A LINEAR STRAIN SEISMOGRAPH

THE PHYSICAL EVALUATION OF
SEISMIC DESTRUCTIVENESS

A METHOD FOR THE INSTRUMENTAL DETERMINATION
OF THE EXTENT OF FAULTING

Thesis
by
Hugo Benioff

In partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California
1935

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by Hugo Benioff

Carnegie Institution of Washington
Seismological Research
Pasadena, California

## SUMMARY

A non-pendular seismograph is described, having a response which depends upon the linear strains between two points of the ground. Essentially, the seismometer consists of two piers separated by an interval of 20 meters, ~~and~~ with a horizontal bar ~~which is~~ rigidly fastened to one pier and ~~xxxxx~~ extend~~s~~ing nearly to the other pier. Relative movements of the two piers thus actuate an electromechanical transducer which operates between the free end of the bar and the adjacent pier. The resulting induced currents are recorded by two galvanometers having different constants. Three different galvanometer combinations have been used successfully, the galvanometer periods being 0.2, 1.3, and 35 seconds. The equivalent pendular magnifications are 80,000, 10,000 and 100 respectively.

A theory is developed for the linear strain seismograph and related instruments.

# A LINEAR STRAIN SEISMOGRAPH (1)

(1) The name first given to this instrument was "wave seismograph" because of its analogy to the wave antenna in radio. However, the analogy is not close and consequently it seemed best to adopt the more accurate name given above. The instrument is mentioned and described in the Carnegie Institution of Washington Year Book No. 29, 1929-30, and subsequent numbers. It was also described in a paper read before the meeting of the Seismological Society of America held in Pasadena, June, 1931.

The response of hitherto known practical forms of seismographs depends upon the relative motion of a pendulum and the moving ground to which its supporting structure is fastened. The various kinds of seismographs differ in the type of pendulum used, such as gravity, spring, torsion, etc. and/or they differ in the type of magnifying and recording elements which they employ. They all measure or indicate the vibratory motion of the ground at a given point. In contrast to these earlier forms, the strain seismograph is a nonpendular instrument. It does not respond directly to vibratory movements. Its operation depends upon variations in the distance between two points of the ground. Such variations or linear strains are set by seismic waves. A schematic drawing of the seismometer is shown in Fig. 1 . A and B are two piers separated by a distance of 20 meters. One end of the rod R is rigidly fastened to the pier B. The other end extends to within a short distance of Pier A. Earth strains resulting from a seismic wave-train produce variations in the separation of the two piers and these variations are observable as changes in the distance between the free end of the rod and pier A. These small movements of the end of the rod relative to the adjacent pier serve to actuate an electro-mechanical transducer which generates an e.m.f. proportional to the rate of change of the relative displacement. Recording is accomplished by means of galvanometers. A photograph of the seismometer is reproduced in Fig. 2.

## EARLIER FORMS

Subsequent to the development and construction of the instrument herein described, it was brought to the writer's attention that the basic principle had been used previously by John Milne and later by E. Oddone. It was Milne's opinion that relative movements of the ground upon which buildings rested were responsible for some of the damage caused by earthquakes. In order to exhibit these relative movements he set up a device (1) consisting

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(1) John Milne, The Relative Motion of Neighboring Points of Ground. Transaction of the Seismological Society of Japan, Vol. XII, 1888, P. 63.

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of two piers separated by an interval of three feet. To one pier he fastened a rod which extended horizontally to within a small distance of the other pier. A lever system having a magnification of six served to record the relative motion of the free end of the rod with respect to the adjacent pier. With this apparatus he obtained traces of a few millimeters maximum amplitude in some thirteen large local earthquakes. It is apparent that his instrument was very insensitive - in fact for earth-periods of one second the ratio of recorded amplitude to actual earth-displacement was 1/30. Oddone's instrument (1) was similar to Milne's. He increased

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(1) E. Oddone, Ricerche Strumentali in Sismometria con Apparati non Pendulari. Bolletina della Societa Sismologica Italiana, 1900-1901, Vol. XI, P. 168.

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the pier separation to three meters and employed a hydraulic device for indication. This hydraulic indicator was, in effect, an iron box of 18 liters capacity filled with water. A hole in one side of the box contained a piston flexibly fastened to the box by means of a diaphragm and at the same time rigidly bolted to the free end of the rod. Movements of the rod relative to the adjacent pier thus changed the level of the liquid in a glass tube of small bore which communicated with the water in the box. The area of the piston was approximately 3600 times that of the section of the glass tube so that the magnification was 3600. The instrument was sensitive enough to show movements of the meniscus resulting from the passage of trains in the vicinity. The magnification for earth displacements having periods of 1 second was approximately 13. Oddone did not succeed in constructing a satisfactory recording mechanism and as a consequence he was unable to observe the behavior of his instrument with respect to earthquakes. G. Agamenonne (2) adversely criticised the instrument in regard to ques-

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(2) G. Agamennone, Sulla Pretesa Insufficienza degli Apparati Pendolari in Sismometria, loc. cit. Vol. XIII, 1902-1903, P. 49

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tions which, in the light of present knowledge, are no longer of consequence.

It is clear that neither of these early instruments was satisfactory for routine operation - the one because of its extremely low sensitivity and the other because of low sensitivity and lack of a suitable recording mechanism. A further difficulty lay in the fact that a theory for this type of instrument was not available.

The development of a highly sensitive electromagnetic transducer [1] for the writer's new vertical seismograph opened the way

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(1) Hugo Benioff, A New Vertical Seismograph, Bulletin of the Seismological Society of America, Vol. 22, No. 2, June 1932.

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for the construction of a satisfactory strain seismograph. This transducer, in connection with suitable galvanometers, provides magnifications up to 1,000,000 with complete stability of operation. For example, with such a magnification, mechanical movements of $10^{-7}$ cm. or ~~1/2000~~ 2/1000 the wave-length of sodium light produce galvanometer deflections of 1 mm. Within its useful range of frequencies this device is considerably more sensitive to displacements than a Michelson interferometer. With the strain seismograph, magnifications of about 300,000 are sufficient to raise the sensitivity to the maximum value allowable by the microseismic activity of the ground in Pasadena.

## CONSTRUCTION

The piers are made of sections of 12-inch standard iron pipe 2 meters in length. These are sunk approximately 1.5 meters into the weathered granite underlying the Laboratory, and are cemented in with concrete. The submerged sections of the pipes are filled with concrete to the level of the floor. (See Fig. ). The distance between the piers is approximately 20 meters. The rod is made of 2-inch inside diameter standard iron pipe and is rigidly attached to one of the piers. In order to reduce short period temperature variations, the rod is surrounded by a 2 cm thick layer of asbestos insulation. It is supported by 12 structures of the type shown in Figs. 3 and 4. These are distributed at equal distances along its length. H (Fig. 3) is constructed of 1.25-inch iron pipe and is bolted securely to the concrete floor. R is a ring of steel with set-screws S bearing on the seismometer rod P. The ring R is supported by three taut bicycle spokes B. These 12 structures effectively constrain the rod so that it can move only in the longitudinal direction. (1) Since the rod is clamped at

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(1) When the rod is unclamped from the pier and allowed to oscillate longitudinally as a pendulum, its period is approximately 0.25 sec. The total mass of the rod is approximately 100 kgs. Hence the restoring-force due to the combined effects of the 12 supporting structures is $m\omega^2 = 6\times10^7$ dynes per cm displacement. ($m$ = mass, $\omega = 2\pi/T$, and $T$ = free period of pendulum). When the bar is clamped to the pier, the force required to elongate it 1 cm is $YA/L$, where $Y$ is Young's Modulus, $A$ the effective section of the bar, in this case 6.6 cm, and $L$ the length of the bar. Thus $YA/L = 5\times10^9$ dynes approximately. The effective restoring-force of the 12 supports acting on the clamped rod is one half the value for the unclamped rod. Hence, the ratio of the restoring-force of the supports to the longitudinal stiffness of the rod is $3\times10^7/5\times10^9 = 6\times10^{-3}$ or approximately one half of one percent. Therefore the longitudinal restraining effects of the supports may be neglected.

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one end, its longitudinal period is equal to the time required for a longitudinal wave to travel four times the length of the rod. Thus for this instrument, the period is 80/5,000 = 0.016 second approximately.

## ELECTRO-MECHANICAL TRANSDUCER

The transducer is a modified form of the one devised for the writer's electromagnetic pendulum seismographs. A detailed description with theory will appear in a future paper and consequently only a brief description is given here. Referring to the schematic drawing of the transducer, Fig. 5 , M is a permanent magnet which supplies magnetic flux to the pole-pieces B, B. From the pole-pieces, the flux crosses the air-gaps dividing equally between the two armatures A, A. The structure consisting of magnet and pole-pieces is bolted to the free end of the seismometer rod, and the armature assembly is fastened rigidly to the adjacent pier. Movement of the rod relative to the pier thus varies the lengths of the air-gaps, one pair increasing while the other is decreasing. The resulting change in flux through the armatures induces an e.m.f. in the coils C, C surrounding the armatures, which is proportional to the rate of change of the relative displacement. The coils are connected in series aiding. Since a flux increase in one pair of gaps is offset by a corresponding decrease in the other pair, the total flux through the magnet remains constant. In this way, difficulties due to the high reluctance and hysteresis of the permanent magnet circuit are entirely avoided. By virtue of the push-pull structure of the transducer, the output e.m.f. is linear up to terms of the third order of the displacement divided by the air-gap length. A photograph of the transducer is shown in Fig. 6.

