RADIATIVE MASS STRUCTURE IN UNIFIED MODELS AND FERMIONS IN THE DESERT

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ABSTRACT

The radiative mass structure of some Grand Unified Models is discussed. They contain fermions with SU(2)-invariant masses in the desert. The possibility of such fermions is examined in detail with the conclusion that their mass can be low enough ($\sim$ 20 GeV) to be found in accelerators today. The mixing of such fermions with ordinary fermions is analysed and their contribution to rare processes calculated. They do not upset standard GUT predictions. Finally an analysis of their contribution to the $\mu \rightarrow e\gamma$ rate is an interesting illustration of decoupling.

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I. INTRODUCTION

The observables of classical physics -- mass, energy, space, time and temperature -- can today be reduced to one dimensionful observable and a set of numbers -- the so-called "universal constants" [1]. Statistical mechanics relates temperature to energy per degree of freedom via Boltzmann's constant k_B. Special Relativity relates energy to mass and space to time via the speed of light c, and Quantum mechanics fixes the time development of an energy eigenstate. We are left with one quantity -- call it mass -- in terms of which the others are related by these universal constants. But can we determine this mass from first principles? Not yet. The mass spectrum of elementary particles is still unexplained. There is no theory that predicts the mass of each particle in terms of a dimensionless number and a fundamental mass scale. However there are a number of approximate relations among particle masses. In the past these were explained by assuming a hierarchy of interactions with the stronger interactions having the greater symmetry. This view is inconsistent with the modern notion of a renormalizable gauge theory because the explicit breaking of the gauge symmetry destroys renormalizability.

Can we understand these mass relations with present quantum field theories? Consider the successful quantum field theory of the electromagnetic interaction, Quantum Electrodynamics, QED [2]. The electromagnetic mass of the photon is zero because of the exact gauge symmetry of local phase transformations $U^{\gamma}(1)$. The electromagnetic mass of the electron is infinite. This infinity must be cancelled by a counterterm. We can renormalize the bare mass of the electron such that the resultant "physical" mass is finite. The electron mass is then a free parameter of the theory. Making sense of QED has prevented us from predicting the electron mass.

However, renormalizable theories with spontaneously broken gauge symmetries allow a different possibility. Suppose that a field has zero mass to lowest order in perturbation theory (tree level), i.e. the Lagrangian does not contain a mass term $m\phi^{*}\phi$ for the field ϕ. Since the theory is renormalizable there cannot be a counterterm for this mass. Thus higher order (radiative loop) corrections to the mass must be finite. We call the mass calculable. The hope in this case is that approximate symmetries are calculable corrections to tree level symmetries.

To classify fermion masses [3] we use the standard model. At energies above ~ 250 GeV we believe physics is described by an effective Lagrangian with the (semi-simple) gauge group $SU_3^C \times SU_2^W \times U_1^Y$. SU_3^c is the symmetry group of unitary transformations among three colours of fundamental fermion fields called quarks. It describes the strong interactions; the corresponding field theory is known as Quantum Chromodynamics (QCD) [4]. $SU_2^W \times U_1^Y$ is the group of weak isospin transformations on the left-handed components of quarks and leptons together with an additional phase transformation [5]. At an energy of ~ 250 GeV, the electroweak theory shifts from a disordered phase, where the effective potential is minimized at zero vacuum expectation value (v.e.v) of the scalar Higgs field, to an ordered phase where the v.e.v. is non-zero. The vacuum (ground state) no longer respects the full symmetry of the theory. The electroweak group $SU_2^W \times U_1^Y$ is spontaneously broken to U_1^γ (QED), with Goldstone bosons being avoided by the Higgs mechanism [6]. The normal fermions (quarks, charged leptons) receive their mass from Yukawa couplings to the Higgs field, i.e. terms in the Lagrangian of the form $y\bar{f}_L f_R \phi$. The Higgs field is an isospin doublet ($I_w = 1/2$) in the standard model -- this is well supported by the closeness of the ρ parameter to its predicted value of 1, [7]. Thus the structure of the normal fermion mass $f_L^+ f_R$ is $\underline{\Delta I_w = 1/2}$. One can also have $\underline{\Delta I_w = 1}$ fermion mass terms such as Majorana masses for the neutrinos. In 2-component notation

$$m\ \nu_L^T \sigma_2 \nu_L$$

Such terms entail maximal C (charge conjugation) violation and lepton number violation ($\Delta L = 2$). There is no strong reason to believe that L is an exactly conserved quantum number but why it would be approximately conserved is puzzling. Finally one can envisage $\underline{\Delta I_w = 0}$ mass terms. These are old fashioned Dirac-like mass terms $mf_L^+ f_R$. They require the left-handed and right-handed fermions to be in the same isospin multiplet. These are then fermions with vector-like or no weak interactions. The low energy effective theories obtained from the spontaneous breaking of Grand Unified Theories (GUTs) usually contain many fermions which are assumed to get their mass at the grand unification mass scale before $SU_2^W \times U_1^Y$ is broken. Thus they have SU_2^W

invariant ($\Delta I_w = 0$) masses.

Grand Unified Theories incorporate the standard model in a gauge theory based on a single simple Lie group G (e.g. SU(5) [8], SO(10) [9], E(6) [10], etc.) which contains $SU_3^C \times SU_2^W \times U_1^Y$. The scale at which the individual coupling constants associated with SU_3^C, SU_2^W and U_1^Y join is the grand unification scale $M_x \simeq 10^{15}$ GeV [11]. At M_x there is a single coupling constant g_G. Minimal grand unification seems to imply that there are no fundamental scales between the scale of weak breaking $\simeq 250$ GeV and $M_x \simeq 10^{15}$ GeV. This gap is called the desert. A desert would be very discouraging experimentally although aesthetically attractive.

Is the desert inevitable in this scheme? No! There are a number of ways in which intermediate mass scales may arise. It may be that two coupling constants merge at a scale below M_x, defining an intermediate scale, and then join with the third coupling constant at M_x [12]. The solution of the strong CP problem with a Peccei-Quinn U(1) chiral symmetry requires that this symmetry be spontaneously broken between 10^9 and 10^{12} GeV if we believe the standard cosmological model [13]. There is another possibility which we discuss in this thesis. We know that there is a large hierarchy of masses in the $\Delta I_w = 1/2$ sector. In particular there is a tremendous suppression of the fundamental mass scale $\langle\phi\rangle \sim 250$ GeV, as evidenced by the ratio $m_e/\langle\phi\rangle \sim 2 \times 10^{-6}$. Can something analagous happen in the $\Delta I_w = 0$ sector?

In Chapter II we present grand unified E_6 and E_6 x (family group) models with rich radiative structure in both the $\Delta I_w = 1/2$ and $\Delta I_w = 0$ sector. This work was done with P. Ramond. For a summary of attempts to understand the $\Delta I_w = 1/2$ masses as radiative effects see Ibañez [14].

In Chapter III we take seriously the existence of $\Delta I_W = 0$ mass fermions with mass below the unification scale. Within the framework of the standard model we examine the phenomenological constraints on the $\Delta I_W = 0$ mass. We also calculate the effect on grand unified model predictions.

In Chapter IV we illustrate the general conclusions of Chapter 2 with a 1-loop calculation of the rate for $\mu \to e\gamma$ in a model in which this is mediated by the exchange of heavy $\Delta I_W = 0$ mass fermions. The work in Chapters III and IV was done with F. del Aguila.

The conclusion is that such fermions, which are residues of a large variety of models, could be added to the standard model with masses as low as the present experimental limit of $\simeq 20$ GeV. If they arise from grand unification there would be little effect on the predictions of minimal models. We must now abandon our pencils and dirty our hands searching for these particles.

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II. CALCULABLE MASSES IN GRAND UNIFIED THEORIES

In the standard model [1] the fermion masses can be characterized by their weak isospin-breaking properties, ΔI_W. The known charged particles have a $\Delta I_W = 1/2$ mass since they form left-handed weak doublets ($I_W = 1/2$) and right-handed singlets. The values of these $\Delta I_W = 1/2$ lepton and heavy quark masses are known experimentally, as well as the light quark mass ratios. On the scale of M_W, which characterizes the strength of the weak isospin breaking, the $\Delta I_W = 1/2$ masses seem to show a perturbative structure: the family consisting of the τ, t and b has the largest masses, followed by μ, c and s with intermediate masses and e, u and d with tiny masses ($\frac{m_e}{m_W} \sim 10^{-5}$). There is at present no quantitative understanding of these values. The neutrinos can form $\Delta I_W = 1$ Majorana masses but are prevented from doing so in the standard model by lepton number conservation. Finally we can envisage $\Delta I_W = 0$ mass fermions, which would either have vector-like weak interactions or be weak isosinglets.

Some attempts to understand the $\Delta I_W = 1/2$ masses have used the concept of a family group [2,3] which has to be gauged in order to avoid Nambu-Goldstone bosons. Indeed the zeroth order family mass matrix (here shown in the e-, μ-, τ- family basis)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1.1)$$

strongly suggests an SU_3 structure with the masses appearing in the sextet representation [4] (the trace condition makes it awkward to use SO_3 or an SU_3 octet since it would require delicate cancellations between representations to guarantee the zeros in the diagonal).

In this chapter we discuss ways in which a radiative structure can arise in a Grand Unified Theory and present an illustrative example based on a one family E_6 model. We then generalize it to include SU_2^f and SU_3^f family groups.

Consider a classical Lagrangian containing a Yukawa term $ff\phi$ where f is a fermion and ϕ is a scalar field. The effective action generated by one particle irreducible graphs will in general contain non-renormalizable terms, compatible with the symmetries of the theory, of the form $ff\phi^n$, $n > 1$. When ϕ is given a vacuum value, such terms will contribute to the fermion masses. Usually they will just change the magnitude of the renormalized coupling $ff\phi$ and will play the role of a correction to an undetermined (arbitrary) coupling. In special cases, however, ϕ^n can contain quantum numbers not present in ϕ. Then the strength of these channels is computable in terms of the input parameters of the theory, and leads to a calculable radiative mass [5]. We will present many such examples.

In Grand Unified Theories, there are at least two scales of symmetry breaking which we denote by their weak isospin breaking properties: a $\Delta I_w = 0$ breaking done by a Higgs field H and a $\Delta I_w = 1/2$ breaking done by a Higgs field h. H breaks the Grand Unified Theory gauge group G down to the standard model and h breaks the standard model down to $SU_3^c \times U_1^\gamma$. When G is broken to the standard model all fermions which were held massless by $G/(SU_2 \times U_1 \times SU_3^c)$ - invariance will acquire masses. These are $\Delta I_W = 0$ masses; they can have values significantly smaller than $\langle H\rangle$ if the $\Delta I_w = 0$ sector itself has a radiative structure, coming from calculable terms of the form FFH^n, $n > 1$, where F is a $\Delta I_w = 0$ fermion. This could generate Grand Unified Theories where the "desert" is populated by fermions.

In addition there will be induced terms of the form $ffH^n h$ which, evaluated in the H vacuum, will give rise to $\Delta I_w = 1/2$ masses when $\langle h\rangle$ is non-zero. We will appeal to this mechanism to generate a radiative structure in the $\Delta I_w = 1/2$ sector. It becomes especially important when we consider it in the light of the family group. If both H and h have family quantum numbers, then $H^n h$ will have quantum numbers different from those of h. Then some of the zeros of the family matrix (1.1) could be filled by these radiative terms. In this case H plays the role of dialler in family space.

Let us give an example of the $ffH^n h$ mechanism in the standard SU_5 model [6] applied to the τ-family only. In this minimal model, $H = H^a{}_b \sim 24$ of SU_5, $h = h^a \sim \overline{5}$ of SU_5 and the fermions are $\overline{5}^a_f$ and 10_{fbc}. To the renormalizable coupling

$$\overline{5}^a_f\ 10_{fbc}\ \delta^b_a\ h^c \tag{1.2}$$

radiative corrections add a term of the form

$$\overline{5}^a_f\ 10_{fbc}\ (H^b_a\ h^c - H^c_a\ h^b), \tag{1.3}$$

when $(24)^3$ is present in the potential (if $(24)^3$ is absent the next induced term is of the form ffH^2h). This term generates the reducible representation $\overline{5} + \overline{45}$. The $\overline{5}$ just corrects the strength of (1.2) which is an unknown input parameter, but the $\overline{45}$ produces a calculable mass. This term is generated by a 2-loop graph. Thus in the standard SU_5 model we have a correction of the form

$$0 = (m_b - m_\tau) + c\,(m_b - 3m_\tau), \tag{1.4}$$

where c is calculable and depends (in this case) on the strength of the cubic coupling.

For our second example, we consider a one family E_6 model [7]. The E_6 family consists of a 27 of left-handed particles

$$27_L = 16 + 10 + 1, \ (SO_{10}) \tag{1.5}$$

$$27_L = (\overline{5} + 10 + 1) + (5 + \overline{5}) + 1, \ (SU_5) \tag{1.6}$$

and consists of one charge 2/3 quark, two charge -1/3 quarks, two charge -1 leptons and 5 neutral leptons (which could be arranged into 2 Dirac and one Majorana). We do as much of the symmetry breaking as possible by means of scalar fields which are themselves 27's of E_6 [8] (27_H). The Yukawa couplings are of the form

$$27_L 27_L 27_H. \tag{1.7}$$

They preserve a global Abelian X-symmetry: 27_L has X = +1 and 27_H has X

= -2. If this X symmetry is respected by the potential, it survives all the symmetry breakings by mixing with broken local charges (in the same way that B-L conservation arises in SU_5). As such it forbids Majorana mass terms for the neutrinos in the theory and leads to ordinary neutrinos with masses of $0\ (m_u, m_c, m_\tau)$. Hence this disastrous symmetry must be explicitly broken in the potential. Examination of the Higgs potential shows that this can be done* by means of the cubic term $27_H 27_H 27_H$ which then breaks X mod 6. This allows for induced terms of the form

$$27_L 27_L \overline{27}_H \overline{27}_H,\ 27_L 27_L 27_H \overline{27}_H 27_H,\ \text{etc...} \tag{1.8}$$

Since

$$27 \times 27 = (\overline{27} + \overline{351'})_S + \overline{351}_A \tag{1.9}$$

these can excite the $\overline{351}'$ Higgs sector in a calculable way and some of the $\overline{27}$, depending on the stability of the chosen vacuum values.

The $\Delta I_W = 0$ breaking of the theory proceeds as follows: we need two Higgs 27_H, $27'_H$ to break E_6 down to SU_5. From the decomposition (1.5) and (1.6) we start with $\langle 27_H \rangle \sim 1$ of SO_{10}: it breaks E_6 down to SO_{10} and a discrete symmetry; it leaves a massless $16_L + 1_L$ of SO_{10} at the tree

*The X symmetry can also be broken by a quartic term in the potential of the form $78(27)^3$. However, a careful examination of the discrete symmetries shows that the right-handed neutrinos become massive only after $\Delta I_W = 1/2$ breaking.

level, giving a $\Delta I_w = 0$ mass to $(5 + \overline{5})$. The 1_L fermion, whose mass has the quantum number of the 351', is massless at the tree level but is given an induced radiative mass by terms of the form (1.8). It occurs at the one loop level by scalar exchange and at two loops by vector and scalar exchange[9]. Thus the E_6 theory can be reduced to the SO_{10} theory with just one 27_H! The second Higgs $27'_H$ takes a vacuum value which is the SU_5 singlet along the 16 of SO_{10}. This breaks SO_{10} down to SU_5 and another discrete symmetry. However the SO_{10} singlet left-handed fermion has a mass lying along the 126 of SO_{10}; it stays massless at the tree level but picks up a radiative mass from terms like $16_L 16_L \overline{16}_H \overline{16}_H$. This is the mechanism advocated by Witten [10]. In our theory it appears naturally since all the required representations are present by E_6 invariance.

Thus with two Higgs, 27_H and $27'_H$, we are left with the SU_5 theory with 15 massless fermions and the usual global U_1 replaced by a discrete symmetry. Out of the original 27 fermions, 10 pick up tree level masses and 2 neutral leptons pick up radiative calculable $\Delta I_w = 0$ masses.

To further break SU_5 down to the standard model, we use a 78 of E_6. The $\Delta I_w = 1/2$ breaking is done by another $27''_H$; it has $\Delta I_w = 1/2$ values either along the 5 or the $\overline{5}$ of SU_5. These directions are distinct in the E_6 theory. If it can be naturally arranged in the Higgs potential that $\langle 27''_H \rangle \sim 5$ alone, then only the charge 2/3 quark will acquire a tree level mass. However the radiative term $27_L 27_L 27_H \overline{27}''_H 27_H$ will induce in the 27_H vacuum a breaking along the $\overline{5}$, giving masses to the charge -1/3 and -1 fermions. This occurs by means of a 1-loop diagram and yields in principle $\frac{m_b}{m_w} \sim (\frac{\alpha}{\pi})$. However such estimates may not be trustworthy since the gauge hierarchy can generate large logarithms. Thus we have managed to give all fermions a mass with a relatively modest Higgs sector. Note that we do not have B-L conservation; it is replaced by a harmless discrete symmetry. This was made possible by the use of the rich radiative structure of E_6.

