TWO TOPICS IN ELEMENTARY PARTICLE PHYSICS:

(1) QUARK GRAPHS AND ANGULAR DISTRIBUTIONS IN THE DECAYS OF THE AXIAL-VECTOR MESONS

(2) UNIVERSAL CURRENT-CURRENT THEORIES AND THE NON-LEPTONIC HYPERON DECAYS

Thesis by

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ABSTRACT

The Thesis is divided into the following two parts:

(1) We examine three aspects of the axial-vector mesons: (i) angular distributions of the $I = 1$ states, (ii) mixing of the $I = 1/2$ states, and (iii) absence of the $I = 0$ states.

Using a model of mesons decaying via production of a quark-antiquark pair with the quantum numbers of the vacuum, we relate the angular distributions in the decays $A_1 \rightarrow \rho\pi$ and $B \rightarrow \omega\pi$, predicting $2(g_1/g_0)_{A_1} = (g_0/g_1)_B + 1$. This relation is consistent with the present, somewhat ambiguous experimental data. Also, we describe satisfactorily, in terms of two parameters, the partial widths of the 0^+, 1^+, and 2^+ mesons decaying into $1^-\ 0^-$ and $0^-\ 0^-$ pairs. The prediction of the model is that $SU(6)_W \times O(2)_{L_z}$ relations hold among all the D waves and among all the S waves, but not between the two groups. In fact, our two-parameter fit to the data entails a ratio of S wave to D wave amplitudes of approximately the same magnitude but opposite sign to that implied by $SU(6)_W \times O(2)_{L_z}$. Unlike the widths, the angular distributions are sensitive to the relative sign and are thus <u>crucial</u> in determining that the fit of our model differs considerably from the $SU(6)_W$ solution.

Parameters of the fit are applied to the 1^+ kaons, which may mix with one another. The results are sensitive to the mixing angle ϕ, and merely assuming lower bounds on widths of both physical states

establishes the limits $10^\circ \leq \phi \leq 35^\circ$. As a result of this mixing, one predicts: (a) the suppression of the $K^*\pi$ mode of the lower peak, (b) the suppression of the ρK mode of the upper peak, and (c) decay distributions in the $K^*\pi$ mode similar to that of the A_1 for the lower state and to that of the B for the higher.

The properties of the missing isoscalar mesons are described with particular emphasis on the ninth 1^{++} state. Expected properties of this meson, the D', include: (a) assignment to a weakly mixed SU(3) singlet, predicted by duality and confirmed by the Gell-Mann-Okubo mass formula; (b) a mass of ~950 MeV, predicted by superconvergence with assumptions about the relative couplings of D and D'; (c) decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$; and (d) the possibility of a suppressed ρ signal in the $\pi^+\pi^-$ spectrum of the $\pi^+\pi^-\gamma$ final state, despite the expectation that the pions are in a state with I = J = 1. These features suggest that a recently reported meson near this mass with decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$ may be a candidate for this state, although $J^{pc} = 1^{+-}$ is also a definite possibility for the new meson.

(2) Because of the limited evidence for the V-A Cabibbo theory in the non-leptonic weak decays, we examine the compatibility with experiment of more general current-current theories. These theories, constrained by universality, are constructed from the neutral and charged currents obtainable in the quark model, i.e., scalar, pseudoscalar, vector, axial-vector, and tensor. Using current algebra and PCAC, a certain class of these theories, including Cabibbo's, is found to be consistent

with the S wave amplitudes for the non-leptonic hyperon decays. The P wave amplitudes remain unexplained. Nevertheless, another class of theories, also including V-A, plus the assumption of a symmetric quark model, predict the $\Delta I = 1/2$ rule.

TABLE OF CONTENTS

PART 1	TITLE	PAGE
I	INTRODUCTION	1
II	ANGULAR DISTRIBUTIONS AND QUARK GRAPHS FOR THE 1^+ ISOVECTOR MESONS	26
III	MIXING OF THE 1^+ KAONS	49
IV	THE NINTH 1^{++} MESON	63
V	CONCLUSIONS	73

PART 2		
I	INTRODUCTION	81
II	UNIVERSAL CURRENT-CURRENT THEORIES	108
III	CONSEQUENCES FOR THE NON-LEPTONIC DECAYS	112
IV	CONCLUSIONS	118
APPENDIX A		119
APPENDIX B		123

PART 1

QUARK GRAPHS AND ANGULAR DISTRIBUTIONS IN THE DECAYS OF THE AXIAL-VECTOR MESONS*

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J. Rosner and E. W. Colglazier, "A Ninth Meson with $J^{pc} = 1^{++}$," Phys. Rev. Letters 26, 933 (1971).

I. INTRODUCTION

A significant advance in understanding the spectrum of mesons and baryons came from the proposal that the group SU(3) is an approximate symmetry of the strong interactions.(1) With an exact symmetry of nature, the particles can be classified into irreducible representations of the symmetry group since the generators commute with the Hamiltonian. The particles belonging to an irreducible representation are degenerate in mass. With an approximate symmetry, it may still be possible to classify the particles into nearly degenerate multiplets. Gell-Mann and Ne'eman proposed that there exist eight operators which (i) have the right commutation rules to be the generators of SU(3), (ii) approximately commute with the Hamiltonian, and (iii) tie the strongly interacting particles into nearly degenerate multiplets. The tremendous success of SU(3) is that all the well-known particles can be classified into its representations.* The existence of the symmetry does not indicate which representations are to be found in nature. The

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*Examples of classification into SU(3) representations are:

Mesons - $\underset{\sim}{8} + \underset{\sim}{1}$ of $J^{pc} = 0^{-+}$ (π, K, η, η');

$\underset{\sim}{8} + \underset{\sim}{1}$ of $J^{pc} = 1^{--}$ $(\rho, K^*, \omega, \Phi)$;

$\underset{\sim}{8} + \underset{\sim}{1}$ of $J^{pc} = 2^{++}$ (A_2, K^{**}, f, f')

Baryons - $\underset{\sim}{8}$ of $J^p = \frac{1}{2}^+$ $(N, \Sigma, \Lambda, \Xi)$, $\underset{\sim}{10}$ of $J^p = \frac{3}{2}^+$ $(\Delta, Y^*, \Xi^*, \Omega^-)$.

only representations which seem to be allowed experimentally are the singlet and octet for mesons and the singlet, octet and decimet for baryons.

Since SU(3) is an approximate symmetry of the Hamiltonian, it should describe the matrix elements as well as the spectrum of the strongly interacting particles. For example, the three particle couplings - or vertices - should be approximately invariant under transformations of the group. These couplings can be determined experimentally from the strong decays of the hadrons. However, the extraction of coupling strengths from widths is subject to ambiguities because of mass factors. Since the particles within a multiplet are not really degenerate, the value for the coupling depends on the assumed form for the variation of the matrix element with mass. Consequently, an unambiguous comparison of SU(3) predictions for matrix elements is more difficult than for masses. Nevertheless, coupling strengths determined from the strong decays of the $\frac{3}{2}^+$ baryon decimet and the 2^+ nonet are in good agreement with the symmetry predictions.*

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*Comparison of SU(3) predictions with the widths of the $\frac{3}{2}^+$ decimet[2] and 2^{++} nonet[3] yields (table also continued on next page):

$\frac{3}{2}^+$ decimet	Γ_{exp} (MeV)	Γ_{theor} (MeV)	2^{++} nonet	Γ_{exp} (MeV)	Γ_{theor} (MeV)
$\Delta(1238) \to \pi N$	120 ± 5	116	$K^{**} \to K\pi$	59 ± 4	59
$\Sigma(1385) \to \pi\Lambda$	33 ± 6	37	$A_2 \to \eta\pi$	16 ± 3	14.5
$\to \pi\Sigma$	3.6 ± 1.5	3.8	$A_2 \to K\bar{K}$	6.5 ± 1.3	7.1

Thus, SU(3) provides insight into both the spectrum and the matrix elements of the hadrons.

The hadronic spectrum in terms of spins, parities, and allowed representations cannot be understood by SU(3). An incorporation of spin with internal symmetry to explain these features of the spectrum was achieved in the quark model of Gell-Mann[4] and Zweig.[5] This model considers the observed hadrons as being constructed out of a fundamental triplet called the quarks. The mesons are made of a quark and antiquark while the baryons are made of three quarks. The quarks form a triplet representation of SU(3) and are spin $\frac{1}{2}$. The spin of a hadron is determined from adding the spin (S) and orbital angular momentum (L) of the constituent quarks. Whether the quarks are real or only a mathematical device is not known since they have yet to be discovered experimentally. Nevertheless, they are extremely useful in classifying the hadrons and describing many of their properties.

The meson spectrum is described by the quantum numbers of $q\,\bar{q}$ pair. The charge conjugation of such a pair is $(-1)^{L+S}$; and, if the parity of an antiquark is opposite to that of a quark as it is for spin $\frac{1}{2}$ particles, the parity is $-(-1)^L$. The lightest mesons

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$\frac{3}{2}^+$ decimet	Γ_{exp}(MeV)	Γ_{theor} (MeV)	2^{++} nonet	Γ_{exp}(MeV)	Γ_{theor} (MeV)
$\Xi(1530) \to \pi\,\Xi$	10 ± 3	13	$f \to \pi\pi$	150 ± 25	154
			$f' \to K\bar{K}$	53 ± 20	49

should correspond to a $q\bar{q}$ pair with $L = 0$, which consists of an octet and singlet ($\underset{\sim}{3} + \bar{\underset{\sim}{3}} = \underset{\sim}{8} + \underset{\sim}{1}$) of $J^{pc} = 0^{-+}$ and $J^{pc} = 1^{--}$. The low lying pseudoscalar and vector mesons do indeed correspond to these eighteen states. The next states in the mass spectrum should correspond to a $q\bar{q}$ pair with $L = 1$. More states are now possible because there are several ways to combine quark spin S and $L = 1$ to obtain the total angular momentum J. In particular, nine states (octet plus singlet) of each of the following J^{pc}: $0^{++}(S = 1)$, $1^{++}(S = 1)$, $2^{++}(S = 1)$, and $1^{+-}(S = 0)$. It is these mesons, expecially the axial-vectors, that we shall concentrate on in this paper. The best known are the nine tensor mesons, whose widths were seen to be in good agreement with SU(3). Note that the quark model predicts the absence of states with $J^{pc} = 0^{--}$, $(\text{odd})^{-+}$, $(\text{even})^{+-}$; and no such "exotic" states have been found.

In order to quarantee the proper isospin and hypercharge (Y = average charge in multiplet, i.e., $Q = e[I_3 + \frac{Y}{2}]$) quantum numbers of the mesons (which are to be associated with the additive quantum numbers of SU(3)), the three quarks must belong to an SU(3) doublet (p,n) and singlet (λ) in isospin with hypercharges $\frac{1}{3}$ and $-\frac{2}{3}$, respectively. Consequently, the quarks are fractionally charged.

The lowest observed baryon states are the $\frac{1}{2}^{+}$ octet of nucleons and $\frac{3}{2}^{+}$ decimet of nucleon resonances. The quark wave functions for the $\frac{3}{2}^{+}$ decimet are completely symmetric in both SU(3) and spin. If the quarks are fermions, the spatial wave function must be antisymmetric which excludes S wave, contrary to what we would expect for the lowest mass states. However, if the quarks are assumed to obey Bose statistics, the S wave q q q states consist of a $\frac{1}{2}^{+}$ $\underset{\sim}{8}$ and $\frac{3}{2}^{+}$ $\underset{\sim}{10}$. The symmetric quark model is assumed because of this agreement with the low-lying baryon spectrum. Spin $\frac{1}{2}$ bosons would seem to violate the spin and statistics theorem except that real quarks have not been seen. The next states, of negative parity, should correspond to L = 1, which consist of $J^{pc} = \frac{5}{2}^{-}$ $(\underset{\sim}{8})$, $\frac{3}{2}^{-}$ $(\underset{\sim}{1}, \underset{\sim}{8}, \underset{\sim}{8}, \underset{\sim}{10})$, and $\frac{1}{2}^{-}$ $(\underset{\sim}{1}, \underset{\sim}{8}, \underset{\sim}{8}, \underset{\sim}{10})$. All these multiplets have been established except for an $\underset{\sim}{8}$ $\frac{3}{2}^{-}$ ——— a tremendous success of the symmetric quark model.

The quark model can give a description of the breaking of SU(3) as well as predicting the spectrum. Symmetry breaking can be understood by the obvious and simple model of mass differences between the quarks. Since isospin is conserved in the strong interactions, SU(3) breaking can only split the isosinglet λ quark from the isodoublet p and n quarks. Assigning a different mass to the λ quark yields the Gell-Mann Okubo mass formula for the mesons:

$$M_K^2 = \frac{3\,M_\eta^2 + M_\pi^2}{4} \quad .$$

This prescription also produces the equal spacing rule for the baryon decimet and an equal spacing rule, not the experimentally valid G-M-O mass formula, for the baryon octet. The mass split between the Λ and Σ baryons, which both contain one strange quark, results from differences in their quark wave functions. The G-M-O mass formula does not work well for the observed vector mesons, but this anomaly can also be understood by the quark model. The quark wave functions for the $I = 0$ members of the SU(3) octet and singlet are $\frac{1}{\sqrt{6}}(p\bar{p} + n\bar{n} - 2\lambda\bar{\lambda})$ and $\frac{1}{\sqrt{3}}(p\bar{p} + n\bar{n} + \lambda\bar{\lambda})$, respectively. When SU(3) is broken by a heavier λ quark, the two states should mix, corresponding to diagonalizing the mass matrix, so that the physical states are pure strange quark and pure non-strange quark, i.e.,

$$\omega = \frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n}) = \cos\Theta\,\omega_1 + \sin\Theta\,\omega_8$$

$$\Phi = -\lambda\lambda = -\sin\Theta\,\omega_1 + \cos\Theta\,\omega_8$$

where ω_8 = octet state, ω_1 = singlet state, and $\tan\Theta = \frac{1}{\sqrt{2}}$. The mass formulae for the physical states which follow from this mixing are: $m^2(\rho) = m^2(\omega)$ and $m^2(\rho) + m^2(\Phi) = 2m^2(K^*)$. Consequently, the isosinglet vector meson states mix naturally in the quark model.* These various examples of symmetry breaking indicate that the quark model is useful for describing the fine structure as well as the gross features of the spectrum.

The incorporation of spin in the quark model may be described in terms of the higher symmetry SU(6), or its orbital extension SU(6) x O(3). In a non-relativistic description there are six states

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*The absence of mixing of the $I = 0$ 0^{-+} states is now obscure, but will be resolved by the spin symmetry implied by the quark model.

of the quark when both SU(3) and spin are included. The symmetry which describes invariance under the unitary transformations of these six states is $SU(6)_S$. The "S" refers to static because this description can only be meaningful when the particles are at rest, as only then are their spins well defined. Thus, $SU(6)_S$, unlike SU(3), cannot be symmetry of the complete Hamiltonian. The separation of spin from intrinsic orbital angular momentum is the reason for the failure of SU(6) as a dynamical symmetry. Attempts at a relativistic generalization by combining SU(3) and the Lorentz group have failed to produce a universal symmetry that could be valid for the strong interactions.[6] Yet, $SU(6)_S \times O(3)$ is very successful in describing the particle spectrum. The pseudoscalar and vector mesons can be classified at rest into a $\underset{\sim}{35}$ (two SU(3) octets - 0^{-+} and 1^{--}, plus one SU(3) singlet - 1^{--}) and a $\underset{\sim}{1}$ (SU(3) singlet - 0^{-+}) of $SU(6)_S$ ($\underset{\sim}{6} \times \underset{\sim}{6} = \underset{\sim}{35} + \underset{\sim}{1}$). (The $I = 0$ 0^{-+} states are not mixed because the SU(3) singlet is in a different $SU(6)_S$ representation from the octet.) The positive parity mesons form a $\underset{\sim}{35}$ $L = 1$. Similarly, the baryons can be classified into a $\underset{\sim}{56}$ $L = 0$ and a $\underset{\sim}{70}$ $L = 1$ representation of $SU(6)_S \times O(3)$ (and there is considerable evidence for a $\underset{\sim}{56}$ $L = 2$). Also, $SU(6)_S$ mass breaking in the spectrum can be introduced as various forms of $S \cdot S$ and $L \cdot S$ couplings ($L \cdot S$ needed for mesons but not for baryons).

We have seen that $SU(6)_S \times O(3)$ is very useful as an approximate symmetry of the spectrum, but not of the dynamical Hamiltonian. Similar situations have occurred before in physics. For example, the energy levels of the hydrogen atom possess an extra degeneracy beyond that given by rotational invariance; states of different angular momentum, but with the same principal quantum number, have the same energy. This phenomenon can be understood from the symmetry group O(4) (generators are the conserved vectors of the coulomb problem: angular momentum $\vec{J}$ plus a new one called the Lenz vector $\vec{L}$*). In the nonrelativistic problem, the generators of O(4) commute with the Hamiltonian of the hydrogen atom. A relativistic generalization of the hydrogen atom (for spinless particles) also yields the same O(4) symmetry of the energy levels.(7) However, the scattering of bound states does not possess this symmetry. Consequently, O(4), like $SU(6)_S$, is useful only for analyzing the spectrum of isolated states, not the dynamics of particle interactions.

The question now arises as to what is the dynamical symmetry of the matrix elements when spin is included. Since a complete unification of SU(3) and the Lorentz group has not succeeded, a less ambitious approach is required. Perhaps certain sets of processes, such as collinear ones, can be described according to some covariant version of SU(6). The quark model, which provides an answer for the symmetry of the spectrum, can possibly do the same for vertices. In the quark model, vertices can be drawn pictorially as "quark graphs,"(5) which are especially useful for visualizing the SU(3) structure. The in-

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*In classical physics, the Lenz vector points to the perihelion of the orbit; and its conservation means that the orbit does not precess.

clusion of spin in quark graphs could provide a clue to the dynamical symmetry of vertices.

Quark graphs are pictures of hadron vertices (Fig. 1) drawn according to the following rules: a) each quark or antiquark is represented by a directed line, b) a baryon is represented by three lines running in the same direction, c) a meson is represented by two lines running in opposite directions, d) each line must begin and end in a different hadron. Each diagram corresponds to a possible way of contracting SU(3) indicies to form an SU(3) scalar. The coupling constant of a meson vertex is proportional to the number of quark graphs, each graph being weighted by the product of normalizations of the meson wave functions. For meson-baryon vertices, the graphs are weighted by an additional factor $(f + d)$ or $(f - d)$ depending upon whether or not a mesonic quark is contracted with a quark that has been symmetrized or antisymmetrized in the baryon wave function. The couplings determined from quark graphs are no more than SU(3) results except for the last rule that no disconnected graphs are allowed. The effect of this rule, postulated by Zweig,[5] is to relate the coupling of the singlet to the octet, and is sometimes called the nonet anzatz.[8] Results that follow from this rule are the observed weakness of the $\Phi \rho \pi$, $\Phi N \bar{N}$, and $f' \pi \pi$ couplings (f' is the pure strange quark 2^+ meson). All the observed two-body strong decays are consistent with the quark graph picture of the decay taking place via creation of an additional quark-antiquark pair in an SU(3) singlet state, with no disconnected graphs allowed.

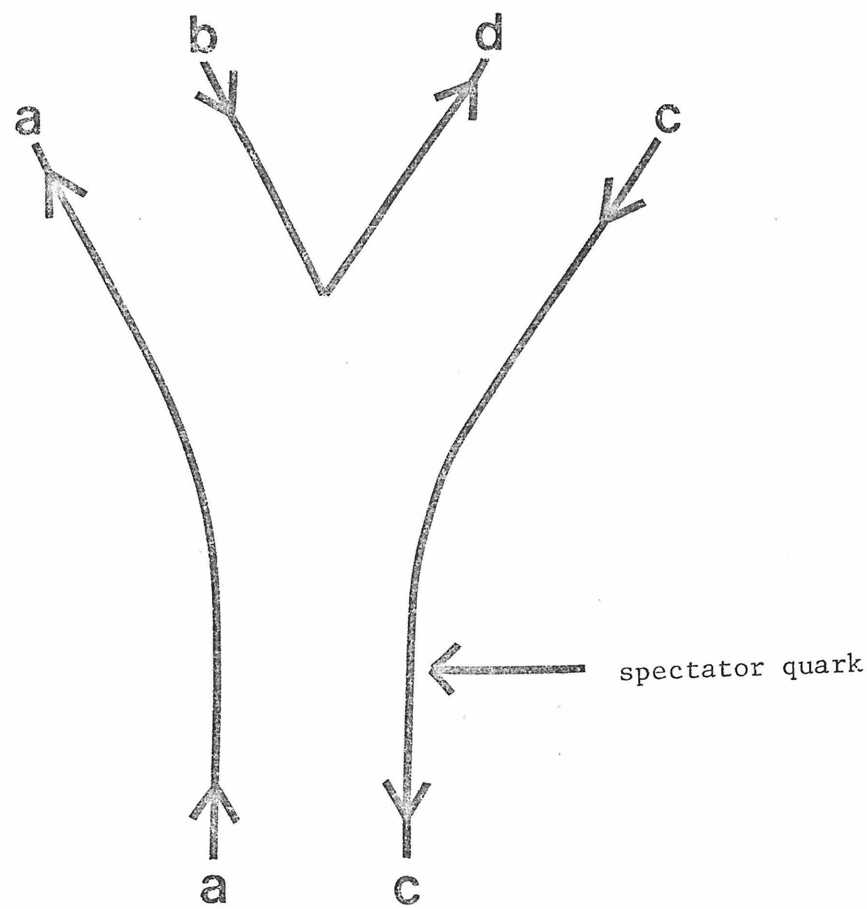

Fig. 1. Quark graph for a meson-meson-meson coupling.

Spin can be added to quark graphs in several ways. $SU(6)_S$ as a dynamical symmetry (which, of course, is not possible) would imply conservation of quark spin; or, in terms of the graphs, the spin of a spectator quark (see Fig. 1) is conserved and the $q\bar{q}$ pair is created in a $^1S_0$ state. Many of the strong decays are now forbidden, such as $\rho \to \pi\pi$ and $\Delta \to N\pi$. A more realistic spin coupling can be generalized from the fact that the $q\bar{q}$ pair in a quark graph is created in an SU(3) singlet state. If the $q\bar{q}$ pair is assumed to be created with the quantum numbers of the vacuum, then the conservation of J and P requires that the pair be produced in a $^3P_0$ state.[9,10,11] If the further restriction is made that in a collinear frame (which is always possible for a two-particle decay) the transverse momenta of quarks in the pair can be neglected compared to their longitudinal momenta, then

$$L_z = S_z = 0 \tag{1}$$

for the $^3P_0$ state.[10] If the spin of a spectator quark is assumed to be conserved in a collinear frame as well, then the prescription of equation (1) is equivalent to the collinear symmetry $SU(6)_W$.[12]

The properties of $SU(6)_W$, or its subgroup $SU(2)_W$ which describes the spin coupling, can be derived from equation (1). $\vec{W}$ spin, rather than regular $\vec{S}$ spin, is the generator of $SU(2)_W$. The z component of W-spin can be identified with S_z which is still conserved in a collinear process (at least for quark graphs connecting $q\bar{q}$ $L = 0$ states). The x and y components of W-spin, however, are different. Since the $q\bar{q}$ pair is created with the quantum numbers of the vacuum, the $^3P_0$ state with $S_z = L_z = 0$ must have W-spin zero. This assignment can be achieved by defining W_x and W_y on a quark or antiquark to be:

$$W_x = P_{int} S_x = P_{int} \frac{\sigma_x}{2} \qquad W_y = P_{int} S_y = P_{int} \frac{\sigma_y}{2}$$

where P_{int} is the intrinsic parity of the quark or antiquark.(13) (Actually, $W_x = \beta \frac{\sigma_x}{2}$ and $W_y = \beta \frac{\sigma_y}{2}$ where β is the Dirac matrix, but β is just the intrinsic parity in the rest frame.) These operators satisfy an SU(2) algebra and are invariant under Lorentz transformations in the z direction. Consequently, the W-spin classification for a particle moving with arbitrary momentum along the z axis is equivalent to its classification at rest, so that this group is a possible candidate for a relativistic symmetry of collinear processes. The difference between the W-spin and S-spin classification of a $q\bar{q}$ pair can be determined by using lowering operators on the highest state:

$$|q\uparrow\bar{q}\uparrow\rangle = |W = 1, W_z = 1\rangle = |S = 1, S_z = 1\rangle$$

$$W^- |q\uparrow\bar{q}\uparrow\rangle = (W_x - iW_y)|q\uparrow\bar{q}\uparrow\rangle = |q\downarrow\bar{q}\uparrow\rangle - |q\uparrow\bar{q}\downarrow\rangle$$

$$S^- |q\uparrow\bar{q}\uparrow\rangle = (S_x - iS_y)|q\uparrow\bar{q}\uparrow\rangle = |q\downarrow\bar{q}\uparrow\rangle + |q\uparrow\bar{q}\downarrow\rangle$$

$$|W = 1, W_z = 0\rangle = |S = 0, S_z = 0\rangle = \frac{1}{\sqrt{2}}(|q\downarrow\bar{q}\uparrow\rangle - |q\uparrow\bar{q}\downarrow\rangle)$$

$$|W = 0, W_z = 0\rangle = |S = 1, S_z = 0\rangle = \frac{1}{\sqrt{2}}(|q\downarrow\bar{q}\uparrow\rangle + |q\uparrow\bar{q}\downarrow\rangle) .$$

The interchange between the singlet and triplet states of W and S spin is called the W-S spin flip. Consequently, a $^3P_0$ $q\bar{q}$ state with $L_z = S_z = 0$ does have $W = 0$. Note that the W-spin and S-spin classifications of baryon are the same. The consequences of W-spin for decays follows immediately from the W-spin classification and the Clebsch-Gordan coefficients for SU(2). For example, the decays of the ρ and Δ are now allowed:

$$\rho^{(0)} \rightarrow \pi\,\pi \qquad\qquad \Delta^{(\frac{1}{2})} \rightarrow N^{(\frac{1}{2})}\,\pi$$

$$\left\langle \begin{matrix} W = 0 & W = 1 & W = 1 \\ W_z = 0 & W_z = 0 & W_z = 0 \end{matrix} \right\rangle \neq 0 \qquad \left\langle \begin{matrix} W = \frac{3}{2} & W = \frac{1}{2} & W = 1 \\ W_z = \frac{1}{2} & W_z = \frac{1}{2} & W_z = 0 \end{matrix} \right\rangle \neq 0 \; .$$

Non-trivial predictions are made for the decay of the Δ into $N\rho$:

$$\Delta^{(\frac{3}{2})} \rightarrow N^{(\frac{1}{2})}\,\rho^{(1)} \qquad\qquad \Delta^{(\frac{1}{2})} \not\rightarrow N^{(\frac{1}{2})}\,\rho^{(0)}$$

$$\left\langle \begin{matrix} W = \frac{3}{2} & W = \frac{1}{2} & W = 1 \\ W_z = \frac{3}{2} & W_z = \frac{1}{2} & W_z = 1 \end{matrix} \right\rangle = 1 \qquad \left\langle \begin{matrix} W = \frac{3}{2} & W = \frac{1}{2} & W = 0 \\ W_z = \frac{1}{2} & W_z = \frac{1}{2} & W_z = 0 \end{matrix} \right\rangle = 0$$

$$\Delta^{(\frac{1}{2})} \rightarrow N^{(-\frac{1}{2})}\,\rho(1)$$

$$\left\langle \begin{matrix} W = \frac{3}{2} & W = \frac{1}{2} & W = 1 \\ W_z = \frac{3}{2} & W_z = \frac{1}{2} & W_z = 1 \end{matrix} \right\rangle = \sqrt{\frac{1}{3}}$$

Not much evidence is known experimentally for this decay. However, if the photon is assigned $W = 1$, then predictions similar to those for the ρ follow for electroproduction and photoproduction of the Δ. The experimental data is in good agreement with $SU(2)_W$.*

$SU(6)_W$ is constructed by combining SU(3) with W-spin. For the 0^{-+} and 1^{--} mesons, the W-S spin flip requires that the $SU(6)_W$ singlet contain the SU(3) singlet, helicity zero vector meson rather than the SU(3) singlet, pseudoscalar meson. The predictions of $SU(6)_W$ can be derived by using this classification and the Clebsch-Gordan coefficients of SU(6) or by using the quark graphs with equation (1). Unfortunately, only a few predictions of the symmetry can be checked for the lowest meson and baryons ($L = 0$ states in quark model) because of their small number of two-body strong decays. The $SU(2)_W$ structure is verified in the electroproduction and photoproduction of the Δ, while the $SU(6)_W$ structure is verified in the ratio of $\omega \to \pi\pi\pi$ to $\rho \to \pi\pi$** and in the F/D for meson-

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*The photoproduction data give

$(\Delta^{(3/2)} \to N^{(1/2)}\gamma^{(1)})/(\Delta^{(1/2)} \to N^{(-1/2)}\gamma^{(1)}) \simeq 1.77 \pm .10$ to be compared with the theoretical value $\sqrt{3}$. The electroproduction data give σ for a longitudinal photon less than 20% of σ for a transverse photon. (14)

**From the Gell-Mann, Sharp, Wagner model for $\omega \to 3\pi$ and the known rate for $\rho \to \pi\pi$, $SU(6)_W$ predicts the width of $\omega \to 3\pi$ equal to 7.0 MeV in comparison with the experimental value of 10.7 MeV. (15)

baryon coupling.* Thus, even with the limited evidence, $SU(6)_W$ looks very good for the low-lying states.

