# STUDIES IN MARKET SIGNALING: THE FIRM'S SELECTION OF FINANCE AND CONSUMER PRODUCT WARRANTIES

Thesis by

James Michael Gerard

In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1985

(Submitted September 19, 1983)

© copyright 1983

James Michael Gerard

All Rights Reserved

for Barbara

It is not certain that everything is uncertain.

- Pascal, Pensees, 1670.

He would like to start from scratch. Where is scratch?

- Elias Canetti

# ACKNOWLEDGEMENT

It is a great pleasure to begin by thanking those who have provided a fledgling scholar generous financial assistance, asking nothing in return but the production of ideas: the Haagen-Smit Tyler Fellowship committee, the John R. and Dora Haynes Foundation, and the National Science Foundation, particularly the administrators of grant number SES 81-17708. Few investments are riskier than the ones made by the organizations mentioned; I can only hope that the finished product justifies their support.

Whatever value resides in that product does so in no small measure through the efforts of my advisors, critics, and colleagues. Edward J. Green contributed the seed of the idea that became the first two chapters of this thesis; I benefited greatly from his keen insight during the early stages of the work. Kim Border and Jennifer Reinganum saw these chapters through to the conclusion, adding many fruitful suggestions and in general helping me to move about on the often slippery surface of economic theory. Alan Schwartz contributed a number of insightful suggestions, particularly in those instances where theory could better illuminate the legal framework, and vice versa. Louis Wilde, my chairman, was the overseer of the last chapter -- and given the subject matter, literally the best I could have -- and a valuable organizing presence, particularly in the last hectic weeks. From these, the past and present members of my committee, I have learned a great deal about the process and satisfactions of good scholarship. I thank them, as well as other members of the faculty, Roger Noll, and David Grether among others, for their time, patience and honest evaluation of my work.

I would also like to express my gratitude to my mother, Kathleen M. Gerard, who lent her unwavering support (despite occasional reservations) as her eldest son made his somewhat erratic way through the graduate schools of America; I trust that my father would have been pleased with the final result as well. Finally, my most sincere and heartfelt thanks to my wife Barbara, who was unavoidably very much a part of this ordeal, but who nevertheless offered me constant encouragement, salved my often contused ego, and above all, put up with me no matter how unbearable I became. To her, with all my love and appreciation, this thesis is dedicated.

# ABSTRACT

This thesis reviews and extends several results stemming from recent developments in the theory of market equilibrium in the presence of asymmetrically-distributed information. Chapters 1 and 2 consider the suggestion that the choice of a firm's financial structure may impart "inside" information about the firm's future return stream to outsider investors. It is found that the formal models of such "incentive signaling" make a strong implicit assumption about the result of the information transfer process; weakening this assumption is shown to disrupt the ability of a wide class of incentive mechanisms to support equilibrium outcomes.

In a related literature, an information-transmitting capacity has also been advanced as a major reason for the existence of warranties on consumer products. One important criticism of this view is that consumer product markets are often characterized by imperfect search, the presence of which might be expected to dilute the effectiveness of warranties as signals of a product's underlying quality. Chapter 3 employs an equilibrium nonsequential search model to demonstrate that the information content of a warranty as a proxy for product quality is not disrupted by the presence of imperfect search; the conditions that

underlie signaling equilibria in perfect markets continue to uphold equilibria in markets with imperfect search. However, when information on product quality is transmitted via warranties in such markets, subtle welfare effects come into play depending upon, *inter alia*, consumers′ inherent willingness to pay for warranties. Some of these effects imply surprising conclusions. For example, it may be the case that competitive market outcomes are easier to support in a market where consumers have no *ex ante* information on product quality before they begin to search (with warranties supplying quality information,) than in a market where information about quality is perfect, but search is not. On the other hand, it is possible that all consumers could be made better off in a "search" world (i.e., where quality information is freely available,) but warranty signals persist in order that a "lemons" equilibrium can be avoided.

# TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

# CHAPTER I

# INCENTIVE CONTRACTING, "CONDENSED" INFORMATION, AND THE AGENCY THEORY OF THE FIRM

## 1.1 INTRODUCTION

Generally speaking, accounts of the production, transmission, and distribution of information have occupied a somewhat precarious position within standard theoretical treatments of market equilibrium. Models of competitive equilibrium under uncertainty (such as Arrow [1964] and Debreu [1959]) implicitly endow economic agents with a nearly limitless capacity to distinguish and appraise states of nature. Moreover, it has been observed (see Radner [1968]) that weakening the strict informational assumptions of the neoclassical models -- for instance, by restricting the agents' access to contingent claims markets -- has a marked disruptive effect on the existence of equilibria.

Radner's early work on markets with asymmetric information represents a watershed in the development of the "economics of information", in that subsequent research has tended to follow one of two sharply demarcated paths. On one hand, general equilibrium theorists, particularly those utilizing rational expectations models, have attempted to isolate conditions on individual agents' utility functions and endowments of private information such that the imperfect information market equilibrium can be shown to coincide with the equilibrium resulting from the interaction of fully-informed agents (for a recent review of this literature, see Jordan and Radner [1982].) The medium of information exchange in a rational expectations model is, of course, the observed sequence of market prices. The second predominant direction of research on the effects of imperfect market information is concerned with a world in which, for whatever reason, the fully-revealing price equilibrium cannot be sustained. As might be expected, the feasible models that can be constructed of such a world are for the most part limited to a partial equilibrium structure, make stronger assumptions on the distribution of private information, and are more conceptual -- more descriptive of a specific market, such as those for labor or securities -- than their rational expectations counterparts. This essay deals with the application of a class of these models, which can be termed "signaling" or <u>incentive</u> <u>contracting</u> models, to the modern agency theory of the firm.

In an important 1937 paper, Coase demonstrated that received theory provided no real explanation of how the firm was bounded -- that is, where the dividing line was drawn between resource allocation decision-making internal to the firm and the interaction of the firm with the external economy via the price mechanism. Coase suggested that intrafirm coordination of production was characterized by a suppression of any price-based allocative mechanism, in favor of less specific, performance-based contracts administered by a special class of factors, the entrepreneur/managers. Thus, the boundary of the firm was dynamically determined by whether, at the margin, the cost of the authoritarian, routinized transactions overseen by managers was lower than the cost of arranging the corresponding transactions on the open market. Coase's theory has been extended and refined in a series of papers which together comprise the "agency" or "property rights" theory of the financial firm (highlights of the literature include Alchian and Demsetz [1972], Jensen and Meckling [1976], and Fama [1980].)

The major contribution of the agency theory is its description of the complex set of contracts binding together management, non-management factors of production, and investors — and its identification of the firm as precisely this "nexus of contracts", nothing more or less. The theory provides an analysis of means to the resolution of the potential conflict between management and investors through a framework of equilibrium contractual relations, while seeking to demonstrate that such a relationship can persist when managers are not themselves large-scale owners of firms, and when investors are not privy to the internal decision making processes of managers.

The agency approach, then, posits that firms are able to solve the problems attending contracting within an environment of imperfect, asymmetrically distributed information, further complicated by the presence of moral hazard. Over the last decade or so, a steady progression of formal models has attempted to characterize the properties of these incentive-compatible contracts, to compare the theoretical results with the terms of actually-existing incentive contracts, and to draw normative conclusions about, among many other things, the socially optimal extent of "insider trading" and information disclosure in capital markets. A relatively recent development in theory, the so-called "incentive signaling" models of financial structure, suggest that incentive contracts contingent upon actions taken by the agent (the manager) may have the property of decentralizing "inside" information about the firm's future return stream, provided that the action the manager takes is routinely observable by the principal (the investor). These models are potentially important, because they offer a formal rehabilitation of the long held but somewhat murky thesis that, for example, dividend or debt/equity ratio policy choices by managers somehow communicate information about the firm to outside investors.[1]

This essay is about the relationship between agency and information transfer in financial markets. In particular, it will be shown in the next section that the financial incentive signaling models proposed to date contain a very strong implicit assumption about the result of the information decentralization process: namely, that when the equilibrium exists, managers in effect transmit an exhaustive report of their inside information. Section 3 lays out a less restrictive signaling model in which the principal, the outside investor, does not require an exhaustive report of all the manager knows, but instead seeks to elicit a suitable, "condensed" version of the manager's private information, while preserving the incentive compatibility of the signaling mechanism. Section 4 extends the formal models of Ross [1977] and Bhattacharya [1980] into this structure, and demonstrates that the signaling equilibrium found in the original models fails to exist in the more general formulation. Section 5 concludes.

## 1.2 AGENCY, INCENTIVES, AND "CONDENSED" INFORMATION

When the economist addresses the relationship between firm managers, shareholders, and the distribution of critical market information, his concern has a well-defined historical source: the transition, centered for the most part in the nineteenth century, from commercial to managerial capitalism. With very few exceptions,[2] the dominant organization of enterprise prior to 1850 was the family-owned and operated, functionally-integrated partnership.[3] The officers of these firms operated at very low levels of specialization, sharing the tasks of management, entrepreneurship, risk-bearing, and ownership between tightly-knit members of small social groups. And even where a firm's activities were geographically widely dispersed, the cohesive nature of relationships within the firm all but eliminated the problem of agency.[4] As Alfred Chandler has demonstrated, however, this was to change dramatically in the second half of the nineteenth century with the invention and commercial application of the railroad and the telegraph.

The joint effect of these two innovations was to make feasible a vast increase in the volume of production and distribution of manufactured goods. In turn, the new, higher tempo of operations fostered the development of radically new kinds of administrative organizations. Initially in the rail and telegraph companies, and then in the manufacturing industries themselves, there evolved hierarchical management systems in which each lower-, middle-, and upper-level manager performed a particular set of specialized functions, and contributed to the decision-making process along a well-defined chain of command.[5] Standardized techniques for reporting and evaluating internal information were perfected. But most important, along with the increased scale of production came an increased demand for financial capital -- a demand which rapidly outstripped the immediate resources of even the largest family-owned firms. Industry's growing reliance on public stock sales for infusions of capital, and the awareness of the relative autonomy of the new class of managerial professionals, together marked the beginning of the modern debate over the welfare effects of this presumed "separation of ownership and control."

Ironically, the uncompromising critical position on the ascendancy of the new managerial class was first set forth by Adam Smith. Referring to the joint stock companies of his day, Smith wrote:[6]

> [Stockholders] seldom pretend to understand anything of the business of the company; and when the spirit of faction happens not to prevail among them, give themselves no trouble about it, but receive contentedly such half-yearly or yearly dividend as the directors think proper to make to them.

More contemporary critics such as Adolph Berle, Gardiner Means, and Carl Kaysen have refined the idea expressed in this passage, emphasizing the potential link between dispersed, "outsider" ownership of the firm and an enhanced latitude for managers to act arbitrarily in their own interests, to the detriment of shareholders.[7] Those who viewed the shareholder as the exploited party in the ostensibly democratic process of corporate control had an important influence on the enactment of far-reaching securities regulation in the 1930s. Stripped of the legal and regulatory complexities, the crux of the dispute between the "orthodox" and the "contractarian" theories of the publicly-held firm lies in what each has to say about the ease and extent to which, in Smith's phrase, "the spirit of faction" may be aroused among the firm's owners.

In order to show how existing theory has inadequately come to grips with the relationship between managers and outside investors in the "nexus of contracts" perspective of the firm, it is necessary to examine two related questions. First, how does casting the manager-investor conflict into a contractual model yield the conclusion, in direct contradiction to the orthodox view, that the separation of ownership and control is in fact the most efficient mode of organization of the firm? And second, what is the role of the revelation of "inside" information in supporting an equilibrium outcome of the implied agency relationship between management and shareholders?

Before a firm can begin production, it must lay claim to a stream of capital. Where capital requirements exceed the wealth of the firm's operators, it would be desirable to have access to a specialized market on which financing of present needs for capital could be exchanged for claims on the firm's future returns. But if this securities market is incomplete -- that is, if investors are unable to guarantee their preferred return stream in every state of the world -- and if investors remain for the most part outside the decision-making process of management, an obvious problem of adverse selection imperils the efficient functioning of the market.[8] The agency theory of the firm asserts that the persistence of outsider-held, residual-claim financial instruments can only be explained by regarding the relationship between managers and outsider investors as a contractual one, and by realizing that the contracts defining the firm contain important provisions designed to limit the ability of managers to divert the firm's returns away from the shareholders. These provisions take a number of forms, but their overall effect is to reinforce the efficiency of <u>both</u> the securities market and the market for management teams.

One important class of contractual provisions, discussed at length in Alchian and Demsetz [1972] and Jensen and Meckling [1976], is concerned with the neutralization of monitoring and coordination costs. When investors advance "front money" to a firm in return for a set of residual claims on the firms' returns, they undertake the risk that the firm's operations may yield no residual. In order to dilute this risk, shareholders require a perfect property right in the shares they hold, so that they can freely transfer their holdings between firms (via the capital markets) without having to incur the costs of attempting to proxy the firm away from the control of the current management team. A similar argument is advanced for the observed existence of limited liability shareholdings — since unlimited liability investors would derive sharply reduced benefits from diversification of their holdings, by virtue of the fact that the unforeseen demise of a single firm could pose a serious threat to an investor's entire wealth.[9] Besides strengthening the efficiency of the market for risk, these contractual provisions are the basis for the informational asymmetry between shareholders and managers. The neutralization of diversification costs translates directly to an increasing separation of ownership and control. Or, as Fama has said:[10]

> Since he holds the securities of many firms precisely to avoid having his wealth depend too much on any one firm, an individual security holder generally has no special interest in personally overseeing the detailed activities of any firm.

It is clear then, that the attributes of manager-shareholder relationships distinguished by the contractarian theory of the firm actively contribute to the efficiency of risk bearing; but it is equally clear that institutions such as limited liability or free access to secondary claims markets do not in themselves guarantee a solution to the agency problem they create. Hand in hand with a contracts-based theory must go a satisfactory account of an agency relationship — which is why the two terms are often used interchangeably in the literature. If a firm is to be validly regarded as a nexus of contracts, it must be demonstrated that the assignment to managers of a property right in "inside" information takes place not through an exercise of raw managerial power, but through a contractual allocation disciplined by competitive markets. So, the questions addressed in this section reduce to a question about how this market discipline can be maintained in a non-rational-expectations world. Or more specifically, how do investors -- characterized by their preferences over risky streams of asset returns -- develop expectations of the market values of risky assets, given the manager's stewardship over the details of the firm's operations?

The incentive contracting literature has made a valuable contribution towards bridging this "expectations gap". The idea of decentralizing private information through agency contracts contingent on actions and outcomes directly observable by the principal can be traced to the "market signaling" models of Akerlof [1970] and particularly, Spence [1974].[11] Over the last five years or so, financial signaling models have been developed which suggest that inside information may be systematically communicated to the market by dividend policy (Bhattacharya [1979], [1980]), a choice of debt insurance (Thakor [1982]), the manager's holdings of stock in his own firm (Leland and Pyle [1977]), and, of immediate concern to this essay, the selection of debt-equity policy (Ross [1977], [1978], Heinkle [1982].) In Ross's two-period, partial-equilibrium model, managers possess "inside" information on the distribution of firms' future returns, while the security market trades of "outside" investors impart none of this inside information to the market. In turn, managers are compensated according to a well-defined incentive schedule which is known by all investors. By announcing a financial structure for the firm (for example, by contracting for a particular level of debt financing), management creates a perception of the firm's expected return among investors. The incentive schedule is arranged so that the manager's compensation depends upon both the value of the firm perceived by shareholders at time 0 and the value actually revealed at time 1; thus, the manager is held accountable at time 1 for a falsely optimistic signal of the firm's value. Ross makes use of Spence's notion of a signaling equilibrium to derive a determinate financial structure for the firm, optimal from the standpoint of the manager in that his chosen signal maximizes his expected compensation, and optimal from the investor's point of view in that, where an equilibrium exists, no firm gives a false signal of its expected value -- that is, investors' perceptions of which firm is which are revealed to be valid. The model therefore provides the necessary link between fulfilled investor expectations (hence, a smoothly-functioning capital market), an equilibrium wage schedule for managers (giving correct signals to the market for managerial services), and optimal finance.[12]

Nevertheless, all of these signaling models -- for they are nearly identical in a formal sense -- make use of a crucial simplifying assumption that is inconsistent with the properties of manager-investor contracting discussed previously. It is generally assumed that both managers and investors know the form of the distribution function of next period's returns, but only managers know the parameters of the distribution particular to their firms. Nothing unusual here; this is a strong assumption, but one typically made in two-period agency models. However, the incentive signaling models are consistently based upon one-parameter families of return distributions. For example, the period 1 returns of Ross's and Bhattacharya's firms are assumed to be uniformly distributed on an interval [0,t], and the manager's inside information is his knowledge of t. Risk-neutral investors would assign a value of $t/2(1 + r_0)$ to these returns, with $r_0$ the one-period rate of interest. It can then be shown -- exploiting a special property of one-dimensional distributions such as the uniform[13] -- that for an attractively simple class of incentive mechanisms, managers give financial signals that are a strictly increasing function of their firm's underlying value. Further, having collected a cross-sectional sample of signals, investors can invert the equilibrium signal function and intuit an expectation of the firm's value which will turn out to be consistent.

But there is a problem with this elegant construct: when the equilibrium exists, outsider investors become informationally equivalent to managers. The investors receive a signal which decentralizes, to within a trivial linear transformation, all of the manager's inside information. Since the incentive mechanism investigated by Ross functions without deadweight welfare loss relative to the full-information equilibrium, there can be effectively no divergence of interests between managers and shareholders, no reason for investors to expend resources to monitor the conduct of managers, and no meaningful separation of ownership and control. Security prices may not reveal any inside information to the market, but there exists a nondissipative revelation mechanism, operating through incentive contracts, which restores informational parity between managers and shareholders.

In seeking to provide a coherent story of information transfer through routine financial decision-making, the existing incentive contracting models plainly overreach themselves. The regulatory and legal remedies for perceived informational defects in the operation of securities markets are the focus of a lively debate within government and academia. It seems generally agreed, however, that an exhaustive characterization of the future return distribution of a firm is highly complex, and its disclosure is certainly not costless. Some of the firm's information, its "material" or "non-public" information in legal parlance, is rightly proprietary to the firm and produces produces economic rent. Full disclosure of such information would damage the firm's competitive position, imposing costs on managers and investors alike.[14] In any event, outsider investors do not need to become informational copies of the firm's chief executive officer in order to make their utility-maximizing investment decisions — indeed, this is the very reason they remain outsiders.

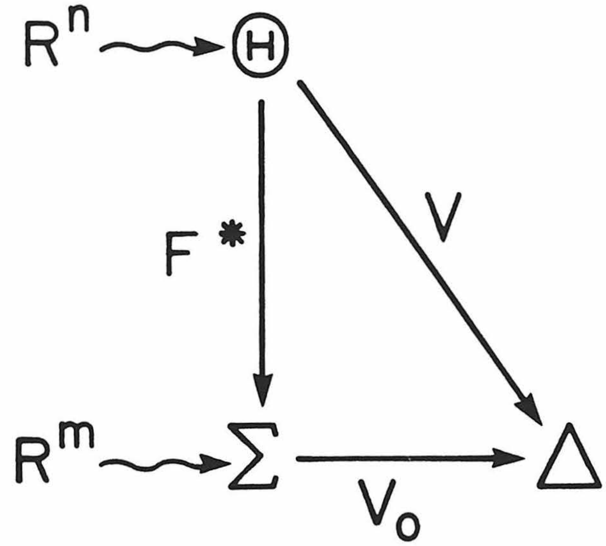
The proper incorporation of incentive contracting into the overall "nexus of contracts" theory of the firm requires a structure enabling investors to form expectations of the firm's "market price of risk" without ever having to incur the costs of distinguishing the firm's <u>state</u>, or full description of the return distribution. The elicitation by the principal of summary data drawn from the manager's state information will be referred to in this essay as <u>condensed</u> information transmission.[15] A less restrictive -- and more realistic -- problem for the principal is to attempt to construct an incentive contract that advances the timing of this condensed revelation to the market, since this is all that is needed by the investors in order for their risk-bearing function to be efficiently carried out. In turn, the manager's incentive compensation will depend on investors' expectations of the firm's value imparted by the condensed information signal, and if an equilibrium exists, the manager's actions on his own behalf will be constrained to those consistent with the interests of shareholders. The onus on the incentive contracting models, therefore, is a demonstration of the robustness of their equilibria to the more refined model of information transmission. The next section lays the formal groundwork for an approach to this task.

## 1.3 SIGNALING WITH "CONDENSED" INFORMATION: FORMAL PRELIMINARIES

Consider a firm which seeks to undertake a series of risky investment projects. From the developmental data on the projects, management knows that the future returns from the projects will be distributed according to a probability density function which can be uniquely specified by n parameters. The state of the firm at time 0 is a vector $(\theta_1, \theta_2, \ldots, \theta_n)$ in $\mathbb{R}^n$. The outsider investors evaluate the firms according to a set of decision relevant variables, $\Delta$, a subset of $\mathbb{R}^m$, $m < n$. For example, it may be the case that the firm finances its capital needs by issuing m different types of financial instruments (common and preferred stock, senior and subordinated bonds, and so on), and investors with diversified portfolios wish to establish the risk- adjusted market value of each security. Of course, the principal's utility function may contain more than just his *ex post* wealth; an investor could have direct preferences over the mean and variance of security returns. For the purposes of this essay, however, it will be assumed that investors seek consistent expectations of "next period's" risk-adjusted market value of securities.

In order to accomplish this, the outsider investors and managers enter into an incentive compensation contract M, contingent on the manager's financial signals, $\sigma \in \sum \subset \mathbb{R}^m$. Managers with knowledge of the firm's state at time 0 release a signal which is observed by investors. If investors could somehow observe $\theta$ directly, they would associate a "true" value to each firm according to the smooth function $V: \Theta \rightarrow \Delta$. But because the signal and state spaces are of different dimension, the outsider investors are clearly unable to invert the signaling function and map the signals smoothly back onto $\Theta$. However, under the correct conditions on M, there may exist a one-to-one function $V_0: \sum \rightarrow \Delta$ such that investors can form fulfilled expectations of the period 1 values of firms.

