ESSAYS ON THE ECONOMICS OF SPONSORED RESEARCH

Thesis by

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ABSTRACT

Essays on the Economics of Sponsored Research concerns the regulation of a firm conducting R&D under contract with a sponsor. Chapter 1 surveys the rich economics and policy literature concerned with R&D contracting. Prior to Balbien and Wilde (1980), the chief weakness of the literature was in the analysis of dynamic contracting incentives and the implications of information assymetry between a researcher and sponsor.

Chapter 2 is an empirical essay describing R&D contracting by the Department of Defense. Based on a sample of DOD R&D contract data from 1979, several hypothesis are tested with multivariate statistics. These hypotheses concern the choice of generic contract type by sponsors, the effect of competition, and the performance of various contractual forms.

The third chapter analyzes a dynamic model of incrementally funded research which is descriptive in nature and not subject to direct econometric estimation. Nevertheless, it provides valuable insight into a firm's behavior in revealing research progress to a sponsor,

through targets set over a sequence of research periods.

Chapter 4's essay focuses on the influence of various types of research assistance on a firm's internal investment in a number of private research projects.

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CHAPTER 1

A LITERATURE SURVEY

Central themes of economic research on R&D have been 1) the relationships between growth in productivity, innovation, and aggregate R&D expenditures, and 2) the extent to which market failures bring about underinvestment in research. During the 1960's, an impressive body of empirical research indicated that technical change associated with R&D accounted for much of the observed increase in gross output per worker during the 20th century. More recently, the work of Grilliches (1980) indicates that a firm's rate of productivity increase is directly related to the amount it has spent on R&D.

With a positive relationship between productivity growth and R&D established at both the micro and macro level, the focus of ecnomists has shifted towards addressing the question of whether a market economy will under-invest or over-invest in R&D relative to some theoretical optimum. Turning to the adequacy of the nation's investment in research and development, Mansfield wrote in 1970:

> "There is too little evidence to support a very confident judgment as to whether or not we are underinvesting in certain types of research and development. However, practically all of the studies addressed to this question seem to conclude with varying degrees of confidence that we may be underinvesting in particular types of R&D in the civilian sector of the economy where estimated marginal returns seem very high." (p.33)

For example, one study of 37 commercial innovations found that the estimated median social rate of return on investment in R&D was 70 percent and the median private rate was 25 percent. (Mansfield,1977) Even the private rate of return is high, suggesting underinvestment in R&D by industry.

Four sources of market failures associated with R&D are summarized by Noll (1975) in a state of the art survey. These sources include:

1) Indivisibility: the minimum efficient scale of R&D operations can be sufficiently large that the market for a particular class of ideas cannot be competitive;

2) Inappropriability: innovators are unable to capture the full economic value made possible by their innovations;

3) Indirect failures: if a good must be produced outside a competitive market, the institutions created to bring this about may lead to inefficiencies in the advancement of knowledge with respect to production and distribution of the good;

4) Uncertainty: the economic uses of the technical ideas that emanate from R&D activities are not

> known in advance so that the search for innovation is a gamble. (Noll, p. 3)

Although recognizing the sources of market failures, Noll cautions that existing public policies (e.g., tax laws and research sponsorship) do on balance promote R&D relative to other investments, at least in industries not subject to public utility regulation. Thus, even if free markets under-invest in research, this effect may be offset by other factors including current levels of government intervention.

The reasons often cited for government sponsorship of research and development are closely related to the market failure hypothesis. In the defense area, Scherer and Peck (1962) observed that after World War I, industry, with few exceptions, did not transform research concepts into workable hardware until demand for a specific weapon system arose. They attributed this industry anomaly to an inverse relationship between the costs of a military research and development effort and the certainty with which R&D results could be foreseen and capitalized. Thus, for private firms it was usually too costly to develop a highly specialized weapon, the demand for which might never be realized.

Since private risk capital has seldom been invested in weapons research beyond initial feasibility demonstrations, the Department of Defense (DOD) funds development, testing, and evaluation of new weapon systems through R&D contracts with private firms. (DOD also funds basic and applied research conducted at government owned research laboratories.) In the case of private contractors, initial contract terms, e.g., weapon specifications, often serve as a baseline for a dynamic quasi-regulatory relationship between a research firm and its agency sponsor.

Once a prototype or test model is considered sufficiently developed, production designs satisfying DOD's specifications and receiving congressional approval are purchased, often by negotiation, but sometimes through competitive biddings. For example, DOD contract data analyzed by the Comptroller General (1979) shows that less than 27 percent of the nearly 50 billion dollars in contract awards in 1977 were based on price competition. Contracts values at 31 billion dollars were awarded to sole source contractors.

Of course, federal sponsorship of research and development is not limited to development of military hardware. For example, DOD funds a significant proportion of all industrial and academic research. These expenditures are directed at improving the nation's technological base by adding to human capital and providing advances in basic

TABLE 1

CONDUCT OF R&D BY MAJOR DEPARTMENTS AND AGENCIES*
(In millions of dollars)

Department or Agency	1979 Actual	1980 Estimate	1981 Estimate
Defense-Miltary functions	11,454	13,253	15,169
National Aeronautics and Space Administration	4,064	4,858	5.277
Energy	4,413	4,871	5,088
Health and Human Services	3,068	3,428	3,661
(National Institutes of Health)	(2,637)	(2,895)	(3,100)
National Science Foundation	775	853	964
Agriculture	611	652	670
Environmental Protection Agency	337	384	413
Interior	393	396	413
Labor	102	277	374
Transportation	340	332	348
Commerce	305	347	394
Nuclear Regulatory Commission	141	182	204
Education	128	151	154
Veterans Administration	117	126	135
All other[1]	331	368	456
Total, outlays	26,578	30,477	33,717

[1]Includes the Departments of Housing and Urban Development, Justice, Treasury and State; the Tennessee Valley Authority, the Agency for International Development, the Institute for Science and Technological Cooperation, the Smithsonian Institution, the Corps of Engineers, the Federal Emergency Management Agency, the Arms Control and Disarmament Agency, the Consumer Product Safety Commission, the Federal Communications Commission, the Advisory Commission on Intergovernmental Relations, the Federal Trade Commission, the Library of Congress, the U.S. Office of Personnel Management, and funds appropriated to the President.

*Source: "Small Business Guide to Federal R&D," National Science Foundation (August 1980):34.

and applied research that maintains America's technological edge over potential adversaries. Other agencies also invest in industrial, environmental and medical research. (See Table 1) For example, a General Accounting Office (GAO) report (1979) found that commercially directed R&D programs, funded by the Departments of Energy, Transportation and Health, grew in importance as a proportion of federal spending. The goal of these programs is the adoption and marketing by private firms of goods and services embodying the results of federally funded R&D.

According to GAO, the government invests in commercially directed R&D because:

> "social returns are believed to be positive, but; perceived private returns are insufficient to justify the investments, and; uncertainty regarding commercial success makes todays payoff too low to justify the investment". (GAO, p. ii)

Although the free market oriented R&D policies of the Reagan Administration may reverse the trend towards direct federal funding of commercially directed R&D, increased military expenditures are likely to translate into a steady growth in total federal spending on contract research.

Four literatures are of central importance in formulating a theory of contract research. The first two, concerned with theoritical aspects of appropriability and uncertainty, require extension to the environment of sponsored research. A third literature which addresses incentives in agency affairs, is vital for integrating R&D models with an economic theory of contracts. A final body of research consists of policy oriented studies of R&D contracting by DOD and other federal agencies. These empirical case studies provide a bridge between economic theory and reality.

In research contracting, the opportunity to sell information is superimposed upon the production of innovation. Therefore, appropriability as an incentive for innovation takes on a new dimension. With respect to the effect of competition on appropriability and a firm's incentive to innovate, two contradictory propositions relevant to internally funded research projects are discussed by Rogerson (1980).

In a contracting environment, the first proposition can be stated as follows: assuming that research progress is unobservable to a sponsor, (except through a firm's research reports) and a firm has market power, it should be more able to extract rents from the revelation or sale of research progress. Therefore, a less competitive contracting environment encourages a contractor to pursue innovation because its research advances are more appropriable.

However once a single firm possesses an innovation, a sponsor may pay more for the revelation of research progress than if several firms have knowledge of an innovation. Thus, an incentive to innovate derived from future market power may lead, expost to a slower dissemination of information.

On the other hand, a competing proposition is that a monopolist might face less of an incentive to both innovate and reveal research progress than if a rival firm is holding or pursuing a research contract with the sponsor. As a result, innovation may be maximized at an intermediate level of competition, where some appropriability exists, but rivalry limits a firm's ability to fully exploit its research position.

When a contractor's research progress is observable to a sponsor, the appropriability of a new innovation can depend on the terms governing ownership rights to new knowledge. Historically, almost all federally sponsored research has been in the public domain (excluding defense R&D).[1] However,the trend in recent legislation and executive branch policy is towards granting the United States a free license but otherwise awarding inventors or their employers exclusive rights to the results of federally sponsored research.[2] The hope is that a more generous patent policy will encourage innovation by contractors and commercial application of government owned patents. This position is supported by presidential science advisor Simon Ramo (Smith, 1980), who notes that in return for surrendering public rights the government would receive half the revenue from an innovation through the corporate income tax.

Despite a recent shift in government policy toward a more generous patent policy, there is considerable controversy in the economics literature with regard to the value of exclusive patent rights as an incentive for innovation in research contracting.

In theory, to the extent that a firm's disclosure of proprietary and other information to a sponsor could compromise its technical position or result in the sponsor asserting property rights over commmercially useful innovations, private firms are less likely to participate in sponsored research programs.[3]

However, a firm that holds a research contract has an incentive to limit observability of its research progress and thereby maximize its control over dissemination of information. This is the essence of a firm's sales strategy which has been neglected in much of the literature.

For example, a researcher may choose to temporarily withhold knowledge in order to sign a new contract with the sponsor when the current contract expires. (Balbien and Wilde 1980) Alternatively, it may be in the firm's long term interest to completely capitalize commercially useful information developed at government expense by treating new knowledge as a trade secret. A decision to capitalize will depend on 1) the extent to which an innovation can be reverse engineered and 2) the likelihood that other firms will claim the same innovation as their own.

One indication that capitalization occurs can be gleaned from a standard defense in patent encroachment suits against the United States. The federal government often argues that the relevant innovation was developed under a government grant or contract, but was not reported to the sponsor as required by federal law.[4] In such a case, the contractor forfeits all rights.[5]

Other evidence is also suggestive of capitalization. Historically, it is reported by Utterback and Murrary (1977) that far fewer patents have resulted from defense or space supported R&D than from commercially funded R&D, and a far smaller proportion of those which have resulted from defense support have had any commercial application. This suggests that either government sponsored research is less productive from a commercial standpoint, perhaps because of inefficiency and adverse selection of projects, or alternatively, firms participating in sponsored research are more likely to prefer capitalization, i.e., industrial secrecy, to a public patent. To the extent that this latter argument is true, a more generous patent and licensing policy may have a minimal impact on perceived levels of innovation. Moreover, policy makers may be underestimating the benefits of existing intervention.

Critics of exclusive patents, for example, Rubenstein (1980), add that in such high technology fields as electronics, the speed of entry, proprietory know-how, ongoing R&D, and other factors may mean much more than the possession of a patent. Rubenstein concludes that it is not clear that manipulation of the patent system as such will make a tremendous impact on the rate of innovation or the adoption and utilization of innovations from federally sponsored research. Noll (1975) notes that the patent system is costly to operate and may encourage wasteful effort to invent around an innovation.

A study by Arthur D. Little (1963) found that in electronics, the two to five year delay between invention and receipt of patent is normal, and by the time the patent is received, the cycle for a product is almost over. Utterback and Murray (1977) add that in electronics the mobility of engineers and scientists has led to technology being rapidly shared among firms and to rapid diffusion of innovations. These factors tend to reduce the value of exclusive patents as an incentive for innovation in both contract and internally funded research.

The second major literature relevant to a theory of R&D contracting is concerned with the impact of uncertainty on the behavior of firms and competitive markets. It is the nature of research work that the amount of progress that can be achieved over some contractual period is uncertain. Nevertheless, economic theory provides the insight that uncertainty should not preclude firms from pursuing risky endeavors provided they have access to insurance.[6]

Unfortunately, there are two reasons one would not expect a firm to be able to privately insure its internal R&D activities, or alternatively, insure its contractual research obligations to a sponsor. Difficulties arise because of both moral hazards and an unwillingness on the part of a researcher to share all information with a potential insurer. A contractor whose potential gains and losses from a research project were fully insured might not have an incentive to succeed. Secondly, potential insurers are likely to receive incomplete information on the research project, because otherwise they might have an opportunity to sell knowledge to others. As a result, the insurer's subjective uncertainty may dominate his lesser aversion to risk and result in no insurance being provided by the marketplace.

The likely failure of insurance markets to develop in the research area has two important implications. First, less R&D (both contractual and internal) than is optimal may occur to the extent that owners of firms are risk averse and cannot self-insure through diversification, nor shift financial risk to potential sponsors. Secondly, a large firm participating in several research programs may conduct more research and accept greater financial risk on any one project, ceteris paribus, than several small firms. This assumes that organizational diseconomies, associated with bureaucratic structure, do not offset risk taking encouraged by diversification.[7]

The Principal Agent Literature (Harris and Raviv (1976), Shavell (1979) and others) provides an analytical framework for studying the impact of risk and appropriability in simple contractual relationships between a research firm and sponsor. Most models assume that both parties to an agreement can observe the output of a single research or production period. While this is a strong assumption for basic and applied research, observability may be a useful assumption for answering certain questions about highly visible hardware development programs. In addition, it is assumed that the firm can act to reduce the cost of making research progress, but this effort may not be monitorable by the sponsor. For example, the firm could assign its brightest or alternatively its least imaginative engineers and scientists to a project and pursue low risk research strategies that offer marginal payoffs to the sponsor. Another integral part of the model is that the final cost of achieving research goals is uncertain. Thus, a contractual commitment by a researcher to achieve a given level of progress for a fixed fee subjects a firm's future profits to considerable risk, e.g., larger than expected research costs and possible damage for breach.

Nevertheless, one of Shavell's (1979) major results implies that if the researcher is risk neutral, a contract that pays a fixed fee conditional on a firm's success is optimal, providing both a maximum incentive for cost control and an optimal allocation of risk.[8]

When an agent is risk averse, but must accept fixed price contract terms in order to obtain the contract, a large premium will be demanded in the fixed fee to compensate for the uncertainty in the firm's research costs. As a result, if the sponsor is the less risk averse of the two parties, both could be made better off if the sponsor insured the firm against cost growth by reimbursing some R&D costs, while paying a lower fixed fee that is conditional upon successful completion of the contract. Of course, if the principal insures the researcher against all financial risk (e.g., by refunding all costs and paying a fixed fee independent of the outcome of its research), no incentive is provided for cost control and the accomplishment of research goals.

In summary, there are two important results from the static Principal Agent Literature. First there is a tradeoff between incentive maintenance and optimal risk sharing. A second best contract must stike a balance between the two. The emphasis of one factor, risk sharing, at the expense of another, incentives for cost control, carries with it the penalty of sharply higher contract price per unit of research progress. Secondly, if the owner of a firm is risk averse, contract terms should require that a firm's profits depend to some extent on the outcome of its research, but the firm never bears all the risk.

Other authors have also contributed to the Principal

Agent Literature. In a revision of their earlier paper, Harris and Raviv (1978) recognized that even imperfect information about an agent's effort can make both parties to a contract better off. This may explain why cost accounting standards and audits of a firm's research costs are often incorporated into a negotiated contract price as crude indicators of cost control. However, one problem facing government research sponsors is that monitoring research cost is in itself costly. Moreover, accounting audits tend to be out of phase with the tracking of a firm's research progress. This is because of division of labor within the contracting organization and imperfect observation of research output and research costs as a project unfolds.[9]

Harris and Raviv also outlined the usefulness of dichotomous fee arrangements, (i.e., those which penalize the agent discontinuously when effort lies in an unacceptable region). Cummins (1973) obtained similar results when he showed that a modified contingent sharing ratio in providing both desirable risk sharing and marginal incentives for cost control. Finally, Lewis (1980) showed that lump sum bonus and penalty payments contingent on performance may be preferable to contracts in which an agent's reward depends continuously on observed performance.

This suggests that a research sponsor's threat to terminate a contract if observed performance falls below some threshhold may be a useful control strategy.

Besides prescribing desirable contract terms for sponsored research, the Principal Agent Literature provides one theory to explain the observed behavior of firms holding government R&D contracts. As predicted in simple agency models, case studies of government research contracting indicate that cost reimbursement contracts let by the Department of Defense (DOD) and the National Aeronautic and Space Administration (NASA) have introduced incentive problems that may be reflected in observed cost growth. For example, in one sample of twelve aircraft and missile development projects sponsored by DOD (5, p. 429) the average ratio of actual to estimated cost was 3.2, and the average ratio of actual to estimated time was 1.4. Likewise, an unpublished report by the National Aeronautics and Space Administration (1981) examining a group of seventeen representative projects found that the average ratio of actual cost to planning estimates was 2.0 during the 1960's, 1.41 from 1970-74, and 1.39 between 1975-79.

Interestingly, cost estimates for civilian R&D planning have also proven inaccurate, especially when projects attempt large technical advances. They may indicate that incentive problems often associated with contracting by government agencies apply to some extent to privately funded research projects. Alternatively, all researchers may be prone towards underestimating R&D costs. For example, in a study involving a proprietary drug company, the average ratio of actual to estimated development costs was 2.1 and the average ratio of actual to estimated development time was 2.9.[10]

Most studies of cost growth have concluded that high average cost ratios for R&D projects undertaken by government agencies result from a combination of three factors: real "overrun" associated with inefficiency, biased estimation of initial project costs (because of technical uncertainty and adverse selection of optimists), and improper use of competitive bidding.[11]

Cost growth associated with overruns and schedule slippages in achieving research and development objectives have been encouraged by the attractiveness of the cash flow aspect of cost reimbursement contracts. Many policy makers have failed to recognize that "economic profits" may be more akin to accounting cash flow than accounting definitions of profit.

For example, it is widely recognized in industry and academics that when a government agency reimburses a company for overhead associated with a research cost overrun, a contractor is encouraged to perform additional research in other areas, write new research proposals, and maintain excess capacity in its technical staff.[12] These investment activities tend to enhance a firm's future research opportunities and chances of survival in the unstable business environment that characterizes the aerospace and defense industry. Perhaps, cost growth in one research project may also benefit firms in future negotiations with the same sponsor by empirically justifying a higher future contract price for similar work.

However, Scherer (1971) notes that both government and industry often find cost plus administrative contracting beneficial. Risk averse firms are insured against potential financial losses associated with unexpected increases in research cost. In addition, the government sponsor is spared the political heat associated with rewarding a clever firm large accounting profits.[13]

Nevertheless, in the early 1960's Defense Secretary McNamara rebelled against what was perceived as wasteful and politically expedient cost plus fixed fee (CPFF) contracting. As a result, DOD revised its procurement regulations to encourage the use of so-called "incentive" contracts. The government's objectives in using this new contract instrument was to improve efficiency and raise industry profitability. The government hoped to redistribute the financial risk of a project through cost (profit) sharing when actual costs exceeded (were less than) targeted costs, and raise targeted profit rates to reward firms for risk taking. (Scherer, 1971, p. 531)

Incentive contracting can be described by the following simple equation: Realized Profit = Target Profit + R (Target Cost - Actual Cost), where R is the sharing ratio. In a CPFF contract, R=O. The government benefits from cost underruns, but pays all cost overruns. In a fixed price contract, R=1, so that the firm has maximum incentive for cost control. Incentive contracts represent a compromise between CPFF and fixed price contract with R often taking on values between .2 and .35. (Scherer, 1971, p. 528)

McNamara's policy did reduce the value of CPFF contracts from thirty-seven percent of military prime contracts in 1961 to less than ten percent in 1965. In addition, incentive contracts, predominantly those with a linear sharing of cost overruns and underruns, rose from fourteen percent to thirty percent of total prime contracts' value, while the value of firm fixed price contracts rose to fifty percent, up from thirty-two percent. (Merow, et al., 1976)

However, most researchers have concluded that incentive contracting during the 1960's had only marginal impact on the growth in total contract price required to achieve government R&D objectives.[14] Nor did it restore defense industry profitability. (Scherer, 1971, p. 582)

One explanation in the literature for the inability of incentive contracts to control cost growth is that policy makers did not recognize the motivation of firms and government contract officers to modify contract terms. At DOD, it is reported that costs, fees and R&D objectives were being frequently altered in renegotiations which often involved little competition. Thus, as R&D costs accumulated in a project, it is hypothesized that technical events and an absence of competition encouraged contract modifications. These contract changes often shifted financial risk away from a firm and towards the sponsor, so as to reflect new knowledge, unforeseen technical problems, premature optimism, and changes in contract objectives. However, these hypotheses concerning the factors influencing modifications have not been empirically tested (Merow, et al., 1976, pp. 153-166)

Although risk sharing ratios between 0 and 1 seemed to have marginal influence on cost growth, Parker (1971), did find a strong relationship between cost growth and cost type, as opposed to fixed price contracts. However, this correlation was attributed to an allocation of projects to contract types according to prior expectations of uncertainty, rather than incentives for cost control inherent to fixed price terms. Perhaps fixed price contract terms also provide the government with a slight advantage in future renegotiations.

