Baryon Number Generation and Mass Relations in SO(10) Unified Models

Thesis by

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Abstract

Two topics are discussed in this thesis. The first is the calculation of cosmological baryon number generation in the early universe. These calculations are performed for a variety of SU(5) and SO(10) unified models. The effects of superheavy fermions and charge conjugation symmetry are discussed in the context of SO(10) models. The second section contains an analysis of natural fermion mass and mixing angle relations in a grand unified model based on SO(10). These relations are used to study neutrino masses and oscillations. Appreciable mixing is found only between μ and τ neutrinos. Spinor representations of the Lorentz group and of SO(10) are described in two appendices.

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I. Introduction

It now appears that we have a consistent description of all elementary particle interactions occurring at energies below $\sim$ 100 GeV. The strong interactions between quarks are described by Quantum Chromodynamics (QCD) [1], while the weak and electromagnetic interactions between quarks and leptons are described by the Glashow-Weinberg-Salam model [2] with quarks incorporated as in the scheme of Glashow, Iliopoulos and Maiani [3].

The fundamental principle underlying both these theories (and also Einstein's General Relativity) is that of gauge invariance. This principle requires the invariance of the theory under symmetry transformations which may be performed independently at each point of space and time. In QCD the symmetry group is $SU(3)_c$ and corresponds to unitary transformations among the three colors of otherwise identical quarks. In the Weinberg-Salam model the symmetry group is $SU(2)_L \otimes U(1)_Y$ and consists of weak isospin transformations on the left-handed components of quarks and leptons and an additional phase transformation.

At present, theories based on gauge symmetries are the only theories consistent with both relativity and quantum mechanics which are capable of both describing the observed interactions and dealing consistently with the infinities which arise in relativistic field theories. Attempts to describe the low-energy interactions as manifestations of a single underlying interaction are thus usually based on gauge theories. The strong, weak, and electromagnetic interactions can be successfully incorporated into a gauge theory based on a single gauge group, G (e.g. SU(5) [4], SO(10) [5], E(6) [6], etc.), which contains the symmetry transformations of $SU(3)_c$ and $SU(2)_L \otimes U(1)_Y$ as a subgroup. Such a theory has the virtue of possessing only a single gauge coupling constant. The difference in the coupling strengths of the strong, weak, and electromagnetic interactions is then accounted for by the phenomenon of asymptotic freedom: the coupling strengths for $SU(3)_c$ and $SU(2)_L$ decrease at large energies (small distance scales) [7], while the coupling strength for $U(1)_Y$ increases at large energies. The current values for the weak mixing angle, $\Theta_W \cong 0.23$, and the strong coupling constant, $\alpha_S \cong 0.2$, suggest that all three coupling constants should become equal at an energy $M_G \cong 10^{15}$ GeV [8]. At energies $\gtrsim M_G$, all gauge couplings are equal and any particles transforming among themselves under the action of G must be degenerate in mass. Unified gauge theories thus present an attempt to describe all elementary particle interactions up to energies at which quantum gravitational effects should become important, $M_P \sim 10^{19}$ GeV.

At presently accessible energies the symmetries associated with the weak interactions and with any possible unifying gauge interactions are not apparent. Exact gauge symmetry requires the presence of massless gauge bosons (e.g. gluons for QCD or the photon for electromagnetism). A treatment of the weak interactions and of unified gauge theories thus requires a mechanism which gives masses to the gauge vector bosons which mediate the interactions and thus "breaks" the gauge symmetry. The simplest mechanism for breaking the symmetries associated with unified gauge interactions involves the introduction of scalar fields termed Higgs bosons [9]. These Higgs bosons have minimum energy configurations in the vacuum which select a particular direction in the internal symmetry group space, and thus break the symmetry; much as the alignment of the individual atoms in a ferromagnet at zero temperature breaks rotational symmetry inside the magnet.

There are three major tests of the idea of unification: nucleon decay, cosmological baryon number production, and relations between fermion masses and mixing angles. Since unified gauge theories generally include symmetry transformations which mix quarks and leptons, the gauge bosons of G with masses $\sim M_G$ will in general mediate both baryon and lepton number violating reactions, which are, however, suppressed at accessible energies by the large masses of the gauge bosons. As a result, these theories predict that nucleons should decay with lifetimes only slightly longer than the present experimental limit of 10^{30} years [10]. At the high temperatures present in the very early universe, the suppression due to the gauge boson masses should have been overcome and baryon and lepton number violating reactions should have proceeded with rates comparable to those for baryon and lepton number conserving reactions. The baryon and lepton numbers of the universe may thus be determined by the structure of such a unified theory [11]. In addition to breaking the gauge symmetry, Higgs bosons are also presumed to be responsible for fermion masses. In most cases the coupling of Higgs bosons to fermions involves only a few independent coupling constants. As a result, unified theories usually give rise to relations between fermion masses and mixing angles [12].

In this thesis we will study cosmological baryon number production and relations between fermion masses and mixing angles in unified theories based on the Lie group $SO(10)$. The group transformations of these theories are rotations in a ten-dimensional internal symmetry space. Fermion fields are usually placed in spinor representations of $SO(10)$, which are analogous to the spinor representations of the angular momentum group, $SO(3)$. There were three main reasons for choosing $SO(10)$ unified theories for these investigations. First, these theories include as a subset the simplest and most studied unified theories based on the group $SU(5)$, and thus incorporate the successful features of these theories. Second, several unappealing features of $SU(5)$ models can be removed in theories based on $SO(10)$. In particular, $SO(10)$ theories present a greater unification of the fundamental fermion fields by including transformations among fermion fields which do not mix under the action of $SU(5)$. Finally, theories based on $SO(10)$ predict a number of new phenomena, such as neutrino masses and oscillations, which are of great importance to our understanding of cosmology and elementary particle physics.

Chapter II of this thesis discusses the constraints on unified gauge theories necessary to account for the apparent excess of baryons over antibaryons in the present universe. Sections 2 and 3 contain considerations which are applicable to any unified theory, while Section 4 contains results for $SU(5)$ models. Section 5 discusses the constraints on baryon number production which are peculiar to $SO(10)$ models and presents results for several typical models.

The first two sections of Chapter III of this thesis present a detailed unified model based on $SO(10)$ which successfully reproduces the observed fermion masses and mixing angles. This model is used in Section 3 to study the neutrino masses and oscillations which are a distinguishing feature of $SO(10)$ models. Section 4 contains a description of the

Higgs bosons necessary in this model to achieve the desired pattern of fermion masses.

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II. Cosmological baryon number generation in unified models

1. Introduction

Grand unified gauge models typically attempt to combine quarks and leptons as elements of the same irreducible representation of some gauge group G (which must contain the observed low-energy symmetry group $G_{LE} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$). The gauge bosons (which transform under the adjoint representation of G) induce transitions between members of an irreducible fermion representation. Hence some of them should mediate baryon (B) and lepton (L) number violating interactions, in which, for example, quarks decay into leptons and antiquarks (e.g., $uu \to \bar{d}e^+$). The limit of 10^{30} years [1] on the lifetime of the proton suggests, however, that any baryon-violating bosons should have masses $\gtrsim 10^{14}\,GeV$. Direct evidence for such B-violating interactions must presumably come from observation of proton decay. However, if any B violation does occur, its suppression at accessible energies due to the large masses of the intermediate bosons, should have been overcome at the extremely high temperatures which existed in the very early universe. We will discuss the constraints on such B-violating processes in the standard hot big-bang cosmological model necessary to allow the apparent excess of baryons over antibaryons in the present universe. Even if the universe initially had a nonzero net baryon number, B-violating interactions at very early times should relax the asymmetry away, leaving equal numbers of baryons and antibaryons. Then, when the universe cooled to a temperature $\lesssim 50\ MeV$, the baryons and antibaryons would have annihilated away and the observed baryon number density $n_B/n_\gamma \simeq 10^{-9}$ could not be accounted for. To reconcile the possibility of rapid B-violating processes at very high temperatures with the apparent nonzero net baryon number of the universe, it may therefore be necessary that a baryon asymmetry should have developed from the symmetrical state present after any initial B had been erased. (The possibility of this phenomenon was suggested by A. D. Sakharov in 1967.) The generation of an asymmetry of the required magnitude places severe constraints on B-violating interactions, and therefore on grand unified gauge models. The purpose of this chapter is to provide a detailed and systematic description of these constraints. The basic physical phenomena involved in the generation of a baryon excess were discussed in [2] where several simple illustrative models were considered. We will treat more realistic and complicated gauge models, in which many of the parameters relevant to baryon number generation are determined by the basic structure of the models, rather than being arbitrary, as in the illustrative models of [2].

The generation of a baryon excess from a B = 0 state requires interactions which violate not only B but also CP (and C, T) invariance. This CP violation is probably not connected with that observed in the K^0 system, and in most grand unified models its magnitude is undetermined. In certain models, no such CP violation is present, while in others, the B generated is insufficient even if the CP violation is maximal. Such models (which include the minimal SU(5) model) may therefore be considered in disagreement with the standard cosmology.

If all the contents of the early universe were in thermal equilibrium, then no baryon asymmetry could arise even in the presence of B and CP violation (since in thermal equilibrium, no "direction of time" is distinguished, and CPT invariance renders the CP, T violations ineffective). However, massive particles, such as those expected to mediate B-violating interactions, do not remain in equilibrium when the temperature of the expanding universe falls below their masses. For certain values of the masses, the resulting deviations from equilibrium may be sufficient to allow generation of the required baryon excess.

Above, we mentioned gauge vector bosons as possible mediators of B-violating interactions. However, it will turn out that unless super-heavy ($m \gtrsim 10^{10}$ GeV) fermions exist, gauge boson interactions alone provide insufficient CP violation to produce a baryon excess. Nevertheless, in most schemes, the spontaneous symmetry breakdown presumably responsible for the boson (and fermion) masses must be implemented by a Higgs mechanism. Usually many Higgs fields must be introduced to provide the required pattern of symmetry breaking, and a large fraction of them survive as physical particles. Typically, the Higgs particles have roughly the same masses and quantum numbers as the gauge bosons for whose masses they are responsible. Hence, some Higgs bosons should be capable of mediating B-violating interactions, which may also exhibit CP violation. However, models often sport huge numbers of Higgs scalar particles with a great variety of couplings: baryon asymmetry generation provides only a small number of constraints in the general case. In this section we consider specific models based on the groups SU(5) and SO(10). We begin by reviewing in Sections 2 and 3 some features of B and CP violation relevant to almost any model. Much of Section 3 represents work done by D. Reiss and S. Wolfram and is included here for completeness. Section 4 contains a brief description of the general procedure used to calculate the evolution of baryon number in realistic unified models and presents the results of these calculations for several SU(5) models. This section was written as a letter for submittal to Physical Review Letters in collaboration with E. Kolb, D. Reiss, and S. Wolfram and thus provides an overview of the main features of baryon number generation. Section 5 deals with the production of baryon number in SO(10) unified models. Many details and extensions of the results presented here are discussed in [3].

2. Forms of baryon number violating couplings

All couplings must respect the $SU(3)_c$ and $U(1)_Q$ gauge invariances corresponding to color and electric charge conservation. In addition, at high energies E, the spontaneous breakdown of $SU(2)_L$ gauge invariance is unimportant, and $SU(2)_L$ weak charge should be conserved up to $O(m_W^2/E^2)$ corrections. On the other hand, the observed conservation of baryon and lepton numbers at low energies is probably not a consequence of any gauge invariance, but rather results from the assignment of global quantum numbers to light particles. In this case, B and L can potentially be violated in the couplings of heavy particles. In this section, we discuss the possible forms of B-, L-violating couplings, and the constraints placed on them by $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariance [4]. We derive conditions under which B and L are separately violated, but some combination, usually B-L, is conserved.

The generic constitution of the three known families of quarks (q) and leptons (l) is summarized in Table 2.1. In considering B, L violation at high energies, the masses of q, l may be neglected, so that the left- and right-handed components of each fermion field may be treated independently. Table 2.1 gives the $SU(3)_c$ and $SU(2)_L$ representations under which each field transforms together with the weak hypercharge $Y = I_3 - Q$ assignment which specifies the final U(1) transformation properties. We assume, for now, that neutrinos are described by massless Weyl fields. As indicated by present experimental results, we take all left-handed components q_L, l_L to transform as doublets under $SU(2)_L$ and q_R, l_R to transform as singlets. It appears that the leptons of each family carry a distinct conserved flavor quantum number, but we shall have no cause to consider this.

Particles	[SU(3),SU(2),U(1)]	Antiparticles	[SU(3),SU(2),U(1)]
$\begin{pmatrix}\nu\\E\end{pmatrix}_L$	[1,2,1/2]	$\begin{pmatrix}\nu^c\\E^c\end{pmatrix}_R$	[1,2,-1/2]
E_R	[1,1,1]	E^c_L	[1,1,-1]
$\begin{pmatrix}U\\D\end{pmatrix}_L$	[3,2,-1/6]	$\begin{pmatrix}U^c\\D^c\end{pmatrix}_R$	[$\bar{3}$,2,1/6]
U_R	[3,1,-2/3]	U^c_L	[$\bar{3}$,1,2/3]
D_R	[3,1,1/3]	D^c_L	[$\bar{3}$,1,−1/3]

Table 2.1: The particles and antiparticles in a family, together with their quantum numbers ($SU(3)_c$ multiplicity, $SU(2)_L$ multiplicity and weak hypercharge $Y = T_3 - Q$).

The quarks in Table 2.1 are assigned baryon number B = 1/3; the corresponding antiquarks are assigned B = -1/3. The leptons are assigned L = +1, and antileptons L = -1. The "baryon" and "lepton" numbers of other particles are determined by their couplings to these quarks and leptons. If all the quark-lepton systems to which a given particle couples have the same B and L, then that particle may usefully be assigned a definite B and L. However, some particles may couple to systems with differing B and L, in which case no single assignment of B or L suffices, hence B and L are violated in the interactions of the particles.

Tables 2.2 and 2.3 give the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers for the possible quark and lepton systems to which vector and scalar bosons may couple. Lorentz invariance requires that renormalizable vector couplings have the form $\psi_a^\dagger \sigma^\mu \psi_b V_\mu$ and that renormalizable scalar couplings have the form $\psi_a^T \sigma_2 \psi_b S$ where V_μ and S are vector and scalar fields, respectively, and $\psi_{a,b}$ are spin 1/2 fields (see Appendix A for notation). In gauge theories such as the simplest Weinberg-Salam GIM scheme, only vector bosons of type V_1 exist (see Table 2.2), since these contain the gauge bosons of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$. Hence each boson may be assigned a definite baryon number, and no B violation occurs. Various Higgs bosons could be added in an ad hoc manner but the usual doublet which corresponds to a scalar boson of type S_1 suffices both to give masses to the fermions and to break $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$. With only these bosons B and L are conserved by the Higgs sector as well. In grand unified gauge theories, it is common to include both fermion and antifermion fields in the same representation of the gauge group. In these cases, bosons with couplings of types 3, 4 and 5 (S_3, V_3, ...) may exist. A boson with couplings of type 3 must be a color singlet: it may therefore

		$[SU(3), SU(2), U(1)]$	B	L	B-L
V_1	$l\bar{l}, q\bar{q}$	[8,3,0]	0	0	0
		[8,1,1]			
		[8,1,0]			
		[1,3,0]			
		[1,1,1]			
		[1,1,0]			
V_2	$q\bar{l}$	[3,3,-2/3]	1/3	-1	4/3
		[3,1,-2/3]			
V_3	ll	[1,2,3/2]	0	2	-2
V_4	lq	[3,2,5/6]	1/3	1	-2/3
		[3,2,-1/6]			
V_5	qq	[6,2,-5/6]	2/3	0	2/3
		[6,2,1/6]			
		[$\bar{3}$,2,−5/6]			
		[$\bar{3}$,2,1/6]			

Table 2.2: Quantum numbers for possible spin 1 (vector) pairs of quarks and leptons. Quantum numbers for individual q and l were given in Table 2.1.

		[SU(3),SU(2),U(1)]	B	L	B-L
S_1	$l\bar{l},q\bar{q}$	[8,2,1/2] [1,2,1/2]	0	0	0
S_2	$q\bar{l}$	[3,2,-7/6] [3,2,-1/6]	1/3	-1	4/3
S_3	ll	[1,3,1] [1,1,2] [1,1,1]	0	2	-2
S_4	lq	[3,3,1/3] [3,1,1/3] [3,1,4/3]	1/3	1	-2/3
S_5	qq	[6,1,-4/3] [6,1,-1/3] [6,1,2/3] [6,3,-1/3] [$\bar{3}$,3,−1/3] [$\bar{3}$,1,−4/3] [$\bar{3}$,1,−1/3] [$\bar{3}$,1,2/3]	2/3	0	2/3

Table 2.3: Quantum numbers for possible spin 0 (scalar) pairs of quarks and leptons.

not participate in couplings 4 and 5, and may thus be assigned a definite B. On the other hand, a boson may simultaneously exhibit couplings of types 4 and $\bar{5}$. Such a boson therefore couples to systems with B = 1/3 and B = -2/3: it may therefore be assigned no definite B, and mediates B-violating interactions between quarks and leptons. However, although the separate B and L for cases 4 and $\bar{5}$ differ, the combination B-L is -2/3 in both cases. Thus, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariance and the restriction to the observed fermion fields prevent couplings of bosons to quarks and leptons from violating B-L. At least for the purposes of these couplings, such bosons may always be assigned a definite B-L. In what follows we will denote the possible B violating-vector bosons by (X,Y) for the $[3,2,-1/6]$ and (X',Y') for the $[3,2,5/6]$. The possible B-violating scalar bosons will be denoted by S ([3,1,1/3]), S_1 ([3,1,4/3]), and S_2 ([3,3,1/3]). Fermi statistics require that S_1 and S_2 couple to fermions antisymmetrically in family space.

All known fermions carry nonzero charges under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. However, there may exist massive fermions which carry no absolutely-conserved quantum numbers. Such fermions (N) may mix with their antiparticles through Majorana mass terms (of the form $m_M \bar{f}^c f$). Clearly, they may not be assigned definite B or L. If the coupling $\chi \to qN$ is present, then $\bar{\chi} \to \bar{q}N$ may be present also. Thus N does not carry a definite B-L: production and decay of N will lead to violations of B-L conservation. The allowed types of B- and L-violating bosons in this case are discussed in Section 5.2 in the context of SO(10) grand unified models.

3. Basic parameters for baryon number generation

3.1 General Results

In this section we describe the calculation of the parameters which govern the generation of a baryon asymmetry from the basic couplings in a grand unified gauge model.

The basic parameter which enters the Boltzmann transport equations of Sections 4 and 5 is the average baryon number produced in the free decays of an equal mixture of particles χ and their CP-conjugate antiparticles $\bar{\chi}$:

$$R_\chi \equiv \sum_f B_f \left\{ \frac{\Gamma(\chi \to f)}{\Gamma_\chi} - \frac{\Gamma(\bar{\chi} \to \bar{f})}{\Gamma_{\bar{\chi}}} \right\} \tag{3.1.1}$$

Here $\Gamma(\chi \to f)$ denotes the partial width for decay of χ to the final state f, Γ_χ is the total χ decay width and B_f is the baryon number of the state f (so that $B_f = -B_{\bar{f}}$).

In treating the statistical mechanics of baryon number production it is convenient to choose a basis so that the χ are mass eigenstates. We assume that the χ have no CP-violating mixing (which is assured if χ and $\bar{\chi}$ have distinct conserved quantum numbers). Hence the decay process itself must exhibit CP violation in order for R_χ to be nonzero. As discussed below (and proved in general in Appendix B of [2]), this requires interference between the Born amplitude for the decay and a one loop correction with an absorptive part. In addition, the couplings of the particles participating in the decay must be relatively complex.

We consider first the simplest nontrivial case: two massive bosons, X and Y, coupled to four fermion species i_1, i_2, i_3 and i_4, through the vertices of Fig. 3.1 and their CP conjugates*. In the Born approximation, these vertices lead to the decay processes $X \to \bar{i}_1 i_2$, $X \to \bar{i}_3 i_4$, $Y \to \bar{i}_3 i_1$, $Y \to \bar{i}_4 i_2$ and the corresponding CP conjugated processes. We denote the coupling in, for example, the vertex Fig. 3.1(a) by $\langle i_2 | X | i_1 \rangle$ so that the CP-conjugate coupling becomes $\langle i_2 | X | i_1 \rangle^* = \langle i_1 | X^\dagger | i_2 \rangle$. The quantity X here may be considered as a matrix of couplings in the space of possible fermion states i_j. Note that the set of vertices in Fig. 3.1 is invariant under the combined transformations $X \leftrightarrow Y$ and $i_1 \leftrightarrow i_4$. This invariance will be used below to obtain results for Y ($\bar{Y}$) decays from those for X ($\bar{X}$) decays. The couplings $\langle i_j | X | i_k \rangle$ do not include Lorentz structure which determines, for example, which helicity states of the fermions i_j may contribute.

Born approximations to the X and Y decay rates may be obtained directly from the vertices of Fig. 3.1. For example

$$\Gamma(X \to i_2 \bar{i}_1)_{Born} = I_X^2 \; |\langle i_2 | X | i_1 \rangle|^2$$

$$\equiv I_X^2 \, \langle i_2 | X | i_1 \rangle \, \langle i_1 | X^\dagger | i_2 \rangle. \tag{3.1.2}$$

Here I_X^2 accounts for the kinematic structure of the process $X \to i_2 \bar{i}_1$; it gives the complete result if all couplings are set to one. From eqn. (3.1.2) it is evident that $\Gamma(X \to i_2 \bar{i}_1)_{Born} = \Gamma(\bar{X} \to \bar{i}_2 i_1)_{Born}$, and hence R_X vanishes in this approximation. To obtain a nonzero result for R_X, one must include corrections arising from interference of the one loop contributions shown in Fig. 3.2 with the Born amplitudes of Fig. 3.1. Consider, for example, the interference of the diagrams of Fig. 3.1(b) and Fig. 3.2(a). The resulting term in the squared amplitude is shown as Fig. 3.3(a). There the

*These vertices may be represented schematically by the interaction Lagrangian

$$L \sim i_2^\dagger X i_1 + i_4^\dagger X i_3 + i_1^\dagger Y i_3 + i_3^\dagger Y i_4 + Herm.conj.$$

where all Lorentz structure has been suppressed.

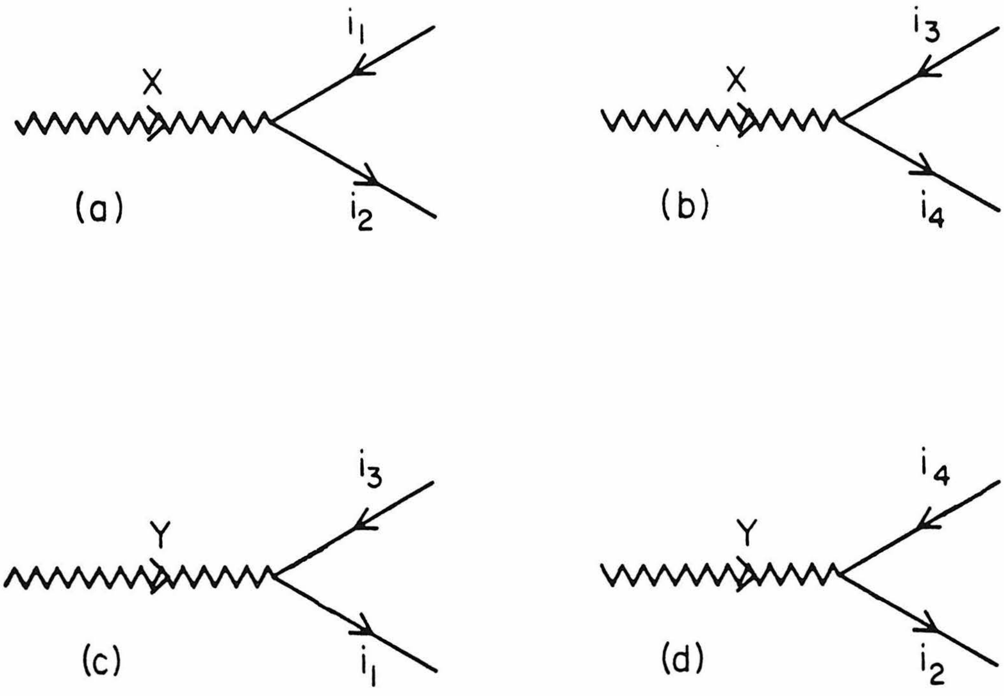

Figure 3.1: Couplings of bosons X and Y to fermion species i_j in the simplest case for which B generation is possible. These couplings correspond to possible decays of X and Y in the Born approximation.

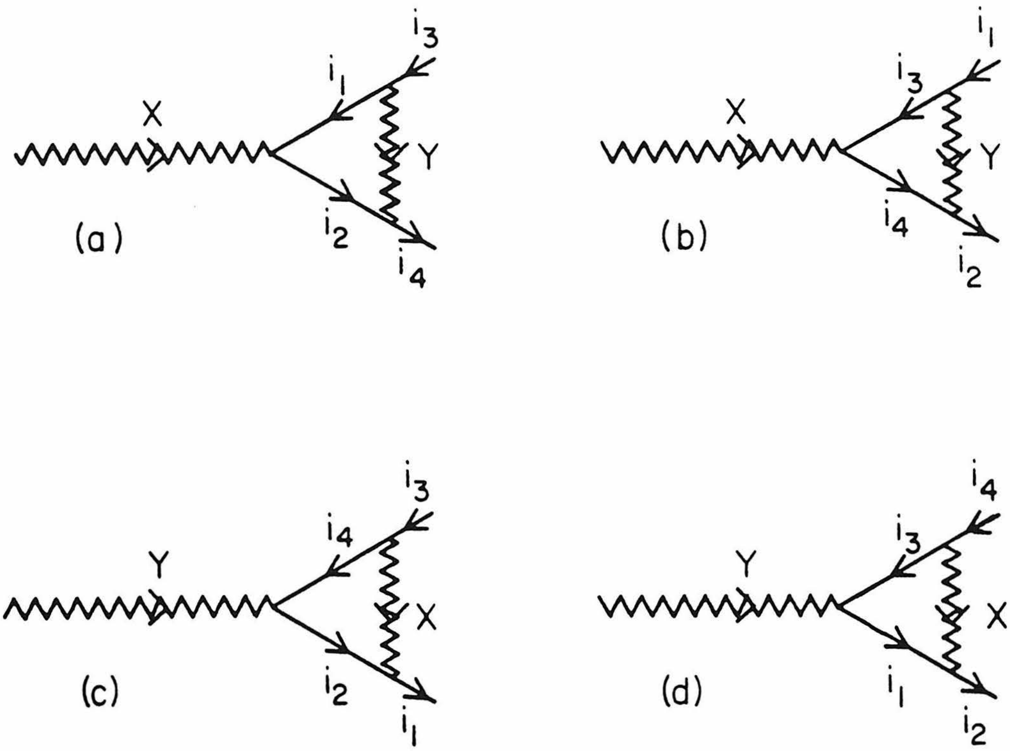

Figure 3.2: One-loop corrections to the decay amplitudes for the bosons X and Y. The couplings of X and Y to the fermions i_j are shown in Fig. 3.1.