## THEORETICAL BEHAVIOR OF THE ROD

During the early stages of the development of this instrument, the theoretical problem of the behavior of the rod was taken up with Dr. P. S. Epstein. As a result of an investigation which he will report in another paper, he showed that the movement of the free end of the rod relative to the bound end depends upon the damping of the rod. Considering seismic waves with periods which are long in comparison with the natural period of the rod, he found that with small damping the free end of the rod moves essentially with the same phase and amplitude as the bound end. In other words, the undamped rod behaves as a rigid body in accordance with our every-day experience. On the other hand, when the damping is not negligibly small, the motion of the free end differs from that of the bound end in both phase and amplitude. Thus a seismometer constructed with a damped rod would exhibit some very interesting properties, one of which would be an asymmetrical response to waves arriving from opposite directions. However, in order to be effective, the damping must be of a type which is independent of frequency and this condition cannot be met except with the aid of a mechanical frame of reference which does not partake of the seismic wave motion. Hence it has not been possible to build a damped instrument. A simple theory of an undamped rod is given in an appendix to this paper.

## THEORY OF THE HORIZONTAL LINEAR STRAIN SEISMOMETER

In the following discussion it is assumed that the rod behaves as a rigid body. The origin of coordinates is taken at the undisturbed position of the free pier. ~~(See Fig. )~~. The line joining the piers defines the x-axis. Let $\xi$ be the horizontal displacement of the ground at the point x. If $\beta$ is the angle between $\xi$ and the direction of x, the component of the displacement parallel to the rod is $\xi \cos \beta$. The linear strain at any point x is, therefore, $\cos \beta \frac{\partial \xi}{\partial x}$. The total strain or relative displacement of the piers parallel to the line joining them is

$$y = \int_0^L \cos \beta \frac{\partial \xi}{\partial x} dx \qquad (1)$$

the e.m.f. induced in the transducer coils is

$$k \frac{\partial y}{\partial t} = k \frac{\partial}{\partial t} \int_0^L \cos \beta \frac{\partial \xi}{\partial x} dx \qquad (2)$$

k is the e.m.f. induced in the coils for unit relative velocity of rod and pier. L is the distance between the piers. Equations (1) and (2) represent the mechanical and electrical response respectively of the undamped strain seismometer for all conditions in which the proper motions of the rod can be neglected. When the disturbance consists of plane waves which are long in comparison to L it is clear that $\beta$ and $\frac{\partial \xi}{\partial x}$ are essentially constant over the interval L. Under these conditions equations (1) and )2) may be integrated immediately so that we may write

$$y = L \cos \beta \frac{\partial \xi}{\partial x} \qquad (3)$$

and

$$k \frac{\partial y}{\partial t} = kL \cos \beta \frac{\partial^2 \xi}{\partial x \partial t} \qquad (4)$$

In common with pendulum seismometers, the strain seismometer does not respond to the true earth waves but rather to the apparent surface waves which appear on the ground as a result of the incidence of the true waves. True waves which propagate horizontally at the surface of the ground, such as Rayleigh and Love waves, are identical with their corresponding apparent waves. Other waves which arrive at the surface with angles of incidence less than $\pi/2$ give rise to apparent waves which in general differ from the originals in type, amplitude, and velocity. For example, an SV wave, that is, a vertically polarized transverse wave, is accompanied by an apparent wave of the longitudinal type. The velocity of the apparent wave is greater than that of the true wave for all angles of incidence less than $\pi/2$. In the case of vertical incidence the apparent velocity is infinite. If C is the true wave velocity and c is the velocity of the apparent wave, we may write $c = C \csc i$, where i is the angle of incidence. In general the apparent velocity is given by the slope of the travel-time curve.

It will be assumed that a seismic disturbance consists of apparent waves of the form

$$\xi = \phi\left(t - \frac{r}{c}\right) \tag{5}$$

where r is the coordinate in the line of propagation, and c is the apparent wave velocity. In longitudinal waves the displacement is parallel to the line of propagation and according to the usual convention the sign is positive when the displacement is in the direction of propagation. Thus, for longitudinal waves $r = x \cos\beta$ and hence the above equation becomes

$$\xi = \phi\left(t - \frac{x\cos\beta}{c}\right) \tag{6}$$

From equation (6) it follows that

$$\frac{\partial \xi}{\partial x} = -\frac{\cos\beta}{c}\frac{\partial \xi}{\partial t}. \tag{7}$$

Substituting the value of $\frac{\partial \xi}{\partial x}$ from equation (7) into equation (3) we find that the response of the strain seismometer to longitudinal apparent waves is

$$y = -\frac{L}{c}\cos^2\beta\,\frac{\partial \xi}{\partial t} \tag{8}$$

In transverse waves the displacement is normal to the direction of propagation and therefore $r = x\cos(\beta - \pi/2) = x\sin\beta$. Thus, for transverse waves equation (5) is written

$$\xi = \phi\left(t - \frac{x\sin\beta}{c}\right) \tag{9}$$

In this case

$$\frac{\partial \xi}{\partial x} = -\frac{\sin\beta}{c}\,\frac{\partial \xi}{\partial t} \tag{10}$$

If this value of $\frac{\partial \xi}{\partial x}$ is substituted into equation (3) we find that the response of the strain seismometer to transverse apparent waves is

$$y = -\frac{L}{c}\sin\beta\cos\beta\,\frac{\partial \xi}{\partial t} \tag{11}$$

In equations (8) and (11) $\beta$ is the angle between the rod and the direction of the earth displacement. The equations are more useful if given in the form containing $\alpha$, the angle in the plane of the ground surface between the rod and the direction of propagation. For longitudinal waves $\alpha = \beta$, and the response of the strain seismometer to longitudinal apparent waves is therefore

$$y = -\frac{L}{c}\cos^2\alpha\,\frac{\partial \xi}{\partial t} \tag{12}$$

In the case of apparent transverse waves $\alpha = \beta - \pi/2$, so that the response of the strain seismometer to these waves is

$$y = \frac{L}{c}\sin\alpha\cos\alpha\,\frac{\partial \xi}{\partial t} \tag{13}$$

Foot note for Page 11

(1) An interesting verification of this behavior was observed on records of an Indian earthquake. Surface waves which travelled over the long arc were recorded, in addition to those which travelled over the short arc. Thus in a single seismogram of a single earthquake, waves were recorded on the short period galvanometer combination which arrived from opposite directions. Upon comparison with the N-S torsion seismogram it was found that the southern group of waves (long arc) were in phase on the two seismograms, whereas in the northern group (short arc) the waves were exactly opposite in phase on the two seismograms.

It is evident from inspection of equations (12) and (13) that the response of the strain seismometer differs in a number of ways from that of the pendulum seismometer. Attention will be given first to differences in the directional characteristics. Fig. 7 shows a polar graph of the function $\cos^2\alpha$, the directional response characteristic of the strain seismometer to longitudinal apparent waves. The response of the pendulum seismometer varies as $\cos\alpha$ and is shown in dotted line for comparison. Similarly, in Fig. 8 are shown polar graphs of the functions $\sin\alpha\cos\alpha$ and $\sin\alpha$ which are the directional characteristics of the strain and pendulum seismometers to transverse apparent waves. It will be noticed that the strain seismometer exhibits four directions of zero response for transverse waves as compared with two for the pendulum instrument. The most striking difference in the directional characteristics of the two instruments is found in the fact that the pendulum response to a given seismic wave, reverses when the direction of propagation of the wave is reversed, whereas the strain response remains the same. Thus, for example, if two identical earthquakes originate at equal distances from the two seismometers but in opposite directions, $0^\circ$ and $180^\circ$ say, the responses of the strain instrument to the two shocks are identical, while those of the pendulum instrument are opposite in sign. ~~This property is useful, since~~ Hence, comparison of a strain seismograph record with a similar component record from a pendulum seismograph results in the elimination of the $180^\circ$ ambiguity in the determination of epicentral azimuth. The initial movement in the pendulum response to an earthquake indicates the direction of the initial earth movement, such as up or down, east or west, north or south, whereas the initial movement of a strain response indicates compression or rarefaction.

Another difference in the behavior of the two types of instruments is that the response of the strain seismometer is inversely proportional to the apparent wave velocity, while the pendulum response is independent of wave velocity. Accurate comparison of the records of the two instruments therefore provides the data for the determination of apparent wave velocities from observations at a single station.

If standing waves are set up in the ground upon which the instruments are placed, the strain response is maximum at a node and zero at an antinode, while the pendulum response is zero at a node and maximum at an antinode.

If the ground executes pendular vibrations such as the movements of a shaking table, there is zero response with a strain seismometer and full response with a pendulum seismometer.

