THREE ESSAYS IN LAW AND ECONOMICS

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Asha Sadanand

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ABSTRACT

This thesis consists of three essays, which are concerned with policies for economic situations characterized by informationally weak buyers.

The first and third are in related areas. They examine how the equilibrium distribution of market prices is affected when consumers are unimformed about various aspects of the market. The classical explanation of how competitive equilibrium can persist relies heavily on all consumers being perfectly informed about the prices offered in the market. The first essay generalizes the model due to Wilde and Schwartz (1979) which introduced the notion that a sufficient proportion of consumers need to be comparing prices in order that a competitive equilibrium obtains. They showed this under strong assumptions about cost and demand functions. Here, the result is generalized to allow downward sloping demand and U-shaped cost curves. Some comparative stitics are developed.

The second essay uses the simple techniques of optimization to assess how well the remedies of lost profits, market damages and specific performance compensate the seller when a buyer breaches a contract. The conclusion is that in general lost profits overcompensates, and market damages undercompensates; while specific performance always compensates exactly. The merits of these remedies on the basis of economic efficiency and implementation costs are

also dicussed.

The final essay explores how heterogeneous product markets behave when consumers are imperfectly informed about quality. Three models are introduced with varying assumptions about the nature of the lack of information about quality among consumers. If consumers can gain information about quality as they shop, then a large enough proportion of shoppers is sufficient to guarantee a competitive outcome. The critical proportion required is less when a larger proportion of consumers is naturally informed. Lastly, if the state of information does not improve with shopping, competitive outcomes can be generated only by educating the consumers.

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A GENERALIZED MODEL OF PRICING FOR HOMOGENEOUS GOODS UNDER IMPERFECT INFORMATION

1. INTRODUCTION

In a paper published in the Review of Economic Studies in 1979, Wilde and Schwartz explored the extent to which buyers must comparison shop in order that the market generate competitive or near-competitive prices. The model developed in that paper made strong assumptions about the nature of consumers' demand curves and firms' production technologies. The primary purpose of this paper is to explore whether the conclusions of Wilde and Schwartz are robust to the relaxation of those assumptions. Formally it is similar to Braverman's (1980) extension of Salop and Stiglitz's (1977) model of monopolistically competitive price dispersion.

The basic Wilde and Schwartz model posits two types of buyers, those who buy from the first store they enter and those who buy from the store offering the lowest price among a sample of n stores (where $n \geq 2$). All buyers demand exactly one unit of the good and all have a common limit price above which they demand zero units. Firms are identical. Each has a fixed cost of production, a constant marginal cost and a capacity constraint. In equilibrium, free entry forces all firms to earn zero expected profits.

Wilde and Schwartz show that if sufficiently many buyers comparison shop (where "sufficient" is defined in terms of the underlying parameters of the model), then a competitive outcome obtains. With slightly fewer comparison shoppers, a few firms deviate to high prices but most remain at the competitive price. Eventually, as the number of comparison shoppers continues to fall, prices are fully dispersed above the competitive price, and ultimately converge to the monopoly price. In their model, the monopoly price coincides with the monopolistically competitive price because of the "step-function" demand curves.

This paper consists of two parts. The first demonstrates that the qualitative properties of the above results hold under more general conditions; in particular, they hold for typical, downward sloping demand curves and for any u-shaped average cost curves. The primary difference is that while the highest price in any noncompetitive equilibrium is again the monopolistically competitive price, in this case it is less than the monopoly price. The second part investigates properties of demand curves and average cost curves which make the competitive outcome more or less likely. In it we argue that the critical proportion of comparison shoppers needed to generate a competitive equilibrium falls as demand becomes more elastic or average costs become more inelastic.

2. THE GENERALIZED MODEL

Each period a large group of buyers enters the market. The group is partitioned into two types, A_1 and A_n where $1-\sigma$ is the proportion of the total who are of type A_1 and σ is the proportion who are of type A_n. The members of A_n sample exactly n firms ($n \geq 2$) and then buy from the firm offering the lowest price among those they have sampled. Members of A_1 do not comparison shop, but instead buy from the first firm they sample. At the start of a new period the previous period's buyers exit with their purchases (or a "rain check") and a fresh group of buyers with the same σ ratio arrive. Each consumer has a demand curve $f(p)$, with $f'(p) < 0$.

Firms exist over time and maximize expected profits by choosing the price at which to sell. All firms are identical, with total variable costs $T(q)$ and fixed costs F. We shall assume increasing marginal costs, $T''(q) > 0$, to generate u-shaped average cost curves. Let $A(q) = [T(q) + F]/q$ be the average cost curve and $\tilde{s}$ be the capacity which minimizes it. In equilibrium all firms earn zero expected profits since higher profit levels are eroded away by entry. Thus the consumer/firm ratio, denoted by α, is endogenous to the model. We define α_1 as the type A_1 to firm ratio, which is thus equal to $\alpha(1-\sigma)$. Similarly $\alpha_n = \alpha\sigma$. Equilibrium is then defined in the usual way -- a price distribution and a consumer/firm ratio such that all firms earn zero expected profits and no firm can make positive profits by changing its price, given the prices of other firms. Two technical assumptions complete the model. We require that it is feasible for firms to be able to enter this market; i.e., there is a consumer/firm ratio α such that $A(\alpha f(p)) \leq p$. We will also assume that for any $\alpha > 0$, $\alpha f(p)$ intersects the average cost curve <u>at most twice</u>. This assumption preserves the essence of the generalization but keeps the mathematics tractable.

The generalized model yields the following theorem, which is proved in the appendix.

<u>Theorem</u>: Under the assumptions described above,

(i) a unique single price equilibrium occurs at $p^* = A(s)$ — the competitive equilibrium -- if and only if

$$\sigma \geq 1 - \frac{f(A(s))}{s}\,\frac{A^{-1}(p)}{f(p)} \qquad \text{for all } p \geq p^*$$

where $A^{-1}(p)$ refers to the left-hand branch of A;

(ii) a unique nondegenerate equilibrium G(p), with a mass point at p^*, occurs if and only if

$$\frac{s}{\alpha^N f(p^*)} - 1 \quad \frac{1}{n-1} < \sigma < 1 - \frac{f(A(s))}{s}\,\frac{A^{-1}(p)}{f(p)} \qquad \text{for some p.}$$

In this case

$$G(p) = \begin{cases} 0 & p < p^* \\ G(p^*) & p^* \leq p < \sim \\ 1 - \quad \dfrac{A^{-1}(p)}{f(p)} - \alpha^N(1-\sigma) \quad \dfrac{1}{\alpha^N \sigma}^{\frac{1}{n-1}} & \sim \leq p \leq p_u \\ 1 & p_u < p \end{cases}$$

where α^N is the largest consumer/firm ratio such that

$$\alpha^N(1 - \sigma)f(p) \leq A^{-1}(p) \quad \text{for all } p \geq p^*$$

and p_u is defined by

$$\alpha^N(1 - \sigma)f(p_u) = A^{-1}(p_u).$$

In the definition of $G(p)$, $G(p^*)$ is the size of the mass point at p^* and $\tilde{p}$ is the maximum price such that $\tilde{p} > p^*$ and $G(p^*) = G(\tilde{p})$; i.e. it is defined by

$$G(p^*) = 1 - {\frac{A^{-1}(\tilde{p})}{f(\tilde{p})} - \alpha^N(1-\sigma) \quad \frac{1}{\alpha^N\sigma}}^{\frac{1}{n-1}}.$$

(iii) a unique nondegenerate equilibrium $G(p)$, with no mass point, occurs if and only if

$$\sigma \lesssim {\frac{s}{\alpha^N f(p^*)} - 1}^{\frac{1}{n-1}}.$$

In this case

$$G(p) = \begin{cases} 0 & p < p_1 \\ 1 - {\frac{A^{-1}(p)}{f(p)} - \alpha^N(1-\sigma) \quad \frac{1}{\alpha^N\sigma}}^{\frac{1}{n-1}} & p_1 \lesssim p \lesssim p_u \\ 1 & p_u < p \end{cases}$$

where α^N and p_u are as in case (ii) and p_1 is defined by

$$(1 - \sigma + n\sigma)\alpha^N f(p_1) = A^{-1}(p_1)$$

where $A^{-1}(p)$ refers to the left-hand branch of $A(q)$.

<u>Proof</u>: See Appendix.

The qualitative properties of the equilibria described in the theorem are much the same as those derived in the earlier Wilde and Schwartz model. The conditions for each case seem complicated but can be easily understood by the use of some simple diagrams.

In the generalized model, the underlying structural features are summarized by the individual demand curve $f(p)$ and the average cost curve $A(q)$. Expected demand facing any firm will always be proportional to $f(p)$. For example, consider the competitive equilibrium. In the competitive equilibrium all firms charge p^* = minimum average cost, and produce s units, where $s = A^{-1}(p^*)$. Firms enter until all make zero expected profits. Since firms all charge p^*, each gets an equal share of the consumers. Hence if the consumer/firm ratio is α, each faces an expected demand curve of $\alpha f(p)$. Zero profits at (p^*,s) thus requires $\alpha f(p^*) = s$. This <u>defines</u> α^C, the competitive consumer/firm ratio (see Figure 1).

Now suppose the consumer firm ratio is α^C and all firms produce s units priced at p^*. Whether this is an equilibrium depends on the potential for making positive profits by deviating. If a firm raises its price above p^*, it loses all shoppers. Since nonshoppers comprise $(1-\sigma)$ of the population, a deviant faces an expected demand curve given by $(1-\sigma)\alpha^C f(p)$ for prices greater than p^*. Figure 1 also illustrates this demand curve. Positive profits are possible for the deviant firm if and only if $(1-\sigma)\alpha^C f(p)$ intersects the average cost curve. The condition that it does not — a necessary condition for competitive equilibrium — is therefore that $A[(1-\sigma)\alpha^C f(p)] \geq p$, or $(1-\sigma)\alpha^C f(p) \geq A^{-1}(p)$, where A^{-1} is defined on the left-hand branch of $A(q)$. But $\alpha^C = s/f(p^*) = s/f(A(s))$. Hence the critical constraint on shoppers is

$$\sigma \geq 1 - \frac{f(A(s))}{s} \frac{A^{-1}(p)}{p} \qquad \text{for all } p \geq p^* . \qquad (1)$$

Now suppose condition (1) is not met. This means $(1 - \sigma)\alpha^C f(p)$ intersects $A(q)$. Firms will then enter at prices above p^*. This will reduce α and shift the curve $(1 - \sigma)\alpha f(p)$ to the left. When it is just tangent to $A(q)$ no further profitable entry is possible. Call the associated consumer/firm ratio α^N (see Figure 2).

It is shown in the appendix that mass points are only possible at p^*. Hence $(1 - \sigma)\alpha^N f(p)$ describes the expected demand of the firm charging the highest price, p_u. Given zero profits, this price must

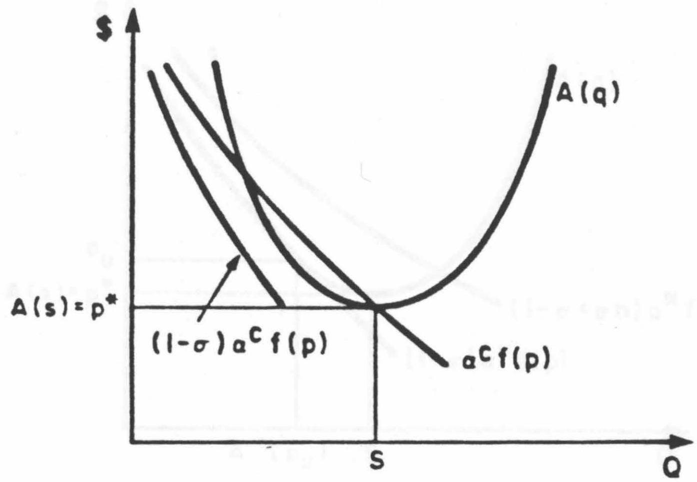

FIGURE 1

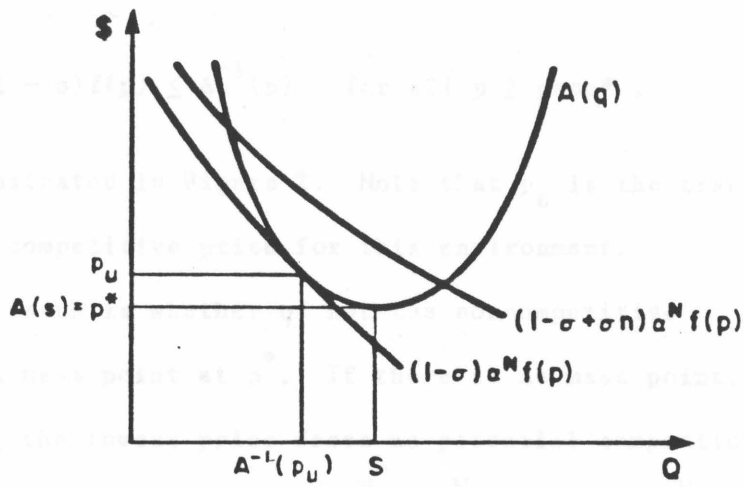

FIGURE 2

by definition be at the tangency point between $(1 - \sigma)\alpha^N f(p)$ and $A(q)$; i.e.,

$$\alpha^N(1 - \sigma) f(p_u) = A^{-1}(p_u)$$

and

$$\alpha^N(1 - \sigma) f(p) \leq A^{-1}(p) \quad \text{for all } p \geq \text{psup}^* .$$

This is also illustrated in Figure 2. Note that p_u is the traditional monopolistically competitive price for this environment.

The next issue is whether or not the noncompetitive equilibrium has a mass point at p^*. If there is no mass point, then the firm offering the lowest price faces no potential competitors. Hence that firm's expected demand is $[\alpha_1^N + n\alpha_n^N]f(p)$ where $\alpha_1^N = (1-\sigma)\alpha^N$ and $\alpha_n^N = \sigma\alpha^N$; i.e., its expected demand is $(1 - \sigma + n\sigma)\alpha^N f(p)$. Figure 2 illustrates this curve. Clearly an equilibrium of this type is impossible if $(1 - \sigma + n\sigma)\alpha^N f(p^*) > s$ (i.e. if the expected demand curve cuts $A(q)$ to the right of s), since in that case zero profits would require the firm offering the lowest price to produce at a quantity greater than s. This cannot be profit maximizing. Thus, the condition $(1 - \sigma + n\sigma)\alpha^N f(p^*) > s$ implies a mass point exists (necessarily at p^*). If it does not hold, i.e. $(1 - \sigma + n\sigma)\alpha^N f(p)$ intersects $A(q)$ to the left of s, or

$$\sigma \leq \left[\frac{s}{\alpha^N f(p^*)} - 1\right] \frac{1}{n-1} , \tag{2}$$

then $G(p_1) = 0$ and p_1 is given by $(1 - \sigma + n\sigma)\alpha^N f(p_1) = A^{-1}(p_1)$.

If (1) and (2) do not hold, then firms "mass up" at p^*. As in the Wilde and Schwartz model, a gap then appears between p^* and the next lowest price, $\tilde{\ }$ (these are prices which are too low to compensate the deviant firm for the loss of most shoppers to firms charging p^*). Above $\tilde{\ }$ firms are continuously distributed up to and including the monopolistically competitive price p_u.

3. CHANGES IN f(p) and A(q).

One of the important features of the model developed by Wilde and Schwartz, and generalized herein, is that it allows for competitive equilibria even if all consumers are not perfectly informed. As a result, the model has turned out to be very useful as the basis for a normative analysis of interventions in consumer product and financial markets. In particular, it provides a set of criteria for determining when a market is likely to be competitive, and a characterization of the form noncompetitive outcomes are likely to take (Schwartz and Wilde, 1979). The usefulness of the present generalization, beyond showing the basic model is robust, is to be able to relate properties of demand curves and average cost curves to the likelihood that a market operating under conditions of imperfect information will be competitive.

Initially, consider changes in average costs. Suppose that s and $A(s) = p^*$ remain constant, but that $|A'(q)|$ increases. In other words, hold minimum efficient scale and the competitive price constant, but let the average cost curve get "steeper." (This is represented by the shift from $A(q)$ to $\bar{A}(q)$ in Figure 3.)

The necessary and sufficient condition for a competitive equilibrium is equivalent to requiring that $\alpha_1^C f(p)$ and $A(q)$ <u>not</u> intersect. Clearly this is more likely when $A(q)$ is steeper (holding s and $A(s)$ constant). Thus a less elastic cost curve makes the single price equilibrium at p^* more likely. It also reduces the dispersion of prices in the noncompetitive equilibrium since p_u will fall (see Figure 4 and recall that p_u is defined by the tangency between $\alpha_1^N f(p)$ and the average cost curve).

If we perturb demand opposite results obtain. In particular the more elastic demand is, the more likely it is that a competitive equilibrium obtains. This can be seen best with an example using constant elasticity demand curves.

Let $f(p) = \delta p^{-\gamma}$. Then the necessary and sufficient condition for a competitive outcome becomes

$$\sigma \geq 1 - \frac{\delta A(s)^{-\gamma}}{s} \cdot \frac{A^{-1}(p)}{\delta p^{-\gamma}} \qquad \text{for all } p \geq p^* ,$$

or

$$\sigma \geq 1 - \left(\frac{p}{A(s)}\right)^{\gamma} \cdot \frac{A^{-1}(p)}{s} \qquad \text{for all } p \geq p^* .$$

For all p such that the right hand side of this inequality is not negative (essentially $p \leq p_u$), it must be that an increase in γ makes the inequality more likely to be satisfied since $p > A(s) = p^*$. Again, a more elastic demand curve also implies less price dispersion in the noncompetitive equilibria.

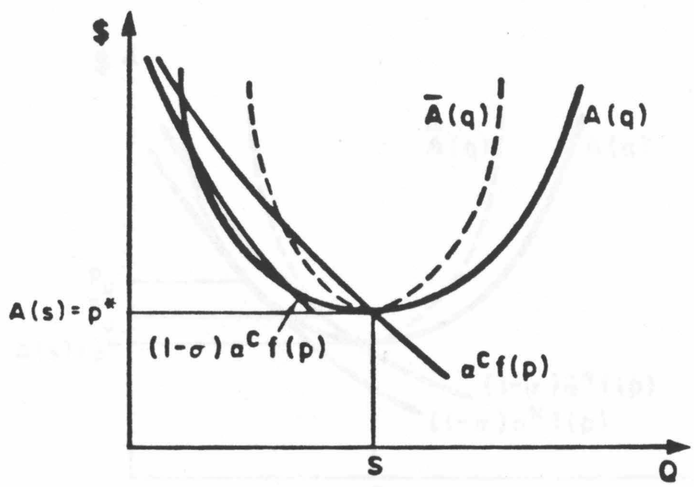

FIGURE 3

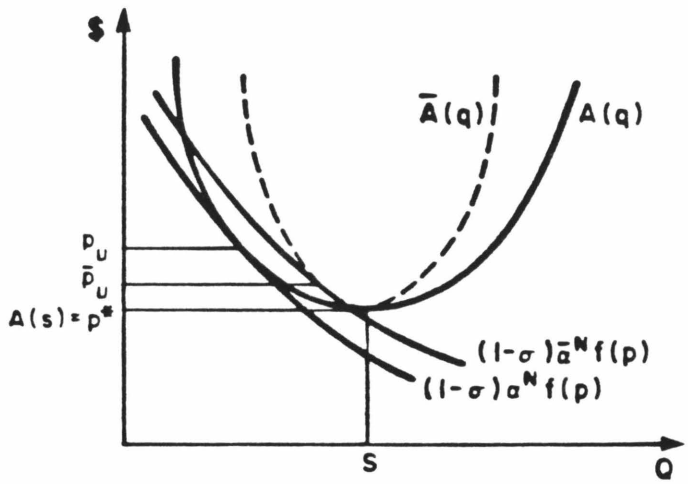

FIGURE 4

4. CONCLUSION

This paper has shown that the qualitative properties of the model developed by Wilde and Schwartz are robust to the strong assumptions regarding the nature of consumers' demand curves and firms' average cost curves used in that model. Furthermore, it has shown that the critical level of comparison shoppers needed to generate a competitive equilibrium falls as a demand becomes more elastic or average costs become more inelastic.

The model also strengthens the connections between equilibrium search models and traditional models of monopolistic competition. When the necessary and sufficient condition for a competitive equilibrium is not met, prices will be dispersed over some range up to and including the monopolistically competitive price. This last result is important because it qualifies the somewhat pessimistic aspects of Wilde and Schwartz (1979). In that model, if there is any price dispersion at all, some firms will necessarily charge the monopoly price. This means that some nonshoppers will be "fully exploited" in the sense that they receive no surplus from purchasing the good. In the generalized model the highest price charged will never exceed the monopolistically competitive price. In other words when imperfect information generates noncompetitive outcomes, they are bounded below, in welfare terms, by the monopolistically competitive equilibrium.

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APPENDIX

This appendix will prove the theorem stated in the text through a sequence of lemmas similar to those found in Wilde and Schwartz (1979). We initially consider some candidate equilibrium distribution of prices $G(p)$ defined on $[p_1, p_u]$.

Lemma 1: $G(p)$ cannot contain any mass points except possibly at p^*.

Proof: The proof of this lemma is stated for the case of $n = 2$. The generalization to arbitrary n will then be obvious.

Suppose there is a mass point at some $p^0 > p^*$. Expected demand at p^0 by members of A_2 is given by

$$2\alpha_2[G(p^0)\frac{1}{2} + 1 - G(p^0)] \cdot f(p^0) \tag{A1}$$

where $G(p^0)$ is the size of the mass point at p^0. Here α_2 is the expected number of shoppers who sample the firm, $1 - G(p^0)$ is the probability that the other firm they sample has a higher price, $G(p^0)$ is the probability that the other firm has the same price and, in this case, $\frac{1}{2}$ represents the probability that the shopper buys from the first firm. Finally, we multiply by two since the draws could occur in either order. Expected demand by members of A_1 is just $\alpha_1 f(p^0)$. Thus

$$D(p^0) = \{\alpha_1 + 2\alpha_2[G(p^0)\frac{1}{2} + 1 - G(p^0)]\}f(p^0) \ . \tag{A2}$$

Now, consider a firm which charges a price of $p^0 - \varepsilon$ where

$\varepsilon > 0$ is such that $p^0 - \varepsilon > p^*$. Since G can have at most a countable number of mass points, it is possible to find ε such that no mass point occurs at $p^0 - \varepsilon$. Hence a firm which offers $p^0 - \varepsilon$ faces an expected demand equal to

$$\begin{aligned} D(p^0 - \varepsilon) &= \{\alpha_1 + 2\alpha_2[1-G(p^0 - \varepsilon)]\}f(p^0 - \varepsilon) \\ &> \{\alpha_1 + 2\alpha_2[G(p^0) + 1 - G(p^0)]\}f(p^0) . \end{aligned} \tag{A3}$$

Let $q = \min\{s,[\alpha_1 + + 2\alpha_2(G(p^0) + 1 - G(p^0))]f(p)\}$. Then, from (A3), $q < D(p^0 - \varepsilon)$.

Now, suppose the firm charging $p^0 - \varepsilon$ sells q units (it might want to sell more). In this case

$$\pi(p^0,D(p^0)) = D(p^0)[p^0 - A(D(p^0))] \tag{A4}$$

$$\pi(p^0 - \varepsilon,q) = q[p^0 - \varepsilon - A(q)] . \tag{A5}$$

Subtracting (A5) from (A4),

$$\begin{aligned} \pi(p^0,D(p^0)) - \pi(p^0 - \varepsilon,q) &= D(p^0)[p^0 - A(D(p^0))] - q[p^0 - \varepsilon - A(q)] \\ &= p^0[D(p^0) - q] + \varepsilon q - D(p^0)A(D(p^0)) + qA(q) \\ &= p^0[D(p^0) - q] + \varepsilon q - A(D(p^0))[D(p^0) - q] + q[A(q) - A(D(p^0))] \\ &= [p^0 - A(D(p^0))][D(p^0) - q] + \varepsilon q + q[A(q) - A(D(p^0))] . \end{aligned}$$

But $p^0 = A(D(p^0))$ since if G(p) is an equilibrium distribution, firms charging p^0 must earn zero profits. Hence

$$\pi(p^0,D(p^0)) - \pi(p^0 - \varepsilon,q) = \varepsilon q + q[A(q) - A(D(p^0))] . \tag{A6}$$

In order for a firm charging p^0 not to prefer charging $p^0 - \varepsilon$

(strictly) it must be that

$$\pi(p^0, D(p^0)) - \pi(p^0 - \varepsilon, q) \geq 0 \ . \tag{A7}$$

Using (A6) the inequality in (A7) is equivalent to

$$A(D(p^0) - A(q) \leq \varepsilon \tag{A8}$$

for $\varepsilon > 0$. Since ε is arbitrary, and the possible number of mass points is at most countable, we can choose a sequence $\{\varepsilon_i\}_{i=1}^{\infty}$ such that $\varepsilon_i \to 0$ and $p^0 - \varepsilon_i > p^*$ is not a mass point of $G(p)$ for any i. Thus, (A8) is equivalent to

$$A(D(p^0)) \leq A(q) \tag{A9}$$

Since $D(p^0) < q < s$, s minimizes average costs and $A' < 0$ on the left-hand branch,

$$A(s) \leq A(q) < A(D(p^0)).$$

which contradicts (A9). This argument fails only when $p^0 = p^*$. Q.E.D.

<u>Lemma 2</u>: In <u>any</u> nondegenerate equilibrium, $\alpha_1 f(p^u) = A^{-1}(p^u)$ and $\alpha_1 f(p) \leq A^{-1}(p)$ for all $p \in [p^*, p^u]$, where p^u is the upper bound on the support of $G(p)$.

<u>Proof</u>: Suppose $\alpha_1 f(p^0) > A^{-1}(p^0)$ for some $p^0 \in [p^*, p^u]$. Then any firm offering p^0 will earn strictly positive profits since $\alpha_1 f(p^0)$ is equal to the expected demand of nonshoppers and $\alpha_1 f(p^0) > A^{-1}(p^0)$

implies $A(\alpha_1 f(p^0)) < p^0$. Hence it must be that $\alpha_1 f(p) \leq A^{-1}(p)$ for all $p - [p^*, p^u]$. Since $G(p)$ has no mass point at p^u, the highest price in the market, any firm charging p_u will sell only to nonshoppers. Hence zero profits implies $A(\alpha_1 f(p_u)) = p_u$. Q.E.D.

<u>Lemma 3</u>: In any nondegenerate equilibrium, α_1 is defined by

$$\alpha_1^N = \max\{\alpha_1 \mid f(p)\alpha_1 \leq A^{-1}(p) \text{ for all } p\}$$

and p_u is defined by

$$f(p_u)\alpha_1^N = A^{-1}(p_u),$$

where $A^{-1}(p)$ is defined on the left hand branch of $A(q)$.

<u>Proof</u>: Immediate from Lemma 2. Q.E.D.

<u>Lemma 4</u>: $G(p_1) = 0$ if and only if $\sigma \leq \dfrac{s}{\alpha^N f(p_1)} - 1 \quad \dfrac{1}{n-1}$, where p_1 is the minimum price in the market.

<u>Proof</u>: If $G(p_1) = 0$, then the firm which offers p_1 sells to all who sample it. Hence $D(p_1) = [\alpha_1^N + n\alpha_n^N]f(p_1)$. But p_1 is defined by $D(p_1) = A^{-1}(p_1)$, and consistency requires $D(p_1) \leq s$; if $D(p_1)$ so defined is greater than s, it cannot be a profit maximizing price/quantity pair since it would be in the firm's interest to hold price constant but reduce output. Thus we need

$$[\alpha_1^N + n\alpha_n^N]f(p_1) \leq s$$

$$\alpha^N(1 - \sigma) + n\alpha^N\sigma \quad f(p_1) \leq s$$

$$1 - \sigma + n\sigma \lesssim \frac{s}{\alpha^N f(p_1)}$$

$$\sigma(n-1) \lesssim \frac{s}{\alpha^N f(p_1)} - 1$$

$$\sigma \lesssim \quad \frac{s}{\alpha^N f(p_1)} - 1 \qquad \frac{1}{n-1} \ .$$

Sufficiency will be proved ror $n = 2$. The validity of the result for $n \gtrsim 3$ will be obvious. We argue by contradiction. Suppose

$$\sigma \lesssim \quad \frac{s}{\alpha^N f(p_1)} - 1 \qquad \frac{1}{n-1} \ . \tag{A9}$$

and $G(p\ ) > 0$. By the same logic as Lemma 1, $p_1 = p^*$ and

$$D(p^*) = \quad \alpha_1^N + 2\alpha_2^N \qquad 1 - G(p^*) + \tfrac{1}{2}G(p^*) \qquad f(p^*).$$

In this case we need $D(p^*) = s$; i.e.

$$\{\alpha_1^N + 2\alpha_2^N[(1 - G(p^*) + \frac{1}{2}\ G(p^*)]\}f(p^*) = s\ .$$

But $G(p^*) = G(p^*)$. Thus we need

$$[\alpha_1^N + \alpha_2^N(2 - G(p^*))]f(p^*) = s$$

$$[(1 - \sigma)\alpha^N + 2\sigma\alpha^N - \sigma\alpha^N G(p^*)] = \frac{s}{f(p^*)}$$

$$\alpha^N[1 + \sigma - \sigma G(p^*)] = \frac{s}{f(p^*)}$$

$$\sigma - \sigma G(p^*) = \frac{s}{\alpha^N f(p^*)} - 1$$

$$1 - G(p^*) = \frac{1}{\sigma}\left[\frac{s}{\alpha^N f(p^*)} - 1\right] , \tag{A10}$$

Where $n = 2$, (A9) implies the right-hand side of (A10) is greater than or equal to 1. Hence $G(p^*)$ must be less than or equal to 0, which is a contradiction.

Q.E.D.

<u>Lemma 5</u>: Suppose $G(p_1) = 0$. Then
$D(p) = [\alpha_1^N + n\alpha_n^N(1 - G(p))^{n-1}]f(p)$.

<u>Proof</u>: The expected number of nonshoppers who sample a firm is α_1^N. Of the shoppers, the expected number who sample a firm is $n\alpha_n^N$ since each takes n observations. The probability of a sale to one of these is just $(1 - G(p))^{n-1}$ since shoppers buy from the lowest priced firm in their sample. Finally, each consumer demands $f(p)$ units. Q.E.D.

<u>Lemma 6</u>: Suppose $G(p_1) = 0$. Then

$$G(p) = 1 - \left[\left(\frac{A^{-1}(p)}{f(p)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}\right]^{\frac{1}{n-1}}$$

where α_1^N and p_n are given in Lemma 3, $\alpha_n^N = (\frac{\sigma}{1-\sigma})\alpha_1^N$, p_1 is given by
$p_1 = \min\{p | f(p)(n\alpha_n^N + \alpha_1^N) = A^{-1}(p)\}$,

where $A^{-1}(p)$ is defined on the left-hand-branch of $A(q)$.

<u>Proof</u>: The zero profit constraint requires that $A(D(p)) = p$ for all

prices which are actually offered. Thus, from Lemma 5,

$$A([\alpha_1^N + n\alpha_n^N(1 - G(p))^{n-1}]f(p)) = p$$

$$\alpha_1^N + n\alpha_n^N(1 - G(p))^{n-1} = \frac{A^{-1}(p)}{f(p)}$$

$$(1 - G(p))^{n-1} = \left(\frac{A^{-1}(p)}{f(p)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}$$

$$G(p) = 1 - \left[\left(\frac{A^{-1}(p)}{f(p)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}\right]^{\frac{1}{n-1}} .$$

In this calculation, $A^{-1}(p)$ is taken to be the left-hand-branch of $A(q)$ since by Lemma 4,

$$s \geq \left(\alpha_1^N + n\alpha_n^N\right) f(p_1) \geq \left(\alpha_1^N + n\alpha_n^N(1 - G(p))^{n-1}\right) f(p)$$

whenever $G(p_1) = 0$.

To find p_1, solve $G(p_1) = 0$:

$$0 = 1 - \left[\left(\frac{A^{-1}(p_1)}{f(p_1)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}\right]^{\frac{1}{n-1}}$$

$$1 = \left(\frac{A^{-1}(p_1)}{f(p_1)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}$$

$$(n\alpha_n^N + \alpha_1^N)f(p_1) = A^{-1}(p_1) .$$

Since this formula can have two solutions, we need to define p_1 as the minimum. The maximum solution can be ruled out since it can be shown to be greater than p_u.

To find p_u, solve $G(p_u) = 1$:

$$1 = 1 - \left[\left(\frac{A^{-1}(p_u)}{f(p_u)} - \alpha_1^N\right)\frac{1}{n\alpha_n^N}\right]^{\frac{1}{n-1}}$$

$$\alpha_1^N f(p_u) = A^{-1}(p_u) \; .$$

Note that this final calculation is consistent with Lemma 3. Q.E.D.

<u>Lemma 7</u>: A necessary and sufficient condition for a single price equilibrium at p^* is

$$\sigma \geq 1 - \frac{f(A(s))}{s} \cdot \frac{A^{-1}(p)}{f(p)} \qquad \text{for all } p \geq p^* \; . \tag{A11}$$

<u>Proof</u>: Assume (A11) holds but there exists a nondegenerate equilibrium. First, we claim such a distribution must have a mass point.

<u>Fact 1</u>: (A11) implies $\alpha_1^C f(p) \leq A^{-1}(p)$ for all $p \geq p^*$ where $A^{-1}(p)$ is the left-hand-branch of $A(q)$ and $\alpha_1^C = (1 - \sigma)\alpha^C$ where α^C is defined by $\alpha^C f(A(s)) \equiv s$ (i.e. α^C is the "competitive" consumer/firm ratio).

<u>Proof of fact</u>: $\alpha^C f(A(s)) \equiv s$ implies $f(A(s))/s = 1/\alpha^C$. Hence (A11) is equivalent to

$$\sigma \geq 1 - \frac{A^{-1}(p)}{\alpha^C f(p)} \qquad \text{for all } p \geq p^* \; ,$$

or,

$$1 - \sigma \leq \frac{A^{-1}(p)}{\alpha^C f(p)} \qquad \text{for all } p \geq p^* \; ,$$

or, finally,

$$p \leq A(\alpha^C (1 - \sigma) f(p)) \qquad \text{for all } p \geq p^* \; . \tag{A12}$$

Now $\alpha^C(1 - \sigma) = \alpha_1^C$, so (A12) establishes the desired fact.

<u>Fact 2</u>: $(\alpha_1^C + n\alpha_n^C)f(p) > \alpha^C f(p)$ for all p such that $f(p) > 0$.

<u>Proof of fact</u>: Since $f(p) > 0$ by assumption, it suffices to show $\alpha_1^C + n\alpha_n^C > \alpha^C$. But $\alpha_1^C + n\alpha_n^C = (1 - \sigma)\alpha^C + n\sigma\alpha^C = \alpha^C + \sigma(n - 1)\alpha^C$. Since $n \geq 2$ by assumption, Fact 2 must hold.

Fact 2 implies that $(\alpha_1^C + n\alpha_n^C)f(p)$ intersects A(q) to the right of s; i.e. while $(\alpha_1^C + n\alpha_n^C)f(p) = A^{-1}(p)$ can be solved for two prices, the minimum of these must satisfy $(\alpha_1^C + n\alpha_n^C)f(p) > s$ in this case.

Now we are assuming a nondegenerate equilibrium exists, so α_1^N satisfies by $f(p_u)\alpha_1^N = A^{-1}(p_u)$ where $A^{-1}(p)$ is the left-hand-branch of A(q). Fact 1 implies $\alpha_1^N > \alpha_1^C$; i.e. α_1 must increase in order for $\alpha_1 f(p)$ to be tangent with $A^{-1}(p)$. Hence $\alpha_1^N + n\alpha_n^N > \alpha_1^C + n\alpha_n^C$, and $(\alpha_1^N + n\alpha_n^N)f(p) > (\alpha_1^C + n\alpha_n^C)f(p)$ for all p such that $f(p) > 0$. Thus, since $A^{-1}(p)$ is increasing on the right-hand-branch, $(\alpha_1^N + n\alpha_n^N)f(p)$ intersects $A^{-1}(p)$ at a point even further to the right of s than $(\alpha_1^C + n\alpha_n^C)f(p)$ does. But G(p) must then have a mass point because this result violates the necessary and sufficient condition for no mass point given in Lemma 4 (see also the proof of that Lemma).

So assume (A11) holds but we have a mass point at $p_1 = p^*$. Proceeding as in Lemma 1 with n = 2 we have

$$\begin{aligned} D(p^1) &= \{\alpha_1^N + 2\alpha_2^N[G(p_1)\tfrac{1}{2} + 1 - G(p_1)]\}f(p_1) \\ &= \{\alpha_1^N + \alpha_2^N[2 - G(p_1)]\}f(p_1) \ . \end{aligned}$$

Again, zero profits requires $D(p_1) = s$ since $p_1 = p^*$, i.e.

$$s = \{\alpha_1^N + \alpha_2^N[2 - G(p_1)]\}f(p_1)$$
$$= \alpha^N(1 - \sigma) \quad 1 + \frac{\sigma}{1-\sigma} \quad 2 - g(p_1) \qquad f(p_1) \ .$$

Thus, solving for $G(p_1)$ gives

$$G(p_1) = 2 - \quad \frac{1-\sigma}{\sigma} \qquad \frac{s}{\alpha^N(1-\sigma)f(p_1)} - 1 \quad . \tag{A13}$$

But using $f(p_1) = f(p^*) = f(A(s))$, (A11) can be written

$$\sigma \geq 1 - \frac{f(A(s))}{s}\frac{A^{-1}(p)}{f(p)} \qquad \text{for all } p \geq p^* \ .$$

Consider $p = p_u$. Then we have

$$\sigma \geq \frac{f(A(s))}{s}\frac{A^{-1}(p_u)}{f(p_u)}$$

or, after some manipulations,

$$\frac{1-\sigma}{\sigma} \qquad \frac{sf(p_u)}{f(p\ )A^{-1}(p_u)} - 1 \qquad \leq 1 \tag{A14}$$

But by definition $\alpha^N(1 - \sigma)f(p_u) = \alpha_1^N f(p_u) = A^{-1}(p_u)$. Hence (A13) implies

$$G(p_1) = 2 - \quad \frac{1-\sigma}{\sigma} \qquad \frac{sf(p_u)}{f(p_1)A^{-1}(p_u)} - 1 \quad .$$

Applying (A14) we have $G(p_1) \geq 2 - 1 = 1$ so any potential mass point must have $G(p_1) = G(p^*) = 1$. Thus (A11) is sufficient for a competitive outcome.

Necessity of (A11) follows in reverse order of the proof of

Fact 1 above, noting that profit maximization requires that no firm earn positive profits by deviating from p^* when all other firms charge p^*, i.e. $p \leq A(\alpha_1^C f(p))$ for all $p \geq p^*$. Q.E.D.

The Theorem follows directly from these lemmas.

LOST PROFITS, MARKET DAMAGES, AND SPECIFIC PERFORMANCE: AN ECONOMIC ANALYSIS OF BUYER'S BREACH

I. INTRODUCTION

A contract is an agreement to exchange, at a later date, a specified amount of goods or services for an agreed upon sum of money. The important characteristic of contracts that distinguishes them from general exchanges is the difference between the time the agreement was made and the time of performance. This difference in timing can often lead to changes in circumstances which can cause one of the parties to withdraw from the contract. Ideally, a contract would contain clauses for every possible contingency that could occur between the time of the agreement and the time of performance. However, such a contract is impossible since all the possible contingencies may not be known to the two parties. Furthermore it may be difficult to monitor states of the world which are known to only one party. Finally, even if the previous two problems could be overcome, the costs of negotiating each clause would be too enormous to make the contract worth while. As a result, contracts are always incomplete and it is frequently the case that one of the parties no longer wishes to fulfill the contract. This breaking of the contract is known as breach. The case that is considered in this work is buyer's breach of contracts for goods.