$SU(6)_W$ has to be generalized to make predictions for $q\,\bar{q}$ states with $L \neq 0$. The most obvious extension, which is called $SU(6)_W \times O(2)_{L_z}$ (17,18,19), assumes that W-spin is still conserved. Assuming $W_z = S_z$ be conserved along with J_z requires that L_z also be conserved (the z axis is the collinear axis of the decay). In order to obtain non-trivial predictions for meson decays, we must turn to the axial-vector mesons which are $^3P_1$ and $^1P_1$ in the quark model.

The axial vector meson states which have been observed experimentally are given in Table 1. These mesons are the only $q\,\bar{q}$ $L = 1$ states that can decay into two $q\,\bar{q}$ $L = 0$ states via two partial waves — S and D. The tensor and scalar mesons decay via pure D wave and pure S wave, respectively. The amplitude describing the 1^+ decays can also be written in terms of two independent helicity amplitudes g_1 and g_0, defined by:

$$g_1 = \text{amplitude for } 1^+ \text{ (helicity 1)} \rightarrow 1^- \text{ (helicity 1)} + 0^- \tag{2a}$$

$$g_0 = \text{amplitude for } 1^+ \text{ (helicity 0)} \rightarrow 1^- \text{ (helicity 0)} + 0^- \tag{2b}$$

Of course, the two helicity amplitudes are linear combinations of the

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*The ratio of F to D coupling for the pseudoscalar meson-nucleon vertex can be inferred from weak interaction data using PCAC. (The baryon matrix elements of the axial-vector current appear in semi-leptonic decays.) The experimental value of F/D determined from a fit (16) is $.66 \pm .03$ which is to be compared with the theoretical value 2/3.

TABLE 1

	State	$A_1(1070)$	$K_A(1240)$	D(1285)	?(D')
$J^{pc} = 1^{++}$	Width*	95 ± 35 MeV	40-130 MeV	33 ± 5 MeV	
	Decay modes	$\rho\ \pi$	$K^*\pi$, $K\rho$	$K\bar{K}\pi$, $\eta\pi\pi$	
	State	B(1235)	$K_B(1250\text{-}1400)$	?(h)	?(h')
$J^{pc} = 1^{+-}$	Width*	102 ± 20 MeV	?		
	Decay modes	$\omega\pi$	$K^*\pi$, $K\rho$		

*The above data is taken from ref. 20.

two partial wave amplitudes. These decay parameters can be determined experimentally from the decay angular distributions, which has been done recently for $A_1 \rightarrow \rho\pi$ and $B \rightarrow \omega\pi$. Thus, the axial-vector mesons provide a test for models of spin coupling in vertices such as $SU(6)_W \times O(2)_{L_z}$.

The predictions of $SU(6)_W \times O(2)_{L_z}$ for $B \rightarrow \omega\pi$ and $A_1 \rightarrow \rho\pi$ can be derived from the conservation of L_z.[17,18,19] Since the B is assumed to be a $^1P_1$ $q\bar{q}$ state, its J_z must equal its L_z. Therefore, according to $SU(6)_W \times O(2)_{L_z}$, the B can decay to $\omega\pi$ only through its $J_z = 0$ state ($g_1 = 0$). Experimentally, the $J_z = \pm 1$ decay seems favored.* A similar reversal occurs for $A_1 \rightarrow \rho\pi$. Since the A_1 is assumed to be a $^3P_1$ $q\bar{q}$ state, its $J_z = 0$ state has no component with $L_z = 0$, i.e.,

$$\langle J = 1 \;\; J_z = 0 \mid L = 1 \;\; L_z = 0;\; S = 1 \;\; S_z = 0 \rangle \;=\; 0 \quad .$$

Therefore, $SU(6)_W \times O(2)_{L_z}$ predicts a decay from the $J_z = \pm 1$ state ($g_o = 0$). Experimentally, the $J_z = 0$ decay seems to predominate,** or at least is significant.*** Therefore, the natural extension of $SU(6)_W$ to $q\bar{q}$ states with orbital angular momentum fails

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* $|g_o/g_1|_B \;=\; .47 \begin{smallmatrix} + .20 \\ - .30 \end{smallmatrix}$ (ref. 21)

** $|g_1/g_o|_{A_1} \;=\; .48 \pm .13$ (ref. 22)

*** $|g_1/g_o|_{A_1} \;=\; .89 \begin{smallmatrix} + .07 \\ - .06 \end{smallmatrix}$ (ref. 23) severely, which means that the incorporation of spin in quark graphs is still an open question. The angular distributions of the axial vector mesons will be the crucial test of future models.

The axial-vector mesons have interesting features besides their angular distributions that set them apart from the other mesons. One is the absence to date of three of the $I = 0$ states predicted by the quark model: one from the A_1 nonet and two from the B nonet. The D, which is the only isoscalar 1^+ meson to be seen, is produced in $\bar{p}\,p$ and $\pi^- p$ reactions. The existence of the other three states is crucial for believing the quark model. The absence of the ninth 1^{++} meson is the most intriguing since the 1^{+-} states could have possibly escaped detection because of extremely large widths predicted by SU(3) and ideal mixing.*

The strange 1^+ mesons are also fascinating. The Q region, which supposedly contains the two $1^+ K^*$ mesons, has not been definitely separated into two resonances.[20] However, the Q peak, which does not have a simple Breit-Wigner shape, can be fitted reasonably well to two Breit-Wigners at all energies.[24,25] The widths determined in these fits are sensitive to assumptions about the background.

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*SU(3) and ideal mixing predict that $M_h^2 \simeq M_B^2 \simeq (1.2 \text{ MeV})^2$, $M_{h'}^2 \simeq 2M_{K_B}^2 - M_B^2 \simeq (1.5 \text{ MeV})^2$, $\Gamma_{h \to \rho\pi} \simeq 300$ MeV, $\Gamma_{h' \to K^*\bar{K} + \bar{K}^*K} \simeq 70$ MeV. The h is certainly wide enough to have been missed. The h' probably should have been seen, but the experimental situation may be confused because of a nearby resonance called the E(1422). The favoured quantum numbers of the E are 0^{-+}.

The fits of ref. 24 (no background) and ref. 25 (deck background) to Q data compilations yield the following resonance parameters:

ref. 24: $M_\alpha = 1250$ MeV $\Gamma_\alpha = 220$ MeV $M_\beta = 1400$ MeV $\Gamma_\beta = 220$ MeV

ref. 25: $M_\alpha = 1240$ MeV $\Gamma_\alpha = 110$ MeV $M_\beta = 1420$ MeV $\Gamma_\beta = 120$ MeV .

In some reactions the Q region can be resolved into two resonances with even narrower widths:[26,27]

ref. 26: $M_\alpha = 1260$ MeV $\Gamma_\alpha = 40$ MeV $M_\beta = 1380$ MeV $\Gamma_\beta = 120$ MeV .

Further complications arise from the decay modes of the Q. The two observed decay modes, $K^*\pi$ (dominant) and $K\rho$, are both $K\pi\pi$ final states, as is the one other possible yet unconfirmed mode $K\sigma$ (σ is an $I = 0$ 0^{++} meson). Consequently, these modes can interfere with each other (in the overlapping region of the Dalitz plot, which has a high concentration of events). Moreover, interference is possible not only between the $K^*\pi$ and $K\rho$ decay modes of a single meson, but also between the decay modes of the two different mesons. Furthermore, the two 1^+ kaons can mix with each other when SU(3) is broken.[28] Mixing phenomena are a common occurence in particle physics - examples being (1) ω - Φ mixing from SU(3) breaking, (2) K^0 - $\bar{K}^0$ mixing from the weak interactions, (3) ω - ρ mixing from electromagnetism. Exact SU(3) would prohibit a 1^{++} kaon from mixing with a 1^{+-} kaon since the two states belong to different SU(3) representations with different $\mathcal{C}$ quantum number. (Exact SU(3) enables a quantum number analogous to G parity to be defined for the kaons.) When SU(3) is broken, however, the kaon states can mix (their G parity, unlike the G parity for non-strange mesons, is invalidated by SU(3) breaking of V-spin). This mixing is very similar to the singlet-octet mixing found in the 1^{--} and 2^{++} mesons (approximately 35^o) and in the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ baryons (approximately 20^o) since all are caused by SU(3) breaking.[20] The simplest and usual assumption is to take a phenomenological mixing of the form:

$$|\alpha\rangle = |K_A\rangle \cos\phi - |K_B\rangle \sin\phi \tag{3a}$$

$$|\beta\rangle = |K_A\rangle \sin\phi + |K_B\rangle \cos\phi \quad . \tag{3b}$$

However, the mixed states are not necessarily orthogonal as they are with the above unitary transformation. The mass matrix $M + i\,\Gamma$, which is not hermetian, must be diagonalized to yield the decaying states. In the narrow width approximation or if M and Γ commute (as $K^o - \bar{K}^o$ with CP conservation), the mass matrix can be diagonalized by unitary transformation so that the physical states are orthogonal. Unfortunately, K_α and K_β, like ω and Φ or f and f', are not guaranteed to be orthogonal since they have finite widths into common final states (and it is not obvious that M and Γ should commute). Nevertheless, mixing from SU(3) breaking into orthogonal states is usually assumed. Thus, the Q region is beset by several experimental and theoretical complications which cloud our understanding of the strange axial-vector mesons.

One further interesting problem faces the 1^+ mesons that are produced diffractively. The Deck model[29] for $\pi p \rightarrow \rho\pi p$, which is a double exchange diagram (Fig. 2) for diffractive processes, interprets the $\pi\rho$ enhancement as a non-resonant kinematic effect rather than A_1 resonance production. Amazingly, the Deck model predicts the $\pi\rho$ mass distribution and momentum transfer dependence of cross-section in good agreement with experiment; the essential feature of the model is the peripheral nature of pion exchange. (The recent concept of duality, which states that the imaginary part of s-channel resonances is equivalent in some average sense to the imaginary part of non-diffractive t-channel exchanges, does not seem to relate the Deck effect to A_1 production since pion exchange is real.) It is very possible that a significant fraction, if not all, of the A_1 bump results from this non-resonant Deck background. The Deck effect also confuses our resonance interpretation of the Q region since a double exchange diagram can be drawn there too. Since the determination of resonance parameters is sensitive to assumptions about the background, the masses and widths, and possibly even the existence, of the A_1 and K_A are subject to doubt. Non-diffractive production of the A_1 is unclear although it has been reported in a few experiments with poor statistics. The 1^{++} D and 1^{+-} B mesons are better established since they are definitely produced in non-diffractive reactions. We shall assume the existence of the 1^{++} mesons, as required by the quark model, while acknowledging the possibility of considerable Deck background in diffractive production.

The main purpose of my paper is to examine the angular distri-

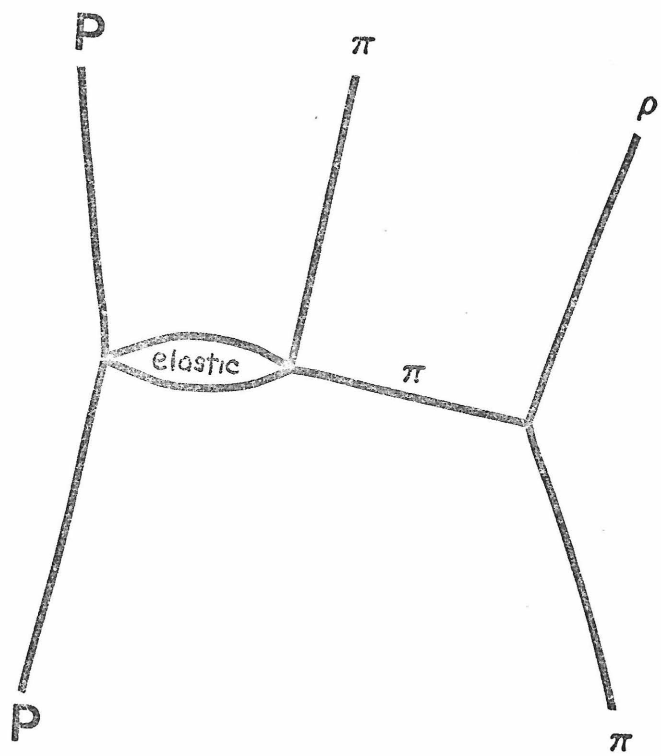

Fig. 2. Deck diagram for $\pi p \rightarrow \pi \rho p$.

butions of the $I = 1$ axial-vector mesons in order to obtain a description of spin couplings in vertices. The mixing of the $I = 1/2$ states and the absence of the $I = 0$ states will also be investigated.

The angular distributions of the $I = 1$ states indicate that $SU(6)_W \times O(2)_{L_z}$ is not a good symmetry. Since $SU(6)_W$ is consistent with the $L = 0$ meson and baryon decays, the orbital generalization rather than the complete symmetry is suspect. The shortcomings of $SU(6)_W \times O(2)_{L_z}$ may be dealt with by simply relaxing the condition $S_z = L_z = 0$ on the $^3P_0$ pair in the quark graph and allowing $L_z = \pm 1$.* It is this approach that we shall take. (The neglect of transverse momenta is probably dubious for decays involving Q-values of no more than a few hundred MeV.) When the $q\,\bar{q}$ pair is allowed to have $L_z = \pm 1$ as well as $L_z = 0$, the A_1 and B decay distributions may be described satisfactorily. Moreover, the angular distributions for $A_1 \to \rho\pi$ and $B \to \omega\pi$ are related by $2(g_1/g_o)_{A_1} = (g_o/g_1)_B + 1$. This prescription for spin couplings is significantly different from $SU(6)_W \times O(2)_{L_z}$, as it involves considerable modification of all S-wave decay amplitudes for the positive-parity mesons (including those of the $0^+ \to 0^- \, 0^-$ transitions). It is then fortunate that we obtain a satisfactory description of the partial widths of all the $q\,\bar{q}$; $L = 1$ mesons into $1^- \, 0^-$ or

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*One may also invoke "recoil terms" in a realistic quark picture.[30,31] Such an approach relies more heavily on details of the wave functions than the picture we describe here, but there may be some overlap of results.

$0^- 0^-$ pairs. This description involves two independent parameters, whereas $SU(6)_W \times O(2)_{L_z}$ involves only one. (A similar description was obtained in ref. 9, but not applied to angular distributions.) It may be summarized as follows: $SU(6)_W \times O(2)_{L_z}$ relates all D wave amplitudes to one another, all S wave amplitudes to one another, and D waves to S waves (since a particular helicity coupling is a linear combination of partial waves). The ^3P_o prescription with $L_z = 0, \pm 1$ uncouples the D waves from the S waves but otherwise preserves all the $SU(6)_W \times O(2)_{L_z}$ relations. As we shall show, the physical solution that emerges has approximately the same magnitude of D wave to S wave, but the opposite sign, from the $SU(6)_W \times O(2)_{L_z}$ solution. Unlike the widths, the angular distributions of the 1^+ mesons are sensitive to the relative sign and are thus crucial in determining that the fit of our model differs considerably from that of $SU(6)_W \times O(2)_{L_z}$.

Next the parameters of the fit are applied to the 1^+ kaons, which may mix with one another (we assume a phenomenological mixing via a unitary transformation and ignore possible interference). The results are sensitive to the mixing angle ϕ, and merely assuming lower bounds on the widths of both physical states establishes the limits $10^o \leq \phi \leq 35^o$. As a result of this mixing, one predicts the following results: (i) the suppression of the $K^*\pi$ mode of the lower peak, (ii) the suppression of the ρK mode of the upper peak, and (iii) decay distributions in the $K^*\pi$ mode similar to that of the A_1 for the lower state and to that of the B for the higher.

Finally, the absence of the three $I = 0$ axial vector mesons is investigated, with particular emphasis on the ninth 1^{++} state. Expected properties of this meson, the D', include: (1) assignment to a weakly mixed SU(3) singlet, predicted by duality and confirmed by the Gell-Mann Okubo mass formula; (b) a mass ~950 MeV, predicted by superconvergence with assumptions about the relative couplings of D and D'; (c) decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$; and (d) the possibility of a suppressed ρ signal in the $\pi^+\pi^-$ spectrum of the $\pi^+\pi^-\gamma$ final state, despite the expectation that the pions are in a state with $I = J = 1$. These features suggest that a recently reported meson(32) near this mass with decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$ may be a candidate for this state, although $J^{pc} = 1^{+-}$ is also a definite possibility for the new meson.

II. ANGULAR DISTRIBUTIONS AND QUARK GRAPHS FOR THE 1^+ ISOVECTOR MESONS

1. $SU(6)_W \times O(2)_{L_z}$ Breaking

$SU(6)_W \times O(2)_{L_z}$ invariant vertex functions can be calculated in a collinear frame with non-relativistic wave functions since the symmetry is invariant under boosts. The same technique can be tried when the $L_z = S_z = 0$ restriction on the ^3P_o state is relaxed. The wave functions for the $L = 0$ and $L = 1$ mesons are:

$$M^{(\alpha a)}_{(\beta b)} = P^{\alpha}_{\beta} \; C_{ab} + V^{\alpha}_{\beta} \; (\epsilon\cdot\sigma \; C)_{ab} \tag{4a}$$

$$M^{(\alpha a)}_{i\;(\beta b)} = B^{\alpha}_{\beta} \; \epsilon_i \; C_{ab} + \frac{1}{\sqrt{3}} S^{\alpha}_{\beta} \; (\sigma_i \; C)_{ab} + A^{\alpha}_{\beta} \; \frac{i}{\sqrt{2}} \epsilon_{ijk} \; \epsilon_j \; (\sigma_k C)_{ab} + T^{\alpha}_{\beta} \; \epsilon_{ij} (\sigma_j \; C)_{ab} \quad . \tag{4b}$$

The symbols P, V_μ, B_μ, S, A^δ, and $T_{\mu\nu}$ are 3 x 3 SU(3) matrices for the 0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++} meson nonets, respectively. The 2 x 2 matrix C in quark spin space is given by:

$$C = i\,\sigma_y = \begin{matrix} & \uparrow \;\; \downarrow \\ \begin{matrix}\uparrow\\ \downarrow\end{matrix} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{matrix} .$$

The index i represents a polarization vector for the one unit of orbital angular momentum. Vertices invariant under $SU(6)_W \times O(2)_{L_z}$ are constructed by contracting indices according to the quark graphs:

$$M^{+\,(\gamma c)}_{(\alpha a)} \quad M^{(\alpha a)}_{(\beta b)} \quad D^{+}_{bd} \quad M^{(\beta d)}_{(\gamma c)} \quad .$$

The matrix $D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \sigma_z C$ quarantees that $\bar{q}_b$ and q_d in Fig. (1) annihilate in a spin state $S = 1$ $S_z = 0$. Relaxing $S_z = 0$ means replacing D by $D_i = \frac{1}{\sqrt{2}} \sigma_i C$ ($\sigma_i C$ represents a ^3P_o state by analogy with the wave function for the scalar meson S which is also ^3P_o). Only one coupling, the $SU(6)_W$ invariant one, can be constructed for L = 0 mesons since D_i must be contracted on e_z (z unit vector) which is the only direction in the problem. However, for L = 1 states, two independent couplings can now be contructed, contracting on e_z and on the orbital index i . The second coupling obviously breaks $SU(6)_W \times O(2)_{L_z}$ as can be seen in $B \to \omega\pi$:

$$\langle C^{-1} \epsilon_i^{B^*} \epsilon_j^{W} \sigma_j C \frac{1}{\sqrt{2}} C^{-1} \sigma_i C \rangle = \sqrt{2} \epsilon^{B^*} \cdot \epsilon^{\omega}$$

i.e., $g_1 \neq 0$. The matrix elements for the decays of all the L = 1 mesons into PP or PV can be calculated in terms of the two couplings. (These predictions are given in Table 2 in terms of two parameters S and D , representing partial wave decay amplitudes, which are just linear combinations of the $SU(6)_W \times O(2)_{L_z}$ conserving and breaking amplitudes discussed above.)

Unfortunately, the validity of a non-relativistic calculation is not obvious when the $SU(6)_W \times O(2)_{L_z}$ symmetry is broken. Consequently, we are forced to calculate our breaking prescription in a manifestly covariant manner. We shall return later to the significance of this simple scheme.

2. Details of the Relativistic Calculation

(a) We guarantee relativistic invariance by using covariant wave functions:[18,33]

$$q\bar{q};\quad L = 0: \qquad M_0(p) = (1 + \not{p}/m_0)(\gamma_5\, P + \gamma^\mu\, V_\mu) \quad , \tag{5a}$$

$$q\bar{q};\quad L = 1: \qquad M_{1\mu}(p) = (1 + \not{p}/m_1)(\gamma_5\, B_\mu + \gamma^\nu\, C_{\mu\nu}) \quad , \tag{5b}$$

where

$$C_{\mu\nu} = \frac{1}{\sqrt{3}}\,(g_{\mu\nu} - p_\mu\, p_\nu/m_1{}^2)\, S + i\ \epsilon_{\mu\nu\gamma\delta}\, p^\gamma\, A^\delta\, /(\sqrt{2}\, m_1) + T_{\mu\nu}\ . \tag{5c}$$

The matrices M_0 and M_1 refer to the 12-dimensional product space of SU(3) matrices (3 indices) and Dirac matrices (4 indices). Normalizations are given in such a way that

$$\langle M_0(p)\, M_0(p)\rangle \sim \langle PP\rangle + \langle VV\rangle$$

and

$$(\bar{M}_{1\mu}(p)\, M_1{}^\mu(p)) \sim \langle BB\rangle + \langle SS\rangle + \langle AA\rangle + \langle TT\rangle$$

where the right-hand side refers to traces of SU(3) matrices. $\bar{M}_0$ and $\bar{M}_{1\mu}$ are given by

$$\bar{M}_0(p) = (1 - \not{p}/m_0)(-\gamma_5\, P^+ + \gamma^\mu\, V_\mu{}^+)$$

and

$$\bar{M}_{1\mu}(p) = (1 - \not{p}/m_1)(-\gamma_5\, B_\mu{}^+ + \gamma^\nu\, C_{\mu\nu}{}^+) \quad .$$

In equations (5a) and (5b), (m_0, m_1) is taken as a common mass of the $q\bar{q}$; $L = (0,1)$ multiplet.

(b) Couplings are constructed as trilinear traces of the matrices M_0 and M_1 obeying charge conjugation invariance and the rule[5,8] that quark and antiquark lines of a single meson should not be connected. For example, the $SU(6)_W \times O(2)_{L_z}$ invariant coupling for the decays of the $q\bar{q}$; $L = 1$ mesons into two $q\bar{q}$; $L = 0$ mesons is $p_2^{\mu}\ \mathrm{Tr}\,(\bar{M}_{1\mu}(p_1)\ [\ M_0(p_2), M_0(p_3)\ ]\ )$. The orbital index μ is coupled to one of the collinear momenta, implying $\Delta L_z = 0$.

(c) Relaxing condition (1) is equivalent to allowing the orbital angular momentum of the $q\bar{q}$; $L = 1$ mesons to couple to that of the $^3P_0$ pair. By analogy with the wave function for the scalar meson S in equations (5b) and (5c), a $^3P_0$ object which carries no four-momentum and is assumed to be an SU(3) scalar transforms like γ_μ. The γ_μ coupled to the orbital index μ of the decaying meson breaks $SU(6)_W \times O(2)_{L_z}$. (For the $SU(6)_W \times O(2)_{L_z}$ symmetric coupling, γ_μ is coupled to one of the collinear momenta.)

