Essays on Early-stage Financing and Firm Behavior

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To my parents, whom I haven't seen for years!

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ABSTRACT

The first chapter of this thesis studies the role of angel finance in the early-stage capital market. Despite anecdotal evidence connecting angel and venture capital (VC) financing, there is little systematic evidence on how the two early-stage capital sources interact. To study this topic, I assemble the first comprehensive dataset on angel financing and characterize its size, scope, and role in the early-stage capital market. I use the population of newly incorporated startups located in California, the largest VC financing state in the United States. Here, the angel capital market is large: approximately 4% of all startups receive angel financing within three years of incorporation. At least five times as many startups receive financing from angels as from VCs in the VC-active industries. Using local individual income as an instrument for angel financing at the zip code level, I show that angels play both supportive and competitive roles in relation to VCs. Angel financing leads to more VC follow-on financing over firms' life cycles (complement), while it crowds out VC financing from the initial financing round (substitute). My results demonstrate the explicit role of angel financing in the early-stage capital market.

In the second chapter, I develop a game-theoretic model to study information asymmetries in the evolving equity crowdfunding market. I assume (1) there are two types of investors: informed ("insiders") and uninformed ("outsiders"); (2) the insiders invest first; and (3) the outsiders observe the aggregate of insiders' actions and then decide whether to invest. Under these assumptions, I prove that there does not exist a crowdfunding market equilibrium in which the insiders' information is aggregated and high quality startups are funded with higher chances. I then use data from Regulation crowdfunding (Title III equity crowdfunding), and provide evidence that is consistent

with the model implications. My results suggest that adverse selection is a primary barrier to equity crowdfunding, and new market designs are required to better develop this market.

The third chapter is joint work with Matt Elliott. We model firms as sets of scarce capabilities, where each capability provides a source of competitive advantage in some markets. Each market is also associated with a set of capabilities that are valued by it. Firm and market hypergraphs represent this information. Our approach provides a new perspective on several industrial organization literatures including merger analysis, strategic alliances and industry dynamics. We argue that merger analysis should be more holistic and that profitable joint ventures increase consumer surplus even when they reduce competition. We also provide formal foundations for a prominent theory of competitive advantage in the management literature.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

WHAT ROLE DOES ANGEL FINANCE PLAY IN THE EARLY-STAGE CAPITAL MARKET?

1.1 Introduction

A well-developed early-stage capital market has long been recognized as crucial in supporting high-growth entrepreneurship and innovation.[1] Among early-stage investors, financial intermediaries venture capitalists (VCs) attract most of the attention. Yet Puri and Zarutskie [84] show that as few as 0.11% of all new firms raise money from VCs. Their work raises a puzzle: how are the rest of startups funded? Anecdotal evidence suggests that the angel capital market plays an important role in financing startups [e.g., 82, 90]. Famous angel investing examples run as far back as Henry Ford, and as recently as Google and Facebook, further suggesting an important role for non-intermediated early-stage capital in funding innovative startups. A lack of systematic data, however, has severely limited our ability to study non-VC early-stage finance. In particular, little is known about the exact role of non-VC early-stage finance: how does it fit into the early-stage capital market? To study this question, using a combination of state and federal government filings, PCRI[2], and Pitchbook, I assemble a comprehensive dataset on private non-VC security[3] issues by incorporated firms less than three years

[1]For example, see Schumpeter [88], Evans and Jovanovic [28], King and Levine [59], Brown et al. [15], and Kerr and Nanda [56].

[2]PCRI is a private capital database, built by academic researchers such as Josh Lerner, with a primary goal to provide a comprehensive and centralized private capital database for academic use. See more at: `http://www.privatecapitalresearchinstitute.org/index.php`

[3]In this paper, I focus on equity financing or, more generally, security financing. In the United States, the definition of security is broad, and security includes any type of investment contracts. However, it does not include formal debt financing such as bank loans. Robb and Robinson [85] use Kauffmann survey data and find that newly founded firms also frequently use formal debt financing.

old (henceforth "startups") in California (CA), which I will refer to as angel financing.[4] With this dataset, I quantify the size and scope of angel financing, and characterize its role in the early-stage capital market through its interaction with VC financing.

I start by quantifying the size of the angel capital market. Consistent with current survey-based estimates of the size of the U.S. angel market, I find that the angel market is large. In California, angels invest about four billion dollars each year in the initial financing for young corporations. About 4% of all incorporated CA firms receive angel financing within three years of their founding. This fraction of firms supported by angels is ten times greater than the 0.37% of firms funded by VCs among California corporations. Even if one focuses on the industries where VCs are active, angels also fund over 90% of initial security issues. Using the VC market as a benchmark, the estimates of the angel capital market using CA data is likely to be a lower bound for the U.S. angel market, because CA is the largest VC financing state in the United States [e.g., 81].

Although the angel capital market is large, it may not support the high-growth, innovative startups that comprise most VC portfolios [see e.g., 35]. For example, survey data show that angels tend to invest in industries that they know, which are not necessarily high-growth, or high-tech industries [89]. To examine whether angels invest in high-growth startups, I calculate the fraction of VC investments preceded by angel capital. For firms that receive VC financing following an angel round, I refer to them as receiving VC follow-on financing. I find that firms receiving VC follow-on

[4] Due to the difficulty in observing social relationships, it is usually hard to distinguish different types of individual investors by social relationship. For example, a professional individual investor could have known the entrepreneur before investing, in which case the boundary between friend investor and so-called business angel investor blurs. So, similar to previous literature (Hellmann, Schure, and Vo, 2013; Kauffman Firm Survey), angel investors in this paper are any non-founder individual investors.

financing account for at least 30% of VC funded firms. So a large fraction of VC funded firms actually start with angel capital. This rejects the idea that angels do not invest in high-growth startups.

Given the large fraction of VC follow-on financing, a natural question arises about the dynamic interaction between the angel and VC markets. Scholars have long argued over the positive impact of angel financing on VC financing over firms' life cycles.[5] Angel investors may play an important role in pre-screening and adding value to startups. So a well-developed angel market may lead to a large number of high quality pre-screened startups for VCs, and thus increase VCs' investment opportunities and deal flows. Therefore, a larger angel capital market should produce a larger complementary VC market. I formalize this argument as the "VC follow-on hypothesis": a larger number of startups financed by angels lead to more VC follow-on financing. The alternative view to the follow-on hypothesis is that angels may not play any complementary role for VCs. For example, angels may invest in startups with limited growth or innovation potential [49]. In this scenario, angels and VCs would operate in separate markets, and we would thus not observe a positive link between them over firms' life cycles. To decide between the two views, I exploit market-level variations, and carry out my analysis at the zip×sector×year level. Using an ordinary least squares (OLS) regression framework, I find that a higher level of angel financing at the initial round is associated with a higher level of VC follow-on financing. This finding provides evidence consistent with the VC follow-on hypothesis, suggesting a positive role angels play in supporting VCs.

VCs also fund initial capital infusion: 70% of first VC investments are not preceded

[5] See e.g., Freear and Wetzel [33], Harrison and Mason [43], and Hellmann and Thiele [44].

by an angel round. This environment creates another angle through which to investigate the interaction between angels and VCs: at the initial financing round, angels and VCs may compete. In this competition, VCs have both advantages and disadvantages over angels. For example, as shown by Hsu [48], startups value VCs' certification and value-adding roles beyond the financial capital provided. So professional VCs could provide better certification and value-adding services than angels. On the other hand, VCs likely face higher transaction costs. For example, as argued by Lerner [69], VCs rarely invest small amounts of money in startups. To investigate the competition between angels and VCs, I formalize a second hypothesis in my paper, the "VC crowd-out hypothesis": angel financing crowds out VC financing at the initial financing round. The alternative view to the crowd-out hypothesis is that angels cannot compete with VCs, and are only able to fund VCs' rejected deals. Were this the case, we would not see a crowd-out effect. Again, to evaluate the crowd-out hypothesis, I conduct a market level analysis at the zip×sector×year level. Consistent with the crowd-out hypothesis, I find that at the initial financing round, a higher level of angel investing is associated with a lower level of VC investing.

The OLS regressions provide evidence consistent with both the VC follow-on and crowd-out hypotheses, yet we cannot draw causal inferences from them. In particular, it is challenging to investigate the causal impact of angel financing on VC financing because angel financing is endogenous. To overcome this difficulty, an ideal experiment would be to randomly select some zip codes, locate additional angel investors there to increase angel financing, and then compare VC financing in the treated zip codes with the control zip codes. Doing so could eliminate any confounding issue (e.g., from the startup capital demand side), and isolate the causal effects of angel financing. In the absence of such variation, I instrument angel financing by a measure of local high income: the fraction of households with annual income greater than $100,000 at each zip code and year (henceforth "share rich").[6]

The main advantage of this instrument is that it is directly related to the supply of angel capital, which is very local. It is also unlikely to be correlated with the capital supply of VCs because VCs' supply mainly comes from geographically dispersed institutional investors such as pension funds [37]. This advantage ensures that the instrument is likely to be valid. In particular, it should satisfy the exclusion restriction: the share rich links to VC financing only through angel financing, and not through either VC supply or demand. As discussed, the supply side is not an issue. The condition that the share rich does not link to VC demand also holds because I examine the geographic proximity of VC investments and find that the demand for VC financing and local economic factors, such as income and wealth, are not positively correlated at the zip code level (see a detailed discussion in Section 1.3.4).

Using this instrumental variables (IV) approach, I find that six additional initial angel financings lead to one additional VC follow-on financing in the next two years. This economic magnitude is even more meaningful when translating into percentages: a 10% increase in angel financing leads to a 13% increase in VC follow-on financing in the next two years. At the same time, I also find that at the initial financing round, angel financing crowds out VC financing: an increase of three initial angel financings leads to a decrease

[6]The "share rich" measure is derived from the IRS individual income tax reports, and $100,000 is the only threshold (cutoff) consistently used by the IRS to report aggregate data in brackets during my data period. According to the 2013 Survey of Consumer Finances (`https://www.federalreserve.gov/econres/scfindex.htm`), $100,000 roughly corresponds to the 80th percentile of family income distribution in the United States. The survey also provides statistics that 36.3% of families within the top 10% income hold business equities, in contrast to less than 5% of families in the bottom 50% doing so. These statistics justify the use of $100,000 as an effective threshold to capture the households with a high likelihood of investing in startups.

of one initial VC financing at the market level. Overall, my IV results confirm those obtained in the OLS regressions, demonstrating that angel financing has causal impact on VC financing. My findings about the VC follow-on and crowd-out hypotheses together suggest that an increase in angel financing generates two simultaneous responses from VCs: VCs increase their follow-on financing of angel-backed startups, while reducing their direct investing in brand new startups. In short, faced with more angel financing, VCs shift to fund the later stages of startups. This shift could improve the efficiency of the whole early-stage financing market, because each source of capital can specialize in segments of the early-stage financing market where they have competitive advantages. As a result, this improved efficiency of the early-stage financing market could benefit the whole entrepreneurial and innovative ecosystem.

To uncover the channels driving the VC follow-on hypothesis, I examine the survival of the startups that raise their initial financing from angels, and the survival of the startups that start with angel financing and then receive VC follow-on financing. I find that the startups raising their initial financing from angels fail more quickly than those that raise their initial financing directly from VCs. On the other hand, the angel-backed startups that receive VC follow-on financing actually survive longer than the startups that raise their initial financing from VCs. This evidence implies that angels invest in riskier startups with a higher failure rate for the initial financing, and then supply a set of less risky startups to VCs for follow-on investing. So angels reduce the risk of startups for VCs.

To provide further confirmation of the VC crowd-out hypothesis, I study how the distribution of angel investments responds to the overall angel investing activity at the zip$\times$year level. I find that as the volume of angel investments increases, the frequency of sophisticated securities (preferred stock and convertible note) being used in angel investments falls, while angels fund older startups with larger amounts of capital. This finding suggests that an increase in the supply of angel capital can lower the cost of angel capital for startups, reduce angels' relative disadvantage to VCs in providing large amounts of capital, and then induce startups to switch funding from VCs to angels at the initial round.

This paper is related to several strands in the literature. First, there is a small literature focusing on the interaction between angels and VCs. The paper closest to mine is Hellmann et al. [45], who use a sample of Canadian firms and show that VCs and angels are dynamic substitutes across financing rounds. My paper differs from theirs by focusing on the early period financing where the role angels play is very different. Hellmann and Thiele [44] also study the interaction between angels and VCs by building a theoretical model. They derive several important market characteristics such as deal flows and market sizes across the two markets. Consistent with their paper, my paper provides empirical evidence that the size of the angel market has a positive impact on the size of the VC market. Goldfarb et al. [34] use a dataset from a bankrupt law firm and show that VCs obtain more aggressive control rights, and startups with both angel and VC investors at the Series A round perform worse than startups with just VCs for Series A funding. Related to their paper, I show that in terms of survival rate, startups starting with angel and then receiving VC funding (at different financing rounds) actually perform better than startups starting with VCs. Second, this paper is also connected to studies on the impact of angel investing. In particular, Kerr et al. [57] and Lerner et al. [68] show that angel groups have a positive impact on the growth and survival of startups. Lindsey and Stein [71] exploit the change of investor accreditation rules in the 2010 Dodd-Frank Act to study the impact of angel financing, and they find that a large reduction in the pool of qualified angel investors negatively affects firm entry and employment levels for small entrants. Unlike these papers, I study the causal impact of angel financing on VCs. Third, this paper also broadly relates to a few other recent papers on angel financing.[7] For example, Bernstein et al. [10] run random experiments on AngelList (an electronic investment platform) and find that the founding team is causally important for access to angel funding.

The crowd-out hypothesis in this paper reveals that, compared with VCs, angels may have certain advantages in investing in startups, e.g., providing smaller amounts of capital better fitting startups' demand. In this regard, the paper is also related to papers that have suggested that the size of VC investments may be too large for startups. For example, Ewens et al. [29] show that it takes less capital to start and run startups in IT sectors.[8] Lerner [69] argues that VCs rarely invest small amounts of money in startups, and VCs' large investment size problem may be a consequence of VC firms' organization and can get worse over time. Finally, this paper also relates to a broad VC literature (see survey papers such as Da Rin et al. [22] and Metrick and Yasuda [74] for a comprehensive coverage of that literature).

The remainder of the paper is structured as follows: Section 1.2 describes the data. Section 1.3 characterizes the angel capital market. Section 1.4 shows that angel financing leads to VC follow-on financing. Section 1.5 shows that angel financing crowds out VCs from initial stages. Section 1.6 concludes.

[7] See Morrissette [77] for a good coverage of some older literature on angel financing.

[8] Outside the academia, the well-known angel investor Paul Graham argues that "because starting a startup is so cheap, VCs now often want to give startups more money than the startups want to take. VCs like to invest several million at a time..." -`http://paulgraham.com/vcsqueeze.html`

1.2 Data

To study the angel capital market, I assemble a dataset that covers angel and VC investments from 2004 to 2014 for startups that are located in California (CA). Collecting a comprehensive dataset on angel investing is challenging because these investments are often made on an individual basis, thus they are not subject to either state or federal disclosure requirements. I take advantage of California's blanket requirement that all securities issues be reported.

The firm-level dataset is assembled from three main data sources, and then matched with additional information about local capital market conditions. I start with CA incorporation records, which provide firm-level information for the universe of corporations located in CA.[9] Next I collect all CA firms' security exemption filings. These filings provide information about CA corporations' early-stage securities issues. I then use VC databases (PCRI and Pitchbook) to identify these securities issues as angel- or VC-backed. Finally, I match these data with zip code level IRS tax reports. In the following section, I review each data source with more details.

1.2.1 CA business registration records: firm founding and industry

I start with the corporation registrations filed with the CA secretary of state[10]. By law, all corporations and LLCs doing business (either generating income or maintaining tangible property) in CA have to register with the California Secretary of State and report their company address, founding date, and entity type in the registration.

I also use a complementary data source of the "statement of information" filings

[9]VCs rarely invest in unincorporated firms. Moreover, Levine and Rubinstein [70] show that entrepreneurs owning incorporated businesses tend to be more educated and earn more, and starting incorporated businesses is a good proxy for entrepreneurship.

[10]See more at `https://businesssearch.sos.ca.gov/`. Guzman and Stern [40] use the same data source to study the founding and growth of entrepreneurial firms.

(also from the CA Secretary of State) to obtain CA firms' sector information. Together, these two sources allow me to characterize all new corporations founded between 2001 and 2014, including both those that raise private capital (from angels and VCs) from 2004 to 2014, and those that do not. The initial dataset contains 1.27 million corporations.

1.2.2 Security exemption notices: form D filings

Under the federal Securities Act of 1933:

> *"In general, all securities offered in the United States must be registered with the SEC or must qualify for an exemption from the registration requirements."*
> (-SEC: `https://www.sec.gov/fast-answers/answersregis33htm.html`)

Young firms almost always rely on exemption rules to avoid the cost of registration with the SEC.[11] The commonly used federal exemption rules include so-called "private placement offering" (Section 4(2)) and "intrastate offerings" (Section 3(a)(11))[12]. Under both federal rules, the SEC does not impose any filing requirements on the issuers and instead leaves each state to decide their own regulation and filing rules.

In California, however, the state securities law states:

> *"It is unlawful for any person to offer or sell in this state (CA) any security in an issuer transaction, whether or not by or through underwriters, unless such sale has been qualified, or unless such security or transaction is exempted."*
> (California Corporations Code Section 25110)

[11] Registering securities with the SEC requires firms to provide financial statements "certified by independent accountants", and the registration files become public shortly after the companies file them. So registering securities is both financially and informationally costly.

[12] For example, if a CA-registered firm sells securities only to CA investors, the offering does not fall under the jurisdiction of the SEC and is automatically exempt from SEC regulations.

So CA regulatory requirement triggers the state's own exemption rule: Section 25102(f).[13] This rule requires firms to file security offering exemption notices (also called "limited offering exemption notice") with the Department of Business Oversight (DBO) in CA. Since most of early-stage security offerings are small and local, they naturally satisfy the federal conditions for private placement or intrastate offering exemptions, and thus do not need to file at the federal level. Without a state requirement, exempt firms would file nothing at all and thus not be observable. CA security exemption notice filings, thus, constitute my core source for private equity placements, and they are abundant. According to the DBO, Section 25102(f) exemption notices are the most popular filing forms with the department across all types of filings (250,460 filings from 2004 to 2014).

Despite their abundance, 25102(f) filings are still a subset of CA firms' security exemption notices. Because it is difficult to classify one offering as either "private" or "public", the SEC adopted Reg D (Rules 504, 505, 506) to provide safe harbor for private placements. Firms that fall under these rules must file a Form D notice on the SEC's Edgar website.[14] Reg D issues are called covered securities, and they are not necessarily subject to state regulations. For this reason, I also collect CA firms' SEC form D filings (41,725 SEC form D filings between 2004 and 2014).

For California firms, CA 25102(f) and Reg D (mainly Rule 506) are the only two major exemption rules used for private securities offerings (see Table C.1 in the Appendix for an exhaustive list of exemption rules available for private securities offerings in the United States). Overall, CA 25102(f) and Rule 506 are very similar: they both prohibit public advertising, and set an upper limit (35) for the number of non-accredited

[13]http://www.dbo.ca.gov/Codes/25102f.asp

[14]https://www.sec.gov/edgar/searchedgar/companysearch.html

investors allowed in each offering. Because of their similarities, henceforth, I will refer to the CA 25102(f) filings as CA form D, and the two pieces CA form D and SEC form D together simply as form D filings.

The form D filings provide rich and comprehensive information on California firms' early-stage financing. However, there is also a limitation: non-compliance. Because filing form D entails some costs and there are no serious legal consequences for firms that fail to file, some firms may not comply with filing requirements. On the other hand, the extent of the non-compliance issue may be also limited because of the benefits associated with filings. In particular, filings reduce complexity when conflicts arise between investors and entrepreneurs. Late filings could be subject to an increased filing fee in CA. To check for non-compliance and related sample selection issues, I compare Pitchbook's VC financing data with form D filings, and find that VC compliers have raised slightly more money and have more rounds of financing than non-compliers (see Table C.2 in the Appendix). If compliance choices are similar between VCs and angels, we will more likely observe angel financing in startups that raise more money with more financing rounds. In any case, the compliance issue will produce a downward bias on the estimates of the size of the angel market.

Taken together, the data provide the most comprehensive information on non-VC early-stage financing available to the literature. In particular, my dataset has at least four advantages compared to other sources. First, the dataset comes from government's filings, and thus is more accurate than self-reported data such as Crunchbase.[15] Second, the dataset dates back to the early 2000s and thus covers a longer time span than other sources. For example, Crunchbase starts coverage of angel financing only from 2013.

[15] Crunchbase is a popular database on angel financing: `https://www.crunchbase.com/`

Third, the dataset has a particularly wide range of coverage across almost all locations in a large region (CA). Commercial VC databases such as Pitchbook also collect some data on angel financing, but they mainly do so for firms that later receive VC funding. Clearly, these data are selected to be in VC active areas and industries. Fourth, the dataset covers the most active U.S. state in equity financing. In terms of VC activity, CA accounts for almost half of total VC activity in the United States [81]. Using the same ratio of CA VC activity accounting for the U.S. VC activity, the angel investments of CA firms in the dataset could account for half of overall angel activity in the United States during the data period.

1.2.3 VC data: PCRI and Pitchbook

I use two important VC databases (PCRI and Pitchbook[16]) to identify the investor type of a certain financing in the security exemption notices. I use both VC data sources to maximize existing data's coverage on VC financing, and minimize the number of false positives in identifying angel financing.

1.2.4 Zip code level aggregate data: IRS and Zillow

My sources for the zip code level data are from IRS and Zillow. These data sources provide several key zip code level economic statistics. The individual income tax data from the IRS (2004-2014) provide information about aggregated income statistics in different adjusted gross income (AGI) brackets. This special structure of tax data (in brackets) is particularly valuable in measuring the income distribution within zip codes, and it allows me to investigate the links between the angel market and income distribution within and across zip codes. The Zillow data provide an alternative estimate of wealth:

[16]PitchBook, founded in 2007, is one of the industry's leading sources for information on private capital markets, including venture capital and private equity.

the zip code level median estimated home value[17].

1.2.5 Define angel vs. VC

The procedure for integrating corporate registrations, form D filings, and the VC data is detailed in the Appendix. In this paper, I assume that the two VC databases combined (PCRI and Pitchbook) have a relatively complete coverage of VC funded firms in California,[18] and then I rely on the VC information to identify each firm in the form D dataset as either angel- or VC-backed. Specifically, my identification proceeds for three sets of firms. First, for the set of firms that file form D but do not appear in the VC data, they raised money from non-VCs, so I identify them as angel-backed. Second, for the set of firms that appear in both form D and VC data, I identify them as either angel- or VC-backed according to the financing dates indicated in form D filings and VC databases. Specifically, if a firm filed a form D 180 days earlier than the financing date of their first VC financing, then I assume that the firm has raised an angel round before its first VC round, and thus identify this firm as angel-backed. Since the firm also receives a VC financing after its angel round, I also say that this firm has a VC follow-on financing. For the rest of the firms in the second set that are not angel-backed, I classify them as VC-backed. According to the security laws, firms have to file security exemption notices within 15 days of their first sale. Therefore, 180 days is enough time to separate different financing rounds in the data. I also used other thresholds to make robustness checks, and the analysis is not sensitive to this threshold. Third, for the set of firms that appear in the VC but not form D data, they are VC non-compliers of security

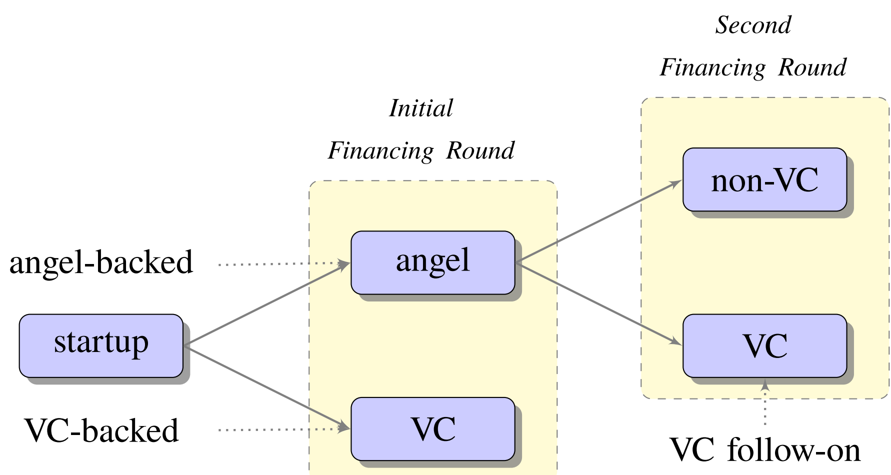
[17] The statistics are estimated by Zillow for all home types, including single-family, condominium, and co-operative homes within a zip code.

[18] Indeed, the total number of VC funded firms is not large, commercial VC databases usually have many resources to collect data, and PCRI and Pitchbook are among the major VC data sources. See more discussion about VC data in Kaplan and Lerner [54].

exemption rules. To make compliance consistent across investor type (VC and angel), I exclude the VC non-compliers from my analysis in this paper, and focus on the sample conditioning on compliance.[19] Figure 1.1 illustrates how a firm is classified as angel- vs. VC-backed according to its financing trajectory.

Figure 1.1: CLASSIFICATION OF STARTUPS AS ANGEL- VS. VC-BACKED

Notes: This figure depicts how a startup is classified as angel- vs. VC-backed according to its possible financing trajectory in multiple rounds of financing. If a startup raises its initial financing ("initial financing round") from at least one VC investor, it is classified as VC-backed. Otherwise, if it raises its first security financing from non-VC investors, it is classified as angel-backed. Additionally, if an angel-backed startup raises its second round of financing from at least one VC investor, it is referred to as receiving a VC follow-on financing.



1.2.6 Sample cleaning

Having classified all form D firms, I take several further steps to clean the sample at the financing level. In particular, I drop financings that occur within 15 days of incorporation to avoid possible stock issuance among founders. To focus on the study of early-stage finance (financing of startups), I also drop financings that occur three years after incorporation. The first two steps drop 31,336 financings from my sample.

[19] At the firm level, the VC non-compliance rate is around 40%. Another reason for dropping VC non-compliers from my sample is that I cannot use security exemption filings to identify whether they have raised angel capital prior to VCs or not.

In addition, I drop all offerings smaller than $10,000[20] because small offerings may not represent a serious financing from outside investors. Nevertheless, small offerings account for a large fraction of all filings, and 87,596 such financings are dropped from my sample. Robustness checks verify that my results are not sensitive to the choice of $10,000 for defining small offerings. Finally, I drop financings that are larger than $50m, because they are not likely to be early-stage, e.g., the 99th percentile of first VC investments is only $38m (see Table C.3 in the Appendix). The cleaning process reduces my sample from 178,838 to 43,912 financings.

1.2.7 Industry classification

I also need to classify CA firms by industries. In the statements of information filings, firms describe their type of business in their own wording. To make this industry information useful, I map these descriptions into a consistent industry categorization. Specifically, I map them into SEC form D industries, a set of predetermined industries by the SEC on the electronic SEC form D filings (from 2009 on). My dataset already includes a set of firms that have filed electronic SEC form Ds with their industry reported in the filings, and then I use this subsample to extend their industry categorizations to the other firms in my sample. In particular, if a firm in my dataset does not have industry categorization but shares a common key phrase in its business description with another firm already being categorized, then it gets the same industry categorization. The key phrases are either one or two words, constructed out of all the firms' business descriptions according to their occurrence frequency. See Table C.4 in the appendix for a correspondence between key phrases and industry categorizations.

[20]The choice of $10,000 as the threshold of small offerings is consistent with conventional wisdom. For example, the well-known angel investor Paul Graham suggests that "the lower bound [of angel investments] is 5-10% of the total [amount the founders want to raise] or $10,000, whichever is greater."

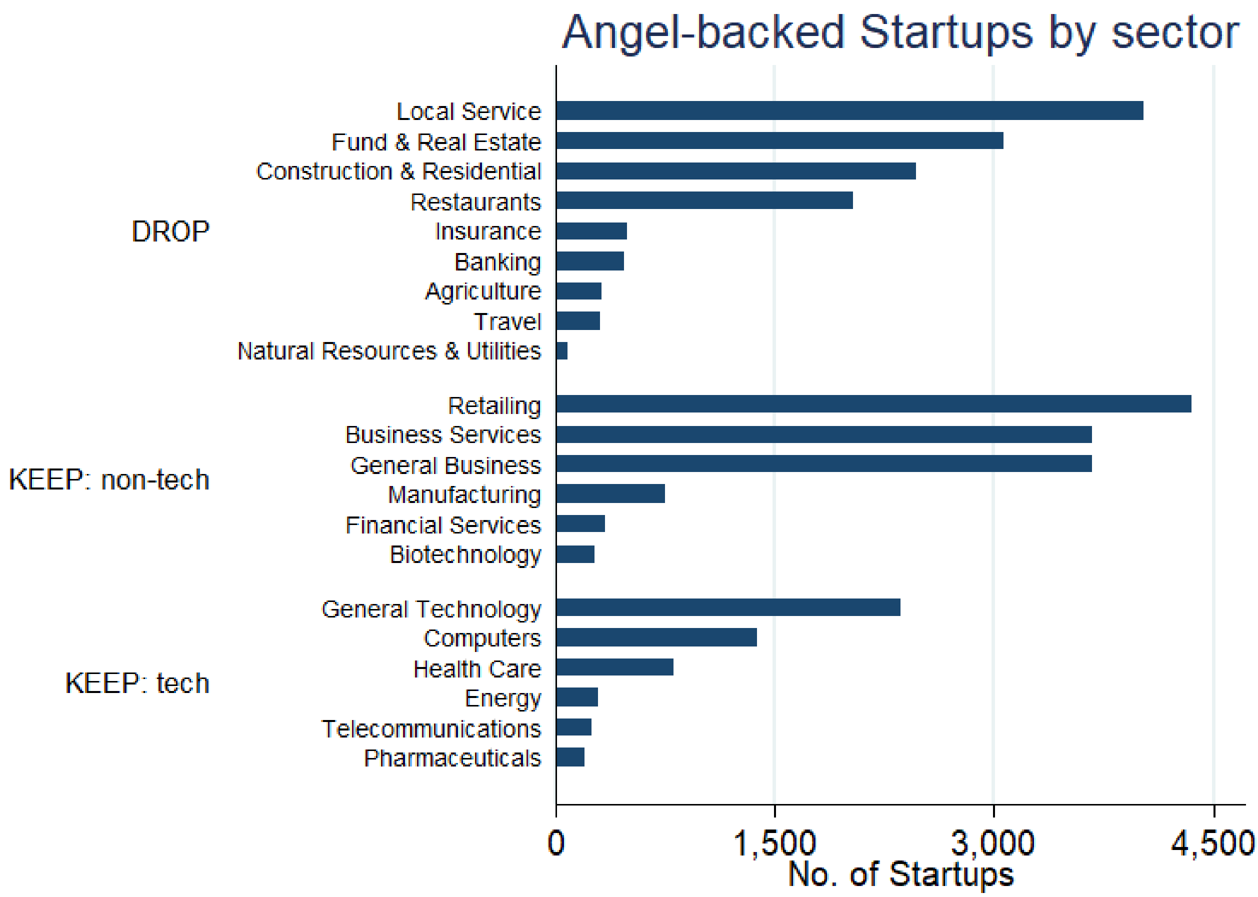
In this paper, I refer to incorporated firms less than three years old as “startups”. Figure 1.2 shows the number of angel-backed startups in my sample by industry classification. Clearly, angels invest in a wide variety of industries, including both high-tech sectors, such as telecommunications and computers, and some traditional sectors, such as restaurants and construction. As is standard in the finance literature, to study the effect of angel investing on the real economy and ease the comparison between angels and VCs, I will drop financial and several other local sectors from my sample, restricting my sample of analysis to the set of industries where VCs are active: retailing, biotechnology, business services, computers, energy, manufacturing, general business, health care, general technology, pharmaceuticals, telecommunications, and financial services. I call these industries “VC-industries”. I also conduct some of my analysis over a subsample of technology sectors including computers, general technology, telecommunications, health care, pharmaceuticals, and energy. I do not include biotechnology in the technology sectors because biotechnology is a capital intensive sector and the fraction of angels investing is low.

1.2.8 Final sample

My sample for statistical analysis contains 18,351 angel-backed startups in CA that raised their first financing between 2004 and 2014. Among these, 16,539 startups were collected from CA form D filings and 1,812 startups from SEC form D filings. The angel-backed startups in my sample on average raise $665,895 for their initial financing and have 1.15 rounds of financing. Summary statistics at the startup and zip×year level are reported in Panel A and B of Table 2.1, respectively. This sample is considerably larger than existing databases on angel financing. For example, Pitchbook details some angel financing for mainly VC funded firms, and they only documented 440 angel

Figure 1.2: SECTORS INVESTED BY ANGEL INVESTORS

Notes: This figure reports the total number of angel-backed startups by sector, and also highlights the set of sectors kept for analysis.



financings between 2004 and 2014 for CA startups.

1.3 The angel capital market

I characterize the angel capital market using the sample of angel-backed startups' initial financing, which I also refer to as "initial angel financing".[21] I focus on initial financing because it allows a convenient startup-level analysis. Moreover, initial financing itself is important in a startup's life cycle because it provides the earliest signal about how

[21] Equivalently, initial angel financings are startups' initial financings where angels are the only outside investors. Similarly, initial VC financings are startups' initial financings where (at least one) VCs are the outside investors.

Table 1.1: SUMMARY STATISTICS

Notes: Panel A reports the summary statistics of my sample at the startup level. Panel B summarizes variables at the zip×year level for the sample of zip codes with at least one angel investment during the sample period 2004-2014. Panel C summarizes market variables at the zip×sector×year level for the sample of zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the data period.

Panel A: startup level variables

	mean	sd	min	p25	p50	p75	max	count
Number rounds	1.16	0.58	1	1	1	1	10	18351
CA Incorporated	0.82	0.39	0	1	1	1	1	18351
DE Incorporated	0.16	0.36	0	0	0	0	1	18351
First capital raised (m)	0.67	2.78	0.010	0.013	0.050	0.22	50	18351
Incorporation year	2008.3	3.30	2001	2005	2008	2011	2014	18351
Year first angel	2008.8	3.23	2004	2006	2009	2012	2014	18351
Age first angel	0.51	0.64	0.041	0.11	0.22	0.64	3	18351
Biotech	0.014	0.12	0	0	0	0	1	18351
Computers	0.075	0.26	0	0	0	0	1	18351

Panel B: market variables at the zip×year level

	mean	sd	min	p25	p50	p75	max	count
Population	29896.8	18364.1	131	16480	27540.5	40360	135791	11490
# of Corporations	76.3	74.7	0	26	56	102	1072	11490
Housing_log	12.9	0.60	11.0	12.6	13.0	13.3	15.3	11490
Income p.c.	38888.7	40601.9	6988.1	19970.4	28836.6	42461.8	789556.3	11490
Salary Share	0.64	0.14	0	0.56	0.66	0.75	0.94	11490
Salary Households Share	0.80	0.26	0	0.82	0.88	0.92	1	11490

Panel C: market variables at the zip×sector×year level

	mean	sd	min	p25	p50	p75	max	count
Angel Financing	1.27	1.03	0	1	1	2	9	2344
VC Financing	0.35	0.63	0	0	0	1	6	2344
VC-year 1	0.093	0.31	0	0	0	0	3	2344
VC-year 2	0.15	0.39	0	0	0	0	3	2344

outside investors view the startup. A positive signal could lead to further investments that critically enhance the startup's chance of survival and growth.

1.3.1 How big is the angel capital market?

Using the VC market as a benchmark, the angel capital market is quite large. To make the comparison to VC investments fair, I focus on angel investments in VC-industries (see Figure 1.2) that removes about half of all angel deals in California. I first compare the angel and VC markets in terms of the number of startups funded. For the angel market, I calculate the number of angel-backed startups. For the VC market, I use data combined from both PCRI and Pitchbook, and compute the total number of VC financed startups (see definition of "VC financed startup" in Table A.1 in the Appendix). In the count of VC financed startups, I included those that are non-compliers to the security exemption rules and those that have raised capital from angels before getting VC financing. Even with this conservative approach, I find that angels funded five times as many startups (18,351) as VCs (3,702) (see Figure 1.3 for a comparison by financing year).[22] I further estimate the size of the angel capital market in terms of ratios. In VC-industries, angels fund about 94% of initial security issues of California startups. If one counts angel investments in all industries, 3.77% of all corporations receive angel financing within three years of incorporation. This fraction contrasts to the 0.37% of CA corporations that receive VC financing within three years of founding.[23] Using the VC market as a benchmark, the estimate of the angel capital market using California data is likely to be a lower bound for the U.S. angel market. The reason is that California is the most VC-active state in the United States, and the VC benchmark estimated using CA data is probably an upper bound for the U.S. market. Indeed, using the census data, Puri and Zarutskie [84] derive a similar but slightly lower estimate of the U.S. VC market: 0.11% of all newly created firms in the United States receive VC financing.