A large part of economic transactions is in the form of contracts. In fact contracts are frequently necessary for economically efficient exchanges to occur. For example a buyer and a seller may be willing to exchange, but suppose the buyer does not have ready cash for the transaction. Suppose all other buyers were willing to pay less for the goods, then it would be economically efficient for the seller to enter into a contract with the first buyer and complete the transaction at a later time. However, if neither of them could rely on the other to fulfill the contract, they would rather engage in less-efficient noncontractual exchanges with other parties. Thus there is a need for legal protection for breach of contract.

It is understood in the legal system that states of the world may arise when it is actually not efficient for a contract to be completed. Consequently, the objective is not to punish the breaching party, but to leave the innocent party unaffected by the breach. This principle is known as the protection of the expectation interest of the innocent party. The principle holds that the court should leave the innocent party as well off as he or she would have been had the contract been completed.

Any action taken by the court to compensate the innocent party in case of breach is known as a remedy. The most commonly used remedy for buyer's breach is market damages. This requires the buyer to pay the seller the difference between the contract price and the price at which the seller would be able to resell, for every unit breached. If the difference is negative, nothing is paid. The reselling price is normally taken to be the current market price and consequently, a market for spot sales is necessary to implement this remedy. Another remedy which does not have this requirement is specific performance.

Here the courts enforce performance as specified in the contract and the buyer is effectively not allowed to breach. The advantage of this remedy is that by definition, it protects the seller's expectation interest exactly. However, specific performance is thought to be economically inefficient, since the buyer is made to accept unwanted goods which he would then attempt to resell. It would be more efficient for the original seller to do the reselling, since he is in the business of selling and would have lower selling costs than the contract buyer. Because of this drawback, specific performance has limited use in practice.

Until recently, the predominant legal practice was to use market damages and only in the rare cases when market damages could not be implemented was specific performance used. Even in the absence of a clear spot market price, if the seller was able to resell and the court was satisfied that a conscientious effort had been made to obtain the highest price, the reselling price was used in the market damages formula. However, it has been argued that market damages insufficiently compensates the seller especially in the case that the spot price is greater than the contract price. The reasoning is as follows: if a buyer repudiates a contract and the seller is able to resell the breached goods to a third party, the seller has still lost the profit that would have been made on the breach contract, because he or she would have sold to the third party whether or not the breach occurred. Compensation based on this argument is called lost profits, and requires the buyer to pay to the seller the extra profit that would be earned if the additional units specified in the contract are produced and sold at the contract price. In the case of constant marginal costs it is simply the contract price less the marginal cost for every unit breached. In practice, this remedy is used when it is clear that the third party would have purchased had there been no breach, and the seller has enough capacity to supply both parties.

This paper compares the various remedies in their effectiveness in protecting the seller's expectation interest under the assumption that there is no reliance; that is, the seller does not take any actions due to the existence of the contract before the breach. It is shown that under very general conditions lost profits overcompensates the seller, and the size of the error is greater the more market power the seller has, and the smaller the difference between the buyer's reselling costs and the seller's selling costs. Similarly, under the condition that the contract price is greater than the spot market price, market damages undercompensates the seller and the size of the error is greater the larger the buyer's extra reselling costs. However, when the condition is not met, it is ambiguous whether market damages protects expectation interest adequately since it offers the seller an advantage in the resale market and a disadvantage in the contract market which may not always offset each other.

Another important area of comparison is economic efficiency. Unfortunately a complete analysis of the efficiency properties of the three remedies would require a model with an endogenous contract market. This has proven to be mathematically intractible. Thus only an informal discussion of economic efficiency is possible. It is argued that specific performance tends to be inefficient whenever the contract buyer incurs extra reselling costs when selling in the spot market. Lost profits tends to be inefficient whenever a thick resale market exists. However a comparison between the economic efficiencies of the three remedies is not possible.

Finally, if we look at the costs of implementing the three remedies, it is clear that specific performance and market damages are straightforward to administer but for lost profits, we require accurate information about the shape of the seller's cost function which will be costly to acquire.

Section II of this paper presents a simple model of buyer's breach, and demonstrates that lost profits overcompensates the seller and market damages usually undercompensates. Section III relates the results of this paper to the existing literature on lost profits remedies, primarily the state-of-the-art legal analysis due to Goetz and Scott [1979]. Section IV contains a discussion of the economic efficiency of the remedies and is followed by the conclusion in Section V.

II. THE MODEL

There are two types of agents in the model: buyers and sellers. They act in a two-period static model in which the two periods are separated by the realization of random variables that influence the buyers' demands. In period one there is a contract market and in period two there is a spot market. The sellers sell in both markets and it is assumed that they take no actions due to the existence of their contract obligations until after the random variables are realized. This means, in legal terms, that there is no reliance on the sellers' part. The sellers' objective is to maximize their expected profits in the two markets.

The buyers' objective is to maximize their expected utility over the two markets. They buy in the contract market or the spot market depending on their preferences. They realize that if they buy in the contract market in period two they will be allowed to resell any amounts they wish in the spot market under a certain market structure that is known to both buyers and sellers. Also, they may breach in period two and incur damages corresponding to the amount breached according to a damage rule that is known to both buyers and sellers. In this model we take the contracts (price and quantities) as given and examine the spot market.

The central issue in the lost profits argument is whether a sale that is made after the breach occurs replaces the contract that was breached. If the buyer, after breaching the contract, waited for a better price and then bought the same product, profits would not be lost; they would simply be realized at a different time, and possibly by a different seller. So it is important in a lost profits claim that the buyer who is breaching really does not want the goods.

To incorporate this idea in the model, it is assumed that there are several buyers (who buy in the contract market) with stochastic demands that are governed by n independent random variables. The buyers make their contracts before the random variables are realized. Afterwards, some buyers may not want all the goods for which they had contracted, because the realizations of their random variables yielded low states of demand. These buyers must decide how to allocate their contracted goods between consumption, resale, and breach. For simplicity we will assume one contract buyer. We denote the buyer's inverse demand curve, a downward sloping function, by $t(Q)$. (Because all the choice variables are quantities, it is most convenient to use inverse demand curves.) Since the contract buyer is not in the business of selling, we assume an extra reselling cost per unit, denoted by r, and it is assumed that $r \geq 0$. We let Q_c be the amount consumed, Q_s be the amount resold, and Q_b be the amount breached by the buyer, if he has contracted for more than he desires. Alternatively, for high realizations of demand, the buyer may have contracted for less than he or she desires and will purchase in the spot market; this quantity is denoted by Q_p. Finally, let Z be the amount contracted by the buyer at the contract price K. Clearly, the amounts consumed, resold and breached should sum to the total amount contracted plus the amount purchased in the spot market.

Consider for the moment a single seller, with constant marginal costs C, who sells in two markets: a contract market in which an initial agreement is made for delivery at a later date, and a spot market. For example, a new car dealer will often sell cars of custom specification which are special-ordered from the factory, but will also sell display models on the spot market. Similarly, faced with uncertain demand, a risk averse buyer may prefer to pay a premium and purchase in the contract market, whereas if the buyer is less risk averse, he or she will wait until the demand is revealed and buy on the spot market. An example is whether to contract for farm labor before the harvest or wait and hire after the size of the crop is known.

The demands in the spot and contract markets will of course be interrelated, since the products in the two markets are slightly differentiated versions of the same commodity, either through their physical properties or by the fact that they are sold at different times. The precise nature of this relationship is unimportant in the analysis of sellers' remedies. We shall simply assume that the realization of the expected inverse demand in the spot market of the buyers who do not buy in the contract market, $P(Q)$, is downward sloping without specifying its relation to the demands of the buyers in the contract market or the contract price. To guarantee that both contract and spot markets exist, we assume that $t(0) > C$ and $P(0) > C$.

In order to complete the model we must describe how the original seller interacts with the buyers from the contract market who wish to resell in the spot market. There is a continuum of possibilities starting with the resellers being complete price takers, and ending with them being equals with the original seller in a Cournot oligopoly. The results hold for most market structures but we shall restrict our attention to the "competitive" case and the

Stackelberg case for the purposes of the paper. The "competitive" assumption is examined in Part A and the Stackelberg in Part B. In each case we will calculate the seller's profits under the three remedies of interest: lost profits, market damages, and specific performance. The final step is to compare the profits under specific performance at the time of breach, which is our benchmark, with the profits under the other two remedies, to determine how well each remedy protects the seller's expectation.

The problem is solved in the following fashion. We begin, after the random variables have been revealed, with the decision of the contract buyers of how to allocate their contract into the three activities of consumption, resale and breach. Then we combine the resale quantity decisions of the contract buyers with the selling quantity decision of the original seller to determine the outcome of the spot market. From this we can calculate the profits of each agent in the spot market.

A. "Competitive" Assumption

Let us assume that the contract buyers are price takers in the resale market. If lost profits is the legal remedy for breach, then the gain to the buyer (surplus plus profit), denoted by π, will be

$$\pi = T(Q_c) + Q_s(P - r) - K(Q_c + Q_s) - PQ_p - (K - C)Q_b, \tag{1}$$

where T(Q) is the surplus associated with the demand t(Q) and P is the price in the spot market. The first term is the surplus from consuming Q_c and the second is the profits from selling Q_s units in the spot market. From that we subtract the cost of purchasing $Q_c + Q_s$ units at the contract price K and Q_p units at the spot market price P; and the lost profits paid to the seller for the Q_b units breached, at the rate of K − C per unit. In order to determine the optimal allocation of the total contract and spot purchases between consumption, resale, and breach, the buyer will maximize profit plus surplus (1) subject to the three uses not exceeding the total contract plus spot purchases

$$Q_c + Q_s + Q_b = Z + Q_p, \tag{2}$$

and

$$Q_c \geq 0,\ Q_s \geq 0,\ Q_b \geq 0,\ Q_p \geq 0. \tag{3}$$

The general solution to the above problem is given in the appendix. It turns out that the buyer will breach when firstly breaching is preferred to selling ($C > P - r$) and secondly the state of demand is low enough ($t(Z) < C$). The amount breached is $Z - t^{-1}(C)$ and the amount consumed is $t^{-1}(C)$, with no units resold and no units purchased. Thus the profit to the original seller under lost profits when the buyer breaches is

$$\begin{aligned}\pi^{\ell} &= \max_{Q}[P(Q) - C]Q + [K - C]t^{-1}(C) + [K - C](Z - t^{-1}(C)) \\ &= \max_{Q}[P(Q) - C]Q + [K - C]Z \end{aligned} \tag{4}$$

where Q is the amount supplied by the original seller in the spot market. The first term represents the profit to the seller from the spot market, the second the profit from actual sales in the contract market and lastly the lost profits on the units breached.

Now we wish to compare the seller's profit under lost profits (4) with the profit made under specific performance, under the same state of demand. Again we begin with the buyer's behavior, how much he consumes, resells, under the same state of demand as before but now specific performance is the legal remedy for breach so the buyer is effectively not allowed to breach. The buyer's objective function is very similar to the previous formulation (1), except for the term which allows breach.

$$\pi = T(Q_c) + Q_s(P - r) - K(Q_c + Q_s) - PQ_p. \tag{5}$$

The buyer maximizes (5) subject to the following constraints which are similar to the previous ones.

$$Q_c + Q_s = Z + Q_p. \tag{6}$$

$$Q_c \geq 0,\ Q_s \geq 0,\ Q_p \geq 0. \tag{7}$$

We show in the appendix that when the buyer would have breached under lost profits ($t(Z) < C$ and $P - r < C$), one of two cases occur. If $t(Z) < P - r$ then the buyer will consume $t^{-1}(P - r)$ units and resell $Z - t^{-1}(P - r)$ units, otherwise he or she will simply consume the entire contract. It is important to notice that the amount resold under specific performance $Z - t^{-1}(P - r)$ is always less than $Z - t^{-1}(C)$, the amount breached under lost profits.

In this case, the profit to the original seller is

$$\pi^s = \max_Q \begin{cases} [P(X) - C]Q + [K - C]Z & t(Z) < P(X) - r \\ [P(Q) - C]Q + [K - C]Z & P(X) - r \leq t(Z) < C \end{cases} \tag{8}$$

where X is the combined supply of the original seller and the contract buyer in the spot market, $X = Q + Z - t^{-1}(P(X) - r)$.

If we compare the seller's profits under the two remedies, we see that when $t(Z) < P(X) - r$ the two expressions are identical, so that lost profits compensates exactly. However, when $t(Z) \geq P(X) - r$, we show in the appendix that $\pi^{\ell} - \pi^s > 0$ and lost profits overcompensates the seller. The intuition for the latter assertion can be understood using the following diagrams.

Let us assume that the state of demand is such that the buyer would breach under lost profits an amount B, and under specific performance he or she would resell some amount less than B depending upon how large r is. Let D be the demand of the other buyers in the spot market. In Figure 1 we see that the total profit to the original seller under lost profits is the sum of the three shaded areas; the profit in the contract market, the lost profits and the profit in the spot market (determined by setting marginal revenue equal to marginal cost). Since production decisions are delayed until after the random variables are revealed, only the amounts supplied in the contract and spot markets are produced and the B units breached are not produced. In Figure 2, we assume the same state of demand occurred, but now specific performance is the legal remedy for breach. With the same demand conditions, under specific performance the optimal amount the contract buyer chooses to resell is an amount between zero and B, depending upon his extra reselling cost. Since the buyer can sell as much as he or she wishes at the going spot price, it is optimal for the seller to choose the spot price so as to maximize the profit from the residual demand (after the contract buyer has resold) which we denote by D'. Clearly the profit to the seller under lost profits is greater than or equal to the profit under specific performance, since $D' \leq D$. It is equal when the buyer resells nothing. The buyer's reselling decision is directly dependent upon the size of the extra reselling costs. The greater the costs, the more is resold. Thus lost profits compensate exactly whenever the extra reselling costs are prohibitively high so that the contract buyer does not resell anything under specific performance. Otherwise lost profits tends to overcompensate the seller.

We analyze the effectiveness of market damages in protecting the seller's expectation interest in a similar fashion. It turns out that market damages undercompensates the seller. The proof is in the appendix. We can show the intuition of the results through the use of some diagrams.

We will let B be the amount the buyer would breach under market damages, D be the demand of the other buyers in the spot

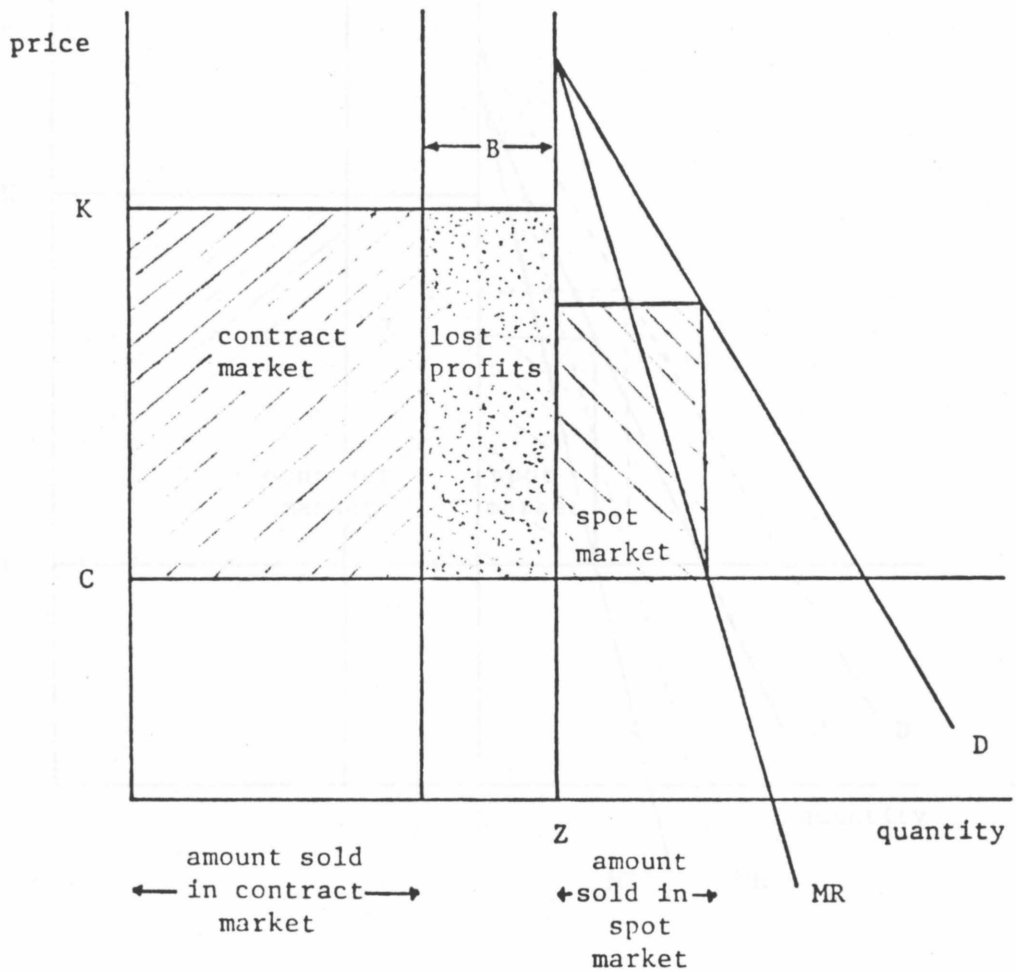
price
B
K
C
contract
market
lost
profits
spot
market
D
Z
quantity
amount sold
in contract
market
amount
sold in
spot
market
MR

FIGURE 1

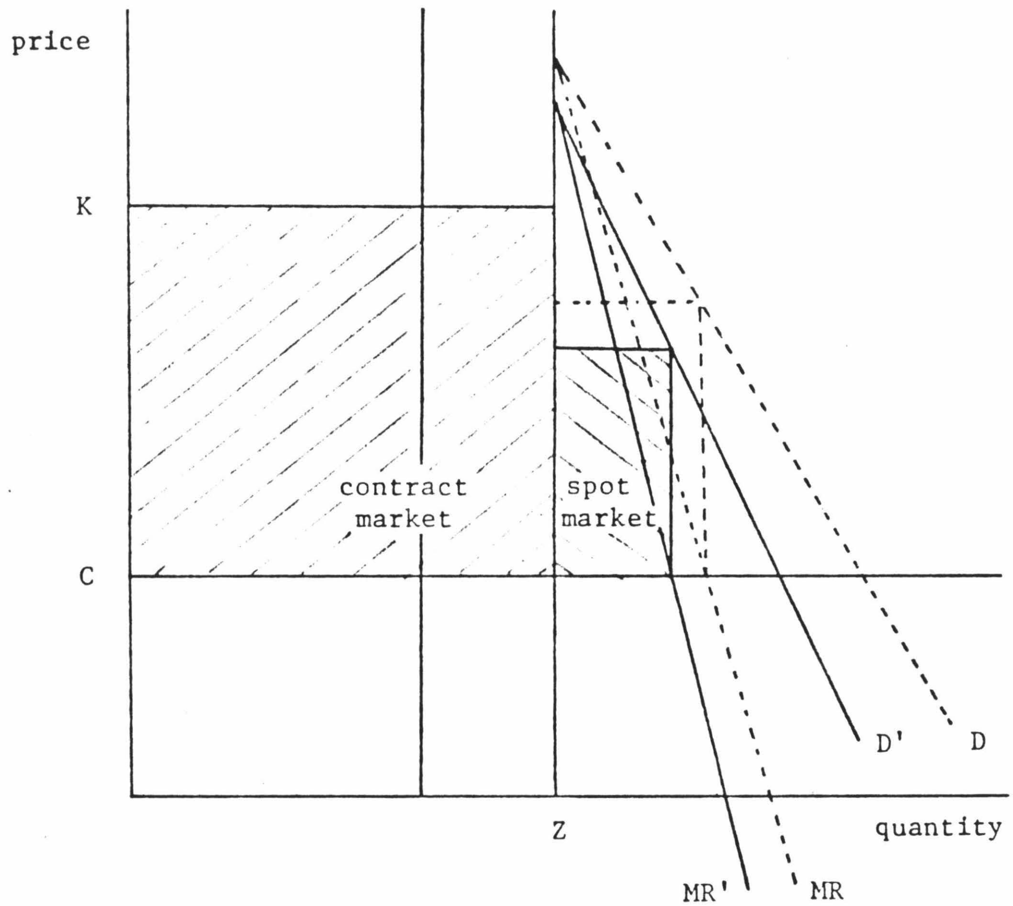

FIGURE 2

market. Then the sum of the three shaded areas in Figure 3 is the profit to the seller. Since the seller gets K - P for each unit breached, it is optimal for him or her to choose a price in the spot market which maximizes the profit on the remaining units (after the first B units).

In Figure 4, the same state of demand has occurred as in the previous figure, but the legal remedy for breach is specific performance. Since breach is not allowed, the profit in the contract market is (K - C)Z. The demand by the other buyers in the spot market will be the same as before; however, the contract buyer will engage in some reselling for prices above t(z) + r. Thus the residual demand facing the original seller is D'. We see that the profit to the seller, which is the total shaded area, is greater in this case than in the case of market damages. Thus market damages tends to undercompensate the seller.

B. Noncompetitive Assumption

The reasoning of the competitive case carries over to the case of the noncompetitive spot markets. However, there is an important difference in the noncompetitive situation for the case of a seller with market power. For both the money damage remedies, the buyer will engage in less reselling since there are more activities in which to allocate the contract. This will result in less loss of market power in the spot market for the original seller, than under specific performance. Thus the two money damage remedies will have an overcompensatory effect, besides the basic effects discussed for the

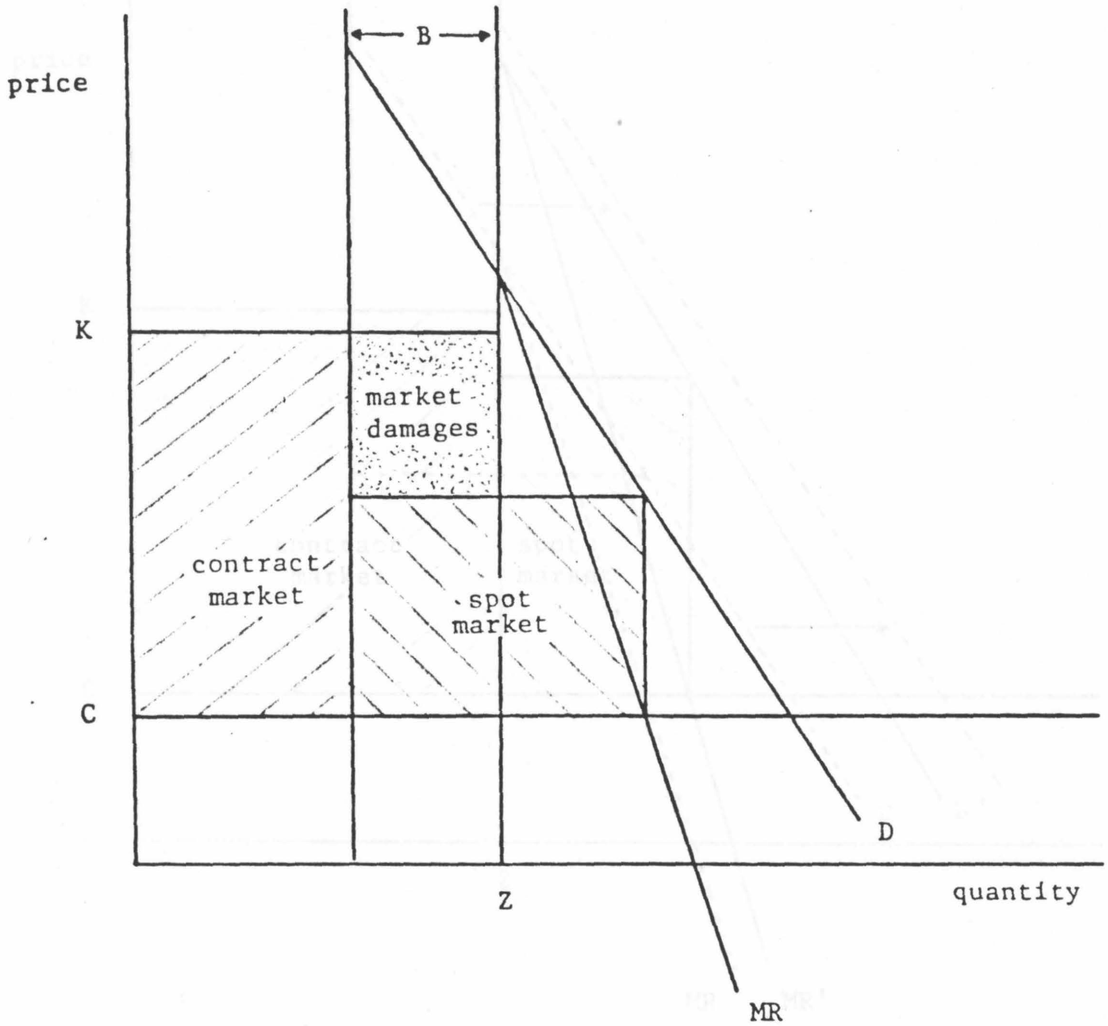
B
price
K
market
damages
contract
market
spot
market
C
D
Z
quantity
MR

FIGURE 3

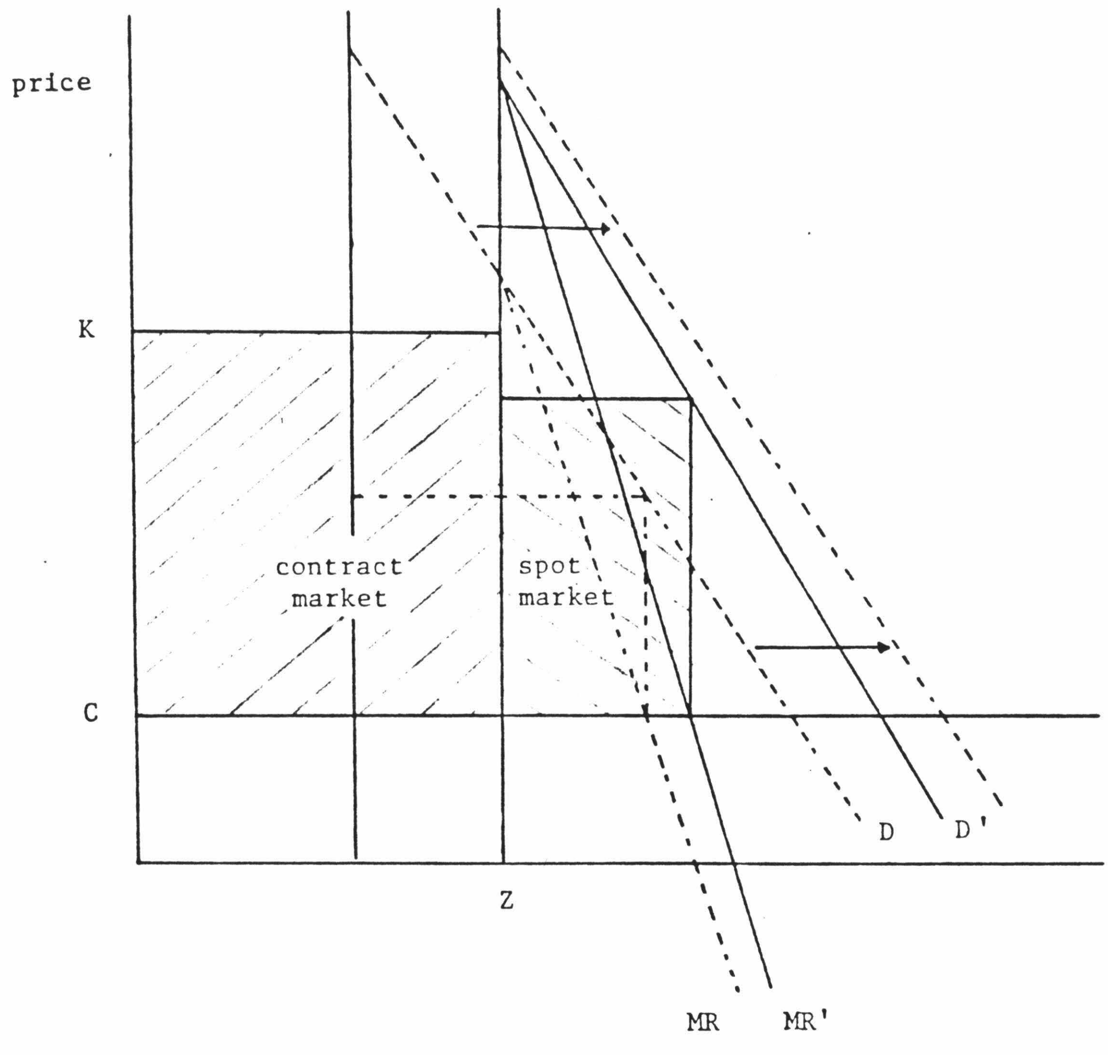

FIGURE 4

competitive case. For lost profits, since both effects are in the same direction, the outcome is that lost profits will continue to overcompensate. However, for market damages the two effects are in opposite directions and the combined effect could be either way depending upon which noncompetitive assumption is made. In the case of a Stackelberg assumption, the basic effect dominates the market power effect, so that the net result is that market damages undercompensates (for $P < K$).

In selecting a noncompetitive assumption the main idea we wish to preserve from the previous section is that the original seller dominates the spot market. The most natural assumption to make is that the original seller is a leader and the resellers are followers in a Stackelberg market structure. The resellers react to the quantity supplied by the original seller in a predictable way, and the original seller chooses the quantity to supply in each market by optimizing against the resellers' reaction functions. This formulation takes into account the weaker position of the resellers with respect to the original seller. In what follows we determine the original seller's profit with the Stackelberg assumption under each remedy. Finally we compare the profit under each money damage remedy with that under specific performance.

1. Lost Profits

In order to determine the seller's profit, we first need to know how the buyers will behave under the lost profits remedy for breach. We start as before with the buyer's objective function π.

Optimizing behavior on the buyer's part will yield a reaction curve describing how much the buyer will supply to the spot market for each quantity the original seller supplies. The original seller will then aggregate all the individual contract buyer's reaction curves into a grand reaction curve, which tells the seller for each quantity it supplies the total amount the others will supply. The quantity the seller chooses to supply in the spot market will maximize its profits given that the buyers behave according to the grand reaction curve.

For simplicity of presentation, we will again assume that there is one contract buyer. As before, the buyer's problem is to maximize

$$\pi = T(Q_c) + P(Q + Q_s) \cdot Q_s + (Z - Q_c - Q_s)C - KZ, \tag{9}$$

where $P(\cdot)$ is the inverse demand curve in the spot market and Q is the amount supplied by the original seller. In this case we assume no resale costs as they are not essential for the existence of buyer's breach and would serve to complicate the expressions. The maximization is subject to the constraint that the amounts consumed and resold should not exceed the total contracted amount, $Q_c + Q_s + Q_b \leq Z$ and the nonnegativity constraints $Q_c \geq 0$, $Q_b \geq 0$. Since resale costs have been eliminated, Q_s refers to both sales and purchases in the spot market; more specifically, negative values of Q_s indicate purchases in the spot market.

The only other term that is different in this objective function from the one previously formulated for the competitive case

(1) is the second term. The reason for the difference is that the noncompetitive case allows the quantity supplied by the reseller to affect the price in the spot market.

It is useful to define some terms which will simplify the notation. Consider the spot market inverse demand curve. If an agent with marginal cost θ was one of several sellers in the spot market, then the agent's objective function would be

$$P(\omega + \Omega) \cdot \omega - \theta\omega, \tag{10}$$

where ω is the amount the agent supplies and Ω the total amount all the other sellers supply. The first order condition for maximizing (10) is

$$P(\cdot) + P'(\cdot)\omega - \theta = 0. \tag{11}$$

Solving for ω as a function of Ω from (13) will give the reaction function of the agent, which we will denote by $R(\cdot)$ as follows:

$$\omega = R(\Omega;\theta). \tag{12}$$

Thus $R(\Omega;\theta)$ is the general reaction curve for the inverse demand function $P(\cdot)$, which describes how much an agent with marginal costs θ will supply given that the others are supplying Ω.

Using this notation, the complete solution to the optimization of the buyer's objective function (11) is

$$[Q_c, Q_s, Q_b] = \begin{cases} [t^{-1}(C), R(Q;C), Z - Q_c - Q_s] & Q > D \\ [Z - Q_s, R(Q;t(Z - Q_s)), 0] & Q \leq D \end{cases} \quad t(Z) > P$$
$$\qquad [t^{-1}(P), t^{-1}(P) - Z > 0] \qquad\qquad t(Z) \leq P \tag{13}$$

where $Q \leq D$ is the condition equivalent to $t^{-1}(C) + R(Q;C) \geq Z$ when we isolate Q. The meaning of the latter condition is that if he or she were unconstrained the amount consumed and resold would exceed Z. Equivalently, given that consumption and reselling are constrained by Z, the condition implies that nothing will be breached. Or in terms ot D, $Q \leq D$ implies the buyer will not breach, because the original seller is supplying so little to the spot market that it is advantageous to resell whatever is not consumed. The amount the buyer supplies in the spot market is indicated by the middle term in each case. The first branch of the buyer's resale decision is simply a normal reaction curve with marginal costs C, and the second branch is the reaction that would occur if consumption plus resale were always constrained to Z. Equation 13 is derived ın the appendix. We can show the reselling decision in a diagram (Figure 5).

The original seller optimizes against the contract buyer's lost profits reaction curve. Let us denote the two part reaction curve illustrated in figure 1 by $\mathbb{R}^{\ell}(Q)$. The original seller's profits are

$$\pi^{\ell} = \max_{Q} K(Q_c + Q_s) + (K - C)(Z - Q_s - Q_c)$$
$$+ P(Q + \mathbb{R}^{\ell}(Q)) \cdot Q - C(Q + Q_c + Q_s). \tag{14}$$

If we separate the components, the sellers' profits are the revenues from contract sales plus lost profits collected from the breach minus the cost of producing the contract sales. The remaining terms reflect what happens in the spot market. The seller supplies Q and the price is determined by the combined supply of the original seller and the contract buyer. The quantity remaining to be produced is Q and we subtract the cost of producing it. Simplifying (14),

$$\begin{aligned} \pi^{\ell} &= \max_{Q} (K - C)Z + P(Q + \mathbb{R}^{\ell}(Q)) \cdot Q - CQ \\ &= \max_{Q} (K - C)Z + [P(Q + \mathbb{R}^{\ell}(Q)) - C]Q \\ &= (K - C)Z + [P(Q^{\ell} + \mathbb{R}^{\ell}(Q^{\ell})) - C]Q^{\ell}, \end{aligned} \tag{15}$$

where Q^{ℓ} refers to the optimal choice of Q for the seller.

2. Market Damages

When market damages is the legal remedy for breach, the buyer's profits are as follows:

$$\pi = \begin{cases} T(Q_c) + Q_sP(Q + Q_s) - K(Q_c + Q_s) - [K - P(\cdot)]Q_b & P < K \\ T(Q_c) + Q_sP(Q + Q_s) - K(Q_c + Q_s) & P \geq K \end{cases} \tag{16}$$

There are two cases. Nothing is paid if the spot price is greater than the contract price. Otherwise, the difference between the contract and spot prices is paid per unit breached.

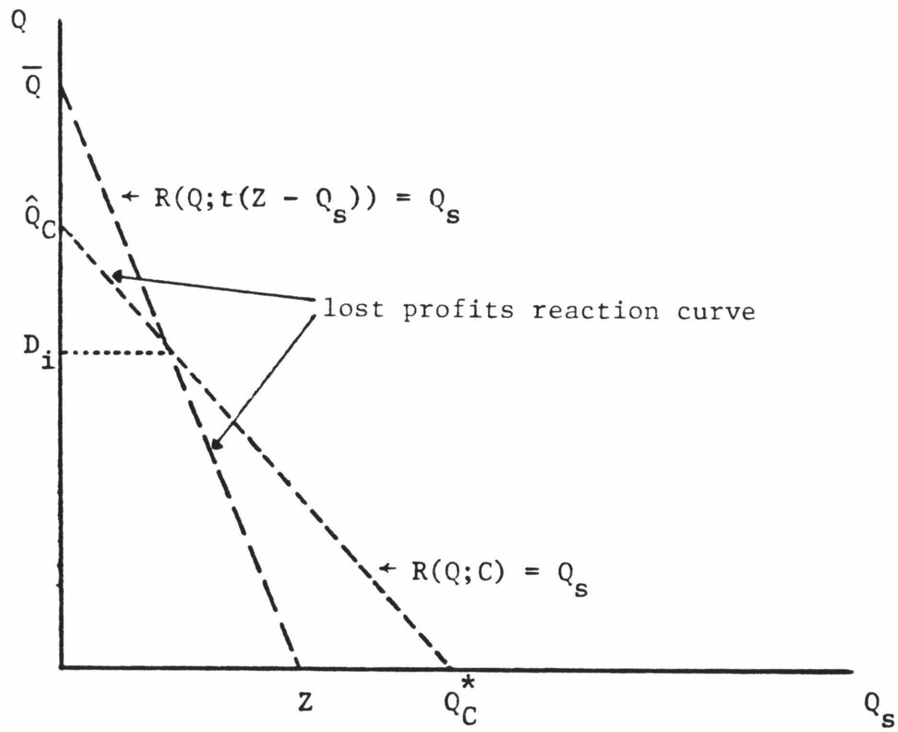

FIGURE 5

Q_C^* is the monopoly output for marginal costs C.

$\hat{Q}_C$ is the competitive output for marginal costs C.

$\overline{Q}$ is the amount the original seller needs to supply to prevent a contract buyer, who cannot breach, from entering.

D is the critical level of the original seller's supply which determines whether the contract buyer will breach or not under lost profits.

The solution to the buyer's problem

$$[Q_c,Q_s,Q_b] = \begin{cases} [t^{-1}(P);0;Z - Q_c] & P < K \\ [t^{-1}(K),R(Q;K),Z - Q_c - Q_s]; \quad Q > E & \\ [Z - Q_s,R(Q;t(Z - Q_s));0]; \quad Q \leq E & P \geq K \\ [t^{-1}(P);t^{-1}(P) - Z;0] & t(Z) \geq P \end{cases} \quad \left.\begin{matrix} \\ \\ \end{matrix}\right\} t(Z) < P \tag{17}$$

where $Q \geq E$ is analogous to the condition $Q \geq D$ in the lost profits calculation.

Using (17) we can again compute the profit of the original seller when the buyer breaches. For $P < K$, the profit is

$$\max_{Q} K \cdot Q_c + [K - P(Q)][Z - Q_c] + P(Q)Q - C[Q + Q_c]. \tag{18}$$

The first two terms in (18) consist of the revenue earned from selling Q_c in the contract market and the market damages collected on the units breached. The remaining terms are the revenue from the spot market minus the entire cost of production.