Another striking difference is the greater sensitivity of the strain seismometer for the case where the instrument period is small compared to the wave period. The approximate ratio of sensitivities is readily calculated. If $\alpha = 0$ , the response of the strain seismometer to longitudinal waves propagating horizontally is

$$y = -\frac{L}{c_p} \frac{\partial \xi}{\partial t} \tag{14}$$

The natural period of the rod is $T_0 = \frac{4L}{c_r}$ where $c_r$ is the velocity of longitudinal waves in the rod. To a first approximation $c_p = c_r = 5 \times 10^5$ cm./sec., where $c_p$ is the velocity of longitudinal waves in the ground. Substituting these values for $\frac{L}{c_p}$ in equation (14), the response of the strain seismometer in terms of the free period of the rod is $y = -\frac{T_0}{4}\frac{\partial \xi}{\partial t}$ approximately. If $\xi = a \sin\frac{2\pi}{T} t$, in which $T$ is the period of the wave, the maximum instantaneous value of the strain response is

$$y_m = \frac{a\pi T_0}{2T} \tag{15}$$

The maximum instantaneous response of a pendulum seismometer having a period $T_0$, short in comparison with the seismic wave period is

$$Y_m = \frac{a T_0^2}{T^2} \tag{16}$$

If the two instruments have the same period, $T_0$, the ratio of the strain response to the pendulum response is

$$\frac{y_m}{Y_m} = \frac{\pi T}{2T_0}, \text{ approximately}. \tag{17}$$

As a concrete example, it will be assumed that $T_0 = 1.5\times10^{-2}$ secs. which is approximately the value for the present strain instrument. With this value for $T_0$ the ratio of the strain response to that of a pendulum having the same period is for earth periods of 1 second, $\frac{\pi}{2\times1.5\times10^{-2}} = 100$, approximately. With longer earth periods the ratio is still greater. Another way of expressing this difference between the two instruments is to note that the response of a strain seismometer with a period of $1.5\times10^{-2}$ secs. to earth waves of 1 second period is equivalent to that of a pendulum having a period of $1.5\times10^{-1}$ secs. approximately.

## THEORY OF THE HORIZONTAL ELECTROMAGNETIC LINEAR STRAIN SEISMOGRAPH

Neglecting the directional factor, the response of the horizontal linear strain seismometer to longitudinal apparent surface waves is from equation (12)

$$y = -\frac{L}{c}\frac{\partial \xi}{\partial t}$$

The e.m.f. induced in the coils of the transducer is

$$E = k\frac{\partial y}{\partial t} = -\frac{Lk}{c}\frac{\partial^2 \xi}{\partial t^2} \tag{18}$$

Neglecting the back m.m.f. of the transducer and also the reactances of the transducer and galvanometer coils, it may be assumed that the current is in phase with and proportional to the e.m.f. Hence the differential equation of a galvanometer connected to the transducer coils is

$$\frac{d^2\theta}{dt^2} + 2\epsilon\frac{d\theta}{dt} + \omega_g^2\theta = \frac{gE}{mr} \tag{19}$$

where

$\theta$ = angular deflection of galvanometer in radians

$\epsilon$ = damping constant

$\omega_g = \frac{2\pi}{T_g}$ , $T_g$ = free period of galvanometer

$g$ = electrodynamic constant of galvanometer - the product of the area of the coil, the number of turns, and the field-strength.

$r$ = the sum of the galvanometer and transducer resistances

The mechanical damping of the galvanometer is neglected in comparison with the electromagnetic damping. Setting

$$b = \frac{kgL}{mrc}$$

and introducing the value of $E$ from equation (18) into (19) the differential equation of the electromagnetic strain seismograph

is
$$\frac{d^2\theta}{dt^2} + 2\epsilon\frac{d\theta}{dt} + \omega_g^2\theta = -b\frac{\partial^2\xi}{\partial t^2} \qquad (20)$$

Equation (20) discloses a most remarkable property of the electromagnetic strain seismograph, for it is evident by inspection that this equation is identical in form with the differential equation of the simple pendulum seismograph.[1] The significance of this

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(1) k is positive or negative depending upon the polarity of the galvanometer connections.

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identity is that the frequency response characteristic of an electromagnetic strain seismograph having a galvanometer with period $T_g$ and damping constant $\epsilon$ is identical with that of a simple pendulum having the same period $T_g$ and the same damping constant $\epsilon$. Thus, for example, a strain seismograph with a critically damped galvanometer of 12 secs. period has a frequency response characteristic which is identical with that of a Milne-Shaw seismograph. With a critically damped galvanometer of 0.8 secs. period the strain frequency characteristic is identical with that of the Wood-Anderson short-period torsion seismograph.

The electromagnetic strain seismograph offers a number of advantages over the equivalent simple pendulum seismograph ~~as follows~~:

1. Higher magnification: for example, one of the instruments in routine operation at this Laboratory has a period of 0.2 seconds and an equivalent static magnification of 80,000. The highest magnification available for routine practice with simple pendulum instruments is approximately 3,000 (Wood-Anderson torsion seismographs).

2. Greater flexibility: instruments of a great variety of charact-

eristics are available by merely changing galvanometers or suspensions.

3. Constructional simplicity: the concentrically balanced galvanometer suspension system with electromagnetic damping is substituted for the excentric system of all types of pendulum seismographs.

4. Complete absence of response to earth-tilt: the strain seismometer itself does not respond to tilt and in addition the transducer-galvanometer system does not respond to such slow movements. As a result of this absence of tilt sensitivity it has been possible to build, for routine operation, an instrument having the characteristics of a pendulum seismograph with a period of 54 seconds, critical damping, and an equivalent static magnification of 100. Detailed descriptions of the various galvanometer combinations which have been tested will be given later.

The linear deflection of the galvanometer light spot is

$$z = 2A\theta \tag{21}$$

where A is the distance from the galvanometer lens to the recording drum. Introducing the value of $\theta$ from (21) into equation (20) and setting $V = 2Ab$, the resulting differential equation of the electromagnetic strain seismograph is

$$\frac{d^2z}{dt^2} + 2\epsilon\frac{dz}{dt} + \omega_g^2 z = -V\frac{\partial^2\xi}{\partial t^2} \tag{22}$$

The quantity $V = 2Ab = \frac{2AkgL}{mrc}$ is thus the equivalent pendulum static magnification of the electromagnetic strain seismograph. When $\xi = a\sin\omega t$, equation (22) is written

$$\frac{d^2z}{dt^2} + 2\epsilon\frac{dz}{dt} + \omega_g^2 z = Va\omega^2\sin\omega t \tag{23}$$

Since equation (23) is identical with the well known pendulum seismograph equation it is unnecessary to discuss the general solution here. The steady-state solution is

$$z = \frac{V a \omega^2 \sin(\omega t + \delta)}{\sqrt{(\omega_g^2 - \omega^2)^2 + 4\epsilon^2 \omega^2}} \tag{24}$$

$$\delta = \tan^{-1} \frac{2\epsilon\omega}{\omega^2 - \omega_g^2} \tag{25}$$

Setting

$$P = \frac{\omega^2}{\sqrt{(\omega_g^2 - \omega^2)^2 + 4\epsilon^2 \omega^2}} \tag{26}$$

(See page 19)

equation (24) can be written

$$z = VPa\sin(\omega t + \delta) \tag{27}$$

P is thus the frequency characteristic of the electromagnetic strain seismograph (and also of the pendulum seismograph). It may be expressed in another form. Thus if

$$\frac{\omega_g}{\omega} = u \quad \text{and} \quad \frac{\epsilon}{\omega_g} = h,$$

Equation (26) may be written,

$$P = \frac{1}{\sqrt{(u^2-1)^2+4h^2u^2}} \tag{28}$$

When the damping is critical, h = 1, and P reduces to the simple form

$$P_1 = \frac{1}{u^2+1} \tag{29}$$

Fig. 9 shows a graph of the function P for h = 1 and h $\frac{1}{2}\sqrt{2} = 0.707$

In the form

$$V = \frac{2AgkL}{mrc},$$

the expression for the equivalent static magnification contains the galvanometer constants g and m. These are usually unknown or difficult to determine and consequently it is desirable to substitute practical constants for them. Thus if D is the linear deflection in cms. of the galvanometer light spot for unit current (1 ampere or 1 e.m.u., say) then it is easy to show that

$$D = \frac{T_g^2 Ag}{2\pi^2 m} \tag{30}$$

~~where~~ in which

$$T_g = \frac{2\pi}{\omega_g} = \text{the galvanometer period.} \tag{31}$$

Hence $V = \frac{4\pi^2 kLD}{T_g^2 rc}$

The constants D, $T_g$, and r may be obtained directly from manufacturer's specifications or from easy measurements. The response of the seismograph with any desired galvanometer can thus be readily calculated. It should be remembered that r is the sum of the galvanometer resistance and the transducer resistance. The value of the latter is chosen to conform with the desired damping constant h. If D is given in cms deflection per ampere, r should be expressed in ohms.

A rather wide variety of galvanometric combinations have been assembled for experimental study of the instrument's characteristics. Only three of the more important assemblies will be described, since the observed characteristics of all the combinations conformed closely with theoretical predictions. A combination which has been found very satisfactory for local earthquakes, uses a galvanometer having a period of 0.25 seconds. The galvanometer and its associated recording apparatus is shown in Fig A.10 (1) and _____. The galvanometer pedestal is made of 6 inch iron pipe and is cut off at an angle of 30 degrees to permit the recording-beam to fall conveniently on the upper surface of the recording drum. This arrangement makes for easy inspection and adjustment of the light-spot during operation. The galvanometer is held to the mounting plate by the tension of a stiff helical spring which hooks into the base of the galvanometer. This type of galvanometer mounting is sufficiently strong to withstand severe earthquakes and yet permits delicate adjustment of the levelling screws. The cylindrical lens near the drum is focused by a rack and pinion. The galvanometer lens is focused by sliding the mounting plate along the cut surface of the pedestal. An automotive type tail lamp mounted at the end of the

(1) This galvanometer was designed jointly by Wm. Miller and the writer.

long cylindrical tube serves as light source. The light beam from the lamp is deflected at right angles toward the galvanometer mirror by a prism which is mounted within the tube. Time marks consist of 1 millimeter deflections of the recording light-spot and are produced by the slight bending of the flat spring mounting of the prism. The bending force is transmitted by a phosphor-bronze ribbon from the armature of a modified Baldwin telephone receiver. The period of the time marker is approximately 1/50 sec. The operating current is 5 milliamperes at 5 volts. In addition to its low power consumption and high precision, this type of time marking mechanism has the further advantage of requiring no readjustment when the recording lamp is changed. Fig. 11 shows a photograph of the time marker. Fig. 12 is a copy of a portion of a seismogram written by this combination with an equivalent static magnification of 80,000.