The three most general couplings with one γ_μ for the decays of the $q\bar{q}$; $L = 1$ mesons are then:

$$c_0\ \frac{p_2^{\mu}}{m_1}\ \mathrm{Tr}\,(\bar{M}_{1\mu}(p_1)\ \ [M_0(p_2), M_0(p_3)]\ )\qquad , \qquad (6)$$

$$c_1\ \mathrm{Tr}\,(\bar{M}_{1\mu}(p_2)\ \{M_0(p_2)\ \gamma_\mu\ M_0(p_3) + M_0(p_3)\ \gamma_\mu\ M_0(p_2)\}\ ),\ (7)$$

$$c_2 \, \mathrm{Tr} \left(\{\gamma^{\mu} , \bar{M}_{1\mu}(p_1)\} \; \{M_0(p_2), M_0(p_3)\} \right) \quad . \qquad (8)$$

The momenta satisfy $p_1 = p_2 + p_3$. The structure of these couplings is illustrated in Fig. 3.

The coupling (6), which conserves $SU(6)_W \times O(2)_{L_z}$, leads to rates for the decays of $q \bar{q}$; $L = 1$ mesons to pairs of $q \bar{q}$; $L = 0$ mesons which all behave as p_f^5 , where p_f is the magnitude of the final three-momentum, for S and D waves alike (because $SU(6)_W \times O(2)_{L_z}$ predicts a definite helicity coupling). Any breaking of $SU(6)_W \times O(2)_{L_z}$ that gives an S-wave contribution to the rate proportional to p_f would then have large effects on the decays of the 0^+ mesons as well as on angular distributions in A_1 and B decays.(17,34) In particular, a small symmetry breaking, which we shall assume, can have a large effect. A contribution proportional to p_f for S waves arises from the coupling in equation (7).

The coupling (8) does not seem to have any deep physical significance. It gives contributions to decay rates proportional to p_f^5 for both S and D waves. Therefore, as it represents an $SU(6)_W \times O(2)_{L_z}$ breaking term whose effects are expected to be relatively limited, we shall ignore it in what follows.

Actually, couplings (6), (7), and (8) are not the most general couplings when $SU(6)_W \times O(2)_{L_z}$ is broken. In particular, γ_μ's contracted on each other can be added to the three couplings to make many more couplings (where the spin of the spectator quark can flip!). However, the inclusion of such couplings would break $SU(6)_W$ for

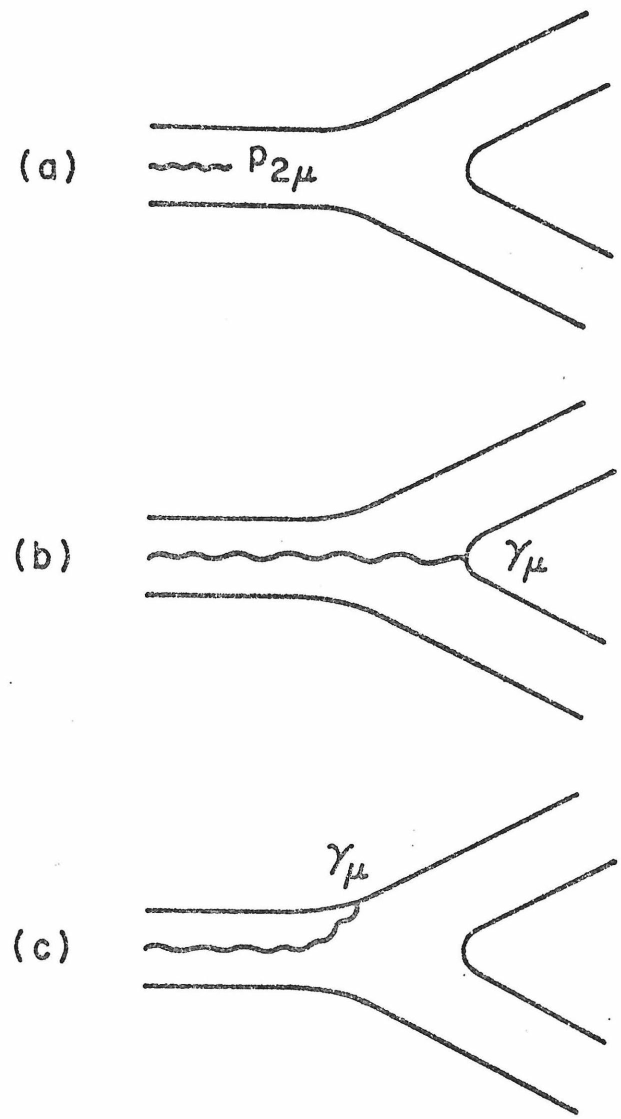

Fig. 3. Quark graphs for spin couplings. (a) Strength c_0. $SU(6)_W \times O(2)_{L_z}$ symmetric. (b), (c) Strengths c_1, c_2, respectively. Both break $SU(6)_W \times O(2)_{L_z}$.

$L = 0$ $q\bar{q}$ states, too. We only wish to break the orbital extension of $SU(6)_W$ since the $L = 0$ predictions agree with the data. Therefore, we assume that coupling a γ_μ to one of the collinear momenta is much larger than coupling it to either the orbital angular momentum or another γ_μ. Therefore, c_o (equation 6) is assumed to be much larger than c_1 (equation 7) and c_2 (equation 8), and the couplings with multiple γ_μ's are smaller still. Consequently, the coupling in equation (7) is the only breaking term that can have a significant effect on the rates and angular distributions.

Typical couplings given by the model are then:

$\underline{\pi_N \rightarrow \eta\pi \quad (0^{++} \rightarrow 0^- 0^-)}$:

$$g_0(0^+) = \left[- \frac{m_1 + 2\, m_0}{3^{1/2}\, m_1\, m_0^{\,2}} \left\{ 4m_1(m_1 + m_0)\, c_1 + (m_1^{\,2} - 4m_o^{\,2})\, c_0 \right\} \right] \langle \pi_N \{\eta, \pi\} \rangle, \tag{9}$$

$\underline{A_1 \rightarrow \rho\pi \quad (1^{++} \rightarrow 1^- 0^-)}$:

$$g_i(1^+) = \left[\frac{1}{2^{1/2}\, m_0^{\,2}} \left\{ \frac{m_1 + 2\, m_0}{m_1} \quad c_0 + 2\, c_1 \right\} \right.$$

$$\left\{ \epsilon_A^{*(i)} \cdot \epsilon_\rho^{(i)} (m_1^{\,2} - 4m_0^{\,2}) + 2\epsilon_A^{*(i)} \cdot p_\pi\, \epsilon_\rho^{(i)} \cdot p_\pi \right\}$$

$$\left. + 4\,(2)^{1/2}\, c_1 (m_1 + 2m_0)\, \epsilon_A^{*(i)} \cdot \epsilon_\rho^{(i)} / m_0 \right] \quad A[\rho, \pi]$$

$$(i = 0, 1)\,, \tag{10}$$

$B \to \omega\pi \ (1^{+-} \to 1^- 0^-)$:

$$g_i(1^+) = \left[\frac{4}{m_1 m_0} (m_1 + 2m_0) \left(\frac{c_0}{m_1} - \frac{c_1}{m_0} \right) \epsilon_B^{*(i)} \cdot p_\pi \, \epsilon_\omega^{(i)} \cdot p_\pi \right.$$

$$\left. - 2 c_1 m_1 (m_1 + 2m_0) \epsilon_B^{*(i)} \cdot \epsilon_\omega^{(i)} / m_0^2 \right] \langle B \{ \omega, \pi \} \rangle \quad (i = 0, 1) \ , \qquad (11)$$

$K^{**} \to K\pi \ (2^{++} \to 0^- 0^-)$:

$$g_0(2^+) = \left[\frac{4}{m_0^2} \left(\frac{m_1 + 2m_0}{m_1} c_0 - 2c_1 \right) (\epsilon_{K^{**}}^{*(0)})^{\alpha\beta} p_{K\alpha} \, p_{K\beta} \right] \langle K^{**} \{ K, \pi \} \rangle , \qquad (12)$$

$K^{**} \to K^* \pi \ (2^{++} \to 1^- 0^-)$:

$$g_1(2^+) = \left[\frac{4}{m_1 m_0^2} \left(\frac{m_1 + 2m_0}{m_1} c_0 - 2c_1 \right) i \, \epsilon_{\alpha\beta\gamma\delta} \, (\epsilon_{K^*}^{(1)})^{\alpha} \, p_{\pi\lambda} \right.$$

$$\left. (\epsilon_{K^{**}}^{*(1)})^{\lambda\beta} \, p_{K^*}^{\gamma} \, p_\pi^{\delta} \right] \langle K^{**} [K^*, \pi] \rangle , \qquad (13)$$

where the brackets $\langle \ \rangle$ denote traces of 3 x 3 SU(3) matrices. All partial widths follow from equations (9)-(13) by substitution of the appropriate SU(3) matrices and polarization vectors. Subscripts on the helicity coupling constants and indices on polarization vectors indicate values of J_z. The partial widths are related to these coupling constants as follows:

$$\tilde{\Gamma}(0^+ \to 0^- 0^-) = \left[g_0(0^+) \right]^2 , \qquad (14)$$

$$\tilde{\Gamma}(1^+ \to 1^- 0^-) = \left\{ \left[g_0(1^+) \right]^2 + 2 \left[g_1(1^+) \right]^2 \right\} / 3 , \qquad (15)$$

$$\tilde{\Gamma}(2^+ \to 0^-0^-) = \left[g_0(2^+)\right]^2/5 \quad , \tag{16}$$

$$\tilde{\Gamma}(2^+ \to 1^-0^-) = 2\left[g_1(2^+)\right]^2/5 \quad . \tag{17}$$

The quantities $\tilde{\Gamma}$ refer to partial widths with a phase space factor divided out:

$$\tilde{\Gamma} = \Gamma(M_i^{\,2}/p_f) \tag{18}$$

In equation (18) M_i is the mass of the initial state and p_f is the magnitude of the final three-momentum in the rest frame of the decaying particle.

3. Angular Distributions in A_1 and B Decays

The A_1 and B angular distribution calculated from equations (10) and (11) yield the following relation:

$$2\,(g_1/g_0)_{A_1} = (g_0/g_1)_B + 1 \quad . \tag{19}$$

Given the present data on B decay[21] we can compare the relation with the two experiments on A_1 decay.[22,23] We shall assume that the observed ratio of helicity couplings is real although the experiments cannot measure the phase accurately.

Let us define $x = 2\,(g_1/g_0)_{A_1} - (g_0/g_1)_B$ and compare first with the SLAC data:[22]

$$0.03 \le x \le 1.05 \quad \text{for} \quad (g_0/g_1)_B > 0 \quad \text{i.e., "}B^{(+)}\text{"} \tag{20}$$

OR

$$0.87 \leq x \leq 1.89 \quad \text{for} \quad (g_0/g_1)_B < 0 \quad \text{i.e., "}B^{(-)}\text{"} \quad . \tag{21}$$

In either case, one must take $(g_1/g_0)_{A_1} > 0$ in order to satisy equation (19). This solution ("$A_1^{SLAC\ (+)}$") is the one in which D-wave decay plays a relatively small role. (For the opposite sign of $(g_1/g_0)_{A_1}$, i.e., "$A_1^{SLAC\ (-)}$", the decay A_1 would be predominantly D wave.)

The BNL data[23] yield:

$$0.99 \leq x \leq 1.75 \quad \text{for} \quad (g_0/g_1)_B > 0 \quad \text{i.e., "}B^{(+)}\text{"} \quad . \tag{22}$$

Here equation (19) can only be satisfied with "$A_1^{BNL(+)}$" and "$B^{(+)}$".

One should stress that the two values of $(g_1/g_0)_{A_1}$ do not agree with one another, and hence comparison of equation (19) with experiment may be premature.

It is also possible that the SLAC (16 GeV) and BNL (6 GeV) experiments are both correct, and that the parameter we call $(g_1/g_0)_{A_1}$ is indeed dependent on the incident energy of the π^- in $\pi^- p \to$ "A_1^-" p. (We use quotation marks since, for a real resonance, the decay angular distribution should obviously be independent of the energy at which the resonance is produced. Any variation with energy indicates an improper separation of non-resonant background or the presence of more than one resonance in the mass range under consideration.)

If the sign of $(g_0/g_1)_B$ were measured, one could remove some of the ambiguity in comparing equation (19) with experiment. A slight preference for $(g_1/g_0)_A > 0$ does emerge from fits to the SLAC data[22] as a result of interference effects in $A_1^- \to \pi^-\pi^-\pi^+$ via two possible ρ bands. However, such effects will be smaller for $B \to \pi^-\pi^-\pi^+\pi^0$ because of the narrowness of the ω.

Equation (19) is of interest, aside from comparison with experiment, in view of the following limits: (a) In the $SU(6)_W$-invariant case, it reads $\infty = \infty + 1$, a correct if useless statement. (b) In the case that A_1 decays via pure S wave, with $(g_1/g_0)_A = 1$, equation (19) predicts that B also decays via pure S wave, with $(g_0/g_1)_B = 1$. This is because the relative effects of D and S waves are related in the two decays. A convenient normalization yields:

$$(g_1)_{A_1^0 \to \rho^-\pi^+} = -\frac{1}{\sqrt{6}}(2S-D) \qquad (g_1)_{B\to\omega\pi} = -\frac{1}{\sqrt{3}}(S+D)$$
$$(g_0)_{A_1^0 \to \rho^-\pi^+} = -\frac{2}{\sqrt{6}}(S+D) \qquad (g_0)_{B\to\omega\pi} = -\frac{1}{\sqrt{3}}(S-2D) \tag{23}$$

where

$$S \equiv -(2/3)^{1/2}(m_1^2 - 4m_0)(m_1 + 2m_0)\, c_0/m_0^2\, m_1 - 4\,(2/3)^{1/2}(m_1 + 2m_0)(m_1 + m_0)\, c_1/m_0^2 \,, \tag{24}$$

$$D \equiv (2/3)^{1/2} (m_1^{\,2} - 4m_0) \, [(m_1 + 2m_0) \, c_0 - 2m_1 \, c_1] \, / m_0^{\,2} \, m_1 \, , \qquad (25)$$

(One can see that equation (19) follows very directly from equation (23).)

The incompatibility of the experimental A_1 and B angular distributions with $SU(6)_W \times O(2)_{L_z}$ is easily illustrated in terms of the above S and D wave amplitudes. $SU(6)_W \times O(2)_{L_z}$ corresponds to $D/S = -1$, while any set of experimental ratios satisfying equation (19) requires $D/S > 0$. The $SU(6)_W \times O(2)_{L_z}$ conserving part of the S-wave amplitude (the c_0 term) is depressed by a factor proportional to $p_f^{\,2} = (m_1^{\,2}/4) - m_0^{\,2}$, so that the small symmetry breaking c_1 term can give a significant S-wave contribution (proportional to p_f rather than $p_f^{\,5}$ in the rate). This contribution actually <u>changes the sign</u> of the net S-wave amplitude.

4. <u>A Fit to Rates and Angular Distributions for 0^+, 1^+, and 2^+ Decays</u>

The above interpretation of a small breaking having a big effect in the S wave suggest a particularly simple limit in which to compare decay rates with the present coupling scheme. Let us assume that: (a) $c_0 \gg c_1$, but that $(m_1^{\,2} - 4m_0^{\,2}) \, c_0/4m_1^{\,2}$ and the breaking term c_1 are of comparable size; and (b) $m_1 \simeq 2m_0$, i.e., we let $p_f^{\,2} \to 0$ but maintain

$$c_0 \;\; p_f^{\,2}/m_1^{\,2} \;\to\; \text{const.} \;\equiv\; c \qquad . \qquad (26)$$

(Empirically, $p_f^{\,2}/m_1^{\,2}$ is quite small.)

In this limit the parameters S and D become

$$S = -16\,(2/3)^{1/2}\,(2c + 3c_1)$$
$$D = 32\,(2/3)^{1/2}\,c \qquad (27)$$

(The effect of a $c_2 = 0(c_1)$ would vanish in this limit.)

All decays of $q\,\bar{q}$; $L = 1$ mesons into pairs of $q\,\bar{q}$; $L = 0$ mesons are now determined by the above two S and D wave amplitudes. The breaking affects only the S wave. Table 2 lists the rates for various decays in terms of S and D. These results are the same as those obtained in Section (1) using the non-relativistic wave functions, but where the physical assumptions were not obvious.* Now, however, the explicit assumptions listed above give a meaning to the simple, non-relativistic calculational scheme.

In Table 2, we include for completeness some processes which are not used in the fit of the present section. These are denoted by the square brackets. Since the 1^+ kaons can mix with each other, their decay properties are treated as derived quantities in the following section. The other processes not included involve partial widths which are very small and open to some uncertainty.

In order to compare the predictions of Table 2 with experiment, one must decide how to take account of centrifugal barrier effects for D wave decays. We regard this problem as essentially unsolved. Any

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*The same is true for the use of recoupling coefficients in ref. 9. Equation (19) was not discussed there.

Table 2
Predictions of decay rates for decays of $q\bar{q}$; $L = 1$ mesons into pairs of $q\bar{q}$; $L = 0$ mesons.

0^+ decays Process	$\tilde{\Gamma}$	1^+ decays Process	$\tilde{\Gamma}$	g_0 or γ_0	g_1 or γ_1
$\pi_N \to \eta\pi$	$\frac{1}{3}S^2$	$A_1 \to \rho\pi$	$\frac{1}{3}(4S^2+2D^2)$	$-\frac{2(S+D)}{\sqrt{6}}$ a)	$-\frac{2S-D}{\sqrt{6}}$ a)
$\sigma \to \pi\pi$	$\frac{3}{2}S^2$	$[K_A(^3P_1) \to K^*\pi]$	$\frac{1}{4}(2S^2+D^2)$	$-\frac{S+D}{\sqrt{3}}$ b)	$-\frac{2S-D}{2\sqrt{3}}$ b)
$S^* \to K\bar{K}$	S^2	$[K_A(^3P_1) \to \rho K]$	$\frac{1}{4}(2S^2+D^2)$	$\frac{S+D}{\sqrt{3}}$ c)	$\frac{2S-D}{2\sqrt{3}}$ c)
$\kappa \to K\pi$	$\frac{3}{4}S^2$	$B \to \omega\pi$	$\frac{1}{3}(S^2+2D^2)$	$-\frac{S-2D}{\sqrt{3}}$	$-\frac{S+D}{\sqrt{3}}$
		$[K_B(^1P_1) \to K^*\pi]$	$\frac{1}{4}(S^2+2D^2)$	$-\frac{S-2D}{\sqrt{6}}$ b)	$-\frac{S+D}{\sqrt{6}}$ b)
		$[K_B(^1P_1) \to \rho K]$	$\frac{1}{4}(S^2+2D^2)$	$-\frac{S-2D}{\sqrt{6}}$ c)	$-\frac{S+D}{\sqrt{6}}$ c)

2^+ decays

$\to 0^-0^-$ Process	$\tilde{\Gamma}$	$\to 1^-0^-$ Process	$\tilde{\Gamma}$
$A_2 \to \eta\pi$	$\frac{2}{15}D^2$	$A_2 \to \rho\pi$	$\frac{6}{5}D^2$
$A_2 \to K\bar{K}$	$\frac{1}{5}D^2$	$[f' \to K^*\bar{K}+\bar{K}^*K]$	$\frac{6}{5}D^2$
$f_0 \to \pi\pi$	$\frac{3}{5}D^2$	$K^{**} \to K^*\pi$	$\frac{9}{20}D^2$
$[f_0 \to K\bar{K}]$	$\frac{1}{5}D^2$	$[K^{**} \to \rho K]$	$\frac{9}{20}D^2$
$f' \to K\bar{K}$	$\frac{2}{5}D^2$	$[K^{**} \to \omega K]$	$\frac{3}{20}D^2$
$K^{**} \to K\pi$	$\frac{3}{10}D^2$		
$[K^{**} \to K\eta]$	$\frac{1}{30}D^2$		

a) for the charge state $A_1^0 \to \rho^-\pi^+$.

b) for the charge state $K_{A,B}{}^+ \to K^{*0}\pi^+$.

c) for the charge state $K_{A,B}{}^+ \to \rho^+K^0$.

For 1^+ mesons, individual helicity amplitudes are also given. Square brackets indicate processes not included in the fit for reasons given in the text.
$\tilde{\Gamma} = (M_i^2/p_f)\Gamma$.

statement about such effects invariably involves an effective interaction radius. If this radius is very small, D-wave amplitudes may be expected to contain a factor p_f^2 no matter how large p_f.* If this radius is very large, on the other hand, the p_f^2 behavior of D-wave amplitudes will be visible only for values of p_f much smaller than those considered here, and may be neglected.

We have thus performed fits for the two extreme cases listed above:

(i) D has no p_f^2 behavior and is common to all processes (28)

and

(ii) $D = \tilde{D}\,(p_f^2/p_0^2)$, and $\tilde{D}$ is common to all processes* (29)

where P_0 is any convenient normalization that we take here to equal 0.5 GeV/c. The physical solution may be expected to lie somewhere between these two extremes.

One then considers a plane with S as the horizontal axis and D or $\tilde{D}$ as the vertical axis. In order that a solution exist which is consistent with the present model, all of the following lines must intersect at a point, corresponding to definite values of S and D or $\tilde{D}$:

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*This approach is adopted in ref. 9.

$\Gamma(0^+ \to 0^-0^-)$ $|S| = \text{const.}$ vertical lines ,

$\Gamma(2^+ \to 1^-0^- \text{ or } 0^-0^-)$ $|D \text{ or } \tilde{D}| = \text{const.}$ horizontal lines ,

$\Gamma(1^+ \to 1^-0^-)$ $a\,S^2 + b(D^2 \text{ or } \tilde{D}^2) = \text{const.}$ ellipses ,

$(g_1/g_0)\ (1^+ \to 1^-0^-)$ $D/S \text{ or } \tilde{D}/S = \text{const.}$ radial lines .

The results in cases (i) and (ii) are shown in Figs. 4a and 4b, respectively. The experimental input to these figures is taken from ref. 20 except in the following cases:

(a) We use A_2 widths based on an unsplit peak, as observed in $\pi^+ p \to A_2^+ p$ at 7.0 GeV/c (ref. 35). This approach is consistent with a split A_2 in $\pi^- p \to A_2^- p$ (ref. 36) if the splitting is due to a narrow, weakly coupled state. The following values* were used:[35]

$$\begin{aligned} \Gamma(A_2 \to \rho\pi) &= 64 \text{ MeV} , \\ \Gamma(A_2 \to \eta\pi) &= 16 \text{ MeV} , \\ \Gamma(A_2 \to K\bar{K}) &= 10 \text{ MeV} . \end{aligned} \tag{30}$$

(b) The "wide-σ" solution[20] is taken:

$$\Gamma(\sigma \to \pi\pi) \simeq 400 \text{ MeV} \tag{31}$$

The σ is assumed to lack strange quarks.

(c) We take f' -like mixing for the S^* and [20]

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*We thank S. Flatte for permission to use these preliminary data, which may have changed slightly in the final analysis.

$$\Gamma(S^* \to K\bar{K}) = 80 \text{ MeV} \tag{32}$$

(d) The $^3P_0$, $I = 1/2$, $|Y| = 1$ state, "κ" is taken to have a mass of 1080 MeV and[37]

$$\Gamma(\kappa \to K\pi) \geq 200 \text{ MeV} \tag{33}$$

(e) The $\pi_N(980)$ (ref. 38, 39), $\delta(962)$ (ref. 36), and $\pi_N(1060)$ (ref. 20) are taken as manifestations of the same state which we identify as the $^3P_0$, $I = 1$, $|Y| = 0$ state. We thus take[38,39]

$$\Gamma(\pi_N(980) \to \eta\pi) = 60 \text{ MeV}^* \tag{34}$$

One notes that the experimental situation regarding the 0^+ mesons and their decay widths is still in considerable confusion. For that reason we cannot take the above assignments and widths too seriously.