For $P \geq K$, the contract buyer is reselling according to the two part reaction curve defined in (19). Let us denote the reaction curve by $\mathbb{R}^m(Q)$. The original seller will optimize against the reaction curve. Thus, the seller's profit is

$$\max_{Q} K(Q_c + Q_s) + P(Q + \mathbb{R}^m(Q))Q - C[Q + (Q_c + Q_s)]. \tag{19}$$

Combining (18) and (19), and simplifying further, we find that the seller's profits under market damages are

$$\pi^m = \max_Q \begin{cases} [P(Q) - C][Q + Q_c] + [K - P(Q)]Z & P(\cdot) < K \\ [P(Q + \mathbb{R}^m(Q)) - C]Q + [K - C][\mathbb{R}^m(Q) + Q_c] & P(\cdot) \geq K \end{cases}$$

$$= \begin{cases} [P(Q^m) - C][Q^m + Q_c] + [K - P(Q^m)]Z & P(\cdot) < K \\ [P(Q^m + \mathbb{R}(Q^m)) - C][Q^m + Q_c + \mathbb{R}(Q^m)] & \\ \qquad + [K - P(Q^m + \mathbb{R}(Q^m))]Z & P(\cdot) \geq K. \end{cases} \tag{20}$$

3. Specific Performance

When the remedy for breach is specific performance, buyers are effectively never allowed to breach. Once a contract is made they can only choose between consumption and resale. Thus buyer's objective function is

$$\pi = T(Q_c) + P(Q + Q_s) \cdot Q_s - KZ, \tag{21}$$

and the problem is to maximize (21) subject to $Q_c + Q_s = Z$. The solution is

$$[Q_c, Q_s] = [Z - Q_s, R(Q; t(Z - Q_s))]. \tag{22}$$

The buyer's behavior in equation (22) is exactly the same as lost profits when $Q < D$, and market damages when $Q < E$.

Figure 6 illustrates the reaction curve for the case when there is only one contract buyer. If there were more than one contract buyer in each case, the seller would aggregate the reaction curves of all the resellers, resulting in a grand reaction curve which tells how much the others will supply in total for each quantity supplied by the original seller. However, that is an unnecessary complication for the present purposes. Denoting $R(Q;t(Z - Q_s))$ by $\mathbb{R}^s(Q)$, the profits to the seller will be

$$\begin{aligned} \pi^s &= \max_Q \quad (K - C)Z + [P(Q + \mathbb{R}^s(Q)) - C]Q \\ &= (K - C)Z + [P(Q^s + \mathbb{R}^s(Q^s)) - C]Q^s. \end{aligned} \tag{23}$$

4. <u>Comparison of the Remedies</u>

In order to determine how well each remedy protects the expectation interest of the innocent party, we compare the profits the innocent party gets in case of breach under each remedy with those it gets under specific performance, our benchmark. We recall the seller's profits under lost profits, market damages, and specific performance.

$$\pi^\ell = (K - C)Z + [P(Q^\ell + \mathbb{R}^\ell Q^\ell)) - C]Q^\ell \tag{24}$$

$$\pi^m = \begin{cases} [P(Q^m) - C][Q^m + Q_c] + [K - P(Q^m)]Z & P < K \\ [P(Q^m + \mathbb{R}^m(Q^m) - C][Q^m + Q_c + \mathbb{R}^m(Q^m)] & \\ \qquad + [K - P(Q + \mathbb{R}^m(Q^m))]Z & P \geq K \end{cases} \tag{25}$$

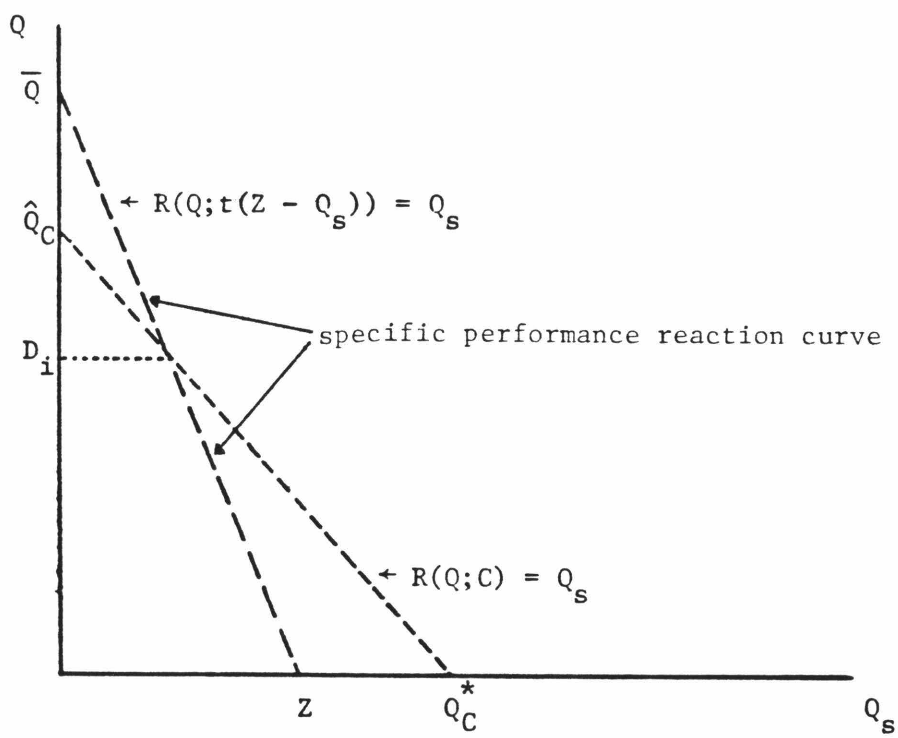

FIGURE 6

Q_C^* is the monopoly output for marginal costs C.

$\hat{Q}_C$ is the competitive output for marginal costs C.

$\overline{Q}$ is the amount the original seller needs to supply to prevent a contract buyer, who cannot breach, from entering.

D is the critical level of the original seller's supply which determines whether the contract buyer will breach or not under lost profits.

$$\pi^{s} = (K - C)Z + [P(Q^{s} + \mathbb{R}^{s}(Q^{s})) - C]Q^{s}. \tag{26}$$

Let us begin with lost profits.

$$\begin{aligned} \pi^{\ell} - \pi^{s} = {} & [P(Q^{\ell} + \mathbb{R}^{\ell}(Q^{\ell})) - C]Q^{\ell} \\ & - [P(Q^{s} + \mathbb{R}^{s}(Q^{s})) - C]Q^{s}. \end{aligned} \tag{27}$$

From (27) it is apparent that the sign of the differences rests fully on the activity in the spot market. Since for the contract buyer, the amount resold under specific performance is less than or equal to that supplied under lost profits, we have that $\mathbb{R}^{\ell}(Q) \leq \mathbb{R}^{s}(Q)$ for all Q. An intuitive explanation of this phenomenon is that under lost profits, the contract buyer can dispose of unwanted goods in two ways, reselling and breaching. Whereas under specific performance, there is only one avenue of disposal, to dump unwanted goods in the spot market. Thus more (or an equal amount) is resold on the spot market under specific performance than under lost profits. Since more is supplied, the original seller will have to share more of the market with the resellers. Consequently, the original seller earns at least as much profit under lost profits as under specific performance. But since we make the comparison when a buyer actually breaches under lost profits, the difference in (24) is actually strictly positive. The situation when a buyer actually breaches restricts our attention to the region where $Q > D$, that is when the lost profits reaction curve is strictly beneath the specific performance reaction curve. Figure 7 will clarify our claims.

As we can see from the diagram, the lost profits reaction curve is identical with the reaction curve for specific performance, below D. Above D, it bends downwards. At $(0,Q_C^*)$ the original seller gets the most profits and as we move outward in any direction profits decrease. Thus if the tangency occurs above D, the isoprofit curve tangent to the lost profits reaction curve will be at a higher profit level than that which is tangent to the specific performance reaction curve. From this we see that π^ℓ is strictly greater than π^s, and so lost profits strictly overcompensates.

Next we shall compare market damages with specific performance. Since we are concerned only about what happens in case of breach, we need to deal with only the cases when $P < K$, and when $P \geq K$ and $Q > E$.

For $P < K$, an argument similar to that used in the competitive case can be used to show that the market damages remedy undercompensates the seller. The proof is in the appendix. However, for $K \leq P$ we show that it is ambiguous whether the market damages remedy protects the seller's expectation interest.

C. EXTENSIONS

In the last sections, we concluded that the lost profits remedy is overcompensatory and the market damages remedy usually undercompensates under the assumptions of our simple model. These results are robust to relaxing certain of these assumptions.

In particular, the conclusions of the simple model are true when we add more contract buyers to the model. Similarly, if we allow

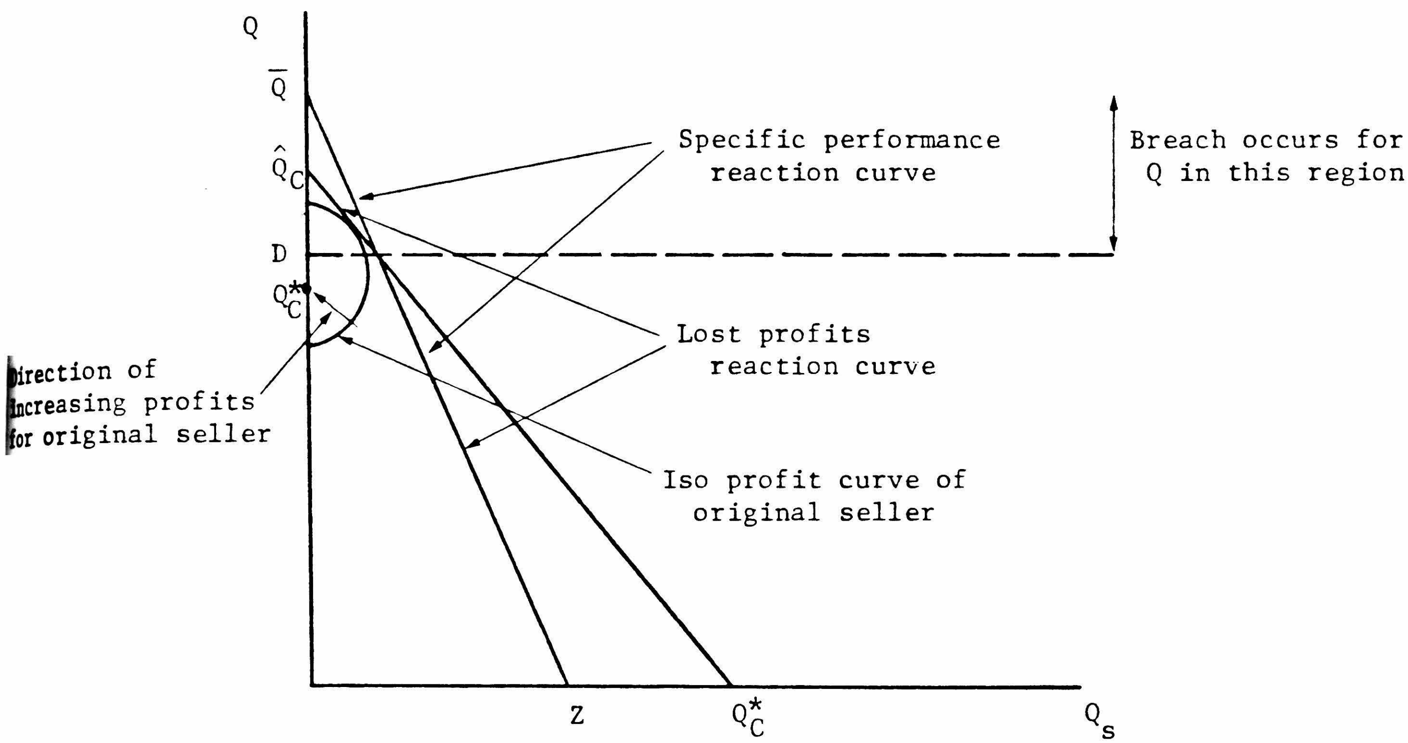
Q
$\overline{Q}$
$\hat{Q}_C$
D
Q_C^*
Specific performance reaction curve
Breach occurs for Q in this region
Lost profits reaction curve
Iso profit curve of original seller
Direction of increasing profits for original seller
Z
Q_C^*
Q_s

FIGURE 7

general convex total cost curves, the results remain unchanged. Moreover, if we add more sellers to the problem, all of which behave in a Cournot fashion with respect to one another (in the Stackelberg case they all behave as leaders with respect to the contract buyers who are selling in the spot market), the generalization does not affect the conclusions. Furthermore, as we add more sellers, the overcompensation under lost profits decreases as does the undercompensation under market damages (for $P < K$). If we retain the assumption of constant marginal costs, and allow the number of sellers to increase, the profit to any seller under each of the three remedies converges to the same amount. Otherwise with non-constant marginal costs, even in the limit, the profit to a seller under the market damages remedy (for $P < K$) is less than that under specific performance, which is less than that under the lost profits remedy. The profits under market damages for $P \geq K$, remain ambiguous when we add more and more sellers. Finally, connection between the size of the resale costs and the overcompensatory and undercompensatory natures of the lost profits and market damages remedies respectively is retained in the Stackelberg case.

III. RELATED LITERATURE

The state of the art literature on buyer's breach is due to Goetz and Scott. The general conclusions of these authors are in agreement with those of this work. In fact, they develop a good intuitive explanation of why these conclusions hold.[2] However, their formal analysis relies mainly on diagrams and does not quite capture the complete argument. Furthermore, the "story" behind the diagrams is never made very explicit. In fact, it is implicitly assumed that when one compares the profits of the seller under lost profits with those under specific performance, profits in the contract market are the same and thus can be ignored, so that only the profits in the spot market matter. This is only true when one correctly interprets the lost profits measure when marginal costs are not constant, since lost profits are always calculated assuming that the last units that would have been produced are breached. Unfortunately, most of their work for the case of the seller with market power deals with increasing marginal costs.

Goetz and Scott consider two cases: sellers with market power and competitive sellers. This is a separation into possibilities on the supply side. However, in the competitive case they also assume that total market demand is competitive. This is evidenced by their claim that every seller can replace breached contracts by selling more in the spot market at the same price. Thus their analysis for the competitive case has limited applicability since total market demand curves are rarely flat.

Finally, in the Goetz and Scott model, buyers who wish to resell always enter the spot market competitively. We draw this conclusion by noting that whenever a buyer breaches, the spot market demand curve in their diagrams shifts outward by the amount the buyers would have resold. This indicates that the buyers are able to sell as much as they wish at the going spot market price. We have examined this case in Section II and realized that under these circumstances, buyers will never breach unless they face a cost disadvantage in the spot market. But this aspect is never explicitly introduced in their model. This certainly is a weakness of their model since the reselling cost is necessary in their model for buyers to ever breach. Furthermore, Goetz and Scott give examples where buyers would have resold virtually all of the breached amount if breach were not allowed, since they had equal access to the resale market. Thus there is a need for a model that does not rely on resale costs as an explanation of breach.

In summary, Goetz and Scott have a very good intuition of why lost profits are overcompensatory. However, their analysis does not support their claim.

IV. OTHER CRITERIA FOR COMPARISON

From the models in the previous sections we have concluded that the lost profits remedy overcompensates the seller and the market damages remedy usually undercompensates. However, there are other criteria for comparing the remedies.

One important consideration is the economic efficiency of the remedies. A complete analysis of economic efficiency requires a model of the two period sequence of contract and spot markets. Then the total expected surplus under each remedy can be compared. However this has proven to be intractable.

The important aspect for efficiency is that all buyers should purchase until their marginal utility equals the marginal production cost from either market, and reselling by buyers should be eliminated since it is inefficient due to the extra reselling costs. Thus in the spot market the price should be marginal cost since that is the last period of the model and so all buyers will be using the straightforward strategy of purchasing until their marginal utility equals price. The contract price can be higher than marginal cost reflecting the added gain to the buyer of being able to resell next period and the loss to the original seller from entry by the contract buyers into the spot market price. But if the spot market price was always going to be marginal cost, no one would make contracts for prices greater than the spot price. Thus, there would be no need for the contract market if the spot market were operating efficiently.

However, this follows when the only reason for the existence of the contract market is due to uncertainty of the spot market price, as it is in the formal model. Contracts can also occur because of convenience. For example in the case of new car sales the normal method of exchange is through a contract, whereby the buyer need not carry cash during the search process, and he or she can have some time in which to back out and only incur the damages. Thus the contract market would not vanish in many realistic cases. Now, for ex ante economy efficiency we need that the expected number of resales by contract buyers be zero. For ex poste efficiency we need that the inefficiency due to the actual number of resales to be compensated by the gain in efficiency due to the convenience of contracts.

Unfortunately, even with many simplyfing assumptions, the mathematical analysis of the two period model is too difficult to permit computation of the spot and contract prices under each remedy. As a result only a discussion of informal observations of the efficiency properties of the remedies is possible. For a fixed contract, specific performance has the disadvantage that more resales occur than under the other two remedies, which results in more inefficiency whenever the reselling costs to the contract buyer are greater than that to the seller. However the effect becomes ambiguous when we notice that less contracts will occur and less will be contracted for under specific performance than under the other two remedies, simply because there are fewer options available under specific performance.

Similarly, lost profits is inefficient in two ways. First, the overcompensation is greater the fewer sellers there are in the market, that is, the more market power the seller has. When there is market power, output is restricted, which is inefficient. The inefficiency is compounded by lost profits, which provides greater overcompensation for sellers that are more inefficient. Second, we need to consider the incentives of a seller, with or without market power, under the lost profits remedy. Since the lost profits remedy reimburses the seller for the profits that would have been earned on the breached contract, the seller can behave as if total sales were the sum of actual sales plus those that were breached. This illusion allows the existence of more sellers than is warranted by the demand. Thus, lost profits is inefficient.

The efficiency properties of the market damages remedy are not clear without endogenizing the contract market. Thus, with the present analysis, some efficiency properties can be discussed, but a comparison of the overall efficiency of each remedy is impossible.

Lastly, for practical purposes it is important that a remedy is not costly to implement. In this regard, market damages and specific performance are favored over lost profits when a thick spot market exists. This is because only the market price is needed for market damages and specific performance has no informational requirements, but for lost profits it is necessary to know the exact production costs of the grievant.

VI. CONCLUSION

We have shown, perhaps not surprisingly, that each remedy has its advantages and disadvantages. As an expectation interest remedy, specific performance is the best, but it can be inefficient. Market damages generally undercompensates the seller and is ambiguous as an expectation interest remedy when the spot market price is greater than the contract price. Lost profits, though it is neither efficient nor protective of expectation interest, is consistent in overcompensating the seller and costly to implement. Thus, it is difficult to decide which remedy is best for all situations.

However, when we consider particular circumstances it is clear in each case that one remedy is better than the rest. If the spot market is thick, lost profits can be ruled out since it is costly to compute and in this case both inefficient and overcompensating.

Specific performance will protect the seller's expectation exactly, but will be inefficient for large reselling costs. Thus for low reselling costs specific performance is best. As the reselling costs become higher, market damages may become a better alternative. Certainly it will be undercompensatory but that will be traded off with the likely gain in efficiency since less is resold under market damages than under specific performance. Finally, lost profits is ideal when the spot market is thin or non-existent. In this case market damages is difficult to compute since it is not clear what the spot market price is. Furthermore, it will be very undercompensatory since there are virtually no resales. Specific performance will tend to be very inefficient since it will frequently force the buyer to keep the goods. The problem of overcompensation of lost profits will vanisn since the thin or non-existent resale market implies the profit is really lost. Similarly the inefficiency due to the overcompensation being greater for sellers with more market power, will no longer exist. This leaves only the costs of implementing lost profits.

REFERENCES

Goetz, Charles J. and Robert E. Scott. (1979), "Measuring Sellers' Damages: The Lost-Profits Puzzle." Stanford Law Review 31, 323-373.

VI APPENDIX

A. COMPETITIVE ASSUMPTION

We will first solve the buyer's problem at the time the random demand is revealed for each remedy and determine the seller's profits under each remedy. Then the seller's profit under specific performance will be compared with those under the other two remedies.

1. Lost Profits.

The buyer's problem is as follows:

$$\max_{Q_c, Q_s, Q_b, Q_p} \quad T(Q_c) + Q_s(P - r) - KZ - PQ_p + CQ_b$$

$$\text{subject to} \quad Q_c + Q_s + Q_b - Z + Q_p$$

$$Q_c \geq 0, Q_s \geq 0, Q_b \geq 0, Q_p \geq 0. \tag{A1}$$

Let us solve for Q_c and substitute into the objective function.

$$\max_{Q_S, Q_B, Q_P} \quad T(Z + Q_p - Q_s - Q_b) + Q_s(P - r) - KZ + CQ_b - PQ_p$$

$$\text{subject to} \quad Q_s + Q_b + \leq Z + Q_p$$

$$Q_s \geq 0, Q_b \geq 0, Q_p \geq 0. \tag{A2}$$

We define

$$L = T(Z + Q_p - Q_s - Q_b) + Q_s(P - r) - KZ + CQ_b - PQ_b$$
$$+ \lambda(Z + Q_p - Q_b - Q_s) + \alpha Q_s + \beta Q_b + \gamma Q_p, \tag{A3}$$

and the first order conditions that follow are

$$\frac{\partial L}{\partial Q_s} = -t(\cdot) + C - \lambda + \alpha = 0$$

$$\frac{\partial L}{\partial Q_b} = -t(\cdot) + P - r - \lambda + \beta = 0$$

$$\frac{\partial L}{\partial Q_b} = \ t(\cdot) - P + \lambda + \gamma = 0$$

$$\begin{array}{lll} \lambda(Z + Q_p - Q_s - Q_b) = 0 & \lambda \geq 0 & Z + Q_p - Q_s - Q_b \geq 0 \\ \alpha Q_s = 0 & \alpha \geq 0 & Q_s \geq 0 \\ \beta Q_b = 0 & \beta \geq 0 & Q_b \geq 0 \\ \gamma Q_p = 0 & \gamma \geq 0 & Q_p \geq 0. \end{array} \qquad \text{(A4)}$$

Looking at the first three conditions,

$$\begin{aligned} t(\cdot) &= \alpha + C - \lambda \\ t(\cdot) &= \beta + P - r - \lambda \\ t(\cdot) &= -\gamma + P - \lambda \end{aligned} \qquad \text{(A5)}$$

we see that no two of α, β, γ can simultaneously be zero for the equations in (A5) to remain consistent. Thus the buyer engages in only one of the three activities of reselling, breaching and purchasing at a time. Since the buyer always consumes a nonzero amount, $\lambda = 0$.

Suppose $Q_s > 0$ then $\alpha = 0, \beta > 0, \gamma > 0$ and $Q_b = 0$, $Q_p = 0$. The system of equations (A4) becomes

$$-t(Z - Q_s) + C = 0$$

or

$$Q_c = t^{-1}(C),\ Q_s = Z - t^{-1}(C),\ Q_b = 0,\ Q_p = 0 \tag{A6}$$

If $Q_b > 0$, then $\alpha > 0$, $\beta = 0$, $\gamma > 0$, $Q_s = 0$, $Q_p = 0$ and (A4) reduces to

$$-t(Z - Q_b) + P - r = 0$$

or

$$Q_c = t^{-1}(P - r),\ Q_s = 0,\ Q_b = Z - t^{-1}(P - r),\ Q_p = 0 \tag{A7}$$

Finally if $Q_p > 0$, then $\alpha > 0$, $\beta > 0$, $\gamma = 0$, $Q_s = 0$, $Q_b = 0$ and (A4) becomes

$$t(Z + Q_p) - P = 0$$

or

$$Q_c = t^{-1}(P),\ Q_s = 0,\ Q_b = 0,\ Q_p = t^{-1}(P)-Z \tag{A8}$$

To determine when each case will occur, we need to consider the conditions on the multipliers associated with each. Let us begin again with $Q_s > 0$, which implies $\beta > 0$, $\gamma > 0$. If we equate the three expressions in (A5), recalling that $\alpha = 0$ and $\lambda = 0$

$$C = \beta + P - r = -\gamma + P, \tag{A9}$$

we deduce that

$$\begin{aligned} &\beta > 0 \;\rightarrow\; C > P - r \\ &\gamma > 0 \;\rightarrow\; C < P \\ &Q_s > 0 \;\rightarrow\; Z - t^{-1}(C) > 0 \;\rightarrow\; t(Z) < C \end{aligned} \tag{A10}$$

Since it is always true that $C < P$, (A6) is characterized by $C > P - r$, and $t(Z) < C$.

If we apply the same procedure to (A7) we find that

$$\alpha + C = P - r = -\gamma + P, \tag{A11}$$

$$\begin{aligned} &\alpha > 0 \;\rightarrow\; P - r > C \\ &\gamma > 0 \;\rightarrow\; P - r > C \\ &Q_B > 0 \;\rightarrow\; Z - t^{-1}(P - r) > 0 \;\rightarrow\; t(Z) < P - r \end{aligned} \tag{A12}$$

and the relevant conditions are $P - r > C$, and $T(Z) < P - r$.

Finally for (A8) we find

$$\alpha + C = \beta + P - r = P \tag{A13}$$

$$\begin{aligned} &\alpha > 0 \;\rightarrow\; C < P \\ &\beta > 0 \;\rightarrow\; P - r < P \\ &Q_p > 0 \;\rightarrow\; t^{-1}(P) - Z > 0 \;\rightarrow\; P < t(Z). \end{aligned} \tag{A14}$$

The first two conditions in (A14) are always true and the last one,

$P < t(Z)$, defines the case.

Combining (A6) to (A14) we get the complete solution.

$$[Q_c, Q_b, Q_s, Q^p] = \begin{cases} \left.\begin{array}{llll} [t^{-1}(C), Z-t^{-1}(C), & 0, & 0] \\ & & C>(P-r) \\ [t^{-1}(P-r), & 0, Z-t^{-1}(P-r), & 0] \\ & & C\leq(P-r) \end{array}\right\} & (t(Z)<\max\{P-r, C\} \\ [\ \ Z, \quad 0, \quad 0, \quad 0] & \max\{P-r, C\} \leq (t(Z) \leq P) \\ [t^{-1}(P), \quad 0, \quad 0, t^{-1}(P)-Z] & P<(t(Z)). \end{cases}$$

The seller's profit when the buyer breaches is

$$\pi^{\ell} = \max_{Q} \; [P(Q) - C]Q + (K - C)Z. \tag{A15}$$

2. Market Damages

In this case the buyer's objective function is

$$\max_{Q_c, Q_s, Q_b, Q_p} \begin{cases} T(Q_c) + (P - r)Q_s - K - Z + PQ_b - PQ_p & P < K \\ T(Q_c) + (P - r)Q_s - KZ + KQ_b - PQ_p & P \geq K \end{cases}$$

$$\text{subject to} \quad \begin{array}{l} Q_c = Z - Q_b - Q_s + Q_p \\ Q_c \geq 0, Q_b \geq 0, Q_s \geq 0, Q_p \geq 0. \end{array} \tag{A16}$$

We will solve each case separately. For $P < K$ we see that breach always dominates resale. Incorporating this fact, substituting for Q_c and adding the constraints with their multipliers we define

$$L = T(Z - Q_b + Q_p) - KZ + PQ_b - PQ_p$$
$$+ \lambda(Z + Q_p - Q_b) + \beta Q_b + \gamma Q_p. \qquad (A17)$$

The first order conditions associated with maximizing (A17) are

$$\frac{\partial L}{\partial Q_b} = -t(\cdot) + P - \lambda + \beta = 0$$

$$\frac{\partial L}{\partial Q_p} = t(\cdot) - P + \lambda + \gamma = 0$$

$$\lambda(Z + Q_p - Q_b) = 0 \qquad \lambda \geq 0 \qquad Z + Q_p - Q_b \geq 0$$
$$\beta Q_b = 0 \qquad \beta \geq 0 \qquad Q_b \geq 0$$
$$\gamma Q_p = 0 \qquad \gamma \geq 0 \qquad Q_p \geq 0.$$
$$(A18)$$

For $P \geq K$, the buyer will prefer to resell than breach

$$L = T(Z - Q_b - Q_s + Q_p) + (P - r)Q_s - KZ + KQ_b - PQ_b$$
$$+ \lambda(Z + Q_p - Q_s - Q_b) + \alpha Q_s + \beta Q_b + \gamma Q_p, \qquad (A19)$$

and the corresponding first order conditions are

$$\frac{\partial L}{\partial Q_s} = -t(\cdot) + P - r - \lambda + \alpha = 0$$

$$\frac{\partial L}{\partial Q_b} = -t(\cdot) + K - \lambda + \beta = 0$$

$$\frac{\partial L}{\partial Q_p} = t(\cdot) - P + \lambda + \gamma = 0$$

$$\begin{array}{rlll}
\lambda(Z + Q_p - Q_s - Q_b) = 0 & \lambda \geq 0 & & Z - Q_p - Q_s \geq 0 \\
\alpha Q_s = 0 & \alpha \geq 0 & & Q_s \geq 0 \\
\beta Q_b = 0 & \beta \geq 0 & & Q_b \geq 0 \\
\gamma Q_p = 0 & \gamma \geq 0 & & Q_p \geq 0
\end{array} \tag{A20}$$

The solution to (A18) and (A20) can be deduced in a similar fashion to the previous lost profits case. The system (A18) reduces to

$$[Q_c, Q_b, Q_s, Q_p] = \begin{array}{lll} [t^{-1}(P), Z - t^{-1}(P), \quad 0, \quad 0] & t(Z) < P & \\ & & P < K, \\ [t^{-1}(P), \quad 0, \quad 0, t^{-1}(P) - Z] & t(Z) \geq P & \end{array}$$

and (A20) becomes

$$\begin{bmatrix} Q_c \\ Q_b \\ Q_s \\ Q_p \end{bmatrix} = \left[\begin{array}{cccc} t^{-1}(P - r) & t^{-1}(K) & Z & t^{-1}(P) \\ 0 & Z - t^{-1}(K) & 0 & 0 \\ Z - t^{-1}(P - r) & 0 & 0 & 0 \\ 0 & 0 & 0 & t^{-1}(P) - Z \\ \multicolumn{2}{c}{\underbrace{K \leq P - r \qquad\qquad K > P - r}_{t(Z) < \max\{P - r, K\}}} & \underbrace{\qquad}_{\substack{P > t(Z) \\ \geq \max\{P - r, K\}}} & \underbrace{\qquad}_{t(Z) \geq P} \end{array} \right]$$

$$P \geq K$$

and combining the two we get

$$\begin{bmatrix} Q_c \\ Q_b \\ Q_s \\ Q_p \end{bmatrix} = \begin{bmatrix} t^{-1}(P) & t^{-1}(P - r) & t^{-1}(K) & Z & t^{-1}(P) \\ Z - t^{-1}(P) & 0 & Z - t^{-1}(K) & 0 & 0 \\ 0 & Z - t^{-1}(P - r) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t^{-1}(P) - Z \end{bmatrix}$$

$$\underbrace{P < K \qquad \underbrace{\underbrace{K \leq P - r \qquad K > P - r}_{t(Z) < \max\{P - r, K\}} \qquad \underbrace{\qquad}_{\substack{\max\{P - r, K\} \\ \leq t(Z) - P}}}_{P \geq K}}_{t(Z) < P} \qquad \underbrace{\qquad}_{t(Z) \geq P}$$

(A21)

The seller's profit when the buyer breaches is given by

$$\pi^m = \max_Q \begin{cases} [P(Q) - C]Q + [K - P(Q)][Z - t^{-1}(P)] + [K - C]t^{-1}(P) & P < K \\ [P(Q) - C]Q + [K - C]t^{-1}(K) & P \geq K. \end{cases}$$

(A22)

3. Specific Performance

Here the buyer's objective function is

$$\max_{Q_c, Q_s, Q_p} \quad T(Q_c) + (P - r)Q_s - KZ - PQ_p$$

$$\text{subject to} \quad Z + Q_p - Q_c - Q_s = 0$$

(A23)

We define L as before

$$L = T(Z + Q_p - Q_s) + (P - r)Q_s - KZ - PQ_p$$
$$+ \lambda(Z + Q_p - Q_s) + \alpha Q_s + \gamma Q_p, \qquad \text{(A24)}$$

and the first order conditions are

$$\frac{\partial L}{\partial Q_s} = -t(\cdot) + P - r - \lambda + \alpha = 0$$

$$\frac{\partial L}{\partial Q_p} = t(\cdot) - P + \lambda + \gamma = 0$$

$$\begin{array}{lll} \lambda(Z + Q_p - Q_s) = 0 & \lambda \geq 0 & Z + Q_p - Q_s \geq 0 \\ \alpha Q_s = 0 & \alpha \geq 0 & Q_s \geq 0 \\ \gamma Q_p = 0 & \gamma \geq 0 & Q_p \geq 0. \end{array}$$

$$\text{(A25)}$$

The solution is

$$[Q_c, Q_s, Q_p] = \begin{array}{llll} [t^{-1}(P - r), Z - t^{-1}(P - r), & 0] & t(Z) < P - r \\ [\quad Z, & 0, & 0] & P - r \leq t(Z) \leq P \\ [t^{-1}(P), & 0, t^{-1}(P) - Z] & & P < t(Z). \end{array}$$

$$\text{(A26)}$$

The profit to the seller under specific performance is

$$\pi^s = \max_Q \begin{cases} [P(X) - C]Q + (K - C)Z & t(Z) < P - r \\ [P(Q) - C]Q + (K - C)Z & P \geq t(Z) \geq P - r \\ [P(Y) - C]Q + (K - C)Z & t(Z) > P \end{cases}$$

where $\quad X = Q + Z - t^{-1}(P(X) - r), Y = Q + Z - t^{-1}(P(X)) \qquad \text{(A27)}$

4. Comparison of Remedies

We will show here that lost profits overcompensates the seller and market damages usually undercompensates the seller. This is achieved by comparing the seller's profit when the buyer breaches under each of the lost profits and market damages with the profit the seller makes under specific performance. Since it is never the case that a buyer would purchase under specific performance when he would have breached under the other two remedies, it is not necessary to consider the last branch of (A27), where the buyer purchases under specific performance. Recall equations (A15) and (A27).

$$\pi^{\ell} = \max_{Q} \; [P(Q) - C]Q + [K - C]Z \tag{A15}$$

$$\pi^{s} = \max_{Q} \begin{cases} [P(X) - C]Q + [K - C]Z & t(Z) < P - r \\ [P(Q) - C]Q + [K - C]Z & t(Z) \geq P - r \end{cases}$$

$$\text{where} \quad X = Q + Z - t^{-1}(P(X) - r). \tag{A27}$$

If we take the difference between π^{ℓ} and π^{s}, for $t(Z) \geq P - r$ it is zero, but for $t(Z) < P - r$ it is positive. To see the latter claim, suppose Q^* and X^*, $(X^* = Q^* + Z - t^{-1}(P(X^*) - r))$, maximized (A27) for the case $t(Z) < P - r$. If we substitute $Q = Q^*$ in (A15) then π^{ℓ} will be greater than or equal to the evaluation of (A15) at $Q = Q^*$ since Q^* may not be the optimal choice of Q for (A15). Thus

$$\pi^{\ell} \geq [P(Q^*) - C]Q^* + [K - C]Z$$
$$> [P(X^*) - C]Q^* + [K - C]Z = \pi^s, \tag{A28}$$

since $X^* > Q^*$ and so $P(X^*) < P(Q^*)$.

We notice that the greater r is, the more unlikely the $t(Z) < P - r$, thus the buyer is less likely to engage in any resale under specific performance. Also this is exactly when lost profits is likely to compensate exactly. Even if $t(Z) < P - r$, the greater r is the smaller $Z - t^{-1}(P(X) - r)$ is (that is, smaller amounts are resold by the buyer under specific performance) and the smaller the overcompensation.

For market damages we will first look at the case when $P < K$. We can rewrite the seller's profit

$$\pi^m = \max_Q [P(Q) - C][Q - t^{-1}(P(Q))] + [K - P(Q)]Z \tag{A29}$$

Under specific performance either $t(Z) < P(X) - r$ or $t(Z) \geq P(X) - r$. Let us first consider the case of $t(Z) < P(X) - r$. We can state the seller's profits for this case with an expression very similar to (A29).

$$\pi^s = \max_Q [P(X) - C]Q + (K - C)Z \quad \text{where } X = Q + Z - t^{-1}(P(X) - r)$$

$$= \max_X [P(X) - C]X - [P(X) - C][Z - t^{-1}(P(X) - r)] + [K - C]Z$$

$$= \max_X [P(X) - C][X + t^{-1}(P(X) - r)] + [(K - C) - (P(X) - C)]Z$$

$$= \max_X [P(X) - C][X + t^{-1}(P(X) - r)] + [K - P(X)]Z. \tag{A30}$$

The optimizations in (A29) and (A30) are very similar. Suppose Q^* solves (A29). Let us evaluate (A30) at $X = Q^*$. Since $P(Q^*) - r < P(Q^*)$, and $t^{-1}(\cdot)$ is downward sloping $t^{-1}(P(Q^*) - r) > t^{-1}(P(Q^*))$. Thus the evaluation of (A30) at $X = Q^*$ will be greater than the evaluation of (A29) at Q^* which is π^m. On the other hand, $X = Q^*$ may not be the optimal choice of X for the problem in (A30) so that π^s will be greater than or equal to the evaluation of (A30) at $X = Q^*$. It clearly follows that $\pi^s > \pi^m$, for the case of $t(Z) < P(X) - r$.

With a parallel argument we can show that for $t(Z) \geq P(X) - r$, it will still be the case that $\pi^s \geq \pi^m$. We can rewrite the expression for π^m in the following manner

$$\pi^m = \max_Q [P(Q) - C]Q + [K - P(Q)][Z - t^{-1}(P)] + [K - C]t^{-1}(P)$$

$$= \max_Q [P(Q) - C][Q - t^{-1}(P(Q))] + [K - C]Z. \tag{A31}$$

Recall

$$\pi^s = \max_Q \; [P(Q) - C]Q + (K - C)Z, \tag{A27}$$

for the case under consideration. Suppose Q^* maximizes (A31), then the evaluation of (A27) at $Q = Q^*$ will be greater than π^m, which is the evaluation of (A31) at $Q = Q^*$ since $Q^* - t^{-1}(P(Q^*)) < Q^*$. But since it is not clear that Q^* maximizes (A27), π^s will be at least as great at the evaluation of (A27) at $Q = Q^*$. Thus $\pi^s > \pi^m$ for both cases, and market damages undercompensates the seller, when $P(Q) < K$.

For $P(Q) \geq K$ the result is not as straightforward. The original seller's profit is given by

$$\pi^m = \max_Q \; [P(Q) - C]Q + [K - C]t^{-1}(K)$$

and the buyer will breach whenever $t(Z) < K$ and $K > P - r$ (where P is the price in that obtains in the spot market when the remedy is market damages). If we compare the profit above with that under specific performance

$$\pi^s = \max_Q \begin{cases} [P(X) - C]Q + [K - C]Z & t(Z) < P - r \\ [P(Q) - C]Q + [K - C]Z & t(Z) \geq P - r \end{cases}$$

we find that if the maximum occurs on the second branch ($t(Z) \geq P - r$) then market damages undercompensates. This can be seen by simply observing that the first term, $[P(Z) - C]Q$, is present in both problems and the second term in each case is independent of Q. Thus both objective functions will be maximized at the same value of Q; the optimized value of the first term in each case will thus be the same.

Since $[K - C]Z$ is larger than $[K - C] + t^{-1}(K)$, π^s will be greater than π^m and market damages will undercompensate. However if π^s is maximized on the first branch $(t(Z) < P - r)$, we cannot determine the sign of $\pi^m - \pi^s$ since the profit in the spot market will be larger for market damages than specific performance but the opposite is true for the contract market.

