(y_p)$, must be non-negative.

In summary, according to the Keynes-Hicks formulation, periods of normalcy exhibit

(43) $EC_1 = C_0$

(44) $[b(y_p)/y_p] - f_0 \leq 0,$

while periods of redundancy exhibit (44) and

(45) $EC_1 > C_0$

(46) $f_0 = C_0 + k'(y_e).$

Moreover, by standard marginal argument, $b'(y_p) \geq f_0$ for both periods. There are two important implications associated with the Keynes-Hicks formulation which appear in works reviewed later. First, if the risk premium is always paid by forward sellers to speculators, then speculative profits from a long forward market position should be consistently observed. The second concerns whether or not the Keynes-Hicks formulation implies any rising trend in futures market prices. A useful servant, when this implication is analyzed later, is the following proposition.

Proposition 4:

Application of the Keynes-Hicks forward market risk premium theory to true futures markets under no basis risk gives the following. During normal periods, contrary to the Keynes-Hicks result of excess forward sales, there is no excess on either side of a futures market. Hence, there is no rising trend in the futures price. During redundant periods, the Keynes-Hicks result of excess sales does carry over into a futures market. In this case, under a risk premium interpretation, there is a rising trend.

Proof: When stocks are at normal levels, the Keynes-Hicks formulation has $C_0 = EC_1$. The result obtained by them is that $f_0 < C_0$ and an excess of forward sales is required to produce this result. By expression (27), however, $B_0 = B_1$ so that $C_0 = EC_1$ is equivalent to an equilibrium where $F_0 = EF_1$ and there is no excess among those with spot commitments on either side of the market. When redundant stocks arise, the Keynes-Hicks formulation has $EC_1 > C_0$ and the result is that $f_0 < EC_1$ again in accordance with an excess of forward sales. Again, by expression (27), $EC_1 > C_0$ is equivalent to an equilibrium where $F_0 < EF_1$ and an excess of sales occurs on the <u>futures</u> market. In this case, if $EF_1 - F_0$ is interpreted as a Keynes-Hicks risk premium, buyers' risks decrease as time of termination approaches and, with EC_1 given, F_0 must rise.

The importance of this proposition will be clear when Telser and Cootner are discussed later.

Kaldor [24] makes the following significant refinement of the Keynes-Hicks formulation. Let $k(y) = w(y) - q(y)$, where $w(y)$ represents the standard carrying costs described thus far. The point of Kaldor's analysis is $q(y)$, which he terms convenience yield. This yield is the value to a holder of stocks from the ability to make use of the stocks at the moment they are needed, rather than having to wait for later delivery. What Keynes called redundant stocks, Kaldor calls speculative stocks (p. 1) and the absence or presence of speculative stocks is defined the same way Hicks chose, represented by expressions (43) and (45). The following intuitive argument about how convenience yield is affected by stock levels is offered (p. 4).

> The amount of stock which can thus be "useful" is, in given circumstances, strictly limited; their marginal yield falls sharply with an increase in stock above "requirements" and may rise very sharply with a reduction below "requirements." When redundant stocks exist, the marginal yield is zero.

In keeping with the risk premium tradition, Kaldor assumes that the following must always hold:

(47) $EC_1 - C_0 - k' > 0$

(48) $f_0 = C_0 + k'$.

The former holds because the expected spot price net of the current spot plus marginal costs of storage must be enough to cover risk payments. The latter, just an old face with a new name, is expression (46), repeated for convenience. Kaldor invokes the following argument to drive the two results mentioned above. While agreeing that risk avoiders are generally forward sellers, he notes (p. 6):

> In the case of certain industrial raw materials, however, where the outside buyers are contractors with given orders for some periods ahead, the "hedgers" may be predominantly forward

> buyers, and the "speculators" spot buyers and forward sellers. Now the "carrying cost" for these speculators may be higher than the carrying costs for the market generally. This is because the yield of stocks of raw materials consists of "convenience," the possibility of making use of them the moment they are wanted, and this convenience is largely lost if the stock held is already sold forward.

There is a definite implication here that when forward buyers are cast as the risk avoiders, those forward sellers earning convenience yield do not sell forward, even though they can sell forward without losing their convenience yield when they are the risk avoiders. In the cases of forward selling risk avoiders and forward buying risk avoiders, respectively, (48) becomes

$$(49) \quad f_0 = C_0 + w'(y_e) - q'(y_e)$$

$$(50) \quad f_0 = C_0 + w'(y_m).$$

Now, according to Kaldor's description, $q' = 0$ when redundant stocks exist and (49) and (50) are identical. Substitution into (47) gives $EC_1 > f_0$ whenever there are redundant stocks and Kaldor is in complete agreement with Keynes and Hicks. The difference in Kaldor's specification occurs when $q' > 0$ in normal periods. First, when forward sellers are the risk avoiders, substitution of (49) into (47) gives $C_0 > f_0$ (since $EC_1 = C_0$) so that $q' > w'$ by (49); $k' = w' - q' < 0$ and Kaldor agrees with the Keynes-Hicks result for normal periods as long as risk avoiders are forward sellers. However, when forward buyers are the risk avoiders, the same method of substitution using (50) gives $C_0 - f_0 + q' > 0$; as long as $q' > f_0 - C_0$, $f_0 > C_0$ is a possibility, and the opposite from what Keynes and Hicks claim will occur.

As previously mentioned, Kaldor's argument for who earns convenience yield when speculators are forward sellers provides some logical inconsistency. If forward selling precludes the earning of convenience yield by speculators, why not so for other stockholders unless Kaldor means for these "ordinary" holders not to sell forward at all. More importantly, when speculators are forward buyers, it must now be the case that the "ordinary" stockholders do sell forward but now Kaldor allows them to earn the convenience yield. He cannot have it both ways and it is precisely this confusion which is the point of departure for the work by Dow [10].

Dow criticizes the previous approaches for their concern with forward markets, since risk avoiders cover their spot positions in futures, as opposed to forward, markets. In this context, Kaldor's artificial separation of market types misses the point; no convenience yield need be lost when spot positions are covered by futures trading since the future position can always be undone by an offsetting trade. Hence, convenience yield must be included for all stockholders or, what is an equivalent statement (since speculative sellers in futures markets do not necessarily have to hold any stocks), storage operators must be allowed to participate at all times.

Aside from his ending comments, Dow assumes the absence of basis risk throughout his analysis. Dow also refers to the risk faced by long hedgers (short in the spot market and long futures, as in the case of processors) as "negative risk" while that faced by short hedgers (long spot, short futures, such as the case of storage operators) is called "positive risk." In this, the first detailed study of the hedging decision in futures trading, Dow finds the result that the risk premium must be paid relative to the net outcome of trading between those with spot commitments. If the trading result is an excess of hedging positive risk over negative (excess short hedging), then $EC_1 > F_0$ is required to induce speculative buyers to make up the difference. If, on the other hand, the excess is hedging negative risk (excess long hedging), $EC_1 < F_0$ must occur. For Dow, the important result is that the risk premium can flow either way depending on the balance of hedging.

With this brief overview, the contents of the theory of the risk premium are fairly clear. However, the context is quite confusing. In forward markets, Kaldor offers a fundamental disagreement with the original theory from Keynes and Hicks. Moving the focus to the true futures markets, Dow leaves the question of who earns the risk premium completely open, theoretically speaking. Using the general framework developed earlier, these points and more can be clarified. The question which concerned these writers was why speculators will necessarily buy forward contracts; why does $EC_1 > f_0$ hold at equilibrium, implying an excess of forward sales among those with spot commitments? The following corollary sheds some light.

Corollary 3.2:

For the normal period described by Keynes and Hicks (pure forward trading, $EC_1 = C_0$, expressions (43) and (44), and all else as in Corollary 3.1), there can be an excess of forward sales only when $y_p > y_m$, at $f_0 = EC_1$ ($\equiv Ef_1$), for $k' > 0$.

Proof: Under the Keynes-Hicks definition of normalcy, (39) - (41) become $R'(y_p) = b'(y_p) - f_0$, $R'(y_e) = k'(y_e)$, and $R'(y_m) = -k'(y_m)$, respectively, by (43). Let $f_0 = EC_1$ ($\equiv Ef_1$). Expression (36) shows that an excess of forward sales occurs when

$$n_p(y_p - y_m) + n_e(y_e - y_m) > 0,$$

since $Q_s = 0$ (by Corollary 1.2) and $D_p = D_e = D_m = 0$ (by Corollary 2.1), while $n_p = n_m$ by hypothesis. By concavity of R, $k' > 0$ implies $y_m > y_e$. Hence, an excess of forward sales can occur only if y_p exceeds y_m.

Corollary 3.2 brings out the important aspects of the theory of the risk premium in two ways. First, it shows that more is required of the Keynes-Hicks formulation than the quantification gleaned in expressions (43) and (44). But whether or not there is an excess of forward sales among spot traders, when $k' > 0$, depends instead upon the level of the relation $y_p > y_m$. Hicks (p. 137) was apparently aware of this deficiency, offering the intuitive argument in the following quote.

> Technical conditions give the entrepreneur a much freer hand about the acquisition of inputs (which are largely needed to start new processes) than about the completion of outputs (whose process of production - in the ordinary business sense - may be already begun). Thus while there is likely to be some desire to hedge planned purchases, it tends to be less insistent than the desire to hedge planned sales. If forward markets consisted entirely of hedgers, there would always be a tendency for a relative weakness on the demand side; a smaller proportion of planned purchases than of planned sales would be covered by forward contracts.

Keynes relied strictly on the intuitive argument that forward sellers wish to avoid the risk associated with their spot positions and must pay to do so.

The second way that Corollary 3.2 highlights the theory of the risk premium is in its intentional disregard of the case where $k' \leq 0$. This sets the stage for the earliest risk premium controversy, between the Keynes-Hicks formulation and that of Kaldor. Keynes and Hicks proceed on the basis of $k' > 0$, even though Hicks was compelled to argue that the desire to "hedge" planned sales is more insistent than the desire to "hedge" planned purchases. There is absolutely nothing in the Keynes-Hicks formulation that even hints that $k' \leq 0$ can occur, since all storage costs mentioned are the standard deterioration, warehousing, and interest charges. Enter Kaldor's notion of convenience yield in normal periods. Using the general framework, Kaldor's logic is clearly stated.

Corollary 3.3:

For the normal period described by Keynes and Hicks, as in Corollaries 3.1 and 3.2, $k' < 0$ implies an excess of forward sales when $f_0 = EC_1 (\equiv Ef_1)$, so long as $y_p \geq y_m$.

Proof: Again, we are interested in when

$$n_p(y_p - y_m) + n_e(y_e - y_m) > 0.$$

Now, $k' < 0$ implies $y_e > y_m$ and $y_e > y_p$. So long as $y_p \geq y_m$, the inequality holds and an excess of forward sales occurs.

Kaldor uses the notion of convenience yield to show that $k' < 0$ can be associated with an equilibrium where $C_0 > f_0$ in normal periods, an important contribution which can be added to Hicks' hedging insistence argument.

The general framework is also insightful in the case of redundant stocks. The following final forward market corollary points out the importance of the Keynes-Hicks assumption, expression (44).

Corollary 3.4:

For the periods of redundant stocks described by Keynes and Hicks (pure forward trading, expressions (44) - (46), $EC_1 > C_0$, and $f_0 = C_0 + k'(y_e)$), and all else as set forth in Corollary 3.1, there can be no excess forward purchases when $f_0 = EC_1$ ($\equiv Ef_1$); equilibrium must occur at $f_0 \leq EC_1$.

Proof: Under the arbitrage enforced relation (46), (39) - (41) become $R'(y_p) = b'(y_p) - f_0$, $R'(y_e) = 0$, and $R'(y_m) = 0$, respectively. When $f_0 = EC_1$, $Q_s = D_p = D_e = D_m = 0$ by Corollaries 1.2 and 2.1.

Now, $b'(y_p) - f_0 \geq 0$ implies

$$y_p \leq y_e = y_m, \text{ or } n_p(y_p - y_m) + n_e(y_e - y_m) \geq 0.$$

Hence, $b'(y_p) - f_0 \geq 0$ implies that there can be no excess forward purchases in redundant stock periods.

The general framework is also informative in Dow's case of true futures trading, in the absence of basis risk. The case of no basis risk, culminating in Proposition 3, reveals a definite shortcoming in Dow's presentation. The absence of basis risk, as described in (27), does not imply $C_1 = F_1$. Hence, Dow's substitution of F_0 and F_1, the futures prices at times 0 and 1, for their forward market counterparts f_0 and f_1 in expressions such as (47) - (49) is invalid. Equilibrium outcomes under the assumption of no basis risk are never found relative to the current futures price and the expectation of the later spot price, and it is for this reason that Dow's use of a relation between futures and spot prices is invalid. Perhaps this was simply due to an endeavor to maintain comparability with previous writers. Whatever the reason, the special case of no basis risk reveals this shortcoming. As a final note, Dow (pp. 194-5) comments on which equilibrium outcome he feels is most likely.

> Normally, one would imagine, the time during which these stocks were moving up to the stage of production where hedging was most perfect, and during which they could profitably be hedged on the futures market, would be longer than the predating of the futures orders; so that, with these alone, positive risks would predominate [excess futures sales]. If, however, the predating were sufficiently great, this would not be so.

In other words, only when the standard grade deliverable on the futures contract is in plentiful supply at the terminal contract date can the risks borne by stockholders be more than offset by the risks confronting long hedgers. Hence, when futures and actuals are most nearly perfect substitutes, there can be an excess of long hedging. Dow, however, finds this possibility extremely remote and his beliefs are depicted in Figure 1, the futures market analog of the Keynes-Hicks position.

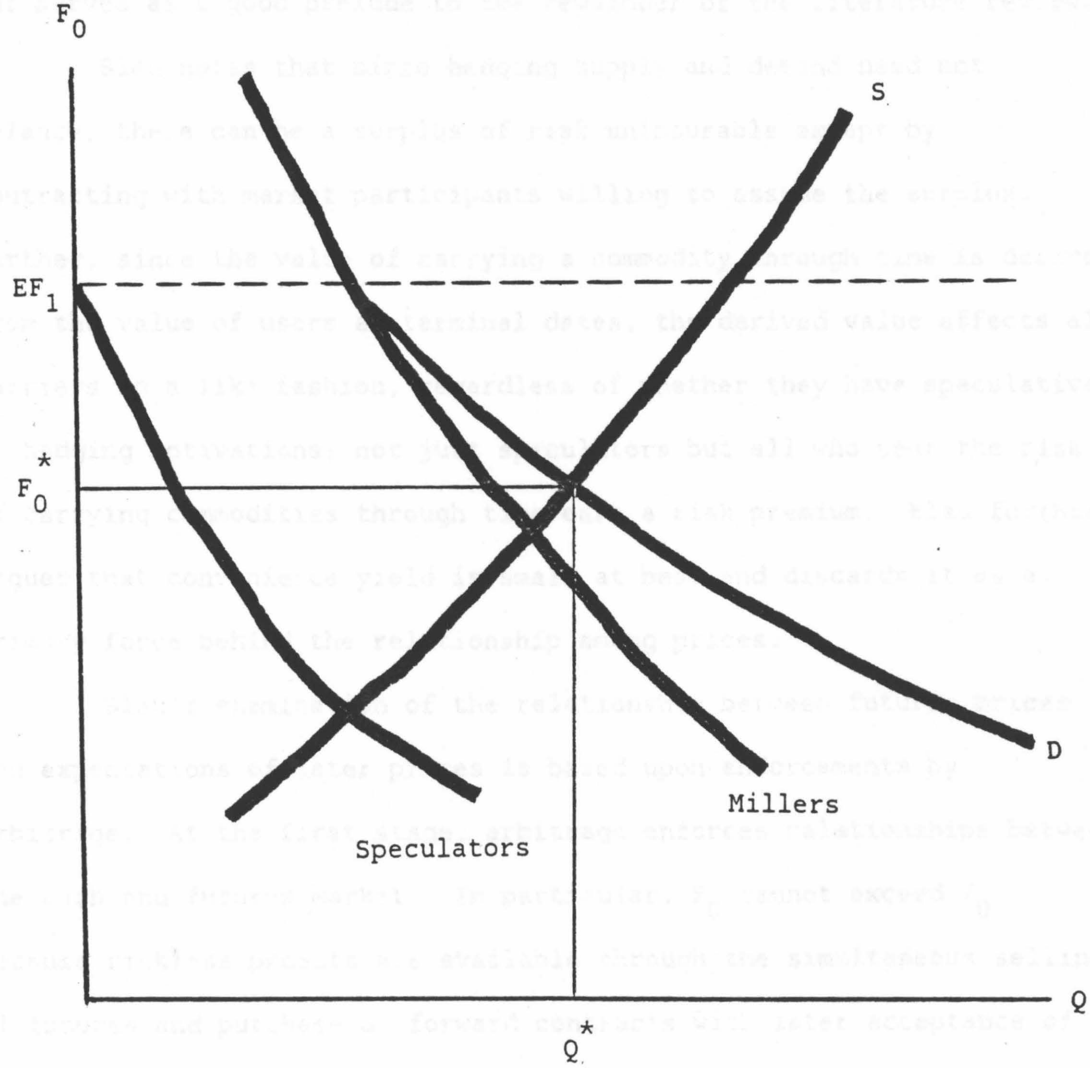

Figure 1. Dow's "Normal" Futures Market Risk Premium.

Review of the theory of the risk premium concludes with the work of Blau [4]. Blau procedes at a level of generality beyond the framework that has served well to this point: true futures markets and their related cash (spot and forward) markets restricted only by arbitrage enforcements. The work is characterized by careful attention to real-world participant behavior and the following summary is brief but serves as a good prelude to the remainder of the literature review.

Blau notes that since hedging supply and demand need not balance, there can be a surplus of risk uninsurable except by contracting with market participants willing to assume the surplus. Further, since the value of carrying a commodity through time is derived from the value of users at terminal dates, the derived value affects all carriers in a like fashion, regardless of whether they have speculative or hedging motivations; not just speculators but all who bear the risk of carrying commodities through time earn a risk premium. Blau further argues that convenience yield is small at best and discards it as a primary force behind the relationship among prices.

Blau's examination of the relationship between futures prices and expectations of later prices is based upon enforcements by arbitrage. At the first stage, arbitrage enforces relationships between the cash and futures market. In particular, F_0 cannot exceed f_0 because riskless profits are available through the simultaneous selling of futures and purchase of forward contracts with later acceptance of the forward delivery to meet the futures contract obligation. Second, since the threat of delivery exists and the choice of which grade to deliver is in the hands of futures sellers, the price of a maturing futures contract cannot stray far from the spot price of deliverable grades at the maturity date.

In addition to these arbitrage considerations, Blau also considers the effect of risk upon participant decisions. For speculators, futures trading is risk-increasing. Hence, futures will be purchased only if $EF_1 - F_0 \geq r$ and sold only if $F_0 - EF_1 \geq r$, where r is the marginal risk factor. Hedgers, on the other hand, trade futures to avoid risk. Arguing that the current cash price can be substituted for hedgers' expectations,[4] Blau states that long hedgers will buy futures as long as $F_0 - C_0 - k' \leq r$ and short hedgers will sell as long as $k' - (F_0 - C_0) \geq r$. Taking the limits on these activities at a strict equality results in the summary in Table 2.

Table 2. Blau's Limits to Futures Market Activity.

	Buy Futures If F_0 Is Less Than Or Equal To	Sell Futures If F_0 Is Greater Than Or Equal To
Speculators	$EF_1 - r$	$EF_1 + r$
Hedgers	$C_0 + k' - r$	$C_0 + k' + r$

In conclusion Blau argues that an excess of expected futures price over current prices need not be the normal state of affairs. For example, a strong predominance of long hedging will have just the opposite effect. Finally, even with strong short hedging, speculative dominance may be the most likely cause of observed price relationships, especially if hedges unequal to spot commitments are considered speculation. In such a case, speculators who desire to sell (rather than assume the short hedgers' risks) require the futures price to exceed the expected later price by the amount of the risk premium, as per the limit in column two for speculators, shown in Table 2.

A summary of the early thinkers covered thus far is in order. First, all consider the primary reason for the existence of futures markets to be their efficacy in the avoidance of risk associated with spot price fluctuations. Not a single writer covered ever questioned the notion that hedgers must pay a risk premium in order to avoid fluctuations in the value of stocks. Whether paid to speculators or hedgers on the other side of the market, the risk premium must be included in a consideration of the relation between spot, forward, and futures prices. This focus on what inventory holders do to avoid risk is seen by later writers to imply that hedging is an afterthought to the cash position. Hicks (p. 138) speaks straight to this point, regarding what is required to generate interest in markets for later delivery, "No forward market can do without the speculative element." This conclusion is the strict result of a consideration which takes the avoidance of risk, and the earning of a profit from risk taking, as the driving forces behind futures trading.

What the theory has to offer so far concerning the distribution of futures prices can be found in the debate over why, and when $EF_1 \gtrless F_0$. In the context of futures markets in particular, one finds the following. Dow, by the argument regarding the substitutability between futures contracts and actual units of the commodity, expects $EF_1 > F_0$ under normal circumstances while Blau, by the speculative dominance argument, holds no priors. For the latter, we are left to wonder when "a strong predominance of long hedging" might occur or just why speculators desiring to sell might come to dominate those buying, even with predominance of short hedging. As shown later, it is precisely this focus upon the changing nature of hedging dominance that is most interesting to a study of futures price distributions.

Interim Mean-Variance Analysis

As noted earlier, the focus of MVA primarily has been the determination of optimal hedging decisions. However, there are two MVA contributions interested in the distribution of futures prices. As seen in the previous section, seasonality in hedging, based upon stock levels, was seen by Dow as a possible (although not likely) outcome. For Blau, which hedging activity would dominate was a more open question. In the two works covered in this section, the relation between hedging and stock levels is made explicit, but the implications for the relationship between F_0 and EF_1 are not approached. However, by using the MVA case of the general framework, some light can be shed upon seasonality of futures prices based on the notions found in these two works. Both concern true futures markets without restricting themselves to the case of no basis risk. The second work actually occurred after some of the literature reviewed in the next section. Placement here is due to the analysis under the MVA case of the general framework.

Telser [35] examines the behavior of long and short hedgers, restricting expected income to exceed a given predetermined disaster level.[5] The hedger chooses the amount of hedging and a non-negative amount of unhedged stocks. Before proceeding to analyze the hedger's decision, Telser notes in passing the equilibrium notions from Keynes. Stating that agricultural commodities are nearly always in redundant supply, Telser assumes that $f_t = C_t + k'(y_e)$; the forward price equals the sum of spot and carrying charges. For futures markets, he argues that futures contracts are not an ideal instrument for long hedgers, due to timing and uncertain delivery characteristics (grade, date, and location), so that $F_t > C_t + k'(y_e)$ is disallowed.[6]

The meat of Telser's work is the hedging decision. Unfortunately, he makes an error that calls for a total restatement of his conclusions. With all due respect, this can be quickly accomplished. In the notation used here, Telser (p. 6) defines the random gains from hedging and unhedged stocks, respectively, as

$$(51)\quad b_1 = F_0 - C_0 + C_1 - F_1 = B_0 - B_1$$

$$(52)\quad b_2 = C_1 - C_0$$

where, as before, B_t is the basis at time t (Telser includes carrying costs in the cash price). The covariance between $B_0 - B_1$ and $C_1 - C_0$ (which Telser labels m_{12}, on p. 7) is

$$(53)\quad \sigma_{b_1 b_2} = EC_1 EB_1 - EC_1 B_1 = - \sigma_{C_1 B_1},$$

the last term being the negative of the covariance between the cash price and the basis. By the definition of a covariance, one further finds

$$(54)\quad -\sigma_{C_1 B_1} = \sigma^2_{C_1} - \sigma_{C_1 F_1},$$

the difference between the cash price variance and the covariance between cash and futures prices. Telser's analytics are impeccable, but he errs in his interpretation of $\sigma_{b_1b_2}$ as the covariance between the cash price and the basis. It is instead the negative of that covariance, a fact which leads Telser to mistaken conclusions. Throughout the development of the optimal hedging strategy, the error in interpretation causes no major problem. The results are: 1) if $\sigma_{b_1b_2} > 0$, short hedged and unhedged stocks will be held only if both actions are expected to be profitable; i.e., $EB_1 > 0$ and $EB_2 > 0$ (p. 9), 2) if $\sigma_{b_1b_2} < 0$, long hedged and unhedged stocks will be held only if both actions are expected to be profitable (p. 9), 3) if $\sigma_{b_1b_2} < 0$, short hedged and unhedged stocks will be held even if a loss is expected on one or the other activities (p. 10), and 4) if $\sigma_{b_1b_2} > 0$, long hedged and unhedged stocks will be held even if a loss is expected on one or the other activities (p. 10).

The mistaken interpretation becomes important when Telser relates the covariance between the cash price and the basis to the covariance between cash and futures prices, aiming to tie hedger behavior to stock levels (pp. 10-11). Telser argues that $\sigma_{C_1F_1} > 0$ is most likely when current stock levels are large, being held for later use (after the harvest and over most of the period to the next harvest). On the other hand, $\sigma_{C_1F_1} < 0$ is most likely when stocks are low and new supplies are imminent (just before the harvest). Under the correct interpretation of $\sigma_{b_1b_2}$, (53) and (54) imply

$$(55)\quad \sigma_{b_1b_2} = \sigma^2_{C_1} - \sigma_{C_1F_1}.$$

Hence, $\sigma_{C_1B_1} > 0$ ($\sigma_{b_1b_2} < 0$) is equivalent to $\sigma^2_{C_1} < \sigma_{C_1F_1}$, with $\sigma_{C_1F_1} > 0$, necessarily. Alternatively, $\sigma_{C_1B_1} < 0$ ($\sigma_{b_1b_2} > 0$) is equivalent to $\sigma^2_{C_1} > \sigma_{C_1F_1}$, which is sure to hold if $\sigma_{C_1F_1} < 0$. By interpreting $\sigma_{b_1b_2}$ as the covariance between the cash price and the basis, Telser's conclusions regarding the relationship between $\sigma_{C_1B_1}$ and $\sigma_{C_1F_1}$ are the oppposite of those just shown.

The corrected versions of Telser's conclusions (p. 12) are as follows. First, if $\sigma_{C_1B_1} > 0$, long hedged and unhedged stocks will be held if both actions are expected to turn a profit. $\sigma_{C_1B_1} > 0$ when $\sigma_{C_1F_1} > 0$, most generally when current stock levels are large. Second, if $\sigma_{C_1B_1} < 0$, short hedged and unhedged stocks will be held if both are expected to turn a profit. $\sigma_{C_1B_1} < 0$ when $\sigma_{C_1F_1} < 0$, most generally at low stock levels just prior to harvest. Third, short hedged and unhedged stocks will be held even when a loss is expected on either activity if $\sigma_{C_1B_1} > 0$ and, again, this is most likely during high stock levels. Finally, long hedged and unhedged stocks will be held even when a loss is expected on either activity if $\sigma_{C_1B_1} < 0$ and, again, this is most likely at low stock levels.

Several questions come to mind after reading Telser, and this should be taken as a compliment. Some are simply due to his reliance upon the covariance between the cash price and the basis. For example,

$\sigma_{C_1B_1} < 0$ can also occur for some $\sigma_{C_1F_1} > 0$.

Another question concerns Telser's belief that $\sigma_{C_1F_1} < 0$ is associated with low stock levels. Instead, could $\sigma_{C_1F_1} > 0$ be the case regardless of stock levels with the covariance simply less strong at low stocks? Focusing upon the covariance between cash and futures prices leads one to ask after typical movements of the cash price at different stock levels in order to examine the relationship between F_0 and EF_1, and hedger behavior. Telser remains mute regarding this relationship, i.e., seasonality in hedging based upon seasonal stocks. Typically, the cash price is viewed as rising throughout most of the post harvest period as stocks are consumed, while falling in anticipation of the harvest and as the harvest arrives. The case of mean-variance analysis developed under the general framework is revealing on these points as seen in the following proposition, based upon Telser's argument that the covariance between cash and futures prices is positive at high stock levels and negative at low stocks.

Proposition 5:

Under the MVA case of the general framework, $\sigma_{C_1F_1} < 0$ is inconsistent with the existence of market clearing prices, regardless of whether a negative covariance occurs at high or low cash prices. However, if $\sigma_{C_1F_1} > 0$, seasonality in net hedging balances is possible. As usual, common expectations are held by all, with common risk aversion parameters as well.

Proof: To prove the proposition, the optimal futures position of a producer, in expression (22), will be used for short hedgers and the optimal futures position of a miller, unstated previously but quite obvious given (22), will be used for long hedgers:

$$Q_p^* = (F_0 - EF_1)/\ \chi\sigma^2_{F_1} + y_p^*\beta_1 \qquad \text{(short hedger)}$$

$$Q_m^* = (EF_1 - F_0)/\ \chi\sigma^2_{F_1} + y_m^*\beta_1 \qquad \text{(long hedger).}$$

Recall that $\chi\sigma^2_{F_1}$ can be thought of as the risk weight associated with the futures position. Let W stand for this risk weight and COV stand for the covariance for the remainder of the proof. When $y_p^* > 0$, $y_m^* > 0$, and COV < 0, Table 3 describes when $Q_p^* > 0$ and $Q_m^* > 0$:

Table 3. MVA Requirements for Positive Short and Long Hedging when The Covariance between Cash and Futures Prices Is Negative.

	$F_0 - EF_1 > 0$	$F_0 - EF_1 < 0$
$Q_p^* > 0$:	$(EF_1 - F_0)/W <$ COV	never
$Q_m^* > 0$:	never	$(F_0 - EF_1)/W <$ COV

Table 3 shows that a negative covariance between cash and futures is inconsistent with market clearing prices. From the first column, even when the covariance is large enough to elicit sales from short hedgers, neither long hedgers nor speculators are willing to buy (recall, Proposition 1 for speculators holds for MVA, and $F_0 - EF_1 > 0$ is equivalent to speculative sales). The opposite holds in column two; the only time long hedgers want to buy is when no other participants wish to sell.

For the remainder of the Proposition, note that when $y_p^* > 0$, $y_m^* > 0$, and COV > 0, Table 4 describes circumstances under which $Q_p^* > 0$ and $Q_m^* > 0$:

Table 4. MVA Requirements for Positive Short and Long Hedging when The Covariance between Cash and Futures Prices Is Positive.

	$F_0 - EF_1 > 0$	$F_0 - EF_1 < 0$
$Q_p^* > 0$:	always	$(EF_1 - F_0)/W < COV$
$Q_m^* > 0$:	$(F_0 - EF_1)/W < COV$	always.

In general, since the magnitude of the covariance will influence net hedging balances, nothing can be said about the pattern of such balances. However, when the cash price is rising, with common expectations and $\sigma_{C_1F_1} > 0$, all traders believe that $F_0 - EF_1 < 0$ (i.e., the futures price is expected by all to rise with the rising cash price). Referring to the second column of Table 4, if the covariance is large enough that $Q_p^* > 0$ exceeds $Q_m^* > 0$, speculators will purchase the excess sales according to Proposition 1. Hence, it is possible that short hedging can dominate when the futures price is expected to rise. On the other hand, when the cash price is falling, all traders believe $F_0 - EF_1 > 0$ (i.e., the futures price is expected by all to fall with the cash price). In the first column of Table 4, if $\sigma_{C_1F_1} > 0$ is large enough that $Q_m^* > 0$ exceeds $Q_p^* > 0$, speculators will satisfy excess demands and it is possible that long hedging can dominate when the futures price is expected to fall. That such differences in net hedging balances can be seasonal follows from the stylized fact that cash prices rise after the harvest is complete, falling in a period prior to and during the harvest.

It should be noted first of all that the MVA case of the general framework departs from the approach based upon the avoidance of disastrous outcomes utilized by Telser. The finding that a negative covariance will not support market clearance is in no way due to any error by Telser but, rather, it is simply a result that the general MVA model provides. Second, while Proposition 5 provides some justification for seasonality in net hedging balances, the proposition depends crucially upon the size of the covariance at different stock levels. At this point, one could resort to the previous line of investigation at a general equilibrium level, identifying sufficient conditions to guarantee the seasonality result. But the point of Proposition 5 is to provide an introduction to the concept of seasonality in net hedging balances which is of great importance later. To this end, the proposition suffices.