The incentive contract binding managers and shareholders is a function $M: \sum \times \Theta \rightarrow \mathbb{R}^1$ which takes the state description of the firm and the firm's signal into the manager's incentive compensation. A well-defined incentive contract depends only on information observable by the investors — at time 0, the signal $\sigma$, and at period 1, the signal plus a (possibly imperfectly measurable) realization of the random variable distributed according to $\theta$. Thus, although the manager knows $\theta$ ex ante, M depends on $\theta$ only implicitly. The manager with utility function $u(\cdot)$ solves the problem $\max_{\sigma \varepsilon \Sigma} E_X[u(M(\sigma;\theta))]$, where X is the random variable -- for example, the firm's period 1 net operating income.




$$M(F^*(\theta);\theta) \geq M(F(\theta);\theta)$$
$$\forall F \in \Sigma$$

Figure 1.1 Sequence of Mappings Supporting an Incentive Signaling Equilibrium with Condensed Information

Formally, the condition for fulfilled investor expectations is clear from Figure 1.1. Under the value mapping V, the set of firms having a particular risk-adjusted value (i.e., a level curve of V) is (locally) a smooth copy of $\mathbb{R}^m$ in $\Theta$. Similarly, if M has the correct properties, the level curves of $F^*$, the managers' compensation maximizing signals, are m-dimensional surfaces in $\Theta$. Investors' expectations are fulfilled if, in a neighborhood of the firm's state $\theta$, the level curves of $F^*$ coincide with those of V. Obviously, if investors find that some signal $\sigma$ is mapped to many elements of $\Delta$, that signal cannot separate firms according to value, and the incentive contract which produced the signal cannot support a separating signaling equilibrium.

The object of this essay is to show that presently-existing incentive signaling models do not extend in a straightforward way when condensed information is to be transmitted. Before turning to the specific model which forms the balance of what follows, a brief word should be given to the relationship between this essay and the growing literature devoted to the study of revelation mechanisms. Green [1982] is a recent study of the practical applicability of direct revelation mechanisms to institutional design problems. Employing a multi-dimensional state space analysis, (where states correspond to the private information possessed by an agent with a given utility function),[16] Green has shown that the report by an agent of <u>summary</u> information -- that is, anything less than an exhaustive report of the entire state vector -- will not be supportable as an equilibrium unless the agent's utility function satisfies a restrictive algebraic identity.

Green's result leaves open the question of the supportability of revelation-like mechanisms on sets of preferences which do satisfy this identity, and which may nevertheless be of considerable interest within economic theory. A conspicuous example might be the set of preferences that can be represented by von Neumann-Morgenstern utility functions. Ross's financial model is defined explicitly over a subset of these utility functions, and so the generalization of Ross undertaken here is aimed at bridging the gap between the conceptual models of market signaling and the highly general (and therefore necessarily abstract) generic existence literature.

It would be misleading, however, for this essay to claim a complete characterization of the properties of condensed information signaling. In the interest of tractability, attention will be confined to the simplest world within which information condensation might take place: one described by a state space lying in $\mathbb{R}^2$, and a signal space (as in the existing signaling literature) of dimension one.

## 1.4 INCENTIVE SIGNALING IN A MEAN-VARIANCE MODEL

This section makes a more concrete examination of the effects of condensed information signaling by extending the model of Ross [1977] into a state space having two dimensions. The two-dimensional framework for analysis, besides being the simplest multidimensional formulation of the problem, also carries with it -- via the capital asset pricing model -- an obvious prescription for the valuation of firms' returns. Several introductory definitions will help to clarify these points. It should also be noted that the revised model will in general restrict itself to Ross's underlying structural assumptions: expected value maximizing managers are engaged in the selection of a debt/equity policy for their firms, abstracting from the problem of activity choice. Further, it is assumed that investors do not make strategic side payments, overt or clandestine, to managers in order to influence signal choices.[17]

The Ross model specified firms with random period 1 returns, X, uniformly distributed on $[0,t] \subset \mathbb{R}^1$. In turn, $t \in [c,d] \subset \mathbb{R}^1$, generating a one-dimensional continuum of firm "types"; manager-insiders were presumed to know the t-value particular to their firms. A correspondingly simple two-parameter state space for firms may be introduced by the following:

<u>Definition 1</u> Let X, a real valued random variable, denote a firm's net operating income. X is assumed to be distributed uniformly in the interval $[t_1,t_2]$, with $t_1 \in [0,c] \subset \mathbb{R}^1$ and $t_2 \in [c,d] \subset \mathbb{R}^1$, $0 < c < d$. The state space of firms is therefore the set

$$\Theta \equiv \left\{ \theta = (t_1,t_2) \in \mathbb{R}^2 \mid t_1 \in [0,c],\ t_2 \in [c,d] \right\} ,$$

and a firm of <u>state</u> $\theta$ (known to managers) has associated with it a net operating income $X_\theta$.

At time 0, each firm plans to undertake a risky project which will be financed through a combination of debt and equity securities. If the capitalization of a firm with state $\theta$ at time 0 is composed of equity shares with a total present discounted money value $S_\theta$ and debt obligations having discounted value $F_\theta$, then the market value of the firm is given by the accounting identity $V_\theta = S_\theta + F_\theta$. Assume that all firms can borrow at the riskless rate of interest $R_0$. Then the firm's (random) rate of return on equity is $R_\theta = (X_\theta - R_0F_\theta)/S_\theta$. In order for the investors to have a motive to diversify their holdings of equity securities, it must be supposed that they are risk averse in <u>ex post</u> wealth. As is well known, with the correct restrictions on utility functions one may express the investors' expected rate of return on equity as $E_X(R_\theta) = R_0 + \lambda\rho(R_\theta,R_M)\sqrt{\text{var } R_\theta}$, where $\lambda$ is the publicly-known CAPM market parameter, and $R_M$ is the return on the market portfolio.[18] Taking the expectation of both sides of the expression for the rate of return on equity yields, for a fixed value of $S_\theta$, and $F_\theta$ (since $S_\theta\sqrt{\text{var } R_\theta} = \sqrt{\text{var } X_\theta}$.)

$$S_\theta R_0 + \lambda\rho(R_\theta,R_M)\sqrt{\text{var } X_\theta} = E(X_\theta) - R_0F_\theta$$

And now it can be seen that the expression for $E_X(R_\theta)$ actually depends on <u>three</u> parameters: the two defining $\theta$, plus one that summarizes the interaction of the firm $\theta$ with the market portfolio. The state space of firms can be restricted to a subset of $\mathbb{R}^2$ by imposing the condition $\rho(R_\theta, R_M) = 1$, (i.e., in effect, constraining the market portfolio to a linear combination of the individual firm returns $R_\theta$.) The sense of such a restriction is that it yields the simplest model within which the effect of information condensation can be analyzed, while preserving the risk aversion of investors. As will shortly be seen, the two-parameter extension of Ross's incentive signaling model will emerge as a special case of the present formulation. Thus, the new structure present in the characterization of equilibria will stem wholly from the presence of condensed information.

With the assumption on $\rho(R_\theta, R_M)$, the expression immediately above is easily seen to yield:

$$V_\theta = \frac{E(X_\theta) - \lambda\sqrt{\text{var } X_\theta}}{R_0}$$

Definition 1 fixes the moments of the return distribution for a firm with state $\theta = (t_1, t_2)$,

$$E(X_\theta) = \frac{t_1 + t_2}{2} \tag{1a}$$

$$\text{var } X_\theta = \frac{t_2 - t_1}{2\ \ 3} \tag{1b}$$

Thus, the assumptions on the investors' preferences, the equilibrium distribution of the firms' returns, and the definition of the state space $\Theta$ can be combined to yield the appropriate risk-adjusted valuation measure for returns, in terms of the underlying state:

Assumption 1  The firm $\theta = (t_1, t_2) \subset \Theta$ has a risk-adjusted value given by the smooth mapping $V: \Theta \rightarrow \Delta \subset \mathbb{R}^1$ defined by (from eqs. (1)):

$$V(\theta) = \frac{1}{R_0}(k^+ t_1 + k^- t_2) \tag{2}$$

with

$$k^{\pm} \equiv \frac{1}{2} \pm \frac{\lambda}{2\sqrt{3}}$$

REMARK: In this version of CAPM, the market parameter $\lambda$ scales the investors' tolerance for trading risk and return. If $\lambda = 0$, investors become risk-insensitive, and the value mapping reduces to the expected value alone. In any case, investors are interested in using the financial signal to form expectations about the period 1 values of firms. Given the fact that they are cut off from knowing the firm's state, however, the issue becomes whether they are capable of decoding the information about the firm's expected value imparted to them via the condensed signal.

As indicated previously, the incentive signaling equilibrium is composed of two parts. First, the financial signal selected by management must maximize the manager's compensation under the investor-enforced incentive schedule. The incentive schedule is itself an object of equilibrium, evolving through a process of arbitrage elimination played out between outsider shareholders and management. Second, the expectation of firm value that the signal creates among shareholders must be fulfilled when the actual values of firms are revealed at time 1. Two further definitions will complete the formal specification of the model.

Assumption 2 (Ross) At time 0, managers with knowledge of their firm's state $\theta$ release a financial signal, the level of debt financing chosen for the firm. If $V_0: \sum \rightarrow \Delta$ is the mapping investors make from signals to firm values, then the incentive schedule $M: \sum \times \theta \rightarrow \mathbb{R}^1$ applied to management by investors can be expressed as:

$$\begin{aligned} M(F,X;\theta) &= c_0 V_0(F) + c_1 \left\{ x_\theta I_{[F,\infty]}(x) + (x_\theta - L) I_{[0,F]}(x) \right\} \\ &= c_0 V_0(F) + c_1 \left\{ x_\theta - L I_{[0,F]}(x) \right\} \end{aligned} \qquad (3)$$

where $c_0$, $c_1$, and L are positive constants, and I(x) is the indicator function.

In setting the incentive schedule (3), investors assume that managers are endowed with a fixed set of claims on the returns of the firm, and that managers are prohibited from trading those claims. The constants $c_0$ and $c_1$ set the relative distribution of compensation between the two periods. For the present, the only restriction put on $F(\theta)$ will be one of non-negativity. Constrained by (3), risk-neutral managers trade off the period 0 enhancement of their compensation obtained by favorably biasing F against the increasing probability of suffering the bankruptcy penalty L should operating revenues be insufficient to service the debt obligation contracted for.

Finally, as defined in the previous section, investors demand a fulfilled expectations condition:

Definition 2 The manager's signal of financial structure $F^*(\theta)$ may be said to fulfill investor expectations at time 1 if the following holds:

$$V_0(F^*(\theta)) = V(\theta) \qquad \theta \in \Theta \tag{4}$$

where $V(\theta)$ is as defined in (2).

Incentive schedules such as (3) can be called the class of linear-dichotomous schedules; as contracts, they have at least three important properties. First, in a fulfilled expectations equilibrium, $V_0(F^*(\theta)) = V(\theta)$, so that the manager's period 0 compensation reflects the firm's actual value. Thus, even if the manager were allowed to trade on his own account at period 0, he could not make systematic "insider" profits at the expense of outsider investors. Second, the incentive contract is contingent on an extremely coarse *ex post* monitor of the firm's operating income, namely, whether the bondholders' claims were satisfied. And third, the process being modelled with (3) is truly one of endogenous equilibrium contracting, as opposed to the exogenous signaling cost structures imposed in most signaling models. The equilibrium results in the setting of a particular $L^*$ (and in general, a $c_0^*$ and a $c_1^*$ ), such that, for each firm in the state space, investor expectations are fulfilled and managers' wages reflect their firms' actual values.

All of this follows, of course, provided that the signaling equilibrium is robust to the presence of condensed information in this simple model. Unfortunately, there is now the following:

Theorem. For each $\theta \varepsilon \Theta$, let the valuation function $V:\Theta \rightarrow \Delta$ and incentive compensation function $M: \sum X \Theta \rightarrow \mathbb{R}^1$ be as defined in equations (2) and (3), respectively. Then an equilibrium signaling schedule $F^*(\theta)$ jointly satisfying:

$$\max_{F(\theta)\varepsilon \Sigma} E_\theta[M(F,X;\theta)]$$

and

$$V_0(F^*(\theta)) \equiv V(\theta)$$

fails to exist.

Proof The proof consists of two parts. First, the necessary condition for the manager's maximization problem will be found, assuming that the fulfilled expectation condition holds. Second, another necessary condition will be derived from the fulfilled expectation condition, and the two will be shown to be inconsistent almost everywhere in $\theta$.

i) Utilizing (3) and the properties of the return distribution (Definition 1), the manager's maximization problem becomes:

$$\max_{F(\theta)\ \varepsilon\ \Sigma} m(F(\theta);\theta) = \max_{F(\theta)\ \varepsilon\ \Sigma} E_\theta[M(F,X;\theta)]$$

$$= \max_{F(\theta)} c_0V_0(F) + c_1\left[\frac{t_1 + t_2}{2} - \frac{L(F - t_1)}{t_2 - t_1}\right]$$

assuming the existence of an interior maximum, the first order condition for $F = F^*(\theta)$ is

$$c_0V_0'(F^*) - \frac{c_1L}{t_2 - t_1} = 0 \qquad (5)$$

and now, to obtain an explicit equation for the optimal signal as a function of the state, the condition for consistent investor expectations is utilized; differentiating (4) in $t_1$, $t_2$:

$$V_0'(F^*(\theta))\frac{\partial F^*(\theta)}{\partial t_1} = \frac{k^+}{R_0} \qquad (6a)$$

$$V_0'(F^*(\theta))\frac{\partial F^*(\theta)}{\partial t_2} = \frac{k^-}{R_0} \qquad (6b)$$

Adding 6(a) and 6(b) and solving for $V_0'(F^*)$ gives:

$$V_0'(F^*(\theta)) = 1/\left\{2R_0\left[\frac{\partial F^*(\theta)}{\partial t_1} + \frac{\partial F^*(\theta)}{\partial t_2}\right]\right\} \qquad (6c)$$

using 6(c) to eliminate $V_0'(F^*)$ in (5) yields an equation for $F^*(\theta)$:

$$\frac{\partial F^*(\theta)}{\partial t_1} + \frac{\partial F^*(\theta)}{\partial t_2} = \frac{c_0(t_2 - t_1)}{2c_1 L} \tag{7}$$

ii) Now a necessary condition on $F^*$ can be derived from the fulfilled expectation condition. Investor expectations about period 1 values are fulfilled iff the identify $V_0(F^*(\theta)) \equiv V(\theta)$ holds across $\theta$. Equation (7), together with the expression for $V_0'(F^*)$ (equation 6(c)), show that $V_0'(F^*) > 0$ a.e., so that $F^*(\theta) = V_0^{-1}(V(\theta))$. Now define the set $\Theta_{\bar{V}} \equiv \{\theta \in \Theta \mid V(\theta) = \bar{V}\}$ -- that is, a level set of V -- and correspondingly, $\Theta_{\bar{F}} \equiv \{\theta \in \Theta \mid F^*(\theta) = \bar{F}\}$. It is then apparent that the fulfilled expectations mapping requires $\Theta_{\bar{F}} \subseteq \Theta_{\bar{V}}$. [Suppose not. Then there can exist a $\hat{\theta} \in \Theta_{\bar{F}}$ such that $V(\hat{\theta}) \neq \bar{V}$. But, $F^*(\hat{\theta}) \in V_0^{-1}(\bar{V})$, so that $F^*(\hat{\theta})$ lies in the preimage set of $\bar{V}$, a contradiction]. In order for $\Theta_{\bar{F}} \subseteq \Theta_{\bar{V}}$ to hold, the slopes of the level curves must be equal, or:

$$\left.\frac{dt_2}{dt_1}\right|_{\bar{F}} = \frac{\partial F^*(\theta)/\partial t_1}{\partial F^*(\theta)/\partial t_2} = \frac{k^+}{k^-} \tag{8}$$

by the definition of $V(\theta)$, provided that $\partial F^*(\theta)/\partial t_2 \neq 0$. Equation (7) describes the surface $F^*(\theta)$, and so it describes all of the level curves of $F^*$. But in order for (7) to be consistent with (8) (up to a linear transformation) along $F^*(\theta) = \bar{F}$, it is clearly necessary for $t_2 - t_1 = 0$ to hold. But by the definition of the state space, this condition holds only at a <u>point</u> in $\Theta$. If every $\theta$ is a regular value of $F^*$, so that the level curves of $F^*$ are smooth copies of $\mathbb{R}^1$, then the necessary conditions (7) and (8) are inconsistent a.e. in $\Theta$. Therefore, the function $F^*(\Theta)$ fails to exist. □

An Illustrative Example

The nature of the equilibrium failure characterized by the Theorem can best be visualized with a specific example. Note that equation (7), which characterizes the equilibrium signaling locus, is a relatively simple member of the class of first-order partial differential equations; taken alone, it has an infinite number of solutions. Generally, a unique solution may be selected by appending to (7) a properly-specified initial condition. The problem in this application is to find an initial condition with the correct technical properties that also has a reasonable interpretation within the model. Such a condition can be derived by noting that the state space of firms, $\Theta$, contains the one-dimensional subset $\Theta_0 = \{(0,t_2),\ t_2 \in [c,d]\}$. This particular set of firms with degenerate (one-dimensional) return distributions is exactly the set of firms treated by Ross in his proof of the existence of the signaling equilibrium. Suppose that the $\Theta_0$ class of firms is known to investors (i.e., at time 0 investors get a fiancial signal and a message that the signaling firm is a member of class $\Theta_0$.) Then it is easy to parallel Ross's construction of $F^*(0,t_2)$ in the model discussed above. The first order condition for the manager's optimal signal under compensation schedule (3) is given by (5) with $t_1 = 0$. For a firm of type $(0,t_2)$, the fulfilled expectations requirement becomes $V_0(F^*(0,t_2)) = (\frac{1}{R_0})k^-t_2$, which is strictly positive as long as $k^-$ is positive -- i.e., for $0 < \lambda < \sqrt{3}$. Now, following Ross [1977], Section 4, considering F as a function of $t_2$ alone, the ordinary differential equation $F'(t_2) = (c_0/2c_1L^*)t_2$ is derived, with $L^* \equiv L/k^*$, and $k^* \equiv k^-/R_0$. Thus the solution is given by:

$$F^*(0,t_2) = \frac{c_0}{4c_1L^*}[t_2^2 + \beta], \quad L^* \equiv L/k^* > 0 \tag{9}$$

with $\beta$ an undetermined constant.

Ross has demonstrated that (9) defines a one-parameter class of equilibrium signals. Equation (7) is to characterize the $F^*(\Theta)$ for firms throughout $\Theta$; as a consistency requirement, it is natural to demand that the solution to (7) reduce to (9) on the set $\Theta_0$. In fact, (9) can be shown to be a well-defined Cauchy initial condition (see John [1982]), which defines a unique solution to the P.D.E. (7) in a neighborhood of the locus of equilibrium-supporting firms. Thus, equations (7) and (9), along with the boundary condition $F(c,c) = 0$ yield a unique $F^*(\Theta)$:

$$F^*(t_1,t_2) = \frac{c_0}{4c_1L^*}[t_2^2 - t_1^2] \tag{10}$$

(Demonstration of this is left to the Appendix.)

REMARK: The "firm" (c,c) is, of course, just a riskless bond with value $V(c,c) = c/R_0$. The boundary condition for this problem is arbitrary[19]; the value adopted here sets the signal of the firm with a riskless return at zero. The locus of optimal signals for managers is depicted in Figure 1.2.

That $F^*(\theta)$ cannot be an equilibrium is abundantly clear from Figure 1.2. Note that a level curve of the "equilibrium" locus (10) has the form $E[X_\theta]\sqrt{\text{var } X_\theta} = \text{const.}$ -- the level curves of $F^*$ are hyperbolae, while the level curves of V are straight lines. As the straight line A'B' is traveled along the gradient vector of V (i.e., along the direction of fastest increase of V in θ), the "equilibrium" signal is following the monotonically decreasing segment AB. But the outsider investors associate a decreasing signal with <u>decreasing</u> firm value. Therefore, (10) cannot support consistent expectations about V in θ, even though it reduces to the equilibrium locus on Ross's set of firms.