B. STACKELBERG ASSUMPTION

As in the competitive case, the buyer's problem under each remedy is solved and an expression for the seller's profit in each case is obtained. Then lost profits and market damages are compared with specific performance to determine how well each protects the seller's expectations.

1. Lost Profits

The buyer's problem

$$\max_{Q^c, Q_s} T(Q^c) + P(Q + Q_s)Q_s + (Z - Q_c - Q_s)C - KZ$$
$$\text{subject to} \quad Q_c + Q_s \leq Z, Q_c \geq 0 \qquad \text{(A32)}$$

is rewritten as follows to accommodate the constraints

$$L = T(Q_c) + P(Q + Q_s)Q_s + (Z - Q_c - Q_s)C$$
$$- KZ + \lambda(Z - Q_c - Q_s) + \mu Q_c. \qquad \text{(A33)}$$

The corresponding first order conditions are

$$\frac{\partial L}{\partial Q_c} = t(Q_c) - C - \lambda + \mu = 0$$

$$\frac{\partial L}{\partial Q_s} = P(Q + Q_s) - C + Q_s P(Q + Q_s) - \lambda = 0$$

$$\lambda(Z - Q_c - Q_s) = 0 \qquad \lambda \geq 0 \qquad Z - Q_c - Q_s \geq 0$$

$$\mu Q_c = 0 \qquad \mu \geq 0 \qquad Q_c \geq 0, \tag{A34}$$

and they yield the solution

$$[Q_c, Q_s, Q_b] = \begin{cases} [t^{-1}(C), R(Q;C) & , Z - Q_c - Q_s] \quad Q > D \\ [Z - Q_s, R(Q;t((Z - Q_s)), & 0] \quad Q \leq D \end{cases} \; t(Z) < P$$

$$ \;\; [t^{-1}(P), Z - t^{-1}(P) \quad , \quad 0] \qquad t(Z) \geq P$$

where $Q \leq D$ is equivalent to $t^{-1}(C) + R(Q;C) \geq Z$. (A35)

The profit to the seller when the buyer breaches is

$$\pi^{\ell} = (K - C)Z + [P(Q^{\ell} + \mathbb{R}^{\ell}(Q^{\ell})) - C]Q^{\ell} \tag{A36}$$

where Q^{ℓ} is the optimal choice of Q for the seller and $\mathbb{R}^{\ell}(\cdot)$ refers to the two part reaction curve

$$\begin{cases} R(Q;C) & Q > D \\ R(Q;t(Z - Q_s)) & Q \leq D \end{cases}$$

describing the buyer's decision.

2. Market Damages

Now the buyer's problem is

$$\max_{Q_c,Q_s,Q_b} \begin{cases} T(Q_c) + Q_sP(Q + Q_s) - K(Q_c + Q_s) - [K - P(\cdot)]Q_b & P < K \\ T(Q_c) + Q_sP(Q + Q_s) - K(Q_c + Q_s) & P \geq K. \end{cases}$$

$$\text{subject to} \quad Z = Q_c + Q_s + Q_b \,, \quad Q_c \geq 0 \,, \quad Q_b \geq 0. \tag{A37}$$

For $P < K$, clearly breach dominates reselling so that the problem becomes

$$\max_{Q_c} T(Q_c) - KQ_c - [K - P(Q)][Z - Q_c]$$

$$\text{subject to} \quad Q_c \leq Z \tag{A39}$$

unless $t(Z) \geq P$, in which case the buyer will not be breaching or reselling, but instead buying in the spot market. The solution for this case is

$$[Q_c,Q_s,Q_b] = \left.\begin{cases} [t^{-1}(P), \quad 0, Z - Q_c] & t(Z) < P \\ [t^{-1}(P), Z - t^{-1}(P), \quad 0] & t(Z) \geq P \end{cases}\right\} P < K \tag{A40}$$

For $P \geq K$ the buyer's problem is

$$\max_{Q_s,Q_c} \; T(Q_c) + Q_sP(Q + Q_s) - K(Q_c + Q_s)$$

$$\text{subject to} \quad Q_c + Q_s \leq Z, \tag{A41}$$

which is very similar to the buyer's lost profits problem. Thus the solution is similar

$$[Q_c, Q_s, Q_b] = \begin{cases} \left.\begin{array}{ll} [t^{-1}(K), R(Q;K), \quad Z - Q_c - Q_s] & Q>E \\ [Z - Q_s, R(Q; t(Z - Q_s)), \quad 0] & Q\leq E \end{array}\right\} t(Z) < P \\ [t^{-1}(P), Z - t^{-1}(P), \quad 0] \qquad t(Z) > P \end{cases} \quad P\geq K.$$

where $Q \leq E$ is equivalent to $t^{-1}(K) + R(Q;K) \geq Z$. (A42)

Combining (A38) and (A40) the complete solution is

$$[Q_c, Q_s, Q_b] = \begin{cases} [t^{-1}(P), \quad 0, \quad Z - Q_c] & P < K \\ [t^{-1}(K), \quad R(Q;K), Z - Q_c - Q_s] \quad Q > E & P \geq K \quad t(Z) < P \\ [Z - Q_s, R(Q; t(Z - Q_s)), \quad 0] \quad Q \leq E & P \geq K \quad t(Z) < P \\ [t^{-1}(P), \quad Z - t^{-1}(P), \quad 0] & t(Z) \geq P, \end{cases}$$

(A43)

and the profit to the seller when the buyer breaches is

$$\pi^m = \max_Q \begin{cases} K \cdot t^{-1}(P) + [K - P(Q)][Z - t^{-1}(P)] + P(Q)Q - C[Q + t^{-1}(P)] \\ \qquad P < K \\ K \cdot [t^{-1}(K) + R(Q;K)] + P(Q + R(Q;K))Q - C[Q + t^{-1}(K) + R(Q;K)] \\ \qquad P \geq K \end{cases}$$

(A44)

and the associated first order conditions are

$$\frac{\partial L}{\partial Q_s} = -t(\cdot) + P(P + Q_s) + P'(Q + Q_s)Q_s - \lambda = 0$$

$$\partial(Z - Q_s) = 0 \qquad \lambda \geq 0 \qquad Z - Q_s \geq 0. \qquad \text{(A46)}$$

The solution to (A46) is

$$[Q_c, Q_s] = \begin{cases} [Z - Q_s, R(Q;t(Z - Q_s)] & t(Z) < P \\ [t^{-1}(P), \quad Z - t^{-1}(P)] & t(Z) \geq P. \end{cases} \tag{A47}$$

Since under the other two remedies, the buyer breaches only for the case $t(Z) < P$, we are interested in the seller's profit under specific performance for $t(Z) < P$.

$$\pi^s = \max_Q \; (K - C)Z + [P(Q + \mathbb{R}^s(Q)) - C]Q \tag{A48}$$

where $\mathbb{R}^s(Q)$ is the solution to $Q_s = R(Q;t(Z - Q_s))$.

4. Comparison of remedies.

We will first compare lost profits with specific performance.

$$\pi^\ell = \max_Q \; (K - C)Z + [P(Q + \mathbb{R}^\ell(Q)) - C]Q \tag{A36}$$

$$= (K - C)Z + \max_Q [P(Q + \mathbb{R}^\ell(Q)) - C]Q$$

$$\pi^s = \max_Q \; (K - C)Z + [P(Q + \mathbb{R}^s(Q)) - C]Q \tag{A48}$$

$$= (K - C)Z + \max_Q [P(Q + \mathbb{R}^s + (Q)) - C]Q$$

The first term is common to both and is constant with respect to Q. The second term differs only in the reaction of the contract buyer to the seller's quantity choice.

$$\mathbb{R}^\ell(Q) = \begin{cases} R(Q;C) & Q > D \\ R(Q;t(Z - Q_s) & Q \leq D \end{cases}$$

$$\mathbb{R}^s(Q) = R(Q;t(Z - Q_s)) \tag{A49}$$

For $Q > D$, $t^{-1}(C) + R(Z;C) < Z$. But $R(Q;C) = Q_s$, so that $t^{-1}(C) + Q_s < Z$ or $t^{-1}(C) < Z - Q_s$ which implies that $C > t(Z - Q_s)$. Thus $R(Q;C) < R(Q;t(Z - Q_s))$ since the reaction to higher costs is a smaller reselling choice. We conclude that $\mathbb{R}^{\ell}(Q) \leq \mathbb{R}^{s}(Q)$ and the inequality is strict for $Q > D$. Since we wish to compare the profits of the seller under the two remedies only when the buyer would have breached under lost profits, the relevant region is when $Q > D$ and thus $\mathbb{R}^{\ell}(Q) < \mathbb{R}^{s}(Q)$.

If Q^* maximizes the seller's profit under specific performance, then

$$\begin{aligned}\pi^s &= [K - C]Z + P(Q^* + \mathbb{R}^{s}(Q^*)) - C]Q^* \\ &< [K - C]Z + P(Q^* + \mathbb{R}^{\ell}(Q^*)) - C]Q^* \\ &\leq \pi^{\ell},\end{aligned} \tag{A50}$$

since Q^* may not be the optimal choice of Q under lost profits. Therefore lost profits overcompensates the seller.

For the case of market damages,

$$\pi^m = \max_{Q} \begin{cases} K\cdot t^{-1}(P) + [K - P(Q)][Z - t^{-1}(P)] + P(Q)Q - C[Q + t^{-1}(P)] & P < K \\ K \cdot [t^{-1}(K) + R(Q;K)] + P(Q + R(Q;K))Q - C[Q + t^{-1}(K) + R(Q;K)] & P \geq K \end{cases} \tag{A44}$$

we will first look at $P < K$.

$$\pi^m = \max_Q \; K \cdot t^{-1}(P) + [K - P(Q)][Z - t^{-1}(P)] + P(Q)Q - C[Q + t^{-1}(P)]$$

$$= \max_Q \; K - P(Q)]Z + [P(Q) - C][Q + t^{-1}(P)]. \qquad (A45)$$

On the other hand, if we arrange terms,

$$\pi^s = \max_Q [K - C]Z + [P(Q + Q_s) - C]Q$$

$$= \max_Q [K - P(Q + Q_s)]Z + [P(Q + Q^s) - C][Q + Z]$$

$$= \max_Q [K - P(X)]Z + [P(X) - C][X + Z - Q_s].$$

$$\text{where } X = Q + Q_s \qquad (A46)$$

The only difference between (A45) and (A46) is that we have $t^{-1}(P)$ instead of $Z - Q_s$. Now $Q_s = \mathbb{R}^s(Q) = R(Q;t(Z - Q_s))$, which means that Q_s satisfies

$$P(Q + Q_s) + P'(Q + Q_s)Q_s = t(Z - Q_s). \qquad (A47)$$

Since $X = Q + Q_s$, we can make the substitution, and we get that

$$P(X) + P'(X)Q_s = t(Z - Q_s)$$

or

$$t^{-1}(P(X) + P'(X)Q_s) = Z - Q_s. \qquad (A48)$$

In (A46), instead of $t^{-1}(P)$ we have $t^{-1}(P + P'Q_s)$. These two terms are different as long as $P'Q_s \neq 0$. In general $P'Q_s = 0$ whenever $Q_s = 0$. From (A48), if $Q_s = 0$ then $t^{-1}(P) = Z$ or $P = t(Z)$. But we are only interested in the case when $P > t(Z)$, since this is when the buyer breaches. Thus $P'Q_s \neq 0$. Since $P > P + P'Q_s$,

$t^{-1}(P) < t^{-1}(P + P'Q_s) = Z - Q_s$.

If Q^* maximizes (A45) then

$$\begin{aligned}
\pi^m &= [K - P(Q^*)]Z + [P(Q^*) - C][Q^* + t^{-1}(P(Q^*))] \\
&< [K - P(Q^*)]Z + [P(Q^*) - C][Q^* + (Z - Q_s))] \\
&= [K - P(X)]Z + [P(X) - C][X + Z - Q_s] \quad \text{for } X = Q^* \\
&\leq \pi^s. \qquad \text{(A49)}
\end{aligned}$$

Thus market damages still undercompensates the seller when $P < K$.

For $P \geq K$,

$$\begin{aligned}
\pi^m &= \max_Q K[t^{-1}(K) + \mathbb{R}^m(Q)] + P(Q + \mathbb{R}^m(Q))Q - C[Q + t^{-1}(K) + \mathbb{R}^m(Q)] \\
&= \max_Q (K - C)[t^{-1}(K) + \mathbb{R}^m(Q)] + [P(Q + \mathbb{R}^m(Q)) - C]Q. \qquad \text{(A50)}
\end{aligned}$$

Unfortunately, this case is ambiguous. If we compare with the profit under specific performance,

$$\pi^s = \max_Q (K - C)Z + [P(Q + \mathbb{R}^s(Q)) - C]Q. \qquad \text{(A46)}$$

As in the lost profits case $\mathbb{R}^m(Q) \leq \mathbb{R}^s(Q)$ so that in the spot market alone, there would be more profit under market damages. However since the optimization in (A50) occurs over both spot and contract market, and the profit in the contract market is less under market damages, the final outcome is ambiguous in general.

EQUILIBRIUM PRICING IN HETEROGENEOUS GOODS MARKETS UNDER IMPERFECT INFORMATION ABOUT PRICE AND QUALITY

I. INTRODUCTION

There is a large body of literature on how the distribution of prices offered in the market is affected by the amount of information consumers have about prices. This area was initially explored in the context of homogeneous products. It was assumed that it is costly for a consumer to become informed. Consumers were divided into two types depending upon whether their preferences were such that they would choose to become informed. The standard result of these models is that the market is badly behaved, that is, there is price dispersion above competitive prices, whenever there is an insufficient proportion of consumers being informed. This is best illustrated in a paper by Wilde and Schwartz [1979] where consumers of one group, the non-shoppers, purchase from the first firm they encounter and each consumer of the other group, the shoppers, randomly sample a fixed number of firms and then buy from the firm with the lowest price.

The problem was then generalized to heterogeneous product markets. Even in this case, the result is that the consumers need to be sufficiently informed for competitive outcomes. An important assumption made in this literature, is that the consumer is able to compare the firms he has sampled in order to decide which to purchase from. In the case of homogeneous goods, this assumption is quite innocuous. It simply means that consumers are able to compare prices.

However, in the heterogeneous market setting, the assumption is substantially stronger. It requires that consumers be able to distinguish between the different types of quality and then be able to compare the price-quality pairs they have sampled, in order to determine which is best.

In this work, the consequences of relaxing this assumption are examined. There are two quality levels. Uninformed consumers are assumed to have a rough idea of the attributes the generic product that they seek ought to have. From this concept of the generic product they have a limit price which reflects the maximum they are willing to pay for the product. They are assumed to have no prior information about the existing distribution of firms, nor have any subjective distribution in mind. Thus, after visiting a store, an uninformed consumer does not perform any updating and therefore there is no revising of his limit price. Instead, given that the product satisfied the basic attributes, he presumes that what he just saw is, in fact, the generic product. Only after consumption is the total set of attributes realized.

This is quite common in the actual consumer experience. For example a consumer buying a used car may perform all the tests required to check that the engine is sound, and after the purchase realize that the car seat gives him a backache. Similarly, positive attributes can also be discovered after the purchase.

Three models are developed which incorporate different assumptions about the initial state of information and the extent to which the state of information can change during the shopping process. In each model, the true preferences of the consumers are that they prefer the high quality product to the low quality product at the same price. Furthermore, it is assumed that at competitive prices (these will be defined in section 2) consumers prefer high quality products to low quality products. Thus, we can judge how badly behaved the market is by comparing the equilibrium distribution of firms to the competitive equilibrium with all firms in the high quality market.

In the first model, initially all consumers are uninformed about quality. However, they can learn by shopping, if they encounter products of different quality levels. The reasoning here is that ordinarily these consumers are insensitive to certain attributes. If they observe two products of the same quality then they ignore the remaining attributes, assuming that these are standard in the generic product. However, if they observe two products of different quality, they realize that there are quality differences, and they become sensitive to the full set of attributes. Then, they will compute a new set of limit prices for each quality level. This behavior can be justified by means of a bounded rationality argument. The consumer is faced with the problem of making decisions about a multi-attribute commodity. To make an optimal decision, incorporating all the attributes is too costly. In some cases, to even list all the attributes may be impossible for the consumer. So the consumer has a subset of attributes on which he concentrates. However, when he discovers that there are variations in other attributes that matter to him, he becomes sensitive to these attributes as well.

A certain proportion of the population of consumers are shoppers, and they sample exactly two firms before purchasing. In the process, some of this group becomes informed about quality variation in the market when they sample from firms of two different quality levels. The remainder are non-shoppers and purchase from the first store that they come to. This group always remains uninformed at the time of purchase.

The qualitative results of this model are that if a sufficient proportion of the consumers are shoppers then the resulting equilibrium is a competitive equilibrium with all firms existing in the high quality market. If the proportion of shoppers falls below a critical level there is necessarily quality deterioration and pricing above competitive prices in both markets. This model is developed in section 2.

Section 3 consists of two models which are variations of the original model. The first allows a certain proportion of all consumers, shoppers and non-shoppers, to be naturally informed. As in the original model, it is possible for the uninformed shoppers to become informed during the shopping process. The results are as expected. For very small proportions of naturally informed consumers, the equilibrium properties of this version are qualitatively similar to the original model. As the proportion of naturally informed consumers increases, the market is less badly behaved and approaches the Schwartz and Wilde warranties model (1982).

Schwartz and Wilde (1982) define comparative advantage to be in the market where the least demand is needed for a firm to break even when it is charging the limit prices. They get the result that when the proportion of shoppers is not large enough to guarantee a competitive equilibrium, quality deterioration will not occur unless the comparative advantages lies in the low quality market. The result in the second version of the model is quite similar. If the proportion of shoppers is too small to give a competitive equilibrium, then there is quality deterioration whenever the comparative advantage lies in the low quality market, or when the proportion of naturally informed consumers is too small and the comparative advantage lies in the high quality market.

The last variation of the model in section 3 is where there is no learning by shopping. A certain proportion of the population is naturally informed and no more information can be gathered by shopping. As this model is computationally very difficult, it has been assumed that all consumers are shoppers. If the proportion of informed consumers is large enough the resulting equilibrium is a competitive equilibrium in the high quality market. If the proportion of informed consumers is too low, there is a competitive equilibrium in the low quality market. For intermediate values of the proportion of informed consumers, a variety of non-competitive equilibria arise, with a competitive price in one market and non-competitive pricing in the other market, or non-competitive pricing in both markets. These latter equilibria are mathematically complicated and some too difficult to compute.

The three models give three realistic situations of the initial state of information of consumers and how it changes in the shopping process. For a given market, which model is most applicable depends on how easy it is to become more informed about the quality variation in the market, while shopping, and whether the market is characterized by some consumers being naturally informed.

Section 4 discusses the implications, conclusions and areas of further research.

2. THE MODEL

In the market, two levels of quality of a product denoted by q_L and q_H, are offered at varying prices. The technology for producing each quality of the product is the same for all firms. However, a single firm may produce only one quality of products. For high quality products, there is a fixed cost F_H, a marginal cost c_H for each unit produced and a capacity constraint of S_H. Similarly, for low quality products, the fixed cost is F_L, the marginal cost c_L and the capacity constraint S_L. The competitive price in each market is defined to be the minimum of the average cost. Thus p_H^*, the competitive price in the high quality market is $\frac{F_H}{S_H} + c_H$, and $p_L^* = \frac{F_L}{S_L} + c_L$. We shall assume that the competitive price for high quality products is greater than the competitive price for low quality products, that the capacity constraint in the low quality market is greater than the constraint in the high quality market, and that the average fixed cost at capacity in the low quality product is less than that in the high quality product.

$$p_L^* < p_H^* \tag{1}$$

$$S_H < S_L \tag{2}$$

$$\frac{F_L}{S_L} < \frac{F_H}{S_H} \tag{3}$$

These are natural assumptions to make about the relationships that exist between the parameters of a low quality product cost function and a high quality product cost function. Clearly no firm can offer prices below the competitive prices.

All consumers demand exactly one unit of the product. They enter the market and sample a fixed number of firms. The population is partitioned into two groups. The members of the group, which we refer to as shoppers, each sample exactly two firms and the members of the other group, known as non-shoppers, sample only one firm. The proportion of shoppers to the whole population is γ, and the proportion of non-shoppers is $1 - \gamma$.

All consumers have the same underlying preferences. There is a common limit price, L, for low quality products, which we shall assume is greater then p_L^*, and a common limit price $H > p_H^*$ for high quality products. However, before the search, consumers are not aware that there are two quality levels being offered in the market. Instead, they have an idea of what the generic product is like, and they have a common limit price, R, for the generic product. We shall assume that R lies between H and L. Even after the search, consumers remain insensitive to quality, unless they have sampled products of both qualities.

Hence, non-shoppers will buy as long as the firm they chose offers a price below or equal to R. Shoppers whose entire sample consists of one quality will purchase from the firm with the lowest price, as long as that price is below or equal to R. The shoppers whose draw consists of both qualities will chose the firm which leaves them with the greatest surplus. The surplus derived from choosing (p_L, q_L) is $L - q_L$. Thus the shopper will be indifferent between (p_L, q_L) and (p_H, q_H) as long as $p_H = p_L + H - L$. In case of indifference, the consumer will choose either firm with probability $\frac{1}{2}$. In this way, the consumers are passive players that behave according to these rules.

We shall assume that at competitive prices informed consumers will prefer high quality to low quality. Thus

$$p_L^* + H - L > p_H^*. \tag{4}$$

Under this assumption, informed and uninformed consumers prefer different quality products at competitive prices.

The feasible set of prices that firms can offer is between p_L^* and R in the low quality market and between p_H^* and H in the high quality market. Below p_L^* and p_H^*, firms are asking a price which is not sufficiently high to cover their average costs. No firm can locate at prices above R and H in the low and high quality markets respectively, and make a non-negative profit, since no consumer will buy at these prices. Firms will choose which price and quality pair to offer from the feasible set in order to maximize their profit. However, there will be free entry, so that as long as there are positive profits to be made firms will continue to enter. This completes the description of the model and its assumptions.

Now we will introduce some concepts and notation which enable us to define an equilibrium. Let $G_H(p)$ be the cumulative distribution function which gives the distribution of firms over prices in the high quality market, $G_L(p)$, the distribution of firms over prices in the low quality market, and n_H and n_L be the proportion of the total firms producing in the high quality market and low quality markets respectively. Clearly $n_L + n_H = 1$. Since firms cannot locate outside the feasible set, the following conditions must hold: $G_H(p) = 0$ for $p < p_H^*$, $G_H(p) = 1$ for $p > H$, when $n_H > 0$, ($G_H(p)$ is not defined for $n_H = 0$); and $G_L(p) = 0$ for $p < p_L^*$, and $G_L(p) = 1$ for $p > R$ when $n_L > 0$, ($G_L(p)$ is not defined for $n_L = 0$). We denote a distribution of firms on the feasible set by $G_L(p)$, $G_H(p)$, n_L, $n_H\rangle$.

<u>Definition</u>. An equilibrium is a consumer to firm ratio α, and a distribution $\langle G_L(p), G_H(p), n_L, n_H\rangle$ such that the Nash condition is satisfied for any existing firm or potential entrant.

In other words, given a consumer firm ratio α and a distribution, there is no location in the feasible set where an entrant can enter and make positive profit or an existing firm can deviate and make higher profit. If we let the support of the distribution to be all points (p, q_i) in the feasible set, such that $G_i(p)$ exists and $G_i'(p) \neq 0$ if it exists, (where i = H, L), then in equilibrium the profit earned at any point on the feasible set must be non-positive, with exactly zero profit being earned on the support of the distribution. Using this reasoning, we will restate the definition of equilibrium in a more useful form.

Let $D(p, q_i)$ denote the demand at (p, q_i), i = H, L, then zero profit is earned at (p, q_i) as long as

$$p\, D(p, q_i) - c_i\, D(p, q_I) - F_i = 0, \quad i = H, L.$$

Equivalently,

$$D(p, q_i) = \frac{F_i}{p - c_i}, \quad i = H, L.$$

The right hand side is the amount of demand needed in order that zero profit is earned at (p, q_i). We will call this the break even demand and denote it by $Z(p, q_i)$, i.e., $Z(p, q_i) = \frac{F_i}{p - c_i}$, i = H, L. It follows that if the demand is greater than the break even demand at a point then positive profit may be earned at that location. Conversely, if the demand is below the break even demand then only negative profit may be earned. Now we can formally restate the definition of an equilibrium using the notion of break even demand.

Definition. An equilibrium is α, $\langle G_H(p), G_L(p), m_H, m_L \rangle$ such that $D(p, q_i) \leq Z(p, q_i)$ for all (p, q_i) in the feasible set and equality holds on the support of the distribution.

We are now equipped with a definition of equilibrium that is easy to verify. Given a distribution, to decide whether or not it is an equilibrium, we simply compare actual demand to break even demand at each point in the feasible set. Equilibrium requires that demand not exceed break even demand on the feasible set, and equality holds on the support of the distribution.

Let us determine the various types of equilibria that can obtain under different configurations of the parameters. An equilibrium is competitive if all firms are offering their products at competitive prices. We will first consider the competitive equilibria and the conditions under which they will exist. Then we will consider the non-competitive equilibria in a similar fashion. In what follows, we shall present intuitive arguments, and the formal proofs may be found in the appendix.

A. Competitive Equilibria

The only competitive equilibrium is one where all the firms offer the high quality product with the consumer firm ratio, α, being S_H. To see this, suppose we had a competitive equilibrium with n_L firms offering low quality, $n_L \neq 0$ and $n_H = 1 - n_L$ firms offering high quality. Then the demand for the products of any firm at (p_L^*, q_L) will be due to i) all non-shoppers who sample the firm, and ii) one half of the shoppers whose other draw was also a low quality firm. Of the shoppers whose other draw was a high quality no one will purchase since at competitive prices shoppers who see both products prefer the high quality product. Thus,

$$D(p_L^*, q_L) = \alpha(1-\gamma) + \alpha\gamma n_L.$$

For a firm offering the high quality product, the demand will be i) all the non-shoppers that sample it, ii) all the shoppers whose other draw was a low quality firm, and iii) one half of the shoppers whose other draw was another high quality firm. Thus,

$$D(p_H^*, q_H) = \alpha(1-\gamma) + 2\alpha\gamma n_L + \alpha\gamma n_H$$

$$= \alpha(1-\gamma) + 2\alpha\gamma n_L + \alpha\gamma.$$

Now note that the demand for a high quality firm is greater than for a low quality firm. The break even demand for the high quality firm is S_H which is less than S_L, the break even demand for the low quality firm. In equilibrium we need that the demand equal the break even demand on the support of the distribution. The only manner in which this can be achieved is if there were no firms at (p_L^*, q_L), since then $D(p_H^*, q_H) = S_H$, by choosing $\alpha = S_H$ and $D(p_L^*, q_L) < S_L$. Thus, the only possible competitive equilibrium is one where all firms are in the high quality market.

Now we will show that a distribution with all firms offering the competitive price in the high quality market is an equilibrium if the proportion of shoppers is sufficiently large. The other condition for equilibrium is that the demand be less than or equal to the break even demand for locations not on the support of the distribution. Now if all the firms are at (p_H^*, q_H), then the demand at any other point will be due to the non-shoppers only. Thus, the demand at any point will be $\alpha(1-\gamma)$. We need that this be less than the break even demand. Let us find the point where break even demand is the least. If demand is less than break even demand at this point, then it will clearly be the case elsewhere since the break even demand is higher elsewhere. The break even demand is the least at (R, q_L). To see this, observe that R is the highest price that can be offered in the low quality market. For a given quality level, the higher the price the lower the break even demand. The highest price that can be offered in the high quality market is also R since the demand is due to the non-shoppers only. Clearly the break even demand at (R, q_L) is lower than the break even demand at (R, q_H) since it costs less to produce a low quality product than a high quality product. Thus the break even demand is lowest at (R, q_L), and if demand is less than the break even demand at (R, q_L), then it will also be true elsewhere in the feasible set. Therefore, we need

$$\alpha(1-\gamma) < Z(R, q_L) = \frac{F_L}{R - c_L}.$$

Since $\alpha = S_H$, this condition is equivalent to

$$\gamma > 1 - \frac{F_L}{S_H(R - c_L)}.$$

This means that as long as the proportion of shoppers is sufficiently large, as defined in the above condition, we have the competitive equilibrium with all firms offering the high quality product at the competitive price. In the Appendix we show that the above condition is also a sufficient condition for a competitive equilibrium at (p_H^*, q_H).

<u>Theorem 1</u>. The only competitive equilibrium is one with all firms at (p_H^*, q_H). The necessary and sufficient condition for this equilibrium to exist is $1-\gamma \leq \frac{F_L}{S_H(R - C_L)}$.

<u>Proof</u>: See Appendix.

B. <u>Non-Competitive Equilibria</u>

There are several types of non-competitive equilibria depending upon the properties of the cost functions and the limit prices. Before we can identify the different types, we will establish some properties that are common to all non-competitive equilibria.

<u>Lemma 1</u>. In equilibrium, there cannot be any mass points except at competitive prices.

<u>Proof</u>: See Appendix.

We will present the intuitive argument behind this lemma. Suppose there was a mass point of size m_i at (p, q_I), where p is not a competitive price for q_i. Then, with the same quality for any price below p, the demand will be greater than the demand at (p, q_i) by at least $\alpha\gamma m_i$, since at (p, q_i) only one half of the shoppers whose other draw was also (p, q_I) will buy, while at any price below, all the shoppers whose other draw was (p, q_i) will buy. Similarly, with the same quality, for any price above p, the demand will be less than the demand at (p, q_i) by at least $\alpha\gamma m_i$, since here none of the shoppers whose other draw is (p, q_i) will buy. Thus, in the q_i quality market demand jumps down discontinuously at (p, q_i) and again jumps down discontinuously just above (p, q_i). Since there are firms at (p, q_i), demand must equal break even demand at (p, q_i). But now since break even demand is continuous in price, for prices below p demand will exceed break even demand, which cannot hold in equilibrium. This argument can be made for all prices p for which there are prices below p in the feasible set. Therefore there cannot be any mass points except at competitive prices.

Lemma 2. No firms can exist at (p, q_i) where $p_L^* \leq p < \frac{F_L}{S_H} + C_L$.

Proof: See Appendix.

Basically, the lemma states that there cannot be any firms below the price $\frac{F_L}{S_H} + C_L$, in the low quality market. This follows easily, when we make the observation that in this range of prices, break even demand is less than S_H, which is the break even demand at (p_H^*, q_H). However, the demand at (p_H^*, q_H) is always higher than that at prices below $\frac{F_L}{S_H} + C_L$ in the low quality market, since all informed consumers prefer (p_H^*, q_H). Thus if there are any firms at prices below $\frac{F_L}{S_H} + C_L$ (earning non-negative profit), then positive profit can be earned at (p_H^*, q_H). Clearly, in equilibrium there cannot be any firms in this range of the low quality market.

Lemma 3. For every non-competitive equilibrum (R, q_L) belongs to the support of the distribution,

$$\alpha = \frac{F_L}{(R - C_L)(1 - \gamma)}, \text{ and } \gamma < \frac{S_H(R - C_L) - F_L}{S_H(R - C_L)}.$$

Proof: See Appendix.

Every non-competitive equilibrium contains (R, q_L) in its support. Notice that no matter what the equilibrium distribution is, the demand at (R, q_L) is $\alpha(1 - \gamma)$. If we had a non-competitive equilibrium distribution without, (R, q_L) in its support, then the least preferred point in the support according to true preferences must either be a high quality firm, or a low quality firm offering a lower price. If it is a low quality firm or a high quality firm offering a price of at most R, then it can always improve its profits by moving to (R, q_L), since its demand will be $\alpha(1 - \gamma)$ in both cases, but its costs will be less at (R, q_L). If it is a firm offering a high quality product at prices greater than R, then its only demand is due to shoppers whose other sample is a low quality firm which is less preferred. Thus the firm offering the lowest surplus cannot be a high quality firm with prices greater than R. Hence every non-competitive equilibrium contains (R, q_L). Since in equilibrium demand equals break even demand on the support, $\alpha(1 - \gamma) = F_L/(R - c_L)$ or $\alpha = F_L/(R - c_L)(1 - \gamma)$. The argument for why γ must be less than $\dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$ is in the Appendix.

Lemmas 1, 2, and 3 contain general properties which are true of all non-competitive equilibria. Where the support of the distribution lies in a particular situation, depends upon two things: i) the relationships among the parameters of the cost function and the limit prices, and ii) the proportion of shoppers in the population of consumers.

These two factors affect the equilibrium distribution of prices in quite different ways. The configuration of the cost parameters and limit prices restrict the support of the distribution to a subset of the feasible set. We call this subset the 'maximum support' of the distribution. In all, five different cases arise and these are depicted in Figures 1 through 5. Lemmas 4, 5, and 6 of the Appendix contain the conditions separating the different cases.

In Lemma 4 the notion of comparative advantage, which was first introduced in this context by Schwartz and Wilde (1982), is generalized. Consider any price p, in the low quality market. In general, the demand at (p, q_L) is the same as the demand at (p + H − L, q_H). The reason for this is that the demand due to the

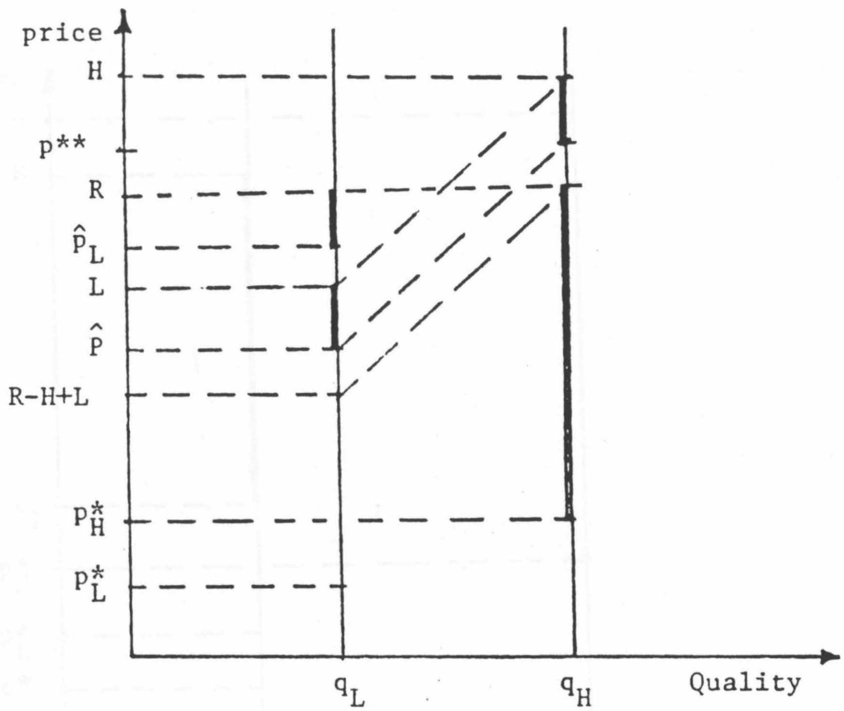

FIGURE 1

denotes 'maximum support' of the distribution.

Here $p^* < c_L$ or $p^* > R\text{-}H\text{+}L$ (Theorem 6 of Appendix)

and $p < L < p^{**}$

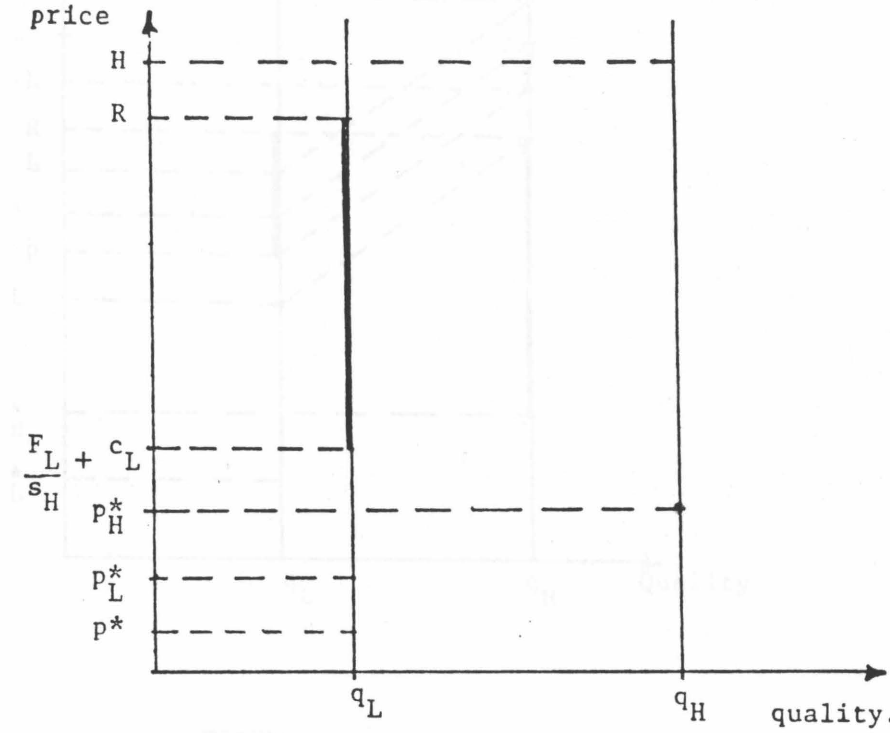

FIGURE 2

•, ═══ Denotes 'maximum support' of the distribution.

Here $\quad c_L < p^* < \frac{F_L}{s_H} + c_L \qquad$ (Theorem 3 of Appendix)

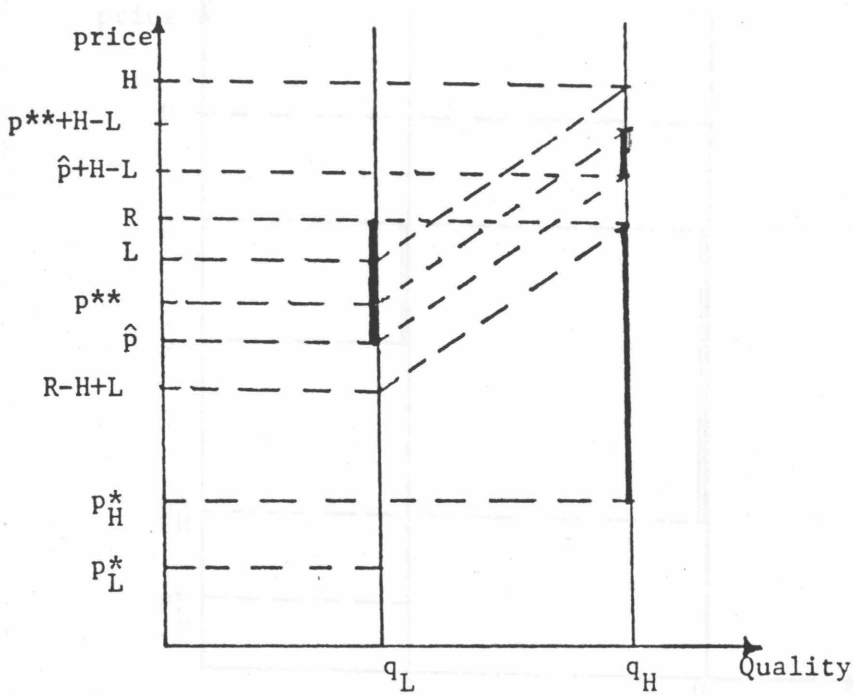

FIGURE 3

═══ denotes 'maximum support' of the distribution

Here $p^* < c_L$ or $p^* >$ R-H+L ⎤ (Theorem 4 of Appendix)
and $\hat{p} < p^{**} <$ L ⎦

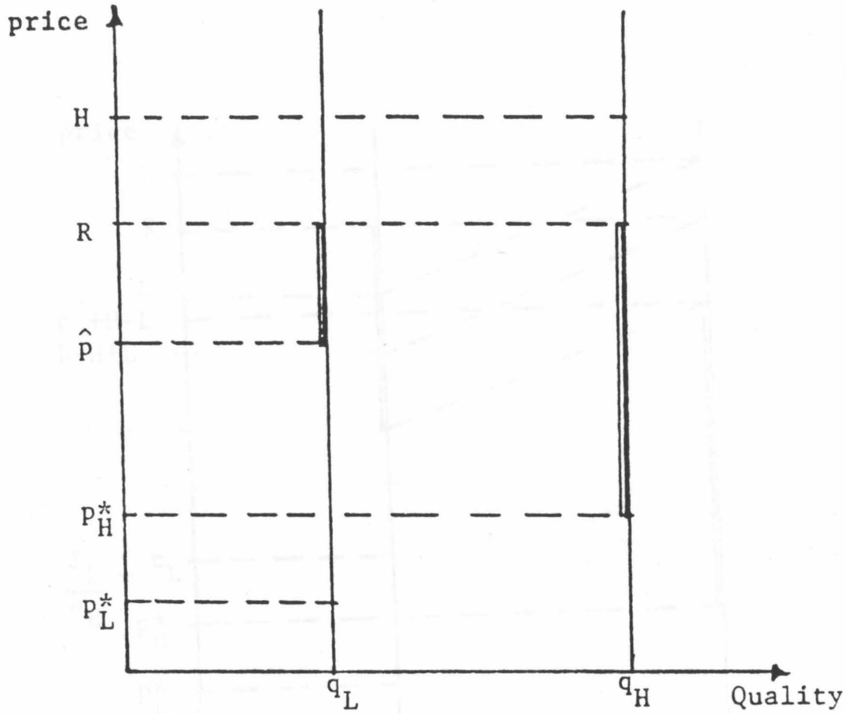

FIGURE 4

═══ denotes 'maximum support' of the distribution.