Another useful combination is made with a galvanometer having a period of 1.3 seconds. The frequency-response characteristic of this combination is very nearly the same as that of the short period torsion seismograph. For routine work it has been operated with an equivalent static magnification of 12,000. A portion of the Rayleigh wave group of the Baffin Bay earthquake of November 20, 1933, is shown in Fig. 13 . Section A was written by the N-S short period torsion seismograph (T = 1 second approximately) while B was written by the N-S strain seismograph with the galvanometer of 1.3 seconds period. Since the periods of the seismic waves are long in comparison to the instrument periods, the theoretical responses of the two instruments are proportional to the ground acceleration and consequently the two records should be identical. The fact that they are so very nearly the same is convincing evidence of the correctness of the assumptions concerning the behavior of the strain seismograph.[1]

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(1) During this test the magnification of the strain seismograph was reduced to conform with that of the torsion seismograph.

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Another very useful combination uses a galvanometer with a period of 35 seconds. With this galvanometer an equivalent static magnification of 100 is readily maintained in very stable form. (2)

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(2) During rain storms water leaks into the seismograph tunnel and renders the instrument inoperative.

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This combination is well suited for registration of teleseisms, especially those with very long waves. Figs. 14 and 15 are reduced copies of portions of teleseisms recorded with this instrument. The surface waves in Fig. 14 have a period of approximately 200 seconds. ~~In Fig. , S and SS with period of 70 and 90 secs. are very prominent as well as the later surface waves with period of 200 secs.~~

## VERTICAL COMPONENT STRAIN SEISMOGRAPH

Although a vertical component strain seismograph has not yet been constructed, an outline of its theory will be given in addition to a description of a proposed structure. A possible form of vertical strain seismometer is shown in the diagram of Fig. 16. However, it is not very satisfactory because it has a zero response for S waves travelling horizontally and for P waves travelling vertically. The S wave response is zero as a result of the fact that the motions at A and B are equal in phase and amplitude. The P wave response is zero because the free surface of the earth is a node of strain for vertically incident waves.

By tilting the seismometer of Fig. 16 some 45 degrees as shown in Fig. 17, a partial response to vertical components is obtained for all waves except those of normal incidence. Such an instrument would be expensive to construct and is therefore not recommended.

Another possible instrument is shown in Fig. 18 . If the rod is sufficiently rigid, the difference in vertical elevation of the two piers may be utilized for the response. When the transverse vibration periods of the rod are short in comparison with the shortest significant seismic periods, the system behaves substantially as though the rod were infinitely rigid. By dividing the rod into two sections as shown in Fig. 19 , the rod periods are much shorter for a given amount of structural material. An instrument of this type is to be built in the near future.

## THEORY OF THE VERTICAL STRAIN SEISMOGRAPH

Let $\zeta$ be the vertical displacement of the ground at the point whose horizontal coordinate in the direction parallel to the rods is x. The difference in vertical displacement of two neighboring points of the ground whose coordinates are x and $x+dx$ is

$$\frac{\partial \zeta}{\partial x}\,dx,$$

and consequently the difference in vertical elevation of the two piers distant L from each other is

$$y = \int_0^L \frac{\partial \zeta}{\partial x}\,dx \qquad (32)$$

If the seismic waves are plane, and if L is small compared to the shortest significant seismic wave length, $\frac{\partial \zeta}{\partial x}$ may be considered constant and equation (32) becomes

$$y = L\,\frac{\partial \zeta}{\partial x} \qquad (33)$$

It is clear that the instrument responds to the vertical transverse component of the apparent surface waves. Transverse apparent waves are generated by both P and S body waves which are incident at the surface. If r is the coordinate along the line of propagation of an apparent surface wave and $\alpha$ is the angle between r and the direction of L we may write

$$\frac{\partial \zeta}{\partial x} = \frac{\partial \zeta}{\partial r} \cos \alpha$$

Hence

(34)

If $\zeta$ is expressed as a plane wave of the form $\phi(t - r/c)$ we have the relation

$$\frac{\partial \zeta}{\partial r} = -\frac{1}{c} \frac{\partial \zeta}{\partial t}$$

and the response of the vertical strain seismometer is therefore

$$y = -\frac{L}{c} \cos \alpha \frac{\partial \zeta}{\partial t} \qquad (35)$$

The output e.m.f. from the transducer is

$$E = k \frac{\partial y}{\partial t} = -\frac{Lk}{c} \cos \alpha \frac{\partial^2 \zeta}{\partial t^2} \qquad (36)$$

It will be noted that for this form of vertical seismometer the response varies with the azimuth of the incoming waves. Furthermore, its response to P waves is quite small because those which have large vertical components arrive at the surface with steep angles of incidence and thus produce apparent surface waves of high velocity.

## THREE-COMPONENT STRAIN SEISMOMETER

With a single instrument of the transverse rigid bar type shown in Fig. 19 , all three components of the ground motion may be derived by means of three properly arranged transducers. The horizontal component parallel to the rods is derived from the relative longitudinal motion of the rods. The horizontal component perpendicular to the rods is derived from the relative horizontal transverse motion of the rods. The vertical component is derived from the relative transverse vertical motion of the rods as described in the preceding paragraph.

## DILATATION SEISMOGRAPH

Another type of instrument which is suggested by the linear strain seismograph is a device which may be designated a volume strain or dilatation seismograph. In the form which the writer proposes to build, a container of any convenient shape is buried preferably in rock in such a manner that firm contact with the surrounding medium is maintained. See Fig. 20 . The container is filled with a liquid and sealed with a diaphragm or silphon. A side tube of capillary dimensions serves to equalize slow pressure variations due to temperature, barometric changes and other causes. Volume strains of the ground change the capacity of the container and thus produce movements of the diaphragm which are recorded by means of an electromagnetic transducer and galvanometer.

## THEORY OF THE DILATATION SEISMOGRAPH

Let the condensation be

$$\sigma = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

in which u, v, and w are the ground displacements parallel to the coordinates axes, x, y, and z, respectively. Consider an elementary rectangular parallelopided with sides dx, dy, and dz. Its quiescent volume is

$$dV = dx\,dy\,dz$$

When strained, its volume is $dV(1+\sigma)$. Thus the volume increment resulting from strain is $\sigma dV$. If $V_0$ is the volume of the unstrained container, its total volume increment is

$$S = \int_0^{V_0} \sigma\, dV \qquad (37)$$

When the seismic disturbance consists of plane waves which are long in comparison with the dimensions of the container, $\sigma$ is constant, and equation (37) is written,

$$S = \sigma V_0$$

Thus a strain $\sigma$ forces a quantity of liquid S into the outlet pipe and displaces the diaphragm through a distance

$$s = \frac{S}{A}$$

A is the effective area of the diaphragm. Hence,

$$s = \frac{V_0 \sigma}{A} \qquad (38)$$

The e.m.f. induced in the transducer coils is

$$k \frac{\partial s}{\partial t} = \frac{k V_0}{A} \frac{\partial \sigma}{\partial t} \qquad (39)$$

Considering a plane condensational wave propagating horizontally with the form

$$u = \phi(t - x/c)$$

it can be shown that

$$\sigma = \frac{\partial u}{\partial x}$$

and

$$\frac{\partial u}{\partial x} = -\frac{1}{c}\frac{\partial u}{\partial t} \; .$$

Consequently

$$s = -\frac{V_0}{Ac}\frac{\partial u}{\partial t}$$

For this case, therefore, the response of the dilatation seismometer is identical in type with that of the linear strain seismometer except that it is independent of the azimuth of the incoming waves. In shear waves $\sigma$ is zero and consequently, they produce no response. However, when they are incident on the surface with angles between $\pi/2$ and zero, shear waves produce reflected longitudinal waves and the instrument responds to these in the same manner as to the primary longitudinal waves. Thus it is impossible, completely, to separate longitudinal and shear waves by means of instruments of this type.

## Appendix

### THEORY OF THE UNDAMPED ROD

The well known differential equation for longitudinal waves in a rod is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

when u is the displacement, x the coördinate parallel to the rod and $c^2$ a constant.

Assume a solution of the form

$$u = \left[ A \cos \frac{\omega x}{c} + B \sin \frac{\omega x}{c} \right] \sin \omega t.$$

For a rod clamped at one end and free at the other end the boundary conditions are

$\frac{\partial u}{\partial x} = 0$ at the free end where $x = l$ and $u = a \sin \omega t$

at the clamped end, when the ground displacement is $a \sin \omega t$.