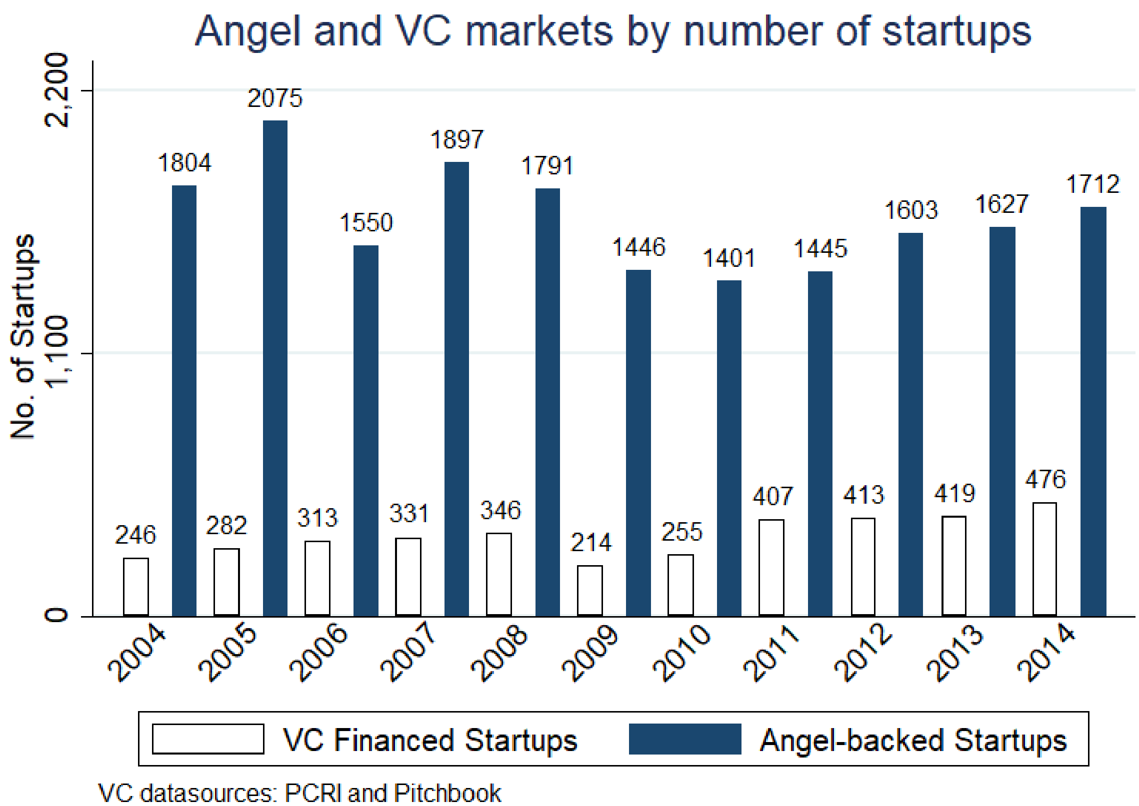
Next, I compare the size of angel and VC markets in terms of the total amount

[22]The Kauffman Firm Survey (including all individual investors other than family members) indicates that 4.5 times as many new firms received equity from angels in their first year of operation as received money from VCs [7].

[23]If counting LLC firms, on average 0.19% of all CA incorporated and LLC firms founded between 2004-2011 receive VC financing within three years of their founding.

Figure 1.3: ANGEL AND VC MARKETS BY NUMBER OF STARTUPS FUNDED

Notes: The figure reports the number of angel-backed startups (startups receiving their first financing from angels) and VC financed startups (startups ever funded by VCs) by financing year.



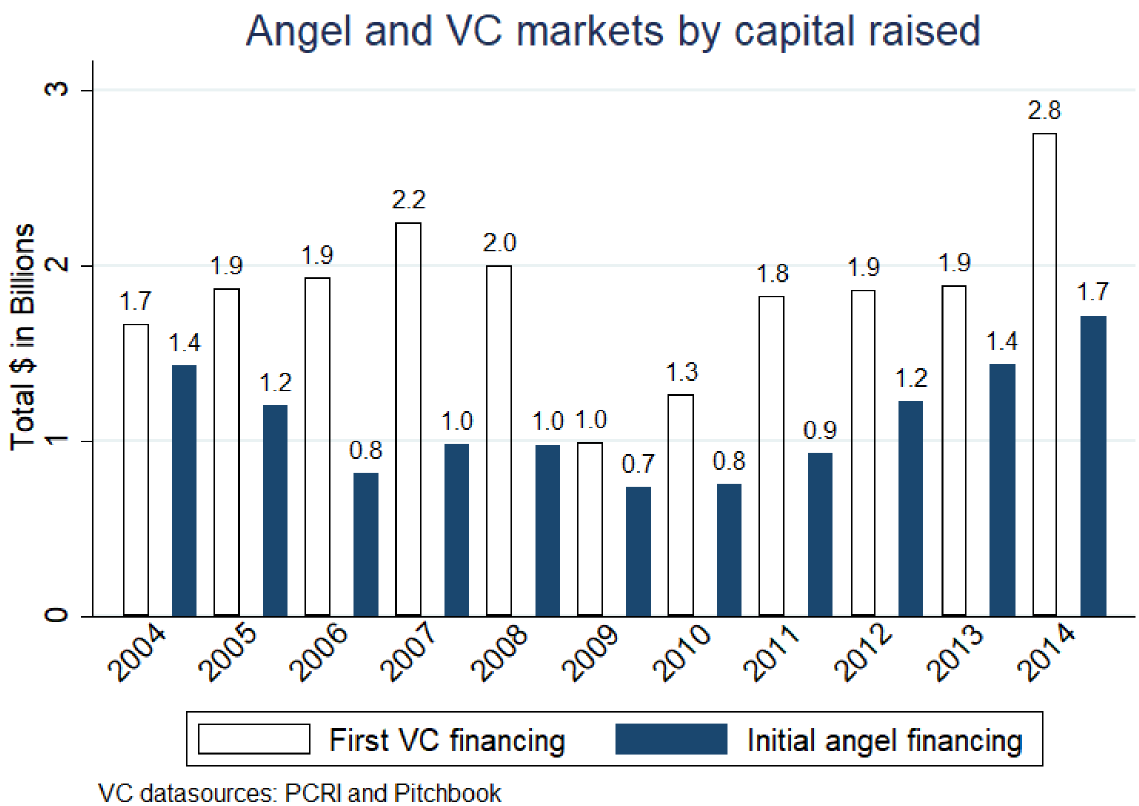
of capital raised in startups' initial angel financing and first VC financing (the first financing round having VC investors, but perhaps being preceded by an angel round). Since angel financing on average occurs at earlier stages of startups than VC financing, this comparison again provides a conservative estimate for angel financing. With this conservative estimate, I find that the angel market's funding in VC-industries is over 60% of VC funding (see Figure 1.4 for a comparison by financing year).[24] Including all industries, the angel market is over 90% of the VC market.[25]

[24]I only have financing size data for VC deals in Pitchbook. For VC deals with missing financing size, I assume the size is the year average of deal size according to Pitchbook.

[25]If I count angel investments in both incorporated and LLC firms, the angel market is 2.45 times as large as the VC market.

Figure 1.4: ANGEL AND VC MARKETS BY CAPITAL RAISED

Notes: The figure reports the total amount of capital raised in initial angel financing and first VC financing by financing year.



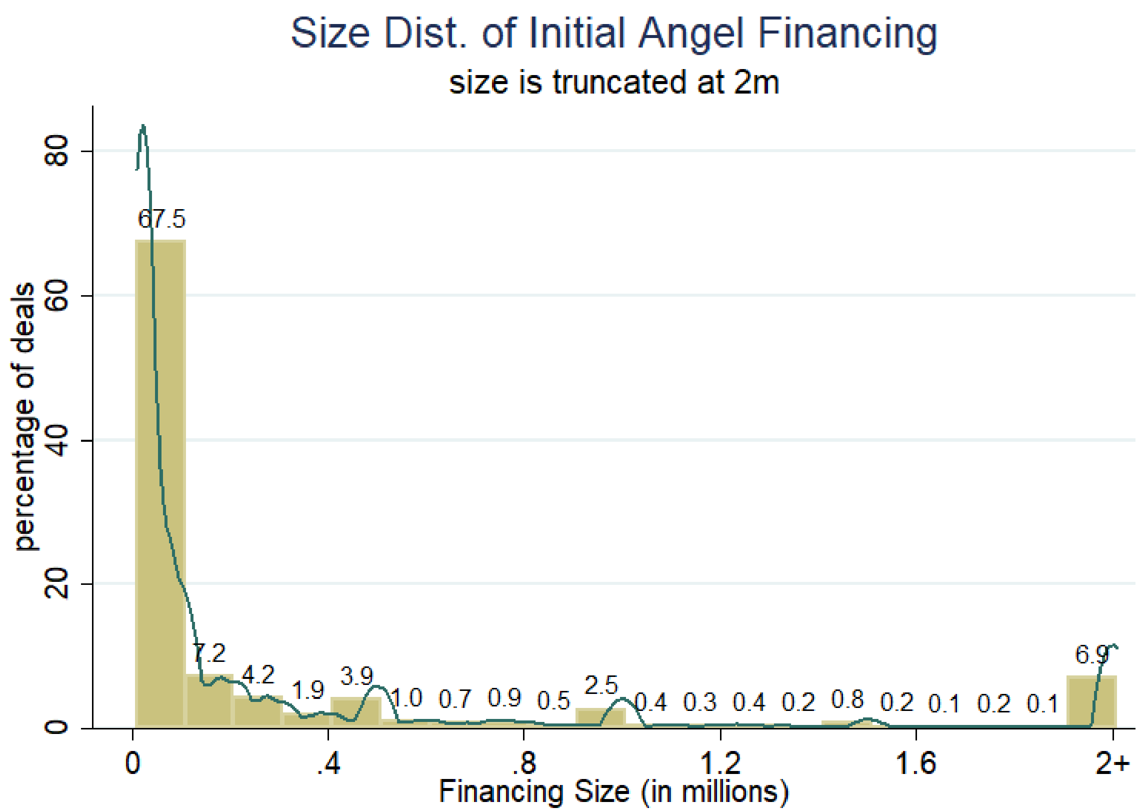
1.3.2 What do angel investments look like?

In the absence of systematic data on angel financing, the literature has relied on surveys or small samples to produce estimates of angel investments [34, 89, 98]. My dataset provides more accurate estimates of angel investments' basic characteristics: size, startup age at financing, and location.

The median size of initial angel investments is $50,000 and the mean size is $665,895, demonstrating that the size of angel investments is highly skewed. Figure 1.5 illustrates the size distribution of initial angel financing.

Figure 1.5: FINANCING SIZE DISTRIBUTION

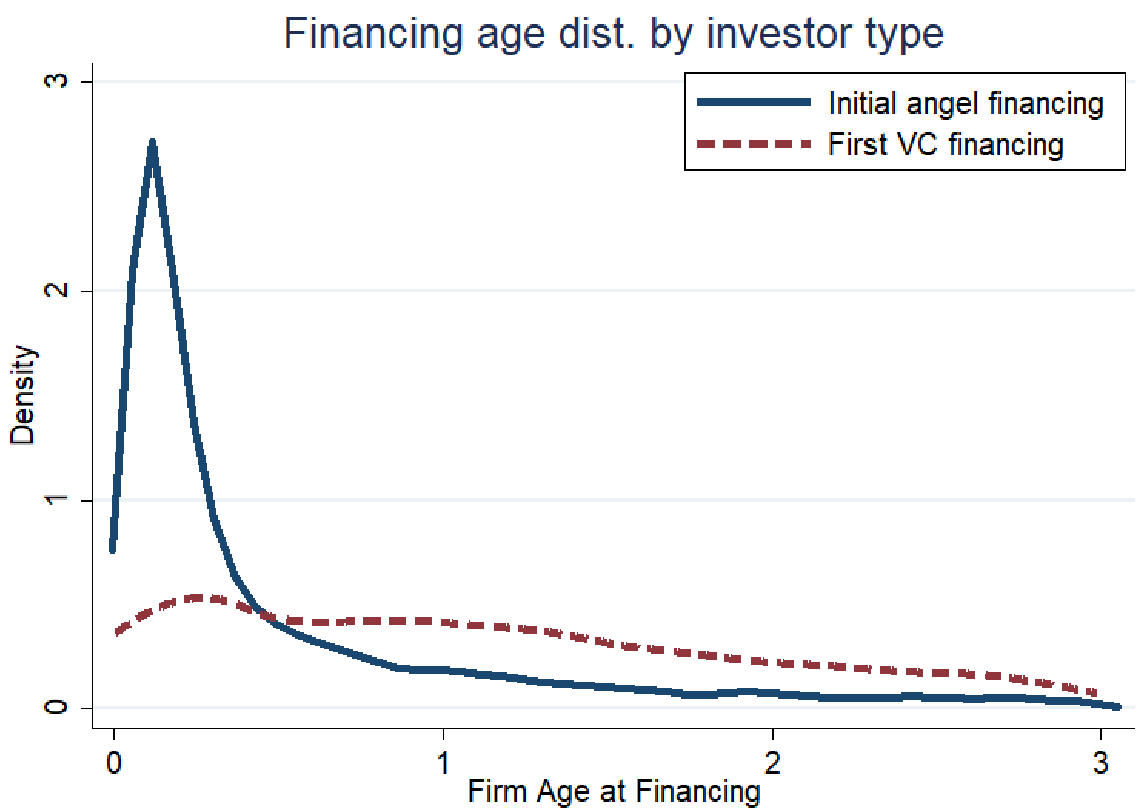
Notes: This figure reports the size distribution of initial angel financing. The size is truncated at two million dollars.



It is widely believed that angel financing occurs early, and plays an important role in helping startups survive the "valley of death". The idea is that nascent firms are unable to access formal financial intermediaries, such as banks and VCs, so they are particularly vulnerable if capital runs out, and only informal investors such as angels can fill their funding gaps. My data support this view. When receiving their initial angel financing, startups are on average six months old, while they are fourteen months old when receiving their first VC financing. So angels invest early, and they invest earlier than VCs (see Figure 1.6 for the startup age distribution at their initial angel financing and first VC financing).

Figure 1.6: STARTUP AGE DISTRIBUTION

Notes: This figure reports the startup age distribution at initial angel financing and first VC financing.

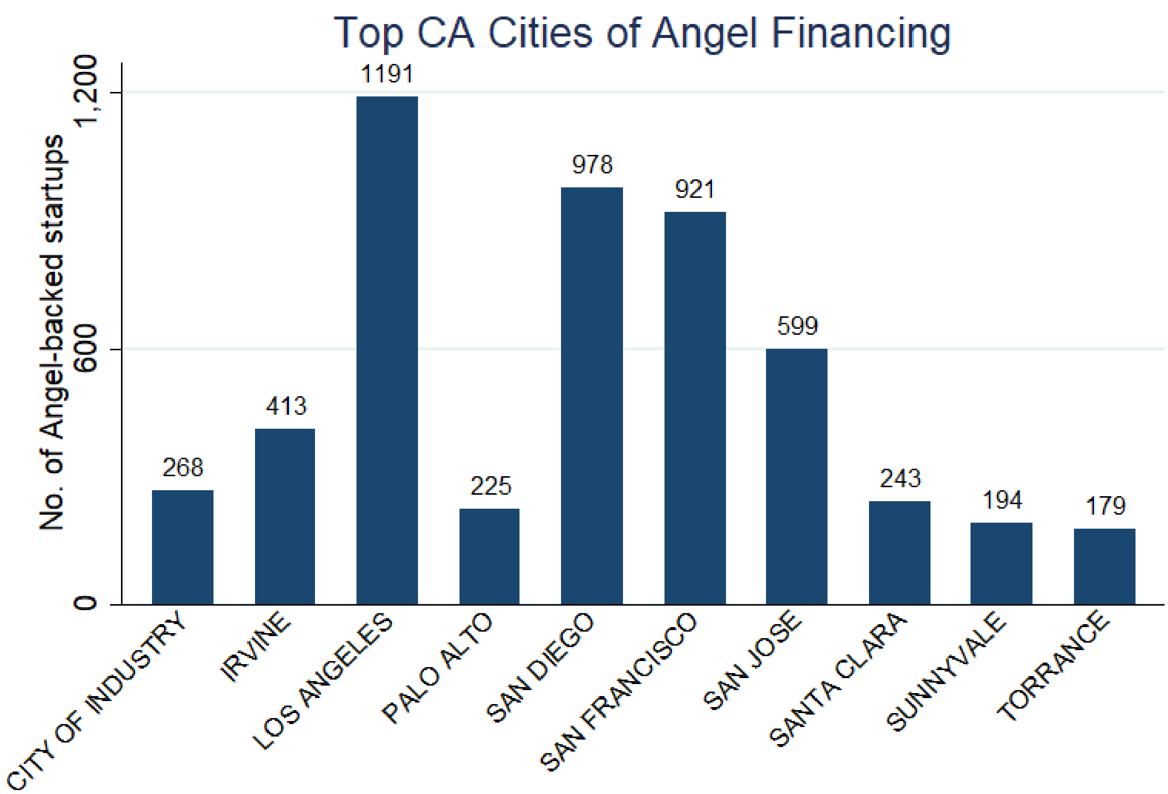


In terms of location, unlike VCs, angel financing is much less geographically concentrated. For example, during 2004-2014, 54% of California zip codes with tax returns have at least one angel financing (see Figure 1.7 for the top cities in angel financing in CA). This figure is in contrast to the 11% of zip codes with at least one VC financing. If counting angel investments in all industries and in both incorporated and LLC firms, the fraction of zip codes with at least one angel financing rises to 66%. This wide geographic distribution of angel investments provides evidence that angel financing may help fill startups' funding gaps in areas not covered by VCs.[26]

[26]For example, the literature has shown that VCs are geographically clustered [e.g., 20].

Figure 1.7: TOP CITIES OF ANGEL FINANCING

Notes: This figure reports the top CA cities in angel financing measured by the number of angel-backed startups.



1.3.3 Angel investment contracts by security type

I now turn to the security types of initial angel financings. I find that debt issues play almost no role, and instead equity (both common and preferred stock) is the overwhelming form of financing.

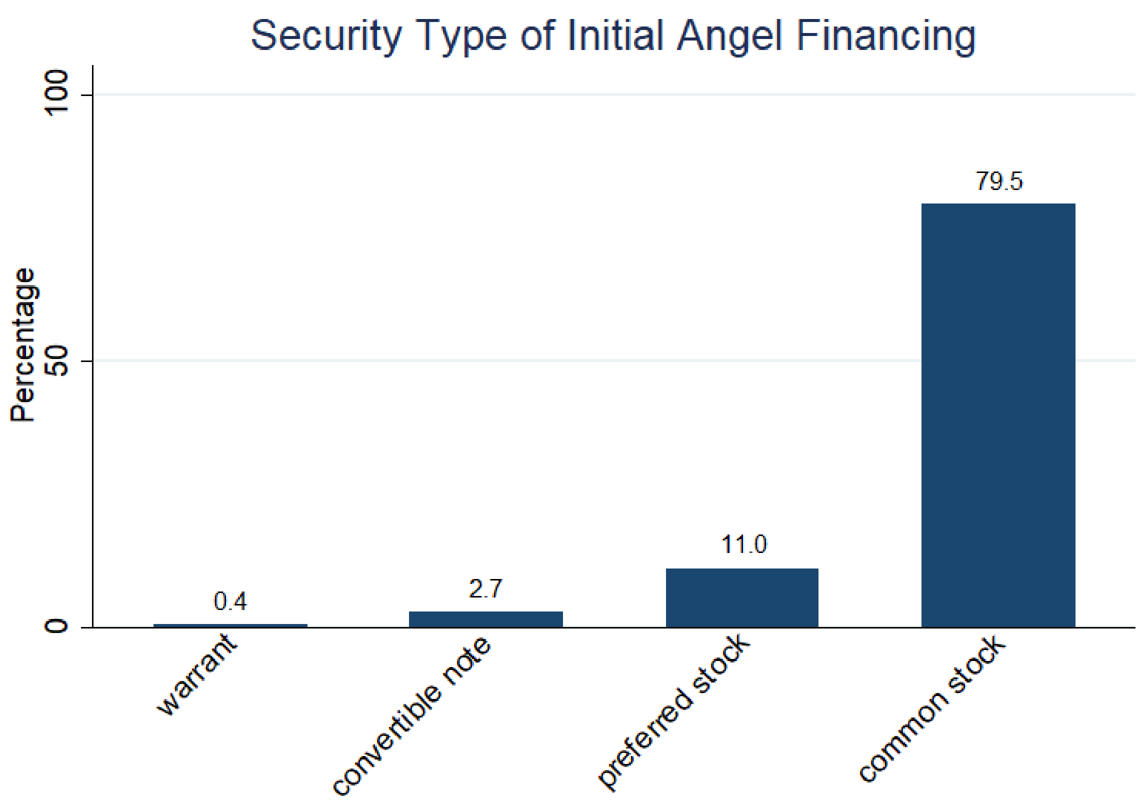
Moreover, common stock, rather than more sophisticated securities such as preferred stock and convertible notes, is the overwhelming security type in angel deals (see Figure 1.8). Anecdotal evidence suggests that angel investors are like VCs,[27] and they often use sophisticated securities in their investment contracts [e.g., 98]. However, I show

[27] For example, Kaplan and Strömberg [55] show that almost all VC contracts use sophisticated securities.

that this is not the case. In initial angel financing, common stock accounts for about 80% of all securities.[28] Although more complex securities provide benefits such as downward protection to investors, their transaction costs may not be worth incurring for the small investments that most angels make. Indeed, my finding is consistent with the theoretical predictions of Casamatta [16], who shows that common stock is best for smaller investments such as angel investments.

Figure 1.8: SECURITY TYPE OF ANGEL FINANCING

Notes: This figure reports the security type of initial angel financing. There are a small number of angel deals in SEC form D filings reporting "equity" as their security type, in which case I assume them as "preferred stock".



[28] Wong et al. [98] use a hand-collected sample and show that common stock accounts for 34% of the angel investments. Since the hand-collected sample is likely to over-represent sophisticated angels, more typical angels would probably be more likely to use common stock than the study suggests.

1.3.4 Localness of the supply of angel capital

Early-stage financing faces high information asymmetries. In particular, the potential for agency conflicts between entrepreneurs and investors is severe. As a result, early-stage investors are frequently involved in their portfolio companies' management [see e.g., 67]. This involvement likely makes investors sensitive to geographic distance. Studies have shown that geographic proximity plays an important role in shaping the interaction between startups and early-stage investors. For example, Chen et al. [20] show that 43% of VC investments are made by VCs within their home combined statistical area (CSA), and Bernstein et al. [8] show that introducing new direct flights reduces the travel time between VCs and their portfolio companies and increases VCs' monitoring. So geographic distance constrains VC investing. Less well understood, however, is at what exact distance VCs are constrained? As financial intermediaries, VCs are supposed to overcome major market frictions in the early-stage market, so VCs may be constrained only when the geographic distance is relatively large, e.g., beyond a metropolitan statistical area (MSA) range. Indeed, Bernstein et al. [8] show that the median distance between a portfolio company and its lead VC is approximately 200 miles.[29] This distance is greater than the distance between Los Angeles and San Diego, two large cities located in different MSAs of California. The Los Angeles MSA alone has a population of over 10 million people and 57 cities with populations larger than 50,000 each, according to the 2010 U.S. Census. By this standard, a typical VC investment is actually far. Taken together, the evidence from VC investments suggest that VCs are constrained by geographic distance, but only to a limited extent.

Studying the role of geographic proximity in early-stage financing, the literature has

[29] The identification strategy of Bernstein et al. [8] also relies on the relatively large distance between VCs and their portfolio companies, so that introducing new airlines could generate enough variations for the treatment.

mainly focused on VCs. As argued by Amit et al. [6], the very existence of VCs relies on their ability to reduce the information asymmetry costs. So if professional VCs are still constrained by geographic distance, non-intermediated early-stage investors such as angels should be more constrained. Notably, angel investors also invest at even earlier stages of startups and may face higher uncertainty and information asymmetry. Then, relative to VCs, do angels invest more locally? Could the angel capital market be integrated at the MSA level? If it is integrated at the MSA level, then the geographic barriers to angel investing within MSAs are negligible, and startups demanding angel capital can raise it from anywhere within the same MSA. If we look at smaller areas, e.g., cities within a MSA, all else equal, we should see no correlation between angel investment activity and the availability of local angel capital at the smaller area level.

To evaluate the extent of angel market integration, I take an extreme version of locality, the zip code,[30] and study the link between a focal zip code's angel investment activity and the income and wealth levels of its neighboring zip codes at different distances. For example, for the zip code 94025, where Facebook is located, I look at the relationship between the number of angel investments in zip 94025 and the income per capita of all zip codes falling within a distance of between 5 and 10 miles. In a regression framework, I explore the following:

$$Y_{it} = \delta_0 + \delta_1 I_{n(i)t} + \delta_2 X_{it} + \alpha_{MSA(i)} \times \alpha_t + \epsilon_{it}. \tag{1.1}$$

Here Y_{it} is the number of initial angel financings per capita in zip code i and year t. $I_{n(i)t}$ is income per capita or median housing price of zip i's neighboring zip codes within a

[30]Goldfarb et al. [34] shows that 18% of all deals in their sample occur within the same zip code, and "investors are closer to the firm in smaller deals, and most likely to be in the same zip code for angel-only deals."

certain distance ring in year t. $I_{n(i)t}$ is a proxy for capital availability in angel investing. X_{it} is a vector of controls for zip i and year t. $\alpha_{MSA(i)} \times \alpha_t$ are MSA by year fixed effects.

Zip codes are indeed very small areas, with an average population of only about 26,000 in CA. If the angel capital market were sufficiently integrated well beyond the zip code level, then all else equal, we should see zero correlation between a zip code's angel investments activity and the availability of local capital. Moreover, we should see no correlation between a zip code's angel investments and the availability of local capital in the focal zip code's neighborhood. Translating into (1.1), the coefficient δ_1 should remain zero for any choice of $n(i)$ either as a focal zip code i, or zip i's any neighborhood. Alternatively, if the angel capital market is not well integrated beyond the zip codes' border, then we should see a positive δ_1 for $n(i)$ close to a focal zip code i. Furthermore, we should also see that the positive coefficients decrease as we examine neighborhoods farther away from the focal zip code.

Table 1.2 reports the regression results of equation (1.1) with income per capita of neighborhoods at different distances as the explanatory variables. The results verify two key predictions: angel investing in a zip code is positively correlated with the zip code's own capital availability measured by income, and this correlation rapidly decreases to zero as we move the neighborhood away from the focal zip. Particularly, at the five miles, the correlation becomes insignificant. This suggests that angel investments are as local as within five miles' distance. I repeat the exercise in Table 1.3 using housing price as a different proxy for capital availability and find the same pattern. The results in Tables 1.2 and 1.3 suggest that angels invest locally.

However, one concern about the regression results may be that the correlation between angel investments and income (or wealth) might be driven by the capital

Table 1.2: ANGEL FINANCING AND INCOME BY DISTANCE

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The dependent variable is "Angel Financing p.c.", the number of initial angel financings per thousand population in zip i and year t. The explanatory variable is "Income_log ($x \sim y$)", the log of income per capita of all zip codes falling in a distance between x and y miles of a focal zip code in year t. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Angel Financing p.c.												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Income_log	0.098***												
	(0.021)												
Income_log (0 ~ 5m)		0.063***											
		(0.014)											
Income_log (5 ~ 10m)			0.031										
			(0.027)										
Income_log (10 ~ 15m)				-0.005									
				(0.009)									
Income_log (15 ~ 20m)					-0.060								
					(0.037)								
Income_log (20 ~ 25m)						-0.040**							
						(0.019)							
Income_log (25 ~ 30m)							-0.000						
							(0.041)						
Income_log (30 ~ 35m)								0.000					
								(0.010)					
Income_log (35 ~ 40m)									-0.008				
									(0.012)				
Income_log (40 ~ 50m)										0.052			
										(0.040)			
Income_log (50 ~ 60m)											0.072*		
											(0.037)		
Income_log (60 ~ 70m)												-0.068*	
												(0.033)	
Income_log (70 ~ 80m)													-0.050
													(0.033)
Adjusted R^2	0.113	0.031	0.007	0.004	0.009	0.007	0.004	0.004	0.004	0.006	0.009	0.008	0.007
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452

demand of startups. In particular, richer areas may simply have more startups seeking angel capital. So although angels do not invest locally, we would still see a positive correlation between angel investments and local income or wealth levels. I take two approaches to deal with this concern. First, I add the number of newly incorporated firms per capita each year in equation (1.1) as a control of overall demand. This mitigates the influence of the capital demand of startups on the estimation of δ_1 in equation (1.1). The results are reported in Table 1.4, and the central findings do not change. Second, if the localness result is driven by demand, replacing angel investments by VC investments as the dependent variable in (1.1), we should see a similar pattern. However, the results in Table 1.5 show the opposite: VC financing is not significantly correlated

Table 1.3: ANGEL-BACKED STARTUPS AND HOUSING PRICE BY DISTANCE

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The dependent variable is "Angel Financing p.c.", the number of initial angel financings per thousand population in zip i and year t. The explanatory variable is "Housing_log ($x \sim y$)", the log of the median housing price among the zip codes falling in a distance between x and y miles of a focal zip code in year t. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Angel Financing p.c.												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Housing_log	0.077***												
	(0.005)												
Housing_log (0 ~ 5m)		0.104***											
		(0.018)											
Housing_log (5 ~ 10m)			0.047**										
			(0.021)										
Housing_log (10 ~ 15m)				0.034									
				(0.031)									
Housing_log (15 ~ 20m)					-0.045								
					(0.030)								
Housing_log (20 ~ 25m)						-0.029							
						(0.040)							
Housing_log (25 ~ 30m)							-0.046*						
							(0.026)						
Housing_log (30 ~ 35m)								-0.029					
								(0.022)					
Housing_log (35 ~ 40m)									-0.019				
									(0.024)				
Housing_log (40 ~ 50m)										0.001			
										(0.032)			
Housing_log (50 ~ 60m)											0.039		
											(0.038)		
Housing_log (60 ~ 70m)												-0.039	
												(0.029)	
Housing_log (70 ~ 80m)													-0.045
													(0.026)
Adjusted R^2	0.036	0.039	0.012	0.009	0.010	0.009	0.010	0.009	0.009	0.008	0.010	0.010	0.011
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5031	5031	5031	5031	5031	5031	5031	5031	5031	5031	5031	5031	5031

with local income. Indeed, the results in Table 1.5 have significant implications for the supply of and demand for early-stage capital. Since VCs invest in relatively large areas [8], the zero correlation between VC investments and local economic factors, such as income and wealth, implies that the demand for early-stage capital is not positively correlated with such factors. Given the elastic supply of VC capital, if the demand for early-stage capital (the demand for VC capital particularly) was correlated with local economic factors, then we would see a positive correlation between VC investments (the VC market equilibrium) and local economic factors. Furthermore, if the demand for early-stage capital is not correlated with local economic factors, then the positive correlation between angel investments and local capital availability implies that local conditions constrain the supply of angel capital.

Table 1.4: ANGEL FINANCING AND INCOME BY DISTANCE WITH DEMAND CONTROL

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The dependent variable is "Angel Financing p.c.", the number of initial angel financings per thousand population in zip i and year t. The explanatory variable is "Income_log ($x \sim y$)", the log of income per capita of all zip codes falling in a distance between x and y miles of a focal zip code in year t. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Angel Financing p.c.												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Income_log	0.049**												
	(0.020)												
Income_log (0 ~ 5m)		0.029**											
		(0.013)											
Income_log (5 ~ 10m)			-0.004										
			(0.014)										
Income_log (10 ~ 15m)				-0.011									
				(0.010)									
Income_log (15 ~ 20m)					-0.034								
					(0.024)								
Income_log (20 ~ 25m)						-0.028*							
						(0.016)							
Income_log (25 ~ 30m)							0.025						
							(0.030)						
Income_log (30 ~ 35m)								0.020*					
								(0.010)					
Income_log (35 ~ 40m)									-0.005				
									(0.009)				
Income_log (40 ~ 50m)										0.019			
										(0.022)			
Income_log (50 ~ 60m)											0.029		
											(0.024)		
Income_log (60 ~ 70m)												-0.041**	
												(0.019)	
Income_log (70 ~ 80m)													-0.023
													(0.020)
# of Corporations p.c.	0.101***	0.106***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***	0.108***
	(0.020)	(0.023)	(0.025)	(0.024)	(0.025)	(0.024)	(0.025)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
Adjusted R^2	0.452	0.432	0.427	0.427	0.428	0.428	0.427	0.427	0.427	0.427	0.427	0.428	0.427
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452

In sum, the results in this section suggest that there may be substantial geography-related market frictions in startup financing. Then unlike VC financing that depends on the general partners' ability to raise money, angel financing relies on local communities' income (wealth) levels. So local capital matters for startups' access to angel financing. This finding supports the popular argument that local wealth plays an important role in funding startups and helps create technology hubs such as Silicon Valley.[31] The

[31]For example, the famous angel investor Paul Graham argues: "I think you only need two kinds of people to create a technology hub: rich people and nerds. They're the limiting reagents in the reaction that produces startups, because they're the only ones present when startups get started." -`http://www.paulgraham.com/siliconvalley.html`

Table 1.5: VC FINANCING AND INCOME BY DISTANCE

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The dependent variable is "VC Financing p.c.", the number of initial VC financings (first financing of VC-backed startups) per thousand population in zip i and year t. The explanatory variable is "Income_log ($x \sim y$)", the log of income per capita of all zip codes falling in a distance between x and y miles of a focal zip code in year t. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	VC Financing p.c.												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Income_log	0.010												
	(0.008)												
Income_log (0 ~ 5m)		0.008											
		(0.005)											
Income_log (5 ~ 10m)			-0.001										
			(0.004)										
Income_log (10 ~ 15m)				0.000									
				(0.002)									
Income_log (15 ~ 20m)					-0.002*								
					(0.001)								
Income_log (20 ~ 25m)						0.002							
						(0.004)							
Income_log (25 ~ 30m)							0.012						
							(0.011)						
Income_log (30 ~ 35m)								0.004					
								(0.005)					
Income_log (35 ~ 40m)									-0.001				
									(0.001)				
Income_log (40 ~ 50m)										0.012			
										(0.008)			
Income_log (50 ~ 60m)											0.013		
											(0.011)		
Income_log (60 ~ 70m)												-0.009	
												(0.009)	
Income_log (70 ~ 80m)													-0.011
													(0.010)
Adjusted R^2	0.054	0.043	0.037	0.037	0.037	0.037	0.039	0.037	0.037	0.038	0.039	0.038	0.039
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452	5452

localness of angel financing has important policy implications as well. For example, attracting wealthy people into a local area could benefit the early-stage financing market and encourage entrepreneurship.

Importantly, the localness result also motivates my further aggregate analysis at the zip code level, and informs my choice of instrumental variable later in this paper.

1.3.5 Are rich households more likely to be angel investors?

Investing in startups is risky. If we believe that richer people have a higher level of risk tolerance, then richer people should be more likely to be angel investors. For this reason, the share of high income households or the share of income earned by high income households should be good proxies for the pool of potential angel investors. Following this intuition, I explore two new measures of local availability of angel capital. The first is “share rich”, the share of households with annual income greater than \$100,000 (also termed as “rich households”), and the second is “income share of rich”, the share of income earned by rich households. I still use the regression model (1.1), but replace the independent variable $I_{n(i)t}$ by these two new measures I define above.

Table 1.6: ANGEL FINANCING AND CAPITAL AVAILABILITY

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The two dependent variables are “Angel Financing p.c.”, the number of initial angel financings per thousand population in zip i and year t, and “Angel Financing in Tech p.c.”, the number of initial angel financings in technology sectors per thousand population in zip i and year t. The two explanatory variables are “Share Rich”, the share of rich households, and “Income Share of Rich”, the share of income earned by rich households in zip i and year t. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Angel Financing p.c.				Angel Financing in Tech p.c.			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Share Rich	0.331***	0.171***			0.069***	0.049***		
	(0.022)	(0.022)			(0.006)	(0.007)		
Income Share of Rich			0.195***	0.081***			0.037***	0.023***
			(0.011)	(0.012)			(0.003)	(0.003)
# of Corporations p.c.		0.126***		0.125***		0.015***		0.015***
		(0.017)		(0.017)		(0.004)		(0.004)
Adjusted R^2	0.051	0.394	0.065	0.392	0.039	0.115	0.041	0.112
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11367	11367	11367	11367	11367	11367	11367	11367

Table 1.6 reports the regression results. Column (1) shows that the share of rich households is strongly positively correlated with angel investment activity. In particular, a 1% increase (100 basis points) in the share rich is associated with a 5% increase in the number of initial angel financings from the mean. This magnitude is economically meaningful. To control the influence of high income on angel activity through the capital demand side, I include the number of all newly incorporated firms as a control, and Column (2) reports the result. The coefficient decreases, but remains statistically

and economically significant. Columns (3) and (4) repeat the exercise using the income share of rich as an independent variable, and the results deliver the same message. Finally, I repeat the exercise over a subsample of technology sectors, and the results change little. The results in the section suggest that rich households are more likely to be angel investors, and confirm that local capital availability matters for angel investment activity.

1.3.6 Are high salary earners more likely to be angel investors?

I further investigate angel investors' demographics. Some studies have argued that angel investors are mainly former entrepreneurs, or retirees [see e.g., 77]. These people have already accumulated enough wealth, so they can afford to invest in risky startups. Alternatively, angel investors can be a broad set of people who mirror the distribution of the whole population [89]. Knowing angel investors' demographics is important for understanding several key dimensions of the angel capital market, e.g., what determines angel capital supply, and how it is connected to other sectors of the economy. To open the black box of angel investors' demographics, I examine the links between the salary income of rich households and the level of angel investing activities at the market level. Since rich households are more likely to be angel investors as suggested above (see details in Section 1.3.5), examining the links between the salary income of rich households and the level of angel investing activities will reveal information about the average angel investor's occupational and economic status.

Specifically, I explore two measures of salary income of rich households: "salary share", the income share of salary for rich households, and "salary households share", the share of households with salaries among all rich households.[32] I still use the

[32] Among the rich households, some earn salaries and wages, and others do not. "Salary households share" is the fraction

regression model (1.1), but replace the explanatory variable with the two new salary income measures. If most angel investors are rich and do not hold regular salary jobs, then angel investments should negatively correlate with the salary income of rich households. Alternatively, we should not see a significant correlation.

Table 1.7: ANGEL FINANCING AND SALARY INCOME OF RICH HOUSEHOLDS

Notes: This table reports regression results of (1.1). A unit of observation is (zip, year). The two dependent variables are "Angel Financing p.c.", the number of initial angel financings per thousand population in zip i and year t, and "Angel Financing in Tech p.c.", the number of initial angel financings in technology sectors per thousand population in zip i and year t. The two explanatory variables are "Salary Share", the income share of salary for rich households, and "Salary Households Share", the share of households with salaries among all rich households in zip i and year t. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Angel Financing p.c.				Angel Financing in Tech p.c.			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Salary Share	-0.316***	-0.133***			-0.055***	-0.033***		
	(0.026)	(0.022)			(0.006)	(0.006)		
Salary Households Share			-0.263***	-0.021			-0.043***	-0.012
			(0.038)	(0.032)			(0.009)	(0.009)
# of Corporations p.c.		0.126***		0.130***		0.015***		0.016***
		(0.017)		(0.018)		(0.004)		(0.004)
Adjusted R^2	0.059	0.391	0.020	0.383	0.034	0.109	0.016	0.102
MSA×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11365	11365	11365	11365	11365	11365	11365	11365

Table 1.7 reports the regression results. Column (1) shows that the salary share is strongly negatively correlated with angel investment activity. Column (2) reports estimation from the same regression, but with a capital demand control, and the result does not change much. Columns (3) and (4) report estimation using the salary households share as the explanatory variable. I repeat the exercise over a subsample of technology sectors (Columns (5)-(8)). In all cases, the results remain similar.