Here $p^* < c_L$ or $p^* > R-H+L$
and $\hat{p} \geq p^{**}$ or $\hat{p} > L$ } (Theorem 5 of Appendix)

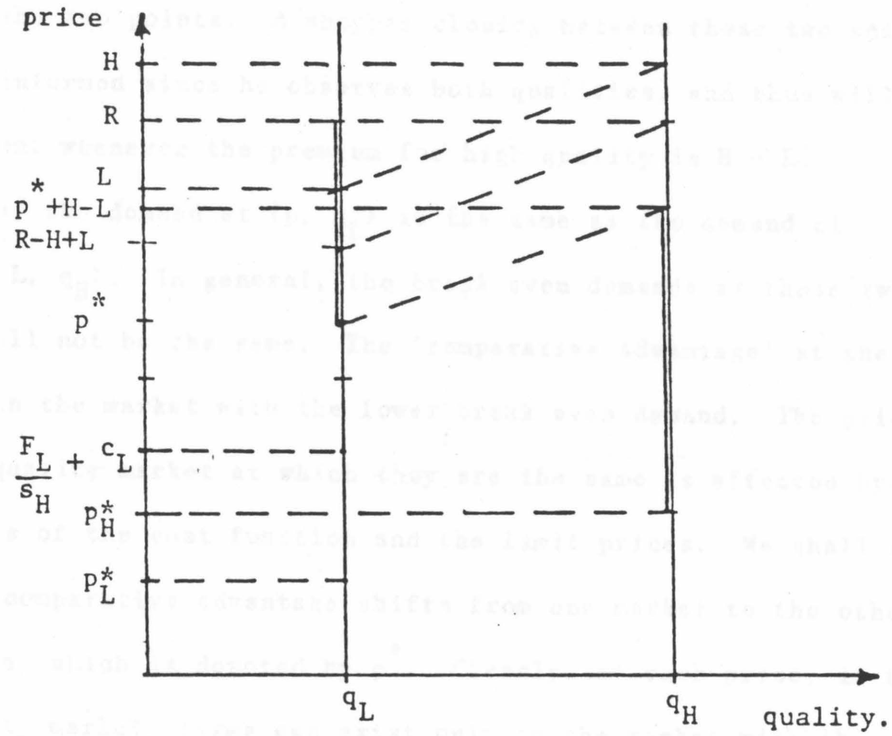

FIGURE 1

═══ maximum support of the distribution.

Here $\frac{F_L}{s_H} + c_L \leq p^* \leq R-H+L$ (Theorem 2 of Appendix) non-shoppers is the same at each point, and the demand due to the shoppers will be the same as long as the shopper is indifferent between the two points. A shopper chosing between these two points must be informed since he observes both qualities, and thus will be indifferent whenever the premium for high quality is H - L. Therefore, the demand at (p, q_L) is the same as the demand at $(p + H - L, q_H)$. In general, the break even demands at these two points will not be the same. The 'comparative advantage' at the price p, lies in the market with the lower break even demand. The price in the low quality market at which they are the same is affected by the parameters of the cost function and the limit prices. We shall say that the comparative advantage shifts from one market to the other at this price, which is denoted by p^*. Clearly, at each price, in the low quality market, firms can exist only in the market with the comparative advantage. To see this, suppose in equilibrium at certain prices there existed firms in the market without the comparative advantage. There firms must be earning zero profit since this is an equilibrium. But then at the corresponding prices in the market with the comparative advantage positive profit can necessarily be earned and this contradicts that this is an equilibrium.

In this way, Lemma 4 introduces the generalized notion of comparative advantage at a price. Lemma 4 applies only to a certain range of prices, and Lemmas 5 and 6 explore the possibilities that arise when the price at which the comparative advantage shifts, lies outside the range of Lemma 4. The fundamental factor governing where firms lie in the feasible set, is that zero profit should be earned on the support and non-positive profit should be earned off the support of the distribution. We use this rule repeatedly in Lemmas 5 and 6 to establish whether firms may lie in the low quality or high quality market in different price ranges. As these lemmas are quite technical and do not add significantly to the understanding of the model, they will not be discussed here.

Once the maximum support of the distribution is determined by the configuration of the cost parameters and limit prices, the proportion of shoppers determines the actual support of the distribution. If the proportion of shoppers is sufficiently large (where "sufficient" is as defined in Theorem 1), then the equilibrium is competitive with all firms offering (p_H^*, q_H). With a slightly smaller proportion of shoppers, a few firms deviate to high prices in the low quality market. As the proportion of shoppers continues to fall, the equilibrium distribution transforms itself, with a smaller mass of firms at the competitive price in the high quality market and more firms dispersed in the maximum support, above (p_H^*, q_H). The order in which points from the maximum support are added to the actual support as γ decreases in the order of increasing utility for the informed consumer. Eventually, the mass point at (p_H^*, q_H) dissapears, and further reduction in the proportion of shoppers results in the shrinking of the actual support, in a manner that is exactly reverse to the initial expansion of the actual support. Ultimately, the equilibrium distribution converges to the single point (R, q_L) and this occurs when the proportion of shoppers is zero.

Thus, the relationships among the cost parameters and limit prices determine the maximum support, and the actual support of the equilibrium distribution is determined by the proportion of shoppers in the population. A complete mathematical classification of all the cases and rigorous proofs of all the necessary and sufficient conditions on γ for each case are given in the Appendix. While this detailed classification is necessary in order to prove the conditions for each equilibrium and to ensure that the set of equilibria is exhaustive, it is more useful now to examine the economic aspects of the equilibria.

The most significant observation is that there is quality deterioration in every non-competitive equilibrium. That is, whenever the proportion of shoppers is not sufficiently large to yield a competitive equilibrium, there will always be some firms selling low quality products. This result differs from the one obtained by Schwartz and Wilde (1982), where all consumers are perfectly informed about quality, in that firms entered at the limit price in the market with the comparative advantage when the proportion of shoppers fell just below the critical level needed for competitive equilibrium. To explain this difference, note that the firms deviating to the high prices from the competitive equilibrium are attracting the non-shoppers. In the Schwartz and Wilde model these consumers are able to distinguish between the quality levels and so it is necessary for firms to enter in the market with the comparative advantage at limit prices. In this model it is advantageous for the firms to enter in the low quality market since the consumers cannot tell the difference between the two quality levels, and since costs are lower for producing low quality goods.

Another similarity among the equilibria is that as we increase the proportion of shoppers, eventually there will be some firms offering the high quality product in equilibrium. This result concurs with Schwartz and Wilde, and is expected since the only competitive equilibrium is in the high quality market.

We can separate the equilibria according to the following economically significant criteria: i) firms exist in the low quality market only, ii) firms exist in the high quality market only at prices below R, and iii) firms exist in the high quality market at prices above R.

Firms existing in the low quality market only, indicates quality deterioration and non-competitive pricing. This can occur under all circumstances, as long as the proportion of shoppers is small enough. However, the critical proportion of shoppers in order that this occurs, differs for different configuration of the cost and demand parameters. For example, if the cost and demand parameters are as depicted in Figure 1, then the proportion of shoppers must be small enough to allow the distribution to lie above p^* in the low quality market. On the other hand, for the situation in Figure 2, it is possible to have a larger proportion of shoppers and still maintain firms in the low quality market only, since in this case the distribution can extend as low as $\frac{F_L}{S_H} + C_L$. In general the likelihood that firms exist in the low quality market only, is greater, the lower the price is at which the comparative advantage changes from the low quality market to the high quality market.

If firms exist in the high quality market at prices below R only, this indicates that there is not complete quality deterioration, and firms in the high quality market are unable to extract all the surplus from the informed consumer. This situation is possible in three of the five configurations of parameters, shown in Figure 1, 2, and 4, as long as the proportion of shoppers is sufficiently large. In the first two configurations, firms cannot exist in the high quality market at prices above R, because the comparative advantage moves to the high quality market at a price below R.

The case of Figure 4 is slightly more complicated. Here the comparative advantage is in the high quality market at prices above R. However, another factor prevents firms from existing at prices above R. In the high quality market, if we compare the demand at prices above R with the demand at R, we find above R, the only demand is due to shoppers whose other draw is a low quality firm, that is lower in the preference of the informed consumer. At R there is a surge in demand due to the non-shoppers. If there are firms at R in the high quality market, then there cannot be firms in an interval above R in the high quality market (and in an interval above R - H + L in the low quality market) since break even demand, which is continuous in prices, would exceed demand for prices in an interval above R. (A similar sort of reasoning explains the corresponding gap in the low quality market for an interval above R – R + L.)

Under the appropriate conditions, this gap can be large enough to preclude the existence of firms above R. The exact condition is quite technical, $Z(p^{**}, q_L) > z(R, q_H)$ or $z(L, q_L) > z(R, q_H)$ but it roughly means that the comparative advantage at R in the high quality market must be sufficiently large that the break even demand in the low quality market exceeds that at (R, q_H), for a large interval of prices above R – H + L. A simple sufficient condition is that the break even demand at (L, q_L) exceeds that at (R, q_H). Thus, this case occurs for very large p^*.

Finally, we consider the situation where firms exist in the high quality market at prices above R. This situation occurs in Figures 3 and 5, when there is a sufficiently large proportion of shoppers, and is a result of the price at which the comparative advantage changes to the high quality market, being greater than R – H + L, but not very large (as was discussed in the previous case). The significance of this case is that some informed consumers are being fully exploited in the high quality market. The intuitive reasoning here is as follows: The consumers who buy at these prices are necessarily informed, since no uninformed consumer would buy at these prices. In order that they are informed, the other draw they made must have been a firm of low quality. That firm must have yielded a lower utility since they chose to buy from the high quality firm. Thus the high price low quality firms create an externality, since their existence allows the existence of firms at high prices in the high quality market. Note in particular that for a firm to exist at (H, q_H), it is necessary that there exist firms in the low quality market that yield a negative utility for the informed consumer.

Thus the three economically relevant cases are governed by two factors: the proportion of shoppers and the price at which the comparative advantage changes from the low quality market to the high quality market. Each case is as likely as the other, in general. However, for a particular industry or type of production, we may be able to predict which cases are more likely, as Schwartz and Wilde (1983) do in the case of warranties.

3. EXTENSION AND GENERALIZATIONS

From the previous section we conclude that the market can be very badly behaved in a world where consumers learn about quality by shopping. If the proportion of shoppers is below a critical level, then a non-competitive equilibrium results, in which there is always quality deterioration and non-competitive pricing. Thus government policy should be aimed at reducing the cost of comparison shopping, since insufficient shopping is the cause of the quality deterioration and non-competitive pricing.

The nature of the equilibria is strongly influenced by the assumption that the only manner in which a consumer can know about quality variation in products is by sampling firms of different quality products. In this section, we examine how sensitive the equilibria are to this assumption. The model moves smoothly from a world of learning by shopping to a world of completely informed shoppers, via a parameter β. In this case, there is quality deterioration whenever the comparative advantage at limit prices lies with low quality, or if a large enough proportion of the consumers are learning by shopping when the comparative advantage lies with high quality production.

The last variation we consider is where a certain proportion of consumers is naturally informed and the remainder is uninformed and cannot learn by shopping. In this formulation, even when there are no non-shoppers we find quality deterioration occurring, when the proportion of informed consumers is low. Thus we realize that adequate comparison shopping is not enough to prevent quality deterioration in markets where some consumers are always uninformed about quality.

A. Some Learning by Shopping

We now modify the original model so that a proportion β of all consumers are naturally aware of the two quality levels and thus are able to make decisions according to informed preferences. The other $(1 - \beta)$ of the consumers are able to learn about the quality differences if they observe products of different quality levels. The impact this change has on the behavior of the non-shoppers is that β of them will not buy at prices above L in the low quality market, but these same consumers will buy at prices between R and H in the high quality market. Similarly, of the shoppers, β of them will not buy if both their draws are prices above L in the low quality market, but they will buy when both their draws are prices between R and H in the high quality market.

The qualitative nature of the equilibria remains unchanged for small enough β. There is a certain proportion of shoppers required for competitive equilibrium in the high quality market to obtain. No other competitive equilibria are possible. If there are too few shoppers then there is necessarily quality deterioration as well as non-competitive pricing in both markets. The only difference is that there is a discontinuous increase in demand at (L, q_L) as we approach from above which is due to the addition of the informed consumers, who will not buy at prices above L in the low quality market. The discontinuity limits the amount of mass that can exist above L since in equilibrium the demand at (L, q_L) must be at most the break even demand at (L, q_L). As a result, there are no firms in the low quality market at prices for an interval above L. The argument here is identical to the arguments used previously when there is a discontinuity in demand.

To find the critical level of β, below which the nature of the equilibrium is unchanged we observe the following. The competitive equilibrium at (p_H^*, q_H) obtains whenever there is a sufficient proportion of shoppers. This means that profitable entry is not possible at any point in the feasible set. We will find qualitatively similar equilibria if the constraint of no profitable entry is binding at (R, q_L). This follows because, if the inequality is not met, firms will enter at (R, q_L) as in the previous model. The critical β is found according to this argument; the exact value, $\bar{\beta}$, is given in Lemma 8 of the Appendix. When $\beta \leq \bar{\beta}$ the equilibria are very similar to those of the previous model.

Now let us see what happens when $\beta > \bar{\beta}$. If there are sufficient shoppers to permit a competitive equilibrium, then the only possible competitive equilibrium is at (p_H^*, q_H). The reason is that all the previous arguments against any other competitive equilibrium still apply. If there are too few shoppers, then α must be chosen so that the demand equals the break even demand at the point in the feasible set where the most profit can be made. Clearly, this will depend on β. At $\beta = 1$, everyone is informed and we are left with the Schwartz and Wilde model [1982]. The maximum support is either the high quality market only, or on prices between p^* and L in the low quality market and $p^* + H - L$ and p_H^* in the high quality market, depending upon where the comparative advantage lies at limit prices. The former case obtains if the comparative advantage at limit prices is in the high quality market, that is $Z(L, q_i) > Z(H, q_H)$, and the latter can obtain if the comparative advantage is in the low quality market.

Now we will examine the outcome for intermediate values of β. There are two cases. Let us begin with the case where the comparative advantage of limit prices is in the low quality market, that is $Z(L, q_L) < Z(H, q_H)$. As β is increased above $\bar{\beta}$ the discontinuous jump in demand at (L, q_L) increases, and so the interval on which firms may not exist in the low quality market, increases. At the same time, the jump in demand at (R, q_H) due to uninformed consumers entering at R is decreasing and so the gap in the maximum support, at prices just above R in the high quality market is reducing. At $\beta = \frac{R - L}{R - c_L}$ the jump in demand at (L, q_L) is so large that no firms can exist at prices above L in the low quality market. This is shown in Lemma 9 of the Appendix. As we continue to increase β, the gap in the high quality market, at prices just above R, becomes smaller until $\beta = 1$ when there is no gap.

If the comparative advantage at limit prices lies in the high quality market, then several possibilities arise. These are discussed in more detail in the Appendix. In general, four processes are taking place. First, there is a discontinuity in demand at (L, q_L) due to informed consumers refusing to buy at prices above L in the low quality market. This creates a gap in the equilibrium distribution for an interval just above L. As we increase β, this gap grows until finally there are no firms between L and R. Second, there is a gap in the distribution at prices above R in the high quality market, and prices above R − H + L in the low quality market, due to the fact that uninformed non-shoppers will not buy at prices above R in the high quality market and this creates a discontinuity of demand at (R, q_H). As β increases, this gap decreases since more and more non-shoppers are becoming informed. Next, the lowest price at which firms can exist in the low quality market is increasing as β increases. As β tends to 1, this price tends to p^*, which is greater than L by the assumption that the compartative advantage at limit prices lies in the high quality market. Finally, the highest price at which firms can exist in the high quality market is increasing as β increases.

Different possibilities occur depending upon the different times at which the above four processes are complete. In some instances, the transition to the high quality market occurs directly. That is, initially the point at which only the non-shoppers purchase is (R, q_L). As β increases the distribution in the low quality market slowly erodes away, and the point at which only the non-shopper purchase is (H, q_H). At other instances, the transition occurs indirectly. The firm catering exclusively to non-shoppers is initially at (R, q_L) then it moves to (L, q_L), or in some cases to (R, q_H), and finally it is at (H, q_H).

The classification of the different maximum supports under the different conditions is not of great interest in itself. It has been included here for completeness. The factor of interest is under what conditions will there be no quality deterioration. In the Appendix, we show that there is a critical value β^* such that when the comparative advantage lies with high quality products, $\beta > \beta^*$ is the condition for no quality deterioration. Thus, we have quality deterioration when the proportion of shoppers is too low and the comparative advantage lies with low quality. We also have quality deterioration when the comparative advantage lies with high quality and we have too few shoppers, if β is below β^*.

B. Naturally Informed Consumers

The final variation of the model is when there is no learning by shopping. A certain proportion, β, of the consumers is naturally informed about quality and the remainder of the consumers is insensitive to quality, at the time of purchase. Since the equilibria in this model are extremely difficult to compute, we will make some simplifying assumptions.

First we will assume that there are no non-shoppers. This will reduce the different types of consumers and thus uncomplicate the equilibria. Next we will assume that the fixed costs are the same in both technologies, $F_H = F_L = F$, and the capacity constraints are also the same, $S_H = S_L = S$. So the only difference in the costs are marginal costs c_H and c_L. The purpose of these assumptions is to demonstrate how badly behaved the market equilibria are, even under these strong simplifying assumptions. The assumptions about how the competitive prices and the limit prices are related are maintained from the first model. Thus

$$p_L^* < p_H^* < p_L^* + H - L,$$

or $$\frac{F}{S} + c_L < \frac{F}{S} + c_H < \frac{F}{S} + c_L + H - L$$

which is equivalent to

$$c_L < c_H < c_L + H - L.$$

Define $c_H - c_L = K_c$ and $H - L - (c_H - c_L) = K_\theta$. This will allow us to write expressions without excess notation.

Now we are prepared to discuss the equilibria of this model. There are several different equilibria that can arise in this model, two of which are competitive.

The competitive equilibria occur for very large values of β and for very small values of β. This follows because if there is a competitive equilibrium at (p_L^*, q_L), then for large values of β, it is possible to profitably deviate to the high quality market. Similarly a competitive equilibrium at (p_H^*, q_H) is not stable for small values of β. The exact conditions are given in Lemma 10 of the Appendix.

Of the different types on non-competitive equilibria two simple ones consist of price dispersion in one market with a competitive price in the other, and the others have price dispersion in both markets. Figure 6 indicates the ranges of β on which the various equilibria occur.

4. CONCLUSION

The most important observation from the models in the preceeding sections is that when there is imperfect information about quality, the incidence of quality deterioration in equilibrium increases. In the first model, quality deterioration can be prevented by there being a sufficiently large proportion of shoppers. In the second model a badly behaved market can be improved by increasing the proportion of shoppers, or to some extent by reducing the proportion

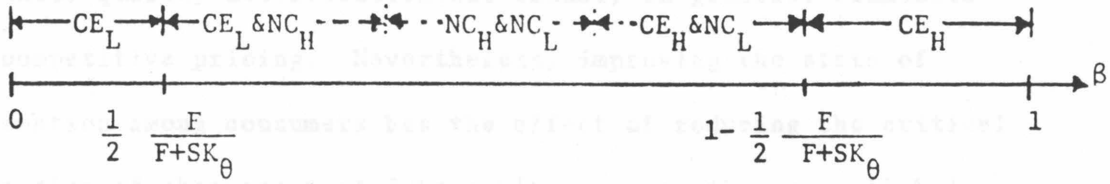

FIGURE 6

CE_L denotes competitive equilibrium in the low quality market.

NC_H denotes noncompetitive equlibrium in the high quality market.

NC_L denotes noncompetitive equilibrium in the low quality market.

CE_H denotes competitive equilibrium in the high quality market.

of consumers that are initially uninformed. The latter serves to eliminate quality deterioration but cannot, in general, eliminate non-competitive pricing. Nevertheless, improving the state of information among consumers has the effect of reducing the critical proportion of shoppers needed to achieve a competitive equilibrium. In the last model, more shopping cannot cure the suboptimal behavior of the market, instead the level of information among consumers must be increased.

These models give us insight on two different types of state intervension: disclosure laws and other policies designed to reduce the cost of comparison and search; and educational plans to inform consumers, of which attributes are important in comparing products in a market. A third type of intervension that is often discussed is the regulation of product quality. Welfare can certainly be improved in all three models by regulation of product quality, when the cost is small enough. However, this idea assumes that the state knows the correct level of quality to allow in the market. Making this assumption is tantamount to assuming that the state knows the competitive prices, and then it might as well dictate both prices and quality. Thus quality regulation is not a realistic remedy for markets with imperfectly informed consumers.

Reducing the cost of comparison shopping through disclosure laws, uniform statement of terms and other standardization will certainly improve the equilibrium distribution whenever the consumer is able to comprehend the information or at least recognize the difference when different quality products are encountered. Thus, it will benefit markets which can be described by the first two models, where there is learning by shopping.

The problem with the consumers in the third model is that even if everything about the quality is disclosed, all specifications are stated in standard terms, the consumer is unable to understand, and more important, he is unable to distinguish between products of two different quality levels at the time of purchase. Examples of such markets might be insurance markets where the statement of the contract is so complicated that the consumer is unable to know which contract is better and which is worse. The appropriate policy for such a market is to educate the consumer so that he is able to make proper decisions.

The most damaging aspect of this model is that the uninformed consumers also shop. Because they shop, and choose lower prices over higher prices it creates more demand in the low quality market where it is easier to offer lower prices. In fact if we observe the competitive equilibria as the sample size of the shoppers gets large we find that the range of β over which the competitive equilibria occur shrinks to just the two endpoints $\beta = 0$ and $\beta = 1$. The increase in shopping activity of the uninformed reduces the likelihood of a competitive equilibrium in the high quality market. Similarly, the increase in shopping among the informed consumers reduces the likelihood of the competitive equilibrium in the low quality markets. An area for further research would be to examine the effects of a policy that would encourage the informed to shop more and the uninformed to shop less. Clearly this would increase the range of β in which the competitive equilibrium in the high quality market occurs. The effect of such a policy on the non-competitive equilibria needs to be examined. Finally it must be determined whether the policy is Pareto-improving.

The last model can be alternatively interpreted as a model of heterogeneous tastes in a heterogeneous product market. That is, instead of the uninformed group having the same tastes as the informed, suppose they were actually informed and their behavior reflected their tastes. Then we have the result that all consumers are perfectly informed about quality and all consumers are shoppers and yet the market is badly behaved, simply because there are heterogeneous tastes. Another area of further research is to determine what sort of policies could improve the behavior of markets with heterogeneous products and heterogeneous tastes.

The policy implications of these models should be taken with a little caution. We assumed quite specific ways in which information could be gained and we simply assumed in some models that a certain proportion of consumers was informed. Instead, a model endogenizing the state of information among consumers by formalizing the transmission of information between consumers, such as reputation, or from firms to consumers, in the form of advertising may produce more accurate policy recommendations.

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APPENDIX

Theorem 1. The only competitive equilibrium is one with all firms at (p_H^*, q_H). The necessary and sufficient condition for this equilibrium to exist is $1-\gamma \leq \frac{F_L}{S_H(R-C_L)}$.

Proof: The other possible competitive equilibria are with n_L firms at (p_L^*, q_L) and n_H firms at (p_H^*, q_H), $n_H \neq 1$ $(n_L + n_H = 1)$.

We will show by contradiction why these cannot occur.

Suppose in equilibrium there are n_L firms at (p_L^*, q_L) and n_H firms at (p_H^*, q_H), $n_H \neq 1$ $(n_L + n_H = 1)$.

Then the following condition must be true at (p_L^*, q_L):

$$S_L = D(p_1^*, q_L) = \alpha(1-\gamma) + 2\alpha\gamma(\frac{n_L}{2}) = \alpha(1-\gamma + \gamma m_L).$$

This implies that $\alpha = \frac{S_L}{1-\gamma + \gamma n_L}$. Now let us see what condition must be true at (p_H^*, q_H).

$$S_H = D(p_H^*, q_L) = \alpha(1-\gamma) + 2\alpha\gamma(n_L + \frac{n_H}{2})$$

$$= \alpha(1-\gamma) + 2\alpha\gamma(\frac{n_L}{2} + \frac{1}{2}) = \alpha(1-\gamma + \gamma + \gamma n_L)$$

$$= \alpha(1+\gamma n_L) = \frac{S_L(1+\gamma n_L)}{(1-\gamma+\gamma n_L)} > S_L \geq S_H.$$

But this is a contradiction. Thus we have shown that the other possible competitive equilibria do not exist.

Now let us determine the necessary and sufficient conditions for competitive equilibrium to exist at (p_H^*, q_H).

$$D(p_H^*, q_H) = Z(p_H^*, q_H) \Rightarrow \alpha = S_H.$$

Suppose a firm attempted to deviate to any other location. Clearly it will receive its share of the non-shoppers. Of the shoppers that sample this firm, no one will purchase because the other firm they would sample would offer (p_H^*, q_H), which is preferred by the shopper whether or not the deviant was offering high or low quality. Thus, the deviant will receive only the non-shoppers. Given that the deviant receives only non-shoppers, he can earn the most profit by selling at the highest price that non-shoppers would pay. Thus, a deviant would either locate at (R, q_L) or (R, q_H). Of these two locations, he would earn the most profit if he chose (R, q_L) since between the two possibilities, the revenue is the same but the cost is greater for producing high quality than low quality.

Hence the most profit is made by a deviant who locates at (R, q_L). It follows that if it is not profitable to enter at (R, q_L), then it is not profitable to enter at any other location in the feasible set.

Thus the configuration with all firms located at (p_H^*, q_H) is an equilibrium if and only if

$$D(R, q_L) \leq Z(R, q_L),$$

$$D(R, q_L) = \alpha(1-\gamma) = S_H(1-\gamma) \leq \frac{F_L}{R-C_L},$$

or equivalently,

$$\gamma \geq 1 - \frac{F_L}{S_H(R-C_L)}.$$

Q.E.D.

<u>Lemma</u> <u>1</u>. In equilibrium, there cannot be any mass points except at competitive prices.

<u>Proof</u>: Suppose in equilibrium there is a mass point at (p,q) of size m, m > 0, and p is not the competitive price for quality q. Then since it is an equilibrium

$$D(p,q) = Z(p,q).$$

Choose a sequence $\{p - \varepsilon_i\}_{i=1}^{\infty}$ such that there is no mass point at $(p - \varepsilon_i, q)$ and $\{p - \varepsilon_i\}$ converges to p. Such a sequence can be chosen since any distribution can have at most a countable number of mass points. Now,

$$D(p - \varepsilon_i, q) \geq D(p,q) + \alpha\gamma m$$

since of all the $2\alpha\gamma m$ shoppers that choose (p,q) as their other draw, only one half will buy from a firm at (p,q), but all of them will buy from a firm at $(p - \varepsilon_i, q)$.

If we take the limit as $p - \varepsilon_i$ tends to p,

$$D(p - \varepsilon_i, q) \rightarrow D(p,q) + \alpha\gamma m, \text{ but}$$

$$Z(p - \varepsilon_i, q) \rightarrow Z(p,q).$$

Thus, in a neighborhood of prices below p, the demand will exceed the break even demand.

Q.E.D.

<u>Lemma 2</u>. No firms can exist at (p,q_L) where $p \varepsilon [p_L^*, \frac{F_L}{S_H} + C_L)$.

<u>Proof</u>: Suppose in equilibrium there is firm at (p,q_L) where $p \varepsilon [p_L^*, \frac{F_L}{S_H} + C_L)$. Then $Z(p,q_L) = D(p,q_L)$ must be true. Now the demand at (p_H^*, q_H) will be at least as large as $D(p,q_L)$ since each will receive their share of the non-shoppers and each shopper that purchases from (p,q_L) would necessarily purchase from (p_H^*, q_H) since the latter is preferred. Thus $D(p_H^*, q_H) \geq D(p,q_L) = Z(p,q_L)$.

Now since $p < \frac{F_L}{S_H} + C_L$ and Z is decreasing in p,

$$Z(p,q_L) > Z(\frac{F_L}{S_H} + C_L, q_L) = S_H.$$

But, this contradicts that we have an equilibrium.

Q.E.D.

<u>Lemma 3</u>. For every non-competitive equilibrium (R,q_L) belongs to the support of the equilibrium distribution, $\alpha = \frac{F_L}{(R-C_L)(1-\gamma)}$, and $\gamma < \frac{S_H(R-C_L) - F_L}{S_H(R-C_L)}$.

<u>Lemma 4</u>. Define $p^* = \frac{F_L(H-L-C_H) - F_H C_L}{F_H - F_L}$. If $p^* > C_L$, then for $p \varepsilon (\frac{F_L}{S_H} + C_L, R-H+L)$, if $p > p^*$ then no firm may locate at $(p+H-L, q_H)$ and if $p < p^*$, no firm may locate at (p, q_L); if $p^* > C_L$ then for $p \varepsilon (\frac{F_L}{S_H} + C_L, R - H + L)$, no firm may locate at (p, q_L).

<u>Proof</u>: Note the following fact. For any $p \varepsilon (\frac{F_L}{S_H} + C_L, R-H+L)$,

$$D(p,q_L) = D(p+H-L,q_H).$$

This is true for the following reasons. First both firms will receive their share of the non-shoppers which is the same. Second, suppose a shopper has sampled (p,q_L), then in order that he purchase from this firm, his other sample must be a firm of low quality with price greater than p, or a firm of high quality with price greater than p+H-L. But now, if a consumer samples $(p+H-L,q_H)$ and decides to buy from this firm the same conditions are required for the other sample. Thus the demand due to the shoppers is also the same for both firms.

If in equilibrium firms may locate at (p,q_L) then it must be the case that

$$Z(p,q_L) = D(p,q_L) = D(p+H-L,q_H) \leq Z(p+H-L,q_H)$$

Thus firms may not locate at $(p+H-L,q_H)$ if $Z(p,q_L) < Z(p+H-L,q_H)$. It follows, similarly, that firms may not locate at (p,q_L) if $Z(p,q_L) > Z(p+H-L,q_H)$.

Now let us determine how $Z(p,q_L)$ and $Z(p+H-L,q_H)$ compare with one another by first determining when they are equal, and denote that price by p^*.

$$Z(p^*,q_L) = \frac{F_L}{p^*-C_L} = Z(p^*+H-L,q_H) = \frac{F_H}{p^*+H-L-C_H}$$

Solving for p^* we find

$$p^* = \frac{F_H C_L + F_L(H - L - C_H)}{F_H - F_L}.$$

If $p^* > C_L$, for $p > p^*$, $Z(p,q_L) < Z(p+H-L,q_H)$ and for $p < p^*$, $Z(p,q_L) > Z(p+H-L,q_H)$. If $p^* < C_L$ then $z(p, q_L) > z(p + H - L, q_H)$. See Figures 6 and 7.

Q.E.D.

<u>Lemma 5</u>. Define p^{**} to be the positive solution to the quadratic equation defined by

$$\frac{F_L}{p-C_L} - \alpha(1-\gamma) = \frac{F_H}{p+H-L-C_H}.$$

For $p \in (R-H+L,L)$, if $p > p^{**}$, firms may not locate at $(p+H-L,q_H)$.

<u>Proof</u>: For $p \in (R-H+L,L)$,

$$D(p,q_L) - \alpha(1-\gamma) - 2\alpha\gamma n_H(1-G_H(p+H-L)) = D(p+H-L,q_H).$$

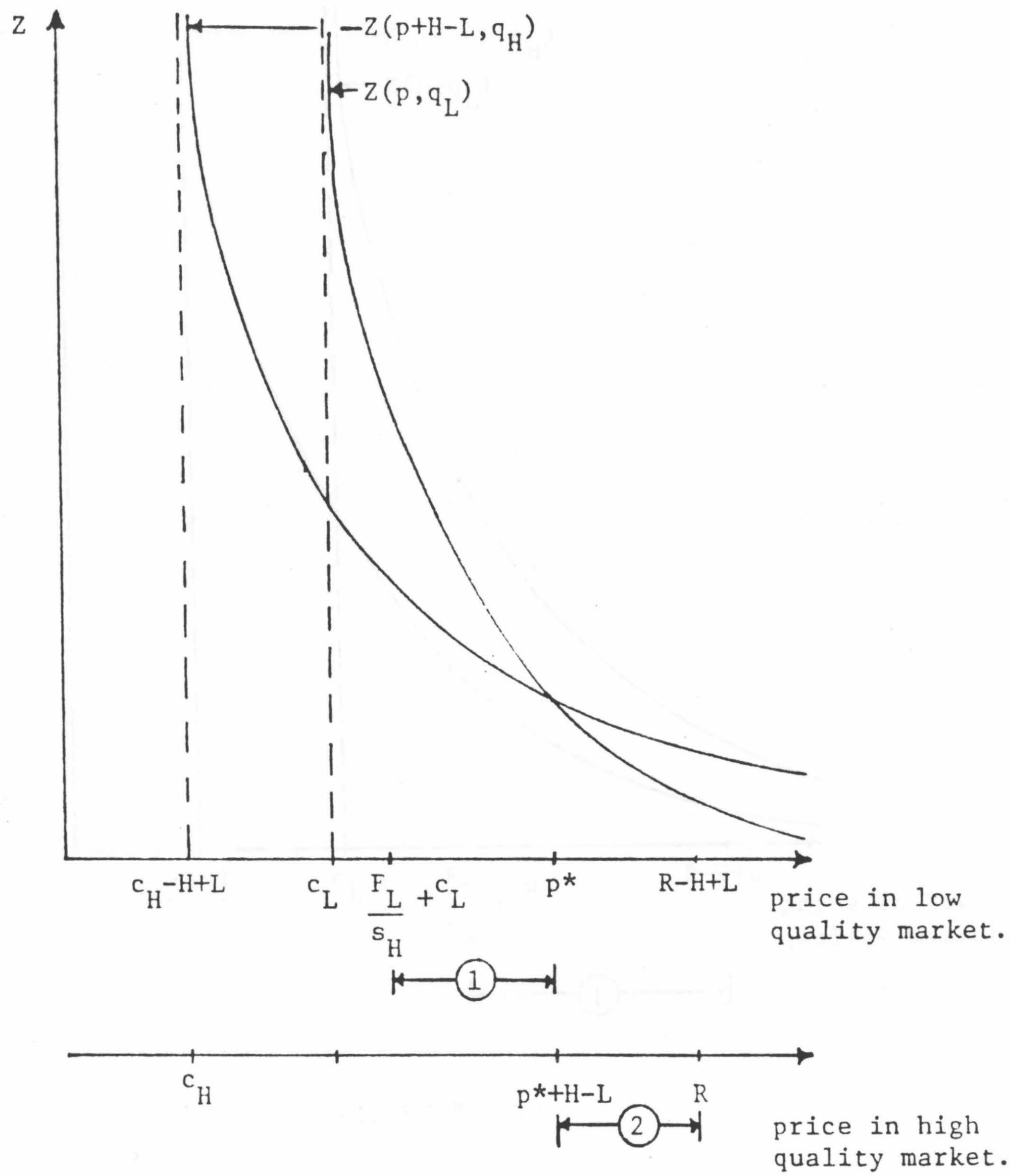


FIGURE 7

(1) : Firms cannot locate in low quality market.

(2) : Firms cannot locate in high quality market.

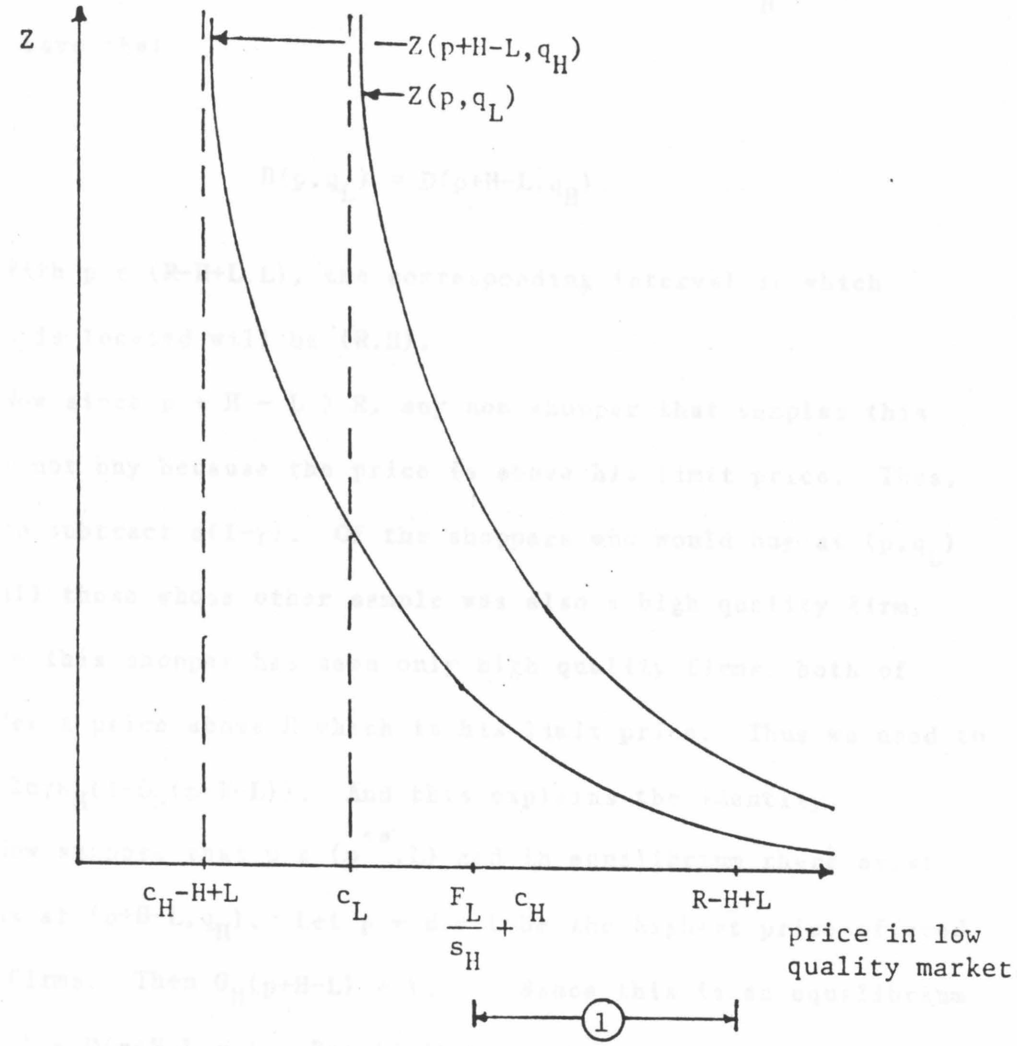


FIGURE 8

(1) : Firms cannot locate in the low quality market.