We now present several generalizations of this model to include three families of fermions [11]. In the first the τ-family is treated as in the one family model, but the e- and μ-families form a family SU_2 doublet. The particle content of the model under $E_6 \times SU_2^f$ is taken to be:

left-handed fermion:

$$(27,2)_L + (27,1)_L \tag{1.10}$$

Higgs particles:

$$(27,2)_H + (27,1)_H + (78,1)_H + (1,2)_H + (27,1)_H'. \tag{1.11}$$

The Yukawa couplings are

$$(27,1)_L\ (27,1)_L\ (27,1)_H + (27,1)_L\ (27,2)_L\ (27,2)_H. \tag{1.12}$$

Note the absence of any $(27,3)_H$. This enables us to develop calculable masses for the e-family as in a previous model [3] based on SU_2^f. However there will be induced terms of the form

$$(27,2)_L(27,2)_L(\overline{27},1)_H(\overline{27},1)'_H,$$

$$(27,2)_L(27,2)_L(\overline{27},2)_H(\overline{27},2)_H, \text{ etc.} \qquad (1.13)$$

The two continuous global symmetries in (1.12) are broken down to discrete symmetries by explicit quartic terms in the Higgs potential. In order to achieve the same rich radiative structure we allow cubic Higgs self coupling terms. In the first stage of symmetry breaking, $(27,1)_H$ acquires an SO_{10} singlet vev, thereby breaking $E_6 \times SU_2^f \rightarrow SO_{10} \times SU_2^f$; this leaves, as in the previous model, 16 massless fermions in the τ-family and all (16+10+1,2) fermions massless in the μ- and e-families. At the second stage, $(27,2)_H$ gets a vev which is a 16 of SO_{10} but an SU_5 singlet. This leaves $SU_5 \times U_1$ invariant, where U_1 is generated by a linear combination of $T_5 = SO_{10}/SU_5$ and F_3 the family charge generator. At this stage we are left with 3 massless $\overline{5}$ + 10 families, one massless* $5 + \overline{5}$ (with $\Delta I_w = 0$ mass) and two massless $I_w = 0$ neutrinos. Next we use $(78,1)_H$ to break SU_5 down to $SU_3^c \times SU_{2L} \times U_1^Y$ but we are still left with the mixed U_1. Hence the fermion spectrum does not change. We use the complex $(1,2)_H$ to break the unwanted U_1 at a scale M'. Interestingly M' need not be as large as $M_{GUTS} \sim 10^{15}$ Gev; it cannot be too small either lest it induce flavor-changing neutral current effects. The hitherto massless $5 + \overline{5}$ fermions and two $I_w = 0$ neutrinos will then acquire masses of 0 (M'). Thus in the desert we can have one

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*At tree level - it could get a radiative mass.

extra massive vector boson and some particles with $\Delta I_w = 0$ masses. The $5 + \overline{5}$ particles have vector-like weak interactions and should be almost degenerate in mass, even after $\Delta I_w = 1/2$ breaking. Hence we are left with the usual 45 massless fermions. When $\Delta I_w = 1/2$ breaking occurs, by means of say $(27,1)'_H$, the τ- and μ- families get tree level masses and the e-family calculable radiative masses. The usual neutrinos now get masses which can be "large" if M' is significantly smaller than M, as per the G-MRS mechanism. Thus this model reproduces the standard SU_5 results without B-L conservation, introduces a new U_1 interaction in the "desert," and explains the smallness of the e-family masses.

Our third and last example generalizes these concepts to include an SU_3 family group [12]. We have already given arguments for choosing SU_3^f. This introduces anomalies. The anomaly number of the simplest SU_3 representations are $A_3 = +1$, $A_6 = 7$, $A_{10} = 27$, $A_{15} = 14$, $A_{15'} = 77$, etc.... Thus we can naively build anomaly-free sets of fermions[13]. For instance we can have such extensions as

$$SU_5 \times SU_3^f\colon\ (\overline{5} + 3,\ 3^f) + (1,\overline{15}^f) + (1,\overline{3}^f) \tag{1.14}$$

$$SO_{10} \times SU_3^f\colon\ (16,3^f) + (1,\overline{15}^f) + 2(1,\overline{3}^f) \tag{1.15}$$

$$E_6 \times SU_3^f\colon\ (27,3^f) + (1,\overline{10}^f). \tag{1.16}$$

All these minimal models add a large number of flavorless leptons. The $E_6 \times SU_3^f$ model is the least reducible -- it contains a very rich radiative structure (if Higgs cubic self couplings are present), and serves as an example for the Higgs dialler mechanism we have just introduced. The Yukawa couplings are taken to be of the form

$$(27,3)_L \cdot (27,3)_L \cdot (27,\bar{6})_H \quad , \tag{1.17}$$

$$(27,3)_L \cdot (1,\overline{10})_L \cdot (\overline{27},6)_H \quad , \tag{1.18}$$

$$(1,\overline{10})_L \; (1,\overline{10})_L \; (1,27)_H \quad , \tag{1.19}$$

where the real $(1,27)_H$ leaves a discrete global symmetry X mod 6; $(27,3)_L$ has $X = 1$, $(1,\overline{10})_L$ has $X = -3$, and $(27,\bar{6})_H$ has $X = -2$. In the presence of Higgs cubic self coupling, $E_6 \times SU_3^f$ symmetry allows for the generation of invariants of the form

$$(27,3)_L(27,3)_L(\overline{27},6)_H(\overline{27},6)_H,\ (27,3)_L(27,3)_L(27,\bar{6})_H(1,27)_H \quad , \tag{1.20}$$

$$(27,3)_L(1,\overline{10})_L(27,\bar{6})_H(27,\bar{6})_H,\ (27,3)_L(1,\overline{10})_L(\overline{27},6)_H(1,27)_H \quad , \tag{1.21}$$

$$(1,\overline{10})_L(1,\overline{10})_L(27,\bar{6})_H(\overline{27},6)_H,\ (1,\overline{10})_L(1,\overline{10})_L(1,27)_H(1,27)_H \quad , \tag{1.22}$$

which is made possible by the fact that both the sextet and the 27-plet of SU_3 have cubic couplings. Similarly one can generate many couplings of dim -6. The procedure is to decompose any product of Higgs into irreducible representations of the original group and match them with the ones appearing in the products of two fermion representations. We add a Higgs field (78,8) which does not couple to fermions directly but is necessary to break SU_5 and a remnant family invariance.

The symmetry breaking goes as follows: we need two different Higgs $(27,6)_H$, $(27,6)'_H$ to break E_6 down to SU_5. We also need to break the family group at the same scale to avoid unwanted flavor changing neutral current effects. The pattern is: (a) $(27,\overline{6})_H$ breaks $E_6 \times SU_3^f \to SO_{10} \times SU_2^f \times U_1$, and leaves invariant a discrete symmetry of the "RU" type [14] (henceforth we neglect the discrete symmetries). All fermions, which can, acquire mass compatible with $SO_{10} \times SU_2^f \times U_1$. (b) $(27,\overline{6})'_H$ breaks $SO_{10} \times SU_2^f \times U_1 \to SU_2^f \times SU_2^f \times U_1$. At this stage we are left with only 15 massless members of the τ-family, but the other two families, being SU_2^f doublets, are still massless, except in the SU_5 singlet sector (i.e. neutral lepton). (c) We now break SU_2^f completely with $(1,27)_H$ (say by means of the SU_2 doublet within the 27^f). Then all fermions which were prevented from having masses by SU_2^f acquire them. We are left with a $SU_5 \times U_1$ theory, where the U_1 is a local symmetry and is generated by a linear combination of flavor and family charges. (d) $(78,8^f)_H$ breaks $SU_5 \times U_1$ down to the standard model. (e) The $\Delta I_w = 1/2$ breaking is now done by one single $(27,\overline{6})''_H$, with a tree level value which breaks only SU_3^f down to SU_2^f. We hope that this can be achieved in a natural way, even though the family group has already been completely broken. This hope is intimately tied in with the gauge hierarchy problem where the feedback between the two sectors (H vs h) of the theory can be controlled. Clearly much calculation is needed to see if such a model reproduces the observed radiative structure.

In summary this model has a radiative structure in both the $\Delta I_w = 0$ and $\Delta I_w = 1/2$ sector. It provides an example of a Grand Unified

Theory with fermions filling the desert, leaving only a bosonic desert. The question is how low can those radiative $\Delta I_w = 0$ masses be and how the presence of such fermions can affect known phenomenology.

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III. THE POSSIBILITY OF NEW FERMIONS WITH $\Delta I=0$ MASS

1. Introduction

Fermions can be classified by their quantum numbers under the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ [1]. These quantum numbers are of two kinds: those belonging to $SU(3)_C \times U(1)_Q$ which are therefore conserved (C) and those belonging to $SU(2)_L \times U(1)_Y$ which are non-conserved (NC). We call the known fermions $(u,d,e,\nu_e; c,s,\mu,\nu_\mu; \ldots)$ normal, the fermions with standard C quantum numbers but with different NC quantum numbers pseudoexotic and the fermions with some non-standard C quantum numbers exotic. In general pseudoexotic fermions will mix with normal ones after the breaking of $SU(2)_L \times U(1)_Y$.

The closeness of the measured parameter $\rho = M_W/M_Z\cos\theta_W$ to 1 establishes that the normal fermions get their mass, when the standard model spontaneously breaks to $SU(3)_C \times U(1)_Q$, primarily from the coupling of $\bar{f}_L f_R$ to a Higgs doublet ($I=1/2$). They have a $\Delta I=1/2$ mass (the left-handed fields are doublets and the right-handed fields singlets). Can the standard model tolerate the addition of new fermions with $SU(2)_L \times U(1)_Y$ invariant ($\Delta I=0$) masses? Does experiment constrain the mass (M) and other properties of such fermions? If the mixing angles between different fermions were Cabibbo-like, for example, as commonly assumed, the contribution of these new fermions to flavour changing neutral current (FCNC) processes would in general exceed the experimental bounds

[2,3]. The angles, however, are not Cabibbo-like. We shall prove that the mixing angles between $\Delta I=0$ and $\Delta I=1/2$ fermions are order η or η^2, where $\eta \sim m/M$ ($m_{\Delta I=1/2} \ll M_{\Delta I=0}$). Rare processes are suppressed and heavy $\Delta I=0$ fermions decouple [4]. $\Delta I=0$ mass fermions can thus be very light, perhaps as low as 20 GeV, depending on the structure of the normal fermion part of the theory (see Section 2). Such fermions would have striking signatures. Masses within isospin multiplets would be nearly degenerate and the ratio of neutral to charged decays amongst normal fermions would be enhanced. Their presence is compatible with the predictions of Grand Unified Theories (GUTS).

Before starting our analysis we note some models with $\Delta I=0$ fermions. Grand Unified Theories (GUTS) based on SO(10) [5] and E_6 [6], GUTS derived from N=8 supergravity and supersymmetric unified models all contain $\Delta I=0$ fermions with masses originating from the extra mass scales. For example, consider the model proposed by Ellis, Gaillard and Zumino (EGZ) [7]. Its SU(5) content is

$$F: \underbrace{3(\overline{5}+10)}_{\substack{\text{normal} \\ \Delta I=1/2}} + \underbrace{9(1) + 3(\overline{5}+5) + 9(10+\overline{10})}_{\text{pseudoexotic}} + \underbrace{4(24) + (45+\overline{45})}_{\text{pseudoexotic and exotic}} \qquad (1.1)$$
$$\Delta I=0$$

In GUTS $\Delta I=0$ fermions are usually assumed to be very heavy. We emphasize, however, that their masses are a priori arbitrary. Examples are known of natural GUTS where these $\Delta I=0$ fermions acquire masses approximately ten orders of magnitude smaller than the grand unification mass [8,9]. In Section 4 we discuss another example based on the EGZ model.

In supersymmetric models [10] the partners of the usual Higgs and gauge bosons can acquire $\Delta I=0$ masses and there can also be genuine $\Delta I=0$ supermultiplets. Our analysis may then be relevant. The situation, however, is very model dependent; in particular it is crucial to know whether supersymmetry is broken at high or low energy. Non-renormalization theorems allow one to arrange some particles to have light mass and/or very small mixing angles; thus our assumptions need not apply.* Our results apply if the effective theory has $\Delta I=0$ fermions much heavier than normal fermions and each class of normal fermion gets mass from only one light Higgs scalar multiplet.**

In Section 2 we give a theorem on the mixing angles between normal and $\Delta I=0$ fermions. The details are given in the Appendices. We also discuss the characteristics of these new fermions. Section 3 discusses the experimental constraints on $\Delta I=0$ fermions from standard phenomenology, GUTS and cosmology. In Section 4 we give an example of a model with low mass $\Delta I=0$ fermions and Section 5 contains final remarks and conclusions.

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*In supersymmetric models with supersymmetry broken at low energy the supersymmetric partners of the known particles do not have masses larger than the scale of breaking. In any model in which their mass is low their phenomenological effects should be carefully studied.

**In particular this means that light scalars couple with typically small Yukawa couplings and only one neutral light scalar, that giving mass, couples to each class of normal fermions.

2. Mixing Angles and Main Characteristics of $\Delta I=0$ Fermions

In this section we elaborate on the crucial point of our analysis, the values of the mixing angles, and study the main characteristics of $\Delta I=0$ fermions. Our starting point is normal fermions with left-handed (LH) doublets and right-handed (RH) singlets. They acquire mass when the standard model (GWS) is broken. $\Delta I=0$ fermions have LH and RH parts in the same type of multiplet. Their mass comes from a GWS-invariant term and is thus arbitrary, but apparently heavier than normal masses.

We must consider two kinds of vertices: fermion-fermion-gauge boson (FFG) and fermion-fermion-Higgs boson (FFH). We begin with the gauge mixing angles. They are obtained by diagonalizing a general mass matrix and rewriting the FFG vertices determined by the fermion content in terms of mass eigenstates. The values of the mass matrix entries are guided by experiment. $\Delta I=0$ entries come from GWS-invariant mass terms which we take of order a large mass M. $\Delta I=1/2$ entries come from the Yukawa terms when GWS is broken and are of order a normal mass m which is much less than M. We assume all $\Delta I \geq 1$ entries are negligible as evidenced by the ratio of the charged to neutral current strengths $\rho \sim 1$ and the smallness of neutrino masses [11]. The details of the diagonalization and mixing matrices are given in Appendix A where we prove the theorem below.

Before GWS is broken all the mixing angles are zero, and weak current and mass eigenstates coincide. When GWS breaks we obtain mixing angles as an expansion in the mass ratio $\eta=m/M$. Because $\eta \ll 1$, the mass eigenstates will coincide with the current eigenstates to order 0 in η.

In this sense we will speak about mass eigenstates having a well-defined isospin. Gauge bosons do not change the fermion helicity. Thus the FFG vertices will involve only LH (RH) fermions.

2.1. FPG Mixing Angle Theorem:

LH(RH) fermions whose isospin quantum numbers differ by 1/2 will mix with an angle $\eta \sim m/M$. Different LH(RH) fermions with the same isospin will mix with angles of order η^2 (except normal fermions in the charged current which mix with Cabibbo angles) and LH(RH) fermions differing in isospin by 1 or more will have mixing smaller than η^2.

Thus normal fermions mix with $\Delta I=0$ fermions and $\Delta I=0$ among themselves with angles η or η^2 depending on the isospin of the new $\Delta I=0$ fermions, and normal fermions mix among themselves in the neutral current with angles η^2. The mixing angles quoted above are upper bounds; they may be smaller or zero for particular sets of fermions (see Tables 2.1, 2.2).

An exception to this theorem appears when two heavy $\Delta I=0$ fermions with the same C quantum numbers are nearly degenerate in mass. This is particularly relevant when such fermions have different NC quantum numbers. To zeroth order in η the mixing angle in this case is maximal (45°). Other fermions mix with the degenerate multiplets with the maximum possible mixing angle (e.g. a doublet will mix with such a maximal mixture of a degenerate doublet and a singlet with an angle order η). All the low energy consequences of the theorem, however, still hold. In particular, the net effect of interchanging these heavy fermions in low energy processes is the same. Leading order mixing effectively cancels. Such exceptions are not detectable in low energy experiments. We will not discuss any further this case, except to note that one must treat carefully the expressions containing differences of large masses in the denominator (see Appendices). When these masses are degenerate the corresponding mixing angles diverge and a more delicate analysis is necessary, the conclusion being that just presented.

As an example of the FFG Mixing Angle Theorem, consider the vertices involved in a typical lepton number changing process such as $\mu \to e\gamma$ via gauge boson exchange. The fermion lines for the one loop diagrams arising from $\Delta I=0$ lepton E exchange are shown in Fig. 2.1. Fig. 2.2 shows the tree level mixings which are of the same order since two $\Delta I=1/2$ vertices are comparable in mixing to one $\Delta I=0$ vertex.