From these boundary conditions we find that

$$A = a$$

and

$$B = a \tan \frac{\omega l}{c}.$$

Hence

$$u = a \cos \frac{\omega}{c}(l - x) \sec \frac{\omega l}{c} \sin \omega t.$$

The displacement at the free end where $x = l$ is therefore

$$u_f = a \sin \omega t \sec \frac{\omega l}{c}$$

In the case of the seismometer rod $l/c$ is small ($Ca\ \frac{1}{100}$) , so that for long waves, $\sec \frac{\omega l}{c}$ is approximately equal to $1$. For waves having a period of 1 second $\sec \omega l/c$ is approximately 1.003. It is clear therefore that for long waves the displacement of the free end of the rod is substantially the same as that of the clamped end.

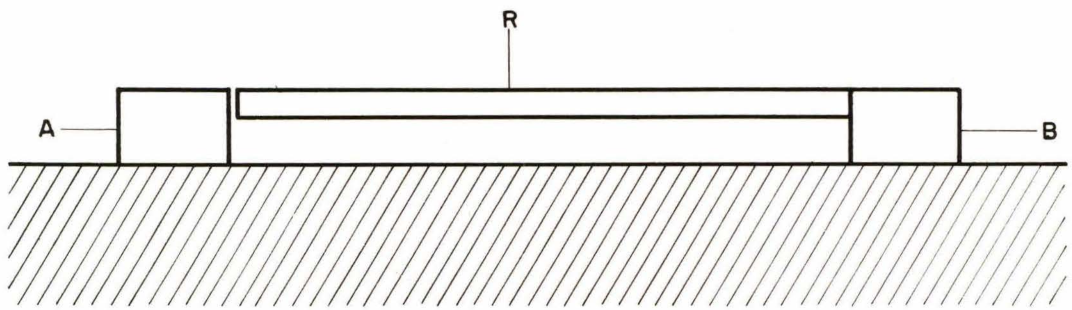



Fig. 1

Fig. 2

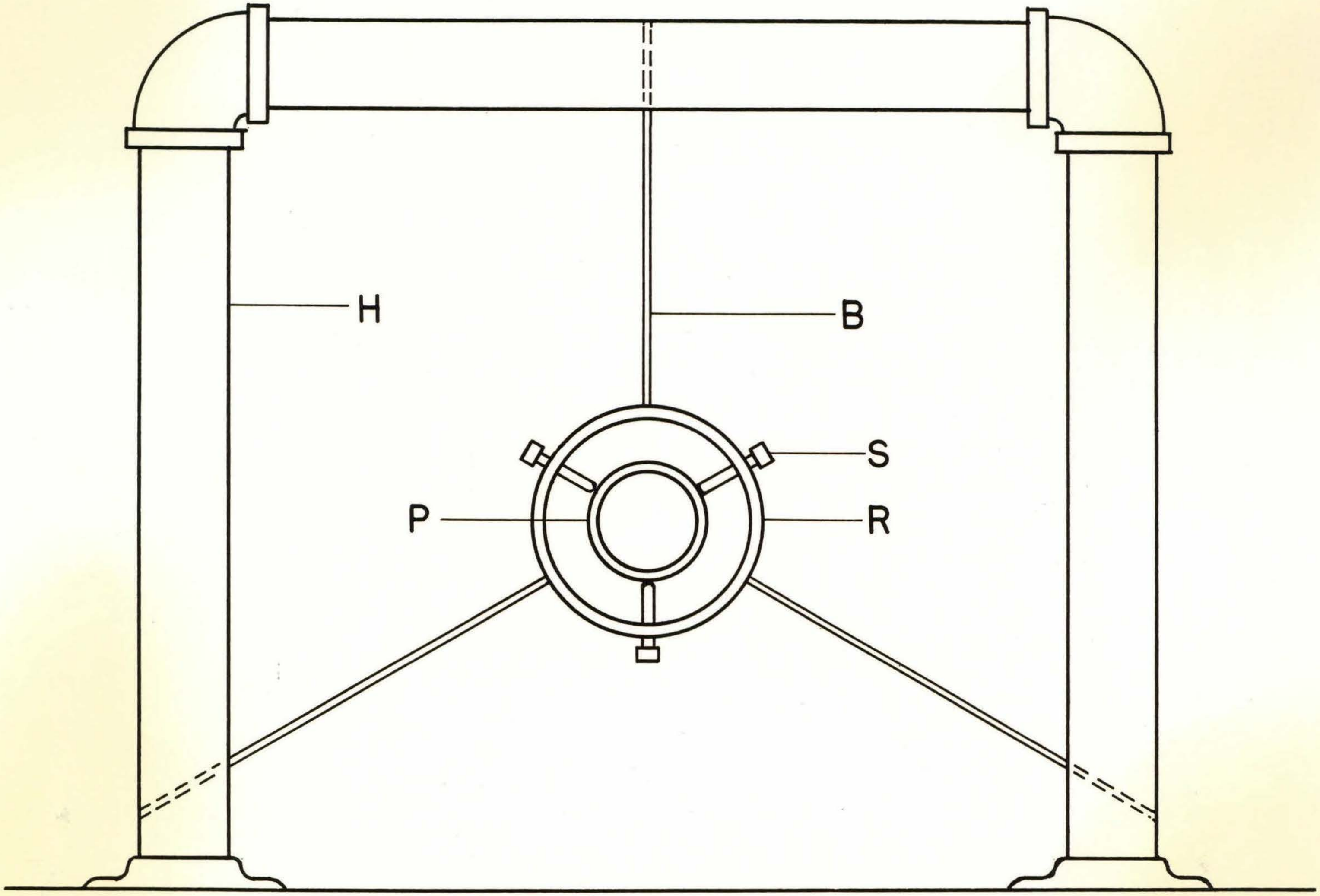