$Z(p,q_L)$ intersects $Z(p+H-L,q_H)$ below c_L .

To see this recall from Lemma 4, that if $p \varepsilon (\frac{F_L}{S_H} + C_L, R\text{-}H\text{+}L)$, we would have that

$$D(p,q_L) = D(p+H-L,q_H).$$

With $p \varepsilon (R\text{-}H\text{+}L,L)$, the corresponding interval in which $p + H - L$ is located will be (R,H).

Now since $p + H - L > R$, any non-shopper that samples this firm will not buy because the price is above his limit price. Thus, we need to subtract $\alpha(1-\gamma)$. Of the shoppers who would buy at (p,q_L) we lose all those whose other sample was also a high quality firm, since, now this shopper has seen only high quality firms, both of which offer a price above R which is his limit price. Thus we need to subtract $2\alpha\gamma n_H(1-G_H(p+H-L))$. And this explains the identity.

Now suppose that $p \varepsilon (p^{**},L)$ and in equilibrium there exist some firms at $(p+H-L,q_H)$. Let $p + H - L$ be the highest price offered by these firms. Then $G_H(p+H-L) = 1$. Since this is an equilibrium $Z(p+H-L,g_H) = D(p+H-L,q_H)$. Recall that

$$\begin{aligned} D(p+H-L,q_H) &= D(p,q_L) - \alpha(1-\gamma) - 2\alpha\gamma n_H(1-G_H(p+H-L)) \\ &\quad \text{by the above argument,} \\ &= D(p,q_L) - \alpha(1-\gamma) \\ &\quad \text{since } G_H(p+H-L) = 1, \\ &\lesssim Z(p,q_L) - \alpha(1-\gamma) \end{aligned}$$

since this is an equilibrium. Thus we need that

$$Z(p+H-L,q_H) \leq Z(p,q_L) - \alpha(1-\gamma).$$

Figures 9 and 10 illustrate the two curves. From the figures we see that $p > p^{**}$ is exactly the region where

$$Z(p+H-L,q_H) > Z(p,q_L) - \alpha(1-\gamma),$$

which is a contradiction.

Q.E.D.

<u>Lemma 6</u>. Define $\hat{p} = C_L + \frac{F_L}{F_H}(R - C_H)$. For $p \in (R - H + L, \hat{p})$, firms cannot locate at (p,q_L) or $(p + H - L, q_H)$.

<u>Proof</u>: Suppose there is a firm at (p,q_L) and $R - H + L < p < \hat{p}$. Then

$$D(p,q_L) = Z(p,q_L) = \frac{F_L}{p - C_L} > \frac{F_L}{\hat{p} - C_L} = \frac{F_H}{R - C_H}.$$

Now $D(R,q_H) \geq D(p,q_L)$. But this implies $D(R,q_H) > \frac{F_H}{R - C_H} = Z(R,q_H)$. This cannot hold in equilibrium.

Suppose there is a firm at $(p + H - L, q_H)$ and $R - H + L < p < \hat{p}$. Then $p^{**} \geq p$. In equilibrium, $D(p + H - L, q_H) = 2\alpha\gamma n_L(1 - G_L(p)) = Z(p + H - L, q_H)$.

If $p^{**} \leq \hat{p}$, then $(1 - G_L(p)) = (1 - G_L(\hat{p}))$ since there are no firms between $R - H + L$ and $\hat{p}$ in the low quality market. But now $\hat{p} > p^{**}$ implies $Z(\hat{p},q_L) - \alpha(1 - \gamma) \leq Z(\hat{p} + H - L, q_H) < Z(p + H - L, q_H)$ since $p < \hat{p}$. Since it is an equilibrium, $D(\hat{p},q_L) = \alpha(1 - \gamma) + 2\alpha\gamma n_L(1 - G_L(\hat{p})) \leq Z(\hat{p},q_L)$ or $2\alpha\gamma n_L(1 - G_L(\hat{p})) \leq Z(\hat{p},q_L) - \alpha(1 - \gamma)$.

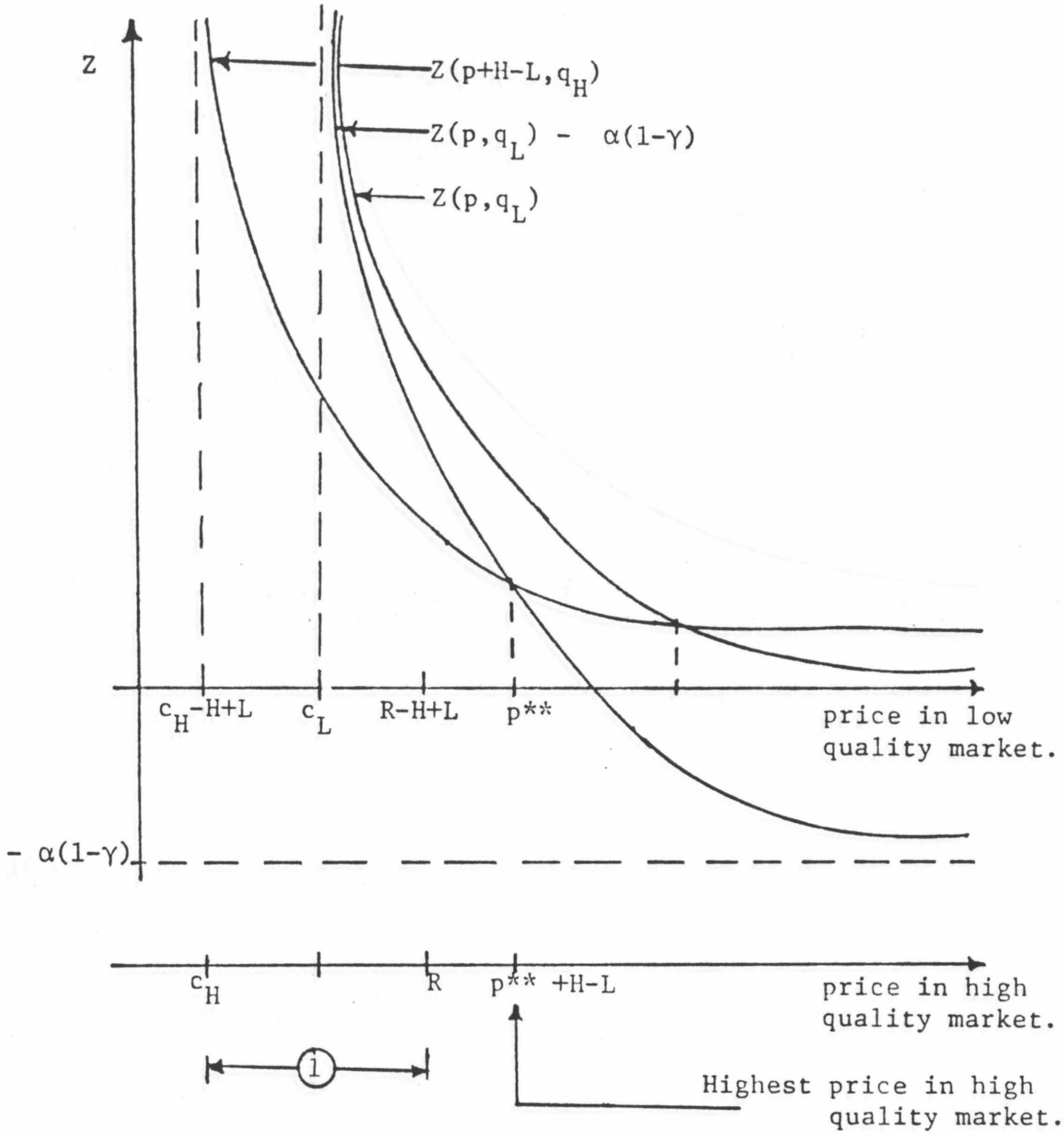


FIGURE 9

(1) : Firms can be in high quality market.

$p^* > R-H+L$

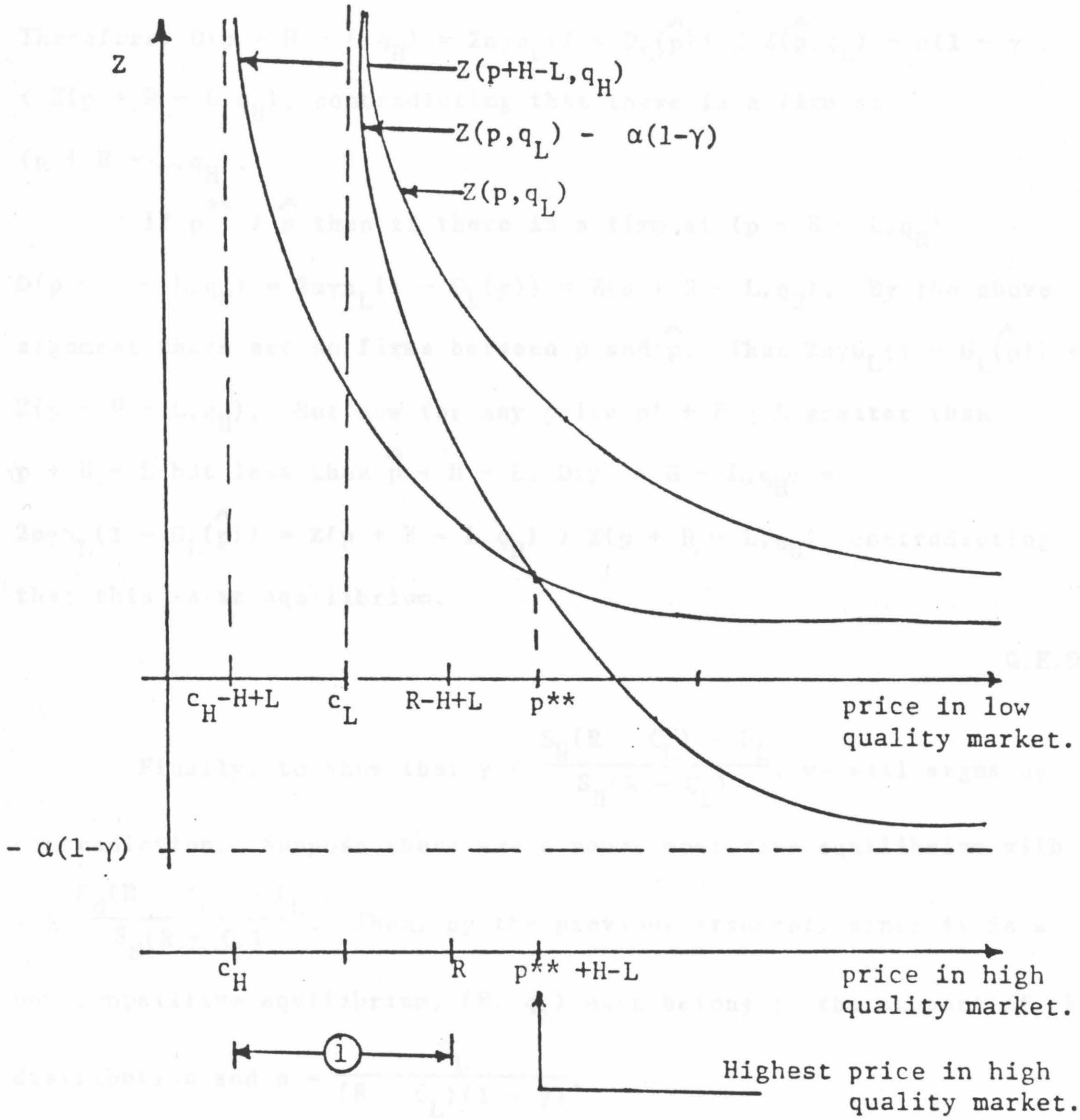


FIGURE 10

1 : Firms can be in high quality market.

$p^* < c_L$.

Therefore, $D(p + H - L, q_H) = 2\alpha\gamma n_L(1 - G_L(\hat{p})) \leq Z(\hat{p}, q_L) - \alpha(1 - \gamma)$ $< Z(p + H - L, q_H)$, contradicting that there is a firm at $(p + H - L, q_H)$.

If $p^{**} > \hat{p}$ then if there is a firm at $(p + H - L, q_H)$ $D(p + H - L, q_H) = 2\alpha\gamma n_L(1 - G_L(p)) = Z(p + H - L, q_H)$. By the above argument there are no firms between p and $\hat{p}$. Thus $2\alpha\gamma n_L(1 - G_L(\hat{p})) =$ $Z(p + H - L, q_H)$. But now for any price $p' + H - L$ greater than $p + H - L$ but less than $\hat{p} + H - L$, $D(p' + H - L, q_H) =$ $2\alpha\gamma n_L(1 - G_L(\hat{p})) = Z(p + H - L, q_H) > Z(p + H - L, q_H)$, contradicting that this is an equilibrium.

Q.E.D.

Finally, to show that $\gamma < \dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$, we will argue by contradiction. Suppose there was a non-competitive equilibrium with $\gamma \geq \dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$. Then, by the previous argument, since it is a non-competitive equilibrium, (R, q_L) must belong to the support of the distribution and $\alpha = \dfrac{F_L}{(R - C_L)(1 - \gamma)}$.

Consider a firm entering at (p_H^*, q_H).

$$D(p_H^*, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma[(1 - m) + \frac{m}{2}]$$

where m is the size of the mass point at (p_H^*, q_H), if any. Note that since it is a non-competitive equilibrium, $m < 1$.

$$D(p_H^*, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma(1 - \frac{m}{2})$$

$$> \alpha(1 - \gamma) + 2\alpha\gamma$$

$$= \alpha$$

$$= \frac{F_L}{(R - C_L)(1 - \gamma)}$$

$$\geq \frac{F_L}{(R - C_L)[1 - \frac{S_H(R - C_L) - F_L}{S_H(R - C_L)}]}$$

since $\gamma \geq \frac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$ by assumption,

$$= S_H = z(p_H^*, q_H).$$

In summary $D(p_H^*, q_H) > z(p_H^*, q_H)$. This cannot hold in equilibrium and so by condraction we show that $\gamma < \frac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$.

Q.E.D.

The next five theorems describe the non-competitive equilibria that arise under five mutually exclusive and totally exhaustive sets of conditions on the technology and limit prices. In each case several different equilibria may occur depending on the proportion of shoppers to non-shoppers. Since they are non-competitive equilibria we shall assume throughout that

$$\alpha = \frac{F_L}{(R-C_L)(1-\gamma)}$$

$$\text{and } \gamma < \frac{S_H(R-C_L) - F_L}{S_H(R-C_L)}.$$

<u>Theorem</u> <u>2</u>. If $R-H+L > p^* > \frac{F_L}{S_H} + C_L$, then one of the following is the equilibrium, depending on γ.

i) $n_L = 1, \quad n_H = 0$

$$G_{L(p)} = \begin{cases} 1 & p > R \\ 1 - \frac{(1-\gamma)}{2\gamma}\left[\frac{R-p}{p-C_L}\right] & R \geq p \geq \tilde{p}_1 \\ 0 & \tilde{p}_1 > p \end{cases}$$

where $\tilde{p}_1 = C_L + \frac{(R-C_1)(1-\gamma)}{\gamma + 1}$,

$\tilde{p}_1$ solves $G_L(p) = 0$

$$\Leftrightarrow \quad \frac{R-p^*}{2(p^*-C_L) + R-p^*} \geq \gamma \geq 0.$$

ii) $$n_L = \frac{1-\gamma}{2\gamma}\left[\frac{R-p^*}{p^*-C_L}\right], \quad n_H = 1 - n_L$$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{1\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > p^* \\ 0 & p^* \geq p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > p^* + H - L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H(R-C_L)}{F_L(p-C_H)} - \dfrac{R-C_L}{p^*-C_L} - 1\right] & p^* + H - L > p \geq \tilde{p}_2 \\ 0 & \tilde{p}_2 > p \end{cases}$$

where $\tilde{p}_2 = C_H + \frac{F_H}{F_L} \cdot \frac{1-\gamma}{1+\gamma}(R-C_L)$, $\tilde{p}_2$ solves $G_H(p) = 0$

$$\Leftrightarrow \quad \frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L} \geq \gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}$$

iii) $$n_L = \frac{1-\gamma}{2\gamma}\left[\frac{R-p^*}{p^*-C_L}\right], \quad n_H = 1 - n_L$$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{(1-\gamma)}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > p^* \\ 0 & p^* \geq p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > p^* + H - L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H(R-C_L)}{F_L(p-C_H)} - \dfrac{(R-C_L)}{(p^*-C_L)} - 1\right] & p^* + H - L \geq p \geq \tilde{p}_3 \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H(R-C_L)}{F_L(\tilde{p}_3-C_H)} - \dfrac{(R-C_L)}{(p^*-C_L)} - 1\right] & \tilde{p}_3 > p \geq p_H^* \\ 0 & p_H^* > p \end{cases}$$

where

$$\tilde{p}_3 = C_H + \frac{(1-\gamma)(R-C_L)\cdot F_H}{2(1-\gamma)(R-C_L)S_H-(1+\gamma)F_L}$$

$$\Longleftrightarrow \quad \frac{2(p^*+H-L-C_H)S_H(R-C_L)-(p^*+H-L-C_H)F_L-(R-C_L)F_H}{2(p^*+H-L-C_H)S_H(R-C_L)+(p^*+H-L-C_H)F_L-(R-C_L)F_H} \geq \gamma > \frac{S_H(R-C_L)-F_L}{S_H(R-C_L)}$$

iv) $n_L = \dfrac{S_H(R-C_L)(1-\gamma) - F_L}{\gamma F_L}, \qquad n_H = 1-n_L$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > \tilde{p}_4 \\ 0 & \tilde{p}_4 \geq p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p \geq p_H^* \\ 0 & p < p^{*H} \end{cases}$$

where

$$\tilde{p}_4 = C_L + \frac{(1-\gamma)(R-C_L)F_L}{(1-\gamma)(R-C_L)2S_H - F_L(\gamma+1)}, \quad (\tilde{p}_4 \text{ solves } G_L(p) = 0)$$

$$\Longleftrightarrow \quad \frac{S_H(R-C_L) - F_L}{S_H(R-C_L)} > \gamma > \frac{2(p^*+H-L-C_H)S_H(R-C_L) - (p^*+H-L-C_H)F_L - (R-C_L)F_H}{2(p^*+H-L-C_H)S_H(R-C_L) + (p^*+H-L-C_H)F_L - (R-C_L)F_H}$$

Proof: i) Suppose $\frac{R-P^*}{2(p^*-C_L)+R-P^*} \geq \gamma \geq 0$. The condition that $\frac{R-p^*}{2(p^*-C_L)+R-p^*} \geq \gamma$ is equivalent to $\tilde{p}_1 \geq p^*$. We will show that the distribution given in i) is an equilibrium. For any equilibrium we need that, on the support of the distribution, the demand equals the break even demand.

Thus, for any $p \in (R,\tilde{p}_1)$

$$D(p,q_L) = \alpha(1-\gamma) + 2\alpha\gamma[1-G_L(p)] = \frac{F_L}{p-C_L} = Z(p,q_L)$$

If we solve for $G_L(p)$ we find that

$$G_L(p) = 1 - \frac{1-\gamma}{2\gamma}\left[\frac{R-p}{p-C_L}\right].$$

Next, for any equilibrium we need that the demand at any point not on the support of the distribution not exceed the break even demand at that point. In the low quality market, the prices not in the support of the distribution are those below $\tilde{p}_1$. For $p < \tilde{p}_1$, $D(p,q_L) = D(\tilde{p}_1,q_L) = Z(\tilde{p}_1,q_L) < Z(p,q_L)$ so firms cannot deviate to these points and earn a positive profit. In the high quality market none of the prices are in the support of the distribution. Let us look at two cases.

First, for $p > p^* + H-L$

$D(p,q_H) \leq D(p-H+L,q_L)$ by Lemmas 4 and 5

$\leq Z(p-H+L,q_L)$ since firms cannot deviate to any point in the low quality market and earn a positive profit,

$\leq Z(p,q_H)$ by the definition of p^*.

Thus, firms may not deviate to the high quality market at prices greater than $p^* + H-L$.

Next for $p \leq p^* + H-L$.

$D(p,q_H) = D(p-H+L,q_L)$ by Lemma 4,

$= D(\tilde{p}_1,q_L)$ since there are no firms in the low quality market $p \varepsilon (p-H+L,\tilde{p}_1)$,

$= Z(\tilde{p}_1,q_L)$

$\leq Z(\tilde{p}_1+H-L,q_H)$ since $\tilde{p}_1 \geq p^*$

$\leq Z(p^*+H-L,q_H)$

and equality holds only if $\tilde{p}_1 = p^*$ and $p = p^*+H-L$. Thus, firms may not deviate to the high quality market at prices less than p^*+H-L. At (p^*+H-L,q_H) they just earn zero profit.

Thus, we have shown that demand equals break even demand on the support of the distribution and at any point not on the support demand does not exceed the break even demand. Therefore this is an equilibrium.

Suppose the distribution described in i) is an equilibrium then we must show that

$$0 \leq \gamma \leq \frac{R-p^*}{2(p^*-C_L) + R-p^*}.$$

The condition that $0 \leq \gamma$ always holds since γ is assumed to lie between 0 and 1.

Suppose $\gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}$, then we will show that the distribution described in i) is not an equilibrium.

$$\gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*} \Rightarrow \tilde{p}_1 < p^*.$$

But this means that there exist firms in the low quality market with prices below p^*. This contradicts Lemma 4. Thus $\gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}$ implies that the distribution in i) is not an equilibrium.

ii) Suppose $\frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + R_L} \geq \gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}$

we will show that the distribution given in case ii) is an equilibrium. Note first that $\frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L} \geq \gamma$ is a necessary and sufficient condition for $\tilde{p}_2 \geq p_H^*$. First we must show that for any point in the support of the distribution the demand equals the break even demand.

Let us begin with the low quality market. For $p \varepsilon (p^*, R)$,

$$D(p, q_L) = \alpha(1-\gamma) + 2\alpha\gamma n_L(1-G_L(p)) = \frac{F_L}{p-C_L}.$$

Solving for $G_L(p)$ we find that

$$G_L(p) = 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-p}{p-C_L}\right].$$

Now from Lemma 4, we know that firms cannot exist in the low quality market at prices below p^*. But, if we substitute for n_L as given, we find that $G_L(p^*) = 0$. Thus, demand equals break even demand on the support of the distribution, in the low quality market.

In the high quality market since $\tilde{p}_2 \geq p_H^*$ there are no mass points in the distribution. Thus, for $p \varepsilon (\tilde{p}_2, p^*+H-L)$

$$D(p, q_H) = \alpha(1-\gamma) + 2\alpha\gamma n_L + 2\alpha\gamma n_H(1-G_H(p)) = \frac{F_H}{p-C_H}.$$

If we substitute for α, n_L and recall that $n_H = 1 - n_L$ we arrive at the distribution given in case ii). Thus demand equals break even demand on the support.

Next, for any point not on the support the actual demand must not exceed the break even demand, or equivalently it should not be profitable to deviate to a point not on the support of the distribution. By the definition of p^*, it is not possible to deviate to the high quality market at prices greater than $p^* + H-L$, or to the low quality market, at prices below p^*. Lastly for $p \in (p_H^*, \tilde{p}_2)$ in the high quality market

$$D(p,q_H) = D(\tilde{p}_2,q_H) = Z(\tilde{p}_2,q_H) < Z(p,q_H).$$

Thus, profitable entry at this point is again not possible. Therefore, the distribution described in case ii) is an equilibrium.

Now suppose the distribution given in case ii) is an equilibrium we will show that

$$\frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L} \geq \gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}$$

by contradiction.

First suppose $\gamma > \dfrac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L}$ then $\tilde{p}_2 < p_H^*$. But we know that firms cannot exist at these prices since they are below the minimum of the average cost. Thus this is clearly not an equilibrium distribution.

Next suppose $\gamma < \dfrac{R-p^*}{2(p^*-C_L) + R-p^*}$ then $n_L > 1$, which is again not possible in equilibrium.

Thus if the distribution given is an equilibrium then

$$\frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L} \geq \gamma > \frac{R-p^*}{2(p^*-C_L) + R-p^*}.$$

iii) Suppose γ lies in the interval given in case iii), we will show that the distribution defined in case iii) is the equilibrium.

Clearly, $G_L(p)$ as defined in case iii) gives us the result that demand equals break even demand in the support of the distribution that lies in the low quality market. Similarly $G_H(p)$ guarantees that on its support demand equals break even demand for $p \geq \tilde{p}_3$.

Now for $p = p_H^*$, we need that

$$S_H = D(p_H^*, q_H) = \alpha(1-\gamma) + 2\alpha\gamma n_L + 2\alpha\gamma n_H\left[1-\frac{G_H(p_H^*)}{2}\right].$$

But now $G(p_H^*) = G(\tilde{p}_3)$. Thus,

$$S_H = \alpha(1-\gamma) + 2\alpha\gamma n_L + \frac{1}{2}\left[2\alpha\gamma n_H(1-G_H(\tilde{p}_3))\right] + \alpha\gamma - \alpha\gamma n_L.$$

Recalling that $n_L = \frac{1-\gamma}{2\gamma}\left[\frac{R-p^*}{p^*-C_L}\right]$ we find

$$S_H = \frac{F_L}{p^*-C_L} + \frac{1}{2}\left[\frac{F_H}{\tilde{p}_3 - C_H} - \frac{F_L}{p^* - C_L}\right] + \frac{\gamma F_L}{(1-\gamma)(R-C_L)} - \frac{1}{2}\,\frac{(R-p^*)F_L}{(p^*-C_L)(R-C_L)}.$$

Solving for $\tilde{p}_3$ we find

$$\tilde{p}_3 = C_H + \frac{(1-\gamma)(R-C_L)F_H}{2(1-\gamma)(R-C_L)S_H - (1+\gamma)F_L}.$$

Thus, as long as $\tilde{p}_3 \leq p^* + H - L$ we have that zero profit is earned on the support of the distribution.

From arguments presented in cases i) and ii), we know that positive profit cannot be made by deviating either to the low quality market at prices below p^* or to the high quality market at prices above p^*+H-L. The only additional region which is not in the support of the distribution is for prices between p_H^* and $\tilde{p}_3$ in the high quality market. For any $p \in (p_H^*, \tilde{p}_3)$,

$$D(p, q_H) = D(\tilde{p}_3, q_H) = Z(\tilde{p}_3, q_H) < Z(p, q_H),$$

and so deviation to these prices is not profitable. Thus, deviation to points not in the support of the distribution is not profitable and hence, the distribution in case iii) is an equilibrium.

Now suppose the given distribution is an equilibrium, we must show that γ must be in the range given in case iii). Suppose γ does not lie in the above interval. First suppose $\gamma \leq \dfrac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L}$. Then $G_H(p_H^*) < 0$ from the proof of ii) and so the distribution described in iii) will not be a proper distribution.

Next, if γ lies above its upper limit, then $\tilde{p}_3 > p^*$ and again $G_H(p_H^*)$ is not properly defined since it is not clear where the mass of size $1-G_H(p_H^*)$ lies. If it lies at p^* then the demand just below p^* will increase discontinously, but the break even demand is continuous in p so that it would be profitable to enter at prices just below p^*.

Thus, if γ is not in the prescribed range, the distribution given in case iii) is not an equilibrium.

iv) Suppose γ lies in the range indicated in case iv) then we will show that the distribution given in case iv) is an equilibrium.

Notice that γ lying above the lower end point of its range is equivalent to $\tilde{p}_4 \geq p^*$. Clearly demand equals break even demand on the support of the distribution in the low quality market. In the high quality market the only point in the support is at (p_H^*, q_H).

$$D(p_H^*, q_H) = \alpha(1-\gamma) + 2\alpha\gamma n_L + \alpha\gamma(1-n_L) = S_H.$$

This is clearly satisfied when $\alpha = \dfrac{F_L}{(R-C_L)(1-\gamma)}$ and n_L is as given in the distribution.

Next, we know that deviation to the high quality market at prices above p^*+H-L is not profitable. For $p \leq p^*$+H-L,

$$D(p, q_H) = D(\tilde{p}_4, q_L) = Z(\tilde{p}_4, q_L) \leq Z(p^*, q_L) \ Z(p^*+H-L, q_H) \leq Z(p, q_H),$$

so firms cannot locate in the high quality market in this range of prices.

Now, in the low quality market we know that firms may not exist below p^*. For $p \in (p^*, \tilde{p}_4)$

$D(p, q_L) = D(\tilde{p}_4, q_L) = Z(\tilde{p}_4, q_L) < Z(p, q_L)$. Therefore the distribution in case iv) is an equilibrium.

Now suppose we assume that the distribution given in case iv) is an equilibrium. We must show that γ lies in the range specified. We show this by contradiction. If γ lies below its lower limit, then $\tilde{p}_4 < p^*$. But $G_L(p)$ is negative when $p < p^*$ and so $G_L(p)$ is no longer a proper distribution, thus cannot be an equilibrium. If $\gamma > \dfrac{S_H(R-C_L) - F_L}{S_H(R-C_L)}$ then $n_L < 0$, so again this is not a proper distribution.

Therefore if the given distribution is an equilibrium, γ lies in the specified interval.

Q.E.D.

<u>Theorem 3</u>. If $\dfrac{F_L}{S_H} + C_L \geq p^* > C_L$ then one of the following is the equilibrium, depending on γ.

i) $\quad n_L = 1 \quad n_H = 0$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p \geq \tilde{p}_1 \\ 0 & \tilde{p} > p \end{cases}$$

where $\tilde{p}_1 = C_L + \dfrac{(R - C_L)(1 - \gamma)}{\gamma + 1}$, $\qquad \tilde{p}_1$ solves $G_L(p) = 0$.

$$\iff \quad \frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L} \geq \gamma \geq 0.$$

ii)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > \tilde{p}_4 \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-\tilde{p}_4}{\tilde{p}_1-C_L}\right] & \tilde{p}_4 \geq p \geq \dfrac{F_L}{S_H} + C_H \\ 0 & \dfrac{F_L}{S_H} + C_H > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p \geq p_H^* \\ 0 & p_H^* > p \end{cases}$$

where $n_L = \frac{1}{\gamma}\left[\frac{S_H(R - C_L)(1 - \gamma) - F_L}{F_L}\right]$, $\tilde{p}_4$ solves $G_L(p) = 0$.

$$\tilde{p}_4 = C_L + \frac{(1 - \gamma)(R - C_L)F_L}{2(1 - \gamma)(R - C_L)S_H - F_L\gamma}$$

$$\Longleftrightarrow \frac{S_H(R - C_L) - F_L}{S_H(R - C_L)} > \gamma > \frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L}.$$

<u>Proof</u>: i) The proof of this case is identical to that of Theorem 2, case i). The only difference is that since $p^* \leq \frac{F_L}{S_H} + C_L$, it is not enough to impose $\tilde{p}_1 \geq p^*$. We instead need that $\tilde{p}_1 \geq \frac{F_L}{S_H} + C_L$.

This condition is equivalent to $\dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L} \geq \gamma$.

ii) This case is identical to Theorem 2 case iv) with the only difference that $\tilde{p}_4 > \frac{F_L}{S_H} + C_L$. This is equivalent to

$$\gamma < \frac{S_H(R - C_L)}{S_H(- C_L) + F_L}.$$

We have $\gamma < \dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L)}$ which is a stricter condition which quarantees that $n_L > 0$. Notice also that $\gamma > \dfrac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L}$ is equivalent to $n_L < 1$.

Q.E.D.

Theorem 4. If

i) $p^* < C_L$ or $p^* > R - H + L$

ii) $R - H + L \leq \hat{p} \leq p^{**} < L$

then one of the following is an equilibrium.

i) $n_L = 1 \qquad n_H = 0$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \frac{1-\gamma}{2\gamma}\left[\frac{R - p}{p - C_L}\right] & R \geq p \geq \tilde{p}_1 \\ 0 & \tilde{p}_1 > p \end{cases}$$

where $\tilde{p}_1 = C_L + \dfrac{(R - C_L)(1 - \gamma)}{\gamma + 1}$, $\qquad \tilde{p}_1$ solves $G_L(p) = 0$.

$$\Longleftrightarrow \frac{R - p^{**}}{2(p^{**} - C_L) + R - p^{**}} \geq \gamma \geq 0.$$

ii)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p > p^{**} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{F_H}{F_L}\,\dfrac{(R - C_L)}{(p + H - L - C_H)}\right] & p^{**} \geq p \geq \tilde{p}_2 \\ 0 & \tilde{p}_2 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > p^{**} + H - L \\ 1 - \dfrac{1 - \gamma}{2\gamma(1 - n_L)}\left[\dfrac{R - p + H - L}{p - H + L - C_L} - \dfrac{F_H(R - C_L)}{F_L(p - C_H)}\right] & p^{**} + H - L \geq p \geq \tilde{p}_2 + H - L \\ 0 & \tilde{p}_2 + H - L > p \end{cases}$$

where $\tilde{p}_2 = C_L + \dfrac{1 - \gamma}{1 + \gamma}(R - C_L)$ and n_L is chosen so that $G_L(\tilde{p}_2) = 0$.

$$n_L = \frac{1-\gamma}{2\gamma} \cdot \frac{F_H}{F_L} \cdot \frac{(R - C_L)(1 + \gamma)}{(R - C_L)(1 - \gamma) + (H - L - C_H + C_L)(1 + \gamma)}$$

$$\Leftrightarrow \frac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} \geq \gamma > \frac{R - p^{**}}{2(p^{**} - C_L) + R - p^{**}}$$

iii)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > p^{**} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{F_H}{F_L}\,\dfrac{(R-C_L)}{(p+H-L-C_H)}\right] & p^{**} \geq p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > p^{**}+H-L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(p-C_H)}\right] & p^{**}+H-L \geq p \geq \hat{p}+H-L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\hat{p}}{\hat{p}-C_L} - \dfrac{F_H(R-C_L)}{F_L(\hat{p}+H-L-C_H)}\right] & \hat{p}+H-L > p > R \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{p-C_H} - \dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & R \geq p > \tilde{p}_3 \\ 0 & \tilde{p}_3 \geq p \end{cases}$$

where $n_L = \dfrac{1-\gamma}{2\gamma}\left[\dfrac{F_H}{F_L}\,\dfrac{(R-C_L)}{(\hat{p}+H-L-C_H)}\right]$

$$\tilde{p}_3 = C_H + \frac{1-\gamma}{1+\gamma}\cdot\frac{F_H}{F_L}(R - C_L)$$

$$\Longleftrightarrow \frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L} \geq \gamma > \frac{(R-C_L)F_H - (R-C_H)F_L}{(R-C_L)F_H + (R-C_H)F_L}$$

iv)

$$G_L(p) = \begin{cases} 1 & p - R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > p^{**} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{F_H}{F_L}\dfrac{(R-C_L)}{(p+H-L-C_H)}\right] & p^{**} \geq p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > H \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(R-C_H)}\right] & H \geq p \geq \hat{p} + H - L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\hat{p}}{\hat{p}-C_L} - \dfrac{F_H(R-C_L)}{F_L(\hat{p}+H-L-C_H)}\right] & \hat{p} + H - L > p > R \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{p-C_H} - \dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & R \geq p > \tilde{p}_4 \\ 2 - \dfrac{1-\gamma}{\gamma(1-n_L)}\left[\dfrac{S_H(R-C_L)}{F_L} - \dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & \tilde{p}_4 \geq p \geq p_H^* \\ 0 & p_H^* > p \end{cases}$$

where $n_L = \dfrac{1-\gamma}{2\gamma}\left[\dfrac{F_H}{F_L}\dfrac{(R-C_L)}{(\hat{p}+H-L-C_H)}\right]$

$$\tilde{p}_4 = C_H + \frac{(1-\gamma)F_H(R-C_L)}{2(1-\gamma)S_H(R-C_L) - (1+\gamma)F_L}$$

$$\iff \frac{2S_H(R-C_L)(R-C_H) - F_L(R-C_H) - F_H(R-C_L)}{2S_H(R-C_L)(R-C_H) + F_L(R-C_H) - F_H(R-C_L)} \geq \gamma > \frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L}$$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p > p^{**} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{F_H}{F_L}\,\dfrac{(R-C_L)}{(p+H-L-C_H)}\right] & p^{**} \geq p \geq \tilde{p}_5 \\ 0 & \tilde{p}_5 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > p^{**}+H \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(p-C_H)}\right] & p^{**}+H-L \geq p \geq \tilde{p}_5+H-L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\tilde{p}_5+H-L}{\tilde{p}_5-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(\tilde{p}_5-C_H)}\right] & \tilde{p}_5+H-L > p \geq p_H^{*} \\ 0 & p < p_H^{*} \end{cases}$$

where $n_L = \dfrac{1-\gamma}{2\gamma}\left[\dfrac{F_H}{F_L}\,\dfrac{(R-C_L)}{(\tilde{p}_5+H-L-C_H}\right]$

$$\tilde{p}_5 = C_L + \frac{F_L(1-\gamma)(R-C_L)}{2S_H(1-\gamma)(R-C_L)-(1+\gamma)F_L}$$

$$\iff \frac{2S_H(p^{**}-C_L)(R-C_L)-(p^{**}-C_L)F_L-(R-C_L)F_L}{2S_H(p^{**}-C_L)(R-C_L)+(p^{**}-C_L)F_L-(R-C_L)F_L} \geq \gamma$$

$$> \frac{2S_H(R-C_L)(R-C_H)-F_L(R-C_H)-F_H(R-C_L)}{2S_H(R-C_L)(R-C_H)+F_L(R-C_H)-F_H(R-C_L)}$$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p \geq \tilde{p}_6 \\ 0 & \tilde{p}_6 \end{cases}$$

$$G_H = \begin{cases} 1 & p \geq p_H^* \\ 0 & p < p_H^* \end{cases}$$

$$n_L = \frac{1}{\gamma}\left[\frac{S_H(R - C_L)(1 - \gamma) - F_L}{F_L}\right]$$

$$\tilde{p}_6 = C_L + \frac{(1 - \gamma)(R - C_L)F_L}{2(1 - \gamma)(R - C_L)S_H - (1 + \gamma)F_L}$$

$$\frac{S_H(R - C_L) - F_L}{S_H(R - C_L)} > \gamma > \frac{2S_H(p^{**} - C_L)(R - C_L) - (p^{**} - C_L)F_L - (R - C_L)F_L}{2S_H(p^{**} - C_L)(R - C_L) + (p^{**} - C_L)F_L - (R - C_L)F_L}$$

<u>Proof</u>: i) This is identical to Theorem 2, case i) except that $\tilde{p}_1 \geq p^{**}$. This is equivalent to

$$\frac{R \quad p^{**}}{2(p^{**} - C_L) + R - p^{**}} \geq \gamma.$$

ii) Suppose $\frac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} \geq \gamma > \frac{R - p^{**}}{2(p^{**} - C_L) + R - p^{**}}$.