Let us emphasize the phenomenological consequences of these results. The mixing angles go to zero with the scale of GWS breaking, as expected. The explicit functional dependences, however, are non-trivial. For contrast, imagine all the mixing angles were $\sqrt{\frac{m}{M}}$ instead of the values we have quoted. FCNC would then almost certainly forbid $\Delta I=0$ mass fermions. Consider the case of $\mu \to e\gamma$ [4]. Typical diagrams where this process is mediated by exchange of $\Delta I=0$ mass fermions are shown in Fig. 2.3. The diagrams with a mass insertion on the internal fermion line (LR) give a contribution proportional to M up to the mixing angles. If these angles were $\sim \sqrt{\frac{m}{M}}$ the net contribution would be $\sim M \left(\sqrt{\frac{m}{M}}\right)^2$, i.e. independent of M, and with the absence of the GIM mechanism [12] $\Delta I=0$ mass fermions would, in general, be forbidden. We have found that the mixing angle behaviour is significantly different. In the example of an intermediate lepton with I=0 one vertex has $\Delta I=1/2$ $(\mu_L \to E_L)$ and the other has $\Delta I=0$ $(E_R \to e_R)$. Therefore the net mixing angle suppression is $(\frac{m}{M})(\frac{m}{M})^2 \sim (\frac{m}{M})^3$ as opposed to $\left(\sqrt{\frac{m}{M}}\right)^2$. The phenomenological consequences are then very different. $\Delta I=0$ fermions

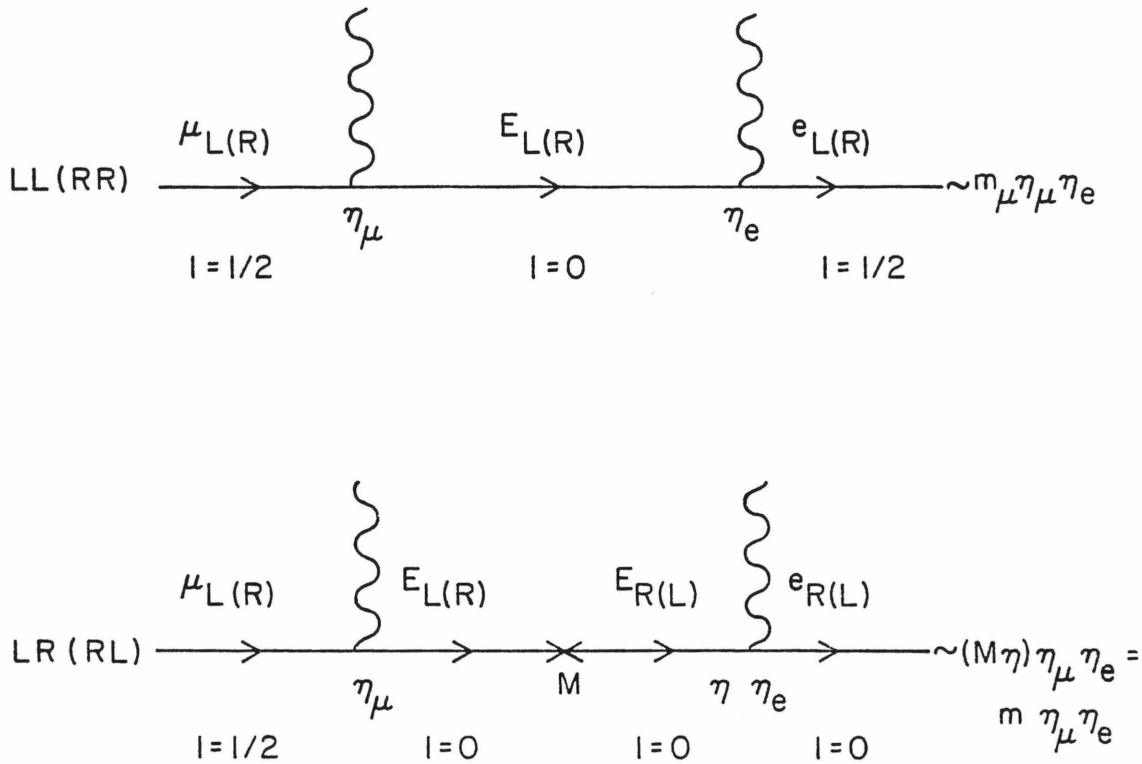

FIG 2.1: Fermion line for one loop $\mu \to e$ transitions. The m_μ in the LL(RR) diagram comes from the external momentum of the μ line.

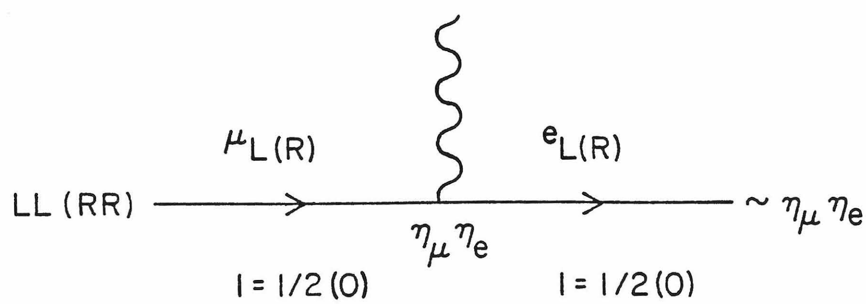

FIG 2.2: Tree level $\mu \to e$ transition.

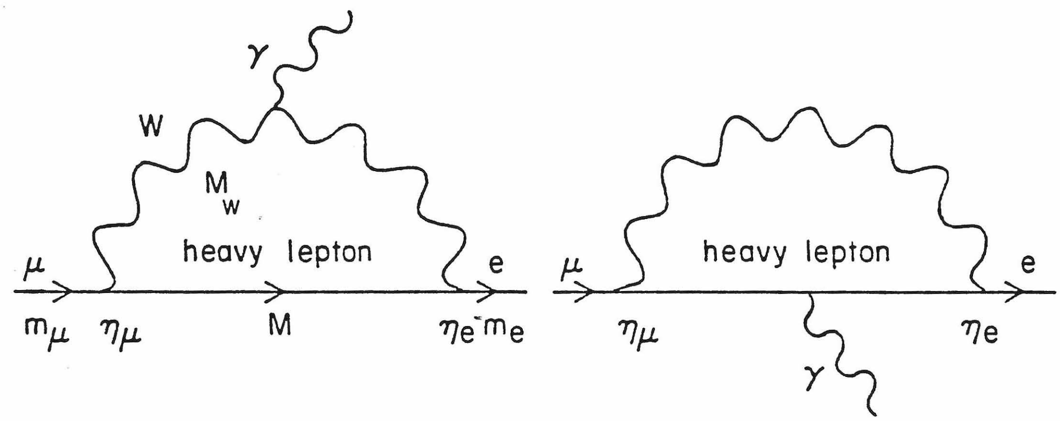

FIG 2.3: One loop $\mu \to e\gamma$ diagrams.

induce FCNC, on which there are strong experimental bounds, at the tree level. These processes have $\Delta I=0$ vertices (see Fig. 2.2) and thus according to our theorem have mixing angle suppression order $(\frac{m}{M})^2$ (the consequent limits on M would be very different for less mixing angle suppression). In summary any normal $\rightarrow$ normal transition forbidden in the absence of $\Delta I=0$ fermions is suppressed by a factor η^2; heavy $\Delta I=0$ fermions decouple.

Our conclusions so far depend only on the assumption that $\Delta I=0$ fermions are much heavier than normal fermions. To make quantitative predictions we must specify the light mass m which enters in the mixing angles. This is important given, for instance, the range in lepton masses from m_e to m_τ. Determining this light mass requires further assumptions about the fermion mass matrix. We will assume that the low energy ($\Delta I=1/2$) spectrum (in particular the light mass hierarchy) does not depend on a special choice of the large mass ($\Delta I=0$) parameters, and will henceforth refer to this assumption as the "Hierarchy principle." The Hierarchy principle implies that the mixing angle η_a for a normal fermion of mass m_a and a $\Delta I=0$ one of mass M is $\sim m_a/M$ (see Appendix A). Grand Unified Models exist where some low mass hierarchies, for instance $m_e \ll m_\mu$, are a consequence of particular relations between large M and small m mass parameters. However one needs to restrict the particle content and the allowed fermion mass-generating mechanisms to enforce these relations. In the cases we know, these relations are lost when one adds extra Higgses or further $\Delta I=0$ fermions (see Section 4). Many models simply do not allow any linear relation among M and m entries. We stress, however, that the Hierarchy principle is not a necessary assumption. The restrictions on $\Delta I=0$ masses are still not severe without it -- the masses can be low. For this reason we present our analysis for both the general case and the specific case resulting from use of the Hierarchy principle.

To discuss the Higgs mixing angles we must first specify the Higgs content whether or not we consider the Higgses elementary. We assume that there is only one Higgs doublet giving $\Delta I=1/2$ mass to each set of fermions with the same C quantum numbers. Owing to model dependence we cannot make any general statement if there are more Higgs particles. Conflict with the experimental limits on rare processes is likely, however, [13] and specific models should be carefully analysed. Often the only way to avoid this conflict is to banish the extra Higgs particles to high mass.

From now on we regard the physical Higgs as neutral but the conclusions for charged Higgses are the same, up to Cabibbo mixing in the normal fermion sector. For a given Higgs field the mixing angles among fermions are obtained by expressing the corresponding Yukawa matrix in the mass eigenstates. This matrix can only be simultaneously diagonalized with the mass matrix if the two are proportional. This requires there to be only one source of fermion masses, with all the Yukawa couplings constrained to reproduce the fermion spectrum. When there is more than one source the Yukawa couplings cannot all be constrained by the fermion spectrum and there can be large FCNC effects. Because we also allow for $\Delta I=0$ mass terms, there will be induced mixing. However we prove in Appendix B a theorem which shows that the mixing angles in this case are adequately small.

Higgs bosons change the fermion helicity. Thus the FFH vertices involve LH and RH fermions.

2.2. FFH Mixing Angle Theorem:

Light normal fermions mix among themselves with mixing angles η^2 (except for charged Higgs vertices in which case the mixing is Cabibbo-like). Light normal fermions and heavy $\Delta I=0$ fermions or heavy $\Delta I=0$ fermions among themselves mix with angles order unity if their LH and RH parts differ in isospin by 1/2, with angles η if they have the same isospin and with angles η or smaller if they differ in isospin by 1 or more.

A corresponding comment to that of the FFG theorem follows when two heavy $\Delta I=0$ fermions with the same C but different NC quantum numbers have nearly degenerate masses.

The Hierarchy principle implies that η, or the Yukawa coupling y, is proportional to the mass of the light fermion that is mixing. In FFG vertices the mixing angles are multiplied by the gauge coupling constant g. In FFH vertices they are multiplied by a Yukawa coupling $y = \frac{m}{v}$ where m is a small normal mass m and $v \sim 250$ GeV is the Higgs vacuum expectation value. Consequently y is typically much smaller than the gauge coupling constant. Since the mixing angle suppression is less in Higgs diagrams, however, their contributions are not negligible. In the cases we study it is possible to set the constraints on $\Delta I=0$ masses by considering only the gauge diagrams. We discuss essentially three classes of diagrams. The first class is one loop diagrams where Fig. 2.1 applies. Here the Higgs diagram corresponds to replacing the gauge bosons by scalars and the ratio of Higgs to gauge contributions in the amplitude is obtained by replacing the factor $m_\mu \eta_\mu \eta_e g^2$ by $m_\mu y_\mu y_e$

according to the FFH theorem. Since

$$\frac{m_\mu \eta_\mu \eta_e g^2}{m_\mu y_\mu y_e} \simeq \frac{g^2 v^2}{M^2} \simeq \left(\frac{2M_W}{M}\right)^2 \sim \left(\frac{155\text{GeV}}{M}\right)^2, \tag{2.2.1}$$

the Higgs contribution is less important than the gauge one for $M < 155$GeV and the Higgs mass order M_W. For small Higgs mass both contributions are at most comparable. Henceforth we use the gauge contributions to estimate the constraints on M. For higher M values both contributions are comparable but well below the experimental limits. (Note that in (2.2.1) we have not written the expression for the one loop diagrams [4].) The second class is tree level diagrams where Fig. 2.2 applies. For these processes the Higgs diagram is suppressed by the same mixing angle and by an extra factor y/g. Finally in discussing the characteristics of $\Delta I=0$ fermions we consider tree diagrams where a single $\Delta I=0$ fermion is produced or decays. In this case the vertex emitting a Higgs is in the ratio

$$\frac{y}{\eta g} \sim \frac{M}{gv} \sim \frac{M}{155\text{GeV}} \tag{2.2.2}$$

to the vertex emitting a gauge boson. For $M > 155$ GeV, then, Higgs contributions will dominate. We note though that if the Higgs and gauge bosons are virtual and decay to a pair of light fermions, the Higgs contribution will be suppressed by an extra factor y/g. In conclusion we consider the Higgs contributions only when we discuss the production and decay of heavy $\Delta I=0$ fermions (see below). The Higgs contributions to FCNC are considered in detail in Chapter IV.

2.3. Characteristics of $\Delta I=0$ Fermions

The weak interactions of $\Delta I=0$ fermions are determined by their multiplet assignments and mixing angles. In Tables 2.1 and 2.2 we gather the salient characteristics of $\Delta I=0$ pseudoexotic leptons and quarks respectively. For each possible multiplet assignment we give the relevant non-diagonal couplings, an estimate of the lifetime (assuming the Hierarchy principle), the first SU(5) representation in which the multiplet occurs and the signatures which distinguish them from normal fermions. These are the mass degeneracy within each multiplet, the typical splitting being $m \frac{m}{M}$, and a larger value of the neutral to charged decay ratio. For estimating the production of these new $\Delta I=0$ fermions and their detection signatures the usual diagonal couplings following from their multiplet structure and the dominant non-diagonal couplings given in Tables 2.1 and 2.2 should be used. The diagonal couplings are of strength unity, the non-diagonal ones are suppressed by mixing angles $\eta \sim \frac{m}{M}$, where m is a typical light mass for the $\Delta I=1/2$ fermion entering or leaving the vertex and M the typical large mass of the new heavy $\Delta I=0$ fermion. If the Hierarchy principle applies, the mixing and then the signals are expected to be larger for processes involving the heaviest normal generations. Higgs and gauge vertices are in the ratio

Non-diagonal couplings with normal generations.
We indicate only the mixing angles ($\eta \approx m/M$) suppressing the gauge and Yukawa couplings.

	LH and RH parts	Masses (M large)	Gauge L	Gauge R	Higgs RL (ℓ is a doublet)	Higgs LR (ℓ is a singlet)	Lifetimes $M < M_{W,Z}$	Lifetimes $M_{W,Z} < M$	Usual SU(5) representations containing these $\Delta I=0$ leptons	Other characteristics
Singlets	N	No constraint	L —η— ℓ, Z,W	(N will be in general a self conjugate Majorana spinor)	L — ℓ, H	(N will be in general a sel conjugate Majorana spinor)			1_F and 24_F	
	E		L —η— ℓ, Z,W	η^2 mixing (no mixing if no other $\Delta I=0$ multiplets)	L — ℓ, H	L —η— ℓ, H	$\tau \le \tau(L\to\ell_\tau e \nu_e) \sim$	$\tau \le \tau(L\to\ell_\tau Z) \sim$	$10_F + \overline{10}_F$	
Doublets	N E		η^2 mixing (no mixing in NC if no other $\Delta I=0$ multiplets)	L —η— ℓ, Z,W	L —η— ℓ, H	L — ℓ, H	$\left[\frac{G^2}{192\pi^3} m_\tau^2 M^3\right]^{-1} \sim$ $10^{-15}\left(\frac{20}{M \text{ in GeV}}\right)^3$ sec	$\left[\frac{G}{8\sqrt{2}\pi} m_\tau^2 M\right]^{-1} \sim$ $10^{-20}\left(\frac{100}{M \text{ in GeV}}\right)$ sec	$5_F + \overline{5}_F$ (used in supersymmetry) and $45_F + \overline{45}_F$	No sequential character or associated nearly massless neutrino or conserved quantum number,
	E .	Particles in the same multiplet are nearly degenerate in mass	η^2 mixing						other representation	ratio of neutral to charge decays large
Triplets	E^c N, N , E, E .		L —η— ℓ, Z,W	Negligible	L — ℓ, H	L —η— ℓ, H			24_F, other representations	in general imply exotics
	E . .	$\Delta M \sim m \frac{m}{M}$	Negligible		Negligible				other representations	
Other Multiplets			Negligible		Negligible		10^{-16} sec $< \tau <$ 1 sec		Difficult to embed	

TABLE 2.1: $\Delta I=0$ mass lepton characteristics

Non-diagonal couplings with normal generations. We indicate only the mixing angles ($\eta\sim m/M$) suppressing the gauge and Yukawa couplings.

	LH and RH parts	Masses (M large)	Gauge L	Gauge R	Higgs RL (q is a doublet)	Higgs LR (q is a singlet)	Lifetimes $M < M_{W,Z}$	Lifetimes $M_{W,Z} < M$	Usual SU(5) representations containing these $\Delta I=0$ quarks	Other characteristics
Singlets	U	No constraint	$Q \xrightarrow{\eta} q$ Z,W	η^2 mixing (no mixing if no other I=0 multiplets)	Q — q H	$Q \xrightarrow{\eta} q$ H	$\tau \lesssim \tau(Q\to q_b e\nu_e) \sim \left[\frac{G^2}{192\pi^3} m_b^2 M^3\right]^{-1} \sim$	$\tau \lesssim \tau(Q\to q_b Z) \sim \left[\frac{G}{8\sqrt{2}\pi} m_b^2 M\right]^{-1} \sim$	$10_F + \overline{10}_F$	
	D								$5_F + \bar{5}_F$ (Used in supersymmetry) and $45_F + \overline{45}_F$	
Doublets	U D	Particles in the same multiplet are nearly degenerate in mass	η^2 mixing (no mixing in NC if no other I=0 multiplets)	$Q \xrightarrow{\eta} q$ Z,W	$Q \xrightarrow{\eta} q$ H	Q — q H	$10^{-17}\left(\frac{50}{M \text{ in GeV}}\right)^3$ sec	$10^{-21}\left(\frac{100}{M \text{ in GeV}}\right)$ sec	$10_F + \overline{10}_F$	
	. D U .		η^2 mixing						$45_F + \overline{45}_F$, 24_H	ratio of neutral to charged decays large
Triplets	U . D U . D	$\Delta M \sim m\,\frac{m}{M}$	$Q \xrightarrow{\eta} q$ Z,W	Negligible	Q — q H	$Q \xrightarrow{\eta} q$ H			$45_F + \overline{45}_F$, other representations	imply exotics
	. D . . U .		Negligible		Negligible		10^{-18} sec $< \tau <$ 1 sec		other representations	
Other Multiplets			Negligible		Negligible				Difficult to embed	

TABLE 2.2 : $\Delta I=0$ mass quark characteristics.

$$\frac{y}{g\eta} \sim \frac{M}{155\ \mathrm{GeV}}.^{*}$$

*The Jade collaboration at PETRA [13] has searched for heavy neutral leptons.

$$e^+e^- \to \nu E^0_L \dashrightarrow e^+\tau^-\nu^0$$

Neutral $\Delta I=0$ fermions will have similar signatures. However, the correct incorporation of mixing angles is essential. In the $\Delta I=0$ case this means that we expect heavy generations to be preferred in the final state.