Scherer (1971) takes the macro viewpoint that continuous renegotiation of incentive contract terms during the 1960's was an indirect symptom of underlying defense and aerospace industry excess capacity rather than an exogenous cause of cost growth. According to this theory, intensive competition for programs forced firms to accept increasingly optimistic and unrealistic cost and technical targets. As a result, most incentive contracts ended in cost overruns accompanied by a failure to achieve initial specifications. (Scherer, 1971, p. 532)

It should also be noted that McNamara's incentive contracting policies encouraged firms and government contract officers to negotiate fixed price contracts and/or incentive provisions which presuppose well-defined sets of baseline specifications. As a result, when uncertainty dictated more general specifications, costly renegotiation of initial contract terms were required, often in a non-competitive environment.

The problems posed by after-the-fact modifications are not limited to R&D contracts let by DOD. A review of the Atomic Energy Commission's (AEC) prototype-reactor-development program confirmed that contract modifications motivated by unforeseen technical problems and industry persuasiveness enabled firms to circumvent risk sharing imposed on the researcher by initial contract terms. (Merow, et al., 1976. pp. 61-66)

It is reported that where the AEC contracted for construction of reactors, four out of five projects suffered major overruns in reactor costs and substantial project delays, regardless of contract type. (Merlow, et al., 1976, p. 64) Three out of five projects included cost-reimbursement-type contracts, and two projects involved fixed price contracts. In all cases, initial contract terms were modified to reduce losses to the firms.

Cummins (1973), in one of the most sophisticated econometric studies of cost overruns, investigated the thesis that "overruns" are not necessarily inconsistent with efficiency, i.e., accomplishing contract objectives at the lowest possible cost.

Cummins analyzed 118 multi-million dollar army contracts completed between 1965 and 1970. His theoretical model assumes that managers of firms have utility functions favoring both high-current profits and high-final project costs, which may enhance a firm's future profitability. Cummins' analysis leads to a simultaneous equation system between overrun and sharing ratio. First, initial risk sharing and then final overrun are determined by expected cost minimization by the government, and a two-stage utility maximization by the firm. In contrast, contract modifications are assumed to be the result of random, exogenous events. Cummins' major result is that the size of cost overruns and the target profit rate on a contract are not directly relevant to the objectives of the firm or government. One can vary both the percentage fee and target cost for a project, which changes the expected cost overrun, without changing the firm's profit or the final cost of the project to the government.

One major problem in Cummins' thesis is the assumption that modifications are exogenous to the degree of risk sharing imposed on a firm. In addition, there is some conflict in the literature in regard to Cummins' assumption that the profit fee on an incentive contract is independent of contractual risk assumed by the firm. This is counter to the assumptions made by Scherer. Nevertheless, Cummins' two-stage model was a major step towards a dynamic theory of R&D contracting. Like Scherer & Peck (1962), Cummons recognized that firms do not always minimize costs in pursuit of maximum profits on a given contract. He incorporated long-term profits implicitly into a model of contract selection and firm behavior.

FOOTNOTES - CHAPTER 1

1. However, DOD retains royalty free use of these inventions for military purposes. (See Title 10 Armed Services, U.S. Code 2273.) In addition, there are provisions for government recoupment of the development cost of commercially valuable inventions that are "reduced to practice" under a DOD contract. (SEE ASPR 1-2400.)

2. For example, H.R. 6933 (Entitled: "To Amend The Patent and Trademark Laws") and submitted March 1980, was in the direction of a more generous patent policy.

3. For a discussion of this issue from industry's perspective, see the following Aerospace Industries Association publications: "Risk Elements In Government Contracting," mimeo, (1970), and also "Proprietary Data: An Essential Asset," in A Report To The Commission On Government Procurement (1970).

4. Analysis was based on interviews with the NASA patent attorney Jon Trevansky (November 1980) and Navy Patent Officer Al Kwitneski (February 1981). Also see U.S.

Court of Claims S76, Reporters Against U.S.

5. See Vorster clause, (30 USC 666), 15 USC 1395 (c), (1964 Supp V).

6. For an excellent review of the literature see Burness, S., R. Cummings, and J. Quirk. "Speculative Behavior and the Operation of Competitive Markets Under Uncertainty: A Survey Article." Staff Paper 80-11, Montana State University (1980).

7. For example, B. Klein argues that when well established firms become large-scale, well organized bureaucracies, inadequate feedback from a "hidden foot" may reduce risk taking. See Klein, B.H. "The U.S. Productivity Slowdown and Its Relation to the Inflation Problem," Social Science Working Paper 286, California Institute of Technology (1979).

8. The fact that government R and D contractors are strongly opposed to "fixed price" contracts for a development effort, provides evidence that the owners

or managers of these firms are very risk averse. For an industry perspective concerning contract selection, see "Types of Contracts and Their Selection," mimeo, Aerospace Industries Association (July 1971).

9. For example, DOD administrative instructions for contract officers note that:

> ...Instead of working with estimates of future activity, the contracting team is soon working with imperfectly measured actual costs to date plus estimates to complete. The actuals are made more difficult to work with intelligently by the imprecision of cost accounting and the difficulty of measuring ongoing work in progress.

Taken from ASPM No. 1, (September 1975).

10. Cited in Mansfield, E. "How Economists See R and D." Harvard Business Review (November-December 1981).

11. Reference 18 suggests that an incompatibility between competitive bidding (as a means of selecting a contractor and determining price) and cost

reimbursement contracting has contributed to observed cost growth in NASA sponsored projects. A. Herman has empirically shown that underestimates of R and D cost would be increasingly likely for more complex and/or technologically advanced systems. "Choice Among Strategies For System Acquisition," mimeo, The Rand Corporation (March 1972). In reference 19, Balbien observed that adverse selection of optimists within a R and D organization may contribute to biased cost estimation.

12. Confidential interviews were conducted with contract officers who had worked both as R and D sponsors and industry negotiators. Their statements about incentives for cost growth were consistent with early arguments made by Scherer and Peck in Reference 5, and later Cummins in Reference 16 and 23.

13. A state of the art discussion appears in Reference 20. One aspect of cost reimbursement contracting that appears to have been missed in the literature is that it may enable firms to receive a higher after tax price for their R and D services while DOD pays a lower

before tax price. Thus, a federal agency may be able to raid the treasury, in part through the tax system rather than appropriations.

14. For example, R. Perry concludes that despite the contracting reforms of the 1960's, typical programs continued to exhibit an average cost growth of about 40 percent, a schedule slip of 15 percent, and a final system performance that would deviate by 30 or 40 percent from initial specifications. Perry, R., G. K. Smith, A. Harman, S. Henrichsen "System Acquisition Strategies," The Rand Corporation (1971):v.

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CHAPTER 2

AN EMPIRICAL STUDY OF R&D CONTRACTING BY THE DEPARTMENT OF DEFENSE

I. INTRODUCTION

Chapter 2 describes the results of an empirical study of R&D contracting at the Department of Defense. It focuses on the microeconomic factors that influence a sponsor and a firm's choice of contract type, and the magnitude of both initial and modified contract price.

There are two unique aspects of this study. First, large quantities of information concerning 8,000 DOD R&D contracts, active during fiscal year 1979, are extracted directly from a government data system used to monitor the contracting activities of all federal agencies. Second, multivariate statistics are used to analyze new contracts and contract changes over a relatively short period of time, twelve to twenty-four months.

II. A THEORETIC FRAMEWORK

Definitions of R&D

Basic and applied research involves the pursuit of new knowledge. Contracts usually call for experimental work directed at tangible goals. At the end of a research period a contractor is often required to provide an end report or written document describing its research findings. In contrast, development contracts may require that a contractor design and develop a new product or process, or demonstrate progress towards cost reduction. Contract terms may also require delivery of a prototype or test model satisfying a set of specifications.

An additional distinction between research and product development is that in the former, the researcher's subjective probability of success at some endeavor can be represented by a random draw from a distribution over possible states of nature, the outcome of which is less sensitive to a researcher's perseverence. As a result, moral hazard is less problematic so that risk sharing, at least in the context of a researcher's contractual obligations to achieve a specified level of progress, would be conterproductive. Therefore, one would expect to observe few incentive and fixed price research contracts structured around a researcher's commitment to achieve research progress.

However, in development, uncertainty tends to be focused on the costs required to achieve more or less feasible technical goals, i.e., there is greater uncertainty that a process or product will eventually work, but less is known about research costs. Furthermore, in development the probability that a researcher can achieve planned milestones during a research period is more sensitive to the researcher's effort. As a result, in development contracts, the classic tradeoff between optimal risk sharing and the maintenance of incentives should play a greater role in the selection of initial contract terms than in basic research.

Intertemporal Incentives

While clear distinctions are often drawn between Research and Development, government purchases of new products and services for the public sector, or as an incentive to industry, sometimes link an early research phase with product development, and production. Support of basic and applied research through grants and contracts provides a foundation for subsequent contracts requiring conceptual, preliminary, and then engineering design followed by the building of a prototype. A successful prototype or demonstration of cost reduction may give a firm an opportunity to obtain more profitable production contracts from a government agency or enter a new commercial market. The former is particularly true for DOD, which is the first and often the only customer for a new weapon technology that results from its sponsored research.

The programmatic context of many R&D contracts and the future business opportunities they provide overshadow contractual incentives to perform to the satisfaction of a research sponsor. Moreover these intertemporal incentives exist independently of any particular contract form.

Yet, several factors may limit the effectiveness of intertemporal contract incentives. Government appropriations are on an annual basis so that the continuity of federally sponsored R&D programs is an important source of uncertainty to a contractor.[1] Furthermore several years can pass between research, development, and application of an innovation.[2] Also, the firm that performs the initial research work may not be the same firm that builds a prototype. As a result, high risk-adjusted discount rates can reduce the importance of a firm's future rewards. Finally, a sponsor's difficulty in measuring a firm's actual research progress over relatively short contract periods, against that of rival firms, can result in a research sponsor unknowingly rewarding the firm that is the most optimistic, rather than the most innovative.[3]

While an ongoing research program, perhaps structured around multi-year contracting might address the problems outlined above, a firm's opportunity to develop a sole source position may cause problems. It is often argued in the literature that a firm seeking to become a new supplier or sole source may risk a short term loss by offering an initial contract price that is below the expected cost of reaching a sponsor's research objectives. This phenomenon is called a "buyin."[4]

A firm has an incentive to bid low or set extremely challenging research goals in order to win a technical or price competition, or encourage government sponsorship of a marginal research project. A company hopes that its current contract will be extended, modified, or result in an entirely new contract negotiated in a more favorable environment.

In summary, there are economic incentives for performance in R&D contracts that are associated with future business opportunities, but external to the specific terms of a firm's current contract. However, these economic incentives are less effective when there are long time periods between research inputs and observable product innovation, there is uncertainty surrounding future contracts, and a firm has an opportunity to develop a monopoly position in an R&D program.

Contract Award

Although intertemporal incentives may on balance provide a positive impact, government R&D sponsors have historically negotiated contract terms that focus on a firm's near term performance incentives. At DOD the contracting process often begins with the selection of a generic contract type followed by a solicitation of potential sources. Then the selection of a contractor, its research obligations, (e.g., to design a prototype, or supply a level of R&D effort), and the initial contract price it is to receive are determined, either by a competition between two or more firms, or alternatively through bilateral negotiations with a sponsor.

Competitive theory suggests that when a technical or price competition is used to select among similar R&D proposals, the attractiveness of an R&D contract to a firm is likely to be reduced relative to a bilaterally negotiated contract. This is because external competition for initial contract awards improves the relative negotiating position of the sponsor. As a result, the winning firm may be forced to accept risk sharing and price terms that make it indifferent between success and failure in obtaining a contract.

However, once a firm is awarded a contract, it enters a submarket made up of contractors conducting R&D in similar areas for the same sponsor. In a submarket firms who already have R and D contracts compete for extensions or renewals of those contracts. In addition contractors working for a budget constrained sponsor may compete for favorable changes in contract terms necessitated by new information not available when a particular contract was negotiated.

The existence of submarkets raises important research issues. For example, what is the possible impact of internal competition on a sponsor's initial selection of contract type? Perhaps the positive incentive effects of internal competition may provide contracting parties with an opportunity to avoid the use of explicit risk sharing clauses that are costly to negotiate and less acceptable to a risk averse firm. A sponsor who can observe a firm's initial research position and future research effort, only imperfectly and at considerable cost, is compelled to structure the contract so as to explicitly shift financial risk towards the researcher. Incentives for performance are thereby maintained. However, risk sharing contracts may have the effect of discouraging risky, but profitable research by the contractor, and encouraging modification of contract terms in the post award period. By substituting internal competition for explicit risk sharing, a sponsor can absorb greater financial risk (and thereby encourage a firm to conduct research), while competition for extensions, renewals, and contract changes maintains incentives for an efficient research effort.

Two alternative hypotheses are that the government's selection of contract type may be more or less institutionalized by a set of standard operating procedures, e.g., the Armed Services Procurement Regulations (ASPR), so that the opportunity to substitute the invisible hand for legal instruments is not acted upon. Secondly, different research projects may not be perceived as close enough substitutes, relative to the sponsor's objectives, to make budgetary competition an effective means of shifting risk.

Contract Modifications

Government procurement regulations define a contract modification as:

> "any unilateral or bilateral written alteration in the specifications, delivery point, rate of delivery, contract period, price, quantity or other provision of an existing contract that obligates or deobligates funds." (ASPM No.1:9A1)

While this definition covers a broad array of contract changes, there is a common element. Modifications involve bilateral negotiations directed at revising initial contract terms, including price, in light of new information revealed by one or both parties since the contract was awarded. These renegotiations occur within a submarket, where other contractors may likewise be engaged in renegotiations with the sponsor, for renewals, extensions, and changes in contract terms.

Analogous to its impact on a new contract, internal competition should provide a sponsor with more information and possible research alternatives. As a result, each contractor's project may become more expendable. These factors may moderate price changes awarded in renegotiations with an individual firm. An alternative possibility is that submarkets may lack depth so that each contract is administered in more or less of a vacuum. This might occur, again, because the sponsor does not view its research projects as viable substitutes or because of fragmented power within a sponsoring agency.

III. DATA

In November 1969, Congress created a Commission on

Government Procurement. Its purpose was to study and recommend reforms that would promote efficiency, economy, and effectiveness in contracting by the federal government.[5]

One of the commission's major recommendations was the establishment of a central office of federal procurement policy that would implement a standardized system of collection and dissemination of statistics on contracting. As a result, in 1978, a Federal Procurement Date System (FPDS) was created by the Office of Management and Budget.[6]

The FPDS was originally designed and operated by the Department of Defense. It reports on contracting by all executive agencies of the federal government entering into contracts with funds appropriated by Congress. Nevertheless, several types of contractual agreements are excluded from the FPDS and should be mentioned. Exclusions include:

1. Grants and subsidies, etc.;
2. Nonappropriated fund activities, e.g., the Import Export Bank of the United States;
3. Transactions between different federal agencies;
4. Contracts let by intergovernmental organizations;
5. The research work of international bodies in which the United States is a participant, e.g., the the United Nations;
6. Subcontracts let by private corporations which hold federal prime contracts. 7

The unit of analysis in the FPDS is called a contract action. There are two general categories: the award of a new contract and a modification of an existing contract.

A government contracting office that awards a new contract or negotiates a modification for more than ten thousand dollars is required to file a Contract Action Report which is the raw input data for the FPDS. These reports provide a standardized format for recording the history of a contract.

Each Contract Action Report contains twenty-seven questions or data entries. Some of the important data entries utilized in this empirical study are discussed below along with a brief explanation of their role in the analysis.

Contract Type

The Federal Procurement Data System provides for eleven categories. However, an analysis of the full sample of R&D contracts indicates that the qualitative results in this study are unaffected by the pooling of similar contract types into three major categories, cost plus fixed fee, incentive contracts, and fixed price contracts.[8] These contract classifications are ordered to reflect a firm's increasing responsiblity to control cost, and therefore the underlying allocation of risk between the sponsor and research firm.

Product Service Codes

This data entry defines the service rendered or product being developed, including optional information about the stage of research and development.[9] Product Service Codes are matched with SIC codes for related industries in order to estimate the number of firms in an industry. In addition, product service codes are used to count the number of contractors in a submarket and thereby measure the extent of internal competition associated with a given contract. Industry size is used in the analysis as a proxy variable for external competition, i.e., the number of firms potentially bidding on a contract.

Extent of Competition in Negotiation

This entry indicates whether a contract follows a research competition between several contractors, is negotiated with a sole source, or is awarded to the best of two or more research proposals. As a result the extent of competition in negotiation can also provide a measure of external competition for a contract award.

Contract Identification Number

These codes are used to identify and merge data from different Contract Action Reports that are associated with the same contract or research project. For example, if a single contract is modified ten times over the reporting period, ten separate Contract Action Reports would be entered into the FPDS. By matching contract identification numbers, 17,000 DOD contract actions could be reduced to a twenty-four month history of 8,000 separate contracts.

IV. ANALYSIS AND RESULTS

Bivariate

Table 2 displays a crosstabulation of data for 5,700 new R&D contracts reported during fiscal year 1979. The data is sorted on one dimension by contract type, and on a second dimension by sponsoring agency. All agencies exclusive of DOD and NASA are collapsed into a third category because of small sample size. The table suggests general similarity, but some differences in research contracting among federal agencies. The null hypothesis of independence between agency and contract type is rejected at the one percent level.

Table 3 shows the same sample of data crosstabulated by four stages of R&D, and by defense and non-defense

TABLE 2

CONTRACT TYPE FOR DEFENSE AND NONDEFENSE SECTORS

	Fixed Price	Incentive	CPFF	Other
DOD	1232	119	3051	100
NASA	203	9	422	29
OTHER	174	14	330	48

Chi Square = 85.6

Sample = 5731

TABLE 3

STAGES OF R&D FOR DEFENSE AND NONDEFENSE SECTORS

	Research	Development 1	Development 2	Development 3
DOD	789	1511	843	1281
OTHER	103	47	7	29

Chi Square = 168

Sample = 4610

sponsorship. The small sample of non-defense contracts reflects the fact that NASA does not report a stage of R&D most applicable to its projects. Research stage is an optional item in the FPDS. The higher frequency of development contracts observed at DOD is consistent with that agency's unique role as both a research sponsor and the principal consumer of products developed at the government's expense. In Table 3, the null hypothesis of independence between stage of research and sponsor is also rejected at the one percent level.

Next, Table 4 illustrates the simple statistical association between the choice of contract type and stage of R&D applicable to a particular project. The cross-tabulation again reveals that most R&D contracts are either CPFF or fixed price. Incentive contracts are relatively rare for R&D. However, as predicted by agency theory, when incentive contracts are used, they are relatively more likely to be selected for development work than are CPFF contracts. Yet, when comparing CPFF and fixed price contracts, the same relationship, while in the predicted direction, is far less pronounced. This suggests that other factors, not controlled for in a simple bivariate analysis are influencing the selection of contract type. In Table 4, the Chi Square test suggests rejection of the null hypothesis of independence between contract selection and stage of research.