We must show that the given distribution is an equilibrium. For $p \in [p^{**}, R)$

$$D(p, q_L) = \alpha(1 - \gamma) + 2\alpha\gamma n_L(1 - G_L(p)).$$

Thus for demand to equal break even demand in the low quality market in this range of prices, we need that

$$G_L(p) = 1 - \frac{1 - \gamma}{2\gamma n_L}\left[\frac{R - p}{p - C_L}\right]$$

For $p \in (\tilde{p}_2, p^{**})$

$$D(p, q_L) = \alpha(1 - \gamma) + 2\alpha\gamma n_L(1 - G_L(p)) + 2\alpha\gamma n_H(1 - G_H(p + H - L))$$

and $D(p + H - L, q_H) = 2\alpha\gamma n_L(1 - G_L(p))$

as long as $\tilde{p}_2 \geq R - H + L$.

Since this is on the support of the distribution, we must have that

a) $D(p + H - L, q_H) = 2\alpha\gamma n_L(1 - G_L(p)) = \frac{F_H}{p + H - L - C_H}$

$$\text{or } G_L(p) = 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{F_H}{F_L}\frac{(R - C_L)}{(p + H - L - C_H)}\right]$$

b) $$\frac{F_L}{p - C_L} = D(p,q_L) = \alpha(1-\gamma) + 2\alpha\gamma n_L(1 - G_L(p)) + 2\alpha\gamma n_H(1 - G_H(p + H - L))$$

$$= \alpha(1-\gamma) + \frac{F_H}{p + H - L - C_H} + 2\alpha\gamma n_H(1 - G_H(p + H - L))$$

$$\text{or} \quad G_H(p + H - L) = 1 - \frac{1-\gamma}{2\gamma(1 - n_L)}\left[\frac{F_L}{p - C_L} - \frac{F_L}{R - C_L} - \frac{F_H}{p + H - L - C_H}\right]\frac{R - C_L}{F_L}$$

$$= 1 - \frac{1-\gamma}{2\gamma(1 - n_L)}\left[\frac{R - p}{p - C_L} - \frac{F_H}{F_L}\frac{(R - C_L)}{(p + H - L - C_H)}\right]$$

for $p \in (\tilde{p}_2, p^{**})$.

Thus, for $p \in (\tilde{p}_2 + H - L, p^{**} + H - L)$

$$G_H(p) = 1 - \frac{1-\gamma}{2\gamma(1-n_L)}\left[\frac{R - p + H - L}{p - H + L - C_L} - \frac{F_H(R - C_L)}{F_L(p - C_H)}\right].$$

Recall that all of the above is true as long as $\tilde{p}_2 \geq R - H + L$. At $\tilde{p}_2$, $G_L(\tilde{p}_2) = 0$ and at $\tilde{p}_2 + H - L$, $G_H(\tilde{p}_2 + H - L) = 0$. Thus

$$D(\tilde{p}_2, q_L) = \alpha(1-\gamma) + 2\alpha\gamma n_L + 2\alpha\gamma(1 - n_L) = \frac{F_L}{\tilde{p}_2 - C_L}.$$

If we solve for $\tilde{p}_2$ we find

$$\tilde{p}_2 = C_L + \frac{1-\gamma}{1+\gamma}(R - C_L).$$

Notice now that $\frac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} \geq \gamma$ is equivalent to

$\tilde{p}_2 \geq C_L + \frac{F_L(R - C_H)}{F_H} = \hat{p} \geq R - H + L$ (by the assumption that $p^* < C_L$ or $p^* > R - H + L$). Thus demand on the support of the distribution equals the break even demand.

Now we must show that for points not on the support the break even demand is at least as large as the actual demand.

For the low quality market, we must look at prices below $\tilde{p}_2$. For $p < \tilde{p}_2$

$$D(p,q_L) = D(\tilde{p}_2,q_L) = Z(\tilde{p}_L,q_L) < Z(p,q_L).$$

In the high quality market, for $R < p < \tilde{p}_2 + H - L$

$$D(p,q_H) = D(\tilde{p}_2 + H - L) = Z(\tilde{p}_2 + H - L) < Z(p,q_H).$$

At (R,G_H),

$$\begin{aligned} D(R,G_H) &= \alpha(1 - \gamma) + 2\alpha\gamma n_L(1 - G_L(R - H + L)) + 2\alpha\gamma n_H(1 - G_H(R)) \\ &= \alpha(1 - \gamma) + 2\alpha\gamma n_L + 2\alpha\gamma(1 - n_L) \\ &= \alpha(1 + \gamma) = \frac{F_L}{\tilde{p}_2 - C_L} \leq \frac{F_L}{\hat{p} - C_L} \text{ since } \hat{p} \leq \tilde{p}_2 \\ &= \frac{F_H}{R - C_H} \text{ by the definition of } \hat{p}. \end{aligned}$$

Thus profitable entry is not possible at (R,q_H). Finally for $p < R$, in the high quality market,

$$D(p,q_H) = D(R,q_H) \leq Z(R,q_H) < Z(R,q_H).$$

Thus at any point not on the support, profitable entry is not possible, and so the distribution described in case ii) is an equilibrium.

Now suppose that the distribution is an equilibrium, we must show that $\dfrac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} \geq \gamma > \dfrac{R - p^{**}}{2(p^{**} - C_L) + R - p^{**}}$. This will be shown by contradiction.

Suppose $\dfrac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} < \gamma$ then $D(R,q_H) > Z(R,q_H)$ and thus firms can profitably enter at (R,q_H). So it is no longer an equilibrium.

$$\text{Suppose } \gamma < \frac{R - p^{**}}{2(p^{**} - C_L) + R - p^{**}} \text{ then } \tilde{p}_2 > p^{**}$$

and the given distributions do not make sense since they assume that $\tilde{p}_2 \leq p^{**}$.

iii) Suppose γ lies in the range described in case iii). We must show that the given distribution is an equilibrium.

$$D(R,q_H) = \alpha(1 - \gamma) + 2\alpha\gamma n_L = \frac{F_H}{R - C_H}$$

But now $\alpha(1 - \gamma) + 2\alpha\gamma n_L = D(\hat{p},q_L) = \dfrac{F_L}{\hat{p} - C_L}$. Thus $\hat{p}$ solves $\dfrac{F_H}{R - C_H} = \dfrac{F_L}{\hat{p} - C_L}$. Solving for $\hat{p}$ we find

$$\hat{p} = C_L + \frac{F_L}{F_H}(R - C_H)$$

From case ii) we know that demand equals break even demand for $p \geq \hat{p}$ in the low quality market, and for $p > \hat{p} + H - L$ in the high quality market.

Since there are no firms between R and $\hat{p} + H - L$ in the high quality market, we will now look at the high quality market for $p \leq R$. For $p \in [p_H^*, R]$

$$D(p, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma(1 - n_H)(1 - G_H(p)) + 2\alpha\gamma n_L = \frac{F_H}{p - C_H}$$

$$\alpha(1 - \gamma) + 2\alpha\gamma n_L = \frac{F_L}{R - C_L} + \frac{F_H}{\hat{p} + H - L - C_H}$$

$$\text{Thus } G_H(p) = 1 - \frac{1 - \gamma}{2\gamma(1 - n_L)}\left[\frac{F_H}{p - C_H} - \frac{F_H}{\hat{p} + H - L - C_H} - \frac{F_L}{R - C_L}\right]\frac{(R - C_L)}{F_L}$$

$$= 1 - \frac{1 - \gamma}{2\gamma(1 - n_L)}\left[\frac{F_H}{F_L} \cdot \frac{R - C_L}{p - C_H} - \frac{F_H}{F_L} \cdot \frac{R - C_L}{\hat{p} + H - L - C_H} - 1\right],$$

as long as $\tilde{p}_3 \geq p_H^*$. Now $\tilde{p}_3$ solves $G_H(p) = 0$.

The condition that $\tilde{p}_3 \geq p_H^*$ is equivalent to the condition that γ lies below the upper bound given in case iii). Thus the demand equals the break even demand at each point on the support.

For points not on the support we need that the demand be less then or equal to the break even demand. Let us begin with the low quality market. For $p \leq R - H + L < p^*$ we cannot have firms, by the definition of p^*. For $R - H + L < p < \hat{p}$,

$$D(p,q_L) = D(\hat{p},q_L) = Z(\hat{p},q_L) < Z(p,q_L)$$

thus firms may not locate here and earn positive profit. In the high quality market, for $R < p < \hat{p} + H - L$,

$$D(p,q_H) = D(\hat{p},q_H) = Z(\hat{p},q_L) < Z(p,q_H)$$

so that profitable entry here is not possible. Finally for $p_H^* < p < \tilde{p}_3$,

$$D(p,q_H) = D(\tilde{p}_3,q_H) = Z(\tilde{p}_3,q_H) < Z(p,q_H).$$

Thus there is no point of the support of the distribution where firms may enter and earn a positive profit. Hence, the given distribution is an equilibrium.

Now if the given distribution is an equilibrium then we must show that γ must lie in the specified range.

Suppose γ lies above the upper end point. Then $\tilde{p}_3 < p_H^*$ and the distribution is no longer proper. If γ lies below the lower end point, then

$$D(\hat{p},q_L) < Z(\hat{p},q_L) \text{ by the argument in case ii).}$$

Since $(\hat{p},q_L)$ belongs to the support of the distribution, the distribution is no longer an equilibrium.

iv). Suppose γ lies in the specified range. From arguments made in the previous cases, we know that demand equals break even demand for all prices in the support of the distribution in the low quality market. We also know this for the high quality market for prices above and including $\tilde{p}_4$. The only point remaining is (p_H^*, q_H).

$$D(p_H^*, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma n_L + 2\alpha\gamma n_H\left(1 - \frac{G_H(p_H^*)}{2}\right) = S_H$$

Thus $$G_H(p^*) = 2\left[1 - \frac{(1 - \gamma)}{2\gamma(1 - n_L)}\left[S_H - \frac{F_L}{R - C_L} - \frac{F_H}{\hat{p} + H - L - C_H}\right]\frac{R - C_L}{F_L}\right]$$

$$= 2 - \frac{1 - \gamma}{\gamma(1 - n_L)}\left[\frac{S_H(R - C_L)}{F_L} - \frac{F_H(R - C_L)}{F_L(\hat{p} + H - L - C_H)} - 1\right]$$

Thus demand equals break even demand on the support of the distribution.

Next, for points not on the support, we need that demand is less than or equal to the break even demand. From previous arguments, we know that this is the case for prices above $\tilde{p}_4$ in the high quality market and for all prices in the low quality market that are not on the support.

Now $\tilde{p}_4$ solves

$$1 - \frac{1 - \gamma}{2\gamma(1 - n_L)}\left[\frac{F_H}{F_L} \cdot \frac{R - C_L}{p - C_H} - \frac{F_H}{F_L} \cdot \frac{R - C_L}{\hat{p} + H - L - C_H} - 1\right] = G_H(p_H^*)$$

$$\tilde{p}_4 = C_H + \frac{(1 - \gamma)F_H(R - C_L)}{2(1 - \gamma)S_H(R - C_L) - (1 + \gamma)F_L}$$

Notice that $\tilde{p}_4 \leq R$ is equivalent to γ being less than or equal to its upper limit. Now for $p_H^* < p \leq \tilde{p}_4$

$$D(p, q_H) = D(\tilde{p}_4, q_H) = Z(\tilde{p}_4, q_H) < Z(\tilde{p}, q_H)$$

and thus profitable entry is not possible. Hence, the given distribution is an equilibrium.

If γ lies above its upper limit then $\tilde{p}_4 > R$ and that will not properly define an equilibrium distribution. If γ lies below its lower limit, then $G_H(p_H^*)$ will be negative which is again not a proper distribution.

v) Again we can verify that demand equals break even demand on the support of the distribution and is at most as large for points not on the support of the distribution.

It is interesting to observe this at (p_H^*, q_H)

$$S_H = D(p_H^*, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma n_L + 2\alpha\gamma(1 - n_L)(1 - \frac{G_H(p_H^*)}{2}).$$

Notice that $G_H(p_H^*) = G_H(\tilde{p}_5 + H - L)$.

Thus $S_H = \alpha(1 - \gamma) + 2\alpha\gamma n_L + 2\alpha\gamma(1 - n_L)\left[1 - \frac{G_H(\tilde{p}_5 + H - L)}{2}\right]$

$$= \alpha(1 - \gamma) + 2\alpha\gamma n_L + \frac{1}{2}[2\alpha\gamma(1 - n_L)(1 - G_H(\tilde{p}_5 + H - L))] + 2\gamma(1 - n_L)$$

$$= \frac{F_L}{R-C_L} + \frac{F_H}{\tilde{p}_5+H-L-C_H} + \frac{1}{2}\left[\frac{F_L}{\tilde{p}_5-C_L} - \frac{F_L}{R-C_L} - \frac{F_H}{\tilde{p}_5+H-L-C_H}\right] + \alpha\gamma - \alpha\gamma n_L.$$

Substituting for n_L (n_L simply solves $G_L(p) = 0$), the above equation becomes

$$S_H = \frac{1}{2}\left[\frac{F_L}{R-C_L}\right] + \frac{1}{2}\left[\frac{F_H}{\tilde{p}_5+H-L-C_H}\right] + \frac{1}{2}\left[\frac{F_L}{\tilde{p}_5-C_L}\right] + \frac{\gamma}{1-\gamma}\left[\frac{F_L}{R-C_L}\right] - \frac{1}{2}\left[\frac{F_H}{\tilde{p}_5+H-L-C_H}\right]$$

Solving for $\tilde{p}_5$ we find

$$\tilde{p}_5 = C_L + \frac{F_L(1 - \gamma)(R - C_L)}{2S_H(1 - \gamma)(R - C_L) - (1 + \gamma)F_L}$$

Clearly we need $\tilde{p}_5 \leq p^{**}$ and that gives us the upper limit for γ.

vi) This case is exactly as case iv) in Theorem 2. The only difference is that we need $\tilde{p}_6 \geq p^{**}$. This is guaranteed by the lower bound on γ. The upper bound ensures that n_L is greater than zero.

<u>Theorem 5</u>. If

i) $p^* < C_L$ or $p > R - H + L$

ii) $\hat{p} > p^{**}$ or $\hat{p} > L$

then one of the following is an equilibrium

i) $n_L = 1 \qquad n_H = 0$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p \geq \tilde{p}_1 \\ 0 & \tilde{p}_1 > p \end{cases}$$

where $\tilde{p}_1 = C_L + \dfrac{(R - C_L)(1-\gamma)}{(\gamma + 1)}$,

$$\iff \frac{R - \hat{p}}{2(\hat{p} - C_L) + R - \hat{p}} \geq \gamma \geq 0$$

ii) $n_L = \dfrac{1-\gamma}{2\gamma}\left[\dfrac{R - \hat{p}}{\hat{p} - C_L}\right] = \dfrac{1-\gamma}{2\gamma}\,\dfrac{F_H(R - C_L) - F_L(R - C_H)}{F_L(R - C_H)}, \qquad n_H = 1 - n_L$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{(1-\gamma)}{2\gamma(1-n_L)}\dfrac{F_H}{F_L}\left[\dfrac{R-C_L}{p-C_H} - \dfrac{R-C_L}{R-C_H}\right] & R \geq p > \tilde{p}_2 \\ 0 & \tilde{p}_2 > p \end{cases}$$

where $\tilde{p}_2 = C_H + \dfrac{1-\gamma}{1+\gamma}\,\dfrac{F_H(R - C_L)}{F_L}$

$$\frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L} \geq \gamma > \frac{R - \hat{p}}{2(\hat{p} - C_L) + R - \hat{p}}$$

iii) $n_L = \frac{1-\gamma}{2\gamma} \frac{F_H(R - C_L) - F_L(R - C_H)}{F_L(R - C_H)} \qquad n_H = 1 - n_L$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-p}{p-C_L}\right] & R \geq p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > R \\ 1 - \frac{1-\gamma}{2\gamma(1-n_L)} \frac{F_H}{F_L}\left[\frac{R-C_L}{p-C_H} - \frac{R-C_L}{R-C_H}\right] & R \geq p \geq \tilde{p}_3 \\ 1 - \frac{1-\gamma}{2\gamma(1-n_L)} \frac{F_H}{F_L}\left[\frac{R-C_L}{\tilde{p}_3-C_H} - \frac{R-C_L}{R-C_H}\right] & \tilde{p}_3 > p \geq p_H^* \\ 0 & p_H^* > p \end{cases}$$

where $\tilde{p}_3 = C_H + \frac{F_H(1-\gamma)(R-C_H)(R-C_L)}{2S_H(1-\gamma)(R-C_H)(R-C_L) + F_L(R-C_H)(1+\gamma)}$

$$\frac{2S_H(R-C_H)(R-C_L) - F_L(R-C_H) - F_H(R-C_L)}{2S_H(R-C_H)(R-C_L) + F_L(R-C_H) - F_H(R-C_L)} \geq \gamma > \frac{S_H(R-C_L) - F_L}{S_H(R-C_L) + F_L}$$

iv)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-p}{p-C_L}\right] & R \geq p \geq \tilde{p}_4 \\ 0 & \tilde{p}_4 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p \geq p_H^* \\ 0 & p < p_H^* \end{cases}$$

$$n_L = \frac{1}{\gamma}\left[\frac{S_H(R - C_L)(1 - \gamma) + F_L}{F_L}\right]$$

$$\tilde{p}_4 = C_L + \frac{(1 - \gamma)(R - C_L)F_L}{2(1 - \gamma)(R - C_L)S_H - (1 + \gamma)F_L}$$

$$\frac{S_H(R - C_L) - F_L}{S_H(R - C_L)} > \gamma > \frac{2S_H(R - C_H)(R - C_L) - F_L(R - C_H) - F_H(R - C_L)}{2S_H(R - C_H)(R - C_L) + F_L(R - C_H) - F_H(R - C_L)}$$

<u>Proof</u>: i) Identical to case i) Theorem 4. Only difference is that $\tilde{p}_1 \geq \hat{p}$.

ii) For $p \leq R$,

$$D(p, q_H) = \alpha(1 - \gamma) + 2\alpha\gamma n_L + 2\alpha\gamma n_H(1 - G_H(p)) = \frac{F_H}{p - C_H}$$

Since $\alpha(1 - \gamma) + 2\alpha\gamma n_L = \frac{F_H}{R - C_H}$, we have

$$G_H(p) = 1 - \frac{(1 - \gamma)}{2\gamma(1 - n_L)}\left[\frac{R - C_L}{p - C_H} - \frac{R - C_L}{R - C_H}\right]\frac{F_H}{F_L}.$$

To find $\tilde{p}_2$ we solve $G_H(p) = 0$.

Thus $\tilde{p}_2 = C_H + \frac{1 - \gamma}{1 + \gamma}\frac{F_H(R - C_L)}{F_L}$

where we have substituted for $n_L = \frac{1-\gamma}{2\gamma} \cdot \frac{F_H(R - C_L) - F_L(R - C_H)}{F_L(R - C_H)}$.

Finally we need $\tilde{p}_2 \geq p_H^*$, and this is equivalent to

$$\gamma \leq \frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L}$$

iii) $D(p_H^*, q_H) = \alpha(1-\gamma) + 2\alpha\gamma n_L + 2\alpha\gamma n_H(1 - \frac{G_H(p_H^*)}{2}) = S_H$

Since $G_H(p_H^*) = G_H(\tilde{p}_3)$,

$$S_H = \alpha(1 - \gamma) + 2\alpha\gamma n_L + \frac{1}{2}(2\alpha\gamma n_H(1 - G_H(\tilde{p}_3))) + \alpha\gamma(1 - n_L)$$

Recall that $n_L = \frac{1-\gamma}{2\gamma} \frac{F_H(R - C_L) - F_L(R - C_H)}{F_L(R - C_H)}$. Therefore,

$$S_H = \frac{F_L}{\hat{p} - C_L} + \frac{1}{2}\left[\frac{F_H}{\tilde{p}_3 - C_H} - \frac{F_L}{\hat{p} - C_L}\right] + \frac{\gamma}{1-\gamma}\frac{F_L}{R - C_L} - \frac{1}{2}\frac{F_H(R-C_L) - F_L(R-C_H)}{(R - C_L)(R - C_H)}$$

Substituting for $\hat{p} = C_L + \frac{F_L}{F_H}(R - C_H)$ we have

$$2S_H = \frac{F_H}{R - C_H} + \frac{F_H}{\tilde{p}_3 - C_H} + \frac{2\gamma}{1-\gamma}\frac{F_L}{R - C_L} - \frac{F_H(R-C_L) - F_L(R-C_H)}{(R - C_L)(R - C_H)}.$$

If we solve for $\tilde{p}_3$, we find

$$\tilde{p}_3 = C_H + \frac{F_H(1 - \gamma)(R - C_H)(R - C_L)}{2S_H(1 - \gamma)(R - C_H)(R - C_L) - F_L(R - C_H)(1 + \gamma)}.$$

Now we need $\tilde{p}_3 \leq R$, and this is equivalent to

$$\frac{2S_H(R - C_H)(R - C_L) - F_L(R - C_H) - F_H(R - C_L)}{2S_H(R - C_H)(R - C_L) + F_L(R - C_H) - F_H(R - C_L)} \geq \gamma$$

iv) This case is identical to Theorem 2 case iv). The only difference is that we need $\tilde{p}_4 \geq \hat{p}$ which accounts for the difference in the lower bound on γ.

<u>Theorem</u> <u>6</u>. If

i) $p^* < C_L$ or $p^* > R - H + L$

ii) $\hat{p} < L < p^{**}$

then the following are equilibria.

i) $n_L = 1$ $\qquad n_H = 0$

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p \geq \tilde{p}_1 \\ 0 & \tilde{p}_1 > p \end{cases}$$

where $\tilde{p}_1 = C_L + \dfrac{(R - C_L)(1 - \gamma)}{(1 + \gamma)}$

$$\iff \quad \frac{R - \hat{\hat{p}}}{2(\hat{\hat{p}} - C_L) + R - \hat{\hat{p}}} \geq \gamma \geq 0,$$

$$\text{where } \hat{\hat{p}} = C_L + \frac{F_L(R - C_H)(R - C_L)}{F_H(R - C_L) + F_L(R - C_H)}$$

ii)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-p}{p-C_L}\right] & R \geq p \geq \hat{\hat{p}} \\ 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-\hat{\hat{p}}}{\hat{\hat{p}}-C_L}\right] & \hat{\hat{p}} > p \geq L \\ 1 - \frac{1-\gamma}{2\gamma n_L}\frac{F_H}{F_L}\left[\frac{R-C_L}{p+H-L-C_H}\right] & L > p \geq \tilde{p}_2 \\ 0 & \tilde{p}_2 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > H \\ 1 - \frac{1-\gamma}{2\gamma n_L}\left[\frac{R-p+H-L}{p-H+L-C_L} - \frac{F_H(R-C_L)}{F_L(p-C_H)}\right] & H \geq p \geq \tilde{p}_2 + H - L \\ 0 & \tilde{p}_2 + H - L > p \end{cases}$$

$$\text{where } \tilde{p}_2 = C_L + \frac{1-\gamma}{1+\gamma}(R - C_L)$$

$$\text{and } n_L = \frac{1-\gamma}{2\gamma}\frac{F_H}{F_L}\frac{(R-C_L)(1+\gamma)}{(R-C_L)(1-\gamma) + (H-L-C_H+C_L)(1+\gamma)}$$

$$n_H = 1 - n_L$$

$$\Longleftrightarrow \quad \frac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L} \geq \gamma > \frac{R - \hat{\hat{p}}}{2(\hat{\hat{p}} - C_L) - R - \hat{\hat{p}}}$$

iii)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-p}{p-C_L}\right] & R \geq p \geq \hat{\hat{p}} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R-\hat{\hat{p}}}{\hat{\hat{p}} - C_L}\right] & \hat{\hat{p}} > p \geq L \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\dfrac{F_H}{F_L}\left[\dfrac{R-C_L}{p+H-L-C_H}\right] & L > p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > H \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(p-C_L)}\right] & H \geq p \geq \hat{p}+H-L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\hat{p}}{\hat{p}-C_L} - \dfrac{F_H(R-C_L)}{F_L(\hat{p}+H-L-C_H)}\right] & \hat{p}+H-L \geq p \geq R \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H}{F_L}.\dfrac{R-C_L}{p-C_H} - \dfrac{F_H}{F_L}.\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & R \geq p \geq \tilde{p}_3 \\ 0 & \tilde{p}_3 > p \end{cases}$$

$$n_L = \frac{1-\gamma}{2\gamma}\left[\frac{F_H}{F_L}\,\frac{(R - C_L)}{(\hat{p}+H-L-C_H)}\right], \quad \tilde{p}_3 = C_H + \frac{1-\gamma}{1+\gamma}\,\frac{F_H}{F_L}(R - C_L)$$

$$\Longleftrightarrow \quad \frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L} \geq \gamma > \frac{(R - C_L)F_H - (R - C_H)F_L}{(R - C_L)F_H + (R - C_H)F_L}$$

iv)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p \geq \hat{\hat{p}} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - \hat{p}}{\hat{p} - C_L}\right] & \hat{p} > p \geq L \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\,\dfrac{F_H}{F_L}\left[\dfrac{R - C_L}{p+H-L-C_H}\right] & L > p \geq \hat{p} \\ 0 & \hat{p} > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > H \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R-C_L)}{F_L(R-C_H)}\right] & H \geq p \geq \hat{p} + H - L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\hat{p}}{\hat{p}-C_L} - \dfrac{F_H(R-C_L)}{F_L(\hat{p}+H-L-C_H)}\right] & \hat{p} + H - L > p > R \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{p-C_H} - \dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & R \geq p > \tilde{p}_4 \\ 2 - \dfrac{1-\gamma}{\gamma(1-n_L)}\left[\dfrac{S_H(R-C_L)}{F_L} - \dfrac{F_H}{F_L}\cdot\dfrac{R-C_L}{\hat{p}+H-L-C_H} - 1\right] & \tilde{p}_4 \geq p \geq p_H^* \\ 0 & p_H^* > p \end{cases}$$

$$\text{where } n_L = \frac{1-\gamma}{2\gamma}\left[\frac{F_H}{F_L}\,\frac{(R - C_L)}{(\hat{p} + H - L - C_H)}\right]$$

$$\tilde{p}_4 = C_H + \frac{(1-\gamma)F_H(R - C_L)}{2(1-\gamma)S_H(R - C_L) - (1-\gamma)F_L}$$

$$\Longleftrightarrow \quad \frac{2S_H(R - C_L)(R - C_H) - F_L(R - C_H) - F_H(R - C_L)}{2S_H(R - C_L)(R - C_H) + F_L(R - C_H) - F_H(R - C_L)} \geq \gamma > \frac{S_H(R - C_L) - F_L}{S_H(R - C_L) + F_L}$$

v)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p \geq \hat{\hat{p}} \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - \hat{\hat{p}}}{\hat{\hat{p}} - C_L}\right] & \hat{\hat{p}} > p \geq L \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\cdot\dfrac{F_H}{F_L}\,\dfrac{(R - C_L)}{(p + H - L - C_H)} & L > p \geq \tilde{p}_5 \\ 0 & \tilde{p}_5 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p > H \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-p+H-L}{p-H+L-C_L} - \dfrac{F_H(R - C_L)}{F_L(p - C_H)}\right] & H \geq p \geq \tilde{p}_5+H-L \\ 1 - \dfrac{1-\gamma}{2\gamma(1-n_L)}\left[\dfrac{R-\tilde{p}_5+H-L}{\tilde{p}_5-H+L-C_L} - \dfrac{F_H(R - C_L)}{F_L(\tilde{p}_5 - C_H)}\right] & \tilde{p}_5+H-L > p \geq p_H^* \\ 0 & p < p_H^* \end{cases}$$

$$\text{where } n_L = \frac{1-\gamma}{2\gamma} \cdot \frac{F_H}{F_L} \cdot \frac{R - C_L}{(\tilde{p}_5 + H - L - C_H)}$$

$$\tilde{p}_5 = C_L + \frac{F_L(1-\gamma)(R - C_L)}{2S_H(1-\gamma)(R - C_L) - (1+\gamma)F_L}$$

$$\frac{2S_H(L - C_L)(R - C_L) - (L - C_L)F_L - (R - C_L)F_L}{2S_H(L - C_L)(R - C_L) + (L - C_L)F_L - (R - C_L)F_L} \geq \gamma$$

$$> \frac{2S_H(R - C_L)(R - C_H) - F_L(R - C_H) - F_H(R - C_L)}{2S_H(R - C_L)(R - C_H) + F_L(R - C_H) - F_H(R - C_L)}$$

vi)

$$G_L(p) = \begin{cases} 1 & p > R \\ 1 - \dfrac{1-\gamma}{2\gamma n_L}\left[\dfrac{R - p}{p - C_L}\right] & R \geq p \geq \tilde{p}_6 \\ 0 & \tilde{p}_6 > p \end{cases}$$

$$G_H(p) = \begin{cases} 1 & p \geq p_H^* \\ 0 & p < p_H^* \end{cases}$$

$$n_L = \frac{1}{\gamma}\left[\frac{S_H(R - C_L)(1-\gamma) - F_L}{F_L}\right]$$

$$\tilde{p}_6 = C_L + \frac{(1-\gamma)(R - C_L)F_L}{2(1-\gamma)(R - C_L)S_H - (1+\gamma)F_L}$$

$$\frac{S_H(R - C_L) - F_L}{S_H(R - C_L)} > \gamma > \frac{2S_H(1-C_L)(R-C_L) - (L-C_L)F_L - (R-C_L)F_L}{2S_H(L-C_L)(R-C_L) + (L-C_L)F_L - (R-C_L)F_L}$$

<u>Proof</u>: i) This case is similar to case i) in all the previous theorems. What is interesting here is that $\tilde{p}_1 \geq \hat{\hat{p}}$, where $\hat{\hat{p}}$ is the lowest price in the low quality market at which firms may exist, without inducing entry at (H, q_H). Thus $\hat{\hat{p}}$ solves

$$\frac{F_L}{p - C_L} - \frac{F_L}{R - C_L} = \frac{F_H}{R - C_H}.$$

$$\hat{\hat{p}} = C_L + \frac{F_L(R - C_H)(R - C_L)}{F_H(R - C_L) + F_L(R - C_H)}.$$

ii) In order that zero profit be made on the support of the distribution for $p \in (R - H + L, L)$

$$D(p, q_L) = \alpha(1-\gamma) + 2\alpha\gamma n_L(1-G_L(p)) + 2\alpha\gamma n_H(1-G_H(p-H+L)) = \frac{F_L}{p - C_L}$$

$$D(p+H-L, q_H) = 2\alpha\gamma n_L(1 - G_L(p)) = \frac{F_H}{p + H - L - C_H}.$$

$$\text{Thus } G_L(p) = 1 - \frac{1}{2\alpha\gamma n_L} \frac{F_H}{p + H - L - C_H} = 1 - \frac{1-\gamma}{2\gamma n_L} \frac{F_H}{F_L} \frac{R - C_L}{p + H - L - C_H}$$

as in Theorem 4 case ii. Also as in that theorem,

$$G_H(p) = 1 - \frac{1 - \gamma}{2\gamma(1-n_L)} \left[\frac{R - p + H - L}{p - H + L - C_L} - \frac{F_H}{F_L} \frac{(R - C_L)}{p - C_H}\right].$$

The value of $\tilde{p}_2$ is computed in an identical fashion and the upper bound on γ is generated by requiring that $\tilde{p}_2 \geq \hat{p}$, which is also the condition required in Theorem 4 case ii).

iii) This case is identical to Theorem 4 case iii).

iv) This is identical to Theorem 4 case iv).

v) Similarly this is identical to Theorem 4 case v). The only difference is that here we need $\tilde{p}_5 \leq L$ while there we need $\tilde{p}_5 \leq p^{**}$.

vi) Again, the arguments are presented in Theorem 4 case vi). The only difference is that $\tilde{p}_6 \geq \hat{\hat{p}}$.

Q.E.D.

This completes the formal proofs associated with the basic model in Section 2. We will now present the formal arguments associated with the extensions of the basic model discussed in Section 3. We begin with the first model in Section 3, where there is some learning through shopping and there are some naturally informed consumers. Next we will examine the final model discussed in Section 3. In each case, the assumptions of the models will not be restated here as they are identical to the assumptions made in Section 3.

<u>Lemma 7</u>. Competitive equilibrium at (p_H^*, q_H) obtains if and only if each of the following holds:

i) $\alpha(1 - \gamma) \leq \dfrac{F_L}{L - C_L}$

ii) $\alpha(1 - \gamma) \leq \dfrac{F_H}{R - C_H}$

iii) $\alpha\beta(1 - \gamma) \leq \dfrac{F_H}{H - C_H}$

iv) $\alpha(1 - \beta)(1 - \gamma) \leq \dfrac{F_L}{R - C_L}$

where $\alpha = S_H$.

Proof: Suppose all firms are at (p_H^*, q_H). This would be an equilibrium as long as no firm could deviate to any point in the feasible set and make a positive profit. Clearly condition i) guarantees that this is not possible for all prices below L in the low quality market. Similarly, condition ii) ensures this for $p \leq R$ in the high quality market, condition iii) for $p \in (R,H)$ in the high quality market and condition iv) for $p \in (L,R)$ in the low quality market.

Q.E.D.

Lemma 8. Define

$$\bar{\beta} = \min\{ \frac{F_H(R - C_L)}{F_H(R - C_L) - F_L(H - C_H)}, \frac{F_H(R - C_L) - R_L(R - C_H)}{R_H(R - C_L)}, \frac{R - L}{R - C_L} \}.$$

For $\beta < \bar{\beta}$, every non-competitive equilibrium contains (R, q_L), and there are no firms in the low quality market at prices between L and $C_L + \dfrac{L - C_L}{1 - \beta}$.

Proof: If we observe conditions i) to iv) of Lemma 7, we find that for any β less than $\bar{\beta}$, condition iv) is binding and the other three are slack. As in the previous model (and by the very same reasoning), (p_H^*, q_H) is the only competitive equilibrium, and if any of the conditions of Lemma 7 do not hold then a non-competitive equilibrium obtains. Since iv) is the strictest condition when $\beta < \bar{\beta}$, for every non-competitive equilibrium, condition iv) is violated. Thus (R, q_L) is in the support of the equilibrium distribution. Finally as we approach from above, there is a discontinuous increase in demand at (L, q_L) due to the entry of the informed consumers, who refuse to buy at prices above L. Thus there is an interval immediately above L where no firms may exist. The argument is identical to the one in Lemma 6, and clearly the interval is (L, p) where p satisfies $z(L, q_L) = z(p, q_L)/(1 - \beta)$. The solution is $p = C_L + \frac{L - C_L}{1 - \beta}$.

Q.E.D.

<u>Lemma 9</u>. Suppose the comparative advantage at limit prices lies in the low quality market. For β greater than $\frac{R - L}{R - C_L}$ there are no firms at prices above L in the low quality market, in equilibrium.

<u>Proof</u>: We argue by contradiction. Suppose $\beta > \frac{R - L}{R - C_L}$ and there were firms at prices above L in the low quality market, in equilibrium. Then as we have argued before, the firm at the highest price would be at (R, q_L). Since this is an equilibrium,

$$D(R, q_L) = (1 - \beta)(1 - \gamma)\alpha = \frac{F_L}{R - C_L)}$$

and thus $\alpha = \frac{F_L}{(R - C_L)(1 - \beta)(1 - \gamma)}$. Now, consider (L, q_L).

$$D(L, q_L) = \alpha(1 - \gamma) + 2\alpha\gamma(1 - G_L(L))$$

$$\geq \alpha(1 - \gamma)$$

$$= \frac{F_L}{(R - C_L)(1 - \beta)}$$

$$> \frac{F_L}{(R - C_L)[1 - \frac{R - L}{R - C_L}]}$$

$$= \frac{F_L}{F - C_L} = z(L, q_L) \ .$$

This contradicts that the distribution was an equilibrium.

Q.E.D.

The proofs of the conditions governing the cases that occur when the comparative advantage at limit prices lies in the high quality market use the same arguments that are used in Lemma 9. In equilibrium, positive profit cannot be made anywhere in the feasible set, and exactly zero profit can be made on the support of the distribution. Instead of proving each of the six cases separately, we shall discuss what creates the differences in each case.