3. Experimental Constraints

What are the experimental constraints on the mass of $\Delta I=0$ fermions? We study first the constraints from the GWS phenomenology of FCNC, second the constraints from the usual GUT predictions and finally those derived from standard cosmology.

3.1. Standard Phenomenology

Since fermions with $\Delta I=0$ masses have not been observed we expect them to be heavier than $\sim$ 20 GeV [14]. We now study their effect on rare processes which are forbidden (or nearly so) in the minimal $SU(2)_L \times U(1)_Y$ model. The strong upper bounds on lepton-number violating processes, such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$ or $\mu N \to eN$, and the high suppression of FCNC in the quark sector, as in $K \to \mu\bar{\mu}$ and the small mass differences between K_L, K_S and D_L, D_S, restrict the possible quantum numbers of fermions embedded in the GWS model. Fermions which mix with the first two families $(e, \nu_e, u, d; \mu, \nu_\mu, c, s)$ with Cabibbo-like mixing angles and moderate mass must have the same C and NC quantum numbers as these families to be consistent with the magnitude of the above processes. Fermions with $\Delta I=0$ mass evade this restriction. Their naturally small mixing angles alone suppress rare processes.

The neutral currents in the GWS model are diagonal at tree level -- this is the GIM mechanism [12]. In our case we have order η and η^2 non-diagonal currents. There are tree level FCNC effects. One loop contributions are down by at least a factor $\frac{\alpha}{\pi}$ with respect to the tree level ones. We neglect CP violation.

3.1.1. Lepton Number Nonconservation

No process violating lepton number has been observed. At present the best bounds on these processes are [15]

$$B(\mu \to e\gamma) < 2 \times 10^{-10},$$

$$B(\mu \to ee\bar{e}) < 2 \times 10^{-9}, \text{ and} \tag{3.1.1.1}$$

$$B(\mu N \to eN) < 4 \times 10^{-10}.$$

Let us examine the most important contributions of $\Delta I=0$ fermions to these processes.

<u>$\mu \to e\gamma$</u>

This is forbidden at the tree level because electromagnetic interactions conserve flavour. The typical one loop gauge diagrams are shown in Fig. 2.3, where M is the $\Delta I=0$ mass, η_μ and η_e are the mixing angles of the muon and electron with the mediating $\Delta I=0$ mass fermion and $M_W, M \gg m_\mu \gg m_e$. The decay rate is

$$\Gamma(\mu \to e\gamma) \sim \Gamma(\mu \to e\nu\bar{\nu}) \frac{\alpha}{\pi} \eta_\mu^2 \eta_e^2 \tag{3.1.1.2}$$

where $\frac{\alpha}{\pi}$ comes from the two vertices and subsequent loop integration. We have calculated this rate for arbitrary M including the Higgs contribution [4] and verified (3.1.1.2) up to a factor order 1.

The branching ratio is then

$$B(\mu \to e\gamma) \sim \frac{\alpha}{\pi} \eta_\mu^2 \eta_e^2 \equiv \frac{\alpha}{\pi} \frac{\delta^4}{M^4} \tag{3.1.1.3}$$

(3.1.1.3) defines the mixing parameter $\delta = \sqrt{\eta_\mu \eta_e}\, M$. For M to be ~ 20 GeV and the branching ratio less than the experimental limit δ must be $\lesssim 0.4$ GeV. The Hierarchy principle gives

$$\eta_\mu \sim \frac{m_\mu}{M}, \quad \eta_e \sim \frac{m_e}{M} \tag{3.1.1.4}$$

and therefore

$$\delta \sim \sqrt{m_e m_\mu} \tag{3.1.1.5}$$

This gives

$$B(\mu \to e\gamma) \sim 4 \times 10^{-17} \left(\frac{20 \text{ GeV}}{M}\right)^4, \tag{3.1.1.6}$$

which is much less than the experimental limit for $M \gtrsim 20$ GeV.

<u>$\mu \to ee\bar{e}$</u>

This process is allowed at the tree level by Z exchange (Fig. 3.1). The decay rate is

$$\Gamma(\mu \to ee\bar{e}) \sim \Gamma(\mu \to e\nu\bar{\nu}) \frac{1}{2} \eta_e^2 \eta_\mu^2 \equiv \Gamma(\mu \to e\nu\bar{\nu}) \frac{1}{2} \frac{\delta^4}{M^4} \tag{3.1.1.7}$$

For M to be ~ 20 GeV and the branching ratio less than the experimental limit, δ must be $\lesssim 0.16$ GeV. The Hierarchy principle gives again (3.1.1.5) and thus

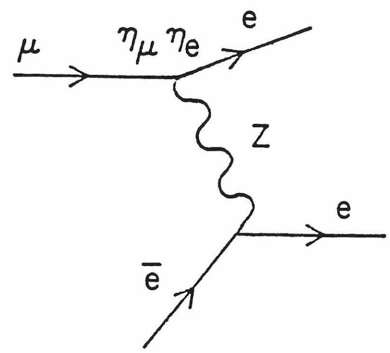

FIG 3.1: Tree level $\mu \to ee\bar{e}$ transition.

$$B(\mu \to ee\bar{e}) \sim 10^{-14} \left(\frac{20\text{ GeV}}{M}\right)^4, \qquad (3.1.1.8)$$

i.e. approximately 5 orders of magnitude below the experimental limit for $M = 20$ GeV.

<u>$\mu N \to eN$</u>

This process gives a branching ratio similar to that of $\mu \to ee\bar{e}$ in (3.1.1.7) and (3.1.1.8). Since the experimental bound is lower (though we think more uncertain) it would give a better constraint on M.

In the last two processes one loop contributions are down by $(\frac{\alpha}{\pi})^2$.

With our assumption on the origin of the normal fermion mass hierarchy the experimental limits on all these lepton number non-conserving processes require considerable improvement to indicate $\Delta I=0$ leptons at 20 GeV. Because mixing angles are proportional to lepton masses experiments on τ and μ non-diagonal decays would be more restrictive.*

---

*Note that there will also be contributions to the electron and muon anomalous magnetic moments (g-2) from the exchange of $\Delta I=0$ mass fermions. Since the amplitude is the relevant quantity here one might expect a better limit on M. However the $\mu \to e\gamma$ rate turns out to be more restrictive [4].

3.1.2. FCNC in the Quark Sector

In the quark sector the best limits on FCNC come from $K \to \mu\bar{\mu}$ decays and the mass differences $m_{K_L} - m_{K_S}$, $m_{D_L} - m_{D_S}$. Since the minimal GWS model is in agreement with experiment we must check that quarks with low $\Delta I=0$ masses do not invalidate these predictions.

The experimental values are [11,16]

$$\begin{aligned} &B(K \to \mu\bar{\mu}) \sim 9 \times 10^{-9}, \\ &\delta m_K/m_K \sim 7\times10^{-15}, \\ &\delta m_D/m_D \leq 10^{-12}. \end{aligned} \tag{3.1.2.1}$$

$K \to \mu\bar{\mu}$

This process goes at the tree level by interchanging a Z via a $\bar{d}sZ$ vertex proportional to $\eta_d\eta_s$ (see Fig. 3.2). This gives a decay rate

$$\begin{aligned} \Gamma(K \to \mu\bar{\mu}) &\sim \Gamma(K^+ \to \mu^+\nu_\mu) \left(\frac{1}{\sin\theta_c} \eta_d\eta_s\right)^2 \\ &\equiv \Gamma(K^+ \to \mu^+\nu_\mu) \frac{1}{\sin^2\theta_c} \left(\frac{\delta^4}{M^4}\right) \end{aligned} \tag{3.1.2.2}$$

For a $\Delta I=0$ mass of 20 GeV, δ must be $\lesssim$ 0.09 GeV to prevent the branching ratio exceeding the experimental value (3.1.2.1). The Hierarchy principle yields

$$\delta \sim \sqrt{m_d m_s} \approx 0.04 \text{ GeV} \quad , \tag{3.1.2.3}$$

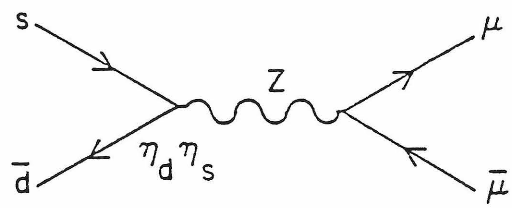

FIG 3.2: Tree level K → $\mu\bar{\mu}$ transition.

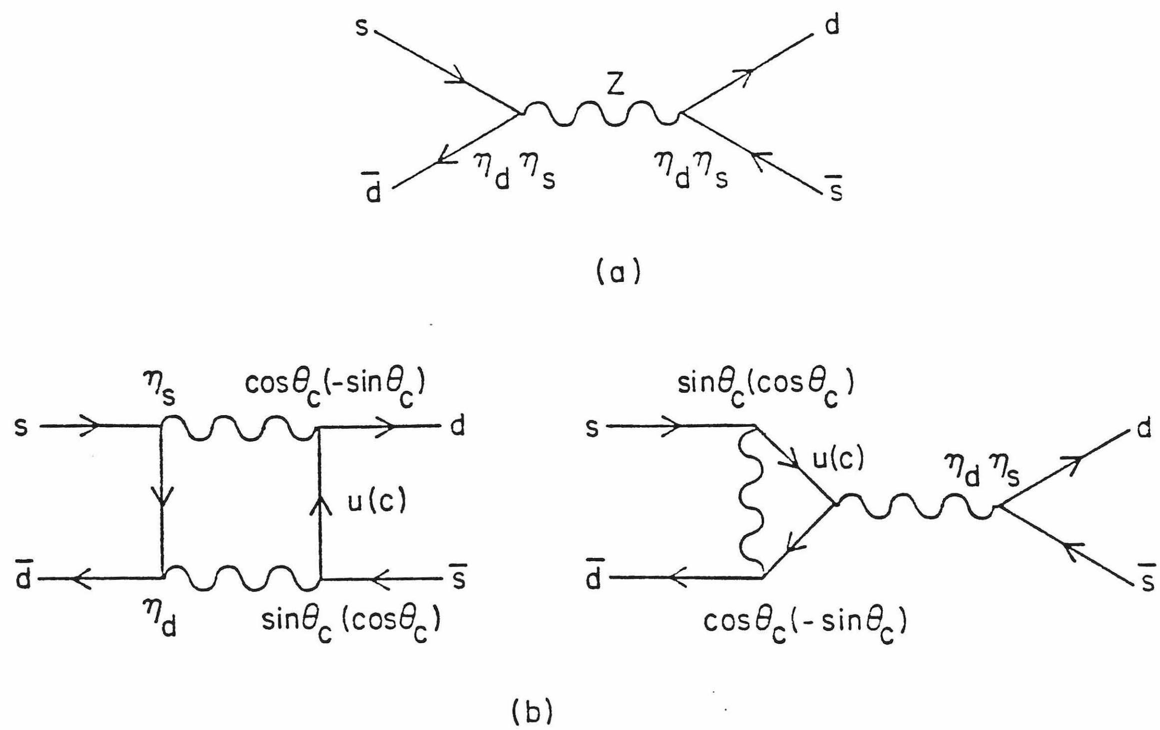

FIG 3.3: $K^\circ \leftrightarrow \bar{K}^\circ$ transition: (a) Tree diagram

(b) Typical one loop diagram

and thus

$$B(K \to \mu\bar{\mu}) \sim 3 \times 10^{-10} \left(\frac{20\ \mathrm{GeV}}{M}\right)^4. \qquad (3.1.2.4)$$

One loop contributions are down by $(\frac{\alpha}{\pi})^2$. As in all hadron processes one needs a model to estimate the effect of non-free quarks inside the hadron.

$\underline{K_L - K_S}$

This mass difference is related to the transition amplitude $K^0 \to \bar{K}^0$. Fig. 3.3a shows the tree level contribution

$$\frac{\delta m_K}{m_K} \sim \frac{2}{3} f_K^2 \frac{G_F}{2\sqrt{2}} (\eta_d \eta_s)^2 \equiv \frac{2}{3} f_K^2 \frac{G_F}{2\sqrt{2}} \frac{\delta^4}{M^4} \qquad (3.1.2.5)$$

With $M \sim 20$ GeV, $\delta \lesssim 1.1$ GeV gives a contribution to $\delta m_K / m_K$ smaller than the measured value. The Hierarchy principle (3.1.2.3) gives

$$\frac{\delta m_K}{m_K} \sim 9 \times 10^{-19} \left(\frac{20\ \mathrm{GeV}}{M}\right)^4 \qquad (3.1.2.6)$$

One loop contributions can be important to the $K_L - K_S$ mass difference since one loop diagrams such as those shown in Fig. 3.3b have only one factor $\eta_d \eta_s$. However, the GIM mechanism [3,12] still works in the line interchanging u or c. Thus the one loop contribution is

$$\frac{\delta m_K}{m_K} \sim \frac{2}{3} f_K^2 \frac{G_F}{2\sqrt{2}} \frac{\alpha}{\pi} \frac{m_c^2}{\sin^2\theta_W M_W^2} \sin\theta_c \cos\theta_c \frac{\delta^2}{M^2} \qquad (3.1.2.7)$$

where $\delta^2 = \eta_d \eta_s M^2$ as above. Thus

$$\frac{\delta m_K}{m_K} \sim 5 \times 10^{-14} \frac{\delta^2}{M^2} , \qquad (3.1.2.8)$$

requiring δ to be $\lesssim 7.5$ GeV for an M of 20 GeV. With the Hierarchy principle

$$\frac{\delta m_K}{m_K} \sim 2 \times 10^{-19} \left(\frac{20 \text{ GeV}}{M}\right)^2 \qquad (3.1.2.9)$$

which dominates (3.1.2.6) for large M but is still well below the experimental value for possible M's.

We have assumed no CP phase is present. The experimental value of CP violation in the K system is very small i.e. $\mathrm{Im}(\delta m_K/m_K) \sim 10^{-7}$ [16]. We note that there is no contribution from tree level diagrams to the $\mathrm{Im}\delta m_K$. The one loop amplitude (Fig. 3.3b) on the other hand is proportional to $\eta_d \eta_s$ alone and gives a contribution to $\mathrm{Im}\delta m_K$ of order (3.1.2.8,9) or smaller.

$\underline{D_L - D_S}$

This is analogous to the $K_L - K_S$ case if we replace d and s by u and c. Then

$$\frac{\delta m_D}{m_D} \sim \frac{2}{3} f_D^2 \frac{G_F}{2\sqrt{2}} \frac{\delta^2}{M^2} \qquad (3.1.2.10)$$

with $\delta^2 = \eta_u \eta_c M^2$. We then have

$$\frac{\delta m_D}{m_D} \sim 9 \times 10^{-8} \left(\frac{\delta}{M}\right)^4 \qquad (3.1.2.11)$$

(i.e. $\delta \lesssim 1.2$ GeV for M = 20 GeV) for the tree level and

$$\frac{\delta m_D}{m_D} \sim 7 \times 10^{-14} \left(\frac{\delta}{M}\right)^2 \tag{3.1.2.12}$$

(i.e. $\delta < 380$ GeV for $M = 20$ GeV)

for the one loop level.

The Hierarchy principle gives

$$\frac{\delta m_D}{m_D} \sim 2 \times 10^{-17} \left(\frac{20 \text{ GeV}}{M}\right)^4 \tag{3.1.2.13}$$

for the tree level and

$$\frac{\delta m_D}{m_D} \sim 10^{-18} \left(\frac{20 \text{ GeV}}{M}\right)^2 \tag{3.1.2.14}$$

for the one loop level.

Note that we have used a $\Delta I=0$ mass M of 20 GeV merely for illustration -- it is a reasonable lower limit since otherwise such fermions would probably already have been seen in experiments. In any given case one must ensure that $M \gg m$ for the mixing angles to follow our theorems.

We have discussed only those FCNC which give the severest constraints on the $\Delta I=0$ masses. We can similarly estimate the contributions to other processes such as $K \to e\mu$ [11], but they are far below the experimental limits.

In conclusion the most restrictive process is $K' \to \mu\bar{\mu}$. Without some additional assumption about the form of the mass matrix we can only place a lower limit on the $\Delta I=0$ mass for a given mixing parameter δ. However worst case estimates ($\delta \approx 1$ GeV) still give a minimum M of the order of hundreds of GeV which is tantalizingly low. If the mass matrix has the form dictated by the Hierarchy principle, then we find that $\Delta I=0$ fermion masses are not constrained by the experimental magnitudes and limits of rare processes. In all cases the masses of the normal fermions involved are small enough to give very small mixing with the heavier $\Delta I=0$ mass fermions. In Table 3.1 we list the limits that the various processes place on M for $\delta = 1$ GeV, and on δ for $M = 20$ GeV.