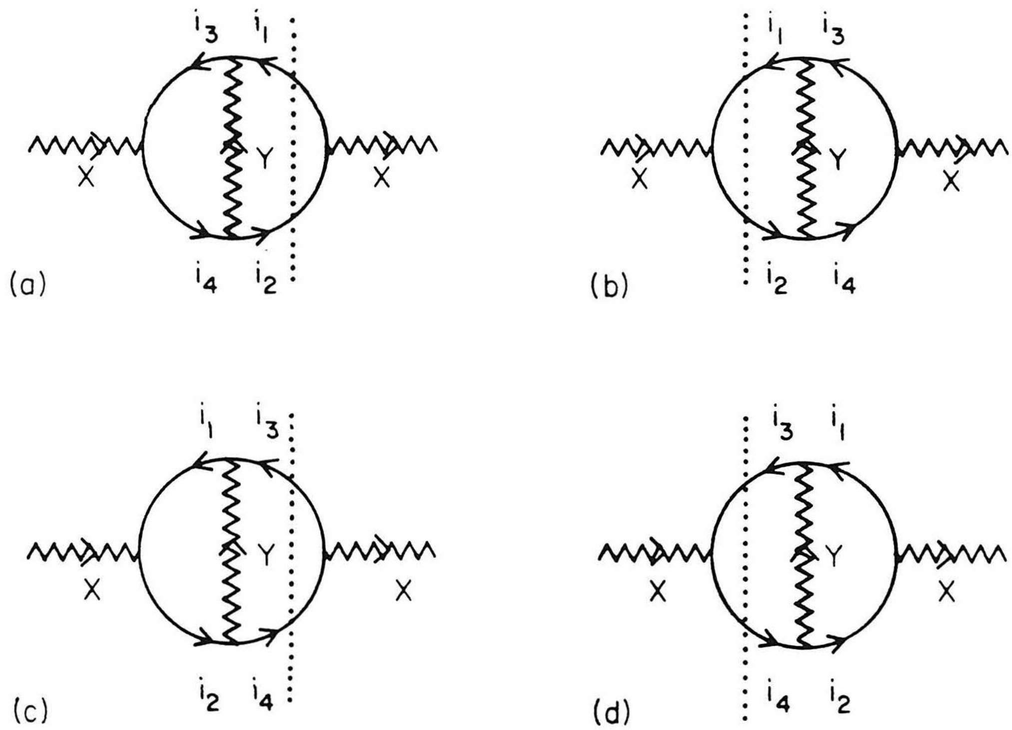

Figure 3.3: Squared amplitudes for one-loop corrections to X and Y decays, obtained as interference terms between the diagrams of Figs. 3.1 and 3.2. The dotted "unitarity cut" specifies the physical final state fermions.

dotted line is a "unitarity cut;" each cut line represents a physical on-mass-shell particle. The amplitude for the diagram Fig. 3.3(a) is then given by

$$I_{XY}^{1234}\,[<i_3|Y^\dagger|i_1><i_4|X|i_3><i_2|Y|i_4>][<i_2|X|i_1>]^*$$

$$= I_{XY}^{1234}\,<i_3|Y^\dagger|i_1><i_4|X|i_3><i_2|Y|i_4><i_1|X^\dagger|i_2>, \qquad (3.1.3)$$

where the kinematic factor I_{XY}^{1234} accounts for integration over the final state phase space of i_2 and $\bar{i}_1$ and over the momenta of the internal i_4 and $\bar{i}_3$. The complex conjugate diagram, Fig. 3.3(b), has the complex conjugate amplitude

$$(I_{XY}^{1234})^*\,[<i_3|Y^\dagger|i_1><i_4|X|i_3><i_2|Y|i_4>]^*<i_2|X|i_1>$$

$$= (I_{XY}^{1234})^*\,<i_2|X|i_1><i_4|Y^\dagger|i_2><i_3|X^\dagger|i_4><i_1|Y|i_3>\,. \qquad (3.1.4)$$

Introducing notations for quadratic and quartic combinations of the couplings of Fig. 3.1

$$\begin{aligned} \Xi_{jk}^{X} &= (\Xi_{jk}^{X})^\dagger \equiv |<i_k|X|i_j>|^2 = <i_k|X|i_j><i_j|X^\dagger|i_k> \\ \Omega_{1234} &= <i_3|Y^\dagger|i_1><i_1|X^\dagger|i_2><i_2|Y|i_4><i_4|X|i_3> \end{aligned} \qquad (3.1.5)$$

one may write the one-loop approximation to the $X\to i_2\bar{i}_1$ decay rate obtained by adding the results (3.1.2), (3.1.3) and (3.1.4) as

$$\Gamma(X\to i_2\bar{i}_1) = I_X^{12}\Xi_{12}^{X} + I_{XY}^{1234}\Omega_{1234} + (I_{XY}^{1234}\Omega_{1234})^*. \qquad (3.1.6)$$

In the Born approximation, the kinematic factors I_X are always real. However, the kinematic factors I_{XY} for loop diagrams may have an imaginary part whenever any internal lines have sufficiently small masses that they may propagate on their mass shells in the intermediate state (and thereby sample the $1/i\varepsilon$ piece of the propagator $1/(p^2-m^2+i\varepsilon)$). In the one-loop diagrams of Fig. 3.3, this occurs when the threshold conditions

$$m_X \geq m_3 + m_4 \tag{3.1.7}$$

and

$$m_X \geq m_1 + m_2. \tag{3.1.8}$$

are satisfied. With light intermediate fermions, I_{XY} thus always exhibits an imaginary part.

We now consider the CP-conjugate decay $\overline{X} \to \overline{i}_2 i_1$. To obtain the CP-conjugate amplitude all couplings must be complex conjugated. The kinematic factors I are, however, unaffected by the CP conjugation (this is manifest in the fact that reversal of the direction of fermion lines in a closed loop does not affect the associated Dirac trace). Thus, to one-loop order, the complete result for $\Gamma(\overline{X} \to \overline{i}_2 i_1)$ becomes

$$\Gamma(\overline{X} \to \overline{i}_2 i_1) = I_X^{12} \Xi_{12}^X + I_{XY}^{1234} \Omega_{1234}{}^* + (I_{XY}^{1234})^* \Omega_{1234}\,. \tag{3.1.9}$$

The diagrams for the decays $X \to i_4 \overline{i}_3$ and $\overline{X} \to \overline{i}_4 i_3$ are shown in Fig. 3.3(c) and 3.3(d) respectively. The loop diagrams differ from those for the decays $X \to i_2 \overline{i}_1$ and $\overline{X} \to \overline{i}_2 i_1$ only in that the unitary cut is taken through the i_3 and i_4 rather than the i_1 and i_2 lines. In analogy with eqns. (3.1.8) and (3.1.9) we thus obtain

$$\Gamma(X \to i_4 \overline{i}_3) = I_X^{34} \Xi_{34} + I_{XY}^{3412} \Omega_{1234}{}^* + (I_{XY}^{3412})^* \Omega_{1234}\,, \tag{3.1.10}$$

and

$$\Gamma(\overline{X} \to \overline{i}_4 i_3) = I_X^{34} \Xi_{34} + I_{XY}^{3412} \Omega_{1234} + (I_{XY}^{3412} \Omega_{1234})^*\,. \tag{3.1.11}$$

Using the results of eqns. (3.1.7) through (3.1.11) together with eqn. (3.1.1) we can compute the average baryon number produced in the free decays of an equal number of X's and $\bar{X}$'s. The one-loop contribution to this asymmetry from the $i_1\bar{i}_2$ and $\bar{i}_1 i_2$ final states is given by

$$R_X^{12} = (B_{i_2}-B_{i_1}) \frac{[\Gamma(X\to i_2\bar{i}_1) - \Gamma(\bar{X}\to\bar{i}_2 i_1)]}{[\Gamma(X\to i_2\bar{i}_1) + \Gamma(X\to i_4\bar{i}_3)]}$$

$$= (B_{i_2}-B_{i_1}) \frac{[I_{XY}^{1234}\Omega_{1234} + (I_{XY}^{1234}\Omega_{1234})^* - I_{XY}^{1234}\Omega_{1234}{}^* - (I_{XY}^{1234})^*\Omega_{1234}]}{[I_X^{12}\Xi_{12}^X + I_X^{34}\Xi_{34}^X]}$$

$$= -4\,\frac{(B_{i_2}-B_{i_1})}{\Gamma_X}\mathrm{Im}[I_{XY}^{1234}]\mathrm{Im}[\Omega_{1234}]\,. \qquad (3.1.12)$$

The analogous result for the 34 final state is

$$R_X^{34} = -4\,\frac{(B_{i_4}-B_{i_3})}{\Gamma_X}\mathrm{Im}[I_{XY}^{3412}]\mathrm{Im}[\Omega_{1234}{}^*]$$

$$= 4\,\frac{(B_{i_4}-B_{i_3})}{\Gamma_X}\mathrm{Im}[I_{XY}^{3412}]\mathrm{Im}[\Omega_{1234}]\,. \qquad (3.1.13)$$

The kinematic factors $\mathrm{Im}[I_{XY}^{1234}]$ and $\mathrm{Im}[I_{XY}^{3412}]$ are obtained from diagrams involving two unitarity cuts (as in Fig. 3.4): one through the i_1 and i_2 lines and the other through the i_3 and i_4 lines. The resulting quantities are invariant under the combined interchanges $i_1 \leftrightarrow i_3$ and $i_2 \leftrightarrow i_4$ and consequently are equal:

$$\mathrm{Im}[I_{XY}^{1234}] = \mathrm{Im}[I_{XY}^{3412}]. \qquad (3.1.14)$$

Hence $R_X^{12}/R_X^{34} = (B_{i_1}-B_{i_2})/(B_{i_4}-B_{i_3})$, as expected. Notice that, if all intermediate fermions have zero mass, then the I_{XY}^{1234} are completely independent of their upper indices; corrections from fermion mass differences are of order $(m_f/m_X)^2$.

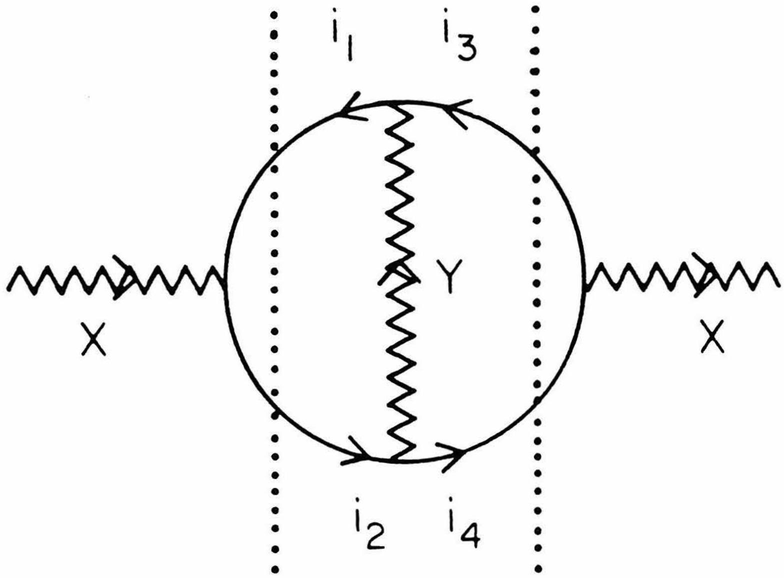

Figure 3.4: "Double-cut" diagram representing the CP-violating combination of amplitudes for X decay. The dotted lines denote "unitarity cuts."

Upon adding the contributions (3.1.12) and (3.1.13) we obtain the final result

$$R_X = \frac{4}{\Gamma_X}\mathrm{Im}[I_{XY}^{1234}]\mathrm{Im}[\Omega_{1234}][B_{i_4}-B_{i_3}-(B_{i_2}-B_{i_1})]\,. \tag{3.1.15}$$

The conditions for the kinematic factor $\mathrm{Im}[I_{XY}^{1234}]$ to be non-vanishing were given in eqns. (3.1.5) and (3.1.6). A further condition for R_X to be non-vanishing is that both X and Y interactions must violate baryon number. If X couplings were B-conserving, the two possible final states in X decay would have the same baryon number, so that

$$B_{i_2}-B_{i_1}=B_{i_4}-B_{i_3} \tag{3.1.16}$$

and R_X would vanish. Similarly, if Y couplings were B-conserving,

$$B_{i_2}-B_{i_4}=B_{i_1}-B_{i_3} \tag{3.1.17}$$

and R_X would again vanish. Thus both X and Y couplings must be B-violating to obtain a non-vanishing R_X. This is as implied by the general theorem given in Appendix B of [2]. Notice that for (3.1.15) to be non-vanishing, at least two of the i_j must be distinct.

The asymmetry R_Y produced in Y and $\bar{Y}$ decays may be obtained from (3.1.15) by the transformation $X \leftrightarrow Y$, $i_3 \leftrightarrow i_4$, yielding

$$R_Y = \frac{4}{\Gamma_Y}\mathrm{Im}[\Omega_{1234}{}^*\mathrm{Im}[I_{YX}^{3142}][B_{i_4}-B_{i_3}-(B_{i_2}-B_{i_1})]$$

$$= -\frac{4}{\Gamma_Y}\mathrm{Im}[\Omega_{1234}]\mathrm{Im}[I_{YX}^{3142}][B_{i_4}-B_{i_3}-(B_{i_2}-B_{i_1})] \tag{3.1.18}$$

and so

$$R_X/R_Y = -\mathrm{Im}(I_{XY}^{1234})/\mathrm{Im}(I_{YX}^{3142}). \tag{3.1.19}$$

It follows that the average baryon number produced in the free decay of an equal number of X, $\overline{X}$, Y and $\overline{Y}$ is

$$R_{X+Y} = 4\left\{\frac{\mathrm{Im}[I_{XY}^{1234}]}{\Gamma_X} - \frac{\mathrm{Im}[I_{YX}^{3142}]}{\Gamma_Y}\right\} \times \mathrm{Im}[\Omega_{1234}][B_{i_4}-B_{i_3}-(B_{i_2}-B_{i_1})]. \quad (3.1.20)$$

Even if the R_X and R_Y are non-vanishing on their own, for the total to be nonzero the terms in the brace must not cancel. This requires that the particles X and Y be distinct either in mass or in the Lorentz structure of their couplings (e.g. one vector and one scalar) and that $\Gamma_X \neq \Gamma_Y$. The brace typically vanishes if X and Y are in the same irreducible representation of an unbroken symmetry group.

If more than the minimal set of four fermion species are present, the result (3.1.20) must be summed over all possible contributing $\{i_j\}$. It must also be summed over all possibly (X,Y) pairs. Whenever particles have equal masses on the scale of m_X, the corresponding kinematic factors may be factored out of the summation.

The individual baryon asymmetry parameters R_X for X decays enter the complete Boltzmann transport equations discussed in Sect. 4. These parameters alone determine the final baryon asymmetry only if back reactions (inverse decays) and $2 \to 2$ scatterings are ignored. The total contribution to the baryon asymmetry from decays of two species X and Y of bosons is thus not in general a simple sum of their corresponding parameters R_X and R_Y: if X and Y have different masses, the extent of back reactions is different in the two cases. If, however, X and Y are degenerate in mass, the sum given in eqn. (3.1.20) represents their total contribution.

In the derivation of eqn (3.1.15) the particles i_j were assumed to be light fermions of definite baryon number. The result nevertheless remains approximately valid for any particles i_j so long as their masses are much smaller than m_X. Some of the i_j may for example be bosons, which enter through a three-boson coupling vertex, as illustrated in Fig. 3.5. The B_j in eqn. (3.1.15) should usually be replaced by the average baryon numbers generated in the decays of the corresponding i_j.

The discussion above concerns the one-loop contributions to baryon asymmetry. In the generic case, an asymmetry occurs at this order if it is to occur at any order. However, in some simple models (such as the minimal SU(5) model treated in sect. 4) the one-loop contribution vanishes, but there are higher loop contributions which are finite: in such cases the detailed analysis given above must be suitably generalized by summing over all possible unitarity cuts through the multiloop diagram.

It should be noted that, although the analysis of this section has focused on baryon number, the expressions that we have derived are not restricted to that quantum number. The expressions may be used to describe the generation of any quantum number in the free decays of X, $\bar{X}$, Y, or $\bar{Y}$. Thus for example, to describe lepton number generation we need to replace the B_i's by the relevant lepton number assignments.

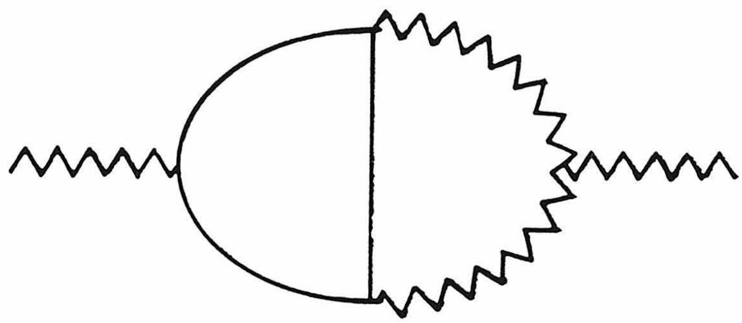

Figure 3.5: A diagram involving three-boson coupling potentially contributing to CP violating X decays.

3.2 Consequences for gauge models

In this section, we give some general results in gauge models for the value of the CP-violating parameter $\mathrm{Im}[\Omega]$ defined by eqn. (3.1.5).

The couplings of gauge vector bosons to fermions (and Higgs bosons) may always be taken real and diagonal. Couplings of Higgs bosons to fermions and to each other may, however, be complex and induce mixing. After spontaneous symmetry breaking, these couplings may give rise to CP violation and mixing in the fermion and Higgs boson mass matrices. If fermion masses are neglected, diagrams involving only fermions and gauge vector bosons (Fig. 3.6) can thus yield no CP violation. For CP violation to occur in the decays of superheavy bosons, it is thus necessary for either explicit Higgs bosons or superheavy fermions with complex mixing angles to be present.

Some CP-violating effects involving Higgs bosons may be investigated before spontaneous symmetry breakdown. If a particular set of Higgs bosons allows CP violation in the unbroken theory, this CP violation will remain possible in the broken theory.

Consider first the case of scalar boson (S) exchange in vector boson (V) decay, as illustrated in Fig. 3.7. The diagonal nature of the gauge couplings requires that the fermions i_1 and i_2 lie in the same irreducible representation $\mathbf{f}_1$ of the gauge group (and i_3 and i_4 in $\mathbf{f}_2$). Scalar bosons contributing to Fig. 3.7 must lie in irreducible representations $\mathbf{s}_a$ such that

$$\bar{\mathbf{f}}_1 \otimes \mathbf{f}_3 \supset \mathbf{s}_a . \tag{3.2.1}$$

In the absence of spontaneous symmetry breakdown, there is no mixing between scalar bosons, and the exchanged S propagator must be

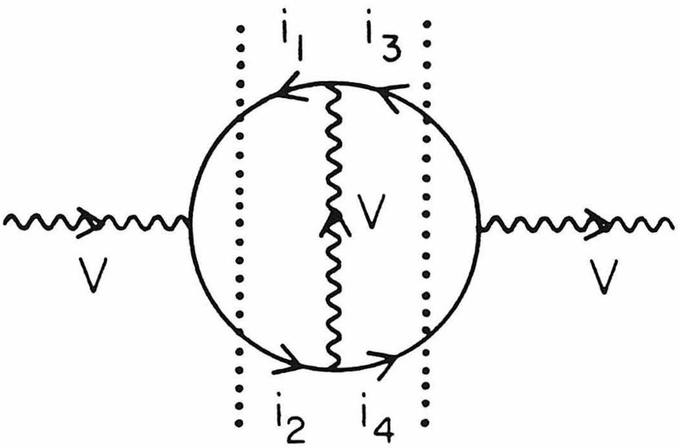

Figure 3.6: Diagram for vector (gauge) boson exchange in vector boson decay.

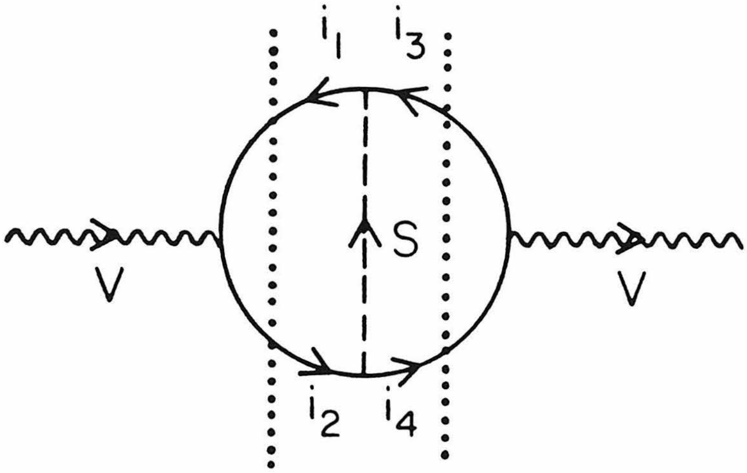

Figure 3.7: Diagram for scalar (Higgs) boson exchange in vector (gauge) boson decay.

diagonal. Hence in the notation of Sect. 3.1, the coupling $<i_2|S|i_4>$ at one end of the exchanged S line is simply the hermitean conjugate of the coupling $<i_3|S^\dagger|i_1>$ at the other end: the product of these couplings is thus real, and no CP violation may occur.

CP violation may be introduced into Fig. 3.7 through mixing terms in the S propagator arising from spontaneous symmetry breakdown. Symmetry breakdown causes the exchanged mass eigenstate scalar boson S to become in general a linear combination of several components with the same conserved charges. These components may occur within the same irreducible representation of the gauge group, or in different irreducible representations. If a model contains only a single B-violating Higgs boson no such mixing is possible, and CP violation cannot occur at the one-loop level through scalar boson exchange in vector boson decay. This is the case for the minimal SU(5) model discussed in Sect. 4. In the general case, we decompose the mass eigenstate field S into its unbroken group eigenstate components according to:

$$S=\alpha_1 S_1+\alpha_2 S_2+\cdots \tag{3.2.2}$$

We shall assume for now that just two components are present; the generalization to an arbitrary number will be immediate. In this case,

$$\begin{aligned}\mathrm{Im}[\Omega^{1234}]&=\mathrm{Im}[\mathit{Tr}[<i_3|S^\dagger|i_1><i_2|S|i_4>]]\\&=\mathrm{Im}[\mathit{Tr}[(\alpha_1{}^*<i_3|S_1^\dagger|i_1>+\alpha_2{}^*<i_3|S_2^\dagger|i_1>)\\&\times(\alpha_1<i_2|S_1|i_4>+\alpha_2<i_2|S_2|i_4>)]]\end{aligned} \tag{3.2.3}$$

where we have dropped the real factor corresponding to the gauge boson couplings, and the trace represents a sum over all fermion representations (usually "families"). Since $i_1, i_2 \subset \mathbf{f}_1$ and $i_3, i_4 \subset \mathbf{f}_2$, the couplings $<i_2|S_a|i_4>$ and $<i_1|S_a|i_3>$ are related by a real Clebsch-Gordan coefficient:

$$<i_2|S_a|i_4> = C_a <i_1|S_a|i_3> \tag{3.2.4}$$

Hence

$$\mathrm{Im}[\Omega]=\mathrm{Im}[Tr[(\alpha_1{}^*<i_3|S_1^\dagger|i_1>\alpha_2{}^*<i_3|S_2^\dagger|i_1>(C_1\alpha_1<i_1|S_1|i_3>+C_2\alpha_2<i_1|S_2|i_3>)]]$$

$$=\mathrm{Im}[Tr[(C_2\alpha_1{}^*\alpha_2<i_3|S_1^\dagger|i_1><i_1|S_2|i_3>+C_1\alpha_1\alpha_2{}^*<i_3|S_2^\dagger|i_1><i_1|S_1|i_3>)]]$$

$$=(C_2-C_1)\mathrm{Im}[Tr[\alpha_1{}^*\alpha_2<i_1|S_1|i_3><i_3|S_2^\dagger|i_1>]] . \tag{3.2.5}$$

Thus, if $C_1=C_2$, Im[Ω] vanishes. This effect occurs when all Higgs bosons coupling to fermions have identical group charges, and are distinguished only by a "family" index. This is inevitable if all relevant Higgs bosons lie in replications of the same irreducible representation of the gauge group, and if this representation contains only one B-violating component. Examples in which $C_1 \neq C_2$ are the SU(5) model with a 5_H and a 45_H (Model III in Sect. 4) and the SO(10) model with a 10_H and a 120_H or a 126_H. In these models, CP violation may occur at the one-loop level from scalar boson exchange in vector boson decay. Notice that since in the absence of spontaneous symmetry breakdown, only one of the α_j is nonzero, the result (3.2.5) yields no CP violation in this case.

The case of vector boson exchange in scalar boson decay (illustrated in Fig. 3.8) is exactly analogous to the case of scalar exchange in vector decay discussed above. When Fig. 3.8 contributes, it is often important by virtue of large value of the vector couplings relative to the scalar ones.

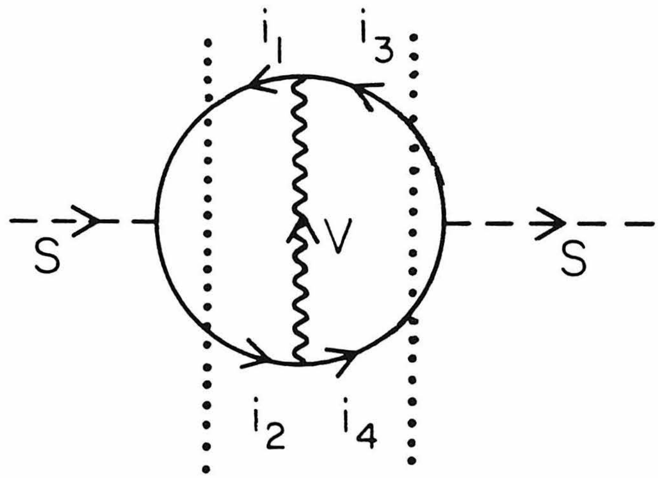

Figure 3.8: Diagram for vector (gauge) boson exchange in scalar (Higgs) boson decay.

We now consider CP violation arising from scalar boson (S') exchange in scalar boson (S) decay, as illustrated in Fig. 3.9. If only one B-violating Higgs boson is present, then the decaying and exchanged bosons must be identical, and the results of Sect. 3.1 show that Fig. 3.9 can give no CP violation. This is the case for the minimal SU(5) model. (However, as described in Sect. 4, CP violation may occur in higher-order diagrams.) We consider for now the case in which all fermions are effectively massless. Then, in analogy with (3.2.1), the contributing scalar bosons must appear in representations $\mathbf{s}_\alpha$ such that

$$\mathbf{f}_1 \otimes \overline{\mathbf{f}}_2 \subset \overline{\mathbf{s}}_\alpha \tag{3.2.6}$$

$$\mathbf{f}_4 \otimes \overline{\mathbf{f}}_3 \subset \mathbf{s}_\alpha$$

$$\mathbf{f}_2 \otimes \overline{\mathbf{f}}_4 \subset \mathbf{s}_\alpha'$$

$$\mathbf{f}_3 \otimes \overline{\mathbf{f}}_1 \subset \overline{\mathbf{s}}_\alpha'$$

If all the left-handed fermions lie in the same complex irreducible representation, $\mathbf{f}$, (or sequence of such identical representations), then $\mathbf{f}_1 = \overline{\mathbf{f}}_2 = \overline{\mathbf{f}}_3 = \mathbf{f}_4 = \mathbf{f}$ and these constraints become

$$\mathbf{f} \otimes \mathbf{f} \subset \mathbf{s}_\alpha,\ \mathbf{s}_\alpha', \overline{\mathbf{s}}_\alpha,\ \overline{\mathbf{s}}_\alpha' \tag{3.2.7}$$

For low-dimensionality representations, this requires that $\mathbf{s}_\alpha$ and $\mathbf{s}_\alpha'$ be real representations. Hence in SO(10) models, where all fermions lie in the 16 representation, only 10_H or 120_H may contribute to Fig. 3.9; the 126_H which appears in $16_f \otimes 16_f$ is complex. (For high-dimensional fermion representations, some complex Higgs representations may satisfy (3.2.8): an example is the 126_H occurring in the symmetric product $144_f \otimes 144_f$ of SO(10).) After spontaneous symmetry breakdown, mixing between scalar

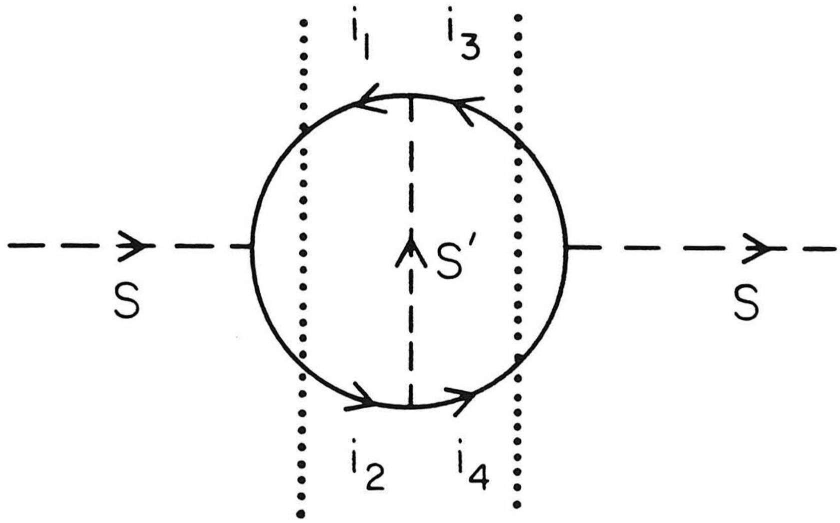

Figure 3.9: Diagram for scalar (Higgs) boson exchange in scalar boson decay.

bosons may occur, and the constraints (3.2.6) are no longer applicable. Thus in both SU(5) models with several Higgs representations coupling to fermions, and in SO(10) models, Fig. 3.9 can yield CP violation.