The most important aspect here, is which firm sells only to the non-shoppers. There are four candidates: (R, q_L), (L, q_L), (H, q_H) and (R, q_L). In each hypothetical case, α would be $\frac{F_L}{(R - C_L)(1 - \beta)(1 - \gamma)}$, $\frac{R_L}{(L - C_L)(1 - \gamma)}$, $\frac{F_H}{\beta(H - C_H)(1 - \gamma)}$

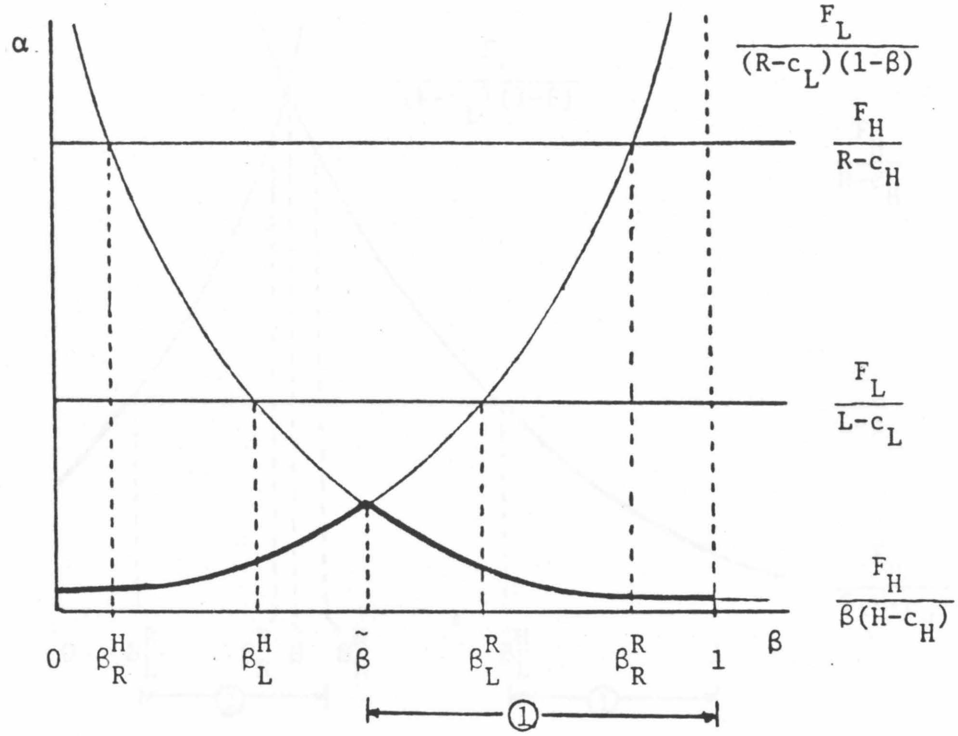


FIGURE 11

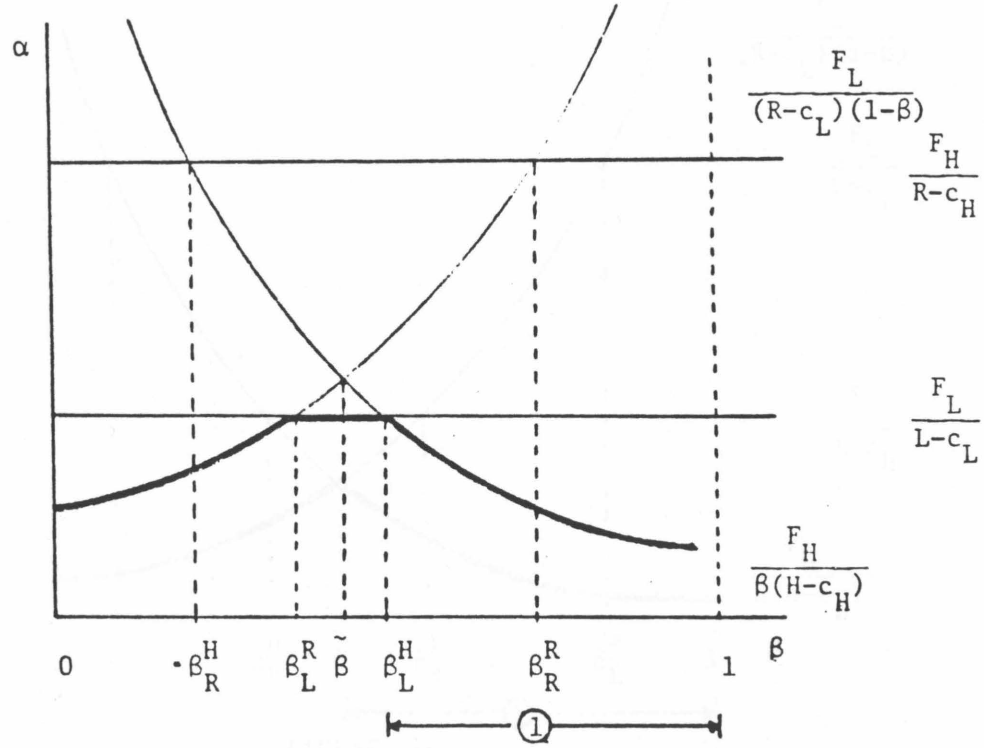


FIGURE 12

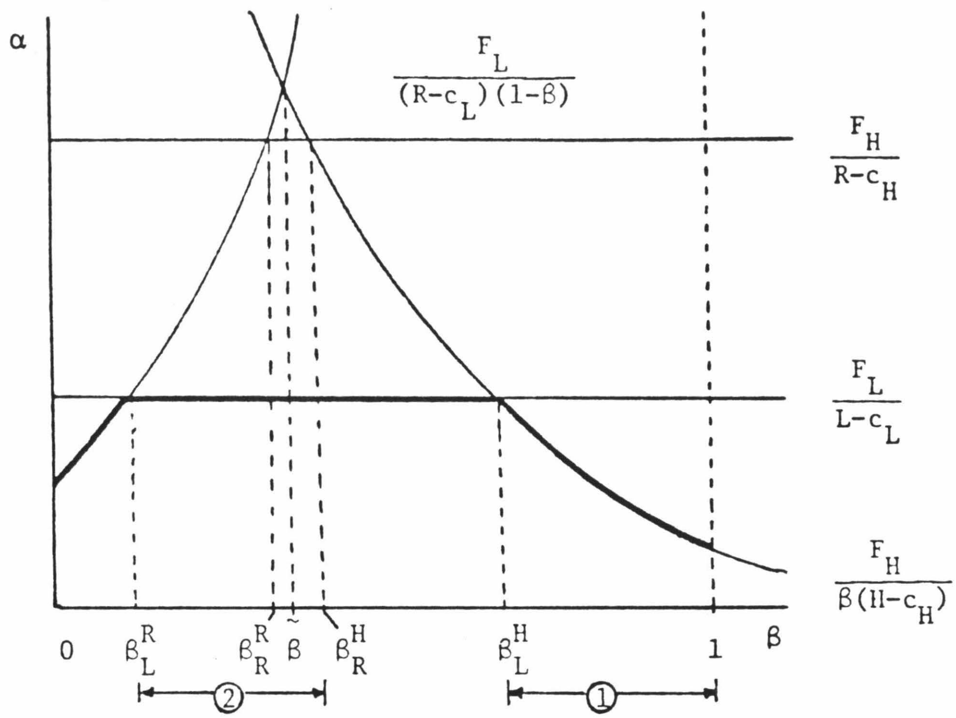


FIGURE 13

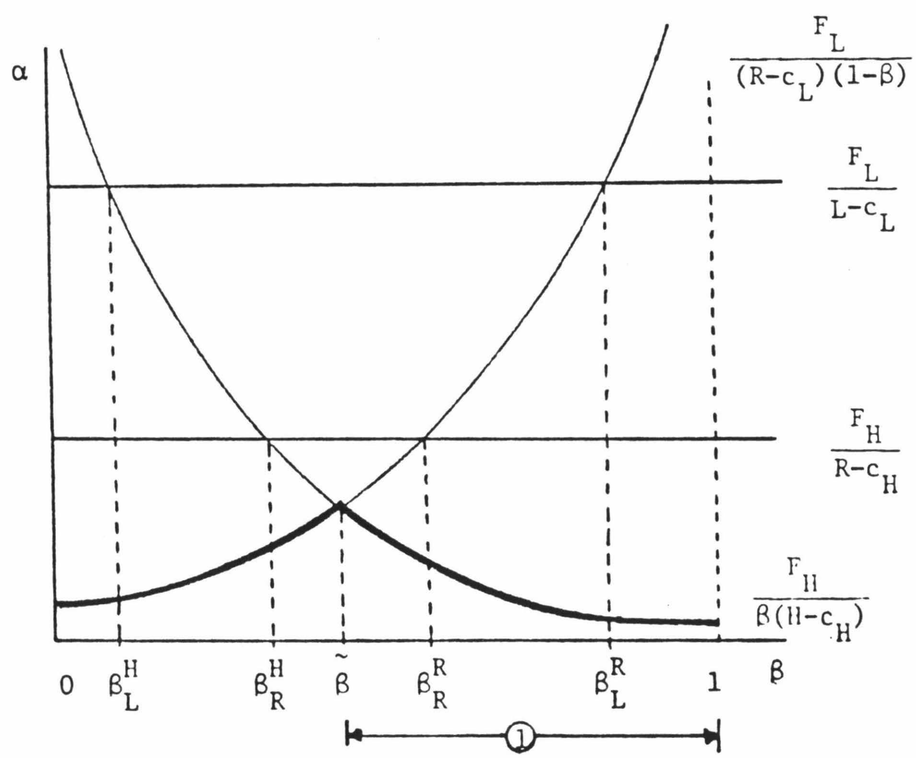


FIGURE 14

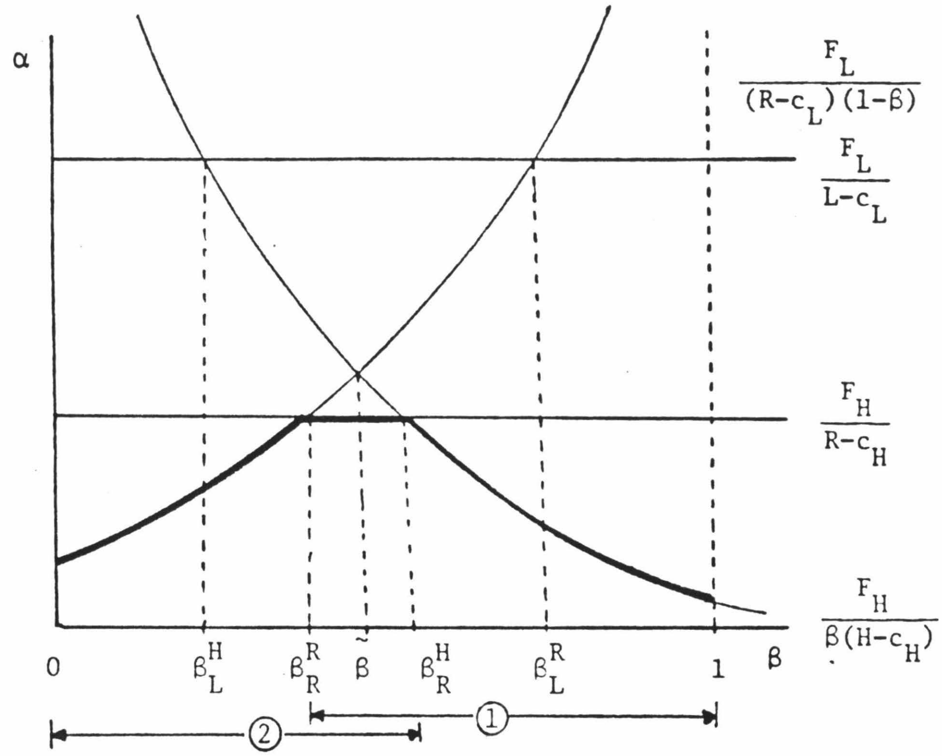


FIGURE 15

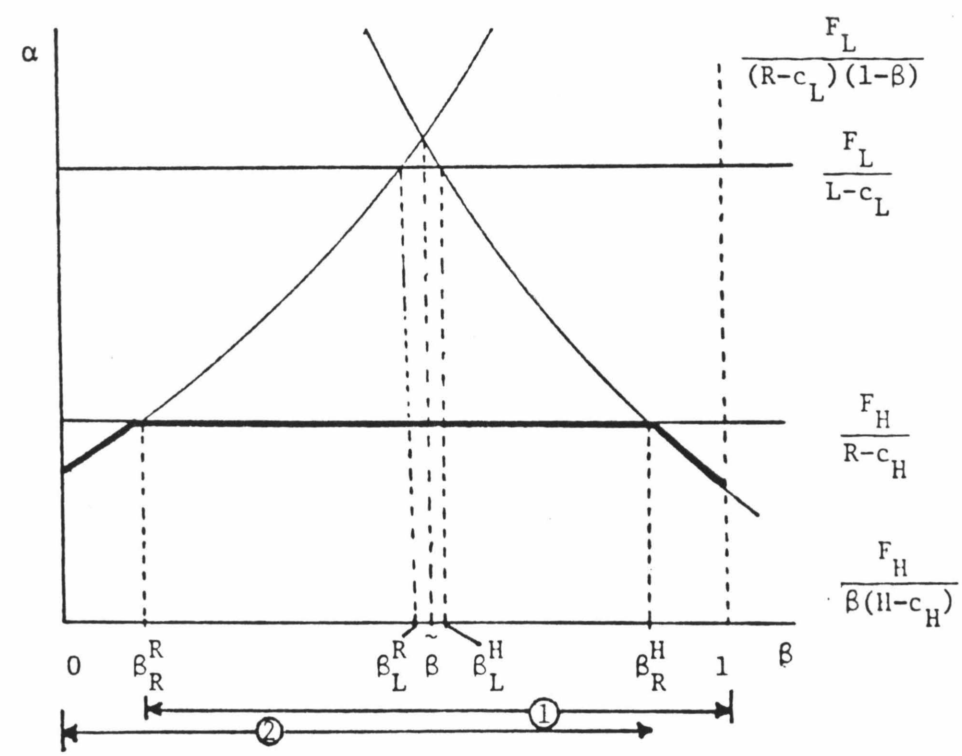


FIGURE 16

and $\frac{F_H}{(R - C_H)(1 - \gamma)}$, respectively. These are illustrated in Figures 11 through 16 (without the factor of $\frac{1}{1 - \gamma}$. In order to satisfy the conditions for equilibrium, the correct 2 would be the minimum of the four candidates, and that in turn determines which firm sells to non-shoppers only. If we observe the figures we see that there are three possible routes: i) (R, q_L) to (H, q_H), ii) (R, q_L) to (L, q_L) to (H, q_H) and iii) (R, q_L) to (R, q_H) to (H, q_H).

Another important aspect is the changes in the gaps in the equilibrium distribution. At (L, q_L) there is a discontinuity in demand due to the entry of the informed consumers and thus there is a gap in the equilibrium distribution for an interval just above L. We can show that this gap increases as β increases. Suppose initially β is small enough so that the firm for which the demand is due to non-shoppers alone is (R, q_L). The lowest price above L where firms can exist satisfies $z(p, q_L)/(1 - \beta) = z(L, q_L)$. This follows from arguments made previously when discontinuities of demand were encountered. Thus, this price is $\frac{L - C_L}{1 - \beta} + C_L$ which is clearly increasing in β. There is a critical β above which this price exceeds R and so there are no firms above L in the low quality market. It follows that this critical β will be at the intersection of $\frac{F_1}{(R - C_L)(1 - \beta)}$ and $\frac{F_L}{L - C_L}$ which is denoted by β_L^R. Thus for $\beta > \beta_L^R$ there will be no firms above L in the low quality market.

Similarly, it can be shown that the gap just above R in the high quality market, due to the uninformed consumers refusing to buy at prices above R, reduces as the consumers become more informed. The gap vanishes completely only at $\beta = 1$, that is, then the discontinuity in demand at (R, q_H) ceases to exist.

Finally, the maximum price at which firms can exist in the high quality market increases with β and the minimum price at which firms can exist in the low quality market eventually increases with β. Let us discuss the first claim. For $p \varepsilon (R - H + L, L]$, in equilibrium

$$D(p,q_L) = \alpha(-\gamma) + 2\alpha\gamma[n_L(1-G_L(p) + n_H(1-G_H(p+H-L))] = \frac{F_L}{p-C_L}$$

$$D(p+H-L),q_1) = \beta\alpha(1-\gamma) + 2\alpha\gamma[n_L(1-G_L(p)) + \beta n_H(1-G_H(p+H-L))] = \frac{F_H}{p+H-L-C_H}\ .$$

Thus, at the highest price in the high quality market, $p^{**} + H - L$, since $G_H(p^{**} + H - L) = 0$, we have

$$D(p^{**}+H-L,q_H) = \frac{F_H}{p^{**}+H-L-C_H} = D(p^{**},q_L)-(1-\beta)\alpha(1-\gamma) = \frac{F_L}{p^{**}-C_L} - (1-\beta)\alpha(1-\gamma).$$

Clearly the price p^{**} which solves

$\dfrac{F_H}{p^{**} + H - L - C_H} = \dfrac{F_L}{p^{**} - C_L} - (1 - \beta)\alpha(1 - \gamma)$ is increasing in β when the comparative advantage at limit prices lies in the high quality market.

Similarly, it can be shown that the minimum price at which firms can exist in the low quality makret eventually increases with β.

Thus, we have examined how changes in β affect three important aspects of the distribution: i) the firm which sells to the non-shoppers alone, ii) the gaps in the distribution and iii) the maximum price in the high quality market and the minimum price in the low quality market. This gives us an intuitive idea of how the maximum support of equilibrium distribution behaves with the comparative advantage at limit prices, in the high quality market.

The discussion that follows concerns the last model discussed in Section 3. The purpose here is not to provide complete rigorous proofs of all the equilibria but instead demonstrate how badly behaved the market can be even when all consumers are shopping.

Lemma 10. The only competitive equilibria are either all firms at (p_L^*, q_L) or (p_H^*, q_H). The necessary and sufficient condition for the competitive equilibrium at (p_L^*, q_L) is

$$\beta \leq \frac{1}{2} \cdot \frac{F}{F + SK_\theta},$$

and the necessary and sufficient condition for the equilibrium (p_H^*, q_H) is

$$\beta \geq 1 - \frac{1}{2} \cdot \frac{F}{F + SK_c} .$$

Proof: Suppose all the firms are located at (p_L^*, q_L) and $\alpha = S$. For this to be an equilibrium, demand cannot exceed break even demand anywhere on the feasible set. At prices greater than p_L^* in the low quality market, and prices above $p_L^* + H - L$ in the high quality market, there is no demand since all shoppers prefer to buy at (p_L^*, q_L). At prices below $p_L^* + H - L$ in the high quality market, the demand is due to the informed consumers. Thus, for p between p_H^* and $p_L^* + H - L$

$$D(p, q_H) = 2\alpha\beta \lesssim Z(p, q_H)$$

in equilibrium. The strongest condition occurs at prices just below $p_L^* + H - L$. Thus, there is a competitive equilibrium at (p_L^*, q_L) if and only if

$$\beta \lesssim \frac{1}{2S} \cdot \frac{F}{\frac{F}{S} + c_L + H - L - c_H} = \frac{1}{2} \cdot \frac{F}{F + SK_\theta}.$$

Similarly, the competitive equilibrium at (p_H^*, q_H) occurs as long as

$$D(p, q_L) = 2\alpha(1 - \beta) \lesssim Z(p, q_L)$$

for p between p_L^* and p_H^*. Since this condition is strongest at prices just below p_H^*, the necessary and sufficient condition for a competitive equilibrium at (p_H^*, q_H) is

$$\beta \gtrsim 1 - \frac{1}{2} \cdot \frac{F}{F + S(c_H - c_L)} = 1 - \frac{1}{2} \cdot \frac{F}{F + SK_c}$$

It is simple to show that there are no other competitive equilibria. Suppose there was a competitive equilibrium with mass m, $0 < m < 1$ at (p_L^*, q_L) and the complementary mass at (p_H^*, q_H). Then

$$D(p_L^*, q_L) = 2\alpha(\frac{m}{2} + (1 - \beta)(1 - m)) = S$$
$$D(p_H^*, q_H) = 2\alpha(\frac{1 - m}{2} + \beta m) = S$$

For both equalities to hold simultaneously, we need $\beta = \frac{1}{2}$, which gives us $\alpha = S$. Furthermore, since in equilibrium non-positive profit must be earned anywhere in the feasible set,

$$D(p, q_L) = 2\alpha\beta m \leq Z(p, q_L) \ , \text{ for } p_L^* < p < p_H^*$$
$$D(p, q_H) = 2\alpha(1 - \beta)(1 - m) \leq Z(p, q_H) \ , \text{ for } p_H^* < p < p_L^* + H - L.$$

These inequalities are equivalent to

$$\beta \leq \frac{1}{2m} \frac{F}{F + SK_0} < \frac{1}{2}$$

and $$\beta \geq \frac{1}{2(1 - m)} \frac{F}{F + SK_c} > \frac{1}{2}.$$

Clearly, β cannot simultaneously satisfy all these conditions and so there are no such equilibria.

Q.E.D.

We will discuss the non-competitive equilibria informally, as the main objective in presenting this model is to demonstrate how badly behaved the market equilibria are even when everyone is shopping. We classify the equilibria according to complexity, and find that there are two simple equilibria with competitive pricing in one market and price dispersion in the other market. Finally there are equilibria with dispersion in both markets. In what follows we discuss two simple non-competitive equilibria and discuss the conditions under which the more complicated equilibria arise.

Let us begin with the equilibrium with mass at (p_H^*, q_H) and price dispersion in the low quality market. Since there is a mass point at (p_H^*, q_H) there will be a discontinuous increase in demand as we move from prices above p_H^* to p_H^* in the low quality market and another discontinuous increase in demand as we move to prices below p_H^* in the low quality market. Since demand cannot exceed the break even demand in equilibrium, and break even demand is continuous in prices, we cannot have any firms in the low quality market at prices at or above p_H^*. Thus the support of the distribution lies within p_L^* and p_H^* in the low quality market.

The demand at the highest price in the low quality market is $\alpha(1 - \beta)n_H$ (where n_H is the size of the mass at (p_H^*, q_H)) as long as the price is less than p_H^*. Thus firms can exist at prices just below p_H^*.

In equilibrium demand equals break even demand on the support of the distribution. We will impose this condition at (p_H^*, q_H), (p_L^*, q_L) and (p_L, q_L), where p_L is just below p_H^* to solve for the parameters of the distributions. Then we can define the distribution $G_L(p)$ between p_L^* and p_L such that demand equals break even demand. Define $\hat{G}_L$ to be the size of the mass point (if any) at (p_L^*, q_L).

Then,

$$D(p_H^*, q_H) = 2\alpha(\beta(n_L + \frac{n_H}{2})) + (1 - \beta)\frac{n_H}{2}) = S$$

$$D(p_L^*, q_L) = 2\alpha(\beta n_L(1 - \frac{1}{2}\hat{G}_L) + (1 - \beta)(n_L(1 - \frac{1}{2}\hat{G}_L) + n_H)) = S$$

$$D(p_L, q_L) = 2\alpha(1 - \beta)n_H = S\frac{F}{F + S(c_H - c_L)}.$$

We can solve these three equations simultaneously using the fact that $n_L + n_H = 1$.

$$n_L = 1 - \frac{2\beta F}{2(F + SK_c)(1 - \beta) + F(2\beta - 1)}$$

$$\alpha = \frac{S[2(F + SK_c)(1 - \beta) + F(1\beta - 1)]}{4\beta(1 - \beta)(F + SK_c)}$$

$$\hat{G}_L = 2\left[1 - \frac{2\beta(1 - \beta)SK_c}{2(F + SK_c)(1 - \beta) - F}\right].$$

Now we can define $G_L(p)$ for $p_L^* < p < p_H^*$.

$$D(p, q_L) = 2\alpha\{n_L(1 - G_L(p)) + (1 - \beta)n_H\} = \frac{F}{p - c_L}$$

Thus,

$$G_L(p) = 1 - \frac{1}{2\alpha n_L}\left[\frac{F}{p - c_L} - 2\alpha(1 - \beta)(1 - n_L)\right].$$

In order that this be a non-competitive equilibrium we need that $0 < n_L < 1$, $0 \leq \hat{G}_L < 1$. If $\hat{G}_H = 0$ then $G_L(p)$ is the above expression whenever it is greater than zero and $G_L(p) = 0$ otherwise. We define $\tilde{p}_L$ to be the highest price such that $G_L(\tilde{p}_L) = 0$. If $\hat{G}_L \neq 0$ we need to find $\tilde{p}_L$ such that $G_H(\tilde{p}) = \hat{G}_L$. Then, we define $G_L(p)$ to be the function given above for $\tilde{p}_L \leq p < p_H^*$ and $G_L(p) = \hat{G}_L$ for $p_L^* \leq p < \tilde{p}_L$.

It is straightforward to show that the condition that $n_L < 1$ is always satisfied. The condition that $0 < n_L$ is equivalent to

$$\beta < 1 - \frac{1}{2}\frac{F}{F + SK_c}.$$

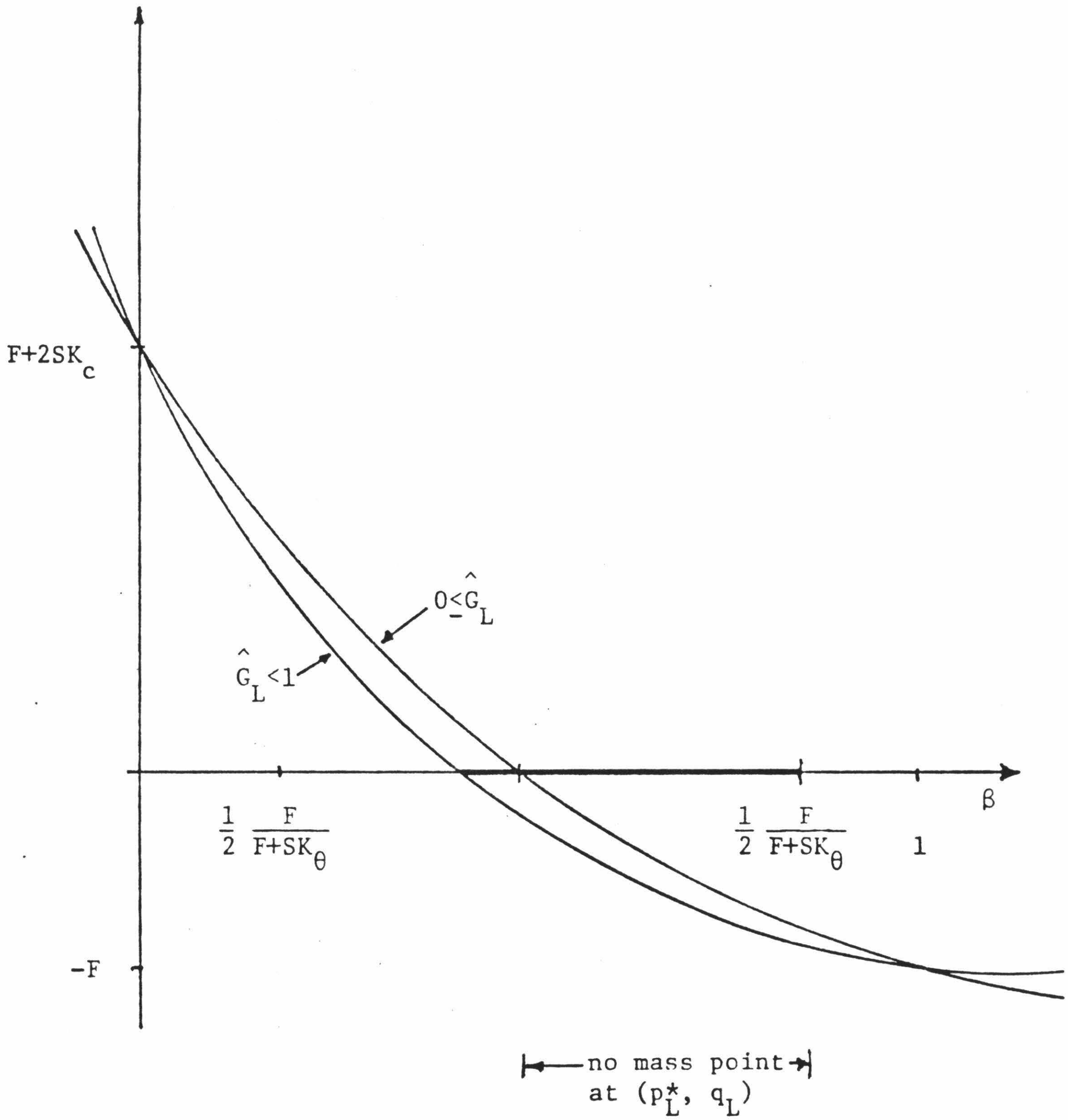
Finally, $0 \leq \hat{G}_L$ and $\hat{G}_L < 1$ give us the following quadratic conditions in β, respectively

$$0 \leq 2SK_c\beta^2 - 2[F + 2SK_c]\beta + F + 2SK_c$$

$$0 < 4SK_c\beta^2 - 2[F + 3SK_c]\beta + F + 2SK_c.$$

These parabolas are illustrated in Figure 17.

If β satisfies these conditions, then the conditions for equilibrium are satisfied for all prices below p_H^* in both markets. The last condition to check is that no firms can enter at prices above p_H^* in either market. Clearly no one will buy at prices above p_H^* in the low quality market and no one will buy at prices above $p_H^* + H - L$ in the high quality market. Thus there are four regions on which we



Range of β for Competitive Price in High Quality Market and Non-competitive Prices in Low Quality Market

FIGURE 17

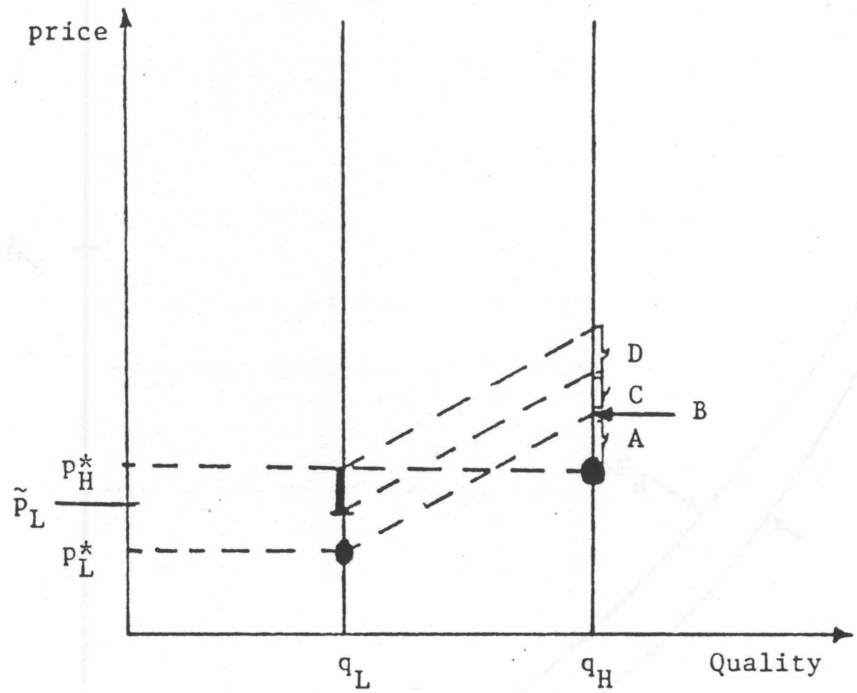
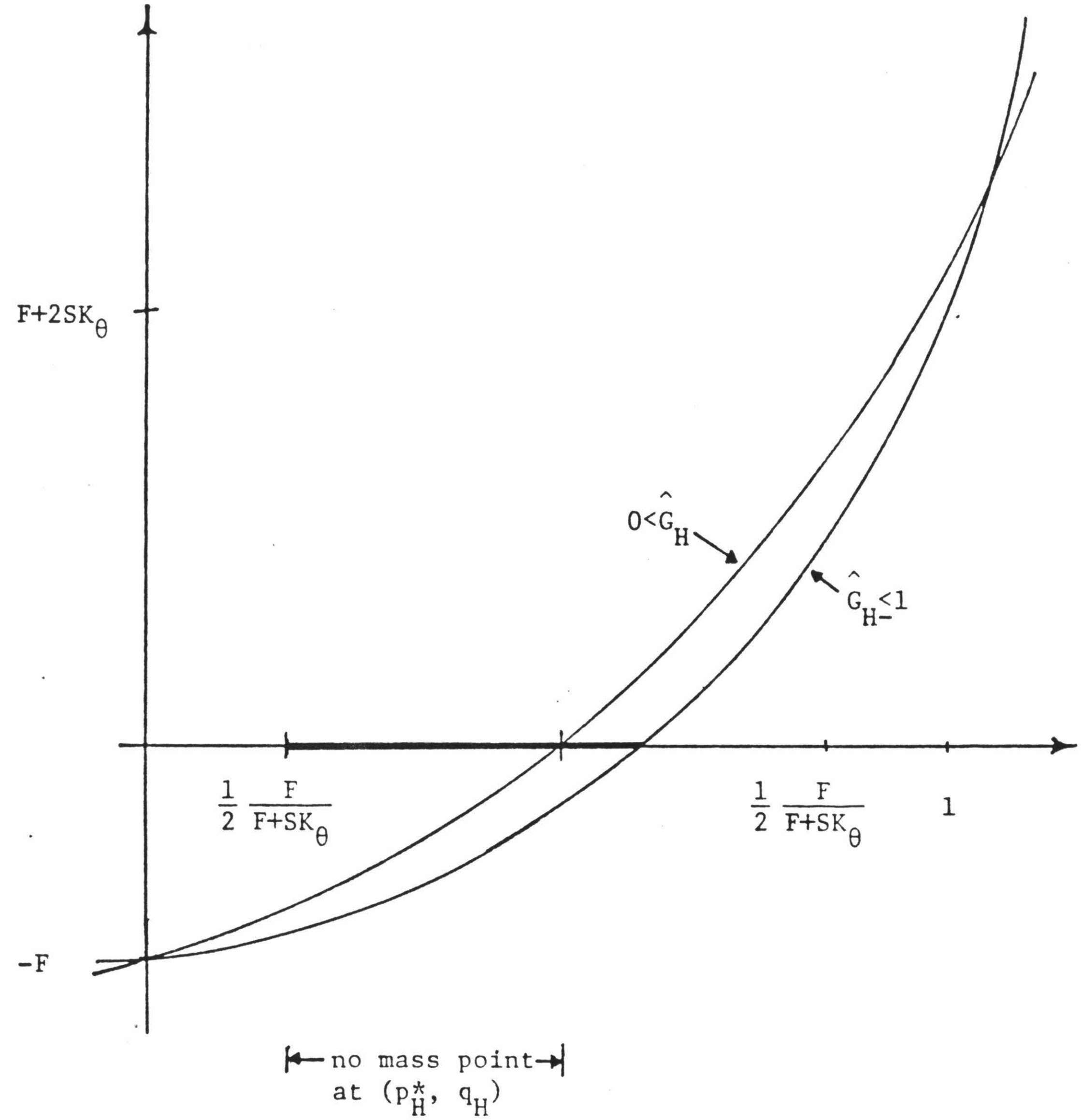


FIGURE 18

●, ▬▬▬ denotes 'maximum support' of the distribution.



Range of β for Competitive Price in Low Quality Market
and Non-competitive Prices in High Quality Market

FIGURE 19

must check whether positive profit can be earned. For the case when there is a mass point at (p_L^*, q_L), the four regions $p_H^* < p < p_L^* + H - L$, $p = p_L^* + H - L$, $p_L^* + H - L < p < \tilde{p}_L + H - L$ and $\tilde{p}_L + H - L \leq p < p_H^* + H - L$ are shown in Figure 18 as regions A, B, C, and D respectively. When there is no mass point, there are only two regions: A, $p_H^* < p < \tilde{p}_L$, and D, $\tilde{p}_L \leq p < p_H^* + H - L$.

Notice that in regions A and C, demand is constant, but break even demand is smallest at the upper end point. Thus, for these regions, it is enough to check that positive profit cannot be made at the end points. For region C this point is $(\tilde{p}_L + H - L, q_H)$ which is in region D. For region A, since demand jumps discontinuous at B, a limiting argument indicates that we should compare the demand in region A with the break even demand at B, to generate the strongest condition. Clearly if this condition is satisfied then positive profit cannot be earned at B also.

Thus,

$$D(\text{region A}) = 2\alpha\beta n_L < \frac{F}{p_L^* + H - L - c_H}.$$

Substituting for α and n_L and solving for β we find

$$1 - \frac{F(F + SK_\theta)}{2(F + SK_c)SK_\theta} < \beta$$

From the assumptions of the model, it is not clear whether this is stronger than $\hat{G}_L < 1$.

Finally we must determine where the strongest condition is imposed in region D. For $\tilde{p}_L < p < p_H^*$,

$$D(p, q_L) = 2\alpha n_L(1 - G_L(p)) + 2\alpha(1 - \beta)(1 - n_L) = \frac{F}{p - c_L}.$$

In equilibrium we need that

$$D(p + H - L, q_H) = 2\alpha\beta n_L(1 - G_L(p)) \leq \frac{F}{p + H - L - c_H}$$

Now $2\alpha(1 - \beta)(1 - n_L) = \frac{SF}{F + SK_c}$, and thus we need that

$$\beta \quad \frac{1}{p - c_L} - \frac{S}{F + SK_c} \quad < \frac{1}{p + H - L - c_h},$$

$$\text{or } \beta \leq \frac{(p - c_L)(F + SK_c)}{(p + H - L - c_H)(F + Sc_H - S_p)}$$

Clearly the right hand side is increasing in p and thus the strongest condition will be imposed at $p = \tilde{p}_L$. This condition is quite complicated but it is always weaker than $\beta \leq 1 - \frac{1}{2F + SK_c}\frac{F}{}$ whenever $p_L^* > c_H$.

A similar analysis for the equilibrium with competitive price in the low quality market and price dispersion in the high quality market gives us the following distribution

$$n_H \quad = 1 - \frac{2(1 - \beta)F}{2(F + SK_\theta)\beta - F(2\beta - 1)}$$

$$\alpha = \frac{S[2(F + SK_\theta)\beta - F(2\beta - 1)]}{4\beta(1 - \beta)(F + SK_\theta)}$$

$$\hat{G}_H = 2 \quad 1 - \frac{2\beta(1 - \beta)SK_\theta}{2(F + SK_\theta)\beta - F}$$

$$G_H(p) = 1 - \frac{1}{2\alpha n_H} \; \frac{F}{p - c_H} - 2\alpha\beta(1 - n_H)$$

If $\hat{G}_H = 0$ then $G_H(p)$ is the above expression whenever it is greater than zero, and zero otherwise. We define $\tilde{p}_H$ such that $G_H(p) = 0$. If $\hat{G}_H > 0$ then we define $\tilde{p}_H$ to be the price at which $G_H(\tilde{p}_H) = \hat{G}_H$. Then we define $G_H(p)$ to be the above expression for $\tilde{p}_H \leq p < p_L^* + H - L$ and $G_H(p) = \hat{G}_H$ for $p_H^* \leq p < \tilde{p}_H$. The conditions $0 < n_H$, $0 \leq \hat{G}_H$ and $\hat{G}_H < 1$ are equivalent to

$$\beta \geq \frac{1}{2} \frac{F}{F + SK_\theta}$$

$$0 \leq 2\beta^2 SK_\theta + 2\beta F - F$$

and

$$0 > 4\beta^2 SK_\theta + 2\beta(F - SK_\theta) - F \text{ ,respectively.}$$

These conditions are illustrated in Figure 19. The condition that $n_H < 1$ is always satisfied.

Finally the condition that non-positive profit be earned for prices above $p_L^* + H - L$ in the high quality market and above p_L^* in the low quality market gives the following conditions on β.

$$\beta < \frac{F(F + SK_c)}{2(F + SK_\theta)SK_c}$$

$$1 - \beta \leq \frac{(p - c_H)}{(p - c_L)} \frac{(F + SK_\theta)}{(F + SK_\theta - S(p - c_H))}$$

Again the right hand side is increasing in p and thus the strongest condition is at $p = \tilde{p}_H$ and is quite complicated. It is always weaker than $\beta \geq \frac{1}{2} \frac{F}{F + SK_\theta}$ if $p_H^* - H + L > c_L$.

The final type of non-competitive equilibrium is where there is price dispersion in both quality levels. The first equilibrium of this sort has a maximum support of (p_L^*, p_H^*) in the low quality market and $(p_H^*, p_L^* + H - L)$ in the high quality market. Clearly it will exist on values of β where entry is possible in this price range; these are exactly those values of β where the previous equilibrium does not exist. There may or may not be mass points at (p_L^*, q_L) and (p_H^*, q_H) depending on β. Again there will be some stability conditions, that is conditions requiring that non-positive profit be earned off the maximum support. If these conditions are binding, then there is another non-competitive equilibrium with dispersion in both quality levels. The difference is that the maximum support is larger. It may include, in addition to the previous maximum support, $(p_H^*, p_L^* + H - L)$ in the low quality market or $(p_L^* + H - L, p_H^* + H - L)$

in the high quality market or both, depending on which of the stability conditions is binding. Notice that in this equilibrium we cannot have a mass point at (p_L^*, q_L) if $(p_L^* + H - L, p_H^* + H - L)$ in the high quality market is in the support, and we cannot have a mass point at (p_H^*, q_H) if $(p_H^*, p_L^* + H - L)$ in the low quality market is in the support of the distribution, since the mass points cause discontinuities in demand in the other market and the support cannot include points of discontinuity of demand in its interior. Clearly this equilibrium will also have stability conditions and if they are binding we will have to find still another non-competitive equilibrium with a larger support. This process will go on until the range of β is exhausted.

Note also that if p_L is the maximum price in the low quality market and p_H is the maximum price in the high quality market, then we must have that $p_L < p_H < p_L + H - L$. If $p_L > p_H$ then for $p \in (p_L, p_H)$ in the low quality market, the demand is zero similarly, there is no demand at prices between p_H and $p_L + H - L$ when $p_H > p_L + H - L$.