Process	Limit on δ	δ=1 GeV	M=20 GeV	Hierarchy Principle		
		$M \gtrsim$	$\delta \lesssim$	δ(GeV)	$B/B_{expt.}$	$M \gtrsim$ (GeV)
$K \to \mu\bar{\mu}$	$\delta < (4.5 \times 10^{-3})M$	220	0.09	0.04	$3 \times 10^{-2}(\frac{20}{M})^4$	8.6
$\mu \to ee\bar{e}$	$\delta < (8 \times 10^{-3})M$	125	0.16	0.007	$10^{-5}(\frac{20}{M})^4$	1.1
$\mu \to e\gamma$	$\delta < (1.7 \times 10^{-2})M$	58.5	0.34	0.007	$2.10^{-7}(\frac{20}{M})^4$	0.45
$m(D_L - D_S)$ (Tree Level)	$\delta < (5.7 \times 10^{-2})M$	17.5	1.15	0.08	$2.10^{-5}(\frac{20}{M})^4$	1.4
$m(D_L - D_S)$ (One Loop)	$\delta < 3.8M$	0.26	7.5	0.08	$10^{-6}(\frac{20}{M})^2$	0.02
$m(K_L - K_S)$ (Tree Level)	$\delta < (6 \times 10^{-2})M$	16.7	1.2	0.04	$1.3 \times 10^{-4}(\frac{20}{M})^4$	2.1
$m(K_L - K_S)$ (One Loop)	$\delta < 0.37M$	2.67	7.5	0.04	$2.9 \times 10^{-5}(\frac{20}{M})^2$	0.1

Table 3.1: The limits on δ,M for various FCNC processes.

3.2. GUT Predictions

Grand Unified Theories [17], apart from incorporating nicely the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, predict $\sin^2\theta_w$, $M_{unification}$ and m_b/m_τ [18,19]. The values of $\sin^2\theta_w$ and m_b/m_τ are in good agreement with experiment. These predictions are unqualified only in the minimal SU(5) model. When new fermions or Higgses are added to SU_5 or the gauge group is enlarged (SO(10), E_6 etc), these quantities can be adjusted and statements are much less rigorous. (For supersymmetric models see Ref. [20].)

$\mathrm{Sin}^2\theta_w$ and M_x

We find that the usual SU(5) predictions of $\sin^2\theta_w$ and M_x can each be separately increased or decreased for different values of the $\Delta I=0$ fermion masses and/or representations. In fact we can obtain any reasonable value consistent with the measured value of $\sin^2\theta_w$ and the lower bound on the proton lifetime. The only general observation is that the addition of $\Delta I=0$ fermions in complete SU(5) representations with the different $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets degenerate does not change $\sin^2\theta_w$ and M_x to first order. Thus the deviations of $\sin^2\theta_w$ and M_x from their standard value are due to mass differences between $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets. Adding new fermions always tends to increase the coupling constant α_{Gum} at the unification mass, and therefore decreases the proton lifetime. The lifetime is much more sensitive to M_x, however, than α_{Gum} and so increases in M_x overwhelm increases in α_{Gum}, giving a longer proton lifetime.

m_b/m_τ

The prediction of the bottom quark mass [19] was a triumph of the minimal SU(5) model. In this model m_b/m_τ is one at the unification mass and there is no extra structure until the GWS scale. More complicated models need not share these features but, given the success of the minimal SU(5) predictions of m_b, one may worry about the effects of new $\Delta I=0$ fermions. We find that m_b/m_τ can be increased or decreased depending on the $\Delta I=0$ fermion masses and/or representations.

In Table 3.2 we list the changes in the SU(5) predictions of $M_{unification}$, $\sin^2\theta_w$, α_{Gum}, τ_p and m_b/m_τ for the extreme case where one $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplet of the SU(5) representations $5_F + \overline{5}_F$ and $10_F + \overline{10}_F$ is at M_W and for the case where the splitting within the $5_F + \overline{5}_F$ representations is an order of magnitude. The quoted values were obtained from one loop formulae [17]. (The $SU(3)_c \times SU(2)_L \times U(1)_Y$ content of these SU(5) representations is given in Table 3.3.) In the extreme cases we see that almost any combination of changes is possible but the changes are usually small. In the nearly degenerate case the changes are even smaller.

$SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets at M_W $(r+\bar{r})$	M'_X/M_X	$\sin^2\theta'_W - \sin^2\theta_W$	$\alpha'_{Gum}/\alpha_{Gum}$	τ'_p/τ_p	m'_b/m_b
$(3,1)_{-2}$	6.6	-0.017	1.03	1.5×10^3	1.06
$(1,2)_3$	0.2	0.015	1.05	1.6×10^{-3}	0.98
$(3,2)_1$	6.6	0.035	1.12	1.2×10^3	1.13
$(\bar{3},1)_{-4}$	1	-0.025	1.09	0.8	1.04
$(1,1)_6$	0.2	-0.009	1.05	1.1×10^{-3}	0.96
$5_F+\bar{5}_F$ with $m_{(3,1)}=10m_{(1,2)} = 10M_W$	0.9	0.001	1.08	0.5	1.04
$5_F+\bar{5}_F$ with $m_{(1,2)}=10m_{(3,1)} = 10M_W$	1.1	-0.001	1.08	1.4	1.03

$\alpha^{-1} = 128$ and $\alpha_3 = 0.12$ at M_W

TABLE 3.2: Illustrative examples of the effects of new $\Delta I=0$ fermions at M_W on $\sin^2\theta_W$, τ_p and m_b/m_τ. Primed denotes new values and unprimed the values for the minimal model.

$$5 = (3,1)_{-2} + (1,2)_{3}$$

$$10 = (3,2)_{1} + (3,1)_{-4} + (1,1)_{6}$$

$$24 = (8,1)_{0} + (3,2)_{-5} + (3,2)_{5} + (1,3)_{0} + (1,1)_{0}$$

$$45 = (8,2)_{3} + (6,1)_{-2} + (3,3)_{-2} + (3,2)_{-7} + (3,1)_{-2} + (3,1)_{8} + (1,2)_{3}$$

Table 3.3: $SU(3)_C \times SU(2)_L \times U(1)_Y$ content of several SU(5) representations. The electric charge is defined $Q = I_3 + Y/6$.

3.3. Cosmological Constraints

The standard model of the early universe [21] does not constrain very short-lived particles i.e. those with lifetime $\lesssim 1$ sec. Thus pseudoexotic fermions, which mix with and decay into normal fermions with lifetimes much smaller than 1 sec (see Tables 2.1 and 2.2), are not constrained by standard cosmology. Exotic fermions like those in the SU(5) representations 45_F and 24_F (see Table 3.3) can be short or long-lived. Those which interact weakly with pseudoexotic fermions and then decay into normal ones have very short lifetimes, comparable to those of the pseudoexotics. For example, there is a quark doublet in the 45_F which contains a U antiquark of charge $-2/3$ and a G antiquark of charge $-5/3$; the decay rate of $G \to t\bar{e}\nu$ is then of the same order as $U \to b\bar{e}\nu$ (see Table 2.2). Those exotic fermions which do not have weak interactions with the pseudoexotic fermions, such as the neutral octet in the 24_F, would be as stable as the proton if their masses were very low. For an asymmetric matter-antimatter universe, the masses of stable fermions must be $\lesssim 10\ m_{nucleon}$ ($\simeq 10$ GeV) otherwise they will contribute too much mass to the universe. Accelerator searches thus seem to rule them out. For a more general discussion and examples see Ref. [22].

4. Example

We illustrate with an example the plausibility of low mass (<1TeV) $\Delta I=0$ fermions that decouple from normal fermions because they mix with very small angles. The example is taken from the EGZ model [7] (see (1.1)), in which we assume that the Yukawa and Higgs sectors respect two continuous symmetries to be identified with B-L and Peccei-Quinn (PQ) [23]. These may be regarded as effective symmetries arising from N=8 supergravity and so perhaps not exact.* We deal with two normal $(\overline{5}_F + 10_F)$ fermion families plus pseudoexotic $5_F + \overline{5}_F$ and $10_F + \overline{10}_F$ fermion families. There are three 5_H, one 10_H and a complex 24_H Higgs representation, all of which appear in the EGZ model. This Higgs sector is minimal for our purposes. The number of normal and $\Delta I=0$ fermion families does not affect the analysis. We restrict their number for clarity and simplicity. One can imagine that the two normal families correspond to the e and μ families and the $\Delta I=0$ families to new fermions. Since nothing depends crucially on having two families we may consider this example as realistic.

The imposition of continuous symmetries is a general technique [8,9]. It can be applied to any fermion content provided one introduces an appropriate Higgs sector. In our case the Yukawa couplings are

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*Domain walls could be a problem for exact global symmetries. In many cases and in particular for axion models [24] the vacuum degeneracy implies the existence of domains in the recent past and hence an excess of energy in domain walls.

$$a_{ij}\overline{5}_{Fi}10_{Fj}\overline{5}_{H} + 10_{Fi}10_{Fj}\,(b_{ij}5'_{H} + c_{ij}5''_{H})$$

$$+ d_j 5_F 10_{Fj} 10_H + g_j \overline{5}_{Fj}\overline{10}_F\overline{10}_H + h\, 5_F\overline{10}_F 5_H, \tag{4.1}$$

where we sum over $i,j = 1,2,3$, and a_{ij}, b_{ij}, c_{ij}, d_j, g_j and h are arbitrary constants. The Higgs sector contains the quartic couplings

$$\overline{10}_H 24_H 5'_H 5''_H \text{ and } 24_H 24_H 5_H \overline{5}''_H \tag{4.2}$$

in addition to the usual potential terms. The coupling $24_H 24_H 5_H \overline{5}'_H$ is also allowed.

The corresponding Lagrangian has two continuous symmetries:

	$\overline{5}_{Fi}$	10_{Fi}	5_F	$\overline{10}_F$	10_H	5_H	$5'_H, 5''_H$	24_H
X_1	1	-1	-4	4	5	0	2	1
X_2	-3	1	3	-1	-4	-2	-2	0

X_1 is a chiral symmetry and is broken at 10^{15} GeV when 24_H acquires a vev. This produces an invisible axion. X_2 corresponds to B-L after the breaking of WS by the $\langle 5_H\text{'s}\rangle$.

Even when 24_H acquires a vev and breaks SU(5) to $SU(3)_C \times SU(2)_L \times U(1)_Y$ the fermions in 5_F and $\overline{10}_F$ are still massless, at the tree level, since the couplings $5_F\overline{5}_F 24_H$ and $10_F\overline{10}_F 24_H$ are forbidden by the symmetry X_1. Radiative corrections give them calculable masses, the most important contributions coming from all-Higgs two loop diagrams (see Fig. 4.1). In addition to the two loop suppression we have an extra

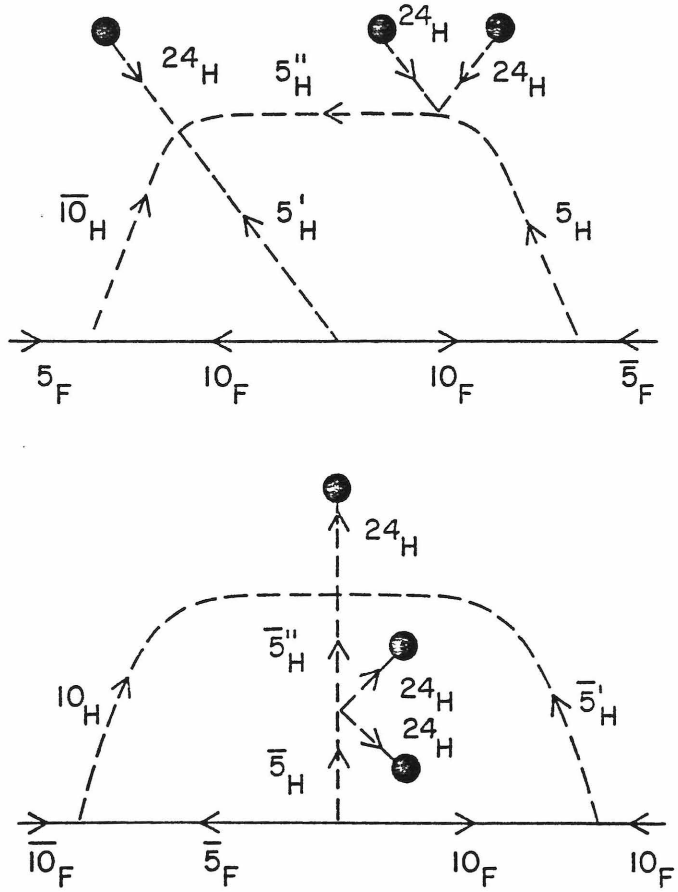


FIG 4.1: Two loop Higgs diagram giving radiative masses.

suppression from the smallness of the Yukawa couplings. Estimating these diagrams [9] we find that the masses of the $\Delta I=0$ fermions can be 1TeV or lower. We have an effective $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory with e and μ families of normal mass m (after GWS breaking) and a group of $\Delta I=0$ fermions with masses M larger than m (they have $SU(2)_L$ invariant mass terms). In this model the different M masses are related since the fermion multiplets couple to the same expectation value $\langle 24_H \rangle$. Also some large M entries are correlated with low m entries. In what follows we assume that M parameters are large but arbitrary.*

The fermion content of the effective theory is

$$\begin{aligned}
&2\begin{pmatrix} N_L \\ E_L \end{pmatrix},\ 2E_R + \begin{pmatrix} N_L \\ E_L \end{pmatrix},\ \begin{pmatrix} N_R \\ E_R \end{pmatrix},\ E_L,\ E_R \\
&2\begin{pmatrix} U_L \\ D_L \end{pmatrix},\ 2U_R,\ 2D_R + D_L,\ D_R,\ \begin{pmatrix} U_L \\ D_L \end{pmatrix},\ \begin{pmatrix} U_R \\ D_R \end{pmatrix},\ U_L,\ U_R
\end{aligned} \tag{4.3}$$

where we have not distinguished the current eigenstates from the $\overline{5}_F$, 10_F, 5_F or $\overline{10}_F$. When necessary we will use subindices for LH doublets and RH singlets. The electroweak interaction Lagrangian for leptons is

*If there was no $10_F + \overline{10}_F$ of fermions in this example and the most important contribution to the $5_F\overline{5}_F$ $\Delta I=0$ mass came from the diagram of Fig. 7, a linear relation would exist between the large M and small m entries in the mass matrix for leptons of charge −1 and quarks of charge −1/3. This relation would give m_e, m_d = 0. It is sufficient to add another 5_H of Higgses coupling to $5_F 10_F$ to destroy this relation.

$$\mathcal{L}_{FFG} + \mathcal{L}_{FFH} = \frac{g}{\sqrt{2}} W_\mu^\dagger (\overline{N}_{Li}\gamma^\mu E_{Li} + \overline{N}_R\gamma^\mu E_R) + \text{H.c.}$$

$$+ \frac{(g^2 + g'^2)^{1/2}}{2} Z_\mu (\overline{N}_{Li}\gamma^\mu N_{Li} - \overline{E}_{Li}\gamma^\mu E_{Li} + \overline{N}_R\gamma^\mu N_R - \overline{E}_R\gamma^\mu E_R \qquad (4.4)$$

$$- 2\sin^2\theta_w \, J^\mu_{EM}) + eA_\mu J^\mu_{EM}$$

$$- [\frac{1}{\sqrt{2}}(v+\chi)(c^e_{ij}\overline{E}_{Li}E_{Rj} + c^e \,\overline{E}_L E_R) + M^e_i \,\overline{E}_{Li}E_R + M^{e'}_j \overline{E}_L E_{Rj}$$

$$+ M^n_i \,\overline{N}_{Li} N_R] + \text{H.c.}$$

where we sum over i, j = 1,2,3 and $M^n_i = M^e_i$ (they correspond to the same bare mass term). χ is the physical Higgs of the single doublet; $m^e_{ij} = \frac{c^e_{ij}}{\sqrt{2}} v$ and $m^e = \frac{c^e}{\sqrt{2}} v$, where v is the vacuum expectation value, are elements of the mass matrix.

The Lagrangian for quarks is obtained by replacing N with U and E with D and by writing the corresponding Yukawa and bare mass terms for U. The fields in (4.4) are current eigenstates. To write the Lagrangian in terms of physical fields we must diagonalize the corresponding mass matrices. These are for leptons (ignoring any CP phase)

$$\begin{array}{cc} & N_R \\ \overline{N}_{Li} & \begin{pmatrix} M^n_i \end{pmatrix} \end{array} \qquad (4.5a)$$

$$\begin{array}{c} \bar{E}_L \\ \bar{E}_{Li} \end{array} \left[\begin{array}{c|c} m^e & M_j^{e\prime} \\ \hline M_i^e & m_{ij}^e \end{array} \right] \qquad (4.5b)$$
$$\qquad\quad E_R \qquad E_{Rj}$$

where $\Delta I=0$ entries are order M and $\Delta I=1/2$ entries order m, with $m \ll M$, and $\Delta I=1$ entries are 0. For the U and D quarks the general mass matrices are of the form (4.5b) with E changed to U and D respectively.