Figure 4 shows only one of the four quadrants of the S-D (or S-$\tilde{D}$) plane. Partial widths alone do not determine the sign of S, D, or $\tilde{D}$, and therefore give no information as to the quadrant. <u>Only the A_1 and B angular distributions</u> tell us that the solution must lie in the first (or third) quadrant. Note the arrow in Fig. 4a corre-

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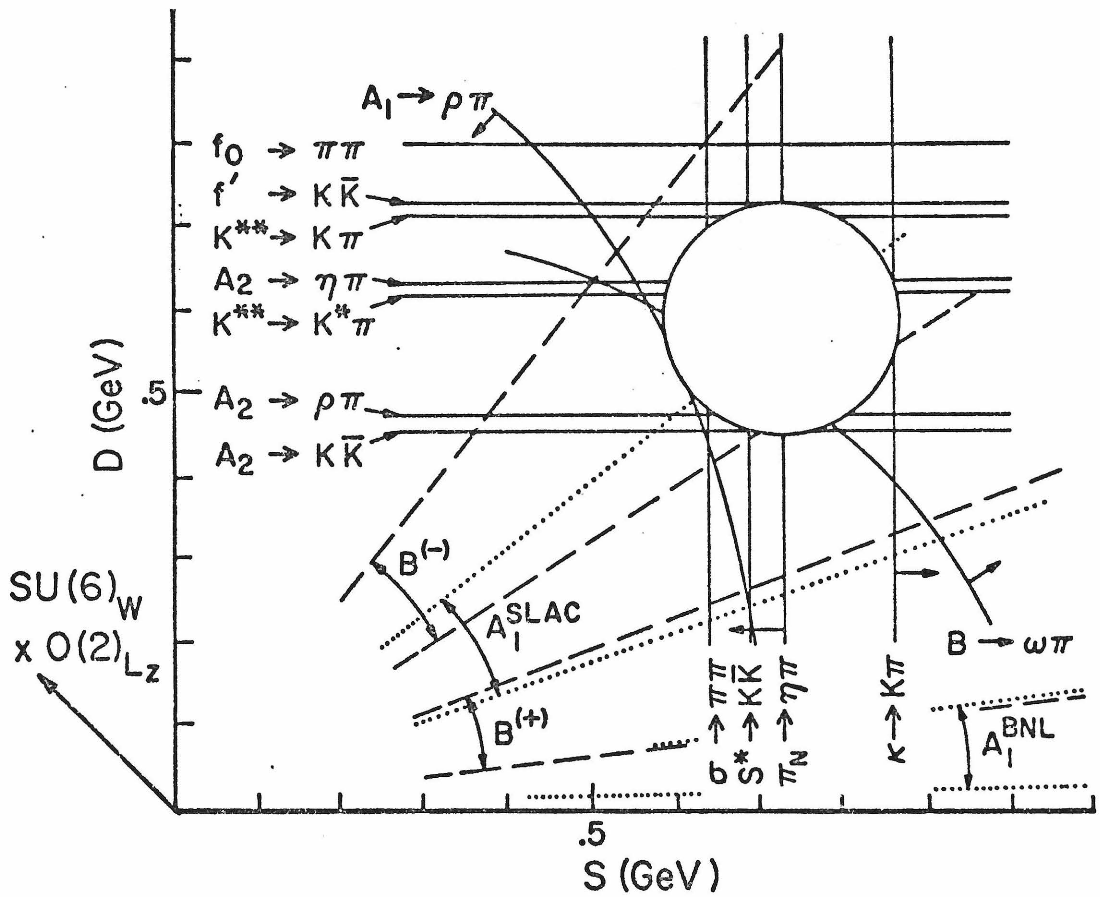
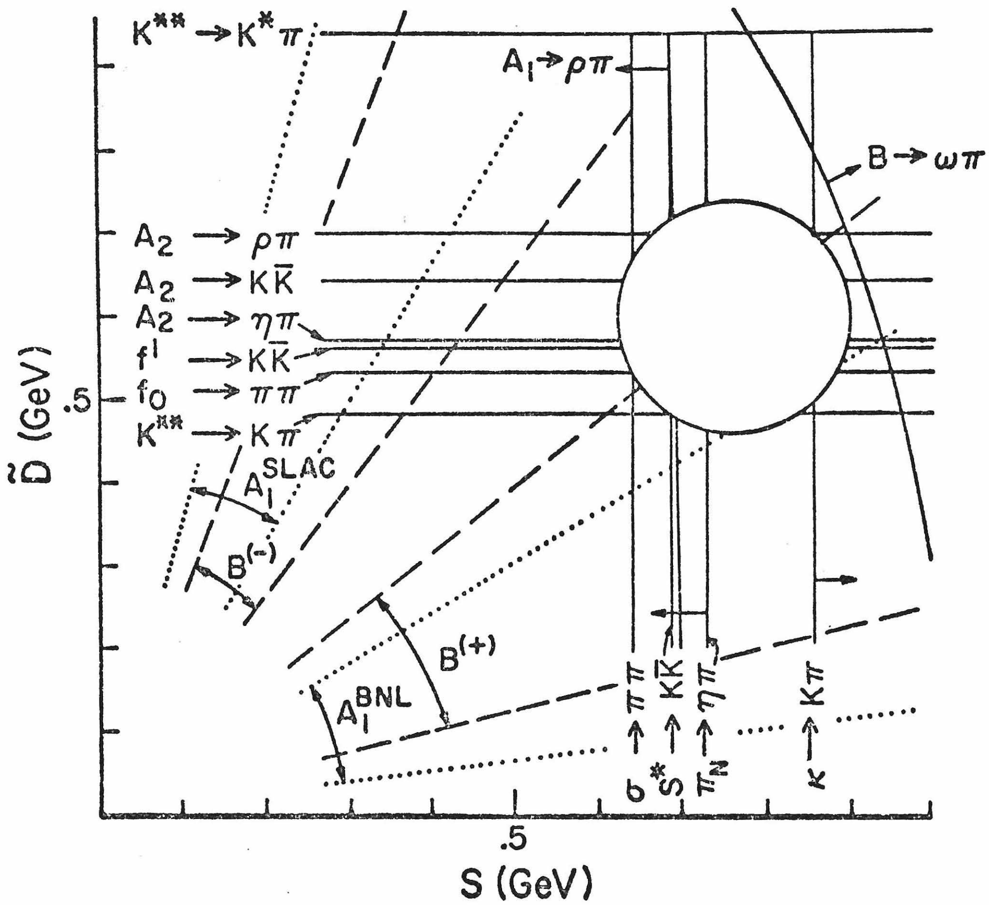
*The arrow in Fig. 4 associated with $\pi_N \to \eta\pi$ allows for the possibility, suggested by some of the experiments quoted in ref. (38), that the width may be somewhat smaller.

Fig. 4. (a) Comparison of predictions of Table 2 with experiment. Radial sectors indicate regions allowed by A_1 and B angular distributions. Negative values of $(g_1/g_o)_A$ disagree with eq. (19) and are not shown. A_1^{SLAC}: ref. (22); A_1^{BNL}: ref.(23); $B^{(\pm)}$: $(g_o/g_1)_B \gtrless 0$ (see ref. (21). The $SU(6)_W \times O(2)_{L_z}$ solution D = -S is shown as an arrow pointed into the second quadrant. Sections of ellipses are based on $\Gamma(A_1) \leq 130$ MeV, $\Gamma(B) \geqslant 80$ MeV (see ref. (20)). Horizontal lines indicate values of $|D|$ implied by 2^+ decays shown (see refs. (20,35)). Vertical lines indicate values of $|S|$ implied by 0^+ decays shown (see refs. (20,37-39)). The circle surrounding the point of intersection represents only an "eyeball estimate" of the probable errors associated with our fit. (b) Same as (a) with $\bar{D} = (0.5\ \mathrm{GeV}/p_{physical})^2\ D$.



A1 → ρπ
f0 → ππ
f′ → KK̄
K** → Kπ
A2 → ηπ
K** → K*π
.5
A2 → ρπ
A2 → KK̄
D (GeV)
SU(6)W
x O(2)Lz
B(−)
A1SLAC
B(+)
σ → ππ
S* → KK̄
πN → ηπ
κ → Kπ
B → ωπ
A1BNL
.5
S (GeV)

(a)


K** → K*π
A₁ → ρπ
B → ωπ
A2 → ρπ
A2 → KK̄
A2 → ηπ
f' → KK̄
f0 → ππ
K** → Kπ
.5
D̃ (GeV)
A1 SLAC
B(-)
B(+)
A1 BNL
σ → ππ
S* → KK̄
πN → ηπ
κ → Kπ
.5
S (GeV)

(b)

sponding to the $SU(6)_W$ invariant solution $D = -S$.* As mentioned before, relaxation of the condition (1) actually allows D/S to <u>change sign</u> relative to its $SU(6)_W$ value of -1. We have seen that this is due to an S-wave contribution whose presence we have every right to expect. This contribution must be compatible with all S-wave decays. In either fit, this is seen to be the case, but again we point out that this success should not be taken too seriously. More data on 0^+ meson decay rates would be welcome.

The experimental uncertainty regarding $(g_1/g_0)_{A_1}$ prevents testing the model conclusively at present. By comparing Figs. 4a and 4b, one sees that no statements can even be made regarding which treatment of centrifugal barrier effects agrees with experiment. The neglect of such effects in D (case (i), equation (28)) yields a fit favoring the low side of the SLAC[22] bounds on $(g_1/g_0)_{A_1}$, as shown in Fig. 4a. The "maximal" inclusion of such effects (case (ii), equation (29)) favors the low side of the BNL[23] bounds on $(g_1/g_0)_{A_1}$, as shown in Fig. 4b. In the former solution, D-wave decay accounts for at least 25% of the $A_1 \to \rho\pi$ partial width, while in the latter, it accounts for at most 3%. However, only the latter value is consistent with an upper bound of 5% obtained for this quantity using partial wave analysis.[40]

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*The $SU(6)_W \times O(2)_{L_z}$ limit does not exist in the case of Fig. 4b. This symmetry dictates a definite correlation of S and D whose effect is hard to evaluate if D is parametrized as in equation (29).

We therefore conclude that considerable note of barrier effects must be taken in the present model. If this is done, we favor the experimental values

$$(g_1/g_0)_{A_1} \simeq 0.8 \tag{35}$$

and

$$(g_0/g_1)_B \simeq 0.2 \tag{36}$$

on the basis of the fit in Fig. 4b. Note that these values do not satisfy the rule (19) exactly. As a result of the barrier factor, D-wave amplitudes in $A_1 \to \rho\pi$ are suppressed more strongly than those in the higher Q reaction $B \to \omega\pi$. Consequently, $(g_1/g_0)_{A_1}$ is slightly larger (less D wave) than the value 0.6 predicted from $(g_0/g_1)_B = 0.2$ and rule (19).

Recently (after publication of the above predictions), more data on the angular distribution of B into $\omega\pi$ have become available. The Illinois group(41) has compiled its data on $\pi^-p \to B^-p$ at 5.0 GeV/c and 7.5 GeV/c, finding

$$|g_0|^2 = .18 \pm .06 \text{ , i.e., } |g_0/g_1|_B = 0.68 \pm .12 \ . \tag{37}$$

This new value reflects a rather larger value of $(g_0/g_1)_B$ in the 7.5 GeV/c sample, which was not included in ref. (21). Although the A_1 - B sum rule is still satisfied with $A_1^{BNL(+)}$, this value for $(g_0/g_1)_B$ does not agree with our prediction. However, the most recent data(42) from $\pi^+p \to B^+p$ at 3 and 5 GeV/c yields:

$$|g_0|^2 = .06 \pm .10, \quad \text{i.e.,} \quad |g_0/g_1|_B = .36 \begin{smallmatrix} +.25 \\ -.36 \end{smallmatrix} \qquad (38)$$

with a relative phase compatible with zero. This result is in complete agreeement with the old B data and with our prediction, including the sign which confirms our solution in Fig. 4b. Also, new data[3] on the partial widths of K^{**} and A_2 have been reported which improve the agreement with Fig. 4b: $K^{**} \to K^* \pi$ 26 ± 6 MeV (35), $K^{**} \to K\pi$ 59 ± 4.4 MeV (47), $A_2 \to \rho\pi$ 67 ± 8 MeV (64), $A_2 \to \eta\pi$ 16 ± 3 MeV (16), $A_2 \to K\bar{K}$ 6.5 ± 1.3 MeV (10). The old data are given in parentheses.

III. MIXING OF THE 1^+ KAONS

We shall apply the results of the previous section to the 1^+ kaons, which may mix with one another as a result of SU(3) breaking. We assume a phenomenological mixing via a unitary transformation and ignore possible interference in the various decay modes.[28] We shall denote the upper and lower of the physical states by α and β which are then related to the unmixed states by:

$$|\alpha\rangle = |K_A\rangle \cos\phi - |K_B\rangle \sin\phi \tag{39}$$

$$|\beta\rangle = |K_A\rangle \sin\phi + |K_B\rangle \cos\phi \quad . \tag{40}$$

In some reactions the physical states can be resolved,[26,27] for example into peaks:[26]

$$M_\alpha = 1260 \pm 10 \text{ MeV} \qquad \Gamma_\alpha = 40 \pm 10 \text{ MeV} \tag{41}$$

$$M_\beta = 1380 \pm 20 \text{ MeV} \qquad \Gamma_\beta = 120 \pm 20 \text{ MeV} \quad . \tag{42}$$

These masses will be assumed in what follows merely for the sake of calculating phase space for the decays. The corresponding widths, which are the narrowest reported, will only be used as lower bounds.

For definiteness, helicity couplings will be defined for given charge states: "K^+" $\rightarrow K^{*o}\pi^+$ or "K^+" $\rightarrow \rho^+ K^o$. These will be denoted as follows:

	K_A:	K_B:	α:	β:
"$K^+ \to K^{*o}$ $(J_z=1)$ π^+:	g_1^A	g_1^B	g_1^α	g_1^β
$\to K^{*o}$ $(J_z=0)$ π^+:	g_0^A	g_0^B	g_0^α	g_0^β
$\to \rho^+$ $(J_z=1)$ K^o:	γ_1^A	γ_1^B	γ_1^α	γ_1^β
$\to \rho^+$ $(J_z=0)$ K^o:	γ_0^A	γ_0^B	γ_0^α	γ_0^β .

(43)

The quantities in equation (43) are related to one another in an obvious manner as a consequence of equations (39) and (40), e.g.

$$g_i^\alpha = g_i^A \cos \phi - g_i^B \sin \phi \qquad (i = 0, 1) \quad . \tag{44}$$

From Table 1 one then can calculate the partial widths and decay angular distributions in terms of the parameters D, S, and the mixing angle ϕ.

In the $SU(6)_W$ limit $S = -D$, the partial widths of α and β into $K^*\pi$ and ρK are all independent of ϕ* and are all equal to

$$\tilde{\Gamma}(\alpha \to K^*\pi) = (3/4)\ \tilde{\Gamma}(B \to \omega\pi) = (3/8)\ \tilde{\Gamma}(A_1 \to \rho\pi) \ . \tag{45}$$

One cannot test these partial width predictions in view of the uncer-

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*The equality of $\Gamma(\alpha \to K^*\pi)$, $\Gamma(\beta \to K^*\pi)$, $\Gamma(\alpha \to \rho K)$, and $\Gamma(\beta \to \rho K)$ follows in any theory involving a $\Delta L_z = 0$ transition. See H. J. Lipkin, ref. (28).

tainty surrounding the total widths of the states $|\alpha\rangle$ and $|\beta\rangle$. Whereas refs. (26) and (27) favor narrow states sitting above considerable Deck-type background, one may also fit the majority of events in the "Q" mass region (1100 - 1500 MeV) with two broad resonances.[24]

The following prediction for the angular distributions in α and β decays follows from $SU(6)_W$ and is independent of ϕ:

$$(g_1^{\alpha}/g_0^{\alpha})(g_1^{\beta}/g_0^{\beta}) = (\gamma_1^{\alpha}/\gamma_0^{\alpha})(\gamma_1^{\beta}/\gamma_0^{\beta}) = -\frac{1}{2} \ . \tag{46}$$

For a purely S-wave decay, $g_0 = g_1$, while for a purely D-wave decay, $g_0 = -2g_1$. Data presented several years ago[42] are consistent with $|g_1/g_0| = 1$ throughout the Q region, but, as we see from equation (46), such a possibility would necessarily violate $SU(6)_W$. However, until the signs of $g_1^{\alpha}/g_0^{\alpha}$ and g_1^{β}/g_0^{β} are known, gross violations of $SU(6)_W$ cannot be demonstrated.

We are thus left with A_1 and B angular distributions as the best indications of serious breaking of $SU(6)_W$. In what follows, we shall assume the validity of the fit of the previous section and derive results for Q decays as a function of ϕ.

One then finds the following results:

<u>$K^*\pi$ mode:</u>

$$g_1^{\alpha} = (D/2)\sin(\phi + \phi_0) - (S/\sqrt{2})\cos(\phi + \phi_0)$$

$$g_0^{\alpha} = -D\sin(\phi + \phi_0) - (S/\sqrt{2})\cos(\phi + \phi_0)$$

$$g_1^{\beta} = -(D/2)\cos(\phi + \phi_0) - (S/\sqrt{2})\sin(\phi + \phi_0)$$

$$g_0^{\beta} = D\cos(\phi + \phi_0) - (S/\sqrt{2})\sin(\phi + \phi_0)$$

(47)

<u>ρK mode:</u>

$$\gamma_1^{\alpha} = (D/2)\sin(\phi - \phi_0) + (S/\sqrt{2})\cos(\phi - \phi_0)$$

$$\gamma_0^{\alpha} = -D\sin(\phi - \phi_0) + (S/\sqrt{2})\cos(\phi - \phi_0)$$

$$\gamma_1^{\beta} = -(D/2)\cos(\phi - \phi_0) + (S/\sqrt{2})\sin(\phi - \phi_0)$$

$$\gamma_0^{\beta} = D\cos(\phi - \phi_0) + (S/\sqrt{2})\sin(\phi - \phi_0)$$

(48)

where

$$\phi_0 \equiv \tan^{-1}(2^{-1/2}) \quad . \tag{49}$$

The quantity ϕ_0 is only a convenience, and does not appear to have any physical significance. However, as a result of the f-type coupling in the decays $J^{pc} = 1^{++} \to 1^-0^-$ as opposed to the d-type coupling in $J^{pc} = 1^{+-} \to 1^-0^-$, a very important change occurs between equations (47) and (48)(would be identical if the two couplings were either both d or both f.) <u>This change leads to qualitative differences between the $K^*\pi$ and ρK modes of the two physical states α and β.</u>

The barrier factors are introduced at this stage. For the $K^*\pi$

mode, we have

$$D(\alpha \to K^{*}\pi) = \tilde{D}\,\rho(\alpha \to K^{*}\pi) \tag{50}$$

where

$$\rho(\alpha \to K^{*}\pi) = (p_f^{2}/[0.5 \text{ GeV/c}]^{2}) = 0.35 \tag{51}$$

and

$$D(\beta \to K^{*}\pi) = \tilde{D}\,\rho(\beta \to K^{*}\pi) \tag{52}$$

where

$$\rho(\beta \to K^{*}\pi) = 0.61 \quad . \tag{53}$$

For the ρK mode, we have

$$D(\beta \to \rho K) = 0.28\,\tilde{D} \quad . \tag{54}$$

We have to be more careful about $D(\alpha \to \rho K)$. The prediction of $\Gamma(\alpha \to \rho K)$ depends very sensitively on M_α, and should really be obtained by integrating over a Breit-Wigner distribution. Taking $M_\alpha \leq 1280$ MeV instead to obtain a conservative upper bound, we find $p_f \leq 100$ MeV, and thus

$$D(\alpha \to \rho K) \simeq 0 \quad . \tag{55}$$

The net result of these considerations is a set of predictions for α and β partial widths as functions of the mixing angle ϕ. We use the central values associated with the fit of the previous section, namely

$$S = 0.77 \text{ GeV} \quad , \tag{56}$$

$$\tilde{D} = 0.60 \text{ GeV} \quad , \tag{57}$$

(with errors of about 20%). These predictions are:

$$\Gamma(\alpha \to K^* \pi) = [82 - 76 \sin^2 (\phi + \phi_0)] \text{ MeV} \quad , \tag{58}$$

$$\Gamma(\beta \to K^* \pi) = [20 + 72 \sin^2 (\phi + \phi_0)] \text{ MeV} \quad , \tag{59}$$

$$\Gamma(\alpha \to \rho K) \leq [28 \cos^2 (\phi - \phi_0)] \text{ MeV} \quad , \tag{60}$$

and $$\Gamma(\beta \to \rho K) = [3 + 59 \sin^2 (\phi - \phi_0)] \text{ MeV} \quad , \tag{61}$$

and are illustrated in Fig. 5.

One notes that equations (58) and (59) predict very small $K^* \pi$ rates for α or β for certain ranges of ϕ. Using the estimates based on the data of refs. 26, 27,*

$$\Gamma(\alpha \to K^* \pi) \geq 15 \text{ MeV} \tag{62}$$

and $$\Gamma(\beta \to K^* \pi) \geq 55 \text{ MeV} \tag{63}$$

one finds that ϕ is constrained by the bounds (62) and (63) to lie within one of the ranges:

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*These allow for likelihood of a $K\sigma$ mode as well as for the observed $K\rho$ mode. (See ref. 26.) Some fits imply considerably larger widths (see ref. 24), but none imply smaller.

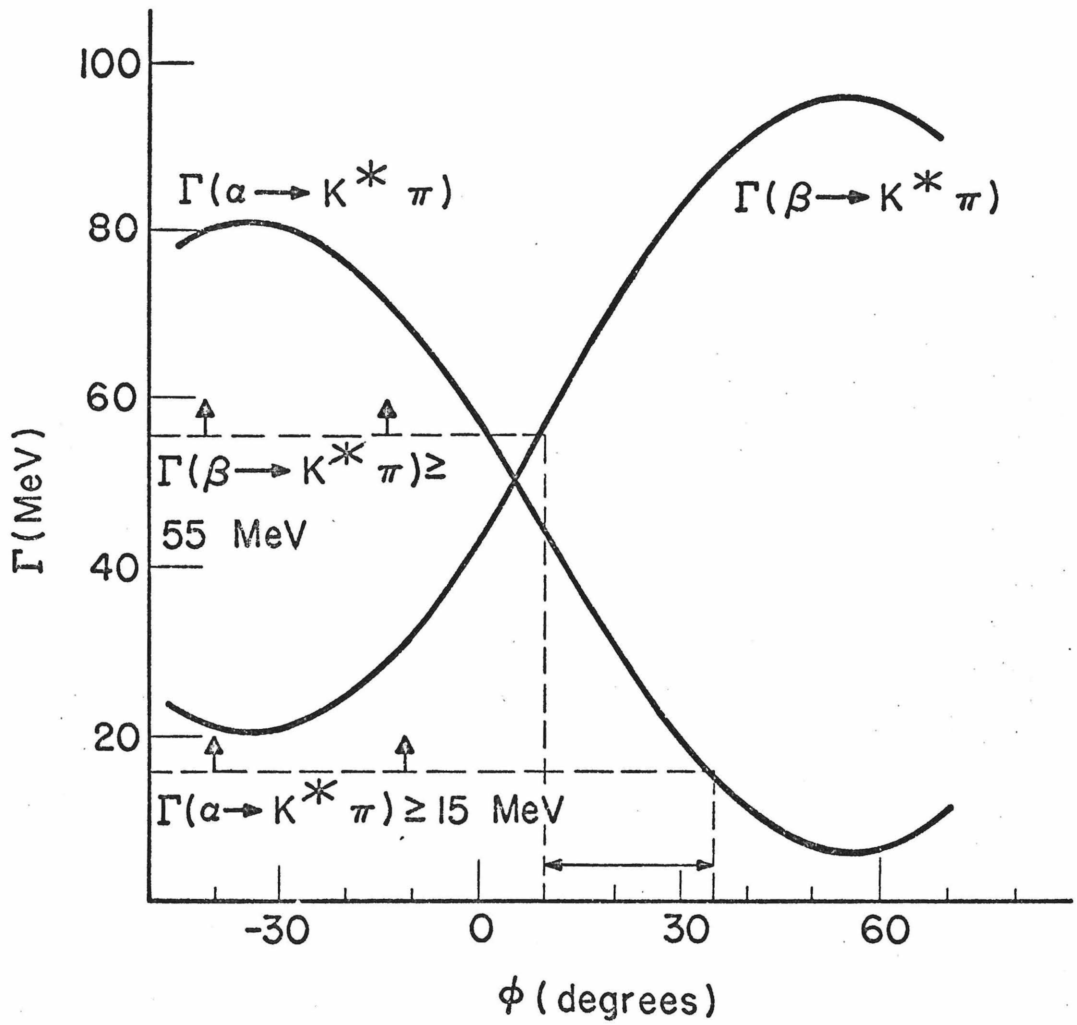

Fig. 5a. Predicted partial widths $\Gamma(\alpha \rightarrow K^*\pi)$ and $\Gamma(\beta \rightarrow K^*\pi)$ as a function of the mixing angle ϕ. The bounds $\Gamma(\alpha \rightarrow K^*\pi) \geq 15$ MeV, $\Gamma(\beta \rightarrow K^*\pi) \geq 55$ MeV (26) restrict ϕ to lie between 10° and 35° (shown by dotted lines) or between 75° and 100° (not shown).

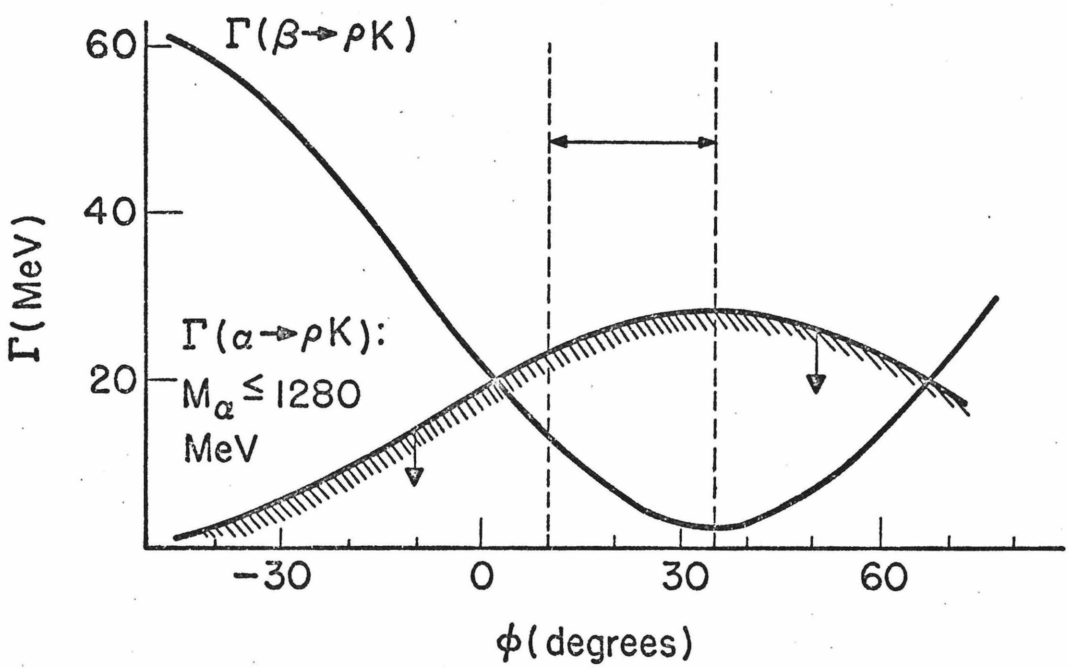

Fig. 5b. Predicted partial widths $\Gamma(\alpha \to \rho K)$ and $\Gamma(\beta \to \rho K)$ as a function of ϕ. The curve for $\Gamma(\alpha \to \rho K)$ represents only an upper bound, based on the assumption $M_\alpha \leq 1280$ MeV. The dotted lines and the arrow denote the range of ϕ allowed from fig. 3a.

$$\text{I:} \quad 10^\circ \leq \phi \leq 35^\circ \tag{64}$$

or

$$\text{II:} \quad 75^\circ \leq \phi \leq 100^\circ \quad . \tag{65}$$

The second solution corresponds to the state β (higher in mass) being very much like the A_1-like object K_A. Since we expect the lower mass α to be more like the A_1, we shall restrict our discussion to solution I in what follows. (Also, broken duality predicts an unmixed 1^{++} octet which agrees with the G-M-O mass formula using the lower mass K^*.)

Somewhat less mixing is favored here than in the approach of Gatto and Maiani,(43) who do not consider the possibility of D-wave admixtures in decay amplitudes.

Solution I leads to the prediction of a rather weak $K^*\pi$ mode for the α. This effect may be responsible for the narrowness of α in the data of ref. 26.

The bounds (64) imply that $\phi - \phi_0$ is small if not zero. In this case:

(a) The decay $\beta \rightarrow \rho K$ will be suppressed, as its S-wave contribution is multiplied by $\sin^2 (\phi - \phi_0)$.