The discussion above has assumed that all relevant fermion species are effectively massless. With gauge groups such as SO(10) or E(6), it is common for fermions with $SU(2)_L$ singlet and thus potentially large mass terms to exist. Such fermions may introduce CP-violating effects into Figs. 3.6 through 3.9. These effects are, however, always suppressed by $O(m_f^2/m_X^2)$ with m_f the mass of the superheavy fermion and m_X the mass of the decaying boson.

4. Baryon number generation in realistic grand unified models

Cosmology is potentially an important source of information on the baryon number (B) violating interactions expected in most grand unified gauge models. Any net B imposed as an initial condition on the universe should have been rapidly destroyed by any B violating interactions. To account for the observed baryon number density to photon number density ratio, $n_B/n_\gamma \simeq 10^{-9}$, a net baryon number must subsequently have been generated. This requires, in addition to B violation, the violation of C and CP (and hence T) invariance, along with departures from thermal equilibrium [5,2]. The magnitude of the baryon excess generated depends sensitively on detailed features of the grand unified model considered. This letter outlines the complete calculation of n_B/n_γ generation in specific grand unified models in the context of the standard hot big bang model of the early universe. The method we present allows for the exact treatment of an arbitrary number of superheavy bosons as well as the presence of non-thermalizing modes [6]. We summarize results for several realistic SU(5) models. Many details and extensions are discussed in ref. [3].

We denote heavy bosons generically by χ and light fermions by a, b,.... The number density n_i of a particle i, and that of its antiparticle $n_{\bar{i}}$ are parameterized by $i_+ \equiv (n_i + n_{\bar{i}})/n_\gamma$ and $i_- \equiv (n_i - n_{\bar{i}})/n_\gamma$. The time development of these quantities is described by a set of coupled Boltzmann transport equations. For the heavy bosons these are [2,3]

$$\dot{\chi}_+ = -\sum_{a,b} \langle \Gamma(\chi \to ab) \rangle \, (\chi_+ - \chi_+^{eq}) \tag{4.1}$$

$$\dot{\chi}_- = -\sum_{a,b} \langle \Gamma(\chi \to ab) \rangle \, (\chi_- - (a_- + b_-)\, \chi_+^{eq}) \ . \tag{4.2}$$

where dots denote time derivatives and the expansion of the universe is accounted for through division by n_γ in the definitions of $i_\pm$. The first terms on the right side of eqns (4.1) and (4.2) correspond to free decays of χ and $\bar{\chi}$ with partial rates $<\Gamma(\chi\to ab)>$ averaged over time dilation factors for the decaying χ energy spectrum. The second terms account for back reactions in which the χ decay products interact to produce χ. The equilibrium number density χ_+^{eq} is obtained by integrating the $\exp[-E_\chi/T]$ equilibrium Maxwell-Boltzmann phase space density (for $T \gg m_\chi$, $\chi_+^{eq} \simeq 1-(m_\chi/2T)^2$, while for $T \ll m_\chi$, $\chi_+^{eq} \simeq \exp[-m_\chi/T]$). In equilibrium, $\chi_+ = \chi_+^{eq}$ and $\dot{\chi}_+ = 0$; the expansion of the universe produces deviations from equilibrium at temperatures, $T \sim m_\chi$.

The densities of fermion species develop according to

$$\dot{f}_- = \sum_{a,b,\chi} <\Gamma(\chi\to ab)> (N_f)_{ab} \{(\chi_+ - \chi_+^{eq})\, R(\chi\to ab) + 2\chi_- - (a_- + b_-)\, \chi_+^{eq}\}$$

$$+ \sum_{a,b,c,d,\chi} n_a [(N_f)_{ab} - (N_f)_{cd}] \{a_- + b_- - c_- - d_-\} <|v|\sigma'_\chi(ab\to cd)> , \qquad (4.3)$$

where $(N_f)_{ab}$ denotes the number of particles of type f in the state ab. $R(\chi\to ab)$ denotes the difference in branching ratios between the CP conjugate decays $\chi\to a\, b$ and $\bar{\chi}\to\bar{a}\,\bar{b}$ divided by the full rate for χ decay; it vanishes if CP is conserved. The first part of the first term on the right side of eqn (4.3) therefore represents the production of an asymmetry in fermion number densities as a result of CP violating decays of a symmetrical χ, $\bar{\chi}$ mixture. The second part causes asymmetries χ_- between χ and $\bar{\chi}$ to be transferred to the fermions when the χ ($\bar{\chi}$) decays. The third part gives a correction to the rate for inverse decays resulting from the deviation of the fermion number densities from their equilibrium value. The second term in eqn (4.3) represents the production and destruction of fermions by two-to-two scattering processes. σ'_χ is the cross-section for this scattering mediated by χ exchange, but with the term corresponding to a real intermediate χ removed (since this is already accounted for by χ decay and inverse decay processes).

The number of independent particle densities to be treated in eqns 4.1 through 4.3 may be reduced by using unbroken symmetries (gauge* and global). For non-Abelian groups, any asymmetries are shared symmetrically among members of each irreducible representation (if $SU(2)_L$ is unbroken $e_{L-}=\nu_{L-}$ but in general $e_{L-}\neq e_{R-}$). The unbroken symmetry may contain U(1) factors; conservation of the corresponding charges reduces the number of independent particle densities. This number may be reduced further by considering only the heaviest family of fermions; since the rates for reactions that produce asymmetries are always proportional to Yukawa couplings of the scalars to fermions (see below), the changes in asymmetries occur fastest in the heaviest family. These asymmetries are quickly shared equally among all families through Higgs couplings between the different families.

If only a subset of the interactions that may potentially contribute to eqn (4.3) are included there may be additional symmetries leading to further conserved combinations of fermion number densities (e.g., Π conservation in the absence of Higgs-fermion couplings for the models discussed below).

Let f^i_- $(i=1,\cdots,N_f)$ be the remaining independent fermion asymmetries and χ^α_- $(\alpha=1,\ldots,N_\chi)$ the independent supermassive boson asymmetries. It is convenient to form a set $\vec{Q}$ which consists of independent

* For SU(5) this will usually be $SU(3)\otimes SU(2)_L\otimes U(1)$, while in other models it may be a larger symmetry (e.g., $SU(4)\otimes SU(2)_L\otimes U(1)_R$ in SO(10)).

quantum number densities B, L, etc.... related to $\vec{F}=\{f^i_-, \chi^\alpha_-\}$ by a unitary transformation, $\vec{Q}=H\,\vec{F}$, $\vec{F}=H^{-1}\,\vec{Q}$.

The thermalization of a quantum number Q_i through reactions of a particular boson χ is given from eqn (4.3) by $\dot{Q}_i=\sum_{\chi} \chi_+^{eq}\, M_{ij}^{\chi}\, Q_j$, where $M_{ij}^{\chi}=\sum_{k,l} \Delta Q_i(\chi\to f^k f^l)\langle\Gamma(\chi\to f^k f^l)\rangle(H_{kj}^{-1}+H_{lj}^{-1})$ and $\Delta Q_i(\chi\to f^k f^l)$ represents the change in the value of Q_i through the reaction $\chi\to f^k f^l$. Boltzmann's H theorem requires that the eigenvalues of M^{χ} are all real and non-positive. Any zero eigenvalues reveal additional symmetries; the corresponding eigenvector of number densities is then conserved in χ reactions (e.g. Π in vector boson exchanges in SU(5)). If this eigenvector is conserved in the reactions of all χ species, then it represents a globally conserved quantum number (e.g. B-L in SU(5)) and results in a further reduction in the number of independent Q_i.

We consider three grand unified models based on SU(5). In all cases each family of fermions transforms as a reducible representation $(\bar{5}\oplus 10)_i$, labeled by the family index i. The following Higgs representations are taken to couple to fermions: in model I (minimal SU(5)), a single 5 of Higgs, H_5; in model II, H_5 and an additional 5 of Higgs, $H_{5'}$; in model III, H_5 and a 45 of Higgs, H_{45}. The Yukawa couplings in these models have the schematic form $(\bar{5}_i\,(D_\alpha)_{ij}\,10_j)\,H_\alpha+(10_i\,(U_\alpha)_{ij}\,10_j)\,\bar{H}_\alpha$. The suppressed (real) group coupling coefficients are different for $\alpha=5$ and for $\alpha=45$ (but may be factored out of the relevant expressions in models I and II where only $\alpha=5$ occurs).

It may be shown that a CP violating nonzero $R(\chi\to ab)$ enters through an imaginary part of the product of the couplings in diagrams in which one boson is exchanged between the ab produced in the χ decay. The sum over a and b in eqn (4.3) runs over all types and families of fermions; thus for fixed fermion types the double cut diagram is proportional to a family space trace of coupling matrices. The gauge boson coupling matrix is proportional to the family space identity matrix; processes involving only vector bosons and massless fermions can thus give no CP violation in perturbation theory. Higgs bosons are required for CP violation and hence for baryon number generation to occur.

In model I the first diagram exhibiting CP violation involves only Higgs bosons and is of eighth order in the Yukawa couplings [7,8,4]. It is proportional to the imaginary part of the family space trace, $Tr[UU^{\dagger}UD^2U^{\dagger}D^2]$, suggesting the rough estimate $R \sim \alpha^3\, (m_F/m_W)^6\, \varepsilon/(128\pi^3)$ $= 4\times10^{-9}\,(m_F/m_W)^6\,\varepsilon$, with $|\varepsilon|\lesssim 1$, where m_F is the mass of the heaviest fermion. (Stability of the effective potential requires that $m_F \lesssim \sqrt{3}m_W$ [9] and hence $R \lesssim 10^{-8}\,\varepsilon$, making the production of an adequate baryon asymmetry implausible in this model.)

In model II, both H_5 and $H_{5'}$ have only the single B-violating component*, (3,1,1/3): since 5 is a complex representation one may form complex linear combinations so that the (3,1,1/3) in both 5 and 5' is separately a mass eigenstate. This suffices to show that no CP violation may occur for gauge boson decay with Higgs scalar exchange or (vice versa) . CP violation may occur at $O(\alpha(m_F/m_W)^2)$ through 5 decay with 5' exchange (and vice versa) [4,10].

In model III, CP violation may occur not only through scalar exchange in scalar decay, but also through vector exchange in scalar decay (and

* In this notation the first entry is the SU(3) multiplicity, the second is the SU(2) multiplicity and the last the value of the weak hypercharge Y normalized so that the charge operator is given by $Q = T_{3L} - Y$.

vice versa). In the vector exchange case it is $O(\alpha)$. H_{45} contains three B-violating components: (3,1,-2/3), ($\bar{3}$,1,-4/3) and (3,3,1/3).

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry allows the 15 independent fermion fields in the heaviest family of an SU(5) model to be reduced to the set U_L, $(U^c)_L$, $(D^c)_L$, E_L and $(E^c)_L$ (the subscript L denotes the left-handed helicity state and c denotes charge conjugation). The model contains a (3,2,5/6) of B-violating vector bosons X (with number densities parameterized by X_- and X_+). We consider the case where there are n_s (=1 or 2) scalars, S_1, S_2,..., S_{n_S}, transforming as (3,1,1/3) (with number densities parameterized by S_{i-} and S_{i+}). These models possess a locally conserved weak hypercharge, $Y=-U_{L-}+2(U^c)_{L-}-(D^c)_{L-}-(E^c)_{L-}-E_{L-}+5X_-+S_{1-}+S_{2-}+\cdots+S_{n_s-}$, whose initial value we assume to be zero. The models exhibit two further zero eigenmodes. The first is $B-L = 2U_{L-}-(U^c)_{L-}-(D^c)_{L-}+(E^c)_{L-}-2E_{L-}-4X_--2S_{1-}-2S_{2-}-\cdots-2S_{n_S-}$ and has zero eigenvalue (is conserved) in all boson interactions. A second zero eigenmode, $\Pi=-3(D^c)_{L-}-2E_{L-}$, is present if scalar-fermion interactions are removed [6]. Π (termed "fiveness") corresponds to the net number density of the fermion species appearing in the 5 representation. A density Π_0 generated through Higgs decays would be distributed as B $= -\Pi_0/10$, $\nu_-=-\Pi_0/5$ through Π-conserving X interactions. Π_0 may be destroyed through exchanges of light Higgs bosons. A convenient choice of independent combinations of fermion densities is $n_B/n_\gamma \equiv$ B $= 2D_{L-}-(U^c)_{L-}-(D^c)_{L-}$, Π and $\nu_-=E_{L-}$.

For model I, according to the estimate for $R(S\to ab)$ given above, an adequate baryon number asymmetry will be generated only if very heavy fermions exist $(m_F \sim m_{\not{v}})$*. Figure (4.1a) shows the baryon asymmetry

* Similar conclusions have recently been reached in ref [11].

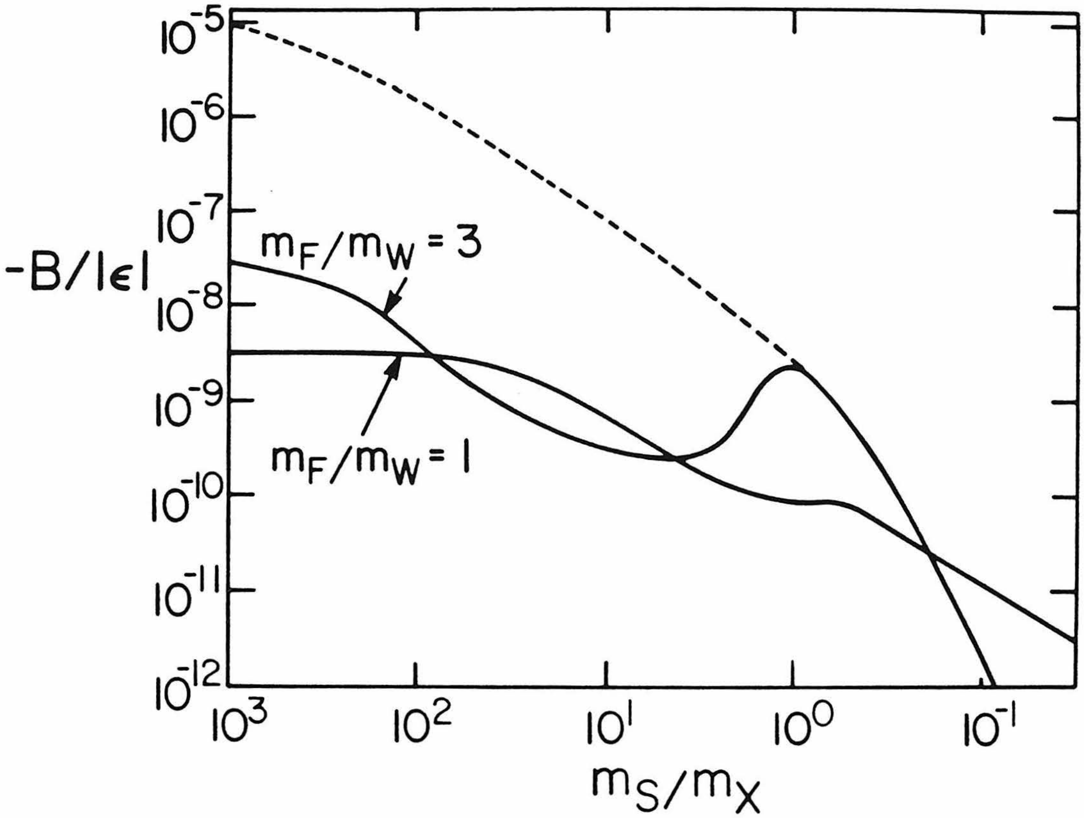

Figure 4.1a: Baryon number density as a function of the Higgs boson (S) mass generated in the minimal SU(5) model in which the heaviest fermion has mass m_F. Results are for $\alpha=1/40$, $m_X=5\times10^{14}$ GeV. The CP violation parameter ε is unknown but less than 1.

(taking $m_X=5\times10^{14}$ GeV and $\alpha=1/40$) as a function of m_S/m_X for $m_F/m_\Psi=1$ and $m_F/m_\Psi=3$ obtained by numerically integrating the Boltzmann transport equations 4.1-4.3. When $m_S/m_X \gg 1$, X exchanges thermalize the B produced in S decay to the value $-\Pi/10$; meanwhile, Π is reduced by light Higgs interactions. The final B attained is determined by the reduction in Π that occurs before X exchanges cease to be important and B becomes fixed. For $m_S/m_X<1$ the X is not effective in destroying the baryon number built up through S decay. The enhancement in the final value of B around $m_S/m_X=1$ is a result of the transition between these two regions. The dotted curve shows the final baryon number if all X interactions are artificially set to zero. Figure (4.1b) shows the temperature (time) development of the quantum number asymmetries B, Π and ν_- for the case $m_F/m_\Psi=1$, $m_S/m_X=10$. The solid and dashed curves for B correspond to two extreme assumptions for X exchange cross-sections at high temperature, $\sigma \sim \alpha^2/m_X^2$ and $\sigma \sim \alpha^2/T^2$ respectively. The final results are independent of this choice. We have also included the effects of the usual light Higgs doublet which can change Π and ν_-, but not B. Figure (4.1c) shows the temperature development for the case $m_F/m_\Psi=3$, $m_S/m_X=10$ with the solid (dashed) curves indicating the effect of including (excluding) the destruction of Π and ν_- by the interactions of the light Higgs doublet.

For model II the final baryon number density as a function of m_{S_1}/m_X is shown in figure (4.2) for different choices of m_{S_2}/m_X. Note that, when $m_1=m_2$, we have (assuming $(\Gamma_{S_1})_{total}=(\Gamma_{S_2})_{total}$ in the Born approximation) $R(S_1\to ab)=-R(S_2\to ab)$ and hence no B is generated. For $m_{S_i}>m_X$ the additional decay mode $S_i\to X+\varphi$ (where φ is a light Higgs boson) decreases the effective CP violation, $R(S_i\to ab)$, in S_i decay. For $m_{S_2}>m_X$ and $m_{S_1}>m_X$,

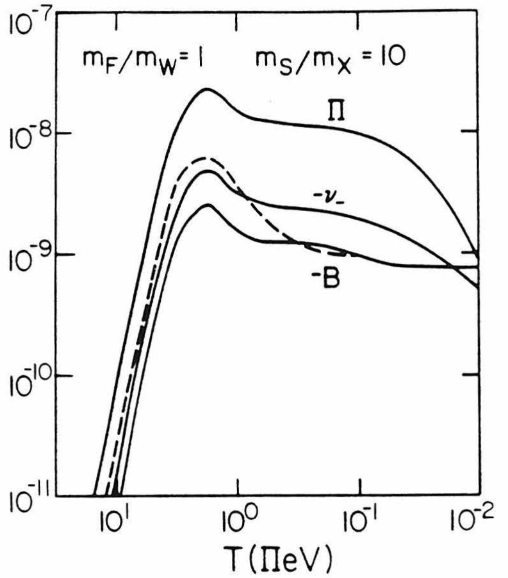

Fig. 4.1b

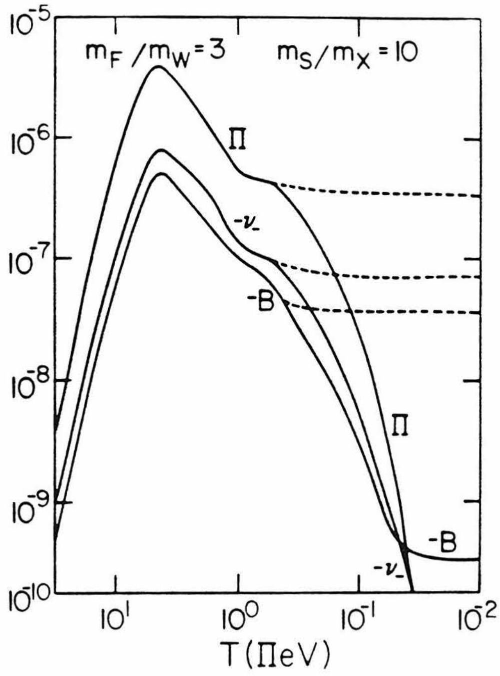

Fig. 4.1c

Figure 4.1(b,c): Evolution of independent quantum number densities as a function of temperature in the minimal SU(5) model. B denotes the net baryon number, ν_- the asymmetry between $\bar{\nu}$ and ν densities and Π the total asymmetry between fermion in the 5 and $\bar{5}$ representations of SU(5). $1\Pi eV = 10^{24} eV$. In these graphs the parameter ε has been scaled out. The dashed curve in (b) shows results with a smaller high-energy X exchange cross-section. The dashed curves in (c) are results obtained by neglecting light Higgs boson exchange processes.

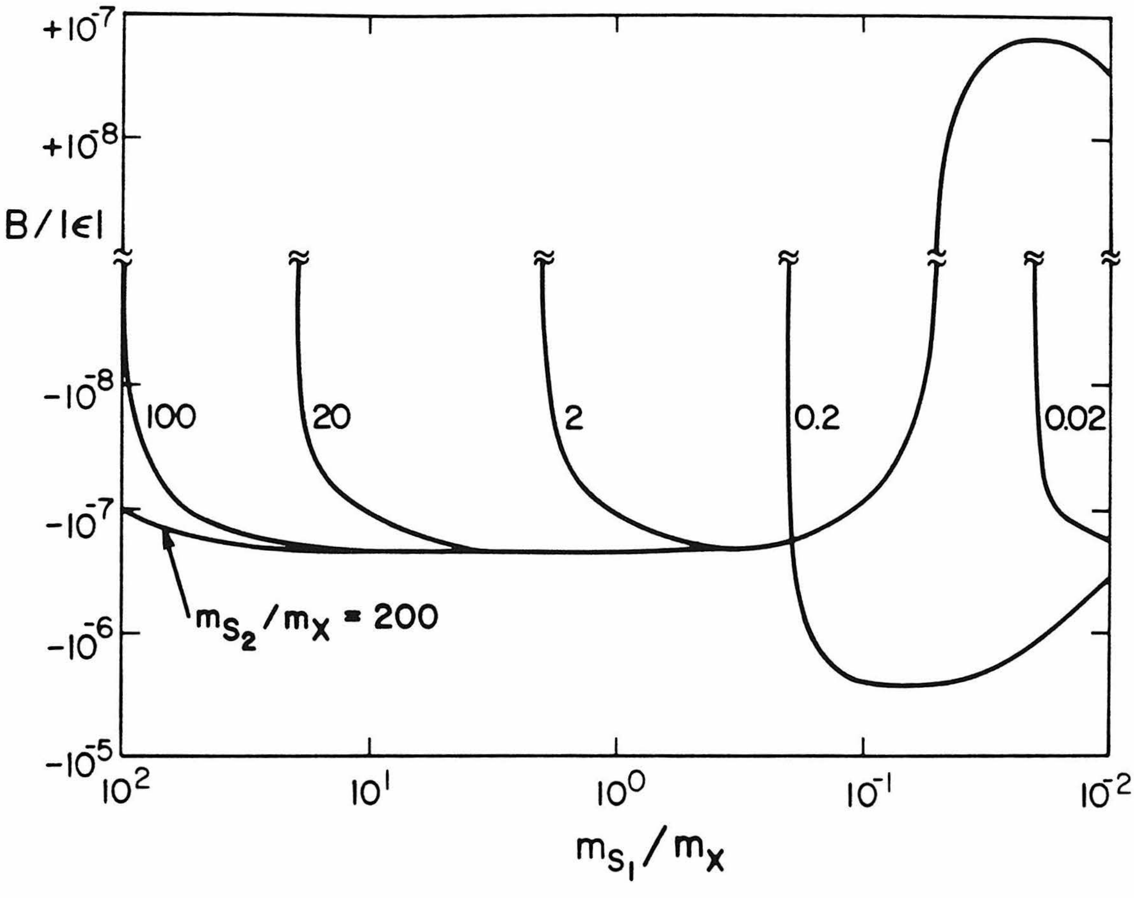

Figure 4.2: Baryon number density for an SU(5) model with two baryon number violating Higgs bosons (S_1, S_2) as a function of the S_1 mass for different choices of the S_2 mass. The results are for $\alpha = 1/40$ and $M_X = 5 \times 10^{14}$ GeV. The CP violation parameter ε is unknown but less than 1.

the final B is negative and determined by vector thermalization of the positive Π produced in S_2 decay. For $m_{S_2}>m_X$ but $m_{S_1}<0.1m_X$, the final baryon number is positive and determined mainly by inverse decays into S_1. The dominant term governing the time evolution of B for $T\gtrsim m_{S_1}$ is $\dot{B}\propto S_{1+}^{eq}<\Gamma_{S_1}>(14\nu_- -12B+7\Pi)$ with similar equations for $\dot{\nu}_-$ and $\dot{\Pi}_-$. Since $\Pi>0$, $\Pi>\nu_-$ and $\Pi>B$, this term tends to drive B positive. In general there are three linear combinations of B, ν_- and Π which decrease as pure exponentials until cut off at temperatures below m_{S_1}. B is a linear combination of these three exponentials, and its final value depends sensitively on the initial values of Π, ν_- and B. For this reason, it is inadequate to assume that B is produced and damped in successive independent stages as in simple models which treat only one quantum number [2,12]. For both $m_{S_2}<m_X$ and $m_{S_1}<m_X$ inverse decays into S_1 are no longer able to change the sign of the negative B produced through S_2 decays and hence the final B produced is negative.

The results for model III are complicated by the presence of additional sources of CP violation but are qualitatively similar to those of model II. The possibility of changes in the sign of B associated with detailed features of the boson spectrum indicates that no generic relation may be found between the definition of "matter" as given for the $K^0-\bar{K}^0$ system and that determined from the cosmological baryon number asymmetry.

5. Baryon Number Generation in SO(10) models

5.1 General Features

Although SU(5) grand unified theories contain the fewest fundamental fields, they exhibit a number of perhaps undesirable features which may be avoided in models based on larger gauge groups. One of these features is the assignment of fermions to the reducible representation $\bar{5}\oplus 10$. As a result of this assignment, some of the particles belong to different irreducible representations than their antiparticles. Also, although the anomalies cancel between the $\bar{5}$ and 10 representations of fermions, this cancellation appears rather artificial from the standpoint of SU(5). In addition, SU(5) models contain a global quantum number corresponding to the baryon number minus the lepton number, $B-L$. These features may be removed by embedding the SU(5) theory into a SO(10) theory with the fermions assigned to the spinor representation, 16 [13]. The explicit forms of the SO(10) representations and couplings discussed below may be found in Appendix B.

To elicit the structure of SO(10) it is useful to decompose SO(10) with respect to SU(5):

$$SO(10)\supset SU(5)\otimes U(1) \tag{5.1.1}$$

while the chiral structure is most easily seen by using the decomposition

$$SO(10)\supset SU(4)\otimes SU(2)_L\otimes SU(2)_R \tag{5.1.2}$$

where SU(4) is the Pati-Salam generalized color group [14],

$$SU(4)\supset SU(3)_c\otimes U(1). \tag{5.1.3}$$

The U(1) factor in this embedding is proportional to $B-L$ so that the global $B-L$ symmetry present in SU(5) models is gauged in SO(10) models. SO(10) is automatically anomaly-free as are all SO(n) groups for n>6. This is due to the fact that the symmetric product of the adjoint representation with itself does not contain the adjoint and hence the d-coefficients vanish.