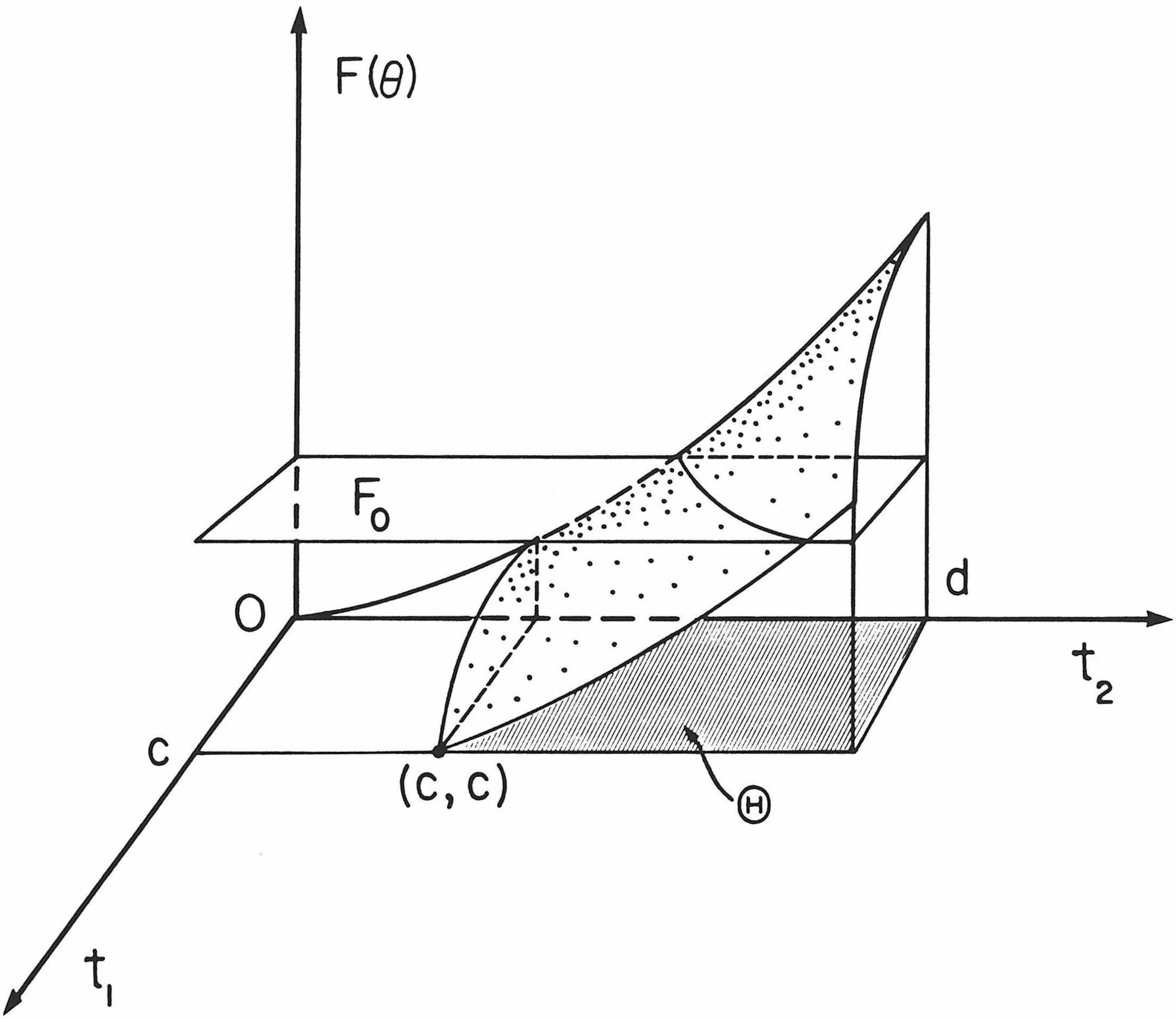



Figure 1.2 Signaling Equilibrium Breakdown for the Illustrative Example

## 1.5 CONCLUSIONS

This essay has offered a critical perspective on the existing agency theory of finance. A central objective of the agency theory is the derivation of contractual conditions under which non-management investors will lend a fraction of their wealth to firms in return for residual claims, while conceding to managers a proprietary interest in the "inside" information of the firm. To the extent that this research program bears fruit, and can be bolstered by empirical evidence, there seems likely a radical reformation of the prevailing climate of securities regulation. It has been contended here, however, that existing incentive contracting theory inadequately captures the informational subordination of outside investors to managers, a prominent result of contractual provisions aimed at minimizing transactions costs. The efficient raising of financial capital requires investors capable of correctly pricing the risky residual claims of firms -- but this is all they need do. Inside information critical to the firm's operations -- and profits -- must remain the property of the corporation, inevitably under the stewardship of the management team. But managers, able to appraise the probable effect of the firm's informational property in the securities markets, will see an incentive to attempt to trade on the information, or worse, to color the market's perception of the firm's future profitability.

Incentive contracting theory puts strict limits on the payoffs to such managerial behavior. The theory asserts that managers do not conduct themselves in a vacuum, but in an environment of incentive contracts where management's routine financial dealings constantly transmit information, allowing outsider investors to intuit the correct value of the firm's residual. This effectively short-circuits the manager's ability to sell any important fraction of the corporation's informational property for his own gain.

As this study has attempted to demonstrate, though, when the constraints on information transmission between management and investors are more properly taken into account, the supportability of incentive contracting (as it is presently understood) is thrown open to question. A single <u>condensed</u> information signal is insufficient to allow shareholders to form fulfilled expectations for the firm's market value, at least for the most often-cited class of agency contracts in the literature. Granted, it may simply be the case that the equilibrium failure investigated in this essay is a pathological example, a contrivance. If one particular type of incentive contract is not robust to the presence of condensed information, others may be, and these equilibrium-supporting contracts may yield testable propositions about agency relationships in financial markets. The emphasis of the investigation now turns, therefore, to the problem of characterizing the properties of candidate condensed information signaling mechanisms.

# FOOTNOTES FOR CHAPTER I

1. See, for example, Black [1976].

2. As Chandler notes, there did exist the British innovation of the joint- or incorporated stock company, dating from the sixteenth century and used primarily in the promotion of trade with far-flung colonies. Prior to the mid-nineteenth century, however, this organizational form was almost never observed in other areas of commerce. Chandler [1977], p. 16.

3. Ibid, p. 36.

4. ". . . the choice of agent had been for centuries one of the most important decisions a merchant had to make. Since loyalty and honesty were still more important than business acumen, even the more specialized merchants continued to prefer to have sons or sons-in-law, or men of long acquaintance, as partners or agents handling their business in a distant city." Ibid, p. 38.

5. Ibid, pp. 94-109.

6. Smith [1937], p. 699.

7. See Berle and Means [1932], Berle [1965], and Kaysen [1965]. An important methodological critique of Berle and Means, anticipating later developments in the agency theory of the firm, is De Alessi [1973].

8. And this is essentially where the orthodoxy leaves the problem. For example, Berle regards the stockholder's role in corporate affairs to have simply withered away -- "Desire to discover an 'owner-entrepreneurship' or 'risk-taking' function in stockholders is basically . . . an emotional desire to find some functional justification for having stockholders at all." See Berle op. cit., p. 37.

9. See Jensen and Meckling [1976], p. 331. The institutions of fully alienable rights to residuals, limited liability, and perpetual life of the underlying organization define the corporate enterprise. See Posner [1972], pp. 177-78.

10. Fama [1980], p. 291.

11. What is to be posed here is an agency model, if the term agency is used in its broadest sense. This is clearly not a model in the mold of Harris and Raviv [1979], in which the agent has a disutility of effort in taking some action. But to the extent that a principal structures a well-defined set of rules within

which agents must communicate and interact, and the outcome of this interaction is the division of some economic prize, an agency problem may be said to exist.

12. Stiglitz (see Stiglitz [1974]) has established a theorem generalizing the famous "irrelevancy proposition" of Miller and Modigliani [1958]. In a multi-period model, Stiglitz demonstrated that, given a general equilibrium solution for bond prices and values of firms following the announcement of a particular financial policy, there exists a second general equilibrium solution wherein some or all of the firms have changed financial policies, and investors have made offsetting portfolio adjustments which leave the values of firms unchanged. Three critical assumptions were made in the proof, leaving the implication that if any of the three were sufficiently weakened, a positive role for "pure" finance might be revealed. These basic assumptions were, first, equal-cost access to the frictionless capital markets by firms and individuals; second, no bankruptcy, reorganization, or other brokerage costs; and third, that investor expectations about a firm's profitability remain invariant to the firm's announcement of a financial policy. The incentive contracting approaches to a theory of optimal financial structure result from discarding the third of Stiglitz's assumptions.

13. The characteristic alluded to is the monotone likelihood ratio property, which, in the one-parameter incentive signaling models, helps to insure the monotonicity of the equilibrium signal in the firm's state parameter. See, for example, Bhattacharya [1979], p. 263.

14. Scott [1980], especially Section I, is a cogent summary of the controversy in the field of disclosure regulation. The legal status of a property right in information itself -- "intellectual" property -- is unclear (see, for example, Cheung [1982].) Viewed from a legal perspective, the issue raised by condensed information incentive contracting is whether shareholders can safeguard their property right to the firm's residual returns while allocating the rights in "material" information to the corporation via management.

15. Condensation of signalled information is not confined to the financial incentive signaling models. It occurs in any incentive contracting relationship where eliciting the state is costly. For example, in Spence's job market models, the employer is interested in a particular applicant's productivity in a given job, not in an exhaustive account of all the accrued knowledge, resources, and related task experience that in fact determine the applicant's productivity. The employer acquires a suitably cheap, manageable condensation of the applicant's state (in the

form of an academic or vocational education record, for example), and if the employment contract is structured according to the signaling paradigm, the applicant "assigns himself" the correct job -- but the employer never knows the applicant's state.

16. Green imposes on agents a decision rule (a smooth mapping from a set of states S to actions); in turn, a reward function maps agent actions to a set of transfers, T. For a given state s, then, a decision rule is strictly supportable if there exists a reward function r such that the decision rule specifies the unique action $a^*$ which maximizes agent utility $u(s,r(a^*))$ at s.

17. Bhattacharya [1980] notes that because shareholders are not subject to the same cost structure as managers in Ross's model, there may exist adverse incentives for shareholders to attempt to influence the signaling process.

18. A standard reference on the underlying assumptions of CAPM is Rubenstein [1973]. A more detailed examination of the relationship between the investor's and the manager's optimization problem appears in Chapter 2 of this thesis.

19. As can be seen from equation (9) and Figure 1.2, the boundary condition given is such that only one firm [the firm (c,c)] signals zero. By the implicit function theorem, the inverse

image of zero under the signaling map is in general a rectifiable curve. This, taken together with the non-negativity constraint on signals implies that if some other firm, (0,c) perhaps, is set to signal zero, then there will exist a set of positive measure in $\mathbb{R}^2$ of zero-signaling firms.

## REFERENCES FOR CHAPTER I

Akerlof, G., "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," Quarterly Journal of Economics 84 (1970), 488-500.

Alchian, A., and Demsetz, H., "Production, Information Costs, and Economic Organization," 62 American Economic Review 5 (1972), 777-792.

Arrow, K., "The Role of Securities in the Optimal Allocation of Risk Bearing," Review of Economic Studies 31 (1964), 91-96.

Berle, A., "The Impact of the Corporation on Classical Economic Theory," 79 Quarterly Journal of Economics 1 (1965), 25-40.

_________, and Means, G. The Modern Corporation and Private Property (New York: Macmillan Publishing Co., 1932).

Bhattacharya, S., "Imperfect Information, Dividend Policy, and the 'Bird in the Hand' Fallacy," Bell Journal of Economics 10 (1979), 259-270.

________________, "Nondissipative Signaling Structures and Dividend Policy," Quarterly Journal of Economics 95 (Aug. 1980), 1-24.

Black, F., "The Dividend Puzzle," 2 Journal of Portfolio Management (Winter 1976).

Chandler, A., The Visible Hand (Cambridge, MA: Harvard University Press, 1977).

Cheung, S., "Property Rights in Trade Secrets," Economic Inquiry 20 (Jan. 1982), 40-53.

Coase, R., "The Nature of the Firm," 4 (n.s.) Economica (1937), 386-405.

De Alessi, L., "Private Property and Dispersion of Ownership in Large Corporations," 28 Journal of Finance (1973), 839-851.

Debreu, G., Theory of Value (New York: Wiley, 1959).

Fama, E., "Agency Problems and the Theory of the Firm," 88 Journal of Political Economy 2 (1980), 288-307.

Green, E., "Decentralizability of Truthful Revelation and Other Decision Rules," Social Science Working Paper #419, California Institute of Technology, June 1982.

Harris, M., and Raviv, A., "Optimal Incentive Contracts with Imperfect Information," 20 Journal of Economic Theory (1979), 231-259.

Heinkle, R., "A Theory of Capital Structure Relevance Under Imperfect Information," 37 Journal of Finance 5 (1982), 1141-1150.

Jensen, M., and Meckling, W., "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure," 3 Journal of Financial Economics (1976), 305-360.

John, F. Partial Differential Equations, 4th ed., (New York: Springer-Verlag, 1982).

Jordan, J., and Radner, R., "Rational Expectations in Microeconomic Models: An Overview," 26 Journal of Economic Theory (1982), 201-223.

Kaysen, C., "Another View of Corporate Capitalism," 79 Quarterly Journal of Economics 1 (1965), 41-51.

Leland, H., and Pyle, D., "Informational Asymmetries, Financial Structure, and Financial Intermediation," 32 Journal of Finance 2 (1977), 371-387.

Modigliani, F., and Miller, M., "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review 48 (June 1958), 261-297.

Posner, R., Economic Analysis of Law (Boston: Little, Brown and Co., 1972).

Radner, R., "Competitive Equilibrium Under Uncertainty," Econometrica 36 (1968), 31-58.

Ross, S. A., "The Determination of Financial Structure: The Incentive

Signaling Approach," Bell Journal of Economics 8 (Spring 1977), 23-40.

_________, "Some Notes on Financial Incentive Signaling Models, Activity Choice and Risk Preferences," Journal of Finance 33 (June 1978), 777-792.

Rubenstein, M., "A Mean-Variance Synthesis of Corporate Financial Theory," Journal of Finance 28 (March 1973).

Scott, K., "Insider Trading Rule 10b-5, Disclosure and Corporate Privacy," 9 Journal of Legal Studies 4 (1980), 801-818.

Smith, A., The Wealth of Nations (New York: Random House, 1937).

Spence, A., Market Signaling: Information Transfer in Hiring and Related Screening Processes (Cambridge, MA: Harvard U. Press, 1974).

Stiglitz, J., "On the Irrelevance of Corporate Financial Policy," 64 American Economic Review 6 (1974), 851-866.

Thakor, A., "An Exploration of Competitive Signaling Equilibria with 'Third Party' Information Production: The Case of Debt Insurance," 37 Journal of Finance 3, (1982) 717-739.

# CHAPTER II

# FINANCIAL INCENTIVE MECHANISM DESIGN WITH "CONDENSED" INFORMATION: CHARACTERIZATION THEOREMS

## 2.1 INTRODUCTION

The preceding essay has shown that the reservation by insider-managers of information proprietary to the firm may have a strong disruptive effect on the existence of the incentive contracting equilibria which have been advanced as explanations of determinate financial policies for firms. A demonstration of the non-existence of equilibria, however, has so far only been provided for a single concrete and highly stylized example drawn from the existing literature. This essay broadens the field of investigation; the sustainability of incentive signaling equilibria based upon the firm's choice of financial structure (as in Ross [1977]) will be examined for a wide class of candidate incentive mechanisms or contracts.

The models previously considered assumed (largely for formal convenience) that the state space of firms was a subset of $\mathbb{R}^2$. Here it will be assumed that the state of a firm lies within some bounded, measurable space of two dimensions. For example, a firm's state at time 0 may be represented as a particular endowment of a production technology along with a marketing strategy for the product. The joint specification of the state parameters, along with the distribution function for the firm's period 1 returns, will imply some realization of net operating income for the firm at time 1. The state information remains proprietary to the firm (specifically, to management) presumably because an exhaustive revelation to the market of the firm's technology or its marketing plans would jeopardize the realization of the firm's returns. Instead, the efficiency of risk bearing and financial capital formation via the securities markets is to be supported by "condensed" information signals emanating from the routine financial decisions of management. Outsider investors and management enter into incentive contracts which enable outsiders to rely on financial structure as a signal of the firm's expected value.

The demonstration of the non-existence of incentive contracting equilibria undertaken here will differ importantly from the strategy adopted in the previous chapter. Rather than attempting to derive an "optimal" signaling schedule $F^*$ and thence reasoning to an internal contradiction within the equilibrium contracting structure, a necessary condition for the stability of the incentive contracting equilibrium in a neighborhood of an arbitrary state $\theta \varepsilon \Theta$ will be found, for an arbitrary incentive mechanism. It will then be shown that if the incentive mechanism is to be non-trivial, that is, if it is to support a separating signaling equilibrium, severe restrictions may have to be placed upon the underlying preferences of outsider investors for returns on risky assets, for a given distribution of firms' returns. Thus, no incentive mechanism having the properties attributed by presently-existing signaling theories is likely to sustain informational equilibria in the presence of condensed information, for arbitrary preferences and return distributions.

## 2.2 A MORE GENERAL MODEL

This section defines a formal model within which it will be possible to investigate the supportability of condensed information signaling over a wide class of two-period incentive contracts. Managers bound by these contracts are assumed to know their firm's position in some measurable, two-dimensional space of states; financial signals released by managers are analyzed by outsider investors for clues about firms' (scalar) risk-adjusted values.

To make a start, some notation is established. Let $\Omega \subset \mathbb{R}^1$ be a bounded, non-empty set on which probability-measurable events (realizations of a random variable $\tilde{X}$) will be defined. Let $\Phi$, a bounded subset of a 2-dimensional space with elements $(\varphi_1, \varphi_2)$, denote the state space. Then a set of two-parameter cumulative distribution functions $\mathbb{H}: \Phi \rightarrow \mathbb{P}_X$ can be defined, where $\mathbb{P}_X$ is the set of all probability measures for the random variable $\tilde{X}$ over $\Omega$.[1] Assume that for any particular element $H(x;\varphi)$ of $\mathbb{H}$, the following exist:

$$\theta_1(\varphi) \equiv \int_\Omega x dH(x;\varphi)$$

$$\theta_2(\varphi) \equiv [\int_\Omega (x - \theta_1)^2 dH(x;\varphi)]^{1/2}, \; 0 < \theta_2 < +\infty$$

(where, of course, $dH(x;\varphi)$ is shorthand for $h(x;\varphi)dx$ ). The probability distributions $H(\cdot)$ can (locally) be written as distributions with parameters $\theta = (\theta_1, \theta_2)$ if the following holds in a neighborhood of $\theta$:

$$\det J_\varphi(\theta) = \det\left(\frac{\partial\varphi_i(\theta)}{\partial\theta_j}\right) \neq 0 \quad \forall\, \theta \in \Theta;\; i,j = \{1,2\}.$$

where $\Theta$ is the equivalent state space derived from $\Phi$ -- note that since $\Phi$ is bounded, and the distributions $\mathbb{H}$ have finite mean and variance, $\Theta$ is also bounded. Denote the transformed distributions by $\Phi(x;\theta)$. Many of the most interesting 2-parameter distributions (such as the uniform and, trivially, the normal) can be transformed from their original parameters to a representation in terms of mean and standard deviation. Such a representation becomes particularly useful in the present model, where a notion of stability between mappings in the state space must be unified with an asset valuation function expressed in terms of mean and variance. Exactly how this comes about must now be discussed in some detail.

The Investor's Problem

Investors are assumed to have preferences over (risky) streams of future wealth, described by utility functions $u_i(\tilde{W}_i)$ for each investor i. At period 0, investors choose, according to their risk preferences, from a portfolio of risky residual claims on the return streams of firms, and a set of riskless bonds. In general, all investors hold both risky and riskless securities -- that is, the securities market has a separation property.

In contrast to the usual maintained assumption of asset pricing models, however, shareholders do not possess at period 0 (nor can they infer from market prices) a full description of the random variables that determine outcomes of period-1 wealth. This market imperfection stems from the existence of private information about the particulars of the firms' return streams, which, as a kind of "business property," remains vested in the hands of management. Shareholders, the risk-bearers and providers of capital, seek to infer the true period-1 risk-adjusted value of equities from the financial signals released by managers at period 0. Upon receipt of the signals (which take the form of the firm's choice of debt financing,) shareholders associate a period-1 value with the residual claims on the signaling firm. The actions of managers, the equilibrium values of firms, and the values of firms perceived by shareholders are in turn linked by an incentive compensation contract, the equilibrium properties of which are to be investigated in this essay.

Suppose, then, that investor i's period 0 disposable wealth is $W_{0i}$, his money value holding of riskless bonds is $F_i$, and his holding of firm θ's risky securities has value $S_{\theta i}$. Then his period 0 objective is:

$$\max_{F_i, S_{\theta i}} E_X[u_i(\tilde{W}_i)]$$

$$\text{s.t. } F_i + \sum_\theta S_{\theta i} = W_{0i}$$

where $\tilde{W}_i = R_0 F_i + \sum_\theta \tilde{R}_\theta S_{\theta i}$, and $R_j = 1 + r_j$, the one-period interest factor for security j. The conditions under which this problem can be solved to yield an expression for the market risk premium $E_X[\tilde{R}_\theta - R_0]$ for security θ are well known;[2] it must be shown that essentially the same structure can be applied in an imperfect markets setting with condensed information signaling.

As indicated in the previous chapter,[3] the firms' mode of finance has the potential to act as an informative signal if changes in finance can alter shareholders' perceptions of the firm's market value. The other standard assumptions of asset pricing models -- no restriction on short sales, equal-cost borrowing by firms and individuals, and absence of bankruptcy costs -- will be maintained. An immediate consequence of these assumptions is that the true market value of the firm, V(θ), is <u>independent</u> of the financial signal, the firm's debt obligation. The signal underlies the investor's expectation of V (i.e., his mapping $V_0\colon \sum \rightarrow \Delta$) but does not in itself constrain the investor's feasible set of portfolio choices. Suppose, for example, that a firm changed its financial structure in a way the market found uninformative (the firm's actual value stayed constant.) Then shareholders who so wished could refinance the firm on their own accounts to restore their preferred portfolios without altering the firm's value. If, on the other hand, increasing the debt obligation of the firm was systematically related to deadweight bankruptcy costs, the firm's value would in general be expressed as $V(\theta,F)$. Therefore, the frictionless market assumptions imply a class of non-dissipative incentive signaling contracts with "non-productive" signals.[4]

The other major assumption needed to reconcile the condensed information signaling approach with asset pricing theory concerns the dimension of the state space. A general expression for the market risk premium accorded the equity of firm $\theta$ is, in the notation developed here:

$$E_X[\tilde{R}_\theta - R_0] = \lambda(\theta, \Pi_\theta) \qquad (1)$$

where $\lambda$ is the "market price of risk," a function of $\theta$ and a random variable $\tilde{\Pi}_\theta$ that depends on the joint distribution of $\tilde{X}_\theta$ and all of the other risky assets. Thus, expressing (1) in terms of the state space $\theta$ alone requires an avowedly partial equilibrium framework. The incentive contract characterization theorem to be presented in this essay investigates the local stability of incentive signaling in the neighborhood of a state $\theta \varepsilon \Theta$. A shareholder receives a signal $F(\theta)$ from the firm and updates his expectation of the firm's value, holding the rest of the portfolio constant. If, for firms in a neighborhood of $\theta$, $\lambda$ changes more rapidly with the firm's mean and own-variance of returns than with the change in the market portfolio, (1) can be approximated by:[5]

$$E_X[\tilde{R}_\theta - R_0] = \lambda(\theta) \qquad (1')$$

Of course, such an approximation cannot be globally extended. Once again, though, the object of the present study is to examine the threshold conditions for the supportability of condensed information signaling. The larger the degree of condensation of information, holding the dimension of the signal space constant, the less likely equilibria may become.