The results in this section show that angel investing activity is negatively correlated with the salary income of rich households, which suggests that the rich households that do not earn regular salary and wages are more likely to be angel investors. Therefore,

of households that earn salaries and wages among all rich households.

this finding supports the argument that a large proportion of angel investors may indeed be former entrepreneurs or retirees who earn their income mainly from capital income sources. These people being angel investors has implications for the role angel investors play in early-stage financing. For example, former entrepreneurs may be able to provide better value-adding services to their portfolio companies than other types of investors, because they are more experienced in managing and growing startups.

1.3.7 Angel investing and startup survival

A large body of literature studies the relationship between startups' financing and their outcomes [see e.g., 13, 56]. However, prior studies have mostly focused on financial intermediaries such as banks and VCs. In this paper, I investigate the connection between angel financing and startups' survival. To study this question, my sample has at least two advantages. First, it includes both startups receiving angel capital and those that did not receive any private capital. Second, the sample is large. In general, it is challenging to collect a large set of startups with their outcomes, because most startups fail quickly after their founding, and they could have disappeared before even being observed. In my case, I am able to exploit the nature of government filings, and infer all firms' survival time for the universe of California corporations. In CA, corporations have to file a statement of information each year as long as they remain active.[33] So the latest statement of information filing year indicates a startup's last year of operation before it exited the market. Using this information, I infer startups' survival time and indicate startups' survival outcomes by a set of dummy variables on whether a startup has survived longer than a certain number of years.

[33] Corporations have their initial filing due 90 days from the entity's registration date, and they update their filing each year. See more at `http://bpd.cdn.sos.ca.gov/corp/pdf/so/corp_so550.pdf`.

To study the relation between startups' angel financing and their survival, I explore the following regression model at the startup-level:

$$Y_i = \psi_0 + \psi_1 A_i + \psi_2 X_i + \alpha_{MSA(i)} + \alpha_{Year(i)} + \epsilon_i. \tag{1.2}$$

Here Y_i is a dummy variable on whether startup i has survived longer than a certain number of years. A_i is a dummy variable on whether startup i received angel financing. X_i are startup-level controls such as the state where it is incorporated. $\alpha_{MSA(i)}$ are MSA fixed effects with respect to the MSA of startup i's location, and $\alpha_{Year(i)}$ are year fixed effects with respect to the incorporation year of startup i.

Table 1.8: ANGEL FINANCING AND STARTUP SURVIVAL

Notes: This table reports regression results of (1.2). The regression is at the startup level. The dependent variable is "x Years", a dummy variable indicating whether a startup has survived more than x years. The explanatory variable is "Angel-backed?", a dummy variable indicating whether a startup is angel-backed or not. The full sample includes CA startups incorporated between 2004 and 2011, but excludes VC-backed startups. Then I run regressions based on 3 subsamples, defined by "Survival > k & Financing < k" ($k = 1, 2, 3$). "Survival > k & Financing < k" is a set of startups conditioning on that they survived longer than k years, and they either received angel funding within k years' founding or never received angel financing. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Survival>1 & Financing<1			Survival>2 & Financing<2		Survival>3 & Financing<3
	(1)	(2)	(3)	(4)	(5)	(6)
	2 Years	3 Years	4 Years	3 Years	4 Years	4 Years
Angel-backed?	0.026***	0.048***	0.048***	0.030***	0.036***	0.012***
	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)
CA Incorporated	0.047***	0.079***	0.102***	0.049***	0.084***	0.051***
	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.005)
DE Incorporated	0.031***	0.054***	0.071***	0.036***	0.062***	0.039***
	(0.005)	(0.006)	(0.006)	(0.005)	(0.007)	(0.006)
Housing_log	0.025***	0.043***	0.050***	0.026***	0.037***	0.018***
	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Adjusted R^2	0.004	0.006	0.012	0.004	0.015	0.023
Incorporation Year FE	Yes	Yes	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	449144	449144	449144	383438	383438	327172

Table 1.8 reports the regression results. Conditioned on startups surviving longer than one year and receiving angel financing within one year's incorporation, Columns

(1)-(3) show angel financing is positively correlated with startups' survival rate measured over different time windows. In particular, Column (1) shows that relative to startups that did not receive any capital, angel-backed startups have a 2.6% higher rate of survival longer than two years, which is about 4% of the average rate of surviving longer than two years. The remaining columns report estimations based on different subsamples, and deliver similar results. As robustness checks, I also run hazards models and have similar findings.

The results in this section show that angel investing is associated with startups' higher survival rate. These results highlight a potentially important role of angel financing, and suggest that angels are able to invest in a set of high quality startups.[34] In particular, the results suggest that family and friends (F&F) financing cannot comprise the bulk of my data, because startups that are funded by F&F investors do not appear to be of higher quality than the average [see e.g., 66].

1.4 Does initial angel financing lead to VC follow-on financing?

1.4.1 VC follow-on hypothesis

As noted above, scholars have long argued that angels and VCs have complementarities over firms' life cycles [e.g., 33]. A well-developed angel market may lead to a large number of high quality pre-screened startups for VCs, and thus increases VCs' investment opportunities and deal flows. Anecdotal evidence also suggests that startups often first raise money from angels, and then move on to VCs for later rounds.[35] However, no systematic evidence has been provided about whether angel financing enhances

[34] In unreported regressions with the same model specification as equation (1.2), I find that angel-backed startups have a significantly higher rate of patenting than the average startup.

[35] `https://www.nytimes.com/2015/07/07/technology/for-startups-how-many-angels-is-too-many.html`

VC financing. If the angel capital market plays an important role in pre-screening, adding value to startups, or providing deal flows for VCs, then a larger angel capital market should produce a larger complementary VC market. To be more specific, *a larger number of initial angel financings should lead to more VC follow-on financings* (for short, the "VC follow-on hypothesis").

Contrary to this hypothesis, angels may not play any positive role for VCs. For example, it could be that angels just invest in a different set of startups that do not want to grow and never get big [49]. It could also be that angels invest in startups VCs would like to follow on, but angels and VCs are different types of investors, and they do not mix well over firms' life cycles. In these cases, angels and VCs would operate as segmented or competitive markets, and we would not observe a positive link between the two markets over firms' life cycles.

To decide between these two views, I examine angel and VC activities at the market level. Indeed, the two views are statements about markets, so they can be best addressed using market-level variation. As I showed above, the angel capital market is local at the zip code level, so I explore the variations of initial angel investments at the zip×sector×year level, and use the following regression model to tie the angel capital market to VC outcomes:

$$VC_{ijt} = \beta_0 + \beta_1 A_{ijt} + \beta_2 X_{it} + \alpha_{L(i)} + \gamma_j + \zeta_t + \epsilon_{ijt}. \tag{1.3}$$

Here the dependent variable VC_{ijt} measures the number of VC follow-on financings. In particular, I explore two measures of VC follow-on financing: "VC-year 1", the number of initial angel financings in zip code i, industry j, and year t that are followed by VC financing within a year; and "VC-year 2", the number of initial angel financings in zip code i, industry j and year t that are followed by VC financing within two years. The variable A_{ijt} is the total number of initial angel financings in zip code i, industry j, and year t. X_{it} contains zip level controls such as population and total number of new corporations. $\alpha_{L(i)}$ are location fixed effects, either at the zip or MSA level, γ_j are industry fixed effects, and ζ_t are financing year fixed effects. I use the sample of zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the sample period. I stop the sample at 2012 in order to observe the VC outcomes occurring within two years of angel financing (summary statistics of the variables are in Panel C of Table 2.1).

Table 1.9: VC FOLLOW-ON HYPOTHESIS: OLS

Notes: This table reports results from the OLS regression (1.3) with zip code fixed effects. I consider two VC dependent variables. The first one is "VC-year 1", the number of initial angel financings in zip code i, industry j, and year t that are followed by VC financing within a year. The second one is "VC-year 2", the number of initial angel financings in zip code i, industry j and year t that are followed by VC financing within two years. The explanatory variable is "Angel Financing", the number of initial angel financings in zip code i, industry j, and year t. The first two columns are regression results on the full set of industries, the last two columns are on a subsample of technology sectors. The sample includes zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the sample period. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Full Sample		Sub Sample: technology sector	
	(1)	(2)	(3)	(4)
	VC-year 1	VC-year 2	VC-year 1	VC-year 2
Angel Financing	0.076***	0.123***	0.121***	0.145***
	(0.011)	(0.013)	(0.025)	(0.028)
Population	-0.008	-0.041	0.069	0.106
	(0.050)	(0.063)	(0.073)	(0.104)
# of Corporations	0.134	0.518	0.531	1.219*
	(0.297)	(0.365)	(0.581)	(0.682)
Size Quartile	0.038***	0.055***	0.043***	0.073***
	(0.006)	(0.007)	(0.011)	(0.013)
Adjusted R^2	0.094	0.158	0.126	0.141
Year FE	Yes	Yes	Yes	Yes
Zip FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
Observations	2344	2344	731	731

Table 1.9 reports the estimation of equation (1.3). Columns (1) and (2) show that

there is a strong positive correlation between initial angel investments and VC follow-on financing, measured over either one or two years' windows. These OLS results provide the first systematic evidence for the hypothesis that there are positive complementarities between the angel and VC markets over firms' life cycles. In particular, the findings imply that like VCs, angels also invest in a set of high-growth and innovative startups, and angels do not just provide "dumb" money for entrepreneurs.[36] Otherwise, professional VCs would not follow angels' investments.

The last two columns repeat the exercises on the subsample of technology sectors, showing a stronger relationship. Since the results using either "VC-year 1" or "VC-year 2" are similar, I will use the latter as the dependent variable for measuring VC follow-on financing in later sections.

1.4.2 Instrumental variable estimation

The results in Table 1.9 are consistent with the follow-on hypothesis that an increase of angel financing leads to more VC follow-on financing. Yet we cannot draw causal inferences from Table 1.9 because angel financing (A_{ijt}) is endogenous.

Endogeneity

One concern from the OLS regression (1.3) is that entrepreneurs may expect they have a higher chance of receiving future VC financing in zip codes with a higher level of VC activity, and thus locate their startups there. Then these startups may raise money first from angels, and then from VCs. In this case, VC activity in the past leads to entrepreneurs' higher expectation of future VC financing, then this higher expectation leads to more angel financing. Because VC activity is usually persistent in local areas,

[36] For example, angels are often referred to as "fool" investors by observers. See e.g., Shane [89].

the naive regression may suffer from reverse causality. Unfortunately, we cannot assign the direction on how reverse causality affects the OLS regression coefficient [see e.g., 86].

Another concern is that the positive correlation between angel investing and VC follow-on financing may be driven by startup side factors, e.g., local labor markets shocks lead to more high quality entrepreneurs, so that the demand for initial angel financing and VC follow-on financing both increase. In this case, although the naive OLS regression result is still interesting, the reasons driving the result are not the supply of angel capital per se, but instead demand side factors that are omitted from the model. These omitted variables produce upward bias for the OLS estimation. Since we are really interested in the causal impact of angel financing from the capital supply side, the demand side omitted variable problems make it difficult to interpret the OLS estimation.

Instrumental variable

To overcome the endogeneity issues of angel financing (A_{ijt}), I instrument it by a proxy of angel capital supply: "share rich", the share of households with annual income greater than \$100,000 within a zip and year. Note that \$100,000 is not a particularly high threshold, for example, on average 26% of households in my sample have income greater than this number. As a result, my instrumental variable defined by this threshold is able to proxy the angel capital supply from a broad set of potential individual investors.

For "share rich" to be a valid instrument of angel financing, it has to satisfy two critical conditions: relevance and the exclusion restriction. The first condition seems straightforward. As the first stage regression (reported below) shows, the share rich is highly correlated with angel investment activity. This is also consistent with the notion that richer households are more likely to invest in risky assets, and thus to be angel investors. The second condition needs more careful examination. To satisfy the exclusion condition, the share rich must link to VC financing only through angel financing. In particular, the share rich cannot directly link to VC financing through either VC supply or demand.

On the supply side, it does not seem likely that the share rich is directly linked to the supply of VC financing at the zip code level. In particular, as discussed in Section 1.3.4, VCs invest in relatively large areas. In addition, institutional investors, such as pension funds, contribute most of VC capital,[37] and the geographic distance between VC firms and their own institutional investors is not necessarily small.

A direct (positive) link between the share rich and the demand for VC financing does not seem likely, either. Indeed, the results in Table 1.5 show that VC financing is not correlated with local economic factors such as income and housing price at the zip code level, and this suggests that the demand for VC financing is not correlated with local economic factors. Moreover, we know that it is usually more expensive to start and run businesses for entrepreneurs in areas with a higher share of rich households. We also know that VCs select startups over a relatively large area to fund [8], and particularly within an MSA (MSA fixed effects are included in regression models), the distance between startups and VC firms is not a major factor in startups' chances of raising VC money. Therefore, for two neighboring zip codes within an MSA, one is cheaper than the other, it does not seem likely that financially constrained startups would prefer to locate in the more expensive zip code while their chances of raising money from VCs in both zip codes are the same. Thus, this further mitigates the concern that a direct

[37]For example, Gompers et al. [37] show that by 1994, individuals contribute less than 12% of VC firms' capital.

link exists between the share rich and the demand for VC financing. As an additional measure to confront this issue, I include overall entrepreneurship defined by the total number of all newly incorporated firms as a control in all regressions.

Identification assumption

One key identification assumption of my IV strategy is that angels indeed invest locally, and particularly, there is a strong home bias of investments at the zip code level. If this assumption holds, my instrument "share rich" is a good predictor of angel financing activity. Without direct information on angel investors, it is hard to verify this assumption, but the results in Tables 1.2-1.4 provide consistent evidence with it.

Empirical model and results

Let the variable S_{it} denote the share rich. The first stage regression is

$$A_{ijt} = \rho_0 + \rho_1 S_{it} + \rho_2 X_{it} + \alpha_{MSA(i)} + \gamma_j + \zeta_t + u_{ijt}. \tag{1.4}$$

A_{ijt} is the number of initial angel financings in zip code i, industry j, and year t. X_{it} contains zip level controls, such as population and the total number of new corporations. $\alpha_{MSA(i)}$ are MSA fixed effects, γ_j are industry fixed effects, and ζ_t are financing year fixed effects. The estimate of ρ_1 reveals whether there is a reduced form correlation between the share rich and the number of initial angel financing. I predict that an increase in the fraction of rich households should lead to a larger number of initial angel financing.

The second stage is now

$$VC_{ijt} = \delta_0 + \delta_1 A_{ijt} + \delta_2 X_{it} + \alpha_{MSA(i)} + \gamma_j + \zeta_t + \epsilon_{ijt}, \quad (1.5)$$

where A_{ijt} is instrumented from equation (1.4).

Table 1.10 contains the results of my instrumental variable regression. Column (1) presents estimates of (1.4). The estimate of the coefficient (ρ_1) on initial angel investments is economically and statistically significant, with the predicted positive sign. The weak instruments F-statistics [e.g., 92] is over 10. The results suggest that angel investing is indeed sensitive to the capital availability at the zip code level.

Column (3) of Table 1.10 presents the second stage estimation for the dependent variable "VC-year 2". The IV regression confirms the finding of the OLS regression. The IV regression produces a slightly larger coefficient than the OLS, suggesting a downward bias by the OLS. This likely results from the existence of a mild measurement error of angel financing. The economic magnitude of the IV estimate is that six additional initial angel financings can lead to one additional VC follow-on financing in the next two years. In the remaining columns of Table 1.10, I repeat the exercises over a subsample of technology sectors. The findings are similar. Thus, I conclude that there is a causal link between initial angel financing and VC follow-on financing.

1.4.3 Mechanisms

The regressions reported in Table 1.10 show a positive causal effect of initial angel financing on subsequent VC follow-on financing. Then what role do angels play so that VCs invest in a set of startups only after angels have invested? There are at least three possible explanations. First, angels help VCs reduce the risk of ("de-risk") startups. For example, information asymmetry may be high enough for early-stage financing,

Table 1.10: VC FOLLOW-ON HYPOTHESIS: INSTRUMENTAL VARIABLE

Notes: This table reports results from IV regressions (1.4) and (1.5). The dependent variable is "VC-year 2", the number of initial angel financings in zip code i, industry j and year t that are followed by VC financing within two years. The first three columns are regression results on the full set of VC-industries, the last three columns are on technology sectors. The sample includes zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the sample period. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Full Sample			Sub Sample: technology sector		
	(1) First Stage	(2) OLS	(3) IV	(4) First Stage	(5) OLS	(6) IV
Angel Financing		0.124***	0.151**		0.154***	0.210*
		(0.021)	(0.063)		(0.044)	(0.122)
Share Rich	0.845***			0.974**		
	(0.155)			(0.320)		
Population	-0.096***	-0.000	0.003	-0.075***	-0.001	0.004
	(0.019)	(0.010)	(0.014)	(0.018)	(0.007)	(0.010)
# of Corporations	3.651***	-0.236*	-0.341	2.832**	-0.079	-0.259
	(1.044)	(0.118)	(0.323)	(1.038)	(0.105)	(0.529)
Size Quartile	0.003	0.054***	0.054***	-0.029	0.061***	0.063***
	(0.017)	(0.006)	(0.006)	(0.019)	(0.009)	(0.009)
Housing_log	-0.173***	0.037*	0.037*	-0.242**	0.010	0.012
	(0.032)	(0.021)	(0.021)	(0.093)	(0.044)	(0.047)
Adjusted R^2	0.139	0.147	0.142	0.144	0.144	0.129
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
1st-Stage F Stat.			29.664			9.279
Observations	2234	2234	2234	695	695	695

so that even sophisticated VCs cannot select and fund the best startups. In this case, many high-quality startups instead have to raise money from angels for the first round financing. Then VCs are willing to invest after angels only when startups grow older and information asymmetry is reduced. It is worth noting that facing brand new startups, angels and VCs could be equally badly informed. The reason that angels are willing to be the first investors but VCs are not could simply be that non-intermediated angel capital is cheaper and angels have a lower expectation of returns. Second, angels may have their own information and networks to access high quality deals, so they pre-screen and certify startups for VCs. For example, angel investors may know the entrepreneurs before investing, or angels may have related industry experience. So compared with

VCs, angels actually have better knowledge of startups. In this case, VCs could use angels' investments as a signal of startups' quality, and follow angels' investing. Third, it could be that at the time of angel financing, startups may not look good enough to meet VCs' investing standards. Startups are made better by angels' value-adding services and then attract VC follow-on financing. Indeed, Kerr et al. [57] show that angel group investors can add value to their invested startups. Obviously, my data do not allow me to test the third possibility, so I test the first two in this paper.

To test the first possibility, I measure startups' risk by their survival rate. Then I study two questions. First, I investigate whether angel-backed startups are on average riskier than VC-backed startups. Second, I investigate whether angel-backed startups that receive VC follow-on financing are still riskier than VC-backed startups. To examine these two questions, I run regressions at the startup level. Table 1.11 reports regression results for answering the first question. The results show that on average, angel-backed startups do not survive as long as VC-backed startups regardless of time windows. Thus, in terms of survival rate, angel-backed startups are riskier than those VC-backed ones. Table 1.12 reports regression results answering the second question. The results show that angel-backed startups receiving VC follow-ons actually survive longer than VC-backed startups. Importantly, as measured in a longer time window, the result is stronger. Taken together, the two results (Tables 1.11 and 1.12) suggest that angels start with riskier startups on average, and then supply VCs with less risky ones for follow-on financing. So in the high-risk early-stage investing environment, angel investors may help VCs de-risk startups.

To investigate whether angels pre-screen and certify startups for VCs, I use the sample of all first VC financings of CA firms, and study the link between the likelihood

Table 1.11: VC FOLLOW-ON MECHANISMS OF DE-RISK: ANGEL- VS. VC-BACKED

Notes: The regression is at the startup level. The dependent variable is "x Year(s)", a dummy variable indicating whether a startup has survived more than x year(s). The explanatory variable is "Angel-backed?", a dummy variable indicating whether a startup is angel-backed or not. The sample includes CA corporations which have raised money from either angels or VCs during 2004-2011. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Startup Survival Years			
	(1)	(2)	(3)	(4)
	1 Year	2 Years	3 Years	4 Years
Angel-backed?	-0.035***	-0.066***	-0.061***	-0.075***
	(0.010)	(0.014)	(0.018)	(0.020)
CA Incorporated	-0.095***	-0.086**	-0.036	-0.009
	(0.024)	(0.038)	(0.047)	(0.050)
DE Incorporated	-0.016	0.002	0.044	0.062
	(0.025)	(0.039)	(0.048)	(0.051)
Housing_log	0.004	-0.010	-0.019*	-0.022*
	(0.008)	(0.010)	(0.012)	(0.012)
Adjusted R^2	0.018	0.020	0.018	0.023
Incorporation Year FE	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
Observations	12991	12991	12991	12991

of a VC investment being preceded by angels and the level of prior VC investing activities at the focal VC investment's zip code. To measure the level of VC activity, I count the total number of firms funded by VCs from the year 1995[38] until the focal VC investment's financing month. The rationale behind my test is that if angels certify startups' quality for VCs, then their certification should be most valuable in places where VCs have not invested a lot before and are perhaps not familiar with the local market. In other words, prior VC investment activity in a zip code should negatively predict the likelihood of a local VC financing being preceded by angels. The regression results are reported in Table 1.13. I use three model specifications: OLS, probit, and logit. All three coefficients have the same negative sign, but they are not statistically significant. Therefore, these results provide little support for the hypothesis that angel investors play certification/pre-screening roles for VCs.

[38] I count VC investments from the year 1995 because the VC data are of higher quality since the 1990s.

Table 1.12: VC FOLLOW-ON MECHANISMS OF DE-RISK: VC FOLLOW-ON VS. VC-BACKED

Notes: The regression is at the startup level. The dependent variable is "x Year(s)", a dummy variable indicating whether a startup has survived more than x year(s). The explanatory variable is "Raised Angel Before?", a dummy variable indicating whether a VC funded startup has raised money from angels prior to VC financing. The sample includes CA corporations which have raised money from VCs during 2005-2011. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Startup Survival Years			
	(1)	(2)	(3)	(4)
	1 Year	2 Years	3 Years	4 Years
Raised Angel Before?	0.015	0.036	0.068**	0.077**
	(0.016)	(0.025)	(0.033)	(0.038)
CA Incorporated	-0.066	-0.136**	-0.197**	-0.361***
	(0.046)	(0.067)	(0.077)	(0.135)
DE Incorporated	-0.040	-0.075	-0.124**	-0.271**
	(0.033)	(0.055)	(0.048)	(0.121)
Adjusted R^2	0.119	0.084	0.076	0.048
First VC Year FE	Yes	Yes	Yes	Yes
Zip FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
Observations	929	929	929	929

Table 1.13: VC FOLLOW-ON MECHANISMS: CERTIFICATION

Notes: The regression is at the startup level. There are three model specifications: OLS, probit and logit. The dependent variable is "Raised Angel Before?", a dummy variable indicating whether a VC funded startup has raised money from angels prior to VC financing. The explanatory variable is "Prior VCs", the total number of VC financed firms in a startup's home zip code counting from 1995 until the financing month of the focal startup's first VC financing. I normalize "Prior VCs" by dividing it by 1000 in the regression. The sample includes CA corporations which have raised money from VCs during 2005-2014. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Raised Angel Before?		
	(1)	(2)	(3)
	OLS	Probit	Logit
Prior VCs	-0.155	-0.444	-0.728
	(0.111)	(0.314)	(0.524)
Adjusted R^2	0.004		
First VC Year FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes
Observations	1497	1497	1497

1.5 Does angel financing crowd out VC financing from initial stages?

1.5.1 VC crowd-out hypothesis

Section 1.4 shows that initial investments by angels create additional VC follow-on investments. That brings up a separate question: do angels compete with VCs at the initial financing round? This question is relevant because VCs often fund a startup's initial capital infusion: 70% of first VC investments are not preceded by an angel round. This suggests that angels and VCs may compete to invest in startups. In this competition, VCs have both advantages and disadvantages. For example, as shown by Hsu [48], startups value VCs' certification and value-adding roles beyond the financial capital provided. So VCs could provide better certification and value-adding services than angels. On the other hand, VCs likely face higher transaction costs. For example, as argued by Lerner [69], VCs are usually unwilling to invest small amounts of money in startups. If VCs compete with angels for early-stage deals, we should observe that *a higher level of angel financing crowds out VC financing at the initial financing round* (for short, the "VC crowd-out hypothesis").

The crowd-out hypothesis is a market-level question, so I also exploit market level variations to test it. I regress the number of initial VC financings (startups' initial financings where VCs are the outside investors) on the number of initial angel financings (startups' initial financings where angels are the outside investors) at a unit of observation zip×sector×year. The model specification is the same as equation (1.3) except that the dependent variable is replaced by the number of initial VC financings. I use the sample of zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the data period (see summary statistics in Panel C of Table 2.1).

Table 1.14 reports the estimation of equation (1.3). Column (1) shows that there is a strong negative correlation between initial angel investments and initial VC investments. The correlation does not change for the subsample of technology sectors (Column (2)). Both results are consistent with the crowd-out hypothesis that angels play a competitive role with VCs at the initial financing round. Yet again, we cannot draw causal inferences

Table 1.14: VC CROWD-OUT HYPOTHESIS: OLS

Notes: This table reports results from the OLS regression (1.3) with zip code fixed effects. The dependent variable is "VC Financing", the number of initial VC financings in zip code i, industry j, and year t. The explanatory variable is "Angel Financing", the number of initial angel financings in zip code i, industry j, and year t. The first column is on the sample of the full set of VC-industries, the second column is on a subsample of technology sectors. The sample includes zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the sample period. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Full Sample	Sub Sample: technology sector
	(1)	(2)
	VC Financing	VC Financing
Angel Financing	-0.078***	-0.150***
	(0.021)	(0.045)
Population	-0.071	-0.310**
	(0.078)	(0.134)
# of Corporations	1.854***	2.095**
	(0.582)	(0.975)
Size Quartile	0.198***	0.169***
	(0.011)	(0.021)
Adjusted R^2	0.237	0.170
Year FE	Yes	Yes
Zip FE	Yes	Yes
Industry FE	Yes	Yes
Observations	2344	731

from Table 1.14 because the number of initial angel investments is endogenous.

1.5.2 Instrumental variable estimation

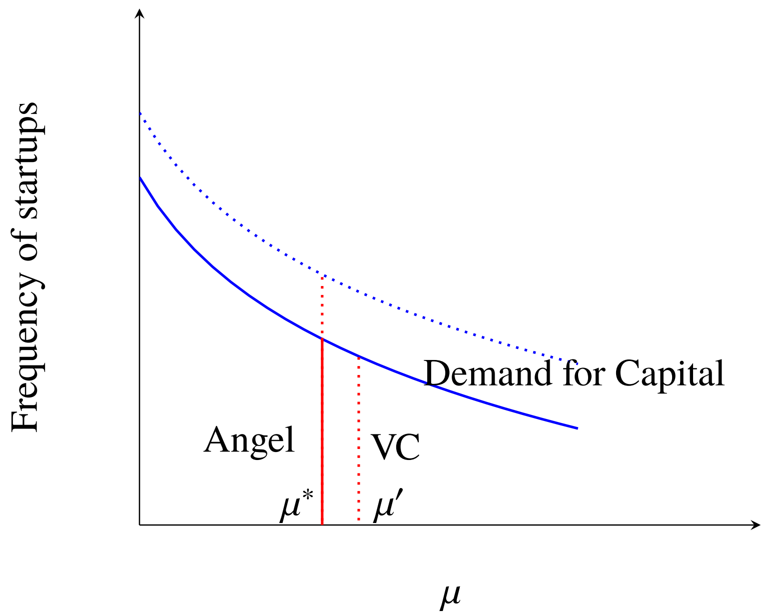
Endogeneity

The endogeneity problems mainly come from two sources. First, we may worry about a reverse causality problem. For example, when VCs face more investment opportunities in later stages, they may intentionally choose not to be the first equity investors in startups. Then the decrease of early stage VC funding leads to a higher demand for the alternative: angel financing.

Second, we may face omitted variable problems. Because angel and VC financing are equilibrium outcomes in the market for capital, it is useful to consider the issues in a supply and demand framework.

Figure 1.9: ANGEL AND VC COMPETITION

Notes: This figure illustrates how angels and VCs may operate as segmented markets, and compete for deals. The x-axis is represented by some startup characteristic variable μ, such as startup quality and size. The "Demand for Capital" curve is the frequency of startups according to μ. The early-stage financing market segments into two parts according to μ: the angel market on the left, and the VC market on the right. Then the competition between the angel and VC markets shifts the market equilibrium μ^* along the x-axis.



To start, suppose that startups' demand for capital is distributed according to some startup characteristics variable μ (e.g., startup quality or capital demand size). Angels and VCs select startups to finance according to μ so that it reaches an equilibrium at μ^*: startups with characteristics $\mu > \mu^*$ get funded by VCs, and startups with characteristics $\mu \leq \mu^*$ get funded by angels (Figure 1.9). Now suppose there are some positive economic shocks on the local entrepreneurial ecosystem, e.g., lower housing prices attract more young entrepreneurs to move to an area. Then these positive shocks would increase demand for early-stage capital for all values of μ, even though the incremental amounts for different μ's could vary. More generally, any shock on demand for early-stage capital is likely to shift the whole demand curve in Figure 1.9 in one direction, either up or down. In this scenario, the demand side shocks may leave the equilibrium μ^*

unchanged, but it would always change the number of initial angel and VC investments in the same direction, either up or down. Therefore, omitted capital demand side shocks would create an upward bias for the coefficient β_1 that I estimate in the OLS regression.

On the capital supply side, the omitted variables are less of a concern because sources of angel and VC capital differ. Angel capital is provided by individuals, while VC capital is provided by institutional investors, such as pension funds and university endowments. Also, as suggested by results in Section 1.3.4, zip code level economic factors, such as individual income, matters for angel capital supply, but not for VC capital supply. Therefore, capital supply side omitted variables should not be a major problem.

Results

Again, I use the share of rich households (share rich) as my instrument for angel financing. The instrument captures the cross-section variation in angel investments that is due to variation in local angel capital supply. Then I estimate two stage models. At the first stage, I regress the number of initial angel financings on the share rich at a unit of zip×sector×year. At the second stage, I regress the number of initial VC financings on the instrumented number of initial angel financings from the first stage. The first stage model specification is the same as equation (1.4), and the second stage is the same as equation (1.5) with the number of initial VC financings as the dependent variable.

Column (1) of Table 1.15 presents the first stage estimates (it is identical to Column (1) of Table 1.10). Column (3) of Table 1.15 presents the second stage estimation. The IV coefficient has the same sign as the OLS coefficient, but it is much smaller. The reason that the IV coefficient is smaller likely stems from the existence of demand side omitted variables, as discussed above. The economic magnitude of the IV estimate is that an increase in three initial angel financings crowds out one initial VC financing at the market level. In the remaining columns of Table 1.15, I repeat the exercises over a subsample of technology sectors. The findings are similar. These results provide evidence in support of the VC crowd-out hypothesis: angel financing crowds out VC financing from initial stages.

Table 1.15: VC CROWD-OUT HYPOTHESIS: INSTRUMENTAL VARIABLE

Notes: This table reports results from IV regressions (1.4) and (1.5). The dependent variable is "VC Financing", the number of initial VC financings in zip code i, industry j, and year t. The first three columns are regression results on the full set of VC-industries, the last three columns are on a subsample of technology sectors.The sample includes zip×sector×year cells from 2004 to 2012 conditioned on each zip×sector in the sample having at least one VC investment during the sample period. Standard errors, clustered at the MSA level, are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Full Sample			Sub Sample: technology sector		
	(1) First Stage	(2) OLS	(3) IV	(4) First Stage	(5) OLS	(6) IV
Angel Financing		-0.057***	-0.332**		-0.132***	-0.415***
		(0.011)	(0.142)		(0.038)	(0.124)
Share Rich	0.845***			0.974**		
	(0.155)			(0.320)		
Population	-0.096***	-0.026*	-0.055*	-0.075***	-0.024	-0.049
	(0.019)	(0.012)	(0.028)	(0.018)	(0.021)	(0.034)
# of Corporations	3.651***	0.623	1.684	2.832**	0.501	1.410
	(1.044)	(0.424)	(1.169)	(1.038)	(0.429)	(1.148)
Size Quartile	0.003	0.215***	0.216***	-0.029	0.196***	0.188***
	(0.017)	(0.015)	(0.018)	(0.019)	(0.009)	(0.008)
Housing_log	-0.173***	-0.072	-0.072**	-0.242**	-0.010	-0.022
	(0.032)	(0.053)	(0.036)	(0.093)	(0.049)	(0.051)
Adjusted R^2	0.139	0.216	0.032	0.144	0.175	0.003
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
1st-Stage F Stat.			29.664			9.279
Observations	2234	2234	2234	695	695	695

The results in Table 1.15 demonstrate a negative causal effect of angel financing on VC financing at the initial round. Then we should ask: why would more angel financings lead to fewer startups going to VC directly? Intuitively, angel capital may be relatively cheaper than VC, so that the increase of angel capital allows some entrepreneurs to switch from VCs to angels. Figure 1.9 illustrates this switch. When the demand for capital is fixed, the increase of angel capital supply reduces the cost of angel capital and switches the market equilibrium from μ^* to μ'. To verify that this is the case, I examine how the distribution of angel investments, in terms of security type, startup age (at financing), and financing size, respond to the volume of angel financing at the market level.

All else equal, common stock is preferred by entrepreneurs over sophisticated security (preferred stock and convertible note), so at market equilibrium, the increase of angel capital supply would lead to a lower likelihood of using sophisticated security in angel investments. So I regress a typical angel investment's security type on the number of initial angel financings at a zip×year level. I control the total demand (all new corporations), and include MSA and year fixed effects in the model. The regression results are reported in Column (1) of Table 1.16. Indeed, a higher level of angel financing is associated with a lower likelihood that a sophisticated security is used in a typical angel investment. This finding provides evidence that an increase in angel capital supply leads to a lower cost of angel funding for entrepreneurs.

Next, I examine how the distribution of angel investments in terms of startup age and financing size changes relative to the volume of angel financing. Since VCs on average invest in older startups with more capital, if the increase of angel supply allows some marginal startups to switch from VC to angel funding, then there should be older startups demanding larger amounts of capital that receive angel funding after the switch. This would lead the startup age and financing size distribution of angel investments to be more skewed to the right. To test these predictions, I regress the 75th and 90th percentiles of startup age and financing size at a zip×year level on the number of initial angel financings. Again, I control the total demand, and include MSA and year fixed effects in the model. The regression results are reported in Columns (2)-(5) of Table 1.16. All results, as predicted, suggest that the distribution of angel investments in terms of startup age and financing size are more skewed to the right when facing an increase of angel funding.

Therefore, the results in Table 1.16 provide evidence that the increase of angel capital supply reduces the cost of angel capital, and induces startups to switch funding from VCs to angels at their initial stages. These findings provide further confirmation for the VC crowd-out hypothesis: the growth of angel financing crowds out VC financing at the initial stages.

Table 1.16: CONFIRMATION OF THE VC CROWD-OUT HYPOTHESIS

Notes: A unit of observation is (zip, year). I consider five dependent variables (derived from the collection of initial angel investments within each zip×year): (1), the median of an indicator that an angel investment uses sophisticated security; (2) & (3), 75th and 90th percentile of startups' age distribution at their initial angel financing; and (4) & (5), 75th and 90th percentile of financing size distribution at startups' initial angel financing. The explanatory variable is "Angel Financing", the number of initial angel financings within a zip×year. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Sophisticated Security	Financing Age		Financing Size	
	(1)	(2)	(3)	(4)	(5)
	Median	Perc 75	Perc 90	Perc 75	Perc 90
Angel Financing	-0.004***	0.028***	0.098***	0.020	0.319***
	(0.001)	(0.004)	(0.006)	(0.015)	(0.033)
# of Corporations	0.033	0.351**	0.674***	1.538***	2.995***
	(0.034)	(0.138)	(0.168)	(0.468)	(0.896)
Adjusted R^2	0.082	0.057	0.186	0.041	0.121
Year FE	Yes	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes	Yes
Observations	6496	6496	6496	6496	6496

1.6 Conclusion

Anecdotal evidence suggests that the angel capital market is important for funding entrepreneurship and innovation. Yet partially due to data scarcity, we have a limited understanding about the market. In this paper, I build a comprehensive dataset on angel financing covering about 18,000 angel-backed startups in California during 2004-2014 by relying on sources of government filings.

Using this novel dataset, I study the size, scope, and role of angel financing. My findings improve our understanding of the angel capital market and the overall early-stage financing market in several aspects. First, the angel financing market is large, and thus is an important segment of the early-stage financing market. Second, the supply of angel capital is sourced locally, so local capital matters. Wealthy individuals may be an indispensable contributing factor for a prosperous and innovative entrepreneurial ecosystem. Third, angel financing has causal impact on VC financing. Specifically, angels play a supportive role for VCs over firms' life cycles, while they also play a competitive role with VCs over the initial financing deals. I also provide evidence that angels support VCs over firms' life cycles by reducing the risk of startups and supplying less risky and more successful startups to VCs.

Overall, my results in the paper demonstrate the explicit role of angel financing, and how it fits into the early-stage capital market. Because California has the most developed VC market in the United States, using California data to study the angel capital market and its interaction with the VC market is likely to attenuate many of my results. For example, one would expect that the VC follow-on result would be stronger in areas with less VC activity, because VCs may be less informed about the local entrepreneurial ecosystem, and angels may thus play a more significant role in supporting VCs. For future research, I plan to collect more information about the profiles of angel investors and the returns of angel investments so that I can evaluate questions such as the determinants of the angel capital supply and the total social welfare of the angel market.