Stein [32] uses mean-variance analysis and a general equilibrium approach in order to examine the relationship between the cash price and the basis. Rather than assuming normally distributed final profits, Stein follows Tobin [37] and generates a mean-variance framework assuming quadratic utility functions and decreasing marginal utility of income. Stein begins by defining hedged and unhedged returns as with Telser. Since he later assumes that carrying costs are constant (although showing that this is not required to provide his results), expressions (51) and (52) again describe returns. Stein then requries clearance in the supply and demand for stocks, clearance in the supply and demand for futures contracts, and examines the comparative statics at equilibrium. Letting $U(C_1 - C_0)$ and $H(B_1 - B_0)$ represent the demand for unhedged and hedged stocks, respectively, the total demand for stocks is

$$(56)\quad D_0 = U(C_1 - C_0) + H(B_0 - B_1),\ U' > 0,\ H' > 0.$$

Letting s_{-1} be carryover stocks from the previous period and $X(C_0; \alpha)$ be current supply in excess of consumption (with α as a shift parameter), the supply of stocks is

$$(57)\quad S_0 = s_{-1} + X(C_0; \alpha),\ \partial X/\partial C_0 > 0.$$

Equilibrium in the market for stocks is then defined by

$$(58)\quad U(C_1 - C_0) + H(B_0 - B_1) = s_{-1} + X(C_0; \alpha).$$

The variables for which we can solve are C_0 and B_0 and, once these are known, F_0 is derived ($F_0 \equiv B_0 + C_0$). Differentiating (58) with respect to C_0, Stein finds $\partial B_0/\partial C_0 > 0$, as for curve SS in Figure 2. In the futures contract market, the supply is simply equal to the demand for hedged stocks, $H(B_0 - B_1)$. Casting speculators as the buyers of futures (p. 1020), Stein defines the demand for futures contracts as $G(EF_1 - F_0)$, $G' > 0$. The futures market clears when

$$(59) \quad H(B_0 - B_1) = G[EF_1 - (C_0 + B_0)],$$

since $F_0 \equiv C_0 + B_0$. Differentiating (59) with respect to C_0, Stein finds $\partial B_0/\partial C_0 < 0$, as for curve DD in Figure 2. Requiring (58) and (59) to hold simultaneously, the result is (C_0^*, B_0^*) in Figure 2. At this point, $F_0^* = B_0^* + C_0^*$ and both cash and futures prices have been simultaneously determined.

In his comparative statics analysis of (C_0^*, B_0^*), Stein chooses to place his discussion in terms of the covariance between <u>changes</u> in the cash price and <u>changes</u> in the basis. It is easily shown, however, that $\text{cov}[(C_1 - C_0), (B_1 - B_0)]$ equals $\sigma_{C_1F_1}$. Similarly, $\text{cov}[(C_1 - C_0),(X_1 - X_0)]$, the covariance between changes in the cash price and changes in excess current production, is equal to $\sigma_{C_1X_1}$. In order to preserve the comparison with Telser and the MVA case of the general framework, Stein's comparative static results will be restated in these terms. They are: 1) $\sigma_{C_1B_1} > 0$ suggests that the market has expected $\sigma_{C_1F_1} > 0$, 2) $\sigma_{C_1B_1} < 0$, along with

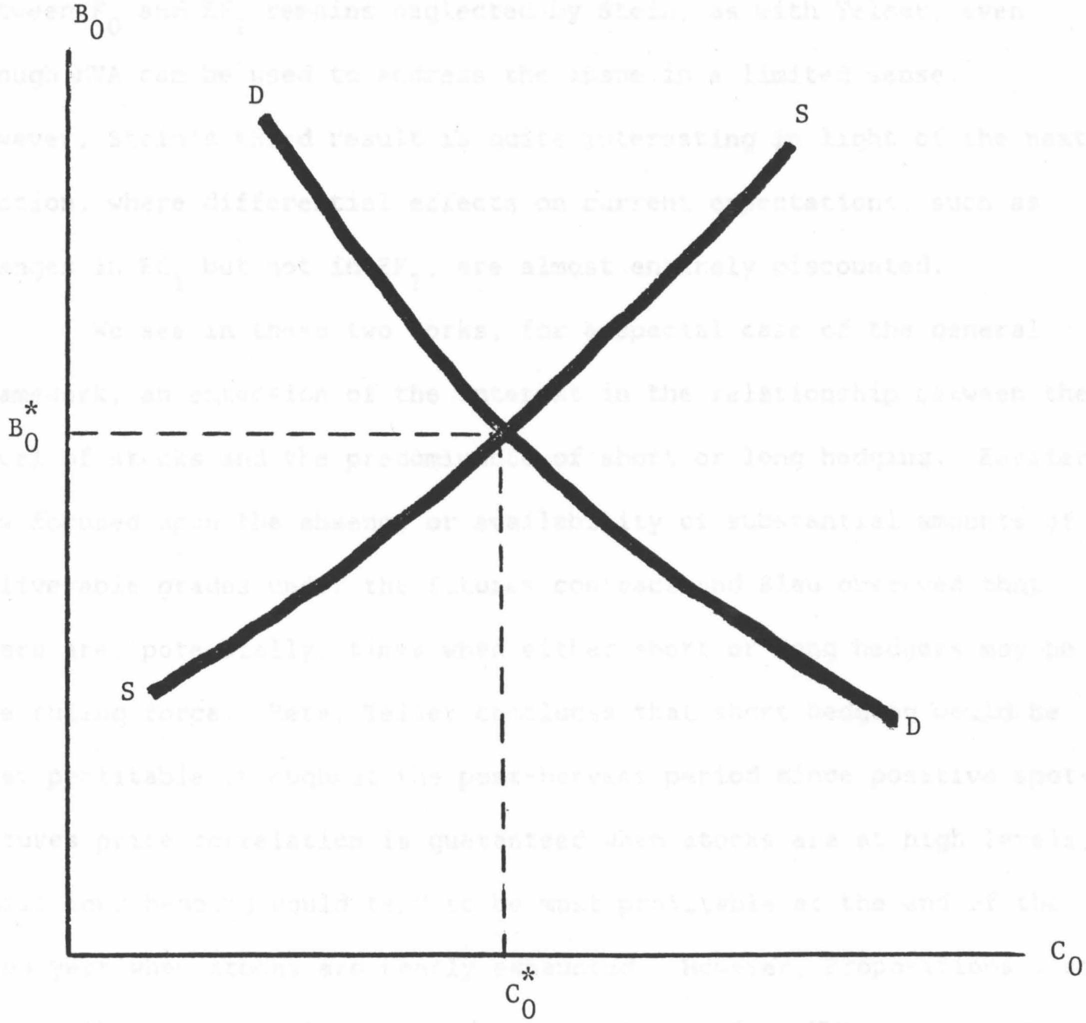

Figure 2. Stein's Simultaneous Price Determination.

$\sigma_{C_1X_1} < 0$ indicates that there has been a change in the excess supply of current production, and 3) $\sigma_{C_1B_1} < 0$, with $\sigma_{C_1X_1} > 0$, indicates that there has been a change in EC_1 but not in EF_1.

While delving deeper into reasons why the covariance between the cash price and the basis may be positive or negative, the relation between F_0 and EF_1 remains neglected by Stein, as with Telser, even though MVA can be used to address the issue in a limited sense. However, Stein's third result is quite interesting in light of the next section, where differential effects on current expectations, such as changes in EC_1 but not in EF_1, are almost entirely discounted.

We see in these two works, for a special case of the general framework, an extension of the interest in the relationship between the level of stocks and the predominance of short or long hedging. Earlier, Dow focused upon the absence or availability of substantial amounts of deliverable grades under the futures contract and Blau observed that there are, potentially, times when either short or long hedgers may be the ruling force. Here, Telser concludes that short hedging would be most profitable throughout the post-harvest period since positive spot-futures price correlation is guaranteed when stocks are at high levels, while long hedging would tend to be most profitable at the end of the crop year when stocks are nearly exhausted. However, Propositions 5 and 6 show that this conclusion, and any conclusion from MVA concerning seasonality in hedging, depend upon the level of the cash-futures covariance at large stocks. In what follows, a full flowering of the predictability of stock levels and their importance in determining net hedging balances occur. In the concluding section, review and comparison of the theoretical implications for a study of futures price distributions find a previously neglected bounty of important influences.

Theory of the Price of Storage

The theory of the risk premium and that reviewed in this section, the theory of the price of storage, hold dissimilar views regarding the prime motivation behind hedging and it is hardly surprising that their views on the relationship between the current futures price and expectations of the later cash price, as well as the seasonality issue, are at odds. For the theory of the risk premium, the avoidance of cash position price risk drives the decision to hedge. Originally (Keynes-Hicks), the risk premium was paid by forward sellers to speculators. This was later amended to permit the possibility of forward buyers paying the risk premium (Kaldor). In futures markets, the original belief was also that sellers pay the risk premium (Dow) while later work left the question open (Blau). In each case, the avoidance of cash price risk was primary and the hedging decision secondary to that purpose. The following theory based upon the predictable variability of stock levels holds that the cash and futures positions are simultaneously determined.

In his comment on the works of Kaldor and Dow, Hawtrey [16] suggests a radical reinterpretation of futures trading in seasonal commodities. Contrary to Keynes and Hicks, he notes that the risk premium is not only paid by hedgers to speculators, but also by all other speculators operating on same side of the market as hedgers. Further, the premium is not received only by speculators, but also by hedgers of like position. In this, he agrees with Dow, Blau, and the general framework which shows that all participants confront a speculative element in their hedging decision. However, in seasonal commodities, Hawtrey argues that the uncertainty attached to stock carrying requires that aarriers, rather than risk buyers, must be paid for their services (pp. 204-5).

> In the case of seasonal products the carrying of the stocks is an onerous function, and the merchant who performs it gets payment in the form of a premium of future over spot prices. Even in the case of non-seasonal products such as minerals, the merchant who sells forward to a manufacturer relieves the latter of the cost of holding stocks.

This is in direct juxtaposition to the risk premium view that stockholders pay to avoid risk. In essence, Hawtrey is arguing that all carriers must be assured enough to cover the costs of carrying inventories, including the costs of uncertainty. The only possibility Hawtrey allows for an excess of spot over futures prices is when the spot commodity is in short supply. In this case, as Kaldor noted originally, stockholders must face the trade-off between convenience yield and carrying charges. This situation has now come to be called an inverse carrying charge, since inventory carriers are paying, rather than being paid, to move stocks through time.

Working [42, 43] addresses the issue of inverse carrying charges in some detail and introduces the theory of the price of storage. There is some overlap in these two works, and the review synthesizes the ideas in both. Further, while Working is concerned with the relationship between the futures prices for near and far delivery, both quoted at a given point in time, no injustice is done by presenting Working's theory in the context of the two period model here. Since arbitrage guarantees that the price of a futures contract at its termination date must equal a cash grade price, the analysis proceeds in the context of the relation between C_0 and F_0.

The first question addressed by Working concerned why, at a given point in time, the price for far delivery can exceed the price for near delivery or, in the context here, $F_0 - C_0 > 0$. One explanation is that F_0 reflects known circumstances that will affect later cash prices, but not current ones. Working argues that this is incorrect from an empirical standpoint, since $F_0 - C_0 > 0$ is usually observed when the near and far prices maintain a constant percentage relationship to each other over time. Indeed, Working argues that [42, pp. 14-15]

> The business of a futures market, so far as it may differ from that of any other, is to anticipate future developments as best it may and give them due expression in present prices, spot and near future as well as distant futures.

Accordingly, it is only supplies currently in existence which have any bearing on inter-temporal prices quoted at a given time.

In an effort to prove the point, Working [43] examined the implication from the Keynes-Hicks formulation that the futures price must rise throughout the duration of the contract. For agricultural commodities, redundant stocks are the rule, rather than the exception, and, as shown in Proposition 4, transportation of the Keynes-Hicks formulation to a futures market context yields the prediction of a rising futures price. Finding no such trend, Working concludes that his evidence is sufficient "to show that no theory founded on a downward bias could explain more than a very small inverse carrying charge" (p. 13). Then, Working argues "that in a perfectly functioning futures market continuous existence of any stocks should prevent the emergence of price differences that depend on expectations" (p. 13).

The theory of the price of storage is presented in two parts. First, when stocks are large and carried in such volume that rewards must go to stock-carrying, $F_0 - C_0$ reflects the cost of carrying inventories between delivery dates. Since costs of carrying stocks may also include excess profit when storage space is in short supply, this price difference reflects necessary returns for storage; $F_0 - C_0 > 0$. From the point of view of short hedging storage providers, Working states [42, p. 1257]:

> In making his decision, the hedger assumes as a first approximation that at the end of April the price of the wheat he owns will stand in the same position relative to the price of the May future as it holds at the end of November relative to the December future. ...it is common to make no adjustment for this possibility [a change in the price relationship over time] because the most reasonable assumption at the time is that no change in relation will occur.

This is simply an argument that there is no basis risk, as in expression (27).[7] Hence, the market provides a good estimate of the returns to hedging stocks and the means to earn the return by trading futures. When stocks are large and carryover causes expectations to bear equally on all current price quotations, $F_0 - C_0 > 0$ is a derived price of storage.

The second part of the theoretical development concerns the sign of $F_0 - C_0$ when stocks are in short supply. Working notes that at first glance there is a paradox because $F_0 - C_0 < 0$ quite often when stocks are low, even though storage still occurs. His explanation rests primarily on convenience yield, with the fact that storage is most usually undertaken by firms that both process or merchandise the commodity, along with storing it. Hence, the convenience yield to the processing operation may induce positive storage even when $F_0 - C_0 < 0$ and storage, in and of itself, is not profitable. Storage now occurs under an inverse carrying charge, or negative price of storage. In this case, Working allows that the carryover linkage may be broken, so that a negative price of storage may indicate some disproportionate influence of expectations upon near and distant prices, but it is as much an economically useful concept as a positive price. The storage supply curve in Figure 3 represents Working's theory. Working [40] later shows that the correlation between the current basis and the later basis is statistically significant. In all essential respects, the basis is a price of storage, competitively determined by those who seek to supply storage.

In the seed from Hawtrey and the full flowering from Working, a radical revision of the theory of futures markets arises. The earlier notion of a risk premium is rejected outright. The implication of a rising futures price toward the maturity date is found to be insignificant, and the foundation of differential expectations effects is replaced by equal effects on spot, near, and distant futures when stocks must be carried. Further, as Working states [42, pp. 1257-8], the motivation behind futures trading has been misunderstood.

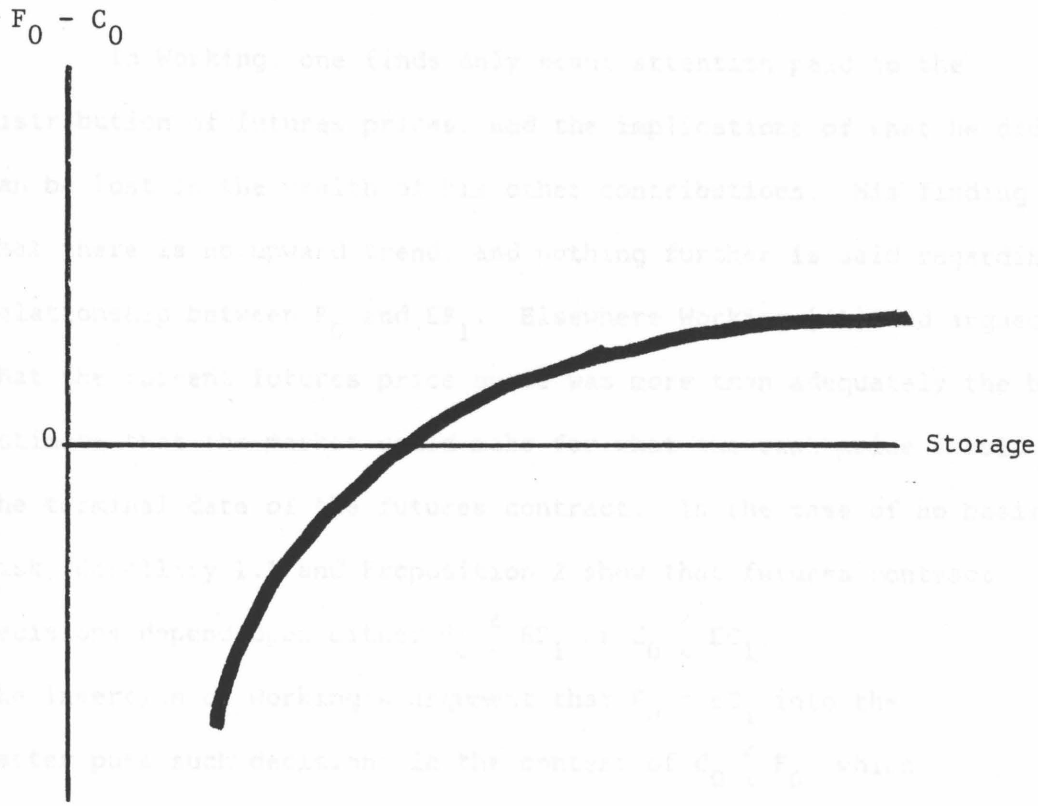

Figure 3. Working's Supply of Storage Curve.

> Thus existence of a futures market, coupled with the practice of hedging, gives potential holders of wheat a precise or at least a good approximate index of the return to be expected from storing wheat. This is an important fact which has been too much neglected in discussion of the economics of futures trading. It is through supplying a direct measure of the return to be expected from storage, and a means, through hedging, of assuring receipt of that return, or of approximately that return, that a futures market makes its most direct and powerful contribution to the economical distribution of supplies of a commodity over time.

In Working, one finds only scant attention paid to the distribution of futures prices, and the implications of what he did find can be lost in the wealth of his other contributions. His finding was that there is no upward trend, and nothing further is said regarding the relationship between F_0 and EF_1. Elsewhere Working [41] had argued that the current futures price quote was more than adequately the best estimate that the market could make for what the cash price should be at the terminal date of the futures contract. In the case of no basis risk, Corollary 1.1 and Proposition 2 show that futures contract decisions depend upon either $F_0 \gtrless EF_1$ or $C_0 \gtrless EC_1$. The insertion of Working's argument that $F_0 = EC_1$ into the latter puts such decisions in the context of $C_0 \gtrless F_0$, which makes the theory of the price of storage and the general expected utility model consistent with one another.

Proposition 6:

By the assumption that $F_0 = EC_1$, we find the following in the case of no basis risk.

(a) $Q_s \gtrless 0$ as $C_0 \gtrless F_0$

(b) $D_p \gtrless 0$ as $C_0 \gtrless F_0$

(c) $D_e \gtrless 0$ as $C_0 \gtrless F_0$

(d) $D_m \gtrless 0$ as $F_0 \gtrless C_0$

(e) Under the assumptions of Proposition 3, with $F_0 = EF_1$, $F_0 = C_0 = EF_1 = EC_1$ is an equilibrium which can be established without speculative participation.

Two things are particularly interesting about Proposition 6. First, in part (c) of the proposition, a positive price of storage occurs in Working's framework if and only if hedging is below the routine level, the price of storage is zero if and only if hedging is at the routine level, and a negative price of storage occurs if and only if hedging is beyond the routine level. These are interesting testable hypotheses, given data on hedging levels relative to total cash commitments. An interesting test of the theory would be to determine whether observed hedging practices conform to parts (b) - (d), and whether speculative positions appear as indicated in (a). The second point of interest results from (e). In an admittedly restrictive situation, speculative participation is not required when $F_0 = EC_1$, as seen in part (e) of the proposition. The current futures price is now a best estimate of its own later price, and the cash price is a best estimate of its own later price (both futures and cash prices perform a martingale).

The contribution of an alternative hypothesis to the theory of a risk premium set the stage for a flurry of empirical efforts aimed at determining whether or not such a premium existed. Since the empirical work and controversy over the results dominate this period, the important theoretical contributions usually receive scant notice. This is not as it should be. The "search for the risk premium" was directed almost entirely through the formalization of the theory reviewed thus far.

First among risk premium searchers to incorporate the contributions of Working was Houthakker [20]. He observes that, while distant futures prices are influenced primarily by general economic factors, the prices of near futures are governed by the magnitude and ownership of deliverable stocks. This implies that large speculators (predominantly professionals), with a comparative advantage over information concerning such stocks, should do better than small, occasional speculators in futures closer to maturity. Empirical evidence is garnered to prove the point. Houthaker further concludes that hedgers are the source of profits for other traders in a very consistent fashion. Monthly, gains accrue to large and small speculators in months of rising futures prices, as Houthakker interprets the prediction of the risk premium theory. Further, when profits are examined by the total net position, both large and small speculators lose when they go short. In general, if speculators assume a long position, they will on balance earn profits. Houthakker concludes that hedgers pay for the risk bearing services of speculators.

Brennan [6] and Telser [33] specify a formal profit maximization framework based upon Working's arguments. Telser develops the theory of the price of storage without accounting for the risk premium, choosing to reject such inclusion empirically, in a manner identical to Working's rejection of the risk premium, prior to a test of his theoretical development. In direct contrast, Brennan includes the risk premium in his theoretical development and then tests for its significance. Their work is presented in a comparative description, pointing out the differences in their two approaches.

Carrying through with the previous notation, Brennan and Telser both posit the following relationships relevant to stored commodities:[8]

(60) $d_0 = S_{-1} + X_0 - S_0$

(61) $C_0 = z_0(d_0)$

(62) $C_1 - C_0 = z_1(S_0 + X_1 - S_1) - z_0(S_{-1} + X_0 - S_0)$,

where d_0 = consumption at time 0
S_{-1} = stocks carried into time 0
X_0 = production at time 0
S_0 = stocks carried into time 1,

and the remaining variables are clear from this much detail. Expression (60) is the definition of consumption at time 0: the amount of stock available at time 0 which is not carried over into time 1. Expression (61) is the demand relationship at a point in time, a function of amounts consumed. The final expression (62) is the derived intertemporal demand for stocks; simple substitution of (60) into (61), taking the difference between the two time periods. To see that (62) is decreasing in stocks, suppose that the amount carried out from time 0, S_0, increases. This must have resulted from a decrease in consumption with its consequent rise in price at time 0. With S_1 and X_1 known, d_1 must increase and C_1 fall. Hence, a rise in carryover causes C_1 to fall relative to C_0. In summary, while the price spread, $C_1 - C_0$, may be positive or negative, the spread is inversely related to the level of stocks.

It is important to note at this point that C_0 is specified as the spot price at time 0. Since participants maximize under uncertainty, $EC_1 - C_0$ will play an important role. The manner in which each chooses to treat EC_1 as the current futures price provides interesting exceptions to the price relationships dictated by the various versions of the general framework, namely $F_0 \gtrless EF_1$ or $C_0 \gtrless EC_1$, as noted shortly.

The supply side is the source of the distinction between the two characterizations of the supply of storage. The distinction is their outlook on the role of a risk premium. Preserving a comparative notation, Brennan (superscripted B) and Telser (superscripted T) write, respectively:

$$(63)\quad k_0^B(s_0) = w_0(s_0) - q_0(s_0) + r_0(s_0)$$

$$(64)\quad k_0^T(s_0) = w_0(s_0) - q_0(s_0),$$

where, as in the discussion of Kaldor, w is standard storage costs and q is convenience yield. The obvious supply side distinction is that Telser develops his theory without a risk premium while Brennan includes it.

With suppliers maximizing expected net revenue, u, the familiar first-order condition for firm revenue maximization is

(65) $u' = k'$.

In a competitive industry, $u' = EC_1 - C_0$, and both authors assume constant returns to scale so that industry supply is the sum of equations such as (65). Both assume that the supply curve is stable over time so that shifts in demand identify the supply of storage. Hence, at equilibrium [by (65)],

(66) $EC_1 - C_0 = w' - q' + r'$

for Brennan, while no entry for the risk premium appears on the R.H.S. for Telser.

The developments by Brennan and Telser thus far concern the choice of stock-holding. As yet, there is no futures trading. Brennan's introduction of futures trading is quite direct. He simply states that, for stocks hedged on active futures markets, the relevant price spread is the basis, $F_0 - C_0$. The result $F_0 = EC_1$ occurs, according to Brennan, in the arbitrage related to establishing equilibrium. As noted earlier, Working argued the same should occur. For stocks with no active futures markets, or those that are characteristically thin, Brennan develops a moving-average price expectation model. He finds that : 1) the estimated expected prices deviate little from the actual price outcome and 2) in actively traded markets, the estimated prices differ little from the futures price quote. He then simply substitutes futures or estimated expected prices in the L.H.S. of (66), further estimates w', and relates the balance, r' - c', to stock levels over the harvest cycle.

Telser's introduction of futures trading is more theoretical. First, the speculative excess demand for futures contracts is posited as $x_i(F_0 - EC_1')$. This is similar to the demand curve posited later (reviewed earlier) by Stein [31], $G(EF_1 - F_0)$. Aside from redefining excess sales ($x_i > 0$ as purchase, vs. $G < 0$), the significant difference is the replacement of EF_1 with EC_1', which Telser describes as the ith speculator's expectation of the later cash price, rather than the market expectation EF_1 used by Stein. The reason that EC_1' appears, rather than, say, EF_1' is due to a further assumption that all speculators believe $EF_1' = EC_1'$, but not a commonly held value. To obtain the market demand curve, Telser assumes perfect competition and free speculative entry so that the market demand is perfectly elastic, and argues that $F_0 = EC_1$, where EC_1 is now a weighted average of speculators' expectations. Telser's justification is that $F_0 = EC_1$ because any other systematic relation would entice entry until the equality is restored.

In an effort to kill two birds with one stone, Telser undertakes an empirical examination of the Keynes-Hicks risk premium implication of a rising futures price, identical to Working's. Again, as shown in Proposition 4, the Keynes-Hicks formulation in the context of a futures market does lead to such an implication. However, there is nothing in that formulation, in a futures market context, concerning the relationship between the current futures price and expectations of the later spot price. Indeed, the general framework has shown that relationships between cash prices and prices for later delivery are only important in pure forward markets. Telser believes that by his empirical test he can evaluate both the existence of a risk premium and the question of whether or not $F_0 = EC_1$. The test can only accomplish the former. As with Working, Telser's test does reject a Keynes-Hicks risk premium (futures sellers paying buyers to bear their risks as indicated by a rising futures price) by finding no trend in the futures price. But Telser's conclusion that this also shows that $F_0 = EC_1$ is mistaken. Had he examined forward, as opposed to futures, prices and found no rising trend, then both rejection of the payment of a risk premium and $f_0 = EC_1$ would be valid conclusions, since they are equivalent in pure forward markets.

Brennan's introduction of $F_0 - C_0$ as the unit of analysis is by real world applicability and assumption and one can argue that the assumption is too heroic ($F_0 = EC_1$). Telser's introduction is by theoretical argument, assumption (perfect competition among speculators), and empirical justification. For the latter, Telser asks more of his hypothesis than it can bear. In conclusion, Brennan finds empirical support for the risk premium and Telser finds his own body of evidence rejecting the risk premium. Brennan shows that r' - q' is indeed greater than zero through a range of storage. By assuming that q' = 0 in peak storage months and that r' is linearly increasing in stocks, he calculates the average monthly return to be between 6.6% and 9.5% with the highest return occurring in semiperishables. Keynes had earlier put the level at 10%.

Telser goes on to estimate the relationship between "$F_0 - C_0$" and stock levels by ordinary least squares, using the prices of successively maturing futures contracts as the dependent variable in the manner suggested by Working. He notes at the outset that these futures price spreads are greatest in absolute value during low stock periods and smallest for futures contracts maturing at the beginning of the crop year when stocks are large. Estimation results show that a regular relation exists between price spreads and stocks for both cotton and wheat, in the predicted positive fashion. Also, slope coefficients are larger when stocks are low; not only are spreads largest in absolute value, but the effects of stock changes are largest during periods of low stocks.

On balance, Brennan's appears the most convincing argument concerning the risk premium. First, Brennan endows his theory with the ability to explicitly test for the level of the risk premium. Telser's conclusion that the estimation of the storage supply curve supports his version of the theory is not convincing. In the first place, judgments concerning support for any theory must come from an evaluation of how well the theory explains the issues involved. The entire point of Working's original formulation was to argue against the risk premium notion, an issue which Telser's theory is not equipped to address. Put another way, if Brennan had been so disposed, it is highly unlikely that estimation of the supply curve in Brennan's model would find vastly different results than those found by Telser. Neither author has any contention with the idea that the seasonal pattern of stocks determines price spreads and Brennan could have shown the same positive relation between spreads and stocks. Instead, he chose to concentrate his attention on the primary issue, the existence of a risk premium.

Perhaps the most telling argument in favor of Brennan concerns the added explanation that it provides for some shortcomings in the theory of the price of storage, originally acknowledged by Working [42]: with only marginal outlay and convenience yield, the theory fails to explain the observations that 1) much storage is supplied by firms that do not hedge, or completely hedge their inventory and 2) many hedged stocks earn per unit returns not equal to marginal outlay. Brennan's model, with its explicit inclusion of a risk term, addressed both points while Telser remains silent about hedging practices. Regarding per unit returns, Brennan points out that the risk premium is an added cost to hedging not reflected by marginal outlay alone. Regarding unhedged storage, Brennan argues that the observed higher return required by speculators in semi-perishables, assuming that marginal risk functions of all suppliers are about the same, would require the semi-perishable holder to accumulate relatively greater stocks before he will hedge. That is, relatively greater stocks must be accumulated before the holder's risk factor exceeds the going rate and he would choose to have speculators bear risks in his stead. Hence, semi-perishable holders will be much more likely to store part, or all, of their stocks unhedged. This harkens back to the observation from the general framework that stock carriers may prefer to hold unhedged stocks and assume, at least partially, the speculative role.

In the most famous exchange on the behavior of futures prices, Cootner [7] and Telser [34] spar over Telser's previous finding that there is no trend. Cootner stresses that a rising contract price should be expected only under certain conditons: net short hedging, instantaneous harvest, and hedges immediately instituted. Only under these conditions must the price rise be continuous over the post-harvest period. Cootner is quick to point out that these conditions are often violated. Just prior to harvest, hedging need not be net short, in which case we require a falling futures price for speculative profits. Further, if the harvest is not instantaneous, even if hedges are immediate, speculators would perceive a price decline as inventories gradually rise and avoid long positions until hedged inventories peaked. With these exceptions to the conditions of Telser's test, one would expect net long speculation only after the peak of hedging has passed; there should be a seasonal trend in the pattern of futures prices, falling prior to and shortly after harvest and rising once the peak of hedging has occurred.

By way of evidence, Cootner designs a linked futures price index for wheat. Assume that the position is initiated in May and continued through to next April in the following fashion. In May, purchase the nearest wheat future, switching to the next contract at maturation of the currently held future. The index is then the average of the month-end prices, calculated over stable general price level years. According to this index, declines are uniform through the first half of the position period (May through October-November) and the index rises thereafter through the post-harvest months. As the crop comes in, rising to peak commercial holding, futures prices fell. During the consumption months, prices rose. Cootner calculates the implied potential rate of return at roughly 8%, which approximates Brennan's findings. Further, Cootner notes that the data on net speculative and hedging positions support his deductions; while speculators are on average net long, the level of net long positions shows a decline during the preharvest months, which is compatible with speculative profits.

In his rebuttal, Telser constructs his own index, the ratio of monthly price to the average price from the duration of a given contract, using only months traded in every year and only years of stable general prices. Based on this index, most months have a mean which includes the value 100 in a plus or minus one standard deviation interval. The value 100 of a monthly average indicates no seasonal trend. According to this index, the hypothesis of no trend cannot be rejected. However, a test of no <u>upward</u> trend reveals that May wheat exhibits such a trend, December wheat and both May and December corn show a positive but insignificant trend, and May and December cotton show a significant positive trend. On the whole, while the monthly means arrange themselves in an upward trend, it is not the pre- and post-harvest seasonal which Cootner expects. In further rebuttal, Telser calculates the relationships between his futures price index and a seasonal index of net hedging commitments. If Cootner's proposed relation, based on a seasonal futures price, is correct, then these two series should vary inversely. Only in the May and December wheat contracts is there a significant inverse relationship.

Cootner has the final round in this exchange, and most relevant to this review, takes exception to Telser's argument that no seasonal trend in the May contract is counter to Cootner's seasonal hypothesis. In the May contract, hedges decline throughout and the trend should be upward but not seasonal. Finally, if the inverse relation between hedging and prices is calculated using deviations from the price trend of the latter, all correlations are negative and of increased significance.

Two other writers who contributed to the risk premium controversy about this time were Houthakker [22] and Gray [13]. In Houtakker's writing, an acute awareness of the role of inventories is clear and, in the vein of Cootner, the role of the risk premium receives special attention. Gray, on the other hand, argues expressly against any role for the risk premium, aligning himself with Working and the belief that there can be no trend in futures prices. We will return to Houthakker later.

Cootner [8] and Telser [36] subsequently reaffirm their disagreement over seasonality in futures prices. Cootner argues that hedgers have good reason to expect that it is more likely for the price of near futures to fall relative to the price of more distant futures. This argument is based on the asymmetry of the relation between spot and futures prices during times of decreasing stocks versus times of increasing stocks. So long as the supply of storage has positive slope, at a decreasing rate, the gain of spot relative to futures price, if stocks are smaller than required to meet expected demands, will always exceed the loss of spot relative to futures price if stocks are greater than expected. The typical storage supply curve is given in Figure 4. Given expectation E_0 about future demands, stock level S_0 is held at basis B_0. Suppose that stockholders detect an error in their expectations, having underestimated future demands. Expectations are revised upward to E_1, requiring greater stock levels S_1 at basis B_1. Given stocks are increased at a decreasing rate, an equal overestimation of future demands would require an adjustment of stocks to the lower level S_2 (commensurate with expectations E_2) at basis B_2. Now, while $S_1 - S_0 = S_0 - S_2$ since under- and overestimation are assumed equal, note that $B_1 - B_0$ is less than $B_0 - B_2$. Hence the most reasonable probability density function concerning changes in the basis is highly skewed right, with mean greater than the median; it is most reasonable to expect the near future (in the limit, the spot) price to lose ground on a distant future much more often than the near future gains on the distant. Given the shape of the storage supply curve, this asymmetry would be much more pronounced at low stock levels, at the end of the crop year.