In discussing the decomposition of SO(10) representations we will use the following notation: representations of $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ will be enclosed in round brackets: (m,n_L,n_R) with m, n_L, and n_R indicating the representations of $SU(4)$, $SU(2)_L$, and $SU(2)_R$ respectively. Representations of $SU(4)\otimes SU(2)_L\otimes U(1)_R$ will be enclosed in angle brackets: $<m,n_L,$ $Q_R>$ with the last entry indicating the value of the $U(1)_R$ charge operator, T_{3R}, normalized to be $\pm 1/2$ when acting on an $SU(2)_R$ doublet. Finally, representations of $SU(3)\otimes SU(2)_L\otimes U(1)_Y$ will be enclosed in square brackets: $[m,n_L,Q_Y]$ with the last entry indicating the value of the hypercharge, Y, normalized so that the electric charge operator is given by $Q=T_{3L}-Y$. Y may also be written in terms of T_{3R} and $B-L$, $Y = -\frac{1}{2}(B-L) - T_{3R}$.

In SO(10) models each family of left-handed fermions is assigned to the 16-dimensional complex spinor representation of SO(10) with the right-handed CP conjugate states assigned to the $\overline{16}$. The 16 has the chiral decomposition

$$16 = (4,2,1) + (\bar{4},1,2) \tag{5.1.4}$$

while with respect to $SU(3)\otimes SU(2)_L\otimes U(1)_Y$ we have

$$16 = [\bar{3},1,2/3] + [3,2,-1/6] + [\bar{3},1,-1/3] + [1,2,1/2] + [1,1,-1] + [1,1,0]. \tag{5.1.5}$$

It is evident that the 16 contains the usual quark and lepton fields per family: $U_L^c, U_L, D_L, D_L^c, E_L, \nu_L, E_L^c$; as well as an additional field which is neutral with respect to $SU(3) \otimes SU(2)_L \otimes U(1)_Y$. This may also be seen from the SU(5) decomposition

$$16 = 10 + \bar{5} + 1. \tag{5.1.6}$$

We will denote this extra field by N_L^c. As can be seen from the decomposition (5.1.4) it provides a charge conjugate partner for the left-handed neutrino and thus allows a $\Delta I_W = 1/2$ Dirac mass term for the neutrino. The potential disaster of neutrino masses of order the $\Delta I_W = 1/2$ breaking may be avoided if the N_L^c acquires a very large $\Delta I_W = 0$ Majorana mass, M_N [15]. The neutral lepton mass matrix will then have the form (assuming three families)

$$\begin{bmatrix} 0 & m_q \\ m_q^T & M_N \end{bmatrix} \tag{5.1.7}$$

with m_q a three by three matrix with entries of order the observed quark masses and M_N the three by three Majorana mass matrix for the N_L^c. For $M_N \gg m_q$ the six eigenvalues of this matrix are given approximately by the three eigenvalues of M_N, which are the masses of the N_L^c; and the three eigenvalues of the matrix $m_q^T M_N^{-1} m_q$, which are the light neutrino masses. As a result of this mechanism, SO(10) models naturally predict the existence of neutrino masses and hence neutrino oscillations.

The vector bosons transform as the 45-dimensional adjoint representation of SO(10) with the chiral decomposition

$$45_V = (6,2,2) + (15,1,1) + (1,3,1) + (1,1,3) \tag{5.1.8}$$

The last three representations correspond to the gauge bosons of $SU(4)$, $SU(2)_L$, and $SU(2)_R$ respectively. The (6,2,2) contains the usual leptoquark-diquarks (X,Y) of SU(5) transforming as [3,2,5/6] under $SU(3)\otimes SU(2)_L\otimes U(1)_Y$, their antiparticles; and an additional doublet of leptoquark-diquarks, (X',Y'), transforming as [3,2,-1/6], and their antiparticles. The gauge bosons of SU(4) contain the gluons of $SU(3)_c$ and an additional color triplet field transforming as [3,1,2/3], which we denote by V. The gauge bosons of $SU(2)_R$ transform as [1,1,-1], [1,1,0], and [1,1,1] and will be denoted by W, W^0, and $\overline{W}$ respectively when no confusion with the gauge bosons of $SU(2)_L$ is possible.

The Higgs content of SO(10) models is dictated by the need to break SO(10) down to $SU(3)_c\otimes U(1)_Q$ and by the desire to obtain the observed masses and mixing angles of the fermions. Higgs fields which can couple to fermions appear in the product

$$16\otimes 16 = (10 + 126)_S + (120)_A \tag{5.1.9}$$

where S (A) indicates that the representation appears in the symmetric (antisymmetric) product. These representations have the chiral decompositions

$$10_H = (6,1,1) + (1,2,2) \tag{5.1.10}$$

$$120_H = (15,2,2) + (6,3,1) + (6,1,3) + (\overline{10},1,1) + (10,1,1) + (1,2,2)$$

and

$$126_H = (15,2,2) + (10,3,1) + (\overline{10},1,3) + (6,1,1).$$

The 10_H contains the weak doublet, φ, plus its antiparticle, as well as a B-violating scalar transforming as [3,1,1/3] , which we denote by S, and the antiparticle of the S. If only the 10_H contributes to fermion masses then the tree level mass relations

$$m_u = m_d = m_e \tag{5.1.11}$$

hold at the unification scale for each family. Attempts to fit the observed mass spectrum more accurately generally lead to models with a rather baroque Higgs sector [16,17]. The couplings of the fields in the 120_H and 126_H will be discussed in Sections 5.2 and 5.3.

Higgs fields which do not couple to fermions are also required in SO(10) models in order to achieve the desired breaking. Representations commonly used are a 45 with the decomposition given in (5.1.8), and a 54 with the chiral decomposition

$$54_H = (20,1,1) + (6,2,2) + (3,3,1) + (1,1,1). \tag{5.1.12}$$

In contrast to SU(5) models, SO(10) models allow the possibility of intermediate mass scales in the region between 300 GeV and 10^{15} GeV. If SO(10) breaks first to $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ or to $SU(4)\otimes SU(2)_L\otimes U(1)_R$ before breaking to $SU(3)\otimes SU(2)_L\otimes U(1)_Y$ then fits to the weak mixing angle suggest that there may exist mass scales as low as 10^{10} GeV. Let M_V and M_S be the masses of typical B- violating vectors and scalars respectively and define M_R to be the scale at which $SU(2)_R$ breaks to $U(1)_R$ and M_c to be the scale at which $SU(4)$ breaks to $SU(3)_c\otimes U(1)$. The nonobservation of proton decay requires that $M_V \gtrsim 4\times10^{14}$ GeV and $M_S \gtrsim 2\times10^{12}$ GeV. The experimental constraints on M_R and M_c are much less stringent. Nonobservation of muon and electron number violating decays such as $K^0\rightarrow\mu^-e^+$ gives a lower bound of about 10^4 GeV for M_c while limits on the strength of right-handed weak currents require only that $M_R \gtrsim 200-300$ GeV. Theoretical fits to α_s and the weak mixing angle give a minimum value for M_c of about 10^{10} GeV with typical values being $\sim10^{12}$ GeV for SO(10) broken first to $SU(4)\otimes SU(2)_L\otimes U(1)_R$. If SO(10) breaks first to $SU(4)\otimes SU(2)_L\otimes SU(2)_R$

then typical values are $M_c \sim 10^{12}$ GeV and $M_R \sim 10^{10}$ GeV.

The production of a net B in SO(10) models may thus take place at a temperature at which the unbroken symmetry is larger than $SU(3) \otimes SU(2)_L \otimes U(1)_Y$. In the following sections we discuss the production of baryon number in models with $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, $SU(4) \otimes SU(2)_L \otimes U(1)_R$, and $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ unbroken gauge symmetries at the temperatures relevant to baryon number production. Since the generator of $U(1)_R$ is proportional to $B-L$, the first two of these possibilities forbid a Majorana mass for the N_L^c. In these cases the N_L^c may carry conserved quantum numbers. In particular, it must be assigned $B-L$ value +1. With $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ effective symmetry the presence of an unbroken charge conjugation symmetry forbids the production of a net baryon number. As discussed in Section 3, this charge conjugation symmetry constrains the temperatures to which $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ symmetry may persist if the theory is to produce an adequate baryon number.

Finally, we mention that use of a 126_H opens up the possibility of spontaneous CP violation at temperatures comparable to the unification mass [17]. This is caused by a phase in the vacuum expectation value of the 126_H due to terms in the Higgs potential of the form

$$\lambda((126_H)^4 + (\overline{126_H})^4) \qquad (5.1.13)$$

The relevance of this mechanism for B production is discussed in [3].

5.2. B, B-L Violation

In this section we extend the analysis of Sect. 2 to include an SU(5) singlet fermion $N_L^c([1,1,0])$. We first suppose that the N_L^c has a large Majorana mass so that it may carry no quantum numbers as would be the case if the effective symmetry were $SU(3) \otimes SU(2)_L \otimes U(1)_Y$. The additional vector and scalar products of fermion fields not appearing in Tables 2.2 and 2.3 are given in Table 5.1 along with their values of B and L. There are, of course, no new B-violating bosons. However, comparison of Tables 2.1, 2.2, and 5.1 shows that the X' vectors and S scalars are now capable of violating $B-L$ due to their interactions with N_L^c. In addition, there are $(B-L)$-violating vectors transforming as [3, 1, -2/3] and [1, 1, 1] which are gauge fields for the $SU(4)$ and $SU(2)_R$ subgroups of $SO(10)$. The additional $(B-L)$-violating scalars transform as [1, 2, 1/2] (the ordinary Higgs doublet of $SU(2)_L \otimes U(1)_Y$), [3, 2, -1/6] and [1, 1, 1]. These scalars appear in the following SO(10) representations which may couple to fermions:

$$[1,2,1/2] \supset 10,\ 120,\ 126 \tag{5.2.1}$$

$$[3,2,-1/6] \supset 126$$

$$[1,1,1] \supset 120,\ 126.$$

If the effective symmetry is $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ or $SU(4) \otimes SU(2)_L \otimes U(1)_R$ then a Majorana mass for the N_L^c is forbidden by the $SU(2)_R$ or $U(1)_R$ symmetry and this analysis must be modified. If the effective symmetry is $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, then as discussed in Section 5.3, the presence of an unbroken charge conjugation symmetry forbids the production of any baryon number. We thus consider B violation in a theory with effective $SU(4) \otimes SU(2)_L \otimes U(1)_R$ symmetry.

		[SU(3),SU(2),U(1)]	B	L	B-L
V_6, S_6	qN	[3,2,-1/6]	1/3	0	1/3
		[3,1,-2/3]			
		[3,1,1/3]			
V_7, S_7	lN	[1,2,1/2]	0	1	-1
		[1,1,1]			
V_8, S_8	NN	[1,1,0]	0	0	0

Table 5.1: Quantum numbers for possible vector and scalar fields which couple to singlet fermions N and either quarks q or leptons l.

As indicated by (5.1.4), the fermions then fall into three irreducible representations per family:

$$16_f = \langle 4,2,0\rangle + \langle \bar{4},1,1/2\rangle + \langle \bar{4},1,-1/2\rangle \tag{5.2.2}$$

which contain the fermion fields in the form

$$\langle 4,2,0\rangle = (\vec{U}_L, \nu_L, \vec{D}_L, E_L) \tag{5.2.3}$$

$$\langle \bar{4},1,1/2\rangle = (\vec{D}^c_L, E^c_L)$$

$$\langle \bar{4},1,-1/2\rangle = (\vec{U}^c_L, N^c_L)$$

with the arrow indicating a color triplet. The unbroken gauge symmetry guarantees the equality of number density asymmetries of members of a given irreducible multiplet. For a given color of each quark we thus have the relations

$$U_{L-} = \nu_{L-} = E_{L-} = D_{L-} \equiv n_{D_L} - n_{D^c_R} \tag{5.2.4}$$

$$E^c_{L-} = D^c_{L-} \equiv n_{D^c_L} - n_{D_R}$$

$$N^c_{L-} = U^c_{L-} \equiv n_{U^c_L} - n_{U_R}$$

with n_i the number density per color of species i. Although $U_{L-} = D_{L-}$ as a consequence of the unbroken $SU(2)_L$ symmetry, we will write both U_{L-} and D_{L-} in order to make the charge conjugation symmetry discussed in the next section evident. Since an asymmetry in E^c_L requires an equal asymmetry in each color of D^c_L, etc., the baryon number is given by

$$B = D_{L-} + U_{L-} - D^c_{L-} - U^c_{L-} \tag{5.2.5}$$

Following the earlier discussion we find that the possible B-violating vectors transform as <6,2,1/2> and correspond to the (X,Y) and (X',Y') doublets discussed earlier. The possible B-violating scalars transform as <6,1,0>, which contains the S and its antiparticle; <6,1,1>, which contains the S_1 and its antiparticle as well as an additional field; and <6,3,0>, which contains the S_2 and its antiparticle. Scalar representations transforming as 10 under $SU(4)$ contain B-violating scalars after breaking to $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ has taken place but these representations may not violate B at the level of $SU(4) \otimes SU(2)_L \otimes U(1)_R$.

In considering the production of baryon number in this model it is useful to define a new quantum number, Θ, defined to be

$$\Theta = D_{L-} + U_{L-} + D^c_{L-} + U^c_{L-} \tag{5.2.6}$$

Θ is the total asymmetry in the left-handed fermion fields. The chiral structure of vector couplings requires that Θ be conserved by all vector interactions. It may, however, be violated by Higgs scalar interactions and is thus analagous to the Π quantum number introduced for the $SU(5)$ models in Sect. 4

5.3. C, CP Violation

The generation of a net baryon number from symmetrical initial conditions requires the presence of both C and CP violation. In $SU(2)_L \otimes U(1)_Y$ weak interaction models and SU(5) grand unified models no C operator may be defined since there is no left-handed antineutrino to form the charge conjugate partner of the left-handed neutrino. In larger models, such as SO(10) or E(6), each fermion has a potential charge conjugate partner or is an eigenstate of C and a C operation may be defined which is a symmetry of the unbroken theory [18]. The production of a C-odd quantum number (such as B or L) in these models therefore depends on the interplay between the sources of C violation and the processes which violate the quantum number under consideration.

The lack of B production in a C-symmetric theory may be seen by considering the decays of B-violating bosons χ and their antiparticles $\bar{\chi}$ as well as the decays of their charge conjugate partners χ^c and $\overline{\chi^c}$. The B produced by the decays of an equal mixture of χ and $\bar{\chi}$ into the specific final state $i_1 i_2$ and the charge conjugate decays of χ^c and $\overline{\chi^c}$ into the state $i_1^c\ i_2^c$ is proportional to the quantity (see eqn. 3.1.12)

$$R_\chi^{12} + (R_\chi^{12})^c = ImI\ \mathrm{Im}\Omega(B_{i_2} - B_{i_1}) + ImI^c\ \mathrm{Im}\Omega^c(B_{i_2^c} - B_{i_1^c}). \qquad (5.3.1)$$

I represents an integral over the intermediate momenta and final state phase space for the decay and Ω is a product of the relevant couplings. The lowest order contributions to I and Ω are discussed in Section 3. I^c and Ω^c are the corresponding quantities for the charge conjugate reaction. In a C-symmetric theory, $I = I^c$ and $\Omega = \Omega^c$, while since B is C-odd, $B_{i_2} = -B_{i_2^c}$ and $B_{i_1} = -B_{i_1^c}$ causing $R_\chi^{12} + (R_\chi^{12})^c$ to vanish.

We now restrict our attention to SO(10) grand unified models. The presence of a C partner for the neutrino, $N_L^c \equiv \nu_L^c$, allows the definition of a C operation for all left-handed fermion fields appearing in the theory. In terms of the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ subgroup of SO(10), C interchanges the two SU(2)'s, as well as conjugating them, and also complex conjugates the representations of SU(4). C invariance may be broken either by the presence of different masses for the ν_L and N_L^c or through mass splittings between bosons and their charge conjugate partners. It may be shown that all C violation in the fermion mass matrix must lie in the part of the 126 representation of SO(10) which gives a huge Majorana mass to N_L^c [18]. This C-violating mass term allows for the production of a nonzero B even if the decaying boson is degenerate in mass with its C partner since $\mathrm{Im}I$ is no longer equal to ImI^c . Expanding I and I^c in powers of M_N / M_X gives

$$R_X^{12} + (R_X^{12})^c = O((M_N / M_X)^2) \tag{5.3.2}$$

where M_N is the Majorana mass of N_L^c and M_X is the mass of the decaying boson.

If all asymmetries can be expressed in terms of C-odd quantum numbers then (5.3.2) constrains the possible values of M_N / M_X if we demand that the theory be able to produce the observed baryon asymmetry [19]. However, in the general case, asymmetries which have no definite behavior under C must be considered. Examples are ν_- and Π in the case of SO(10) broken to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ (see Sec. 5.4) or Θ in the case of SO(10) broken to $SU(4) \otimes SU(2)_L \times U(1)_R$ (see Sections 5.3 and 5.5). Large asymmetries in such quantum numbers may be produced even if the theory is in a C-conserving phase (e.g. SO(10) broken to

$SU(4)\otimes SU(2)_L\otimes SU(2)_R$). These asymmetries may later be converted into a baryon asymmetry by B-violating reactions which occur in a C-violating phase of the theory. These reactions will be able to produce a sufficient baryon asymmetry only if there exist B- violating bosons with masses less than the transition temperature between the C-conserving and C-violating phases of the theory. For $SO(10)\rightarrow SU(4)\otimes SU(2)_L\otimes SU(2)_R$ the $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ symmetry must not persist to temperatures below $\sim 10^{12}$ GeV if an adequate B is to be produced.

C may also be violated in phases in which the N_L^c is effectively massless through C-violating boson mass splittings. An example of this is discussed in Sec. 5.5 where B generation is considered in an SO(10) model broken to $SU(4)\otimes SU(2)_L \otimes U(1)_R$. In order for a nonzero B to be produced in such models, the bosons with masses different from their charge conjugate partners must also be B-violating. These bosons may then convert asymmetries in non-C-odd quantum numbers, such as Θ, into asymmetries in B at a rate proportional to their mass splitting. In the case of SO(10) broken to $SU(4)\otimes SU(2)_L\otimes U(1)_R$, this requirement places nontrivial constraints on the Higgs content of the model as we now discuss.

Under the embedding $SO(10)\supset SU(4)\otimes SU(2)_L\otimes SU(2)_R$ the 45_V adjoint representation of gauge vector bosons has the branching given in (5.1.8). The color triplet $SU(2)_L$ doublet B-violating bosons (X,Y) and (X',Y') and their antiparticles combine to form the (6,2,2) representation. With our conventions the (X,Y) have electric charge (-4/3,-1/3) and the (X',Y') have electric charge (-1/3,2/3). Charge conjugation takes $X\rightarrow\overline{X}$, $Y\rightarrow\overline{X'}$, $X'\rightarrow\overline{Y}$, and $Y'\rightarrow\overline{Y'}$. B production through vector boson reactions therefore requires a mass splitting between the (X,Y) and (X',Y') doublets. This will in general be the case if SO(10) is broken to $SU(3)\otimes SU(2)_L\otimes U(1)_Y$.

However, if SO(10) is broken only to $SU(4)\otimes SU(2)_L\otimes U(1)_R$, then the (6,2,2) splits into <6,2,1/2> and the CP-conjugate state <6,2,-1/2> and as a result there is no mass splitting. The B-violating vector bosons will therefore be unable to produce a net B in their decays or to convert an asymmetry in Θ into an asymmetry in B.

The Higgs representations which may couple to fermions form the representations of $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ given in (5.1.10). With $SU(2)_R$ broken to $U(1)_R$ the bosons which can have masses different from their C partners are the (6,3,1) and (6,1,3) appearing in the 120_H, and the (10,3,1) and $(\overline{10},1,3)$ appearing in the 126_H. The usual color triplet, B-violating boson S appears along with its antiparticle in the (6,1,1) representation and is thus an eigenstate of C (under C, $S\to\overline{S}$). The (10,3,1) contains a boson which may violate B if the effective symmetry is $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$. However, as discussed previously, these bosons do not violate B if the effective symmetry is $SU(4)\otimes SU(2)_L\otimes U(1)_R$. The only Higgs bosons which may violate B and have masses different than their charge conjugate partners are the (6,3,1) and (6,1,3) which occur in the 120_H. With $SU(2)_R$ broken to $U(1)_R$ these fields break up into <6,1,±1>, which we denote by $\widetilde{S}_1$; <6,1,0>, which we denote by $\widetilde{S}$; and <6,3,0>, which we denote by $\widetilde{S}^c$. Note that for these fields to be present there must exist more than one family of fermions since the 120_H couples to the antisymmetric product of $16_f\otimes 16_f$.

We now discuss the possible CP-violating decays for SO(10) models broken to $SU(3)\otimes SU(2)_L\otimes U(1)_Y$ or to $SU(4)\otimes SU(2)_L\otimes U(1)_R$. If the effective symmetry is $SU(3)\otimes SU(2)_L\otimes U(1)_Y$, then the N_L^c has a large Majorana mass. If this mass is significantly less than the mass of the decaying boson, χ, then the quantity R_χ^B governing the production of B through χ decays will be negligible due to (5.3.2).If transitions between N_L^c and N_R due to the Majorana mass term occur at a faster rate than other transitions involving the N_L^c then the N_L^c will have an effective lepton number of zero and R_χ^{B-L} will be non-zero.In addition to B and $B-L$, it is convenient to consider the quantum numbers Π and ν_- defined in Section 6. R_χ^{Π} and $R_\chi^{\nu_-}$ are not constrained by the charge conjugation symmetry and will in general be non-zero. The R_χ will in general recieve contributions from scalar exchange in vector decay and vice versa and also from scalar exchange in scalar decay and vector exchange in vector decay in a generic SO(10) model. For simplicity we have included only the contributions from scalar exchange in scalar decay in the calculations of Section 5.4. The inclusion of other sources of CP violation increases the complexity of the results, but introduces no qualitatively new results.

5.4 B Generation for $SO(10) \to SU(3) \otimes SU(2)_L \otimes U(1)_Y$

In this section we describe the calculation of baryon number generation in SO(10) models where $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ is the effective gauge symmetry at temperatures relevant to baryon number production. We assume that all B production occurs in this phase but this need not be the case. If SO(10) breaks first to a larger group and then to $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ at a temperature at which B violating processes are still important, then the equations presented here may be used with the proper initial conditions to track the subsequent production and thermalization of the baryon number. If SO(10) breaks first to SU(5) and then to $SU(3) \otimes SU(2)_L \otimes U(1)_Y$, then fits to the weak mixing angle suggest that $M_X \cong 0.5\ \Pi eV \equiv 5 \times 10^{14}\ GeV$ but do not constrain the values of $M_{X'}, M_{W_R}$ or M_V. Below we will usually choose the values $M_{W_R} = M_V = M_{X'} = 10\ \Pi eV$.

N decay is potentially an important mechanism for production of B, L, etc. in these models. The N have two distinct types of decay modes. The first are two-body decays

$$N \to e\varphi,\ \bar{e}\bar{\varphi} \tag{5.4.1}$$

with $e = \begin{pmatrix} \nu \\ e \end{pmatrix}$, $\varphi = \begin{pmatrix} \varphi^0 \\ \varphi^+ \end{pmatrix}$ where φ is the usual $SU(2)_L \otimes U(1)_Y$ weak doublet. The width for this decay is

$$\Gamma_{N \to e\varphi} \simeq \alpha (M_q / M_W)^2 M_N \tag{5.4.2}$$

where M_q is the mass of the relevant charge 2/3 quark and M_W is the mass of the usual weak boson. The N may also undergo three body decays mediated by exchange of a supermassive boson, Ξ, in SO(10) but not in SU(5)

$$N \to qqq,\, q\bar{q}l,\, \cdots \qquad (5.4.3)$$

with q a quark and l a lepton. These decays have typical widths given in analogy to μ decay by

$$\Gamma_{N \to qqq} \sim \alpha^2 \frac{m_N^5}{384\pi} m_{\Xi}^{4}. \qquad (5.4.4)$$

As long as $m_N \lesssim m_{\Xi}$ these decays will be completely swamped by the decays $N \to e\varphi,\, \bar{e}\bar{\varphi}$. Since the decays given by (5.4.1) violate L but not B, N decay may contribute to the production of a net L but will be completely ineffective in producing baryon number. In models containing SU(5) singlet fermions where the decays given by (5.4.3) dominate, N decay may produce a net B and may also produce entropy which will tend to dilute any existing B. The requirement that these decays should not generate excessive entropy provides constraints on the masses and lifetimes of such fermions [20].

We shall consider a definite but presumably typical SO(10) model in which two 10_H couple to fermions. The mass eigenstate B-violating Higgs bosons will be denoted by S and S'. We include CP violation only for exchanges of S in S' decay and vice-versa.

For comparison with the SU(5) results, we choose to track the quantum numbers B, $B-L$, Π and ν_- with Π defined to be -1 for fermions in the $\bar{5}$ representation of SU(5) (D_L^c, E_L, ν_L) and zero for all other fermion fields and ν_- defined to be +1 for ν_L and zero for all other fermion fields. Π is conserved by the gauge bosons of SU(5) since they couple only to $\bar{5}_f \otimes 5_f$ and $\overline{10}_f \otimes 10_f$, but will be violated by the gauge bosons of SO(10) not in SU(5). The quantum number assignments for the various fields are given in Table 5.2. $SU(2)_L$ invariance requires that $X'_- = Y'_-$, $X_- = Y_-$ and

Field	$SU(3)\otimes SU(2)_L\otimes U(1)_Y$	B	B-L	Π	E
U_L^c	$[\bar{3},1,2/3]$	-1/3	-1/3	0	0
D_L, U_L	$[3,2,-1/6]$	1/3	1/3	0	0
D_L^c	$[\bar{3},1,-1/3]$	-1/3	-1/3	-1	0
E_L, ν_L	$[1,2,1/2]$	0	-1	-1	1
N_L^c	$[1,1,0]$	0	0	0	0
X, Y	$[3,2,-5/6]$	-	-2/3	0	-
X', Y'	$[3,2,-1/6]$	-	-	-	-
V	$[3,1,-2/3]$	1/3	-	-	-
W_R	$[1,1,-1]$	0	-	-	0
S	$[3,1,1/3]$	-	-	-	-
φ	$[1,2,-1/2]$	0	-	-	-

Table 5.2: Quantum numbers for fields relevant to baryon number production for $SO(10)\rightarrow SU(3)\otimes SU(2)_L\otimes U(1)_Y$.

$D_{L-} = U_{L-}$, in what follows we write only X and X' and treat $SU(2)_L$ as an additional degeneracy for the vector bosons.