(b) The decay $\alpha \rightarrow \rho K$ will have its largest possible S-wave contribution, explaining its observation despite nearly total exclusion by phase space.(26)

One may eliminate ϕ from equation (47) to obtain the following constraint on α and β angular distributions in the $K^*\pi$ mode:

$$\frac{1 - g_1^{\alpha}/g_0^{\alpha}}{1+2(g_1^{\alpha}/g_0^{\alpha})} \cdot \frac{1 - g_0^{\beta}/g_1^{\beta}}{2 + g_0^{\beta}/g_1^{\beta}} = \frac{\tilde{D}^2}{2S^2} \quad \rho(\alpha \to K^* \pi) \quad \rho(\beta \to K^* \pi) \quad . \tag{66}$$

As a result of the fit of the previous section, the right-hand side of equation (66) obeys

$$0.034 \leq (\tilde{D}^2/2S^2) \quad \rho(\alpha \to K^* \pi) \quad \rho(\beta \to K^* \pi) \leq 0.123 \quad . \tag{67}$$

Hyperbolae corresponding to these bounds are plotted in Fig. 6, with only the branch corresponding to solution I shown. The lines cutting across the hyperbolae indicate contours of constant ϕ for varying $\tilde{D}/S$. The constraints (64) and (67) restrict the predicted ratios to the shaded area.

At present, as mentioned above, the published angular distributions in both the low and high mass Q regions appear consistent with purely S-wave decays.[42] The closest approach to this solution allowed in Fig. 6 corresponds to $g_1^{\alpha}/g_0^{\alpha} \simeq g_0^{\beta}/g_1^{\beta} \simeq 0.6$, values whose effects should be detectable if background is not an appreciable fraction of the Q signal.

The quantities g_0 and g_1 give a distribution in ξ, the angle between the outgoing kaon and the momentum of the Q (1^+ kaon) in the K^* rest frame, proportional to $g_0^2 \cos^2\xi + g_1^2 \sin^2\xi$. We thus predict distributions in ξ varying as $a_\alpha + b_\alpha \cos^2\xi$ and $a_\beta + b_\beta \sin^2\xi$ $(a,b > 0)$ for $\alpha \to K^* \pi$ and $\beta \to K^* \pi$, respectively. As ϕ increases, b_α/a_α increases while b_β/a_β decreases. These angular distributions thus provide a sensitive test for the mixing angle ϕ if the present

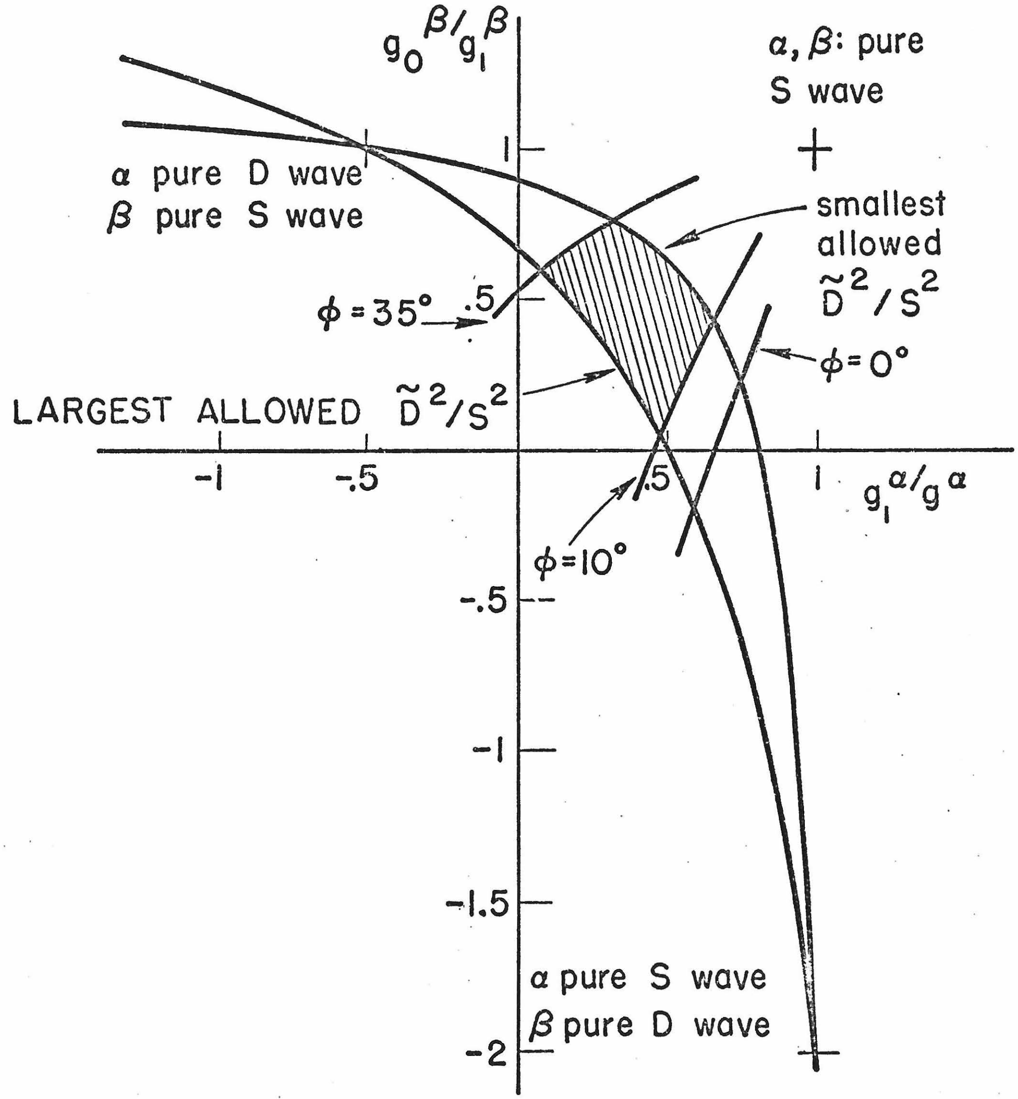

Fig. 6. Relations between (g_1^α/g_0^α) and (g_0^β/g_1^β) implied by the present model. The shaded area represents the range of values allowed by bounds on ϕ and $\tilde{D}/S$. The sharply curved hyperbola represents the boundary $\tilde{D}^2\rho_\alpha\rho_\beta/2S^2 = .034$ while the one passing closer to the origin represents $\tilde{D}^2\rho_\alpha\rho_\beta/2S^2 = .123$.

model is correct.*

In view of the predicted suppression of $\beta \rightarrow \rho K$, predictions about the angular distribution in this decay would be difficult to test. (The ratio $\gamma_0^{\beta}/\gamma_1^{\beta}$ will be a sensitive function of ϕ.) It is also doubtful that enough events of $\alpha \rightarrow \rho K$ can be collected to confirm the prediction (implied by equations (64) and (67)) that $\gamma_1^{\alpha}/\gamma_0^{\alpha}$ should be very close to 1.

Interference between α and β has not been considered here. The present solution implies $g_1^{\alpha}, g_0^{\alpha} < 0$ as well as $g_1^{\beta}, g_0^{\beta} < 0$. If α and β are produced via diffractive dissociation one might expect them to arise in $K^+p \rightarrow Q^+p$ primarily from K_A, in analogy with A_1 production from pions. In this case the positive relative sign in equations (39) and (40) combines with the positive relative sign just mentioned to give a coherent sum in the $K^*\pi$ final state. The interference region should then consist of a well-defined minimum, and events taken below and above this minimum might be expected to provide reasonably pure samples of α and β decays, respectively.

Interference effects in the ρK mode should be destructive, by similar arguments. This opposite sign for $K^*\pi$ and ρK modes is quite general.[44]

The mixing angle ϕ cannot be too small if diffractive pro-

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*We thank D. Lissauer for informing us that the predicted $\cos^2\xi$ modulation may be occurring in $\alpha \rightarrow K^*\pi$. The $\sin^2\xi$ modulation in $\beta \rightarrow K^*\pi$ is not seen. This would tend to weigh in favor of a larger ϕ within the bounds (64).

duction of K_A is the only mechanism by which α and β are produced in $K^+p \rightarrow Q^+p$. Otherwise the β peak would be anomalously suppressed. It is not, however, as indicated by the prominent $K^*\pi$ signal in this reaction at the mass of the β peak.

The α state, sometimes call "C", has been seen in $p\bar{p}$ reactions at rest,(45) in the final state $K\bar{K}\pi\pi$. If one naively treats the initial $^3S_1$ state as a virtual ρ if $I = 1$, or ω if $I = 0$ (annihilation from the $J^P = 0^-$ $^1S_0$ state is forbidden to produce KQ by angular momentum and parity conservation),(44) then the present model, predicting a suppression of $\beta \rightarrow \omega K$ and $\beta \rightarrow \rho K$ by virtue of the specific value of ϕ, would predict a suppression of β production in $N\bar{N} \rightarrow K\bar{K}\pi\pi$ (for those incident $\bar{N}$ energies which allow β production* but for which the N and $\bar{N}$ still annihilate in a relative S wave an appreciable fraction of the time.) To our knowledge the comparison of α and β production by antiprotons in flight has not yet been performed.

To conclude, we favor a mixing angle for states in the Q region lying somewhere between 10^o and 35^o, a range which gives rise to: (a) suppression of the $K^*\pi$ mode of the lower peak, (b) suppression of the ρK mode of the upper peak, and (c) angular distributions in the dominant $K^*\pi$ mode close to those characteristic of pure S wave, for both peaks, with the lower more longitudinal and the higher more transverse. The first two predictions, which are the opposite for the

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*The β is too massive to be produced at rest in $N\bar{N} \rightarrow (\beta K$ or $\beta\bar{K})$.

Deck effect, have been subsequently confirmed.* The absence of one $I = 0$ partner of the A_1, and of both $I = 0$ partners of the B, prevents us from confirming the predicted mixing angle via studies of mass formulae at present.

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*See, for example, U. E. Kruse.[29]

IV. THE NINTH 1^{++} MESON

A second isoscalar meson with $J^{pc} = 1^{++}$ has not been seen experimentally. Together with the isoscalar D(1285), the isotriplet $A_1(1070)$, and the lower axial-vector kaon isodoublets $K_A(1240)$, such a meson would complete the set of nine $^3P_1$ states expected from the quark model. In this section we stress some aspects of this meson, which we call the D', that are expected on theoretical grounds. These properties of the D' include: (a) assignment to a weakly mixed SU(3) singlet; (b) a mass lower than the A_1 (~ 950 MeV with assumptions about the relative coupling of D and D'); (c) decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$, and (d) a possible suppression of the ρ signal in the $\pi^+\pi^-$ effective mass spectrum of the $\pi^+\pi^-\gamma$ final state.

These predictions are of special relevance at present because of the recent claim for a new meson with properties similar to (b) - (d).(32) This meson, called M(953),(32) is distinguished from the $\eta'(958)$ only by the absence of any appreciable ρ in its $\pi^+\pi^-\gamma$ final state. The η', in contrast, appears to decay to $\pi^+\pi^-\gamma$ predominantly via $\rho\gamma$.(32,46) While one feels uneasy about a claim for two states so close in mass sharing common decay modes except for the difference just mentioned, our theoretical expectation of a $D'(1^{++}, I = Y = 0)$ near the $\eta'(0^{-+}, I = Y = 0)$ suggests that this degeneracy may be the case.* It should be noted, however, that a

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*The possibility of the D' near the η' was mentioned in ref. 47.

$J^{pc} = 1^{+-}$, $I = 1$ assignment instead of 1^{++}, $I = Y = 0$ for the M(953) is not ruled out by the data of ref. 32, and will be discussed as well.

1. Unitary Singlet Nature

A hierarchy of constraints based on duality — in which the most reliable follow from $PP \rightarrow PP$, the next most reliable from $PV \rightarrow PV$, and the least reliable from $VV \rightarrow VV$ ($P = 0^-$, $V = 1^-$ mesons) — has been used to predict exchange degeneracies.[48,49] In such a system the omission of the "worst" VV constraints predicts nonet structure for mesons of $J^{pc} = 2^{++}$, 1^{--}, 2^{--}, and 1^{+-}, but only octet structure for 0^{-+} and 1^{++}. The unitary singlets of 0^{-+} and 1^{++} do not couple to PP or PV, and hence are not involved in the "good" constraints, which are assumed to determine the observed exchange degeneracies. The remaining eight 0^{-+} and 1^{++} mesons are then expected to form weakly mixed octets. This is certainly the case for the π, K, and η; it seems to hold as well by consideration of the Gell-Mann Okubo mass formula for the $A_1(1070)$, the $K_A(1240)$, and the D(1285):

$$M^2(D) = \frac{4\, m^2(K^*) - m^2\,(A_1)}{3}$$

$$1.65 \simeq 1.67 \quad .$$

The "missing" D' is then identified as a unitary singlet member, as is the η'.

2. Mass of ~ 950 MeV

The existence of a low-lying (1^{++}, $I = Y = 0$) meson was predicted several years ago from the application of superconvergence relations (SCR) to $\pi\pi$, $\pi\rho$, $\pi\delta$, and πA_1 scattering.[50] The extreme saturation assumptions adopted in this work led to many useful predictions, among them $m_{A_1}{}^2 \simeq 2m_\rho{}^2$, $m_\sigma \simeq m_\rho$, and so on. A $(1^{++}, I = Y = 0)$ state was required for superconvergence in $\pi\delta$ scattering, in which it was required to cancel the contribution of the η' pole, and in πA_1 scattering, in which it was needed to saturate the appropriate Adler-Weisberger relation. The predicted mass was $m(1^{++}, I = Y = 0) \simeq m_{A_1}$.

A low-lying D' and the D(1285) may combine to give the "effective" D of ref. 50. The algebraic approach considered there is concerned with states lacking strange quarks. As we have seen, both D and D' have components with this property. If they are both incorporated into the SCR of ref. 50, the superconvergent sum rule for $\pi A_1{}^{\text{Transverse}}$ scattering predicts

$$(m_{A_1}{}^2 - m_{D'}{}^2)\, g^2_{D'A_1\pi} + (m_{A_1}{}^2 - m_D{}^2)\, g_{DA_1\pi}{}^2 = 0 \tag{68}$$

when <u>all other constraints of the model</u> are taken into account. (A single $(1^{++}, I = Y = 0)$ meson coupling to nonstrange quarks would then have to be at $m_D \simeq m_{A_1}$.) Equation (68) predicts that the D' lies lower than the A_1.

A priori there is no relation between the couplings of the D and the D' since they are not mixed. Zweig's rule for quark graphs (eliminating disconnected graphs) relates the singlet and octet couplings for <u>mixed</u> nonets (such as $J^{PC} = 1^{--}$ or 2^{++}). A simple but naive assumption would be to extend the quark graph rule to the case of no mixing. Then the coupling of D and D' to states lacking strange quarks would be in the ratio: $|g_D|/|g_{D'}| = 1/\sqrt{2}$, which via equation (69) predicts the mass of the D' to be:[51]

$$m_{D'} = \sqrt{(3m_{A_1}^{\ 2} - m_D^{\ 2})/2} \simeq .95 \text{ GeV} \quad . \qquad (69)$$

A smaller value of the D' coupling would require an even lower mass for the D'.

If the D' coupling is given by the quark graph prescription, then the apparent failure of the D' to be identified in reactions where the D is produced can only be understood if the D' is actually being seen but is confused with something else. The predominant decay mode of D is $\eta\pi\pi$.[46,52] It is interesting that every published experiment showing an $\eta\pi\pi$ mode of the D also shows an $\eta\pi\pi$ peak around 960 MeV.[46,52] This peak may contain some D' as well as the η' generally assumed. In such cases the study of "η'" $\rightarrow \pi\pi\gamma$ is of particular importance.

3. <u>Decay modes $\eta\pi\pi$ and $\pi^+\pi^-\gamma$</u>

The dominant hadronic decay mode of the D' should be $\eta\pi\pi$, as $m_{D'} < 1.13$ GeV forbids the $K\bar{K}\pi$ channel seen in D decay. Four-pion modes require at least two units of ℓ between various pion pairs (as in $D' \rightarrow \rho\rho \rightarrow 4\pi$), and should be suppressed.

The M1 transition $\eta' \rightarrow \rho\gamma$ competes favorably with the hadronic mode $\eta' \rightarrow \pi\pi\eta$, which can proceed without angular momentum barriers.

The transition $D' \to \rho\gamma$ can be, *a priori*, both E1 and M2,[47] while the $D' \to \pi\pi\eta$ mode involves at least a unit of ℓ between some particle and the other pair. Thus one might expect:

$$\frac{\Gamma(D' \to \pi^+\pi^-\gamma)}{\Gamma(D' \to \eta\pi\pi)} \gtrsim \frac{(\eta' \to \pi^+\pi^-\gamma)}{(\eta' \to \eta\pi\pi)} , \qquad (70)$$

with approximate equality holding if a suppression of $E1(D' \to \rho\gamma)$ (to be discussed) takes place.

Measurements of $\gamma\gamma/\eta\pi^+\pi^-$ or $\gamma\gamma/\pi^+\pi^-\gamma$ ratios in the 960 MeV peak, if found to vary among experiments,[53] could indicate the presence of a variable D' "contaminant" in the η' signal. (The D' cannot decay to $\gamma\gamma$.)[54]

4. Apparent suppression of ρ in $\pi^+\pi^-\gamma$ mode

The E1 and M2 transitions $D' \to \rho\gamma$ may be of comparable magnitude.[47,55] An example is provided by the vector dominance model (VDM). The matrix elements for $D' \to \rho\gamma$ must have a ρ pole in the square of p_2, the photon 4-momentum, at $p_2^2 = m_\rho^2$. The residue of this pole must vanish, however, for $D' \to \rho^{\text{Transverse}}\gamma$, since a 1^+ particle cannot decay into a pair of identical transverse vector mesons.[54] To the extent that this effect may be extrapolated to $p_2^2 = 0$, we then expect

$$T(D' \to \rho^T \gamma) = E1 + M2 \simeq 0 \qquad (71)$$

so that in the matrix element for longitudinal ρ production,

$$T(D' \rightarrow \rho^L \gamma) \quad = \quad E1 - M2 \tag{72}$$

the E1 and M2 constributions are equal. Since $M2 = O(k^2)$, we then expect $E1 = O(k^2)$, where k is the photon energy in the D' rest frame. In contrast, $\eta' \rightarrow \rho\gamma$ proceeds via $M1 = O(k)$. Hence $D' \rightarrow \pi^+\pi^-\gamma$ will favor lower $\pi^+\pi^-$ masses than $\eta' \rightarrow \pi^+\pi^-\gamma$, and one may expect distortion of the ρ peak.[32]

To illustrate this effect, we have compared the Dalitz plot projections in $m_{\pi\pi}$ for the matrix elements

$$T_{\eta}' \quad = \quad g_{\eta'} \quad \epsilon_{\alpha\beta\gamma\delta} \quad P_1^{\alpha} \quad \epsilon_2^{*\beta} \quad P_2^{\gamma} \, (q_1 - q_2)^{\delta} \tag{73}$$

and

$$T_{D'} \quad = \quad ig_{D'} \quad \epsilon_{\alpha\beta\gamma\delta} \, \epsilon_1^{\alpha} \quad \epsilon_2^{*\beta} \quad P_2^{\gamma} \quad P_1^{\delta} \quad P_2 \cdot (q_1 - q_2)/m_{D'}^{\ 2} \tag{74}$$

where $q_{\pi^+} \equiv q_1$, $q_{\pi^-} \equiv q_2$, and other indices (1,2) refer to (η' or D', γ), respectively. The matrix element (74) is the simplest gauge-invariant one satisfying equation (71). If $g_{\eta'}$ and $g_{D'}$ have a similar dependence on $m_{\pi\pi}$, one finds

$$\frac{d\,\Gamma(\eta' \rightarrow \pi^+\pi^-\gamma)/dm_{\pi\pi}}{d\,\Gamma(D' \rightarrow \pi^+\pi^-\gamma)/dm_{\pi\pi}} \quad = \quad W(m_{\pi\pi}^{\ 2}/m_{\eta'}^{\ 2}) \tag{75}$$

where
$$W(\xi) \quad = \quad A\xi/(1 - \xi)^2 \tag{76}$$

and A is a suitable normalization constant.

We find the $\eta' \rightarrow \pi\,\pi\,\gamma$ and "M(953)" $\rightarrow \pi^+\pi^-\gamma$ shapes of ref. 32 are strikingly similar after weighting according to equation (75).

This fit is shown in Fig. 7. The normalization A was fixed by minimizing the sum of squares of

$$N_m(m_{\pi\pi}) - A^{-1} W^{-1}(m_{\pi\pi}^2/m_{\eta'}^2) \; N_{\eta'}(m_{\pi\pi}) \qquad (77)$$

in $0.56 \leq m_{\pi\pi} \leq 0.80$ GeV (R in Fig. 7), the mass range for which both processes show sufficient events. Here $(N_{\eta'}, N_M)$ are the number of $\pi^+\pi^-\gamma$ decays (above background) of (η', M) in 40 MeV bins[32], centered on $m_{\pi\pi}$.[56]

While this example of the distortion is crude (we assumed equivalent $\pi^+\pi^-$ final-state interactions in η' and M(953) decay, for example) it confirms that <u>one need not assume C(M) = -</u>[32] to explain the apparent suppression of the $\rho\gamma$ mode. Many such choices of matrix elements can lead to this suppression. Our particular model, motivated by VDM, predicts a $\cos^2\Theta$ distribution in the γ-π^+ angle in the $\pi\pi$ rest frame. Distortion of the type mentioned can occur, however, for <u>any</u> distribution in Θ. To illustrate this, we present in Table 3 four gauge-invariant matrix elements $T_i^{\lambda_2\lambda_3}$ for $D' \to [\pi^+\pi^-]_{\lambda_3}\,\gamma_{\lambda_2}$ and their sum $T^{\lambda_2\lambda_3}$ for $\pi\pi$ helicities $\lambda_3 = 0, 1$. For $g_1 + g_4 = 0$, both T^{10} and T^{11} are $O(k^2)$. They depend on $g_2 - g_4$ and g_3, respectively, whose relative strengths govern the respective $\cos^2\Theta$ and $\sin^2\Theta$ contributions to the angular distribution. VDM gives $g_3 = g_1 + g_4 = 0$.

Despite the arguments just given in favor of a D' near 950 MeV, we are not able to identify the effect of ref. 32 (the M(953)) as definitely associated with this state. No significant difference in

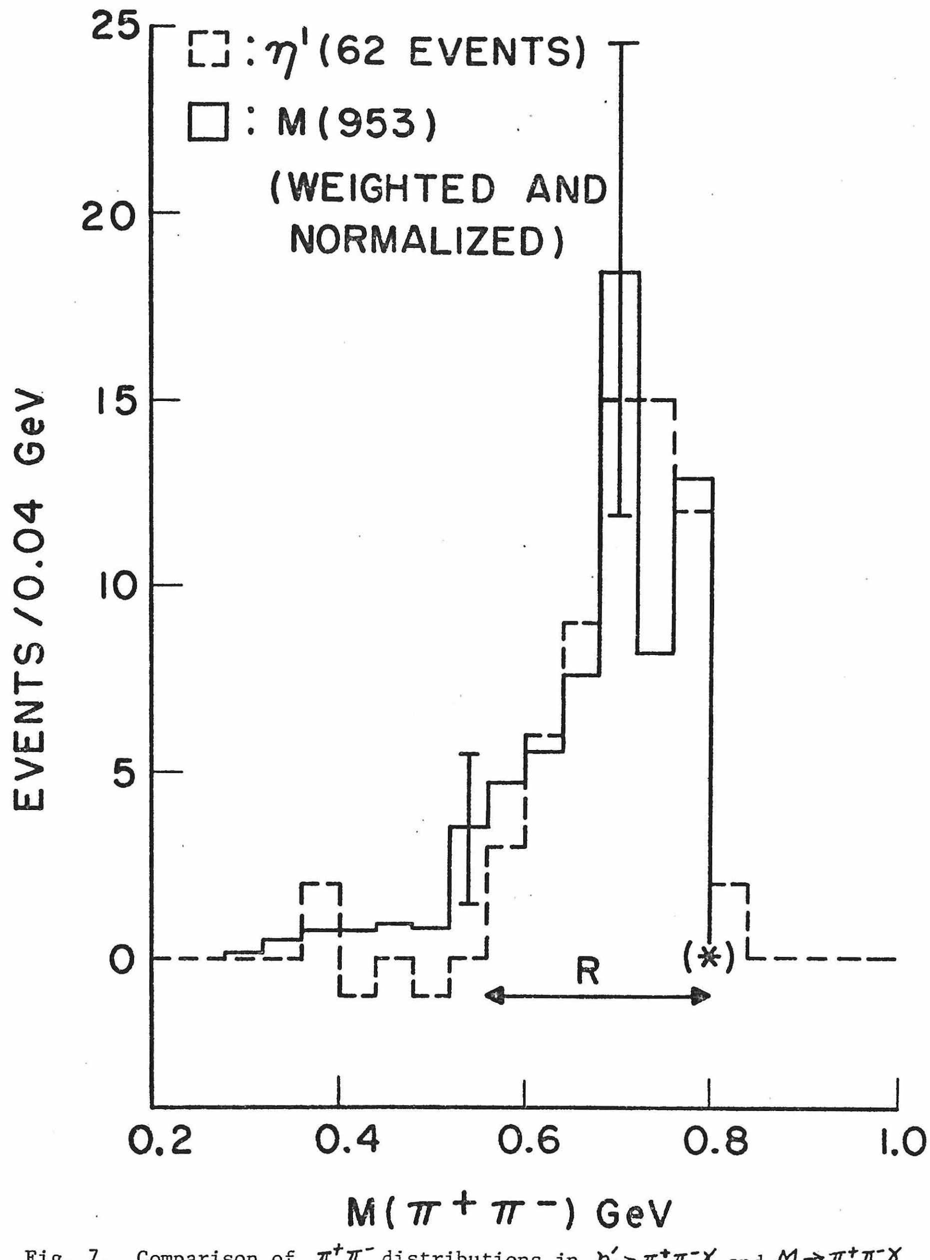

Fig. 7. Comparison of $\pi^+\pi^-$ distributions in $\eta' \to \pi^+\pi^-\gamma$ and $M \to \pi^+\pi^-\gamma$ The latter distribution is weighed for comparison by the factor $m_{\eta'}^2 m_{\pi\pi}^2 / (m_{\eta'}^2 - m_{\pi\pi}^2)^2$ and normalized by a least-squares fit in the range R, described in the text. The distribution for $M \to \pi^+\pi^-\gamma$ above (*) is based on bins with no more than one event and is not shown.