The bivariate analysis illustrated in Tables 2-4, which is representative of much of the policy literature concerned with R&D contracting, is of limited explanatory value. For example, the defense contracting literature discussed in Chapter 1 suggests that incentive contracts are associated with hardware development projects and are also plagued by cost overruns and changes in contract terms. But are modifications motivated by the use of incentive contracts, the stage of R&D, or other factors in the procurement environment perhaps correlated with these variables? In the next section multivariate regression analysis, N-Chotomous Probit, and a Tobit model are used to analyze factors influencing the selection of contract type, contract price, and the price of contract modifications. These powerful statistical tools enable one to test a number of hypotheses about R&D contracting.

TABLE 4

STAGES OF R&D AND CONTRACT TYPE

	Research	Development 1	Development 2	Development 3
Fixed Price	185	406	240	415
Incentive Contract	11	9	21	77
Cost Plus Fixed Fee	666	1107	562	794
Other	30	36	23	24

Chi Square = 135.3

Sample = 4606

A Multi-variate Approach

While a number of important factors may influence the choice of contract type, Table 3 indicates that sixty-eight percent of all DOD R&D contracts reported to the FPDS during 1979 were CPFF. As hypothesized earlier, a substitution of internal competition for contractual risk sharing may contribute to the widespread use of a contract, that in a legal sense, shifts most financial risk to the government.

Table 5 presents the results of a multi-variate analysis of contract selection at DOD, using a N-Chotomous Probit model developed by McKelvey and Zavonia (1975). A crucial statistical assumption for the application of Probit analysis is that the dependent variable be a choice among K ordered categories, where K is greater than two. Thus, the stepwise progression from a fixed price contract to a CPFF contract must represent an underlying continuum with respect to risk assumption or a firm's responsibility to control costs.[10]

The analysis reported in Table 5 supports the hypothesis that internal competition is used as a substitute for contractual risk sharing. The positive and statistically significant coefficient for the number of firms in a submarket, means that the more firms a sponsor has under

TABLE 5

FACTORS INFLUENCING A SPONSOR'S SELECTION OF CONTRACT TYPE

Independent Variables	Maximum Likelihood Estimate	Discrete Values Dependent Variable
		Contract Types
Research Period	.028** (7.54)	Fixed Price = 1
Internal Competition	.002** (3.42)	Incentive = 2
Agency's Residual Budget	-.00001** (-3.49)	Cost Plus Fixed Fee = 3
Industry Size	-.09* (-2.46)	
Noncompetitively Negotiated	.43* (2.15)	
Army	.63** (3.84)	
Navy	.34 (1.90)	
Development 1	-.34** (-2.98)	
Development 2	-.46** (-3.98)	
Development 3	-.66** (-4.40)	
Follows Research Competition	.11 (.59)	
Constant	.142	

Significance Level

*5%

**1%

N = 1432

R^2 = .21

% Predicted = 72

Rank Order Correlation = .36

contract in a particular research area, the more likely a new contractor is to receive favorable contract terms with respect to contractual risk bearing, all other factors being equal.

Several other factors also appear to have a statistically significant effect on the selection of contract type. The positive and significant coefficient for the variable measuring the length of a firm's research period may reflect increasing uncertainty about project cost and success, which forces a firm to forecast further into the future. As uncertainty increases, the government tends to bear a larger share of the contract risk.

The size of the research budget to be spent in a submarket may affect contract selection. The Residual Budget variable is defined as the sum of the dollar value of contracts in a submarket excluding the contract associated with the value of the dependent variable, i.e., contract type.[11] The negative and significant coefficient indicates that an increase in a sponsor's research budget tends to force firms to accept increased risk through written contract terms. This may be explained by either greater monopsony power resting with the sponsor or closer agency scrutiny of large budget areas.

In contrast, when contracts are awarded in the absence of a technical or pricing competition between research proposals, Table 5 suggests that contractors are able to shift the risk of cost growth back towards the government. Two variables measure the extent of competition in a contract award. They are "Industry Size" and "Noncompetitively Negotiated." The latter takes on a value of zero when a contract is awarded through competition, and 1.0 when a contract is awarded in the absence of competition. As discussed earlier, Industry Size is a proxy variable for the number of firms making research proposals. Both indicators of competition in the award of a contract are significant and have the expected signs.

Organizational factors may also influence the selection of contract type. The FPDS records the branch of the Armed Services that let the contracts analyzed in Table 5. The dummy variables identifying Army and Navy contracts indicate the contracting preferences of these two services relative to a reference category, the Air Force. Both the Army and Navy appear less likely to use fixed price contracts than the Air Force. However, this organizational effect is statistically significant only for the Army.

A final factor influencing the choice of contract type, and by implication contractual risk assumption by the firm, is the stage of R&D associated with a project. Dummy variables taking on values of zero and one depending on which of three phases of development can best be associated with a contract, are labeled in ascending order according to proximity to final product or process design. The dummy variable identifying basic research contracts has been omitted so that the coefficients for the other R&D variables indicate their effect on contract selection relative to that of a basic research contract. The sign and relative magnitude of the development coefficients indicate that the more development oriented is a project, the more likely it is that a fixed price or incentive contract will be used.

One potential objection to the model specification in Table 5 is the omission of a right-hand variable explicitly measuring a sponsor's expectations about future modifications in contract terms. However, if the observed frequency of changes for each contract (or their monetary value), are included as right-hand variables in the Probit model, potential similtanaeity between contract selection and realized contract changes would invalidate the statistical results.

A defense of the specification chosen by the author rests on two premises. First a sponsor's expectations concerning future contract changes is considered indirectly through other variables included in the model, e.g., the length of the period covered by the contract and the stage of research. Secondly, the analysis presented in the next section suggests that, ceteris parabus, all types of R&D contracts are equally subject to economically significant modifications in contract terms.

Having addressed the selection of contract type, one can turn to the factors that influence intitial contract price and the price of subsequent modifications or revisions of contract terms. An immediate dilemma arises. What is the unit of output for R&D, i.e., what normalization can be used to determine price consistently across different contracts? Ideally, one could observe and then measure research milestones in a consistent way across projects, and thereby develop both an ex ante and realized price to the sponsor for innovation. However, interpretable data concerning research milestones is difficult to acquire.

Therefore, the multi-variate analysis of contract pricing that appears in Tables 6 and 7 is based on a normalization by research period, which is measured in months. This means that the independent variables that influence the dependent variable (the dollar amount obligated in a contract action) are multiplied by the length of the research period inherent to a contract. In a statistical sense, one is estimating coefficients for economic factors that affect the flow of revenue to an R&D contractor.

Two other issues need to be addressed before discussing the results in Tables 6 and 7. The statistical model shown in Table 6 assumes that the contract type appropriate for an R&D project is selected prior to the negotiation of a contract price. Therefore the three contract types can be introduced as control or dummy variables which may or may not influence the initial flow of revenue to a contractor. Similtanaeity between price and other contract terms would be a more of a problem in an alternative specification where contract prices are regressed on negotiated risk sharing ratios and other variables associated with a sample of contracts. Such a specification is beyond the scope of this study.

Likewise any similtanaeity between initial contract price and the selection of other contract terms should not violate the statistical assumptions of a multivariate model designed to explain the variation in revenue from contract modifications. Contract revisions occur after the determination of contract terms. Therefore contract type and initial price can be treated as pre-determined factors that possibly affect the future price of contract modifications.

Finally, while multiple regression is a helpful tool for studying the flow of revenue on new contracts, linear regression may not be useful for analyzing a history of contract modifications. In a sample of new contracts, observations of the dependent variable, contract price, take on positive dollar values. In contrast, one may observe zero valued contract changes because some contracts are completed without modifications. Other contract actions result in price changes valued under ten thousand dollars, which are not recorded by the government's procurement data system.

When a significant number of observations concerning a dependent variable are censored from a positive valued sample, application of a linear regression model will result in biases in the estimated coefficients and a violation of the assumptions that underly statistical tests of significance. According to the FPDS, less than fifty percent of the contracts analyzed in this study were awarded positive price changes over a twelve month period.

One way to address the statistical problems associated with the censored observation of the dependent variable is to truncate the sample of contracts and only analyze positive values of contract changes. However this approach would throw away valuable information and possibly bias the results in a systematic direction.

To address the objections to the use of a regression model discussed above, a Tobit or Censored Regression Model, (Tobin, 1958), is used for the analysis of two samples of contract changes reported in Table 7. With the exception of dummy variables, the maximum likelihood estimates reported for the Tobit Model can be interpreted in the same way as regression coefficients. However, they are estimated with the intercept term forced through the origin. In contrasts, the regression results reported in Table 6 include estimates of constant terms. As a result, a co-efficient for a dummy variable reported in Table 6, represents

TABLE 6

FACTORS INFLUENCING INITIAL CONTRACT REVENUE

FY 1979

Independent Variables	New Contracts
Research Period	.56** (8.51)
Internal Competition	-.20** (-5.1)
Agency's Residual Budget	.0014** (8.5)
Industry Size	-.119** (-3.74)
Non-competitively Negotiated	7.99 (1.2)
Fellows Research Competition	17.12* (2.04)
Development 1	-7.53 (-1.4)
Development 2	-1.47 (-.24)
Development 3	8.97 (1.38)
Incentive	573.95 (3.97)
Cost Plus Fixed Fee	33.53 (.66)
Constant	64.5

Significance Level

**1%
*5%

N = 2109
R^2 = .13
F = 29.3

TABLE 7

FACTORS INFLUENCING REVENUE FLOW FROM MODIFICATIONS

Independent Variables	First Modification	Second Modification
Research period	.066 (1.82)	.64** (9.14)
Initial contract term	-.101** (-3.3)	-.38** (-6.20)
Internal Competition	-.017* (-2.19)	-.037** (-3.81)
Industry Size	-.0076* (-2.08)	-.012* (-2.78)
Non-competitively negotiated	.03 (.04)	-.63 (-.57)
Stage of Research		
Development 1	-2.0 (-.012)	.69 (.008)
Development 2	-1.6 (-.001)	6.7 (.07)
Development 3	-3.6 (-.02)	3.16 (.031)
Research	2.5 (.01)	-4.9 (-.054)
Follows Research Competition	3.02** (2.78)	5.78** (3.29)
Contract Type		
Incentive	7.5 (.04)	-6.7 (-.074)
Cost Plus Fixed Fee	4.95 (.03)	-6.9 (-.075)
Fixed Price	2.5 (.015)	-.128 (-.14)
Initial Dollar Award	.0007** (6.75)	-.00016* (-2.05)
Agency's Residual Budget	.000042* (2.02)	.00005* (2.05)
Significance Level	N = 1380	N = 1587
*5% **1%	Nonlimits = 497	Nonlimits = 288

the difference between the intercept term of the identified group and a reference category. Suppression of the constant term in Table 7 implies that the coefficients of the dummy variables represent the intercept term for the identified category.

The principle result that can be derived from Tables 6 and 7 is that both internal and external competition between firms working in similar industries or product areas seems to reduce the flow of revenue to a contractor below what it would be in a less competitive environment. More importantly, this result seems to apply to both new contract awards and subsequent contract changes.

However, the role played by the other independent variables in determining the price of contract actions as reported in Tables 6 and 7, appears to differ among new contracts and subsequent contract changes.

The positive and significant coefficients for research period, illustrated in Tables 6 and 7, imply that the longer the time period covered by either a contract or modification, the larger is the flow of revenue to a firm. Perhaps long term research efforts are insulated from competitive pressures, that would otherwise reduce a contractor's revenue.

In contrast, Table 7 suggests that the longer is the initial term of a contract, the lower is the firm's revenue derived from future contract revisions. This result is illustrated by the statistically significant and negative coefficients for Initial Contract Term. It may be the case that a contractor's monopoly power is restrained by the sponsor's option of terminating an incomplete contract at government convenience. Then the longer is the initial term of a contract, the greater profit a firm, on average, may lose as a result of contract termination by a sponsor.

Another important result pertains to the "buyin hypothesis" discussed earlier. Consider the behavior of the three variables measuring the level of competition in the award of a new contract, i.e., Industry Size, Non-Competitively Negotiated, and Follows Research Competition. Their coefficients do not exhibit a pattern of sign reversals, when comparing new contracts with modifications, that would support the view that greater competition in the award of a contract is more likely to result in untruthful bidding and rewriting of contract terms.

For example, in Table 6, Industry Size has a negative sign and is significant at the five percent level. As expected, external competition appears to reduce the initial dollar flow associated with the award of a new contract. But Industry Size also has a negative and significant sign in the Tobit model explaining dollar flow associated with contract modifications. A better informed sponsor and potential competition from other firms in the same industry may restrain the pricing of contract revisions.

The behavior of the dummy variable measuring whether a contract follows a research or design competition between different contractors, also is not supportive of a "buyin hypothesis." The two coefficients have positive and significant signs for both new contracts and subsequent contract changes. Perhaps the winner of a research competition possesses a superior technology or level of knowledge, and as a result, is in a strong position to negotiate both initial contract price and future contract changes. Alternatively, contracts that follow a research competition may systematically involve higher cost and more intensive research efforts, the nature of which are not completely captured by the dummy variables that control for stage of research.

While the dummy variables that control for phase of R&D are not statistically significant at the five percent level, their exclusion from the Tobit and Regression models reduces the explanatory power of the other variables studied. Nevertheless the statisical weakness of the R&D variables may reflect a common level of labor intensity.

Tables 6 and 7 provide conflicting evidence with respect to the impact of contract selection on contractor revenues. Table 6 suggests that incentive contracts tend to have a larger initial dollar flow than fixed price and CPFF contracts. However, evidence of a relationship between revenue and contract type is not supported in the two sampels of modifications analyzed in Table 7. On the contrary, the statistically insignificant coefficients for the three contract categories supports the view that all types of R&D contracts are modified to some degree to reflect research outcomes. In addition this result enhances the statistical validity of the multi-variate Probit model of contract selection discussed earlier.

Two final variables influencing dollar flows are Agency Residual Budget and Initial Dollar Award. It appears that the larger is the sponsoring agency's Residual Budget in a particular research area, (defined by a FPDS Product

Service Code and research stage) the higher is the dollar flow to a contractor from both new contracts and modifications. If a government research sponsor is acting as a monopsonist, one might expect to observe a negative coefficient for Residual Budget. The opposite result may imply that certain research areas have been assigned priorities for more intensive R&D funding.

The initial dollar award on a contract appears to have a significant, but ambiguous effect on the pricing of contract modifications. (See Table 7)

Conclusions and Policy Implications

The impact of competition on R&D contracting appears pervasive and beneficial, influencing the contract selection process, the initial determination of price, and negotiation of modifications. It is useful to differentiate between two types of competition: external competition among firms desiring government contracts, and internal competition among existing contractors seeking modifications, extensions of their contracts, and renewal. High levels of external competition tend to lower a contractor's revenue from a new contract and encourage a sponsor to select a contract type that shifts financial risk towards industry. In contrast, the expectation of internal competition among contractors working in similar research areas, may cause the government to substitute competition for contractual risk sharing, as a means of encouraging both risky research and an efficient R&D effort.

The empirical work in this chapter was made possible because the author was able to enhance the statistical power of the Federal Procurement Data system through extensive data processing. Thus the FPDS was not designed to answer the questions analyzed in this chapter. In view of the size of projected Defense outlays over the next decade and the importance to national security of monitoring the performance of the government's procurement system, the FPDS ought to be improved. In particular, modifications of the system should focus on 1) obtaining a more precise description of the competitive environment affecting a contract action, 2) the reasons for a contract modification, e.g., extension, renewal, overrun, incentive payment, etc., 3) a qualitative record of contract outcomes, e.g. R&D objectives were achieved, exceeded or scaled back. These changes would enable future researchers to more accurately determine the characteristics of optimal procurement systems.

FOOTNOTES TO CHAPTER 2

1. The question of program continuity and termination of contracts is a vital issue to industry. See the discussion in "Risk Elements In Government Contracting," mimeo, Aerospace Industries Association (October 1970).

2. For example, it is reported by N. Rosenberg, Perspectives on Technology, Cambridge, University Press 1976, pp. 69-70, that the interval varies considerably among innovations. The flourescent lamp took 79 years while streptomycin took 5.

3. See the discussion pertaining to General Electric's optimism in a prototype development project funded by the Department of Energy, in Balbien, J.

   "A Probilistic Cost Study of Solar Thermal Power Systems," mimeo, Jet Propulsion Laboratory (1981).

4. For example, see the discussion by J. Gansler, Defense Industry Consultant, in Science Vol. 212 (April 1981), p. 312

5. A brief history of the Office of Federal Procurement Policy is contained in the first volume of a Reporting Manual published by the Federal Procurement Data Center. Further insight was obtained through interviews with FPDS employees in Washington D.C. during Spring 1981. See "Federal Procurement Data System REPORTING MANUAL, Volume 1," Office of Management and Budget, (October 1979).

6. As above see "FPDS REPORTING MANUAL, Volume 1," (October 1979).

7. The most important exclusions from the stand point of sponsored research are R&D grants, mostly to universities, offered by DOD, The National Science Foundation, and NASA. In addition federal assistance programs directed at subsidizing private demonstrations of new technology are not under the jurisdiction of the Office of Federal Procurement Policy.

8. The 11 types of contracts that can be coded into the federal procurement data system include:
    i) Fixed Price Redetermination
    ii) Firm Fixed Price
    iii) Fixed Price Economic Price Adjustment
    iv) Fixed Price Incentive
    v) Cost Plus Award Fee
    vi) Cost Plus Incentive Fee
    vii) Cost Plus Fixed Fee
    viii) Cost No Fee
    ix) Labor Hour
    x) Time & Materials
    xi) Cost Sharing

   Ninety-seven percent of the sample of new defense contracts were either a type of fixed price contract, cost plus incentive fee or cost plus fixed fee. The small sample of award fee contracts, which link a contractor's performance on several dimensions (including overrun) to its profit rate, were pooled with the larger sample of incentive contracts. In view of their scarcity, and the objectives of this study, the residual contract types were dropped from the sample. Their selection by a sponsor is likely to be associated with factors beyond the scope of this study.

9. For example product service codes for defense applications recognize the following areas:

   i) Defense Aircraft R&D
   ii) Defense Missle & Space Systems R&D
   iii) Defense Ships R&D
   iv) Defense Tank - Automotive R&D
   v) Weapons R&D
   vi) Defense Electronics & Communications Equipment R&D
   vii) Ammunition R&D
   viii) Services R&D
   ix) Subsistence R&D
   x) Textiles R&D
   xi) Construction

   In addition, each contract receives an R&D code identifying the phase of research and development associated with each product or service being rendered. The categories of R&D relevant to the sample of contracts analyzed in this study are defined below.

   i) "Research - includes all effort of scientific and experimentation directed toward increasing knowledge and understanding in those fields of the physical, engineering, environmental and life sciences related to long term national security needs. It provides fundamental knowledge required for the solution of social, economic, political, physical or military problems. It forms a part of the base for subsequent exploratory and advanced developments in the various technologies, and new or improved functional capabilities in areas such as communications, propulsion, medicine, mobility, tracking, surveillance, propulsion, medicine, mobility, guidance and control, navigation, energy conversion, materials and structures, transportation, personnel support, and social services.

ii) Exploratory Development - includes all effort directed toward the solution of specific problems, short of major development projects. This type of effort may vary from fairly fundamental applied research to quite sophisticated bread-board hardware, study, programming and planning efforts. It would thus include studies, investigations, and minor development efforts. The dominant characteristic of this category of effort is that it be pointed toward specific problem areas with a view toward developing and evaluating the feasibility and practicability of proposed solutions and determining their parameters.

iii) Advanced Development - includes all effort directed toward projects which have moved into the development of hardware for test. The prime result of this type of effort is proof of design concept rather than the development of hardware for service use. Projects in this category have a potential application.

iv) Engineering Development - includes those projects in full-scale engineering development for Government use..," in mimeo, Office of Management and Budget, "Federal Procurement Data System-Product Service Codes," (April 1980).

10. Ideally one could observe the exact risk sharing factor

for each contract. Then fixed price contracts could be coded as 0, CPFF as 1.0, and the sharing ratio for each incentive contract would fall somewhere in between. However, because the FPDS does not record the precise risk sharing ratio associated with an incentive contract, the three categories have been assigned consecutive integer values as illustrated in Table 5.