Chapter 2

CROWDFUNDING WITHOUT INTERMEDIATION?

2.1 Introduction

Since crowdfunding first launched in 2008 in the U.S., it has attracted a lot of attention as a financial innovation to alleviate small businesses and startups' financial constraints.[1] The market has also enjoyed rapid expansion (see Figure 2.1). To further exploit crowdfunding to increase small businesses and startups' access to capital, the JOBS Act of 2012 legalized equity crowdfunding (a special type of crowdfunding): investors can now invest online to buy equity shares of startups. The legalization of equity crowdfunding is perceived to be revolutionary for the early-stage financing market because it allows not just accredited investors[2] to buy un-registered securities online (also referred to as "Title II equity crowdfunding"), but also millions of non-accredited investors to do so ("Title III equity crowdfunding"). This opening of private business investment opportunities to a large number of non-accredited investors is the first such instance since the Security Act of 1933. Distinct from traditional early-stage financing sources, such as bank loans and venture capital (VC), the key feature of the equity crowdfunding model is that the capital is directly raised from a large number of investors in relatively small amounts,[3] and no financial intermediation is involved.

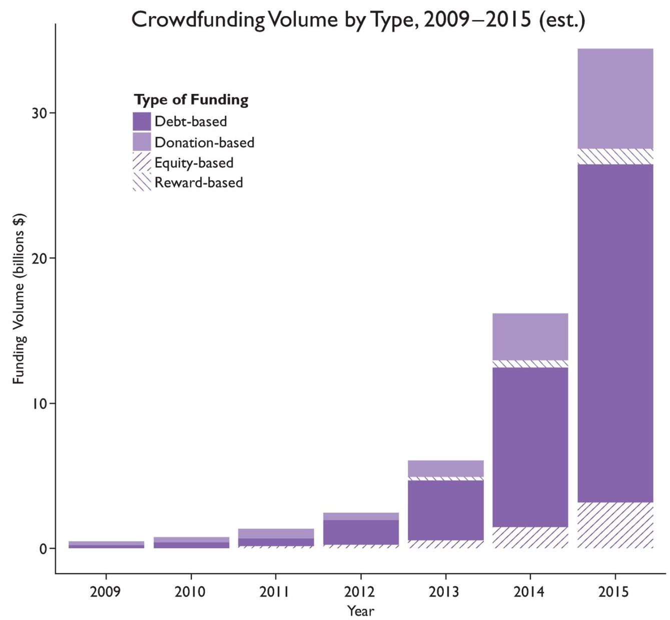
[1] There is a large literature showing that startups and small businesses are financial constrained [e.g., 28, 47], and that access to capital is key to entrepreneurship and innovation [e.g., 15, 59, 88].

[2] According to the SEC, an individual will be considered an accredited investor if he or she (1) earned income that exceeded \$200,000 (or \$300,000 together with a spouse) in each of the prior two years, and reasonably expects the same for the current year, or (2) has a net worth over \$1 million, either alone or together with a spouse.

[3] In particular, Title III of the JOBS Act explicitly sets a maximum amount of dollars each investor can annually invest, which should keep the "big players" away from the market and attract mainly small investors. See more at `https://www.sec.gov/oiea/investor-alerts-bulletins/ib_crowdfunding-.html`

Figure 2.1: THE GROWTH OF CROWDFUNDING

Notes: the following figure [source: 31] reports the worldwide growth of crowdfunding volume by type.



The absence of financial intermediation in the equity crowdfunding market, however, raises questions about how information asymmetries between startups and investors are resolved. In particular, in an environment where the entrepreneurs cannot credibly communicate with the investors, the resolution of information asymmetries requires that information be aggregated from the informed market participants and then conveyed to the uninformed ones. This transmission of information among investors usually relies on price variation in markets [e.g., 39]. However, in the equity crowdfunding market, the offering price is fixed by entrepreneurs ex-ante. Therefore, the price cannot adjust to revealed information. This may thus hinder information aggregation in the equity crowdfunding market. In this paper, I examine the information aggregation issue, and study whether the current equity crowdfunding market rules can resolve one major type of information asymmetry that is common in early-stage financing: adverse selection (the "lemons" problem).

Adverse selection, and information asymmetry more generally, has long been a central issue in economics and finance. For example, Akerlof [4] first addressed how adverse selection can lead to market failures in automobile and insurance markets. More specific to the environment of early-stage financing, a large literature has focused on the study of how financial intermediaries such as VCs solve information asymmetries.[4] In the setting of equity crowdfunding, adverse selection is a key concern for academics [17] and the regulatory authorities. For example, the SEC made this issue clear in the final implementation rule of Regulation Crowdfunding (Title III)[5]:

> *"The statute and the final rules related to entrepreneur disclosures are intended to reduce the information asymmetries that currently exist between small businesses and investors[...]. These considerations may give rise to adverse selection and moral hazard in offerings in reliance on Section 4(a)(6)."*

To study the adverse selection issue in equity crowdfunding, I develop a simple game-theoretic model. In the model, I assume that there are two types of investors: informed ("insiders") and uninformed ("outsiders"). The insiders would include entrepreneurs' family and friends ("F&F"), social network friends, and maybe some local angel investors. The informational advantage of F&F and social network friends has been well documented in early-stage financing.[6] The outsiders in my model are investors

[4]For example, Amit et al. [6] argue that the very existence of VCs relies on their ability to reduce the costs of information asymmetry. Chan [18] develops a theory of financial intermediation that highlights the contribution of intermediaries as informed agents in a market with imperfect information. To overcome the information asymmetry problem, the intermediaries also usually develop some special mechanisms. For example, the VC firms send their representatives to sit on the boards of the private firms they have invested in [67], and also usually structure their investments in stages [36].

[5]https://www.sec.gov/rules/final/2015/33-9974.pdf

[6]See e.g., Agrawal et al. [3], Cumming and Johan [21], Engelberg et al. [27] and Nanda and Khanna [78].

who are outside the entrepreneurs' personal and social circles, and only learn about startups from the crowdfunding platforms. I exogenously fix the order of how insiders and outsiders take investment actions. In particular, I assume that the insiders take their investment actions first, and the outsiders observe the aggregate of the insiders' actions and then decide whether to invest or not. Empirical evidence supports this assumption. For example, Agrawal et al. [3] show that F&F accounts for a big proportion of the early investors. In fact, it has become an important practice for entrepreneurs on the Kickstarter platform to leverage their own network:

> *"The first thing they tell you at Kickstarter is to leverage your own network. But people are like, I came to Kickstarter so they would give me free money!"*
> -Page 82, Steinberg [91], The Kickstarter Handbook

Then using my model, I investigate whether there is a *crowdfunding market equilibrium* in which an entrepreneur's expected utility is maximized, the insiders' information is effectively aggregated, and the outsiders' participation constraint is satisfied. At the crowdfunding market equilibrium, the market would be able to differentiate high quality startups from low quality ones, and fund high quality ones with a significantly higher probability, thus alleviating the adverse selection problem. However, my main result in this paper shows that there does not exist such a crowdfunding market equilibrium. The primary driver of my result is the incompatibility of aggregating insiders' information and satisfying outsiders' participation constraint when the share price is fixed for all investors.

The non-existence of a crowdfunding market equilibrium has several implications for the equity crowdfunding market. First, a decentralized equity crowdfunding market may fail to overcome the "lemons" problem that plagues early-stage financing. More specific to Title III equity crowdfunding in which investment vehicles are prohibited, the adverse selection problem may be a key factor limiting the growth of that market. Second, to promote the development of the equity crowdfunding market, both practitioners and policy makers may need to focus on the adverse selection problem, and develop new market mechanisms to address it. For example, as discussed in Section 2.5 below, mechanisms allowing for varying prices during the offering process solve the outsiders' participation constraint problem. Additionally, it may be helpful to encourage the crowdfunding portals to take more responsibility in screening startups listed in their platforms.

This paper provides the first rigorous study of the adverse selection issue in the equity crowdfunding market. In particular, the equity crowdfunding market involves transactions of equity stakes, and thus is more formal than other types of crowdfunding models such as reward-based crowdfunding. The relatively formal nature of the equity crowdfunding market allows me to model its market participants as rational agents, and examine the participants' interaction from an information aggregation perspective. With this framework, I find that one of the major early-stage financing market frictions, adverse selection, persists in the equity crowdfunding market, and may not be solved by market mechanisms under current market designs. My paper provides insights on understanding the equity crowdfunding market in a rigorous economic framework, and also sheds light on policy implications for how to promote the development of the equity crowdfunding market.

My paper relates to several literatures. First, it directly relates to a small literature studying information asymmetries in the environment of equity crowdfunding. For example, Agrawal et al. [2] use data from AngelList and argue that syndicates of investors are effective tools in reducing information asymmetries in equity crowdfunding. My paper complements theirs by providing a rigorous analysis for the setting in which no financial intermediation, such as syndicates, are involved. In this regard, my paper speaks more directly to Title III equity crowdfunding where the law prohibits financial intermediation such as syndicates.

Second, my paper is broadly related to the literature studying the role of price in conveying information from the informed market participants to the uninformed. For example, Grossman and Stiglitz [39] argue that price cannot perfectly reflect the information that is available in the market, because the price must be such that the informed participants receive compensation for their cost of acquiring information. Similar to theirs, my paper shows that when the market price is fixed for all participants regardless of their information, no market equilibrium exists that can effectively convey the information from the informed to the uninformed.

Third, my paper is also related to papers investigating the "wisdom of the crowd" in crowdfunding markets. For example, Hakenes and Schlegel [41] argue that the "all or nothing" mechanism can be used by (homogenous) household investors to aggregate information so that at equilibrium they acquire information and the aggregation of their information enables more good projects to receive funding. The focus of their paper is on how firms and investors interact to facilitate crowdfunding. Different from their paper, I focus on the study of information asymmetry/adverse selection and the interaction between heterogeneous investors. Another related paper is Brown and Davies [14], who build a one-period model with naive and sophisticated investors. In their model, identical naive investors have weak information and act on the information, and sophisticated investors receive private information and behave strategically. The main conclusion of their paper is that naive investors rather than sophisticated ones communicate the "wisdom of the crowd" and improve financing efficiency.

Fourth, my paper also connects to the literature studying quality signaling of online markets with asymmetric information. For example, Bernstein et al. [9] show that average investors respond strongly to the founding team. Mollick [75] argues that the entrepreneurs' social capital and preparedness are associated with an increased chance of project success, suggesting that quality signals play a role in project outcomes. These papers are related to mine in the sense that I assume that even if the entrepreneurs have information concerning the quality of their startups, they will not have credible channels to signal this information to the investors. Therefore, the remaining information asymmetries are still high. Unlike other online markets, a crowdfunding campaign is usually a one-shot game, which also invalidates many signaling mechanisms, such as reputation signaling, that are effective in other markets (Agrawal, Catalini, and Goldfarb, 2014).

Finally, my paper also speaks to the finance literature on information cascades and investor herding. A major focus of that literature is to explain the occurrence of herding and its consequences [see e.g., 12, 96]. In the crowdfunding market, some studies also show the existence of investors' herding behavior [see e.g., 3, 99]. In particular, Agrawal et al. [3] show that investors are much more likely to invest in startups that have reached a higher percentage of their funding goal. My paper differs from these papers in several aspects. First, unlike the case in which investors make decisions sequentially in the information cascade literature, in my model the investors with private information do not move sequentially, preventing information cascades from happening. Second, in my model it is part of the uninformed investors' investing strategy to invest in those startups that have already attracted more investors. For the uninformed investors, the number of existing investors is a public signal from which they infer the quality of startups. In this sense, the uninformed investors' behavior cannot be viewed as herding in the classical case.

The paper is organized as follows. In Section 2.2, I introduce the formal model. In Section 2.3, I analyze the market equilibrium, and prove the main result. In Section 2.4, I provide empirical evidence for my model's implications. In Section 2.5, I suggest an improving market mechanism. I conclude in Section 2.6.

2.2 The model

Consider an equity crowdfunding campaign in which an entrepreneur is raising capital for her startup. The startup is of high or low quality, denoted by H and L, respectively. The common prior on the startup's quality is $\mathbb{P}(H) = \mathbb{P}(L) = 1/2$, known to the entrepreneur and all investors. The entrepreneur wants to sell a fixed number of shares of her startup. In practice, when issuing equity, the entrepreneur can choose two things to reflect how much she evaluates her startup: share price and number of shares to sell. For simplicity, in my model, I fix the number of shares (or the fraction of the equity placed on the market), and only allow the entrepreneur to choose share price (denoted by p).

I assume a so-called "all or nothing" rule in my model for determining the final outcome of an equity crowdfunding campaign: the entrepreneur receives capital from the crowdfunding campaign if and only if there are enough investors to buy all the shares offered for sale. This rule is consistent with Title III equity crowdfunding implementation rule.[7] So the entrepreneur either sells all or none of the shares she offers. I refer to the former case as "a successful crowdfunding campaign" (denoted by S). Only upon a successful crowdfunding campaign are the entrepreneur and investors' payoffs in the crowdfunding market realized.

I assume that the entrepreneur does not know the quality of her startup. This assumption is technically equivalent to the scenarios in which either the entrepreneur does not have credible mechanisms to signal her startup's quality even if she knows it, or the entrepreneur cannot accurately estimate the market prospects of her startup. The entrepreneur is an expected utility maximizer with Bernoulli utility $u(x) = x^\gamma$, $\gamma \in (0, 1]$. Because the entrepreneur has a fixed number of shares to sell, as long as she maximizes her expected utility from selling just one share, she maximizes her utility for the whole sale. So the entrepreneur solves the following optimization problem:

$$\max_p \mathbb{P}(S) \cdot p^\gamma = \max_p \frac{\mathbb{P}(S|H) + \mathbb{P}(S|L)}{2} \cdot p^\gamma. \tag{2.1}$$

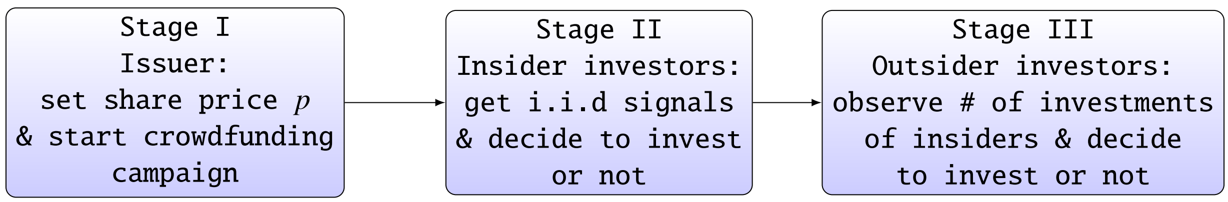
There are two types of investors: $n + 1$ insiders and infinitely many outsiders. The two types of investors are first differentiated by their information concerning the quality of the startup: the insiders are informed, and each of them receives an i.i.d binary signal: either "Good" (G) or "Bad" (B) with precision $\mathbb{P}(G|H) = \mathbb{P}(B|L) = \alpha$ for $\alpha > 1/2$. The outsiders are not informed, so they receive no signal. Secondly, the two types of investors are differentiated by when they take investment actions: insiders observe their private information and the share price p, and then simultaneously take investing actions. Outsiders observe the aggregate of the insiders' investing actions, and then

[7] The Title III final rule states that: "...including a statement that if the sum of the investment commitments does not equal or exceed the target offering amount at the offering deadline, no securities will be sold in the offering, investment commitments will be cancelled and committed funds will be returned."

simultaneously take their investing actions.[8] See Figure 2.2 for the timeline of the model.

Figure 2.2: THE MODEL TIMELINE

Notes: This figure depicts the timeline of my model.



All the investors are rational, and the insiders behave strategically. For simplicity, I assume that each investor can buy at most one share. Then each investor' strategy boils down to a pure strategy: either invest or not, or a mixed strategy of investing and not investing. Individual investors' investing strategies depend on their information. The insiders' investing strategy depends on their private signals and the share price set by the entrepreneur. The outsiders' investing strategy depends on the number of insiders who invest and also the share price. The return of one share is 1 when the startup quality is "High", and 0 otherwise. All investors are risk neutral with Bernoulli utility $u(x) = x$, and they are expected utility maximizers. The investors are assumed to be risk neutral, because in the crowdfunding setting, each investor is required to just invest a small amount of her money into the market, and to be prepared for the possible loss of all that money.[9] Let Y denote the action that an investor invests, and let $\mathbb{E}^I(Y|F, S)$ denote the expected utility (payoff) of an insider who receives an F signal (also termed

[8]Since there are infinitely many identical outsiders, they are determined randomly to invest in a startup if too many of them want to invest.

[9]The SEC sets a clear rule on the limit an investor can invest in the equity crowdfunding market each year. The limit depends on the investors' income, but the general principle is that an investor can absorb the risk of losing all the money she invests.

as an "F-signal insider") ($F = G, B$). So for $F = G$ or B,

$$\begin{aligned}\mathbb{E}^I(Y|F,S) &= \mathbb{P}(H|F,S)\times 1 + \mathbb{P}(L|F,S)\times 0 - p \\ &= \mathbb{P}(H|F,S) - p. \qquad (2.2)\end{aligned}$$

Let n_I denote the number of insiders who invest. The outsiders' expected utility (payoff) is

$$\begin{aligned}\mathbb{E}^O(Y|n_I,S) &= \mathbb{P}(H|n_I,S)\times 1 + \mathbb{P}(L|n_I,S)\times 0 - p \\ &= \mathbb{P}(H|n_I,S) - p. \qquad (2.3)\end{aligned}$$

2.3 The crowdfunding market equilibrium

2.3.1 Main result

In this section, I analyze the formal model and prove my main result. I focus on *symmetric strategies* equilibria. By playing symmetric strategies, the same type of investors (insiders and outsiders) use identical strategies if they have the same information set. Then I define a market equilibrium that is desired for a well-functioning crowdfunding market:

Definition 1. *A crowdfunding market equilibrium is an equilibrium where*

1. *the entrepreneur chooses share price p to maximize her expected utility,*
2. *the insiders with bad (B) signals do not invest, and*
3. *the outsiders' participation constraint is satisfied.*

I require that the B-signal insiders do not invest, because only when not all insiders invest can the insiders' information be effectively aggregated, and high quality startups can achieve significantly higher success rates of raising capital than low quality ones. Also, only in this case can the adverse selection problem in the crowdfunding market be overcome. As Lemma 3 below shows, as the insiders' likelihood of investing increases (e.g., more and more B-signal insiders start to invest), high quality startups' chances of crowdfunding success fall relative to low quality ones'.

It has long been recognized that adverse selection could lead to markets failure [4], which also makes overcoming this issue essential for the crowdfunding market. However, my main result shows that in the absence of a financial intermediary, the decentralized crowdfunding market may not be able to do so.

Theorem 1. *There does not exist a crowdfunding market equilibrium.*

It has been widely expected that crowdfunding democratizes the investing opportunities to all investors, and reduces geography-related frictions in early-stage investing [e.g., 58]. However, my result in Theorem 1 shows that the crowdfunding market could fall short of people's expectations. In the rest of this section, I analyze the formal model and present the main steps to prove Theorem 1.

2.3.2 Outsiders' equilibrium strategies

I first characterize outsiders' equilibrium strategies. I say that the outsiders play *cutoff strategies* if there exists a threshold number $n_0 + 1$ such that outsiders invest in a startup if and only if they observe that at least $n_0 + 1$ insiders have invested. Then I prove the following result.

Proposition 1. *At equilibrium, outsiders play cutoff strategies.*

The proof of Proposition 1 is in Appendix B.1. The key to proving Proposition

1 is to first narrow down insiders' strategy space, and then show that, conditioning on the insiders' strategy space, the outsiders play cutoff strategies. To narrow down insiders' strategy space, I show that insiders always receive a higher expected payoff from investing if the private signal is more positive.

Lemma 1. $\mathbb{E}^I(Y|G,S) > \mathbb{E}^I(Y|B,S)$.

The proof of Lemma 1 follows from the Bayes' rule and the independence of private information among insiders. Lemma 1 separates the G-signal insiders' investing behavior from the B-signal ones', and it restricts the insiders' strategy space to five cases.

1. *Case I:* no insider invests.
2. *Case II:* G-signal insiders use mixed strategy to invest, B-signal insiders do not invest.
3. *Case III:* G-signal insiders invest, B-signal insiders do not invest.
4. *Case IV:* G-signal insiders invest, B-signal insiders use mixed strategy to invest.
5. *Case V:* all insiders invest.

Once the insiders' strategy space is specified, it suffices to prove that the outsiders play a cutoff strategy conditioning on each possibility of insiders' strategy. See details in the proof of Proposition 1.

2.3.3 Insiders' equilibrium strategies

Next I analyze insiders' equilibrium investing strategy. I show that conditioning on the outsiders' cutoff strategies $n_0 + 1$ and a fixed share price p, the insiders have unique symmetric equilibrium strategies. Furthermore, I compute insiders' equilibrium strategies in terms of share price p.

Proposition 2. *Conditioning on the outsiders' cutoff strategies $n_0 + 1$ and a fixed share price p, there exist unique insiders' symmetric equilibrium strategies. Moreover, for fixed parameters α, n, and n_0, the equilibrium strategies can be characterized according to the share price p. Denote $e(n, n_0, \alpha) = \frac{\sum_{k \geq n_0} \binom{n}{k}(1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \binom{n}{k}\alpha^k (1-\alpha)^{n-k}}$. The insiders' equilibrium strategies have the following forms:*

I. If $p \geq \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{n_0+1}}$, then at the equilibrium, no insider invests (referred to as Equilibrium I).

II. If $\frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)} < p < \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{n_0+1}}$, then at the equilibrium, the insiders with G signals use a mixed strategy to invest and those with B signals do not invest (referred to as Equilibrium II).

III. If $\frac{1}{1+\frac{\alpha}{1-\alpha}\cdot e(n,n_0,\alpha)} \leq p \leq \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$, then at the equilibrium, all insiders with G signals invest and those with B signals do not (referred to as Equilibrium III).

IV. If $1 - \alpha < p < \frac{1}{1+\frac{\alpha}{1-\alpha}\cdot e(n,n_0,\alpha)}$, then at the equilibrium, the insiders with G signals invest and those with B signals use a mixed strategy to invest (referred to as Equilibrium IV).

V. If $p \leq 1 - \alpha$, then at the equilibrium, all the insiders invest regardless of their private signals (referred to as Equilibrium V).

The proof of Proposition 2 is in Appendix B.1. Because Cases I and V can be considered special cases of Cases II and IV, respectively, and Case III is an intermediate case, I just need to prove Proposition 2 for Cases II and IV. At equilibrium, when the insiders play mixed strategies, they are indifferent between investing and not investing. So at Case II, a G-signal insider receives zero expected payoff from investing, i.e.,

$$\mathbb{E}^I(Y|G, S) = \mathbb{P}(H|G, S) - p = 0. \tag{2.4}$$

Similarly, at Case IV, a B-signal insider receives zero expected payoff from investing, i.e.,

$$\mathbb{E}^I(Y|B, S) = \mathbb{P}(H|B, S) - p = 0. \tag{2.5}$$

Then conditioning on the outsiders' cutoff strategies $n_0 + 1$ and a fixed share price p, to prove the uniqueness of insiders' equilibrium strategies, I just need to prove that there is a unique solution of insiders' strategies to the equilibrium conditions (2.4) and (2.5). Equivalently, I just need to prove that one insider's gross payoff ($\mathbb{P}(H|G, S)$ and $\mathbb{P}(H|B, S)$) is monotone in her mixed probability of investing when assuming all other investors (both insiders and outsiders) play equilibrium strategies. For Case II, let r_G denote the mixed probability that a G-signal insider invests. Similarly, for Case IV, let r_B denote the mixed probability that a B-signal insider invests. Then I prove the following result.

Lemma 2. *1. At Equilibrium II, one G-signal insider's gross payoff* $\mathbb{P}(H|G, S)$ *is strictly decreasing in* $r_G \in (0, 1]$.

2. At Equilibrium IV, one B-signal insider's gross payoff $\mathbb{P}(H|B, S)$ *is strictly decreasing in* $r_B \in [0, 1]$.

The proof of Lemma 2 is in Appendix B.1. Although the proof of Lemma 2 is a bit involved, the intuition is simple. For example, at Equilibrium II, when the insiders with good signals increase their investing probability, the probability that at least $n_0 + 1$

insiders invest in a low quality startup increases relatively more than that probability in a high quality startup. Therefore, a low quality startup has relatively higher probability of being successfully funded when the G-signal insiders invest more aggressively, thus reducing G-signal insiders' gross expected payoff at equilibrium.

2.3.4 Effective screening

As argued before, whether the crowdfunding market can avoid market failure relies on its ability to solve the adverse selection problem. More specifically, it depends on whether the informed investors' information can be aggregated effectively. In this section, I show that the effectiveness of insiders' information aggregation decreases in insiders' likelihood of investing.

Definition 2. *Define the effectiveness of insiders' information aggregation as the ratio of the probability that a high quality startup gets funded over the probability that a low quality one does so:*

$$IA = \frac{\mathbb{P}(S|H)}{\mathbb{P}(S|L)}.$$

Lemma 3. *The effectiveness of insiders' information aggregation IA is decreasing in insiders' likelihood of investing. In particular, IA is decreasing in r_G at Equilibrium II and decreasing in r_B at Equilibrium IV.*

The proof of Lemma 3 is in the Appendix. Lemma 3 demonstrates that as insiders become more likely to invest, the crowdfunding market's relative ability to fund high quality startups weakens, and the crowdfunding market becomes less effective in screening startups. This also motivates the second condition in the definition of crowdfunding market equilibrium that B-signal insiders do not invest. Under the condition that B-signal insiders do not invest, I just need to focus on the first three cases of insiders' equilibrium strategies (Equilibrium I, II, III) when investigating the existence of a crowdfunding market equilibrium and solving the entrepreneur's problem.

2.3.5 The entrepreneur's equilibrium strategy

From the standpoint of the entrepreneur, when conditioning on the outsiders' equilibrium strategies, the event S (a successful crowdfunding campaign) is equivalent to that at least $n_0 + 1$ insiders invest. So the entrepreneur's problem becomes

$$\max_p \frac{\mathbb{P}(n_I \geq n_0 + 1|H) + \mathbb{P}(n_I \geq n_0 + 1|L)}{2} \cdot p^\gamma.$$

Let $n_i = n_I - 1$. Fix one insider's investing decision. Then from this insider's point of view, the event $n_I \geq n_0 + 1$ is also equivalent to the event that at least n_0 other insiders invest, i.e. $n_i \geq n_0$. Because the share price p is a function of $\mathbb{P}(n_i \geq n_0|H)$ and $\mathbb{P}(n_i \geq n_0|L)$ at equilibrium (see details in the proof of Proposition 2), it is convenient to solve an equivalent problem of the entrepreneur:

$$\max_p \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^\gamma. \tag{2.6}$$

Because the expressions of $\mathbb{P}(n_i \geq n_0|H)$, $\mathbb{P}(n_i \geq n_0|L)$ and p vary across different insiders' equilibrium strategy cases, to solve the entrepreneur's problem, it is useful to take a two-step procedure: (1) find an optimal price at each insiders' equilibrium strategy case, and (2) look for the global maximum across all insiders' equilibrium strategy cases. As discussed before, it suffices to focus on the first three cases of insiders' equilibrium strategies (Equilibrium I, II, III). Therefore, (2.6) is equivalent to the following problem:

$$\max_{\text{Equilibrium I, II, III}} \max_{p} \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^{\gamma}.$$

I can further simplify the above problem. In the first case of the insiders' equilibrium strategies, no insider invests, thus the entrepreneur's expected utility is zero. In the third case, the insiders who invest are those with G signals. Because the number of G signals does not depend on the price p, $\mathbb{P}(n_i \geq n_0|H)$ and $\mathbb{P}(n_i \geq n_0|L)$ do not depend on p. Thus, in the third case, the entrepreneur's expected utility is maximized at the highest price possible $p^* = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$. At p^*, the insiders' equilibrium strategies can also be considered a special case of Equilibrium II at $r_G = 1$. Therefore, I just need to solve the entrepreneur's problem at the insiders' equilibrium strategy at Equilibrium II. Then I prove the following result.

Proposition 3. *For any $\gamma \in (0, 1]$, at insiders' equilibrium strategies II (Equilibrium II), the entrepreneur's expected utility is maximized at the share price $p^* = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$, which corresponds to the insiders' equilibrium strategy $r_G = 1$.*

The proof of Proposition 3 is in Appendix B.1. Proposition 3 implies that conditioning on insiders' equilibrium strategy at Equilibrium II and outsiders' equilibrium strategies at n_0+1, the entrepreneur's expected utility is maximized at $p^* = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$. At p^*, G-signal insiders invest for sure and B-signal insiders do not. Then taking all three cases (Equilibrium I, II, III) together, we have that when B-signal insiders do not invest and all investors play equilibrium strategies, the entrepreneur will set the share price at p^* to maximize her expected utility.

2.3.6 Violation of outsiders' participation constraint

At a market equilibrium, the outsiders' participation constraint has to be satisfied. In other words, the outsiders' expected payoff has to be non-negative at equilibrium. Because the outsiders play a cutoff strategy at equilibrium, it suffices to guarantee that the outsiders receive a non-negative expected payoff at the threshold $n_0 + 1$. However, I show that if the entrepreneur sets the share price at p^*, the outsiders have a strictly negative expected payoff at the threshold of any equilibrium strategy.

Proposition 4. *For fixed parameters n, n_0, and α, if the entrepreneur sets the share price at $p^* = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$, then the outsiders' expected payoff at the threshold $n_0 + 1$ of any equilibrium strategy is strictly negative.*

The proof of Proposition 4 is in Appendix B.1. Proposition 4 implies that for a given funding threshold $n_0 + 1$ of the outsiders, when the entrepreneur chooses the optimal price and insiders play equilibrium strategies, it is not profitable for the outsiders to actually participate right at the threshold. Proposition 4 leads to the proof of the main result of Theorem 1.

2.4 Empirical evidence

In this section, I use data from Regulation Crowdfunding (Regulation CF, or Title III equity crowdfunding), and provide evidence that is consistent with my model's implications. The data period is between the start of Title III equity crowdfunding in May 2016 until September 2017. My final sample includes 461 firms' crowdfunding filings[10] and 133 successful crowdfunding offerings. See Table 2.1 for summary statistics of the data.

[10]The data are collected from the SEC edgar website: `https://www.sec.gov/edgar/searchedgar/companysearch.html`. A few firms have multiple crowdfunding campaigns, in which case I only include the first one.

Analyzing the data, I find several pieces of evidence that are consistent with my model's implications. First, I find that the total number of Title III crowdfunding offerings is small. In contrast to Regulation CF, Regulation D, the current dominant private offering regulation, attracted 27,725 firms to use it during the same period (05/16/2016-9/3/2017). So in terms of number of firms using each regulation, the Regulation D market is currently 60 times larger than the Regulation CF market. In this regard, after being legal more than one year, the Regulation CF market has grown much more slowly than expected. We can also look at the Regulation CF market in terms of sectors. The sector with the greatest number of successful crowdfunding offerings is the food and beverage industry, especially small breweries and restaurants.[11] Upon launching crowdfunding offerings, the firms from the food and beverage industries also have higher success rates of completing their offering than the technology sectors.[12] Therefore, the overall development of the Regulation CF market suggests that the market size is small, and the market has favored the industries that potentially have lower information asymmetries, such as restaurants. This pattern is consistent with my model's implication that the Regulation CF market is not able to solve the "lemons" problem, and thus performs worse in industries with higher information asymmetries.

Second, in terms of financials, the Regulation CF market does not appear to fund high quality firms with higher probability. To examine whether the current Regulation CF market is able to fund high quality firms, I investigate the link between firms' financials and their rate of succeeding in their crowdfunding offerings. In particular, all

[11]Using the first six months of data, 28.13% of all funded campaigns are from food and beverage companies, see `https://blog.vcexperts.com/2016/12/20/regulation-crowdfunding-a-six-month-update/`

[12]In the first six months, the food and beverage sector accounts for 17.5% of all launched campaigns, and 28.13% of all funded campaigns, in contrast to the technology sector which accounts for 25% of all launched campaigns, and 12.5% of all funded campaigns, see `https://blog.vcexperts.com/2016/12/20/regulation-crowdfunding-a-six-month-update/`

Table 2.1: SUMMARY STATISTICS

Notes: The table reports the summary statistics of Regulation Crowdfunding filings up to September 3, 2017. The sample includes each firm's first crowdfunding campaign. The variables include: "Compensation %", the percentage of offering size charged by funding portals upon a successful offering; "Total asset", the total asset of a firm in the most recent fiscal year; "Cash", the total cash and cash equivalents in the most recent fiscal year; "Corporation", a dummy variable indicating whether a firm is incorporated; "Campaign Success", a dummy variable whether a crowdfunding offering is successful. The units of some variables are in million dollars (M).

	mean	sd	min	p25	p50	p75	max	count
Age at financing	2.48	3.55	0.01	0.36	1.38	3.28	45.07	459
Compensation %	5.36	1.86	0.00	4.00	5.00	6.00	12.00	459
Offering amount (M)	0.10	0.13	0.01	0.03	0.05	0.10	1.00	459
Number of employees	5.24	8.85	0.00	1.00	3.00	6.00	100.00	459
Total asset (M)	0.38	1.52	0.00	0.00	0.04	0.23	21.28	459
Cash (M)	0.07	0.19	-0.06	0.00	0.00	0.05	2.32	459
Revenue (M)	0.30	0.93	0.00	0.00	0.00	0.08	7.45	459
Net income (M)	-0.22	0.61	-5.78	-0.15	-0.01	0.00	0.39	459
Common stock	0.34	0.47	0.00	0.00	0.00	1.00	1.00	459
Debt	0.23	0.42	0.00	0.00	0.00	0.00	1.00	459
Corporation	0.71	0.46	0.00	0.00	1.00	1.00	1.00	459
Incorporated in CA	0.14	0.34	0.00	0.00	0.00	0.00	1.00	459
Incorporated in DE	0.46	0.50	0.00	0.00	0.00	1.00	1.00	459
Campaign Success	0.29	0.45	0.00	0.00	0.00	1.00	1.00	459

else equal, firms with better financials, e.g., higher profitability, should have a higher rate of completing their crowdfunding offerings. To test this, I use firms' net income in the most recent fiscal year to measure firms' profitabiltiy, and explore the following regression framework:

$$Y_i = \delta_0 + \delta_1 I_i + \delta_2 X_i + \alpha_p + \alpha_L + \epsilon_i. \quad (2.7)$$

Here, Y_i is a dummy variable indicating whether firm i has successfully completed its crowdfunding offering. The variable of interest I_i is the net income of firm i in the most recent fiscal year. X_i includes firm level controls such as age at crowdfunding, number of employees, and so on. These controls take care of firms' heterogeneity effects in the corresponding dimensions that relate to firms' success rate of crowdfunding offerings.

Importantly, equation (2.7) also allows me to control crowdfunding portal fixed effects (α_p). Different crowdfunding portals appeal to different sets of investors, and also have different volumes of active investors, so launching crowdfunding campaigns in different portals could significantly impact firms' chances of success. Therefore, controlling portal fixed effects is important in the regression. I also include several other fixed effects, such as firms' residence and incorporation location (α_L) in the regression.

Table 2.2: FIRMS' FINANCIALS AND SUCCESS OF CROWDFUNDING

Notes: This table reports results from OLS regressions of (2.7). The dependent variables are "Success": a dummy variable indicating whether a firm has successfully completed its crowdfunding offering, and "Big Success", a dummy variable indicating whether a firm has successfully completed its crowdfunding offering with capital raised greater than $500000. The main explanatory variable is "Net income", the net income of a firm in the most recent fiscal year. Several controls are "Revenue", the revenue of a firm in the most recent fiscal year, "Total asset", total assets of a firm in the most recent fiscal year, "Cash", cash and cash equivalents in the most recent fiscal year. For some of the explanatory variables, the units are in million dollars (M). The sample includes firms' first crowdfunding campaign between 05/16/2016-9/3/2017. Robust standard errors are reported in parentheses. Significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Success		Big Success	
	(1)	(2)	(3)	(4)
Net income (M)	0.064*	0.063	0.005	-0.008
	(0.038)	(0.039)	(0.033)	(0.032)
Revenue (M)		0.005		0.079***
		(0.036)		(0.025)
Total asset (M)	-0.002	-0.003	0.014	0.010
	(0.012)	(0.012)	(0.010)	(0.008)
Cash (M)	0.326***	0.321***	0.277***	0.192**
	(0.108)	(0.118)	(0.088)	(0.076)
Number of employees	0.006**	0.006*	0.007**	0.003
	(0.002)	(0.003)	(0.003)	(0.002)
Age at financing	0.004	0.004	-0.010***	-0.017***
	(0.005)	(0.006)	(0.003)	(0.003)
Adjusted R^2	0.231	0.228	0.325	0.374
Entity Type FE	Yes	Yes	Yes	Yes
Incorporation State FE	Yes	Yes	Yes	Yes
Location State FE	Yes	Yes	Yes	Yes
Intermediary FE	Yes	Yes	Yes	Yes
Observations	461	461	461	461

Table 2.2 reports regression results of (2.7). Although Column (1) shows a sig-

nificant correlation between firms' net income and their success rate of crowdfunding, the significance disappears once I control for revenue (Column (2)). Moreover, if we focus on successful crowdfunding that raised more than 0.5 million dollars in Columns (3) and (4), the positive correlation completely disappears. These results suggest that firms' profitability does not predict firms' success in crowdfunding. In other words, the crowdfunding market does not seem to be financing high quality startups. This evidence is consistent with the main implication of my model that the Regulation CF market in current design may not be able to overcome the lemons problem and fund high quality startups.