The quantum number densities divided by the photon number density are given in terms of the particle number asymmetries by

$$B-L = D_{L-} + U_{L-} - U^c_{L-} - D^c_{L-} + E^c_{L-} - E_{L-} - \nu_{L-} - 4X_- \tag{5.4.5}$$

$$B = D_{L-} + U_{L-} - U^c_{L-} - D^c_{L-} + V_- \tag{5.4.6}$$

$$\Pi = -3D^c_{L-} - E_{L-} - \nu_{L-} \tag{5.4.7}$$

$$\nu_- = E_{L-} \tag{5.4.8}$$

We use the constraint that the total hypercharge be zero

$$Y = 0 = -\frac{1}{2}D_{L-} - \frac{1}{2}U_{L-} + 2U^c_{L-} - D^c_{L-} - E^c_{L-} + \frac{1}{2}E_L + \frac{1}{2}\nu_{L-} + 5X_- - X'_- \tag{5.4.9}$$

$$- 2V_- - W_- + S_- + S'_-$$

and $SU(2)_L$ invariance to reduce the number of quantities that we need consider. Solving for the fermion asymmetries in terms of the quantum numbers and the boson asymmetries gives

$$U_{L-} = D_{L-} = \frac{1}{3}[(B-L)+B-\Pi - \frac{1}{2}\nu_- - X_- + X'_- + V_- + W_- - S_- - S'_-] \tag{5.4.10}$$

$$U^c_{L-} = \frac{1}{3}[2(B-L)-B-\Pi-2X_-+2X'_-+5V_-+2W_--2S_--S'_-] \tag{5.4.11}$$

$$D^c_{L-} = \frac{1}{3}[-\Pi-\nu_-] \tag{5.4.12}$$

$$E^c_{L-} = (B-L) - B + \nu_- + 4X_- + V_- \tag{5.4.13}$$

$$\nu_{L-} = E_{L-} = \nu_-. \tag{5.4.14}$$

These relations along with the decay modes of the X, X', W_R, V, S, and S' bosons given in Table 5.3 give the following set of equations for the time rate of change of the number densities.

$$\dot{X}_+ = -\langle\Gamma_X\rangle(X_+ - X_+^{eq}) \tag{5.4.15}$$

$$\dot{X}_- = -\langle\Gamma_X\rangle[X_- - \frac{1}{8}X_+^{eq}(X'_- - 5X_- + W_- + 2V_- - S_- - S'_-)]$$

$$\dot{X}'_+ = -\langle\Gamma_{X'}\rangle(X'_+ - X'^{eq}_+)$$

$$\dot{X}'_- = -\langle\Gamma_{X'}\rangle[X'_- - \frac{1}{8}X'^{eq}_+(-X'_- + X_- - W_- - 2V_- + S_- + S'_- - (B-L))]$$

$$\dot{V}_+ = -\langle\Gamma_V\rangle(V_+ - V_+^{eq})$$

$$\dot{V}_- = -\langle\Gamma_V\rangle[V_- - \frac{1}{8}V_+^{eq}(4X_- + (B-L))]$$

$$\dot{W}_+ = -\langle\Gamma_W\rangle(W_+ - W_+^{eq})$$

$$\dot{W}_- = -\langle\Gamma_W\rangle[W_- - \frac{1}{8}W_+^{eq}(-2X'_- + 6X_- - 2W_- - 4V_- + 2S_- + 2S'_- - (B-L))]$$

$$\dot{S}_+ = -\langle\Gamma_S\rangle(S_+ - S_+^{eq})$$

$$\dot{S}_- = -\langle\Gamma_S\rangle[S_- - \frac{1}{8}S_+^{eq}(-2X_- - \frac{1}{2}(B-L))]$$

$$\dot{S}'_+ = -\langle\Gamma_{S'}\rangle(S'_+ - S'^{eq}_+)$$

$$\dot{S}'_- = -\langle\Gamma_{S'}\rangle[S'_- - \frac{1}{8}S'^{eq}_+(-2X_- - \frac{1}{2}(B-L))]$$

$$\dot{B} = \langle\Gamma_X\rangle[(X_+ - X_+^{eq})R_X^B - 2X_- + \frac{1}{2}X_+^{eq}(X'_- + 3X_- + W_- + 6V_- - S_- - S'_- + \nu_- + 2(B-L) - 4B)]$$

$$+ \langle\Gamma_{X'}\rangle[(X'_+ - X'^{eq}_+)R_{X'}^B - 2X'_- - \frac{1}{2}X'^{eq}_+(X'_- - X_- - S_- - S'_- + W_- + \nu_- + (B-L) + 2B)]$$

$$+ \langle\Gamma_S\rangle[(S_+ - S_+^{eq})R_S^B - S_- + \frac{1}{8}S_+^{eq}(4X_- + 6V_- + (B-L) - 6B)]$$

$$+ \langle\Gamma_{S'}\rangle[(S'_+ - S'^{eq}_+)R_{S'}^B - S'_- + \frac{1}{8}S'^{eq}_+(4X_- + 6V_- + (B-L) - 6B)]$$

$$+ 4n_b\langle|v|\sigma_X\rangle[X'_- + 11X_- + W_- + 10V_- - S_- - S'_- + 2\nu_- + 4(B-L) - 8B]$$

Boson	Decay Mode	Partial Width	B	B-L	Π	E
	$E_L D_R, \nu_L D_R$	1/4	1/3	-2/3	0	1
X, Y	$U_L^c U_R^c, U_L^c D_R^c$	1/2	-2/3	-2/3	0	0
	$D_L E_R, U_L E_R$	1/4	1/3	-2/3	0	0
	$E_L U_R, \nu_L U_R$	1/4	1/3	-2/3	-1	1
X', Y'	$D_L N_R, U_L N_R$	1/4	1/3	1/3	0	0
	$D_L^c U_R^c, D_L^c D_R^c$	1/2	-2/3	-2/3	-1	0
	$D_R E_L^c$	1/4	1/3	4/3	1	0
V	$\nu_R^c U_L, E_R^c D_L$	1/2	1/3	4/3	1	-1
	$U_R N_L^c$	1/4	1/3	1/3	0	0
W_R	$U_R D_L^c$	3/4	0	0	-1	0
	$N_R E_L^c$	1/4	0	1	0	0
	$D_L \nu_L, U_L E_L$	1/4	1/3	-2/3	-1	1
	$E_R U_R$	1/8	1/3	-2/3	0	0
S	$U_R^c D_R^c$	1/4	-2/3	-2/3	0	0
	$U_L^c D_L^c$	1/4	-2/3	-2/3	-1	0
	$D_R N_R$	1/8	1/3	1/3	1	0

Table 5.3 Quantum numbers and partial widths for supermassive boson decay modes

$$- 4n_b <|\nu|\sigma'_{X'}>[X'_- - X_- - S_- - S'_- + W_- - 2V_- + 2\nu_- + (B-L) + 4B]$$

$$+ 12h^4 n_b <|\nu|\sigma'_{S+S'}>[4X_- + 4V_- + (B-L) - 4B]$$

$$(\dot{B}-\dot{L}) = <\Gamma_{X'}>[(X'_+ + X'^{eq}_+)R^{B-L}_{X'} - 5X'_- - \frac{1}{2}X'^{eq}_+(3X'_- - 3X_- - 3S_- - 3S'_-$$

$$+3W_- + 5V_- - \Pi - \frac{1}{2}\nu_- + 3(B-L) + B)]$$

$$+ <\Gamma_V>[(V_+ - V^{eq}_+)R^{B-L}_V + \frac{13}{2}V_- - \frac{1}{4}V^{eq}_+(2X'_- + 14X_- + 2W_- + 5V_- - 2S_-$$

$$-2S'_- - \Pi + 6(B-L) - B)]$$

$$+ <\Gamma_X>[(W_+ - W^{eq}_+)R^{B-L}_V + \frac{1}{2}W_- - \frac{1}{4}W^{eq}_+(4X_- + V_- + \nu_- + (B-L) - B)]$$

$$+ <\Gamma_S>[(S_+ - S^{eq}_+)R^{B-L}_S - S_- + 3S^{eq}_+(4X_- + 6V_- + (B-L) - 6B)]$$

$$+ <\Gamma_{S'}>[(S'_+ - S'^{eq}_+)R^{B-L}_{S'} - S'_- + 3S'^{eq}_+(4X_- + 6V_- + (B-L) - 6B)]$$

$$- 2n_b <|\nu|\sigma'_{X'}>[7X'_- - 7X_- - 7S_- - 7S'_- + 7W_- + 10V_- - 4\Pi - 2\nu_- + 7(B-L) + 4B]$$

$$- n_b <|\nu|\sigma'_V>[8X'_- + 4X_- + 8W_- + 20V_- - 8S_- - 8S'_- - 4\Pi + 11(B-L) - 4B]$$

$$- 2n_b <|\nu|\sigma'_V>[2X'_- + 10X_- + 2W_- + 8V_- - 2S_- - 2S'_- + 4\nu_- + 5(B-L) - 4B]$$

$$- n_b h^4 <|\nu|\sigma'_{S+S'}>[12X_- + 8\Pi + 8\nu_- + 3(B-L)]$$

$$+ n_b h^2 <|\nu|\sigma_\varphi>\nu_-$$

$$\dot{\nu}_- = <\Gamma_X>[(X_+ - X^{eq}_+)R^{\nu_-}_X + 3X_- - \frac{1}{4}X^{eq}_+(2\Pi + 5\nu_-)]$$

$$+ <\Gamma_{X'}>[(X'_+ + -X'^{eq}_+)R^{\nu_-}_{X'} + 3X'_- + \frac{1}{2}X'^{eq}_+\ (2X'_- - 2X_-$$

$$-2S_- - 2S'_- + 2W_- + 5V_- - \Pi - \frac{3}{2}\nu_- + 2(B-L) - B)]$$

$$+ <\Gamma_V>[(V_+ - V^{eq}_+)R^{\nu_-}_V + \frac{3}{2}V_- + \frac{1}{2}V^{eq}_+(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi - 2\nu_- + (B-L) + B)]$$

$$+ <\Gamma_S>(S_+ - S^{eq}_+)R^{\nu_-}_S + \frac{3}{2}S_- - \frac{1}{4}S^{eq}_+(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi + \nu_- + (B-L) + B)]$$

$$+ <\Gamma_{S'}>(S'_+ - S'^{eq}_+)R_{S'}^{\nu_-} + \frac{3}{2}S'_- - \frac{1}{4}S'^{eq}_+(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi + \nu_- + (B-L) + B)]$$

$$+ 2n_b <|\nu|\sigma'_X>[3X'_- - 15X_- + 3W_- + 6V_- - 3S_- - 3S'_- - 4\Pi - 10\nu_-]$$

$$+ 2n_b <|\nu|\sigma'_{X'}>[5X'_- - 5X_- - 5S_- - 5S'_- + 5W_- + 14V_- - 4\Pi - 6\nu_- + 5(B-L) - 4B]$$

$$+ 2n_b <|\nu|\sigma'_V>[4X'_- - 4X_- - 4S_- - 4S'_- + 4W_- + 7V_- - 4\Pi - 5\nu_- + 4(B-L) + B]$$

$$- 2n_b h^4 <|\nu|\sigma'_{S+S'}>[8X'_- + 4X_- + 8V_- - 8S_- - 8S'_- - 8\Pi + 8\nu_- + 11(B-L) + 8B]$$

$$- 2n_b h^2 <|\nu|\sigma_\varphi>[4\nu_- + (B-L) - B]$$

$$\dot{\Pi} = <\Gamma_X>[(X'_+ - X'^{eq}_+)R_X^\Pi - 9X'_- - \frac{1}{2}X'^{eq}_+(4X'_- - 4X_- - 4S_- - 4S'_-$$

$$+4W_- - 7V_- - \Pi - \frac{1}{2}\nu_- + 4(B-L) + B)]$$

$$+ <\Gamma_V>[(V_+ - V^{eq}_+)R_V^\Pi + \frac{9}{2}V_- - \frac{1}{4}V^{eq}_+(2X'_- + 10X_- + 2W_- + 5V_- - 2S_- - 2S'_- - \Pi + 5(B-L) - B)]$$

$$+ <\Gamma_W>[(W_+ - W^{eq}_+)R_W^\Pi - \frac{3}{2}W_- - \frac{1}{4}W^{eq}_+(2X'_- - 2X_- - 2S_- - 2S'_- + 2W_- + 5V_- + \nu_- + 2(B-L) - B)]$$

$$+ <\Gamma_S>[(S_+ - S^{eq}_+)R_S^\Pi - \frac{9}{4}S_- + \frac{1}{8}S^{eq}_+(6X'_- - 6X_- - 6S_- - 6S'_- + 6W_- + 12V_- - 7\Pi - \nu_- + 6(B-L))]$$

$$+ <\Gamma_{S'}>[(S'_+ - S'^{eq}_+)R_{S'}^\Pi - \frac{9}{4}S'_- + \frac{1}{8}S'^{eq}_+(6X'_- - 6X_- - 6S'_- + 6W_- + 12V_- - 7\Pi - \nu_- + 6(B-L))]$$

$$- 2n_b <|\nu|\sigma'_{X'}>[7X'_- - 7X_- - 7S_- - 7S'_- + 7W_- + 10V_- - 4\Pi - 2\nu_- + 7(B-L) + 4B]$$

$$- n_b <|\nu|\sigma'_V>[8X'_- + 4X_- + 8W_- + 20V_- - 8S_- - 8S'_- - 4\Pi + 11(B-L) - 4B]$$

$$- 2n_b <|\nu|\sigma'_W>[2X'_- + 10X_- + 2W_- + 8V_- - 2S_- - 2S'_- + 4\nu_- + 5(B-L) - 4B]$$

$$+ n_b h^4 <|\nu|\sigma'_{S+S'}>[32X'_- + 4X_- + 32W_- + 80V_- - 32S_- - 32S'_- - 40\Pi + 4(B-L) - 16B]$$

$$- 4n_b h^2 <|\nu|\sigma_\varphi>[\Pi - (B-L) - \frac{1}{2}\nu_-]$$

The total widths for vector and scalar decay are given by

$$\Gamma_V = \alpha M_V/3 \tag{5.4.16}$$

$$\Gamma_S = M_S h^2/2\pi$$

with $\alpha = g^2/4\pi \cong 1/40$ where g is the SO(10) coupling constant and h is the Yukawa coupling of the Higgs scalar to the heaviest family of fermions. For simplicity we take the Yukawa couplings of S and S' to be equal with

$$h = \frac{gM_F}{\sqrt{2}M_W} \tag{5.4.17}$$

where M_F is an effective quark mass of the heaviest family at the scale $\sim M_X$. The cross sections for two to two scattering processes not already included as successive inverse decay and decay processes are given by

$$|v|\sigma_V' = \pi\alpha^2\left[\frac{s}{M_V^2(s+M_V^2)} + \frac{1}{3}\,\frac{s(s-M_V^2)^2}{[(s-M_V^2)^2+M_V^2\Gamma_V^2]^2} + \frac{2}{s} + \frac{1}{M_V^2}\right. \tag{5.4.18}$$

$$\left. - 2\left(\frac{s+M_V^2}{s^2}\right)\log\left(\frac{s+M_V^2}{M_V^2}\right)\right]$$

$$|v|\sigma_{S+S'}' = |v|\sigma_S' + |v|\sigma_{S'}' + |v|\sigma_{int} \tag{5.4.19}$$

with

$$|v|\sigma_S' = \frac{1}{8\pi s^2}\left[s + M_S^2 - \frac{M_S^4}{(s+M_S^2)} - 2M_S^2\log\left(\frac{s+M_S^2}{M_S^2}\right)\right. \tag{5.4.20}$$

$$\left. + \frac{1}{2}\,\frac{s^3(s-M_S^2)^2}{[(s-M_S^2)^2\, M_S^2\Gamma_S^2]}\right]$$

and the interference term given by

$$|v|\sigma_{int} = \frac{1}{16\pi s^2}\left[s + \frac{M_S^4\log(\dfrac{s-M_S^2}{M_S^2}) - M_{S'}^4\log(\dfrac{s-M_{S'}^2}{M_{S'}^2})}{M_S^2 - M_{S'}^2}\right]. \tag{5.4.21}$$

$$|v|\sigma_\varphi = \frac{3\alpha h^2}{4s}. \tag{5.4.22}$$

In these formulae, subscripts indicate the type of exchanged boson, and $\sqrt{s}$ is the c.m.s. energy in the collision. The widths and cross-sections in (5.4.14) are averaged over an equilibrium Boltzmann distribution for the initial particles, as indicated by the angle brackets. The effects of screening on the two to two cross-sections due to a background gas are discussed in Section 4. Here we assume that the cross-sections have their free form. The effective CP violation parameters, R_χ^Q, are given by a sum of the decay modes for χ, weighted by the value of Q created in the decay and multiplied by an overall factor corresponding to the multiplicity of the decaying boson. This gives

$$R_X^B = 2R(X\to E_L D_R) - 4R(X\to U_L^c D_R^c) + 2R(X\to D_L E_R) \tag{5.4.23}$$

$$R_{X'}^B = 2R(X'\to E_L U_R) - 4R(X'\to D_L^c D_R^c) + 2R(X'\to D_L N_R)$$

$$R_S^B = 2R(S\to D_L \nu_L) + R(S\to E_R U_R) - 2R(S\to U_R^c D_R^c) - 2R(S\to U_L^c D_L^c) + R(S\to D_R N_R)$$

$$R_{X'}^{B-L} = -4R(X'\to E_L U_R) + 2R(X'\to D_L N_R) - 4R(X'\to D_L^c D_R^c)$$

$$R_V^{B-L} = 4R(V\to D_R E_L^c) + 4R(V\to E_R^c D_L) + R(V\to U_R N_L^c)$$

$$R_W^{B-L} = R(W\to N_R E_L^c)$$

$$R_S^{B-L} = -4R(S\to D_L \nu_L) - 2R(S\to E_R U_R) - 2R(S\to U_R^c D_R^c)$$

$$- 2R(S\to U_L^c D_L^c) + R(S\to D_R N_R)$$

$$R_X^{\nu-} = 6R(X\to E_L D_R)$$

$$R_{X'}^{\nu-} = 6R(X'\to E_L U_R)$$

$$R_V^{\nu-} = -3R(V\to E_R^c D_L)$$

$R_S^{\nu_-} = 6R(S \to D_L \nu_L)$

$R_{X'}^{\Pi} = -6R(X' \to E_L U_R) - 6R(X' \to D_L^c D_R^c)$

$R_V^{\Pi} = 3R(V \to D_R E_L^c) + 3R(V \to E_R^c D_L)$

$R_W^{\Pi} = -R(W \to U_R D_L^c)$

$R_S^{\Pi} = -6R(S \to D_L \nu_L) - 3R(S \to U_L^c D_L^c) + 3R(S \to D_R N_R)$

If we ignore the effect of the N_L^c mass then for a 10_H we have

$$R(S \to E_R U_R) = R(S \to D_R N_R) = \frac{1}{2} R(S \to U_R^c D_R^c) \tag{5.4.24}$$

$$= \frac{-1}{2} R(S \to U_L^c D_L^c) = -R(S \to D_L \nu_L)$$

so that R_S^B and $R_{S'}^B$ vanish in this limit as expected. Using (5.4.24.) we find

$$R_S^{B-L} = -3R(S \to D_L \nu_L)$$

$$R_S^F = 6R(S \to D_L \nu_L) \tag{5.4.25}$$

$$R_S^{\Pi} = -15R(S \to D_L \nu_L)$$

with the same relations holding for S replaced by S'. $R(S \to D_L \nu_L)$ and $R(S' \to D_L \nu_L)$ may be written in terms of an unknown CP violating phase ε using the results and notation of Sect. 3 as

$$R(S \to D_L \nu_L) = \frac{\pi\alpha}{2} \left(\frac{M_f}{M_W}\right) \varepsilon \mathrm{Im} J_{SS'} \tag{5.4.26}$$

$$R(S' \to D_L \nu_L) = -\frac{\pi\alpha}{2} \left(\frac{M_f}{M_W}\right) \varepsilon \mathrm{Im} J_{S'S}$$

where α is the gauge coupling constant at the unification scale, $\alpha \sim 1/40$,

$M_W \sim 80$ GeV is the mass of the weak gauge boson and M_f is the effective mass of the heaviest family at the unification scale. We take $M_f / M_W \cong 1/20$.

In Sect. 5.3 we showed that the $SO(10)$ model discussed here can generate directly only asymmetries in $B-L$, ν_- and Π; asymmetries in B may arise only indirectly through conversion of these quantum numbers by inverse decay and $2 \to 2$ scattering processes. Figure 5.1 shows the final baryon number and B-L generated in this model, together with the values of ν_- and Π obtained by neglecting low temperature light Higgs boson exchanges. The results assume $M_{S'} = 1\Pi eV$. For $M_S > M_{S'}$, $B-L, \Pi$, and E are produced dominantly through the CP-violating decays of the S with their signs and magnitudes determined by the relations (5.4.24). At $M_S = M_{S'}$ the contributions from S decay and S' decay exactly cancel and no asymmetries are produced. For $0.1\ \Pi eV \lesssim M_S \lesssim M_{S'}$, S' decays dominate and since $R(S \to D_L E_L)$ is opposite in sign to $R(S' \to D_L E_L)$ the values of the quantum numbers produced differ in sign from the case $M_S > M_{S'}$. For $M_S < 0.1 \Pi eV$, inverse decays into S tend to damp the asymmetries produced through S' decay. The final values of the quantum numbers in this case depend sensitively on the values initially produced through S' decay. A similar phenomenon was noted in Sect. 4.

For $M_S \gtrsim 5\Pi eV$, B production in Fig. 5.1 is dominated by inverse decay and $2 \to 2$ scattering processes mediated by X'. Inspection of the X' inverse decay terms in (5.4.15) (or of the decay modes given in Table 5.3) reveals that the combination of quantum numbers $(B-L)-B+\nu_-$ is conserved in these processes. Hence if only X' exchange occurred, the final equilibrium values of quantum numbers would be nonzero and given by

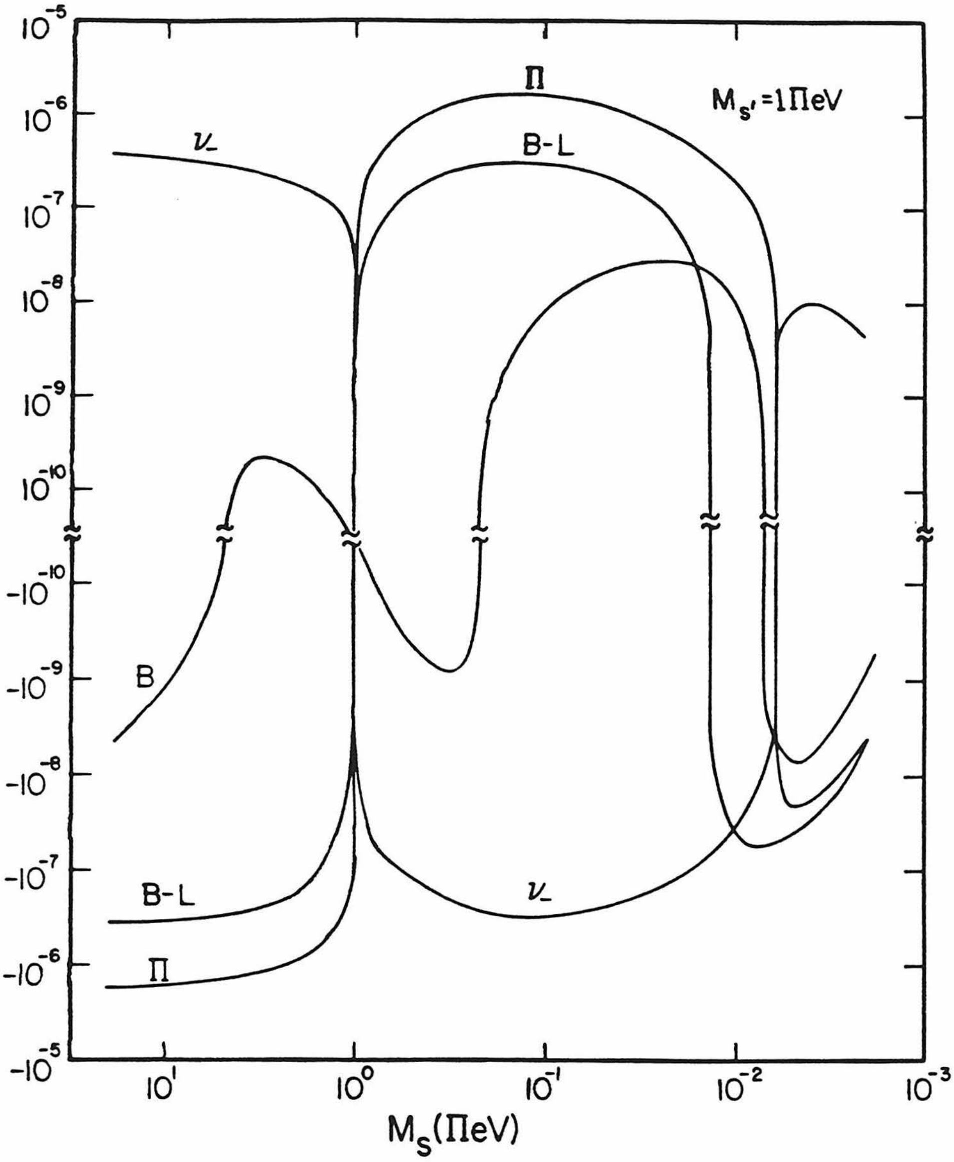

Figure 5.1: Final quantum number densities (scaled by the CP violation parameter ε) generated in an SO(10) model with no intermediate effective symmetry larger than $SU(3)_c \otimes SU(2)_L \otimes U(1)$. Results for Π and ν_- are obtained neglecting effects of light Higgs boson exchange at low temperatures. S and S' are mass eigenstate 10_H Higgs bosons.

$$B = \frac{\Pi_0}{2} \tag{5.4.27}$$

$$\nu_- = -\frac{\Pi_0}{2}$$

$$(B-L) = 0$$

where Π_0 is the initial value of Π generated through S decays. Since $\Pi_0<0$, the X' processes tend to produce a negative B. For $0.2\,\Pi eV < M_S < 5\,\Pi eV$, asymmetries are produced through S and S' decays at temperatures below the X' mass where X' reactions are negligible. In this case B is dominantly produced through processes involving the X boson. Conservation of Π in X reactions leads to the equilibrium values (c.f. Sect.4)

$$\nu_- = -\frac{1}{5}\Pi_0 \tag{5.4.28}$$

$$B = -\frac{1}{10}\Pi_0 + \frac{1}{2}(B-L)_0$$

Since $R_S^{\Pi} = 5R_S^{B-L}$ the contributions to B tend to cancel and the resulting B is small. The fact that the X and X' tend to produce B of the opposite sign is a consequence of charge conjugation symmetry. As discussed in Sect. 5.3, unbroken C invariance would yield $M_X{=}M_{X'}$ and would cause the contributions of X and X' to B production to cancel. For $M_S < 0.2\,\Pi eV$, B production is dominated by inverse decays into S. When M_S is sufficiently small, all asymmetries are reduced to zero.

If both the S and S' are sufficiently light then B may also be produced directly since in this case the cancellation due to the charge conjugation symmetry is less effective.

The results of Fig. 5.1 demonstrate that the model considered in this section can generate sufficient B to accord with present observations, even though no B is produced directly through CP-violating decays. The magnitude and sign of the resulting baryon number depend sensitively, however, on the Higgs structure and the masses of the B violating-bosons.

5.5 B production for $SO(10)\to SU(4)\otimes SU(2)_L\otimes U(1)_R$

As described in Section 5.3, the production of baryon number in a SO(10) model with $SU(4)\otimes SU(2)_L\otimes U(1)_R$ effective symmetry requires the presence of a 120_H with a C-violating mass splitting between two of its B-violating components. Since the 120_H cannot on its own account for observed fermion masses*, we include also a 10_H. We shall consider only those components in 120_H which may attain a C-violating mass splitting, and may thus contribute directly to B production.

The equations presented here may also be used to track the evolution of assymetries produced in earlier stages. In particular, with effective $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ symmetry no B may be produced due to the unbroken charge conjugation symmetry. This restriction does not apply to asymmetries in Θ. The equation used here may be used to consider the subsequent conversion of Θ to B when C is broken. With effective $SU(4)\otimes SU(2)_L\otimes U(1)_R$ symmetry, B may be produced directly through decays of $\tilde{S}$ and $\tilde{S}^c$. C symmetry implies that S decays may produce no net B (since $S\to S$ under C), while the B produced through $\tilde{S}$ decays must be opposite in sign to that produced in $\tilde{S}^c$ decays. To illustrate the conversion of Θ to B we will suppose that no B is produced directly through boson decays. This would be the case if assymetries are produced dominantly through S decays but thermalized by the $\tilde{S}$ and $\tilde{S}^c$ bosons.

The quantum number assignments for the various fields are given in Table 5.4. In this table a field stands for the asymmetry per member of an irreducible multiplet of $SU(4)\otimes SU(2)_L$. We will assume that the total

* Since the 120_H couples antisymmetrically to fermions, it must yield an antisymmetric fermion mass matrix with a zero eigenvalue for at least one out of three families.