11. The total dollar value of the contract was excluded from the specification in Table 5, because it is assumed that the sponsor selects the general contract type prior to choosing a contractor and/or negotiating the final contract price. Nevertheless, the qualitative results in Table 5, i.e., the signs of the coefficients and their statistical significance levels, are robust to the addition of other variables including contract price.

CHAPTER 3

A DYNAMIC MODEL OF TARGETING IN R AND D CONTRACTS

I. INTRODUCTION AND PRELIMINARIES

This chapter analyzes a model in which a sponsor contracts with a single firm to engage in research over a sequence of periods. At the beginning of each period the firm inherits a research performance target it hopes to meet by the end of the period. This target might represent a reduction in expected cost for some manufacturing process or the increased potency of an anti-cancer drug. The firm selects a level of research effort, conducts research, and observes the output of the research process. If the current target is achieved a new target is selected for the next period. The old target becomes the new state of sponsor knowledge for the next period and therefore a baseline for measuring further advances.

The model introduced below will necessarily be highly stylized — it makes several strong assumptions about the nature of the research process and the contractual reward to the firm. Focusing on the dynamic nature of the research environment requires that one simplify as far as possible the dimensions of the problem that are not intimately connected with qualitative aspects of the firm's choice of research effort and targeting decision. The idea is to create a filter for evaluating alternative contractual relationships. Contractual forms taken from more complex environments which do not perform well in a simplified research environment are unlikely to operate well when various complications are added.

One simplifying assumption concerns the nature of the R and D process within the firm. In order to focus on the dynamic aspects of the firm's research strategy, production of knowledge is modeled as random draws from a probability distribution over some measure of ''performance.'' The implication of relaxing this assumption is discussed in the conclusion.

Three crucial assumptions will also be made about the nature of the research contract. These assumptions capture the essence of the incentive problem from the firm's perspective. First it is assumed that in any given period, the firm's reward depends on fulfillment of the current performance target, X. If that goal is met, then the reward is a function of the target and the current state of sponsor knowledge, a level of performance R. If the target is not met, the firm's contract with the sponsor is not renewed. More formally, the instantaneous reward for fulfilling the current target is W(X,R). The firm receives no additional bonus for reporting progresses beyond the target.

It is also assumed that $W_X(X,R) > 0$ and $W_R(X,R) < 0$; the greater the difference between the current target and sponsor known performance, the higher the payoff to the firm if the target is achieved. Furthermore it is assumed that W_{RR} and W_{XX} are negative, which simply follows from diminishing returns.

It turns out that the properties of the firm's behavior under the stylized contract modeled here are sensitive to the sign of the cross partial derivative $W_{XR}(X,R)$. In the first section of the paper the model is analyzed under the assumption that $W_{XR}(X,R) \geq 0$. Letting W_{XR} be non-negative implies that the marginal return to setting higher performance targets remains high as sponsor knowledge increases.

II. THE MODEL

Let $V_t(\sigma,R,X)$ be the discounted expected profits from pursuing an optimal R and D plan when there are t periods remaining in the firm's planning horizon. The time index t can be thought of as the number of contracts the firm believes are available to it if all future targets are fulfilled. The firm's level of privately held knowledge at the beginning of a research period is represented by σ.

One issue that arises in a model of this sort is the nature of expectations over research potential. In principle placing no particular structure on expectations is ideal (e.g., Balbien and Wilde, 1980). However in practice there are two problems with the most general approach. Interviews with R and D engineers suggest they seldom have a strong notion of the shape of the distribution of research potential but feel confident about its upper and lower bounds. Secondly certain technical problems arise, further complicating the model, if no structure is placed on expectations. Given these facts it will be assumed herein that research potential is described by the uniform distribution. For notational convenience the distribution is normalized to be over [0,1].

In describing the research firm's strategy (in this case the choice of whether or not to conduct research in the current period and a choice of a target for the next period), it is useful to distinguish between two cases. In case I, $\sigma < X$; that is the level of privately held knowledge yields a level of performance (essentially the technology a firm has in inventory), which falls short of the currently active research target (which the firm set in the previous period). In this situation the firm has a ''risky target'' and must either conduct research or forfeit both the current reward and future contract opportunities on this particular project.

Thus for $\sigma < X$,

$$V_t(\sigma,R,X) = \max \begin{cases} -c + [W(X,R) + \underset{z \geq X}{E} \; \beta \underset{1 \geq x \geq X}{\max} V_{t-1}(Z,X,x)](1-X) \\ 0 \end{cases} \tag{1}$$

The logic of (1) is as follows. If the firm does research it incurs a cost c. If the outcome of its research, a random variable denoted by Z, is less than X (the currently active target) the firm gets no reward and its contract with the sponsor is not renewed. If $Z \geq X$ then it earns a reward for meeting the current target, $W(X,R)$, and gets to sign a new contract which specifies a new target, x. Of course x is set to maximize $V_{t-1}(Z,X,x)$. Note that the new target is set after the random variable representing research output is observed. Therefore the expectation, E, of maximized discounted profits when t-1 contracts remain is evaluated conditional upon the random performance variable, Z, being greater than or equal to the currently active target, X.

Finally if $Z \geq X$ the firm has the choice of setting the new target at a level of performance either above, equal to, or below its level of private knowledge. The firm will be said to set a ''risky target'' if $x > \max\{\sigma, Z\}$, and a ''safe target'' if $x \leq \max\{\sigma, Z\}$.

In case II where $\sigma > X$, the firm can fulfill the current target by drawing upon its technology inventory; i.e., research is not compulsory.

In this case, for $\sigma \geq X$,

$$V_t(\sigma,R,X) = \max \begin{cases} -c + W(X,R) + E\,\beta \max\limits_{1 \geq x \geq X} V(\max\{\sigma,Z\},X,x) \\ W(X,R) + \beta \max\limits_{1 \geq x \geq X} V_{t-1}(\sigma,X,x) \end{cases} \tag{2}$$

The first term in (2) again reflects discounted expected profits when the firm conducts research and then, depending on the results of that research, decides whether to set a risky or safe target. The second term reflects expected profits when the firm does not conduct new research in the current period and merely meets the current target ''out of inventory.'' Nevertheless, depending on σ, either a risky target or a safe target may be set for the next contract.

In both case I and II, the relevant discount rate is $\beta \in (0,1)$. Equations (1) and (2) hold for $t \geq 1$. For $t = 0$ define $V_0(\sigma,R,X) = 0$ for all σ,R,X. If $\sigma > R$ there might be some profit to the firm from selling the residual information to other private parties, but it is assumed that penalties for such action are so severe as to eliminate this possibility.

One final assumption important to a firm's targeting strategy concerns whether the sponsor will renew the firm's contract if the research firm sets a ''safe target'' with respect to the level of sponsor knowledge, i.e. the firm sets X = R. Under one scenario, a sponsor might require that a firm demonstrate some minimal improvement in sponsor knowledge as a condition for contract renewal. Such a policy would encourage the setting of risky targets when a firm exhausted its inventory of knowledge, but might lead to premature cancellation of research projects if targets are not achieved. An alternative policy, implicit in equations (1) and (2) permits contract renewal when the firm sets X = R. However W(R,R) is still assumed to be equal to 0.

The formal analysis of this model focuses on two aspects of a firm's research strategy over a sequence of contracts: (i) a firm's choice of research effort in the current period as determined by the firm's level of private knowledge, the level of sponsor known performance, the currently active target, the cost of research, and the number of remaining contracts in which the firm expects to participate; and (ii) the decision to set a safe target versus a risky target for the next research period as determined by the level of privately known performance at the end of the research period, the level of sponsor knowledge, research costs, and again the length of the firm's planning horizon. The analysis proceeds by induction, working backwards from the end of the horizon; i.e., beginning with t = 1.

In case I (where $\sigma < X$)

$$V_1(\sigma, R, X) = \max \begin{cases} -c + W(X,R)(1 - X) \\ 0 \end{cases} \quad (3)$$

The firm conducts research if and only if $W(X,R)(1-X) > c$. The function $W(X,R)(1-X)$ is concave in X. It has a negative slope at $X = 1$ and positive slope at $X = R$ (since $R \leq 1$ and $W_X(R,R) > W(R,R)$. Among the set of targets that produce non-negative expected returns one can define the least and most ambitious profitable targets by

$$a_1 = \min\{X | W(X,R)(1-X) \geq c;\ 1 \geq X \geq R\} \ .$$

$$b_1 = \max\{X | W(X,R)(1-X) \geq c;\ 1 \geq X \geq R\} \ .$$

If $X > b_1$, then the likelihood of meeting the current target is so low that no research is conducted. If $X < a_1$, the probability of meeting the current target is high but the payoff is so low that again, no research is conducted. The boundary targets are illustrated in Figure 1.

Lemma 1 examines the sensitivity of the least and most ambitious profitable targets to changes in the cost of research and the levels of sponsor known performance. The proof of this result (and all which follow) can be found in an appendix.

<u>Lemma 1</u>: $\frac{da_1}{dc} > 0$, and $\frac{db_1}{dc} < 0$;

$\frac{da_1}{dR} > 0$ and $\frac{db_1}{dR} < 0$.

This lemma implies that when a firm faces a risky target in the final research period, the worse the state of sponsor knowledge (i.e. the lower is R) the more likely the firm is to conduct research. This result follows from the assumption that the reward to the firm for fulfillment of the current target increases as the difference between sponsor knowledge and the current target increases (i.e., $W_R(X,R) < 0$). Raising R reduces the profitability of conducting research without affecting the likelihood of achieving the target. As expected, Lemma 1 also implies that the lower the cost of research, the more likely the firm is to conduct research. As can be seen in Figure 1, increasing the cost of research decreases the size of the set defined by the least and most ambitious profitable targets and reduces the profitability of conducting research for all levels of the performance target.

All that remains in the one period problem is the case of $\sigma \geq X$. Since $V_0(\sigma,R,x,)$ is defined to be zero, for $\sigma \geq x$

$$V_1(\sigma,R,X,) = W(X,R) \quad . \tag{4}$$

No research is conducted and at least a level of performance X is delivered to the sponsor.

In order to establish results for $t \geq 2$, certain cross partials and second derivatives of $V_1(\sigma,R,X)$ need to be analyzed. The formal details can be found in the appendix. The important point is that $V_1(\sigma,R,X)$ is concave in its arguments.

III. THE MODEL WITH t = 2

The two period problem is richer than its one period analogue since the firm now sets an optimal target for the last period's research after deciding whether to conduct research during the second to the last period.

From (1) and (2) one has for $X > \sigma$

$$V_2(\sigma,R,X) = \max \begin{cases} -c + [W(X,R) + \beta \underset{Z\geq X}{E} \underset{X\leq x\leq 1}{\max} V_1(Z,X,x)](1-X) \\ \\ 0 \end{cases} \tag{5}$$

and for $X \leq \sigma$

$$V_2(\sigma,R,X) = \max \begin{cases} -c + W(X,R) + \beta E \underset{X\leq x\leq 1}{\max} V_1(\max\{\sigma,Z\},X,x) \\ \\ W(X,R) + \beta \underset{X\leq x\leq 1}{\max} V_1(\sigma,X,x) \end{cases} \tag{6}$$

Recall that the target for the final period is set after observing the current period's research output. The analysis of the two period problem begins by taking that output as given and examining the target setting decision of the firm as a function of it. If the current target was not met, then the planning horizon is over. Otherwise

$$V_1(\sigma,X,x) = \max \begin{cases} W(x,X)(1-x)-c \\ \\ W(\sigma,X) \end{cases} \tag{7}$$

In (7), σ is treated as the firm's privately held level of performance (at the opening of the final research period); whether it represents inventory held over from earlier research periods or the result of new research is unimportant at this point.

The firm's problem is to choose a performance target for the final period which maximizes $V_1(\sigma,X,x)$. Since $W_X > 0$ and

$V_0(\sigma,R,X) = 0$, it never pays the firm to withhold information in setting the final period's target; i.e. the firm either reveals all it knows or sets a risky target. Let x_2^* be the optimal target i.e., $x_2^* = \underset{X \leq x \leq 1}{\text{argmax}}\ V_1(\sigma,X,x)$. Then $x_2^* = \sigma$ or $x_2^* = r_2$ where

$$r_2 = \underset{\sigma < x \leq 1}{\text{argmax}}\ W(x,X)(1-x) - c\ .$$

That is r_2 is the optimal risky target. Whether $x_2^* = r_2$ or $x_2^* = \sigma$ depends on which yields higher expected profits. In other words,

$$x_2^* = \begin{cases} \sigma \text{ if } W(\sigma,X) \geq W(r_2,X)(1-r_2) - c \\ r_2 \text{ if } W(\sigma,X) < W(r_2,X)(1-r_2) - c \end{cases} \tag{8}$$

Note that the optimal target for the final research period is subscripted by t = 2 because it is reported to the sponsor at the close of the second to the last research period. Equation (8) follows directly from (7) where $V_1(\sigma,X,x)$ is evaluated either at $x_2^* = r_2$ or $x_2^* = \sigma$. It shows that whether $x_2^* = r_2$ or $x_2^* = \sigma$ depends on σ. Define $\bar{\sigma}_2$ as the level of private knowledge that makes a firm indifferent between a safe and risky target, i.e., the value of σ that satisfies:

$$c = W(r_2,X)(1 - r_2) - W(\bar{\sigma}_2,X)\ . \tag{9}$$

However let $\bar{\sigma}_2 = 0$ if $c > W(r_2,X)(1 - r_2) - W(\sigma,X)$ for all $\sigma \geq X$. When $\sigma \geq \bar{\sigma}_2$, the firm sets $x_2^* = \sigma$, which is a safe target. No new research will be conducted and all information will be revealed. If $\sigma < \bar{\sigma}_2$ then $x_2^* = r_2$, which is a risky performance target. Research will be conducted in the final period and at least r_2 revealed, if the target is met. Thus $\bar{\sigma}_2$ is a critical level of private knowledge making the firm indifferent between a risky and safe targeting strategy in the two period problem.

Figure 2 illustrates the definition of $\bar{\sigma}_2$. In addition Figure 3 illustrates the relationship between r_2, c, $\bar{\sigma}_2$, and X.

Using $\bar{\sigma}_2$, (7) can be rewritten in a more useful form.

$$V_1(\sigma, X, x_2^*) = \begin{cases} W(r_2, X)(1 - r_2) - c & \sigma < \bar{\sigma}_2 \\ W(\sigma, X\ ) & \sigma \geq \bar{\sigma}_2 \end{cases} \tag{10}$$

Before continuing, consider the properties of $\bar{\sigma}_2$ as a function of research costs, c, and the new state of sponsor knowledge, X.

<u>Lemma 2</u>: X and c determine a unique $\bar{\sigma}_2$. Furthermore:

(i) $\frac{d\bar{\sigma}_2}{dc} < 0$

(ii) $W_{XR} \geq 0$ implies $\frac{d\bar{\sigma}_2}{dX} \geq 0$

(iii) $\bar{\sigma}_2 \geq X$

That $\bar{\sigma}_2$ is uniquely defined by c and X obtains because the optimal risky target r_2 is independent of the firm's level of private knowledge, while W(σ,R) is increasing in σ. Lemma 2 also implies that the lower the cost of research and the better the new state of public knowledge, the more likely the firm is to set a risky target for the final research period. The first result is obvious (see Figure 2) since a decision to set a risky target implies that research is compulsory in the final period, and a larger value of c makes this research more costly. The second result obtains because if X, the currently active target at t = 2, is met, it becomes the final period's level of sponsor knowledge. As a result, a higher level of sponsor known performance must be accompanied by a higher level of privately known performance, if the firm is to remain indifferent between the two targeting strategies.

<u>Lemma 3</u>: (i) $W_{XR} \geq 0$ implies $\frac{dr_2}{dX} > 0$

(ii) $\sigma \geq \bar{\sigma}_2$ implies $\frac{dx_2^*}{dX} = 0$

This lemma suggests that the better the state of sponsor knowledge the higher the risky target set for the last research period. Again this result holds because the current target, X, if achieved, becomes the level of sponsor knowledge in the last research period. Finally Lemma 4 implies that if a firm's level of private knowledge reaches at least the critical level of $\bar{\sigma}_2$, the firm's optimal target, x_2^* is independent of the new level of sponsor known performance. This is an obvious result since $\sigma \geq \bar{\sigma}_2$ implies that the firm reveals everything it knows.

Using $\bar{\sigma}_2$ it is now possible to back up to the beginning of the second to the last period and write $V_2(\sigma,R,X)$ in a more useful form. If σ is less than the current target in the second to the last period, $\bar{\sigma}_2$ determines whether a risky or safe target will be set for the last period.

Hence

$$V_2(\sigma,R,X) = \max\begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz \\ \qquad\qquad\qquad\qquad + \beta\int_{\bar{\sigma}_2}^{1} V_1(z,X,z)dz \\ 0 \end{cases} \tag{11}$$

But for $\sigma \geq X$ it is impossible to know whether $\sigma < \bar{\sigma}_2$ or $\sigma \geq \bar{\sigma}_2$. Thus the most one can say is that

$$V_2(\sigma,R,X) = \max\begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_1(\sigma,X,x_2^*)dz \\ \qquad\qquad\qquad + \beta\int_{\sigma}^{1} V_1(z,X,x_2^*)dz \\ W(X,R) + \beta V_1(\sigma,X,x_2^*) \end{cases} \tag{12}$$

Consider first the case of $\sigma \geq X$. From (12) the firm does research when two research periods remain in the firms planning horizon if and only if the expected net benefits are positive, i.e.

$$c < \beta\int_{\sigma}^{1} \left[V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)\right]dz \tag{13}$$

Define σ_2^* to be 0 if

$$c > \beta\int_{\sigma}^{1} \left[V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)\right]dz \ \forall\ \sigma .$$

Otherwise σ_2^* is defined by

$$c = \beta\int_{\sigma_2^*}^{1} \left[V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)dz \tag{14}$$

The reservation level σ_2^* is a level of private knowledge that makes a firm indifferent between conducting research and not conducting research. In general one can assert that if σ_2^* is greater than 0, then the reservation level for conducting research is also greater than $\bar{\sigma}_2$, the critical level of private knowledge that makes the firm indifferent between setting a ''risky'' versus a ''safe'' target at the end of the second to the last research period. This implies that when the firm begins the second to the last research period with a level of private knowledge below the cutoff point for setting a safe target, it always conducts research hoping to avoid the need to set a risky performance target for its last contract. Therefore if $\sigma_2^* > 0$, equation (14) becomes

$$c = \beta \int_{\sigma_2^*}^{1} \left[W(z,X) - W(\sigma_2^*,X) \right] dz \; . \qquad (15)$$

The next lemma establishes properties of σ_2^*.

<u>Lemma 4</u>: σ_2^* is uniquely defined. Furthermore

(i) $\dfrac{d\sigma_2^*}{dc} < 0$ and

(ii) $W_{XR} > 0$ implies $\dfrac{d\sigma_2^*}{dX} > 0$

The definition of σ_2^* enables one to rewrite (12) in a more useful form.

$$V_2(\sigma,R,X) = \begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_1(\sigma,X,x_2^*)dz \\ \qquad + \beta\int_{\sigma}^{1} V_1(z,X,x_2^*)dz & \sigma < \sigma_2^* \\ W(X,R) + \beta V_1(\sigma,X,x_2^*) & \sigma \geq \sigma_2^* \end{cases} \tag{16}$$

All that remains in the analysis of the two period problem is to consider under what conditions the firm might voluntarily drop out by failing to conduct research when $\sigma < X$. The firm conducts research for $\sigma < X$ if

$$c < [W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^{1} V_1(z,X,z)dz] \tag{17}$$

The right hand side of (17) is analogous to $W(X,R)(1-X)$ in the one period problem. It turns out that this function is quasiconcave so that results similar to those derived in the one period problem can be stated. Among the set of targets that produce non-negative expected returns, define the least and most ambitious by

$$a_2 = \min\{X/W(X,R)(1-X) + \beta(\int_X^{\bar{\sigma}_2} V_1(z,X,x_2^*)dz$$

$$+ \int_{\bar{\sigma}_2}^{1} V_1(z,x,z)dz) \geq C;\ 1 \geq X \geq R\}$$

$$b_2 = \max\{X/W(X,R)(1-X) + \beta(\int_X^{\bar{\sigma}_2} V_1(z,X,x_2^*)dz$$

$$+ \int_{\bar{\sigma}_2}^{1} V_1(z,X,z)dz) \geq C;\ 1 \geq X \geq R\}$$

The least and most ambitious profitable targets a_2 and b_2, are illustrated in Figure 4.