Telser observes that there is also asymmetry in the opportunities confronting short and long hedgers which results in net short hedging and, further, that this explanation has nothing to do with periodic production. Short hedgers want to keep the opportunities for use of their spot holdings as open as possible while reducing risks due to price changes. While they could simply sell spot, this closes potentially lucrative spot transactions later. Given that they will hold spot inventories, they could sell forward rather than future. But this also closes the door on spot transactions. So, short hedgers hold

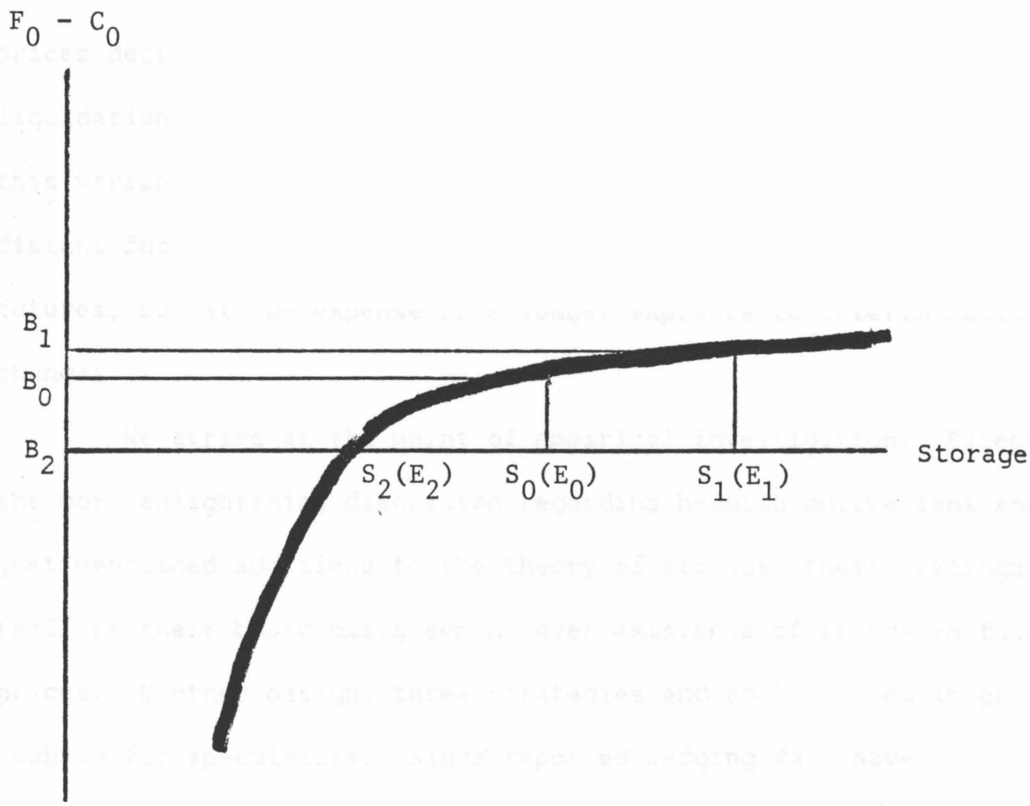

Figure 4. Cootner's Argument for Asymmetry in Basis Changes.

spot and sell futures. On the other hand, for long hedgers who sell their output forward, purchase of futures is riskier than immediate spot acquisition but specific grade-location combinations may be unavailable. Hence, while they would rather purchase spot, the specific nature of their spot needs may not be met by spot stocks at some particular point in time. The futures market is their alternative, but they would always rather buy spot.

On a final note, Cootner argues that, if the variance of futures prices decreases with the length of time remaining for inventory liquidation, and speculators are risk averse (risk measured in terms of this variance), then speculators would require a lower premium on more distant futures. Hedgers could then hedge more cheaply in distant futures, but at the expense of a longer exposure to interim basis changes.

We arrive at the point of empirical investigation. Except for the more enlightening discussion regarding hedging motivations and the just mentioned additions to the theory of storage, these writings reaffirm their basic disagreement over existence of trends in futures prices. Cootner designs three strategies and applies them in an "as if" fashion for speculators. Since reported hedging data have characteristic problems, Cootner decides that if reported short hedging falls below 3000 contracts, then actual net short hedging was less, and hedging was probably long in the May and July wheat contracts. His result is that both long and short hedgers pay a risk premium as evidenced by significant positive gains for speculators in wheat and soybeans. Hence, there must be futures price trends to facilitate these risk payments.

Telser first examines for trend using a computed seasonal price index and ordinary least squares to identify any upward trends and argues that there are none. Then he regresses futures prices on hedging commitments; the price index ought to vary inversely with short hedging and directly with long, both types of hedging calculated as an index. The expected relationships are not supported.

In summary, these two exchanges are fraught with contention. The major issues are 1) the existence of price trends and 2) the question of whether or not the pattern of hedging is consistent with speculative profits. The evidence is clearly mixed. Further fuel to the fire was added by Rockwell [29] in his refutation of Houthakker's [20] finding in favor of long-run speculative profits from consistently long positions.

Regarding the implications for a study of futures price distributions, the works of Brennan, Telser, and Cootner can be summarized by the following question. Does the pattern of stock levels, to which hedging behavior is keyed, have an effect on the observed pattern of futures prices and the relationship between cash and futures prices? The most explicit statement thus far is found in Cootner: the behavior of hedgers results in the seasonal pattern of prices which induces speculators to be long and short during particular, identifiable phases of the interharvest period. When short hedgers are most active at the hedging peak, speculators earn profits by staying long. As the hedging peak passes, long hedgers can come to dominate the market and speculative profits are earned on the short side of the market. It should be noted that the concern over the relationship between cash and futures prices is not the unit of analysis indicated in the general framework, and this focus results strictly from the view held among these writers that $F_0 = EC_1$.

The final contributor to the early theory of futures markets was Houthakker [21]. In futures markets, the relevant unit of observation, in the absence of assumption, is the series of futures prices, and Houthakker is the first to consider it explicitly. He also describes the crucial link between hedging, stock levels, and the pattern of futures prices. Houthakker introduced the notion of the risk premium in terms of futures markets, namely, when the futures price is a biased estimator of its own price at maturity. For the required dominance of short over long hedging, resulting in long run speculative viability on the long side of the market, he gives two arguments. One emphasizes the asymmetry of arbitrage between long and short hedgers. While short hedgers face limited risk because the futures price cannot exceed the spot price by more than carrying charges, long hedgers have no such protection. This limited risk situation encourages short hedging relative to long.

The second argument rests on the notion that the correlation between spot and futures prices depends upon stock levels. The price correlation is highest when spot grades are close substitutes for grades deliverable under the futures contract. Note the parallel to Dow's beliefs some thirty years prior. Due to the flexibility of grade and location characterizing futures contracts, futures are close substitutes for actuals when there is no shortage or surplus of particular spot grades and locations. These circumstances occur when large inventories exist. Hence, it should be expected that the cash-futures price correlation will rise as the crop reaches commercial hands and that the correlation should fall as the inventories are consumed. Examining wheat and cotton price correlations, Houthakker concludes that both long and short hedging are favored in the middle of the crop year but not before and after harvest. While correlation is important, so is the basis, which Houthakker argues is seasonal from the wheat and cotton data. The conclusion is that short (long) hedging is favored (discouraged) when stocks in commercial hands are largest and conversely for small inventories. The overall findings are summarized in Table 5. Accordingly, Houthakker expects short hedging to be seasonally largest when stocks are large, with little difference between the level of positions when stocks are small. Again, Houthakker argues that the data are supportive. Further analysis of Cootner and Houthakker is undertaken in Chapter 3, where the theoretical formulation for the empirical work in this thesis is described.

Table 5. Houthakker's Favored Hedging Positions Indicated by Stock Levels, Price Correlation, and the Basis.

Large Stocks		Small Stocks	
Basis	Correlation	Basis	Correlation
Short	Short	-	-
-	Long	Long	-

The goals of this chapter were twofold. On the first, bringing cohesion to a diverse theoretical literature, the different approaches and their underlying assumptions have been identified. It is in the comparison between them that the chapter makes its contribution; the tie that binds, from risk premium theory through the theory of the price of storage, is the effect of stock levels upon participant activity through the behavior of the relevant price for later delivery. The second goal was to identify theoretical implications for the analysis of commodity futures price distributions and such were highlighted. A more thorough undertaking comprises Chapter 3. In the next chapter, the empirical work on price distributions is reviewed. The richness of the theoretical implications for such work is virtually ignored in the literature as it now stands.

Footnotes for Chapter 1

[1] Lien and Quirk [26], in work found after this writing, apply a rational expectations frameowrk to a T-period, single contract model similar to the one developed here. They find that the futures market becomes a forward market in all periods prior to T-1, indicating that one might just as well examine a two-period model.

[2] For example, an analogy to Proposition 1 for those with a spot commitment is inexpensively derived since all have one first-order condition identical to (8). However, the relation between the level of the spot commitment and the futures position is inaccessible at this level of generality. One can only find the following. In (9), let y_p^* be the optimal choice for producers. Following the method in the proof of Proposition 1, one finds $y_p^* \geq 0$ iff $R'(y_p) + EC_1 \geq b'(y_p)$. Likewise, for elevator operators (expression (11)), $y_e^* \geq 0$ iff $R'(y_e) + EC_1 \geq C_0 + k'(y_e)$ and for millers (expression (13)) $y_m^* \geq 0$ iff $R'(y_m) + k'(y_m) + C_0 \geq EC_1$. With the added structure imposed below, the relation between spot and futures positions is much more interesting.

[3] It is verifiable through standard comparative statics analysis that $\partial y_p^*/\partial \rho^2 > 0$. Hence, for given $\sigma^2_{C_1}$, with risk aversion ($\chi > 0$), the partial derivative of the risk premium term with respect to ρ^2 is $-[\chi\sigma^2_{C_1}\rho^2(\partial y_p^*/\partial \rho^2) + \chi\sigma^2_{C_1}y_p^*] < 0$.

[4] Blau argues that hedgers use futures markets to buy security, rather than engage in trading for profit; hedgers desire to base their hedging decision on C_0, the current cash price, rather than their expectations about C_1. Hence, in analyzing the hedging decision, the current spot price may be substituted for hedgers' "non-effective" expectations about the later spot price. This substitution for expectations leads to the hedger buy and sell limits in Table 2, based upon $F_0 - EC_0$, rather than $F_0 - EC_1$.

[5] As observed in the title of Telser's paper, his approach is a variant on the "safety first" approach which has its own set of problems. In particular, the safety first approach is not consistent with the concept of Von Neumann-Morgenstern expected utility maximization, since it is essentially a lexicographic ordering of alternatives.

6 The fact that $F_t > C_t + k'(y_e)$ is disallowed also has an arbitrage interpretation, one which Telser does not state.

7 Working [38], [39] later notes that things are not so simple as the case of no basis risk, providing an argument for why hedgers might be considered as speculating on changes in the basis.

8 There are minor differences in the assumptions between the two works. Brennan assumes no time lag between sale and utilization, current and future production are known, and future carryovers are known. Also, Brennan eventually specifies the time period of interest at one month. Instantaneous harvest is implicit in Brennan, but explicit in Telser. Further, Telser specifies the time period of interest as that time spanned by the maturity dates of two adjacent futures contracts.

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CHAPTER 2

Review of the Empirical Work on Price Distributions

Introduction

The vast majority of empirical studies on the distributions of prices in so-called "speculative markets" have not concerned commodities. Instead, they have concentrated upon other assets such as common stocks. While the distributions across these different types of assets have been observed to share many characteristics, the markets themselves differ greatly and theoretical endeavors regarding the markets often differ as well. However, the work in other markets provides some insight into commodities and the work strictly concerning commodities provides an extensive array of issues even though the applications are few.

The chapter follows the historic evolution of ideas concerning empirical price distributions fairly closely from tests of the stable Paretion hypothesis against the original normality hypothesis through subordinated stochastic process alternatives. The works are evaluated for their relevance to theoretical considerations, especially those works concerning commodity markets. The only breaks with history occur in the reviews of work regarding serial independence in observed price series and, in a later section, where works of particular theoretical relevance are presented. These latter are singled out as examples of the important contribution that sound theoretical structure makes to understanding observed phenomena.

The empirical analysis of price behavior on organized markets has a long history and competing ideas abound. The evolution of this history begins with Bachelier [2], who posited the notion that prices perform a random walk. The assumptions underlying the random walk hypothesis are two: price changes are independent random variables and these variables conform to some common distribution. A major issue in subsequent work is the common distribution assumption, and it is the distributional aspect which is of most interest here. However, the independence assumption cannot be neglected since it is basic to the random walk. Before continuing on to the distributional aspects, the works concerned with serial independence are reviewed.

Serial Independence in Commodity Market Prices

Under the random walk hypothesis, prices are generated by the process

(1) $\varepsilon_t = p_t - p_{t-1}$,

where $p_t - p_{t-1}$ is the change in the random variable between the subscripted time periods and $\{\varepsilon_t\}$ is a sequence of independent identically distributed random variables. The random walk is a special case of a martingale where the changes are iid; i.e., the model in (1) has $E(\varepsilon_t | p_{t-1}, p_{t-2}, \ldots) = 0$ (Samuelson [42]).

Analysis of the independence assumption in commodities markets is nearly thirty years standing, and even a longer history exists in other markets. The techniques used typically involve tabulation of serial correlation coefficients to various lag periods, "runs" tests (nonparametric tests of whether the number of observed consecutive changes in the same direction violates the number dictated by chance), spectral analysis, and the profitability of "filter rules." These last are often called stop-loss rules, in which a trader's position is liquidated as soon as the price reaches a specified value or, in percentage terms, as soon as the price changes a specified percentage. For the changes to be independent, 1) serial correlation coefficients should be zero, 2) there should be no excessive number of runs, 3) there should be no excess contributions to the overall variance of ε_t by any observed price-change frequency (spectral analysis), and 4) no filter rule should result in a better profit outcome than a simple buy and hold strategy. The results in the literature on commodity prices are mixed regarding the existence and type of any serial dependence.

Working [50] found positive serial correlation in grain futures prices at lags exceeding one day. Houthakker [25], while finding little evidence of positive serial correlation to lags of 120 days, found that some stop-loss rules outperformed buy and hold strategies in spot and futures cotton prices, indicating non-randomness. Smidt [43] found negative serial correlation in soybean futures and Brinegar [7] found that grain futures exhibited behavior consistent with negative first-order serial correlation at lags of one to two weeks. Brinegar also found an excessive number of runs at four to sixteen week intervals. Stevenson and Bear [44] found both positive and negative serial correlation (5 day lags), excessive numbers of runs, and filter rules that outperformed buy and hold in cotton and soybean futures. Examining all futures contracts in wheat, corn, and soybeans, Dusak [12] found that serial correlation coefficients (lags to five months on semimonthly futures price observations) were usually insignificant and fluctuated about zero. Cargill and Rausser [8], using a variety of the above tests, found grain futures prices often to be non-random and copper prices even more so.

In markets other than commodities, the evidence is equally as mixed. The earliest analysis was by Cowles [11] who found positive serial correlation in stock market prices. Later, Working [51] showed that Cowles' use of average price changes could theoretically introduce this result and in his revision Cowles [10] admits to finding little in the way of dependence. Granger and Morgenstern [23] argue that the random walk is upheld by spectral analysis of stock market prices and, in subsequent work using a different sample, Godfrey, Granger, and Morgenstern [22] reach the same conclusion by the same technique. In his work on stock prices, Fama [15] analyzes serial correlation coefficients and runs finding no serious departure from independence in stock prices. Ying [52], using spectral analysis, finds dependence in stock price indexes and Poole [38] finds evidence of non-randomness in foreign exchange rates in the form of first-order serial correlation and abnormally profitable filter rules. Using spectral analysis, Upon [46] also found dependence at 32, 3.8, and 2.5 week periods in foreign exchange prices while Boness, Chen, and Jatusipitak [5] found some, generally negative, serial correlation and that about one-third of their electric utilities' stock price series showed excessive runs. Boness, <u>et</u> <u>al</u>, judged their evidence unconvincing on the issue of serial dependence or independence.

The efficacy of the above techniques will be discussed in the next chapter. All in all, the verdict is not clear regarding the issue of serial independence but, given the variety of results just shown, suffice it to say that any analysis based upon a random walk model must investigate the independence assumption thoroughly. Independence is basic to the random walk and there is no clear empirical precedence for simply assuming it to hold in general. Moving on to the identical distribution aspect of the random walk, the earliest work was Bachelier's in which normality reigned.

<u>The Stable Paretian Hypothesis V. The Normal Hypothesis</u>

According to Bachelier, if price changes from transaction to transaction, in a given trading period, are independent and identically distributed random variables with finite variance and the transactions are uniformly spaced, then we can appeal to the classic central limit theorem for the following powerful result. The central limit theorem dictates that price changes across trading periods of a day, week, or month will be normally distributed because they are sums of changes from transaction to transaction. Bachelier's stationary normal random walk is usually written:

(2) $L(t,T) = \log p(t,T) - \log p(t)$

where t = beginning of the trading period,
T = end of the trading period,
$p(t)$ = price at the outset,
$p(t,T)$ = price at the end,

with L(t,T) being random, independent, and identically distributed successive price increments. The L(t,T) increments are assumed to have normal marginal distributions with mean zero. Kendall [29] and Osborne [37] argued that the data conform to the model, approximately. The qualification was due to the finding that the sample variance of L(t,T) was not constant over time and the tails of the empirical distributions were thicker than for the case of the normal distribution.

Mandelbrot [33] was the first to take serious issue with the contention of normality. Since the qualifications just mentioned were consistently observed, Mandelbrot argued that agreements with normality were overemphasized while departures were neglected. Using data on spot cotton prices, Mandelbrot reached the following conclusions. While the L(t,T) increments are unimodal, the tails of the empirical distributions are so long that second moments vary erratically and the empirical distribution is too peaked relative to the normal distribution; L(t,T) exhibits a leptokurtotic distribution.

Mandelbrot offered the class of stable Paretian (SP) distributions as an alternative to the assumption of normality as the limiting distribution of successive price changes. In general, SP distributions do not have finite second moments and do not conform to any classical central limit theorem. Only in special cases, addressed below (including the normal distribution), is the variance finite. However, SP distributions do obey asymptotic laws which make them a statistically viable tool. The following short description of SP distributions is taken from Fama [16]. Since density functions are not necessarily defined for SPs, we refer to the natural logarithm of their characteristic function:

$$(3)\quad \log f(t) = \log \int_{-\infty}^{\infty} e^{iut} dP(\tilde{u} < u)$$

$$= i\delta t - \gamma|t|^{\alpha}[1 + i\beta(t/|t|)\tan(\alpha\pi/2)],$$

where $i = (-1)^{\frac{1}{2}}$
t = an arbitrary real number
$\tilde{u}$ = the random variable under consideration.

There are four parameters, α, β, δ, γ. The parameter α is the characteristic exponent that determines the total probability contained in the tails of the distribution, $0 \leq \alpha \leq 2$. An index of skewness is $-1 \leq \beta \leq 1$. The distribution is symmetric when $\beta = 0$. The location parameter is δ, akin to the expected value of $\tilde{u}$ but not always finite. The SP analog of the variance is the scale parameter, γ.

Finding that price distributions are SP has far reaching theoretical and empirical implications. If the population variance is infinite, then the sample variance is an inappropriate measure of dispersion and other useful statistical tools such as least-squares regression, which are based on the assumption of finite variance, are considerably weakened, at best. On the theoretical side, the often used tool of Markowitz mean-variance analysis is rendered inappropriate when variance of returns are infinite, and many models of capital asset pricing and portfolio choice use mean-variance analysis. The SP hypothesis clearly is of great importance.

The most important shortcoming of the SP family is that there are only three known explicit expressions for density functions. The normal density is characterized by $\alpha = 2$ (in which case δ and γ become the expected value of $\tilde{u}$ and half the variance of $\tilde{u}$ respectively), the Cauchy density by $\alpha = 1$ and $\beta = 0$, and the binomial density by $\delta = 1/2$, $\beta = 1$, $\delta = 0$, and $\gamma = 1$. In the normal case, we write (3) as: $\log f(t) = i\mu t - (\frac{1}{2})\sigma\tau^2$. Due to the lack of explicit density expressions, it is difficult to develop and prove propositions concerning the sampling behavior of any parameter estimates. Essentially, empirical efforts have been concerned with determining whether prices follow the normal distribution or the more general SP family, based on estimates of the characteristic exponent, α. If $\alpha = 2$, prices follow the normal, while for $0 < \alpha < 2$ the finite variance property is lost and prices follow the SP.

Due to the lack of explicit densities, in general nothing can be determined about the sampling error of any given estimate of &alpha&. However, attempts have been made to bracket the true value of using differing estimators. Estimators can be based on the properties of fractile ranges of variables that are distributed SP, behavior of the sample variance, or a weak form of the asymptotic law of Pareto. The most widely used estimator is one version based upon fractile ranges by Fama and Roll [17]. However, in order to discuss Mandelbrot's results, the asymptotic law of Pareto estimator must be described since that is the one he both developed and used. For the development of other fractile estimators and an estimator based upon the sample variance, the reader is referred to Fama [15].

Mandelbrot's estimator of the characteristic exponent is summarized as follows. Basically, for $\alpha < 2$, the tails of SP distributions obey the following asymptotic law (Gnedenko and Kolmogoroff [21]):

$$(4) \quad \lim_{u\to\infty} P(\tilde{u} > u) = (u/V_1)^{-\alpha}, \quad u > 0, \text{ and}$$

$$(5) \quad \lim_{u\to-\infty} P(\tilde{u} < u) = (|u|/V_2)^{-\alpha}, \; u < 0,$$

where the constants V_1 and V_2 are defined by

$$(6) \quad \beta = (V_1^\alpha - V_2^\alpha)/(V_1^\alpha + V_2^\alpha)$$

and β is the skewness parameter. In logarithms, we have

$$(7) \quad \lim_{u\to\infty} \log P(\tilde{u} > u) = -\alpha(\log u - \log V_1), \; u > 0, \text{ and}$$

$$(8) \quad \lim_{u\to-\infty} P(\tilde{u} < u) = -\alpha(\log|u| - \log V_2), \; u < 0.$$

The implications of expressions (7) and (8) are that if the $P(\tilde{u} > u)$ and $P(\tilde{u} < u)$ are plotted against $|u|$ on double-log paper, the two curves should become asymptotically straight with slope approaching $-\alpha$ as $|u|$ approaches infinity. However, the true asymptotic slope will only be observed within a tail area containing a total probability which is a decreasing function of α. This is due to the fact that α determines the height of, or total probability under, the extreme tails of the distribution. Letting $F_0(\alpha)$, $F_0' < 0$, be this total probability, derivation of the estimate of α is best shown using Mandelbrot's Figure 3, reproduced here as Figure 1, which sets $\delta = 0$, $\gamma = 1$, and $\beta = 0$. Horizontally, $\log|u|$ is plotted and, vertically, $\log P(\tilde{u} < u)$ which now equals $\log P(\tilde{u} > u)$ is plotted.

True asymptotic slopes are only reached in the linear portions of the curves, approximately at $F_0(1.5) = .015$ for $\alpha = 1.5$ (i.e., $P(\tilde{u} > u) \leq .015$), $F_0(1.8) = .0011$ for $\alpha = 1.8$, and at $F_0(1.99) = .00006$ for $\alpha = 1.99$. Suppose we choose a sample size

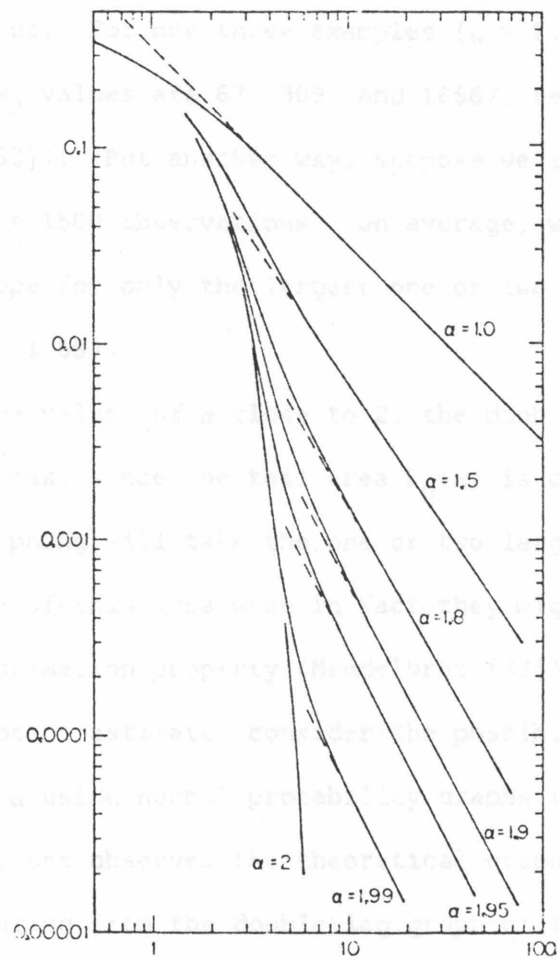

Figure 1. Double-Log Graphs (Mandelbrot [32], p. 402).

$N_0(\alpha)$ for each α in order to set the expected number of extreme values which exhibit the true asymptotic slope equal to one; choose the absolutely smallest sample size in which we can still expect to observe the true asymptotic slope. The "expected value" formula is $N_0(\alpha)F_0(\alpha) = 1$ so that we must have $1/F_0(\alpha)$ observations before the slope will even begin to approach $-\alpha$, the true asymptotic value. For our three examples ($\alpha = 1.5$, $\alpha = 1.8$, and $\alpha = 1.99$), the N_0 values are 67, 909, and 16667, respectively (Fama[15, p. 63]). Put another way, suppose we estimate $\alpha = 1.8$ from a sample of $N = 1500$ observations. On average, we will observe the true asymptotic slope for only the largest one or two observations in each tail ($NF_0(\alpha) = 1.65$).

For the values of α close to 2, the double-log graphing technique is weak, since the tail area $F_0(\alpha)$ is quite small, and double-log graphing will take the one or two largest values as representative of this area when in fact they might not be. This is not a desirable estimation property (Mandelbrot [32]) but all is not lost. Instead of a point estimate, consider the possibility of bounding the true value of α using normal probability graphs (Fama [15, pp. 63-5]). In the figure, one observes the theoretical graph for the case of $\alpha = 1.99$ deviating from the double-log graph of the normal distribution ($\alpha = 2$) at approximately the point where $P(\tilde{u} > u) = .001$. At another characteristic exponent value, $\alpha = 1.95$, we observe the deviation from the graph for $\alpha = 1.99$ at about $P(\tilde{u} > u) = .01$. This means that if $1.99 \leq \alpha \leq 2.00$, curvature in normal probability graphs would begin somewhere beyond the point

$P(\tilde{u} > u) = .001$ and if $1.95 \leq \alpha \leq 1.99$, the curvature would begin where $.001 \leq P(\tilde{u} > u) \leq .01$. We can similarly bracket the range of curvature origin in normal probability graphs for the other values of α, as in Table 1. This technique is superior to using double-log graphs exclusively since it accounts for more of the total area in the tails of the distributions. On the other hand, the most that can be accomplished is to bound the true value with this estimation technique.

Table 1. Curvature Origins and Estimated Intervals for the Characteristic Exponent.

Estimated Interval	Curvature Origin
$1.99 \leq \alpha \leq 2.00$	$0 \leq P(\tilde{u} > u) \leq .001$
$1.95 \leq \alpha \leq 1.99$	$0.001 \leq P(\tilde{u} > u) \leq .01$
$1.90 \leq \alpha \leq 1.95$	$0.01 \leq P(\tilde{u} > u) \leq .05$
$1.80 \leq \alpha \leq 1.90$	$0.05 \leq P(\tilde{u} > u) \leq .10$
$1.50 \leq \alpha \leq 1.80$	$0.10 \leq P(\tilde{u} > u) \leq 1.00$

Fama and Roll [17] provide a far simpler estimator of the characteristic exponent than that obtained by double-log graphing and normal probability graphs. In addition, they offer an estimator of the scale parameter in an earlier work (Fama and Roll [18]). Since the former can be stated as a function of the latter, the scale parameter estimator is described first. Based upon their derivation of cumulative distribution functions for standardized symmetric stable distributions, Fama and Roll [18, Table 1] show that the following estimator of the scale parameter has less than a 0.4 percent asymptotic bias:

$$(9) \quad c = (x_{.72} - x_{.28})/1.654,$$

where x_f refers to the f(N + 1)st order statistic which is used to estimate the .28 and .72 fractiles of the underlying distribution of x, and N is the sample size.

Along with their derivation of cumulative distribution functions, Fama and Roll [18] also derive a table of fractiles of standardized stable symmetric distributions. Based upon the behavior of higher fractiles, they suggest a simple estimator of α, also obtained from order statistics (Fama and Roll [17]). To obtain the estimator, first calculate

$$(10) \quad z_f = (x_f - x_{1-f})/2c.$$

They show, through the use of Monte Carlo techniques, that f = .95 or f = .97 are generally only trivially biased and that f = .99 is a better estimator only when $\alpha \geq 1.9$. The second step in arriving at the estimator is to read across the table of fractiles of standardized stable symmetric distributions, in the row associated with the chosen value of f, and choose the α_f value at the top of the column where the tabular value is closest to z_f. As mentioned before, by virtue of its simplicity and the apparent accuracy of its approximation, the Fama and Roll estimation technique is the most widely used in estimating the characteristic exponent.

There are two final notes. Fama and Roll [18] also show that the 0.5 truncated mean, where the sample mean is calculated only for the central 50 percent of the ordered observations, "performs very well for most values of α and N (p. 832)" as an estimator of location. Lastly, they show [17] that the studentized range, defined by

$$(11)\quad SR = (x_{max} - x_{min}) \Big/ \left[\frac{1}{N-1} \sum_{j=1}^{N} (x_j - \bar{x})^2\right]^{\frac{1}{2}}$$

where: x_{max} = sample maximum

x_{min} = sample minimum

$\bar{x}$ = sample mean,

had higher power than a broad array of distribution-free goodness-of-fit statistics (e.g., Kolmogoroff-Smirnov, χ^2) for testing normality against non-normal stable alternatives and generally outperformed the Shapiro-Wilk test which was designed specifically for a normal null hypothesis.

Tests of the SP Hypothesis

In this section, the results of work directed at testing the SP hypothesis against the normality hypothesis are presented. This literature is open to criticism for its fascination with determining which distributional requirements are satisfied without regard to the economic theory of price determination in the markets analyzed. Regarding commodity prices, the conclusion is that modeling economic processes involves more than curve fitting or simply mentioning the nature of information generation. While this is recognized by Mandelbrot ([31], p. 158), it remains a fundamental and largely ignored challenge to the empirical work on price distributions, especially in commodity futures markets. The primary contributions to tests of the SP hypothesis against the normality hypothesis as an explanation of price behavior are many. They will be discussed extensively when they directly concern commodity markets and receive summary acknowledgement when concerned with other markets.

Mandelbrot's [33] original evidence concerned spot prices of cotton, some obtained from personal communication with the U.S. Department of Agriculture and others obtained from Hendrik Houthakker. The argument is based on double-log graphs of the following three price series:

$$(12)\quad L_1(t,1) = \log p_1(t+1) - \log p_1(t)$$

$$L_2(t,1) = \log p_2(t+1) - \log p_2(t)$$

$$L_3(t,M) = \log p_3(t+M) - \log p_2(t).$$

Subscript "1" refers to the daily closing spot price of cotton in New York, 1900-1905 (USDA). Subscript "2" refers to an index of daily closing spot prices for cotton in the United States, 1944-1958 (originally, Houthakker's data). Subscript "3" refers to the closing spot prices of cotton on the 15th of each month in New York, 1880-1940 (USDA). The second argument in $L_i(t,1)$, $i = 1, 2$, denotes one-day intervals and the argument M in L_3 denotes a monthly interval; i.e., in the notation of expression (2), set $T = 1$ (day) in series 1 and 2 and set $T = M$ (one month) for series 3.

If the three series followed an SP distribution, their double-log graphs should resemble those in the previous figure, asymptotically approaching linearity. The graphs conform closely to the model's predictions (p.405), with evidence of a slightly negative skew ($\beta < 0$). Furthermore, for the daily series L_1 and L_2, the graphs are horizontal translations of one another indicating that between 1900 and 1950 (the years which these series best cover) the cotton price-generating process has changed only to the extent that its scale (parameter γ) has decreased.