Field	$SU(4)\otimes SU(2)_L\otimes U(1)_R$	B	Θ
D_L, U_L	<4,2,0>	1/4	1/4
D_L^c	<$\bar{4}$,1,1/2>	-1/4	1/4
U_L^c	<$\bar{4}$,1,1/2>	-1/4	1/4
X	<6,2,-1/2>	-	0
W_R	<1,1,1>	0	0
S	<6,1,0>	-	-
$\tilde{S}_1$	<6,1,1>	-	-
$\tilde{S}$	<6,1,0>	-	-
$\tilde{S}^c$	<6,3,0>	-	-
φ	<1,2,1/2>	0	-

Table 5.4: Quantum numbers for fields relevant to baryon number production for $SO(10)\to SU(4)\otimes SU(2)_L\otimes U(1)_Y$

We will take $\tilde{S}_1$ to be degenerate in mass with $\tilde{S}$ in what follows. Since B is determined by the mass splitting between $\tilde{S}$ and $\tilde{S}^c$ this choice should have little effect on the final results.

Figure 5.2(a) shows the final baryon number generated in this model, for a variety of values of M_S, $M_{\tilde{S}}$ and $M_{\tilde{S}^c}$. Figures 5.2(b) and (c) show the development of B and Θ in two characteristic cases, and indicate the dominant processes in each temperature range. An asymmetry in Θ is produced by S, $\tilde{S}$ and $\tilde{S}^c$ decays. An asymmetry in B must then be generated by conversion of this asymmetry. Only $\tilde{S}$ and $\tilde{S}^c$ interactions violate C and thus may contribute to B.

In Figure 5.2(b), $M_S > M_{\tilde{S}^c} > M_{\tilde{S}}$, so that $\tilde{S}^c$ inverse decays first convert the positive Θ produced in S decay into a negative B. As the temperature falls below the $\tilde{S}$ mass, inverse decays into $\tilde{S}$ dominate and B is driven positive. When Θ is driven negative by $\tilde{S}$ and $\tilde{S}^c$ decays the $\tilde{S}$ inverse decays drive B negative again yielding a negative final baryon number. For $M_{\tilde{S}^c} < M_{\tilde{S}}$, the roles of $\tilde{S}$ and $\tilde{S}^c$ are reversed and the final baryon number is positive.

Figure 5.2(c) shows the development of Θ and B when $M_{\tilde{S}} > M_S = M_{\tilde{S}^c}$. The final B produced is positive since $M_{\tilde{S}^c} < M_{\tilde{S}}$. The B produced in $\tilde{S}^c$ decays is reduced by S inverse decays. For $M_{S^c} < M_S$, B is produced after the effects of S inverse decays are important and as a result the final baryon number is large than in the previous case.

Although the sign and magnitude of the final B depend sensitively on the masses and couplings of the Higgs bosons, as long as there exists a 120_H with a small mass splitting between its $\tilde{S}$ and $\tilde{S}^c$ components, a baryon number compatible with present observations may be produced.

Boson	Decay Mode	Partial Width	B	Θ
X	$D_R^c\ U_L^c$	1/2	-1/2	0
	$D_R\ U_L$	1/2	1/2	0
W_R	$D_L^c\ U_R$	1	0	0
S	$D_L\ U_L$	1/4	1/2	1/2
	$D_L^c\ U_L^c$	1/4	-1/2	1/2
	$D_R^c\ U_R^c$	1/4	-1/2	-1/2
	$D_R\ U_R$	1/4	1/2	-1/2
$\tilde{S}_1$	$D_L^c\ D_L^c$	1/2	-1/2	1/2
	$U_R\ U_R$	1/2	1/2	-1/2
$\tilde{S}$	$D_L^c\ U_L^c$	1/2	-1/2	1/2
	$D_R\ U_R$	1/2	1/2	-1/2
$\tilde{S}^c$	$D_L\ D_L$	1/6	1/2	1/2
	$U_L\ U_L$	1/6	1/2	1/2
	$D_L\ U_L$	1/6	1/2	1/2
	$D_R^c\ D_R^c$	1/6	-1/2	-1/2
	$U_R^c\ U_R^c$	1/6	-1/2	-1/2
	$D_R^c\ U_R^c$	1/6	-1/2	-1/2

Table 5.5: Quantum numbers and partial decay widths for supermassive bosons for $SO(10) \rightarrow SU(4) \otimes SU(2)_L \otimes U(1)_R$.

$$R_S^{\Theta} = 3R(S \to D_L U_L) + 3R(S \to D_L^c U_L^c) - 3R(S \to D_R^c U_R^c) - 3R(S \to D_R U_R) \quad (5.5.2)$$

$$R_{\widetilde{S}}^{\Theta} = 3R(\widetilde{S} \to D_L^c U_L^c) - 3R(\widetilde{S} \to D_R U_R)$$

$$R_{\widetilde{S}_1}^{\Theta} = 3R(\widetilde{S}_1 \to D_L^c D_L^c) - 3R(\widetilde{S}_1 \to U_R U_R)$$

$$R_{\widetilde{S}^c}^{\Theta} = 3R(\widetilde{S}^c \to D_L D_L) + 3R(\widetilde{S}^c \to U_L U_L) + 3R(\widetilde{S}^c \to D_L U_L)$$

$$- 3R(\widetilde{S}^c \to D_R^c D_R^c) - 3R(\widetilde{S}^c \to U_R^c U_R^c) - 3R(\widetilde{S}^c \to D_R^c U_R^c)$$

Using the partial widths given in Table 5.5 gives

$$R_S^{\Theta} = 12R(S \to D_L U_L) \quad (5.5.3)$$

$$R_{\widetilde{S}}^{\Theta} = 6R(\widetilde{S} \to D_L^c U_L^c)$$

$$R_{\widetilde{S}_1}^{\Theta} = 6R(\widetilde{S}_1 \to D_L^c U_L^c)$$

$$R_{\widetilde{S}^c}^{\Theta} = 12R(\widetilde{S}^c \to D_R^c D_R^c)$$

Since light Higgs, φ, exchange violates Θ, it presumably dominates these CP violation parameters. For simplicity we take the Yukawa couplings of the 10_H and 120_H to be equal in magnitude and given by $\frac{g}{2\sqrt{2}} \frac{M_f}{M_W}$. We then have

$$R(S \to D_L U_L) = \pi\alpha \frac{M_f}{M_W} \varepsilon \mathrm{Im} I_{S\varphi} \quad (5.5.4)$$

$$R(\widetilde{S} \to D_L^c U_L^c) = -2\pi\alpha \frac{M_f}{M_W} \varepsilon \mathrm{Im} I_{\widetilde{S}\varphi}$$

$$R(\widetilde{S}_1 \to D_L^c D_L^c) = -2\pi\alpha \frac{M_f}{M_W} \varepsilon \mathrm{Im} I_{\widetilde{S}_1\varphi}$$

$$R(\widetilde{S}^c \to D_R^c D_R^c) = \frac{-2\pi\alpha}{3} \frac{M_f}{M_W} \varepsilon \mathrm{Im} I_{\widetilde{S}^c\varphi}$$

charge associated with $U(1)_R$ is zero. The decay modes of the $X, W, S, \tilde{S}, \tilde{S}_1$, and $\tilde{S}^c$ bosons given in Table 5.5 give the following set of equations for the time evolution of the various number densities:

$$\dot{X}_+ = -\langle\Gamma_X\rangle(X_+ - X_+^{eq}) \tag{5.5.1}$$

$$\dot{S}_+ = -\langle\Gamma_S\rangle(S_+ - S_+^{eq})$$

$$\dot{\tilde{S}}_{1+} = -\langle\Gamma_{\tilde{S}_1}\rangle(\tilde{S}_{1+} - \tilde{S}_{1+}^{eq})$$

$$\dot{\tilde{S}}_+ = -\langle\Gamma_{\tilde{S}}\rangle(\tilde{S}_+ - \tilde{S}_+^{eq})$$

$$\dot{\tilde{S}}_+^c = -\langle\Gamma_{\tilde{S}^c}\rangle(\tilde{S}_+^c - \tilde{S}_+^{ceq})$$

$$\begin{aligned}
\dot{B} = {} & -3\langle\Gamma_X\rangle X_+^{eq} B - \frac{3}{4}\langle\Gamma_S\rangle S_+^{eq} B - \frac{9}{4}\langle\Gamma_{\tilde{S}^c}\rangle \tilde{S}_+^{ceq}(\Theta + B) \\
& + [\frac{6}{4}\langle\Gamma_{\tilde{S}_{1}}\rangle \tilde{S}_{1+}^{eq} + \frac{3}{4}\langle\Gamma_{\tilde{S}}\rangle \tilde{S}_+^{eq}](\Theta - B) - 12 n_b \langle|v|\sigma_X'\rangle B \\
& - 12 n_b \langle|v|\sigma_S'\rangle B + 6[n_b\langle|v|\sigma_{\tilde{S}_1}'\rangle + n_b\langle|v|\sigma_{\tilde{S}}'\rangle](\Theta - B) - 12 n_b \langle|v|\sigma_{\tilde{S}^c}'\rangle(\Theta + B)
\end{aligned}$$

$$\begin{aligned}
\dot{\Theta} = {} & \langle\Gamma_S\rangle(S_+ - S_+^{eq})R_S^\theta + \langle\Gamma_{\tilde{S}_1}\rangle(\tilde{S}_{1+} - \tilde{S}_{1+}^{eq})R_{\tilde{S}_1}^\theta + \langle\Gamma_{\tilde{S}}\rangle(\tilde{S}_+ - \tilde{S}_+^{eq})R_{\tilde{S}}^\theta \\
& + \langle\Gamma_{\tilde{S}^c}\rangle(\tilde{S}_+^c - \tilde{S}_+^{c\,eq})R_{\tilde{S}^c}^\theta - \frac{3}{4}\langle\Gamma_S\rangle S_+^{eq}\Theta \\
& - \frac{9}{4}\langle\Gamma_{\tilde{S}^c}\rangle \tilde{S}_+^{c\,eq}(\Theta + B) - [\frac{6}{4}\langle\Gamma_{\tilde{S}_1}\rangle \tilde{S}_{1+}^{eq} + \frac{3}{4}\langle\Gamma_{\tilde{S}}\rangle \tilde{S}_+^{eq}](\Theta - B) \\
& - 12 n_b\langle|v|\sigma_S'\rangle\Theta - 6[n_b\langle|v|\sigma_{\tilde{S}_1}'\rangle + n_b\langle|v|\sigma_{\tilde{S}}'\rangle](\Theta - B) \\
& - 12 n_b\langle|v|\sigma_{\tilde{S}^c}'\rangle(\Theta - B) - 4 n_b\langle|v|\sigma_\varphi\rangle\Theta.
\end{aligned}$$

The averaged widths and cross-sections appearing in these equations are given in Section 5.4. The effective CP violation parameters are

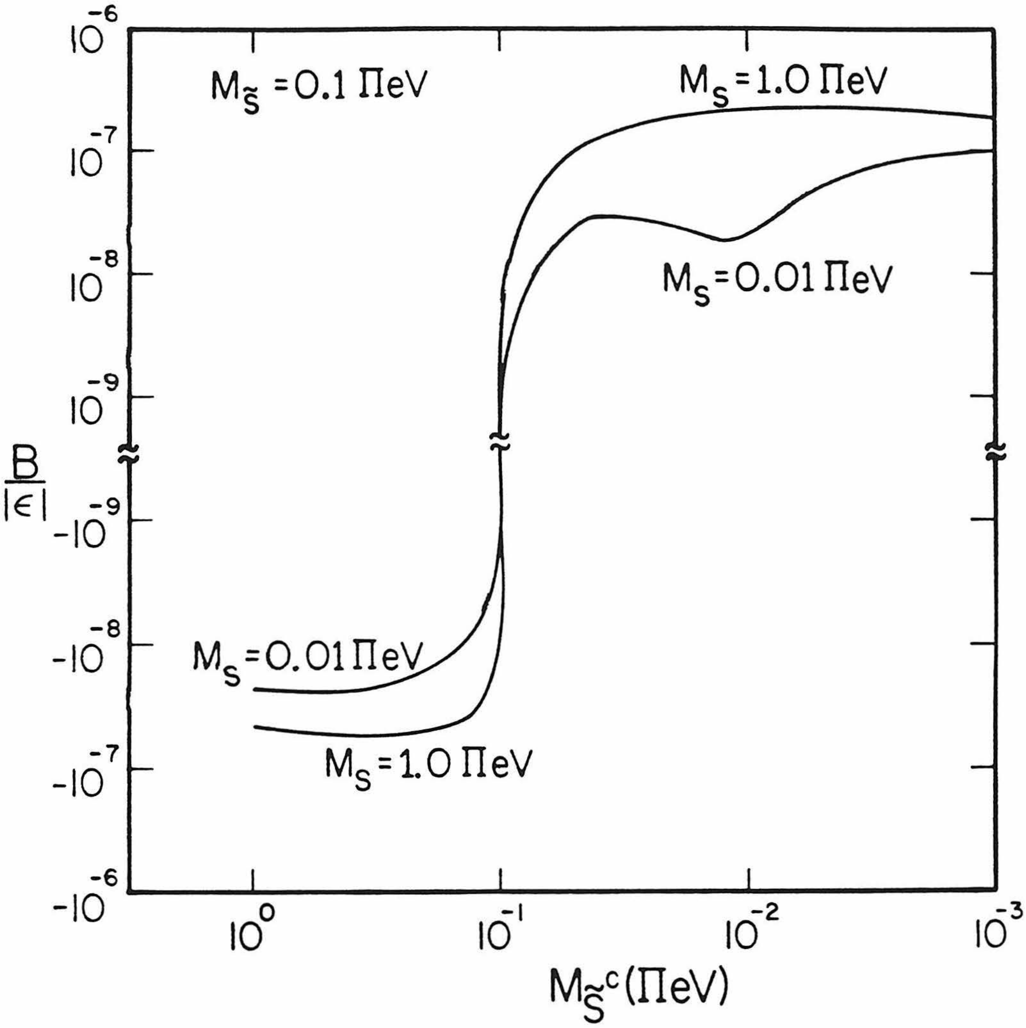

Figure 5.2a: Baryon number density (scaled by the CP violation parameter ε) generated in an SO(10) model with an $SU(4)\otimes SU(2)_L\otimes U(1)_R$ intermediate effective symmetry for a range of S, $\tilde{S}$ and $\tilde{S}^c$ masses. $\tilde{S}$ and $\tilde{S}^c$ are mass eigenstate Higgs bosons occuring in 120_H, while S is a Higgs boson from 10_H.

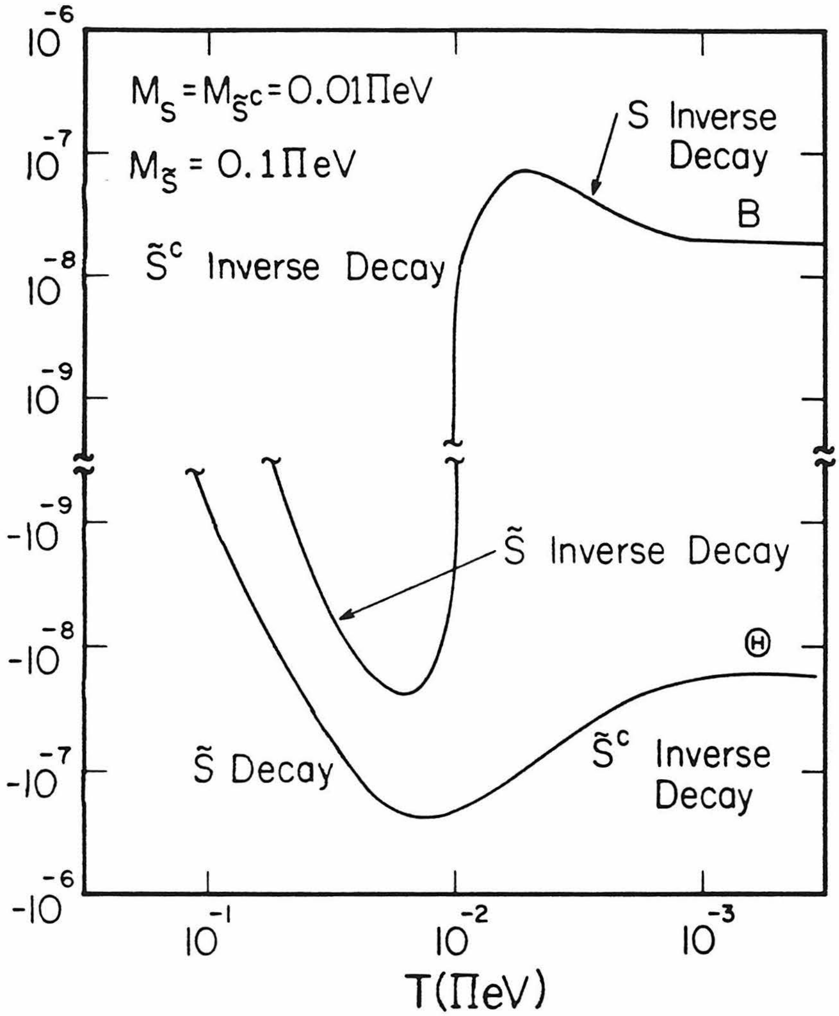

Figure 5.2b: Processes governing the temperature evolution of quantum number densities for $M_S = 1.0 \Pi eV \equiv 10^{15} GeV$, $M_{\tilde{S}} = 0.1 \Pi eV$ and $M_{\tilde{S}^c} = 0.2 \Pi eV$.

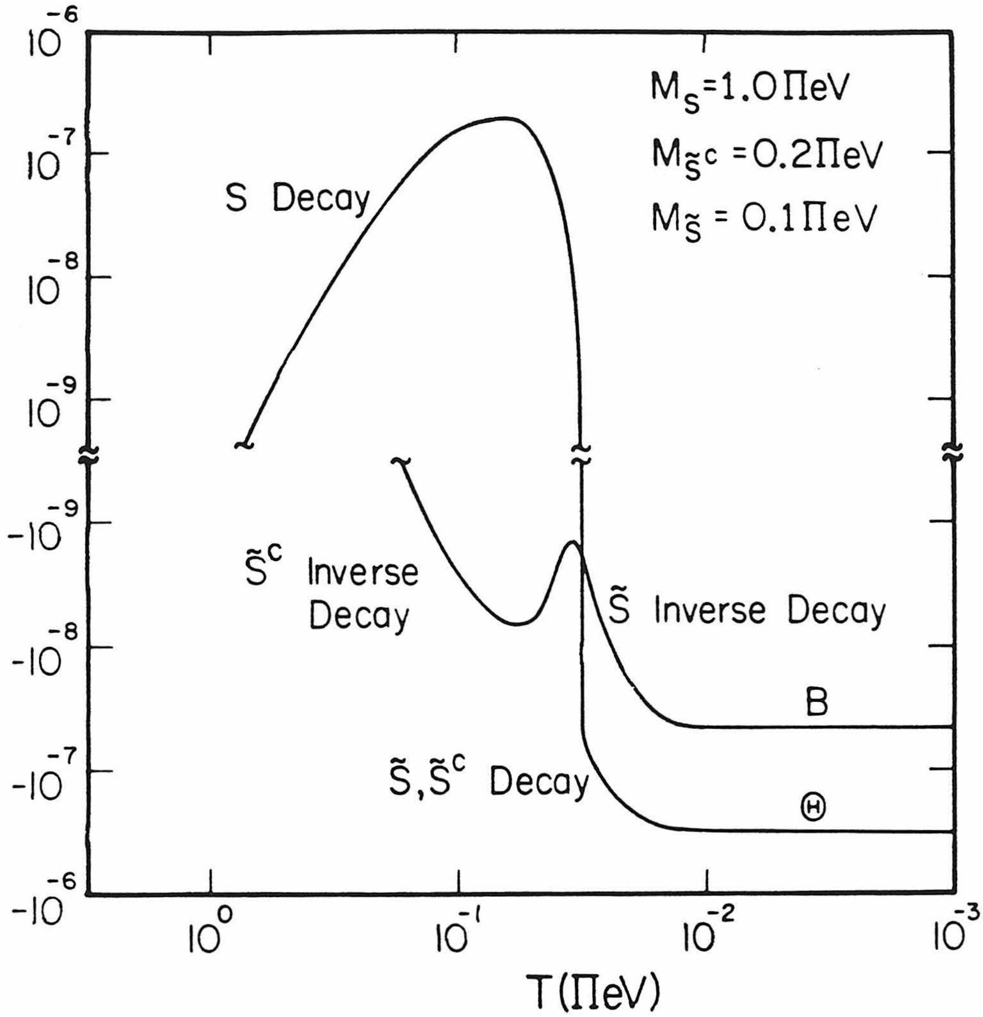

Figure 5.2c: Processes governing the temperature evolution of quantum number densities for $M_{\tilde{S}}$=0.1ΠeV and $M_S=M_{\tilde{S}^c}$=0.01ΠeV.

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III. Masses and mixings in an SO(10) unified model

1. Description of the model

The main weakness of all grand unified theories is the specification of the symmetry breaking mechanism which is responsible for the fermion masses and mixing angles. Here we will rely on explicit Higgs bosons for this purpose. In general, the couplings of such bosons to fermions and to each other are completely undetermined. In order to obtain testable predictions from the model to be considered one must impose constraints on these couplings. One of the few ways of imposing such constraints is to demand that the model be natural in the technical sense. That is, the results of the model must depend only on the symmetries and representation content of the model and thus be insensitive to small changes in the fundamental parameters of the theory. In practice this requires the imposition of various discrete symmetries which forbid certain couplings to all orders of perturbation theory. If these symmetries are broken by terms in the Lagrangian of dimension less than four, then corrections to any relations obtained from the discrete symmetries will be finite and calculable but not necessarily small. In what follows we will find such "soft" breaking of discrete symmetries to be unnecessary.

One may also obtain relations between physical parameters by requiring that only some of the possible vacuum expectation values (v.e.v.'s) of the Higgs fields be realized. In this case naturalness demands that the assumed pattern of v.e.v.'s be possible for a finite range of parameters in the Higgs potential.

In grand unified theories with explicit Higgs bosons one must demand that certain combinations of parameters in the Higgs potential cancel to an accuracy of 1 part in 10^{30} in order to explain the difference between the scale of $SU(2)_L \otimes U(1)_Y$ breaking ($\sim 10^2$ GeV) and the unification scale ($\sim 10^{15}$ GeV) [1]. This cancellation obviously violates the requirement of naturalness. This may betoken the bankruptcy of grand unified models with explicit Higgs breaking or of the requirement of naturalness. On the other hand, such a delicate cancellation seems to be required in an apparently different context in order to explain the smallness of the cosmological constant when compared to the naturally expected scale. In any event, the explanation of such a gauge hierarchy remains as the outstanding problem facing grand unified model enthusiasts.

In what follows we present a unified model based on the gauge group $SO(10)$ which reproduces the observed fermion masses and mixing angles in a technically natural way. The price for this success is high. The Higgs sector of our theory will contain 830 independent degrees of freedom! Nevertheless, the model makes a number of predictions which depend on having a phenomenologically successful fit to the fermion masses. Here we will concentrate on an analysis of the charged fermion mass matrices (Section 2) and on the predictions for neutrino masses and neutrino oscillations (Section 3). In Section 4 we present an analysis of the Higgs potential and comment on the constraints that naturalness imposes. Other features of the model are discussed in detail in [2]. A general introduction to SO(10) may be found in Section 5.1 of Chapter II. Appendix B contains explicit forms for the couplings described here.

The particle representations appearing in the model are as follows*:

Spin 1: adjoint of vector bosons $\sim 45_V$

Spin 1/2: three families of fermions ~ 16: 16_1, 16_2, 16_3

Spin 0: 54, two real 10's (10_1 and 10_2), three 126's : 126_1, 126_2, 126_3

The Lagrangian contains the usual gauge couplings, the Higgs self couplings described in Section 4, and the following set of Yukawa couplings:

$$L_Y = (A\,16_1 16_2 + B\,16_3 16_3)\overline{126}_1 + (a\,16_1 16_2 + b\,16_3 16_3)(10_1 + i\,10_2) \qquad (1.1)$$

$$+ (c\,16_2 16_2)\overline{126}_1 + (d\,16_2 16_3)\overline{126}_3$$

where A, B, a, b, c, and d are undetermined Yukawa coupling constants. We take them to be real so as to avoid hard CP violation. This form for the Yukawa couplings is maintained by two continuous global phase symmetries, X and Y, which are summarized in the following table.

	16_1	16_2	16_3	10	126_1	126_2	126_3
X	-3/2	1/2	-1/2	1	-1	1	0
Y	1	-1	0	0	0	-2	-1

In order to avoid massless Goldstone-Nambu bosons when these symmetries are broken by the Higgs vacuum expectation values (v.e.v.'s), X and Y must be broken by explicit terms in the Higgs potential. In order to maintain the form of the Yukawa couplings, this breaking must leave remnant discrete symmetries which forbid the terms not already included in L_Y. We find it possible to arrange this due to a marvelous property of the 126 Higgs field: the four times symmetrized product $(126)^4_S$

*We thank H. Georgi for pointing out to us that naturalness in this model requires the use of a 54_H rather than a 45_H.

contains one $SO(10)$ singlet. By including the terms $(126_1)^4$ and $(126_2)^4$ in the Higgs potential we can break X to a mod-4 discrete symmetry and Y to a mod-8 symmetry. These two discrete symmetries suffice to maintain the naturalness of the Yukawa couplings. The additional Higgs self couplings are chosen so as to honor these symmetries.

We take the Higgs fields to develop the following v.e.v.'s.

$$\Delta I_W = 0: \quad <54> \sim 24 \text{ of } SU(5) \tag{1.2}$$

$$<126_1> \sim 1 \text{ of } SU(5)$$

$$\Delta I_W = 1/2: \quad <10_1 + i10_2> \sim 5 + \bar{5} \text{ of } SU(5) \tag{1.3}$$

$$<126_1> \sim \bar{5} \text{ of } SU(5)$$

$$<126_2> \sim 45 \text{ of } SU(5)$$

$$<126_3> \sim \bar{5} \text{ of } SU(5)$$

This choice is made for the specific purpose of reproducing the observed fermion mass spectrum and achieving the symmetry breaking of $SO(10)$ down to $SU(3)_c \otimes U(1)_Q$. However, these v.e.v.'s must be realized for a finite range of parameters in the Higgs potential. Since the form of the Higgs potential is dictated in part by the discrete symmetries discussed above, this turns out to be a nontrivial constraint. Section 4 contains the explicit form of the Higgs potential and the demonstration that the above v.e.v.'s can be naturally maintained.