Table 7. Matrix elements for $D'(p_1) \to [\pi^+(q_1)\pi^-(q_2)]_{\lambda_3}\,\gamma(p_2,\lambda_2)$. In the expressions for $T^{\lambda_2\lambda_3}$, we use $Q \equiv (m_{\pi\pi}^2 - 4m_\pi^2)^{1/2}$, $k \equiv (m_{D'}^2 - m_{\pi\pi}^2)/2M_{D'}$, (θ,φ) = angles of π^+ with respect to γ in $\pi\pi$ rest frame.

$$T_1^{\lambda_2\lambda_3} = ig_1\epsilon_{\alpha\beta\gamma\delta}\epsilon_1^{\alpha}\epsilon_2^{*\beta}p_2^{\gamma}(q_1-q_2)^{\delta}$$

$$T_2^{\lambda_2\lambda_3} = ig_2\epsilon_{\alpha\beta\gamma\delta}\epsilon_1^{\alpha}\epsilon_2^{*\beta}p_2^{\gamma}p_1^{\delta}p_2\cdot(q_1-q_2)/m_{D'}^2$$

$$T_3^{\lambda_2\lambda_3} = ig_3\epsilon_{\alpha\beta\gamma\delta}p_1^{\alpha}\epsilon_2^{*\beta}p_2^{\gamma}(q_1-q_2)^{\delta}\epsilon_1\cdot p_2/m_{D'}^2$$

$$T_4^{\lambda_2\lambda_3} = ig_4\epsilon_{\alpha\beta\gamma\delta}\epsilon_1^{\alpha}[\epsilon_2^{*\beta}p_1\cdot p_2 - p_2^{\beta}\epsilon_2^{*}\cdot p_1]p_1^{\gamma}(q_1-q_2)^{\delta}/m_{D'}^2$$

$$T^{\lambda_2\lambda_3} \equiv \sum_{i=1}^{4} T_i^{\lambda_2\lambda_3}$$

$$T^{10} = Qk(m_{D'}/m_{\pi\pi})\cos\theta[g_1+g_4+k(g_2-g_4)/m_{D'}]$$

$$T^{11} = Qk\sin\theta e^{-i\varphi}[g_1+g_4-kg_3/m_{D'}]/\sqrt{2}$$

$\eta\pi\pi$ Dalitz plots apparently exists between the η' and the M(953),[57] whereas one would expect suppression of high $\pi\pi$ masses for the latter since the η and $\pi\pi$ system must be in a relative P wave. An alternative assignment ($J^{pc} = 1^{+-}$, I = 1, Y = 0) is indeed as tenable as that discussed above. The decay $1^{+-} \to \sigma\gamma$ can easily give lower $m_{\pi\pi}$ values than $\eta' \to \rho\gamma$, as the σ is much broader than the ρ. Furthermore, the Dalitz plot for $1^{+-} \to \eta\pi\pi$ automatically favors high $m_{\pi\pi}$ (as here the two pions must be in a relative P wave) and thus could resemble that of $\eta' \to \eta\pi\pi$.*[58]

A G = +, $J^{pc} = 1^{+-}$ state has the quantum numbers of the B(1235). With $\pi\omega$ the expected dominant decay mode,[59] such a state should have been seen by now (but it nonetheless worth looking further for). It would be as embarrassing to the harmonic-oscillator spectrum of the naive quark model as a split A_2.[60]

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*I = 1 assignments[59] for the M(953) other than $J^{pc} = 1^{+-}$ look less likely: $J^{pc} = 1^{--}$ suggests a dominant (unobserved) $\pi\pi$ mode, while $J^{pc} = 0^{--}$ implies considerable structure in the $\eta\pi\pi$ Dalitz plot (unobserved)(57) and suppression of the ratio $\pi\pi\gamma/\eta\pi\pi$. (The two pions in $0^{--} \to \pi\pi\gamma$ must have relative L of at least 2 and must be produced by at least an M2 transition, involving more centrifugal barrier factors than the $\eta\pi\pi$ mode.) I = 0 assignments other than $J^{pc} = 1^{++}$ or 0^{-+} also look less likely: $J^{pc} = 1^{-+}$ suggests considerable structure in the $\eta\pi\pi$ mode.

V. CONCLUSIONS

We have seen that the lowest-lying 0^+, 1^+, and 2^+ mesons seem to act as if decaying via production of a $q\,\bar{q}$ pair in a $^3P_0$, SU(3) singlet state, as long as one does not restrict this pair to have $L_z = S_z = 0$. The angular distributions of the $I = 1$ axial-vector mesons played a crucial role in obtaining and verifying this description of the spin couplings. Our model can be interpreted as a small breaking of the collinear symmetry $SU(6)_W \times O(2)_{L_z}$ even though the predictions for the S wave decay amplitudes are considerably different (same magnitude, but opposite sign).

The model was then applied to the $I = 1/2$ axial-vector kaons which are predicted to mix by an angle between 10° and 35° as determined from lower bounds on the widths of the physical states. The mixing predicts a suppression of both the $K^*\pi$ mode of the lower peak and the ρK mode of the upper peak — results that have been subsequently confirmed.[29]

Lastly, we examined the expected properties of the missing $I = 0$ axial-vector meson needed to complete the nine $J^{pc} = 1^{++}$ states predicted by the quark model. Our theoretical expectation of this meson having a mass lower than the A_1 and decay modes $\eta\pi\pi$ and $\rho\gamma$ (with a suppressed ρ signal) suggests identification with a recently reported state having similar properties. However, this new state cannot be definitely associated with the missing axial-vector meson.

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PART 2

UNIVERSAL CURRENT-CURRENT THEORIES AND THE NON-LEPTONIC HYPERON DECAYS

I. INTRODUCTION

The "V-A theory"[1,2] of the weak interactions has been very successful in explaining both leptonic processes, such as muon decay ($\mu \rightarrow e\nu\bar{\nu}$), and semi-leptonic processes, such as neutron beta decay ($n \rightarrow pe\nu$). The "V-A theory" is a phenomenological description of the weak interactions by an effective Lagrangian, the matrix elements of which are calculated in first order. The effective Lagrangian is written in terms of currents:

$$L_{eff} = \sqrt{2}\, G \, (J_\lambda^{+} J^\lambda + J_\lambda J^{+\lambda}) \quad . \tag{1}$$

The universal coupling G is approximately $10^{-5}/m_p^{\,2}$. The currents are a sum of both a lepton and a hadron piece:

$$J_\lambda = J_\lambda^{\ell} + J_\lambda^{h} \quad , \tag{2}$$

and each piece is the difference of a vector current and an axial-vector current. For example, the lepton current is:

$$J_\lambda^{\ell} = \bar{\nu}_\mu \gamma_\lambda \left(\frac{1-\gamma_5}{2}\right) \mu + \bar{\nu}_e \gamma_\lambda \left(\frac{1-\gamma_5}{2}\right) e \tag{3}$$

where μ and ν_μ (e and ν_e) are the Dirac fields for the muon and its neutrino (electron and its neutrino). The operator "$1-\gamma_5$" insures a two component neutrino theory, i.e., that the massless neutrino has only one helicity. Consequently, the V-A theory (at least for the leptons) is, in a sense, maximally parity violating.

Using the lepton current, the matrix element for muon decay is:

$$\langle e\,\bar{\nu}_e\;\nu_\mu\,|H_{wk}|\,\mu\rangle = 2\sqrt{2}\,G\,(\bar{e}\,\gamma^\lambda\,(\frac{1-\gamma_5}{2})\,\nu_e)(\bar{\nu}_\mu\,\gamma_\lambda\,(\frac{1-\gamma_5}{2})\,\mu)\quad . \qquad (4)$$

The matrix elements of the hadronic current are not as simple because of complications from the strong interactions (renormalization effects). Consequently, the hadronic current is written formally as:

$$J_\lambda^{\ h} = \frac{1}{2}\,(V_\lambda^{\ h} - A_\lambda^{\ h})\quad , \qquad (5)$$

and the matrix element for neutron beta decay becomes:

$$\langle pe\bar{\nu}\,|H_{wk}|\,n\rangle = \sqrt{2}\,(\bar{e}\,\gamma^\lambda\,(\frac{1-\gamma_5}{2})\nu)(g_V\,\bar{U}_p\,\gamma_\lambda\,U_n - g_A\,\bar{U}_p\,\gamma_\lambda\,\gamma_5\,U_n). \qquad (6)$$

(the coupling G has been absorbed in g_V.) So far, the scale of the hadron current has not been fixed.

The remarkable equality (within 2%) of G, which describes the strength of muon decay, and g_V, which describes the strength of vector coupling in neutron beta decay, led Feynman and Gell-Mann[1] to propose the strangeness conserving hadronic vector current is not renormalized by the strong interactions. Such a situation has occurred before in particle physics. The electric charge is not renormalized by any interaction because the electric current is conserved. Feynman and Gell-Mann proposed that $V_\lambda^{\ h}$ be identified with the i-spin raising part of the conserved isotopic spin current (CVC hypothesis). The near equality of G and the unrenormalized g_V reinforces the concept of universality, that the weak interactions have a universal strength.

The hadronic axial-vector current is not conserved or otherwise the massive pion would not decay. The matrix element for $\pi \rightarrow \mu\nu$ is:

$$\langle \mu^- \nu \; | H_{wk} | \; \pi \rangle = \sqrt{2}\, G \, (\bar{\mu} \, \gamma^\lambda (\frac{1-\gamma_5}{2}) \, \nu_\mu) \, \langle 0 \; | A_\lambda^{\,h} | \; \pi^- \rangle \qquad (7)$$

where

$$\langle 0 \; | A_\lambda^{\,h} | \; \pi^- \rangle = i \, g_\lambda \, f_\pi \qquad f_\pi \simeq .96 \, m_\pi \qquad (8)$$

($\nabla_\lambda A^\lambda = 0$ would imply $f_\pi = 0$ or $m_\pi = 0$.) In other words, the divergence of the axial-vector current is a pseudoscalar operator that connects the pion to the vacuum, with a proportionality constant that goes to zero in the limit $M_\pi \rightarrow 0$. Thus $\nabla_\lambda A^\lambda$ can be interpreted as a pion field. If the matrix elements of $\nabla_\lambda A^\lambda$ in the interval $0 \leq q^2 \leq M_\pi^{\,2}$ (q^2 = momentum transfer) are slowly varying as a function of q^2, then they can be replaced by corresponding matrix elements involving the pion (the PCAC hypothesis).(3) This hypothesis leads to the famous Goldberger-Treiman relation(4) which connects the pion-nucleon coupling constant ($g_{\pi NN}$), the axial-vector coupling constant in neutron beta decay (g_A), and the decay constant of the pion (f_π):

$$f_\pi = \frac{\sqrt{2} \; M_N \; M_\pi^{\,2} \; g_A}{g_{\pi NN}} \; . \qquad (9)$$

the relation is satisfied to within 10%.

The hadronic current also includes a strangeness changing part since both the strange mesons and baryons decay via the weak interactions — $K \to \mu\nu$, $\Lambda \to p\ e\ \nu$, etc. The CVC hypothesis and SU(3) symmetry suggest the vector currents belong to an octet, with their charges being the generators of the symmetry group. (A charge is the spatial integral of J^{o} and is conserved if $\nabla_{\lambda} J^{\lambda} = 0$.) The $\Delta S = 1$ vector currents are conserved only in the limit of exact SU(3). If the axial-vector currents are identified with an octet too, then the following commutation relations are a consequence of SU(3):

$$[Q_i, Q_j] = i f_{ijk} Q_k \tag{10}$$

$$[Q_i, Q_j^5] = i f_{ijk} Q_k^5 \quad . \tag{11}$$

The symbols Q_i and Q_i^5 represent the charges of the vector and axial-vector currents, respectively; f_{ijk} represents the structure constants of SU(3). In a quark model, even with symmetry breaking via a different mass for the strange quark, commutation relations (10) and (11) are valid plus one more:

$$[Q_i^5, Q_j^5] = i f_{ijk} Q_k \quad . \tag{12}$$

The vector and axial quark currents are:

$$V_{\lambda}^{i} = \bar{q}\ \gamma_{\lambda} \frac{\lambda_i}{2}\ q \tag{13}$$

$$A_\lambda^i = \bar{q}\, \gamma_\lambda \gamma_5 \frac{\lambda_i}{2} q \quad . \tag{14}$$

Gell-Mann[5,6] proposed that equations (10), (11), and (12) are valid independent of the quark model and SU(3) breaking. These relations then give a meaning to SU(3) even when the symmetry is broken. Using equation (12) and PCAC, Adler and Weisberger[7] calculated $g_A/g_V = 1.24 \pm .03$ in agreement with the experimental number $1.23 \pm .01$. (Actually, this is Adler's corrected result.)

One problem remained with the $\Delta S = 1$ semi-leptonic decays — they are one-tenth the size predicted by universality. This problem was solved by generalizing the concept of universality. Gell-Mann noticed that the leptonic charges satisfy an SU(2) algebra, i.e., defining

$$Q_\ell^+ = \int d^3x\, J_\ell^o = (Q_\ell^-)^\dagger \tag{15}$$

and

$$2Q_\ell^3 = [Q_\ell^+, Q_\ell^-] \tag{16}$$

implies

$$[Q_\ell^3, Q_\ell^\pm] = \pm Q_\ell^\pm \quad . \tag{17}$$

He then postulated that the hadronic charges satisfy the same SU(2) commutation relations.[8] This new version of universality fixes the scale of the hadronic current through the nonlinear commutation relations. The most general, charged V-A current that satisfies this postulate is:

$$J_\lambda^{h} = \cos\theta \; J_\lambda^{T^+} + \sin\theta \; J_\lambda^{V^+} \tag{18}$$

where the superscripts "T^+" and "V^+" refer to the SU(3) indices, i.e.,

$$J_\lambda^{T^+} = \frac{1}{2}(V_\lambda^{T^+} - A_\lambda^{T^+}) = \frac{1}{2}(V_\lambda^{1} + i\,V_\lambda^{2} - A_\lambda^{1} - i\,A_\lambda^{2}) \tag{19}$$

$$J_\lambda^{V^+} = \frac{1}{2}(V_\lambda^{V^+} - A_\lambda^{V^+}) = \frac{1}{2}(V_\lambda^{4} + i\,V_\lambda^{5} - A_\lambda^{4} - i\,A_\lambda^{5}) \,. \tag{20}$$

The angle θ is not fixed by the commutation relations. Cabibbo[9] showed that the predictions of this current (with the angle θ determined experimentally) agree extremely well with the meson and baryon semi-leptonic decays. Actually, the effective Cabibbo angle is not necessarily the same for the vector current and the axial current because of SU(3) breaking. (The Ademollo-Gatto theorem[10] states that the matrix elements of the strangeness-changing vector current, whose charge is a generator of SU(3), are renormalized only in second order of SU(3) breaking. The matrix elements of the axial current can be renormalized in first order.) Recent determinations[11] of the effective Cabibbo angle are: (1) $\theta_A = .2688 \pm .0006$ from $K^+ \to \mu^+\nu/\pi^+ \to \mu^+\nu$, (2) $\theta_V = .222 \pm .003$ from $K^+ \to \pi^o e\nu/\pi^+ \to \pi^o e^+\nu$, and (3) $\theta_V = .233 \pm .012$ and $\theta_A = .238 \pm .018$ from a fit to the baryon decays.* Moreover, the small discrepancy between G and g_V is explained by the fact that $g_V = G \cos\theta \approx .98\ G$ in this theory.

The mesons and baryons also decay weakly into non-leptonic final states, e.g. $K^o \to \pi\pi$, $\Lambda \to p\pi$, $\Sigma \to p\pi$, $\Xi \to \Lambda\pi$. Although the Cabibbo theory expects such decays, the matrix elements are almost impossible to calculate because the product of the two currents cannot be separated as in the leptonic and semi-leptonic decays. Before examining the limited evidence for the Cabibbo theory in the non-leptonic hyperon decays, we will discuss the experimental data on a strictly phenomenological level.

The most general matrix element for the baryon decays is:

$$\bar{U}(p_2)\,(A + B\,\gamma_5)\,U(p_1) \quad , \qquad (21)$$

where p_1 and p_2 are the momenta of the initial and final baryon (see Fig. 1). If CP is conserved (as in the Cabibbo theory) and there is no final state interaction, then A and B are relatively real. Experimentally, A and B have a small phase difference which is consistent with CP conservation and the predicted final state interaction from low energy phase shifts (approximately 7^o for $\Lambda \to p\pi^-$).[11] The matrix element may also be written in terms of two component

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*With F/D for $\langle B|A^h|B\rangle$ equal to $.66 \pm .03$ in agreement with PCAC and F/D = 2/3 predicted by $SU(6)_W$ for PBB coupling.

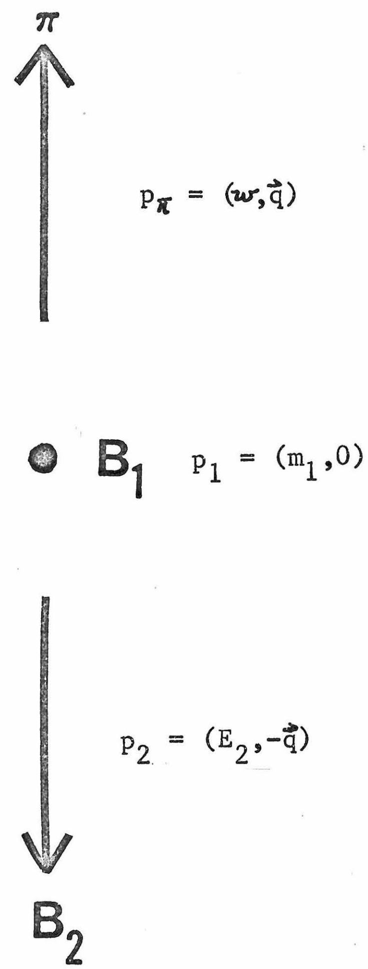

Fig. 1. Non-leptonic decay of a baryon: $B_1 \to B_2 \pi$

spinors:

$$\chi_2^{\dagger} \; (S + P \, \vec{\sigma} \cdot \vec{q}) \; \chi_1 \tag{22}$$

where $S = \sqrt{E_2 + M_2} \; \sqrt{2M_1} \; A$ and $P = \dfrac{q\sqrt{2M_1}}{\sqrt{E_2 + M_2}} \; B$. Thus, S (or A) corresponds to an S-wave interaction while P (or B) corresponds to a P-wave interaction. Because of the negative intrinsic parity of the pion, the S wave is parity violating while the P wave is parity conserving. The interference between the two waves appears in the angular distribution of the decay from a polarized particle:

$$W(\theta) = 1 + \alpha \; P \cos \theta \qquad P = \text{polarization of initial baryon} \qquad \alpha = \frac{2\,\mathrm{Re}\,(S^* P)}{|S|^2 + |P|^2} \; . \tag{23}$$

The most recent experimental data are given in Table 1.[11] (Λ^o_- represents the decay $\Lambda^o \to p\pi^-$, Σ^+_+ represents the decay $\Sigma^+ \to n\,\pi^+$, etc.) The most striking fact is the excellent agreement of the data with an empirical $\Delta I = 1/2$ rule for the non-leptonic weak Hamiltonian. The $\Delta I = 1/2$ rule implies:

$$\Lambda^o_- + \sqrt{2}\,\Lambda^o_o = 0 \tag{24}$$

$$\Xi^-_- - \sqrt{2}\,\Xi^o_o = 0 \tag{25}$$

Table 1

Rates, decay parameters and decay amplitudes for hyperon hadronic decays

	Λ°_{-}	Σ^{-}_{-}	Σ^{+}_{+}	$\Sigma^{+}_{\circ}$	Ξ^{-}_{-}	$\Xi^{\circ}_{\circ}$
RATE	.397 ± .005	.610 ± .021	1.235 ± .020	1.235 ± .020	.602 ± .013	.330 ± .020
BR	.640 ± .014	1	.472 ± .015	.528 ± .015	1	1
α	.645 ± .016[a]	-.060 ± .047	.026 ± .042	-.960 ± .067	-.425 ± .035[a]	-.346 ± .075
ϕ [degr]	-9 ± 6	-5 ± 17	161 ± 22	—	-3 ± 6	22 ± 20
γ	> 0	> 0	< 0	?	> 0	> 0
A	1.52 ± .02	1.87 ± .03	.02 ± .04	1.53 ± .14 1.15 ± .18[b]	2.07 ± .03	1.53 ± .05
B	10.44 ± .33	-.55 ± .43	19.08 ± .35	-11.52 ± 1.85 -15.36 ± 1.40[b]	-7.42 ± .65	-4.54 ± 1.02
C_{AB}[c]	.122	.021	-.004	.959	.239	.102

[a] α_{Λ} and α_{Ξ^-} fitted to $\alpha_{\Lambda}\alpha_{\Xi^-} = -.285 \pm .026$

[b] second solution since sign of γ unknown

[c] $C_{AB} = \dfrac{\langle \Delta A \; \Delta B \rangle}{\sqrt{\Delta A^2 - \Delta B^2}}$

RATE and BR. from Rosenfeld tables Aug. 68, α and ϕ Jan. 69 world averages

$$\Sigma^-_- - \sqrt{2}\ \Sigma^+_0 = \Sigma^+_+ \quad . \tag{26}$$

One other empirical rule (with some theoretical justification) has been noticed — the Lee-Sugawara relation:[12]

$$2\ \Xi^-_- - \Lambda^0_- = \sqrt{3}\ \Sigma^+_0 \tag{27}$$

The agreement of equations (26) and (27) with the experimental data is indicated in Fig. 2.[11] (The sign convention of ref. (11) is used.)

The neutral K^0 decays are also in good agreement with the $\Delta I = 1/2$ rule. The only real evidence for a $\Delta I = 3/2$ component comes from the decay $K^+ \to \pi^+\pi^0$ which cannot go via $\Delta I = 1/2$. The ratio

$$\frac{\Gamma_{K^+ \to \pi^+\pi^0}}{\Gamma_{K^0 \to \pi\ \pi}} \simeq \frac{1}{500}$$

indicates that the $\Delta I = 3/2$ component in kaon decays is no more than 5% in the amplitude. (Recent accurate data[13] on $K^0 \to \pi\pi$ and $K \to \pi\pi\pi$ indicates a comparably small amount of $\Delta I = 3/2$.) The neutral kaon decays exhibit one other striking phenomena — a small violation of CP conservation on the order of .2%.

The Cabibbo theory predicts that the non-leptonic Hamiltonian is given by:

$$H_{\Delta S = 0} = \sqrt{2}\, G \left[\cos^2\theta \left\{J^{T^+}, J^{T^-}\right\} + \sin^2\theta \left\{J^{V^+}, J^{V^-}\right\}\right] \tag{28}$$

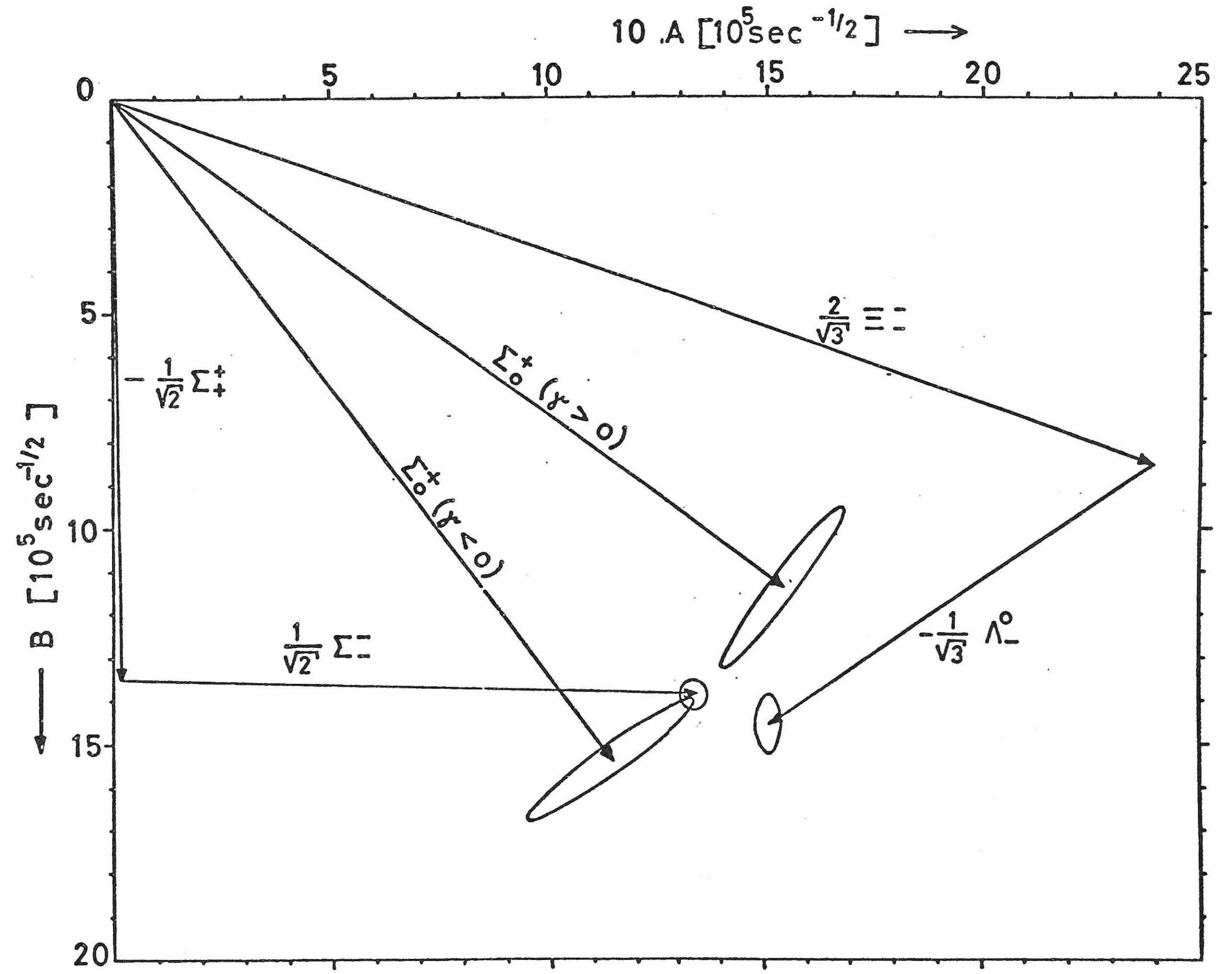

Fig. 2. Test of the $\Delta I = 1/2$ and Lee-Sugawara triangle relations by the measured decay amplitudes of Λ , $\Sigma^{\pm}$ and Ξ^- .

$$H_{\Delta S = 1} = \left(H^{\Delta S = -1}\right)^{\dagger} = \sqrt{2}\, G \sin\Theta \cos\Theta \left\{ J^{T^-}, J^{V^+} \right\} \qquad (29)$$

(The notation $\{A,B\}$ means $AB + BA$. Also, the 4-vector indices on the currents have been dropped for convenience.) These Hamiltonians may be divided into parity conserving and parity violating parts, e.g.,

$$H^{p.c.}_{\Delta S = 1} = \frac{G}{2\sqrt{2}} \sin\Theta \cos\Theta \left[\left\{ V^{T^-}, V^{V^+} \right\} + \left\{ A^{T^-}, A^{V^+} \right\} \right] \qquad (30)$$

$$H^{p.v.}_{\Delta S = 0} = -\frac{G}{2\sqrt{2}} \sin\Theta \cos\Theta \left[\left\{ V^{T^-}, A^{V^+} \right\} + \left\{ V^{V^+}, A^{T^-} \right\} \right] \qquad (31)$$

The $\Delta S = 1$ Hamiltonian contains both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces, and there is no "a priori" reason to expect a suppression of the $\Delta I = 3/2$. In terms of SU(3), the Hamiltonian contains only $\underset{\sim}{8}$ and $\underset{\sim}{27}$ because it is symmetric in the unitary spin of the two currents. Octet dominance, which is assumed in various dynamical schemes, is sufficient but not necessary for $\Delta I = 1/2$. The Cabibbo $\Delta S = 1$ Hamiltonian has another symmetry property, which for the octet part corresponds to being the sixth (λ_6) component.(14) For the baryon decays, the sixth component of an octet (plus CP conservation) is sufficient to guarantee the Lee-Sugawa relation for the S waves, but <u>not</u> for the P waves. Moreover, the sixth component of an octet forbids the dominant decay $K_1^o \rightarrow \pi\pi$ (K_1^o is the linear combination of K^o and $\overline{K}^o$ that decays into $\pi\pi$ via CP conservation). This failure is not too serious as the decay is only forbidden with exact SU(3), so that the matrix element could be quite large if proportional to the mass difference of K and π. The small CP violation in kaon decays, however, is strictly forbidden by Cabibbo's Hamiltonian. A fair conclusion is that the Cabibbo theory fails to explain many features of the non-leptonic hyperon decays.