Turning to a longer time span, Mandelbrot changes to another series of average monthly prices in place of series 3 since the series of averages provides a longer sample. He divides the period 1816-1940 into two subperiods, excluding the years 1862-1879. The two subperiods are 1816-1860 and 1880-1940. He further divides these subperiods as follows: 1816-1860 into 1816-1832, 1832-1847, 1847-1860; 1880-1940 into 1880-1896, 1896-1916, 1916-1931, 1931-1940. The first subperiod, then, is divided into three units of between 14 and 16 years. The second is divided into four units of between nine and 20 years. There is a 19 year gap between the subperiods, covering the Civil War years and reconstruction (1861-1880). Mandelbrot feels that his case is made:

> If cotton prices were indeed generated by a stationary stochastic process, our graphs should be straight, parallel, and uniformly spaced . . . The graphs (of his Figure 6, p. 407) are, indeed, not quite as neat as those relating to longer periods; but, in the absence of accurate statistical tests, they seem adequately straight and uniformly spaced except for the period 1880-1896 (p. 406).

In a later paper, Mandelbrot [34] extends his model to wheat at Chicago, 1883-1936, noting that there is at best a span of closing spot prices for the various grades of wheat, rather than one closing price for the standardized cotton commodity. At the close of a trading day, there is an interval holding the various cash prices associated with the different grades of wheat. Isolating "the" daily cash price becomes a problem, since closing intervals on consecutive days often overlap. Mandelbrot chooses one week as the shortest period for which use of a single cash price is reasonable. Mandelbrot uses both the normal probability/double-log graphing and analysis of sample variance methods to derive estimates of the characteristic exponent. First, graphing on probability paper reveals distinctly non-linear outcomes for weekly and lunar monthly price changes (pp. 402-3) but a more linear plot for annual price changes (p. 406). Second, referring to double-log graphs, wheat prices show the expected SP curvature and asymptotic linearity, but not to the extent that cotton prices showed; wheat prices have a characteristic exponent less than two but closer to two than the exponent for cotton prices. Finally, based on an analysis of the sample variance behavior, since the characteristic exponent is closer to two for wheat than for cotton it would be expected that the sample variance of wheat prices should increase at a lower rate than the sample variance of cotton prices as the sample size is increased. Indeed, this was the case (p. 406). Also, the distributions were found to be markedly stable over time.

Additional evidence of SP distributions in commodity prices is found in Dusak [12]. Her data cover futures prices for corn, wheat, and soybean contracts, 1952-1967, on a semi-monthly basis. The variable of interest in the study is the percentage change in the futures price, which Dusak (p. 1393-4) argues is representative of the risk inherent in ownership of the spot commodity. Given the nature of the data used, percentage changes are calculated on a two-week basis. All returns are computed separately for each contract. Thus, return series were discontinuous since a given futures contract did not run an entire year, in Dusak's sample.

Plotting the cumulative distributions of sample returns for each of the five wheat contracts, five corn contracts, and six soybean contracts on normal probability paper, Dusak (pp. 1396-7) finds that all contracts exhibit the non-linear S-shape characteristic of SP distributions (linearity is equivalent to normality on probability paper). The interfractile technique was used for estimating the characteristic exponent and, for the sixteen contracts over the sample period, the estimated values of α reported range from 1.44 to 1.84, with half of the estimates below 1.56. Dusak states, "It would seem safe to conclude that the distributions of returns of futures contracts conform better to the stable non-Gaussian family than to the normal distribution" (p. 1398).

In studies related to tests of the independence of successive price changes in commodity markets, Houthakker [25] reports that the daily closing spot prices for cotton, 1944-1958, and the prices of the six nearest futures contracts do not have day-to-day changes in the logs of prices that conform to normality. He observed more very large and very small changes than would be dictated by a normal distribution with the same mean and variance. In addition, the variance of price changes was not constant, exhibiting high variability in subintervals of the data. Stevenson and Bear [44] also report that their data on corn and soybeans are extremely non-normal.

In markets other than commodities, Mandelbrot [34] also found support for SP distributions in railroad stock prices (1857-1936), and interest and exchange rates (1857-1936). Earlier, Fama [15] performed all three estimation techniques on the thirty stocks comprising the Dow

Jones Industrial Average and argues that, while the different techniques offer different estimates, the results are sufficiently in favor of characteristic exponents less than two, consistently across all stocks. He states, "This would seem to be conclusive evidence in favor of the Mandelbrot hypothesis" (p. 68). Additional evidence favoring the SP distribution has been found in other stock prices (Teichmoeller [45]), Government Treasury Bills (Roll [41]), and foreign exchange markets (J. Westerfield [48]). Fieltz [20] finds evidence of an unstable variance in 200 New York Stock Exchange listings, daily and weekly, 1963-1968. However, he is among the first to note that instability does not necessarily imply an infinite variance. Another potential explanation is discrete steps in the variance as prices perform a random walk between the steps. This is an important observation, as shall be seen in later portions of this chapter, following an evaluation of the literature reviewed thus far. Before proceeding to the evaluation, it is worth noting that work on the detection of discrete steps in the variance due to exogenous factors has also been undertaken. Hsu [27] develops a test for discrete steps in the variance over time due to exogenous shocks and performs the test on air traffic flows and stock prices. In the latter, the test detects such a shift in 1973, which is attributed to a combination of effects from the Watergate scandal and rises in the prime lending rate. In related work, Hsu [26] develops tests to detect variance shifts in a Bayesian framework and Ali and Giaccotto [1] devise nonparametric tests of such shifts.

As one reviews this early empirical work testing the SP hypothesis, the most striking characteristic is the lack of theoretical economic underpinnings; the work concerns prices generated by functioning markets, yet makes no attempt to apply theoretical economic considerations to observed price series. Especially in the area of concern to this thesis, commodity futures markets, the preceding Chapter 1 documented the well-developed theory of the joint determination of cash and futures prices, but the work on the behavior of commodity prices is completely divorced from it. For example, the theory focuses quite extensively upon how the harvest cycle affects the behavior of short hedgers, long hedgers, and speculators, indicating a distinct frame of time reference for these participants tied to the interharvest period; short hedgers predominate during most of the post-harvest period while long hedgers may play an important role later in the period. Further, the theory gives an explicit account of the unit of observation: market participants deal in contracts of fixed duration with a multiplicity of contracts existing at any given time. In this sense, only Dusak pays attention to the unit of importance to commodities participants by calculating returns separately for each contract under observation. However, Dusak utilized capital asset pricing theory which addresses none of the issues relevant to the theory of commodity markets based upon hedging needs. Regarding Mandelbrot's evidence in cotton and wheat, the theory leads to questions concerning his choice of time intervals and the use of price indices without regard to the relevant duration of contracts. Furthermore, Mandelbrot's evidence on both cotton and wheat concerns only cash prices; Dusak's is the only treatment of futures prices.

It was noted that instability need not necessarily imply infinite variance. There is a substantial literature regarding this point, to which we now turn. As with the work just reviewed, this literature will also be presented in depth when directly concerned with commodity futures markets. Further, another evaluation follows its presentation, again with an eye toward the application of theoretical considerations in empirical work.

Subordinated Stochastic Process Alternatives

Before proceeding with the work on unstable variances, it is worth noting that the work on common stock prices by Granger and Morgenstern [23] and Godfrey, Granger and Morgenstern [22] found no instability. This finding was later reinforced by the analyses of Officer [36] and Upon [46]. Using spectral analysis, first Granger and Morgenstern and then Godfrey, Granger, and Morgenstern, in weekly and monthly price series from different New York Stock Exchange and London Stock Exchange data, found the normal random walk model to perform well. Regarding the problem of infinite variance, Godfrey, Granger, and Morgenstern state (p. 13):

> No evidence was found in any of these series that the process by which they were generated behaved as if it possessed an infinite variance.

With mixed evidence concerning the stability of the variance, and the fact that even instability does not necessarily imply an infinite variance, it is not surprising that alternative explanations to the SP hypothesis arose. These alternatives have been generally characterized as distributions arising from subordinated stochastic processes (SSP).

The notion of an SSP is now described in general (Clark [9]) and then particular specifications from the literature are presented.

Let $T(t)$ be a positive stochastic process; $T(t_1) \leq T(t_2) \leq T(t_3) \leq \ldots$ Feller [19] has proven that the following stochastic process can be formed: $X[T(t)]$. This later process, $X[T(t)]$, is said to be <u>subordinated</u> to $X(t)$ through the <u>directing process</u> $T(t)$. In the context of price generation, $X[T(t)]$ is the end result of the price generating process itself over calendar time t. $T(t)$ represents a "clock" measuring the speed at which the price process evolves over the calendar period t. For example, in a functioning market, calendar time t may be one trading day while the "clock" measures the rate of information accumulation in the market during that calendar day. High rates of information accumulation are associated with a greater evolutionary speed in the price generation process.

Moving to price change distributions, the distribution of $\Delta[T(t)]$ is said to be subordinate to the distribution of $\Delta X(t)$ through whatever distribution governs the speed of price evolution, $T(t)$. The distribution of observable price changes, $\Delta X(t)$, now measures the accumulated events of the past day without any adjustment for the effects of the distribution of price process evolutionary speed, $T(t)$, during that day. When a series of daily price changes is analyzed, the distribution of evolutionary speed affects the price generating process, resulting in the distribution of actual price changes that are observed. Following the example of a directing process related to information accumulation, if no new information is available on a given day, the price process may evolve slowly while on days when new information appears prices can evolve briskly. The variance of observed price changes will not be the same in these two cases not because the price generating process itself has changed but because the directing process is different. Hence, if one could condition on the variance of price changes, the distribution of the pricing process could be isolated. Put another way, let L(t,T) be the change in the log of price, as before. Let σ^2 be the variance of L(t,T), possessing density $g(\sigma^2)$. Let h(L) be the observed density of L(t,T) and $f(L|\sigma^2)$ be the density of L(t,T) conditioned on σ^2. Then

$$(13)\quad h(L) = \int_0^\infty f(L|\sigma^2)g(\sigma^2)d\sigma^2,\ 0 \leq \sigma < \infty.$$

Several alternatives for $g(\sigma^2)$ have been offered which, when coupled with a normal $f(L|\sigma^2)$, produce thick-tailed, higher peaked at the mean, i.e. leptokurtotic, distributions of observed prices h(L). This makes SSP models testable alternative to the SP hypothesis. All alternatives found in the literature have finite variances for h(L), although the variances do shift according to increments in the directing process. In the remaining works to be reviewed, the key to unlocking the unconditional distribution of price changes lies in: 1) finding a proxy measure for the directing process and 2) analyzing the distribution of this proxy. Not all of the works make such an attempt and, once again, the lack of theoretical considerations behind this literature is a glaring omission, especially in the one application to commodity markets.

Tests of Subordinated Stochastic Process Models

Press [40] was the forerunner in stochastic processes, before the mixture of a normal process and a distribution for the variance of that process received wide use. In his compound events model, the change of log price relatives was specified as the sum of two elements. First, a Poisson counting process representing the number of random events occurring during the given time interval was imposed on a sequence of mutually independent, normally distributed random variables with known, finite mean and variance. The second element of the sum was the change in a Wiener stochastic process which occurred over the given interval, also normally distributed with mean zero and known variance. The resulting distribution is leptokurtotic, relative to the normal distribution, with parametrically defined underlying variances in the two component stochastic processes, and a closed-form density.

On ten stocks in the Dow Jones Industrial Average, Press finds that agreement between the model and the data is not always close, but sometimes startlingly so. However, explicit estimators in the model were not available from maximum likelihood techniques and Press resorted to a method called cumulant matching.

The application of SSPs, as outlined above, began with the work of Praetz [39]. He shows that a gamma distribution for $g(\sigma^2)$, mixed with a normal distribution for $f(L|\sigma^2)$, yields a scaled t-distribution for $h(L)$. Again, this distribution is characteristically leptokurtotic compared to the normal. Praetz offers the following intuitive reasoning behind the instability of the variance (p. 52). The gamma distribution for $g(\sigma^2)$ represents changing expectations about interest rates, the state of the industry relative to a particular company, the state of the economy in general, and company earnings, dividends, and risk. All of these reflect information related to the price of a given stock for which the market must account. However, relating these variables is "a very difficult problem" and Praetz makes no such attempt.

Using seventeen share price index series, weekly observations from 1958-1966, Praetz finds that the t-distribution always outperforms the normal, SP, and compound process alternatives. Indeed, the other alternatives are rejected in fourteen cases. Praetz calls for a formal linkage of the variance distribution to variables determining changes in expectations.

Clark's [9] application is the only one concerning commodity futures prices. Further, Clark provides a link between the distribution of price variance and a variable regarding expectations, as suggested by Praetz. Calling the resulting unconditional distribution the "lognormal-normal distribution," Clark assumes $g(\sigma^2)$ to be lognormal and $f(L|\sigma^2)$ to be normally distributed. The resulting unconditional price distribution has no closed-form solution and must be solved using numerical techniques. Linking the distribution of the variance to a variable regarding expectations and information accumulation is accomplished through the use of trading volume. There is no theoretical relation between the variance of price changes and trading volume derived, but Clark performs a number of tests of this proposition.

After constructing two continuous time series of cotton futures prices, in index form for 1947-50 and 1951-5, Clark subdivides the two samples into groups of observations arbitrarily judged to exhibit similar trading volumes. Then the sample variance and kurtosis are calculated for each subgroup in the two samples. A curvilinear relationship between price change variance and volume is observed and on the basis of kurtosis outcomes closer to three (the value indicating normality), the subgroups are judged to be more normally distributed than the overall samples. A regression analysis of the relation between variance and volume, followed by tests of the lognormality of the directing process based on trading volume, leads Clark to conclude that 1) volume is lognormally distributed, 2) the variance of price changes is well-estimated using a function of trading volume, and 3) once price changes are conditioned on trading volume, there is a fairly strong case for the normal distribution. The evidence favors a finite variance, SSP model whose unconditional distribution should be lognormal-normal rather than stable Paretian. Additional empirical support for the relation between prices and volume was offered by Ying [52].

Blattberg and Gonedes [4] and R. Westerfield [49] provide evidence reinforcing the conclusion of SSP superiority over the SP and normal distributions. Blattberg and Gonedes (pp. 255-6) state a number of shortcomings in Praetz's approach and use both daily and weekly observations on the thirty stocks comprising the Dow Jones Industrial Average to show the superiority of the t-distribution over the SP. Westerfield extends the model of Clark to a much larger data set on common stocks (315 stocks on the New York Stock Exchange, daily for 412 trading days, January 1968-September 1969) in order "to reach clearer and more reliable conclusions about the validity of the subordinated model of security returns (p. 745)." The results support the SSP model relative to the normal and SP distributions.

Other results regarding alternatives to the SP distribution include the following. Brada, Ernst, and Van Tassel [6] found support for finite-variance SSP models while Mandelbrot and Taylor [35] argued in favor of an infinite variance version. The latter argument appears again in the work of Barnea and Downes [3] and McFarland, Pettit, and Sung [30] provide empirical support for the SP hypothesis in foreign exchange. Hsu, Miller, and Wickern [28] find in favor of a mixture of normal distributions within periods of homogeneous variance, Hagerman [24] refutes the SP hypothesis at both the portfolio and individual security levels, and Upton and Shannon [47] provide a test for shifts in the variance of price changes and apply the test to common stocks as well.

An evaluation of the work regarding the tests of alternatives to the SP model reaches the same conclusion as the previous evaluation of the SP alternative itself: there is still no economically guided theoretical content. While Clark's intuitive reasoning regarding the relation between price change variance and trading volume is a step in the direction of theoretical application, little of the richness of the economic theory of futures trading is exploited. Since Clark's is the only work related to commodity markets in the SSP literature, it is worth reviewing his intuition further to point out some questions that reveal where the theory has a role in analyzing price behavior in commodity markets.

In his analysis of the dependence of price change variance on volume, Clark (pp. 144-5) asks us to "consider how the futures market actually works." At any time, traders' expectations about the price of a given contract are reflected in their market position. Some are holding contracts (long), some have sold contracts (short), and some traders have no net position. When information relevant to traders flows to the market, prices and expectations change. Clark distinguishes information as being perceived as a cause for disagreement in expectation revision or being perceived as a cause for traders to revise expectations in unison.

> If the information is uncertain (i.e., some traders shift expectations up and others down on the basis of information), or if only "inside" traders get the information first, then large price changes will be coincident with high volumes. On the other hand, very large price changes will probably be due to information that is perceived by all traders to move the price in one direction. News of widespread insect problems might be an example of this sort of information in the cotton market. In this case, all traders would revise their expectations in the same direction, and the price change would have relatively low volume.

Clark is clearly interested in reasons behind the variation in trading volume. In commodities markets, the sorts of information that cause disagreement in expectations revision among traders include uncertain foreign demands, the unpredictable acts of governments, and insider information regarding crop status (early weather reports, early pest damage reports). Information that is cause for revision in unison include widespread crop reports such as those released periodically by government agencies and other reporting services. However, of overriding concern in commodity markets are the needs of short and long hedgers throughout the interharvest period, needs that are predictable and well-explained by the theory of futures trading. This observation leads naturally to conjectures about whether competing, theoretically justified reasons for shifts in price change variance might reveal more about the behavior of prices on commodity futures markets than is exhibited by the current state of the literature.

At this point, we have exhausted the literature strictly concerned with the distribution of prices on commodity futures markets. The primary conclusion is that the lack of theoretical concerns in empirical applications begs many questions that the theory has been designed to analyze. The following part of the section, although not strictly about commodity markets, outlines works that have started from a theoretical foundation in their quest after empirical content. If for no other reason, this makes them worth reviewing as an example of how theoretical richness contributes to empirical endeavors.

<u>Theoretical Relevance and Empirical Price Studies</u>

Boness, Chen, and Jatusipitak [5] begin their examination of stock market price distributions from a theoretical development of the relationship between a company's financial leverage and price behavior. They define the stochastic rate of return on common equity shares (stocks) as

$$(14) \quad R_e = (R_a A - iB)(1 - t)/C$$

where: R_a = stochastic rate of return on total assets, with known mean and variance

A = market value of total assets

i = interest rate on bonds

B = market value of bonds

t = average corporate tax rate

C = market value of equity share.

R_e is interpreted as the net return per dollar equity. Supposing that investors are interested only in the mean and variance of R_e, the authors show that both variables are increasing in B/C, the firm's financial leverage. Define the one-period rate of return on one share of equity as the log of price changes calculated over the period. Postulating that this one-period rate of return on equity is positively linearly correlated with R_e leads to their hypothesis that successive changes in the logs of prices may be affected by alterations in the firm's financial leverage, i.e., capital structure. More specifically, price changes follow a random walk with finite mean and variance but capital alterations cause the parameters of the random walk to change.

Much as Clark argued that sample partitions according to trading volume levels isolate the conditional distribution of prices, Boness, Chen, and Jatusipitak argue that sample partitions based upon capital alterations will isolate the conditional price distribution. The two arguments are distinguished by the latter's firm theoretical justification for the capital alteration criterion. While making no distributional assumptions about the occurrence of capital alteration, carrying out the price series partitioning on the basis of capital alterations by electric utilities companies yields subsamples of these companies' stock prices which the authors contend are much more often normally distributed than the overall, non-partitioned samples. The authors rightfully point out that, if their evidence is regarded as convincing, then partitioning price series to remove the effects of capital structure changes should be done before using the data for mean-variance and capital asset market model applications.

In papers by Epps [13] and Epps and Epps [14], the relation between price changes and volume is provided with theoretical justification. Using mean-variance analysis at the individual level, investor demands are aggregated by Epps to show how excess market demands jointly determine price and volume. Also, the effect of information flows on excess demands is analyzed to show how changes in expectations alter the price-volume equilibrium. The basic theoretical result is that the number of shares exchanged per transaction due to a price rise exceeds the number exchanged accompanying a price fall of the same magnitude. Epps tests the proposition using individual transaction data (volume and price for each transaction), from the New York Stock Exchange and the American Exchange, finding support for the theory.

In Epps and Epps, the original theory of Epps is extended to show the dependence between transaction volume and the change in the logarithm of security price between transactions. The log of price changes is then shown to follow a mixture of normal distributions with transaction volume as the mixing factor, using the same sample used earlier by Epps. The overall finding is that stock price changes over fixed intervals of time follow mixtures of finite-variance distributions.

While none of these works deal with commodities, each attempts to investigate the empirical distribution of price series from a strong theoretical basis. The results go beyond simply determining what form the distributions take and provide insight into the perplexing nature of these distributions, based upon theoretical reasoning. Rather than finding only that distributions appear to follow a subordinated process rather than a Stable Paretian, the theory allows for an explanation of the mixing process itself. In commodities, the theoretical explanation of the mixing process remains a largely unexplored topic. The suggestion from Chapter 1 is that the level of commodity stocks will have some directing influence on the distribution of commodity prices. In the next chapter, after developing the theoretical implications from Chapter 1 for empirical application, a simpler "first-try" attempt to account for the directing influence of stocks is identified. The peak in commercially held stocks is the key to this more modest attempt.

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CHAPTER 3

Theory, Empirical Implications and Methodology

Introduction

The theory reviewed in Chapter 1 showed that the behavior of futures market participants was inexorably tied to the relationship between current and expected futures prices, which is in turn bound to the cash price by the need to carry stocks through time. Chapter 2, the empirical review, pointed out a marked disregard of these important factors. In this chapter, the following are set out. First, from the arguments of Cootner and Houthakker, one implication of the theory of futures trading is identified; namely, that while futures prices may follow a random walk, the parameters of the price-generating process may change over the interharvest period. Second, from the arguments of Houthakker concerning the behavior of the correlation between cash and futures prices, a second empirical application is identified. The two are not completely independent and the final section of the chapter presents an empirical methodology designed to cover both.

The Implications of Short Hedging Dominance

In the theory review of Chapter 1, there are basically three views identified, regarding the behavior of futures prices. The first view holds that the futures price should rise throughout the duration of the contract whenever there are stocks in excess of the level required to maintain production at normal levels (Keynes-Hicks-Dow). The argument goes that commodity grades in production at any given point in time are not perfect substitutes for the grade deliverable on the futures contract so that long hedgers will not be able to hedge all of the risks that confront them (Dow), or that the desire to hedge planned sales is somehow less insistent than the desire to hedge planned purchases (Hicks). The second view is that there is no trend in futures prices since expectations are brought to bear equally on all prices in futures trading (Hawtrey-Working-Telser). This is due to the connection over time provided by carryover stocks from harvest to harvest. The third view admits the possibility of seasonal trends in the futures price, based upon the behavior of hedgers relative to stock levels over the harvest cycle (Cootner-Houthakker). It is the latter view which admits the most possibilities and provides the most interesting set of empirical propositions.

Cootner argues that, when inventories are small, short hedging will be light and, if the output commitments of long hedgers are large, then hedging can be net long. The time when inventories are likely to be small is just before the harvest. Hence, Cootner expects a falling futures price until hedged inventories reach their peak (i.e., stocks in commercial hands are at their peak) and a rising price only after this peak. He concludes that the requirements for a rising futures price may not hold over a substantial portion of the duration of some contracts. This can be illustrated with the aid of Figure 1, where the durations of the various Chicago wheat contracts, pre-1977, are shown. Before 1977, contracts did not run year-round and were typically of nine or ten months' duration. The harvest season for wheat varies geographically from late May in the southwest to September in the north, with actual arrival at storage facilities lagging approximately three months. In Figure 1, the dotted lines bracket an approximate period which would witness the peak of commercial stocks and the subsequent peak in hedging, i.e., the arrival of the northern harvest at storage facilities. All contracts in Figure 1 are drawn so that they expire in year 2.

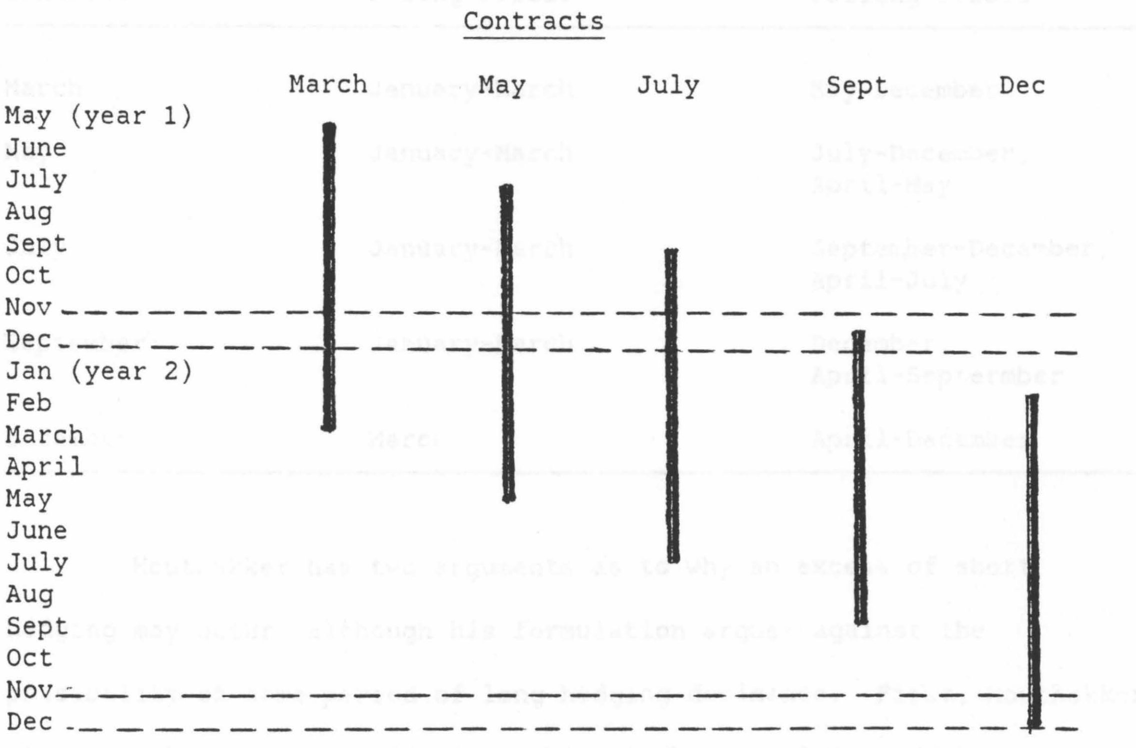

Figure 1. Shifts in Net Hedging Balances

According to Cootner, the pattern of futures prices exhibits two stages. Inside, and at some point prior to, the harvest period (May through December) futures prices should be falling. After the peak in commercial stocks, the futures price should be rising. The precise point in time where the futures price begins to fall is vague, and Cootner tells us only that "speculators must be expected to foresee this price decline at harvest time and are likely either to avoid long positions or to be short." Table 1 shows a Cootner-type breakdown of the futures price pattern for each contract.

Table 1. Price Pattern Break Down

Contract	Rising Prices	Falling Prices
March	January-March	May-December
May	January-March	July-December, April-May
July	January-March	September-December, April-July
September	January-March	December, April-Septermber
December	March	April-December

Houthakker has two arguments as to why an excess of short hedging may occur, although his formulation argues against the possibility of some period of long hedging dominance. First, Houthakker stresses the asymmetry of price arbitrage between long and short hedgers. While short hedgers face limited risk because the futures price cannot exceed the spot price by more than carrying charges, or costs of storage, long hedgers have no such protection. This limited risk situation encourages short hedging relative to long hedging.

Houthakker's second argument for the predominance of short hedging rests on the notion that the correlation between cash and futures prices depends on the stocks of the commodity. When inventories of a commodity are large, cash and futures prices for the commodity tend to be highly correlated, while at low inventory levels, cash and futures prices are less highly correlated. The link between short hedging dominance and this pattern of price correlation is that large inventories tend to be associated with low cash prices. Since short hedgers endeavor to avoid the risks associated with low cash prices and the correlation between cash and futures prices is large with large inventories, the futures contract offers a desirable instrument for their purposes. On the other hand, long hedgers try to avoid the risks of high cash prices, but the low correlation between cash and futures prices at high cash prices limits the effectiveness of the futures contract for long hedging purposes.

Recalling Proposition 5 of Chapter 1, this argument about the magnitude of the correlation between cash and futures prices is more than just intuitive. In Proposition 5, based upon mean-variance analysis, it was shown that seasonality in net hedging balances could occur under specific circumstances regarding the covariance between cash and futures prices. First, the covariance must be large enough to make short hedging profitable at high stock levels. Second, the covariance must be large enough to provide long hedging profits at low stock levels. Houthakker has simply argued that the second circumstance need not occur; however, theoretically speaking, there is no justification for this argument. Thus, whether at low stock levels the covariance diminishes sufficiently so as to to preclude long hedging, is an empirical matter.

Houthakker's second argument can be seen as highly reminiscent of Dow's earlier argument regarding the substitution opportunities between grades deliverable under the futures contract, as seen in the following example. Figure 2 depicts a situation where there are three cash grades of wheat (X, Y, Z) and three products (A, B, C) produced using wheat as an input. Product A requires wheat of grade X, product B requires wheat of grade Y, and product C can be produced with any of the three grades. D_A, D_B, and D_C are the factor input demand curves for the three products so that the x-axes measure quantities of wheat demanded for production of each output, (W_A, W_B, W_C). When stocks of all grades are large (superscript 1), just after harvest, grades X and Y are used to produce C, as well as A and B, and all grades sell at a common price, $P = p_X^1 = p_Y^1 = p_Z^1$; at price p_Z^1, the excesses of X and Y (E_X and E_Y, respectively) are bid away and the going price for any wheat input is P. Since the price at time t of a futures contract maturing at time t must equal the minimum cash price at t by arbitrage, the price of a futures contract maturing at time 1 is also equal to P. Later in the crop year (superscript 2), the smaller available supplies of X and Y are allocated to production of A and B, and the cash prices for different grades of wheat deviate from one another. Thus cash prices and futures prices tend to be more highly correlated at low rather than at high cash prices, because various grades of wheat are closer substitutes for one another at low rather than at high cash prices.

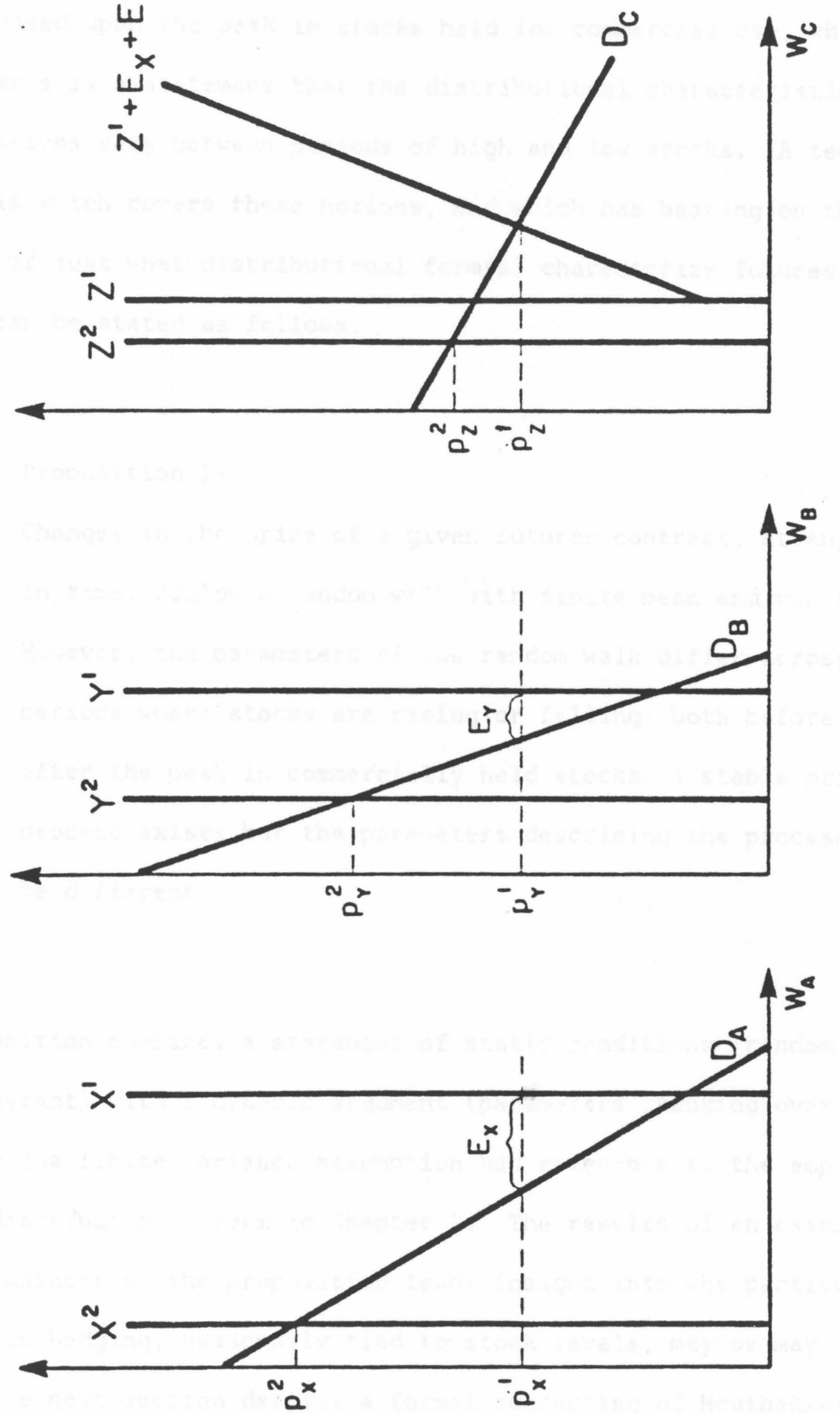

Figure 2. Inventories and Price Patterns.