In the two period problem, a_2 might equal the current level of sponsor knowledge, R, particularly if the firm was hopeful of making progress in its next sampling of the distribution over research potential.

Analogous to Lemma 1, comparative statics results can be derived for a_2 and b_2.

<u>Lemma 5</u>: (i) $\frac{da_2}{dR} < 0$ and $\frac{db_2}{dR} > 0$,

(ii) $\frac{da_2}{dc} > 0$ and $\frac{db_2}{dc} < 0$.

This lemma implies that the analysis concerning the least and most ambitious profitable targets in the one period problem generalizes to the case of two periods. When the firm sets a risky target in the third to the last research period, it is less likely to voluntarily drop out of the project, the worse the level of sponsor known performance and the lower the cost of research.

<u>Lemma 6</u>: $a_2(R) < a_1(R)$ and $b_2(R) > b_1(R)$.

This lemma implies that when the currently active target is risky and the level of sponsor knowledge is held constant, the longer the firm's planning horizon the less likely the firm is to drop out by failing to conduct research.

This result follows directly from (17) which has all positive terms. The size of the set of targets providing non-negative returns, (which is bounded by the least and most ambitious profitable targets), increases in the two period model relative to the single period case. When two contracts remain, failure to do research not only implies the loss of the current reward but a future contract opportunity as well.

The definition of the least and most ambitious profitable targets enables one to rewrite (11).

For $\sigma > X$

$$V_2(\sigma,R,X) = \begin{cases} -c + [W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz \\ \quad + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z_2)dz] & X \in [a_2,b_2] \\ 0 & X < a_2,\ X > b_2 \end{cases} \tag{18}$$

To summarize the analysis up to this point, working backwards from the end of the second to the last period, the definition of $\bar{\sigma}_2$ as a critical level of private knowledge for setting a risky vs a safe performance target, enables one to simplify an inherently complicated research strategy. As a result it is possible to back up to the beginning of the research period and analyze the firm's decision to conduct research for the two cases of $\sigma \geq X$ and $\sigma < X$. In the former case a reservation level of private knowledge σ_2^* determines whether it is profitable to conduct research when facing a safe target. In the latter case, where $\sigma < X$ and future contracts are at risk, the decision to conduct research as opposed to dropping out of the project depends on the current target X belonging to a set defined by the last and most ambitious targets producing non-negative returns.

IV. THE MODEL WITH $t > 2$

Now that the firm's research strategy in the two period case is fully characterized, it is possible to consider the firm's selection of targets and decision to conduct research when three or more periods remain in the planning horizon. The multiperiod problem is more complicated than a two period R and D project because when $t = 3$ the firm's best safe target at the end of the period is not necessarily to reveal everything it knows. Instead the firm may have an incentive to temporarily withhold some of its private knowledge. This knowledge inventory can then be depleted over the remaining research periods so as to maximize the net present value of profits. A conservative strategy of sequentially setting higher safe performance targets also insures the firm against the risk of losing profits on the two remaining contracts if it encounters a bad draw from the distribution over research potential.

When $t = 3$ and $\sigma < X$ equation (1) gives

$$V_3(\sigma,R,X) = \begin{cases} -c + [W(X,R) + \beta \underset{Z>X}{E} \underset{X\leq x\leq 1}{\max} V_2(Z,X,x)](1-X) \\ 0 \end{cases} \tag{19}$$

and when $\sigma \geq X$ equation (2) implies

$$V_3(\sigma,R,X) = \max \begin{cases} -c + W(X,R) + \beta E \underset{X\leq x\leq 1}{\max} V_2(\max\{\sigma,Z\},X,x) \\ W(X,R) + \beta \underset{X\leq x\leq 1}{\max} V_2(\sigma,X,x) \end{cases} \tag{20}$$

Analogous to the two period problem, one begins by taking the output of the third to the last research period as given, and examining the target setting decision of the firm as a function of it. Once the targeting decision is fully characterized one can again back up to the beginning of the third to the last research period and embed the solution to the two period targeting problem in a three period setting.

The firm's objective at the end of the third to the last research period is to choose a target for the next contract which maximizes $V_2(\sigma,X,x)$. In general all one can say about $V_2(\sigma,X,x)$ is that it (1) has a discontinuity at $x = \sigma$, (2) is quasiconcave with respect to x, and (3) has negative slope at $x = 1$.

Let x_3^* be the optimal performance target with respect to $V_2(\sigma,X,x)$. Then

$$x_3^* = \underset{X \leq x \leq 1}{\text{argmax}}\ V_2(\sigma,X,x) .$$

Analogous to r_2 define

$$r_3 = \underset{\sigma < x \leq 1}{\text{argmax}}\ V_2(\sigma,X,x)$$

However since the optimal safe target is not necessarily σ in the three period problem, one must also define

$$s_3 = \underset{X \leq x \leq \sigma}{\text{argmax}}\ V_2(\sigma,X,x) .$$

Then

$$x_3^* = \begin{cases} r_3 & \text{if} \quad V_2(\sigma,X,r_3) > V_2(\sigma,X,s_3) \\ s_3 & \text{if} \quad V_2(\sigma,X,r_3) \leq V_2(\sigma,X,s_3) \end{cases} \tag{21}$$

Of course whether the firm sets a safe or risky performance target depends on its level of private knowledge at the end of the research period. Therefore analogous to the definition of the critical level of private knowledge $\bar{\sigma}_2$, define $\bar{\sigma}_3$ to be 0 if

$$V_2(\sigma,X,s_3) > V_2(\sigma,X,r_3) \text{ for all } \sigma \geq X \ .$$

Otherwise define $\bar{\sigma}_3$ to be that level of private knowledge which makes the firm indifferent between setting a safe and risky target; e.g. $\bar{\sigma}_3$ is that value of σ which satisfies

$$V_2(\bar{\sigma}_3,X,s_3) = V_2(\bar{\sigma}_3,X,r_3) \ . \tag{22}$$

If $\sigma < \bar{\sigma}_3$ then the firm sets a risky target, while $\sigma \geq \bar{\sigma}_3$ implies that a safe target is set. Lemma 10 establishes properties of $\bar{\sigma}_3$ as a function of research costs and the state of sponsor knowledge.

<u>Lemma 7</u>: $\bar{\sigma}_3$ is uniquely defined. Furthermore

(i) $\frac{d\bar{\sigma}_3}{dc} \leq 0$ and

(ii) $W_{XR} \geq 0$ implies that $\frac{d\bar{\sigma}_3}{dX} > 0$.

Analogous to the case of $t = 2$, the better the new state of sponsor knowledge at the end of the third to the last research period, or the lower the cost of research, the more likely the firm is to set a risky target for the second to the last contract.

If the firm knows it will set a safe target ($\sigma \geq \bar{\sigma}_3$) the question arises whether the firm will reveal everything it knows by setting $S_3 = \sigma$, or temporarily withhold some private knowledge. It turns out that one can define a second critical level of private knowledge which makes the firm indifferent between revealing everything it knows and withholding some private knowledge, i.e., a level of σ, $\underline{\sigma}_3$ which satisfies

$$\left. \frac{\partial V_2(\underline{\sigma}_3, R, x)}{\partial x} \right|_{\substack{R=X \\ x=\underline{\sigma}_3}} = 0 \; . \tag{23}$$

In effect $\underline{\sigma}_3$ is a level of private knowledge which just satisfies the first order condition implicit in the firm's selection of an optimal safe target. The properties of $\underline{\sigma}_3$ are described below.

<u>Lemma 8</u>: $\underline{\sigma}_3$ is uniquely defined. Furthermore

(i) $\dfrac{d\underline{\sigma}_3}{dc} = 0$

(ii) $V_2(\sigma, X, x)$ concave in x and $W_{XR} > 0$ implies $\dfrac{d\underline{\sigma}_3}{dX} > 0$

(iii) $\bar{\sigma}_3 \leq \underline{\sigma}_3 \leq X$

The better the new state of sponsor knowledge, the more likely the firm is to reveal all it knows at the end of the research period by setting a target which equals its current state of private knowledge. Surprisingly the cost of research does not affect the likelihood the firm reveals all it knows.

The definitions of $\underline{\sigma}_3$ and $\bar{\sigma}_3$ enable one to rewrite (21) in a more useful form.

$$x_3^* = \begin{cases} s_3 & \text{if} \quad \sigma \geq \underline{\sigma}_3 \\ \sigma & \text{if} \quad \underline{\sigma}_3 \geq \sigma \geq \overline{\sigma}_3 \\ r_3 & \text{if} \quad \overline{\sigma}_3 > \sigma \end{cases} \qquad (24)$$

where $s_3 < \sigma$.

To summarize, in the three period problem there are three instead of two target setting modes for a research firm working under contract. When private knowledge is poor and probabilities of research progress are high, the firm sets a risky target for its research. As both sponsor and privately known performance improves, setting a risky target makes contract renewal more uncertain. As a result the firm becomes more conservative and reveals the current state of private knowledge as the target for the next contract. Finally, if very high levels of performance are achieved the firm insures contracting continuity and maximum profits by setting safe targets that temporarily withhold some private knowledge and thereby reveal the results of the firm's research over a sequence of contracts.

In order to complete the analysis of the firm's research strategy in a three period setting, comparative dynamic properties of s_3, r_3, and $\overline{\sigma}_3$ as well as the relationship between optimal targets, research cost and sponsor knowledge must be described. In addition σ_3^*, a level of private knowledge that makes the firm indifferent between conducting research and not doing research at the beginning of the third to the last research period, must be defined and its properties described. Finally the three period analysis needs to be generalized to t periods.

Lemma 9: When s_3 is an interior solution:

(i) $\frac{ds_3}{dc} = 0$

(ii) $V_2(\sigma,X,x)$ concave and $W_{XR} > 0$ implies

a) $\frac{ds_3}{dX} > 0$ and

b) $\frac{ds_3}{d\sigma} > 0$.

Furthermore when r_3 is an interior solution:

(iii) $\frac{dr_3}{dc} = 0 = \frac{dr_3}{d\sigma}$

(iv) $V_2(\sigma,R,x)$ concave and $W_{XR} \geq 0$ implies $\frac{dr_3}{dX} > 0$.

Lemma 9 implies that in the multiperiod setting, the level of a risky target and the amount of information revealed through a safe target are both independent of the cost of acquiring new information. However, as noted earlier, the decision whether to set either a risky or safe target is determined in part by research costs.

Furthermore the Lemma implies that the better the state of sponsor knowledge, the higher is the risky performance target set by the firm and the larger is the share of the firm's private knowledge revealed through a safe target.

The next two lemma focus on comparative dynamic properties of a firm's targeting strategy.

In particular Lemma 10 compares the firm's optimal safe targets (s_3 and s_2), optimal risky targets (r_3 and r_2), and the critical levels of private knowledge ($\bar{\sigma}_3$, $\bar{\sigma}_2$), holding the levels of sponsor and privately known performance constant.

<u>Lemma 10</u>: (i) $r_3(X) < r_2(X)$

(ii) $s_3(X) < x_2^*(X)$

(iii) $\bar{\sigma}_3(X) \gtrless \bar{\sigma}_3(X)$

The first result implies that the longer the firm's planning horizon, the lower the optimal risky target that is set. When there are two as opposed to one remaining contract, the firm is concerned about the impact of the new target on its chances for contract renewal and the value of its research reports in the last research period.

Similarly, the second result implies that the longer the firm's planning horizon the more likely the firm is to withhold information uncovered in its research. This result obtains because a firm setting a safe target for the last contract reveals everything it knows. This is not necessarily the case when the firm expects that two or more contracts remain.

Finally the third part of the lemma establishes that, in general, one cannot say whether the firm is more or less likely to set a risky target as the firm's planning horizon increases from two to three periods. The firm's preference for a big immediate payoff and the discounting of future rewards encourages the setting of risky targets as the planning horizon increases, while a more conservative strategy of setting safe targets or temporarily withholding some private knowledge to be reported over a sequence of periods, insures contract renewal and a higher expected reward.

Now, one can return to the beginning of the third to the last research period, and analyze the firm's decision to conduct research when its current target is safe, i.e. $\sigma \geq X$. The firm does research if its level of private knowledge and cost of research satisfy the following inequality

$$c < \beta \int_{\sigma}^{1} \left[V_2(z,X,x_3^*) - V_2(\sigma,X,x_3^*)\right]dz \ .$$

If the inequality is not satisfied for $\sigma \geq 0$, define σ_3^* to be 0. Otherwise σ_3^* is defined by

$$c = \beta \int_{\sigma_3^*}^{1} \left[V_2(z,X,s_3^*) - V_2(\sigma_3^*,X,x_3^*)\right]dz \ .$$

Analogous to the definition of σ_2^*, σ_3^* is a level of private knowledge which makes a firm indifferent between conducting research and not conducting research, when three periods remain. One can again assert that $\sigma_3^* \geq \bar{\sigma}_3$, which implies that when the firm begins the research period with a level of private knowledge less than the cut off point for setting a risky target, i.e. $\sigma < \bar{\sigma}_3$, it always conducts research hoping to avoid the need to set a risky performance target. The next two lemmas describe the properties of σ_3^*.

<u>Lemma 11</u>: c and X define values of σ_3^*, which may or <u>may not</u> be unique.

If σ_3^* is unique

(i) $\frac{d\sigma_3^*}{dX} > 0$ is implied by $W_{XR} > 0$

(ii) $\frac{d\sigma_3^*}{dc} < 0$

Analogous to σ_2^*, the higher the level of sponsor known performance and the lower are research costs, the more likely is the firm to conduct research.

<u>Lemma 12</u>: Necessary conditions for $\sigma_3^* \geq \sigma_2^*$, are

(i) $W_{XR} > 0$

(ii) $\beta > \hat{\beta} > 0$

In order for the firm to be more likely to conduct research the longer its planning horizon, holding the level of sponsor knowledge and research costs constant the marginal return to revealing higher levels of performance must increase as sponsor knowledge increases. In addition the firm's discount rate must be sufficiently low.

Next, an induction argument is utilized to place the analysis of the firm's research strategy for t = 2,3 in a more general framework.

<u>Theorem 1</u>: $V_t(\sigma,R,X)$ is strictly quasiconcave in X.

(a) Suppose $X > \sigma$. If there exists levels of the current target producing nonnegative expected returns, one can define least and most ambitious targets acceptable to the firm by

$$a_t = \min\{X | W(X,R)(1-X) + \underset{Z \geq X}{E} \beta \max_{1 \geq x \geq X} V_{t-1}(z,X,x)(1-X) \geq c\}$$

and

$$b_t = \max\{X \mid W(X,R)(1-X) + \underset{Z \geq X}{E}\ \beta \max_{1 \geq x \geq X} V_{t-1}(z,X,x)(1-X) \geq c\}$$

Furthermore, for a_t and $b_t > 0$,

(i) $\frac{da_t}{dR} > 0$ and $\frac{db_t}{dR} < 0$

(ii) $\frac{da_t}{dc} \geq 0$ and $\frac{db_t}{dc} < 0$

(iii) $a_t(R) \leq a_{t-1}(R)$ and $b_t(R) > b_{t-1}(R)$.

(iv) The firm voluntarily drops out of the research program for all $X \in [0,a_t)$ and $X \in (b_t,1]$

(b) Suppose $X < \sigma$. If there exist levels of privately known performance low enough so that conducting research provides non-negative expected returns, one can define a reservation level of private knowledge making a firm indifferent between conducting research and not conducting research, i.e.

$$\{(\sigma,X,R) \mid -c + E\ \beta \max_{X \leq x \leq 1} V_{t-1}(\max\{\sigma,z\},X,x) > \max_{X \leq x \leq 1} V_{t-1}(\sigma,X,x)\} =$$

$$\{(\sigma,X,R) \mid \sigma < \sigma_t^*(c,X)\}$$

Furthermore, for $\sigma_t^* > 0$ and unique:

(i) $\frac{d\sigma_t^*}{dc} > 0$

(ii) $W_{XR} > 0$ implies $\frac{d\sigma_t^*}{dX} > 0$.

(iii) $W_{XR} > 0$ and $\beta > \hat{\beta}$ implies $\sigma_t^*(c,X) \geq \sigma_{t-1}^*(c,X)$, where $\beta > \hat{\beta} > 0$

(c) At the end of the current research period, if the target is met the firm has an opportunity to set a new target for the next contract. Define the optimal performance target

$$x_t^* = \underset{X \leq x \leq 1}{\text{argmax}}\ V_{t-1}(\sigma, X, x).$$

Also let

$$s_t = \underset{X \leq x \leq \sigma}{\text{argmax}}\ V_{t-1}(\sigma, X, x)$$

and

$$r_t = \underset{\sigma < x \leq 1}{\text{argmax}}\ V_{t-1}(\sigma, X, x)$$

Then,

(i) $\frac{dr_t}{dc} = 0 = \frac{ds_t}{dc}$

(ii) $W_{XR} \geq 0$ implies $r_t(X) < r_{t-1}(X)$ and

$W_{XR} > 0$ implies $s_t(X) < s_{t-1}(X)$

Furthermore if $V_{t-1}(\sigma, X, x)$ is strictly concave,

(i) $W_{XR} \geq 0$ implies $\frac{dr_t}{dX} > 0$

(ii) $W_{XR} > 0$ implies $\frac{ds_t}{dX} > 0$

(iii) $W_{XR} > 0$ implies that $\frac{ds_t}{d\sigma} > 0$

(d) If there exist values of σ such that a firm setting a risky as opposed to a safe target expects increased returns, one can define a critical level of σ, $\bar{\sigma}_t$, that makes a firm indifferent between setting a safe and risky target. That is $\bar{\sigma}_t$ is defined such that

$$V_{t-1}(\bar{\sigma}_t, X, r_t) = V_{t-1}(\bar{\sigma}_t, X, s_t) \ .$$

Furthermore

(i) $W_{XR} \geq 0$ implies $\dfrac{d\bar{\sigma}_t}{dX} > 0$

(ii) $\dfrac{d\bar{\sigma}_t}{dc} \leq 0$

(iii) $\bar{\sigma}_t(X) \gtrless \bar{\sigma}_t(X)$

$$\text{(iv)} \quad x_t^* = \begin{cases} r_t & \text{if} \quad \sigma < \bar{\sigma}_t \\ \sigma & \text{if} \quad \bar{\sigma}_t \leq \sigma \leq \underline{\sigma}_t \\ s_t & \quad\quad \sigma > \underline{\sigma}_t \end{cases}$$

where $s_t < \sigma$ and $\underline{\sigma}_t$ is a second critical level of private knowledge satisfying

$$\frac{\partial V_{t-1}(\underline{\sigma}_t, X, \underline{\sigma}_t)}{\partial X} = 0$$

(v) If $V_{t-1}(\sigma, X, x)$ is strictly concave and $W_{XR} \geq 0$

$$\frac{d\underline{\sigma}_t}{dX} > 0 \quad \text{and} \quad \frac{d\underline{\sigma}_t}{dc} = 0 \ .$$

V. EXTENSIONS

The model developed in this paper can be extended to address particular problems in research contracting and to reflect more complex research environments. For example there might be learning by the firm as research unfolds. This can be modeled by letting $F(z;\sigma)$ be the distribution over research potential, given the firm's level of privately known performance σ. Learning is represented by assuming that $\frac{\partial F(z;\sigma)}{\partial \sigma} < 0$, i.e., the higher the level of privately known performance the better the chances of making further progress.

In addition the firm's choice of research effort might be a continuous variable defined in terms of multiple draws from the same distributions, each one associated with a unique sampling cost and expected performance. Although adding these complications would enhance the realism of the model, perhaps making it more likely that the firm conducts research and sets higher targets, one would not expect the basic qualitative results to change.

One extension that is likely to have a major impact on the firm's behavior is the introduction of (i) a second research firm who also sets targets and conducts research for a sponsor, or (ii) several firms any one of which can bid away the bilateral monopoly position of the firm relative to the sponsor, by revealing a level of knowledge superior to the current performance target promised by the firm. Although the competitive case has not been analyzed in detail, it appears that the presence of a second firm may discourage the withholding of private knowledge and perhaps encourage the setting of risky targets. However competition might discourage research by leading the firm to discount the availability of future contracts and the reward for target fulfillment.