Provided that investors assign risk premia to the returns of firms in accord with (1'), the equilibrium value for the firm of state $\theta$ can be derived from the definition of the rate of return on equity,[6] $\tilde{R}_\theta = (\tilde{X}_\theta - R_0F^*)/S_\theta$; taking the expectation,

$$\lambda(\theta) + R_0S_\theta = E[\tilde{X}_\theta] - R_0F^*$$

where $S_\theta$ has been absorbed into $\lambda$. Now, defining $V_\theta = F^*_\theta + S_\theta$ as before,

$$V(\theta) = \frac{1}{R_0}[\theta_1 - \lambda(\theta)] \qquad (2)$$

The properties of $\lambda(\theta)$ -- and therefore, of $V(\theta)$ -- are apparent from the results of capital asset theory. In the more usual notation, the market price of risk is expressed as:

$$\lambda(\theta) = \frac{\rho(\tilde{X}_\theta, \tilde{R}_M)\theta_2}{\sqrt{\text{var}\tilde{R}_M}}[E_X(\tilde{R}_M - R_0)]$$

where $\tilde{R}_M$ is the return on the market portfolio. Clearly, then, the effect of a change in the mean of a firm's returns, holding variance constant, is zero,[7] while for any security having nonzero correlation with the market portfolio, $\frac{\partial\lambda(\theta)}{\partial\theta_2} \neq 0$. Therefore, as would be expected, the equilibrium value of the firm (equation (2)) rises with increasing return mean, and changes (with varying sign)[8] as the variance of returns changes.

As Rubenstein has pointed out, the expression for $\lambda(\theta)$ has a direct relation to the risk preferences of investors; hence the term "market price of risk." To see this, note that the general expression for the equilibrium expected return on an arbitrary, nonempty portfolio p of securities, subject to the previously-mentioned conditions, is $E(\tilde{R}_p) - R_0 = (\theta/I)\rho(\tilde{R}_p, \tilde{W}_M)\sqrt{\text{var}\tilde{R}_p}\sqrt{\text{var}\tilde{W}_M}$ where I is the number of (identical) investors, $\tilde{W}_M$ is the future value of all securities, the "wealth" of the market, and $\theta \equiv -E[u''(\tilde{W})]/E[u'(\tilde{W})]$, the Arrow-Pratt measure of individual risk aversion. Now let p = M, the entire market portfolio. Then the above becomes

$$\lambda^* = \frac{E(\tilde{R}_M) - R_0}{\sqrt{\text{var}\tilde{R}_M}} = (\theta/I)\sqrt{\text{var}\tilde{W}_M}$$

and the expression for $\lambda(\theta)$ can be rewritten as

$$\lambda(\theta) = \lambda^* \rho(\tilde{X}_\theta, \tilde{R}_M)\theta_2$$

in a way which makes clear the dependence of $\lambda(\theta)$ on the risk preference of individuals, given a specification of the state space $\theta$. The direction the subsequent inquiry will take will be to investigate whether the local necessary conditions for a nontrivial incentive signaling mechanism may imply restrictions on $\lambda(\theta)$.

## The Manager's Problem on a Class of Incentive Contracts

The wealth-maximizing manager's task is relatively simple: select the mode of finance for the firm that maximizes his incentive compensation, given the terms of his incentive contract and his knowledge of the firm's state.[9] The issue at hand is whether an incentive contract can be found that elicits an equilibrium-supporting condensed information signal. The terms of a prospective contract are constrained by the timing of information flow to the principal. At period 0, all that is observable by the shareholders is the financial signal; shareholders derive a period 0 expectation of the firm's value $V_0\colon \sum \rightarrow \Delta$, and the securities market clears. At time one, the realization of each firm's cash flow occurs, and investors can compare their perception of a firm's value with the value justified by the firm's earnings. If the manager's compensation package is tied to the market value of the firm, the manager will in general receive incentive pay in both periods. A broad class of two-period incentive contracts might then be characterized:

Assumption 1. Let $\mathbb{M}$ denote the class of two-period incentive mechanisms with elements $M: \sum \times \Omega \rightarrow \mathbb{R}^1$ such that:

$$M(\sigma,x) = c_0V_0(\sigma) + c_1C(\sigma,x)$$

where $\sigma$ is the firm's financial signal, $c_0$ and $c_1$ are constants, and $C: \sum \times \Omega \rightarrow \mathbb{R}^1$ is a bounded, continuously differentiable function of its arguments.

Thus, a relatively "smooth" class of incentive mechanisms has been assumed. This in turn implies the presence of a reliable technology for monitoring realizations of $\tilde{X}_\theta$ at period 1. The nonexistence of such a technology brings a new set of problems;[10] note, however, that the class of incentive functions $\mathbb{M}$ can be extended -- at the cost of some formal complexity -- to a situation in which X can be resolved only over discrete intervals of $\Omega$.

The incentive contracts $\mathbb{M}$ are feasible for the principal, since they nowhere depend on $\theta$. The period-1 reward function $C(\sigma,x)$ is to be interpreted as the incentive signaling cost function. The incentive portion derives from the assumed property that, for $\theta$

resulting in higher values of $V(\theta)$, $\frac{\partial C(\sigma,x)}{\partial x} \geq 0$, with the strict inequality for some subset of $\Omega$ of positive probability measure. In other words, managers who correctly report higher expected cash flows should receive enhanced compensation. On the other hand, it will also be found, in keeping with previous signaling models, that $\frac{\partial C(\sigma,x)}{\partial \sigma} < 0$; for a fixed level of expected cash flow, it is increasingly expensive for a manager to increase his financial signal. This is because the signal is a fixed obligation on the firm -- a set of debt claims payable at period 1 -- and therefore, if the manager increases the firm's debt obligation unduly, he increases the probability of the firm's bankruptcy at period 1. Clearly, shareholders will want to discipline this sort of managerial behavior -- the firm is being "stolen" by the bondholders. These properties of $C(\sigma,x)$ will be seen to be consistent with the following:

Assumption 2 $V_0'(\sigma) > 0$.

If $V_0$ is many-to-one, but $\partial C/\partial \sigma < 0$ for some $X_\theta$, the manager will select the minimum signal $\sigma$ consistent with his firm's equilibrium value; to do otherwise exposes him to unnecessary bankruptcy risk. Given that, investors are naturally led to associate increasing financial leverage with increasing value.

## 2.3 CHARACTERIZATION THEOREMS

Continuing to follow most of the existing literature, it will be assumed that managers are risk-neutral compensation maximizers. A typical contract $M \varepsilon \mathbb{M}$ will then enter the manager's objective function in the form:

$$m(\sigma,\theta) = E_X[M(\sigma,\tilde{X}_\theta)] = c_0 V_0(\sigma) + c_1 \int_\Omega C(\sigma,x)\phi(x;\theta)dx \qquad (3)$$

where $\phi(x;\theta)$ is the p.d.f. corresponding to $\Phi(x;\theta)$.

Then we have:

<u>Theorem 2.1</u> If the firm's period 1 cash flow $\tilde{X}_\theta$ is distributed according to the continuously differentiable c.d.f. $\Phi(x;\theta)$ for each $\theta \varepsilon \Theta$, then a necessary condition for the signal $\sigma = F(\theta)$ given by a compensation-maximizing manager constrained by (3) to be locally consistent with fulfilled investor expectations is:

$$\int_\Omega \left(\frac{\partial C(\sigma,x)}{\partial x}\right)\left[\frac{\partial \lambda(\theta)}{\partial \theta_2}\left(\frac{\partial \Phi(x;\theta)}{\partial \theta_1}\right) + \frac{\partial \Phi(x;\theta)}{\partial \theta_2}\right]dx = 0.$$

for all $\theta \varepsilon \Theta$.

<u>Proof</u> The first order condition for a manager of a firm of type $\theta$ to maximize his incentive compensation under contract (3) is:

$$\left(\frac{\partial m(\sigma;\theta)}{\partial \sigma}\right) F'(\theta) + \frac{\partial m(\sigma;\theta)}{\partial \theta} = 0 \qquad (4)$$

Here $\frac{\partial m}{\partial \sigma}$ is a scalar; $F'(\theta)$ and $\frac{\partial m}{\partial \theta}$ are $1 \times 2$ vectors with components

$\frac{\partial F(\theta)}{\partial \theta_i}$ and $\frac{\partial m(\theta)}{\partial \theta_i}$, respectively. In addition to satisfying (4), the manager's signal $\sigma = F(\theta)$ must support a fulfilled expectations equilibrium for investors, summarized by the identity $V_0(\sigma) \equiv V(\theta)$ for all $\theta \in \Theta$. Differentiating this identity yields another expression for $F'(\theta)$ which must hold in a neighborhood of $\theta$:

$$F'(\theta) = \frac{1}{V_0'(\sigma)} \frac{dV}{d\theta}$$

since, by Assumption 2, $V_0'(\sigma) \neq 0$. Thus (4) becomes:

$$\frac{1}{V_0'(\sigma)} \frac{dV}{d\theta} \left( \frac{\partial m(\sigma;\theta)}{\partial \sigma} \right) + \frac{\partial m(\sigma;\theta)}{\partial \theta} = 0 \tag{5}$$

Now each term of (5) must be calculated. First, it is found that (from (2)), the components of $dV/d\theta$ are:

$$\frac{\partial V(\theta)}{\partial \theta_1} = \frac{1}{R_0} > 0 \tag{6}$$

and

$$\frac{\partial V(\theta)}{\partial \theta_2} = - \frac{1}{R_0} \frac{\partial \lambda(\theta)}{\partial \theta_2} \neq 0$$

then,

$$\frac{\partial m(\sigma,\theta)}{\partial \sigma} = c_0 V_0'(\sigma) + c_1 \int_\Omega \left( \frac{\partial C(x,\sigma)}{\partial \sigma} \right) \phi(x;\theta) dx \tag{7}$$

while:

$$\frac{\partial m(\sigma,\theta)}{\partial \theta_i} = c_1 \int_\Omega C(\sigma,x) \left( \frac{\partial \phi(x;\theta)}{\partial \theta_i} \right) dx$$

Now the expressions for $\frac{\partial m(\theta)}{\partial \theta_i}$ can be integrated by parts once, noting that:

$$\int_0^x \frac{\partial \phi(u;\theta)}{\partial \theta_i} du = \frac{\partial}{\partial \theta_i} \int_0^x \phi(u;\theta) du = \frac{\partial \Phi(x;\theta)}{\partial \theta_i}$$

one readily obtains:

$$\frac{\partial m(\sigma,\theta)}{\partial \theta_i} = -c_1 \int_\Omega \frac{\partial C(\sigma,x)}{\partial x} \left( \frac{\partial \Phi(x;\theta)}{\partial \theta_i} \right) dx \qquad (8)$$

and so the first-order condition (5) becomes, for $\theta_1$:

$$\frac{[c_0 V_0'(\sigma) + c_1 \int_\Omega \frac{\partial C(\sigma,x)}{\partial \sigma} \phi(x;\theta) dx]}{c_1 R_0 V_0'(\sigma)}$$

$$= \int_\Omega \frac{\partial C(\sigma,x)}{\partial x} \left( \frac{\partial \Phi(x;\theta)}{\partial \theta_1} \right) dx \qquad (9)$$

and for $\theta_2$:

$$\frac{-(\partial \lambda(\theta)/\partial \theta_2) \left[ \frac{\partial m(\sigma;\theta)}{\partial \sigma} \right]}{c_1 R_0 V_0'(\sigma)} = \int_\Omega \frac{\partial C(\sigma,x)}{\partial x} \left( \frac{\partial \Phi(x;\theta)}{\partial \theta_2} \right) dx \qquad (10)$$

Note that, if the derivatives of $V(\theta)$ are cleared to the right hand side, the left hand sides of (9) and (10) are identical. Thus, adding (9) and (10), and bringing the functions of $\theta$ inside the integral, the necessary condition becomes:

$$\int_\Omega \left( \frac{\partial C(\sigma,x)}{\partial x} \right) \left[ \frac{\partial \lambda(\theta)}{\partial \theta_2} \left( \frac{\partial \Phi(x;\theta)}{\partial \theta_1} \right) + \frac{\partial \Phi(x;\theta)}{\partial \theta_2} \right] dx = 0 \qquad (11)$$

$\square$.

Necessary condition (11) has been derived, not to characterize the optimal managerial signal $F^*$ for a given incentive function, but to test for joint restrictions on the C, V and $\Phi$ functions which are necessary to support an equilibrium incentive structure. Evidence that these restrictions both in general exist and may be quite severe is provided by the following:

Theorem 2.2 If necessary condition (11) is to hold on a local neighborhood of an arbitrary state $\theta \varepsilon \Theta$, then there exists a family of return distributions $\Phi(x,\theta)$ such that a necessary condition for $\frac{\partial C(\sigma,x)}{\partial x} \neq 0$ almost everywhere is $\left|\frac{\partial V}{\partial \theta_2}\right| < \frac{\sqrt{3}}{R_0}$.

Proof: Consider the uniform distribution with parameters (a,b), and cumulative distribution function:

$$H(x;a,b) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

Along with the familiar moments (in state space notation),

$$\theta_1 = \frac{a+b}{2} \qquad \theta_2 = \frac{b-a}{2\sqrt{3}}$$

Since $H \varepsilon \mathbb{H}$, an equivalent (local) representation in terms of $\theta$ can be made; the above immediately give:

$$a = \theta_1 - \sqrt{3}\,\theta_2 \qquad b = \theta_1 + \sqrt{3}\,\theta_2$$

and,

$$\Phi(x;\theta) = \frac{x - (\theta_1 - \sqrt{3}\,\theta_2)}{2\sqrt{3}\,\theta_2} \qquad a \lesssim x \lesssim b.$$

A straightforward calculation now yields:

$$\frac{\partial\Phi(x;\theta)}{\partial\theta_1} = -\frac{1}{2\sqrt{3}\,\theta_2}$$

$$\frac{\partial\Phi(x;\theta)}{\partial\theta_2} = \frac{\theta_1 - x}{2\sqrt{3}\,\theta_2^2}$$

observe that (11) will yield a trivial incentive mechanism, with $\frac{\partial C(\sigma,x)}{\partial x} = 0$ except on sets of probability measure zero in $\Omega$, if the expression in large brackets within the integral is either positive or negative for all $x \varepsilon \Omega$, in the neighborhood of some arbitrary $\theta \varepsilon \Theta$. In order to derive necessary conditions for non-trivial $C(\sigma,x)$, the conditions giving positive or negative values of the term in brackets are derived and jointly negated. The required conditions are to be derived from (by (11)):

$$\frac{1}{R_0}\left[\frac{\theta_1 - x}{2\sqrt{3}\,\theta_2^2}\right] - \frac{1}{2\sqrt{3}\,\theta_2}\left(\frac{\partial\lambda(\theta)}{\partial\theta_2}\right) \gtrless 0 \quad \forall x \varepsilon \Omega. \qquad (12)$$

(i) Suppose that in the neighborhood of some $\theta \varepsilon \Theta$, $\frac{\partial\lambda(\theta)}{\partial\theta_2} > 0$. Then the second term of (12) is negative. The first term is positive at $x = a$ and monotonically decreases with $x$, becoming negative for $x > \theta_1$. Therefore if:

$$\frac{1}{R_0}\left[\sup_{x\varepsilon\Omega} \frac{\theta_1 - x}{2\sqrt{3}\ \theta_2^2}\right] - \frac{1}{2\sqrt{3}\ \theta_2}\left(\frac{\partial\lambda(\theta)}{\partial\theta_2}\right) < 0 \tag{13}$$

then (12) is negative for all $x \varepsilon \Omega$. Evaluating the supremum and simplifying gives:

$$\frac{\partial\lambda(\theta)}{\partial\theta_2} = \frac{\partial V(\theta)}{\partial\theta_2} > \frac{\sqrt{3}}{R_0} \tag{14}$$

(ii) For a neighborhood of $\theta \varepsilon \Theta$, assume $\frac{\partial\lambda(\theta)}{\partial\theta_2} < 0$. Then (12) is positive for $x \varepsilon \Omega$ if:

$$\frac{1}{R_0}\left[\inf_{x\varepsilon\Omega} \frac{\theta_1 - x}{2\sqrt{3}\ \theta_2^2}\right] - \frac{1}{2\sqrt{3}\ \theta_2}\left(\frac{\partial\lambda(\theta)}{\partial\theta_2}\right) > 0$$

or

$$- \frac{\partial V(\theta)}{\partial\theta_2} > \frac{\sqrt{3}}{R_0} \tag{15}$$

Thus, the negation of (14) and (15) imply the restriction:

$$\left|\frac{\partial V(\theta)}{\partial\theta_2}\right| < \frac{\sqrt{3}}{R_0}$$

□.

Theorems 2.1 and 2.2 are in the form of an "impossibility theorem by example." Notice that the restriction given in Theorem 2.2 is not a sufficient condition for (11) to be satisfied with $\partial C/\partial x \neq 0$ for the uniform distribution. The restrictions merely guarantee that $\partial C/\partial x \equiv 0$ is not forced in order for (11) to hold. It is known that the equilibrium market price of risk in the partial equilbrium framework assumed here is of the form $\lambda(\theta) = \lambda^{*}\rho(\tilde{X}_{\theta}, \tilde{R}_{M})\theta_{2}$. If the necessary condition for supporting the incentive signaling equilibrium with a nontrivial $C(\sigma,x)$ is to hold in a neighborhood of some state $\theta$, for uniformly-distributed security returns, $\lambda^{*}\rho(\tilde{X}_{\theta}, \tilde{R}_{M})$ must be restricted to a fixed, bounded interval around $\theta$, independent of $\theta$. If the state space of firms is specified, satisfying the restriction will in general place a constraint on the allowable range of risk preferences of investors (their utility functions are, of course, already limited to quadratic form). On the other hand, if coefficients of risk aversion are unrestricted, one cannot guarantee that any nontrivial $C(\sigma,x)$ will support an informational equilibrium at every $\theta \in \Theta$. These restrictions must be viewed as extremely troublesome, in view of how little they actually guarantee about the existence of a nontrivial function $C(\sigma,x)$.

## 2.4 CONCLUSIONS

An important part of the program of theoretical finance seeks to establish the relationship between the reservation of inside or proprietary information by management, the efficiency of risk-bearing, and the nature of agency relationships between management and the suppliers of financial capital. It seems clear that a central characteristic of efficient risk bearing is the capacity of investors to distinguish the value of risky claims without the necessity of duplicating the management function. But by the same token, not all of the inside information possessed by the firm can be divulged to the market without destroying its proprietary value to the firm.

This essay has provided additional support for the assertion that received incentive contracting theory may be inadequate to the task of accommodating these divergent properties of capital markets. Elaborating on the suggestive example of the previous essay, it has been found that even the weakest local conditions consistent with the supportability of condensed information signaling in general may require unpalatable restrictions on the state spaces of firms or the risk preferences of individuals.

While this preliminary investigation seems to indicate that the prospects for robust incentive contracting equilibria are quite bleak, the notion of condensed information signaling may in fact provide fresh directions for the incentive contracting literature. Two paths of inquiry appear particularly important; both are concerned with the properties of markets in which inside information may coexist with a suitable notion of efficient asset price equilibrium.

First, the model can be respecified so that <u>ex post</u> monitoring of realizations of the firm's net operating income is costly. By paying a monitoring cost on a fixed schedule known to all market participants, investors select the "fineness" of the partition of the event space over which they can distinguish outcomes. Adoption of monitoring technology by outsiders would be a new object of equilibrium. Because outcomes of random variables would no longer be perfectly distinguishable, it would be necessary to examine epsilon-equilibria in firm valuations, where presumably, $\varepsilon = \varepsilon(c, \theta)$. One object of the model would be the derivation of the equilibrium bounds on $\varepsilon(c, \theta)$, which would scale the efficiency loss of risk-bearing in the presence of condensed information transmission and imperfect monitoring. Note that the restriction $\partial C/\partial x\big|_{\Omega_i} = 0$ on the element $\Omega_i$ of the partition of $\Omega$ is now consistent with the principal's inability to distinguish outcomes within the selected fineness of the partition. For an equilibrium-supporting incentive schedule, the investor's tradeoff cuts between draining off too much of the equity value of the firm in the form of monitoring costs versus giving managers too much leeway to misrepresent the value of the firm.

The second line of attack would involve attempting to restore incentive equilibrium by increasing the number of (condensed) information signals transmitted to the market. A difficulty with agency models of the firm based on equilibrium signaling is the relative paucity of testable propositions that can be derived. The small empirical literature on signaling (see, for example, Downes and Heinkle [1982],) is hard-pressed to identify patterns of dividend policy or insider equity ownership that might be consistent with an underlying incentive contract structure, and are, in addition, distinguishable from patterns predicted by competing theories. A model based on multiple condensed-information signals may yield sharper and more directly testable hypotheses. The present study has established, informally speaking, that difficulties arise in supporting informational equilibria with feasible, non-trivial incentive functions when the agent's inside information is "richer" (in the sense of dimension) than the information the principal seeks to process. The key to restoring a fulfilled-expectations equilibrium, then, may lie in increasing the number of (condensed information) signals observed by the principal.

The easiest way to introduce a new condensed information signal into the present model is to remove the restriction on manager's trading on their own accounts. At time zero, managers may choose a portfolio of securities including those of their own firms, provided that their trades are disclosed to the market. Investors are therefore able to observe jointly the firm's selection of financial policy and insider trades. The signal space is now two-dimensional, affording investors a more detailed mapping to the (two) determinants of underlying firm value. But since the state and signal spaces are equidimensional, investors may be able to invert the equilibrium signaling correspondence and obtain an exhaustive copy of the manager's state information. This is merely a revelation mechanism result rendered in higher-dimensional clothing.

The most demanding test of this interpretation of incentive contracting stems from preserving the fundamental informational disparity between managers and outsider investors. In a general model, the formal condition that is likely to be needed is the existence of a one-to-one mapping from the signal space onto the space of decision-relevant variables for the principal. That is, investors map condensed-information signals to values of the underlying variables which determine asset prices (e.g., in a k-factor arbitrage pricing model, a set in $\mathbb{R}^k$). Such an equilibrium model, if successfully formulated, would have important implications for efficient market theory: through the incentive contract mechanism, a securities market could be shown to closely approximate strong-form efficiency in spite of the fact that firms retain proprietary information.