From Cootner, then, one finds a distinction in the behavior of hedgers based upon the peak in stocks held for commercial use, while Houthakker's is a statement that the distributional characteristics of futures prices vary between periods of high and low stocks. A testable hypothesis which covers these notions, and which has bearing on the question of just what distributional form(s) characterize futures prices, can be stated as follows.

Empirical Proposition 1:

> Changes in the price of a given futures contract, at any point in time, follow a random walk with finite mean and variance. However, the parameters of the random walk differ across periods where stocks are rising or falling; both before and after the peak in commercially held stocks, a stable price process exists but the parameters describing the process may be different.

The proposition combines a statement of static conditions (random walk at any instant) with a dynamic argument (parameters changing over time). Examining the finite variance assumption has relevance to the empirical work on distributional form in Chapter 2. The results of an examination of the remainder of the proposition lends insight into why particular patterns in hedging, seasonally tied to stock levels, may or may not occur. The next section details a formal accounting of Houthakker's argument concerning the correlation between cash and futures prices.

Therein lies the other empirical application receiving attention in the next chapter.

The Houthakker Effect

Fort and Quirk [2] develop an analysis of the Houthakker Effect in two stages. First, turning to the futures contract itself, they show that a fundamental difference between commodity futures contracts and forward contracts is the flexibility provided to sellers (promising delivery) under the contract. For a wheat futures contract, the seller has the choice of the date during the delivery month to actually make delivery, the grade of wheat to actually deliver (at set penalties or premiums for nonstandard grades), and the delivery location itself (from a set of locations available under the contract). This flexibility on the seller's side provides certain arbitrage relations that characterize the joint probability distribution between cash and futures prices at the delivery date. Second, the Houthakker Effect can be cast as a property of the joint pdf between cash and futures prices that reflects high correlation between these prices at low values of the cash price. Building on this work, it is shown in this section that the Houthakker Effect results in a probability distribution at low cash prices which stochastically dominates the probability distribution at high cash prices for short hedgers. The two-period framework and, as nearly as possible, the notation of Chapter 1 are both used here.

Fort and Quirk assume that there are two cash grades, one delivery location under the futures contract, and both grades deliverable with no penalties or premiums. The prices of the two deliverable grades at time 1 are C_1^1 and C_1^2. To derive the joint pdf of cash and futures prices, they make use of the following arbitrage relationship:

$$(1)\quad F_1 = \min(C_1^1,\ C_1^2).$$

Because the seller of the contract has the choice of the grade-location combination to deliver, the arbitrage condition in expression (1) is insured, since he would deliver the lowest priced alternative should delivery become a reality. Hence, the joint pdf over the futures price and the grade 1 cash price is

$$(2)\quad h(F_1,\ C_1^1) = \begin{cases} 0, & \text{for } F_1 > C_1^1, \\ \int_{C_1^1}^{\infty} f(C_1^1,\ C_1^2)dC_1^2, & \text{for } F_1 = C_1^1, \\ f(C_1^1,\ F_1), & \text{for } F_1 < C_1^1, \end{cases}$$

where $f(C_1^1,\ C_1^2)$ is the joint pdf over the cash prices at time 1. An exactly symmetrical story can be told for a joint density between the futures price and the grade 2 cash price. Henceforth, let it be understood that C_1 stands for the grade 1 cash price in order to ease the notational burden.

Houthakker argued that the joint density in (2) should be characterized by a high correlation between C_1 and F_1 at low values of the cash price, and by a low correlation between these prices at high values of the cash price. Fort and Quirk interpret the Houthakker Effect in a somewhat different manner. They posit that the joint pdf is characterized by a Houthakker Effect if C_1 and F_1 are "close together" at low values of C_1. Formalizing, a Houthakker Effect is present (in the Fort and Quirk sense) if there exists $\varepsilon > 0$ and S sufficiently small such that

$$(3) \quad \Pr(C_1 - F_1 \le \varepsilon \mid C_1 < S) > \Pr(C_1 - F_1 \le \varepsilon \mid C_1 > S)$$

or, what is the same thing,

$$(4) \quad \int_0^S \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1 > \int_S^\infty \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1.$$

Note that (4) can be thought of as a comparison of two cumulative distribution functions for the futures price, one for $C_1 < S$ (L.H.S.) and one for $C_1 > S$ (R.H.S.).

Fort and Quirk continue their investigation at the more general level of the model used here in Chapter 1. Recall that the whole point of Houthakker's argument revolves around why short hedging will dominate futures trading, with the result that $EF_1 > F_0$ so that the futures price is expected to rise. Examining the decisions of short and long hedgers, their definitions of revenue functions follow expressions (5) and (6) in Chapter 1 quite closely. The main difference occurs in the effect of arbitrage on the expected utility function used to examine hedger behavior. Recall the arbitrage condition (1) in this chapter. Taking this into account, and using the short hedger as an example, the expected utility function becomes

$$(5) \quad \int_{F_1}^\infty \int_0^\infty U(V_e)h(F_1, C_1)dF_1dC_1 + \int_0^\infty U(V_e)h^*(C_1)dC_1,$$

where $h^*(C_1)$ is the pdf holding when C_1 is the minimum cash price, i.e., $C_1 \equiv F_1$. The first term is expected utility occurring when C_1 is not the minimum cash price. All other notation is familiar from Chapter 1. Upon integration of the short hedger's first order conditions by parts, Fort and Quirk find (with respect to y_e and Q_e, respectively):[1]

$$(6)\quad -\int_{F_1}^{\infty}\int_{0}^{\infty} H(F_1, C_1)[U''y_e(C_1 - C_0 - k + R') + U']dF_1 dC_1$$
$$+ \int_{0}^{\infty} U'[F_0 - C_1]h^*(C_1)dC_1 = 0,$$

$$(7)\quad -\int_{F_1}^{\infty}\int_{0}^{\infty} H(F_1, C_1)[U''y_e(F_0 - F_1)]dF_1 dC_1$$
$$+ \int_{0}^{\infty} U'[F_0 - C_1]h^*(C_1)dC_1 = 0,$$

$$\text{where } H(F_1, C_1) = \int_{F_1}^{C_1} h(F_1, x)dx$$

$$H^*(C_1) = \int_{0}^{C_1} h^*(x)dx.$$

Fort and Quirk introduce a Houthakker Effect into the decision problems of participants by perturbing the pdf $h(F_1, C_1)$ in a manner consistent with (3). They write

$$(8)\quad h(F_1, C_1, \theta) = h(F_1, C_1) + \alpha\theta(F_1, C_1),$$

where α is simply a shift parameter, e.g., $h(F_1, C_1, 0) = h(F_1, C_1)$. Note that $\partial h/\partial\alpha$, evaluated at $\alpha = 0$, is $\theta(F_1, C_1)$. This function can be thought of as a "shift function" imposed on the pdf $h(F_1, C_1)$. Their idea is to produce a Houthakker Effect like that in (3) through an appropriate choice of $\theta(F_1, C_1)$. In particular, they examine a "marginal" shift function of the form

$$(9)\ \Theta(F_1, C_1) = \int_{F_1}^{C_1} \theta(F_1, x)dx.$$

$\Theta(F_1, C_1)$ is a marginal function in the sense that it accounts for effects on the joint density of cash and futures prices when they are "close" to each other. Fort and Quirk proceed to examine sufficient conditions under which a Houthakker Effect, based on (9), can lead to short hedging dominance with $EF_1 > F_0$. They find that a perturbation of the following type produces just such an outcome:[2]

$$(10)\ \Theta(F_1, C_1) \geq 0 \text{ for } F_1 < F_0,\ C_1 < C_0 + k - R' + (1/\rho Q),$$

$$\Theta(F_1, C_1) \leq 0 \text{ for } F_1 > F_0,\ C_1 > C_0 + k - R' + (1/\rho Q).$$

Expression (10) identifies how "high" and "low" cash prices are identified in the context of expected utility maximization. Under the asymmetry in $h(F_1, C_1)$ produced by (10), short hedging [according to (6) and (7)] increases, long hedging decreases, and the resulting equilibrium has $EF_1 > F_0$.

Fort and Quirk summarize the implications fo their analysis. First, the conditions in (10) are sufficient conditions only. Further, while less restrictive sufficient conditions can be derived, they will be sensitive to the utility function chosen to characterize long and short hedgers. Second, it is clear from their analysis why Houthakker's original intuitive argument based upon partial correlation coefficients at low versus high cash prices does not provide simple proofs of short hedging dominance with $EF_1 > F_0$. Such correlation coefficients aggregate over ranges of cash and futures prices which are too coarse for proofs based upon arbitrary concave utility functions.

A third implication has greater impact upon empirical endeavors aimed at determining the importance of short hedging dominance as an explanation of price patterns on futures markets. A utility function independent description of a Houthakker Effect such as (3) is insufficient by itself to induce the short hedging dominance that Houthakker envisioned. Designing perturbations of the joint pdf $h(F_1, C_1)$ so that an effect like (3) will occur, i.e., an increased probability that F_1 is close to C_1 for low values of C_1 and a decreased probability that F_1 is close to C_1 for high values of C_1, requires more than the separation of high and low cash prices. Cash and futures prices are closer together when <u>both</u> are low and farther apart when <u>both</u> are high. Furthermore, while (3) does not depend upon the utility function chosen for long and short hedgers, the sufficient conditions in (10) are found in the context of expected utility maximization. The upshot of all this is that (3) must be specified in a consistent fashion with the conditions in (10) before a simple comparison of cdfs at low versus high cash prices can provide any insight into the conditions for short hedging dominance under a Houthakker Effect.

A Houthakker Effect as reasonably defined by Fort and Quirk is certainly an important occurence in the derivation of an equilibrium with short hedging dominance ($EF_1 > F_0$). Even empirical endeavors aimed at analyzing sufficient conditions are worthwhile. Based upon the idea of a Houthakker Effect in (3), the following section presents a cumulative density function interpretation of sufficient conditions for short hedging dominance. Whether price distributions have the characteristics proposed by Fort and Quirk, in the spirit of Houthakker, can shed light on the important issue of hedging dominance and the pattern of commodity futures prices.

Short Hedging Dominance and CDFs: An Empirical Framework

An empirical frameowrk for testing the existence of sufficient conditions for short hedging dominance can be designed as follows. The idea is to bring the descriptive statement of a Houthakker Effect in (3) into conformity with the sufficiency conditions for short hedging dominance in (10). Prior to any perturbation of the joint pdf $h(F_1, C_1)$, equilibrium is characterized by $EF_1 = F_0$ and the joint density is symmetric. Hence, y satisfies $R'(y) = 0$. With participants sufficiently risk averse ($\rho > 0$ becomes large), the term $1/\rho y$ in (10) becomes small. Hence, at the equilibrium prior to perturbation, with participants sufficiently risk averse, the conditions in (10), approximately, are

$$\text{(11)}\quad \theta(F_1, C_1) \geq 0 \text{ for } F_1 < F_0 \text{ and } C_1 < C_0 + k,$$

$$\theta(F_1, C_1) \leq 0 \text{ for } F_1 > F_0 \text{ and } C_1 > C_0 + k.$$

Now, write (3) as

$$\text{(12)}\quad \Pr(C_1 - F_1 \leq \varepsilon \mid C_1 \varepsilon R) > \Pr(C_1 - F_1 \leq \varepsilon \mid C_1 \varepsilon T),$$

where $R = [0, S)$ and $T = (S, \infty]$. For (12) to conform with the sufficiency conditions in (11), simply choose S so that $F_1 = F_0$ and $C_1 = C_0 + k$. Let $B = C_1 - F_1$, and note that the L.H.S. of (12) is just the distribution function for B, say, $H_B^R(\varepsilon)$, conditioned on low values of C_1 while the R.H.S. is the distribution function for B at high values of C_1, say, $H_B^T(\varepsilon)$. The definition of high and low is in accord with the restrictions from (10): R is such that $F_1 < F_0$ and $C_1 < C_0 + k$ while T is such that $F_1 > F_0$ and $C_1 > C_0 + k$.

In the same way that (3) can be written as (4), (12) can be written as

$$(13)\quad \int_0^{C_0+k} \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1 > \int_{C_0+k}^{\infty} \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1.$$

Using the cumulative density function notation, (13) can also be stated as

$$(14)\quad H_B^T(\varepsilon) - H_B^R(\varepsilon) < 0,$$

where the limits on C_1 are inserted to be consistent with (10). The question remains as to whether the L.H.S. of (13) has $F_1 < F_0$ or the R.H.S. has $F_1 > F_0$. Looking at the "inside" integrals on both sides, it is clear that the L.H.S. of (13) has $F_1 < F_0$ only when $C_1 \geq F_0$ while the R.H.S. has $F_1 > F_0$ only when $C_1 \geq F_0$. On the R.H.S., the arbitrage restriction in (1) allows no area of positive probability for $F_1 > F_0$ when $F_0 \geq C_1$, i.e., if $F_1 \leq C_1$ (according to (1)), then $\Pr(F_0 \geq C_1) = 0$. Hence, the L.H.S. must include an additional term when $F_0 \geq C_1$. Adding that term, the L.H.S. is

$$(15)\quad \Pr(C_1 \geq F_0) \int_0^{C_0+k} \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1 + \Pr(F_0 \geq C_1) \int_0^{C_0+k} \int_{C_1-\varepsilon}^{F_0} h(F_1, C_1)dF_1dC_1.$$

On the R.H.S. of (13), the sufficiency conditions in (10) are covered for $C_1 \geq F_0$, and arbitrage disallows any other outcome. Hence, the R.H.S. becomes

$$(16)\quad Pr(C_1 \geq F_0)\int_{C_0+k}^{\infty} \int_{C_1-\varepsilon}^{C_1} h(F_1, C_1)dF_1dC_1.$$

Combining (15) and (16), using the cumulative density function notation stated earlier, (12) can be rewritten as

$$(17)\quad Pr(F_0 \geq C_1) \int_0^{C_0+k} \int_{C_1-\varepsilon}^{F_0} h(F_1, C_1)dF_1dC_1 + H_B^R(\varepsilon)Pr(C_1 \geq F_0) > H_B^T(\varepsilon)Pr(C_1 \geq F_0).$$

One final rearrangement, along with some renaming of terms, produces

$$(18)\quad H_B^T(\varepsilon) - H_B^R(\varepsilon) < \omega[H_B^R(\varepsilon)]^*,$$

$$\text{where}\qquad \omega = Pr(F_0 \geq C_1)/Pr(C_1 \geq F_0),$$

$$[H_B^R(\varepsilon)]^* = \int_0^{C_0+k} \int_{C_1-\varepsilon}^{F_0} h(F_1, C_1)dF_1dC_1.$$

Comparing (14) and (18), it is clear that the R.H.S. of (18) represents the added adjustment required to account for the fact that cash and futures prices are closer together when <u>both</u> are low and farther apart when <u>both</u> are high. Expression (14) already accounts for the definition of low and high cash prices and (18) differs from (14) only in its R.H.S.. Since $\omega[H_B^R(\varepsilon)]^* > 0$, (14) is a more restrictive comparison of distributions at high and low cash prices than (18). Accounting for $F_1 < F_0$ at low cash prices and $F_1 > F_0$ at high cash prices increases the likelihood that a Houthakker Effect can result in short hedging dominance.

In what follows, the more restrictive comparison that does not account for the fact that $F_1 < F_0$ at low cash prices and $F_1 > F_0$ at high cash prices will be adopted. The difficulty with adopting the comparison in (18), instead, lies with the ω term. For descriptive purposes, suppose that the time zero futures price is a good estimate of the later cash price, so that C_1 and F_0 are not far apart. Then, one could approximate (18) by $H_B^T - (1 + \omega)H_B^R < 0$. Determination of the weight attached to the cumulative distribution at low cash prices requires analysis of the conditional (on $F_1 \leq C_1$) <u>cash</u> price density and greatly complicates any empirical work. Further, in a test of sufficient conditions, a fairly demanding test is preferrable; if short hedging dominance is found under a comparison such as (14), it will also hold for the comparison in (18). It cannot be denied that by choosing to ignore the R.H.S. of (18) some theoretical direction over the empirical endeavor is sacrificed.

A further refinement for the empirical work in the next chapter is to rewrite (12) as either of the following:

(19) $\Pr(F_1 \leq \phi \mid C_1 \varepsilon R) < \Pr(F_1 \leq \Phi \mid C_1 \varepsilon T)$,

(20) $H_{F_1}^T(\Phi) - H_{F_1}^R(\phi) > 0$,

where $\phi = C_1 - \varepsilon$ for $C_1 \varepsilon R$, $\Phi = C_1 - \varepsilon$ for $C_1 \varepsilon T$, and $H_{F_1}^R$ and $H_{F_1}^T$ are cdfs of the futures price, rather than $C_1 - F_1$. Note that the $C_1 \varepsilon R$ never exceed the $C_1 \varepsilon T$. Hence for any $\varepsilon > 0$, $\Phi \geq \phi$.

Using a comparison of futures price distribution functions as in (20) has advantages over a comparison of distribution functions for

$C_1 - F_1$ as in (14). The advantage centers around the type of quesitons one wishes to ask. Expression (14) lends itself to questions regarding how close the cash and futures prices must be in order for a Houthakker Effect to result in short hedging dominance. Expression (20) can be used to address the question of how far apart must high and low cash prices be for such an outcome. While (10) gives the theoretical result for the latter question, empirical possibilities ask for more.

To see this, take the benchmark empirical case in Figure 3. In part (a), corresponding to (14), the sufficient conditions from Fort and Quirk for a Houthakker Effect to result in short hedging dominance are met for any $\varepsilon > 0$. Similarly, in part (b), corresponding to (20), short hedging dominance can occur for any choice of $\Phi \geq \phi$. Should all empirical cdfs conform to the case in Figure 3, there is little reason to prefer one comparison over the other.

The advantages of one comparison over the other can be seen with reference to Figure 4. Part (a) is associated with (14) and part (b) with (20). Turning first to part (a), the requirement from (14) is $H_B^R > H_B^T$. For $\varepsilon < \varepsilon^0$, the sufficiency conditions for short hedging dominance are met. An example is shown for ε^1. Beyond ε^0, the cdfs do not satisfy the requirements. In a sense, $\varepsilon > 0$ sufficiently small is an empirical amendment to the requirements embodied in (14); nearness of the cash and futures price takes on an added dimension in the case of Figure 4. In part (a), the cumulative probability of meeting the sufficiency conditions can be seen as $H_B^T(\varepsilon^0) = H_B^R(\varepsilon^0) = H_B(\varepsilon^0)$.

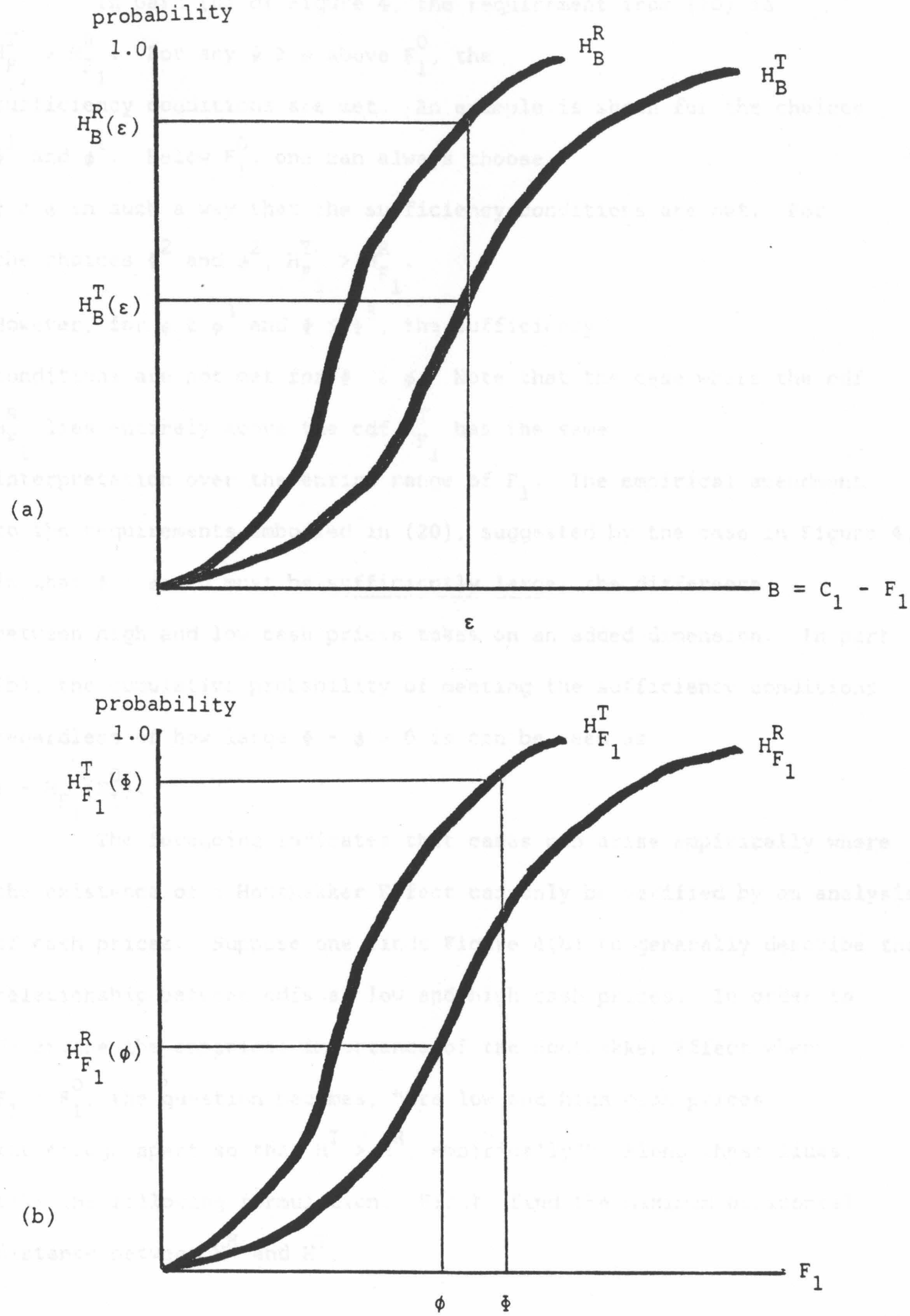

Figure 3. Empirical Distributions: Benchmark Case.

In part (b) of Figure 4, the requirement from (20) is $H^T_{F_1} > H^R_{F_1}$. For any $\Phi \geq \phi$ above F^0_1, the sufficiency conditions are met. An example is shown for the choices Φ^1 and ϕ^1. Below F^0_1, one can always choose $\Phi \geq \phi$ in such a way that the sufficiency conditions are met. For the choices Φ^2 and ϕ^2, $H^T_{F_1} > H^R_{F_1}$. However, for $\phi \geq \phi^3$ and $\Phi \leq \Phi^3$, the sufficiency conditions are not met for $\Phi \geq \phi$. Note that the case where the cdf $H^R_{F_1}$ lies entirely above the cdf $H^T_{F_1}$ has the same interpretation over the entire range of F_1. The empirical amendment to the requirements embodied in (20), suggested by the case in Figure 4, is that $\Phi - \phi > 0$ must be <u>sufficiently large</u>; the difference between high and low cash prices takes on an added dimension. In part (b), the cumulative probability of meeting the sufficiency conditions regardless of how large $\Phi - \phi > 0$ is can be seen as $1 - H_{F_1}(F^0_1)$.

The foregoing indicates that cases can arise empirically where the existence of a Houthakker Effect can only be verified by an analysis of cash prices. Suppose one finds Figure 4(b) to generally describe the relationship between cdfs at low and high cash prices. In order to determine the empirical importance of the Houthakker Effect when $F_1 < F^0_1$, the question becomes, "Are low and high cash prices far enough apart so that $H^T > H^R$, empirically?" Along these lines, take the following formulation. First, find the maximum horizontal distance between H^R and H^T,

$$(21)\quad \Delta F = \max(\bar{F} - \underline{F}), \text{ for } F_1 < F^0_1,$$

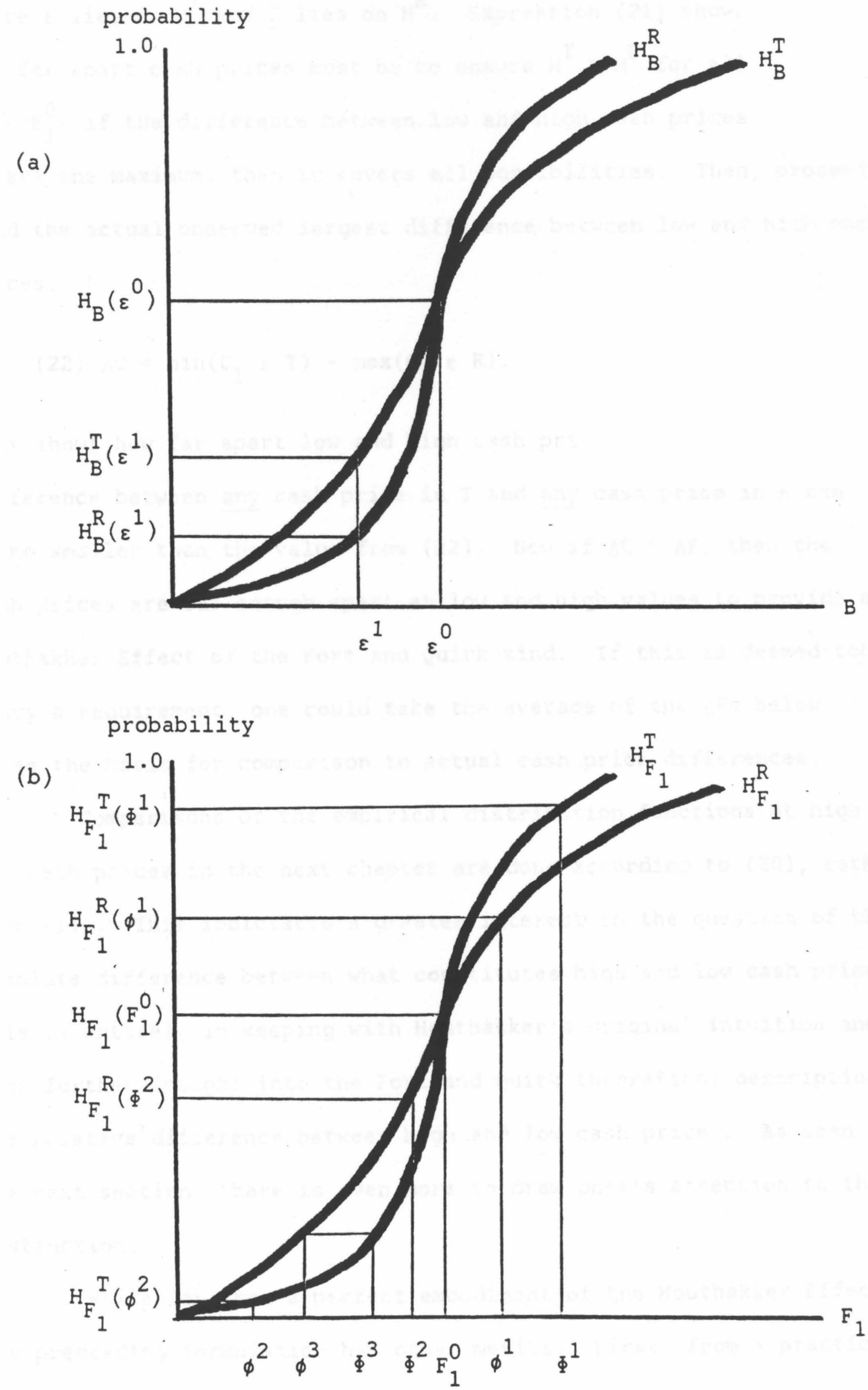

Figure 4. Cumulative Distributions: $C_1 - F_1$ Versus F_1.

where $\bar{F}$ lies on H^T and $\underline{F}$ lies on H^R. Expression (21) shows how far apart cash prices must be to ensure $H^T > H^R$ for all $F_1 < F_1^0$; if the difference between low and high cash prices covers the maximum, then it covers all possibilities. Then, proceed to find the actual observed largest difference between low and high cash prices,

$$(22)\ \Delta C = \min(C_1 \varepsilon T) - \max(C_1 \varepsilon R).$$

This shows how far apart low and high cash prices actually are; the difference between <u>any</u> cash price in T and <u>any</u> cash price in R can be no smaller than the value from (22). Now if $\Delta C > \Delta F$, then the cash prices are far enough apart at low and high values to provide a Houthakker Effect of the Fort and Quirk kind. If this is deemed too heavy a requirement, one could take the average of the ΔFs below F_1^0 as the basis for comparison to actual cash price differences.

Comparisons of the empirical distribution functions at high and low cash prices in the next chapter are done according to (20), rather than (14). This indictates a greater interest in the question of the absolute difference between what constitutes high and low cash prices. This is entirely in keeping with Houthakker's original intuition and adds further insight into the Fort and Quirk theoretical description of the relative difference between high and low cash prices. As seen in the next section, there is even more to draw ones's attention to this distinction.

While far from a perfect embodiment of the Houthakker Effect, the preceeding formulation has other merits. First, from a practical standpoint, it is quite easy to obtain empirical cumulative distributions, once high and low cash prices are separated, in order to see if (20) has any observable verification. How such separation can be accomplished will be described in the next section. Second, cash price series present their own set of problems for empirical work. While a series for a grade deliverable under the futures contract is available, expression (1) makes it clear that only a particular cash price has significant bearing on the distribution of futures prices. The cash price series available is not necessarily the one of interest. Third, while Houthakker's conclusions favored short hedging dominance, Cootner argued that switching between short and long hedging dominance would occur around the peak in commercially held stocks. If, as also detailed in the next section, high and low cash prices occupy the periods before and after the peak in stocks, respectively, empirical examination of (20) can show whether there is any observable sufficiency for short hedging dominance and contribute some insight into the conflict between Cootner and Houthakker. Basically, it would be interesting to see if the cumulative distribution foundation supports any empirical superstructure.

Empirical Proposition 2:

> The importance of a Houthakker Effect to short hedging dominance, and the theoretical conflict between Cootner and Houthakker, can be examined empirically on the basis of (20); are there sufficiency results dictated by the cdfs at low versus high cash prices?

Empirical Methodology

The theoretical summaries and new developments in this chapter reveal a common element in the Cootner-Houthakker argument that shifts in net hedging balances may occur, and Houthakker's arguments concerning short hedging dominance, the latter being referred to as the Houthakker Effect. The common element is the level of stocks. The peak in stocks determines the point of possible net hedging shifts and the behavior of the joint density over cash and futures prices at low versus high stocks is the foundation of the Houthakker Effect. This suggests the following methodology.

By identifying the peak in commercially held stocks and dividing time series samples into sets of observations relevant to the period before the peak and after the peak, the following questions can be addressed: 1) are the distributions the same in each subsample, 2) how do the subsamples dictated as important by the theory of hedging and price behavior differ from the overall samples from which they were obtained, 3) what statements can be made regarding the empirical controversy over the type of price-generating process which operates in commodity futures markets, 4) what are the implications for other theoretical applications to observed price series, and 5) do empirical cdfs conform to sufficiency conditions for short hedging dominance.

To address the first four questions, the unit of observation chosen for analysis is the relative change in the futures price, defined as

$$(23)\quad P_t = (F_t - F_{t-1})/F_{t-1}.$$

This choice has the virtues of comparability with the other work specifically interested in the distributional aspects of futures prices (Dusak), the removal of time trends due simply to rises in the general price level that might affect a time series of futures prices, and a natural interpretation as the rate of return on holding a futures contract from time t-1 to time t. Subsamples are obtained by identifying the peak in commercially held stocks at Chicago, found in the Statistical Annual of the Chicago Board of Trade [1], and simply dividing the observed sample into two subsamples at the date on which the peak occurred.

This subsampling technique also facilitates an empirical analysis of the Houthakker Effect and Cootner's notion of a switch in hedging dominance. The result common to both Houthakker and Cootner is that short hedging will dominate at low cash prices. They do not agree over the existence of any long hedging dominance at high cash prices. Cootner argues that the requirements for a rising futures price ($EF_1 > F_0$) and short hedging need not always be met, particularly prior to the peak in commercial stocks. The subsample technique can be used as a method of conditioning on low and high cash prices since it is reasonable that prices are low when stocks have reached commercial hands. In this sense the cumulative distribution function of the futures price at low values of the cash price is derived from the period after the peak in commercial stocks, while the cdf relevant to high cash prices covers the period prior to the peak. In the spirit of (10), $C_1 \lessgtr C_0 + k$ after (before) the peak in commercial stocks.