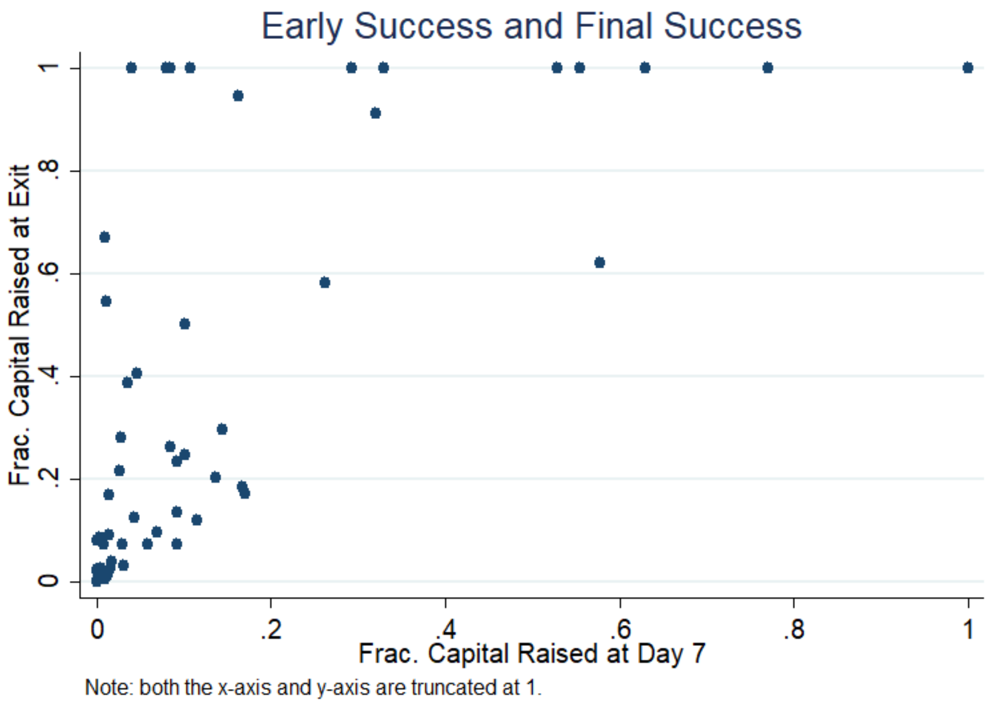
2.5 Discussion

In Proposition 1, I proved that outsiders play cutoff strategies conditioning on insiders' strategy space. One direct implication of this result is that if a startup achieves early success by attracting enough insiders at the beginning of its crowdfunding campaign, then it should eventually succeed with high probability. To test this, I collected funding dynamics data from a Title III equity crowdfunding portal called Wefunder (the most popular Title III crowdfunding portal) (October 2017-April 2018). Figure 2.3 plots the fraction of capital raised over minimum funding goal at exit against the fraction of capital raised at Day 7. It shows that crowdfunding campaigns that are able to raise more than 20% of minimum funding goal within 7 days will succeed with 75% probability. In contrast, crowdfunding campaigns that raise less than 20% of minimum funding goal within 7 days will succeed with only less than 10% probability. This evidence is consistent with the implication of Proposition 1.

Figure 2.4 shows a more complete picture of the funding dynamics of crowdfunding

Figure 2.3: EARLY SUCCESS AND FINAL SUCCESS

Notes: The x-axis represents the fraction of capital raised over the minimum funding goal at the end of Day 7 since the beginning of a crowdfunding campaign, and the y-axis represents the fraction of capital raised over the minimum funding goal when a crowdfunding campaign ended (was taken down from the funding portal).

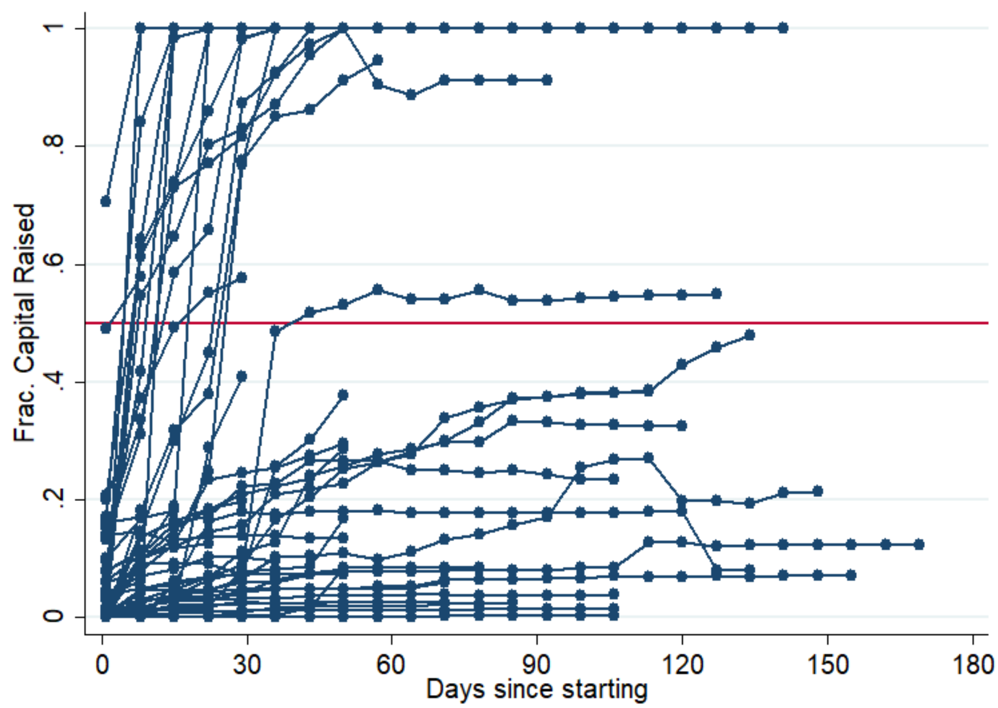


campaigns. Three patterns are salient in Figure 2.4: (1) most funding activities occurred within the first month of the crowdfunding campaigns; (2) the campaigns that eventually succeeded (the capital raised exceeds the minimum funding goal) raised capital quickly at the beginning; and (3) the campaigns that eventually failed raised little capital till the end. These patterns are again consistent with the implication of Proposition 1: outsiders' investment decisions depend on how insiders react positively towards a crowdfunding campaign at the beginning, and campaigns achieving early success will have high probability to succeed in the end.[13]

[13]Figure 2.4 can be also consistent with investors' herding behavior [see e.g., 3, 99]. To possibly disentangle the information aggregation story studied in this paper and investor herding story, it would be useful to observe the long-term outcomes of crowdfunding startups and to learn whether the startups succeeding in crowdfunding campaigns are indeed of higher quality, e.g. being able to achieve high survival and growth rate. Clearly, this is beyond the scope of this paper.

Figure 2.4: FUNDING DYNAMICS

Notes: The x-axis represents the number of days since a crowdfunding campaign is posted on the crowdfunding portal, and the y-axis represents the fraction of capital raised over the minimum funding goal (set by the entrepreneur). The y-axis is truncated at 1. For simplicity, the sample includes only observations at Day $7n + 1$ ($n = 0, 1, 2, \ldots$) since the start of crowdfunding campaigns.



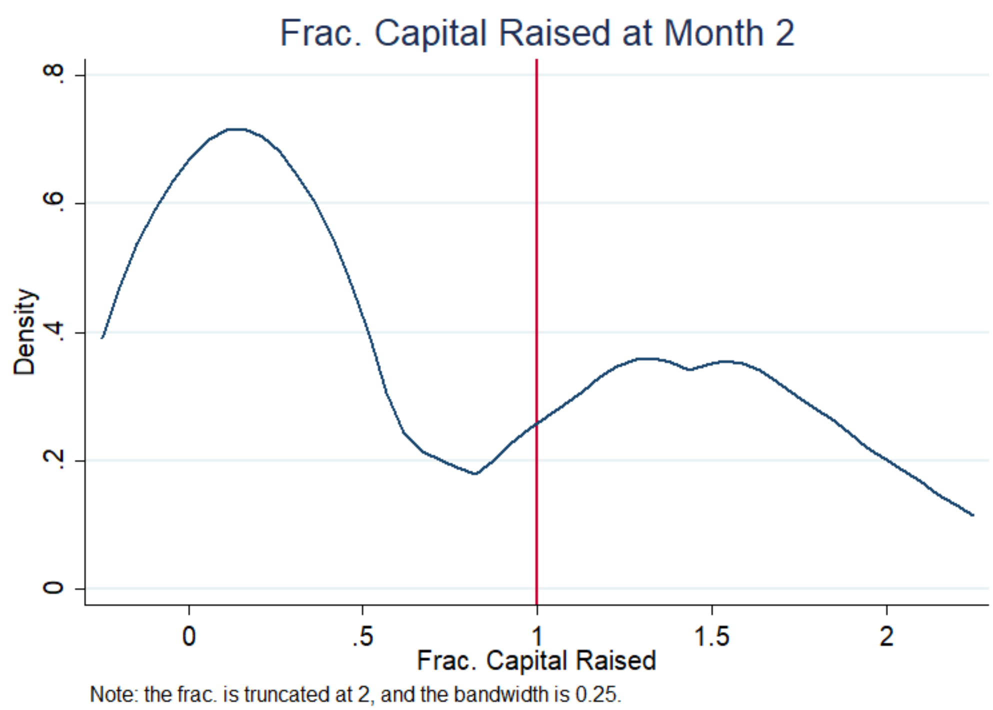
One particularly interesting pattern emerging from Figure 2.4 is that there is a wide wedge between campaigns that eventually succeeded and those failed in terms of capital raised at the time of exit. In other words, there are very few crowdfunding campaigns that almost reached the (minimum) funding goal but did not. The pattern can also be observed in a density plot (Figure 2.5).

2.5.1 Improving market mechanisms

Theorem 1 proves that there does not exist a crowdfunding market equilibrium. Here, I suggest an improving market mechanism that would support a crowdfunding market equilibrium.

Figure 2.5: OUTCOME AT THE END OF MONTH 2

Notes: The figure plots the kernel density estimation of the fraction of capital raised over minimum funding goal at the end of month 2 for all crowdfunding campaigns.



The main reason for the non-existence of a crowdfunding market equilibrium is that the outsiders' participation constraint cannot be satisfied. At a crowdfunding market equilibrium, because only the insiders with good signals invest, the outsiders can essentially observe the aggregate of all the insiders' private signals by observing the aggregate of all the insiders' actions. As a result, the outsiders have more information than any single insider, thus giving the outsiders information advantage in inferring the quality of the startup, and thus requiring a stronger participation constraint. As studied in the IPO literature [87], underpricing is an effective method for solving the uninformed investors' participation constraint problem. However, in the crowdfunding market, underpricing may not be so feasible. As argued before, the crowdfunding market needs to aggregate the insiders' information to overcome the adverse selection problem. When underpricing, on one hand, could solve outsiders' participation constraint, on the other hand, it could also destroy the insiders' information aggregation. Indeed, the insiders can collectively aggregate information only when insiders with different private information make different investment decisions (G-signal insiders invest and B-signal insiders do not). But because the information difference between G-signal and B-signal insiders is small if they are strategic, very little room exists for the share price to be set so that insiders with different levels of information invest differently. This limits the practicality of underpricing in the crowdfunding market: lowering the share price by any significant amount will make it worthwhile for B-signal investors to deviate, leading insiders' information aggregation to fail.

To remedy the non-existence of a crowdfunding market equilibrium, one simple mechanism is to set different prices for the insiders and outsiders. More specifically, the entrepreneur can set a *state-contingent share price for the outsiders*. Consider n_I, the number of insiders who invest, as the *states*. Then a state-contingent price for the outsiders is a price function $p(n_I)$ with $n_0 + 1 \leq n_I \leq n + 1$. The entrepreneur solves a new optimization problem:

$$\max_{p(n_I)} p^{\gamma}(n_I) \quad \text{s.t. } \mathbb{P}(H|n_I, S) - p(n_I) \geq 0,$$

where $\mathbb{P}(H|n_I, S) - p(n_I) \geq 0$ is the outsiders' participation constraint conditioning on n_I insiders invest. The entrepreneur's optimization problem can be easily solved.

Proposition 5. *If the entrepreneur were allowed to set a state-contingent price for the outsiders, the entrepreneur would optimally set the share price* $p'(n_I) = \frac{1}{1+\left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}}$ *(*$n_0 + 1 \leq n_I \leq n + 1$*) for the outsiders. When the entrepreneur sets the optimal price*

p^* for the insiders, and $p'(n_I)$ for the outsiders, there exists a crowdfunding market equilibrium.

Proposition 5 implies that were the entrepreneur allowed to set a state-contingent price for the outsiders, she can offer a discount on the share price for the outsiders if the realized n_I is low, and charging a premium if the realized n_I is high. Clearly, the state-contingent price in Proposition 5 satisfies the outsiders' participation constraint. Under the state-contingent price scheme, the insiders' information can be conveyed to the outsiders at a market equilibrium, and high quality startups end up with higher probability of being funded.

2.6 Conclusion

Equity crowdfunding is a new evolving market for startups to raise capital. Like other early-stage financing models, equity crowdfunding also faces high information asymmetries. In this paper, I provide a rigorous analysis for one main type of information asymmetries: adverse selection. I find that no simple market equilibrium exists that could solve the adverse selection problem. Under the current fixed offering price rule, the equity crowdfunding market does not seem able to screen high quality projects. Using Regulation CF filings, I also provide empirical evidence that is consistent with my theoretical findings. The inability to overcome the lemons problem could lead to market failure [4]. My finding suggests that to promote the development of the equity crowdfunding market, introducing new market mechanisms that can remove the adverse selection barrier may be necessary.

Chapter 3

FIRM CAPABILITIES AND INDUSTRY STRUCTURE

3.1 Introduction

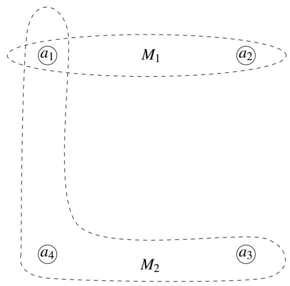
There are two predominant and competing theories of competitive advantage in the management literature. With strong economic and industrial organization foundations, Porter [2008] argues that the industrial environment drives competitive advantage and that firms must exploit and manipulate this to make profits. In contrast, the resource-based view of the firm pioneered by Wernerfelt [1984] argues that firms should be viewed as sets of immutable and scarce resources and that firms derive their competitive advantage from these resources, broadly defined. This idea is simple and intuitive but does not have the mathematical and formal foundations of Porter [2008]. As a result it suffers from critiques of tautology. Resources are defined to include everything that can generate a competitive advantage, while its objective is to provide a theory of competitive advantage (see Kraaijenbrink, Spender and Groen, 2010).

In this paper we provide formal foundations for the resource-based view of the firm. This resolves the tautology and builds a framework that ties firms' capabilities to their relative competitiveness within markets. Our approach utilizes similar industrial organization foundations to Porter [2008], and so provides a framework that can accommodate the two theories of competitive advantage. It also delivers practical economic insights. We argue that merger analysis and regulations should be more holistic, considering markets beyond those in which the merging firms compete and find that strategic alliances are welfare enhancing even though they can reduce competition. We also explain industry consolidation as a profitable and possibly consumer surplus increasing change in equilibrium industry structure.

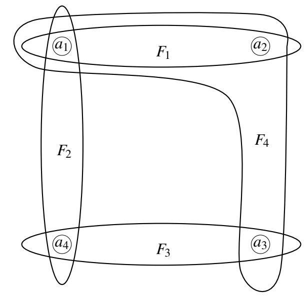
Our model is simple. Each firm has a finite set of capabilities and each market values a finite set of capabilities. The firms' capabilities are their potential sources of competitive advantage, while markets' capabilities reflect which sources of competitive advantage are useful in which markets. Firm and market capabilities are fixed in the short run, reflecting their scarcity. If one firm has more relevant capabilities than another for a given market, then it is a stronger competitor in that market. Stronger competitors have higher price-cost margins. We model this through reduced marginal costs, but it could instead be modeled through consumers' higher willingness to pay without much changing. Firms choose how much to produce in each market. Thus we model Cournot competition market by market, where the marginal cost of a firm is determined by the intersection of that firm's capabilities and that market's capabilities.

As firms are associated with sets of capabilities and markets are also associated with sets of capabilities, we let a firm hypergraph describe firms and a market hypergraph describe markets. In both hypergraphs the nodes of the graph are attributes/capabilities. In the firm hypergraph each firm is then represented by an edge which comprises of a subset of the available capabilities. Similarly, in the market hypergraph each market is represented by an edge, which is a subset of the available capabilities. See for example Figure 3.1 below.

In this example there are four underlying attributes. There are two markets shown in Panel (A) of Figure 3.1. In market M_1 attributes a_1 and a_2 are valued. In market M_2 attributes a_1, a_3 and a_4 are valued. There are four firms. Their capabilities are shown in Panel (B) of Figure 3.1. In the first market firms 1 and 4 have all the useful attributes



(a) The market hypergraph



(b) The firm hypergraph

Figure 3.1: REPRESENTATION OF FIRMS AND MARKETS BY HYPERGRAPHS.

and are strong competitors. Firm 2 is a weaker competitor with just one attribute and firm 3 is the weakest competitor with none of the useful attributes. In the second market firms 2, 3, and 4 all have (a different) two of the three useful capabilities and will be strong competitors. Firm 1 has only one of the three useful capabilities and is a weaker competitor.

We apply our model to address a couple of different, but related questions. First we use our model to reevaluate merger guidelines and analysis. We assume that a merged firm inherits the combined capabilities of merging firms but that it is costly to maintain additional capabilities and more costly per capability the more capabilities a firm has. This limits the profitability of mergers, particularly those that do not generate any relevant synergies by enhancing the competitiveness of the merged firm in some market. These costs might, for example, be associated with limitations on management time and expertise.

Our capability centric approach allows us to formalize (production) synergies. While this formalization is valuable in itself when considering the benefits of a merger within a market in which the merging firms compete, it also emphasizes the efficiency benefits mergers can have in other markets. If only one of the merging firms competes in a given market prior to the merger, the merger may create a stronger competitor in that market. We show that in equilibrium this always increases consumer surplus. The merged firm may also enter new markets, thereby increasing consumer surplus. Altogether these benefits can be substantial. Blocking a merger based on anticompetitive effects in overlapping markets (or prescribing remedies that render the merger unprofitable) can be socially inefficient. Indeed, we find that a decision which maximizes consumer surplus in overlapping markets by blocking a merger can lead to an arbitrarily high loss of consumer surplus in comparison to the counterfactual in which the merger is permitted. Moreover, in contrast to the holistic approach our model suggests, merger guidelines prescribe a market-by-market analysis:[1]

> *"The Agencies will not challenge a merger if cognizable efficiencies are of a character and magnitude such that the merger is not likely to be anticompetitive in any relevant market. To make the requisite determination, the Agencies consider whether cognizable efficiencies likely would be sufficient to reverse the merger's potential to harm customers in the relevant market, e.g., by preventing price increases in that market."*
> (Horizontal Merger Guidelines, US department of Justice and the Federal Trade Commission, 2010.)

We also note that if merger guidelines achieve their objective of making the merger review process transparent, then mergers that would reduce consumer surplus in overlapping markets will not get to the review stage, even if they would also substantially

[1] See Farrell and Shapiro [2001] for an excellent discussion of the treatment of synergies in the merger guidelines within a market.

increase consumer surplus in non-overlapping markets.[2] This makes it hard to estimate the efficiency gains that could be obtained by a more holistic merger review approach.

Strategic alliances are an alternative means through which firms can combine their capabilities to better compete in a market. We relate the profitability of joint ventures to the current competitiveness of the parent firms in the market being targeted, and the capabilities the joint venture is endowed with. While not all joint ventures will be profitable, those that are reduce the market price and increase consumer surplus even when the parent firms compete less intensively or exit the market as a result.

Finally, we show that a market structure with many low capability firms can be stable, while an alternative market structure in which firms with different capabilities have merged to create fewer more competitive firms is also stable. We provide an example in which the equilibrium with fewer but more competitive firms generates higher consumer surplus as well as higher profits. We can therefore explain waves of industry consolidation as an efficiency and profitability enhancing change in equilibrium, in which firms become more capable within the market through their acquisitions. A change in beliefs that others will merge is sufficient to trigger such a change in equilibrium, but exogenous events that reduce profitability can cause the fragmented market structure to no longer be an equilibrium and also trigger the consolidation.

3.1.1 Related Literature

Beyond the management literature we discuss above, which does not provide formal mathematical foundations, we are unaware of any economic paper that proposes modeling firms as sets of capabilities or formulates a hypergraph representation of firms

[2] Nevertheless, a possible example of a merger that was blocked but might have had substantial efficiency benefits outside of overlapping markets is the proposed merger between General Electric and Honeywell which was blocked by the European antitrust authorities in 2001.

or markets. There is one exception.[3] Malamud and Rostek [2015] model financial exchanges, and competition across them, as a hypergraph. In their setting the set of nodes are the exchanges and the edges represent which traders are active on which of these exchanges. This is fundamentally different from the hypergraphs we consider where the nodes are attributes/capabilties.

There is a huge industrial organization literature that considers mergers and different aspects of the merger review process. We do not attempt a comprehensive review here and instead only mention a couple of papers we consider more relevant. Like us Nocke and Whinston [79] and Nocke and Whinston [80] evaluate the merger review process, although they focus on a different aspect—they reconsider merger review policy in the context of endogenous merger choices. Perhaps most closely related to us are papers that focus on synergies. Larsson and Finkelstein (1999) argue that the success of a merger depends on the degree of synergies that are realized, and that not only the similarities across firms matter, but also their production and marketing complementarities. Chatterjee [2002] provides a framework for categorizing different types of synergies. Nevertheless, our formulation of synergies in capabilties/attributes space is novel to the best of our knowledge. Also, as argued by Jensen and Ruback (1983), mergers in general have been demonstrated to create value, and the gains created do not appear to come from the creation of market power. By explicitly modeling firms as sets of capabilities we are able to formally capture synergies which can be achieved through combining firms' key sources of competitive advantage.

There is a large amount of literature in management that studies joint ventures. A main focus of this literature is to better understand the motivation for creating joint ven-

[3] Dziubinski and Goyal [2013] also use hypergraphs, but in the very different setting of attack and defence networks.

tures (see Kogut, 1988). Hennart (1988), for example, argues that equity joint ventures can be explained by the presence of transactions costs. In contrast, Kogut (1991) argues that joint ventures are developed by their parent firms for strategic motivations, and are created as options to expand in response to future technological and market developments. Tsang (2000) argues that the formation of joint ventures can be explained using a resource-based perspective of firms, and that this complements the transaction cost approach. This resource-based view of joint ventures is closest to us. Our formalization permits us to prove results about profitability and consumer surplus. In the economics literature the focus has been somewhat different. For example, Kamien et al. (1992) analyze the effects of research motivated joint ventures on the set of firms that compete in the relevant product market. Perhaps the closest paper to ours in the economics literature is Goyal et al. (2008). They develop a model of R&D collaboration in which firms undertake joint projects on non-core activities with other firms.

On industry consolidation, the literature most related to our paper is the work of Sutton (1991, 2001).[4] Sutton shows that exogenous changes in production technology or consumers' willingness to pay, can trigger a period of industry consolidation. Endogenous investments in sunk costs reduce marginal costs and the number of firms the market can support, causing some to exit. A classic example of such a change in market structure is the tire industry (Klepper and Simons, 2000). We offer a complementary but related mechanism through which consolidation can occur. Firms acquire scarce capabilities through mergers, thereby reducing marginal costs. This directly models consolidation as being achieved through mergers and further explains how marginal cost reductions are obtained. It also demonstrates that efficient consolidation can be

[4] See also Dasgupta and Stiglitz [1980].

triggered simply by changing expectations and the resulting change in the equilibrium being played. Conceptually, our results on stable industry structures are also related to the networks literature (Jackson and Watts, 2001).

The organization of our paper is as follows: in Section 3.2, we introduce our model and prove the existence of Nash equilibrium given a fixed set of firms. In Section 3.3, we apply our framework to study merger guidelines. Section 3.4 considers joint ventures as an alternative means by which firms might combine their capabilities. In Section 3.5 we define and analyze stable industry structures. Section 3.6 concludes.

3.2 Firms as sets of capabilities

There is a finite set of attributes A, a finite set of firms $\{1, \ldots, n\}$ and a finite set of markets $\{1, \ldots, m\}$. Each firm i is associated with a set of capabilities $F_i \subseteq A$. This represents the sources of competitive advantage for firm i. Each market j is also associated with a finite set of capabilities $M_j \subseteq A$. This represents the attributes that are useful in market j.

We sometimes represent this information in a hypergraph. A hypergraph is defined by a set of nodes, in this case the set of attributes A, and a set of edges, each of which constitutes a subset of the nodes. The firm hypergraph is $H_F = (A, \{F_1, F_2, \ldots, F_n\})$. Abusing notation, we sometimes write $H_F = \{F_1, F_2, \ldots, F_n\}$. The market hypergraph is $H_M = (A, \{M_1, M_2, \ldots, M_m\})$ and similarly we abuse notation by writing $H_M = \{M_1, M_2, \ldots, M_m\}$. We let $\mathcal{H}(A)$ be the set of all possible hypergraphs fixing attributes A.

Firms that have more relevant capabilities for a given market are stronger competitors in that market. We assume that each firm's marginal cost in a given market is constant and does not depend on the firm's output in that market. This constant marginal cost of firm i in market j does depends on how well i's capabilities, F_i, match the capabilities associated with market j, M_j. For simplicity we define the variable $\theta_{ij} \equiv |F_i \cap M_j|$ and assume that i's marginal cost in market j is a decreasing function of θ_{ij}. We denote i's marginal cost in market j by $c(\theta_{ij})$, but abusing notation will sometimes write c_{ij} instead.[5]

All firms simultaneously decide how much to produce in all markets. We take a zero output decision of firm i in market j to mean that the firm i does not enter market j. The output choice of firm i in market j is given by $q_{ij} \geq 0$ and the vector $\mathbf{q}_i \in \Re^m_+$ represents i's entry and output choices in all markets.

We let $Q_j = \sum_{i=1}^{n} q_{ij}$ be the total output of all firms in market j, and let $P_j(Q_j)$ be the inverse demand function for market j. For simplicity we consider a linear inverse demand curve and set $P_j(Q_j) = \alpha_j - \beta_j Q_j$ with $\alpha_j, \beta_j > 0$. Firm i's profits in market j are

$$\pi_{ij}(q_{ij}) = \left(\alpha_j - \beta_j Q_j - c(\theta_{ij})\right) q_{ij}.$$

Let $Q_{-ij} = Q_j - q_{ij}$ be the output of firms other than i in market j. Firm i's overall profitability is given by

$$\pi_i(\mathbf{q}_i) \equiv \sum_{j=1}^{m} \pi_{ij}(q_{ij}) - \kappa(|F_i|),$$

[5] Although we assume that the match of a firm's capabilities with a market affects the firm's profitability by reducing their marginal costs, our results would not change if the match instead increases consumers willingness to pay for i's product. Specifically, we could let the demand parameter β_j, defined below, instead be a function $\beta_j(\theta_{ij})$ that is strictly decreasing in θ_{ij}. The price obtained by firm i in market j would then be $P_{ij}(Q_j, \theta_{ij}) = \alpha_j - \beta_j(\theta_{ij})Q_j$.

where we assume that κ is a weakly increasing and convex function. This captures the conglomeration costs associated with maintaining many unrelated capabilities. It may reflect the scarcity of management time or inability of the firm to tailor their corporate culture towards maintaining a broad set of capabilities. The firm therefore solves the problem:

$$\max_{\mathbf{q}_i \in \Re^m_+} \pi_i(\mathbf{q}_i).$$

Each firm's overall profitability is the sum of its profitability over all markets, less a penalty for the number of capabilities it maintains. Thus, fixing capabilities, a firm's output decision in each market can be made in isolation. Taking the output decisions of the other firms as given, firm i therefore maximizes its profits by selecting in each market j, an output

$$q^*_{ij} = \max\left\{\frac{\alpha_j - \beta_j \sum_{k\neq i} q_{kj} - c(\theta_{ij})}{2\beta_j}, 0\right\}.$$

3.2.1 Existence and uniqueness of Nash equilibrium

Standard results apply to our setting and, fixing the capabilities of firms, there is a unique equilibrium output decision for all firms in all markets.

Proposition 6. *There is a unique Nash equilibrium. In equilibrium firms who are more capable in a given market produce more: If $q^*_{ij} > q^*_{kj}$ then $\theta_{ij} > \theta_{kj}$.*

The proof is in Appendix C.1. It is a standard result that in the $m = 1$ case there is a unique equilibrium. This result is easily extended to our setting because firms' profitability in each market is independent of decisions in other markets. It is also intuitive that more capable firms in a given market have higher output because they have lower marginal cost. An immediate implication is that the x firms who enter a given market are the x most capable with the x lowest marginal costs for that market.

3.2.2 Discussion

To the best of our knowledge, there are no other papers in the economics literature that represent firms and markets as sets of capabilities. Doing so enables us to analyze competition across many markets in which different firms are better suited to compete in different markets.[6] Firm acquisitions and joint ventures can then be viewed in this light, providing a new perspective on corporate strategy. It also provides valuable foundations for modeling the dynamic evolution of markets and undertaking empirical work. We explore some of these applications in the subsequent sections.

While the economics literature has not previously taken a capability centric view towards markets and firms, the management literature has long viewed firms in this way. Indeed, the resource based view of the firm is one of the preeminent theories of strategic competition. In relation to this literature our model provides simple formal foundations. In doing so it is able to address a long standing critique of tautology—while key competencies or capabilities are broadly defined to include anything that yields a competitive advantage, they are also the determinants of competitive advantage. Moreover, the set of capabilities might include the ability to use other capabilities creating the potential for an infinite hierarchy of capabilities. Modeling capabilities as a fixed set allows the separation of capabilities and the actions of the firm in bringing these capabilities to bear in a given market.

[6] It does not, nor is it intended to, provide a theory of the firm. We do not explain what is made within a firm and what is bought in through the market from suppliers.

Our view of capabilities is a slightly narrower view than that originally envisaged, but one that provides a separate role for managers in terms of capability acquisition and the decisions of which markets to enter. Moreover, by putting the resource based view of the firm into a standard industrial organization (IO) setting, it can be combined with the IO foundations underlying Porter.

3.3 Mergers guidelines

In this section we consider mergers as a way in which firms can combine their capabilities. We compare the combined profits of two firms pre-merger with their profits post merger, assuming that all other firms' capabilities remain fixed. We also consider market prices pre-merger and post-merger, again assuming that all other firms' capabilities remain fixed. In Section 3.5 we extend our focus to include the equilibrium response of other firms and look for stable industry structures in which no two firms want to merge and no firm wants to demerge by spinning-off another firm.

If firms i and k merge then these firms cease existing individually and a new firm l is created with attributes $F_l = F_i \cup F_k$. On the hypergraph, it is equivalent to the union of the two edges representing i and k. We refer to the set of firms present in a given market as the market structure and collectively to these market structures across all markets as the industry structure. Several factors determine whether a merger between two firms i and k is profitable, and how it affects consumer surplus in various markets. We separate these changes into six effects. As an intermediate step it is helpful to differentiate between: (i) *overlapping markets* in which both i and k compete (such that pre-merger $q^*_{ij} > 0$ and $q^*_{kj} > 0$); (ii) *non-overlapping markets* in which either i or k compete, but not both (such that pre-merger $\max\{q^*_{ij}, q^*_{kj}\} > 0$ but $\min\{q^*_{ij}, q^*_{kj}\} = 0$);

and (iii) *newly entered markets* in which neither i nor k compete pre-merger (such that $q^*_{ij} = q^*_{kj} = 0$). We then have the following effects of the merger:

1. *Concentration in overlapping markets:* In an overlapping market j, firms i and k no longer compete against each other reducing competition but capturing an overall smaller market share.

2. *Synergies in overlapping markets:* In an overlapping market j, if $\left(F_i \cap M_j\right) \subset \left(F_l \cap M_j\right)$ and $\left(F_k \cap M_j\right) \subset \left(F_l \cap M_j\right)$ then firm l is a stronger competitor in market j than either i or k was alone and we say that the merger has synergies in market j.

3. *Synergies in non-overlapping markets:* In a non-overlapping market j with $q^*_{ij} > 0$ and $q^*_{kj} = 0$ pre-merger, if $F_i \cap M_j \subset F_l \cap M_j$ such that l is a stronger competitor in market j than i was alone, then we say that the merger has synergies in market j.

4. *Synergies in new markets:* There are synergies generated whenever the merged firm enters new markets. As post-merger $q^*_{lj} > 0$ but pre-merger $q^*_{ij} = q^*_{kj} = 0$, by Proposition 6 we must have that $\left(F_i \cap M_j\right) \subset \left(F_l \cap M_j\right)$ and $\left(F_k \cap M_j\right) \subset \left(F_l \cap M_j\right)$.

5. *Economies of Scale:* If capabilities overlap sufficiently, the combined cost of maintaining capabilities can decrease. In particular, if $F_i = F_k$ then combined maintenance costs will decrease from $2\kappa(|F_i|)$ to $\kappa(|F_i|)$ post merger.

6. *Diseconomies of Scope:* If capabilities do not overlap sufficiently, the combined cost of maintaining capabilities can increase. In particular, if $F_i \cap F_k = \emptyset$ then by

the convexity of κ, combined maintenance costs will increase from $\kappa(|F_i|)+\kappa(|F_k|)$ to $\kappa(|F_i|+|F_k|)$ post merger.

This list is exhaustive and mutually exclusive—we can decompose the profitability of a merger between firm i and k to create l into these effects. To do so, we denote the sets of overlapping markets, non-overlapping markets in which i but not k competes, non-overlapping markets in which k but not i competes and newly entered markets by J_o, J_i, J_k and J_e respectively. To separate market concentration and synergy effects in overlapping markets, let $\hat{\pi}_{ij}$ and $\hat{\pi}_{kj}$ be the profits i and k would get in market j were they to both reduce their marginal cost to the post merger level, but without actually merging so that they both still compete against each other. By definition

$$\pi_l - (\pi_i + \pi_k) \;=\; \sum_{j=1}^{m} \pi_{lj} - (\pi_{ij} + \pi_{kj}) - [\kappa(|F_l|) - (\kappa(|F_i|) + \kappa(|F_k|))]. \qquad (3.1)$$

The right hand side of equation (3.1) can be decomposed into the following terms:

$$\begin{aligned}
&\underbrace{\sum_{j\in J_0} \pi_{lj} - (\hat{\pi}_{ij} + \hat{\pi}_{kj})}_{\textit{(1)}} + \underbrace{\sum_{j\in J_0} (\hat{\pi}_{ij} + \hat{\pi}_{kj}) - (\pi_{ij} + \pi_{kj})}_{\textit{(2)}} \;+ \underbrace{\sum_{j\in J_i} \left(\pi_{lj} - \pi_{ij}\right)}_{\textit{(3a)}} \\
&+ \underbrace{\sum_{j\in J_k} \left(\pi_{lj} - \pi_{kj}\right)}_{\textit{(3b)}} + \underbrace{\sum_{j\in J_e} \pi_{lj}}_{\textit{(4)}} + \underbrace{\kappa(|F_i| + |F_k|) - \kappa(|F_i \cup F_k|)}_{\textit{(5)}} \\
&- \underbrace{[\kappa(|F_i| + |F_k|) - \kappa(|F_i|) - \kappa(|F_k|)]}_{\textit{(6)}},
\end{aligned}$$

where the number below each term refers to the enumerated list of effects above and the *synergies in non-overlapping markets* are split into those occurring in markets in which i competes pre-merger (3a), and those in which k competes pre-merger (3b). Having decomposed the effects of a merger we can identify markets in which consumer surplus always increases.

Proposition 7. *Following a merger the equilibrium market price weakly decreases and consumer surplus weakly increases in all non-overlapping and all newly entered markets.*

The proof is in Appendix C.1. Consider competition in a non-overlapping market in which a merger has synergies. After the merger the merged firm will be a stronger competitor and other firms may optimally exit the market reducing competition. Nevertheless consumer surplus always increases. The intuition is that for consumer surplus to be reduced the market price needs to increase following the merger, but in this case no firms would optimally exit.

While the competitive effects of a merger on newly entered and non-overlapping markets are unambiguously good, consumer surplus can increase or decrease in overlapping markets. This will depend on the magnitude of synergies in these markets versus increased concentration and the associated reduction in competition.

It is helpful to sometimes consider a special case of our model in which firms can only (effectively) compete in a market once they have all the attributes associated with that market. Under this *full cover* assumption, for a market j, firm i's marginal cost is

$$c_{ij} = \begin{cases} c_j < \alpha_j & \text{if } M_j \subset F_i; \\ \infty & \text{if } M_j \not\subset F_i. \end{cases}$$

Thus a firm enters a market only if it has all the capabilities required by that market. The full cover assumption considerably simplifies the analysis. First, all firms entering a given market must make the same profits. Second, the effects of a merger are simpler. There can be no synergies in overlapping markets or non-overlapping markets so all synergies are confined to newly entered markets. Another implication is that a merger reduces consumer surplus in overlapping markets.

Proposition 8. *Under the full cover assumption, following a merger the equilibrium market price increases and consumer surplus decreases in all overlapping markets.*

Mergers can improve competition in some markets while reducing it in others. However, in combination Proposition 8 and Proposition 7 show that consumer surplus benefits are likely to accrue disproportionately in non-overlapping and newly entered markets. Merger policy typically focuses only on overlapping markets, placing little weight on the competitive benefits that might accrue in other markets. This can lead to the prohibition of mergers that are overall beneficial for society. Moreover, as our next result shows, the losses from such a policy can be arbitrarily large in terms of consumer surplus.

Proposition 9. *A merger policy focusing only on consumer surplus in overlapping markets can lead to unbounded losses of consumer surplus in comparison to a holistic approach that considers all markets.*

We now provide an example to demonstrate that a merger policy aiming to maximize overall consumer surplus should consider all markets when evaluating the effects of a merger.