# FOOTNOTES FOR CHAPTER II

1. That is, $(\Omega, B, P_X)$ constitutes a probability space for some $P_X \varepsilon \mathbb{P}_X$, where B is the set of $\sigma$-subsets of $\Omega$. See Laha and Rohatgi [1979].

2. Rubenstein [1973] provides a compact derivation of the general relationship between the aggregation of individual measurable utility functions and the concept of the "market price of risk," which will play an important role in the present work. The mean-variance analysis carried out here is valid for arbitrary distributions of security returns provided that the $u_i(\cdot)$ are quadratic; alternatively, any measurable, twice differentiable utility function $u_i(\cdot)$ may be used in conjunction with normally-distributed security returns.

3. See footnote 12, p. 37 <u>infra</u>.

4. The terminology derives from Rothschild and Stiglitz [1976] and Bhattacharya [1980]. "Nondissipative" refers to the characteristic that the realization of informational equilibrium takes place with no deadweight loss relative to the equilibrium with full information about security returns. In the present model, this follows from the assumption of no deadweight

bankruptcy or reorganization costs--all flows in the the event of bankruptcy are pure transfers between bondholders, equity claimants and managers. "Nonproductive" means that here, in contrast to, for example, job market signaling models, the signal itself does not contribute to the value of the asset being signaled. This property holds because of investors' access to "homemade leverage," along with the absence of bankruptcy costs.

5. Formally, the partial equilibrium assumption takes the following form. Consider an initial value of a firm's state $\theta^0$ and a configuration of the remainder of the market yielding a realization of the market portfolio variable $\Pi^0$. Then locally, $\lambda(\theta,\Pi)$ is expressed as:

$$\lambda(\theta,\Pi) = \lambda(\theta^0,\Pi^0) + (\theta - \theta^0)\frac{\partial\lambda}{\partial\theta}\bigg|_{(\theta^0,\Pi^0)} + (\Pi - \Pi^0)\frac{\partial\lambda}{\partial\Pi}\bigg|_{(\theta^0,\Pi^0)}$$

and $\lambda(\theta,\Pi) = \lambda(\theta)$ if the last term is held at zero (i.e., it is assumed that $\Pi$ remains constant).

6. Note that in deriving the expression for $\tilde{R}_\theta$, use has been made of the quite reasonable assumption that the manager's incentive compensation is much smaller than the firm's debt obligation.

7. A change in $\theta_1$ with $\theta_2$ held constant implies the transformation $\tilde{X}' = \tilde{X} + \alpha$, and $\rho(\tilde{X}',\tilde{\Pi}) = \rho(\tilde{X},\tilde{\Pi})$.

8. That the sign of $\frac{\partial\lambda}{\partial\theta_2}$ is generally ambiguous in mean-variance models was established by Rothschild and Stiglitz [1970].

9. In other words, the manager treats his task as one of decision-making under uncertainty, not (as he might in general) one of finding his optimal Nash equilibrium strategy given the other managers' actions.

10. See Townsend [1979].

## REFERENCES FOR CHAPTER II

Bhattacharya, S., "Nondissipative Signaling Structures and Dividend Policy," Quarterly Journal of Economics 95 (Aug. 1980), 1-24.

Downes, D., and Heinkle, R., "Signaling and the Valuation of Unseasoned New Issues," 37 Journal of Finance 1 (1982), 1-10.

Laha, R., and Rohatgi, V., Probability Theory (New York: Wiley, 1979).

Ross, S., "The Determination of Financial Structure: The Incentive Signaling Approach," Bell Journal of Economis 8 (Spring 1977), 23-40.

Rothschild, M., and Stiglitz, J., "Increasing Risk: I. A Definition," Journal of Economic Theory 2 (1970), 225-243.

__________, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics 90 (1976), 629-650.

Rubenstein, M., "A Comparative Statics Analysis of Risk Premiums," 46 Journal of Business 4 (1973), 605-615.

__________, "A Mean-Variance Synthesis of Corporate Financial Theory," Journal of Finance 28 (March 1973).

Townsend, R., "Optimal Contracts and Competitive Markets with Costly State Verification," Journal of Economic Theory 21 (1979), 265- 293.

# CHAPTER III

# CONSUMER WARRANTIES AS SIGNALS OF PRODUCT QUALITY WHEN SEARCH IS IMPERFECT

## 3.1 INTRODUCTION

A major area of concern in theoretical and empirical law and economics is the extent to which imperfect information about contract terms may affect the performance of consumer product markets. An important market imperfection arises when producers and consumers are asymmetrically informed about the market's true distribution of offered prices, product qualities, and purchase terms—as is the case when, for example, the collection and comparison of information about products is a costly activity for consumers. The conventional response to this perceived departure from the competitive ideal has been the direct regulation of contract terms, often coupled with judicial and legislative measures aimed at reducing information acquisition costs.[1] The developing theoretical literature critical of the current direction of regulation seeks to develop analytical tools capable of measuring the extent to which market performance, rather than the individual decision-making process, is adversely affected by the presence of imperfect information.

Much of this new theory is devoted to the study of equilibria in markets for search goods (i.e., where the quality of the goods is readily distinguishable by consumers at the time of purchase), under the assumption that consumers can obtain (only) an imperfect sample of the existing price distribution in the market (Rothschild [1974], Salop and Stiglitz [1977], Wilde and Schwartz [1979], and Schwartz and Wilde [1982a], among others). Recently, ancillary contract terms such as warranties and security interests--which have a bearing on the quality dimension of the consumer's purchase decision--have been introduced into the equilibrium search framework (Schwartz and Wilde [1982b], [1983]).

Some of the most far-reaching consumer product legislation is concerned with the provision of warranties. This essay deals with a leading theory of the function of consumer warranties, the so-called signaling theory (after Spence [1974], [1977]), in consumer product markets where search may be imperfect. The interpretation of warranties as potential signals of product quality stems from a straightforward but vital observation: as the intrinsic reliability of a product falls, it becomes relatively more expensive to warrant the product against failure. Therefore, if quality cannot be directly ascertained by consumers at the time of purchase, firms offering goods of varying quality may see an incentive to attempt to differentiate themselves in the eyes of consumers by attaching warranties with terms growing more generous with higher product reliability. Empirical evidence for such an informative capacity associated with warranties is perhaps best described as quite weak, but often positive.[2]

An important reason advanced as an explanation for the less than dramatic empirical verification of the signaling theory is the presence of imperfect search. Derivations of warranty signaling equilibria typically assume either a single seller in the market (Grossman [1981]) or that consumers can exhaustively and costlessly compare contract terms (as in the various contributions of Spence). It would seem plausible that as the frequency of search undertaken by consumers falls, firms see a reduced incentive to undertake the costly process of distinguishing themselves via warranties. It will be shown here, however, that there is in general only a weak interaction between the information equilibrium (wherein consumers derive fulfilled expectation of product quality from warranty signals) and the determination of the equilibrium prices of goods of various qualities (which, as Schwartz and Wilde have shown, is directly affected by the intensity of consumer search). In other words, as long as the underlying conditions which support consistent warranty signals continue to hold, a falling level of consumer search eventually results in the onset of noncompetitive prices for goods, but <u>not</u> in a disruption of the equilibrium pattern of warranty coverage. Schwartz and Wilde [1982b], on the other hand, isolated instances in which noncompetitive pricing coincided with a deterioration of warranty coverage.

The differences in these equilibrium structures flow directly from the differences between the markets studied in Schwartz and Wilde [1982b] and this essay. Schwartz and Wilde examined a market for a homogeneous search good, assuming that consumers had well-defined preferences for warranty coverage. In turn, firms were characterized by a measure of comparative advantage[3] in supplying warranties, defined according to the level of break-even demand required by a firm charging the highest price the market would bear, for a given warranty status. If, then, a particular firm charging the limit price required less demand to cover its fixed costs by dispensing with warranty coverage, the firm was said to have a comparative advantage at selling without warranties. It could then be shown that if the comparative advantage at selling without warranties was sufficiently strong (and consumer search was sufficiently scarce), firms exploiting the low incidence of search by raising their prices would also see an incentive to drop their warranties, even if all consumers preferred warranties.

Clearly, though, the potential for the informational role of consumer warranties cannot be evaluated in a market for search goods. Warranties can be expected to have a signaling function only in markets for heterogeneous experience goods--goods whose intrinsic reliability or quality cannot be directly assessed by consumers at the time of purchase. But now, in the presence of an equilibrium structure of expectations, that is, a consistent mapping from warranty signals to expected product quality, a decision by a firm to alter its warranty is met by a very different consumer reaction. A change in warranty implies a change in markets. And if the correct conditions on the costs of production (including the production of warranties) prevail across markets, inconsistent warranty signals render firms into inhospitable markets, independent of the intensity of search. Sections 3.2 and 3.3 demonstrate these results for an experience good market with two qualities of goods and two types of consumers.

A second area of concern is the welfare comparison between search and experience goods markets. In view of the fact that experience goods markets may be converted into search goods markets through regulation designed to require the disclosure of quality information to consumers, it is of interest to policymakers to know if such a conversion is necessarily welfare-improving, net of regulatory costs. Section 3.5 demonstrates that, in many instances, the necessary and sufficient conditions for competitive equilibrium with imperfect search in a world of perfect quality information are <u>more</u> restrictive than the corresponding conditions in an experience goods world with a warranty-signaling equilibrium. Thus, when search is imperfect, competitive equilibrium may be easier to realize in markets where consumers are ignorant of product quality <u>per se</u>, but utilize warranties as quality proxies.

## 3.2 WARRANTY SIGNALING WITH PERFECT SEARCH

This section extends the equilibrium search models of Schwartz and Wilde [1982a], [1982b] to a market for a good available at two quality levels, indistinguishable by consumers prior to purchase. Consumers form expectations of product quality by observing whether the product is sold with or without a warranty. Following Schwartz and Wilde [1982b], a warranty is assumed to be a perfect promise made by the seller to replace any and all units of a good which fail in service. Under these conditions, with the further assumption of perfect, costless search, a familiar restriction on the production technologies of firms can be derived which enables consumers to form consistent expectations of product quality based upon observation of warranty "signals." The efficiency properties of the informational equilibria depend in part on consumers' relative preferences for warranty coverage and the underlying quality of goods.

## Firms

Otherwise homogeneous goods are produced at two quality levels, measured by a unidimensional failure probability $\pi_i$: high quality goods, with per-period failure probability $\pi_H$, and low quality goods, with failure probability $\pi_L$, such that $1 > \pi_L > \pi_H > 0$. In equilibrium, a total of N firms are engaged in the production of one type of good (N is assumed to be large). $N_H$ of the firms produce high quality goods (and will be denoted H-type firms); $N_L$ produce low quality goods ( L-type firms.) It should be noted that throughout the models to follow, upper case subscripts will refer to characteristics of firms, while lower case subscripts will pertain to the characteristics of consumers. The proportion of firms producing each type of good is $n_i = N_i/N$, $i \in \{L,H\}$.

All firms face a binding capacity constraint at s units. For production rates lower than capacity, marginal costs are constant within each quality class. Firms which choose not to offer warranties face fixed costs of production $F_i$ -- assume for the sake of convenience that $F_L = F_H = F$. If the marginal costs of the two products are $c_i$, then the total and average cost schedules for firms producing without warranties can be written:

$$T_i(x) = F + c_i x \qquad A_i(x) = c_i + \frac{F}{x} \quad 0 < x < s$$

The assumption of a capacity constraint and the presence of fixed costs of production make possible the existence of a competitive equilibrium with a determinate number of firms in each market. If entry and exit are free, the competitive equilibrium price is just the average cost for each type of firm at full capacity,

$$p^*_{i(\sim W)} = A_i(s) = c_i + \frac{F}{s} \qquad i \in \{L,H\}$$

Now suppose a firm offers units for sale with a warranty. In doing so, the firm contracts to replace defective units it sells; thus, in arriving at a decision about the number of units to be offered for sale, the firm must allow for a reserve of replacement units. It can easily be shown that each unit sold must be backed up by $1/(1 - \pi_i)$ reserve units, so that the firms providing warranties face an effective capacity constraint $s_{iW} = (1 - \pi_i)s$. The competitive price for goods offered with warranties becomes:

$$p^*_{iW} = A_i(s_{iW}) = \frac{1}{1 - \pi_i}[c_i + \frac{F + \bar{F}}{s}] \qquad i \in \{L,H\}$$

The expression $c_{iW} = c_i/(1 - \pi_i)$ can be thought of as the effective cost of producing a good of type i with a warranty. It is also assumed that the registration and administration costs of a warranty program impose additional fixed costs $\overline{F}$. Apart from its alteration of the firm's production schedule, the warranty program adds no new marginal costs. Before attempting to derive conditions on the $c_i$ and $\pi_i$ supporting a signaling equilibrium, the assumptions which define the buyer's side of the market must be laid out.

## Consumers

Consumers' preferences are to be characterized by Von Neumann-Morgenstern utility functions. Each period, consumers enter the market and purchase one unit or none of one of the two types of goods. The total number of consumers in the market in A; of these, $A_\ell$ prefer low-quality goods, while $A_h$ are high-quality-preferring. Let the initial incomes of the two types of consumers be $Y_j$, $j \in \{\ell, h\}$. A reservation level of utility can be assured each class of consumer if no units are purchased:

$$u_j(Y_j, 0) = u_{0j} \qquad j \in \{\ell, h\}$$

Now suppose that a low-quality-preferring consumer purchases a good of type i, for which he pays $p_i$. Since $\partial u_\ell(Y_\ell - p_i, 1)/\partial p_i < 0$, there will exist well-defined reservation prices $\ell_{i(\sim W)}$ for goods purchased without warranty coverage, given by:

$$(1 - \pi_i)u_\ell(Y_\ell - \ell_{i(\sim W)}, 1) + \pi_i u_\ell(Y_\ell - \ell_{i(\sim W)}, 0) = u_{0\ell}$$

And for goods supplied with warranties, reservation prices $\ell_{iW}$:

$$u_\ell(Y_\ell - \ell_{iW}, 1) = u_{0\ell} \qquad i \in \{L,H\}$$

since the warranty guarantees the presence of a functioning unit at the end of the period. Exactly analogous expressions can be derived for the reservation prices $h_{i(\sim W)}$, $h_{iW}$ of consumers who prefer high quality. If utility is increasing in product reliability, it will generally be true that $\ell_L < \ell_H$ and $h_L < h_H$, for a given warranty status. The expressions for the reservation prices are well-defined if all consumers know the failure probabilities $\pi_i$ of the goods,[4] but it is assumed that the qualities of goods are not apparent to consumers in a pre-purchase inspection.

Instead, consumers attempt to form expectations of product quality based upon observations of warranty coverage. The "signal space" in this model is the two-element set $\{W, \sim W\}$. When a consumer with preference index j, $j \in \{\ell, h\}$ encounters a good with a warranty and infers it to be of type i, $i \in \{L,H\}$, his reservation price, conditional on the signal, may be written $r_j(i|W)$; if the good is offered without a warranty, the consumer assignes a reservation price $r_j(i|\sim W)$. In equilibrium, warranties are to be positively correlated with product quality, so that all high-quality products will be offered with warranties, and all low-quality products will appear without them. For high-quality-preferring individuals, then, consistent expectations are given by $r_h(L|\sim W) = h_L$ and $r_h(H|W) = h_H$, while for low-quality-preferring consumers, $r_\ell(L|\sim W) = \ell_L$ and $r_\ell(H|W) = \ell_H$ are consistent. Therefore, when equilibria can be characterized, the signal index can be supressed from the consumers' reservation prices.

The welfare properties of the signaling equilibrium derive from what can be established about consumer welfare in the absence of warranty signals. The most important initial observation to be made is that, if differential warranty coverage is not present across markets, the only equilibria that can exist, regardless of the incidence of consumer search, are "pooling" equilibria. That is, unlike the search good case, there will never exist discrete markets for H and L goods <u>both</u> offered with or without warranties. In the absence of reputation or brand-name effects, consumers have no means of distinguishing pure experience goods by quality levels if warranty coverage is uniform. To see what is likely to happen in a pooling equilibrium, consider the case in which no warranties are offered. Assume further that the equilibrium is competitive--goods transact at prices $p^*_{i(\sim W)}$. Then it is clear that the pooling equilibrium will involve only a single good—if quality has an incremental marginal cost, so that $c_L < c_H$, the only good offered will by the low-quality one. Any firm offering a good at a higher price than $p^*_{L(\sim W)}$ (in particular, an H-type firm charging its competitive price) will be shunned by consumers who search, and will not attract enough demand from nonsearching consumers to cover costs. Of course, <u>price</u> cannot be used by consumers as a proxy for quality, because offered prices can costlessly be biased by firms. The resulting equilibrium is the imperfect search analogy to the "lemons" equilibrium (see Akerlof [1970]), and is grossly inefficient, since an entire class of consumers (in this case, the h-types), get none of the goods they prefer.

Contrast this with a market outcome in which warranties are offered with all products. Note that it matters not to the consumer whether the purchased good is H- or L-type, since the warranty transforms either good into a homogeneous good with a zero failure probability. To save on notation, then, let $\ell_W \equiv \ell_{HW} = \ell_{LW}$, and, consistent with the previous definition, $h_H \equiv h_{HW} = h_{LW}$. The only sustainable competitive equilibrium price in this case is $p^* = \min[p^*_{LW}, p^*_{HW}]$. If all consumers prefer warranties, such an equilibrium is, somewhat paradoxically, strictly efficient.

It can now be made clear how $\ell$- and h-type consumers can be defined, and how warranty preferences can be incorporated. Consumer types are defined by which product (offered at competitive prices) maximizes the welfare of a given type. Accordingly, if <u>no</u> consumers prefer warranties, the $\ell$- and h-types are:

$$\ell_L - p^*_{L(\sim W)} > \ell_{H(\sim W)} - p^*_{H(\sim W)} > \ell_W - p^* \tag{1a}$$
$$h_{H(\sim W)} - p^*_{H(\sim W)} > h_{L(\sim W)} - p^*_{L(\sim W)} > h_H - p^*$$

respectively; that is, $\ell$-types prefer L goods to H goods without warranties, while h-types prefer H goods to L. Relations (1a) represent a preference ordering consistent with a lemons equilibrium in an experience good world. Consider, however, the following slight alteration:

$$\ell_L - p^*_{L(\sim W)} > \ell_{H(\sim W)} - p^*_{H(\sim W)} > \ell_W - p^* \qquad (1b)$$
$$h_{H(\sim W)} - p^*_{H(\sim W)} > h_H - p^* > h_{L(\sim W)} - p^*_{L(\sim W)}$$

then the possibility arises of a pareto-optimal pattern of differential warranty coverage. Here, the opening of the HW market is <u>relatively</u> <u>efficient</u>: h-types get H goods (albeit at the cost of a warranty), and $\ell$-types are indifferent to the change. There is an unavoidable welfare loss relative to the (unattainable) full-information equilibrium without warranties. The warranty-signaling outcomes is more likely for consumers with sharply divergent preferences for quality, per dollar spent, but with relatively homogeneous and comparatively weak preferences for "comprehensive" warranty protection.[5] This, in turn, is consistent with the generally-observed lower saliency of warranty terms for consumers in comparison with the basic quality attributes of goods.[6]

The remainder of this section isolates a familiar set of conditions under which only H-type firms offer goods with warranties, and $p^* = p^*_{HW}$. It will then be shown that these conditions can be generalized to markets which, unlike those of existing signaling models, are characterized by imperfect search.

A Warranty Signaling Theorem

It remains to be shown that the informational equilibrium can be attained. As has been demonstrated previously, without restrictions on the firms' technologies there may exist "pooling" equilibria in which, for example, all firms issue warranties -- Figure 3.1 portrays a graphical example. In such equilibria, warranty signals carry no information about product quality. Theorem 1 sets forth the conditions under which a warranty signaling equilibrium forms when consumers have costless access to price information. In this model, a price- expectations equilibrium exists when, for some set of prices and warranty coverage for each good, and a ratio of firms to consumers in each market: i), all firms earn zero profits; ii), no firm can raise its profits by changing its price or altering its warranty coverage; and iii), consumers, associating offered warranties to the underlying reliability of goods, maximize their net welfare.

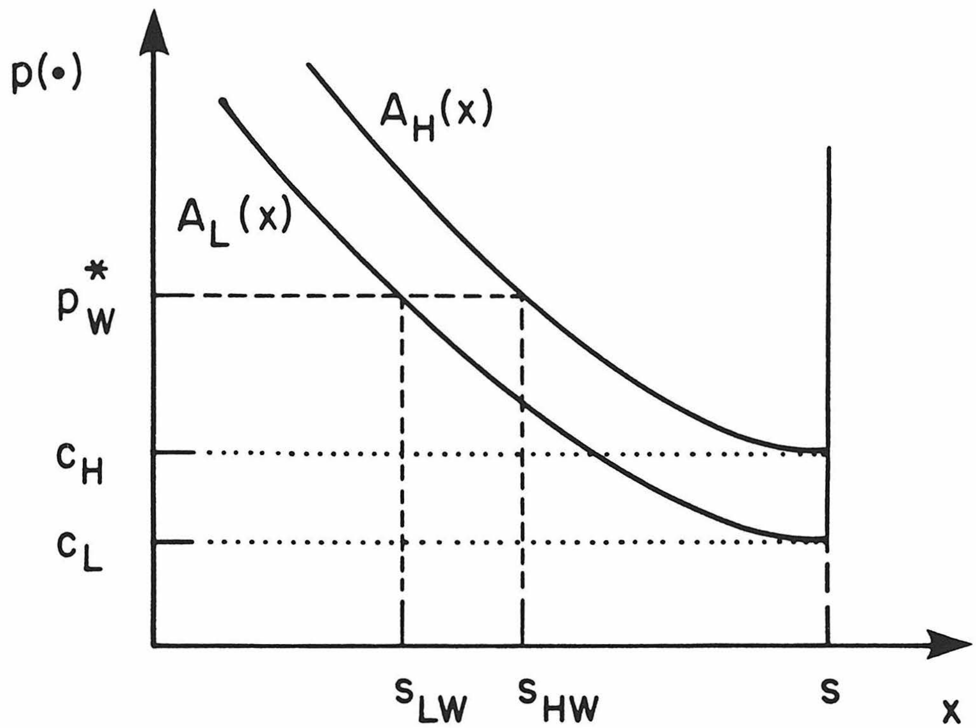



Figure 3.1 Warranty Signaling Equilibrium Breakdown

<u>Theorem 1</u> Let consumer preferences be given by relations (1b). Under conditions of perfect, costless search, warranties serve as unambiguous signals of product quality (i.e., $n_{HW} = 1$, $n_{H(\sim W)} = 0$; $n_{LW} = 0$, $n_{L(\sim W)} = 1$), if and only if:

$$\text{i)} \quad c_L < c_H \tag{2a}$$

$$\text{ii)} \quad c_L/(1 - \pi_L) > c_H/(1 - \pi_H) \tag{2b}$$

and the equilibrium consumer/firm ratio is $\sigma \equiv \frac{A}{N} = s[1 - \frac{\alpha\pi_h}{\alpha + (1 - \pi_h)}]$ where $\alpha \equiv (A_H/A_\ell)$.