Since the Houthakker Effect is only a theoretically sufficient condition for $EF_1 > F_0$, such an analysis does not identify it as <u>the</u> reason for any rising trend in the futures price, but the empirical presence of (20) would be evidence that net short hedging can occur at low cash prices, as Houthakker originally proposed. It is important to note that this portion of the empirical work concerns the distribution of futures prices proper, rather than relative price changes specified in (23).

The method just developed can accomplish the following. First, the empirical importance of sufficient conditions for short hedging dominance at times of low cash prices receives theoretically guided attention. It must be stressed that only sufficient conditions for short hedging dominance at low cash prices ($EF_1 > F_0$) can be addressed. However, if little support is found, this does not indicate that short hedging dominance <u>will</u> not occur. Instead, it indicates that high and low cash price distributions do not offer the incentives that Houthakker suggested. Second, we are able to delve further into the contribution of the absolute difference between high and low cash prices to a Houthakker Effect and short hedging dominance. Third, and finally, the approach developed here avoids two problems with Houthakker's original notion based on the correlation between cash and futures prices. It avoids problems which may be due to badly behaved higher moments of the futures price distribution and there is no problem of aggregating over ranges of the cash and futures price, or "coarseness," in the Fort and Quirk sense.

Footnotes for Chapter 3

1 Fort and Quirk do not specify costs as a function of stock levels chosen, i.e. $k(y_e)$. Instead marginal costs are constant, k.

2 The assumptions underlying (10) are: except as specified in (10), $\theta(F_1, C_1) = 0$, $\theta(F_1, \infty) = \theta(F_1, F_1) = 0$ for all F_1, y and Q (cash and futures positions, respectively) are complementary in the sense that marginal expected utility of hedging is an increasing function of the cash commitment, speculative excess demands are less than perfectly elastic, an equal number of short and long hedgers with equal cash commitments and equal futures commitments, all participants have identical concave utility functions and nondegenerate pdf $h(F_1, C_1)$, and the original equilibrium prior to the perturbation had $EF_1 = F_0$.

REFERENCES FOR CHAPTER 3

[1] Chicago Board of Trade, Statistical Annual, 1968-1982.

[2] Fort, R. and Quirk, J. "Normal Backwardation and the Houthakker Effect." Social Science Working Paper No. 467. Division of Humanities and Social Sciences, California Institute of Technology, Pasadena, California, (Revised) October 1985.

CHAPTER 4

The Data and Empirical Results

Introduction

As shown in the preceding chapter, two testable implications can be drawn from the theory of futures trading: 1) difference in the parameters of an hypothesized random walk in futures returns, before and after the peak in commercial stocks and 2) the Houthakker Effect implication that short hedging dominates at low values of the cash price. Further, while both of these items pertain to the identical distribution aspect of the random walk, another point of empirical interest is serial independence of successive relative price changes.

Regarding the two theoretical implications, the methods of analysis chosen for investigating the presence of a Houthakker Effect and the difference in distributions, before and after the peak in commercially held stocks, were described in the last chapter. The question of serial independence will be addressed, following a description of the data, and the two distributional analyses round out the chapter.

The Data

The futures price data analyzed are percentage changes calculated for the closing price of the March wheat contract, daily at Chicago, over the period April 3, 1968 (opening of the contract) to

March 22, 1982 (when the price was observed for the 1982 contract). The data were kindly supplied by the Center for the Study of Futures Markets, at Columbia University (New York), and verified by cross-checking in the Wall Street Journal. Sample descriptives for relative futures price changes appear in Table 1. There are no missing data; observations are present for every trading day (trading occurs every weekday, except holidays or when weather precludes it). Only two trading interruptions occurred. The Commodities Futures Trading Commission suspended trading in the 1979 contract due to a suspected corner on March 21, 1979, with four trading days remaining. The move was overruled by the courts and the remaining open contracts were allowed to close. The Board of Trade itself was closed again at the insistence of the Commission, on January 7 and 8, 1980, due to the embargo on exports to the Soviet Union in response to the invasion of Afghanistan. Another interesting occurrence, though not resulting in

Table 1. Sample Descriptives

Contract Yr.	Sample Period	Number of Observations	Mean	Variance
1968-1982	4/3/67-3/22/82	4016	.000203	.000235
1968	4/3/67-3/20/68	245	-.000770	.000080
1969	4/1/68-3/20/69	244	-.000549	.000114
1970	4/1/69-3/19/70	243	.000622	.000089
1971	3/23/70-3/22/71	252	.000466	.000094
1972	3/23/71-3/21/72	253	.000075	.000102
1973	3/29/72-3/21/73	247	.001354	.000298
1974	4/2/73-3/20/74	242	.003901	.000726
1975	4/10/74-3/19/75	234	-.000648	.000567
1976	3/19/75-3/22/76	255	.000446	.000415
1977	2/2/76-3/22/77	287	-.000947	.000223
1978	1/4/77-3/21/78	305	-.000193	.000130
1979	1/5/78-3/21/79	304	.000767	.000192
1980	1/8/79-3/20/80	301	.000722	.000231
1981	12/19/79-3/20/81	311	-.000460	.000208
1982	1/22/81-3/22/82	293	-.001258	.000126

interrupted trade, was Nixon's opening of trade with China in 1974.

Referring to Table 1, the sample period column refers to the first and final days on which price quotes were observed during the run of the given contract (often, trading is officially opened by the Board, but no trading actually occurs until later). There are two interesting aspects regarding the Chicago March wheat contracts as data. First, while trading always ceases at approximately the same time during March, there is an interesting split regarding the opening of trades. Prior to the opening of the 1977 contract, trading in a succeeding contract never opened until after trading in its predecessor had ended. For example, the 1973 contract stops on March 21, 1973, and trade in the 1974 contract begins on April 3. In this manner, the data through the 1976 contract are discontinuous, as were Dusak's data. However, with the beginning of the 1977 contract, this is no longer the case. For example, the 1976 contract stops on March 22 and the 1977 contract begins on February 2. There is no mention in any popular trade source regarding this clear split in the data.[1]

The second aspect is the imposition of limits on futures price movements. The Board sets the maximum allowable change in the futures price at 20 cents over, or under, the previous day's closing price. This occurs in order to minimize defaults in the event of drastic price changes on a given day. Basically, this means that the observed data are open to the possibility of "censoring" problems, the importance of which will be analyzed in a later section.

Finally, the Chicago wheat contracts have remained unchanged over the sample period in terms of grade, delivery location, and delivery time, with the exception of the addition of Toledo, Ohio to the list of acceptable delivery locations in 1974. All-in-all, the data exhibit a high degree of integrity as a valid representation of prices for later delivery. For example, regarding unpredictable changes in the action of governments, after the suspension due to the export embargo, futures prices of the 1980 and 1981 contracts fell the limit on the first day, dropped ten more cents on the second day, and rose back to their previous levels on the third and fourth day, with no relatively abnormal fluctuations thereafter.

The March contract was chosen for analysis because it spans only one harvest, with little overlap into preceding or succeeding tails of other harvests. Hence, the March contract holds only one point of maximum commercial stocks, greatly facilitating an uncomplicated treatment of the empirical implications derived in Chapter 3.

An Examination of Serial Independence

With percentage changes as defined in (35), Chapter 3, the sample autocovariances from a sample of size T are defined by

$$(1)\quad c(\tau) = (1/T) \sum_{t=\tau+1}^{T} (P_t - \bar{P})(P_{t-\tau} - \bar{P}),\ \tau = 1, \ldots, T,$$

where $\bar{P}$ is the mean relative price change. Note that the variance of the sample is c(0). Sample serial correlation coefficients, or autocorrelations, are now defined as

$$(2)\quad r(\tau) = c(\tau)/c(0),\ \tau = 1, \ldots, T.$$

Serial independence is basic to the random walk, and analysis of autocorrelations is indicative of the presence of dependence, although, as seen presently, not decisively (in-depth discussion of all the elements in this analysis of serial independence can be found in Harvey [5]). A useful tool to this end is a plot of $r(\tau)$ against non-negative values of τ, known as the correlogram. An example correlogram is shown in Figure 1, for the 1973 March wheat contract, based on the estimates of $r(\tau)$, $\tau = 1, \ldots, 20$ in Table 2.

While the sample autocorrelations tend to mirror their theoretical counterparts, they only do so approximately since the estimates are subject to sampling variability. Hence in order to interpret the correlogram, something about the sampling variability of these estimates must be known and there is a simple result available for large samples (Harvey, pp. 146-7). For $\tau \neq 0$, the $r(\tau)$s are iid normal with mean zero and variance $1/T$. Hence the hypothesis that, for some lag specified in advance of the test, the true autocorrelation is zero can be tested by treating $T^{\frac{1}{2}}r(\tau)$ as a standardized normal variable (at .05 level, the null is rejected if $|T^{\frac{1}{2}}r(\tau)| > 1.96$). While this implies some knowledge of the nature of the series beforehand, in order to specify a particular τ, it is useful to plot bounds at $\pm\ 2/T^{\frac{1}{2}}$ on the correlogram as an indicator of departures from serial independence. An example of placing these bounds appears in Figure 1 and a summary of the procedure on all contracts is in Table 3. As a yardstick, if the underlying process were white noise, about one in every twenty estimates would be outside the bounds, i.e., about 5%.

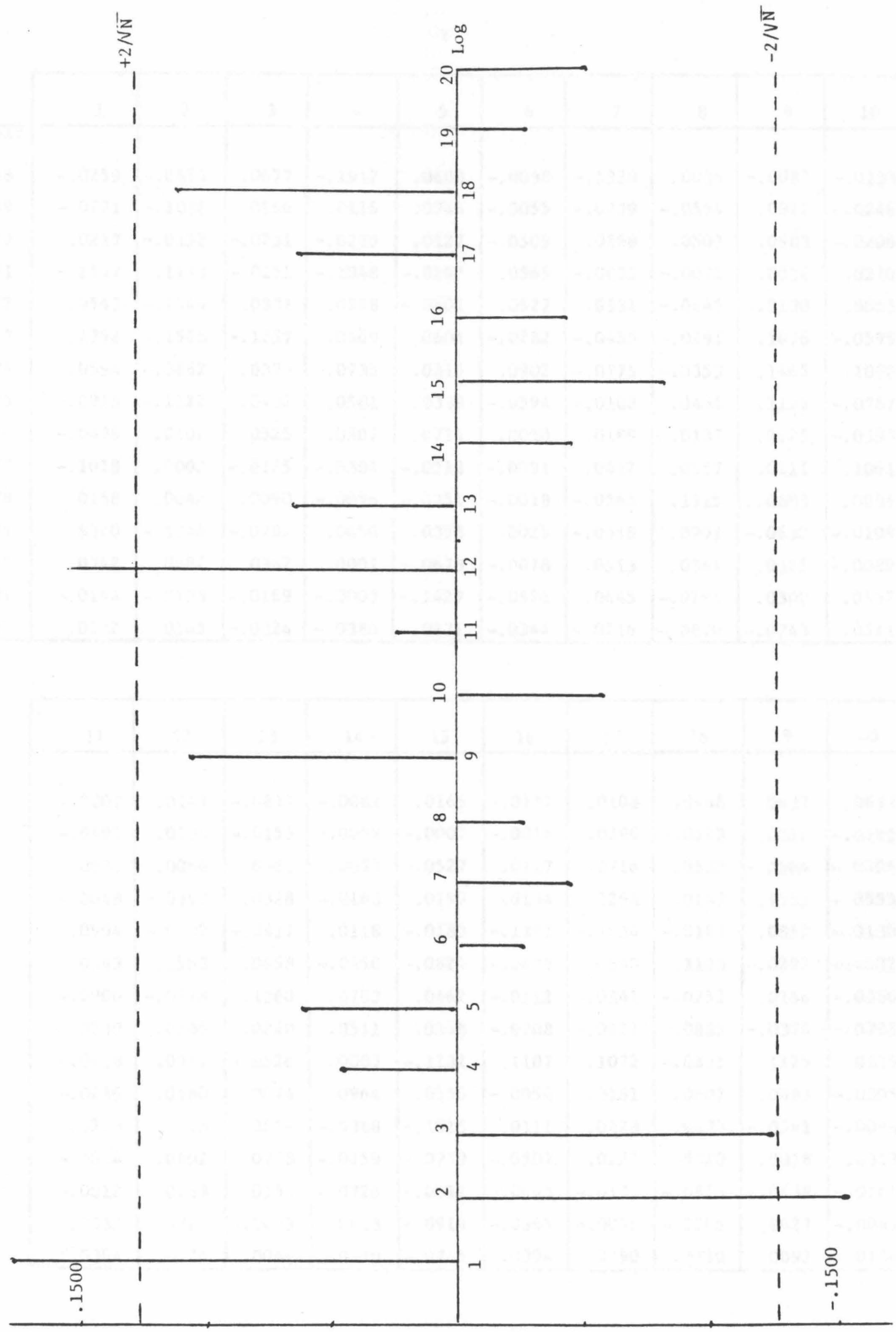

Figure 1. Correlogram, 1973 (N = 247).

Table 2. Estimated Autocorrelations, τ = 1, ... ,20 days.

τ

Year	1	2	3	4	5	6	7	8	9	10
68	-.0259	-.0455	.0677	-.1912	.0608	-.0038	-.1330	.0036	-.0082	-.0133
69	-.0271	-.1036	-.0566	.0116	.0744	-.0055	-.0739	-.0554	.0922	-.0246
70	.0217	-.0152	-.0231	-.0289	.0127	-.0509	.0368	.0507	.0983	-.0206
71	-.2147	.1173	-.0251	-.1048	-.0197	.0865	-.0631	-.0022	.0034	.0249
72	.0543	-.1544	.0331	.0268	-.0401	.0527	.0331	-.0645	-.0130	.0055
73	.1792	-.1566	-.1257	.0469	.0604	-.0282	-.0465	-.0291	.1076	-.0595
74	.0594	-.0862	.0323	.0735	.0815	.0902	-.0775	-.0353	.1465	.1092
75	-.0918	-.1122	.0432	.0581	.0339	-.0594	-.0108	.0456	.1126	-.0767
76	-.0476	.0102	.0525	.0507	-.0714	.0010	.0189	.0137	.0125	-.0393
77	-.1018	.0002	-.0125	-.0304	-.0518	-.0091	.0412	.0557	.0111	.1061
78	.0158	.0042	.0090	-.0056	-.0352	-.0018	-.0565	.1515	.0803	.0084
79	.8320	-.1348	-.0782	.0450	.0358	.0025	-.0516	.0997	-.0632	-.0109
80	.0342	.0482	.0342	.0007	-.0673	-.0078	.0513	.0364	.0325	-.0089
81	-.0194	-.0508	-.0169	-.0003	-.1427	-.0526	.0645	-.0291	.0300	.0532
82	.0232	.0165	-.0324	-.0380	-.0271	-.0344	-.0216	-.0870	-.0743	.0511

11	12	13	14	15	16	17	18	19	20
-.0007	-.0147	-.0877	-.0081	.0165	-.0132	.0108	.0446	.0437	.0637
-.0492	.0739	-.0155	-.0099	-.0007	-.0014	.0289	-.0523	.0016	-.0193
-.0636	.0068	-.0363	-.0053	-.0527	.0117	.0716	.0530	-.0564	-.0504
-.0048	-.0397	.0328	-.0163	.0195	.0134	.1294	.0147	.1551	-.0593
.0904	-.0202	-.0417	.0118	-.0183	-.1391	-.1034	-.0147	.0852	-.0130
.0343	.1563	.0658	-.0450	-.0824	-.0439	.0640	.1103	-.0297	-.0502
-.0900	-.0378	.1360	.0783	.0462	-.0112	-.0841	-.0252	.0166	-.0386
.0230	.0286	.0240	.0511	.0548	-.0708	-.0121	.0835	-.0376	-.0798
-.0418	.0327	-.0526	.0085	-.1232	.1107	.1072	-.0231	.1125	.0315
-.0256	.0180	-.0074	-.0964	.0556	-.0054	.0181	.0607	.0483	-.0305
.0228	.0045	-.0628	-.0388	-.1015	-.0111	.0428	.0477	-.0541	-.0098
-.0624	.0402	.0758	-.0159	-.0719	-.0507	.0227	.0320	.0018	-.0313
-.0012	-.0259	.0195	-.0726	.0012	-.0643	-.0177	-.0825	-.0438	-.0161
.0332	.0727	-.0033	.0613	-.0914	-.0365	-.0056	-.1205	.0627	-.0045
.0394	.0174	.0044	.0370	-.0745	-.0394	.0290	.0750	.0092	-.0126

Table 3 indicates that one-third of the contracts exhibit some dependence (1968, 1971-74, 1981) with evidence of consistent first- or second-order positive or negative serial dependence, respectively, in the period 1971-4; 1971 and 1973 exhibit some first-order positive serial correlation while 1972 and 1973 exhibit some second-order negative dependence. However, over all of the contracts, the largest percentage of bounds violations is only 10% above the level expected of white noise. Formal statistical testing of first-order autocorrelation, i.e., $r(1) \neq 0$, is available using the Von Neuman Ratio (Harvey, pp. 147-8). The statistic is approximately $2[1 - r(1)]$ and the small sample distribution is known if the observations are iid normal, $(0, \sigma^2)$. Critical values are available in the tables from Hart (1942). A one-sided test against positive serial correlation is carried out by rejecting the null of independence if $2[1 - r(1)]$ lies below the critical value and conversely for negative dependence. For every contract, and in particular for 1971 and 1973, the test fails to reject independence against either type of first-order dependence.

In an indicative nature, from the correlogram analysis, serial correlation problems do not appear large and independence, formally, is not rejected at the first-order. However, to examine longer lag periods which are relevant to an analysis of relative changes in futures prices from a participant's point of view, the correlogram cannot be pushed any further; for formally passing judgment on serial correlation problems at higher lags, the correlogram is not much help. The technique of spectral analysis is a likely candidate for analyzing longer-term periodicity in the price generating process. While analogous techniques

Table 3. Correlogram Analysis.

Yr	Bounds	Bounds Violations	Lag of Violations (Coeff. Sign)
68	±.1278	10%	4, 7 (both neg.)
69	.1280	0	
70	.1283	0	
71	.1260	15	1 (pos.), 17 (neg.), 19 (neg.)
72	.1257	10	2, 16 (both neg.)
73	.1272	15	1 (pos.), 2 (neg.), 12 (pos.)
74	.1285	10	9, 17 (both neg.)
75	.1307	0	
76	.1252	0	
77	.1181	0	
78	.1145	5	8 (pos.)
79	.1147	5	2 (neg.)
80	.1153	0	
81	.1134	10	5, 18 (both neg.)
82	.1168	0	

exist based upon analysis of autocorrelations, under the Portmanteau test statistic, the choice of how far back to go in time is somewhat arbitrary. Spectral analysis has the virtue of requiring no such choice.

The following brief description of spectral analysis is taken from Harvey, Chapter 3. Let y_t represent the stochastic process at hand. The autocovariance function of y_t is

$$(3)\quad \gamma(\tau) = E(y_t y_{t-\tau}),\ \tau = 1, \ldots,$$

and the complex Fourier transform of (3) is

$$(4)\quad f(\lambda) = (1/2\pi) \sum_{t=-\infty}^{\infty} \gamma(\tau)e^{-i\lambda\tau},$$

where λ is the frequency in radians, $-\pi \le \lambda \le \pi$. Expression (4) is called the power spectrum of the process y_t. For y_t real, (4) becomes

$$(5)\quad f(\lambda) = \gamma(0) + 2 \sum_{\tau=1}^{\infty} \gamma(\tau)\cos\lambda\tau\ .$$

As a standard for comparison, the power spectrum of a white noise process [$r(\tau) = 0$ for $\tau \neq 0$] is

(6) $f(\lambda) = \sigma^2/2\pi$,

since $r(0) = \sigma^2$ is just the variance of y_t. The important feature of the spectrum is seen in the fact that

(7) $\int_{-\pi}^{\pi} f(\lambda)d\lambda = \gamma(0) = \sigma^2$.

The interpretation of (7) is that the area under the power spectrum is equal to the variance of y_t, so that, for any given frequency, the spectrum gives the contribution of that frequency to the total variance of the process. Hence, the power spectrum provides a description of the cyclical movements in a series.

Fourier analysis provides a simple representation of the power spectrum. Now, let y_t be observations from a time series, t = 1, ..., T. The Fourier representation of y_t fits T trigonometric terms to the series, and is given by (T even):

$$(8)\quad y_t = T^{-\frac{1}{2}}a_0 + T^{-\frac{1}{2}}a_n(-1)^t + (2/T)^{\frac{1}{2}} \sum_{j=1}^{n-1} (a_j\cos\lambda_j t + b_j\sin\lambda_j t),$$

where $n = T/2$ for T even, and $(T - 1)/2$ for T odd. The frequency, λ_j, is now defined as

(9) $\lambda_j = 2\pi j/T$, $j = 1, \ldots, n$.

Defining frequencies according to (9) renders simple expressions for the coefficients a_0, a_j, and a_n based upon orthogonality properties (Harvey, pp. 60-1). It is also clear from (9) that π is the limit beyond which no frequency is defined. For T even, $\lambda_n = 2\pi n/T = \pi$ and, for T odd, $\pi > \lambda_n = \pi(T - 1)/T$. Rearranging (9), one finds

$$(10) \quad T/j = 2\pi/\lambda_j, \; j = 1, \ldots, n.$$

The factor 2π is all the keeps period and frequency from being reciprocals. The spectrum can show the power of either frequency, or period, as contributors to the variance of y_t.

The important result of Fourier representations is that the conributions to overall variance can be measured by a single quantity,

$$(11) \quad p_j = a_j^2 + b_j^2.$$

An obvious estimator of the power spectrum in (5) is

$$(12) \quad I(\lambda) = c(0) + 2 \sum_{\tau=1}^{T-1} c(\tau)\cos\lambda\tau, \; 0 \le \lambda \le \pi,$$

where the true autocovariances in (5) are replaced by their sample estimates, as defined in (1). Expression (12) is called the sample spectral density, and it is closely related to (11) (Harvey, pp. 83-4):

$$(13) \quad I(\lambda_j) = p_j/4\pi, \; j = 1, \ldots, n.$$

In passing, the sample spectral density is an unbiased, but not a consistent, estimator of $f(\lambda)$.

A plot of each p_j against its corresponding frequency or, by (10), its corresponding period, is known as the periodogram. The periodogram is the frequency analog of the correlogram. The periodograms for the sample of fifteen March wheat contracts appear in Figures 1 - 5, and are based upon the spectral estimates in Table 1. Particular attention should be given to the scale of the y-axis for the spectral estimates. Peaks in the estimated spectral density are indicative of relatively large contributions to the overall variance by a given frequency or period. Put another way, a peak in the estimated spectrum at a four-day period means that large contributions to the overall variance occur in cycles of four days. From the point of view of the independence assumption, the flatter the better, and the estimated spectra for 1968-70 (Figure 1), 1971-2 (Figure 2), and 1977-82 (Figures 4 and 5) appear flat relative to 1973-6 (Figures 2 and 3). However, this statement is only a relative one and, in a sense, analogous to the indecisiveness of the correlogram, a more formal judgment is to be preferred.

The cumulative periodogram provides just such a formal basis (Harvey, pp. 150-1). The cumulative periodogram irons out the unstable behavior of the spectral estimates through a process of accumulation by defining the following test statistic:

$$(14) \quad s_i = \sum_{j=1}^{i} p_j^2 / \sum_{j=1}^{n} p_j^2, \; i = 1, \ldots, n.$$

For a white noise process, a graph of s_i against i approximates the 45 degree line. Alternatively, the cumulative periodogram for a process with an excess of low (high) frequency, relative to high (low), will

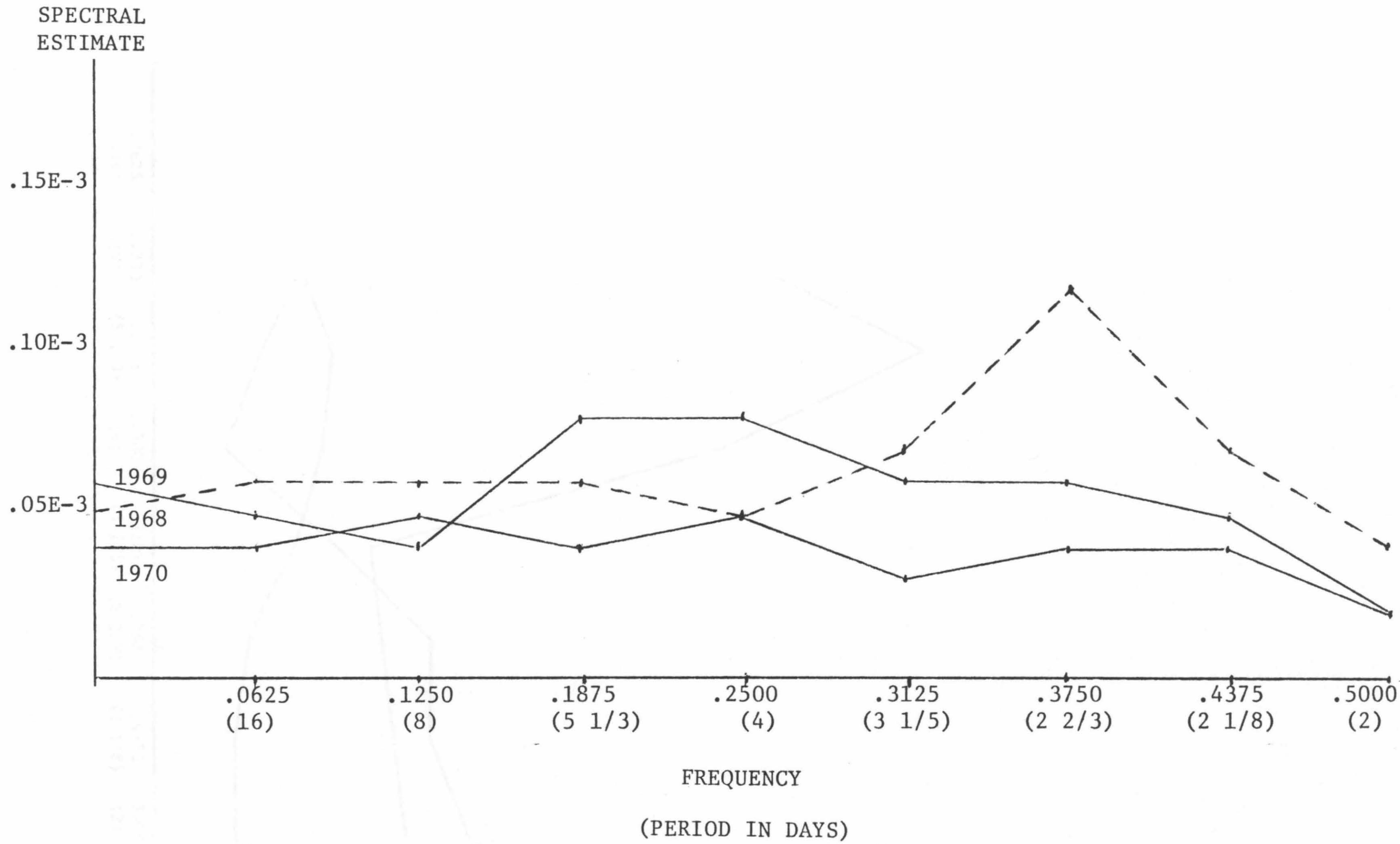

Figure 2. Periodogram, 1968 - 1970.

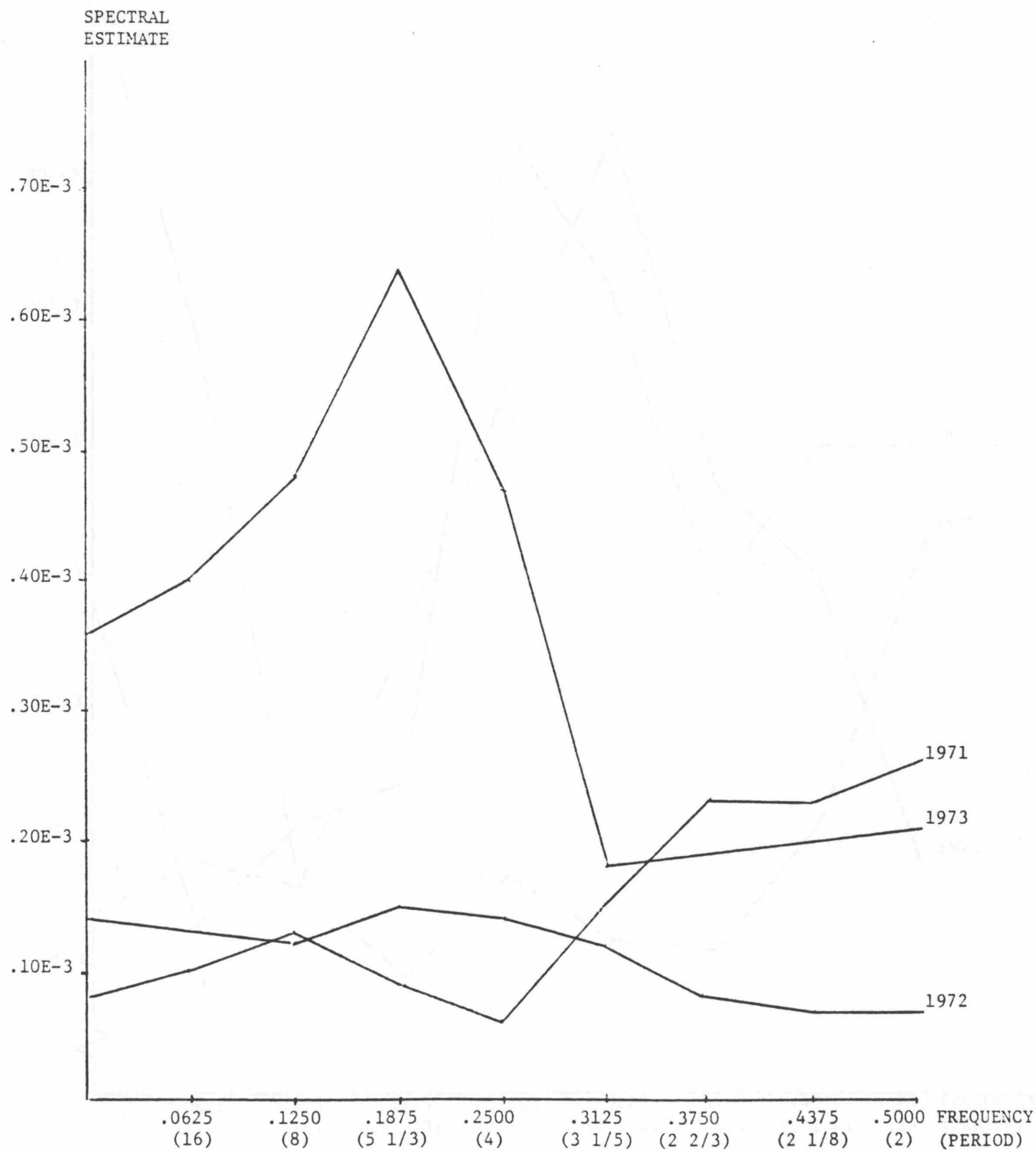

Figure 3. Periodogram, 1971 - 1973.