Example 1. *Suppose that there are attributes* $\mathbf{A} = \{a_1, \ldots, a_6\}$*, Let the market and firm hypergraphs be*

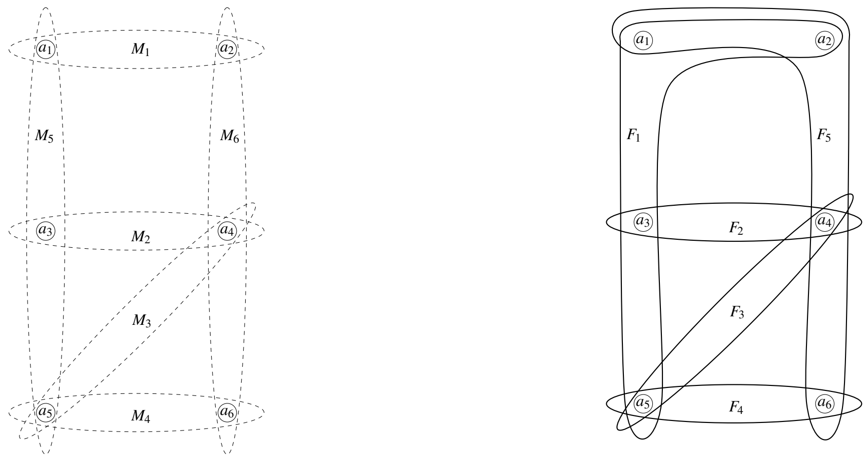
$$\begin{aligned} H_M &= \{M_1 = \{a_1, a_2\}, M_2 = \{a_3, a_4\}, M_3 = \{a_4, a_5\}, M_4 = \{a_5, a_6\}, \\ &\qquad M_5 = \{a_1, a_3, a_5\}, M_6 = \{a_2, a_4, a_6\}\} \\ H_F &= \{F_1 = \{a_1, a_2, a_3, a_5\}, F_2 = \{a_3, a_4\}, F_3 = \{a_4, a_5\}, F_4 = \{a_5, a_6\}, F_5 = \{a_1, a_2, a_4, a_6\}\} \end{aligned}$$

See Figure 3.2. We make the full cover assumption. If a merger between F_1 *and* F_5 *occurs, then the number of firms in each market changes as below:*

$$\begin{bmatrix} & M_1 & M_2 & M_3 & M_4 & M_5 & M_6 \\ \textit{Before merger:} & 2 & 1 & 1 & 1 & 1 & 1 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \textit{After merger:} & 1 & 2 & 2 & 2 & 1 & 1 \end{bmatrix}.$$

It is easy to see that after the merger market M_1 *gets more concentrated while* M_2, M_3, M_4 *get less concentrated. Assuming that the inverse demand function in all markets is given by* $P_j = \alpha - Q_j$ *and that the marginal cost of producing is the same in all markets, overall consumers benefit considerably from the merger between* F_1 *and* F_5*.*

Regulators in the US, EU, and UK, when deciding whether to approve a merger, focus on anticompetitive effects in overlapping markets on a market by market basis. Based only on an analysis of overlapping markets, and with an objective of maximizing consumer surplus, the merger of F_1 *and* F_5 *would be blocked. An intermediate action a regulatory authority might take in the above case is to approve the merger subject to a remedy requiring that a firm with attributes* a_1 *and* a_2 *be spun off prior to the merger to compete in market* M_1*. Although this would increase consumer surplus further* *conditional on the merger going through, it would also reduce the profitability of the merger and could make it unprofitable.*



(a) The market hypergraph (b) The firm hypergraph

Figure 3.2: A MERGER OF F_1 AND F_5 INCREASES THE NUMBER OF COMPETITORS IN MARKETS M_2, M_3, M_4, WHILE DECREASING IT IN MARKET M_1.

3.4 Joint ventures

In this section we present a very simple and stylized model of joint ventures as a means for firms to share their capabilities other than merging. Nevertheless, our model is able to capture a key feature of joint ventures: their ability to better utilize the combined capabilities of multiple firms to compete in a given market.

> *"(A) virtue of joint ventures is that often they make use of a resource which hitherto has been left dormant because it was not coupled with the necessary handmaiden."*
>
> (Harrigan and Newman, 1990.)

For simplicity we restrict attention to joint ventures formed by two (parent) firms i and k, suppose i and k restrict the joint venture to operate in a single predetermined market, give i and k equity stakes in the joint venture, and endow the joint venture with some of i and k's capabilities (which we assume does not diminish the parent firms' capabilities). As in the previous section, we turn off the equilibrium response of other firms to a joint venture, and only look at the effects of a joint venture in isolation. Section 3.5 analyzes stable industry structures.

Definition 3. *A joint venture is a tuple* $\mathcal{J} = (\{i,k\}, F_{\mathcal{J}}, j, \lambda_i) \in \{1,\ldots,n\}^2 \times 2^{F_i \cup F_k} \times \{1,\ldots,m\} \times (0,1)$ *specifying the pair of firms forming the joint venture, the capabilities of the joint venture, the market it will operate in and the parent firms' respective profits shares.*

The profit shares (λ_i, λ_k) of a joint venture $\mathcal{J}$, its capabilities and the market it operates are fixed upon its formation. The joint venture then chooses $q_{\mathcal{J}j}$ to maximize its profits in the market j it operates in, while the parent firms i and k choose their outputs q_{ij} and q_{kj} to maximize their profits and their share of $\mathcal{J}$'s profits in this market. Thus firm i chooses

$$q_{ij}^* = \underset{q_{ij}}{\operatorname{argmax}}\, \pi_{ij}(q_{1j},\ldots,q_{nj},q_{\mathcal{J}j}) + \lambda_i \pi_{\mathcal{J}j}(q_{1j},\ldots,q_{nj},q_{\mathcal{J}j}).$$

We refer to $\pi_{ij} + \lambda_i \pi_{\mathcal{J}j}$ as i's profits in market j following the creation of $\mathcal{J}$.

We assume a joint venture $\mathcal{J}$ operates independently from its parent firms as its parent firms would typically want to choose different output levels for it. By being more or less aggressive profits can be transferred between the joint venture and the parent firms, potentially to the benefit of one parent and the cost of the other. Suppose, for example, that only one of the parent firms operates in market j. If the joint venture competes less aggressively the profits accruing to it will decrease but the profits of the competing parent firm will increase. To avoid incentivizing the parent firms to manipulate the decisions of the joint venture, we make the simplifying assumption that a joint venture operates independently and maximizes its own profits only. Moreover, *committing* to produce more aggressively by making the joint venture independent might sometimes increase the combined profits of the parents firms and joint venture.

As before, once joint ventures are formed, there is a unique equilibrium market outcome.[7]

Lemma 4. *For any set of firms, including joint ventures, there is a unique Nash equilibrium output of each firm in each market.*

Without loss of generality we will restrict attention to joint ventures $\mathcal{J}$ which operate in equilibrium in their designated markets (i.e., joint ventures that once created will produce a strictly positive output in the unique Nash equilibrium).[8] Like the previous section, we differentiate the markets according to firm i's and k's pre joint venture market participation: (i) overlapping markets, (ii) non-overlapping markets, and (iii) new markets. With this differentiation in hand the conditions for a joint venture to be profitable can be analyzed.

Proposition 10. *There exist profit shares* (λ_i, λ_k) *such that a joint venture* $\mathcal{J} = (\{i, k\}, F_{\mathcal{J}}, j, \lambda_i)$ *strictly increases i's profit and k's profit in:*

(i) A new market j*.*

(ii) A non-overlapping market j *that* i *enters if and only if* $c_{\mathcal{J}j} < c_{ij}$*.*

[7] This is more tricky to establish than before because the optimal output decisions of the parent firms depend on the market share of the joint venture and their profit shares in it.

[8] This just requires that the joint venture has a marginal cost in market j less than the equilibrium market price prior to its creation.

(iii) An overlapping market that both i and k enter only if $c_{\mathcal{J}j} < \max\{c_{ij}, c_{kj}\}$.

Proposition 10 considers the additional profits that accrue to the parent firms in a given market following the creation of a joint venture. For the joint venture to be profitable overall, additional market profits needs to be traded off against the fixed costs incurred by the joint venture in maintaining its capabilities. In this sense, Proposition 10 identifies necessary, but not sufficient conditions for a joint venture to be profitable for both parent firms for some equity shares. For joint ventures that operate in distinct markets from the parent firms, all that matters are the net profits of $\mathcal{J}$. For joint ventures that operate in a non-overlapping market, the joint venture must be more capable than both parent firms, while for joint ventures that operate in overlapping markets the joint venture must be more capable than at least one of the parent firms.

A crucial difference between a merger and joint venture is that a joint venture will compete with its parent firms in overlapping markets. This changes the welfare implications of forming joint ventures as opposed to mergers. While mergers can have ambiguous welfare effects, joint ventures always weakly increase welfare.

Proposition 11. *The creation of any joint venture* $\mathcal{J} = (\{i, k\}, F_{\mathcal{J}}, j, \lambda_i)$ *weakly decreases the equilibrium price in all markets and weakly increases consumer surplus in all markets.*

Proposition 11 shows that adding a joint venture to a market (weakly) increases competition even though the parent firms compete less aggressively in that market and might both exit.

3.5 Stable industry structures and consolidation

We now endogenize the industry structure by allowing firms' to merge and demerge with each other. We do so by considering a two stage game. In stage 1 firms endogenously develop their capabilities through merging with each other and spinning off new companies. In stage 2 firms compete as perviously described.

First we define a *demerger*. A demerger of a firm l into firms i and k is equivalent to the bi-partition of its set of capabilities so that $F_l = F_i \cup F_k$ and $F_i \cap F_k = \emptyset$. On the hypergraph, it is equivalent to the bi-partition of the edge representing the initial firm.

We treat mergers and demergers asymmetrically. If two firms merge with overlapping capabilities, duplicate capabilities are, in effect, lost. Suppose, for example, that two firms both have good marketing departments before they merge and that these marketing teams constitute a key source of competitive advantage. After the merger only one marketing department is required and so the two departments are consolidated and some workers leave. If, after this consolidation has been completed, this firm then demerges to create two new firms, only one inherits the strong marketing department. While in practice some attributes may be inherited by both companies following a demerger, we rule this out for simplicity while noting that if we were to assume that all capabilities could be inherited by both companies following a demerger there would exist a sequence of mergers and demergers that would result in all firms acquiring all capabilities. As capabilities are intended to represent sources of competitive advantage they need to be scarce and this modeling choice would be inconsistent with that.

3.5.1 Stable industry structures and their existence

We wish to analyze the set of *stable industry structures* in which there are no further pairwise profitable mergers or demergers. As a benchmark, and except where otherwise stated, we assume that firms can merge or demerge freely without regulation. We define an industry structure to be unstable if there exists a merger that is jointly profitable or a demerger that is jointly profitable holding fixed the capabilities of other firms. This is similar to concepts used when studying stable networks (see, for example, Jackson, 2008). One notable difference is that in our setting the set of decision makers changes following a merger or demerger. We also view each capability as a vertex of a hypergraph and each firm is represented by an edge. So in our model, each decision maker is an edge, instead of a node as is typical in network models.

Now we formally define the stability of a firm hypergraph, which by Proposition 6 uniquely pins down entry decisions into all markets and so also defines a stable industry structure.

Definition 4. *A firm hypergraph (distribution of firms over the set of capabilities A) $H_F \in \mathcal{H}(A)$ is stable if and only if*

1. *[merger] For all firms i and k with capabilities $F_i, F_k \in H_F$, a merger that generates $F_l = F_i \cup F_k$ is strictly unprofitable:*

$$\pi_l(H'_F) < \pi_i(H_F) + \pi_k(H_F),$$

   where H'_F is the firm hypergraph following the merger.

2. *[demerger] For any $F_l \in H_F$ with $|F_l| > 1$, any demerger generating firms $F_i \subset F_l$*

and $F_k = F_l \setminus F_i$ is strictly unprofitable:

$$\pi_i(H'_F) + \pi_k(H'_F) < \pi_l(H_F),$$

where H'_F is the firm hypergraph following the demerger.

So an industry structure is stable if and only if (i) there is no pair of firms which can earn higher profits after a merger than their combined profits before it, and (ii) there is no firm which can be demerged into two smaller firms thereby increasing overall profits.

To simplify our analysis, we maintain the full cover assumption that a firm must have all the relevant capabilities for a market to be able to compete effectively in it. To prove the existence of stable equilibria (at which there is no profitable merger or demerger), we need the following lemma.

Lemma 5. *If there exists a sequence of profitable mergers/demergers generating firm hypergraphs $H_F^{(1)}, H_F^{(2)}, \ldots$ that only ends when there is no profitable merger/demerger, and a function $f : \mathcal{H}(A) \to \Re$ that is weakly increasing such that $f(H_F^{(t)}) \leq f(H_F^{(t+1)})$, then either there exists a stable hypergraph or else there exists a sequence of profitable mergers and demergers that cycles and holds the function f constant.*

The proof is in Appendix C.1. It extends a similar and widely utilized result for networks in Jackson and Watts [2001]. Next we use Lemma 5 to show that a stable firm hypergraph/industry structure exists.

Proposition 12. *Under the full cover assumption there exists a stable industry structure.*

The proof is in Appendix C.1. The key statistic we use to apply Lemma 5 is the sum over all firms of all their attributes. We show that in all sequences of profitable mergers / demergers this statistic weakly decreases, and that no cycle exists holding this statistic constant, allowing us to apply Lemma 5 and conclude that a stable hypergraph exists. It is not straightforward to show that there does not exist a cycle of profitable mergers and demergers. Business stealing effects mean that negative externalities can be imposed on others by merging and demerging firms, so there could in principle exist cycles of mergers and demergers that strictly increase the profits of the firms merging or demerging at each step. The proof shows that this is impossible by proving that the total profits induced by all the mergers and demergers over a cycle have to be zero when holding the total number of attributes in the economy fixed.

We now consider which industry structures will be stable. First we show that if there are no lost synergies, demergers are profitable. We formalize this below.

Proposition 13. *If a firm l's set of capabilities can be partitioned into two non-empty sets S_1, S_2 such that (i) $F_l = S_1 \cup S_2$, (ii) $S_1 \cap S_2 = \emptyset$, and (iii) either $F_l \cap M_j = S_1 \cap M_j$ or $F_l \cap M_j = S_2 \cap M_j$ for all markets j such that $q_{lj} > 0$, then the demerger of l into firms i and k such that $F_i = S_1$ and $F_k = S_2$ is profitable.*

The proof is in Appendix C.1. It follows from the convexity of κ which captures diseconomies of scope. Proposition 13 implies that if a firm can be split into two firms without reducing its competitiveness in any market it operates in, then such a demerger is profitable. By the convexity of the capability maintenance cost κ, demerged firms will be better able to focus their efforts on the markets they operate in than the conglomerate was able to do before the demerger and as no synergies are lost in any relevant market, there is no downside to the demerger.

3.5.2 Stability and Efficiency

We now turn to the questions of whether our stable industry structures will be efficient, and if not why not, by considering an example. We start with the following definition.

Definition 5. *We say an industry structure is efficient if it maximizes the social surplus, i.e. the unweighted sum of firms' profits and consumer surplus.*

Example 2. *Suppose there are 6 attributes $\{a_1, \ldots, a_6\}$. Let the market hypergraph be*

$$\begin{aligned} H_M &= \{M_1 = \{a_1, a_2\}, M_2 = \{a_2, a_3\}, M_3 = \{a_3, a_4\}, M_4 = \{a_4, a_5\}, \\ &\qquad M_5 = \{a_5, a_1\}, M_6 = \{a_1, a_2, a_3, a_4, a_5\}\}, \end{aligned}$$

and consider the following three firm hypergraphs:

$$\begin{aligned} H_1 &= \{F_{11} = \{a_1, a_2\}, F_{12} = \{a_2, a_3\}, F_{13} = \{a_3, a_4\}, F_{14} = \{a_4, a_5\}, F_{15} = \{a_5, a_1\}\}; \\ H_2 &= \{F_{21} = \{a_1, a_2, a_3, a_4, a_5\}\}; \\ H_3 &= \{F_{31} = \{a_1, a_2, a_3, a_4, a_5\}, F_{32} = \{a_1, a_2, a_4, a_5\}\}. \end{aligned}$$

See Figure 3.3. We make the full cover assumption, and also assume that all markets have equivalent demand curves ($P_j = \alpha - Q_j$) and the marginal costs of production is the same in all markets. Then a firm's profits in a given market depend only on the number of firms entering. Abusing notation, we let $\pi(k)$ be the profits a firm makes in a market with k firms. We assume that $\pi(1) > \kappa(2)$ so that a monopolist can cover their fixed cost when serving a market M_k, for $k = 1, \ldots, 5$. Consider first the firm hypergraph H_1 (corresponding to Panel (B) of Figure 3.3). Any demergers would prevent the firms in question from competing in any market and are unprofitable as $\pi(1) > \kappa(2)$, so this industry structure is stable if and only if no two adjacent firms want to merge and no *two other firms want to merge. This requires that*

$$\kappa(3) > 2\kappa(2) \tag{3.2}$$

$$\pi(2) < \kappa(4) - 2\kappa(2). \tag{3.3}$$

Consider now the second firm hypergraph H_2 (corresponding to Panel (C) of Figure 3.3). No mergers are possible as there is a single firm, so this hypergraph is stable if and only if it is not profitable for F_{21} to demerge: (i) into two firms with attributes (a_1, a_2, a_3) and (a_4, a_5); (ii) into two firms with attributes (a_1, a_2, a_3, a_4) and (a_5); (iii) into three firms with attributes (a_1, a_2), (a_3, a_4), and (a_5); or (iv) into three firms with attributes (a_1, a_2, a_3), (a_4), and (a_5). Although there are many other possible demergers they are all weakly less profitable than one of these four demergers. Thus industry structure H_2 is stable if and only if

$$3\pi(1) > \max\{\kappa(5) - \kappa(2) - \kappa(3), \kappa(5) - \kappa(4) - \kappa(1)\} \tag{3.4}$$

$$4\pi(1) > \max\{\kappa(5) - 2\kappa(2) - \kappa(1), \kappa(5) - \kappa(3) - 2\kappa(1)\}. \tag{3.5}$$

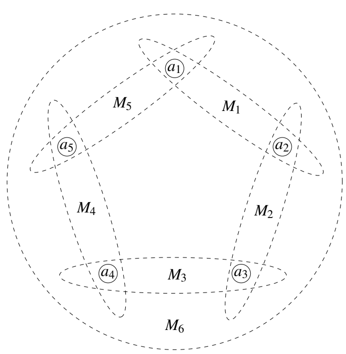
Firm hypergraph H_3 (corresponding to Panel (D) of Figure 3.3) is not stable. A merger between firms F_{32} and F_{31} reduces capability maintenance costs by $\kappa(4)$ while increasing profits in markets M_1, M_4 and M_5.

Consumer surplus in a market with k firms is $\frac{k^2}{2}\pi(k)$. Thus, summing consumer and producer surplus across all markets, the industry structure represented by H_3 is more efficient than H_1 and H_2 respectively if

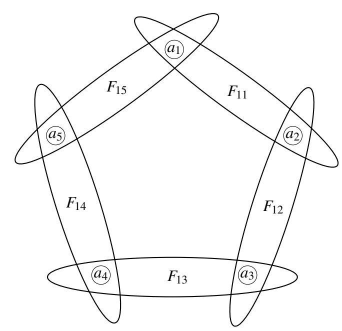
$$12\pi(2) - 3\pi(1) > \kappa(5) + \kappa(4) - 5\kappa(2). \tag{3.6}$$

$$12\pi(2) - \frac{9}{2}\pi(1) > \kappa(4). \tag{3.7}$$

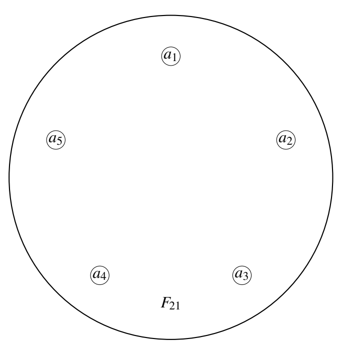
We now identify parameter values for which the industry structures H_1 and H_2 are simultaneously stable, while industry structure H_3 is more efficient than both but unstable. This requires satisfying inequalities (3.2–3.7). Let $\kappa(q) = \phi(q-1)^2$ where $\phi > 0$ and $q \geq 1$. In the above hypergraphs, $\kappa(1) = 0$ implies that a firm with a single capability, who is therefore unable to enter any market is able to freely dispose their capability and make a net profit of 0. *We also let $\pi(1) = 12\phi$. Since there is Cournot competition in the second stage of our game with linear demand, $\pi(2) = (4/9)\pi(1)$, and so $\pi(2) = (16/3)\phi$. It is straightforward to verify that for these parameter values inequalities 3.2–3.7 are satisfied.*



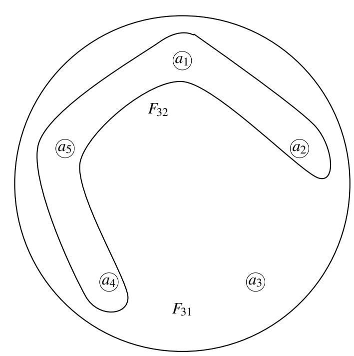
(a) Market hypergraph H_M



(b) Firm hypergraph H_1



(c) Firm hypergraph H_2



(d) Firm hypergraph H_3

Figure 3.3: STABILITY AND EFFICIENCY.

Example 2 shows that for the same parameter values there can be stable industry structures that are too concentrated and not concentrated enough while a more efficient intermediate industry structure is unstable. There could be too many small firms or too few big firms in equilibrium from a social planner's perspective. This motivates regulations that restrict mergers, as is currently practiced, but also potentially intervention to encourage mergers when the industry structure is fragmented.

3.5.3 Industry Consolidation

Example 2 demonstrates that industry structures can be both too fragmented or too concentrated. Mergers are sometimes complementary and sometimes substitutes. Mergers are more profitable in more concentrated markets, but as we have seen, mergers can both increase and decrease competition in markets. A merger might, for example, increase competition in a newly entered market while simultaneously reducing competition in an overlapping market.

When there is a single relevant market, mergers reduce the number of competitors and are complements. This goes some way to explaining how there can be multiple stable market structures that have very different levels of concentration. Moreover, one of these market structures may be considerably more efficient than the others. We show this in the simple example below. In this example a very fragmented market structure is stable, in which no firms are especially strong or capable competitors, and a much more concentrated market structure with strong and more capable competitors is also stable. The less concentrated market structure generates lower aggregate profits and lower consumer surplus. Moreover, starting form this fragmented industry structure, rapid consolidation can occur if there is a change in beliefs about whether other firms are going to merge.

Example 3. *Suppose there are u attributes $\{a_1, \ldots, a_u\}$ and for simplicity we suppose* *there is a single market with inverse demand curve $P = \alpha - Q$. The market hypergraph is*

$$H_M \;=\; \{M_1 = \{a_1, \ldots, a_u\}\}.$$

Consider now the following two firm hypergraphs:

$$H_1 \;=\; \big(F_{si} = \{a_i\}\big)_{s=1,\ldots,k,\ i=1,\ldots,u};$$
$$H_2 \;=\; \big(F_s = \{a_1, \ldots, a_u\}\big)_{s=1,\ldots,k}.$$

The firm hypergraph H_1 consists of ku small firms each with a single capability and the firm hypergraph H_2 consists of k firms each with u capabilities. Firms' marginal costs for the unique market M_1 are decreasing in their capabilities. For simplicity let F_i's marginal cost be $c_i = \alpha/(|F_i| + u)$ and let the fixed cost of maintaining q capabilities be $\kappa(q) = (q-1)^e$ for $q \geq 1$ and some $e > 1$.

We assume that merger regulations prevent any further mergers when the firm hypergraph is H_2. Then for e close to 1 and $k \geq 3u$, the following can be established:

(i) H_1 is stable;

(ii) H_2 is stable;

(iii) H_2 generates higher consumer surplus than H_1.

The calculations are in Appendix C.2 but some intuition follows. Consider first the fragmented hypergraph. Suppose two firms with the same capabilities merge. As production costs remain unchanged, the combined profits of the firms after the merger will be the same as their combined profits were one of them to exit the market. If one exits the other's profits increase a little, but not much because there are still many firms *present, and the exiting firm loses the profits they would otherwise have received. When the market is sufficiently fragmented the net effect is negative. Mergers between firms with different capabilities induce some additional profits through lowering marginal cost, but also require maintaining additional capabilities which is costly. Overall these mergers are not profitable either.*

Consider now hypergraph H_2. By assumption additional mergers are prohibited. So the hypergraph will be stable as long as demergers are unprofitable. This is the case because demerged firms are weaker competitors (have higher marginal costs) in the market than the other firms, and collectively extract lower profits. The demerger will reduce combined fixed costs, but when e is close to 1 these effects are small and the former effect will dominate making demergers unprofitable. Finally, the consumer surplus generated by the two hypergraphs is

$$\begin{aligned} CS(H_1) &= \frac{1}{2}\left(\frac{ku}{ku+1}\right)^2\left(1-\frac{1}{1+u}\right)^2\alpha^2, \\ CS(H_2) &= \frac{1}{2}\left(\frac{k}{k+1}\right)^2\left(1-\frac{1}{2u}\right)^2\alpha^2. \end{aligned}$$

As $u > 1$, when k is sufficiently large $CS(H_1) < CS(H_2)$ and H_2 generates more consumer surplus than H_1. Total profits are also higher for H_2 than H_1.

As both H_1 and H_2 are stable, sudden transitions are possible. Starting form H_2, if firms anticipate that the industry will consolidate through a sequence of mergers these beliefs are self fulfilling and the mergers anticipated will be profitable. In this way, through rapid industry consolidation we can move from hypergraph H_1 to hypergraph H_2. Much empirical evidence documents waves of industry consolidation. Viewing firms as sets of capabilities provides a simple and natural formal explanation for this.

Example 3 illustrates that viewing firms as sets of capabilities might help explain waves of industry consolidation as the transition between multiple equilibria in which firms become more capable competitors through their acquisitions. This is closely related to the work of Sutton (1991, 2001). The main difference is that we do not require exogenous changes in technology or demand to precipitate the changes in industry structure and that additional entry is limited by the scarcity of capabilities rather than through the accumulation of sunk costs as a barrier to entry. Further, in our model, mergers lower marginal costs incentivizing further mergers, while in Sutton's work sunk cost accumulation lowers marginal cost and incentivizes mergers.

3.5.4 Conglomeratization

There is a literature that documents a sharp change of the form of industrial firm in the United States over the course of the twentieth century (Davis and Diekmann, 1994). Following the conglomerate merger wave of the late 1960s and 1970s, Fligstein [1991] reports that by 1980 growth through diversification had become the most widely used corporate strategy among large firms. Fewer than 25 percent of *Fortune* 500 companies made all their sales within a single broadly-defined (2-digit SIC) industry. However, during the 1980s, a wave of "deconglomeration" restructured American industry, resulting in the largest US firms becoming considerably less diversified by 1990. Many papers seek to explain this transition in industry structure (see Amihud and Lev [1981], Fligstein [1991], Bhagat et al. [1990], Davis and Stout [1992], LeBaron and Speidell [1987], Morck, Shleifer, and Vishny [1990]). Almost all suggest that an enormous "collective error" was made in the conglomeration period and represent the subsequent breaking up of these firms as a correction. Our model suggests an alternative explanation.

In our model, mergers that are followed by demergers can be efficiency enhancing if they redistribute capabilities in a way that creates stronger competitors. Consider the markets shown below in Figure 3.4. There are two markets which are unrelated in the sense that they require disjoint capabilities. There are also initially two firms who are both weak competitors in both markets (Panel (A), Figure 3.5). A merger between these two firms creates a new conglomerate firm that competes strongly in both markets (Panel (B), Figure 3.5), but has to maintain an inefficiently large number of capabilities. A subsequent demerger generates two firms that are both strong competitors in their respective markets (Panel (C), Figure 3.5).

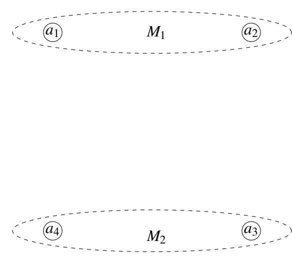
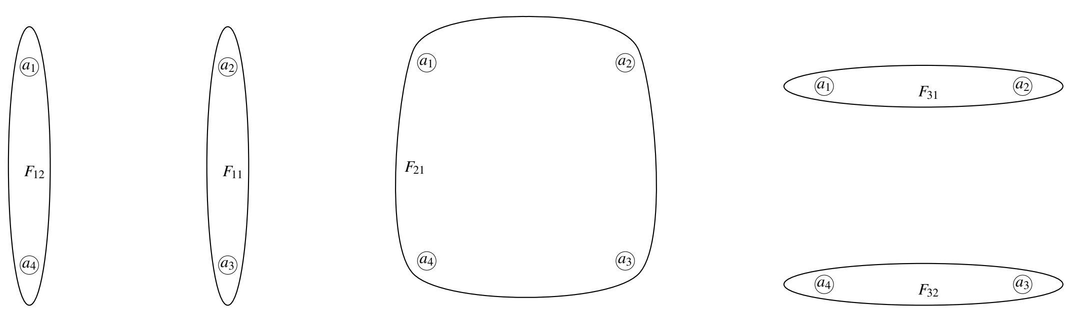


Figure 3.4: MARKET HYPERGRAPH



(a) Initial firms (b) The merged firm (c) The demerged firms

Figure 3.5: EFFICIENT CONGLOMERIZATION AND DEMERGERS

3.6 Conclusions

In this paper we provide formal foundations for one of the main theories of competitive advantage from the management literature. We do so by modeling firms and markets as sets of capabilities where a firm's marginal cost in a given market is decreasing in the intersection of its capabilities with those that would be useful in the market. Given these heterogeneous marginal costs firms decide which markets to enter and how much to produce in each market. The equilibrium is unique, pinning down market prices and firm profits. This representation of firms and markets as hypergraphs helps formalize the notion of synergies obtained through mergers. By linking firms' marginal costs in each market to their relevant capabilities we also model natural entry barriers: Firms need to acquire a suitable set of capabilities before they can compete effectively in a given market. Our approach to merger analysis emphasizes the potential efficiency gains that can be obtained in non-overlapping markets and calls for a more holistic analysis than is currently set out in the US merger guidelines and widely practiced. Joint ventures provide an alternative way in which firms might combine their capabilities. Although the parent firms of a joint venture will compete less aggressively in a market if they create a joint venture that enters that market, we show that consumer surplus increases in all markets whenever a joint venture is profitable. Finally, by defining what it means for an industry structure to be stable, we explain industry consolidation as the transition from a less profitable but stable market structure to a more profitable (and possibly higher consumer surplus) market structure with fewer but more capable firms.

Our model is simple and intended as a first step. On the theoretical side there is much that can be generalized. Firms' competitiveness can more generally be defined by the set of relevant capabilities they have for a market rather than the number. Such an approach would lead to a partial ordering (in set inclusion with respect to capabilities) of the competitiveness of firms in each market. A more dynamic approach than has been taken in this paper is also called for. Markets evolve over time in terms of the capabilities they value and in terms of their size. Explicitly modeling this would further emphasize the role of management in terms of acquiring and developing the capabilities of firms. On the empirical side there is also much to do. Are mergers that generate more complementary sets of capabilities more profitable in the long run? Do newly merged firms use their new found capabilities to enter new markets? Do firms become stronger competitors after mergers in non-overlapping markets?

The model can also be applied to study many problems we have not mentioned. For example, government procurement contracts can be viewed as demanding a set of capabilities. In some cases no one firm has all the required capabilities and bidding for the contract occurs through endogenously formed consortia of firms. An interesting theoretical and empirical question is then “what can be said about the competitiveness of these bidding processes?” Viewing firms as sets of capabilities might help address questions like this.

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Appendix A

APPENDIX TO CHAPTER 1

A.1 Variable and term definitions

A.2 Data integration

This section details the integration of the form D and VC data with business registration.

Form D filings and business registration

I rely on a standard company name merge to combine form D filings and business registration records. To my advantage, full legal name of companies is required to be unique within a single state's border, so this largely guarantees the accuracy of company name merge. To further improve the matching outcomes, I use additional information available in the two datasets including company address, incorporation year and incorporation state to clean the matching. In particular, I drop a matching either (1) if the two datasets indicate different county location of the matched company,[1] or (2) if the two datasets indicate different incorporation state.

The merge of form D filings and business registration is useful in several aspects. First, business registration records provide more detailed company-level information for the startups which have raised private money in CA. Second, the merge allows to filter out some startups which raised money and filed form D in CA but do not actually operate in CA, and thus creates a cleaner sample of CA startups that base their operation activity in CA. Third, the merge assigns each form D filer a unique company identifier

[1] Young companies are mobile, so I match companies at the county level by taking into account the possibility that companies may re-locate within county.

Table A.1: VARIABLE AND TERM DEFINITIONS

Panel A: Variable definitions	
Variables	Definition
CA/DE Incorporated	dummy variable whether a startup is incorporated in California/Delaware
Year/Age first Angel	year/age at initial angel financing
Biotech/Computers	dummy variable whether a startup is in biotech/computer sector
# of Corporations p.c.	number of new corporations per capita (normalized by multiplying by 100 in regressions)
# of Corporations	number of new corporations (normalized by dividing by 1000 in regressions)
Population	number of exemptions in tax returns, which approximates population according to the IRS (normalized by dividing by 10,000 in regressions)
Size Quartile	quartiles of the financing size distribution
Capital Raised	financing size of angel investments (normalized by dividing by 10 million in regressions)

Panel B: Term definitions	
Terms	Definition
angel-backed startup	a startup receiving its initial financing from angels
initial angel financing	startups' initial financing where angels are the only outside investors
VC-backed startup	a startup receiving its initial financing from VCs
initial VC financing	startups' initial financing where (at least one) VCs are the outside investors
first VC financing	a financing that is backed by (at least one) VCs for the first time over a startup's financing history
VC funded/financed startup	a startup which ever receives a financing from VCs
Rich households	households with annual income greater than $100,000

from the business registration records, which facilitates further data integration with the VC data.

VC data and business registration

Similarly, I also rely on a company name merge to combine VC data (both PCRI and Pitchbook) and business registration records. Specifically, for the PCRI data, I got valuable help from PCRI on the matching process, and they manually checked all the name merge outcomes using additional information such as specific company address, incorporation date, and so on. For Pitchbook data, I also performed a similar matching process. Specifically, to clean the name matching outcomes, I drop a matching from the name merge outcomes if Pitchbook and business registration indicate a different county location of the matched company.

The merge between VC data and business registration is useful in the sense that (1) it only keeps the VC funded startups which actually operate in CA,[2] (2) it assigns each VC funded startup a unique company identifier from business registration records.

Have merged form D and VC data with business registrations, it is an easy process to use the common registration identifiers of the two matching outcomes and integrate the two datasets at the company level.

A.3 Additional tables

[2] Many VC funded startups have company address in CA, but do not actually operate in CA.

Table C.1: SUMMARY OF SECURITY EXEMPTION RULES

Notes: The table presents an exhaustive list of security exemption rules available for private firms in the United States [73].

Exemption Rules	Type of Investors	Number of investors	Advertising Requirements	Geography	Capital
Traditional Rule 506 (federal)	Accredited and sophisticated, answers by investors can be relied upon unless information to the contrary	Unlimited accredited, up to 35 sophisticated	No public advertising but informational advertising OK Targeted contacts with investors reasonably believed to qualify OK	Anywhere in the U.S.	Unlimited
Advertised Rule 506 (federal)	Accredited only; status must be verified	Unlimited	Public advertising allowed	Anywhere in the U.S.	Unlimited
Crowdfunding (federal)	Investor with less than $100K in net assets or $100K income limited to greater of $2,000 or 5% of income. Others can invest up to 10% of annual income or net worth, up to $100K	Unlimited	Public advertising but only through an intermediary registered with the SEC	Anywhere in the U.S. when allowed	$1,000,000 maximum including amounts in the prior 12 months; financials must be audited
Model Accredited Investor Exemption, aka MAIE (approx. 35 states)	Accredited investors only	Unlimited	Only brief “tombstone” ad allowed	Only in states that have adopted the MAIE	Unlimited if restricted to one state. otherwise $5 million (and CA 25102(n) required)
CA 25102(f) (California's variation on Rule 506)	1) Accredited, 2) sophisticated and 3) friends, family and colleagues. Limited to California investors	Unlimited accredited, up to 35 on the others	No public advertising but informational advertising OK Targeted contacts with investors reasonably believed to qualify OK	California investors only can be used by non-California companies if combined with federal Rule 505 and friends, family and colleagues category removed	Unlimited
CA 25102(n) (California's version of the MAIE)	Accredited only except CA corporations may have “half accredited” investors. Limited to California investors unless combined with MAIE offering in about 35 other states.	Unlimited	No public advertising but informational advertising OK. Targeted contacts with investors reasonably believed to qualify OK	California investors only. Can be used by a non-California entity if more than 50% of the average of its property, payroll and sales of tangible personal property (if any) are in California.	Unlimited if California only. $5 million limit if other states involved

Table C.2: T-TEST OF VC COMPLIANCE

Notes: The table reports the T-test for VCs' compliance of security exemption rules using Pitchbook data.

	No Form D	Filed Form D	Diff/s.e.
Year firm founded	2008.7	2007.9	0.872***
			0.167
Year first VC	2010.5	2009.3	1.217***
			0.135
First capital raised (m)	4.998	5.638	-0.640
			0.352
Total capital raised (m)	135.0	202.4	-67.33
			48.73
Total rounds	2.428	3.133	-0.705***
			0.0893
First syndicate size	2.667	2.608	0.0584
			0.0724
Has unknown investors	0.233	0.298	-0.0646***
			0.0194
Information Tech.	0.572	0.524	0.0477*
			0.0215
Healthcare	0.0990	0.183	-0.0835***
			0.0159

Table C.3: SUMMARY STATISTICS OF PITCHBOOK DATA

Notes: The table summarizes the sample of first VC financings of CA startups using Pitchbook data.

	mean	sd	min	p25	p50	p90	p95	p99	max	count
Age first financing	1.07	0.79	0	0.40	0.90	2.30	2.70	3	3	5434
First capital raised (m)	4.55	8.58	0.010	1	2	10	15	38.3	217	5434
Investor not angel or VC	0.12	0.32	0	0	0	1	1	1	1	5434
Has VC investor	0.71	0.45	0	0	1	1	1	1	1	5434
Has angel investor	0.44	0.50	0	0	0	1	1	1	1	5434

Table C.4: KEY PHRASES AND SECTOR ASSIGNED

Notes: The table reports the top two key phrases in each sector used to identify that sector.