<u>Proof</u> By virtue of the perfect shopping assumption, the zero profit equilibrium will form at the competitive prices for the two types of goods. Warranties are to proxy for product quality, so the equilibrium price set is $\{p^*_{L(\sim W)}, p^*_{HW}\}$. It must be shown that a firm producing a given product cannot switch its warranty signal, enter the other market, and break even (i.e., cover its fixed costs.) Suppose an L-type firm warrants its product and attempts to enter the H market. The structure of expectation then dictates that the deviant firm will lose all of its former customers, gaining business from h-type consumers. Because the deviant offers a warranty, its effective capacity will be $s_{LW} = (1 - \pi_L)s$ and $s_{LW} < s_{HW}$ because of the L-type product's lower intrinsic reliability. Before the entry of the L-type firm, the consumer-firm ratio in the H market was $(A_h/N_H) = s_{HW}$; after entry the number of h-type consumers becomes $A_h' = s_{HW}N_H + s_{LW}$. Note

that since $N_H >> 1$,

$$A_h' = s_{HW}N_H[1 + \frac{1}{N_H}(s_{LW}/s_{HW})] = s_{HW}N_H(1 + \varepsilon) \simeq A_h$$

The deviant fails to break even at $p_{LW} = p_{HW}^*$ if:

$$(A_h'/N_H)[p_{HW}^* - (c_L/(1 - \pi_L))] < F + \overline{F} \tag{3}$$

But since $(A_h'/N_H) = (1 + \varepsilon)s_{HW}$ and, by definition of $p_{HW}^*$, $F + \overline{F} = [p_{HW}^* - (c_H/(1 - \pi_H))]s_{HW}$, (3) can be written:

$$p_{HW}^* - (c_L/(1 - \pi_L)) < \frac{p_{HW}^* - (c_H/(1 - \pi_H))}{1 + \varepsilon}$$

$$< p_{HW}^* - (c_H/(1 - \pi_H))$$

or,

$$c_H/(1 - \pi_H) < c_L/(1 - \pi_L)$$

which is condition (2b). An exactly parallel argument in the L market with an H-type deviant yields the analogous negative-profit constraint $(A_\ell'/N_L)[p_{L(\sim W)}^* - c_H] < F$, which requires $c_H > c_L$, or condition (2a).

Finally, with signal-switching precluded by conditions (2), the consumer-firm ratios in the two-markets, $(A_h/N_H) = s_{HW}$ and $(A_\ell/N_L) = s$ yield:

$$A = A_\ell + A_h = N_H s(1 - \pi_H) + N_L s$$

$$= s(N - \pi_H N_H)$$

$$\frac{A}{N} = s(1 - \pi_H(A_h/Ns_{HW}))$$

or, with $Ns_{HW} = A - \pi_h A_\ell$,

$$\sigma \equiv \frac{A}{N} = s[1 - \frac{\pi_h A_h}{A - \pi_h A_\ell}]$$

and the expression given in the theorem follows from the definition $\alpha \equiv (A_h/A_\ell)$. □

Conditions (2) are the familiar within the framework of signaling models. When consumers are fully aware of firms' offered prices, warranties serve as signals of product quality if: i), "quality" costs something at the margin; and ii), the cost of providing a warranty is inversely related to the product's underlying quality or reliability.

To summarize this initial section: an analysis of the informational role of consumer warranties requires a slightly different placement of emphasis in the relationship between warranty terms and underlying goods than that existing in the search good literature. When the reliability of a good is unknown to a consumer prior to the purchase decision, the consumer in effect faces either a homogeneous goods market or a heterogeneous one, depending upon the observed variation in warranty terms. Even if consumers engage in enough search to enforce a competitive market price, a homogeneous good outcome is likely to be inefficient for some class of consumers. To the extent that goods have experience characteristics, then, there may exist a welfare-inproving pattern of (costly) warranty signals. Conditions (2) are the necessary and sufficient conditions for such signals when consumers have full access to price/warranty information.

The formal differences between the signaling and comparative advantage interpretations of consumer warranties can now be drawn into sharper focus. A firm in a search good world will see an opportunity to raise its offered price as the incidence of consumer search falls. Such a firm will warrant or fail to warrant its product based upon its comparative advantage for issuing warranties at the highest price the market will bear. A firm in an experience good market, however, cannot be quite so sanguine about its decision to alter its warranty coverage. As the incidence of search falls, opportunities for firms to raise their prices will still present themselves. But since consumers' expectation of product quality are now conditioned on their observations of warranty coverage, a decision by a firm in a particular market to switch warranty coverage transforms the firm into an inhabitant of the <u>other</u> market (in the eyes of consumers). In particular, the entrant firm will garner exactly the same expected demand at any price as would a "native" firm in that market. The perfect search signaling conditions (2) ensure that the entrant firm faces a cost disadvantage compared to a native firm at <u>any</u> entry price, by virtue of the parity in expected demand.

Intuition suggests, therefore, that conditions (2) may be robust to the presence of imperfect search. Nevertheless, one cannot immediately rule out the possibility that the conditions for the various price equilibria in the presence of imperfect search might conflict with the cost constraints which support consistent warranty signals. The next sections examine the interaction between price and information equilibria.

## 3.3 EQUILIBRIA WITH IMPERFECT SEARCH: COMPETITIVE

This section reanalyzes the competitive equilibria of the previous section in a market where consumer search is imperfect. Consumer populations of each type are to be further partitioned into nonshoppers, who randomly sample only one firm's price and warranty offer, purchasing if the price is lower than the nonshopper's reservation price given his expectation of quality, and shoppers, who compare the price and warranty terms of exactly two randomly-encountered firms before making a purchase decision. Since search intensity (the number of price comparisons made before purchase) is in general positively related to the likelihood of competitive equilibria,[7] adoption of the minimum search intensity most clearly brings out the effects of imperfect search, and, in this model, the interaction between imperfect search and signaling equilibria.

Superscripts will denote the search variable; hence, the number of H-preferring nonshoppers is $A_h^1$; the number of H-preferring shoppers, $A_h^2$, and $A_h = A_h^1 + A_h^2$. In a similar fashion, the total number of nonshoppers in the market can be written $A^1 = A_h^1 + A_\ell^1$, and so on. As before, for any exogenously specified mix of consumers, catagorized by quality preference and search behavior, equilibria will be defined by consumer-firm ratios in each market such that firms earn zero profits. Firms merely select a price-warranty combination, gauge the demand that appears given the mix of consumers, and alter the price-warranty offer if doing so raises their expected profits.

A crucial aspect of market equilibrium lies in the fact that the markets for the two types of goods are <u>interactive</u>. Consumers of different types maximize their net welfare by purchasing goods of differing quality, if those goods are offered at their competitive prices. If firms in a particular market attempt to exploit the presence of nonshoppers and raise prices, shoppers who see the high price <u>and</u> a maintained competitive price for the other market's goods will eventually switch to the other market. For example, if the price of low-quality goods rises above $\bar{p}_L$ (while competitive prices prevail in the H market), where $\ell_L - \bar{p}_L = \ell_H - p^*_{HW}$, $\ell$-type shoppers who see $p^*_{HW}$ will buy high quality. Similarly, define $\bar{p}_H = p^*_{L(\sim W)} - h_L + h_H$. The presence of these <u>switch prices</u>[8] is a constraint on the upward movement of prices; note that although some consumers purchase the "wrong" good, no violation of the signaling equilibrium has necessarily occurred. All consumers have used warranty signals to make welfare-maximizing purchases, given the structure of prices.

Finally, it is helpful to make an assumption about the relationship between reservation prices across consumer types. Probably the most intuitively reasonable of these is $h_L > \ell_L$, and $h_H > \ell_H$ -- that is, a kind of "income effect" prevails in each market.[9] The assumed, derived, and defined features of the two markets can best be summarized on a "map" such as Figure 3.2. The relationships between the market prices follow from the definitions of the previous section and conditions (2a) and (2b) of Theorem 1. Notice that although only two markets appear in Figure 3.2 (high and low quality with consistent signals), there are actually two other incipient markets (where low quality goods are offered with warranties, and high quality goods without). Conditions must be derived such that competitive prices prevail in the consistent signals market while firms are blockaded from the signal switching markets.

Three kinds of competitive market equilibria are possible: (i) a price $p^*_{HW}$ in the H-market with the L-market nonexistent; (ii) $p^*_{L(\sim W)}$ charged in L with H nonexistent; and (iii) $p^*_{L(\sim W)}$ in L <u>and</u> $p^*_{HW}$ in H. Each will now be considered in turn.

Free entry markets such as those studied here can fail to exist if so few consumers prefer a good of a given type that firms offering the good cannot break even. This does not mean, however, that firms producing the nondemanded goods do not exist. With imperfect search, there is the possibility that these firms might give a misleading signal, enter the existing market, and prey on nonshoppers. The following, however, show that this cannot happen when sufficient search is carried out to bring about competitive pricing.

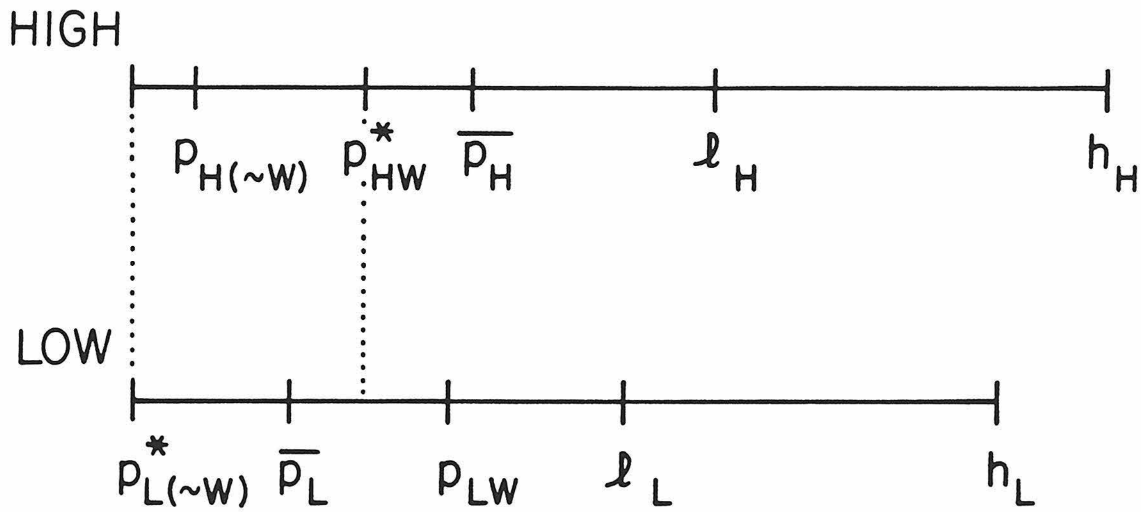



Figure 3.2 Two-Market Equilibrium

Theorem 3.2: Under conditions of imperfect search, a competitive equilibrium with H-good price $p^*_{HW}$ and no L-good market, such that $n_{HW} = 1$, $n_{H(\sim W)} = 0$, $n_{LW} = 0$, $n_{L(\sim W)} = 0$; $\sigma = s_{HW}$ will exist iff:

$$c_H > c_L$$

$$c_{HW} < c_{LW}$$

$$a^1 < \frac{F + \bar{F}}{s_{HW}(\ell_H - c_{HW})}$$

$$a^1_h < \frac{F + \bar{F}}{s_{HW}(h_H - c_{HW})}$$

and, in the L-market,

$$a^1 + 2a^2_\ell < \frac{F}{s_{HW}(\bar{p}_L - c_L)}$$

$$a^1 < \frac{F}{s_{HW}(\ell_L - c_L)}$$

$$a^1_h < \frac{F}{s_{HW}(h_L - c_L)}$$

Proof: If the only price to be observed is $p^*_{HW}$, the equilibrium consumer-firm ratio must be $(A/N) = s_{HW}$. The N firms charging $p^*_{HW}$

will see an expected demand of $(A^1/N)$ from nonshoppers. Shoppers will see a particular firm with probability (2/N); an offsetting factor of 1/2 allocates ties among consumers, since all units transact at the same price. Thus, the expected demand from shoppers (both high- and low-quality preferring) at $p^*_{HW}$ will be $(\frac{A^2}{N})$. Zero profits at the equilibrium price, with all consumers seeing a warranty and inferring high quality, requires:

$$[(A^1/N) + (A^2/N)](p^*_{HW} - c_{HW}) - F + \bar{F} = 0.$$

Or, using $\sigma = (A/N) = s_{HW}$,

$$s_{HW}[a^1 + a^2](p^*_{HW} - c_{HW}) - F + \bar{F} = 0$$

rearranging, noting that $a^1 + a^2 = 1$, gives the zero-profit condition. Further, it cannot be possible for an H-type firm to raise its price and break even. If a firm prices above $p^*_{HW}$, it loses all of the shoppers; up to the price $\ell_H$ it retains its equal share of nonshoppers. At $\ell_H$,

$$s_{HW}a^1(\ell_H - c_{HW}) - F + \bar{F} < 0$$

insures negative profits and implies the second condition. Beyond $\ell_H$, only H-preferring nonshoppers remain. Above $h_H$, they too drop out; so at the highest price the market can bear,

$$s_{HW}a_h^1(h_H - c_{HW}) - F + \bar{F} < 0$$

now consider the L-market. If an L-offering firm, (with no warranty) charges a price in the interval $[p^*_{L(\sim W)}, \bar{p}_L]$, he will acquire the patronage of nonshoppers and L-preferring shoppers. With the prevailing equilibrium $\sigma$ in the H-market, the L-type firm fails to break even at $\bar{p}_L$ if:

$$s_{HW}[a^1 + 2a_\ell^2](\bar{p}_L - c_L) - F < 0$$

the fourth condition. The last two conditions prohibit an L-type firm from raising its price in its "home" market. Thus, the L-market fails to exist.

Notice now that the zero-profit equilibrium in the H market deters the entry of signal-switching entrants even with the nonshoppers present. To enter the H-market, an L-type firm must add a warranty. When it does so, it is treated by consumers as an H-type firm; in particular, the deviant garners the same expected demand at any price as an H-type. So, at $p^*_{HW}$,

$$\Pi_{LW}(p^*_{HW}) = s_{HW}[a^1 + a^2](p^*_{HW} - c_{LW}) - F + \bar{F}$$

But, by zero profits in the H-market:

$$\Pi_{HW}(p^*_{HW}) = s_{HW}[a^1 + a^2](p^*_{HW} - c_{HW}) - F + \bar{F} = 0.$$

Thus, necessary and sufficient for $\Pi_{LW}(p^*_{HW}) < 0$ is $c_{HW} < c_{LW}$. A similar argument with the condition $c_H > c_L$ prohibits an H-type from dropping its warranty and entering the L-market; no L-type firm can survive there.

□

<u>Theorem 3.3</u>: A competitive equilibrium with L-good price $p^*_{L(\sim W)}$ and no H-good market, such that $n_{L(\sim W)} = 1$, $n_{LW} = 0$, $n_{H(\sim W)} = 0$, and $n_{HW} = 0$, with $\sigma = s$ will exist iff: cost conditions (2a) and (2b) hold while:

$$a^1 < \frac{F}{s(\ell_L - c_L)}$$

$$a^1_h < \frac{F}{s(h_L - c_L)}$$

and, in the H-market,

$$a^1 + 2a^2_h < \frac{F + \bar{F}}{s(\bar{p}_H - c_{HW})}$$

$$a^1 < \frac{F + \bar{F}}{s(\ell_H - c_{HW})}$$

$$a^1_h < \frac{F + \bar{F}}{s(h_H - c_{HW})}$$

<u>Proof</u>: Exactly analogous to Theorem 2. □

The most interesting competitive equilibrium results when both markets are active; such an equilibrium structure for the case of search goods has been explored in Schwartz and Wilde [1982a]. The analysis undertaken here, with warranties serving as quality signals, will differ somewhat from Schwartz and Wilde's, but the final results will be comparable. The imperfect search equilibrium is more complicated than the perfect-search two-market equilibrium of Theorem 3.1, because shoppers can now become "stranded" in the market for the good they would not otherwise prefer. An initial lemma will sort out the properties of a two-price equilibrium under these conditions; then, as in the earlier theorems of this section, price increases in the two markets will be ruled out by additional constraints.

Lemma 3.1: In the presence of imperfect search, a necessary condition for competitive prices in both markets, $\{p^*_{L(\sim W)}, p^*_{HW}\}$ with $n_{LW} = 0$, $n_{L(\sim W)} > 0$; $n_{HW} > 0$, $n_{H(\sim W)} = 0$, and a fulfilled expectations equilibrium for warranty signals, is:

$$\pi_H < a_\ell^2 - a_h^2 < \pi_H/(1 - \pi_H) \tag{3}$$

Proof: At the outset, the principal unknowns are the equilibrium proportions of firms offering each type of good, $n_H$ and $n_L$, and the consumer-firm ratio, $\sigma$. If goods are offered at their competitive prices, each firm can expect an equal share of the nonshoppers, $(A_1/N)$.

Now consider an H-type firm. Either H- or L-preferring shoppers have a nonzero probability of locating an H-type firm (by warranty inference) on <u>both</u> shopping trips; the $\ell$-types will thus be stranded. Expected demand from this source is equal to the probability of a given firm being encountered (2/N), times the proportion of H-type firms, $(N_H/N)$, times the number of shoppers seeing high quality twice, $(A_\ell^2 + A_h^2)/2 = A^2/2$, where the factor 1/2 allocates ties. The H-type firm will also gain demand from h-type consumers who see one H-type and one L-type firm as they shop. Expected demand here is $(2/N)(A_h^2)(N_L/N)$. Total demand for the H-type firm is thus:

$$Q_H(p_{HW}^*) = \frac{A^1}{N} + \frac{2}{N}\,[A_h^2 n_L + (1/2)A^2 n_H]$$

zero profits at $p_{HW}^*$ then imply (extracting a factor of $(A/N) = \sigma$):

$$\sigma[a^1 + 2a_h^2 n_L + a^2 n_H](p_{HW}^* - c_{HW}) - F + \bar{F} = 0$$

or, by definition of $p_{HW}^*$,

$$\sigma[a^1 + 2a_h^2 n_L + a^2 n_H] = s_{HW} \qquad (4a)$$

tracing the same argument for an L-type firm readily yields:

$$\sigma[a^1 + a^2 n_L + 2a_\ell^2 n_H] = s \qquad (4b)$$

Equations (4a) and (4b), along with the identity $n_H + n_L = 1$ yield solutions for $n_H$, $n_L$, and $\sigma$.

$$\sigma = \frac{s - s_{HW}}{a_\ell^2 - a_h^2} = \frac{s\pi_H}{a_\ell^2 - a_h^2}$$

$$n_L = \frac{1}{a_\ell^2 - a_h^2} - \frac{1 - \pi_H}{\pi_H}$$

$$n_H = \frac{1}{\pi_H} - \frac{1}{a_\ell^2 - a_h^2}$$

since $s > s_{HW}$ if $\pi_H > 0$, $\sigma > 0$ implies $a_\ell^2 - a_h^2 > 0$. But the nonnegativity constraints on $n_L$ and $n_H$ are more restrictive -- $n_H > 0$ gives:

$$(a_\ell^2 - a_h^2) > \pi_H$$

and $n_L > 0$ gives:

$$(a_\ell^2 - a_h^2) < \frac{\pi_H}{1 - \pi_H}$$

or in combination, condition (3). When (3) holds, $\Pi_H(p^*_{HW}) = 0$ and $\Pi_L(p^*_{L(\sim W)}) = 0$; given conditions (2), no signal switching entrant can break even. □

Now, to prohibit within-market price increases:

<u>Lemma 3.2</u>: The equilibrium price set $\{p^*_{L(\sim W)}, p^*_{HW}\}$ will be stable with respect to price increases in each market, and quality expectations

fulfilled, if, in the H-market:

$$a^1 + 2a_h^2 n_L < \frac{(F + \bar{F})(a_\ell^2 - a_h^2)}{\pi_H s(\bar{p}_H - c_{HW})}$$

$$a^1 < \frac{(F + \bar{F})(a_\ell^2 - a_h^2)}{\pi_H s(\ell_H - c_{HW})} \tag{5}$$

$$a_h^1 < \frac{(F + \bar{F})(a_\ell^2 - a_h^2)}{\pi_H s(h_H - c_{HW})}$$

and, in the L-market:

$$a^1 + 2a_\ell^2 n_H < \frac{F(a_\ell^2 - a_h^2)}{\pi_H s(\bar{p}_L - c_L)}$$

$$a^1 < \frac{F(a_\ell^2 - a_h^2)}{\pi_H s(\ell_L - c_L)} \tag{6}$$

$$a_h^1 < \frac{F(a_\ell^2 - a_h^2)}{\pi_H s(h_L - c_L)}$$

while as before, conditions (2a) and (2b) hold.

Proof: Similar to the proofs of Theorems 3.2 and 3.3 (see also Schwartz and Wilde [1982a]). For example, suppose an H-type firm seeks to raise its price into the interval $(p_{HW}^*, \bar{p}_H]$. Such a firm will get its usual share of nonshoppers, but will lose the patronage of any h-type shoppers who see it and an H-type firm charging $p^*_{HW}$. But because the deviant's price has not risen above $\bar{p}_H$, any h-type shoppers who sample the deviant firm and some L-type firm will purchase from the deviant. Negative profits for such a firm are guaranteed by the first condition in (5); the others follow similarly. As before, the restrictions (2) prohibit signal-switching.