Fig. 3

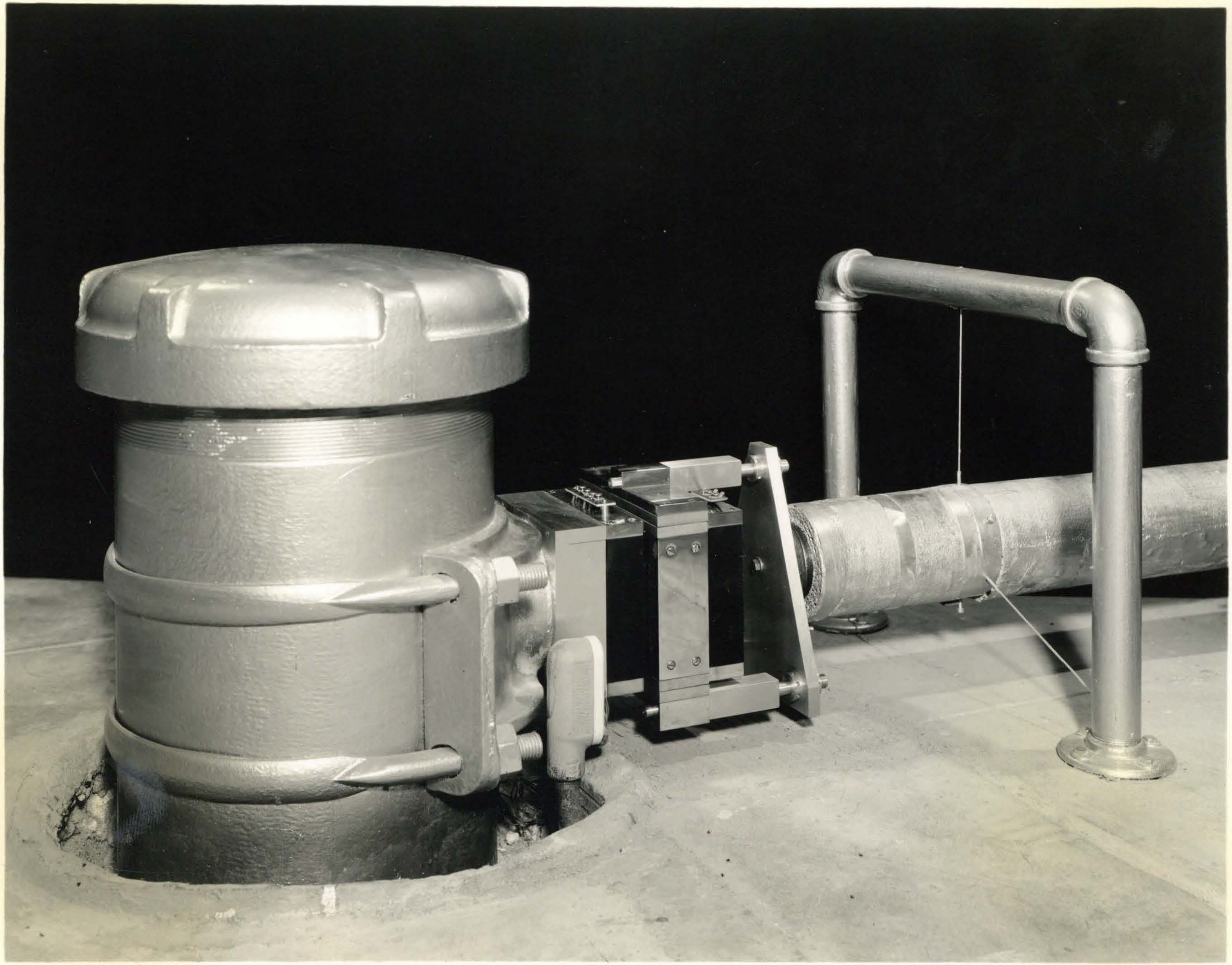


Fig. 4

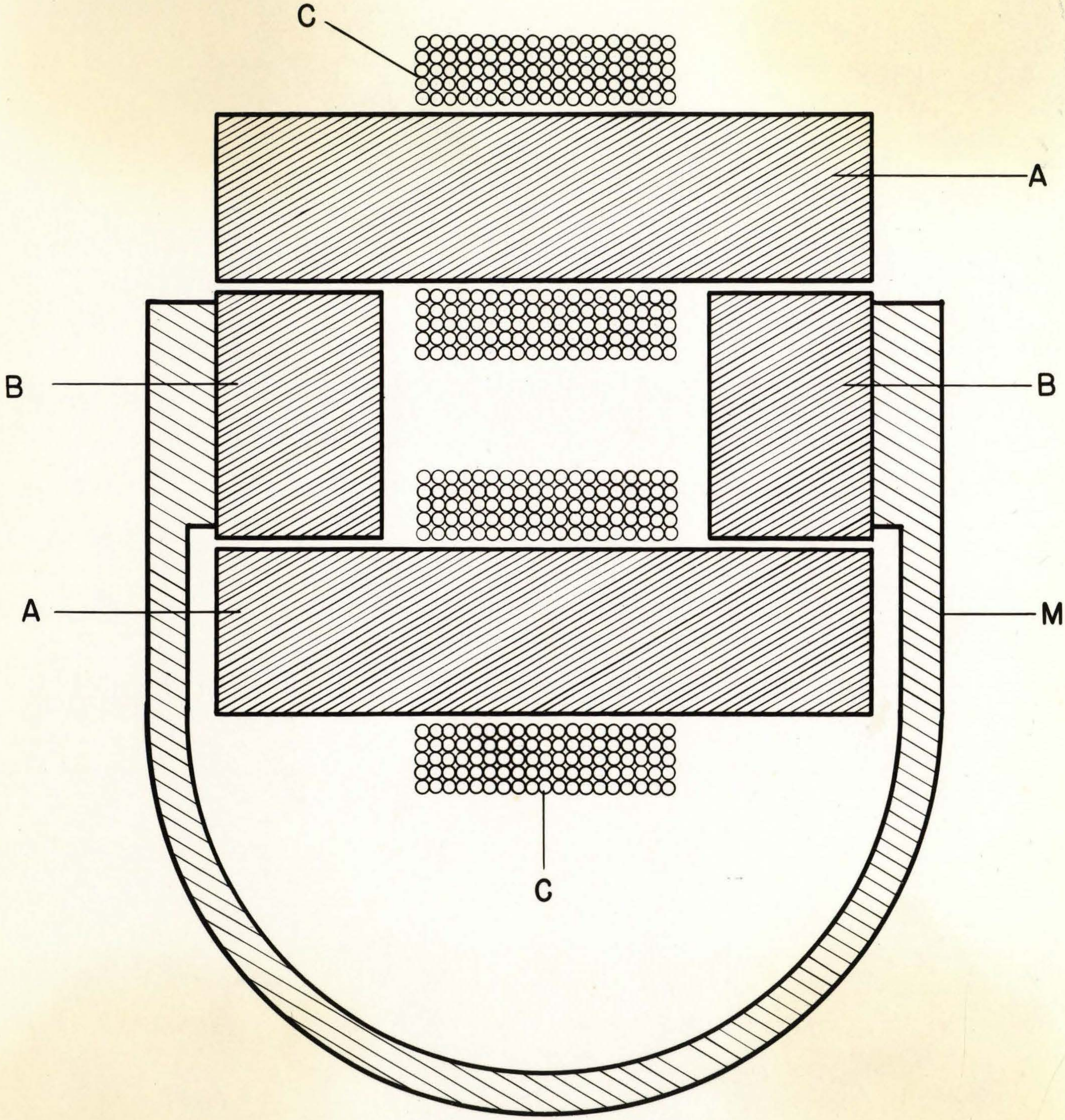



Fig. 5

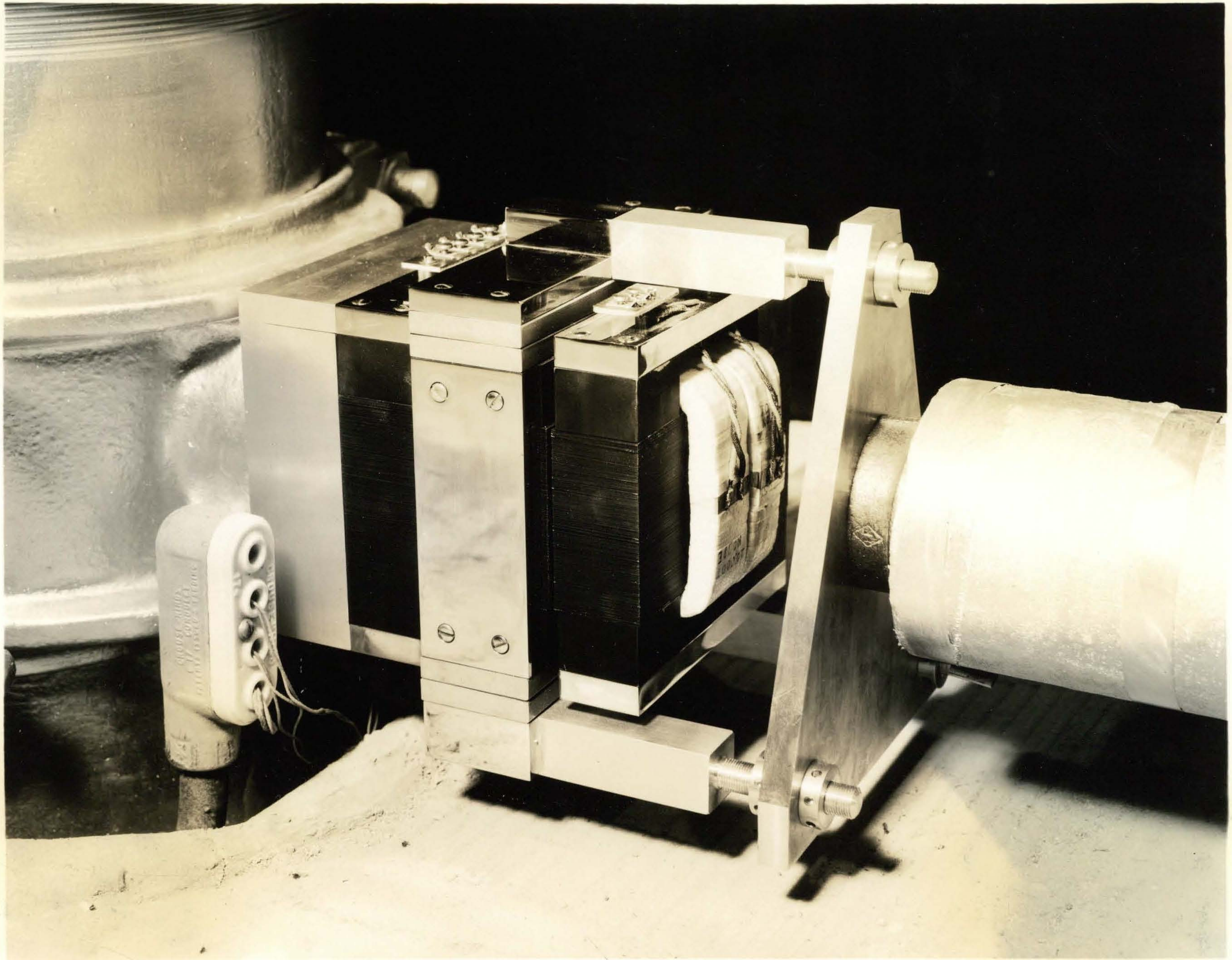


Fig. 6

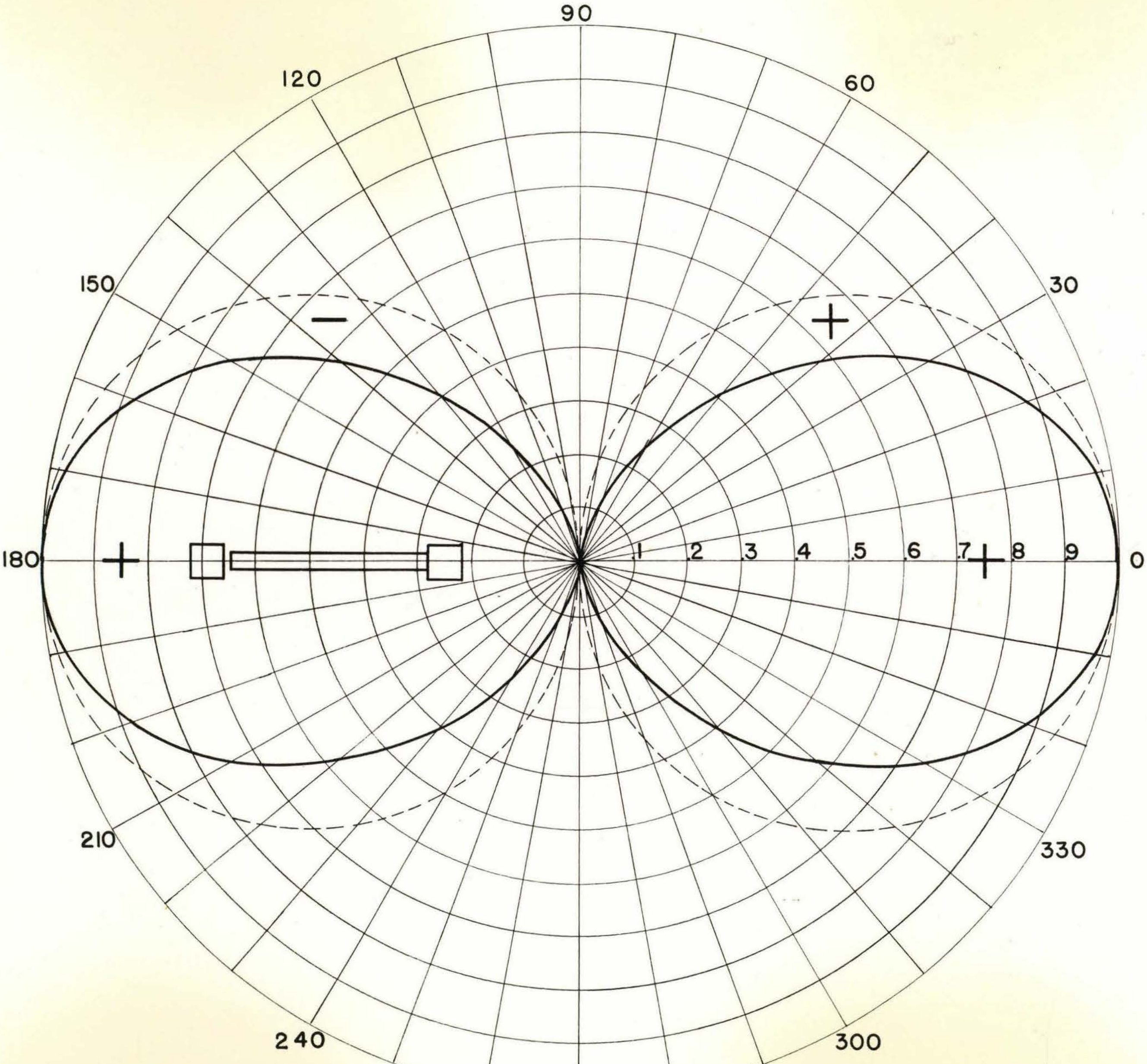



Fig. 7

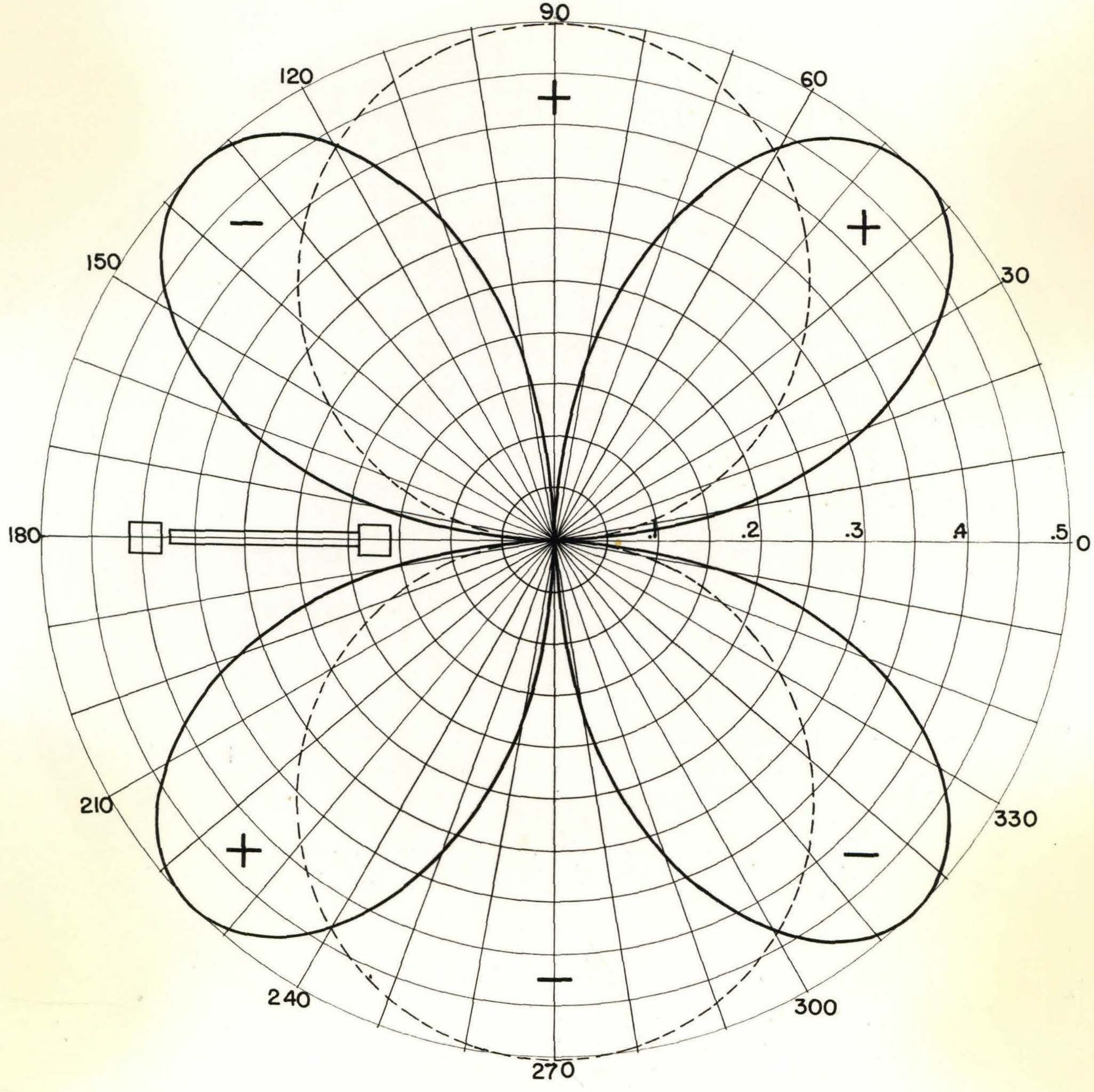
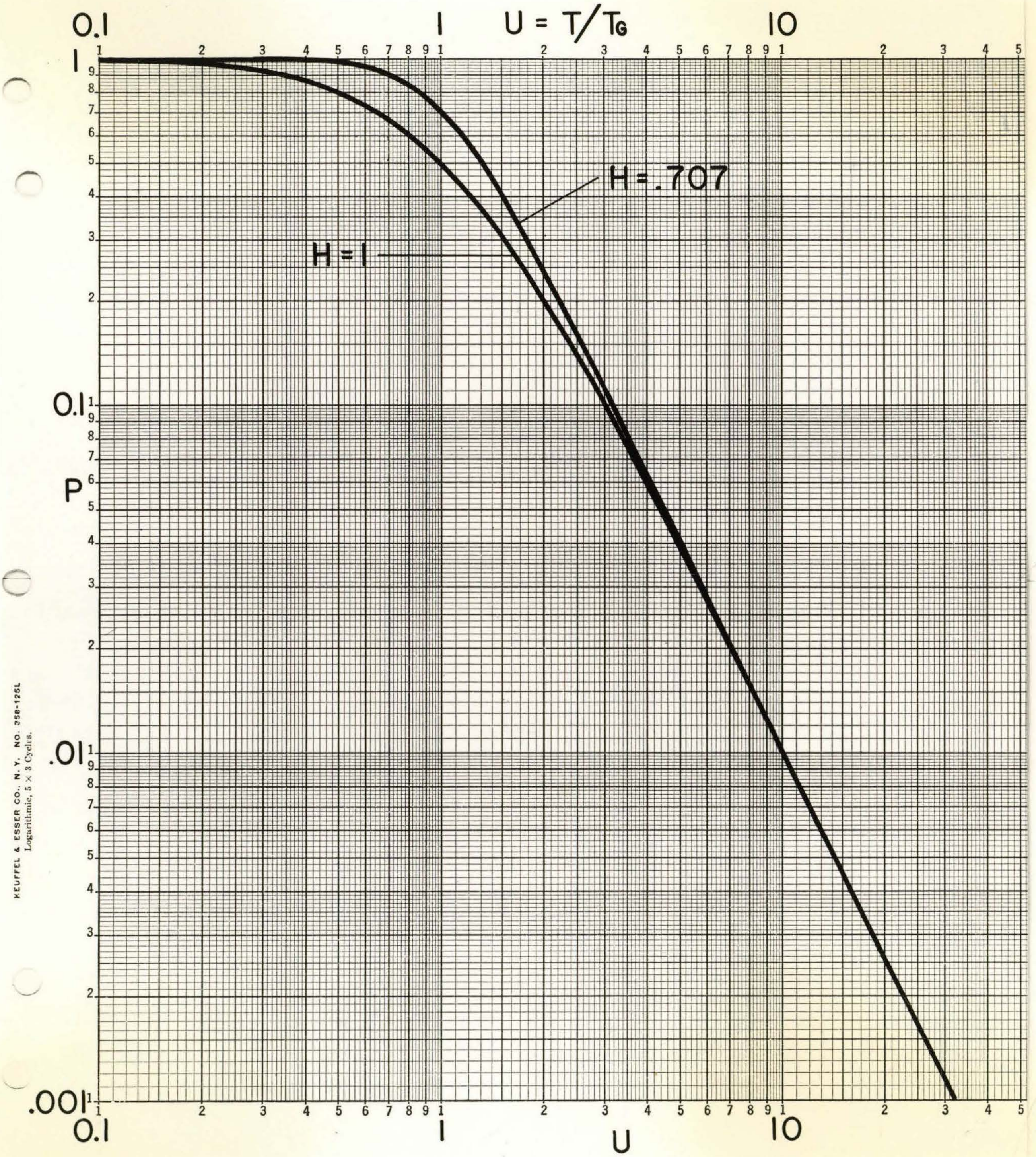



Fig. 8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Logarithmic, 5 × 3 Cycles.