A final extension involves the addition to the firm's profit function of a term reflecting the value of human capital built up over a sequence of periods. Early studies of weapons procurement acquisitioned through various forms of cost reimbursement contracts have suggested capitalization of R and D advances in federal system acquisitions and the training of personnel as an important motivation for cost growth. In the model presented in this paper, capitalization of R and D advances and the build up of human capital would encourage the setting of safe targets in order to insure contract renewal.

VI. CONCLUSION

The model analyzed in this paper again confirms that incentive problems may arise in contracting for research when one goes from a static to a dynamic setting. A firm's targeting behavior and choice of research effort over a sequence of periods is sensitive in a predictable fashion to the changing information assymetries that exist between a sponsor and contractor, and to the length of the firm's planning horizon.

APPENDIX I: PROPERTIES OF $V_1(\sigma, X, x_2^*)$

$$V_1(\sigma,X,x_2^*) = \begin{cases} w(r_2,X)(1-r_2) - c & \sigma < \bar{\sigma}_2 \\ w(\sigma\ ,X) & \sigma \geq \bar{\sigma}_2 \end{cases}$$

(i) $$\frac{\partial V_1(\sigma,X,x_2^*)}{\partial \sigma} = \begin{cases} 0 & \sigma < \bar{\sigma}_2 \\ W_X(\sigma,X) > 0 & \sigma \geq \bar{\sigma}_2 \end{cases}$$

(ii) $$\frac{\partial V_1(\sigma,X,x_2^*)}{\partial R} = \begin{cases} W_R(r_2,X)(1-r_2) < 0 & \sigma < \bar{\sigma}_2 \\ W_R(\sigma,X) < 0 & \sigma \geq \bar{\sigma}_2 \end{cases}$$

(iii) $$\frac{\partial^2 V_1(\sigma,X,x_2^*)}{\partial R^2} = \begin{cases} W_{RR}(r_2,X)(1-r_2) - \left[W_R(r_2,X) - W_{XR}(r_2,X)(1-r_2)\right]\frac{dr_2}{dX} \gtrless 0 \\ W_{RR}(\sigma,X) < 0 \end{cases}$$

(iv) $$\frac{\partial^2 V_1(\sigma,X,x_2^*)}{\partial R \partial X} = \begin{cases} (1-r_2)W_{XR}(r_2,X) - W_R(r_2,X) > 0 & \sigma < \bar{\sigma}_2 \\ W_{XR} \geq 0 & \sigma \geq \bar{\sigma}_2 \end{cases}$$

(v) $$\frac{\partial^2 V_1(\sigma,X,x_2^*)}{\partial X^2} = \begin{cases} W_{XX}(r_2,X)(1-r_2) - 2W_X(r_2,X) < 0 & \sigma < \bar{\sigma}_2 \\ W_{XX}(\sigma,X) < 0 & \sigma \geq \bar{\sigma}_2 \end{cases}$$

(vi) $$\frac{\partial^2 V_1(\sigma,X,x_2^*)}{\partial X \partial c} = 0 \qquad \sigma \geq \bar{\sigma}_2,\ \sigma < \bar{\sigma}_2\ .$$

APPENDIX II: PROPERTIES OF $V_2(\sigma, X, x_3^*)$

$$V_2(\sigma,X,x_3^*) = \begin{cases} V_2(\sigma,X,r_3) & \sigma < \bar{\sigma}_3 \\ V_2(\sigma,X,S_3) & \sigma \geq \bar{\sigma}_3 \end{cases}$$

where

$$V_2(\sigma,X,r_3) = \begin{cases} -c + \left[w(r_3,X)(1-r_3) + \int_{r_3}^{\bar{\sigma}_2} \beta V_1(z,r_3,z)dZ + \int_{\underline{\sigma}_2}^{1} \beta V_1(z,r_3,r_2)dZ\right] & r_3 \in \left[a_2, b_2\right] \\ 0 & \end{cases}$$

and

$$V_2(\sigma,X,s_3 = \begin{cases} -c + W(s_3,X) + \int_0^{\sigma} \beta V_1(\sigma,s_3,x_2^*)dZ + \int_{\sigma}^{1} \beta V_1(Z,s_3,x_2^*)dZ & \sigma < \sigma_2^* \\ w(s_3,X) + \beta V_1(\sigma,s_3,x_2^*) & \sigma \geq \sigma_2^* \end{cases}$$

(i) $\partial V_2(\sigma,X,r_3)/\partial R = W_R(r_3,X)(1-r_3) < 0$

$\partial V_2(\sigma,X,s_3)/\partial R = W_R(s_3,X) < 0$

(ii) $\partial^2 V_2(\sigma,X,r_3)/\partial R^2 = W_{RR}(r_3,X)(1-r_3) - \frac{dr_3}{dX} \left[W_R(r_3,X) - W_{RX}(r_3,X)(1-r_3)\right]$

$\partial^2 V_2(\sigma,X,s_3)/\partial R_2 = W_{RR}(s_3,X) + \frac{ds_3}{dX} W_{RX}(s_3,X) \gtrless 0$

(iii) $\partial^2 V_2(\sigma,X,r_3)/\partial X \partial R = W_{XR}(r_3,X) - W_R(r_3,X) > 0$

$\partial^2 V_2(\sigma,X,s_3)/\partial X \partial R = W_{XR}(s_3,X) \geq 0$

(iv) $\partial^2 V_2(\sigma, X, r_3)/\partial X^2 \gtrless 0$

$\partial^2 V_2(\sigma, X, s_3)/\partial X^2 \gtrless 0$

(v) $\partial V_2(\sigma, X, r_3)/\partial\sigma = 0$

$$\partial V_2(\sigma, X, s_3)/\partial\sigma = \begin{cases} 0 & \sigma < \bar{\sigma}_2 \\ \sigma\beta W_X(\sigma, s_3) & \bar{\sigma}_2 \leq \sigma < \sigma_2^* \\ \beta W_X(\sigma, s_3) & \sigma \geq \sigma_2^* \end{cases}$$

(vi) $$\partial^2 V_2(\sigma, X, s_3)/\partial\sigma\partial X = \begin{cases} \sigma\beta W_{XR}(\sigma, s_3) & \bar{\sigma}_2 \leq \sigma < \sigma_2^* \\ \beta W_{XR}(\sigma, s_3) & \sigma \geq \sigma_2^* \end{cases}$$

(vii) $$\frac{\partial^2 V_2(\sigma, X, s_3)}{\partial X \partial c} = \frac{\partial^2 V_2(\sigma, X, r_3)}{\partial X \partial c} = 0$$

APPENDIX III

Proof of Lemma 1: Taking the total derivatives of $W(a_1,R)(1-a_1) = c$ and $W(b_1,R)(1-b_1) = c$, one gets

$$\frac{da_1}{dR} = \frac{1}{W_X(a_1,R)(1-a) - W(a_1,R)} > 0$$

and

$$\frac{db_1}{dR} = \frac{1}{W_X(b_1,R)(1-b_1) - W(b_1,R)} < 0 .$$

The definition of the least and most ambitious acceptable targets and the concavity of $W(X,R)(1-X)$ imply the needed result.

Similarly, taking total derivatives again implies that

$$\frac{da_1}{dR} = \frac{-W_R(a_1,R)}{W_X(a_1,R)(1-a_1) - W(a_1,R)} > 0$$

and

$$\frac{db_1}{dR} = \frac{-W_R(b_1,R)}{W_X(b_1,R)(1-b_1) - W(b_1,R)} < 0 .$$

Proof of Lemma 2: Taking the total derivative of equation (9) which defines $\bar{\sigma}_2$,

$$\frac{d\bar{\sigma}_2}{dc} = \frac{-1}{W_X(\bar{\sigma}_2,X)} < 0 .$$

Similarly,

$$\frac{d\bar{\sigma}_2}{dX} = \frac{(1-r_2)W_R(r_2,X) - W_R(\bar{\sigma}_2,X)}{W_X(\bar{\sigma}_2,X)} ,$$

where X is the new state of sponsor knowledge at the end of the research period. Thus $r_2 > \bar{\sigma}_2$ and the assumption that $W_{XR} \geq 0$ implies that $W_R(r_2,X) - W_R(\bar{\sigma}_2,X) \geq 0$ which in turn implies $\frac{d\bar{\sigma}_2}{dX} > 0$. Furthermore $\bar{\sigma}_2 \geq X$ since by the definition of the instantaneous reward function $W(X,R)$, $X \geq R$.

<u>Proof of Lemma 3</u>: Totally differentiating the first order condition $(1-r_2)W_X(r_2,X) - W(r_2,X) = 0$ implies

$$\frac{dr_2}{dX} = \frac{-W_{XR}(1-r_2) + W_R(r_2,X)}{W_{XX} - 2W_X} \, .$$

A sufficient condition for $\frac{dr_2}{dX} > 0$ is $W_{XR} \geq 0$. Also $\sigma \geq \bar{\sigma}_2$ implies $x_2^* = \sigma$ and therefore $\frac{dx_2^*}{dX} = 0$.

<u>Proof of Lemma 4</u>: Define $T_2(\sigma,X) = \beta\int_\sigma^1 \left[V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)\right]dz$. Then

$$x_2^* = \begin{cases} r_2 & \text{if} \quad \sigma < \bar{\sigma}_2 \\ \sigma & \text{if} \quad \sigma \geq \bar{\sigma}_2 \end{cases}$$

implies that

$$T_2(\sigma,X) = \begin{cases} \beta\int_{\bar{\sigma}_2}^1 \left[W(z,X) - W(r_2,X)(1-r_2) + c\right]dz & \text{if} \quad \sigma < \bar{\sigma}_2 \\ \beta\int_{\sigma}^1 \left[W(z,X) - W(\sigma,X)\right]dz & \text{if} \quad \sigma \geq \bar{\sigma}_2 \end{cases}$$

Note that the top expression is always greater than or equal to the bottom expression as illustrated in Figure A1. Also $T_2(1,X) = 0$.

Differentiating $T_2(\sigma,X)$ with respect to σ one gets

$$\frac{\partial T_2(\sigma,X)}{\partial\sigma} = \begin{cases} 0 & \text{if } \sigma < \bar{\sigma}_2 \\ -\beta(1-\sigma)W_X(\sigma,X) < 0 & \text{if } \sigma \geq \bar{\sigma}_2 \end{cases}$$

As noted in the text, one need not consider the case of $\sigma < \bar{\sigma}_2$ since $T_2(\bar{\sigma}_2,X) > c$ and $\frac{\partial T_2(\sigma,X)}{\partial\sigma} < 0$ are sufficient conditions for σ_2^* to be uniquely defined, and $0 < \bar{\sigma}_2 < \sigma_2^*$. Taking the total derivative of $T_2(\sigma_2^*,X) = C$ implies that

$$\frac{d\sigma_2^*}{dc} = \frac{1}{-\beta(1-\sigma)W_X(\sigma,X)} < 0 .$$

Similarly,

$$\frac{d\sigma_2^*}{dX} = \frac{\int_{\sigma_2^*}^{1}\left[W_R(z,X) - W_R(\sigma_2^*,X)\right]dz}{(1-\sigma_2^*)W_X(\sigma_2^*,X)}$$

which has a sign opposite to W_{XR}. Therefore $W_{XR} \geq 0$ implies $\frac{d\sigma_2^*}{dX} \leq 0$.

<u>Proof of Lemma 5</u>: For the most ambitious acceptable target, taking the total derivative of

$$c = \left[W(b_2,R)(1-b_2) + \beta\int_{b_2}^{1} V_1(z,b_2,x_2^*)dz\right]$$

one gets $\frac{db_2}{dc} = \frac{1}{A}$ where

$$A = \left[W_X(b_2,R)(1-b_2) - W(b_2,R) + \beta\int_{b_2}^{1}\frac{\partial V_1(z,b_2,x_2^*)}{\partial R}dZ - \beta V_1(b_2,b_2,x_2^*)\right]$$

Similarly $\frac{db_2}{dR} = \frac{-W_R(b_2,R)(1-b_2)}{A}$. That $\frac{db_2}{dc} < 0$ and $\frac{db_2}{dR} < 0$ follows directly from the definition of b_2 and the quasiconcavity of (17) with respect to X.

A similar analysis implies that if a_2 is the least ambitious acceptable target and $a_2 > R$, then $\frac{da_2}{dc} > 0$ and $\frac{da_2}{dR} > 0$.

Proof of Lemma 6: Recalling the definition of the most ambitious acceptable targets, b_1 and b_2, one knows that

$$c = W(b_2,R)(1-b_2) + \beta \int_{b_2}^{1} V_1(z,b_2,x_2^*)dz \text{ and}$$

$$c = W(b_1,R)(1-b_1 .$$

Therefore it must be the case that $W(b_1,R)(1-b_1) > W(b_2,R)(1-b_2)$ which in turn implies $b_2(R) > b_1(R)$. A similar argument with respect to the least ambitious acceptable targets iplies that $a_2(R) \leq a_1(R)$.

Proof of Lemma 7: To establish the uniqueness of $\bar{\sigma}_3$ one needs to analyze the behavior of

$$Q_3(\sigma,X) = \left[V_2(\sigma,X,r_3) - V_2(\sigma,X,s_3)\right],$$

which requires (i) an understanding of $\frac{\partial V_2(\sigma,X,x)}{|\partial\sigma}$, for all $X \leq \sigma \leq 1$ and $X \leq x \leq 1$, and (ii) proof of the claim that the only time a firm can be indifferent beween a risky vs a safe targeting strategy is when its optimal safe target reveals everything that it knows, i.e. $Q_3(\sigma,X) \geq 0$ implies $s_3 = \sigma$. The properties of $\frac{\partial V_2(\sigma,X,x)}{\partial\sigma}$, developed in

Appendix II, which together with (ii) imply the uniqueness of $\bar{\sigma}_3$, include:

(i) If r_3 is an interior maximum, $\frac{\partial V_2(\sigma,X,r_3)}{\partial\sigma} = 0$.

(ii) If r_3 is a corner solution, i.e. $r_3 = \sigma$, then

$$\left.\frac{\partial V_2(\sigma,X,x)}{\partial\sigma}\right|_{x=\sigma}^{-} < 0 .$$

(iii) If s_3 is a corner solution then

$$\left.\frac{\partial V_2(\sigma,X,x)}{\partial\sigma}\right|_{x=\sigma}^{+} > 0$$

(iv) $\left.V_2(\sigma,X,x)\right|_{x=\sigma}^{+} > \left.V_2(\sigma,X,x)\right|_{x=\sigma}^{-}$

Now proof of the claim requires that one show that the following properties are inconsistent.

(i) $$\left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{x=\sigma}^{-} < 0$$

(ii) $$\left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{x=\sigma}^{+} > 0$$

(iii) $$\left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{\substack{x=0\\R=0}} > 0$$

(iv) $$\left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{x=1} \leq 0$$

(v) $$\left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{x=r_3} =0$$

$$\text{(vi)} \quad \left.\frac{\partial V_2(\sigma,R,x)}{\partial X}\right|_{x=s_3} = 0$$

$$\text{(vii)} \quad V_2(\sigma,R,x)\Big|^{+}_{x=\sigma} < V_2(\sigma,R,x)\Big|^{-}_{x=\sigma}$$

Now (i) implies that

$$(1-\sigma)W_X(\sigma,X) < -\sigma\beta W_R(r_2,\sigma)(1-r_2)(1-\sigma) -$$

$$(1-\sigma)\ \beta \int_{\sigma}^{\bar{\sigma}_2} \frac{\partial V_1(z,\sigma,x_2^*)}{\partial R} dz - (1-\sigma)\ \beta \int_{\bar{\sigma}_2}^{1} W_R(z,\sigma)\,dz\ ,$$

and (ii) implies that

$$(1-\sigma)W_X(\sigma,X) > W(\sigma,X) + \beta W(r_2,\sigma)(1-r_2) -$$

$$\beta \int_{\sigma}^{\bar{\sigma}_2} \frac{\partial V_1(z,\sigma,x_2^*)}{\partial R} dz - \int_{\bar{\sigma}_2}^{1} \beta W_R(z,r)\,dz\ .$$

But (i) and (ii) together imply that

$$\Big[W(\sigma,X) + \beta W(r_2,\sigma)(1-r_2) - \beta \int_{\sigma}^{\bar{\sigma}_2} \frac{\partial V_1(z,\sigma,x_2^*)}{\partial R} dz -$$

$$\beta \int_{\bar{\sigma}_2}^{1} \beta W_R(z,\sigma)\,dz\Big] < W_X(\sigma,X)(1-\sigma) <$$

$$-\sigma(1-\sigma)\beta W_R(r_2,\sigma)(1-r_2) - (1-\sigma)\beta \int_{\sigma}^{\bar{\sigma}_2} \frac{\partial V_1(z,\sigma,x_2^*)}{\partial R} dz$$

$$- (1-\sigma)\beta \int_{\bar{\sigma}_2}^{1} W_R(z,\sigma)\,dz\ ,$$

which in turn implies that

$$0 < W(\sigma,X) + \beta W(r_2,\sigma)(1-r_2) <$$

$$-\sigma(1-\sigma)\beta W_R(r_2,\sigma)(1-r_2) + \beta\ \sigma\Big[\int_{\sigma}^{\bar{\sigma}_2} \frac{\partial V_1(z,\sigma,x_2^*)}{\partial R}dz +$$

$$\int_{\bar{\sigma}_2}^{r_2} \frac{\partial V_1(z,\sigma,z)}{\partial R}dz + \int_{r_2}^{1} W_R(z,\sigma)dz\Big]$$

The first conradiction is that if $W_{XR} \leq 0$ then $-\sigma(1-\sigma)\beta W_R(r_2,\sigma)(1-r_2) + \beta \int_{r_2}^{1} W_R(z,\sigma)dz < 0$. A second contradiction which holds for $W_{XR} > 0$ follows from the fact that $0 < W(\sigma,X) + \beta W(r_2,\sigma)(1-r_2) < -\sigma(1-\sigma)\beta W_R(r_2,\sigma)(1-r_2)$. However if $W(r_2,\sigma)(1-r_2)$ is <u>concave</u> in σ, then $W(r_2,\sigma)(1-r_2) > \sigma W_R(r_2,\sigma)(1-r_2)$ since it is a well known result that $f'(x)x < f(x)$ where $f'(x) < 0$ and $f''(x) < 0$.

All that remains in the proof of Lemma 7 is to describe the comparative statics properties of $\bar{\sigma}_3$. Taking the total derivative of

$$V_2(\bar{\sigma}_3,X,\bar{\sigma}_3) = V_2(\bar{\sigma}_3,X,r_3)$$

and rearranging terms gives

$$\frac{d\bar{\sigma}_3}{dX} = \frac{W_R(r_3,X)(1-r_3) - W_R(\bar{\sigma}_3,X)}{\left.\frac{\partial V_2(\bar{\sigma}_3,X,x)}{\partial X}\frac{dx}{d\sigma}\right|_{\substack{x=\sigma_3 \\ R=X^3}}}$$

which is always positive if $W_{XR} \geq 0$.