2. Mass matrices for charged fermions

The Dirac mass terms for charged fermions all transform under $SU(2)_L$ as $\Delta I_W = 1/2$ since the left-handed components of the fields transform as $SU(2)_L$ doublets while the right-handed components are singlets. The $\Delta I_W = 1/2$ v.e.v.'s in our model are

$$<10_1+i10_2> = re^{i\vartheta}\,(along\ \bar{5}) + pe^{i\delta}\,(along\ 5) \tag{2.1}$$

$$<126_1> = te^{-i\sigma}\,(along\ \bar{5}) \tag{2.2}$$

$$<126_2> = se^{-i\chi}\,(along\ 45) \tag{2.3}$$

$$<126_3> = qe^{-i\mu}\,(along\ \bar{5}) \tag{2.4}$$

It is helpful to remember that in terms of the $SU(5)$ decompositions a v.e.v. along the 5 contributes equally to charge 2/3 masses and neutral Dirac masses, a v.e.v. along the $\bar{5}$ contributes equally to charge $-1/3$ and charge -1 masses, and a v.e.v. along the 45 contributes to charge $-1/3$ and charge -1 masses with relative weight -3 for leptons. This last factor of -3 is due to the fact that the 45 v.e.v. lies along the I_{15} generator of $SU(4)$ in the chiral decomposition of $SO(10)$ (see Chapter II, Section 5.1) which is proportional to $B-L$. We thus obtain the following mass matrices:

$$M_{-1/3} = \begin{bmatrix} 0 & Re^{i\vartheta} & 0 \\ Re^{i\vartheta} & Se^{i\chi} & 0 \\ 0 & 0 & Te^{i\vartheta} \end{bmatrix} \quad \text{for } charge\ -1/3\ quarks \tag{2.5}$$

$$M_{-1} = \begin{bmatrix} 0 & Re^{i\vartheta} & 0 \\ Re^{i\vartheta} & -3Se^{i\chi} & 0 \\ 0 & 0 & Te^{i\vartheta} \end{bmatrix} \quad \text{for } charge\ -1\ leptons \tag{2.6}$$

$$M_{2/3} = \begin{pmatrix} 0 & Pe^{i\lambda} & 0 \\ Pe^{i\lambda} & 0 & Qe^{i\mu} \\ 0 & Qe^{i\mu} & Ve^{i\tau} \end{pmatrix} \quad \text{for } \mathit{charge}\ 2/3\ \mathit{quarks} \tag{2.7}$$

Above we have set

$$R = ar, \quad T = br, \quad S = cs, \quad Q = dq \tag{2.8}$$

and

$$Pe^{i\lambda} = ape^{i\delta} + Ate^{i\sigma} \tag{2.9}$$

$$Ve^{i\tau} = bpe^{i\delta} + Bte^{i\sigma}. \tag{2.10}$$

In order to diagonalize these matrices it is convenient to first remove the phases through a redefinition of the left and right-handed components of the fermion fields. We thus set

$$M_{-1/3} = e^{i\Phi_L} \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix} e^{i\Phi_R} \tag{2.11}$$

$$M_{-1} = e^{i\Phi_L} \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix} e^{i\Phi_R} \tag{2.12}$$

$$M_{2/3} = e^{i\Delta_L} \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & V \end{pmatrix} e^{i\Delta_R} \tag{2.13}$$

where Φ_L, Φ_R, Δ_L, and Δ_R are three by three real diagonal matrices with entries determined by the phases appearing in $M_{-1/3}$, M_{-1}, and $M_{2/3}$. To diagonalize these matrices we set

$$\overline{M}_{-1/3} = R^T_{-1/3}\ diag\,(-m_d, m_s, m_b)\ R_{-1/3} \tag{2.13}$$

$$\overline{M}_{-1} = R^T_{-1}\ diag\,(m_e, -m_\mu, m_\tau)\ R_{-1} \tag{2.14}$$

$$\overline{M}_{2/3} = R^T_{2/3}\ diag\,(m_u, -m_c, m_t)\ R_{2/3} \tag{2.15}$$

where the bar indicates the matrices with the phases removed and the R's are three by three orthogonal matrices. We find that

$$-m_d = (S - \sqrt{S^2 + 4R^2})/2, \quad m_s = (S + \sqrt{S^2 + 4R^2})/2, \quad m_b = T \tag{2.16}$$

and

$$m_e = (-3S + \sqrt{9S^2 + 4R^2})/2, \quad -m_\mu = (-3S - \sqrt{9S^2 + 4R^2})/2, \quad m_\tau = T \tag{2.17}$$

which gives the $SU(5)$ relation $m_b = m_\tau$ [1,3], and the Georgi-Jarlskog relations, $m_d m_s = m_e m_\mu$, $(m_d - m_s) = 3(m_e - m_\mu)$ [4] which lead to the approximate relation

$$\frac{m_e}{m_\mu} = \frac{m_d}{9m_s} \tag{2.18}$$

which should be essentially independent of scale since it involves a ratio of quark masses. The rotation matrices in (2.13-15) are given by

$$R_{-1/3} = \begin{pmatrix} \cos\vartheta_c & -\sin\vartheta_c & 0 \\ \sin\vartheta_c & \cos\vartheta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.19}$$

with $\tan\vartheta_c = \sqrt{m_d/m_s}$ and

$$R_{-1} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.20}$$

with $\tan\beta = \sqrt{m_e/m_\mu}$. The rotation matrix $R_{2/3}$ has a more complicated form which is given exactly in [5]. To $O(m_u/m_c)$ it is given by

$$R_{2/3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \tag{2.21}$$

with $\tan\alpha = \sqrt{m_c / m_t}$.

We define the mass eigenstates by

$$E_L^m = R_{-1} E_L, \quad D_L^m = R_{-1/3} D_L, \quad U_L^m = R_{2/3} U_L \tag{2.22}$$

with E_L, D_L, and U_L being weak eigenstate three-component vectors in family space. Using (2.11-2.15) we find that the weak charged current is given in terms of the mass eigenstates by

$$j_\mu = N_L^\dagger \sigma_\mu e^{i\Phi_L} R_{-1}^T E_L^m + U_L^{m\dagger} \sigma_\mu R_{2/3} e^{i(\Phi_L - \Delta_L)} R_{-1/3} D_L^m \tag{2.23}$$

The phase matrix $e^{i(\Phi_L - \Delta_L)}$ appearing in the charged quark current is in general nonzero so our model exhibits CP violation.

Due to the presence of the <$\bar{5}$> v.e.v. in the 126_1, the model makes no prediction for the t quark mass. However, it is measured indirectly since the strength of the $b \to c$ transition depends on the ratio m_c / m_t. Our mixing matrix gives a lifetime for the B meson of $\tau_B = m_t / m_c \; 4.45 \times 10^{-15}$ sec. to be compared with the existing experimental limit $\tau_B \leq 10^{-11}$ sec. This model gives the successful Oakes relation for the Cabibbo angle, $\tan^2\vartheta_c = m_d / m_s$. The effects of the mixing angle β appearing in the lepton sector will be discussed in the next section where we explore the consequences of these mass and mixing relations for the neutral sector of the theory.

3. Mass matrices for neutral fermions

Perhaps the most interesting consequence of $SO(10)$ grand unified models is the prediction of nonzero neutrino masses and hence the possibility of neutrino oscillations. In this section we present an analysis of neutrino masses and mixings which is applicable to any $SO(10)$ model and then consider the predictions that our model makes for these phenomena.

In what follows we will assume for definiteness that fermions come in three families that are simple replications of the lowest mass family. The generalization of the analysis presented here to an arbitrary number of families is immediate. Appendix A contains a brief review of the possible fermion mass terms allowed by Lorentz invariance. In the notation used there we can write the mass term for the three light left-handed neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and the three superheavy $SU(5)$ singlet fermions (N_e, N_μ, N_τ) in the form

$$\Psi_L^T M_o \Psi_L \tag{3.1}$$

where Ψ_L is a six-component vector $\Psi_L^T = (N_e^T, N_\mu^T, N_\tau^T, \nu_e^T, \nu_\mu^T, \nu_\tau^T)$ with the superscript T indicating transposition, and M_o is a six by six complex symmetric matrix:

$$M_O = \begin{bmatrix} M^{(0)} & M^{(1/2)} \\ M^{(1/2)T} & M^{(1)} \end{bmatrix} \tag{3.2}$$

where the $M^{(\Delta I_w)}$ are three by three matrices, the superscript standing for their transformation properties under $SU(2)_L$. Since M_O is symmetric, $M^{(0)}$ and $M^{(1)}$ are also symmetric matrices.

$M^{(1)}$ corresponds to direct Majorana mass terms for the light neutrinos and in general may receive contributions from 126_H's with vacuum expectation values along the 15 of $SU(5)$. In order to be in agreement with the experimental limits on neutrino masses, such vacuum values must be much less than the $\Delta I_W = 1/2$ breaking terms and are thus usually taken to be zero. If $M^{(0)} \neq 0$, then $M^{(1)}$ may in general receive contributions from radiative corrections. It may be shown that these corrections are usually much less than the $\Delta I_W = 1/2$ values divided by the $\Delta I_W = 1$ values and hence may be safely ignored.

In $SO(10)$, the $\Delta I_W = 1/2$ mass matrices of the charge 2/3 and charge 0 sectors are related by Clebsch-Gordan coefficients: the 10, having its v.e.v. along the (1,2,2) in the chiral decomposition gives equal weight to leptons and quarks, while the 126 with its $\Delta I_W = 1/2$ v.e.v. along the (15,2,2) gives leptons a factor of -3 relative to the quarks. We thus identify

$$M^{(1/2)} = \begin{pmatrix} 0 & Pe^{i\lambda} & 0 \\ Pe^{i\lambda} & 0 & -3Qe^{i\mu} \\ 0 & -3qe^{i\mu} & Ve^{i\tau} \end{pmatrix} \tag{3.3}$$

by comparison with the matrix in (2.13). In terms of the light quark masses we have

$$V = m_t - m_c + m_u \tag{3.4}$$

$$P = \left(\frac{m_t m_c m_u}{V}\right)^{1/2}$$

$$Q = \left(\frac{(m_t - m_c)(m_c - m_u)(m_t - m_u)}{V}\right)^{1/2}$$

The form of $M^{(0)}$ is dictated solely by the v.e.v. of the 126_1 along the $\Delta I_w = 0$ direction. Using the couplings given in (1.1) we find

$$M^{(0)} = \begin{bmatrix} 0 & \bar{A}e^{i\zeta} & 0 \\ \bar{A}e^{i\zeta} & 0 & 0 \\ 0 & 0 & \bar{B}e^{i\zeta} \end{bmatrix} \tag{3.5}$$

where

$$\bar{A} = Ak \qquad \bar{B} = Bk \tag{3.6}$$

and the $\Delta I_w = 0$ v.e.v. of the 126_1 is

$$<126_1> = ke^{i\zeta} \qquad (along\ 1) \tag{3.7}$$

The six by six neutral lepton mass matrix is thus symmetric and in general complex, allowing for CP violation. Its diagonalization proceeds as outlined in Appendix A. We write

$$M_O = U^T D U, \tag{3.8}$$

where D is a diagonal matrix with real positive entries and U is a unitary matrix, $U^\dagger U = 1$.

The formidable algebraic task of handling a six by six matrix is somewhat alleviated by the fact that the entries in $M^{(1/2)}$ are much smaller than the entries in $M^{(0)}$, as a result of the gauge hierarchy. Let us set

$$M^{(1/2)} = \varepsilon \hat{M}^{(1/2)} \tag{3.9}$$

where $\hat{M}^{(1/2)}$ is of the same order of magnitude as $M^{(0)}$ and ε measures the relative strength of the $\Delta I_w = 1/2$ to $\Delta I_w = 0$ breakings,

$$\varepsilon = \frac{\Delta I_w = 1/2}{\Delta I_w = 0} \ll 1 \tag{3.10}$$

We then set

$$U = \begin{bmatrix} U_{11} & \varepsilon U_{12} \\ \varepsilon U_{21} & U_{22} \end{bmatrix} \tag{3.11}$$

where the three by three matrices U_{11}, U_{22}, U_{12}, and U_{12} are of the same order of magnitude , and

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & \varepsilon^2 D_s \end{bmatrix} \tag{3.12}$$

where D_1 and D_2 are three by three diagonal matrices of the same order of magnitude. Then, as a consequence of the unitarity of U and of the unitary congruence (3.8), we obtain the set of three by three matrix equations

$$M^{(0)} = U_{11}^T D_1 U_{11} + O(\varepsilon^2) \tag{3.13}$$

and

$$\hat{M}^{(1/2)T} M^{(0)\,-1} \hat{M}^{(1/2)} = -U_{22}^T D_2 U_{22} \tag{3.14}$$

with U_{11} and U_{22} being unitary matrices to $O(\varepsilon^2)$. This last equation is the matrix equivalent of the Gell-Mann, Ramond, Slansky mechanism. From (3.13) we can solve for U_{11} and D_1. Next, we solve the three by three congruence problem (3.14) to find U_{22} and D_2.

Clearly, D_1 gives the masses for the heavy neutrinos while $\varepsilon^2 D_2$ gives the masses for the light neutrinos. In terms of physical mass eigenstates, ψ^m, it is easy to see that the leptonic charged current is given by

$$j_\mu = \nu_L^{\dagger m} \sigma_\mu U_{22} e^{i\Phi_L} R_{-1}^T E_L^m + O(\varepsilon^2) \tag{3.15}$$

where ν_L is a three-component vector with entries corresponding to the light neutrinos.

From (3.12) we see that

$$D_1 = \begin{pmatrix} \bar{A} & 0 & 0 \\ 0 & -\bar{A} & 0 \\ 0 & 0 & \bar{B} \end{pmatrix} \quad U_1 = \frac{e^{i\zeta/2}}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \tag{3.16}$$

so that we have two heavy right-handed neutrinos of mass $M_1 = \bar{A}$ and one of mass $M_2 = \bar{B}$. (The negative sign in front of the second eigenvalue can be absorbed by a chiral transformation on N_μ , the heavy neutrino eigenstate associated with the second family.)

We now consider the diagonalization of the light neutrino mass matrix,

$$M_\nu = M^{(1/2)T} M^{(0)-1} M^{(1/2)} = -U_{22}^T \varepsilon^2 D_2 U_2. \tag{3.17}$$

By taking the determinant of M_ν we find that the light neutrino masses must obey the constraint

$$m_{\nu_e} m_{\nu_\mu} m_{\nu_\tau} = \frac{m_u^2 m_c^2 m_t^2}{M_{N_e} M_{N_\mu} M_{N_\tau}} \tag{3.18}$$

where the overall phase has been absorbed in U_{22}. Using (3.3) and (3.5) we find that

$$M_\nu = e^{i\Phi_\nu} \begin{pmatrix} 0 & P^2/\bar{A} & 0 \\ P^2/\bar{A} & 9Q^2 e^{i\mu}/\bar{B} & -3Q(P/\bar{A} + V/\bar{B}) \\ 0 & -3Q(P/\bar{A} + V/\bar{B}) & V^2/\bar{B} \end{pmatrix} \tag{3.19}$$

where all but one of the phases appearing in M_ν have been absorbed by redefinition of the light neutrino fields by a factor $e^{i\Phi_\nu}$ with Φ_ν a three by three real diagonal matrix. The remaining phase appearing in the 2-2 entry cannot be removed without complete diagonalization of M_ν. For simplicity we will take $\mu = 0$. In general, the light neutrino masses and mixing angles will be functions of μ. For $\mu = 0$, U_{22} is an orthogonal matrix to $O(\varepsilon^2)$ which we parametrize as

$$U_{22} = \begin{bmatrix} c_1 & s_1c_2 & s_1s_2 \\ -s_1c_3 & c_1c_2c_3-s_2s_3 & c_1s_2c_3+c_2s_3 \\ -s_1s_3 & c_1c_2s_3+s_2c_3 & c_1s_2s_3-c_2c_3 \end{bmatrix} \tag{3.20}$$

with $s_i = \sin\vartheta_i$, $c_i = \cos\vartheta_i$, $i = 1,2,3$.

An analytic solution for U_{22} and D_2 is rather complicated and unenlightening. Instead, we will analyze the eigenvalues and mixing angles for two limiting cases that illustrate the general features and then present the results of a numerical solution. It is convenient to take the light neutrino masses to depend on m_u, m_c, and m_t through P, Q, and V, and on the ratio $r = M_{N_e}/M_{N_\tau} = \bar{A}/\bar{B}$ with the overall scale set by $\bar{A}$.

We first suppose that the right-handed neutrino masses obey a family hierarchy similar to that for the observed leptons and quarks so that $r \sim m_u/m_t \ll 1$. In the approximation that

$$V \cong m_t \quad P \cong \sqrt{m_u m_c} \quad Q \cong \sqrt{m_t m_c} \tag{3.21}$$

we then find that

$$m_{\nu_e} \cong \frac{r m_t m_u}{9\bar{A}}, \quad m_{\nu_\mu} \cong m_{\nu_\tau} \cong \frac{3m_c\sqrt{m_t m_u}}{\bar{A}} \tag{3.22}$$

with the mixing angles given approximately by

$$\vartheta_1 \cong (m_u/m_t)^{1/2}/3, \quad \vartheta_2 \cong \pi/2 - r m_t (m_t/m_u)^{1/2}/3m_c, \quad \vartheta_3 \cong 3\pi/4 \tag{3.23}$$

so that appreciable mixing exists only in the μ–τ sector. If m_{ν_μ} and m_{ν_τ}

saturate the cosmological bound $\sum m_\nu \lesssim 100\, eV$ [6], then we find $m_{\nu_e} \sim 3.75\, eV$ for $m_t = 25\ GeV$.

On the other hand, if the right-handed neutrino masses are all equal, so that $r = 1$, we then find in the same approximation that

$$m_{\nu_e} \cong \frac{\sqrt{m_c m_u^3}}{18\bar{A}}, \quad m_{\nu_\mu} \cong \frac{18 m_t \sqrt{m_c^3 m_u}}{\bar{A}(m_t + 9m_c)}, \quad m_{\nu_\tau} \cong \frac{m_t(m_t + 9m_c)}{\bar{A}} \tag{3.24}$$

with the mixing angles given by

$$\vartheta_1 \cong \frac{\sqrt{m_u(m_t + 9m_c)}}{18\sqrt{m_t m_c}}, \quad \vartheta_2 \cong \cos^{-1}\left(\frac{m_t}{m_t + 9m_c}\right), \quad \vartheta_3 \cong \pi \tag{3.25}$$

so that again appreciable mixing exists only between the μ and τ neutrinos. If m_{ν_τ} saturates the cosmological bound then for $m_t = 25\ GeV$ we find that $m_{\nu_e} \cong 1\times10^{-8}\, eV$. Note that for $r = 1$, m_{ν_μ} differs from the case $r \ll 1$ by a factor of $\frac{6\sqrt{m_t m_c}}{m_t + 9m_c}$ which is independent of r and which varies only from 0.98 to 0.82 for m_t between 20 and 50 GeV. We thus find that for all values of r, m_{ν_μ} is essentially constant and therefore provides the best measurement of the mass scale for the right-handed neutrinos. Numerical results for the masses and mixing angles are given in Table 3.1 for r varying from 10^{-4} to 1, and for $m_t = 20, 25$, and 30 GeV. We find that the previous approximations are good to about 10% when applicable. In Table 3.1 the mass scale for the neutrinos is determined by the parameter $\bar{a} = M_{N_e} / 10^{10}\ GeV \equiv \bar{A} / 10^{10}\ GeV$. The value of $\bar{a}$ in SO(10) theories depends both on the breaking scheme and on the values of the Yukawa couplings. It has been suggested that the right-handed neutrinos may acquire their masses only through radiative corrections [10]. In this case $\bar{a}$ depends also on undetermined parameters appearing in the Higgs potential.

$m_t\,(GeV)$	$\log_{10} r$	$\bar{a} m_{\nu_e}$	$\bar{a} m_{\nu_\mu}$	$\bar{a} m_{\nu_\tau}$	ϑ_1	ϑ_2	ϑ_3
20	-4	9.91×10^{-8}	0.148	0.154	0.0054	1.548	2.378
20	-2	2.02×10^{-6}	0.147	0.759	0.0041	0.9217	2.965
20	0	2.50×10^{-6}	0.147	61.3	0.0041	0.7277	3.138
25	-4	1.24×10^{-7}	0.164	0.173	0.0048	1.538	2.386
25	-2	2.09×10^{-6}	0.160	1.05	0.0039	0.8223	3.015
25	0	2.48×10^{-6}	0.159	89.0	0.0039	0.6644	3.136
30	-4	1.49×10^{-7}	0.178	0.190	0.0043	1.527	2.395
30	-2	2.14×10^{-6}	0.170	1.39	0.0037	0.7462	3.048
30	0	2.47×10^{-6}	0.168	122	0.0037	0.6154	3.134

Table 3.1: Masses and mixing angles for light neutrinos as a function of $r = M_{N_e}/M_{N_\tau}$ and top quark mass m_t. M_{N_i} is the mass of the i'th supermassive right-handed neutrino field. m_{ν_i} is the mass in electron volts of the i'th neutrino. $\bar{a} = M_{N_e}/10^{10}GeV$. The mixing angles are given in radians.

Neutrino oscillations are governed by the mixing matrix

$$K = U_{22}e^{i\Phi_L}R^T_{-1} \tag{3.26}$$

which appears in the leptonic charged current (see e.g. [7]). The probability of finding a weak eigenstate neutrino ν_l (i.e. the neutrino that would accompany the lepton l in a weak decay) at a time t in a beam of momentum p which consisted of $\nu_{l'}$ at $t=0$ is given by

$$P_{l'l}(t) = \sum_{i,j} K_{l'i}K^*_{li}K^*_{l'j}K_{lj}\,e^{-i(E_i-E_j)t} \tag{3.27}$$

In the relativistic limit, $(E_i-E_j)t = \frac{2\pi R}{L}$ where R is the distance from the source of the beam and

$$L = \frac{4\pi p}{|m^2_{\nu_i} - m^2_{\nu_j}|} \tag{3.28}$$

is the oscillation length.

In principle, the CP violating phases appearing in (3.27) are measurable since they affect the cosine dependence of the probability for neutrino oscillations [8]. In practice, such phases will be very difficult to measure and we will ignore them in what follows. To a good approximation we can set ϑ_1=0 so that

$$K = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta\cos\vartheta & \cos\beta\cos\vartheta & \sin\vartheta \\ \sin\beta\sin\vartheta & \cos\beta\sin\vartheta & -\cos\vartheta \end{pmatrix} \tag{3.29}$$

with $\vartheta = \vartheta_2 + \vartheta_3$ and $\tan\beta = \sqrt{m_e/m_\mu}$ as before. In this model the neutrino oscillations are thus parametrized by the three squared mass differences of the neutrinos and by the mixing angles ϑ and β. Since $m_{\nu_e} \ll m_{\nu_\mu}, m_{\nu_\tau}$, we give in Table 3.2 only the difference $m^2_{\nu_\tau} - m^2_{\nu_\mu}$ and the value of ϑ for the

$m_t\,(GeV)$	$\log_{10} r$	$\bar{a}^2 \delta m^2$	ϑ
20	-4	0.002	0.785
20	-2	0.554	0.747
20	0	3758	0.722
25	-4	0.003	0.785
25	-2	1.077	0.696
25	0	7921	0.659
30	-4	0.004	0.785
30	-2	1.903	0.653
30	0	14884	0.608

Table 3.2: Parameters for neutrino oscillations. δm^2 is the difference of the squared masses of ν_μ and ν_τ. ϑ is the mixing angle connecting weak and mass eigenstates.

same range of r and m_t as in Table 3.1.

The results obtained here depend to some degree on the simple form of $M^{(0)}$. However, the general features of small ν_e mass and appreciable mixing only between ν_μ and ν_τ are tied to the dependence of the neutrino masses and mixing on the charge 2/3 quark mass matrix. This dependence is a general feature of all $SO(10)$ grand unified theories.

4. Analysis of the Higgs potential

In our model, the Higgs potential consists of all the $SO(10)$ invariant quadratic, cubic and quartic interactions among the $10, 54, 126_1, 126_2$, and 126_3 Higgs fields which also preserve the global X and Y symmetries defined in Sec. 1 of the Yukawa terms mod-4 and mod-8 respectively. For convenience we split the potential up into terms that involve only one type of field and terms with several Higgs fields.

The simplest term is the one involving only the 54 . From the products

$$(54 \otimes 54)_S = 1 + 54 + 660 + 770 \tag{4.1}$$

and

$$(54 \otimes 54 \otimes 54)_S = 1 + 54 + 54 + \cdots \tag{4.2}$$

where the dots stand for representations other than 1 or 54, we see that the most general form for this term is

$$V_{54} = [\ 54 \otimes 54,\ 54 \otimes 54 \otimes 54,\ (54 \otimes 54)^2_1, (54 \otimes 54)^2_{54})] \tag{4.3}$$

Here $(54 \otimes 54)^2_{54}$ denotes the square of the projection of $54 \otimes 54$ onto the 54. The cubic term can be forbidden by the discrete symmetry $54 \to -54$.

The form of the term involving the 10, V_{10} , is dictated by $SO(10)$ invariance and by the (X,Y) value of (1,0) for the complex 10. We use the products

$$10 \otimes 10 = (1 + 54)_S + (45)_A \tag{4.4}$$

and

$$10 \otimes 10 \otimes 10 = (120)_A + (210 + 10)_S + (320 + 320 + 10 + 10)_M \tag{4.5}$$

where the subscript M denotes mixed symmetry. Since there are two real 10's (or one complex 10), we have at most two quadratic invariants and six quartic invariants. Imposing X mod-4 reduces this to one quadratic invariant ($10 \otimes \overline{10}$) and four quartic invariants. These are of the form

$$V_{10} = [(10 \otimes \overline{10})_1, (10 \otimes \overline{10})_1^2, (10 \otimes \overline{10})_{54}^2, (10 \otimes 10)_1^2, (10 \otimes 10)_{54}^2] \tag{4.6}$$

It is easy to see that the other possible quartic invariant $(10 \otimes \overline{10})_{45}^2$ can be expressed in terms of the other four. The last two quartic invariants respect X mod-4 and serve to avoid the massless Goldstone-Nambu boson that would otherwise ensue when the global X symmetry is spontaneously broken.

The terms in the potential that involve only one kind of 126 are more complicated. For the moment we neglect the (X,Y) symmetries and construct the most general potential containing only one type of 126. We will have use of the products

$$126 \otimes 126 = (54 + 1050 + 2772 + 4125)_S + (945 + 6930)_A \tag{4.7}$$

and

$$126 \otimes \overline{126} = 1 + 45 + 210 + 770 + 5940 + 8910. \tag{4.8}$$

Note that although the 126 is complex, only the 1050, 2772, and 6930 are complex. The number of independent quartic invariants is given by the number of times the $\overline{126}$ appears in the the products $126 \otimes 126 \otimes 126$ and $\overline{126} \otimes \overline{126} \otimes 126$. The techniques used to calculate the relevant part of these products are discussed in [7]. The task is easier when only one 126 is present. The result is that the $\overline{126}$ appears once in the symmetric product $126 \otimes 126 \otimes 126$ and four times in $\overline{126} \otimes \overline{126} \otimes 126$. We thus have

$$V_{126} = [(126 \otimes \overline{126})_1, (126 \otimes \overline{126})_1^2, (126 \otimes 126)_{54} \otimes (\overline{126} \otimes \overline{126})_{54}, \tag{4.9}$$

$$(126 \otimes 126)_{1050} \otimes \overline{(\overline{126} \otimes \overline{126})}_{\overline{1050}}, (126 \otimes 126)_{4125} \otimes (\overline{126} \otimes \overline{126})_{4125}, (126 \otimes 126)_{54}^2]$$

There will be a set of such terms for each of the 126 's. The 126_1 with $(X,Y) = (-1,0)$ will have a $(126_1)^4$ term which breaks X mod-4 and preserves Y; the 126_2, with $(X,Y) = (1,-2)$ has a term $(126_2)^4$ which turns out to be the only term in the potential that breaks Y mod-8, thus avoiding a massless Goldstone-Nambu boson associated with the breaking of the global Y symmetry. The 126_3 with $(X,Y) = (0,-1)$ does not have a quartic term of this form since this term does not conserve Y mod-8.