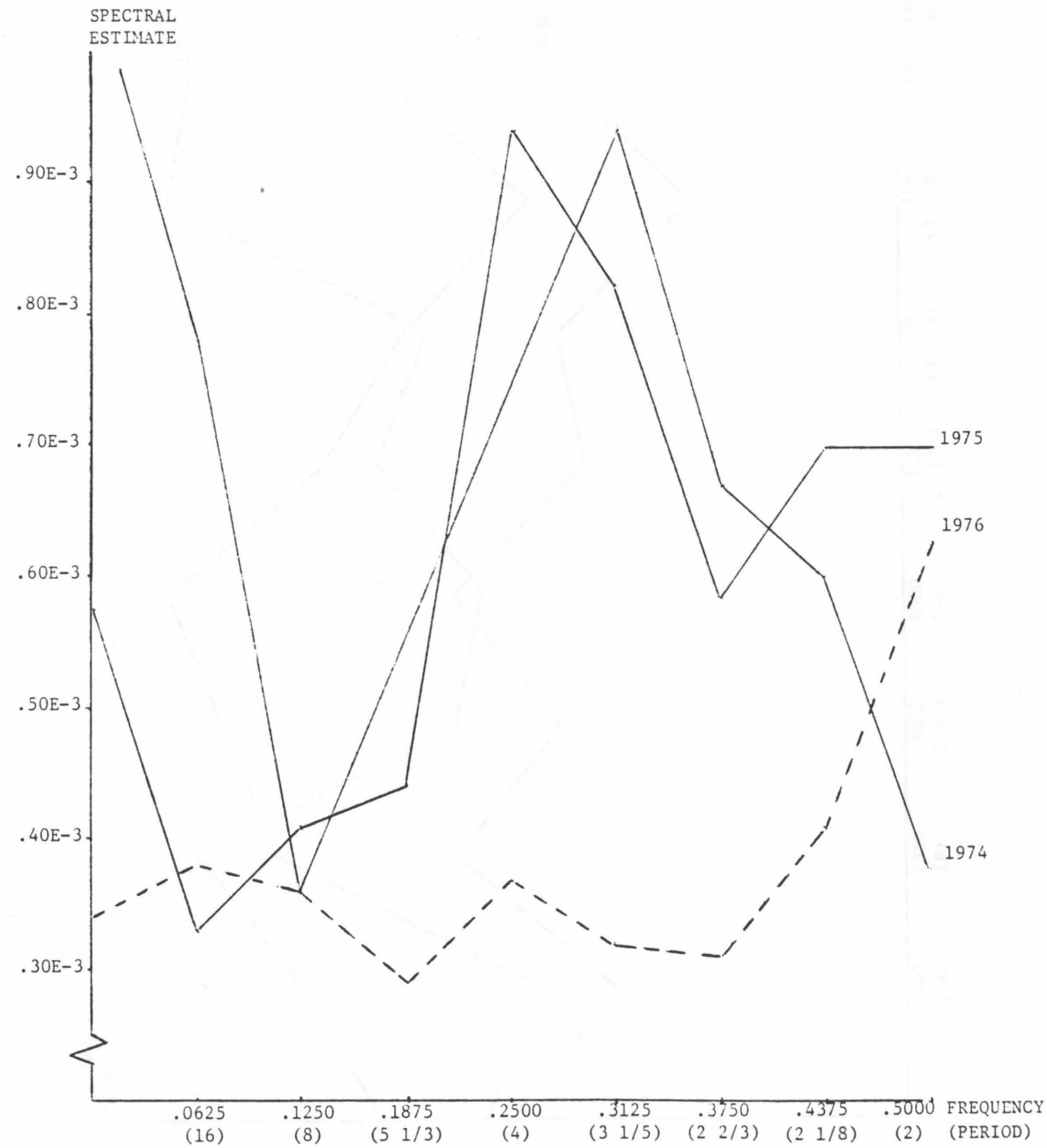

Figure 4. Periodogram, 1974 - 1976.

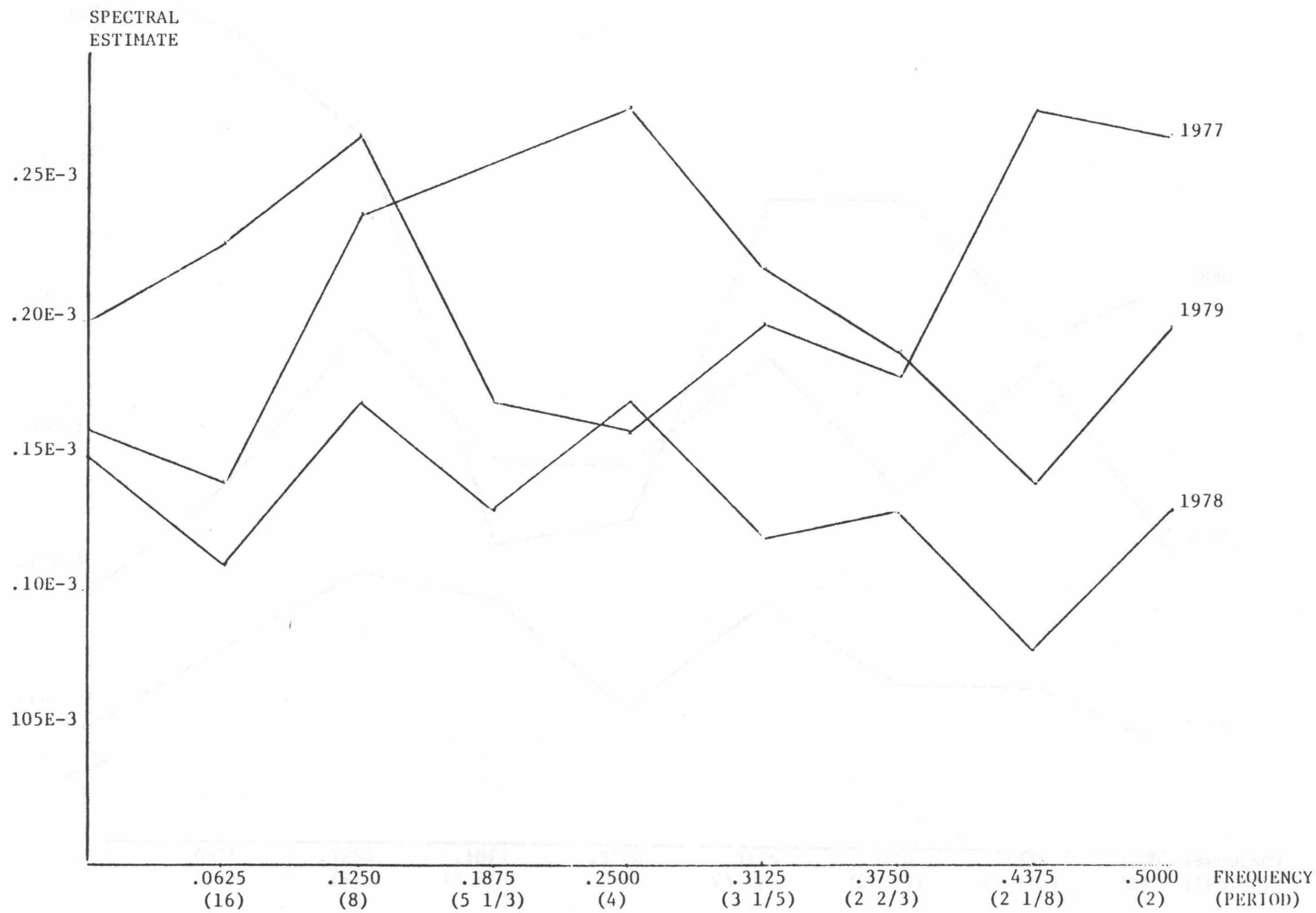

Figure 5. Periodogram, 1977 - 1979.

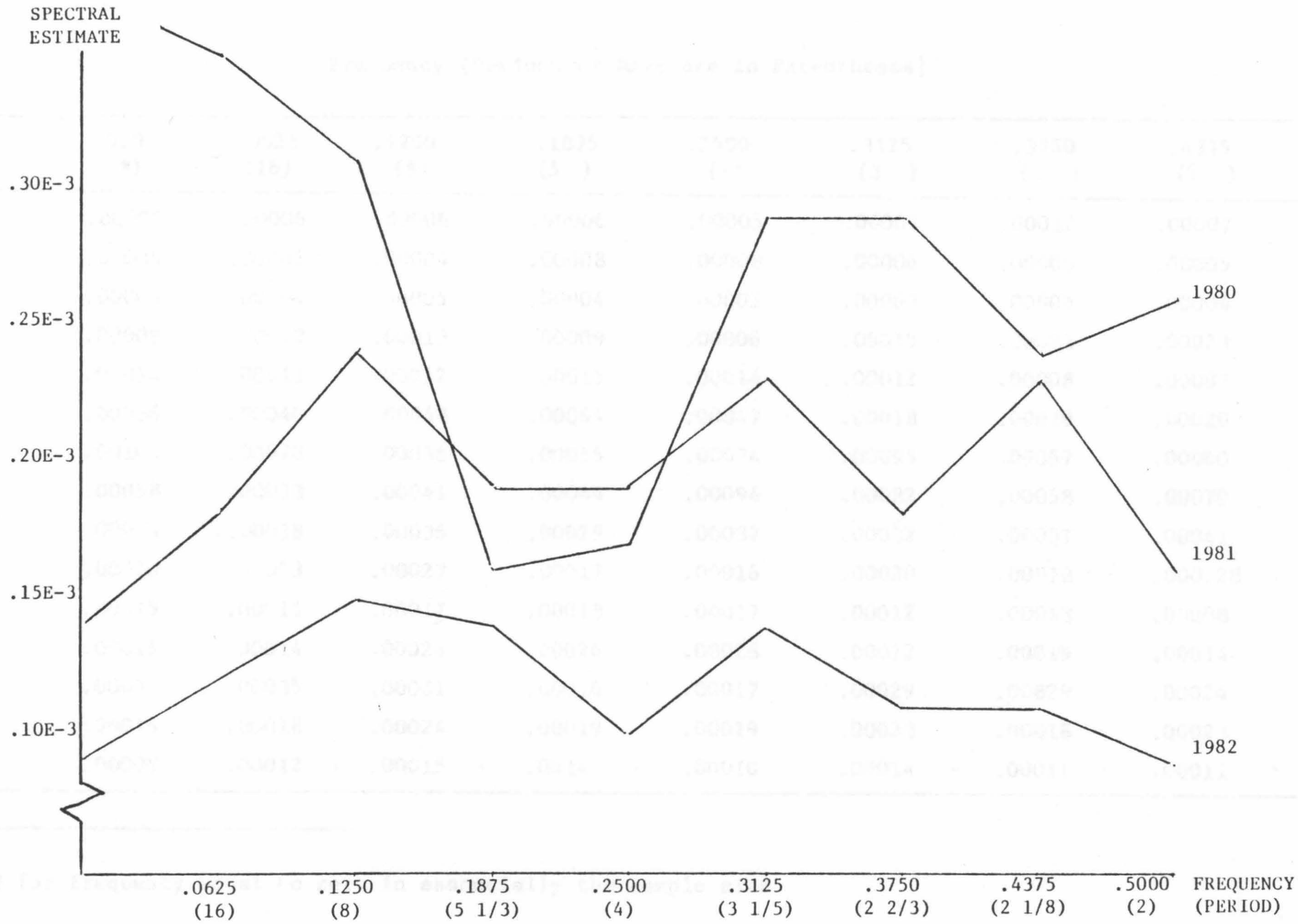

Figure 6. Periodogram, 1980 - 1982.

Table 4. Spectral Estimates.

Frequency [Periods or Days are in Parentheses]

Contract Year	0.0 (*)	.0625 (16)	.1250 (8)	.1875 (5)	.2500 (4)	.3125 (3)	.3750 (2)	.4375 (2)	.5000 (2)
68	.00005	.00006	.00006	.00006	.00005	.00007	.00012	.00007	.00004
69	.00006	.00005	.00004	.00008	.00008	.00006	.00006	.00005	.00002
70	.00004	.00004	.00005	.00004	.00005	.00003	.00004	.00004	.00002
71	.00008	.00010	.00013	.00009	.00006	.00015	.00023	.00023	.00026
72	.00014	.00013	.00012	.00015	.00014	.00012	.00008	.00007	.00007
73	.00036	.00040	.00048	.00064	.00047	.00018	.00019	.00020	.00021
74	.00105	.00078	.00036	.00055	.00074	.00095	.00067	.00060	.00038
75	.00058	.00033	.00041	.00044	.00094	.00082	.00058	.00070	.00070
76	.00034	.00038	.00036	.00029	.00037	.00032	.00031	.00041	.00063
77	.00020	.00023	.00027	.00017	.00016	.00020	.00018	.000.28	.00027
78	.00015	.00011	.00017	.00013	.00017	.00012	.00013	.00008	.00013
79	.00016	.00014	.00024	.00026	.00028	.00022	.00019	.00014	.00020
80	.00037	.00035	.00031	.00016	.00017	.00029	.00829	.00024	.00026
81	.00014	.00018	.00024	.00019	.00019	.00023	.00018	.00023	.00016
82	.00009	.00012	.00015	.0014	.00010	.00014	.00011	.00011	.00009

*Period for frequency equal to zero in essentially the sample size.

tend to lie above (below) the 45 degree line. An excess of low frequency, i.e., recurrent cycles in the process over long periods, is expected in the presence of positive serial correlation.

A formal test of departures from randomness (Durbin [2]) is derived by drawing two lines parallel to the 45 degree line, defined by:

$$(15) \quad s = \pm c_0 + i/n.$$

The term c_0 is a significance value which depends on n, the number of frequencies at which the spectral estimates are calculated, available from a table in Durbin. The null hypothesis of independence is rejected, at the chosen significance level, if the sample path $s_1, \ldots, s_n$ crosses either of the lines in (15). An example cumulative periodogram, complete with confidence bounds, appears in Figure 7. Since graphs need not be drawn in order to perform the test (the null is rejected, for a two-sided test, if $\max_i |s_i - i/n| > c_0$), no further pictures are drawn and the results of the cumulative periodogram analyses appear in Table 5. Since an excess occurrence of high frequency (cycles of low periodicity) is especially interesting, given the time frame of futures trading, only the results of the one-sided test at the lower boundary on (15) is presented in the table. The test fails to reject independence in every case. Based upon the results of the first-order serial correlation and cumulative periodogram analysis, it appears safe to proceed to an analysis of the distributional assumptions of the random walk hypothesis with the conclusion of random successive increments.

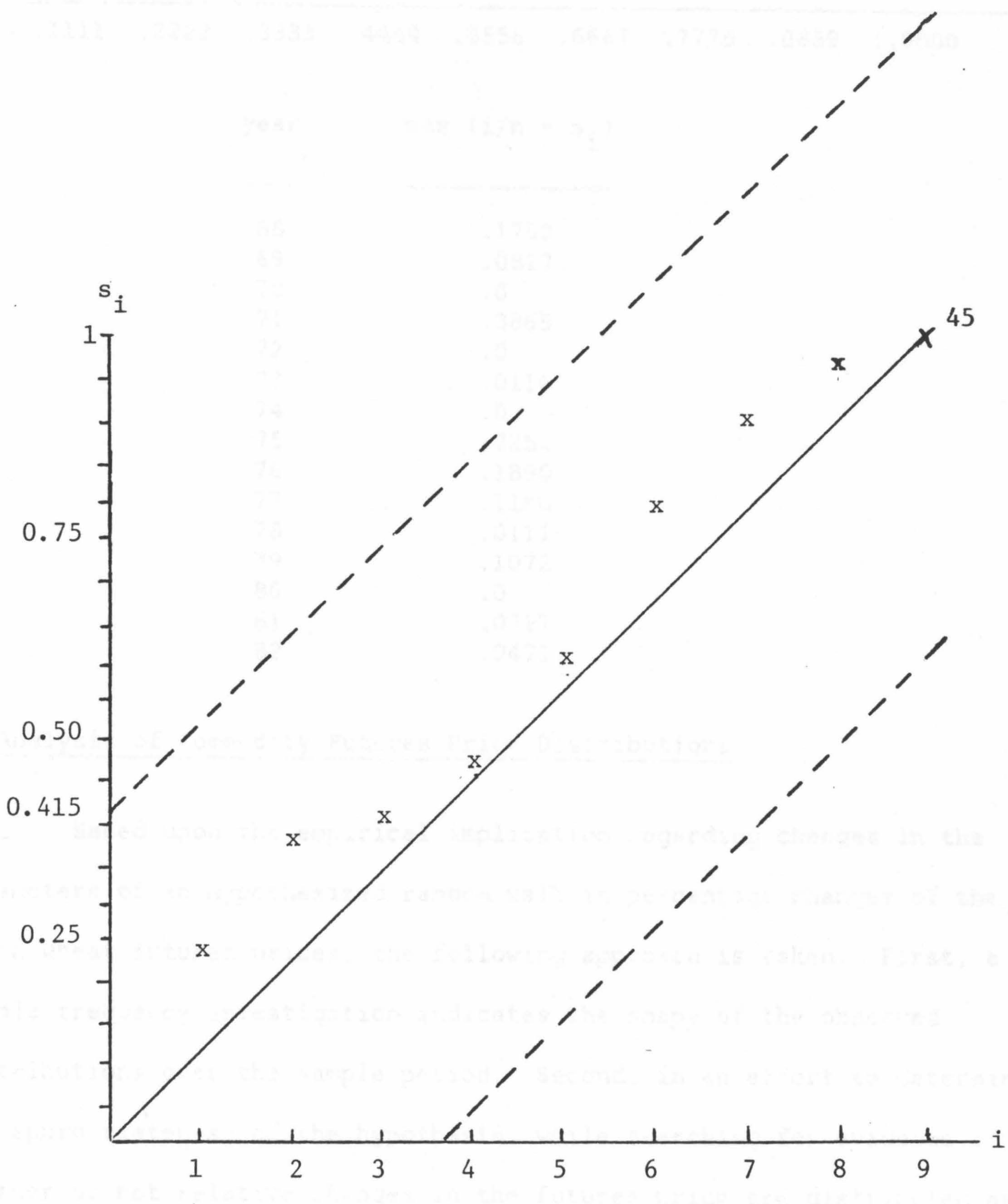

Figure 7. Cumulative Periodogram, 1974.

Table 5. Cumulative Periodogram Analysis [C_0 (.99 level) = .4152]

i=	1	2	3	4	5	6	7	8	9
i/n=	.1111	.2222	.3333	.4444	.5556	.6667	.7778	.8889	1.0000

year	max $(i/n - s_i)$
68	.1758
69	.0817
70	.0
71	.3865
72	.0
73	.0116
74	.0
75	.2251
76	.1890
77	.1186
78	.0111
79	.1072
80	.0
81	.0717
82	.0471

An Analysis of Commodity Futures Price Distributions

Based upon the empirical implication regarding changes in the parameters of an hypothesized random walk in percentage changes of the March wheat futures prices, the following approach is taken. First, a simple frequency investigation indicates the shape of the observed distributions over the sample period. Second, in an effort to determine the appropriateness of the hypothesis, while searching for evidence of whether or not relative changes in the futures price are distributed as stable Paretian variables with characteristic exponent below two, the descriptive statistics suggested by Fama and Roll are calculated for each of the 15 March wheat contracts. In addition, sample kurtosis and Studentized Range are analyzed for conformance of the distributions with normality. Third, according to the implication concerning distributions before and after the peak in commercially held stocks, the observations on each contract are divided into subsamples, one relevant to the period prior to the peak and one to the period after the peak. The subdivision occurs as stated in the previous Chapter 3.

In the subsample analysis, descriptive parameters are reestimated with attention to the possibility that exchange-imposed price movement limits may introduce censoring bias into the observations. Kurtosis and Studentized Range are also examined in the subsamples and compared with the results for the overall samples. The subsamples then are examined for stationarity in the price-generating process across the pre- and post-stock-peak periods. Implications of the results are noted throughout.

Throughout this section, distributional comparisons are betweeen normal and stable-Paretian alternatives. Admittedly, and as shown in Chapter 2, this comparison is one among many. For example, in a subordinated stochastic process analysis, one would try to identify the distribution of the peak in stocks as a directing process on commodity prices (an outline of such an approach appears in the concluding chapter). However, examining whether or not Mandelbrot was right is useful for two reasons. First, the verdict is not yet in on which distributional form best fits commodity futures prices, and one point of the thesis is that previous findings have neglected the theoretical implications of futures trading. Second, by conditioning on the peak in stocks, the problem of poorly behaved higher moments my no longer plague the resulting subsample distributions. While more precise specifications of the price-generating process where the important role of stock levels receives more in-depth treatment may prove useful, the point of the arguments in this thesis is that theoretical implications matter. Empirical extensions beyond the modest aims of this chapter can always be tried. In any event, the behavior of random walk parameters receives adequate treatment.

The observed proportions of relative price changes within a given number of standard deviations of the mean, compared with the proportions one would expect if the distributions were exactly normal, appear in Tables 6 and 7. A positive entry is an excess of relative frequency in the observed distribution over what would be expected in the given interval if the distribution were normal. A negative entry is a deficiency in observed frequency. Table 6 contains the comparison for both the average of observed proportions minus normal proportions, across all 15 series for the March contract, and observed minus normal proportions treating the observations from all series as one large sample.

The pattern exhibited in both comparisons in Table 6 is an excess relative to the normal within 1.5 standard deviations, deficiency over the 2.0 through 5.0 standard deviation intervals and an excess of observations in the extreme tails (beyond 5.0 standard deviations). This is precisely the result found by Fama [3] in his analysis of thirty stock price series. Indeed, the comparison at the average across all series exhibits the precise sign pattern as found by Fama for the average of thirty stock price series, as shown in the last row of Table 6 (in the extreme tails, the excess here is equal to the excess found by

Table 6. Relative Price Change Frequencies Compared to the Normal Distribution.

	INTERVAL								
	.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	>5.0
Unit Normal	.3830	.6826	.8664	.9545	.9876	.9973	.999938	.9999994	.0000006
March Wheat, 1968-82	.4940	.7612	.8733	.9255	.9597	.9878	.999502	.9995019	.0004980
Observed-Normal, 1968-82	.1110	.0876	.0069	-.0290	-.0329	-.0095	-.000436	-.0004975	.0004975
Observed-Normal, Average Across 1968-82	.0678	.0637	.0189	-.0037	-.0105	-.0042	-.001806	-.0011168	.0011168
Observed-Normal, Average Across Fama´a 30 Stocks	.0837	.0636	.0183	-.0066	-.0120	-.0086	-.002979	-.0011632	.0011632

Fama, to four decimal places). Figure 8 pictures this leptokurtotic result.

The similarity with Fama's findings cannot be pushed very far, however. Table 7 shows the frequency comparison for each series of relative price changes. From the table, only the 1968-71 March wheat contracts conform to the leptokurtotic result. In the remaining contracts, the series exhibit a ..us deficiency of frequency beyond 5.0 standard deviations due to the fact that all of the observations within each series are within 5.0 standard deviations of their individual means. In the series for 1972-82, except for 1978, all observations lie within 4.0 standard deviations and 1973-75 have all observations within 3.0, 2.5, and 2.0 standard deviations, respectively. In Fama's sample of 30 stocks, eight had all observations within 5.0 standard deviations and only one had all observations within 4.0 standard deviations. Figure 9 is generally representative of the post-1971 series analyzed here. The case for distributions with excess observations in the tails and higher kurtosis relative to the mean is not as convincing here as elsewhere, and all evidence appears prior to 1972. This is not to say, when it does appear, that excess frequency in the tails of the observed distributions is inconsequential. For example, 1971 has the smallest excess beyond 5.0 standard deviations, about .397%, while the normal would have .00006%; there is an observed excess frequency about 6600 times that expected under the normal distribution.

In an effort to further investigate the properties of the distributions of the March contracts, the descriptive statistics suggested by Fama and Roll -- 0.5 truncated mean, estimated scale

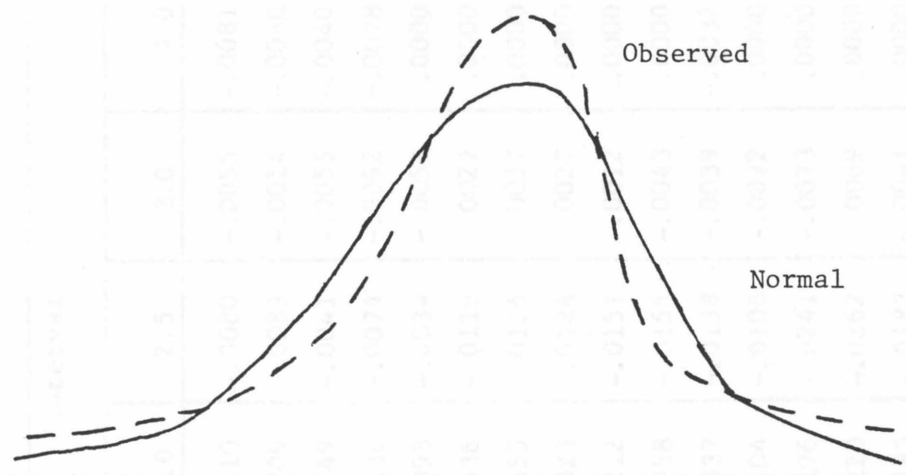

Figure 8. Frequency Comparisons, 1968 - 1971.

Table 7. Relative Price Change Frequencies, Observed versus Normal, by Contract Year.

Contract Year	Interval								
	.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	>5.0
68	.0456	.0888	.0479	.0210	-.0080	-.0055	-.008101	-.0040811	.0040811
69	.1786	.1781	.0844	.0209	.0083	-.0014	-.004036	-.0040978	.0040978
70	.2754	.1487	.0883	.0249	-.0041	-.0055	-.004053	-.0041147	.0041147
71	.1210	.0952	.0463	.0138	-.0074	-.0092	-.007875	-.0039677	.0039677
72	-.0154	.0249	-.0166	-.0098	-.0034	-.0052	.000062	.0000006	-.0000006
73	.1069	.0979	-.0121	-.0436	-.0119	.0027	.000062	.0000006	-.0000006
74	-.0979	-.1041	.0551	.0455	.0124	.0027	.000062	.0000006	-.0000006
75	.0444	-.0031	-.0630	.0023	.0124	.0027	.000062	.0000006	-.0000006
76	.0680	.0507	-.0154	-.0212	-.0151	-.0012	.000062	.0000006	-.0000006
77	.0316	.0700	.0082	-.0068	-.0155	-.0043	.000062	.0000006	-.0000006
78	.0006	.0322	.0254	-.0037	-.0138	-.0039	-.003217	.0000006	-.0000006
79	.0709	.0871	.0086	-.0104	-.0106	-.0072	.000062	.0000006	-.0000006
80	.1153	.0616	.0074	-.0176	-.0241	-.0073	.000062	.0000006	-.0000006
81	.0093	.0184	.0178	-.0220	-.0262	-.0069	.000062	.0000006	-.0000006
82	-.0485	.0307	-.0063	-.0193	-.0183	-.0041	.000062	.0000006	-.0000006

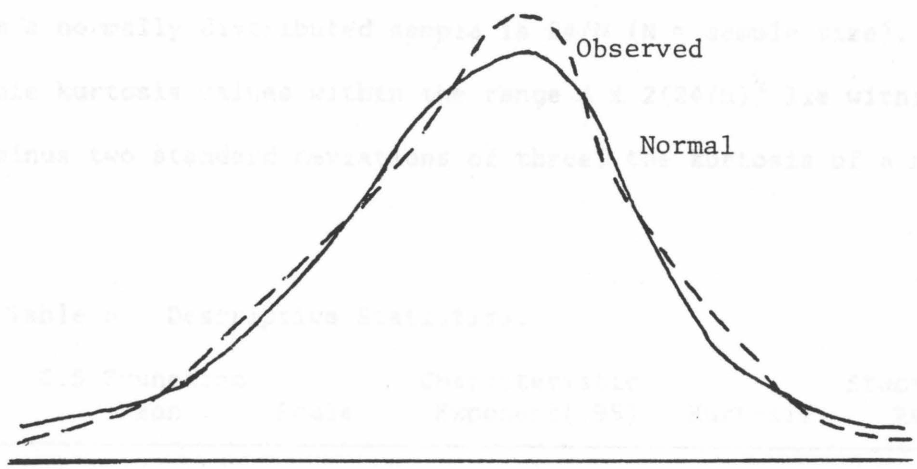

Figure 9. Frequency Comparisons, 1972 - 1982.

parameter (expression (9) in Chapter 2), and estimated characteristic exponent (derived using expression (10) in Chapter 2) -- appear in Table 8, along with estimated kurtosis values and Studentized Range statistics (expression (11) in Chapter 2). Fama and Roll recommend the Studentized Range test for goodness-of-fit of normality against non-normal stable alternatives. Kurtosis is defined by the ratio of the expected value of the fourth power of a random variable in deviation from mean form to the square of the random variable's variance. The variance of the kurtosis from a normally distributed sample is 24/N (N = sample size). Hence, sample kurtosis values within the range $3 \pm 2(24/n)^{\frac{1}{2}}$ lie within plus or minus two standard deviations of three, the kurtosis of a normal.

Table 8. Descriptive Statistics.

Yr.	0.5 Truncated Mean	Scale	Characteristic Exponent(.95)	Kurtosis	Studentized Range
Dusak	.000600	.012300	1.75	--	--
68	.001417	.005441	1.92	8.909[1]	11.000[3]
69	.000111	.004837	1.74	56.715[1]	12.909[3]
70	.000451	.004232	1.82	60.513[1]	15.222[3]
71	.000834	.005441	1.68	8.991[1]	11.100[3]
72	.000442	.007255	1.82	.423[2]	6.700
73	-.004669	.009069	1.44	.543[2]	5.353
74	.004205	.025998	*	-1.237[2]	3.519
75	-.000385	.015115	1.60	-3.27[2]	4.500
76	-.001089	.012092	1.51	.596[2]	5.700
77	.000399	.009674	1.67	.844[2]	6.200
78	-.000664	.007680	*	1.625[2]	7.818[3]
79	-.000134	.009069	1.74	1.173[2]	7.571[3]
80	-.003582	.007860	1.44	1.278[2]	6.133
81	.000492	.009674	1.93	.880[2]	6.143
82	-.001121	.007255	1.71	.853[2]	6.636

*Indicates that no estimate could be derived (see text).
[1]Exceeds two standard deviations above three.
[2]More than two standard devidations below three.
[3]Significant at the .01 level.

Dusak's estimates of the descriptive statistics, from semi-monthly observations of the March contract over the years 1952-67, are also listed in Table 8. The 0.5 truncated mean estimates from the sample at hand vary around the estimate from Dusak, while the scale factors are generally comparable (recall that the scale factor is the non-normal stable analog of the variance). Turning to the investigation of normality, both kurtosis and Studentized Range show that the evidence supports a finding of leptokurticity and thick tails, relative to the normal distribution, for the 1968-71 series. The estimated kurtosis values exceed the upper two standard deviation cutoff and the Studentized Range rejects the hypothesis that the data are normal, as opposed to non-normal stable. However, after 1971, the results are quite different. Kurtosis values are all below the minus two standard deviation cutoff and non-normality is detected for only two years, 1978-79.

The estimated characteristic exponents provide some interesting results. First, estimates for 1974 and 1978 could not be derived by the method of Fama and Roll. Calculated z values were beyond the tabled values for the .95 fractile of the standardized stable symmetric distributions, at $\alpha = 2.0$. For those series for which estimates could be obtained, none equal 2.0 (only two series have estimates above 1.90) and their average value is 1.69 which makes the estimates comparable to Dusak's findings (Dusak also found an average value of 1.66 across the wheat contracts with different delivery dates -- March, May, July, September, December -- with a range from 1.55 to 1.75). The implication of these estimates is that the observed series for which estimates could be derived have excess frequency in the tails of their distributions, relative to the normal. However, as shown in Table 7, the excess is not always in the extreme tails. Indeed, in the post-1972 annual series, there are never any excesses in observed minus normal frequencies. Most excesses occur for four and five standard deviations. Also, the results of the Studentized Range tests do not universally reject normality, which leaves somewhat of a paradox. Given that the point of the distributional analysis is to determine whether or not the distributions of relative price changes vary over the peak in stock levels, and the fact that no sampling theory regarding the estimates of characteristic exponents enables judgment to be passed, the results of the Studentized Range tests are the most useful alternative. This is somewhat lamentable, since statements regarding the efficacy of the Fama and Roll estimate are of great interest.

The evidence thus far is mixed regarding the predominance of non-normal stable versus normal distributions as the best description of the 15 series. Non-normality would appear to be the best description of the early sample years, 1968-71, while normal distributions cannot be rejected throughout most of the post-1971 series. But the theory of shifting net hedging balances suggests that the analysis of distributions can be pushed further. Using the maximum stock criterion as a separation point, Table 9 describes the results of subsample construction, referred to as the "pre-max" and "post-max" periods. Three of the series deserve special mention, regarding the behavior of stock levels. For the 1972 series, stocks peak on July 20, 1971, which is fairly early relative to the harvest. Regarding the second noteworthy series, unlike the remaining series, stocks do not decline throughout the post-max period for the 1974 March wheat contract. Finally, for the 1979 series, identifying the point of maximum stock-carrying was not as easy as for the others. Other than the slightest rise in stocks from December 22, 1978 to January 12, 1979, stocks declined throughout the duration of the 1979 March wheat contract.

Table 9. Summary of Subsample Separation.

Yr.	Separation Date	Pre-Max Observations	Post-Max Observations
68	10/27/67	146	98
69	09/06/68	109	134
70	09/26/69	124	118
71	09/25/70	130	121
72	07/20/71	82	170
73	09/29/72	128	118
74	09/07/73	110	131
75	11/15/74	149	84
76	10/17/75	148	106
77	10/22/76	184	102
78	10/07/77	191	113
79	12/29/78	247	56
80	11/02/79	208	92
81	12/05/80	239	71
82	10/09/81	180	112

The point of the subsample approach is to examine the theoretical implication that the parameters of a random walk in relative price changes may change over the periods spanning the maximum stock level. To this end, Table 10 shows descriptive statistics and the results of kurtosis analysis and Studentized Range tests of normality for the subsamples. Kurtosis analysis of the pre-max periods gives exactly the same result as found in the overall samples: 1968-71 exhibit leptokurticity, but the opposite is true of the remaining pre-max series. In the post-max series, the result is greatly altered. The kurtosis estimates for the 1970 and 1978 post-max series cannot be distinguished from the case of normality and all of the other years have estimated kurtosis values below the minus two standard deviation cutoff.