Similarly, again taking a total derivative and using Appendix II one gets

$$\frac{d\bar{\sigma}_3}{dc} = \frac{\left[\left.\frac{\partial V_2(\bar{\sigma}_3,X,x)}{\partial c}\right|_{x=\bar{\sigma}_3} - \frac{\partial V_2(\bar{\sigma}_3,X,r_3)}{\partial c}\right]}{\left.\frac{\partial V_2(\bar{\sigma}_3,X,x)}{\partial X}\frac{dx}{d\sigma}\right|_{x=\bar{\sigma}_3}} \leq 0 .$$

<u>Proof of Lemma 8</u>: Recall that $\underline{\sigma}_3$ is defined by the first order condition

$$\left.\frac{\partial V_2(\underline{\sigma}_3,X,x)}{\partial X}\right|_{x=\underline{\sigma}_3} = 0 .$$

Taking the total derivative, gives

$$\frac{d\underline{\sigma}_3}{dX} = - \frac{\partial V_2(\underline{\sigma}_3,X,s_3)}{\partial X \partial R} \quad \frac{\partial V_2(\underline{\sigma}_3,X,s_3)}{\partial X^2}$$

which from Appendix II is always positive if $V_2(\sigma,X,x)$ is concave and $W_{XR} \geq 0$. Next

$$\frac{d\bar{\sigma}_3}{dc} = - \frac{\partial^2 V_2(\bar{\sigma}_3,X,s_3)}{\partial c \partial X} \quad \frac{\partial^2 V_2(\underline{\sigma}_3,X,s_3)}{\partial X^2} = 0$$

since

$$\frac{\partial^2 V_2(\underline{\sigma}_3,X,s_3)}{\partial c \partial X} = 0$$

<u>Proof of Lemma 9</u>: Taking the total derivatives of

$$\frac{\partial V_2(\sigma,X,r_3)}{\partial X} = 0 \quad \text{and}$$

$$\frac{\partial V_2(\sigma, X, s_3)}{\partial X} = 0 \quad \text{gives}$$

$$\frac{dr_3}{dX} = - \frac{\partial V_2(\sigma, X, r_3)}{\partial X \partial R} \Big/ \frac{\partial V_2(\sigma, X, r_3)}{\partial X^2}$$

and

$$\frac{ds_3}{dX} = - \frac{\partial V_2(\sigma, X, s_3)}{\partial X \partial R} \Big/ \frac{\partial V_2(\sigma, X, s_3)}{\partial X^2} .$$

Then if $V_2(\sigma, X, x)$ is concave, $W_{XR} \geq 0$ implies that $\frac{dr_3}{dX} > 0$ and $W_{XR} > 0$ implies $\frac{ds_3}{dX} > 0$.

Similarly

$$\frac{ds_3}{d\sigma} = - \frac{\partial V_2(\sigma, X, s_3)}{\partial X \partial \sigma} \Big/ \frac{\partial V_2(\sigma, X, s_3)}{\partial X^2} .$$

Again, if $V_2(\sigma, X, x)$ is concave, $W_{XR} > 0$ implies $\frac{ds_3}{d\sigma} > 0$.

Finally, again taking the total derivative of the first order condition implies

$$\frac{ds_3}{dc} = \frac{dr_3}{dc} = 0 \quad \text{since}$$

$$\frac{\partial V_2(\sigma, X, s_3)}{\partial X \partial c} = \frac{\partial V_2(\sigma, X, r_3)}{\partial X \partial c} = 0 .$$

<u>Proof of Lemma 10</u>:

(i) $V_2(\sigma, X, x)$ is strictly quasi-concave with respect to x since it is the sum of a strictly concave function $W(x, R)$, and a function $V_1(\sigma, x, x_2^*(x))$ which is strictly decreasing in x. From Lemma 3 one knows that

$$W_X(r_2, X)(1-r_2) - W(r_2, X) = 0$$

which implies from Appendix II that

$$\left.\frac{\partial V_2(\sigma, X, x)}{\partial X}\right|_{x=r_2(X)} < 0 .$$

Therefore $V_2(\sigma, X, x)$ strictly quasi-concave implies $r_3(X) < r_2(X)$.

(ii) Since $x_2^*(X) \geq \sigma$ for all X and $x_2^*(X) = \sigma$ for all $\sigma \geq \bar{\sigma}_2(X)$, and $s_3(X) \leq \sigma\ \forall\ X$ it must be that $s_3(X) \leq x_2^*(X)$.

(iii) Define

$$Q_2(\sigma, X) = W(r_2, X)(1-r_2) - c - W(\sigma, X)$$

and

$$Q_3(\sigma, X) = W(r_3, X)(1-r_3) + \beta \int_\sigma^1 V_1(z, \sigma, x_2^*) dz$$

$$-W(\sigma, X) - \beta \int_0^\sigma V_1(\sigma, \sigma, x_2^*) dz - \beta \int_\sigma^1 V_1(z, \sigma, s_2^*) dz$$

Then

$$\frac{\partial Q_2(\sigma, X)}{\partial \sigma} = -W_X(\sigma, X) < 0$$

and

$$\frac{\partial Q_2(\sigma, X)}{\partial \sigma^2} = -W_{XX}(\sigma, X) > 0 .$$

Similarly,

$$\frac{\partial Q_3(\sigma,X)}{\partial\sigma} = -W_X(\sigma,X) - \sigma\beta\frac{\partial V_1(\sigma,\sigma,x_2^*)}{\partial R} -$$

$$\beta\int_{\sigma}^{1}\frac{\partial V_1(z,\sigma,x_2^*)}{\partial R}dz .$$

And if $V_2(\sigma,X,x)$ is concave

$$\frac{\partial Q_3(\sigma,X)}{\partial\sigma^2} > 0 .$$

Since

$$\frac{\partial V_2(\sigma,R,X)}{\partial R} < 0 ,$$

$$\frac{\partial Q_2(\sigma,X)}{\partial\sigma} < \frac{\partial Q_3(\sigma,X)}{\partial\sigma} , \text{ for all } \sigma,X .$$

This result implies that as the firms' planning horizon increases, the difference beween the value of an optimal risky vs safe program becomes less sensitive to the firm's level of private knowledge. This tends to make it more likely that a firm sets a risky target, (all other things equal) — the longer the firms' planning horizon. In other words,

$$\frac{\partial Q_2(\sigma,X)}{\partial\sigma} < \frac{\partial Q_3(\sigma,X)}{\partial\sigma}$$

implies that in going from a 2 to a 3 period R and D project, if $\bar{\sigma}_3 > 0$, the set of values for σ over which the firm will set a risky target, increases. However an ambiguity in the sign of $\left[\bar{\sigma}_3 - \bar{\sigma}_2\right]$ evaluated at X = 0 arises because the magnitude of

$$Q_3(X,X)\Big|_{X=0}$$

relative to

$$Q_2(X,X)\Big|_{X=0}$$

is indeterminate. (See Figure A1) One must impose stronger conditions on the reward function to resolve this ambiguity.

For example if

$$Q_3(X,X)\Big|_{X=0} \geq Q_2(X,X)\Big|_{X=0}$$

then one can unambiguously state that $\bar{\sigma}_3(X) \geq \bar{\sigma}_2(X)$ for all c, X.

<u>Proof of Lemma 11</u>:

Let $T_3(\sigma,X) = \int_{\sigma}^{1}\left[V_2(z,X,s_3^*) - V_2(\sigma,X,x_3^*)\right]dz.$

Totally differentiating $T_3(\sigma_3^*,X) = c$ and rearranging terms one gets

$$\frac{d\sigma_3^*}{dX} = -\frac{\partial T_3(\sigma_3^*,X)}{\partial X}\Bigg/\frac{\partial T_3(\sigma_3^*,X)}{\partial \sigma}$$

$$\frac{d\sigma_3^*}{dc} = 1\Bigg/\frac{\partial T_3(\sigma_3^*,X)}{\partial \sigma}$$

Since $\partial^2 V_2(\sigma,X,s_3)/\partial X\partial R = W_{XR}(s_3,X) \geq 0$, $\partial V_2(\sigma,R,X)/\partial\sigma > 0$ and $\partial V_2(\sigma,X,s_3)/\partial R = W_R(s_3,X) < 0$; $W_{XR} > 0$ implies that $\frac{d\sigma_3^*}{dX} > 0$. Next $\frac{d\sigma_3^*}{dc} < 0$ is implied by $\frac{\partial V_2(\sigma,R,X)}{\partial \sigma} > 0$.

Proof of Lemma 12:

Recall that

$$T_2(\sigma,X) = \beta\int_\sigma^1 \left[V_1(z,X,s_2^*(z)) - V_1(\sigma,X,x_2^*(\sigma))\right]dz \ .$$

This can be rewritten as

$$T_2(\sigma,X) = \begin{cases} \beta\int_{\bar{\sigma}_2}^1 W(z,X)dz - \beta(1-\bar{\sigma}_2)\left[W(r_2,X)(1-r_2) - c\right] & \sigma < \bar{\sigma}_2 \\ \beta\int_\sigma^1 \left[W(z,X) - W(\sigma,X)\right]dz & \text{if } \sigma \geq \bar{\sigma}_2 \ . \end{cases}$$

Then

$$\frac{\partial T_2(\sigma,X)}{\partial\sigma} = \begin{cases} 0 \quad \text{if} \quad \sigma < \bar{\sigma}_2 \\ -\beta(1-\sigma)W_X(\sigma,X) & \text{if} \quad \sigma \geq \bar{\sigma}_2 \end{cases}$$

Also $T_2(1,X) = 0$ and

$$\frac{\partial T_2(\sigma,X)}{\partial\sigma^2} = \begin{cases} 0 & \text{if} \quad \sigma < \bar{\sigma}_2 \\ \beta W_X(\sigma,X) - \beta(1-\sigma)W_{XX}(\sigma,X) > 0 & \text{if} \quad \sigma \geq \bar{\sigma}_2 \end{cases}$$

Similarly

$$T_3(\sigma,X) = \int_\sigma^1 \left[V_2(z,X,x_3^*(z)) - V_2(\sigma,X,x_3^*)\right]dz$$

which can be rewritten as

$$T_3(\sigma,X) = \begin{cases} \beta\int_{\bar{\sigma}_3}^1 \beta\left[V_2(Z,X,x_3^*(z)) - V_2(\sigma,X,r_3)\right]dz & \text{if} \quad \sigma < \bar{\sigma}_3 \\ \int_\sigma^1 \beta\left[V_2(z,X,z) - V_2(\sigma,X,\sigma)\right]dz & \text{if} \quad \bar{\sigma}_3 \leq \sigma \leq \underline{\sigma}_3 \\ \int_\sigma^1 \beta\left[V_2(z,X,s_3(Z)) - V_2(\sigma,X,s_3(\sigma)\right]dz & \text{if} \quad \sigma \geq \underline{\sigma}_3 \end{cases}$$

Note that $T_3(1,X) = 0$. Moreover if the firm's level of private knowledge, currently active target, and research costs are held constant, then

$$T_3(X,X) \geq T_2(X,X)$$

i.e. the value of conducting new research when a firm has exhausted its inventory of private knowledge cannot decrease as the planning horizon increases.

Taking the derivative of $T_3(\sigma,X)$ with respect to private knowledge one gets

$$\frac{\partial T_3(\sigma,X)}{\partial \sigma} = \begin{cases} 0 & \text{if} \quad \sigma < \bar{\sigma}_3 \\ -\beta(1-\sigma)\dfrac{\partial V_2(\sigma,X,x)}{\partial x}\dfrac{\partial X}{\partial \sigma}\Big|_{x=\sigma} & \bar{\sigma}_3 \leq \sigma \leq \underline{\sigma}_3 \\ -\beta(1-\sigma)\dfrac{\partial V_2(\sigma,X,s_3(\sigma))}{\partial \sigma} & \sigma > \underline{\sigma}_3 \end{cases}$$

But $\dfrac{\partial V_2(\sigma,X,x)}{\partial X}\dfrac{\partial x}{\partial \sigma}\Big|_{x=\sigma}$

$$= W_X(\sigma,X) + \beta \int_0^{\sigma} \frac{\partial V_1(\sigma,\sigma,x_2^*)}{\partial R} dz$$

$$+ \beta \int_{\sigma}^{1} \frac{\partial V_1(z,\sigma,x_2^*}{\partial R} dz \geq 0$$

and

$$\frac{\partial V_2(\sigma, X, s_3)}{\partial \sigma} = \begin{cases} 0 & \sigma < \bar{\sigma}_2(s_3) \\ -\sigma\beta W_X(\sigma, s_3) & \bar{\sigma}_2 \leq \sigma < \sigma_2^* \\ -\beta W_X(\sigma, s_3) & \sigma \geq \sigma_2^* \end{cases}$$

Define $\hat{\beta}$ as a threshhold values of a firm's discounting factor, where $0 < \hat{\beta} < 1$.

Suppose that $\bar{\sigma}_3 \geq \bar{\sigma}_2$. Then

(i) $\sigma_3^*(X) > \sigma_2^*(X)$

for all c sufficiently large. Also

(ii) $\sigma_3^*(X) > \sigma_2^*(X)$

for all X, c is implied by

$$\beta W_X(\sigma, s_3) > W_X(\sigma, X)$$

which holds if $W_{XR} \geq 0$ and $1 > \beta > \hat{\beta}$. Otherwise $\sigma_2^* \gtrless \sigma_3^*$.

Now if $\bar{\sigma}_3(X) < \bar{\sigma}_2(X)$ then $\sigma_3^*(X) > \sigma_2^*(X)$ for all X,c is implied by

$$T_3(X,X) - T_2(x,X) > \mathfrak{t}$$

and $\beta W_X(\sigma, s_3) > W_X(\sigma, X)$. Otherwise, $\sigma_3^* \gtrless \sigma_2^*$.

<u>Proof of Theorem 1</u>: All parts of the induction are immediate from any analysis of t = 1,2,3 except part c(iii). For all $t > 2$

$$\left.\frac{\partial V_{t-1}(\sigma, X, s_t)}{\partial X}\right|_{R=X} = 0 \ .$$

Therefore $s_{t+1} < s_t$ is implied by

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} > 0 .$$

Consider $\sigma > \sigma^*_t \geq \sigma^*_{t-1}$. Note that

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} = W_X(s_{t+1}, X) + \beta \frac{\partial V_{t-2}(\sigma, s_{t+1}, x^*_{t-1})}{\partial R}$$

and

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} = 0 .$$

Then

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} = \beta\left[\frac{\partial V_{t-2}(\sigma, s_{t+1}, x^*_{t-1})}{\partial R} - \frac{\partial V_{t-1}(\sigma, s_{t+1}, x^*_t)}{\partial R}\right] .$$

If $\sigma \geq \bar{\sigma}_{t-2}$ then

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} = \beta\left[W_R(x^*_{t-1}, s_{t+1}) - W_R(x^*_t, s_{t-1})\right]$$

By induction $x^*_{t-1} > x^*_t$. Therefore $W_{XR} > 0$ implies

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} > 0 .$$

If $\sigma < \bar{\sigma}_{t-2}$, then

$$\frac{\partial V_{t-1}(\sigma, X, s_{t+1})}{\partial X} = \beta\left[W_R(x^*_{t-1}, s_{t+1})(1-x^*_{t-1}) - W_R(x^*_t, s_{t+1})\right]$$

which is positive for $W_{XR} \geq 0$. An analogous argument shows that $x^*_{t-1} > x^*_t$ and $W_{XR} \geq 0$ implies $r_{t+1} < r_t$.

CHAPTER 4

A MODEL OF R AND D ASSISTANCE BY A SPONSOR

I. INTRODUCTION

Over the last decade there has been concern among some scholars that the rate of innovation in key sectors of the U.S. economy is slowing down.[1] High income tax rates, overregulation, market failures, emphasis on less productive environmental and weapons research, and even diminishing returns to improvements in existing technologies have all been cited as potential culprits in a slow down of American innovation.[2]

Faced with higher research and marketing costs of new products, industry has looked to federal agencies, particularly DOD and NASA to fund an increasing share of the nation's industrial research and development.[3] In addition Congress has provided federal agencies with the funds to accelerate socially useful innovations, where progress by the private sector is perceived to be too slow.[4] However if government agencies and private research sponsors are to encourage industrial innovation, they must understand the impact upon private firms of alternative strategies for sponsoring research.

This essay contributes to this need by modeling the influence of various types of research assistance on a firm's internal investment in a number of alternative research projects. Research 'contracts' where the government sponsor is buying knowledge and/or technology for direct government use, and assistance relationships with nonprofit organizations will not be considered here.

II. THE MODEL

It is useful to begin with a simple two period investment model which considers privately funded R and D, particularly applied research and engineering development, a risky but profit oriented activity. The model makes a number of simplifying assumptions about the nature of the research process and means of sponsorship which enables one to focus on the impact of alternative funding policies on a firm's commitment of private resources to a project.

The manager or owner of a firm is assumed to have continuous subjective probability distributions conditional upon R and D funding levels which represent his/her beliefs about the future market value of a finite number of alternative research projects. The distribution functions, which represent the researcher's assessment of both technical and market uncertainty, are continuous, with first and second moments twice differentiable in the firm's research effort. In the first period the firm selects a level of research support for each project. In the second period the innovating firm conducts research and observes the realization of the random variable determining the value of an R and D venture. It then earns a return either by auctioning off proprietary rights, or marketing the innovation itself. Let $F^i(\pi_i/R_i)$ be the innovator's subjective probability distribution function over the market value of project i, conditional upon research expenditures R_i. Second order stochastic dominance in R_i is assumed which implies diminishing returns to the expected market value associated with increased research support.[5]

Furthermore let the owner of the firm be risk neutral regarding uncertainty over the future value of each project in the firm's portfolio. This is consistent with the concept of self insurance through diversification. Additional assumptions implicit to the model include:

A1. Research effort on project i does not affect the likelihood of success of project j, i.e. there are no externalities to research.

A2. Given any R and D allocation the value of each project is stochastically independent across projects, that is, the technologies being developed by the firm are not close economic substitutes and macroeconomic correlations have been netted out of the profit distributions.

A3. A finite number of projects can continuously absorb R and D funds and new projects are not added to the firm's portfolio over the two period planning horizon.

A4. When a firm participates in government sponsored research it does not destroy its proprietary position vis a vis a technology or research area targeted for government subsidy. In other words the detailed findings of a firm's research are not available to competitors without reimbursement of the firm. Therefore the firm's probability distribution over the market value of an innovation does not shift because of the award of a grant by a sponsor.

The assumption that participation in sponsored research does not damage a firm's proprietary position is less appropriate for government contract research. Here detailed information may be revealed to a sponsor who often assumes ownership rights, or reveals new public knowledge, thereby weakening a researcher's competitive position. In an assistance relationship, the sponsor only needs the details of a firm's research, to the extent that this information enables one to track the firm's research progress, audit expenditure, and efficiently award grants to promising projects.

Despite the model's strong assumption about the nature of the research process and the institutional characteristics of sponsored research, it is a reasonable representation of government assistance programs which seek to encourage innovation through grants, tax credits, cost shared research, and other non-contractual subsidies of private research ventures.

Model 1 - Inelastic Supply of Research Funds

It is useful to begin with a research firm that has a fixed research budget to allocate between k projects. Its objective in the first period is to allocate funds so as to maximize the total expected discounted value of its portfolio of research projects.

Therefore, the manager

$$(1) \quad \underset{R_1..R_k}{\text{MAX}} \quad L = \sum_{i=1}^{k} \frac{\int_0^{\bar{\pi}_i} (1-F^i(\pi_i/R_i))d\pi_i}{1 + r} + \lambda(\sum_{i=1}^{k} R_i - R)$$

The logic of one is as follows:

R_i = total research expenditures on project i, i=1, k and $R_i \geq 0$.

π_i = a random variable representing the market value of project i in period 2.

$\bar{\pi}_i$ = most favorable outcome of π_i, $0 < \pi_i \leq \bar{\pi}_i$.

r = real discount rate, $0 < r < 1$.

$F^i(\pi_i/R_i)$ = Cumulative probability distribution associated with the market value of project i , which is conditional upon research expenditures R_i.

λ = Langrangian multiplier, $\lambda < 0$.

R = The firm's total research budget.

The analytical form of the expectation, as it appears in equation 1, is derived from a fundamental probability theorem obtained by integrating $\int_0^{\bar{\pi}_i} \pi_i dF_i$ by parts, i.e., that the area between a cumulative distribution function and a unit constant function is the mean of the distribution.

The first order conditions associated with an interior optimum for equation (1) are

$$(2) \quad \int_0^{\bar{\pi}_i} F_{R_i} (\pi_i/R_i^*) \, d\pi_i = \int_0^{\bar{\pi}_j} F_{R_j} (\pi_j/R_j^*) \, d\pi_j = \lambda \text{ for all } i \neq j.$$

$$(3) \quad \sum_{i=1}^{k} R_i^* = R, \text{ where } R_i^* \text{ is the optimal level of funding for the } i^{th} \text{ project.}$$

However if $\int_0^{\bar{\pi}_i} F_{R_i} (\pi_i/R_i) \, d\pi_i < \lambda$ for all $R_i > 0$, then $R_i^* = 0$.

In otherwords, if the marginal expected value of spending a dollar more on a project is always less than the decrease in added value of not spending an additional dollar elsewhere, the firm should not fund the project.

The first order conditions imply that the firm determines the best allocation of research funds, by equating marginal expected discounted values across projects. In addition, equations (2) and (3) are sufficient for a unique maximum, given the assumptions of diminishing returns and stochastic independence between projects.