Diagonalizing as indicated in the Appendices and writing the mass eigenstates as $n_{L(R)}$, $e_{L(R)}$, $u_{L(R)}$, $d_{L(R)}$, we find the interaction Lagrangian for leptons to first order in $\eta \sim m/M$ is

$$\begin{aligned} &\frac{g}{\sqrt{2}} W_\mu^+[-(\eta_{L1}^e \bar{n}_{L1} + \eta_{L2}^e \bar{n}_{L2} + \eta_{L3}^e \bar{n}_{L3}) \gamma^\mu e_{L4} \\ &+ \bar{n}_R \gamma^\mu (\eta_{R1}^e e_{R1} + \eta_{R2}^e e_{R2} + \eta_{R4}^e e_{R4})] \\ &+ \frac{(g^2 + g'^2)^{1/2}}{2} Z_\mu[(\eta_{L1}^e \bar{e}_{L1} + \eta_{L2}^e \bar{e}_{L2} + \eta_{L3}^e \bar{e}_{L3}) \gamma^\mu e_{L4} \\ &-(\eta_{R1}^e \bar{e}_{R1} + \eta_{R2}^e \bar{e}_{R2} + \eta_{R4}^e \bar{e}_{R4}) \gamma^\mu e_{R3}] \\ &+ \chi[\bar{e}_{L1}(-y_{13}^e \eta\, e_{R3} + y_{14}^e\, e_{R4}) + \bar{e}_{L2}(-y_{23}^e \eta\, e_{R3} + y_{24}^e\, e_{R4}) \\ &+ \bar{e}_{L3}(y_{31}^e\, e_{R1} + y_{32}^e\, e_{R2} + y_{34}^e\, e_{R4}) \end{aligned} \qquad (4.6)$$

$$- \bar{e}_{L4}(y^e_{41}\eta\, e_{R1} + y^e_{42}\,\eta\, e_{R2} + y^e_{43}\, e_{R3})] + \text{H.c.}$$

The subindex 1 stands for electron, 2 for muon, 3 for $\Delta I=0$ mass leptons with $I=1/2$ and 4 for $\Delta I=0$ mass leptons with $I=0$. The η parameters are of order $\frac{m}{M}$ and the y parameters are typical Yukawa couplings order $\frac{m}{v}$. (4.6) exemplifies the mixing angle theorems of section 2. There is no Cabibbo angle in the charged sector because the e,μ neutrino masses are zero (degenerate).

FCNC are thus significantly suppressed in this model. If we adopt the Hierarchy principle then

$$\eta^e_1 \sim \frac{m_e}{M},\ \eta^e_2 \sim \frac{m_\mu}{M} \tag{4.7}$$

and the y parameters with subindices 1 and 2 go as $\frac{m_e}{v}$ and $\frac{m_\mu}{v}$ respectively.

For quarks the interaction Lagrangian to order η is

$$\begin{aligned}\mathcal{L} \sim\ & \frac{g}{\sqrt{2}}\, W^+_\mu\, [\bar{u}_{L\alpha}\gamma^\mu A^{\alpha\beta} d_{L\beta} + \bar{u}_{R\alpha}\gamma^\mu B^{\alpha\beta} d_{R\beta}] + \text{H.c.} \\ & + \frac{\sqrt{g^2 + g'^2}}{2}\ Z_\mu\, [\bar{u}_{L\alpha}\gamma^\mu C^{\alpha\beta}_u u_{L\beta} - \bar{d}_{L\alpha}\gamma^\mu C^{\alpha\beta}_d d_{L\beta} + \\ & \qquad \bar{u}_{R\alpha}\gamma^\mu D^{\alpha\beta}_u u_{R\beta} - \bar{d}_{R\alpha}\gamma^\mu D^{\alpha\beta}_d d_{R\beta} - 2\sin^2\theta_w J^\mu_{EM}] \\ & + eA_\mu J^\mu_{EM} + \chi\, [\bar{u}_{L\alpha} F^{\alpha\beta}_u u_{R\beta} + \bar{d}_{L\alpha} F^{\alpha\beta}_d d_{R\beta}] + \text{H.c.}\end{aligned} \tag{4.8}$$

where

$$A^{\alpha\beta} = \left[\begin{array}{cc|c|c} c\theta_c & s\theta_c & & -\eta^d_{L1}c\theta_c - \eta^d_{L2}s\theta_c \\ -s\theta_c & c\theta_c & & \eta^d_{L1}s\theta_c - \eta^d_{L2}c\theta_c \\ \hline & & 1 & -\eta^d_{L3} \\ \hline -\eta^u_{L1}c\theta_c + \eta^u_{L2}s\theta_c & -\eta^u_{L1}s\theta_c - \eta^u_{L2}c\theta_c & -\eta^u_{L3} & \end{array}\right]$$

$$B^{\alpha\beta} = \left[\begin{array}{cc|c|c} & & \zeta^u_{R1} & \\ & & \zeta^u_{R2} & \\ \hline \zeta^d_{R1} & \zeta^d_{R2} & 1 & \zeta^d_{R4} \\ \hline & & \zeta^u_{R4} & \end{array}\right] \tag{4.9}$$

$$C^{\alpha\beta}_{U,D} = \left[\begin{array}{cc|c|c} 1 & & & -\zeta^{u,d}_{L1} \\ & 1 & & -\zeta^{u,d}_{L2} \\ \hline & & 1 & -\zeta^{u,d}_{L3} \\ \hline -\zeta^{u,d}_{L1} & -\zeta^{u,d}_{L2} & -\zeta^{u,d}_{L3} & \end{array}\right]$$

$$D^{\alpha\beta}_{U,D} = \left[\begin{array}{cc|c|c} & & \zeta^{u,d}_{R1} & \\ & & \zeta^{u,d}_{R2} & \\ \hline \zeta^{u,d}_{R1} & \zeta^{u,d}_{R2} & 1 & \zeta^{u,d}_{R4} \\ \hline & & \zeta^{u,d}_{R4} & \end{array}\right]$$

$$
F_{u,d}^{\alpha\beta} = \left[\begin{array}{cc|cc}
-\dfrac{m_{u,d}}{v} & & -Y_{13}^{u,d}\eta & Y_{14}^{u,d} \\
 & -\dfrac{m_{c,s}}{v} & -Y_{23}^{u,d}\eta & Y_{24}^{u,d} \\
\hline
Y_{31}^{u,d} & Y_{32}^{u,d} & -Y_{33}^{u,d}\eta & Y_{34}^{u,d} \\
-Y_{41}^{u,d}\eta & -Y_{42}^{u,d}\eta & -Y_{43}^{u,d} & -Y_{44}^{u,d}\eta
\end{array}\right]
$$

In equations (4.9) subindices 1 and 2 are family indices, and subindices 3 and 4 indicate the heavy $\Delta I=0$ quarks with $I=1/2$ and $I=0$ respectively. $A^{\alpha\beta}$ contains the usual Cabibbo mixing in the charged current. The orders of the parameters parallel those in the lepton case.

5. Final Remarks and Conclusions

Fermions with SU(2)-invariant ($\Delta I_{weak}=0$) mass arise in many current theories that introduce new mass scales. Such theories include GUTs and supersymmetric models. These fermions must be heavier than $\sim$ 20 GeV since they have not been detected in accelerator experiments [14].

Assuming $\Delta I=1/2$ breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_Q$ and that each class of normal fermions acquires mass from only one Higgs doublet we show that the mixing of $\Delta I=0$ fermions of mass M with conventional fermions of mass order $m_{\Delta I=1/2}$ is order $m_{\Delta I=1/2}/M_{\Delta I=0}$. $\Delta I=0$ Majorana masses for neutral fermions are covered by our analysis. Mixing then suppresses the amplitude of all the weak processes that $\Delta I=0$ fermions induce or mediate. Section 2 lists the non-diagonal couplings and the salient characteristics of $\Delta I=0$ fermions. The most restrictive flavour changing neutral current process is $K \to \mu\bar{\mu}$ which goes at the tree level. In terms of the mixing parameter δ which modulates the strength of the $\bar{d}s$ vertex ($\delta = M_{\Delta I=0}\sqrt{\eta_d \eta_s}$) we find that $M_{\Delta I=0}$ must be $\gtrsim 220\ \delta$. At worst δ is of order the mass of the heaviest $\Delta I=1/2$ fermion in the problem, allowing $\Delta I=0$ fermions in the Tev range. Adopting the Hierarchy principle $\left(\eta_f \sim \frac{m_f}{M_{\Delta I=0}}\right)$ we find that all FCNC effects are well below the measured values or limits as the case may be.*

Since these $\Delta I=0$ fermions may populate any part of the desert it is exciting to suggest that they may be found in the next generation of accelerators. If produced they will have a distinct signature - very small mass splitting among the members of a given multiplet and enhanced ratios of neutral to charged current decays. More restrictive bounds on their mass would follow from improvements on the limits of FCNC processes involving the heaviest families (τ,b,t?,...). If $\Delta I=0$ fermions are very massive it will be difficult to establish their existence. Grand Unified Models with these fermions can reproduce the predictions of the minimal models.

The strength and pattern of the mixing angles dictated by our theorems is responsible for the effective decoupling of heavy $\Delta I=0$ fermions [4]. If present at low energies $\Delta I=0$ fermions could induce CP violation in the K system. Thus they may connect CP violation at the unification scale to that at low energy, as originally suggested by Sanda [27]. We discuss a specific model elsewhere [28].

---

*The analysis for the fermionic partners of the usual Higgs and gauge bosons in supersymmetric models is sensitive to the particular model. If the scalar partners of the normal fermions are light (low energy supersymmetry breaking) one should worry about their contributions to rare processes, particularly since they can have gauge, in contrast to Yukawa, couplings. Off-diagonal FCNC processes constrain the masses of scalar fermions of different generations to be nearly degenerate [25]. Diagonal processes (g-2) may constrain more severely the gaugino masses [26].

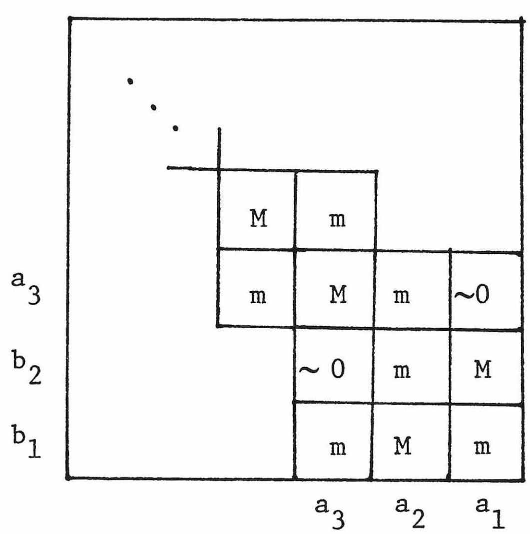
Appendix A

In this Appendix we prove the theorem stated in Section 2 on the mixing angles between $\Delta I=0$ mass and $\Delta I=1/2$ mass fermions for the fermion-fermion-gauge vertices. These mixing angles are determined by diagonalizing a general mass matrix $\mathcal{M}$ for each set of particles with the same conserved (C) quantum numbers. For a set of n Dirac particles $\mathcal{M}$ is an n x n matrix where the rows and columns correspond to LH and RH parts respectively. An arbitrary matrix $\mathcal{M}$ can always be diagonalized by two unitary matrices O_L and O_R,

$$D = O_L \mathcal{M} O_R^+ \tag{A.1}$$

where D is diagonal and positive.

We will take $\mathcal{M}$ to be real and O_L and O_R orthogonal, assuming that no CP violation is present. In the current eigenstates, the general $\mathcal{M}$ is



$$n = a_1 + a_2 + a_3$$
$$= b_1 + b_2 + a_3$$
$$(a_1 > b_2) \tag{A.2}$$

where b_1 is the number of LH doublets, b_2 the number of LH singlets, a_1 the number of RH singlets, a_2 the number of RH doublets, a_3 the number of LH and RH triplets and so on. (A.2) has three kinds of entries; those which correspond to $\Delta I=0$ and are order M, those which correspond to $\Delta I=1/2$ and are order $m \ll M$, and zero entries. (We are assuming that the $SU(2)_L$ breaking is mainly $\Delta I=1/2$, as is experimentally known.) We will diagonalize $\mathcal{M}$ to order $\eta = m/M$.

$\mathcal{M}$ can be trivially diagonalized to first order. The matrix obtained from $\mathcal{M}$ by setting the $\Delta I=1/2$ entries to zero is diagonalized by orthogonal matrices which commute with the isospin generators since $SU(2)_L$ is unbroken; hence the weak current Lagrangian is preserved in this new basis. $\mathcal{M}$ has the form

$$\mathcal{M} = \begin{array}{cccccc c} \ddots & & & & & & \\ & D & m & & & & \\ & m & D & 0 & m & 0 & a_3 \\ & & 0 & D & m & 0 & b_2 \\ & & m & m & D & m & a_2 \\ & & m & m & 0 & m & b_1-a_2 \\ & & a_3 & b_2 & a_2 & a_1-b_2 & \end{array} \qquad (A.3)$$

To order zero in $\eta \sim \frac{m_{\Delta I=1/2}}{M_{\Delta I=0}}$, $b_1 - a_2$ and $a_1 - b_2$ will be the light LH doublets and RH singlets respectively (these correspond to the $\Delta I=1/2$ mass fermions), and a_2, b_2, a_3, ... will be the heavy $\Delta I=0$ doublets, singlets, triplets, ... D stands for diagonal matrices with large M eigenvalues.

The matrices O_L and O_R diagonalizing (A.3) and giving the mixing angles are obtained by solving

$$D^2 = O_L \mathfrak{m}\mathfrak{m}^T O_L{}^T = O_R \mathfrak{m}^T \mathfrak{m} O_R{}^T \tag{A.4}$$

We discuss O_L; the argument for O_R is analogous. We address later the possibility of degeneracy in the large eigenvalues and the presence of zero eigenvalues to first order (corresponding to the light fermions). Expanding in η,

$$\mathfrak{m}\mathfrak{m}^T = \left[\begin{array}{c|ccccc} \ddots & & & 0 & & \\ \hline & D^2+m^2 & Dm & 0 & m^2 & m^2 \\ & Dm & D^2+m^2 & m^2 & Dm & Dm \\ 0 & 0 & m^2 & D^2+m^2 & Dm & Dm \\ & m^2 & Dm & Dm & D^2+m^2 & m^2 \\ & m^2 & Dm & Dm & m^2 & m^2 \end{array}\right] \begin{array}{c} \\ \vdots \\ a_3 \\ b_2 \\ a_2 \\ b_1-a_2 \end{array} \equiv A = A_0 + \eta A_1 + \eta^2 A_2, \tag{A.5}$$

where

$$A_0 = \left[\begin{array}{c|ccccc} & & & & & \\ \hline & D^2 & & & & \\ & & D^2 & & & \\ & & & D^2 & & \\ & & & & D^2 & \\ & & & & & 0 \end{array}\right], \quad A_1 = \left[\begin{array}{c|ccccc} & & & & & \\ \hline & & M^2 & & & \\ & M^2 & & & M^2 & M^2 \\ & & & & M^2 & M^2 \\ & & M^2 & M^2 & & \\ & & M^2 & M^2 & & \end{array}\right]$$

$$A_2 = \left[\begin{array}{c|ccccc} & & & & & \\ \hline & M^2 & & & M^2 & M^2 \\ & & M^2 & M^2 & & \\ & & M^2 & M^2 & & \\ & M^2 & & & M^2 & M^2 \\ & M^2 & & & M^2 & M^2 \end{array}\right] \qquad \text{(A.6)}$$

where we indicate only the orders of the entries.

$$D^2 = B_0 + \eta B_1 + \eta^2 B_2 + \ldots \qquad \text{(A.7)}$$

$$O_L = C_0 + \eta C_1 + \eta^2 C_2 + \ldots \qquad \text{(A.8)}$$

In this basis $C_0 = \mathbf{1}$ by definition $(B_0 = A_0)$ since we have done the zeroth order diagonalization.

From $D^2 O_L = O_L \mathfrak{M}\mathfrak{M}^T$ and $O_L^T O_L = \mathbf{1}$:

$$A_0 C_1 + B_1 = A_1 + C_1 A_0, \qquad \text{(A.9)}$$

$$C_1 + C_1^T = 0 \qquad \text{(A.10)}$$

In components

$$(C_1)_{ij} = \frac{(A_1)_{ij}}{(A_0)_{ii} - (A_0)_{jj}}, \quad i \neq j \qquad \text{(A.11)}$$

$$(B_1)_{ii} = (A_1)_{ii} = (C_1)_{ii} = 0 \qquad \text{(A.12)}$$

Now C_1 gives the order η mixing. From (A.11) C_1 is proportional to A_1, whose only non-zero entries are $\Delta I=1/2$ (A.6). Thus $\Delta I=1/2$ mixing is of order η. $\Delta I \neq 1/2$ mixing is at most order η^2. We see also that the large eigenvalues get corrections of order η m and not m (A.12).

When two large eigenvalues are degenerate (A.11) is divergent. We have analyzed this case more carefully with the result given in Section 2. The physical consequences of the theorem are unchanged.

We next prove that the mixing among light fermions induced by the new $\Delta I=0$ mass fermions is order η^2. In (A.3) one can make a rotation in the light LH and RH fields to diagonalize the $(b_1 - a_2) \times (b_1 - a_2)$ box and then make the perturbation expansion as before. This rotation gives, for instance, the usual Cabibbo mixing in the charged sector. Alternatively one can first make the order η rotations $(\mathbb{1} + \eta C_1, \ldots)$ and at the end rotate the LH and RH fields of the light sector $(b_1 - a_2) \times (b_1 - a_2)$. It is easy to convince oneself that both rotations in the light sector are equal because of the block structure of the order η rotations. This means that an initial diagonalization of the light sector is not undone by the order η rotations which mix $\Delta I=0$ and $\Delta I=1/2$ fermions. This is equivalent to corrections in the light sector being of order η^2. It implies also that the small eigenvalues get corrections only of order η^2m.

Our analysis includes also $\Delta I=0$ Majorana masses for neutral fermions.

The m entries in the light box $(b_1 - a_2) \times (b_1 - a_2)$ of (A.3), which give the physical light masses to order η^2, are combinations of different m entires (A.2). These combinations depend on the large M entries.

If the small mass eigenvalues (in particular their hierarchy ($m_e \ll m_\mu \ll m_\tau$, ...) are not to be the result of a special choice of large M entries, the Hierarchy principle, then all the elements in a row (column) of the $b_1 - a_2$ lines (A.3) must be of the same order. Thus the rows (columns) are in the same hierarchy. This immediately implies that each light fermion mixes in proportion to its mass, $\eta_f \sim \frac{m_f}{M}$.