Sector	Key Phrase	Frequency
Agriculture	farming	124
Agriculture	agriculture	24
Banking	holding company	151
Banking	investment business	117
Biotechnology	biotechnology	98
Biotechnology	medical device	51
Business Services	management service	115
Business Services	service	114
Computers	software development	683
Computers	software	308
Construction & Residential	construction	646
Construction & Residential	property management	134
Energy	electrical contractor	66
Energy	renewable energy	28
Financial Services	financial service	153
Financial Services	equipment leasing	31
Fund & Real Estate	real estate	2019
Fund & Real Estate	investment	193
General Business	consulting	333
General Business	consulting service	198
General Technology	technology	141
General Technology	social media	64
Health Care	medical device	299
Health Care	health care	82
Insurance	insurance agency	55
Insurance	physical therapy	54
Local Service	food service	224
Local Service	trucking service	112
Manufacturing	manufacturing	181
Manufacturing	manufacturer	37
Natural Resources & Utilities	gas exploration	12
Natural Resources & Utilities	aviation	8
Pharmaceuticals	pharmacy	79
Pharmaceuticals	pharmaceutical	32
Restaurants	restaurant	1025
Restaurants	fast food	139
Retailing	retail sale	231
Retailing	general merchandise	203
Telecommunications	telecommunication	30
Telecommunications	online video	27
Travel	transportation	118
Travel	travel service	32

Appendix B

APPENDIX TO CHAPTER 2

B.1 Proofs

B.1.1 Proof of Theorem 1

Proof. The proof of Theorem 1 directly follows from Proposition 4. Proposition 4 suggests that when the first two conditions of the crowdfunding market equilibrium are satisfied, the third one cannot be satisfied. □

B.1.2 Proof of Lemma 1

Proof. Because $\mathbb{E}^I(Y|F,S) = \mathbb{P}(H|F,S) - p$ $(F = G, B)$, it suffices to prove that $\mathbb{P}(H|G,S) > \mathbb{P}(H|B,S)$. By Bayes' rule and the common prior $\mathbb{P}(H) = \mathbb{P}(L) = 1/2$,

$$\mathbb{P}(H|G,S) = \frac{\mathbb{P}(G,S|H)}{\mathbb{P}(G,S|H) + \mathbb{P}(G,S|L)}.$$

Now fix one insider, and assume that this insider invests. Then the event S is determined by all other insiders' private information realization and other investors' investing strategies. Let S' denote the event that the other investors buy all but one shares (n out of total $n+1$ shares) offered in the crowdfunding campaign. Then we can write $\mathbb{P}(F,S|H) = \mathbb{P}(F,S'|H)$ and $\mathbb{P}(F,S|L) = \mathbb{P}(F,S'|L)$ for $F = G, B$. So,

$$\mathbb{P}(H|G,S) = \frac{\mathbb{P}(G,S'|H)}{\mathbb{P}(G,S'|H) + \mathbb{P}(G,S'|L)}.$$

Then by the independence of insiders' information,

$$\begin{aligned}
\mathbb{P}(H|G,S) &= \frac{\mathbb{P}(G|H)\mathbb{P}(S'|H)}{\mathbb{P}(G|H)\mathbb{P}(S'|H) + \mathbb{P}(G|L)\mathbb{P}(S'|L)} \\
&= \frac{\mathbb{P}(S'|H)}{\mathbb{P}(S'|H) + \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)}\mathbb{P}(S'|L)}.
\end{aligned}$$

Since $\frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)} < \frac{\mathbb{P}(B|L)}{\mathbb{P}(B|H)}$ for $\alpha > 1/2$, we have

$$\begin{aligned}\mathbb{P}(H|G,S) &> \frac{\mathbb{P}(S'|H)}{\mathbb{P}(S'|H) + \frac{\mathbb{P}(B|L)}{\mathbb{P}(B|H)}\mathbb{P}(S'|L)} \\ &= \mathbb{P}(H|B,S).\end{aligned}$$

This completes the proof. □

B.1.3 Proof of Proposition 1

Proof. I prove that the outsiders play cutoff strategy for each possibility of insiders' strategy profile. Since Cases I and IV can be considered special cases of Cases II and IV, and Case III is an intermediate case, I just need to prove the proposition for Cases II and IV.

Case II: in Case II, G-signal insiders use mixed strategy (with mixed probability r_G), and B-signal insiders do not invest. Let m denote the number of insiders that outsiders observe. Then the outsiders' posterior is

$$\mathbb{P}(H|m) = \frac{\mathbb{P}(m|H)\mathbb{P}(H)}{\mathbb{P}(m|H)\mathbb{P}(H) + \mathbb{P}(m|L)\mathbb{P}(L)} = \frac{1}{1 + \frac{\mathbb{P}(m|L)}{\mathbb{P}(m|H)}}.$$

To prove that outsiders play cutoff strategies, it suffices to prove that $\mathbb{P}(H|m)$ is monotonically increasing in m. To do so, I am going to prove that $\frac{\mathbb{P}(m|L)}{\mathbb{P}(m|H)}$ is decreasing in m. It is also equivalent to prove that $\mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L) > 0$. Because

$$\mathbb{P}(m|L) = \sum_{k\geq m}\binom{n}{k}(1-\alpha)^k\alpha^{n-k}\binom{k}{m}r_G^m(1-r_G)^{k-m}$$

and

$$\mathbb{P}(m|H) = \sum_{k\geq m}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\binom{k}{m}r_G^m(1-r_G)^{k-m},$$

I denote $p_k = \binom{n}{k}(1-\alpha)^k \alpha^{n-k}$, $q_k = \binom{n}{k}\alpha^k(1-\alpha)^{n-k}$ and $x_k(m) = \binom{k}{m} r_G^m (1-r_G)^{k-m}$. Then

$$\begin{aligned}
& \mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L) \\
= & \left[\sum_{k\geq m} p_k x_k(m)\right] \cdot \left[\sum_{k\geq m+1} q_k x_k(m+1)\right] - \left[\sum_{k\geq m} q_k x_k(m)\right] \cdot \left[\sum_{k\geq m+1} p_k x_k(m+1)\right] \\
= & \sum_{i\geq m, j\geq m+1} p_i q_j x_i(m) x_j(m+1) - \sum_{i\geq m, j\geq m+1} q_i p_j x_i(m) x_j(m+1) \\
= & \sum_{i\geq m, j\geq m+1} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1) \\
= & \underbrace{\sum_{i\geq m+1, j\geq m+1} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1)}_{(1)} + \underbrace{\sum_{i=m, j\geq m+1} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1)}_{(2)}.
\end{aligned}$$

Then it suffices to prove that both (1) and (2) are positive. To prove that (1) is positive, fix a pair i, j with $i < j$. Observe that

$$\begin{aligned}
x_i(m)x_j(m+1) &= \binom{i}{m} r_G^m (1-r_G)^{i-m} \binom{j}{m+1} r_G^{m+1} (1-r_G)^{j-m-1} \\
&> \binom{j}{m} r_G^m (1-r_G)^{j-m} \binom{i}{m+1} r_G^{m+1} (1-r_G)^{i-m-1} \\
&= x_j(m) x_i(m+1).
\end{aligned}$$

Also note that $p_i q_j - q_i p_j > 0$ for $i < j$. Thus,

$$\left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1) > \left(p_i q_j - q_i p_j\right) x_j(m) x_i(m+1).$$

So

$$\begin{aligned}
& \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1) + \left(p_j q_i - q_j p_i\right) x_j(m) x_i(m+1) \\
> & \left(p_i q_j - q_i p_j\right) x_j(m) x_i(m+1) + \left(p_j q_i - q_j p_i\right) x_j(m) x_i(m+1) = 0. \\
= & \ 0.
\end{aligned}$$

Then by the symmetry of pairs of summands in (1), (1) is positive. Since $p_i q_j - q_i p_j > 0$ for $i < j$, (2) is also positive. Thus, $\mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L) > 0$. This proves Case II.

Case IV: in Case IV, G-signal insiders invest, and B-signal insiders use mixed strategy (with mixed probability r_B). Similarly, I just need to prove that $\mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L) > 0$. Because

$$\mathbb{P}(m|L) = \sum_{k\le m} \binom{n}{k}(1-\alpha)^k \alpha^{n-k} \binom{n-k}{m-k} r_B^{m-k}(1-r_B)^{n-m}$$

and

$$\mathbb{P}(m|H) = \sum_{k\le m} \binom{n}{k}\alpha^k (1-\alpha)^{n-k} \binom{n-k}{m-k} r_B^{m-k}(1-r_B)^{n-m},$$

I denote $p_k = \binom{n}{k}(1-\alpha)^k\alpha^{n-k}$, $q_k = \binom{n}{k}\alpha^k(1-\alpha)^{n-k}$ and $x_k(m) = \binom{n-k}{m-k} r_B^{m-k}(1-r_B)^{n-m}$. Then

$$\begin{aligned}
&\mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L)\\
= \;&\left[\sum_{k\le m} p_k x_k(m)\right]\cdot\left[\sum_{k\le m+1} q_k x_k(m+1)\right] - \left[\sum_{k\le m} q_k x_k(m)\right]\cdot\left[\sum_{k\le m+1} p_k x_k(m+1)\right]\\
= \;&\sum_{i\le m, j\le m+1} p_i q_j x_i(m) x_j(m+1) - \sum_{i\le m, j\le m+1} q_i p_j x_i(m) x_j(m+1)\\
= \;&\sum_{i\le m, j\le m+1} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1)\\
= \;&\underbrace{\sum_{i\le m, j\le m} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1)}_{(1)} + \underbrace{\sum_{i\le m, j=m+1} \left(p_i q_j - q_i p_j\right) x_i(m) x_j(m+1)}_{(2)}.
\end{aligned}$$

It suffices to prove that both (1) and (2) are positive. To prove (1) is positive, fix a pair

i, j with $i < j$. Observe that

$$\begin{aligned} x_i(m)x_j(m+1) &= \binom{n-i}{m-i} r_B^{m-i}(1-r_B)^{n-m} \binom{n-j}{m+1-j} r_B^{m+1-j}(1-r_B)^{n-m-1} \\ &> \binom{n-j}{m-j} r_B^{m-j}(1-r_B)^{n-m} \binom{n-i}{m+1-i} r_B^{m+1-i}(1-r_B)^{n-m-1} \\ &= x_j(m)x_i(m+1). \end{aligned}$$

Also note that $p_i q_j - q_i p_j > 0$ for $i < j$. Thus,

$$\left(p_i q_j - q_i p_j\right) x_i(m)x_j(m+1) \;>\; \left(p_i q_j - q_i p_j\right) x_j(m)x_i(m+1).$$

So

$$\begin{aligned} &\left(p_i q_j - q_i p_j\right) x_i(m)x_j(m+1) + \left(p_j q_i - q_j p_i\right) x_j(m)x_i(m+1) \\ > \;&\left(p_i q_j - q_i p_j\right) x_j(m)x_i(m+1) + \left(p_j q_i - q_j p_i\right) x_j(m)x_i(m+1) = 0. \end{aligned}$$

This implies that (1) is positive. Since $p_i q_j - q_i p_j > 0$ for $i < j$, (2) is positive. Thus, $\mathbb{P}(m|L)\mathbb{P}(m+1|H) - \mathbb{P}(m|H)\mathbb{P}(m+1|L) > 0$. This proves Case IV. □

B.1.4 Proof of Proposition 2

Proof. Lemma 1 implies that if there exists an insiders' equilibrium, it must be one of the five possible forms stated in Proposition 2. I just need to focus on Equilibrium II and IV. At Equilibrium II, since the G-signal insiders play mixed strategy, they should be indifferent between investing and not investing. This gives the equilibrium condition that a G-signal insider receives zero expected payoff from investing, i.e. $\mathbb{E}^I(Y|G,S) = \mathbb{P}(H|G,S) - p = 0$. Thus, the share price p is simply equal to a G-signal insider's gross payoff $\mathbb{P}(H|G,S)$. Similarly, at Equilibrium IV, p is equal to a B-signal insider's gross payoff $\mathbb{P}(H|B,S)$.

By Lemma 2 below, $\mathbb{P}(H|G,S)$ and $\mathbb{P}(H|B,S)$ are decreasing in r_G and r_B, respectively. So there is a one-to-one correspondence between the share price p and the insiders' equilibrium strategy. This particularly implies that there is a unique symmetric equilibrium strategy for the insiders. Now, to prove Proposition 2, it suffices to find the ranges of $\mathbb{P}(H|G,S)$ and $\mathbb{P}(H|B,S)$, which identify the price p such that there exists a corresponding insiders' equilibrium strategy. The rest of this proof is to find the ranges of $\mathbb{P}(H|G,S)$ and $\mathbb{P}(H|B,S)$.

(1) The proof of Part I trivially follows from Part II.

(2) Notice that $\mathbb{P}(H|G,S)$ is not continuous at $r_G = 0$. Now, to prove Part II, it suffices to prove that

$$\lim_{r_G \to 0^+} \mathbb{P}(H|G,S) = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{n_0+1}}.$$

By Lemma 2 below, this holds because

$$\lim_{r_G \to 0^+} \xi_{II}(r_G, n, n_0, \alpha) = \left(\frac{1-\alpha}{\alpha}\right)^{n_0}.$$

(3) I just need to prove $\mathbb{P}(H|B,S)|_{r_B=0} < \mathbb{P}(H|G,S)|_{r_G=1}$. Notice that $r_G = 1$ corresponds to the insiders' equilibrium strategy that G-signal insiders invest with mixed probability $r_G = 1$ and B-signal insiders do not invest, and $r_B = 0$ corresponds to the insiders' equilibrium strategy that G-signal insiders invest and B-signal insiders invest with mixed probability $r_B = 0$. In both cases, only G-signal investors invest, and thus the two quantities $\mathbb{P}(n_i \geq n_0|L)$ and $\mathbb{P}(n_i \geq n_0|H)$ are the same. So to prove $\mathbb{P}(H|B,S)|_{r_B=0} < \mathbb{P}(H|G,S)|_{r_G=1}$, it suffices to prove $\frac{\alpha}{1-\alpha} > \frac{1-\alpha}{\alpha}$. This clearly holds for any $\alpha > 1/2$.

(4) Since $\mathbb{P}(H|B, S)$ is continuous in $r_B \in [0, 1]$, it is straightforward to identify the range of $\mathbb{P}(H|B, S)$, which then proves Part IV.

(5) The proof of Part E trivially follows from Part IV.

□

B.1.5 Proof of Lemma 2

Proof. **Part (1):** I first calculate $\mathbb{P}(H|G, S)$, a G-signal insider's gross payoff from investing when all other insiders play equilibrium strategies of Equilibrium II. By Bayes' rule and the common prior $\mathbb{P}(H) = \mathbb{P}(L) = 1/2$,

$$\mathbb{P}(H|G, S) = \frac{\mathbb{P}(G, S|H)}{\mathbb{P}(G, S|H) + \mathbb{P}(G, S|L)}.$$

Since the outsiders are playing equilibrium strategy $n_0 + 1$, the event S is equivalent to that at least $n_0 + 1$ insiders invest. For one insider, let n_i denote the number of other insiders who invest. Then for one insider, conditioning on her own investing decision, $n_i = n_I - 1$, and S is also equivalent to the event that there are at least n_0 other insiders who invest, i.e. $n_i \geq n_0$. So,

$$\mathbb{P}(H|G, S) = \frac{\mathbb{P}(G, n_i \geq n_0|H)}{\mathbb{P}(G, n_i \geq n_0|H) + \mathbb{P}(G, n_i \geq n_0|L)}.$$

By the independence of insiders' private information, I can rewrite the above equation as follows:

$$\mathbb{P}(H|G, S) \;=\; \frac{1}{1 + \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)} \cdot \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)}}.$$

At Equilibrium II,

$$\begin{aligned}\mathbb{P}(n_i \geq n_0|H) &= \sum_{k\geq n_0} \mathbb{P}(n_i \geq n_0|k)\mathbb{P}(k\ G \text{ signals } |H) \\ &= \sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i} r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}\alpha^k(1-\alpha)^{n-k},\end{aligned}$$

and

$$\mathbb{P}(n_i \geq n_0|L) = \sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i} r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}(1-\alpha)^k\alpha^{n-k}.$$

so $\mathbb{P}(H|G,S)$ can be expressed a function of parameters n, n_0, α, r_G:

$$\mathbb{P}(H|G,S) = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot\frac{\sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i}r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}(1-\alpha)^k\alpha^{n-k}}{\sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i}r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}\alpha^k(1-\alpha)^{n-k}}}. \tag{B.1}$$

For fixed parameters n, n_0, α, $\mathbb{P}(H|G,S)$ is a function of r_G, which reflects how the gross payoff of a G-signal insider varies with his equilibrium strategy. Let

$$\xi_{II}(r_G, n, n_0, \alpha) = \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)} = \frac{\sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i}r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}(1-\alpha)^k\alpha^{n-k}}{\sum_{k\geq n_0}\left(\sum_{n_i\geq n_0}\binom{k}{n_i}r_G^{n_i}(1-r_G)^{k-n_i}\right)\binom{n}{k}\alpha^k(1-\alpha)^{n-k}}.$$

To prove the first part of Lemma 2, it suffices to prove that for any fixed n, n_0, α, $\xi_{II}(r_G, n, n_0, \alpha)$ is strictly increasing in r_G. I will prove that $\frac{\partial \xi_{II}(r_G,n,n_0,\alpha)}{\partial r_G} > 0$.

Rewrite $\xi_{II}(r_G, n, n_0, \alpha)$ as

$$\xi_{II}(r_G, n, n_0, \alpha) = \frac{\sum_{k\geq n_0}\left[n_0\binom{k}{n_0}\int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0}dt\right]\binom{n}{k}(1-\alpha)^k\alpha^{n-k}}{\sum_{k\geq n_0}\left[n_0\binom{k}{n_0}\int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0}dt\right]\binom{n}{k}\alpha^k(1-\alpha)^{n-k}}.$$

Let $a_k = n_0\binom{k}{n_0}\int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0}dt$. Then a_k's derivative with respect to r_G is $a_k' = n_0\binom{k}{n_0}{r_G}^{n_0-1}(1-r_G)^{k-n_0}$. Take partial derivative of $\xi_{II}(r_G, n, n_0, \alpha)$ with respect to

r_G:

$$
\begin{aligned}
&\frac{\partial \xi_{II}(r_G, n, n_0, \alpha)}{\partial r_G} \\
= \;& \frac{\left[\sum_{k\geq n_0} \binom{n}{k} a'_k (1-\alpha)^k \alpha^{n-k}\right]\left[\sum_{k\geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k}\right]}{\left[\sum_{k\geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k}\right]^2} \\
& - \frac{\left[\sum_{k\geq n_0} \binom{n}{k} a'_k \alpha^k (1-\alpha)^{n-k}\right]\left[\sum_{k\geq n_0} \binom{n}{k} a_k (1-\alpha)^k \alpha^{n-k}\right]}{\left[\sum_{k\geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k}\right]^2} \\
= \;& \frac{\sum_{i,j\geq n_0, i\neq j} \binom{n}{i}\binom{n}{j} a_i a'_j \left[\alpha^i (1-\alpha)^{n-i} (1-\alpha)^j \alpha^{n-j} - (1-\alpha)^i \alpha^{n-i} \alpha^j (1-\alpha)^{n-j}\right]}{\left[\sum_{k\geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k}\right]^2} \\
= \;& \frac{\sum_{i,j\geq n_0, i\neq j} \binom{n}{i}\binom{n}{j} a_i a'_j \alpha^n (1-\alpha)^n \left[\alpha^{i-j} (1-\alpha)^{j-i} - \alpha^{j-i} (1-\alpha)^{i-j}\right]}{\left[\sum_{k\geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k}\right]^2}.
\end{aligned} \tag{B.2}
$$

To prove (B.2) is positive, I just need to prove that the nominator is positive. Fix a pair i, j with $i < j$. Observe that

$$
\begin{aligned}
a_i a'_j &= n_0 \binom{i}{n_0} \int_0^{r_G} t^{n_0-1} (1-t)^{i-n_0} dt \cdot n_0 \binom{j}{n_0} {r_G}^{n_0-1} (1-r_G)^{j-n_0} \\
&= n_0 \binom{i}{n_0} \int_0^{r_G} t^{n_0-1} (1-t)^{i-n_0} (1-r_G)^{j-i} dt \cdot n_0 \binom{j}{n_0} {r_G}^{n_0-1} (1-r_G)^{i-n_0} \\
&< n_0 \binom{i}{n_0} \int_0^{r_G} t^{n_0-1} (1-t)^{j-n_0} dt \cdot n_0 \binom{j}{n_0} {r_G}^{n_0-1} (1-r_G)^{i-n_0} \\
&= a_j a'_i.
\end{aligned}
$$

Then for any fixed pair i, j with $i < j$, the inequality holds

$$
\begin{aligned}
& a_i a'_j \alpha^n (1-\alpha)^n \left[\alpha^{i-j} (1-\alpha)^{j-i} - \alpha^{j-i} (1-\alpha)^{i-j}\right] \\
& + a_j a'_i \alpha^n (1-\alpha)^n \left[\alpha^{j-i} (1-\alpha)^{i-j} - \alpha^{i-j} (1-\alpha)^{j-i}\right] \\
> \;& a_j a'_i \alpha^n (1-\alpha)^n \left[\alpha^{i-j} (1-\alpha)^{j-i} - \alpha^{j-i} (1-\alpha)^{i-j}\right] \\
& + a_j a'_i \alpha^n (1-\alpha)^n \left[\alpha^{j-i} (1-\alpha)^{i-j} - \alpha^{i-j} (1-\alpha)^{j-i}\right] \\
= \;& 0.
\end{aligned}
$$

So the nominator of (B.2) is positive. This proves $\frac{\partial \xi_{II}(r_G,n,n_0,\alpha)}{\partial r_G} > 0$, and thus part (1) of Lemma 2. Since $\xi_{II}(r_G, n, n_0, \alpha)$ is monotone, we can easily calculate its limits with respect to r_G:

$$\begin{aligned}
\lim_{r_G \to 0^+} \xi_{II}(r_G, n, n_0, \alpha) &= \lim_{r_G \to 0^+} \frac{\sum_{k \geq n_0} \left[n_0 \binom{k}{n_0} r_G^{n_0-1} (1-r_G)^{k-n_0} \right] \binom{n}{k} (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \left[n_0 \binom{k}{n_0} r_G^{n_0-1} (1-r_G)^{k-n_0} \right] \binom{n}{k} \alpha^k (1-\alpha)^{n-k}} \\
&= \frac{\sum_{k \geq n_0} \left[n_0 \binom{k}{n_0} \right] \binom{n}{k} (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \left[n_0 \binom{k}{n_0} \right] \binom{n}{k} \alpha^k (1-\alpha)^{n-k}} \\
&= \frac{\sum_{k \geq n_0} \binom{n-n_0}{k-n_0} (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \binom{n-n_0}{k-n_0} \alpha^k (1-\alpha)^{n-k}} = \left(\frac{1-\alpha}{\alpha} \right)^{n_0},
\end{aligned}$$

and

$$\lim_{r_G \to 1^-} \xi_{II}(r_G, n, n_0, \alpha) = \frac{\sum_{k \geq n_0} \binom{n}{k} (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \binom{n}{k} \alpha^k (1-\alpha)^{n-k}}.$$

Part (2): Similarly as before, I can express $\mathbb{P}(H|B,S)$ as a function of parameters n, n_0, α, r_B:

$$\mathbb{P}(H|B,S) = \frac{1}{1 + \frac{\alpha}{1-\alpha} \cdot \frac{\mathbb{P}(n_i \geq n_0 | L)}{\mathbb{P}(n_i \geq n_0 | H)}}. \tag{B.3}$$

For fixed parameters n, n_0, α, $\mathbb{P}(H|B,S)$ as a function of r_B, reflects how the gross payoff of a B-signal insider varies with his equilibrium strategy. Similarly, let $\xi_{IV}(r_B, n, n_0, \alpha) := \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)}$ be a function of n, n_0, α, r_B. Then to prove Part (2) of Lemma 2, it suffices to prove that for any fixed n, n_0, α, $\xi_{IV}(r_B, n, n_0, \alpha)$ is strictly increasing in r_B. I will prove that $\frac{\partial \xi_{IV}(r_B,n,n_0,\alpha)}{\partial r_B} > 0$.

Let $b_k = \mathbb{P}(n_i \geq n_0 | k\ B \text{ signals})$, and b_k' be its derivative with respect to r_B. Notice that $b_k = 1$ for $k \leq n - n_0$ at Equilibrium IV. So $b_k' = 0$ for $k \leq n - n_0$. Taking partial derivative of $\xi_{IV}(r_B, n, n_0, \alpha)$ with respect to r_B, I have

$$\begin{aligned}
& \frac{\partial \xi_{IV}(r_B, n, n_0, \alpha)}{\partial r_B} \\
= \; & \frac{\left[\sum_{k\geq 0}\binom{n}{k} b'_k \alpha^k (1-\alpha)^{n-k}\right]\left[\sum_{k\geq 0}\binom{n}{k} b_k (1-\alpha)^k \alpha^{n-k}\right]}{\left[\sum_{k\geq 0}\binom{n}{k} b_k (1-\alpha)^k \alpha^{n-k}\right]^2} \\
& - \frac{\left[\sum_{k\geq 0}\binom{n}{k} b'_k (1-\alpha)^k \alpha^{n-k}\right]\left[\sum_{k\geq 0}\binom{n}{k} b_k \alpha^k (1-\alpha)^{n-k}\right]}{\left[\sum_{k\geq 0}\binom{n}{k} b_k (1-\alpha)^k \alpha^{n-k}\right]^2} \\
= \; & \frac{\sum_{i,j\geq 0, i\neq j}\binom{n}{i}\binom{n}{j} b_i b'_j \left[(1-\alpha)^i \alpha^{n-i} \alpha^j (1-\alpha)^{n-j} - \alpha^i (1-\alpha)^{n-i} (1-\alpha)^j \alpha^{n-j}\right]}{\left[\sum_{k\geq 0}\binom{n}{k} b_k (1-\alpha)^k \alpha^{n-k}\right]^2} \\
= \; & \frac{\sum_{i\geq 0, j\geq n-n_0+1, i\neq j}\binom{n}{i}\binom{n}{j} b_i b'_j \alpha^n (1-\alpha)^n \left[(1-\alpha)^{i-j}\alpha^{j-i} - (1-\alpha)^{j-i}\alpha^{i-j}\right]}{\left[\sum_{k\geq 0}\binom{n}{k} b_k (1-\alpha)^k \alpha^{n-k}\right]^2}.
\end{aligned} \tag{B.4}$$

It suffices to prove that the nominator of (B.4) is positive.

Let $n_1 = n - n_0$. For $k \geq n - n_0 + 1$, write $b_k = (n_0 - (n-k))\binom{k}{n_0-(n-k)} \int_0^{r_B} t^{n_0-(n-k)-1}(1-t)^{k-n_0+(n-k)} dt = (k-n_1)\binom{k}{k-n_1} \int_0^{r_B} t^{k-n_1-1}(1-t)^{n_1} dt$. Then $b'_k = (k-n_1)\binom{k}{k-n_1} {r_B}^{k-n_1-1}(1-r_B)^{n_1}$. Fix a pair of i, j with $n - n_0 + 1 \leq i < j$. Then,

$$\begin{aligned}
b_i b'_j &= (i-n_1)\binom{i}{i-n_1}\int_0^{r_B} t^{i-n_1-1}(1-t)^{n_1} dt \cdot (j-n_1)\binom{j}{j-n_1} {r_B}^{j-n_1-1}(1-r_B)^{n_1} \\
&= (i-n_1)\binom{i}{i-n_1}\int_0^{r_B} t^{i-n_1-1} {r_B}^{j-i}(1-t)^{n_1} dt \cdot (j-n_1)\binom{j}{j-n_1} {r_B}^{i-n_1-1}(1-r_B)^{n_1} \\
&> (i-n_1)\binom{i}{i-n_1}\int_0^{r_B} t^{j-n_1-1}(1-t)^{n_1} dt \cdot (j-n_1)\binom{j}{j-n_1} {r_B}^{i-n_1-1}(1-r_B)^{n_1} \\
&= b_j b'_i.
\end{aligned}$$

Then for any fixed pair i, j with $n - n_0 + 1 \leq i < j$, the sum of the two terms indexed by

(i, j) and (j, i) in the nominator of (B.4) is positive:

$$
\begin{aligned}
& \text{Summand}_{(i,j)} + \text{Summand}_{(j,i)} \\
= \;& \binom{n}{i}\binom{n}{j} b_i b'_j \alpha^n (1-\alpha)^n \left[(1-\alpha)^{i-j}\alpha^{j-i} - (1-\alpha)^{j-i}\alpha^{i-j}\right] \\
& + \binom{n}{j}\binom{n}{i} b_j b'_i \alpha^n (1-\alpha)^n \left[(1-\alpha)^{j-i}\alpha^{i-j} - (1-\alpha)^{i-j}\alpha^{j-i}\right] \\
> \;& \binom{n}{i}\binom{n}{j} b_j b'_i \alpha^n (1-\alpha)^n \left[(1-\alpha)^{i-j}\alpha^{j-i} - (1-\alpha)^{j-i}\alpha^{i-j}\right] \\
& + \binom{n}{j}\binom{n}{i} b_j b'_i \alpha^n (1-\alpha)^n \left[(1-\alpha)^{j-i}\alpha^{i-j} - (1-\alpha)^{i-j}\alpha^{j-i}\right] \\
= \;& 0.
\end{aligned}
$$

For $i < n - n_0 + 1$ and $j \geq n - n_0 + 1$, it also holds that

$$\binom{n}{i}\binom{n}{j} b_i b'_j \alpha^n (1-\alpha)^n \left[(1-\alpha)^{i-j}\alpha^{j-i} - (1-\alpha)^{j-i}\alpha^{i-j}\right] > 0.$$

So the nominator of (B.4) is positive, and $\frac{\partial \xi_{IV}(r_B, n, n_0, \alpha)}{\partial r_B} > 0$. This proves part (2) of Lemma 2. □

B.1.6 Proof of Lemma 3

Proof. Write

$$IA = \frac{\mathbb{P}(S|H)}{\mathbb{P}(S|L)} = \frac{\mathbb{P}(n_i \geq n_0|H)}{\mathbb{P}(n_i \geq n_0|L)}.$$

Then at Equilibrium II, $IA = \frac{1}{\xi_{II}(r_G, n, n_0, \alpha)}$. As proved in the proof of Lemma 2, $\xi_{II}(r_G, n, n_0, \alpha)$ is strictly increasing in r_G, so IA is strictly decreasing in r_G at insiders' Equilibrium II. Similarly, at Equilibrium IV, $IA = \frac{1}{\xi_{IV}(r_B, n, n_0, \alpha)}$, and IA is strictly decreasing in r_B. □

B.1.7 Proof of Propositions 3

Proof. Let $R_\gamma(r_G)$ denote the entrepreneur's expected utility. So for $\gamma \in (0, 1]$,

$$R_\gamma(r_G) := \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^\gamma(r_G).$$

To prove Propositions 3, it suffices to prove that $R_\gamma(r_G)$ is increasing in r_G. I will prove that $\frac{\partial R_\gamma(r_G)}{\partial r_G} > 0$. Calculate the partial derivative of $R_\gamma(r_G)$ with respect to r_G:

$$\begin{aligned}
&\frac{\partial R_\gamma(r_G)}{\partial r_G} \\
= \;& \frac{\mathbb{P}'(n_i \geq n_0|H) + \mathbb{P}'(n_i \geq n_0|L)}{2} \cdot p^\gamma(r_G) \\
&+ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot \gamma p^{\gamma-1}(r_G) \cdot p'(r_G) \\
= \;& \frac{\mathbb{P}'(n_i \geq n_0|H) + \mathbb{P}'(n_i \geq n_0|L)}{2} \cdot p^\gamma(r_G) \\
&+ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot \gamma p^{\gamma-1}(r_G) \cdot \left(-p^2(r_G)\frac{1-\alpha}{\alpha} \cdot \xi'_{II}(r_G, n, n_0, \alpha)\right).
\end{aligned}$$

Let $t = \frac{\mathbb{P}'(n_i \geq n_0|L)}{\mathbb{P}'(n_i \geq n_0|H)}$. Observe that $t > 0$. Then

$$\begin{aligned}
&\frac{\partial R_\gamma(r_G)}{\partial r_G} \\
= \;& \frac{p^{\gamma+1}(r_G)\mathbb{P}'(n_i \geq n_0|H)}{2} \\
&\cdot \left[\frac{1+t}{p(r_G)} - (1 + \xi_{II}(r_G, n, n_0, \alpha)) \cdot \gamma \cdot \left(\frac{1-\alpha}{\alpha} \cdot \frac{\mathbb{P}(n_i \geq n_0|H)\xi'_{II}(r_G, n, n_0, \alpha)}{P'(n_i \geq n_0|H)}\right)\right].
\end{aligned}$$

By the equation $\frac{\mathbb{P}(n_i \geq n_0|H)\xi'_{II}(r_G,n,n_0,\alpha)}{P'(n_i \geq n_0|H)} = t - \xi_{II}(r_G, n, n_0, \alpha)$,

$$\begin{aligned}
\frac{\partial R_\gamma(r_G)}{\partial r_G} = \;& \frac{p^{\gamma+1}(r_G)\mathbb{P}'(n_i \geq n_0|H)}{2} \\
&\cdot \left[\frac{1+t}{p(r_G)} - (1 + \xi_{II}(r_G, n, n_0, \alpha)) \cdot \gamma \cdot \left(\frac{1-\alpha}{\alpha} \cdot (t - \xi_{II}(r_G, n, n_0, \alpha))\right)\right].
\end{aligned}$$

Then by the insiders' equilibrium condition at Equilibrium II,

$$p(r_G) = \frac{1}{1 + \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)} \cdot \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)}} = \frac{1}{1 + \frac{1-\alpha}{\alpha} \cdot \xi_{II}(r_G, n, n_0, \alpha)}.$$

Replace $p(r_G)$ in the expression of $\frac{\partial R_\gamma(r_G)}{\partial r_G}$:

$$\begin{aligned}
\frac{\partial R_\gamma(r_G)}{\partial r_G} &= \frac{p^{\gamma+1}(r_G)\mathbb{P}'(n_i \geq n_0|H)}{2} \\
&\quad \cdot \left[(1+t)\left(1+\frac{1-\alpha}{\alpha}\xi_{II}\right) - (1+\xi_{II})\cdot\gamma\cdot\left(\frac{1-\alpha}{\alpha}\cdot(t-\xi_{II})\right)\right] \\
&= \frac{p^{\gamma+1}(r_G)\mathbb{P}'(n_i \geq n_0|H)}{2} \\
&\quad \cdot \left[1+(1+\gamma)\frac{1-\alpha}{\alpha}\xi_{II} + \gamma\frac{1-\alpha}{\alpha}\xi_{II}^2 + \left(1-\gamma\frac{1-\alpha}{\alpha}\right)t + (1-\gamma)\frac{1-\alpha}{\alpha}\xi_{II}t\right] \\
&> 0.
\end{aligned}$$

So this completes the proof. □

B.1.8 Proof of Proposition 4

Proof. After observing the number of insiders who invest (denoted by n_I), the outsiders update their beliefs about the quality of the startup:

$$\begin{aligned}
\mathbb{P}(H|n_I) &= \frac{\mathbb{P}(n_I|H)\mathbb{P}(H)}{\mathbb{P}(n_I|H)\mathbb{P}(H)+\mathbb{P}(n_I|L)\mathbb{P}(L)} = \frac{\mathbb{P}(n_I|H)}{\mathbb{P}(n_I|H)+\mathbb{P}(n_I|L)} \\
&= \frac{\binom{n+1}{n_I}\alpha^{n_I}(1-\alpha)^{n+1-n_I}}{\binom{n+1}{n_I}\alpha^{n_I}(1-\alpha)^{n+1-n_I}+\binom{n+1}{n_I}(1-\alpha)^{n_I}\alpha^{n+1-n_I}} \\
&= \frac{\alpha^{n_I}(1-\alpha)^{n+1-n_I}}{\alpha^{n_I}(1-\alpha)^{n+1-n_I}+(1-\alpha)^{n_I}\alpha^{n+1-n_I}}.
\end{aligned}$$

Then if investing, the outsiders' expected gross payoff is

$$\mathbb{P}(H|n_I) = \frac{1}{1+\left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}}.$$

First notice that $\mathbb{P}(H|n_I)$ is monotone increasing in n_I. Moreover, since $\alpha > 1/2$, the following holds:

$$\mathbb{P}(H|n_I) = \frac{1}{1+\left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}} \approx \begin{cases} 0 & \text{if } n_I < \frac{n+1}{2}; \\ 1 & \text{if } n_I > \frac{n+1}{2}. \end{cases}$$

Then clearly, when $n_0 \le \frac{n+1}{2}$, $\mathbb{P}(H|n_0+1) - p < 0$. So it suffices to prove that when $n_0 > \frac{n+1}{2}$, $\mathbb{P}(H|n_0+1) < p$. At Equilibrium II, when the entrepreneur chooses the optimal price, at the threshold $n_I = n_0 + 1$ the outsiders will have expected payoff

$$\mathbb{P}(H|n_0+1) - p = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{n+1-2(n_0+1)}} - \frac{1}{1 + \frac{1-\alpha}{\alpha} \cdot e(n, n_0, \alpha)}. \tag{B.5}$$

To prove $\mathbb{P}(H|n_0+1) < p$, it suffices to prove $e(n, n_0, \alpha) = \frac{\sum_{k \ge n_0} \binom{n}{k}(1-\alpha)^k \alpha^{n-k}}{\sum_{k \ge n_0} \binom{n}{k}\alpha^k (1-\alpha)^{n-k}} < \left(\frac{1-\alpha}{\alpha}\right)^{2n_0-n}$. Indeed, this inequality holds because $\frac{\binom{n}{k}(1-\alpha)^k \alpha^{n-k}}{\binom{n}{k}\alpha^k (1-\alpha)^{n-k}} = \left(\frac{1-\alpha}{\alpha}\right)^{2n_0-n}$ for $k = n_0$ and $\frac{\binom{n}{k}(1-\alpha)^k \alpha^{n-k}}{\binom{n}{k}\alpha^k (1-\alpha)^{n-k}} = \left(\frac{1-\alpha}{\alpha}\right)^{2k-n}$ is strictly decreasing in k for $k \ge n_0$. □

B.1.9 Proof of Proposition 5

Proof. Fix any n_0 such that $n_0 > \frac{n+1}{2}$. Then there exists a crowdfunding market equilibrium in which (1) the entrepreneur sets the share price $p^* = \frac{1}{1+\frac{1-\alpha}{\alpha}\cdot e(n,n_0,\alpha)}$ for the insiders, and $p'(n_I) = \frac{1}{1+\left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}}$ for the outsiders, (2) the insiders with good signal invest and with bad signal do not invest, and (3) the outsider play equilibrium strategy $n_0 + 1$. The completes the proof. □

Appendix C

APPENDIX TO CHAPTER 3

C.1 Omitted proofs

C.1.1 Proof of Proposition 6

Proof. Firm i's total profits are given by

$$\pi_i(\mathbf{q}_i) = \sum_{j=1}^{m} \pi_{ij}(q_{ij}) - \kappa(|F_i|).$$

As F_i is fixed, there is nothing that links firm i's decisions across markets and for all markets j,

$$\frac{d\pi_i(\mathbf{q}_i)}{dq_{ij}} = \frac{d\pi_{ij}(q_{ij})}{dq_{ij}}.$$

Thus i's profit maximizing output decision in market j is independent of i's output decisions in other markets. This implies that i's output decision in each market can be considered in isolation and it is sufficient to show that there is a unique Nash equilibrium in a given market. Although this result is standard, we prove it below for completeness.