It is a straight forward comparative statics result that if the firm's total research budget is somehow increased from R to R' (perhaps because of an unexpected increase in company sales or a general tax credit for research), all projects funded under the original regime, R, receive increased funding at R'.

More formally,

<u>Lemma 1</u>: If $R_i^* > 0$, $\frac{dR_i^*}{dR} > 0$ for all i.

This lemma follows from the assumption that $F_{R_iR_i}(\pi_i/R_i) < 0$.

Now that the basic model has been outlined, sponsored research, directed at specific projects, can be introduced by assuming that expenditures on a project come from two sources, one internal to a company (P_i), and a subsidy (S_i) provided by a sponsor. Of course $R_i = P_i + S_i$. Define $P_i^*(S_i)$ to be the optimal level of private funding for the i^{th} project when the firm receives a vector of categorical grants from a sponsor $S_1 \ldots S_k$. Now in equation 1, $S_i = 0$ for all i, which implies that $P_i^*(0) = R_i^*$, for, i=1,k, i.e. all ventures are internally funded.

Now suppose a sponsor decides to subsidize venture j by awarding a categorical grant, where the size of the grant is significantly less than the size of the firm's total research budget. Lemma 2 describes the firm's response to the grant.

<u>Lemma 2</u>:

i) $P_j^*(0) \leq S_j$ implies $-1 < \frac{dP_j^*(S_j)}{dS_j} < 0$

ii) $P_i^*(0) > 0$, $P_j^*(0) > 0$ implies $1 > \frac{dP_i^*}{dS_j} > 0$ for all $i \neq j$

iii) $P_j^*(0) = 0$ implies $R_j^*(S_j) = S_j$ and $\frac{dP_i^*}{dS_j} = 0$ for all $i \neq j$.

A categorical research grant to a project which would have received positive private funding, in any case, is likely to increase the total expenditures on the venture, but by less than the amount of the categorical grant. In addition all other projects which would have been funded in the absence of the grant will receive increased funding once the grant is awarded.

However if a research venture would not be funded by a firm in the absence of a grant, a corner solution occurs, where the firm spends precisely the amount of the grant on the project targeted by the sponsor.

In addition, the firm's level of effort on projects it considers viable for internal funding will not be affected by the award of a grant to a venture that the firm perceives as less cost effective at the margin.

That Lemma 2 obtains can be seen by the fact that after a grant is awarded, planned research expenditures on project j become $P_j^*(0) + S_j$. As a result the first order conditions no longer hold and a firm's internal funds are reallocated to the other k-1 projects, until a new equilibrium is restored.

Before relaxing the assumption that the supply of capital for the firm's research budget is inelastic with respect to grants or interest rates, a further simplification of the model may be helpful for planning purposes. Suppose that the owner's subjective probability distributions over the market value of each project, conditional upon research spending, are identical across projects. Then, as expected, the equilibrium allocation without government subsidy is $P_i^*(0) = R/k$ for all $i = 1,k$. A categorical grant awarded to an individual project or alternatively a block grant for the same amount of money, awarded to the firm's total research budget, is divided equally among the k projects.

Model 2 - Elastic Supply of Research Funds

The first model assumed that the supply of private research funds was inelastic with respect to interest rates. This is consistent with the funding of a company's research division on the basis of an annual percentage of sales or profits. In addition, it was assumed that research grants fell as manna from heaven, and did not influence, nor were the grants influenced by, a firm's total internal R and D budget and allocation of funds between projects. These assumptions will now be relaxed. The research firm will be able to sell research and development bonds at the start of the first period and thereby privately fund its research by borrowing up to some $\bar{R}$. These bonds plus an interest rate b will be payable at the end of the second period, after the random market value of all the firm's research projects is realized. The firm's new objective function becomes

$$(4) \quad \max_{P_1 \ldots P_k} \sum_{i=1}^{K} \int_0^{\bar{\pi}_i} \frac{[1-F^i(\pi_i/P_i + S_i)]}{1+r} \, d\pi_i - \sum_{i=1}^{k} \frac{P_i(1+b)}{(1+r)}$$

This gives the first order conditions

$$(5) \quad - \int_0^{\bar{\pi}_i} F^i_{R_i} \, (\pi_i/P_i + S_i) \, d\pi_i = 1+b \qquad i = 1,k.$$

Therefore, the firm borrows to finance its research portfolio until the expected marginal value of the least attractive project, i.e., the project to which the firm makes the smallest financial commitment, equals one plus the rate of interest on the bonds. Now let a sponsor award a categorical grant S_j to project j. Again define $P_j^*(0)$ to be the firm's optimal level of internal funding for a venture, when it anticipates no research sponsorship.

<u>Lemma 3</u>:

i) If $P_j^*(0) \geq S_j$, then $\frac{dP_j^*(S_j)}{dS_j} = -1.0$

ii) If $P_j^*(0) < S_j$ then $\frac{dP_j^*(S_j)}{dS_j} = 0$ and $P_j^*(S_j) = 0$

iii) $\frac{dR_i^*}{dS_j} = 0$ for all $i \neq j$

Lemma 3 suggests that a categorical grant awarded to a venture which would have been privately funded without a grant, at a level of effort greater than or equal to the subsidy, is likely to encourage a complete substitution of public for private financing of research and development, conducted by the firm. In addition, the lemma suggests that it doesn't matter whether the grant is targeted at a specific project or simply supplements the firm's total research budget. The absolute level of spending on the research portfolio, and each individual project will be independent of the subsidy so long as the project being targeted for subsidy was perceived by the firm to be viable as a private venture. This result is robust under fairly general conditons, e.g., any class of continuous probability distributions that exhibit second order stochastic dominance in research effort. It suggests that government efforts to accelerate innovations already being financed by the private sector may be in vain.

The critical assumptions driving this analytical result are i) the firm faces a perfectly elastic supply of working capital and ii) diminishing returns in the research production function.

Suppose there exists a lower range of research expenditures on a project such that diminishing returns is not valid. Certainly, a large corporation with a multi-million dollar research budget would not operate in such a range for economically viable projects. However, it can be argued that a small business or individual inventor, concentrating on one or two projects, may have insufficient resources to reach a scale of operations where diminishing returns to perceived research gains sets in. The firm or inventor may also find private capital difficult and costly to raise. As a result, it has been suggested by policy makers that small businesses are excellent candidates for R and D assistance from a government sponsor. No doubt, under the conditions described above, government research grants might actually increase private commitments to a research venture.

It is also straight forward to show that if the firm's supply of working capital is upward sloping in interest rates (rather than perfectly elastic), a categorical grant will decrease total private expenditures on a project, but by less than the amount of the grant.

Model 3 - Incentive Grants

An alternative means of research sponsorship which may have certain advantages over lump sum categorical grants is based on the concept of matching funds. Cost sharing or matching funds formulas are utilized, particularly in assistance relationships with profit-oriented firms, where it is perceived by the sponsor that financial gains may flow to business over relatively short planning horizons. A simple matching funds relationship can be easily incorporated into the basic model. Let the firm

$$(6)\quad \underset{P_1 \ldots P_k}{\text{maximize}} \sum_{i}^{k} \int_0^{\bar{\pi}_i} \frac{[1-F^i(\pi_i/P_i + \alpha_i(P_i)P_i)]d\pi_i}{1+r} - \frac{(1+b)}{(1+r)} \sum_{i=1}^{k} P_i.$$

Here $\alpha(P_j)$ is the proportion of private funds which will be matched by the sponsor. If the firm's internal support of the venture j is P_j, for every additional dollar spent by the firm, the sponsor awards $\alpha(P_j) + \alpha'(P_j)P_j \geq 0$. The expression $\alpha'(P_j)$ refers to the first derivative of the matching function $\alpha(P_j)$.

The optimization leads to the first order condition

$$(7) \quad -\int_0^{\bar{\pi}_i} \left[F^i_{P_i} \; (\pi_i/P_i^* + \alpha_i(P_i^*)P_i^*)\right] \left[1 + \alpha_i'(P_i^*)P_i^* + \alpha_i(P_i^*)\right] d\pi_i = 1+b \text{ for all } i=1,k.$$

A sufficient condition for a global maximum is that $\alpha_i''(P_i) \leq 0$ and $\alpha_i'(P_i) \leq 0$. However, even if $\alpha_i'(P_i) > 0$, P_i^* is a maximum if $\alpha_i''(P_i^*)P_i^* + 2\alpha_i'(P_i^*) < 0$. Of course, if a project does not receive matching funds the first order condition reduces to

$$\int_0^{\bar{\pi}_i} F^i_{P_i} \; (\pi_i/P_i^*) d\pi_i = (1+b).$$

Alternatively if the matching proportion is a constant for all levels of research support

$$\int_0^{\bar{\pi}_i} F^i_{P_i} \; (\pi_i/P_i^* + \alpha P_i^*)(1 + \alpha_i) d\pi_i = 1+b.$$

Also since $1 + \alpha_i'(P_i^*)P_i^* + \alpha_i(P_i^*)$ is independent of the realization of the random variable, it can be removed from the integral, and placed on the right side of equation (6). Because $1 + \alpha_i'(P_i^*)P_i^* + \alpha_i(P_i^*) > 1$ a matching funds formula has the affect of subsidizing the interest rate the firm faces on individual projects, without affecting the rest of the firm's portfolio. This encourages the firm to increase its internal support of a venture preferred by a sponsor.

III. CONCLUSION

The impact of various forms of research sponsorship on a firm's internal investment in research ventures can be summarized with the aid of three diagrams. Figure 2 shows how a matching funds formula encourages increased private support of research at the margin, by lowering the interest rate faced on individual projects.

Similarly, the impact of a categorical grant on a firm's internal investment in a research venture is illustrated in Figure 3, again, for the case of a perfectly elastic supply of working capital. Here a lump sum award shifts the research production possibility curve to the left, encouraging the firm to reduce internal funding of a project and rely on a subsidy to finance research progress. As a result the sponsor's attempt to 'push' a specific venture may be in vain.

Finally, when a firm's total internal budget for all research ventures is fixed, categorical grants shift the research production possibility curve, up to the left. (See Figure 4.) However, there is less displacement of private funding in the constrained optimization than the unconstrained, where the supply curve for working capital was elastic. In addition, for every internal dollar not spent on a project targeted for a subsidy, a dollar is reallocated to other research projects considered viable by the firm. Thus while the sponsor's attempt to 'push' a specific innovation is not fully effective when categorical lump sum grants are awarded, research and development in general, is encouraged.

FIGURE 2

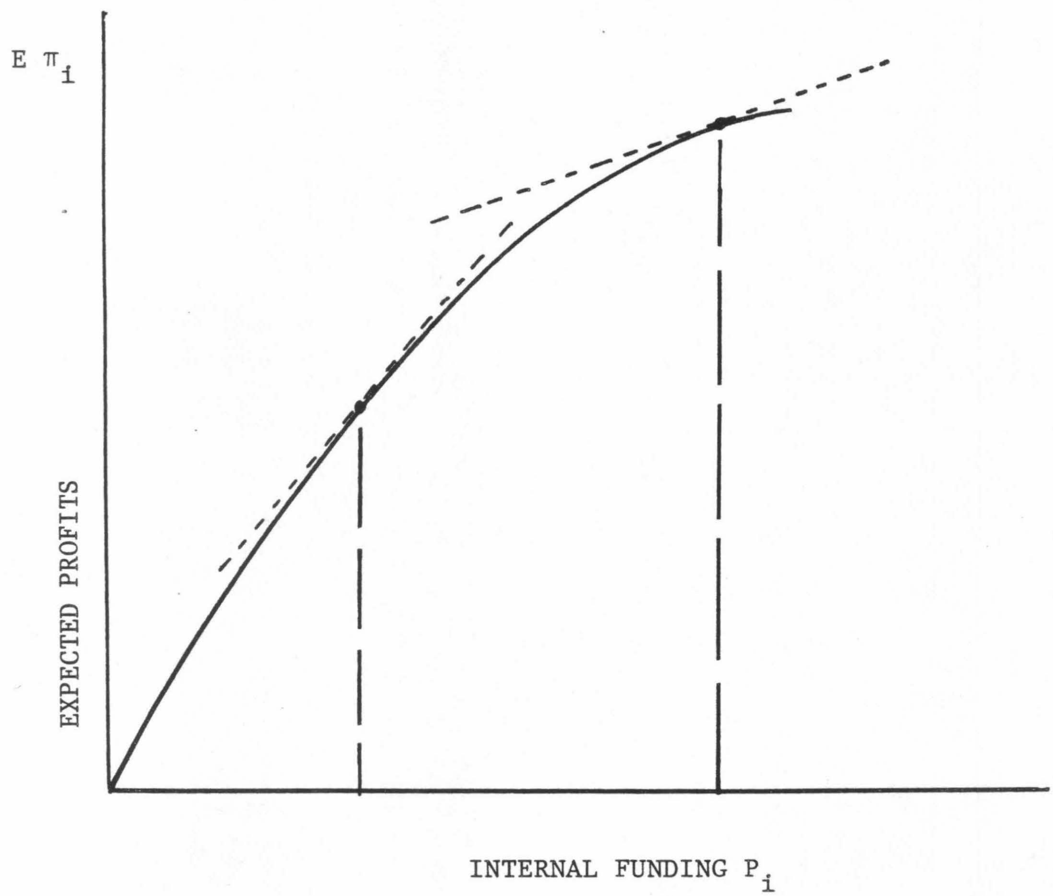


FIGURE 3

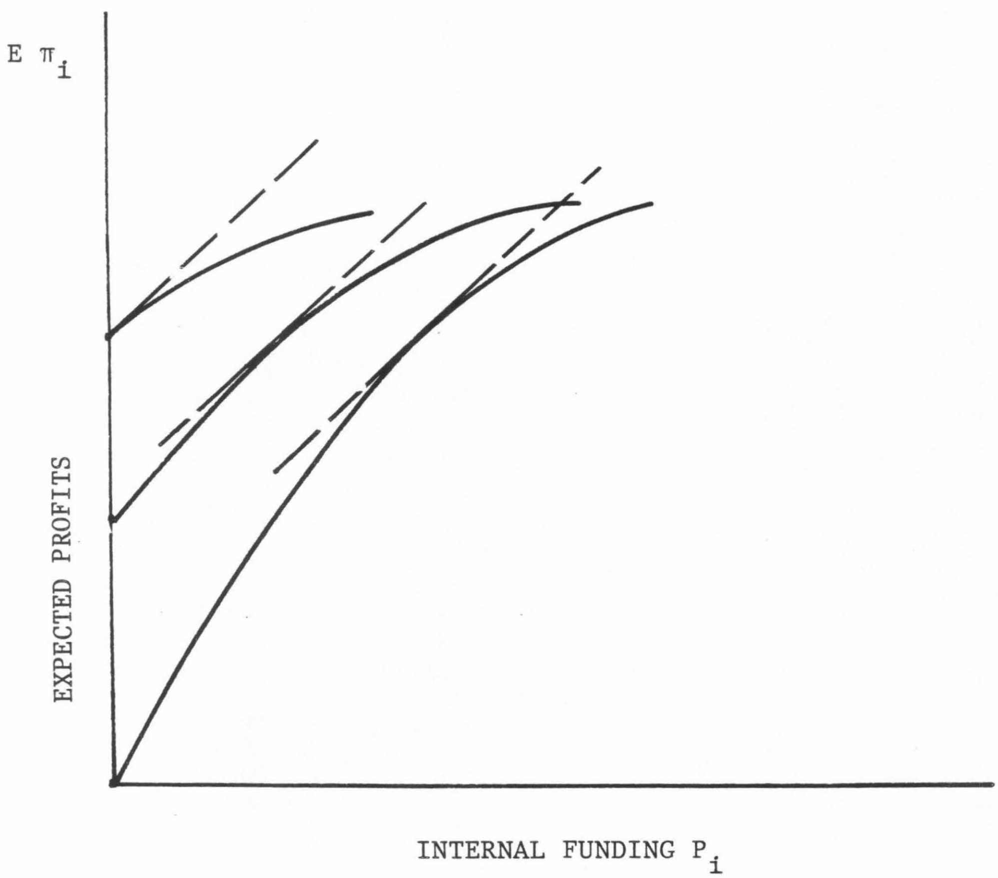
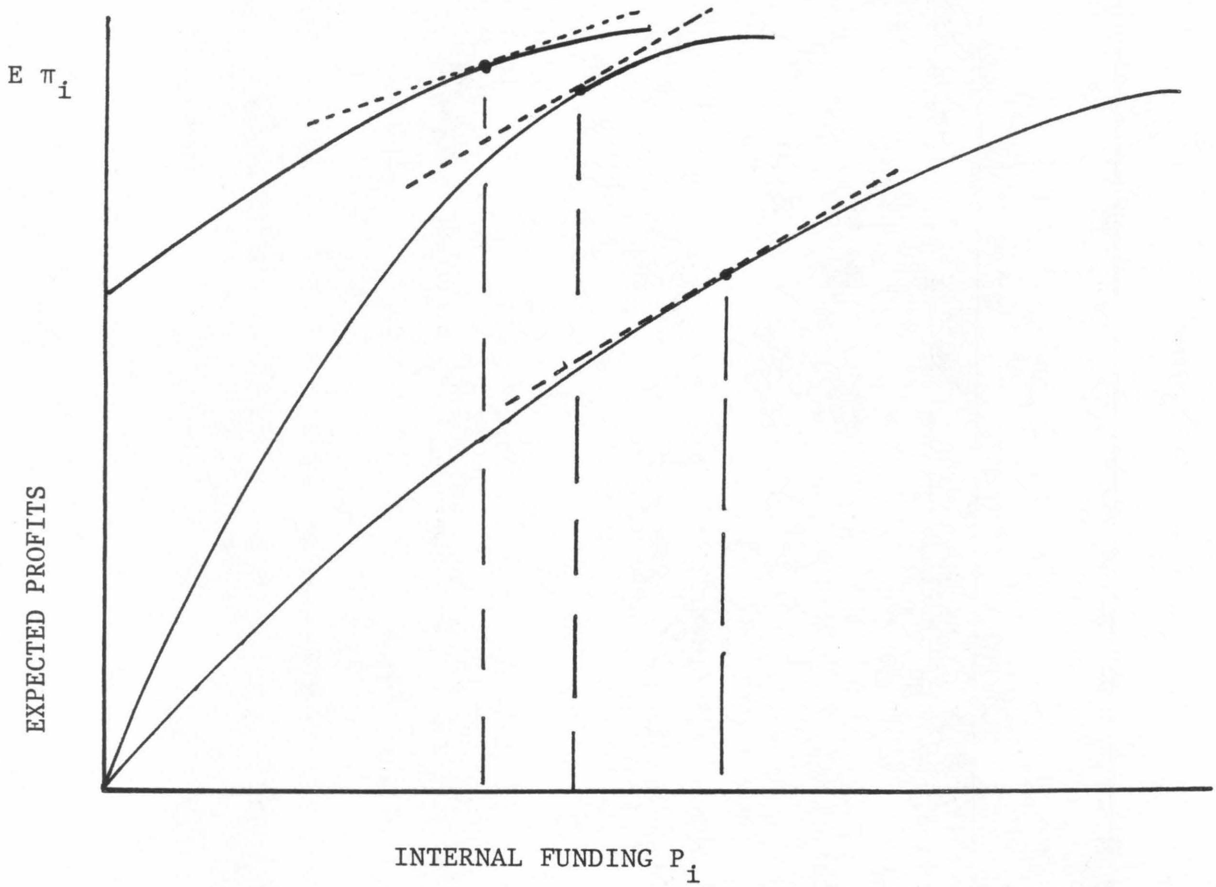


FIGURE 4



FOOTNOTES - CHAPTER 4

1. See Dennison, E. "Is R and D the Key to the Productivity Problem?," _Science_, (February 1981). Also Klein, B.H. "The Productivity Slowdown, and Its Relation to the Inflation Problem" Social Science Working Paper 286, California Institute of Technology (1979).

2. G.C. Eads, "Achieving 'Appropriate' Levels of Investment in Technological Change: What Have We Learned?," mimeo, The Rand Corporation (1977).

3. The Aerospace Industries Association, Inc., "Research and Development, A Foundation for Innovation and Economic Growth," mimeo (1980).

4. Report by the Comptroller General of the United States, "Assessing the Output of Federal Commercially Directed R and D" PAD-79-69 (1979).