As a simple example consider the case of two LH SU(2) doublets (call them e_L^0 and μ_L^0) and a LH singlet (ℓ_L^0) with all their RH partners $(e_R^0, \mu_R^0$ and $\ell_R^0)$ being singlets. The mass matrix has the form $\overline{\psi}_L M \psi_R$ where

$$\psi_{L,R} = \begin{pmatrix} e^0 \\ \mu^0 \\ \ell^0 \end{pmatrix}_{L,R} \tag{A.8}$$

and

$$M \simeq \begin{pmatrix} m & m & m \\ m' & m' & m' \\ M & M & M \end{pmatrix} = \begin{pmatrix} m_i \\ m_i' \\ M_i \end{pmatrix} \tag{A.9}$$

The first two steps in the diagonalization process described above cast M in the form

$$\begin{pmatrix} m_e & 0 & \delta_e \\ 0 & m_\mu & \delta_\mu \\ 0 & 0 & M_\ell \end{pmatrix}$$

$$= \begin{pmatrix} c\theta & s\theta & \\ -s\theta & c\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} m_i \\ mi' \\ M_i \end{pmatrix} \begin{pmatrix} \vec{u} = & \vec{u}' = & \vec{e} = \\ c\phi\vec{e}_1 & -s\phi\vec{e}_1 & M_i/_M \\ +s\phi\vec{e}_2 & +c\phi\vec{e}_2 & \end{pmatrix} \quad (A.10)$$

where θ and ϕ are the rotations necessary to cast M in the form above and $\vec{e}_1$, $\vec{e}_2$ are two arbitrary orthonormal vectors orthogonal to $\vec{e}$.

The initial mass matrix in the new basis is

$$\begin{pmatrix} m_i \\ m_i' \\ M_i \end{pmatrix} \equiv \begin{pmatrix} m'_e\vec{u} & + & m''_e\vec{u}' & + & \delta'_e\vec{e} \\ m'_\mu\vec{u} & + & m''_\mu\vec{u}' & + & \delta'_\mu\vec{e} \\ & & M_\ell\vec{e} & & \end{pmatrix} \quad (A.11)$$

(this defines m'_e, m''_e, etc.)

Now

$$m_e = \frac{m'_e m''_\mu - m'_\mu m''_e}{\sqrt{m''^2_e + m''^2_\mu}}, \qquad \delta_e = \frac{\delta'_e m''_\mu - \delta'_\mu m''_e}{\sqrt{m''^2_e + m''^2_\mu}}$$

$$m_\mu = \sqrt{m''^2_e + m''^2_\mu}, \qquad \delta_\mu = \frac{\delta'_e m''_e + \delta'_\mu m''_\mu}{\sqrt{m''^2_e + m''^2_\mu}} \quad (A.12)$$

Suppose now we keep $\vec{m}$ and $\vec{m}'$ fixed but realign $M\vec{e}$ to $M\vec{u}$. Then

$$m_e \leftrightarrow \delta_e$$

and $$\delta_\mu \rightarrow \frac{m'_e m''_e + m'_\mu m''_\mu}{\sqrt{m''^2_e + m''^2_\mu}} \quad (A.13)$$

Hence we require $\delta_e \sim m_e$ if m_e is to be stable under the realignment of $\vec{Me}$ to $\vec{Mu}$. In a similar way one can prove $\delta_\mu \sim m_\mu$. In fact the assumption that the normal masses do not depend crucially on the large mass parameters implies that $|\vec{m}| \sim m_e$ and $|\vec{m}'| \sim m_\mu$ up to an arbitrary rotation.

Appendix B

Here we prove the theorem on the mixing angles of the fermion-fermion-Higgs vertices stated in Section 2. We draw on the results of Appendix A. For the set of n fermions of Appendix A and the neutral physical Higgs χ the Yukawa matrix $\mathcal{Y}$ is, in the current eigenstates,

$$\mathcal{Y} = \frac{1}{v} \begin{array}{cccccc|l} \ddots & & & & & & \\ & 0 & m & & & & \\ & m & 0 & 0 & m & 0 & a_3 \\ & & 0 & 0 & m & 0 & b_2 \\ & & m & m & 0 & m & a_2 \\ & & m & m & 0 & m & b_1-a_2 \\ \hline & & a_3 & b_2 & a_2 & b_1-a_2 & \end{array} \tag{B.1}$$

($\mathcal{L}_{FFH} = - \sum_{i,j=1} \frac{m_{ij}}{v} \overline{F}_{Li} F_{Rj} \chi$ + H.c., where we assume that only one Higgs doublet gives $\Delta I=1/2$ mass to these fermions.) The matrix $\mathcal{Y}$ is proportional to (A.3) if we set the $\Delta I=0$ entries M of (A.3) to zero. In the mass eigenstates $\mathcal{Y}$ will be

$$Y = O_L \mathcal{Y} O_R^+ , \tag{B.2}$$

where the O_L and O_R are those of Appendix A. Defining

$$\mathcal{M}_0 = \mathcal{M} - v\mathcal{Y} \ , \tag{B.3}$$

the Yukawa matrix is

$$Y = \frac{1}{v} D - \frac{1}{v} O_L \mathcal{M}_0 O_R^+ . \tag{B.4}$$

Using O_L and O_R determined from the steps of Appendix A one finds the generic structure of Y to be

$$\begin{array}{cc|c|c|c|c|c}
\ddots & & & & & & \\
 & \eta m & m & & & & \vdots \\
 & m & \eta m & \eta m & m & \eta m & a_3 \\
 & & \eta m & \eta m & m & \eta m & b_2 \\
 & & m & m & \eta m & m & a_2 \\
 & & m & m & \eta m & \begin{smallmatrix} \ddots & & 0 \\ & m_2 & \\ 0 & & m_1 \end{smallmatrix} & b_1 - a_2 \\
 & \cdots & a_3 & b_2 & a_2 & b_1 - a_2 &
\end{array} \tag{B.5}$$

This proves theorem B. Note that the off-diagonal entries in the light sector are zero to order η^2. As in the gauge case, the mixing angle η will be proportional to the mass of the light fermion involved in the FFH vertex if we make the mass matrix assumption (Hierarchy principle) outlined in Appendix A.

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IV. SUPPRESSION OF LEPTON NUMBER VIOLATION MEDIATED BY $\Delta I=0$ MASS FERMIONS

In this chapter we consider the effect of adding a lepton doublet with $\Delta I=0$ mass M to the minimal Weinberg-Salam model. We then calculate the transition rate for $\mu \to e\gamma$ for all M as a function of an arbitrary mixing parameter δ of order a normal lepton mass. For $\delta \leq 0.4$ GeV M can be as low as 20 GeV. We show that heavy $\Delta I=0$ fermions decouple through their highly but naturally suppressed mixing angles with normal fermions and not through diagrammatic cancellations. Models with heavy $\Delta I=0$ fermions evade the commonly used conditions for natural suppression of rare processes in gauge theories.

In theories with two mass scales m and M, $M \gg m$, heavy particles usually decouple [1]. For particular physical cases it is interesting to confirm this explicitly and to demonstrate the mechanism of decoupling. For instance, consider the Glashow-Weinberg-Salam (GWS) model [2] which is a good symmetry above M_W. To this one can add new gauge interactions mediated by heavy gauge bosons with mass M_X, as in grand unified theories (GUTS) [3,4]. Although these new gauge bosons decouple for $M_X \gg M_W$ it does not mean that heavy particles effects are unobservable. Proton stability requires $M_X \geq 10^{15}$ GeV in SU(5) [4].

In the GWS model one can also add new heavy fermions with SU(2) invariant $\Delta I=0$ masses. We expect to find decoupling here also and a lower limit on the $\Delta I=0$ masses M from the limits on rare processes. This chapter studies this limit for $\mu \to e\gamma$. We note first that $\Delta I=0$ mass fermions are present in many non-minimal GUTS [5], and in SO(10) [6] and E_6 [7]. Their masses are in general arbitrary [8,9]. They also exist in supersymmetric models [10].

We consider a specific $SU(2)_L \times U(1)_Y$ model containing two normal generations of leptons, to be identified with the electron and muon generations, and an extra doublet with $\Delta I=0$ mass M^*. We choose this model because it is the simplest one containing all the interesting features which generalize to any $\Delta I=0$ fermion content. We then calculate the rate of the lepton number violating process $\mu \to e\gamma$ for all M as a function of a mixing parameter δ which is of order a normal lepton mass but otherwise arbitrary. We use this calculation to discuss the decoupling of heavy $\Delta I=0$ fermions, $M \gg m$, where m is a typical normal fermion mass, showing that their small mixing angles are the key reason for their decoupling [9]. The present experimental limit on the branching ratio for $\mu \to e\gamma$ allows M to be as low as 20 GeV for $\delta \leqslant 0.4$ GeV. In fact δ is $\sim \sqrt{m_e m_\mu} \sim 0.007$ GeV, if one assumes that the normal fermion masses do not change significantly with changes in the large $\Delta I=0$ mass parameters.

The content of our $SU(2)_L \times U(1)_Y$ model is two normal lepton generations and a new heavy charged -1 and heavy neutral lepton both with their left-handed (LH) and right-handed (RH) parts in an SU(2) doublet.

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*This corresponds to the addition of the fermion multiples 5_f and $\bar{5}_f$ to the minimal SU(5) GUT.

$$2\begin{pmatrix}N_L\\E_L\end{pmatrix}, 2E_R + \begin{pmatrix}N_L\\E_L\end{pmatrix}, \begin{pmatrix}N_R\\E_R\end{pmatrix} \quad (1)$$

We have also a Higgs doublet. The corresponding interaction Lagrangian is

$$\mathcal{L}_{Int} + \mathcal{L}_{Yuk} = \frac{g}{\sqrt{2}} W_\mu^+ (\overline{N}_{Li}\gamma^\mu E_{Li} + \overline{N}_R\gamma^\mu E_R) + \text{H.c.}$$

$$+ \frac{(g^2+g'^2)^{1/2}}{2} Z_\mu (\overline{N}_{Li}\gamma^\mu N_{Li} - \overline{E}_{Li}\gamma^\mu E_{Li} + \overline{N}_R\gamma^\mu N_R - \overline{E}_R\gamma^\mu E_R$$

$$- 2\sin^2\theta_w J^\mu_{EM}) \quad (2)$$

$$+ eA_\mu J^\mu_{EM}$$

$$- [\frac{1}{\sqrt{2}} c_{i\alpha}\overline{E}_{Li}E_{R\alpha}(v + \chi) + M_i\overline{E}_{Li}E_R + M_i\overline{N}_{Li}N_R] + \text{H.c.},$$

where $i = 1,2,3$, $\alpha = 1,2$, χ is the neutral physical Higgs and $v \sim 250$ GeV is the Higgs vacuum expectation value. The physical fermion mass eigenstates are obtained by diagonalizing the lepton mass matrices with large $\Delta I=0$ entries M_i and small $\Delta I=1/2$ entries $m_{i\alpha} = \frac{c_{i\alpha}}{\sqrt{2}} v$. Their diagonalization and the corresponding mixing angles will be introduced later. We denote the lepton mass eigenstates as e for the electron, μ for the muon, $\nu_{e,\mu}$ for the corresponding neutrinos, L^- for the charged heavy lepton and L° for the heavy neutral lepton. The transition $\mu \to e\gamma$ can go at one loop via the diagrams shown in Fig. 1.

To calculate the amplitude $A(\mu \to e\gamma)$ we follow closely early work by Lee and Shrock [11]. Apart from small discrepancies in some partial

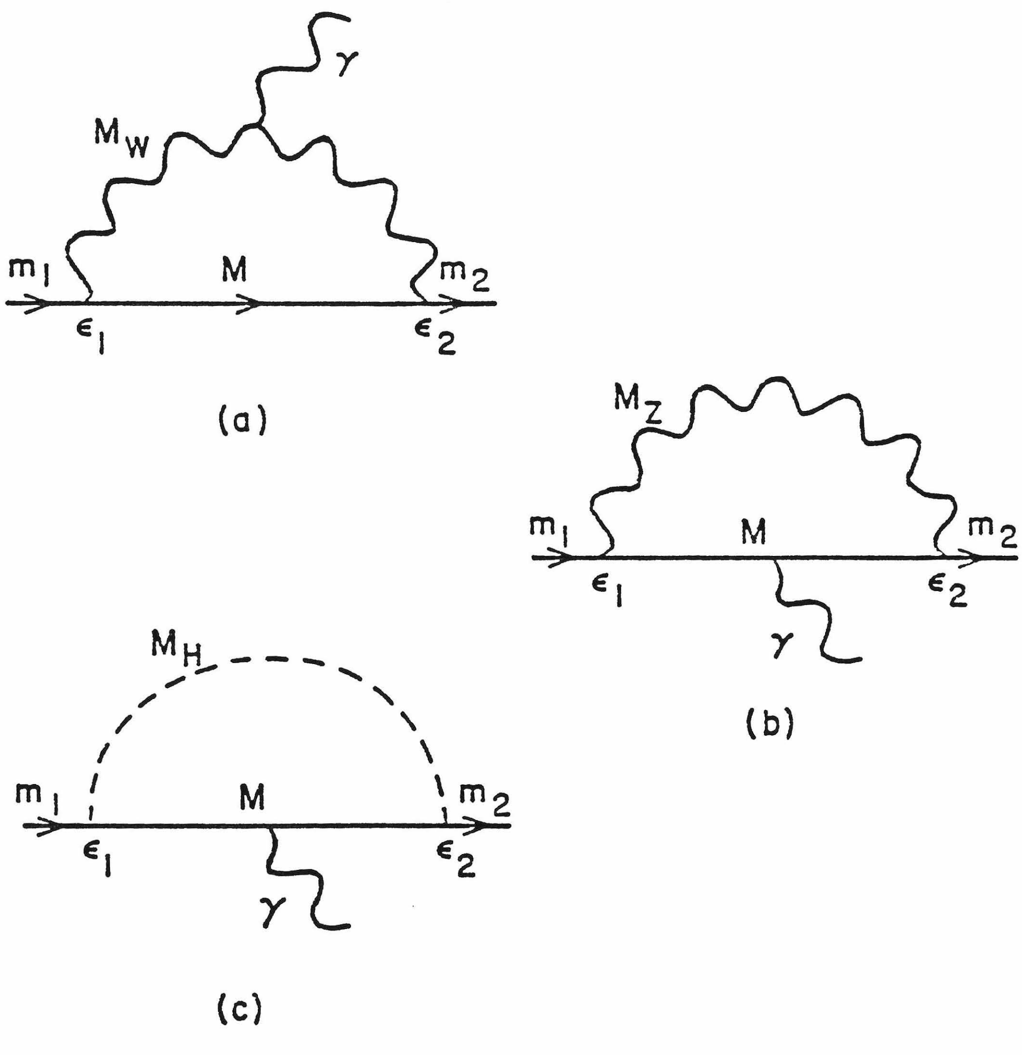

FIG 1: One loop diagrams for $\mu \to e\gamma$.

results the main difference is that we include the physical Higgs contribution and do not constrain M to be less than M_W*. As in Ref. [11] we use the ζ-limiting procedure formulated for spontaneously broken non-Abelian gauge theories by Fujikawa [14]. In this formulation, there are no vertices of the type $A_\mu W_\mu^\mp \phi^\pm$ where $\phi^\pm$ are unphysical scalar fields. Thus diagrams like Fig. 1(a) but with one internal gauge line replaced by an unphysical scalar are absent. Furthermore diagrams like those of Figs. 1(a) and (b) but with all internal gauge lines replaced by unphysical scalars give zero contribution in the limit $\zeta \to 0$. Thus only the diagrams of Fig. 1 need to be evaluated. The procedure is standard, the required Feynman rules being given in Refs. [11] and [15]. We note that the limit $\zeta \to 0$ is taken after all integrations have been done. We write

$$A[\mu(p_1) \to e(p_2) + \gamma(q)] = - i\bar{e}(p_2) \frac{i\sigma^{\alpha\beta} q_\beta \varepsilon_\alpha}{m_\mu + m_e} (F_2^V + \gamma_5 F_2^A) \mu(p_1) \tag{3}$$

where

$$F_2^{V,A} = (F_2^{V,A})_{LL,RR} + (F_2^{V,A})_{LR,RL} \tag{4a}$$

$$(F_2^V)_{LL,RR} = (m_\mu + m_e)^2 \, C_{2,a}^{LL,RR}(i) \, (\varepsilon^a_{e_L i} \, \varepsilon^a_{i\mu_L} + \varepsilon^a_{e_R i} \, \varepsilon^a_{i\mu_R}) \tag{4b}$$

*The process $\mu \to e\gamma$ has been evaluated in many different models; in particular in Ref. [12] general results were presented with special emphasis on the Chang-Li model [13].