We now consider terms involving two types of Higgs bosons. First, from (4.1) and (4.4) and using (X,Y) conservation we easily see that

$$V_{10-54} = [(10 \otimes \overline{10})_{54} \otimes 54, (10 \otimes \overline{10})_1 \otimes (54 \otimes 54)_1, (10 \otimes \overline{10})_{54} \otimes (54 \otimes 54)_{54}] \tag{4.10}$$

Again, the cubic terms can be removed by a discrete symmetry for the 54. Second, from (4.1) and (4.8) it is easy to see that we have

$$V_{126-54} = [(126 \otimes \overline{126})_1 \otimes (54 \otimes 54)_1, (126 \otimes \overline{126})_{770} \otimes (54 \otimes 54)_{770}] \tag{4.11}$$

for each 126. Other possibilities are ruled out by the discrete (X,Y) symmetries. Third, using (4.4), (4.7), and (4.8) and

$$10 \otimes 126 = 1050 + 210 \tag{4.12}$$

we have

$$V_{10-126_1} = [(10 \otimes \overline{10})_1 \otimes (126_1 \otimes \overline{126})_1, (10 \otimes \overline{10})_{45} \otimes (126_1 \otimes \overline{126}_1)_{45}, \tag{4.13}$$

$$(10 \otimes 10)_{54} \otimes (\overline{126}_1 \otimes \overline{126}_1)_{54}]$$

$$V_{10-126_2} = [(10\otimes\overline{10})_1\otimes(126_2\otimes\overline{126_2})_1,\ (10\otimes\overline{10})_{45}\otimes(126_2\otimes\overline{126_2})_{45}] \tag{4.14}$$

$$V_{10-126_3} = [(10\otimes\overline{10})_1\otimes(126_3\otimes\overline{126_3})_1,\ (10\otimes\overline{10})_{45}\otimes(126_3\otimes\overline{126_3})_{45}] \tag{4.15}$$

as well as quartic terms linear in the 10_H:

$$(\overline{10}\otimes 126_1)_{210}\otimes(126_i\otimes\overline{126_i})_{210} \qquad i = 1,2,3 \tag{4.16}$$

and

$$(10\otimes 126_3)_{210}\otimes(126_3\otimes\overline{126_2})_{210} \tag{4.17}$$

Finally, we have terms that involve the different 126_H's; they are of the form

$$(126_i\otimes 126_j)\otimes(\overline{126_i}\otimes\overline{126_j}) \qquad i\neq j,\ i,j = 1,2,3. \tag{4.18}$$

There are three such terms corresponding to different values of (i,j) and from (4.7) and (4.8) we see that we have at most six different coupling schemes for each, for a total of eighteen terms! There is also one other term which has the form

$$(126_1\otimes 126_2)\otimes\ (\overline{126_3}\otimes\overline{126_3}) \tag{4.19}$$

The potential thus consists of five quadratic invariants, one for each Higgs representation, and fifty-nine independent quartic invariants, not counting those linear in the 10_H and the 54_H.

Without entering into the details of the potential to a greater degree, we can check whether or not our set of vacuum expectation values can be maintained in perturbation theory. The procedure is the following: expand any Higgs field about its v.e.v. and check that the magnitude of the v.e.v's can be adjusted so that the potential does not contain any terms linear in the expanded fields. This must shown to be possible for a finite range of the parameters in the Higgs potential. If a particular linear term comes only from one invariant, or if the above constraints are incompatible with one another, the postulated set of vacuum expectation values is not natural. It should be mentioned that arranging a natural set of v.e.v's that depends on the interplay between the discrete symmetries of the potential and the couplings allowed by gauge invariance is a non-trivial task. We will demonstrate the consistency of our v.e.v's only to lowest order in the gauge hierarchy.

In our case it is convenient to expand the fields in terms of their $SU(5)$ components. As an example we set

$$10_H = <5>_{10} + <\bar{5}>_{10} + \zeta_5 + \zeta_{\bar{5}} \tag{4.20}$$

$$126_1 = <1>_{126_1} + <\bar{5}>_{126_1} + \chi_1 + \chi_{\bar{5}} + \chi_{10} + \chi_{\overline{15}} + \chi_{45} + \chi_{50} \tag{4.21}$$

etc. and check that it is possible to choose the v.e.v.'s so that terms linear in the ζ's , χ's ,... vanish. This is a terrible task which is somewhat alleviated by enforcing a gauge hierarchy: some of the v.e.v.'s are much smaller than others. Call the small (large) v.e.v.'s v (V). When we expand the potential and look at the linear terms we demand that they be made to vanish to each order in v and V . As an example, the terms of $O(vV^2)$ coming from the invariant $\overline{10} \otimes 126_1 \otimes 126_1 \otimes 126_1$ contains a term linear in $\chi_{\bar{5}}$ and to this order it is the only such term in the potential. Hence it would have been unnatural to require that the 126_1 have a v.e.v. along the $SU(5)$ singlet only. The term linear in χ_{45} coming from this term is of $O(v^3)$ and there are other terms of this order in the potential so that we need not require that the 126_1 have a component along the 45 of $SU(5)$ as well. The presence of the $\langle\bar{5}\rangle$ in the 126_1 is crucial since it prevents the model from predicting too low a mass for the t quark.

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Appendix A: Conventions for Fermion Fields

We describe spin-1/2 fermions by two-component fields of definite chirality: left-handed fields are denoted ψ_L and right-handed fields ψ_R. For massless fermions, chirality and helicity are equivalent and the two chirality states are independent. Only one of the states need therefore be present in a model (for massless neutrinos ν_R is absent).

For the two-component fields, ψ_L^c denotes the left-handed antiparticle of ψ_R, while ψ_R^c denotes the right-handed antiparticle of ψ_L. For fields in which both helicity states are present, parity (P) serves to interchange L and R components, while charge conjugation (C) interchanges particles with antiparticles of the same chirality, according to:

P: $\psi_L \to \psi_R \qquad \psi_R \to \psi_L$

C: $\psi_L \to \psi_L^c = \sigma_2 \psi_R^* \qquad \psi_R \to \psi_R^c = -\sigma_2 \psi_L^*$

CP: $\psi_L \to -\sigma_2 \psi_L^* \qquad \psi_R \to \sigma_2 \psi_R^*$

σ_2 is a Pauli matrix. These transformations are summarized in Figure A.1. Note the important feature that while the definition of individual C and P transformation properties require the presence of both L and R states, CP transformation properties may be defined for massless particles with only a single helicity state.

The two-component fermion fields may be collected into a four-component Dirac spinor describing a fermion of arbitrary helicity: $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$. It is convenient to take the Dirac gamma matrices which act on this spinor in the Weyl representation:

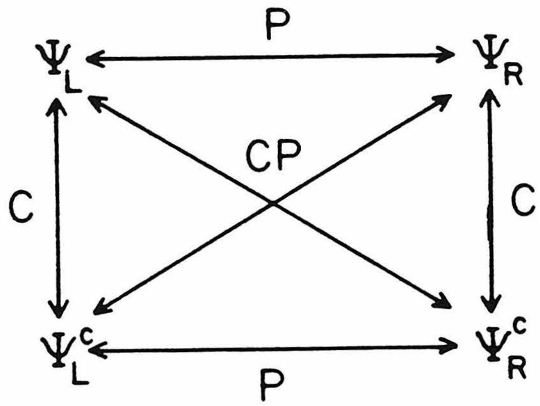

Figure A.1: Charge conjugation (C), Parity (P), and CP transformations among left and right-handed components of particles (Ψ), and antiparticles (Ψ^c).

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \tag{A.1}$$

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

with σ^i ($i=1, 2, 3$) the usual Pauli matrices. (This representation differs from the more usual Dirac representation simply by the interchange $\gamma^0 \leftrightarrow \gamma^5$.)

The kinetic energy term in the fermion Lagrangian is given by

$$\overline{\Psi} \partial\!\!\!/ \Psi = \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + \psi_R^\dagger \overline{\sigma}^\mu \partial_\mu \psi_R \tag{A.2}$$

with $\sigma^\mu = (1, \sigma^i)$, $\overline{\sigma}^\mu = (1, -\sigma^i)$, and $\overline{\Psi} = \Psi^\dagger \gamma^0$.

Fermion fields for which both helicity states are present may give a Dirac mass term

$$m\overline{\Psi}\Psi = m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R) \ . \tag{A.3}$$

If only one helicity is present, say ψ_L, no Dirac mass term may be constructed, but a Majorana mass term is still possible:

$$m\overline{\Psi^c} \frac{(1+\gamma_5)}{2} \Psi = m\psi_L^T \sigma_2 \psi_L \ . \tag{A.4}$$

Here the charge-conjugate four-component spinor Ψ^c is given by

$$\Psi^c = \begin{pmatrix} \psi_L^c \\ \psi_R^c \end{pmatrix} = \begin{pmatrix} \sigma_2 \psi_R{}^* \\ -\sigma_2 \psi_L{}^* \end{pmatrix} \ . \tag{A.5}$$

For a fermion field with only a single helicity state, it is sometimes convenient to define a four-component Majorana spinor

$$\Psi_M = \begin{pmatrix} \psi_L \\ -\sigma_2 \psi_L^* \end{pmatrix} \tag{A.6}$$

in terms of which the Majorana mass term becomes $\frac{m}{2}\bar{\Psi}_M \Psi_M$. Since Ψ_M involves only four real degrees of freedom, one can find a representation of the γ matrices, termed the Majorana representation, in which Ψ_M and the Dirac equation are purely real.

Note that fields with Majorana mass terms may not carry any $U(1)_Q$ charges since the mass term is not invariant under the phase transformation $\psi_L \to e^{i\alpha Q}\psi_L$. Fermion fields with Majorana masses therefore lead to violation of lepton number and hence the possibility of neutrino oscillations.

In grand unified theories it is convenient to deal only with left-handed spinors $\psi_{L1}, \psi_{L2}, \cdots$ with some of the ψ_{La} corresponding to left-handed particles while others correspond to left-handed anti-particles which are related to the right-handed components of particles through the operation of charge conjugation. It is therefore clear that a Dirac mass term of the form $\psi_L^\dagger \psi_R + h.c.$ can be reinterpreted as an off-diagonal Majorana mass term involving only left-handed fields of the form $\psi_L^T \sigma_2 \psi_L' + h.c.$ with $\psi_L' = \sigma_2 \psi_R^*$.

In theories with both Dirac and Majorana masses present it is convenient to write all mass terms in terms of left-handed fields so that the most general mass term is given by

$$\psi_{La}^T M_{ab} \sigma_2 \psi_{Lb} + h.c. \qquad a,b = 1 \ldots N \tag{A.7}$$

where M_{ab} is in general a complex symmetric N by N matrix. The diagonal entries of M correspond to true Majorana mass terms while the off-diagonal entries may be reinterpreted as Dirac masses.

Under the CP transformation

$$\psi_{La} \to \sigma_2 \psi^*_{La} \tag{A.8}$$

so that (7) is CP-invariant only if M is purely real. In the general case M may be diagonalized using Schur's theorem [1] which says that a complex symmetric matrix may be diagonalized by means of a unitary congruence:

$$M = U^T D U \tag{A.9}$$

with U a unitary matrix and D a diagonal matrix with real positive entries. The entries of D may be obtained as the square roots of the eigenvalues of the matrix $M^* M$ since

$$M^* M = U^\dagger D^2 U \tag{A.10}$$

which has manifestly positive eigenvalues. From (9) we see that the fermion mass eigenstates are given by

$$\psi^m_{La} = U_{ab} \psi_{Lb} \tag{A.11}$$

Appendix B: Spinor Couplings

Explicit construction of the gauge and Yukawa couplings in $SO(10)$ gauge models requires a study of how the various irreducible representation (irreps) are built up from products of the spinor irreps. Since there has been recent interest in the use of orthogonal groups for model building [2], we first present a general analysis for $SO(N)$ [3] before specializing to $SO(10)$.

1. SO(N) for N even, N = 2n

The spinor irreps of $SO(2n)$ are most easily studied by introduction of generalized Dirac γ matrices. We introduce $2n$ quantities $\Gamma_i, i=1...2n$ that convert the basic quadratic form into the square of a linear form:

$$x_1^2 + x_2^2 + \cdots + x_{2n}^2 = (\Gamma_1 x_1 + \Gamma_2 x_2 + \cdots + \Gamma_{2n} x_{2n})^2. \tag{B.1}$$

This requires that

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \tag{B.2}$$

(We indicate commutation by square brackets and anticommutation by wavy brackets.) For $SO(2n)$ the Γ_i may be represented by $2^n \times 2^n$ matrices.

If $R \in SO(2n)$ then the $2^n \times 2^n$ matrices $S(R)$ such that

$$S(R)\Gamma_i S(R)^{-1} = R_{ij}\Gamma_j \qquad i = 1...2n \tag{B.3}$$

form a 2^n-dimensional representation of $SO(2n)$ which we denote by Δ. The generators of this representation are given by

$$\Sigma_{ij} = \frac{1}{2}[\Gamma_i, \Gamma_j] \tag{B.4}$$

as can be seen by writing for an infinitesimal rotation $R_{ij} \cong \delta_{ij} + \omega_{ij}$, with

$\omega_{ij} = -\omega_{ji}$. We then have $S(R) \cong 1+i\frac{\omega_{ij}}{2}\Sigma_{ij}$ so that

$$S(R)\Gamma_i S(R)^{-1} \cong \Gamma_i + i\omega_{kl}[\Sigma_{kl},\Gamma_i] = \Gamma_i + \omega_{ij}\Gamma_j = R_{ij}\Gamma_j \tag{B.5}$$

where we have used (B.2) and (B.4). The generators in (B.4) satisfy the commutation relations

$$[\Sigma_{ij},\Sigma_{kl}] = i(\delta_{ik}\Sigma_{jl} + \delta_{jl}\Sigma_{ik} - \delta_{il}\Sigma_{jk} - \delta_{jk}\Sigma_{il}). \tag{B.6}$$

Since all the S(R) commute with $\Gamma_{2n+1} = (-i)^{2n}\Gamma_1 \cdots \Gamma_{2n}$, Δ is reducible into two irreps, Δ_+ and Δ_-, each of dimension 2^{n-1} formed by projection with $\frac{1 \pm \Gamma_{2n+1}}{2}$. For n odd, Δ_+ and Δ_- form complex irreps and are equivalent to the complex conjugates of each other. For n even, Δ_+ and Δ_- form two inequivalent irreps with the complex conjugate of each being equivalent to itself. For n even, if we write $n = 2m$ then for m even, Δ_+ and Δ_- form real representations (also called orthogonal like representations) with

$$(\Delta_+ \otimes \Delta_+)_S \supset 1, \qquad (\Delta_- \otimes \Delta_-)_S \supset 1. \tag{B.7}$$

where the subscript S indicates the symmetrized product. For m odd, Δ_+ and Δ_- form pseudoreal representations (also called symplectic like representations) with

$$(\Delta_+ \otimes \Delta_+)_A \supset 1, \qquad (\Delta_- \otimes \Delta_-)_A \supset 1 \tag{B.8}$$

where the subscript A indicates the antisymmetrized product.

The Γ matrices for $SO(2n)$ may be built up inductively from those for $SO(2n-2)$. A convenient basis in which Γ_{2n+1} is diagonal with +1 in the first n diagonal entries and -1 in the last n is given by

$$\Gamma_i = \sigma_1 \otimes \hat{\Gamma}_i \qquad i = 1,2,\ldots,2n-2 \tag{B.9}$$

$$\Gamma_{2n-1} = \sigma_1 \otimes \hat{\Gamma}_{2n-1}$$

$$\Gamma_{2n} = \sigma_2 \otimes 1$$

$$\Gamma_{2n+1} = (-i)^n \Gamma_1 \cdots \Gamma_{2n} = \sigma_3 \otimes 1$$

where $\hat{\Gamma}_i$, $i=1,2,\ldots 2n-2$ are the Γ matrices for $SO(2n-2)$, $\hat{\Gamma}_{2n-1} = (-i)^n \hat{\Gamma}_1 \cdots \hat{\Gamma}_{2n-2}$ and σ_a $a=1,2,3$ are the usual Pauli matrices. The induction starts at $n = 1$ with $\Gamma_1 = \sigma_1, \Gamma_2 = \sigma_2$.

In the above representation of the Γ matrices the generators are given by

$$\Sigma_{ij} = \begin{pmatrix} \sigma_{ij}^+ & 0 \\ 0 & \sigma_{ij}^- \end{pmatrix} \tag{B.10}$$

where σ_{ij}^+ and σ_{ij}^- are the $2^{n-1} \times 2^{n-1}$ matrix generators of Δ_+ and Δ_- respectively. In terms of $\hat{\Sigma}_{rs}$ defined by

$$\hat{\Sigma}_{rs} = \frac{1}{2i}[\hat{\Gamma}_r, \hat{\Gamma}_s] \quad r,s = 1\ldots 2n-1 \tag{B.11}$$

we have

$$\sigma_{rs}^+ = \sigma_{rs}^- = \hat{\Sigma}_{rs} \qquad r,s = 1\ldots 2n-1 \tag{B.12}$$

$$\sigma_{r\,2n}^+ = -\sigma_{r\,2n}^- = \hat{\Gamma}_r \quad r = 1\ldots 2n-1$$

To construct the scalar, vector, second-rank tensor, etc. representations from the spinor representations we consider a 2^n-component covariant spinor ψ^a transforming under the representation Δ:

$$\psi \to S\,\psi \tag{B.13}$$

and a 2^n-component contravariant spinor φ_a transforming under the contragredient representation $\hat{\Delta}$:

$$\varphi \rightarrow (S^{-1})^T \varphi \tag{B.14}$$

The representations Δ and $\hat{\Delta}$ are unitarily equivalent as is evidenced by the existence of a matrix C such that

$$CSC^{-1} = (S^{-1})^T \tag{B.15}$$

or using (B.3), $C\Gamma_i C^{-1} = \Gamma_i^T$. In our representation of the Γ matrices

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = C^{-1}. \tag{B.14a}$$

The scalar is formed from ψ^a and φ_a as

$$\varphi^T \psi = \sum_{a=1}^{2n} \varphi_a \psi^a \tag{B.16}$$

while the vector is formed by

$$\varphi^T \Gamma_i \varphi \tag{B.17}$$

To see that this is indeed the vector note that under $\psi \rightarrow S\psi$, $\varphi^T \rightarrow \varphi^T S^{-1}$ we have

$$\varphi_T \Gamma_i \psi \rightarrow \varphi^T S^{-1} \Gamma_i S \psi = R_{ij} \varphi^T \Gamma_j \psi \tag{B.18}$$

using (B.3). The second-rank antisymmetric tensor is formed by

$$\varphi^T \Gamma_i \Gamma_j \psi \quad i \neq j \tag{B.19}$$

etc. Note that we have

$$C\psi \rightarrow CS\psi = (S^{-1})^T C\psi \tag{B.20}$$

so that $C\psi$ transforms as a contravariant spinor and we may form the antisymmetric tensor representations as

$$\psi^T C^T \psi, \quad \psi^T C^T \Gamma_i \psi, \quad \psi^T C^T \Gamma_i \Gamma_j \psi \tag{B.21}$$

etc. In general we have

$$\Delta \otimes \hat{\Delta} = T_0 + T_1 + \cdots + T_{2n} \tag{B.22}$$

where T_i is the antisymmetric tensor representation of rank i. Any representation T_f gives rise to another representation T_{2n-f} through the $SO(2n)$ invariant relation

$$\alpha_{i_1 \cdots i_{2n-f}} = \frac{1}{f!} \varepsilon_{i_1 \cdots i_{2n-f}}^{j_1 \cdots j_f} \alpha_{j_1 \cdots j_f} \tag{B.23}$$

where $\alpha_{j_1 \cdots j_f}$ transforms under T_f and $\varepsilon_{i_1 \cdots i_{2n}}$ is the totally antisymmetric symbol on $2n$ indices. We thus have $T_{2n-f} \sim T_f$ upon restriction to proper orthogonal transformations. For improper orthogonal transformations, $T_{2n-f} \sim -T_f$. The tensor representation T_n of dimension $\binom{2n}{n}$ splits into two irreps T_n^+ and T_n^- of dimension $\frac{1}{2}\binom{2n}{n}$ according to the $SO(2n)$ invariant decomposition of a tensor of rank n, $\alpha = \alpha^+ + \alpha^-$, with

$$\alpha^{\pm}_{i_1 \cdots i_n} = \frac{1}{2}(\delta_{i_1 j_1} \cdots \delta_{i_n j_n} \pm \frac{1}{n!} \varepsilon_{i_1 \cdots i_n j_1 \cdots j_n}) \alpha_{j_1 \cdots j_n} \tag{B.24}$$

The representations T_n^+ and T_n^- are real for n even and the complex conjugates of each other for n odd.

Upon splitting Δ into Δ_+ and Δ_- we have the following decomposition for the products appearing in $\Delta \otimes \Delta$:

$$\Delta_+ \otimes \Delta_+ = T_0 + T_2 + \cdots \tag{B.25}$$

$$\Delta_- \otimes \Delta_- = T_0 + T_2 + \cdots \qquad (n\ even)$$

$$\Delta_+ \otimes \Delta_- = T_1 + T_3 + \cdots$$

$$\Delta_+ \otimes \Delta_+ = T_1 + T_3 + \cdots \tag{B.26}$$

$$\Delta_- \otimes \Delta_- = T_1 + T_3 + \cdots \qquad (n\ odd)$$

$$\Delta_- \otimes \Delta_+ = T_0 + T_2 + \cdots$$

2. SO(N) for N odd, N = 2n + 1 .

In this case we take the $2n+1$ quantities Γ_i, $i=1..2n+1$ defined in (B.9) and define the representation $S(R)$ through the correspondence (B.3) as before but now with $i=1...2n+1$. This 2^n-dimensional spinor representation Δ is irreducible and real (orthogonal like) for n=3,4,7,8,11,12,... and pseudoreal (symplectic like) for n=5,6,9,10,13,14,.... The matrix C defined in (B.14) satisfies

$$C\Gamma_i C^{-1} = \Gamma_i^T \qquad i=1...2n \tag{B.27}$$

$$C\Gamma_{2n+1}C^{-1} = (-1)^n \Gamma_{2n+1}^T$$

and thus serves to relate covariant and contravariant spinors only for n even. For n odd the matrix $C\Gamma_{2n+1}$ must be used for this purpose. The various antisymmetric tensor representations are then constructed as before and we have

$$\Delta \otimes \Delta = T_0 + T_1 + \cdots + T_n \tag{B.28}$$

3. SO(10) .

In accordance with our previous discussion we take the 32 by 32 Γ matrices for $SO(10)$ to be

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \qquad \text{(B.29)} \\
\Gamma_2 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \\
\Gamma_3 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \\
\Gamma_4 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes 1 \\
\Gamma_5 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes 1 \\
\Gamma_6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes 1 \otimes 1 \\
\Gamma_7 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes 1 \otimes 1 \\
\Gamma_8 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \\
\Gamma_9 &= \sigma_1 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes 1 \\
\Gamma_{10} &= \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1
\end{aligned}$$

with $\Gamma_{11} = \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1$. The generators for the 32-dimensional spinor representation are given by

$$\Sigma_{ab} = \frac{1}{2i}[\Gamma_a, \Gamma_b] = \begin{pmatrix} \sigma_{ab}^+ & 0 \\ 0 & \sigma_{ab}^- \end{pmatrix} \qquad \text{(B.30)}$$

Since $SO(10)$ has rank five there are five diagonal commuting generators which form the Cartan subalgebra of $SO(10)$. In our basis they are given by

$$\Sigma_{12} = 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \qquad \text{(B.31)}$$

$$\Sigma_{34} = 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3$$

$\Sigma_{56} = 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1$

$\Sigma_{78} = 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1$

$\Sigma_{910} = \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes 1$

Upon projection with $\frac{1}{2}(1 \pm \Gamma_{11})$, the 32-dimensional spinor representation breaks up into two irreps, 16 and $\overline{16}$, with the $\overline{16}$ being equivalent to the complex conjugate of 16. If we follow the conventional assignment of the left-handed fermion states to the 16 then the $\overline{16}$ contains the right-handed CP conjugate states.

The product $16 \otimes 16$ then transforms as a Lorentz scalar and contains the $SO(10)$ antisymmetric tensor representations of odd rank:

$$16 \otimes 16 = T_1 + T_3 + T_5^+ = 10 + 120 + 126 \tag{B.32}$$

while the product $16 \otimes \overline{16}$ transforms as a Lorentz vector and contains the antisymmetric tensor representations of even rank:

$$16 \otimes \overline{16} = T_0 + T_2 + T_4 = 1 + 45 + 210. \tag{B.33}$$

If ψ_L is a column vector containing the 16 left-handed two component fermion fields, $\psi_L \sim 16$, then the coupling to the gauge fields is given by

$$\psi_L^\dagger \sigma^\mu D_\mu \psi_L \tag{B.34}$$

with $\sigma^\mu = (1, \sigma_i)$ acting on Lorentz indices and $D_\mu = \partial_\mu + \frac{ig\,\sigma_{ab}}{2} A_\mu^{ab}$ where $A_\mu^{ab} = -A_\mu^{ba}$, $a,b = 1...10$ are the 45 vector gauge fields.

The $SO(6) \sim SU(4)$ subgroup of $SO(10)$ is generated by σ_{mn} $m,n = 1..6$. The $SO(4) \sim SU(2)_L \otimes SU(2)_R$ subgroup has the generators

$$T^i_{L(R)} = \frac{1}{8}\varepsilon^{ijk}\sigma^{6+j,6+k} \pm \frac{1}{4}\sigma^{6+i,10} \qquad i,j,k = 1,2,3 \tag{B.35}$$

where with this normalization $T^3_{L(R)}$ is $\pm 1/2$ when acting on a $SU(2)_{L(R)}$ doublet. The $SU(3)_c$ subgroup of $SU(4)$ is generated by

$$\sigma_{12} + \sigma_{34},\ \sigma_{13} + \sigma_{24},\ \sigma_{14} + \sigma_{23},\ \sigma_{12} - \sigma_{26} \tag{B.36}$$

$$\sigma_{16} + \sigma_{25},\ \sigma_{35} + \sigma_{46},\ \sigma_{36} - \sigma_{45},\ \frac{1}{\sqrt{3}}(\sigma_{12} - \sigma_{34} + 2\sigma_{56})$$

while the $U(1)$ generator in $SU(4)$ not in $SU(3)_c$ is given by

$$B-L = -\frac{1}{3}(\sigma_{12} - \sigma_{34} - \sigma_{56}). \tag{B.37}$$

The hypercharge, Y, is a linear combination of $B-L$ and T^3_R,

$$Y = -\frac{1}{2}(B-L) - T^3_R \tag{B.38}$$

The electric charge is given as usual by $Q = T^3_L - Y$. With these assignments and the representation of the Γ matrices given by (B.29) the fermions are embedded in the 16 as

$$\psi^T_L = (U_b, U_r, \nu, U_g, U^c_r, U^c_b, U^c_g, N^c, D_b, D_r, E, D_g, D^c_r, D^c_b, D^c_g, E^c)_L \tag{B.39}$$

where b, r, and g are color labels.

In order to write the Lorentz scalar, $SO(10)$-invariant Yukawa couplings we first introduce a 32-component spinor $\Psi = \begin{pmatrix}\psi_L \\ 0\end{pmatrix}$. If φ_a, $a = 1...10$ is a scalar transforming as a 10 under $SO(10)$ then according to (B.16) the Yukawa coupling of φ_a is given by

$$\Psi^T C^T \Gamma_a \sigma_2 \Psi \varphi^a + h.c. \tag{B.40}$$

where σ_2 is a Pauli matrix acting on Lorentz indices. The coupling to $\varphi_{[a,b,c]} \sim 120$ is given by

$$\Psi^T C^T \Gamma_a \Gamma_b \Gamma_c \sigma_2 \Psi \varphi^{[a,b,c]} + h.c. \tag{B.41}$$

while following (B.21) the coupling of $\varphi^{+}_{[a,b,c,d,e]} \sim \overline{126}$ is given by

$$\Psi^T C^T (\frac{1-\Gamma_{11}}{2}) \Gamma^a \Gamma^b \Gamma^c \Gamma^d \Gamma^e \sigma_2 \Psi \varphi^{+}_{[a,b,c,d,e]} + h.c. \tag{B.42}$$

In the representation (B.29) we have

$$\Gamma_a = \begin{pmatrix} 0 & \gamma_a^{\dagger} \\ \gamma_a & 0 \end{pmatrix} \tag{B.42}$$

with γ_a a 16 by 16 matrix and $\gamma_a^{\dagger}$ the hermitean conjugate of γ_a. We can then write the couplings of the 10, 120, and 126 as

$$\psi_L^T \gamma_a \sigma_2 \psi_L \varphi^a \tag{B.43}$$

$$\psi_L^T \gamma_a \gamma_b^{\dagger} \gamma_c \sigma_2 \psi_L \varphi^{[a,b,c]}$$

and

$$\psi_L^T \gamma_a \gamma_b^{\dagger} \gamma_c \gamma_d^{\dagger} \gamma_e \sigma_2 \psi_L \varphi^{+[a,b,c,d,e]}$$

References for Appendices

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