As we are considering a fixed market we suppress the market index j. Without loss of generality, we order the marginal cost of all firms: $c_1 \leq c_2 \leq \ldots \leq c_n$. Now we will show that there exists a unique Nash equilibrium $(q_i^*)_{1 \leq i \leq n}$ such that

1. if $\alpha < c_1$, then $q_i^* = 0$ for all $1 \leq i \leq n$.

2. if $\alpha > c_1$, then

$$q_i^* = \begin{cases} \dfrac{P - c_i}{\beta}, & \text{if } i \leq i^*; \\ 0, & \text{if } i > i^*, \end{cases} \tag{C.1}$$

where $i^* \in \{1, \ldots, n\}$ is uniquely identified by the marginal costs[1], and P is the market equilibrium price:

$$P = \frac{\alpha + c_1 + \ldots + c_{i^*}}{i^* + 1}. \tag{C.2}$$

As the inverse demand function is linear, each firm has a quadratic (and concave) profit function.

$$\pi_i = (\alpha - \beta Q - c_i)q_i = -\beta q_i^2 + (\alpha - \beta Q_{-i} - c_i)q_i,$$

and a marginal profit function

$$\frac{d\pi_i}{dq_i} = -\beta q_i + \alpha - \beta Q - c_i.$$

If $(q_i^*)_{1 \leq i \leq n}$ is a Nash equilibrium, then $q_i^* > 0$ if and only if $\pi_i(q^*) > 0$. Moreover, at $(q_i^*)_{1 \leq i \leq n}$, it follows from the first order conditions that

$$q_i^* > 0 \quad \text{implies} \quad \left.\frac{d\pi_i}{dq_i}\right|_{q_i = q_i^*} = 0, \tag{C.3}$$

$$q_i^* > 0 \text{ if and only if } \left.\frac{d\pi_i}{dq_i}\right|_{q_i = 0} > 0, \tag{C.4}$$

and

$$q_i^* = 0 \text{ if and only if } \left.\frac{d\pi_i}{dq_i}\right|_{q_i = 0} \leq 0. \tag{C.5}$$

[1]Let $P(i) = \frac{\alpha + c_1 + \ldots + c_i}{i+1}$. It is easy to prove: (1) if $P(i) > c_i$, then $P(k) > c_k$ for any $k < i$; (2) if $P(i) \leq c_i$, then $P(k) \leq c_k$ for any $k > i$. Then this implies that there exists a unique $i^* \in \{1, \ldots, n\}$ such that either $i^* \in \{1, \ldots, n-1\}$ satisfies $P(i^*) > c_{i^*}$ and $P(i^* + 1) \leq c_{i^*+1}$, or $i^* = n$ satisfies $P(i^*) > c_{i^*}$.

If $\alpha \leq c_1$, then $\left.\frac{d\pi_i}{dq_i}\right|_{q_i=0} \leq \alpha - c_i \leq 0$. By (C.5), $q_i^* = 0$ for any i. This is then the unique Nash equilibrium.

Let us now consider the case $\alpha > c_1$. First it is easy to check that outputs satisfying equation (C.1) constitute a Nash equilibrium. Indeed, for $i \leq i^*$, (C.4) holds, and hence $q_i^* = (P - c_i)/\beta$ is a best response; for $i \geq i^*$, (C.5) holds, and hence $q_i^* = 0$ is a best response. Next we need to prove that (C.1) is the unique Nash equilibrium. By (C.3), we can write any Nash equilibrium as $q_i = \max\{0, (P - c_i)/\beta\}$. It is easy to see that for a given P, there exists a unique $(q_i)_{1 \leq i \leq n}$. Now suppose there are two equilibria $(q_i^1)_{1 \leq i \leq n}$ and $(q_i^2)_{1 \leq i \leq n}$ corresponding to P^1 and P^2 respectively. Without loss of generality, we can assume $P^1 > P^2$. Then

$$q_i^1 = \max\left\{0, \frac{P^1 - c_i}{\beta}\right\} \geq \max\left\{0, \frac{P^2 - c_i}{\beta}\right\} = q_i^2, \ \forall i.$$

Therefore, $Q^1 = \sum_{i=1}^n q_i^1 \geq \sum_{i=1}^n q_i^2 = Q^2$. This implies $P^1 \leq P^2$, which is a contradiction. □

C.1.2 Proof of Proposition 7

Proof. We first prove a key lemma. For a given market j consider what happens when the relevant capabilities of some firms improve. Recall that $\theta_{ij} = |F_i \cap M_j|$. We say there is an improvement in the capabilities of the firms in market j if θ_{ij} weakly increases for all i.

Lemma 6. *For market j, if there is an improvement in the capabilities of the firms in it, then the market price weakly decreases.*

Proof. As c_{ij} is a decreasing function of θ_{ij}, firm i's marginal cost decreases when θ_{ij} increases. Let c_{ij} be i's marginal cost before the improvement in i's capabilities and let

c'_{ij} be i's marginal cost afterwards. Since we consider a fixed market, we can suppress the market index j. Then holding entry fixed at a set of firms $\{1, \ldots, i^*\}$, the new market price is

$$\tilde{P} := \frac{\alpha + c'_1 + \ldots + c'_i + \ldots + c'_{i^*}}{i^* + 1}.$$

Thus holding entry fixed the market price decreases:

$$\frac{\alpha + c'_1 + \ldots + c'_{i^*}}{i^* + 1} - \frac{\alpha + c_1 + \ldots + c_{i^*}}{i^* + 1} = \frac{\sum_{i=1}^{i^*} c'_i - c_i}{i^* + 1} < 0.$$

However, entry may not remain fixed in equilibrium. Then there are two cases to consider. First there may exist a firm who entered the market before the change and chooses not to enter after the change. Let i' be such a firm. By definition, before the improvement in capabilities, we have that $P > c_{i'}$. If i' no longer finds it profitable to enter then we must have $P > c_{i'} \geq c'_{i'} > P'$ and so the market price decreases, increasing consumer surplus.

The second case is the one in which entry increases in the strong set order, so that new firms participate in the market as well as all those that previously participated. Let $\hat{i} > i^*$ be the new marginal firm. We then have that

$$
\begin{aligned}
P' &= \frac{\alpha + c'_1 + \ldots + c'_{\hat{\imath}}}{\hat{\imath} + 1} \\
&= \frac{\alpha + c'_1 + \ldots + c'_{i^*}}{i^* + 1} \left(\frac{i^* + 1}{\hat{\imath} + 1} \right) + \frac{c'_{i^*+1} + \ldots + c'_{\hat{\imath}}}{\hat{\imath} + 1} \\
&= \tilde{P} \left(\frac{i^* + 1}{\hat{\imath} + 1} \right) + \frac{c'_{i^*+1} + \ldots + c'_{\hat{\imath}}}{\hat{\imath} + 1} \\
&= \tilde{P} \left(\frac{i^* + 1}{\hat{\imath} + 1} \right) + \frac{c'_{i^*+1} + \ldots + c'_{\hat{\imath}}}{\hat{\imath} - i^*} \left(\frac{\hat{\imath} - i^*}{\hat{\imath} + 1} \right).
\end{aligned}
$$

Thus P' is a weighted average of the market price $\tilde{P}$ that would obtain if entry was held fixed and the average marginal cost of the new firms that enter. As all of these firms must have marginal costs below P' (for entry to be profitable), if follows that $\tilde{P} > P'$. Thus as $P > \tilde{P}$ we conclude that $P > P'$ and the market price must decrease. Thus, regardless of whether entry changes the market price decreases following an improvement in the capabilities of firms. This also increases consumer surplus. □

The proof of Proposition 7 follows almost immediately from Lemma 6. A merger weakly improves the merged firm's set of capabilities in both a newly entered and non-overlapping market. Then by Lemma 6 the market price will weakly decrease and consumer surplus weakly increases regardless of whether there is exit. □

C.1.3 Proof of Proposition 8

Proof. Suppose firms i and k merge to firm l. Consider an overlapping market j. Under the full cover assumption for any firm x competing in market j, $F_x \cap M_j = M_j$. Moreover, in equilibrium all firms x with capabilities $F_x \supseteq M_j$ will enter market j. Thus, if pre-merger there are y firms competing in market j and these y firms have the same marginal cost, post merger there will be $y - 1$ firms competing in market j with the same marginal cost as before. Thus the market price will increase and consumer surplus will be reduced. □

C.1.4 Proof of Proposition 9

Proof. We will prove the proposition by constructing an example.

Fix a positive integer m, and suppose there are $2m$ attributes $\{a_1, \ldots, a_{2m}\}$. We consider a market hypergraph with m markets: $H_M = (M_j = \{a_{2j-1}, a_{2i}\})_{\{j=1,\ldots,m\}}$, and a firm hypergraph with two firms: $H_F = \{F_1 = \{a_1, a_2, a_3, \ldots, a_{2m-1}, \ldots, a_{2m-1}\}, F_2 = \{a_1, a_2, a_4, \ldots, a_{2i}, \ldots, a_{2m}\}\}$.

We assume that for some fixed $c > 0$, the marginal cost of production in all markets is

$$c_{ij} = \begin{cases} 2c & \text{if } |F_i \cap M_j| = 1; \\ c & \text{if } |F_i \cap M_j| = 2, \end{cases}$$

and that all markets have equivalent demand curves ($P_j = \alpha - Q_j$) with $c < \alpha < 2c$.

We now consider a merger between firm 1 and 2. Let Δ_j denote the change of consumer surplus following a merger between firm 1 and 2 in market j. Since $c < \alpha < 2c$, both firm 1 and 2 only operate in market 1 pre-merger, and the merged firm will operate in all markets post merger. Then the total consumer surplus change is

$$\begin{aligned} \sum_{j=1}^{m} \Delta_j &= \sum_{j=1}^{m} \frac{1}{2}\left(\alpha - \frac{\alpha + c}{2}\right)^2 - \frac{1}{2}\left(\alpha - \frac{\alpha + c + c}{3}\right)^2 \\ &= \frac{(\alpha - c)^2}{8} \cdot m - \frac{2(\alpha + c)^2}{9}. \end{aligned}$$

Thus, as $m \to \infty$, $\sum_{j=1}^{m} \Delta_j \to \infty$. This implies that a merger between firm 1 and 2 can increase the overall consumer surplus by an arbitrarily large amount. However, note that the price in market 1 would increase following the merger. Therefore, a merger policy only focusing on overlapping markets (market 1) would block the merger, thus leading to unbounded loss of consumer surplus. □

C.1.5 Proof of Lemma 4

Proof. The proof is similar to Proposition 6. We also just consider one market, and thus can suppress the market index j. Suppose firm i and firm k are going to form a joint venture $\mathcal{J}$. Let $c_{\mathcal{J}}$ be the marginal cost of $\mathcal{J}$. We first prove the existence of a Nash equilibrium.

Notice that with a joint venture competing in the market, the same equlibrium output conditions (C.3) (C.4) (C.5) still hold for all firms including $\mathcal{J}$. In particular, this implies that for the firms with the same profit function $\pi_s = (P - c_s)q_s$ as before, they enter the market with $q_s^* > 0$ if and only if $P > c_s$. Thus, the minimum entry price for them is their marginal cost c_s. Things are a bit different for firm i and k, since they have different profit functions. Now they have marginal profit functions

$$\frac{d\pi_f}{dq_f} = -\beta q_f + \alpha - \beta Q - c_f - \lambda_f \beta q_{\mathcal{J}}, \quad f = i, k. \tag{C.6}$$

By the first order conditions, firm i and k's minimum entry price depends on whether their joint venture $\mathcal{J}$ enters the market, i.e. $q_{\mathcal{J}}^* > 0$. If $q_{\mathcal{J}}^* = 0$, the minimum entry price is their marginal cost. If $q_{\mathcal{J}}^* > 0$, then

$$q_f^* > 0 \Leftrightarrow \left.\frac{d\pi_f}{dq_f}\right|_{q_f=0} > 0 \Leftrightarrow P > \frac{c_f - \lambda_f c_{\mathcal{J}}}{1 - \lambda_f}.$$

Thus when $q_{\mathcal{J}}^* > 0$, the minimum entry price for firm i and k is a modified version of their marginal cost

$$\hat{c}_f = \frac{c_f - \lambda_f c_{\mathcal{J}}}{1 - \lambda_f}, \quad f = i, k.$$

Notice that $c_f > c_{\mathcal{J}}$ if and only if $\hat{c}_f > c_{\mathcal{J}}$. In particular, this implies that when $c_f < c_{\mathcal{J}}$, if $\mathcal{J}$ enters the market, then its parent firm f also enters the market.

Having found the minimum entry prices for all firms, we would like to order them. Then for any fixed price (or firm) threshold, we will be able to compute the market equilibrium price resulting from the competition of the firms with minimum entry prices lower than or equal to that threshold. However, such an order depends on whether $\mathcal{J}$ enters the market, since firm i and k's minimum entry price does so. Thus, the relative magnitude of $c_{\mathcal{J}}, c_i, c_k$ matters. To deal with this difficulty, we will proceed our argument in different cases. Without loss of generality, we also assume that $\hat{c}_i \leq \hat{c}_k$. Let the minimum entry price of firm s be e_s, and let $P(s)$ be the market equilibrium price resulting from the competition from firm s and the firms with minimum entry price lower than or equal to e_s. Then depending on whether $\mathcal{J}$ and its parent firms simultaneously compete in the market, $P(s)$ can take the following three forms:

$$P(s) = \begin{cases} P_1(s) := \frac{\alpha + \sum_{t:c_t \leq e_s} c_t}{|\{t:c_t \leq c_e\}|+1} & \text{if } q^*_{\mathcal{J}} = 0 \text{ or } q^*_i = q^*_k = 0; \\ P_2(s) := \frac{\alpha + \sum_{t \neq i:c_t \leq e_s} c_t + c_i - \lambda_i c_{\mathcal{J}}}{|\{t \neq i:c_t \leq e_s\}|+2-\lambda_i} & \text{if } q^*_{\mathcal{J}} > 0, q^*_i > 0, q^*_k = 0; \\ P_3(s) := \frac{\alpha + \sum_{t \neq i,k,\mathcal{J}:c_t \leq e_s} c_t + c_i + c_k}{|\{t \neq i,k:c_t \leq e_s\}|+2} & \text{if } q^*_{\mathcal{J}} > 0, q^*_i > 0, q^*_k > 0. \end{cases} \tag{C.7}$$

Now according to the relative magnitude of $c_{\mathcal{J}}, c_i, c_k$, we can specify which form $P(s)$ takes as a function of minimum entry price e_s of the marginal firm (i.e. the firm that enters with the highest minimum entry price):

1. $c_{\mathcal{J}} \leq \min\{c_i, c_k\}$. If the firm with the highest minimum entry price to enter has a minimum entry price $e_s < \hat{c}_i$, then all the firms possibly including $\mathcal{J}$ with marginal cost $c_t \leq e_s$ also compete in the market but no firms with marginal cost $c_t > e_s$, including i and k, compete. Thus, $P(s) = P_1(s)$. If the marginal firm s

has $e_s \in [\hat{c}_i, \hat{c}_k)$, then all the firms with marginal cost $c_t \leq e_s$, including $\mathcal{J}$ and i, also compete in the market while all firms with marginal cost $c_t > e_s$, including k, don't. Thus, $P(s) = P_2(s)$, if the marginal firm s has $e_s \geq \hat{c}_k$ then all the firms with $c_t \leq e_s$, including $i, \mathcal{J}$ and k, also compete in the market. Thus, $P(s) = P_3(s)$.

2. $\min\{c_i, c_k\} \leq c_{\mathcal{J}} \leq \max\{c_i, c_k\}$. If the marginal firm s has $e_s < c_{\mathcal{J}}$ then $q^*_{\mathcal{J}} = 0$ and $P(s) = P_1(s)$. If the marginal firm s has $e_s \in [c_{\mathcal{J}}, \hat{c}_k)$ competes in the market, then all the firms with $c_t \leq e_s$, including i and $\mathcal{J}$, also compete while all firms with $c_t > e_s$, including k, do not. Thus, $P(s) = P_2(s)$; If the marginal firm s has $e_s \geq \hat{c}_k$, then all the firms with $c_t \leq e_s$, including $i, \mathcal{J}$, and k, also compete in the market and $P(s) = P_3(s)$.

3. $c_{\mathcal{J}} \geq \max\{c_i, c_k\}$. If the marginal firm s has $e_s < c_{\mathcal{J}}$ then $q^*_{\mathcal{J}} = 0$ and $P(s) = P_1(s)$. If the marginal firm s has $e_s \geq c_{\mathcal{J}}$ then all the firms with $c_t \leq e_s$, including $i, \mathcal{J}$ and k, also compete in the market. Thus, $P(s) = P_3(s)$.

To prove the existence of a Nash equilibrium, in each of the above three cases, we solve the following problem:

$$\max_{s \in \{1,\ldots,n\} \cup \{\mathcal{J}\}} e_s \cdot 1_{P(s) > e_s} \tag{C.8}$$

and argue that the solution to (C.8) identifies a unique Nash equilibrium.

By definition, (C.8) has a unique solution since the total number of firms is finite. Let x^* be the solution and let s^* be a marginal firm such that $e_{s^*} = x^*$. Then for any firm t with minimum entry price $e_t \leq e_{s^*}$, $e_t \leq e_{s^*} < P(s^*)$ and so $q^*_t > 0$. For the firms with $e_t > e_{s^*}$, without loss of generality, we can assume that $e_{t'} = \min_{t: e_t > e_{s^*}} e_t$.

Then by the definition of (C.8), $P(t') \leq e_{t'}$. Using equation (C.7) and reorganizing the inequality $P(t') \leq e_{t'}$, we have $P(s^*) \leq e_{t'}$. Thus, it is not profitable for firm t' to enter the market when the market price is $P(s^*)$. Therefore, it is also not profitable for any firm with $e_t \geq e_{t'}$ to enter the market. So for any firm t with $e_t > e_{s^*}$, $e_t \geq P(s^*)$, thus $q_t^* = 0$. Now we have pinned down the entry decisions of all firms. Given these entry choices, output decisions are uniquely determined by the first order conditions of firms' profit maximization problems. Thus we have a Nash equilibrium with market equilibrium price $P(s^*)$. □

C.1.6 Proof of Proposition 10

Proof. First we introduce some notation. Let q'_{ij}, q'_{kj}, Q'_j and P'_j denote the equilibrium output of i, output of k, total output and market price respectively, in market j before the creation of $\mathcal{J}$. Let q_{ij}, q_{kj}, $q_{\mathcal{J}j}$, Q_j, and P_j denote the equilibrium output of i, output of k, output of $\mathcal{J}$, total output, and market price respectively, in market j after the creation of $\mathcal{J}$.

Part (i) By assumption $\mathcal{J}$ enters market j after created and so $q_{\mathcal{J}j} > 0$. This implies that in equilibrium $P_j > c_{\mathcal{J}}$ and $\pi_{\mathcal{J}j} > 0$. Since before $\mathcal{J}$ is created, $\pi'_{ij} = \pi'_{kj} = 0$. So the creation of $\mathcal{J}$ strictly increases both i and k's profits in market j for all $\gamma_i \in (0, 1)$.

Part (ii). As i but not k is present in market j prior to $\mathcal{J}$'s creation $q'_{ij} > q'_{kj} = 0$. Setting $\lambda_i = 1$, i chooses q_{ij} to maximize $\pi_{ij}+\pi_{\mathcal{J}j}$ while $\mathcal{J}$ chooses $q_{\mathcal{J}j}$ to maximize $\pi_{\mathcal{J}j}$. As shown in the proof of Lemma 4, i optimally chooses output $q_{ij} = \max\{\frac{P_j-c_{ij}}{\beta}-q_{\mathcal{J}j}, 0\}$. We consider two cases.

<u>Case 1:</u> Suppose that $c_{ij} < c_{\mathcal{J}j}$. Then $\frac{P_j-c_{ij}}{\beta} \geq q_{\mathcal{J}j} = \frac{P_j-c_{\mathcal{J}j}}{\beta}$ and $q_{ij} \geq 0$. It is straightforward to then verify that there is an equilibrium in which the market price does

not change ($P_j = P_j'$), $q_{ij} = q_{ij}' - q_{\mathcal{J}j}$ and $q_{lj} = q_{lj}'$ for all $l \neq i, \mathcal{J}$. By Lemma 4 this equilibrium is unique. The change in i's profits in market j is thus

$$\begin{aligned}
& (P_j - c_{ij})q_{ij} + (P_j - c_{\mathcal{J}j})q_{\mathcal{J}j} - (P_j' - c_{ij})q_{ij}' \\
= \; & (P_j - c_{ij})q_{ij} + (P_j - c_{\mathcal{J}j})q_{\mathcal{J}j} - (P_j - c_{ij})(q_{ij}') \\
= \; & P_j(q_{ij} + q_{\mathcal{J}j} - q_{ij}') - c_{ij}(q_{ij} - q_{ij}') - c_{\mathcal{J}j}q_{\mathcal{J}j} \\
= \; & (c_{ij} - c_{\mathcal{J}j})q_{\mathcal{J}j}.
\end{aligned}$$

As by assumption $c_{ij} < c_{\mathcal{J}j}$ and $q_{\mathcal{J}j} > 0$ by assumption, the above expression is negative.

<u>Case 2:</u> Suppose that $c_{ij} \geq c_{\mathcal{J}j}$. Then $\frac{P_j - c_{ij}}{\beta} \leq q_{\mathcal{J}j}$ and $q_{ij} = 0$. The market outcome is identical to the case in which $\mathcal{J}$ was not created, but i's marginal cost changed from c_{ij} to $c_{\mathcal{J}j}$. By Lemma 6, reducing i's marginal cost in market j strictly increases i's profits in market j if $c_{ij} > c_{\mathcal{J}j}$. Moreover, if $c_{ij} = c_{\mathcal{J}j}$ then i's profits in market j remain constant.

The above two cases together show that while k's profits remain zero, i's profits strictly increase from the creation of $\mathcal{J}$ if and only if $c_{ij} > c_{\mathcal{J}j}$. By the continuity of payoffs in λ_i and λ_k it follows that there exist $\lambda_i < 1$ and $\lambda_k = 1 - \lambda_i$ such that both i and k do strictly better in market j after the creation of $\mathcal{J}$ if and only if $c_{ij} > c_{\mathcal{J}j}$.

Part (iii) We will show the following, which is logically equivalent to the statement made: If $\max\{c_{ij}, c_{kj}\} \leq c_{\mathcal{J}j}$ then there does not exist a λ_i such that both i and k's profits weakly increase following the creation of $\mathcal{J}$. Fix any λ_i. We showed in the proof of Lemma 4 that i and k optimally enter the market after the creation of $\mathcal{J}$ if: (i) $\mathcal{J}$ optimally enters and (ii) $\max\{c_{ij}, c_{kj}\} \leq c_{\mathcal{J}j}$. As by assumption $\mathcal{J}$ optimally enters and as $\max\{c_{ij}, c_{kj}\} \leq c_{\mathcal{J}j}$, i and k both enter after the creation of $\mathcal{J}$. Thus, as shown in the proof of Lemma 4, we must have $\frac{P_j - c_{ij}}{\beta} \geq \lambda_i q_{\mathcal{J}j}$ and $\frac{P_j - c_{kj}}{\beta} \geq (1-\lambda_i) q_{\mathcal{J}j}$. In such case there exists an equilibrium in which the market price does not change $P_j = P'_j$. To see this, note that if $P_j = P'_j$, then $q_{ij} = q'_{ij} - \lambda_i q_{\mathcal{J}j}$, $q_{kj} = q'_{kj} - (1-\lambda_i) q_{\mathcal{J}j}$ and $q_{lj} = q'_{lj}$ for all $l \neq i, k, \mathcal{J}$. Thus $Q_j = Q'_j$ and so $P_j = P'_j$. Moreover, by Lemma 4 this equilibrium is unique.

The change in i's profits in market j is therefore

$$\begin{aligned} \pi_{ij} + \lambda_i \pi_{\mathcal{J}j} - \pi'_{ij} &= (P_j - c_{ij}) q_{ij} + \lambda_i (P_j - c_{\mathcal{J}j}) q_{\mathcal{J}j} - (P'_j - c_{ij}) q'_{ij} \\ &= (c_{ij} - c_{\mathcal{J}j}) \lambda_i q_{\mathcal{J}j} \leq 0. \end{aligned}$$

Thus the creation of the joint venture strictly reduces i's profits in market j if $\lambda_i > 0$. An equivalent argument shows that the creation of the joint venture also strictly reduces k's profits in market j if $1 - \lambda_i > 0$. Thus there are no profit shares that make the joint venture weakly profitable in market j for both i and k. □

C.1.7 Proof of Proposition 11

Proof. Consider any market j. As we are considering just this market, we suppress the market index in what follows. Suppose firm i and firm k are going to form a joint venture $\mathcal{J}$. Let P' and P be the market prices before and after the creation of $\mathcal{J}$, respectively. We will prove $P' \geq P$ by contradiction.

First suppose $P > P'$. By Proposition 6, we can write any firm's equilibrium output before the formation of $\mathcal{J}$ as $q'_s = \max\{0, (P' - c_s)/\beta\}$ for $s \in \{1, \ldots, n\}$. Let q_s be the equilibrium output of firm s after the formation of $\mathcal{J}$. Then by Lemma 4, for any firm

$s \neq i, k$, we can write $q_s = \max\{0, (P - c_s)/\beta\}$. For firm $s = i, k$, we can write their equilibrium output as

$$q_s = \max\left\{\frac{P - c_s}{\beta} - \lambda_s q_{\mathcal{J}}, 0\right\}.$$

Note that $q_{\mathcal{J}} = 0$ if the market being considered does not correspond to the one $\mathcal{J}$ is created to enter and otherwise $q_{\mathcal{J}} = (P - c_{\mathcal{J}})/\beta > 0$.

Then $P > P'$ implies that

$$q_s = \max\left\{\frac{P - c_s}{\beta}, 0\right\} \geq \max\left\{\frac{P' - c_s}{\beta}, 0\right\} = q'_s, \ \forall s \neq i, k.$$

By an elementary inequality[2], we also have

$$\begin{aligned} q_i + q_k + q_{\mathcal{J}} &= \max\left\{\frac{P - c_i}{\beta} - \lambda_i q_{\mathcal{J}}, 0\right\} + \max\left\{\frac{P - c_k}{\beta} - \lambda_k q_{\mathcal{J}}, 0\right\} + q_{\mathcal{J}} \\ &\geq \max\left\{\frac{P - c_i}{\beta}, 0\right\} + \max\left\{\frac{P - c_k}{\beta}, 0\right\} \\ &\geq \max\left\{\frac{P' - c_i}{\beta}, 0\right\} + \max\left\{\frac{P' - c_k}{\beta}, 0\right\} = q'_i + q'_k. \end{aligned}$$

Therefore, $Q = \sum_{s=1}^{n} q_s + q_{\mathcal{J}} \geq \sum_{s=1}^{n} q'_s = Q'$, which implies that $P \leq P'$. This is a contradiction. □

C.1.8 Proof of Lemma 5

Proof. We use an idea similar to the *energy function* in physics. As there is a finite set of attributes, there is also a finite set of possible hypergraphs. Since f is weakly increasing along the sequence of hypergraphs, $f(H_F^{(t)})$ must converge to a constant in finite steps. When the convergence happens, there are two possibilities. Either the sequence ends or it cycles. Suppose it ends at step τ. Then there can be no profitable merger or demerger and the hypergraph $H_F^{(\tau)}$ is stable. Alternatively the sequence includes a cycle. There

[2] For $b \geq 0$, $\max\{a, 0\} + b \geq \max\{a + b, 0\}$.

exists a sequence of profitable mergers and demergers that cycles. Moreover, as f is weakly increasing each step of the cycle must hold f constant. □

C.1.9 Proof of Proposition 12

Proof. Our proof consists of two steps:

1. Step 1: we identify a function that is weakly increasing at each step in any sequence of profitable mergers and demergers and applying Lemma 5 argue that either a stable hypergraph exists or else a cycle of profitable mergers/demergers exists.
2. Step 2: we show that such a cycle cannot exist.

Define a function

$$\Phi_t = \sum_{F_i \in H_F^{(1)}} |F_i| - \sum_{F_i \in H_F^{(t)}} |F_i|. \tag{C.9}$$

Thus Φ_t is the total number of capabilities summed over all firms in period 1 less the total number of capabilities summed over all firms in period t. Notice that Φ_t weakly increases for any sequence of mergers or demergers. If a merger between two firms with overlapping capabilities occurs, Φ_t strictly decreases and, by definition, a demerger doesn't change Φ_t. We can therefore apply Lemma 5 to conclude that either there exists a stable hypergraph or else there exists a sequence of profitable mergers and demergers that cycles such that $\Phi_t = \Phi_{t+1}$ for all t. Moreover, in this case

(i) there must be both mergers and demergers in the cycle.

(ii) in any merger in this cycle, the merging firms must have disjoint set of capabilities.

We now prove Step 2 by contradiction. Suppose that the set of firms reaches a cycle with length c by step t such that $H_F^{(c+t)} = H_F^{(t)}$. We denote by $N_j^{(\tau)}$ the number of firms in market j at step τ. Let $N^{(\tau)} = (N_1^{(\tau)}, \ldots, N_m^{(\tau)})$. Observe that $N^{(\tau)}$ has the following properties for $\tau \geq t$:

1. if it is a merger that causes the transition from step τ to step $\tau+1$, then $N_j^{(\tau+1)} - N_j^{(\tau)} = 1$ if and only if the new merged firm enters market j;

2. if it is a demerger that causes the transition from step τ to step $\tau+1$, then $N_j^{(\tau+1)} - N_j^{(\tau)} = -1$ if and only if neither of the demerged firms continue to participate in market j.

Let ΔC_τ be the set of coordinates (markets) over which $N^{(\tau)}$ is different from $N^{(\tau+1)}$, i.e. $\Delta C_\tau = \{j | N_j^{(\tau)} \neq N_j^{(\tau+1)}\}$. Let $\Delta\kappa_\tau$ be the fixed cost change of either a merger or demerger from period τ to $\tau+1$. As $\Phi_\tau = \Phi_{\tau+1}$ and κ is convex, $\Delta\kappa_\tau$ is positive for a merger and negative for a demerger. Let $T_M \subset \{t, \ldots, t+c\}$ be the set of steps τ such that there is a merger from τ to $\tau+1$, and $T_D \subset \{t, \ldots, t+c\}$ be the set of steps τ such that there is a demerger from τ to $\tau+1$.

As in the cycle all mergers are between firms with disjoint capabilities, a merger is profitable if and only if the profits obtained from entering new markets is greater than the increase in fixed costs. That is,

$$\Delta\kappa_\tau < \sum_{j \in \Delta C_\tau} \pi_j(N_j^{(\tau+1)}), \qquad \text{for all } \tau \in T_M, \tag{C.10}$$

where $\pi_j(N_j^{\tau+1})$ is the profit each firm earned in market j when there are $N_j^{(\tau+1)}$ firms in that market. Notice that in (C.10), we have suppressed the index of the firms since under the full cover assumption the profit of a firm only depends on the total number of firms in a market.

Similarly, for a demerger, the decreased fixed cost is greater than profit loss of exit. That is,

$$-\Delta\kappa_\tau > \sum_{j\in\Delta C_\tau} \pi_j(N_j^{(\tau)}), \qquad \text{for all } \tau \in T_D. \tag{C.11}$$

Summing up (C.10) over all mergers and (C.11) over all demergers in a cycle, we have that total change of (increased) fixed cost following mergers is less than total profits of entry:

$$\sum_{\tau\in T_M} \Delta\kappa_\tau < \sum_{\tau\in T_M} \sum_{j\in\Delta C_\tau} \pi_j(N_j^{(\tau+1)}) \tag{C.12}$$

and total change of (decreased) fixed cost following demergers is greater than total profit loss of exit:

$$-\sum_{\tau\in T_D} \Delta\kappa_\tau > \sum_{\tau\in T_D} \sum_{j\in\Delta C_\tau} \pi_j(N_j^{(\tau)}). \tag{C.13}$$

Since the total change of fixed cost following mergers on a cycle is equal to the total change of fixed cost following demergers:

$$\sum_{\tau\in T_M} \Delta\kappa_\tau + \sum_{\tau\in T_D} \Delta\kappa_\tau = 0,$$

by (C.12) and (C.13), the total profit loss of exit is less than total profits of entry:

$$\Psi \equiv \sum_{\tau\in T_M} \sum_{j\in\Delta C_\tau} \pi_j(N_j^{(t+1)}) - \sum_{\tau\in T_D} \sum_{j\in\Delta C_\tau} \pi_j(N_j^{\tau}) > 0. \tag{C.14}$$

Next, we will prove that total profit loss of exit is actually equal to total profits of entry, and then this gives us a desired contradiction.

To prove that total profit loss of exit is equal to total profits of entry, we compute their difference:

$$\Psi = \sum_{j=1}^{m} \left[\sum_{\tau : N_j^{(\tau+1)} - N_j^{\tau} = 1} \pi_j(N_j^{(\tau+1)}) - \sum_{\tau : N_j^{(\tau+1)} - N_j^{(\tau)} = -1} \pi_j(N_j^{(\tau)}) \right] = 0. \tag{C.15}$$

The second equality is because, for a fixed market k, $N_j^{(c+t)} = N_j^{(t)}$ and the values of $N_j^{(\tau+1)} - N_j^{(\tau)}$ are taken from a set $\{-1, 0, 1\}$. For each firm that realizes a profit from entering the market k with N firms following a merger, there is some firm that realizes an equal loss through a demerger that reduces the number of firms in market k from $N + 1$ to N. Equation (C.15) contradicts equation (C.14). We thus conclude that there cannot exist a cycle of profitable mergers and demergers. Lemma 5 therefore implies that there exists a stable hypergraph. □

C.1.10 Proof of Proposition 13

Proof. First consider the effect of the demerger on market prices. For a given market j consider the set of capabilities $\{\theta_{hj}\}_h$ such that $\theta_{hj} > 0$. By Proposition 6 this uniquely determines the market price in market j. This set of capabilities is the same before and after the demerger of firm l into i and k for all markets j. Hence, the demerger does not change the market equilibrium in any market. Therefore, without counting changes in fixed costs, firm l's profits are equal to the combined profits of firms i and k. However, by the convexity of κ the total fixed costs following the demerger decrease: $\kappa(|F_l|) > \kappa(|F_i|) + \kappa(|F_k|)$ and the demerger of l into firms i and k increases profits, taking fixed costs into account. □

C.2 Calculations supporting example 3

Any two firms $F_{si}, F_{ti} \in H_1$ do not merge because they lose revenue (profits without counting fixed cost) when ku is large (the business stealing effect is large), and do not save fixed cost ($\kappa(|F_{si}|) = \kappa(|F_{ti}| = 0)$. Any two firms $F_{si}, F_{tj} \in H_1$ ($i \neq j$) do not merge because the merger loses profits for large u (the business stealing effect is large, the synergy is small and the increased fixed costs from merger is large). So H_1 is stable.

To prove H_2 is stable, we just need to check that any firm does not want to demerge. Suppose that a firm of H_2 demerges to two smaller firms with u_1 and $u_2 = u - u_1$ number of attributes respectively. Then the profit of the two demerged firms without counting fixed costs (denoted by R) can be calculated:

$$\frac{R}{\alpha^2} = \left(\frac{1 + (k-1)\frac{1}{2u} + \frac{1}{u_1+u} + \frac{1}{u_2+u}}{k+2} - \frac{1}{u_1+u} \right)^2 + \left(\frac{1 + (k-1)\frac{1}{2u} + \frac{1}{u_1+u} + \frac{1}{u_2+u}}{k+2} - \frac{1}{u_2+u} \right)^2 .$$

Then we can estimate:

$$\frac{R}{\alpha^2} \leq \left(\frac{2 + (k-1)\frac{1}{u} - k\left[\frac{1}{u_1+u} + \frac{1}{u_2+u}\right]}{k+2} \right)^2 \leq \left(\frac{2 - (\frac{k}{3}+1)\frac{1}{u}}{k+2} \right)^2 .$$

Further, for $k \geq 3u$,

$$R \leq \left(\frac{2 - (\frac{k}{3}+1)\frac{1}{u}}{k+2} \right)^2 a^2 < \left(\frac{1 - \frac{1}{2u}}{k+1} \right)^2 \alpha^2.$$

This implies that without counting fixed costs, the aggregated profit of demerged firms is less than the firm's profit pre demerger. When e is close to 1, the demerger cannot save much fixed cost. Therefore, the net effect of the demerger is negative and it is not profitable for the firm to demerge. This proves that H_2 is stable.