Lemmas 3.1 and 3.2 jointly characterize the necessary and sufficient conditions for two-price competitive equilibria with consistent warranty signals. The remarkable property of this equilibrium (as well as those found in Theorems 3.2 and 3.3) is that the same conditions necessary to support a warranty signaling equilibrium in the perfect search case (conditions (2a) and (2b)), also suffice when search is imperfect. Therefore, the first question posed by this essay has been answered: the perfect-search signaling equilibrium conditions translate intact to competitive equilibria in imperfect search worlds.[10]

Price/Information Equilibrium Interaction

It appears at first glance that the two-price competitive equilibrium conditions in a world of experience goods with warranty signals consist of the costless-search signaling conditions (2) grafted onto the price equilibrium conditions for search goods. But in fact, the informational and price equilibrium conditions do interact, although subtly; the equilibrium shopper/nonshopper balance conditions differ from those prevailing in a pure search-good world. The two sets of constraints interact through the parameters they have in common: the marginal costs $c_i$, $c_{iW}$, and the product reliabilities $\pi_i$, $i \in \{L,H\}$. To see this, examine the conditions for a zero-profit equilibrium in the HW market, with the L(~W) market nonexistent (Theorem 3.2). Observe the effect on the market for L goods of increasing $c_L$. As the marginal cost of the L good rises, the RHS of the equilibrium conditions grow larger, slackening the constraints on the equilibrium-supporting combinations of shoppers and nonshoppers. This occurs because firms who would raise prices to exploit the presence of nonshoppers require more expected demand to cover the same level of fixed costs as the marginal cost of their product rises. ceterus paribus. But if $c_L$ rises sufficiently, the signaling condition $c_H > c_L$ will be violated. A similar argument holds for the effect of raising $c_{HW}$ in the HW market--moving a market parameter in a direction which reinforces the price equilibrium eventually brings about a violation of the complimentary information equilibrium, and vice versa.

Changing the reliability of the goods has another interactive effect. Again consider the HW-only market. Suppose that $\pi_H$ increase with all other parameters fixed. Then $s_{HW} = (1 - \pi_H)s$ falls, and again the price equilibrium constraints relax. This time the slackening occurs because the firms are getting smaller (in terms of capacity). Thus, if a price-raising firm failed to cover fixed costs at a higher capacity, it will surely fail as the firm's maximum output shrinks. Notice now that as the failure probability of the H good rises, the signaling conditions $c_H/(1 - \pi_H) < c_L(1 - \pi_L)$ must eventually be broken.

Of course, these same effects hold in the two-price equilibrium. For example, as $\pi_H$ falls, reinforcing the signaling equilibrium condition, the range of price-equilibrium supporting $a_\ell^2 - a_h^2$ steadily shrinks, by the results of Lemma 3.1. It should not be surprising, of course, that the signaling equilibrium conditions imply new restrictions on the range of competitive equilibrium market outcomes; this merely indicates that the informational equilibrium restrictions are nontrivial. What is vital is that warranties remain useful as instruments of information transfer in the presence of imperfect search. Granting this, it is important to assess how efficient warranties are as quality information conduits, relative to a world in which consumers are endowed with perfect information about product quality.

### 3.4 SEARCH VS. EXPERIENCE GOODS: COMPARATIVE EQUILIBRIA

Having established the relative robustness of warranty signaling equilibria to problems of imperfect search, inquiry naturally turns to a comparison of competitive equilibria in search and experience goods settings. This section seeks to shed light on two related questions. First, under the assumption of heterogeneous warranty preferences consistent with information equilibrium (i.e., h-type consumers prefer warranties, $\ell$-types do not), are the conditions for a competitive two-price equilibrium more restrictive in an experience good world or in a search good world? Second, can anything be said about the comparative welfare of consumers facing competitive market outcomes in the two settings?

The first questions is meant to address the issue of the most efficient means of imparting information about product quality to consumers. It is tempting to think that the optimum state for consumers is one of perfect _ex ante_ quality information. But this reflex intuition may not hold in markets with imperfect search. As the consumer's ability to distinguish product quality at the time of purchase grows, so does the likelihood that a given consumer pursuing a fixed-sample-size shopping strategy will in effect be a nonshopper in any particular market. In other words, to cite an established result of equilibrium search theory, product heterogeneity dilutes the effectiveness of search.

In order to assess the effects of different levels of consumer information about quality on competitive equilibria, it is necessary to construct a "parallel" search good world to the experience good markets of the previous section. This is accomplished by means of the following assumptions:

_Assumption 1_. Let two goods, types L and H, be traded under conditions of perfect quality information. Let h-type consumers be defined by the ordering $h_H - p^*_{HW} > h_{H(\sim W)} - p^*_{H(\sim W)} > h_{L(\sim W)} - p^*_{L(\sim W)}$

and $\ell$-type consumers by $\ell_{L(\sim W)} - p^*_{L(\sim W)} > \ell_{H(\sim W)} - p^*_{H(\sim W)} > \ell_{HW} - p^*_{HW}$.

<u>Assumption</u> <u>2</u>. Let the product reliabilities $\pi_i$, capacities s and $s_{iW}$, and cost functions F, $c_i$, and $c_{iW}$ be as defined previously. In addition, assume that conditions (2) hold.

Now consider the necessary and sufficient conditions to establish the two-price equilibrium $\{p^*_{L(\sim W)}, p_{HW}\}$, $n_{HW} > 0$, $n_{H(\sim W)} = 0$; $n_{L(\sim W)} > 0$, $n_{LW} = 0$ in the market for <u>search</u> goods. Notice first that Assumptions 1 and 2 imply the same equilibrium consumer/firm ratio $\sigma$ for the search good market as that established in Lemma 3.1 above. The cost functions, capacities, and prices of the firms in the two markets are identical with the corresponding quantities of the firms in the previous section. Given the equilibrium $\sigma$, then, the analogous necessary conditions to relations (5) and (6) can be written, which restrain firms from raising prices above competitive levels in the search goods markets. For example, in order for $p^*_{HW}$ to be in zero-profit equilibrium in the HW market, the following must hold:

$$a^1 + 2a^2_n n_{L(\sim W)} < \frac{F + \bar{F}}{\sigma(\bar{p}_{HW} - c_{HW})} \qquad (5')$$

$$a^1 < \frac{F + \bar{F}}{\sigma(\ell_{HW} - c_{HW})}$$

$$a^1_h < \frac{F + \bar{F}}{\sigma(h_H - c_{HW})}$$

where care has been taken to fully specify reservation prices by product type and warranty coverage, since both are now distinguishable by consumers.

As demonstrated in the previous section, the signaling conditions (2) along with (5), (6) Lemma 3.1 constituted necessary and sufficient conditions to blockade entry into the LW and H(~W) markets for experience goods. In particular, if (5) holds, then condition (2b) serves to close the LW market to profitable entry. To see if this result holds in a search-good market, compare each expression in (5') with the analogous expression in the incipient LW market. The "switch" price in the LW market is the same as $\bar{p}_{HW}$, since $h_{LW} = h_{HW} = h_H$. But since $p^*_{LW} > p_{HW}$, it may happen that $\bar{p}_{HW} < p^*_{LW}$, in which case all h-type shoppers who see $p^*_{LW}$, and an L-type firm will buy L. If this does not occur, then zero profits at $\bar{p}_{HW}$ in the LW market requires $a^1 + 2a^2_h n_{L(\sim W)} < (F + \bar{F})/\sigma(\bar{p}_{HW} - c_{LW})$. This, in turn, is more restrictive than the first condition in (5') only if:

$$\frac{F + \bar{F}}{\sigma(\bar{p}_{HW} - c_{LW})} < \frac{F + \bar{F}}{\sigma(\bar{p}_{HW} - c_{HW})}$$

or, if $c_{LW} - c_{HW} < 0$, which contradicts (2b). This, (2b) and (5') are sufficient to deter entry into the LW market at $\bar{p}_{HW}$ if $\bar{p}_{HW} > p^*_{LW}$. A similar argument holds for higher prices in the LW market (i.e., at $\ell_{HW}$ and $h_H$)--asserting that the LW market zero-profit condition is more restrictive than its counterpart in (5') contradicts (2a), which was assumed to hold.

But now consider potential entry into the H(~W) market. Zero profit conditions for the L(~W) market are, analogous to (6):

$$a^1 + 2a_\ell^2 n_{HW} < \frac{F}{\sigma(\bar{p}_{L(\sim W)} - c_L)} \tag{6'}$$

$$a^1 < \frac{F}{\sigma(\ell_{L(\sim W)} - c_L)}$$

$$a_h^1 < \frac{F}{\sigma(h_{L(\sim W)} - c_L)}$$

once again, an $\ell$-type shopper who encounters an HW firm will defect to the HW market if the price he is offered in the H(~W) market exceeds $\bar{p}_{H(\sim W)} = \ell_{H(\sim W)} - \ell_{HW} + p^*_{HW}$. And, since $\ell_{H(\sim W)} > \ell_{L(\sim W)}$, it is clear that $\bar{p}_{H(\sim W)} > \bar{p}_{L(\sim W)}$. So the zero-profit restriction at $\bar{p}_{H(\sim W)}$ in the H(~W) market is <u>more</u> restrictive than the corresponding element of (6') if:

$$\frac{F}{\sigma(\bar{p}_{H(\sim W)} - c_H)} < \frac{F}{\sigma(\bar{p}_{L(\sim W)} - c_L)}$$

or, if $\bar{p}_{H(\sim W)} - \bar{p}_{L(\sim W)} - c_H + c_L > 0$. By the definition of the switch prices, this is equivalent to:

$$\ell_{H(\sim W)} - \ell_{L(\sim W)} - c_H + c_L > 0 \tag{7}$$

But now, by the definition of $\ell$-type individuals from Assumption 1 and the fact that $p^*_{i(\sim W)} = c_i + \frac{F}{s}$, $i \in \{L, H\}$,

$\ell_{H(\sim W)} - \ell_{L(\sim W)} - c_H + c_L < 0$, contradicting (7); again, no new restriction arises. The same holds true for prices up to $\ell_{H(\sim W)}$ in the H(~W) market—condition (7) is reproduced.

At the highest price the H(~W) market will bear, $h_{H(\sim W)}$, only h-type nonshoppers remain. Hence,

$$a_h^1 < \frac{F}{\sigma(h_{H(\sim W)} - c_H)} < \frac{F}{\sigma(h_{L(\sim W)} - c_L)} \tag{8}$$

implies that ensuring zero profit at the maximum price in the H(~W) market requires a new restriction. Relation (8) above is equivalent to $h_{H(\sim W)} - c_H > h_{L(\sim W)} - c_L$. But by Assumption 1 and the definition of the h-type individual, $h_{H(\sim W)} - h_{L(\sim W)} - c_H + c_L > 0$, which is consistent with (8). Therefore, the search good two-price equilibrium is relatively more constrained than an analogous experience good equilibrium.

The reason for the additional restriction is clear: h-type individuals are willing to pay up to $h_{H(\sim W)}$ for an H good without a warranty. In a search good world, an H-type firm can drop its warranty, charge $h_{H(\sim W)}$, and get demand from h-type nonshoppers who see that the offered good is H. Hence, an additional restriction is required to prohibit such an entrant from breaking even.

It is worth noting that the additional restrictions on equilibria in the search goods market are not in general entirely the fault of h-type nonshoppers. The above analysis implicitly carried along a rather strong assumption which has been made to simplify the exposition so far—that the capacity of the L-type firms is equal to the capacity of H-type firms, when no warranties are offered. Weakening this assumption adds some new structure. Let the nonwarranty capacities of the L- and H-type firms be $s_L$ and $s_H$, respectively, with $s_L \neq s_H$. Then the equilibrium consumer-firm ratio for both markets shifts to $\sigma' = (s_L - s'_{HW})/(a_\ell^2 - a_h^2)$, with $s_L > s'_{HW}$ and $s'_{HW} = (1 - \pi_H)s_H$ (this is clear from solving equations (4a) and (4b) with the new capacities).

Because the experience and search goods markets have been set up with a common $\sigma$, the shift in $\sigma$ to $\sigma'$ does not change the inequalities which determine the relative restrictiveness of the competitive equilibrium conditions, such as (7). But with $s_H \neq s_L$, the definition of $\ell$-type consumers becomes:

$$\ell_{H(\sim W)} - \ell_{L(\sim W)} - c_H + c_L + \frac{F}{s_L} - \frac{F}{s_H} > 0 \qquad (9)$$

which <u>may</u> be consistent with (7) in the H(~W) search good market if $(F/s_L) - (F/s_H) < 0$, or, if $s_L > s_H$. Notice that this restriction is consistent with the requirement that $s_L > s_{HW}$. Therefore, whenever (9) and (7) are mutually consistent, the conditions to close the H(~W) search good market are more restrictive than the corresponding experience good conditions, for <u>all</u> limit prices.

The following has thus been established:

<u>Proposition 1</u>. In a search good world of two quality types where consumers have heterogeneous preferences for warranties consistent with those in an experience good world having a warranty signaling

equilibrium, the conditions for a two-price, competitive equilibrium in the search-good world are more restrictive than the corresponding experience-good conditions.

The answer to the first question is somewhat counterintuitive. When problems of imperfect search are present, competitive equilibria may be more readily attained (i.e., more shopper/nonshopper combinations are consistent with competitive equilibrium) if consumers are utterly ignorant of product quality *ex ante*, but instead rely upon an equilibrium structure of quality expectations supported by the signaling conditions (2). The reason for this is that the structure of expectations in effect limits the number of potential markets to the number of equilibrium signals, therefore cutting out the possibility of consumers "spilling over" into the remaining markets.

Welfare Comparisons

Finally, it is of interest to be able to make an assessment of the relative welfare of each type of consumer in search and experience goods worlds. In the analysis leading to Proposition 1, it was assumed that each type of good transacted at the same price in each world; it could then be established that the markets in each setting had the same $\sigma$ and the same equilibrium distribution of firms of each type. Thus, the expected welfare of consumers of a given type in the two worlds was the same. In order to have different levels of consumer welfare in search and experience goods markets, it must now be assumed that, in the absence of problems stemming from a lack of information about quality, consumers have homogeneous preferences for warranty protection (i.e., both types of consumers either prefer or eschew warranties).

Suppose, for example, that all consumers prefer warranties, and that preferences for warranties as a quality attribute dominate consumer's relative preferences for the two types of goods without warranties. Then in competitive equilibrium, all consumers will get goods with warranties, but no two-price equilibrium will exist. This is because the addition of a warranty transforms the two heterogeneous goods into a single homogeneous good, and a homogeneous good cannot transact simultaneously at two different prices without violation of the zero-profit condition. The signaling conditions imply that the H-type firms have a "comparative advantage" at supplying warranties, and so the resulting equilibrium will have only the HW market open in both worlds.

Alternatively, consider the case in which warranties are not preferred by either class of consumer. In a world of perfect quality information, the competitive equilibrium would then be $\{p^*_{L(\sim W)}, p_{H(\sim W)}\}$, $n_{L(\sim W)} > 0$, $n_{LW} = 0$; $n_{H(\sim W)} > 0$, $n_{HW} = 0$. In the corresponding experience good world, however, warranties would emerge in the H market, allowing h-type consumers to avoid the lemons equilibrium—with the equilibrium $\{p^*_{L(\sim W)}, p^*_{HW}\}$, $n_{L(\sim W)} > 0$, $n_{LW} = 0$; $n_{H(\sim W)} = 0$, $n_{HW} > 0$. It is clear that the consumer/firm ratio is different for the two equilibria, and that the equilibrium distribution of H- and L-type firms is also different. Who gains, and who loses in a transition from one world to the other? One intuitive answer might be that h-type consumers must lose in a transition from a search to an experience good setting, since they must shoulder the unwanted warranty, while $\ell$-type consumers should be indifferent to the change. Once again, however, our intuition will be seen to fail.

The first step in making a rigorous welfare comparison is to ensure that the necessary condition for a zero-profit equilibrium can be simultaneously satisfied in the two worlds. That is, a range of $a_\ell^2 - a_h^2$ must be found such that the analogy to Lemma 3.1 can be simultaneously established in both worlds. Once again, let Assumption 2 hold, with the added proviso that $s_L > s_H$; to reflect the different preference ordering, Assumption 1 is slightly altered:

Assumption 1′. Let the preference ordering of the two consumer types be given by relations (1b).

Then there immediately follows:

Lemma 3.3. A necessary and sufficient condition for the search and experience good equilibria to have in common an interval of $a_\ell^2 - a_h^2$ of positive measure is $(s_L - s_H)/(s_L - s_{HW}) > s_H/s_L$.

Proof: Label the variables for experience and search goods markets by superscripts e and s, respectively. Then the same calculation carried out in Lemma 3.1 gives for the two sets of equilibrium proportions of firms:

$$n_H^e = \frac{s_L}{s_L - s_{HW}} - \frac{1}{a_\ell^2 - a_h^2}; \qquad n_L^e = \frac{1}{a_\ell^2 - a_h^2} - \frac{s_{HW}}{s_L - s_{HW}} \tag{10}$$

$$n_H^s = \frac{s_L}{s_L - s_H} - \frac{1}{a_\ell^2 - a_h^2}; \qquad n_L^s = \frac{1}{a_\ell^2 - a_\ell^2} - \frac{s_H}{s_L - s_H} \tag{11}$$

Therefore, a necessary condition for two-price zero profit equilibria in the two worlds is:

$$\frac{s_L - s_{HW}}{s_{HW}} > a_\ell^2 - a_h^2 > \frac{s_L - s_{HW}}{s_L} \tag{12}$$

in the experience good market, and:

$$\frac{s_L - s_H}{s_H} > a_\ell^2 - a_h^2 > \frac{s_L - s_H}{s_L} \tag{13}$$

in the search good market. Relations (12) and (13) have a common interval if the upper limit of one lies above the lower limit of the other, or if:

$$\frac{s_L - s_{HW}}{s_L} < \frac{s_L - s_H}{s_H}$$

which is:

$$\frac{s_L - s_H}{s_L - s_{HW}} > \frac{s_H}{s_L} \tag{14}$$

□

Condition (14) is not overly strong. Note that if $\pi_H \Rightarrow 0$, $s_{HW} \Rightarrow s_H$, and the LHS goes to one, so that (14) becomes $s_L > s_H$, which is always true. Therefore, there will exist an interval of $\pi_H > 0$ where the intervals of $a_\ell^2 - a_h^2$ will overlap, and hence, where the two equilibria will satisfy the necessary condition for zero profits.

The joint necessary and sufficient conditions for the two-price competitive equilibria in the two worlds are now just the obvious analogous expressions to (5') and (6'), preventing price rises above competitive levels. The direct comparisons between these conditions will in general be ambiguous, since limit prices, product prices, and consumer/firm ratios differ across the two sets of markets. It is not necessary to perform these comparisons, however, in order to calculate the welfare effects.

Knowing that there exists a common range of $a_\ell^2 - a_h^2$ for both two-price equilibria, we can order the equilibrium distribution of the two type of firms. Assume that (14) holds, and pick some arbitrary value of $a_\ell^2 - a_H^2$ in the common interval. Then (10) and (11) imply that $n_H^e < n_H^s$, and $n_L^e > n_L^s$. Now if an h-type consumer encounters an H-type good and purchases it in the search world, his realized surplus is $S^s(h|H) = h_{H(\sim W)} - p^*_{H(\sim W)}$; if the same consumer must purchase an L-type good his surplus is $S^s(h|L) = h_{L(\sim W)} - p^*_{L(\sim W)}$. In the experience good world, $S^e(h|H) = h_{HW} - p^*_{HW}$, while $S^e(h|L) = S^s(h|L)$. The same kinds of expressions can be defined for the $\ell$-type individuals, $S^e(\ell|i)$, $S^s(\ell|i)$, $i \in \{L,H\}$.

Now consider an p-type nonshopper. The probability that this consumer will encounter an H-type firm on his single shopping trip in the search good market is simply $n_H^s$; he will encounter an L-type firm with probability $n_L^s$. The h-type nonshopper's expected welfare in the search good market is thus $W^s(h,1) = n_H^s S^s(h|H) + n_L^s S^s(h|L)$ $= S^s(h|L) + n_H^s[S^s(h|H) - S^s(h|L)]$. The corresponding expression for the experience good market is

$W^e(h,1) = S^e(h|L) + n_H^e[S^e(h|H) - S^e(h|L)]$. Now, since $S^e(h|L) = S^s(h|L)$, and the expression in brackets is larger in the search world than in the experience good world by Assumption 1', it is found that $W^s(h,1) - W^e(h,1) > 0$, h-type nonshoppers are unambiguously better off in the search world. The "surplus" and "capacity" effects reinforce each other.

An h-type shopper will purchase from an L-type firm only if both of his shopping trips turn up L-type goods. In the search good world, this occurs with probability $(n_L^s)^2$. The probability that the shopper encounters at least one H-type firm is thus $1 - (n_L^s)^2$. So,

$$W^s(h,2) - W^e(h,2) = [1 - (n_L^s)^2]S^s(h|H) - [1 - (n_L^e)^2]S^e(h|H)$$

$$+ (n_L^e)^2 S^s(h|L) - (n_L^e)^2 S^e(h|L)$$

Adding and subtracting one from the final two terms transforms them to $[-1 + (n_L^s)^2]S^s(h|L) + [1 - (n_L^e)^2]S^s(h|L) + S^s(h|L) - S^e(h|L)$; rearranging gives:

$$[W^s - W^e](h,2) = (1 - (n_L^s)^2)[S^s(h|H) - S^s(h|L)]$$

$$- (1 - (n_L^e)^2)[S^e(h|H) - S^e(h|L)]$$

since $S^s(h|L) = S^e(h|L)$. Further, it is known that $S^s(h|H) > S^e(h|H)$, (from the preference ordering,) and $1 - (n_L^s)^2 > 1 - (n_L^e)^2$. Therefore, $W^s(h,2) - W^e(h,2) > 0$. The following has been established:

Proposition 2. If competitive two-price equilibria exist in markets for search and experience goods, subject to Assumptions 1′ and 2, then h-type consumers enjoy unambiguously higher expected welfare in the market for search goods.