$$(F_2^V)_{LR,RL} = (m_\mu + m_e)\, M_i\, C_{2,a}^{LR,RL}(i)\, (\varepsilon^a_{\bar{e}_L i}\, \varepsilon^a_{\bar{i}\mu_R} + \varepsilon^a_{\bar{e}_R i}\, \varepsilon^a_{\bar{i}\mu_L}) \tag{4c}$$

$$(F_2^A)_{LL,RR} = (m_\mu^2 - m_e^2)\, C_{2,a}^{LL,LR}(i)\, (\varepsilon^a_{\bar{e}_L i}\, \varepsilon^a_{\bar{i}\mu_L} - \varepsilon^a_{\bar{e}_R i}\, \varepsilon^a_{\bar{i}\mu_R}) \tag{4d}$$

$$(F_2^A)_{LR,RL} = (m_\mu + m_e)\, M_i\, C_{2,a}^{LR,RL}(i)\, (\varepsilon^a_{\bar{e}_L i}\, \varepsilon^a_{\bar{i}\mu_R} - \varepsilon^a_{\bar{e}_R i}\, \varepsilon^a_{\bar{i}\mu_L}) \tag{4e}$$

where $\varepsilon_{\bar{f}_1 f_2}$ denotes the mixing between fermions f_1 and f_2 and a sum on i (intermediate lepton) and a (diagrams (a), (b), and (c)) is understood. We find*

$$C_{2,1}^{LL,RR}(i) = -\frac{1}{32\pi^2}\frac{eg^2}{M_W^2}(Q_\mu - Q_{Fi})\Big[-\frac{1}{3} + \frac{11}{4}w_i + \frac{15}{4}w_i^2 + \frac{3}{2}w_i^3$$

$$+ \left(\frac{3}{2}w_i + \frac{9}{2}w_i^2 + \frac{9}{2}w_i^3 + \frac{3}{2}w_i^4\right)\ell n\,\frac{w_i}{1+w_i}\Big] \tag{5a}$$

$$C_{2,2}^{LL,RR}(i) = -\frac{1}{32\pi^2}\frac{e(g^2+g'^2)}{M_Z^2}\, Q_{Fi}\Big[\frac{5}{12} + \frac{1}{2}z_i + \frac{9}{4}z_i^2 + \frac{3}{2}z_i^3$$

$$+ \left(\frac{3}{2}z_i^2 + 3z_i^3 + \frac{3}{2}z_i^4\right)\ell n\,\frac{z_i}{1+z_i}\Big] \tag{5b}$$

$$C_{2,3}^{LL,RR}(i) = -\frac{1}{32\pi^2}\frac{eY_eY_\mu}{M_H^2}\, Q_{Fi}\,\frac{1}{2}\left[-\frac{1}{6}h_i + \frac{1}{2}h_i^2 + h_i^3 + (h_i^3 + h_i^4)\,\ell n\,\frac{h_i}{1+h_i}\right] \tag{5c}$$

$$C_{2,1}^{LR,RL}(i) = -\frac{1}{32\pi^2}\frac{eg^2}{M_W^2}(Q_\mu - Q_{Fi})\left[\frac{1}{2} - \frac{9}{2}w_i - 3w_i^2 - 3(w_i + 2w_i^2 + w_i^3)\,\ell n\,\frac{w_i}{1+w_i}\right] \tag{5d}$$

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*Our results for the limit $M \ll M_W$ reproduce those of Ref. [11] except for $C_{2,2}^{LR,RL}$ which differs by a piece which we can identify as coming from one part of a gauge propagator.

$$C_{2,2}^{LR,RL}(i) = -\frac{1}{32\pi^2}\frac{e(g^2+g'^2)}{M_z^2} Q_{F_i}\left[-\frac{1}{2}-\frac{3}{2}z_i - 3z_i^2 - 3(z_i^2+z_i^3)\ln\frac{z_i}{1+z_i}\right] \quad (5e)$$

$$C_{2,3}^{LR,RL}(i) = -\frac{1}{32\pi^2}\, e\, \frac{Y_e Y_\mu}{M_H^2} Q_{Fi}\left[-\frac{1}{2}h_i + h_i^2 + h_i^3 \ln\frac{h_i}{1+h_i}\right] \quad (5f)$$

where $w_i = \frac{M_w^2}{M_i^2 - M_w^2}$, and similarly for z and h. (6)

We point out that the amplitude A, modulo mixing angles ε, does not vanish for $M \to \infty$ in the gauge diagrams (a) and (b). LR(RL) contributions grow as M and LL(RR) ones remain finite and non-zero as $M \to \infty$. Higgs LR(RL) contributions decrease as 1/M for $M \to \infty$ and LL(RR) as $1/M^2$.

This behavior is quite general. We have chosen this case because there are no complications from renormalization or strong interactions. Also the experimental limit on this decay, BR($\mu \to e\gamma$) < 1.9 x 10^{-10} [16], is at least as stringent as any other on rare processes.

The behavior of A is completely different when the mixing angles are inserted [9]. These are found by diagonalizing the lepton mass matrices $\mathcal{M}^{e,n}$ with left and right unitary matrices $O_L^{e,n}$ and $O_R^{e,n}$. (We assume M to be real and expand in powers of m/M.) Rewriting the Lagrangian in the mass eigenstates,

$$n_{L(R)} = O_{L(R)}^{n} N_{L(R)}, \quad e_{L(R)} = O_{L(R)}^{e} E_{L(R)}. \quad (7)$$

We find (9)

$$\mathcal{L}_{FFG} + \mathcal{L}_{FFH} = \frac{g}{\sqrt{2}} W^{+}_{\mu} \{ \bar{n}_{Li} \gamma^{\mu} A_{ij} e_{Lj} + \bar{L}^{0}_{R} \gamma^{\mu} B_{i} e_{Ri} \} + \text{H.c.}$$

$$+ \frac{(g^2+g'^2)^{1/2}}{2} Z_{\mu} \{ \bar{\nu}_{eL} \gamma^{\mu} \nu_{eL} + \bar{\nu}_{\mu L} \gamma^{\mu} \nu_{\mu L} + \bar{L}^{0}_{L} \gamma^{\mu} L^{0}_{L} - \bar{e}_{L} \gamma^{\mu} e_{L} \tag{8}$$

$$- \bar{\mu}_{L} \gamma^{\mu} \mu_{L} - \bar{L}^{-}_{L} \gamma^{\mu} L^{-}_{L} + \bar{L}^{0}_{R} \gamma^{\mu} L^{0}_{R} - \bar{e}_{Ri} \gamma^{\mu} C_{ij} e_{Rj} - 2 \sin^2\theta_{w} J^{\mu}_{EM} \}$$

$$+ e A_{\mu} J^{\mu}_{EM} - \frac{\chi}{v} \cdot \bar{e}_{Li} Y_{ij} e_{Rj}$$

where*

$$A_{ij} = \begin{pmatrix} 1 & 0 & -\frac{m_e}{M}\eta_1 \\ 0 & 1 & -\frac{m_\mu}{M}\eta_2 \\ \frac{m_e}{M}\eta_1 & \frac{m_\mu}{M}\eta_2 & 1 \end{pmatrix} , \tag{9a}$$

$$B_i = \left(\eta_1 \quad \eta_2 \quad 1 - \frac{\eta_1^2 + \eta_2^2}{2} \right) , \tag{9b}$$

$$C_{ij} = \begin{pmatrix} \eta_1^2 & \eta_1 \eta_2 & \eta_1 \\ \eta_2 \eta_1 & \eta_2^2 & \eta_2 \\ \eta_1 & \eta_2 & 1-(\eta_1^2+\eta_2^2) \end{pmatrix} \tag{9c}$$

and

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*In chapter 2 we study a slightly more general model in which we include a charge -1 $\Delta I=0$ singlet. The mixing angles for the case here are obtained by suitably reducing the relevant matrices. Note that the mixing angles must be accurate to second order in m/M for a consistent calculation of the $\mu \to e\gamma$ rate.

$$Y_{ij} = \begin{pmatrix} m_e & 0 & -m_e\eta_1 \\ 0 & m_\mu & -m_\mu\eta_2 \\ -M\eta_1 & -M\eta_2 & M(\eta_1^2+\eta_2^2) \end{pmatrix} \tag{9d}$$

with $M^2 = \Sigma M_i{}^2$,

$$\eta_1 = \frac{1}{\sqrt{N}} \{[m_\mu^2 - \Sigma m_{i1}^2 + (\frac{M_i m_{i1}}{M})^2] \frac{M_j m_{j1}}{M^2} - [m_{i1}m_{i2} - \frac{M_i m_{i1} M_j m_{j2}}{M^2}] \frac{M_k m_{k2}}{M^2}\}$$

$$= \sin\theta \frac{M_j m_{j1}}{M^2} - \cos\theta \frac{M_j m_{j2}}{M^2} \tag{10a}$$

$$\eta_2 = \frac{-1}{\sqrt{N}} [m_{i1}m_{i2} \frac{M_j m_{j1}}{M^2} + (m_\mu^2 - \Sigma m_{i1}^2) \frac{M_j m_{j2}}{M^2}]$$

$$= -\cos\theta \frac{M_j m_{j1}}{M^2} - \sin\theta \frac{M_j m_{j2}}{M^2} \tag{10b}$$

$$\text{with } N = [m_{i1}m_{i2} - \frac{M_i m_{i1} M_j m_{j2}}{M^2}]^2 + [m_\mu^2 - \Sigma m_{i1}^2 + (\frac{M_i m_{i1}}{M})^2]^2, \tag{10c}$$

$M_{L^0} = M$ and $M_{L^-} = M\,(1 + \frac{\eta_1^2 + \eta_2^2}{2})$.

We introduce the angle θ in 10a,b merely to make it manifest that η_1 and η_2 are of order $\frac{m}{M}$, but a priori arbitrary.

Inserting the mixing angles in (4) we obtain

$$F_2^V = -F_2^A = m_\mu\eta_1\eta_2\ \{\frac{1}{2} m_\mu[C_{2,1}^{LL,RR}(M) + C_{2,1}^{LR,RL}(M)]$$

$$+ \frac{1}{4} m_\mu\ [-2\sin^2\theta_w\ (C_{2,2}^{LL,RR}(m_e{\sim}0) + C_{2,2}^{LL,RR}(m_\mu{\sim}0)) \tag{11}$$

$$+ C^{LL,RR}_{2,2}(M) + (1-2\sin^2\theta_w)\, C^{LR,RL}_{2,2}(m_\mu \sim 0)\Big]$$

$$+ \frac{M^2}{m_e}\Big[C^{LL,RR}_{2,3}(\sim M) + C^{LR,RL}_{2,3}(\sim M)\Big]\Big\}$$

(The V-A structure of (11) is particular to this model and order*, ignoring the electron mass.) Thus the final result of incorporating mixing angles is to make each contribution suppressed by the same factor $\delta^2/M^2 \equiv \eta_1\eta_2$. In Table 1 we present the corresponding branching ratios $B(\mu \to e\gamma) = \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\bar{\nu}_e\nu_\mu)$ for various values of the heavy $\Delta I=0$ fermion mass M and Higgs mass M_H as a function of δ. We find that for $\delta \leqslant 0.4$ GeV, M can be as low as 20 GeV. In general δ, although of order a normal lepton mass, is only specified with further assumptions about the mass matrix. We argue later that a plausible value of δ is given by $\sqrt{m_e m_\mu} \sim 0.007$ GeV, in which case M is only constrained by the lower bound on its direct production [17].

Equations (9) exhibit the important features of this model, namely the dependencies on m/M of the different mixing angles which lead to the relatively small lower bounds on the $\Delta I=0$ masses M. They are: (i) flavour changing neutral current vertices involving normal fermions are suppressed by at least m^2/M^2; (ii) gauge vertices involving normal and

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*We see that in [11] the mixing angles appear as an overall factor with no arbitrary parameters. In a more general case there will appear different mixing angles in different terms with arbitrary parameters to be fixed. The simplicity of the situation here leaves no possibility of intricate cancellations. We have studied the model with a $\Delta I=0$ singlet and doublet [9] but the extra arbitrariness makes the final results not so clear.

heavy $\Delta I=0$ fermions which change isospin by 1/2 (here right-handed (RH) normal and heavy $\Delta I=0$ fermions) are suppressed by a factor m/M whereas those which do not change isospin (here left-handed (LH) normal and heavy $\Delta I=0$ fermions) are suppressed by a factor m^2/M^2; (iii) Higgs vertices involving normal and heavy $\Delta I=0$ fermions which change isospin by 1/2 (here those with a RH normal and a LH $\Delta I=0$ fermion) are order a Yukawa coupling whereas those which do not change isospin (LH normal and RH $\Delta I=0$ fermion) have an extra m/M suppression.

In the light of the above features we discuss the most important contributions of each diagram. The dangerous contribution from diagram (a) is that due to the exchange of the heavy neutral $\Delta I=0$ lepton since it is proportional to the heavy lepton mass M up to mixing angles. When these are incorporated the net dependence is $M(\frac{m}{M})(\frac{m^2}{M^2}) \sim m\,\frac{m^2}{M^2}$, according to (ii). Diagram (b) can exchange a muon - in this model there is an $\bar{e}\mu Z$ vertex - but according to (i) the net contribution goes also as $m\,\frac{m^2}{M^2}$. Finally the Higgs diagram (c) with exchange of the heavy $\Delta I=0$ charged lepton has the same suppression $m\,\frac{m^2}{M^2}$ which comes not from the mixing angles (see (iii)) but the diagrams themselves (see eqns. (5c,f).

Motivated by this model and the results of a general study on mixing angles [9] we observe that any $SU(2)_W \times U(1)_Y$ model obtained by adding $\Delta I=0$ mass fermions to the GWS model can be written as follows

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_M + \mathcal{L}^G_{m/M} + \mathcal{L}^H \qquad (12)$$

where $\mathcal{L}_m$ is the light Lagrangian obtained by deleting the heavy fermions fields and in our case is the minimal GWS model; $\mathcal{L}_M$ is the rest of $\mathcal{L}$ when GWS is unbroken and no mixing exists; $\mathcal{L}^G_{m/M}$ contains all terms which result from gauge mixing between light and heavy fields and so vanish when the heavy fields become infinitely massive, and $\mathcal{L}^H$ contains all the Yukawa interactions which are not in $\mathcal{L}_m$. The tree level interaction Lagrangian (8-10) of our model can be written directly in the form (12).

We now describe how the decoupling of heavy fermions arises. In terms of the effective low energy theory $\mathcal{L}_M$ simply renormalizes bare parameters. At one loop the vertices in $\mathcal{L}_{m/M}$ appear in two ways, those in which the intermediate heavy fermion line conserves chirality (giving a constant diagrammatic contribution m) and those which change chirality (giving a contribution proportional to M). In the first case there is a suppression $m(\frac{m}{M})^2$ from the two gauge vertices. In the second case we find an extra suppression. This happens because the low energy theory has LH doublets and RH singlets whereas the heavy fermions have LH and RH parts in the same type of multiplet giving an extra suppressed mixing with one of the chiralities [9]. Hence the net suppression is $M(\frac{m}{M})^3 \sim m(\frac{m}{M})^2$ as before. In the Higgs case the origin of the suppression is in the diagrams as opposed to the mixing angles with the same net suppression.

This result should generalize to higher orders [18]. Our analysis, which can be extended to any $\Delta I=0$ fermion content and rare process, relies on having only one Higgs giving mass to each class of normal fermions [9]. However this is a general constraint for models to naturally suppress rare processes [19,20]*.

It should be noted that any $SU(2)_L \times U(1)_Y$ model containing new $\Delta I=0$ fermions violates the common rules for natural suppression of rare processes [11,19]. Nevertheless these $\Delta I=0$ fermions can naturally have low masses**. Using the upper bound for the $BR(\mu \to e\gamma)$ listed in Table 1 we see that $M \geq 50\ \delta$ will always give a branching ratio less than the experimental limit. To place an absolute lower bound on $\Delta I=0$ masses M from rare processes one must consider tree level processes such as $\mu \to \bar{e}ee$ [9]. This gives $M \geq 125\ \delta$***. To specify how low these masses can be then depends on fixing δ. Assuming that the normal fermion mass spectrum does not change significantly with changes in the large mass parameters (M_i in eqn. (2)), one can show that $\delta \sim \sqrt{m_e m_\mu} \sim 0.007$ GeV. To see this consider the limiting case $m_e=0$. According to our assumption above m_{i1} must be parallel to m_{i2} in the mass matrix

$$M^e = \begin{pmatrix} M_i & m_{i1} & m_{i2} \end{pmatrix} \tag{13}$$

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*Supersymmetry violates these constraints, introducing many new Higgses. The particular couplings and masses used in many supersymmetric models allows them to have naturally small flavour changing neutral currents.

**A model with the same conclusion was proposed by Cheng and Li [13]. There also the mixing angles play a key role. In this model the $\Delta I=0$ masses are small and the $\Delta I=1/2$ masses large, the opposite of our case, but the diagonalization and consequent mixing follows parallel lines.

***These processes have the same mixing angle suppression as the one loop processes but they do not have the characteristic one loop suppression factor $\sim \alpha$.

This gives $\eta_1 = 0$ in (4.10a) since now

$$m_\mu^2 = \Sigma\, m_{i1}^2 - \left(\frac{M_i m_{i1}}{M}\right)^2 + \Sigma\, m_{i2}^2 - \left(\frac{M_i m_{i2}}{M}\right)^2 \qquad (14)$$

In general m_{i1} and m_{i2} will be proportional to order $\frac{m_e}{m_\mu}$ and then $\eta_1 \sim \frac{m_e}{m_\mu}$. The $m_{i\alpha}$ entries are generally of order m_μ so we estimate $\eta_2 \sim \frac{m_\mu}{M}$. Thus $\delta = M\sqrt{\eta_1\eta_2} \sim \sqrt{m_e m_\mu}$. This case gives a lower bound on M smaller than the experimental limit of direct production.

	M=20 GeV	50 GeV	100 GeV	M large
M_H = 10 GeV	2×10^{-9}	9×10^{-12}	9×10^{-13}	$10^{-3}\ M^{-4}$
M_W	6×10^{-9}	3×10^{-11}	3×10^{-13}	$10^{-3}\ M^{-4}$

Table 1. $B(\mu \to e\gamma)$: All numbers are multiplied by δ^4, where δ (in GeV) is a model dependent mixing parameter of order a normal lepton mass.

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