The results of Studentized Range tests provide some striking results concerning the distributions in subsample, summarized in Table 11. For the years which appear to be described best by non-normal distributions in their overall series, 1968-71 and 1978-79, departures from normality are accounted for, in every case except for 1978, in the pre-max period. This is perhaps not surprising since the pre-max period is wrought with uncertainty concerning the harvest outcome (size of the harvest, time of arrival at the storage facilities) and the variance can be expected to be ill-behaved. Another interesting result in Table 11 occurs in the comparison of 1978 and 1980. Both are judged non-normal in the post-max period, and 1978 appears non-normal overall while no departure from normality is detected for 1980. The most striking result revealed in Table 11 is that the post-max period is quite well-characterized as conforming with normality.

At least three conclusions can be drawn from the subsample analysis, thus far. First, the theoretical implication is that the parameters of the random walk in relative futures price changes may change over the pre- and post-max periods. According to the empirical analysis, they may change, and they may not. In eight of the fifteen March wheat contracts, 1972-77 and 1981-2, normality cannot be rejected

Table 10. Sample Descriptives, Pre- and Post-Max.

Year	Mean		Variance		Scale		Kurtosis		Studentized Range	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
68	-.000773	-.000718	.000109	.000039	.006651	.004232	7.805[a]	-.025[b]	9.438[c]	5.320
69	.001080	-.000143	.000177	.000064	.005441	.005441	54.686[a]	1.412[b]	10.438[c]	6.324
70	.000721	.000555	.000132	.000045	.004232	.003628	53.987[a]	2.306	11.813[c]	6.429
71	.001274	-.000458	.000123	.000062	.004837	.004837	9.677[a]	.439[b]	10.044[c]	5.502
72	-.000046	.000226	.000117	.000094	.007255	.006651	.597[b]	.377[b]	4.925	5.964
73	.002333	.000276	.000191	.000414	.007860	.015115	2.019[b]	-.410[b]	6.286	4.296
74	.007662	.000421	.000852	.001117	.025393	.022370	-1.195[b]	-1.201[b]	3.371	3.562
75	.001410	-.004490	.000792	.000524	.013906	.016929	-.487[b]	.151[b]	4.177	4.762
76	.001578	-.000975	.000469	.000375	.012092	.012092	.327[b]	1.004[b]	5.132	5.700
77	-.001034	-.000708	.000284	.000114	.011487	.007860	.384[b]	.710[b]	5.469	5.525
78	-.000697	.000774	.000111	.000159	.007860	.007860	.285[b]	2.460	5.898	6.791[c]
79	.000599	.001598	.000206	.000130	,008464	.009069	1.213[b]	-.476[b]	7.321[c]	4.266
80	.001939	-.000390	.000270	.000173	.006651	.010278	1.170[b]	1.197[b]	5.628	6.608[c]
81	.000311	-.002944	.000196	.000288	.009674	.008464	.687[b]	1.247[b]	6.008	5.293
82	-.000814	-.001963	.000119	.000137	.007255	.007255	.691[b]	1.080[b]	6.150	6.188

a. Exceeds 2 std. devs. above 3.
b. More than 2 std. devs. below 3.
c. Significant at .01 level.

Table 11. Studentized Range Comparison, Overall, Pre-, and Post-Max (significance at the 0.01 level occurs as indicated by an x).

Year	Overall	Pre	Post
68	x	x	–
69	x	x	–
70	x	x	–
71	x	x	–
72	–	–	–
73	–	–	–
74	–	–	–
75	–	–	–
76	–	–	–
77	–	–	–
78	x	–	x
79	x	x	–
80	–	–	x
81	–	–	–
82	–	–	–

in the overall series or in the subsamples. Given this, parametric tests of equal means (t - test) and equal variances (F - test) across the subsamples provide a valid judgment of stationarity for these eight contracts.

Before discussing the results of the stationarity analysis, the observation that exchange-imposed limits on price movements deserves attention. The exchange (Chicago Board of Trade) stops any further trade in a given futures contract whenever the price moves more than twenty cents above or below the preceding day's closing price. Thus, in a series where the limit is hit, we do not observe relative price changes over their entire range. This is known as drawing observations from censored distributions and was first addressed by Tobin [7]. Translating the problem to observations on relative price changes, the limit at time t becomes (plus or minus):

$$(16) \quad A_t = 20/F_{t-1},$$

where F_{t-1} is the closing price of the given contract at time t - 1 (yesterday). Hence, relevant to relative price changes, the limit varies from observation to observation in a given series. The implications of the censoring problem can be seen as follows and the description is an adaptation of that found in Johnson and Kotz [6] to the case at hand.

Suppose we observe P_t^*, where

$$(17) \quad P_t^* = \begin{cases} -A_t, \text{ if } P_t^i < -A_t, & P_t^i \varepsilon P^I,\ i = 1, \ldots, I \\ P_t^j, \text{ if} -A_t \le P_t^j \le A_t, & P_t^j \varepsilon P^J,\ j = 1, \ldots, J \\ A_t, \text{ if } P_t^k > A_t, & P_t^k \varepsilon P^K,\ k = 1, \ldots, K. \end{cases}$$

With P_t iid normal, mean μ and variance σ^2, the likelihood function is

$$(18) \quad L(\mu, \sigma^2) = \prod_{P_t^i \varepsilon P^I} \int_{-\infty}^{-A_t} \frac{1}{\sigma} g\left(\frac{P_t - \mu}{\sigma}\right) dP_t^i$$

$$\times \prod_{P_t^j \varepsilon P^J} \frac{1}{\sigma} g\left(\frac{P_t - \mu}{\sigma}\right)$$

$$\times \prod_{P_t^k \varepsilon P^K} \int_{A_t}^{\infty} \frac{1}{\sigma} g\left(\frac{P_t - \mu}{\sigma}\right) dP_t^k,$$

where g is the standard normal density. Since the random variables are independent, letting G denote the cumulative distribution function, (18) can be written as

$$(19)\quad L(\mu,\sigma^2) = \exp\left[-(1/2)\sum_{j=1}^{J} u_j^2\right]$$

$$X \quad \left[1 - G\left(\frac{A_t + \mu}{\sigma}\right)\right]^I \left[1 - G\left(\frac{A_t - \mu}{\sigma}\right)\right]^K$$

where $u_j = (P_t^j - \mu)/\sigma$.

Taking the derivatives of (19) with respect to μ and σ and setting them to zero,

$$(20)\quad m = (s/J)\left[\frac{IG'\left(\frac{A_t - m}{s}\right)}{1 - G\left(\frac{A_t - m}{s}\right)} - \frac{KG'\left(\frac{A_t - m}{s}\right)}{1 - G\left(\frac{A_t - m}{s}\right)}\right] + \bar{P}^J,$$

where m = maximum likelihood estimate of μ

s = maximum likelihood estimate of σ

$$\bar{P}^J = (1/J)\sum_{j=1}^{J} P_t^j,$$

$$(21)\quad s^2 - s\left[\frac{(A_t - m)IF'\left(\frac{A_t - m}{s}\right)}{J\left[1 - F\left(\frac{A_t - m}{s}\right)\right]} + \frac{(A_t - m)KF'\left(\frac{A_t - m}{s}\right)}{J\left[1 - F\left(\frac{A_t - m}{s}\right)\right]}\right] + s_J^2 = 0,$$

where $s_J^2 = (1/J)\sum_{j=1}^{J} (P_t^j - m)^2$.

Expressions (20) and (21) show the effects of the censoring upon the estimates of the mean and variance. Basically, there is always an adjustment to be made to the usual maximum likelihood estimates, involving the ratio of marginal to total probability evaluated at the imposed limit.

In the fifteen series covered by the sample, five of the March contracts contained observations at the imposed limits: 1974-6, 1980-1.[2] In the 1974-6 and 1981 series, observations at the limits occurred in both the pre-max and post-max periods, while, for 1980, observations at the limits occurred only in the pre-max period.[3] Accordingly, the entries for the means and variances, in the indicated subsamples for these series, in Table 10 are the maximum likelihood estimates from a two-limit probit computer program. Probit estimation assumes that the observations are normally distributed, and such seems justifiable given the results of the Studentized Range tests for these nine subsamples.

Returning to a parametric analysis of stationarity in the price-generating process over the subsamples judged as normally distributed, F-tests revealed that the variances were equal (.05 level) and t-tests showed the same result for the means (.05 level). It is extremely interesting to note that, had the simple sample means and variances rather than the two-limit probit estimates been used in these parametric tests, the results would show nonstationarity in the means for the 1974 subsamples (.05 level). The lesson is that close attention should be paid to real-world impositions affecting observed distributions; the fact that censored observations occur can alter the perception of distributional characteristics.

In the remaining contracts, the evidence regarding changes in the parameters is clear, since the distributions are different over the subsamples. Hence, there are seven March wheat series that exhibit parameter changes between the pre-max and post-max periods and eight which exhibit a high level of stationarity in their parameters. It was mentioned that there were three conclusions, before the censoring interlude, and the second conclusion follows from the first. While it may, or may not, be the case that the distribution of relative price changes varies over periods that span the peak in commercial stocks, the issue should receive careful attention in applications which assume stationarity in probability distributions. An important case in point is the application of the mean-variance framework to the hedging behavior of futures market participants. The results suggest that nonstationary elements in the data should be identified and removed before they are used to form probability distributions underlying such applications. A second important case occurs under applications of the equilibrium capital asset pricing model to futures markets, such as in Dusak, since stationarity is just as important there.

The third conclusion is that the world of futures market participants can be described as substantially less complicated after the peak in stocks. Elements of non-normality in the pre-max period can be associated with uncertainty over the harvest outcome. The results here suggest that the market sorts itself out in a manner which is amenable to simple probability description, after the harvest has arrived. This conclusion has implications for the belief that there are seasonal shifts in net hedging balances, since at times of large stocks, i.e., the post-max period, it is short hedgers who face substantial risks at low values of the cash price. The fact that the distribution with which they must deal is an uncomplicated one is yet another of the elements one can add to the list of factors contributing to the efficacy of the short hedge at high stocks. On the other hand, at low stocks and high values of the cash price, it is long hedgers who face the more substantial risks to their cash positions. The distribution confronting long hedgers is quite often the wilder non-normal one, and their attempt to avoid cash price risk becomes an unenviable task in such situations. But the task is not universally an onerous one. In over fifty percent of the sample years, the pre-max distribution has been judged identical in all respects to the post-max distribution and in an additional two years (1978 and 1980) it was the long hedgers who faced the less complicated state of the world.

The result, that distributions are generally less complicated during the post-max period in the March wheat contract, has a definite Houthakker flavor. However, a more formal analysis of the Houthakker effect can be had, and that examination concludes the empirical portion of the thesis.

The Houthakker Effect and Short Hedging Dominance

In the preceding chapter, the use of empirical cumulative distribution functions to analyze sufficient conditions for short hedging dominance was discussed. A variety of potential relationships between the cdfs at high and low cash prices can meet the sufficiency conditions, some requiring care in further refining the notion of how far apart low and high cash prices must be.

An example of how the analysis is undertaken can be described using Figure 10, the observed cdfs for the pre-max and post-max periods of the 1976 March contract. Sufficiency conditions for short hedging dominance require $H^T_{F_1}(\Phi) > H^R_{F_1}(\phi)$, $\Phi \geq \phi$, and are met for any $\Phi \geq \phi$ below F^0_1. Above F^0_1, approximately 352 cents per bushel, the sufficiency conditions will be met for $\Phi \geq \phi$ such that low and high cash prices are far enough apart, for example, $\Phi^1 \geq \phi^1$. Hence, above F^0_1, the distribution of cash prices must ultimately receive attention. Regarding short hedging dominance, the important items derived from cdf comparisons like Figure 10 are: 1) whether sufficient conditions for short hedging dominance can be found to characterize futures price distributions, in general, 2) if not a general phenomenon, when could low and high cash prices be far enough apart to make a case for such sufficiency, and 3) what is the cumulative probability that the sufficiency conditions are met for any given March wheat contract.

Since there are some contracts where resorting to the cash price will be required, it is worth reviewing the method from Chapter 3, based on expressions (21) and (22) in that chapter, for these cases. In Figure 10, the question concerning the relationship between low and high cash price distributions below 352¢ becomes, "Are low and high cash prices far enough apart so that $H^T > H^R$, empirically?" In the strongest case, one can take $\max(\bar{F} - \underline{F})$, for F_1 below 352¢, $\bar{F}$ on H^T and $\underline{F}$ on H^R, and compare it to the largest observed difference between high and low cash prices. If the cash price difference covers the maximum requirement then it covers all possibilities. If this is deemed too heavy a requirement, one could take the average of the values $\bar{F} - \underline{F}$, at various cumulative probability levels in the range where the difference between low and high cash prices matters as the basis of comparison to actual low and high cash price differences.

Table 12 is a summary of the cdf comparison for the March wheat contract series, 1968-1982. The table provides values for $\bar{F} - \underline{F}$ at selected cumulative probabilities and the maximum of these values is clearly seen. The average of the $\bar{F} - \underline{F}$ values in the table is calculated for each contract. Also, the table reveals the total probability that the sufficiency conditions for short hedging dominance are met for any contract. For example, the total probability of meeting the sufficiency conditions in the 1968 contract is 1.00, while the total probability is about .60 for the 1980 contract. The results in Table 12 show that the sufficiency conditions for short hedging are met for any choice of $\Phi \geq \phi$ in five of the contracts (1970, 1971, 1973, 1974, and 1979). The conditions are met for portions of the cdf comparison below specific cumulative probability levels in three contracts (at or below probability .40 in 1976, .50 in 1978, and .70 in 1980). Finally, above the cumulative probability levels for the contracts just mentioned, and in the remaining seven contracts (1968, 1969, 1972, 1975, 1977, 1981, and 1982) at any cumulative probability level, only reference to the cash price distribution can provide the requisite information to pass judgment on the presence of sufficiency conditions for short hedging dominance.

For contracts where sufficiency could be satisfied for low and high cash prices far enough apart, over some or all of the range of cumulative probability (1968-69, 1972, 1975-78, 1980-82), one finds the strongest requirements to range from 9¢ for the 1972 contract to 134¢ in

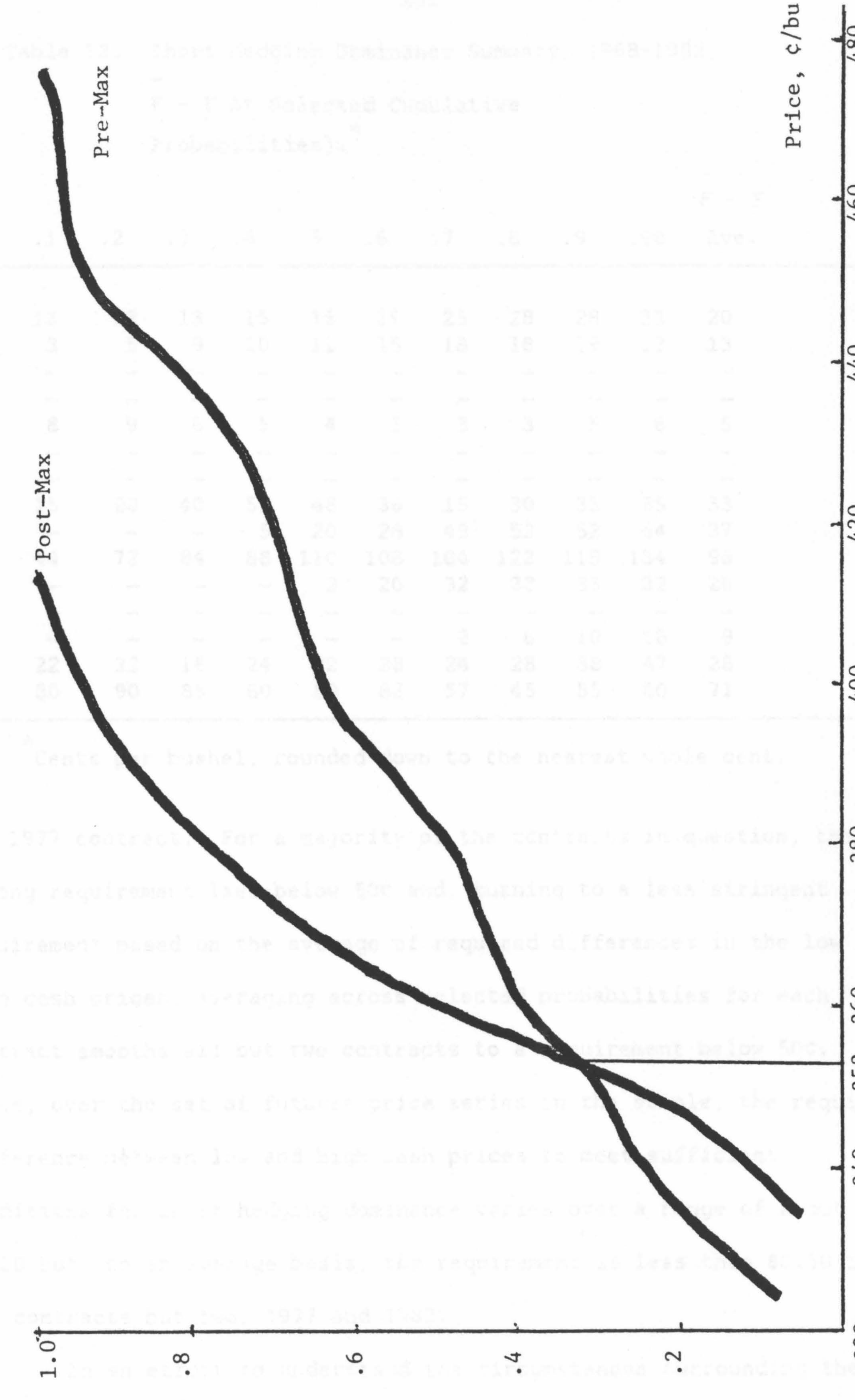

Figure 10. Observed Cumulative Distributions, 1976.

Table 12. Short Hedging Dominance Summary, 1968-1982, ($\bar{F} - \underline{F}$ At Selected Cumulative Probabilities).*

Yr.	.1	.2	.3	.4	.5	.6	.7	.8	.9	.98	$\bar{F} - \underline{F}$ Ave.
68	12	12	13	15	16	19	25	28	28	33	20
69	3	5	9	10	11	15	18	18	19	23	13
70	–	–	–	–	–	–	–	–	–	–	–
71	–	–	–	–	–	–	–	–	–	–	–
72	8	9	6	5	4	3	3	3	5	6	5
73	–	–	–	–	–	–	–	–	–	–	–
74	–	–	–	–	–	–	–	–	–	–	–
75	15	20	40	52	48	36	15	30	35	35	33
76	–	–	–	5	20	28	48	52	52	54	37
77	44	72	84	88	110	108	104	122	118	134	98
78	–	–	–	–	2	20	32	32	35	32	26
79	–	–	–	–	–	–	–	–	–	–	–
80	–	–	–	–	–	–	2	6	10	18	9
81	22	32	16	24	22	28	24	28	38	47	28
82	80	90	85	80	80	82	57	45	55	60	71

*Cents per bushel, rounded down to the nearest whole cent.

the 1977 contract. For a majority of the contracts in question, the strong requirement lies below 50¢ and, turning to a less stringent requirement based on the average of required differences in the low and high cash prices, averaging across selected probabilities for each contract smooths all but two contracts to a requirement below 50¢. Hence, over the set of futures price series in the sample, the required difference between low and high cash prices to meet sufficient conditions for short hedging dominance varies over a range of about \$1.20 but, on an average basis, the requirement is less than \$0.50 in all contracts but two, 1977 and 1982.

In an effort to understand the circumstances surrounding the variety of results found for the March wheat contract, the information regarding the eventful years mentioned in the chapter introduction can be put to use. Recall that trade with China was opened in 1974, trading in wheat futures was stopped in 1979 (suspected corner situation) and 1980 (due to the Soviet embargo), and stocks did not follow their usual pattern during two of these episodes. In 1974, stock levels did not decline continuously throughout the post-max period and, in 1979, a peak in commercially held stocks was barely detectable. In addition, the peak in stocks occurred at an uncharacteristically early date in 1972. Relationships between these occurrences and observed short hedging dominance results provide some interesting insight.

In each of these years, stocks behaved uncharacteristically, or an important national or international event occurred, or both. With respect to sufficient conditions for short hedging dominance, 1974 and 1979 meet the requirements over the entire range of cumulative probability, while 1972 and 1980 are the most likely, of all contracts where it matters how far apart low and high cash prices are over some range of cumulative probability, to have actual cash price differences meet even the strong requirement. The stronger requirement is 9¢ (5¢ on an average basis) in 1972 and 18¢ (9¢ on an average basis) in 1980. Thus, even when important events occur and/or stocks behave uncharacteristically, the satisfaction of sufficiency conditions for short hedging dominance is pervasive.

Focusing upon the more major exogeneous "shocks" of 1974 and 1980, the pattern of stocks is quite consistent with events, although an interesting observation can be made concerning short hedging dominance. In 1974, stocks rose again in the period after the harvest had arrived. This could be due to the nature of the shock in 1974, with its associated uncertainty over when, how much, and what type of wheat might be exportable to China. Hence, the standard single-peak in commercial stocks no longer is the case, and the crucial link between stocks and a Houthakker Effect seems to be broken. However, sufficiency conditions are met over the entire cumulative probability range in 1974. On the other hand, in 1980, an embargo does not carry the same level of uncertainty. In effect, it is contacts already made which are affected and, while uncertainty would exist over responses to the embargo, it is more likely that the rate of stock depletion would slow in the post-max period but still hold the standard pattern. Referring to the data on stocks, while the decline in the rate of stock depletion was slight, this is approximately what happened. While sufficient conditions are met over an extensive range in 1980 (total probability at .70), over the remainder of the cdf comparison they are not, in the absence of restrictions on low and high cash prices. Hence, stock behavior is in accord with events but sufficiency is more pervasively satisfied, in the absence of further restriction, in a year where the crucial link between stocks and short hedging cannot occur because stocks do not behave in the manner predicted by Houthakker.

With the exception of the early peak in commercial stocks for 1972, there is no uncharacteristic stock behavior in any of the series where sufficiency conditions are met throughout the entire range of cumulative probability. Turning to their distributional aspects, no deviation from normality can be detected in any of the post-max distributions of relative price changes in these five series, but there is no exhaustive correspondence between short hedging dominance and well-behaved post-max distributional parameters. For example, 1973 and 1974 have identical post-max distributional outcomes to those in 1977 and 1982, but the latter series are the most unlikely candidates to satisfy the added requirements that low and high cash prices are far apart. The satisfaction of sufficient conditions for short hedging dominance does not appear to be connected to how well-behaved the post-max distributions are in any consistent fashion.

All-in-all, sufficient conditions for short hedging dominance are a recurrent empirical phenomenon, and on an average basis, the empirical requirements on how far apart low and high cash prices must be are not overly demanding. While some interesting anomalies occur, sufficient conditions for a Houthakker Effect of the Fort and Quirk kind to produce short hedging dominance appear pervasively met by empirical futures price distributions. A comparison of cdfs at low and high cash prices tends to support Houthakker's notion that short hedging dominance will occur.

Footnotes for Chapter 4

[1] Opening of trade in a given contract is determined by a committee, based upon the perceived desire amongst traders to trade it. In written correspondence with Professor Roger Gray, Food Research Institute (Stanford University), he offers the conjecture— and that he stresses it as such must be acknowledged— that the earlier opening dates may reflect a concern that the long term gain holding period had increased. The author would like to thank Professor Gray for even this much insight into the occurrence.

[2] The limit was also hit one time in 1977. This year is not mentioned because the program used to calculate maximum likelihood estimates required observations at both the upper and lower limits.

[3] Again, this statement is made relative to the abilities of the estimation program. In 1980, there were also two observations at the limit, but only at the upper limit, and the package could not derive maximum likelihood estimates.

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CHAPTER 5

Conclusions and Suggestions for Further Research

Conclusions and Summary

In Chapter 1, a cohesive picture of the evolution of the theory of futures trading was presented, with the aid of the general expected utility model. The review traced the development from the theory of the risk premium to the theory of the price of storage and on to arguments regarding trends in hedging and the futures price. The diverse elements of this theory were shown to be comparable under the general model, new theoretical findings were discovered, and the theory was shown to be rich in implications for the empirical analysis of commodity futures prices.

The review extends the theoretical behavior of speculators to the case of arbitrary concave utility functions in a true futures market and shows that the separability of optimal cash and futures positions can be extended to the case of a true futures market, absent basis risk. To this writing, both speculative behavior and the separability result were only known in the case of pure forward markets. Highlights of the remaining findings in the review include 1) clarification of what it means to apply the Keynew-Hicks forward market risk premium theory in a futures market setting, 2) putting theoretical flesh to the bones of the controversy between the Keynes-Hicks view and Kaldor's notion of convenience yield, 3) identification of flaws in some past works: the first true futures market work by Dow and the mean-variance analysis of Telser, and 4) a demonstration of the limited ability of mean-variance analysis to contribute to the controversy over trends in hedging.

Regarding empirical endeavors, Chapter 1 made it clear that the effect of stock levels upon participant activity through prices for later delivery was the common thread in the theory of futures trading. Basically, the behavior of futures market participants is tied to the relationship between current and expected futures prices, which is in turn bound to the cash price by the need to move stocks through time. The works of Cootner and Houthakker focus upon an especially crucial time, the peak in commercially held stocks. Specification of an empirical frameowrk, based upon participant behavior around the peak in stocks, comprises Chapter 3. Chapter 2 demonstrated the reasons why the theory review was important. There is a fundamental lack of accounting for theoretical implications in empirical work on the distribution of futures prices. Also in Chapter 2, two important considerations for analysis of futures prices under a random walk hypothesis were clear, serial independence and the controversy over the form of the limiting distribution of observed price series.

The empirical propositions developed in Chapter 3 account for the theoretical implications of Chapter 1. First, from the arguments of Cootner and Houthakker, one implication of the theory is that while futures prices may follow a random walk, the parameters of the price-generating process may change from their values prior to the peak in commercial stocks once the peak has occurred. Second, from a specification by Fort and Quirk, based on notions by Houthakker, an investigation of sufficiency conditions for short hedging dominance is designed. Both elements in the empirical specification depend crucially upon the behavior of hedgers around the peak in stocks.

Finally, Chapter 4 is the empirical undertaking. The first finding was the absence of any serial dependence in the March wheat contract prices. Second, examining distributional form from the perspective dictated by theory adds much to the understanding of price behavior. The parameters of an hypothesized random walk in futures prices often change relative to the peak in stocks. This is a clear indication that such instability should be removed, as with the use of subsamples, before any further theoretical applications to the data are performed. Further, in a number of years where distributions appear normal in both subsamples, their parameters are stable across the peak in stocks. There is also evidence that exchange imposed limits on futures price movements have impacts on the parameters of the distributions. Regarding short hedging dominance, the analysis in Chapter 4 finds recurrent evidence that sufficient conditions for this result are met by observed price distributions. Houthakker's original notion tends to be supported.

Suggestions for Further Research

Additional research efforts suggest themselves in two directions, theoretical and empirical. The theory review in Chapter 1 made plain the current emphasis on forward markets or, at best, the case of no basis risk in futures markets. Furthermore, the number of actors accounted for is limited. For example, board members of the exchange have incentives to keep futures prices from hitting the limit and, pesumably, can take actions in accord with these incentives. However, the incentives and the activity of the exchange directors receives no attention in the literature. Another example of the limited scope of theoretical analysis is its perfeclty competitive nature. The role of government regulators is not covered and the problems of imperfect competition receive the same treatment. Corners and squeezes are much more likely at the termination date of any futures contract, and the orderly liquidation of open positions is quite vulnerable at such times. This could have interesting effects on subsample analysis, especially at the close of trading.

An especially timely theoretical analysis would be the examination of participant behavior under the probability of default on contractual obligations. Such occurrences, in the case of elevator operators, are the current subject of a substantial amount of policy scrutiny. For the sake of descriptive simplicity, suppose short and long hedgers incur identical debt, D, in a futures market without basis risk ($F_1 - C_1 = F_0 - C_0$). In the notation of Chapter 1, default occurs when $V_e < D$ for short hedgers or $V_m < D$ for long hedgers. Hence, a short hedger receives $V_e - D > 0$ if default does not occur and zero if it does occur. From (2) and (5) for short hedgers and (3) and (6) for long hedgers, Chapter 1, in the absence of basis risk, default occurs as follows.

(1) $C_1 < [D + k(y_e) - R(y_e)]/(y_e - Q_e) + C_0$

(2) $C_1 < [D - k(y_m) - R(y_m)]/(Q_m - y_m) + C_0$.

Letting the R.H.S. of (1) and (2) be d_e and d_m, respectively, short hedgers default when $C_1 < d_e$ and long hedgers when $C_1 < d_m$. Both d_e and d_m are taken to be positive for $C_1 \geq 0$. Taking the short hedger, with pdf $h(F_1, C_1)$ over time 1 prices, arbitrage dictating $F_1 \leq C_1$, and default as in (1), the probability of default is

$$(3) \quad Pr(V_e < D) = \int_0^{d_e} \int_0^{C_1} h(F_1, C_1) dF_1 dC_1.$$

What effect does default have on hedging decisions? Intuitively, the indirect utility function, $U(C_1)$, is concave only beyond the value d_e of C_1, where default occurs. Because payoff is zero for $0 \leq C_1 \leq d_e$, default risk introduces a nonconcavity into the indirect utility function. Consequently, at values of the cash price below the default level, hedging is discouraged; short hedgers are more likely to take the chances associated with unhedged stocks. Similarly, at values of the cash price above d_m, long hedgers are less likely to hedge against changes in the cash price. The problem for lenders is that they bear the losses associated with this increased risk taking, while borrowers earn increased rewards from favorable outcomes. It would be interesting to develop the theoretical foundations of the lender's behavior in the face of default risk.

Further empirical research also is suggested, in both the vein of the work done here and other related areas. Concerning the former, the subsampling technique used in Chapter 4 is quite unrefined. If, as hypothesized and evidenced, futures prices follow the distribution of stocks, then an in-depth investigation of commodity stock levels over the harvest cycle is important. After all, the ultimate aim is to provide a link between the distribution of the variance of futures prices and variables accounting for participants' expectations (recall the discussion of Praetz, or Clark, in Chapter 2). Choosing the distribution of commodity stocks as the directing process for futures prices asks the question, "What are the characteristics of the distribution of stocks?"

Another continuation of the work in this thesis concerns investigations into cash prices. The results in Chapter 4, regarding the Houthakker Effect and short hedging dominance, beg questions of an associated cash price series. First, the empirical nalysis failed to account for the theoretical result that cash and futures prices are closer together when <u>both</u> are low and farther apart when <u>both</u> are high. This requires an adjustment to the straightforward cdf comparison based upon the conditional distribution of cash prices. Second, with the correct cash price series, one could compare the cdfs of $B = C_1 - F_1$ at low and high cash prices in order to further analyze the quesiton of how close must cash and futures prices be to fith the Houthakker description. Finally, while the results in Chapter 4 tend to support Houthakker's notion that short hedging dominance can occur, finding actual cash price differences at low and high cash prices to compare with the empirical requirements for short hedging dominance would lend more precision to that claim.

While the questions of serial independence and distributional form received extensive treatment for one wheat contract, the analysis of more contracts and commodities is certainly called for. Of special interest are non-agricultural commodities since the importance of a peak in stocks and the harvest cycle are not apparent for these types of commodities. Whether the notion of a directing process based on stock levels has any general applicability, for example in the metals, might depend on a theory of stock level behavior for those commodities. However, other sample separation criteria for non-agricultural commodities may be more relevant.

Two other questions or, more properly, data anomalies remain unanswered. Prior to 1977, trading in a given March wheat contract never overlapped trading in its predecessor March contract. But beginning with the 1977 contract, trade in all succeeding contracts begins to overlap. No explanation of this occurrence could be found. Also, there is a fairly clear schism in the distributional characteristics of contract prices before 1972 and from 1972 through 1977. For the 1968-1971 contracts, distributions are nonnormal overall and in the pre-max period but normal after the peak in stocks. For the 1972-1977 contracts, distributions are always normal. Such schisms are often indications of a fundamental structural change but, again, no explanation presents itself.

The implications of the work done here for empirical efforts in related areas include two items from Chapter 1 and one item not appearing in the thesis in any form. The risk premium notions of Keynes, Hicks, and Kaldor concerning price patterns in <u>forward</u>, as opposed to futures, markets have received little attention in the literature. While the works of these writers are standard citations, seldom is the context of their work recognized. Price patterns in actual forward markets are the proper unit of analysis for tests of the risk premium theory. Also, whether or not deviations from routine hedging would occur relative to positive or negative prices of storage is a testable implication of Working's theory of the price of storage. On a final note, while complete in its analysis under an hypothesized random walk, as far as it goes, an additional interesting question not addressed here is what effect analysis under the subsample technique would have for efficient market tests. It would certanly seem that whether or not the expected value of a price change today, given all knowledge of the price in the days before, is equal to zero would depend upon the sample period chosen, if the work in this thesis is judged to be valid.