Some success for the Cabibbo theory has been achieved by the use of the current commutation relations and PCAC. Low energy theorems are derived which relate $\langle B_2 \, \pi^i \, | H_{wk} | \, B_1 \rangle$ to $\langle B_2 | \, [Q_i^5 \, (0), \, H_{wk}] \, | \, B_1 \rangle$ in the limit of vanishing pion four-momentum.[15,16] (A detailed derivation of these relations and the ones that follow is given in Appendix A.) In particular, Suzuki[15] was able to show that the parity violating S wave in non-leptonic baryon decays is approximately given by:

$$\langle B_2 \, \pi^i \, | H_{p.v.}^{\Delta S=1} \, (0) | \, B_1 \rangle \approx - \frac{\sqrt{2}}{f_\pi} \, i \, \langle B_2 \, | [Q_i^5 \, (0), \, H_{p.v.}^{\Delta S=1} \, (0)] | \, B_1 \rangle \quad . \tag{32}$$

A smooth extrapolation of the physical amplitude to zero four momentum for the pion can be justified for the S wave, but not for the P wave. If the Cabibbo Hamiltonian is assumed, then the commutator with the axial charge is easily evaluated, e.g., for $Q_3^5 \, (0)$ (π^0 emission in the original decay):

$$\left[Q_3^5(0),\ H_{p.v}^{\Delta S=1}(0)\right] = -\frac{G}{2\sqrt{2}} \sin\theta \quad \cos\theta \left[Q_3^5(0),\ \left\{V^{T^-},\ A^{V^+}\right\} + \left\{V^{V^+}, A^{T^-}\right\}\right]$$

$$= \frac{1}{2} \cdot \frac{G}{2\sqrt{2}} \sin\theta \quad \cos\theta \left[\left\{V^{T^-},\ V^{V^+}\right\} + \left\{A^{T^-},\ A^{V^+}\right\}\right] \tag{33}$$

The right side is just proportional to $H_{p.c.}^{\Delta S=1}$. Similar results follow for commutation with

$$\frac{1}{\sqrt{2}}(\mp Q_1^5 + i\, Q_2^5) \quad (\pi^{\pm} \text{ emission in the original decay})$$

except that $H_{p.c.}^{\Delta S=1}$ is in a different SU(3) direction:

$$\left[\frac{1}{\sqrt{2}}(-Q_1^5(0) + i\, Q_2^5(0)),\ H_{p.v.}^{\Delta S=1}(0)\right] = -\frac{1}{\sqrt{2}} \cdot \frac{G}{2\sqrt{2}} \sin\theta \quad \cos\theta$$

$$\times \left[\left\{V^{T^-},\ V^{U^+}\right\} + \left\{A^{T^-},\ A^{U^+}\right\}\right] \tag{34}$$

$$\left[\frac{1}{\sqrt{2}}(Q_1^5(0) + i\, Q_2^5(0)),\ H_{p.v.}^{\Delta S=1}(0)\right] = \sqrt{2} \cdot \frac{G}{2\sqrt{2}} \sin\theta \quad \cos\theta$$

$$\times \left[\left\{V^{T_3},\ V^{V^+}\right\} + \left\{A^{T_3},\ A^{V^+}\right\}\right] \tag{35}$$

where $V^{U^+} = V^6 + i\,V^7,\ V^{T_3} = V^3,\ V^{T^-} = V^1 - i\,V^2$, etc.

We can summarize these results for the non-leptonic baryon decays as follows:

$$\pi^0 \text{ emission } \alpha \ \langle B_2 \mid \tfrac{1}{2} H_{p.c.}^{T^-,\ V^+} \mid B_1 \rangle \tag{36}$$

$$\pi^+ \text{ emission } \alpha \ \langle B_2 \,|\, -\frac{1}{\sqrt{2}} H_{p.c.}^{T^-,\,U^+} \,|\, B_1 \rangle \tag{37}$$

$$\pi^- \text{ emission } \alpha \ \langle B_2 | \sqrt{2}\, H_{p.c.}^{T_3,\,V^+} \,|\, B_1 \rangle \quad . \tag{38}$$

Since $H^{\Delta S=1}$ contains $\underset{\sim}{8}$ and $\underset{\sim}{27}$ parts, the matrix elements $\langle B_2 \,| H_{p.c.}^{\Delta S=1}(0) |\, B_1 \rangle$ can be expressed in terms of three parameters — two for the $\underset{\sim}{8}$ part (D,F) and one for the $\underset{\sim}{27}$ part (α). The seven non-leptonic baryon decays are then determined by these three parameters. The four relations(15) that follow are:

$$\Lambda^o_- + \sqrt{2}\,\Lambda^o_o = 0 \tag{39}$$

$$\Xi^-_- - \sqrt{2}\,\Xi^o_o = 0 \tag{40}$$

$$\Sigma^-_- - \sqrt{2}\,\Sigma^+_o = -\Sigma^+_+ \tag{41}$$

$$2\,\Xi^-_- - \Lambda^o_- = \sqrt{3}\,\Sigma^+_o - \sqrt{\tfrac{3}{2}}\,\Sigma^+_+ \quad . \tag{42}$$

The first two relations are the $\Delta I = 1/2$ rule for Λ and Ξ decays, which follows from the fact that only the $\Delta I = 1/2$ part of $H_{p.c.}^{\Delta S=1}$ can connect Λ with the N and Ξ isodoublets. However, <u>only if</u> <u>$\Sigma^+_+ = 0$</u> is the third relation the $\Delta I = 1/2$ rule for Σ decays and the fourth relation the Lee-Sugawara rule. Note that the Σ^+_+ amplitude is proportional to α since Σ^+ and n cannot be connected by the $\underset{\sim}{8}$ part of $H_{p.c.}^{\Delta S=1}$. Empirically, $\Sigma^+_+ \approx 0$ implying $\alpha \approx 0$, so that the fit to the data requires octet dominance. Equations (41) and (42) are then equivalent to the $\Delta I = 1/2$ rule for Σ decays and the Lee-Sugawara relation, respectively. However, octet dominance for the Cabibbo Hamiltonian is sufficient to guarantee the $\Delta I = 1/2$ and Lee-Sugawara rules for the S waves without the use of current algebra. Current algebra and PCAC do imply one more relation (using octet dominance, of course):

$$\Sigma_+^+ = 0 \quad . \tag{43}$$

Thus, the S wave amplitudes for non-leptonic baryon decays are reasonably well explained by the Cabibbo Hamiltonian although the reason for octet dominance is still mysterious.

The P wave amplitude is not given by the commutator of Q_i^5 and $H_{p.c.}^{\Delta S=1}$ because, for the parity conserving amplitude, the extrapolation to zero four-momentum of the pion is not valid.[17] Baryon pole terms, or Born terms as they are called (examples illustrated in Fig. 3), contribute to the parity conserving amplitude and vary rapidly in the extrapolation. The Born terms are the contributions of single particle intermediate states approximately degenerate in mass with either B_1 or B_2 (which in the case of SU(3) includes any member of the $\frac{1}{2}^+$ octet). The contribution of a typical Born term to the physical amplitude is given by:

$$g_{BB_2\pi} \frac{1}{M_{B_1} - M_B} \langle B | H^{\Delta S=1} | B_1 \rangle \tag{44}$$

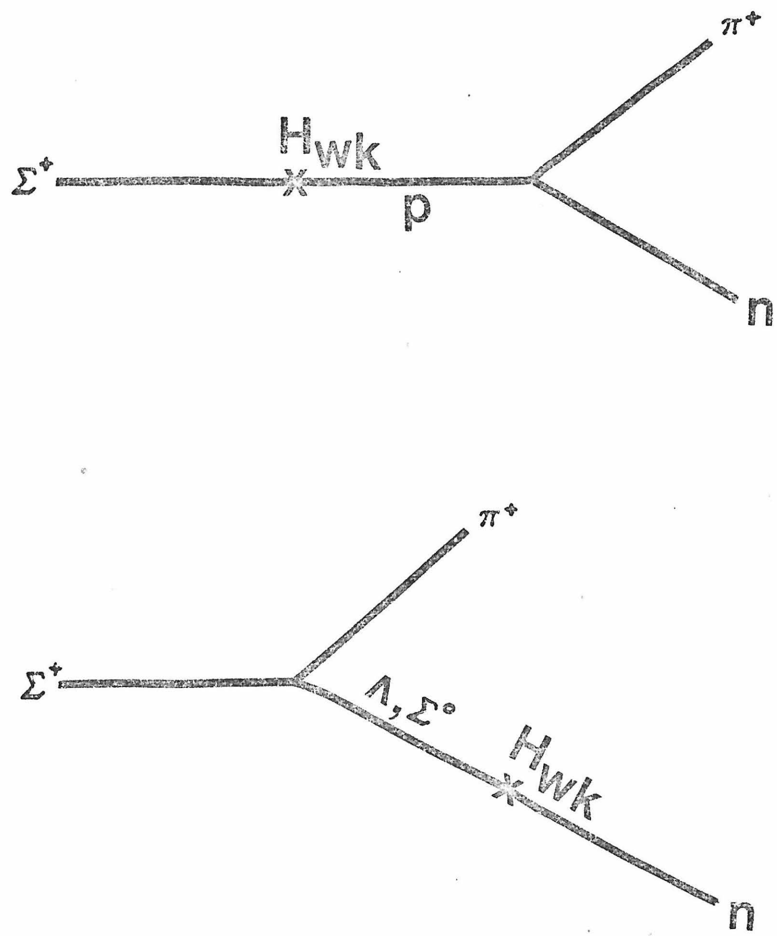

Fig. 3. Born terms for $\Sigma^+ \rightarrow n\pi^+$

The matrix element of $H_{p.v.}^{\Delta S=1}$ between octet baryon states vanishes in the limit of exact SU(3), so that the pole terms contribute only to the P wave amplitude. Note that the vanishing of $\langle B | H_{p.v.}^{\Delta S=1} | B_1 \rangle$ has another consequence — the commutator terms $\langle B_2 | [Q_i^5(0), H^{\Delta S=1}] | B_1 \rangle$ contribute only to the S wave because

$$[Q_i^5(0), H_{p.c.}^{\Delta S=1}(0)] \;\alpha\; H_{p.v.}^{\Delta S=1}(0) \quad . \tag{45}$$

Thus, we have the following picture of the non-leptonic baryon decays: (i) S wave amplitudes determined by $\langle B_2 | [Q_i^5(0), H_{p.v.}^{\Delta S=1}] | B_1 \rangle$ and (ii) P wave amplitudes determined by Born terms. The Born terms seen essential for obtaining the large size of the P wave amplitudes, which otherwise would have been suppressed by the angular momentum barrier. Since the baryon matrix elements of $H_{p.c.}^{\Delta S=1}$, which are determined by the three parameter fit to the S waves, appear also in the formulae for the Born terms, the current algebra analysis indirectly determines the P wave amplitudes too.

The D/F ratio for $\langle B_2 | H_{p.c.}^{\Delta S=1} | B_1 \rangle$ determined in the fit to S-wave decay amplitudes is:

$$D/F \simeq -.31 \quad . \tag{46}$$

This D/F ratio is amazingly close to that for the baryon matrix elements of the SU(3) breaking part of the strong interactions, which is the eighth component of an octet

$$D/F \simeq -.31 \pm .02 \tag{47}$$

If we assume SU(3) for the strong coupling $BB\pi$ in the formulae for the Born terms, we can evaluate explicitly the P wave amplitudes. Three baryon pole terms contribute to the decay with the largest experimental P wave — $\Sigma^+ \to n\pi^+$. However, when the three terms are evaluated, their sum is zero.[16] The D/F from the S-wave fit predicts a vanishing P-wave for Σ^+_+ in complete disagreement with experiment. Meson pole terms contribute along with baryon pole terms to the other decays, but when they are included the other P-wave amplitudes vanish, too. (We have used the value for $\langle \pi | H_{p.c.}^{\Delta S=1} | K \rangle$ determined from a current algebra analysis of $K \to \pi\pi$.) The reason for the identically zero predictions for the P waves can be understood from equations (46) and (47). The simplest explanation for the similarity of the D/F ratios is that $H_{p.c.}^{\Delta S=1}$ $(\sim\lambda_6)$ and $H_{SU(3)}$ mass breaking $(\sim\lambda_8)$ are different components of the same octet. (The proportionality constant between the two in baryon matrix elements is approximately the same as in meson matrix elements.)[16] However, if this is true, there will be no parity conserving non-leptonic decays because an SU(3) rotation of the Hamiltonian $(H_o + gH_8 + \epsilon H_6)$ can remove the parity conserving weak interaction $(H_o + g'H_8')$. The rotated states would then be stable except for the parity violating decays, so that the relation between $H_{p.c.}^{\Delta S=1}$ and $H_{SU(3)}$ mass breaking cannot really be true. However, the relation is correct for the Born terms because the D/F ratios are the same;and, consequently,the Born terms must cancel identically. Thus, the P-wave amplitudes for the baryon non-leptonic decays are not understood by the current algebra approach.

Recently, further success for the Cabibbo Hamiltonian has been achieved through another approach — the symmetric quark model. Feynman[18] (and others previously)[19] noticed that the V-A current-current interaction in a Bose quark model (quarks which obey symmetric statistics) guarantees the $\Delta I = 1/2$ rule for the non-leptonic decays. The reason for this fact is the Fierz transformation properties of the V-A interaction. The Fierz transformation is the permutation of $1 \leftrightarrow 2$ of $3 \leftrightarrow 4$ in the following current-current (point) interaction:

$$\bar{\Psi}_3 \; \Gamma_i \; \Psi_1 \; \bar{\Psi}_4 \; \Gamma_i \; \Psi_2 \qquad (48)$$

where Γ_i are Dirac matrices. For $\Gamma_i = \gamma_\mu (1-\gamma_5)$, the interaction is antisymmetrical under the Fierz transformation. In the quark model, Ψ_1, Ψ_2, Ψ_3, and Ψ_4 are the dirac spinors of the λ, p, p, and n quarks, respectively. Because of the antisymmetry under the Fierz transformation, the V-A point interaction is antisymmetric in space and spin of quarks 1 and 2 or 3 and 4. Since the quarks obey Bose statistics, quarks 1 and 2 or 3 and 4 must then be antisymmetric in unitary spin (either SU(2) or SU(3)). The antisymmetric isospin state of p and n (3 and 4) is $I = 0$. The isospin state of λ and p (1 and 2) is automatically $I = 1/2$. Consequently, the Hamiltonian is pure $\Delta I = 1/2$. Moreover, with SU(3) symmetry, the Hamiltonian is pure octet. (λ-p and p-n are in $\bar{\underset{\sim}{3}}$ representations which can be connected by $\underset{\sim}{8}$, but not $\underset{\sim}{27}$.) Thus, the symmetric quark model implies the $\Delta I = 1/2$ rule for both S waves and P waves and the Lee-Sugawara rule for the S waves (but not the P waves).

The current algebra analysis should apply to the weak interactions in the Bose quark model too. The results for the S waves follow as before except that octet dominance is guaranteed. Two diagrams contribute to the matrix element $\langle B_2 | H^{\Delta S=1}_{p.c.} | B_1 \rangle$: one where the weak interaction takes place on one quark which is pure F and one where it takes place on two quarks which is $D/F = -1$. The relative size of these two diagrams is not predicted, but they must contribute in roughly equal amounts to give the experimental D/F. Thus, the quark model needs another parameter besides the overall normalization to determine the S-wave amplitudes. Unfortunately, the analysis of the P waves via the Born terms also follows as before, which means that the Bose quark model does not solve the enigma with the P waves even though it guarantees $\Delta I = 1/2$.

Because of the limited evidence for the V-A Cabibbo theory in the non-leptonic decays, we shall examine the compatibility with experiment of more general current-current theories. We shall include a wider class of currents while retaining the J^+J form of the interaction. The most obvious source of more general currents is the quark model, where it is possible to construct neutral and charged currents with various Lorentz properties — scalar (S), pseudoscalar (P), vector (V), axial-vector (A), and tensor (T). Since the quark model has been a successful source of intuition in the past, perhaps all the currents obtainable with quarks are observable. If these additional currents do indeed exist, the most logical place for them to appear is in the non-leptonic decays. Since the lepton current is charged (e and ν_e or μ and ν_μ form the current) and of the form "V-A" (right-handed neutrinos), the hadron current in semi-leptonic decays must be a charged 4-vector, i.e., some combination of vector and axial. However, no such restriction is imposed on the weak decays without leptons. Consequently, currents with other Lorentz properties and charges may appear in the non-leptonic decays. An interesting question is whether the presence of these currents is compatible with or eliminated by the present data.

We shall impose the constraint of universality on these more general currents, as was suggested by Zachariasen and Zweig.[20] Universality will be enforced by requiring the charge of each current to be the $\sigma_x + i\,\sigma_y$ component of an SU(2) algebra, a generalization of Gell-Mann's definition of universality. Specifically, the Hamiltonian is assumed to be of the form

$$H_{wk} = \sqrt{2}\, G \sum_a j_a\, j_a^+ \tag{49}$$

where

$$j_a = \sum_\alpha j_a^\alpha \quad \text{and} \quad Q_a^\alpha(0) = \int d^3x\, j_a^\alpha(0, \vec{x})^* \tag{50}$$

with

$$[Q_a^\alpha(0), [Q_a^\alpha(0), Q_a^{\alpha\dagger}(0)]] = -2\, Q_a^\alpha(0). \tag{51}$$

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*Actually, the definition of the charge needs to be made more precise. The above definition suffices for Lorentz scalars; for Lorentz 4-vectors the time component of the current is integrated to give the charge; for Lorentz tensors the best choice is to integrate the xz or yz component.

The index "a" refers to the electric charge and Lorentz properties of each current, while the index "α" represents the lepton and hadron pieces. Of course, we are assuming only the charged 4-vector current has a lepton piece. Equation (51) is just the statement that the charges form an SU(2) algebra. Since the hadron currents are components of an SU(3) octet, two independent SU(2)'s exist: one neutral and one charged. Therefore, the possible hadron currents are charged and/or neutral S,P; and/or V,A; and/or T,$\overline{T}$. $\overline{T}$ is the dual tensor to T, i.e., $\overline{T}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\lambda} T_{\sigma\lambda}$. The Hamiltonian now appears as follows:

$$\begin{aligned} H_{wk} = & \sqrt{2}\, G \left\{ \sum_i (\alpha_i^o S_i + \beta_i^o P_i), \sum_i (\alpha_i^o S_i + \beta_i^o P_i)^+ \right\} \\ & + \sqrt{2}\, G \left\{ \sum_i (\alpha_i^2 V_{i\mu} + \beta_i^1 A_{i\mu}), \sum_i (\alpha_i^1 V_{i\mu} + \beta_i^1 A_{i\mu})^+ \right\} \\ & + \sqrt{2}\, G \left\{ \sum_i (\alpha_i^2 T_{i\mu} + \beta_i^2 \overline{T}_{i\mu\nu}), \sum_j (\alpha_i^2 T_{i\mu\nu} + \beta_i^2 \overline{T}_{i\mu\nu})^+ \right\} . \end{aligned} \tag{52}$$

The index "i" refers to the SU(3) quantum numbers of the currents. The coefficients in front of the currents are severely constrained by the universality condition.

The commutation relations of the S, P, and T currents are not known "a priori," but we will again use the quark model as a source of intuition. The various currents in the quark model and their commutation relations are given in Table 2. We shall assume these algebras independent of the quark model. Several things should be

TABLE 2

$$V_\lambda{}^i = \bar{q}\,\gamma_5\,\frac{\lambda_i}{2}\,q \qquad\qquad A_\lambda{}^i = \bar{q}\,\gamma_5\,\frac{\lambda_i}{2}\,q$$

$$S_\lambda{}^i = \bar{q}\,\frac{\lambda_i}{2}\,q \qquad\qquad P_\lambda{}^i = \bar{q}\,i\,\gamma_5\,\frac{\lambda_i}{2}\,q$$

$$T_{\lambda\nu}{}^i = \bar{q}\,\sigma_{\lambda\nu}\,\frac{\lambda_i}{2}\,q \qquad\qquad \bar{T}_{\lambda\nu}{}^i = \bar{q}\,i\,\sigma_{\mu\nu}\,\gamma_5\,\frac{\lambda_i}{2}\,q$$

Commutation Relations

Type 1: $[Q^i, Q^j] = i\, f_{ijk}\, Q^k$

Q^i	Q^j	Q^k
V_o^i	V_o^j	V_o^k
V_o^i	A_o^j	A_o^k
A_o^i	A_o^i	A_o^k
V_o^i	S^j	S^k
V_o^i	P^j	P^k
S^i	S^j	V_o^k
P^i	P^j	V_o^k
V_o^i	T_{23}^j	T_{23}^k
V_o^i	$\bar{T}_{23}^j$	$\bar{T}_{23}^k$
T_{23}^i	T_{23}^j	V_o^k
$\bar{T}_{23}^i$	$\bar{T}_{23}^j$	A_o^k

Type 2: $[Q^i, Q^j] = i\, d_{ijk}\, Q^k$

Q^i	Q^j	Q^k
A_o^i	S^j	P^k
A_o^i	P^j	$-S^k$
S^i	P^j	A_o^k
A_o^i	T_{23}^j	$\bar{T}_{23}^k$
A_o^i	$\bar{T}_{23}^j$	$-T_{23}^k$
T_{23}^i	$\bar{T}_{23}^j$	A_o^k

Note: Conventions for γ matrices, λ matrices, d_{ijk}, and f_{ijk} as in ref. 6.

$$\lambda_o = \sqrt{\tfrac{2}{3}}\,\left({}^{1}{}_{1}{}_{1}\right) \quad \text{and} \quad d_{oii} = \sqrt{\tfrac{2}{3}}$$

noticed about the commutation relations of the scalar, pseudoscalar, and tensor charges: (i) d_{ijk} as well as f_{ijk} type, (ii) nonets to complete the algebra, (iii) same relations for S and P as for T and $\overline{T}$. The universality condition, with the aid of Table 2, produces non-linear algebraic equations for the coefficients of the currents. The solutions yield the universal theories to be discussed in Section II.

We shall use current algebra and PCAC to investigate the implications of these more general universal current-current theories. First, however, one more restriction will be imposed. The "V-A" Cabibbo theory works extremely well for the semi-leptonic decays, but universality alone is not sufficient to force the "V-A" form for the charged 4-vector current. We want to place a constraint on the universal theories so that the hadronic current in semi-leptonic decays is "V-A". The "V-A" theory was originally guessed by assuming a property of the current-current interaction known variously as "chirality invariance," "maximal parity violation," or "the two component theory." The essence of these hypotheses is the assumption that the hadrons appear in the $J^{+}J$ interaction with the same projection operator "$1-\gamma_5$" as the neutrinos ("$1-\gamma_5$" is the negative helicity, or chirality, projection operator for a massless neutrino). The "$1-\gamma_5$" projection allows the particles to be described by two component spinors rather than four component ones in the usual Dirac theory. Actually, the names "chirality invariance" and "maximal parity violation" come from the equivalent assumption that the interaction is invariant under replacement of any wave function by γ_5 times itself (which changes the parity).[2] This requirement excludes everything except V ± A (corresponding to $1 \pm \gamma_5$ as the projection operator). Obviously, we need a similar but weaker constraint since we are allowing scalar, pseudoscalar, and tensor currents as well. We shall generalize "maximal parity violation" by the assumption of "parity symmetric currents." Parity symmetry of a current is defined to be invariance (up to a phase) of the current under the interchange $S \leftrightarrow P$, or $V \leftrightarrow A$, or $T \leftrightarrow \overline{T}$. Parity symmetry plus universality guarantees that the charged, hadronic 4-vector current is the same as in the Cabibbo theory.* Moreover, parity symmetric currents might be useful for duplicating the success of the Cabibbo theory with the non-leptonic S-wave amplitudes. This success depends on the fact that the commutator of the axial charge and $H_{p.v.}^{\Delta S=1}$ is proportional to $H_{p.c.}^{\Delta S=1}$, a result that must come from some sort of parity symmetry.

In Section III, we shall see if the parity symmetric, universal, current-current theories are compatible with the non-leptonic decays. Current algebra, PCAC, and the Fierz transformation will be used to determine the consequences of the theories.

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*Actually, an even weaker constraint will do. Universality plus the assumption that V and A have the same SU(3) structure is sufficient.