Analogously for $\ell$-type nonshoppers:

$$W^s(\ell,1) = S^s(\ell|L) + n_H^s[S^s(\ell|H) - S^s(\ell|L)]$$

$$W^e(\ell,1) = S^e(\ell|L) + n_H^e[S^e(\ell|H) - S^e(\ell|L)]$$

so,

$$W^s(\ell,1) - W^e(\ell,1) = n_H^s S^s(\ell|H) - n_H^e S^e(\ell|H)$$

and now, $n_H^s > n_H^e$, but $S^s(\ell|H) > S^e(\ell|H)$; again, $\ell$-types are unambiguously better off in the search world! The same expression holds for $\ell$-type shoppers, with the n's replaced by $n^2$.

Proposition 3. For the markets subject to Assumptions 1′ and 2, $\ell$-type consumers are also unambiguously better off in a search good market.

Interestingly, even though the $\ell$-type consumer derives exactly the same consumer surplus from his preferred purchase in both worlds, the change in capacity, and hence the change in the likelihood of encountering an H-type firm, contributes an effect which raises the $\ell$-type consumer's expected welfare in the transition from experience to search goods.

## Caveats on the Welfare Results

The preceding sections have explored the properties of market equilibria in "parallel" search and experience goods markets with warranties, conditional on the orderings of consumers' willingness to pay for goods with and without warranties. The motivation for this exercise has been to judge whether the "channel" by which quality information is presented to consumers has an impact on consumer welfare. In a market for search goods, the reliability of a product is distinguishable by inspection, presumably because a consumer is able to infer reliability directly from some preexisting information at his command — for example, his knowledge of the mapping from brand names to quality reputations across firms. In an experience goods market, however, the equivalent kind of quality discrimination is made by observing warranty signals, which, as we have seen, retain their fidelity as signals when search is imperfect. Analysis of the various types of competitive market equilibria suggests that the welfare question turns on the (h-type) consumer's relative tolerance for warranty coverage.

But this may not be the whole story. Although warranties continue to serve as quality signals in a world of imperfect search, the transition from search to experience goods markets involves subtle capacity effects when h-type consumers exhibit (1b)-type preferences (i.e., they have a relatively low tolerance for warranties). These capacity effects alter the consumer/firm ratio across markets, and therefore may affect the character of market equilibrium observed when search and experience goods are transacted.

This effect can be observed most clearly in a single-price equilibrium where only H-type goods are traded (a similar, though more complicated, argument could be constructed in a two-price equilibrium setting). In a world of experience goods, the observed equilibrium is $\{p^*_{HW}\}$, $n_{HW} = 1$, $\sigma = s_{HW}$; in the corresponding search equilibrium, $\{p^*_{H(\sim W)}\}$, $n_{H(\sim W)} = 1$, $\sigma = s$. Note the direction of the capacity effect: as the information mode is diverted from direct inspection to warranty comparison, the size of firms decreases (the equilibrium consumer/firm ratio falls). Thus we might hope that, if a particular competitive equilibrium exists in a market for search goods, the conditions for equilibrium will be reinforced if warranties are made to conduct the quality information to consumers. Therefore, any mix of consumers consistent with a particular market equilibrium in a search world will find themselves in the qualitatively similar equilibrium in an experience good world, and the previously-derived welfare comparisons will stand. Unfortunately, such a straightforward result cannot in general be established.

To see this, examine the necessary and sufficient conditions for a single-price (H-good) equilibrium in the search/equilibrium world setting. Begin with the search world as a basis for comparison. First, entry into the L(~W) market must be blockaded. If only the H market is open, then the consumer mix is such that no L-type entrant can charge a price up to $\bar{p}_L^s$ in the L(~W) market and break even, or, $\Pi^s_{L(\sim W)}(\bar{p}_L^s = s(a^1 + 2a^2_\ell)[\bar{p}_L^s - c_1] < 0$. But now the following holds:

<u>Claim</u>: $\bar{p}_L^e > \bar{p}_L^s$.

<u>Proof</u>: By the definition of switch prices in the two markets, $\bar{p}_L^e - \bar{p}_L^s = \ell_{H(\sim W)} - p^*_{H(\sim W)} - (\ell_{HW} - p^*_{HW}) > 0$, where the inequality follows from the preference ordering (1b).

Clearly, then, two effects occur in the transition to the experience good market: the effective capacity of the market falls, but the switch price <u>rises</u>, reflecting the fact that as warranties are added (when warranty tolerance is low), the H good becomes a less attractive substitute for the L good. Thus, the entrant has more latitude to raise his price before he losses demand from $\ell$-type shoppers. This implies that closing the L(~W) market to entry in the experience good world requires a new restriction.

Closure of the L(~W) market in the search world implies closure in the experience good world if $\Pi^{e}_{L(\sim W)}(\bar{p}^{e}_{L}) - \Pi^{s}_{L(\sim W)}(\bar{p}^{s}_{L}) \leq 0$ for a distribution of consumers associated with a $\{p^{*}_{H(\sim W)}\}$ single price equilibrium in a search market, or,

$$s_{HW}[\bar{p}^{e}_{L} - c_{L}] \leq s[\bar{p}^{s}_{L} - c_{L}]$$

equivalently,

$$\bar{p}^{e}_{L} - \bar{p}^{S}_{L} \leq \pi_{H}[\bar{p}^{e}_{L} - c_{L}]$$

A slight rearrangement makes the interpretation of the new restriction clear -- multiply both sides by s, and note that $\pi_{H}s = s - s_{HW}$. Then the restriction requires that the pure incremental revenue effect of the switch price rise in the experience good market must be offset by the decline in effective capacity caused by the addition of the warranty.

This is the only new restriction generated in the L(~W) market in the transition to an experience good setting. It is easy to see that at higher prices in the L(~W) market, for example, at $\ell_{L}$ where only non-shoppers remain, negative profits in the search world imply negative profits in the experience world.

A similar, slightly more complicated situation holds in the (open) market for H-type goods. Again, the necessary and sufficient conditions for competitive H-good equilibrium in a search good market do not in general imply the existence of the corresponding equilibrium in a market where warranties carry quality information. The two kinds of markets in effect transact different "versions" of the H good, and so limit prices, costs, and capacities change in the transition.

To derive the new restrictions in the H market, consider the $\ell$-type consumer's limit price in the search world. Assume the search-world consumer proportion $a^1$ is such that no firm can raise its offered price to $\ell_{H(\sim W)}$ and break even, that is, $\Pi^s_{H(\sim W)}(\ell_{H(\sim W)}) < 0$. Then the same holds in the experience good world if:

$$\Pi^e_{HW}(\ell_{HW}) - \Pi^s_{H(\sim W)}(\ell_H) < 0$$

or,

$$s_{HW}[\ell_{HW} - c_{HW}] - s[\ell_{H(\sim W)} - c_H] \leq \bar{F}/a^1$$

but now, from the definition of type (1b) preferences,

$$\ell_{HW} - p^*_{HW} - (\ell_{H(\sim W)} - p^*_{H(\sim W)} \leq 0$$

which can be easily rearranged to yield:

$$s_{HW}[\ell_{HW} - c_{HW}] - s[\ell_{H(\sim W)} - c_H] < [\bar{F} + \pi_H F]/(1 - \pi_H)$$

a comparison of the profit condition and the above implication of the preference ordering indicates that zero profits at the limit price for $\ell$-type consumers in the experience good world requires:

$$a^1 < (1 - \pi_H)/[1 + \pi_H(F/\bar{F})]$$

This new constraint is less likely to bind the higher the reliability of the H-type good, for a given set of fixed costs. Clearly, though, to the extent it is binding, it may modify the shopper/nonshopper combinations consistent with competitive equilibria in the search versus experience market.

The capacity and switch price effects associated with adding or taking away warranty signals may have an important effect on the nature of equilibrium across markets. And, because consumer welfare is plainly affected by any tendency for prices to rise above those prevailing in competitive markets, the "channel" by which quality information is presented to consumers may make a difference, for a reasonable subset of combinations of consumer types, whatever the consumers' relative willingness to pay for warranty protection.

As a final point, it is even possible to show that the capacity effects explored above can result in a transformation between distinct "species" of competitive equilibria; that is, a single-price equilibrium in, for example, a search world may evolve into a two-price equilibrium in the directly corresponding experience good world. Such an occurrence, which does not seem pathological, has nevertheless quite ambiguous welfare effects. Consider:

Example. Let the firms operating in the two markets be defined by the following parameters: $\pi_h = 0.05$, $\pi_L = 0.25$; $s_L = 20$, $s_H = 16$; $c_L = \$13.00$, $c_H = \$15.00$; $F = \$100.00$, $\overline{F} = \$10.00$. The signaling conditions are clearly satisfied, since $c_{HW} = \$15.79$, while $c_{LW} = \$17.33$; the competitive market prices are $p^*_{L(\sim W)} = \$18.00$, $p^*_{H(\sim W)} = \$21.25$, $p^*_{HW} = \$23.03$, $p^*_{LW} = \$24.67$. Let the set of limit prices defining consumers be: $\ell_{L(\sim W)}$ = \$33.00, $\ell_{H(\sim W)}$ = \$36.00, $\ell_{HW}$ = \$37.50, and $h_{L(\sim W)}$ = \$47.00, $h_{H(\sim W)}$ = \$50.50, $h_{HW}$ = \$52.20. It is easily verified (referring to the competitive market prices), that these consumers exhibit type (1b) preferences.

Now in the search world, the conditions for a single price competitive equilibrium with the L(~W) market only are: $a^1 < 0.24$, $a_h^1 < 0.14$, and $a^1 + 2a_h^2 < 0.77$ (this is, of course, the subset of the market constraints that are binding).

Next consider the requirements for a <u>two-price</u> competitive equilibrium in a market where warranties distinguish the quality levels of goods. Because the L market firms have higher capacities than their H-type counterparts, the two-price equilibrium "balancing" constraints (equation [3] from Section 3.3) will entail a restriction on $a_\ell^2 - a_h^2$. For the parameters fixed here, $0.24 < a_\ell^2 - a_h^2 < 0.32$ is necessary for experience-good two-price equilibrium. If a value less than approximately 0.301 is selected for $a_\ell^2 - a_h^2$, the consumer/firm ratio in the experience good market will be less than 16.00. Therefore, the necessary balancing condition in the experience good market will <u>not</u> translate to the analogous condition in the search market. The only competitive equilibrium possible there will be a one-price equilibrium.

It remains only to show that there exists a nontrivial partitioning of consumers across types and shopping intensities such that the same partitioning is consistent with an L-good only competitive equilibrium in the search world <u>and</u> with a two-price competitive equilibrium in the "parallel" experience good world. With a value of 0.301 chosen for $a^2_\ell - a^2_h$, $\sigma$ is fixed at 15.95, which in turn implies (equations [10]), that $n^e_L = 0.16$ and $n^e_H = 0.84$; the zero-profit conditions within the experience markets now add the restrictions $a^1 + 0.32a^2_h < 0.89$, $a^1 + 1.68a^2_\ell < 1.135$, $a^1 < 0.31$, and $a^1_h < 0.18$. It is not difficult to check that the choice: $a^2_\ell = 0.550$, $a^2_h = 0.249$ (implying $a^1 = 0.201$), with $a^1_h < 0.14$, simultaneously satisfies all necessary and sufficient conditions in the two worlds. Further, the equilibria continue to hold within a (small) neighborhood of $a^2_\ell - a^2_h = 0.301$, and for a considerably larger neighborhood of the selected value of $a^1$, holding $a^2_\ell - a^2_h$ relatively constant. Therefore, the conditions for this type of equilibrium transition, while certainly restrictive, are not in general pathological. Note in particular that the experience world two-price competitive equilibrium associated with an L-type only search good market has, in this case, a large majority of _H-type_ firms!

## 3.5 CONCLUSIONS

This essay has attempted to demonstrate three points. First, the ability of consumer warranties to act as informative quality signals is not in itself compromised by the presence of imperfect search. The empirically observed lack of consistent evidence for a signaling function for consumer warranties cannot be explained by an appeal to the notion that search is a costly activity for consumers. However, the necessity of transmitting quality information via warranties can, depending on the specifics of consumer preferences, affect the capacities of markets so as to alter (in some cases profoundly) the nature of market equilibrium. The welfare effects of costly warranty signals turn on the interaction between their value as information and their potential disruption of markets where consumer search is imperfect.

The second and third results of this essay are closely related, and contain the common thread of an attempt to draw together the equilibrium search literature and the long-standing signaling literature, heretofore grounded in an assumption of costless search. The second result is that the necessary and sufficient conditions for competitive equilibrium in the presence of imperfect search may be more readily attained in a market for experience goods with warranties acting as proxies for product quality than in a market for search goods. Thus, provided that the signaling cost conditions hold (and they are relatively innocuous), competitive equilibrium may be more readily reinforced by the simple expedient of making consumer warranties more readily accessible and understandable to consumers. However, if consumers' preferences for the underlying relative qualities of goods dominate their desires for comprehensive warranties as an added quality feature, the third result states that <u>all</u> consumers will tend to strictly prefer the equilibrium resulting in the search good world (if it is attainable) over the experience good equilibrium. A possible consequence of this might be that as consumers become more informed about product quality through venues other than warranty signals (such as a process of reputation validation) the pattern of warranty-as-product-signal would tend to erode, as the good takes on more search characteristics.

This essay has generally treated consumer markets as if the goods transacted within them manifest search or experience characteristics exclusively. In reality, of course, a given consumer durable good (such as an integrated stereo amplifier) may present search and experience attributes to consumers simultaneously -- for example, consumers can tell by pre-purchase inspection whether the tuner section has a multipath elimination circuit, and hence an inference can be made about the amplifier's performance in receiving remote FM stations. Other aspects of the unit's performance clearly remain in the experience category. The welfare comparison carried on above between "parallel" search and experience goods markets implicitly rests on the existence of alternative information channels open to consumers at the time of purchase, which allow them to make inferences about quality, and which might substitute for warranties in that regard.

The most important "alternative channel" is the process of brand-name identification bolstered by advertising. Through advertising, firms make investments in brand-name reputation; the cost of misleading or deceptive advertising is the erosion of this capital investment. Thus, as Nelson [1974] proposes, the intensity of a firm's advertising is a signal of sorts, representing a firm's continuing investment in its good name. But it is clear that this is a different kind of signal than a warrantly, more diffuse, and perhaps less product-specific. One result of successful advertising is a fulfilled expectation about a product's quality in the minds of consumers. Such expectations may create external benefits, in the form of a favorable reception (initially, at least), for newly-launched products carrying the firm's name. Such external benefits are not conferred by product-specific comprehensive warranties; indeed, a more contractual promise of quality may be more expensive to the firm than initially reckoned if a new product takes time to attain a standard of quality.

Therefore, this study can give a tentative prediction about the kinds of consumer goods markets in which warranties might be expected to serve as quality signals, and in which they would not. Warranties may perform well as quality indicators in markets where the leading attribute of product quality is reliability, and where there is some objective, measurable basis for gauging it. Further, consumers should place a high premium on enhanced reliability, so much so that they are willing to pay premium prices for the extra reliability implied by comprehensive warranties. Of course, performance on the contractual terms of warranties must be swift and frictionless. On the other hand, warranty terms are likely to degenerate across firms if i), consumers are unwilling to pay for the reliability premium of comprehensive warranties, or if reliability is not a leading element of perceived quality; and ii), brand name advertising is pervasive, has high credibility with consumers, and is regarded by firms as conferring positive benefits across (as well as within) product lines.

If direct policy prescriptions are not forthcoming from this tentative exploration, it should be clear that important connections can be and need to be drawn between goods with search and experience characteristics. To the extent that most consumer goods have attributes of both, leaving either out of the analysis of market equilibrium gives an incomplete picture. The theories of consumer product warranties associated with search and experience goods are not supplanting, but supplementary.

# FOOTNOTES FOR CHAPTER III

1. For a review of the current regulatory framework and an important critique from the vantage point of equilibrium search theory, see Schwartz and Wilde [1979].

2. Two empirical studies are Gerner and Bryant [1981] and Priest [1981]. Both studies, however, are subject to similar difficulties: adjustment must be made for the fact that warranties are typically <u>multidimensional</u> contracts (including, for example, separate provisions for parts and labor coverage); data is aggregated across brand names for the various goods investigated, so that no connection can be made between a level of warranty coverage and some measure of a good's intrinsic reliability, and so on.

3. A theory of the consumer warranty based upon a notion of comparative advantage is set forth in Priest [1981]. The theory holds that the observed pattern of warranty coverage is dictated by which party to a sales contract, the consumer or the firm, faces the lower cost of insuring against product-related defects or failures. Thus firms will warrant against breakdowns of refrigerator motors, but not refrigerator door hinges, failures of which are more dependent upon the pattern of consumer use than on factory quality control

4. Unlike the sequential search models, it is not required that consumers know the distribution of product types and prices before beginning to shop; the weaker assumption made here is that consumers are generally aware of the range of product reliabilities available, but are unsure as to which products correspond to which reliabilities.

5. As expressed in Schwartz and Wilde [1982b], it is convenient to consider warranties as available in two "flavors," limited and comprehensive -- the model specified seeks to associate comprehensive warranties with quality signals.

6. See, for example, Gerner and Bryant [1981].

7. Wilde and Schwartz [1979].

8. This means of modeling the interaction of competitive search markets originated in Schwartz and Wilde [1982a].

9. One could, of course, work out all of the cases stemming from the feasible permutations of limit and switch prices, but the equilibria described would vary only in details from the case selected here.

10. That is, this is a case which (somewhat artifically) reduces to

the world analyzed in Schwartz and Wilde [1982b]. In this simple model of a warranty contract, adding a comprehensive warranty to any product creates a homogeneous good with a zero failure probability.

## REFERENCES FOR CHAPTER III

Gerner, J. and Bryant, W., "Appliance Warranties as a Market Signal?," 15 Journal of Consumer Affairs 1 (1981), 75-86.

Grossman, S., "The Informational Role of Warranties and Private Disclosure About Product Quality," The Journal of Law and Economics 24 (1981), 461-84.

Nelson, P., "Advertising As Information," 82 Journal of Political Economy 4 (1974), 729-754.

Priest, G. L., "A Theory of the Consumer Product Warranty," The Yale Law Journal 90 (1981), 1297-1352.

Salop, S., and Stiglitz, J., "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," Review of Economic Studies 44 (1977), 493-510.

Schwartz, A., and Wilde, L. L., "Competitive Equilibria in Markets for Heterogeneous Goods Under Imperfect Information: A Theoretical Analysis with Policy Implications," Bell Journal of Economics 12 (1982), 181-93.

__________, "Consumer Markets for Warranties," SSWP No. 445, Caltech, (Dec. 1982).

Spence, A. M., Market Signaling: Informational Transfer in Hiring and

Related Screening Processes (1974).

Wilde, L. L., and Schwartz, A., "Equilibrium Comparison Shopping," The Review of Economic Studies 46 (1979), 543-53.

# APPENDIX

In the example of Section 1.4, the financial signal satisfying the neccessary condition for compensation maximization under the incentive rule (3) for the manager of a firm $\theta = (t_1, t_2)$ is given by

$$\frac{\partial F(\theta)}{\partial t_1} + \frac{\partial F(\theta)}{\partial t_2} = \frac{c_0(t_2 - t_1)}{2c_1 L^*}$$

$$F(0, t_2) = \frac{c_0}{4c_1 L^*}[t_2^2 + \beta]$$

$$F(c, c) = 0$$

which yields:

$$F(t_1, t_2) = \frac{c_0}{4c_1 L^*}[t_2^2 - t_1^2]$$

<u>Proof</u>: The standard solution method for first order P.D.E.s with linear partial derivative terms will be applied; the same method holds for more general Von Neumann-Morgenstern preferences (where the P.D.E. may involve terms nonlinear in F.) The P.D.E. and the initial condition are written as an equivalent system of ordinary differential equations, and then, under the correct conditions, uniqueness of the P.D.E. follows from the general uniqueness theorem for ordinary differential equations.

First, the initial path can be written in parametric form.

For $\tau$ defined in the interval $0 \leq \tau \leq 1$, The initial condition becomes

$$t_1 = 0 \qquad t_2 = \tau \qquad F(0,t_2) = \frac{c_0}{4c_1L^*}[\tau^2 + \beta]$$

Now, fix some $\tau$ and move off the initial path onto the integral surface in the characteristic direction. Parameterize the characteristic curve by $\sigma$. Provided that the initial path is nowhere characteristic,* then the P.D.E. is decomposed into the following autonomous system of O.D.E.s in $\sigma$ (see John [1982]):

$$\frac{dt_1(\sigma)}{d\sigma} = 1$$

$$\frac{dt_2(\sigma)}{d\sigma} = 1$$

$$\frac{dF(\theta(\sigma))}{d\sigma} = \frac{c_0}{2c_1L}[t_2(\sigma) - t_1(\sigma)]$$

The solution of the first two O.D.E.s, with the initial conditions in $\tau$ above, is easily seen to be $t_1(\sigma,\tau) = \sigma$ and $t_2(\sigma,\tau) = \sigma + \tau$, so that the third becomes $F'(\sigma,\tau) = (c_0/2c_1L^*)\tau$; adding the initial condition and solving gives:

$$F(\sigma,\tau) = \frac{c_0}{2c_1L^*}[\tau\sigma + \frac{(\tau^2 + \beta)}{2}]$$

* The initial path is nowhere characteristic if the Jacobian of the transformation from the $(\tau,\sigma)$ parameterization to the $(t_1,t_2)$ coordinates is non-singular. Here,

$$J_{\tau,\sigma}(t_1,t_2)\big|_{\sigma=0} = \frac{\partial t_1}{\partial \tau} - \frac{\partial t_2}{\partial \tau} = -1$$

and, converting back to the original variables,

$$F(t_1, t_2) = \frac{c_0}{2c_1L^*}[t_1(t_2 - t_1) + \frac{1}{2}((t_2 - t_1)^2 + \beta)]$$

the boundary condition $F(c,c) = 0$ sets $\beta = 0$; simplifying the above gives equation (9).

The fact that the initial curve is continuously differentiable and non- characteristic guarantees that the solution (10) holds in some neighborhood of the initial path. In this case, the characteristic curves are lines of constant slope, and thus, the solution holds for an arbitrary bounded set $\theta$. □