II. UNIVERSAL CURRENT-CURRENT THEORIES

Universality plus parity symmetry forces the charged, 4-vector current to be the Cabibbo form:*

$$J_\lambda^{Q=1} = \cos\Theta \; J_\lambda^{T^+} + C^{i\phi_c} \sin\Theta \; J_\lambda^{V^+} \tag{53}$$

where

$$J_\lambda^{T^+} = \frac{1}{2}\left(V_\lambda^{1} + i\, V_\lambda^{2} - A_\lambda^{1} - i\, A_\lambda^{2}\right) \tag{54}$$

$$J_\lambda^{V^+} = \frac{1}{2}\left(V_\lambda^{4} + i\, V_\lambda^{5} - A_\lambda^{4} - i\, A_\lambda^{5}\right) \quad . \tag{55}$$

The arbitrary phase ϕ_c allowed by universality is not measurable as it can be absorbed in the definition of the phase of strangeness. Thus, the Cabibbo theory for the semi-leptonic decays is unaltered.

The neutral, 4-vector current is required by parity symmetry and universality to be:

$$J_\lambda^{Q=0} = \frac{1}{2}(\cos\Theta_N \pm 1)\, e^{-i\phi_N} \, J_\lambda^{U^+} + \frac{1}{2}(\cos\Theta_N \mp 1) e^{i\phi_N} \, J_\lambda^{U^-} + \sin\Theta \; J_\lambda^{U_3} \tag{56}$$

where

$$J_\lambda^{U^+} = (J_\lambda^{U^-})^+ = \frac{1}{2}(V_\lambda^{6} + i\, V_\lambda^{7}) \pm \frac{1}{2}(A_\lambda^{6} + i\, A_\lambda^{7}) \tag{57}$$

$$J_\lambda^{U_3} = \frac{1}{4}(-V_\lambda^{3} + \sqrt{3}\, V_\lambda^{8}) \pm \frac{1}{4}(-A_\lambda^{3} + \sqrt{3}\, A_\lambda^{8}) \tag{58}$$

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*Parity symmetry of the combined lepton plus hadron current does not allow V + A for the hadron current.

(The ± sign in equations (57) and (58) is not related to the ± sign in equation (56).) This neutral current (plus the Cabibbo charged current) leads to the Hamiltonian given by Zachariasen and Zweig.[20] The part of the Hamiltonian coming from the neutral current transforms like a pure $\underset{\sim}{27}$ and includes a $\Delta S = 2$ contribution. The relative phase difference $\Delta\phi = \phi_N - \phi_c$ between the neutral and charged currents produces a CP violation. Zachariasen and Zweig determined limits on $\Delta\phi$ and θ_N from the experimental CP violating parameters ($\Delta\phi$ and θ_N are then required to be quite small). Because CP violation is a small part of the poorly understood non-leptonic decays and because we will have enough phases in our universal theories to fit the meager data, we shall neglect CP violation in our discussion. The remainder of our currents will have the phases removed so that the Hamiltonians are CP conserving.

The commutation relations for T and $\bar{T}$ currents are completely analogous to those for the S and P currents. Consequently, the universal tensor theories are isomorphic to the universal scalar-pseudoscalar theories. For that reason, only the charged and neutral S and P theories need to be discussed.

The charged S and P current is required by parity symmetry, universality and CP conservation to be:

$$J^{Q=1} = \cos\theta \; J^{T^+} + \sin\theta \; J^{V^+} \tag{59}$$

where $$J^{T^+} = \frac{1}{2}(S^1 + i\,S^2) \pm \frac{1}{2}\,i\,(P^1 + i\,P^2) \tag{60}$$

$$J^{V^+} = \frac{1}{2}(S^4 + i\,S^5) \pm \frac{1}{2}\,i\,(P^4 + i\,P^5) \qquad (61)$$

Note that CP conservation requires the pseudoscalar current to have a purely imaginary coefficient.*

The neutral S and P theory is more complicated because of the nonet structure of the currents. Several universal, parity symmetric, CP conserving solutions are possible. If the current is written as:

$$J^{Q=0} = \alpha\, J^{U^+} + \beta J^{U^-} + \gamma J^{U_3} + \delta J^{U_o} + \lambda J^{o} \qquad (62)$$

where
$$J^{U^+} = \frac{1}{2}(S^6 + i\,S^7) \pm \frac{1}{2}\,i\,(P^6 + i\,P^7) \qquad (63)$$

$$J^{U^-} = \frac{1}{2}(S^6 - i\,S^7) \pm \frac{1}{2}\,i\,(P^6 - i\,P^7) \qquad (64)$$

$$J^{U_3} = \frac{1}{4}(-S^3 + \sqrt{3}\,S^8) \pm \frac{1}{4}\,i\,(-P^3 + \sqrt{3}\,P^8) \qquad (65)$$

$$J^{U_o} = \frac{1}{4}(\sqrt{3}\,S^3 + S^8) \pm \frac{1}{4}\,i\,(\sqrt{3}\,P^3 + P^8) \qquad (66)$$

$$J^{o} = \frac{1}{2}S^{o} \pm \frac{1}{2}\,i\,P^{o} \qquad (67)$$

then the following six solutions are possible:

(A) $\alpha = \beta = \cos\nu, \quad \gamma = 2\sin\nu, \quad \delta = \lambda = 0$ (68)

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*Bailin[(19)] studied the consequences of S and P theories via current algebra in the non-leptonic decays, but he took the form S ± P (as a generalization of V ± A) which is strongly CP violating. Universality and CP conservation leads us to the form S ± i P.

(B) $$\alpha = \beta = \cos \nu, \quad \gamma = 2 \sin \nu, \quad \delta = \pm \frac{2}{\sqrt{3}}, \quad \lambda = \pm \sqrt{\frac{2}{3}} \tag{69}$$

(C) $$\alpha = -\beta = \cos \nu, \quad \gamma = 0, \quad \delta = -\frac{2}{\sqrt{3}} \sin \nu, \quad \lambda = \frac{4}{\sqrt{6}} \sin \nu \tag{70}$$

(D) $$\alpha = -\beta = \cos \nu, \quad \gamma = 0, \quad \delta = \frac{2}{\sqrt{3}} (\pm 1 - \sin \nu),$$
$$\lambda = \frac{2}{\sqrt{6}} (\pm 1 + 2 \sin \nu) \tag{71}$$

(E) $$\alpha = \frac{1}{2} (\cos \nu' + \cos \nu), \quad \beta = \frac{1}{2} (\cos \nu' - \cos \nu),$$
$$\gamma = \sin \nu', \quad \delta = -\frac{1}{\sqrt{3}} \sin \nu, \quad \lambda = \sqrt{\frac{2}{3}} \sin \nu \tag{72}$$

(F) $$\alpha = \frac{1}{2} (\cos \nu' + \cos \nu), \quad \beta = \frac{1}{2} (\cos \nu' - \cos \nu)$$
$$\gamma = \sin \nu', \quad \delta = \frac{1}{\sqrt{3}} (\pm 2 - \sin \nu), \quad \lambda = \sqrt{\frac{2}{3}} (\pm 1 + \sin \nu) \tag{73}$$

We have also calculated currents which are universal but not parity symmetric. These currents are listed in Appendix B, except for the S and P neutral case which has not been determined.

III. CONSEQUENCES FOR THE NON-LEPTONIC DECAYS

We shall use the low-energy theorem (equation (32)) to determine the predictions of the parity symmetric, universal, J^+J theories for the S-wave amplitudes. Of course, we are assuming that part of the non-leptonic Hamiltonian results from the charged "V-A" current. At least for this part, the $\Delta I = 1/2$ rules, the Lee-Sugawara relation, and $\Sigma_+^+ = 0$ follow from a fit of three parameters to the data.

The CP conserving "V-A" neutral current (equation (56) without the phase ϕ_N) produces a Hamiltonian which, like that for the "V-A" charged current, has the property:

$$\left[Q_i^5(0) \, , \, H_{p.v.}^{\Delta S=1} \right] \; \alpha \; H_{p.c.}^{\Delta S=1} \quad . \qquad (74)$$

In this case, however, the Hamiltonian transforms like a pure $\underset{\sim}{27}$. Consequently, the baryon matrix elements of the neutral current Hamiltonian, which appear in the low energy theorem, are proportional to the parameter "α" (the $\underset{\sim}{27}$ reduced matrix element) which is determined to be zero. (The S-wave decay of $\Sigma^+ \to n\pi^+$, which is proportional to α, vanishes experimentally.) Thus, the success of Suzuki's analysis is unaffected by the presence of the "V-A" neutral current. (Of course, the coefficients of Σ_+^+ in equations (39) - (42) are changed by the neutral current, but $\Sigma_+^+ = 0$.)

The "S ± i P" charged current (equation (59)) produces a Hamiltonian which has the commutation property given in equation (74) for two choices of the axial charge (corresponding to π^0 or π^+

emission in the decay):

$$\left[Q_3^5 (0), \, H_{p.v.}^{\Delta S=1} (0) \right] = \frac{1}{2} \cdot \frac{G}{2\sqrt{2}} \sin\Theta \cos\Theta \left[\left\{ S^{T^-}, S^{V^+} \right\} + \left\{ P^{T^-}, P^{V^+} \right\} \right] \tag{75}$$

$$\left[\frac{1}{\sqrt{2}} (-Q_1^5 (0) + i \, Q_2^5 (0)), \, H_{p.v.}^{\Delta S=1} (0) \right] = - \frac{1}{\sqrt{2}} \cdot \frac{G}{2\sqrt{2}} \sin\Theta \cos\Theta \left[\left\{ S^{T^-}, S^{U^+} + P^{T^-}, P^{U^+} \right\} \right] \tag{76}$$

Equations (75) and (76) are completely analogous to equations (33) and (34) for the Cabibbo current. The results can again be summarized:

$$\pi^0 \text{ emission } \alpha \ \langle B_2 \, | \tfrac{1}{2} H_{p.c.}^{T^-, V^+} | \, B_1 \rangle \tag{77}$$

$$\pi^+ \text{ emission } \alpha \ \langle B_2 \, | - \frac{1}{\sqrt{2}} H_{p.c.}^{T^-, U^+} | \, B_1 \rangle \quad . \tag{78}$$

However, the commutator with the axial charge corresponding to π^- emission is changed because of the nonet structure of the currents:

$$\left[\frac{1}{\sqrt{2}} (Q_1^5 (0) + i \, Q_2^5 (0)), \, H_{p.v.}^{\Delta S=1} (0) \right] = \sqrt{2} \cdot \frac{G}{2\sqrt{2}} \sin\Theta \cos\Theta \left[\left\{ S^{\omega}, S^{T^+} + P^{\omega}, P^{T^+} \right\} \right] \tag{79}$$

The superscript "ω" denotes a particular combination of the SU(3) indices "0" and "8", e.g. for S^{ω}

$$S^{\omega} = \sqrt{\tfrac{2}{3}}\ S^{o} + \sqrt{\tfrac{1}{3}}\ S^{8} \quad . \qquad (80)$$

(Although there is no connection, the symbol ω is chosen because the vector meson ω is the analogous combination of singlet and octet.) Thus, we have:

$$\pi^{-} \text{ emission } \alpha \ \langle B_2 \,|\, \sqrt{2}\, H_{p.c.}^{\omega, V^{+}} \,|\, B_1 \rangle \quad . \qquad (81)$$

Since the current with the superscript ω contains both singlet and octet, the product of currents in $H_{p.c.}^{\omega, V^{+}}$ contains an $\underset{\sim}{8}$ part from $\underset{\sim}{1} \times \underset{\sim}{8}$ and an $\underset{\sim}{8}$ and $\underset{\sim}{27}$ part from $\underset{\sim}{8} \times \underset{\sim}{8}$. Thus, five parameters are necessary to describe the baryon matrix elements of $H_{p.c.}^{\omega, V^{+}}$ — corresponding to D and F for each $\underset{\sim}{8}$ and α for the $\underset{\sim}{27}$ (no relation to D, F, and α for the V-A case). Consequently, only two relations are possible among the seven amplitudes. However, it is curious that if we treat the S and P currents as nonets (combining the singlet and octet into one 3 x 3 matrix in SU(3)) and assume that two currents couple to form an octet according to the nonet anzatz[22] (the singlet is not split off as the trace), then the octet part of $H_{p.c.}^{\omega, V^{+}}$ is identical to that of $H_{p.c.}^{T_3, V^{+}}$. If equation (81) had been analogous to equation (38) (as equation (77) is to equation (36) and equation (78) is to equation (37)), then π^{-} emission for the "S ± i P" case would have been proportional to the baryon matrix elements of $H_{p.c.}^{T_3, V^{+}}$ instead of $H_{p.c.}^{\omega, V^{+}}$. However, the nonet anzatz for the S and P currents implies that the baryon matrix elements of the octet part of $H_{p.c.}^{T_3, V^{+}}$

are the same as those of the octet part of $H_{p.c.}^{\omega,\, V^+}$. Consequently, the seven amplitudes are now determined by three parameters instead of five. The four relations that follow are:

$$\Lambda^{o}_{-} + \sqrt{2}\,\Lambda^{o}_{o} = -\frac{\sqrt{6}}{5}\,\Sigma^{+}_{+} \tag{82}$$

$$\Xi^{-}_{-} - \sqrt{2}\,\Xi^{o}_{o} = +\frac{\sqrt{6}}{5}\,\Sigma^{+}_{+} \tag{83}$$

$$\Sigma^{-}_{-} - \sqrt{2}\,\Sigma^{+}_{o} = \frac{1}{5}\,\Sigma^{+}_{+} \tag{84}$$

$$2\Xi^{-}_{-} - \Lambda^{o}_{-} = \sqrt{3}\,\Sigma^{+}_{o} + \frac{11}{10}\sqrt{6}\,\Sigma^{+}_{+} \tag{85}$$

Again because $\Sigma^{+}_{+} \approx 0$, equations (82) - (85) are equivalent to the $\Delta I = 1/2$ rules and the Lee-Sugawara relation. Consequently, the results for the charged $S \pm i P$ current are equivalent to those for the charged V-A current. If we had not made the nonet anzatz, we would have had only two relations instead of the four. However, the fit to the data would have required the S and P currents in $H_{p.c.}^{\omega,\, V^+}$ to couple according to the nonet anzatz. Although we do not have a good reason why the S and P currents do couple this way, it is certainly true that the charged $S \pm i P$ current is compatible with the non-leptonic S wave data.

The current algebra analysis follows analogously for the neutral "$S \pm i P$" current (equation (62)). Although the Hamiltonian is not pure $\underset{\sim}{27}$ as in the "V-A" case, four relations similar to equations (82) and (85) (only the coefficients of Σ^{+}_{+} are different) follow with the addition of the nonet anzatz.* (Again only two relation are implied without the nonet anzatz.) Consequently, the neutral "S ± i P" current is also compatible with the experimental data.

The next question is whether these parity symmetric universal theories imply something more than the Cabibbo theory, especially with regard to the P waves. Both the parity conserving and parity violating parts of all these Hamiltonians have the symmetry which for the octet piece corresponds to being the sixth component (even though the p.v. part of the "S ± i P" Hamiltonians consist of $\underset{\sim}{8}$, $\underset{\sim}{10}$, and $\overline{\underset{\sim}{10}}$, rather than $\underset{\sim}{8}$ and $\underset{\sim}{27}$). Consequently, octet dominance and this symmetry imply the Lee-Sugawara relation for the S waves, but not for the P waves. (Also, $K_1^0 \to \pi\pi$ is again forbidden with exact SU(3).) Moreover, the P wave analysis via the Born terms follows exactly as before — which is an utter failure. Thus, the parity symmetric universal theories imply nothing more than the Cabibbo theory. There exist more general universal theories which are not parity symmetric (see Appendix B), but it is almost hopeless for these theories to reproduce the success in the S waves.

With the assumption of Bose quarks, the Fierz transformation property of the "V-A" Hamiltonian (antisymmetry in space and spin) leads to an exact $\Delta I = 1/2$ rule for both S and P wave amplitudes. The "S ± i P" and "T ± i $\overline{T}$" Hamiltonians, however, have no particular

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*Except for the trivial solutions (A) and (B) (equations (68) and (69)) where the $\Delta S = 1$ Hamiltonian has no parity violating piece.

symmetry under the Fierz transformation. Consequently, they cannot reproduce the $\Delta I = 1/2$ rule with either Bose quarks or Fermi quarks. There exist two other current-current Hamiltonians besides the "V-A" one which are antisymmetric under the Fierz transformation (as well as two which are symmetric).[23] The parity-conserving parts of the three which are antisymmetric are of the form:

$$VV + AA \tag{86}$$

$$SS - PP - TT \tag{87}$$

$$SS + PP + AA \tag{88}$$

(The parity-violating parts of these Hamiltonians, which are also antisymmetric under the Fierz transformation, have a similar structure.) Only the first one (equation (86)), which comes from "V-A", arises from a current which is universal. Consequently, the "V-A" Cabibbo theory is the only universal theory that leads to the $\Delta I = 1/2$ rule with Bose quarks.

IV. CONCLUSIONS

We have derived the general current-current theories which are parity symmetric and universal and have shown that they lead effectively to the same good predictions as the Cabibbo theory for the S-wave amplitudes in the non-leptonic baryon decays. The low-energy theorem from current algebra and the commutation relations from the quark model were used to obtain these results. Consequently, the hadronic currents with other Lorentz properties and charges may actually be present in the non-leptonic weak interaction. (The semi-leptonic decays are unaffected by these currents.) However, the presence of more general currents does not solve any of the existing problems with the non-leptonic decays, nor is it indicated by any feature of the data. The most significant argument for the Cabibbo theory being the only source of non-leptonic decays is that it is only universal current-current theory that leads to the $\Delta I = 1/2$ rule with Bose quarks.

APPENDIX A

Application of Current Algebra and PCAC to the Non-Leptonic Decays

We shall review the derivation of the low-energy theorem for non-leptonic decays.[15,16] The off-mass shell amplitude for $B_1 \to B_2\, \pi_i$, using the notation in Fig. 1, can be defined by:

$$M(q,\, i,\, P_1;\, P_2) = i\,(\mu^2 - q^2) \int d^4x\, e^{-iq\cdot x} \langle B_2 | T\{\phi_i(x),\, H_w(0)\} | B_1 \rangle \qquad \text{(A1)}$$

where the physical amplitude $\langle B_2\, \pi^i | H_w(0) | B_1 \rangle$ equals $M(q = P_1 - P_2,\, q^2 = \mu^2,\, i;\, P_1;\, P_2)$. (The index i denotes the charge of the pion.) If the PCAC equation is used to define the off-mass shell pion field

$$\partial_\lambda A_i^\lambda(x) = \frac{1}{\sqrt{2}} f_\pi\, \mu^2\, \phi_i(x) \quad , \qquad \text{(A2)}$$

then

$$M(q,\, i;\, P_1;\, P_2) = i\sqrt{2}\, \frac{\mu^2 - q^2}{f_\pi\, \mu^2} \int d^4x\, e^{-iq\cdot x} \times \langle B_2 | T\{\partial_\lambda A_i^\lambda(x),\, H_w(0)\} | B_1 \rangle \quad . \qquad \text{(A3)}$$

The following identity

$$\partial_\lambda\, T\{A_i^\lambda(x),\, H_w(0)\} = T\{\partial_\lambda A_i^\lambda(x),\, H_w(0)\} + \delta(x_o)[A_i^o(x),\, H_w(0)] \qquad \text{(A4)}$$

implies

$$M(q,\ i;\ P_1,\ P_2) \;=\; -\sqrt{2}\, q_\lambda \frac{\mu^2 - q^2}{f_\pi\, \mu^2} \int d^4 x\, e^{-iq\cdot x}$$

$$\times \langle B_2 | T \left\{ A_i^\lambda (x),\ H_w (0) \right\} | B_1 \rangle \qquad \text{(A5)}$$

$$- i\sqrt{2} \frac{\mu^2 - q^2}{f_\pi\, \mu^2} \int d^3 x\, e^{iq\cdot x} [A_i^o (0,x),\ H_w(0)] \quad .$$

Assuming that H_w (0) is a local operator so that

$$[A_i^o (0,\underset{\sim}{x}),\ H_w (0)] \ \alpha\ \delta^3 (\underset{\sim}{x}) \qquad \text{(A6)}$$

we have the relation

$$\frac{f_\pi}{\sqrt{2}} M(q,\ i;\ P_1;\ P_2) \;=\; -q_\lambda \frac{\mu^2 - q^2}{\mu^2} \int d^4 x\, e^{-iq\cdot x}$$

$$\times \langle B_2 | T \left\{ A_i^\lambda (x),\ H_w (0) \right\} | B_1 \rangle \qquad \text{(A7)}$$

$$- i \frac{\mu^2 - q^2}{\mu^2} \langle B_2 | [Q_i^5 (0),\ H_w (0)] | B_1 \rangle \quad .$$

Taking the limit $q_\lambda \rightarrow 0$ yields the low-energy theorem:

$$\lim_{q_\lambda \rightarrow 0} \frac{f_\pi}{\sqrt{2}} M(q,\ i;\ P_1;\ P_2) \;=\; -\lim_{q_\lambda \rightarrow 0} q_\lambda \int d^4 x\, e^{-iq\cdot x}$$

$$\langle B_2 | T \left\{ A_i^\lambda (x),\ H_w (0) \right\} | B_1 \rangle \; - i \langle B_2 | [Q_i^5 (0),\ H_w (0)] | B_1 \rangle \qquad \text{(A8)}$$

The term proportional to q_λ vanishes unless it has a singularity as $q_\lambda \to 0$, which can happen with the contribution of a single particle intermediate state degenerate in mass with either B_1 or B_2.

The low-energy theorem is only useful if the physical amplitude, i.e., $M(q, i; P_1; P_2)$ at $q^2 = \mu^2$, can be approximated by $M(q, i; P_1; P_2)$ at $q_\lambda = 0$. However, the Born terms in $M(q, i; P_1, P_2)$ (example illustrated in Fig. 2) do not vary smoothly as $q_\lambda \to 0$. The contribution of a typical Born term to the physical amplitude is given by:

$$g_{BB_2\pi} \frac{1}{M_{B_1} - M_B} \langle B | H^{\Delta S=1} | B_1 \rangle \quad .$$

The matrix element for $H^{\Delta S=1}_{p.v.}$ between octet baryon states vanishes in the limit of exact SU(3), so that the pole terms contribute only to the parity conserving amplitude. (Even with broken SU(3) for the masses, the parity violating part of the Born term is small, i.e., $O(1)$ rather than $O(\frac{1}{\Delta M})$ like p.c. part.)

The smoothness difficulty disappears if we subtract the physical Born contribution from both sides of equation (A8). Therefore, let us define:

$$R(q, i; P_1; P_2) = M(q, i; P_1; P_2) - M_{BORN}(q, i; P_1; P_2) \qquad \text{(A9)}$$

then

$$\lim_{q_\lambda \to 0} \frac{f_\pi}{\sqrt{2}} R(q, i; P_1; P_2) = -\lim_{q_\lambda \to 0} [q_\lambda \int d^4 x \, e^{-iq\cdot x}$$

$$\times \langle B_2 | T \{ A_i^\lambda (x), H_w (0) \} | B_1 \rangle - M_{BORN}(q,i; P_1; P_2)]$$

$$- i \langle B_2 | [Q_i^5 (0), H_w (0)] | B_1 \rangle \quad . \qquad (A10)$$

The remainder $R(q, i; P_1; P_2)$ is automatically smoothly varying because the singular part has been removed. Moreover, the term in brackets, which we shall call $\tilde{R}$, has a well defined limit as $q_\lambda \to 0$ that can be calculated. The parity violating part of $\tilde{R}$ $(q_\lambda = 0)$ is small and vanishes, like $M^{p.v.}_{BORN}$ $(q^2 = \mu^2)$, in the limit of SU(3) for $\langle B | H^{p.v.}_{\Delta S=1} | B \rangle$. The parity conserving part of $\tilde{R}$ $(q_\lambda = 0)$ is of the order $\frac{\Delta M}{2M}$, as compared $M^{p.c.}_{BORN}$ $(q^2 = \mu^2)$ and can be neglected. Consequently, the fundamental equation for non-leptonic decays becomes:

$$M(q=P_1 - P_2, q^2=\mu^2, i; P_1;P_2) \simeq M^{p.c.}_{BORN} (q=P_1 - P_2, q^2=\mu^2, i; P_1;P_2)$$

$$-i \langle B_2 | [Q_i^5 (0), H_w (0)] | B_1 \rangle \quad . \qquad (A11)$$

Note that the parity violating part of the decay is given entirely by the commutator. Moreover, the parity conserving part of the decay is given entirely by the Born term since $[Q_i^5 (0), H_w^{p.c.}(0)]$ is a parity violating operator whose matrix element between baryon states vanishes in the limit of SU(3).

APPENDIX B

Non-Parity Symmetric, Universal J^+J Theories

1. V, A charged

$$J_\lambda^{Q=1} = \alpha V_\lambda^{T^+} + \beta e^{i\theta} A_\lambda^{T^+} + e^{i\chi} (\gamma V_\lambda^{V^+} + \delta e^{i\theta'} A_\lambda^{V^+}) \tag{B1}$$

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1 \tag{B2}$$

$$\alpha \beta \cos \theta = -\gamma \delta \cos \theta' \tag{B3}$$

where $V_\lambda^{T^+} = V_\lambda^1 + i V_\lambda^2$, $V_\lambda^{V^+} = V_\lambda^4 + i V_\lambda^5$, etc.

2. S, P charged

$$J^{Q=1} = \alpha S^{T^+} + \beta e^{i\theta} P^{T^+} + e^{i\chi}((\gamma S^{V^+} + \delta e^{i\theta'} P^{V^+}) \tag{B4}$$

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1 \tag{B5}$$

$$\alpha \beta \sin \theta = -\gamma \delta \sin \theta' \tag{B6}$$

where $S^{T^+} = S^1 + i S^2$, $S^{V^+} = S^4 + i S^5$, etc.

3. V, A neutral (CP conserving)

$$(a)\ J^{Q=0} = \frac{1}{4}(\cos\Theta + \cos\Theta' \pm 2)\, V_\lambda^{U^+} + \frac{1}{4}(\cos\Theta - \cos\Theta')\, A_\lambda^{U^+}$$

$$+ \frac{1}{4}(\cos\Theta + \cos\Theta' \pm 2)\, V_\lambda^{U^-} + \frac{1}{4}(\cos\Theta - \cos\Theta')\, A_\lambda^{U^-}$$

$$+ \frac{1}{2}(\sin\Theta + \sin\Theta')\, V_\lambda^{U_3} + \frac{1}{2}(\sin\Theta - \sin\Theta')\, A_\lambda^{U_3}$$

where $V_\lambda^{U^+} = (V_\lambda^{U^-}) = V_\lambda^6 + i\, V_\lambda^7$, $V_\lambda^{U_3} = \frac{1}{2}(-V_\lambda^3 + \sqrt{3}\, V_\lambda^8)$, etc.

(b) same as solution (a), except $V \leftrightarrow A$.

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