Fig. 9

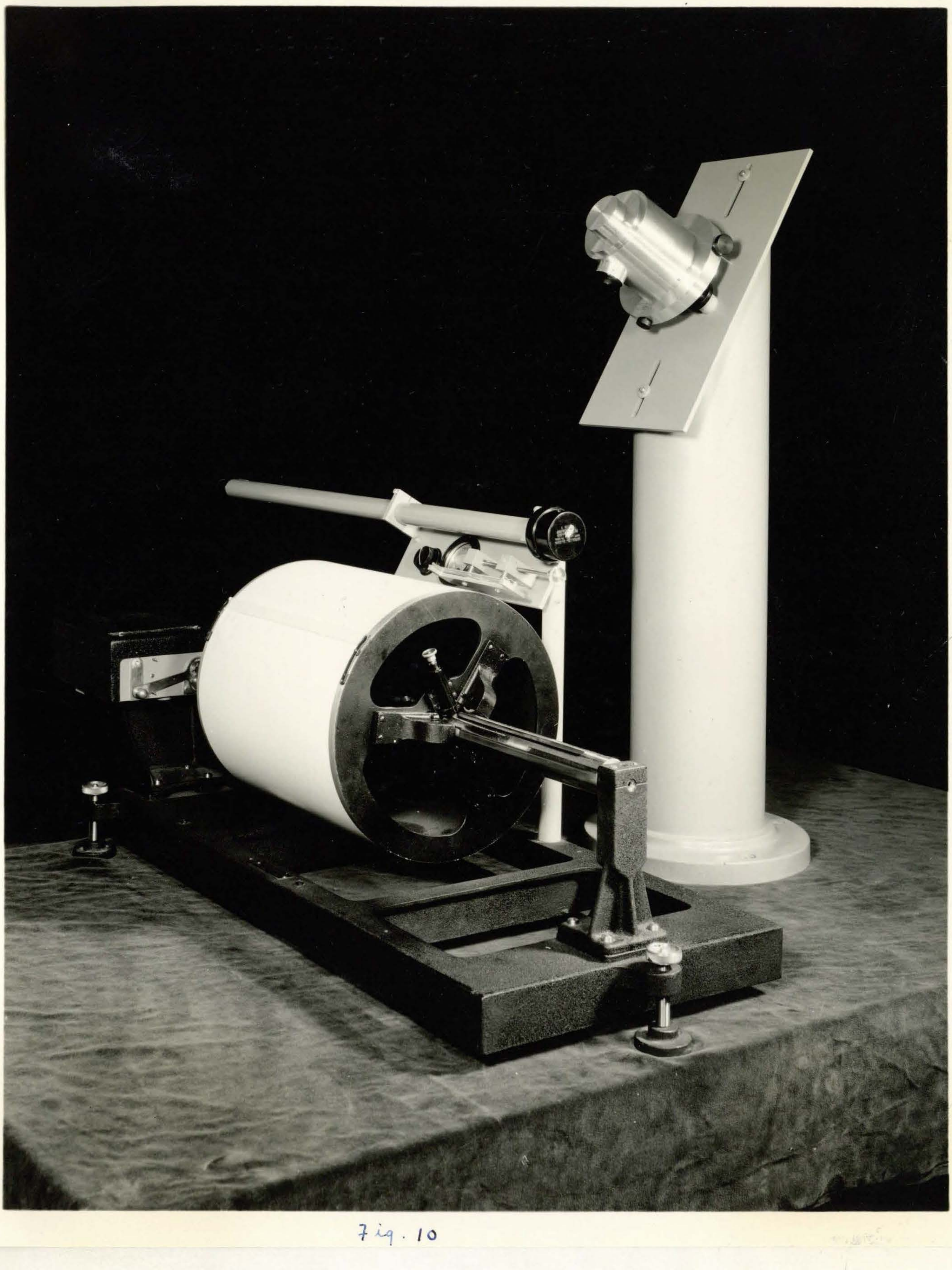

Fig. 10

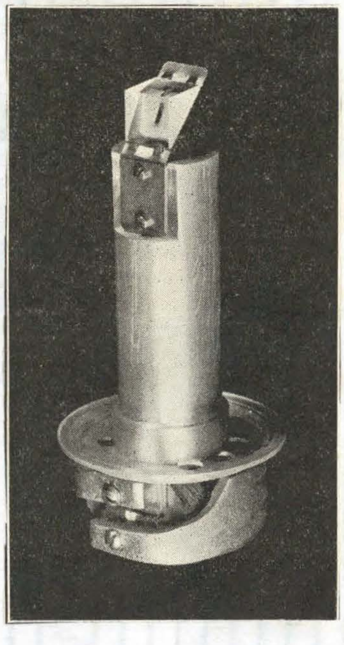

Fig 11

7.

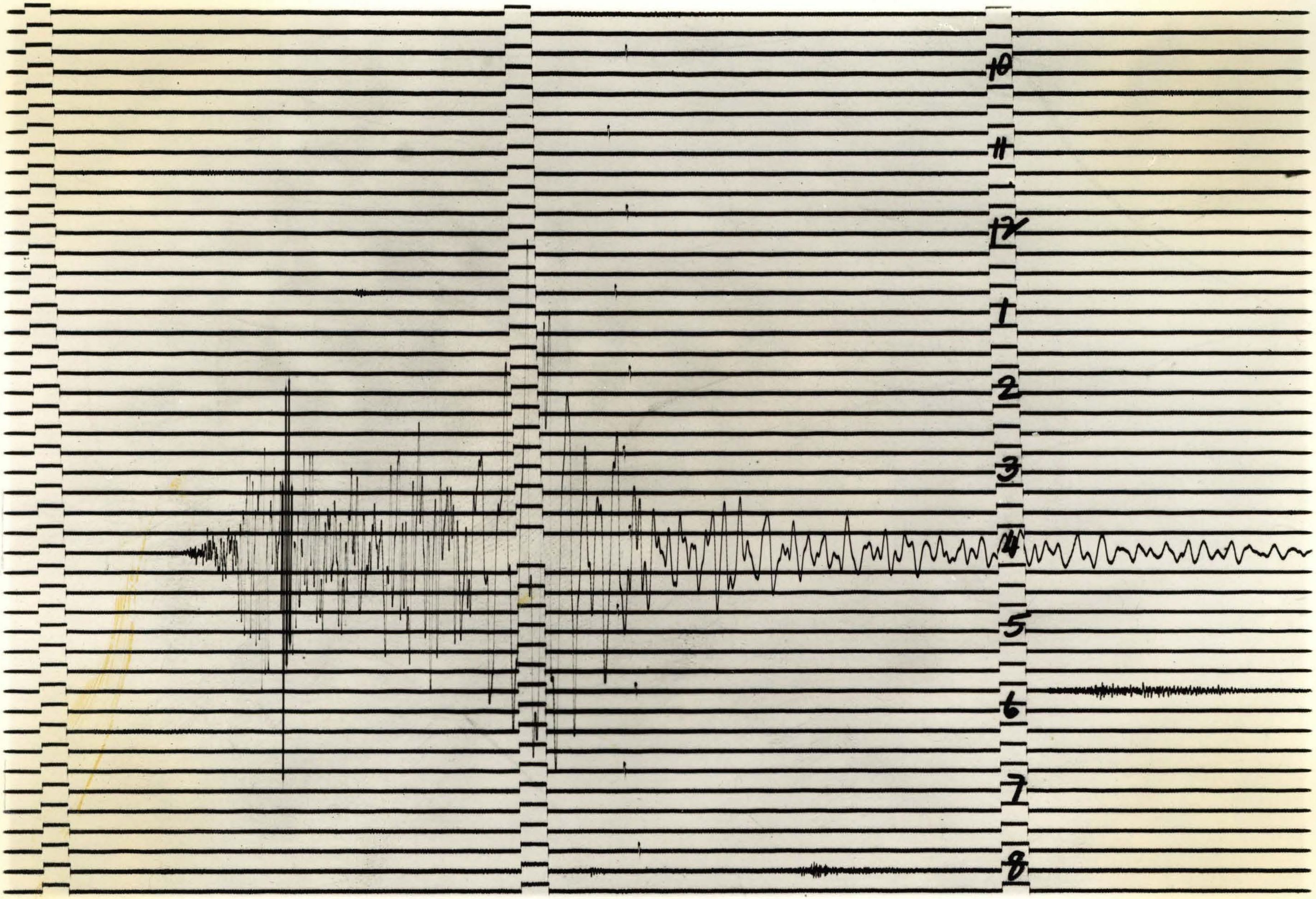


Fig 12

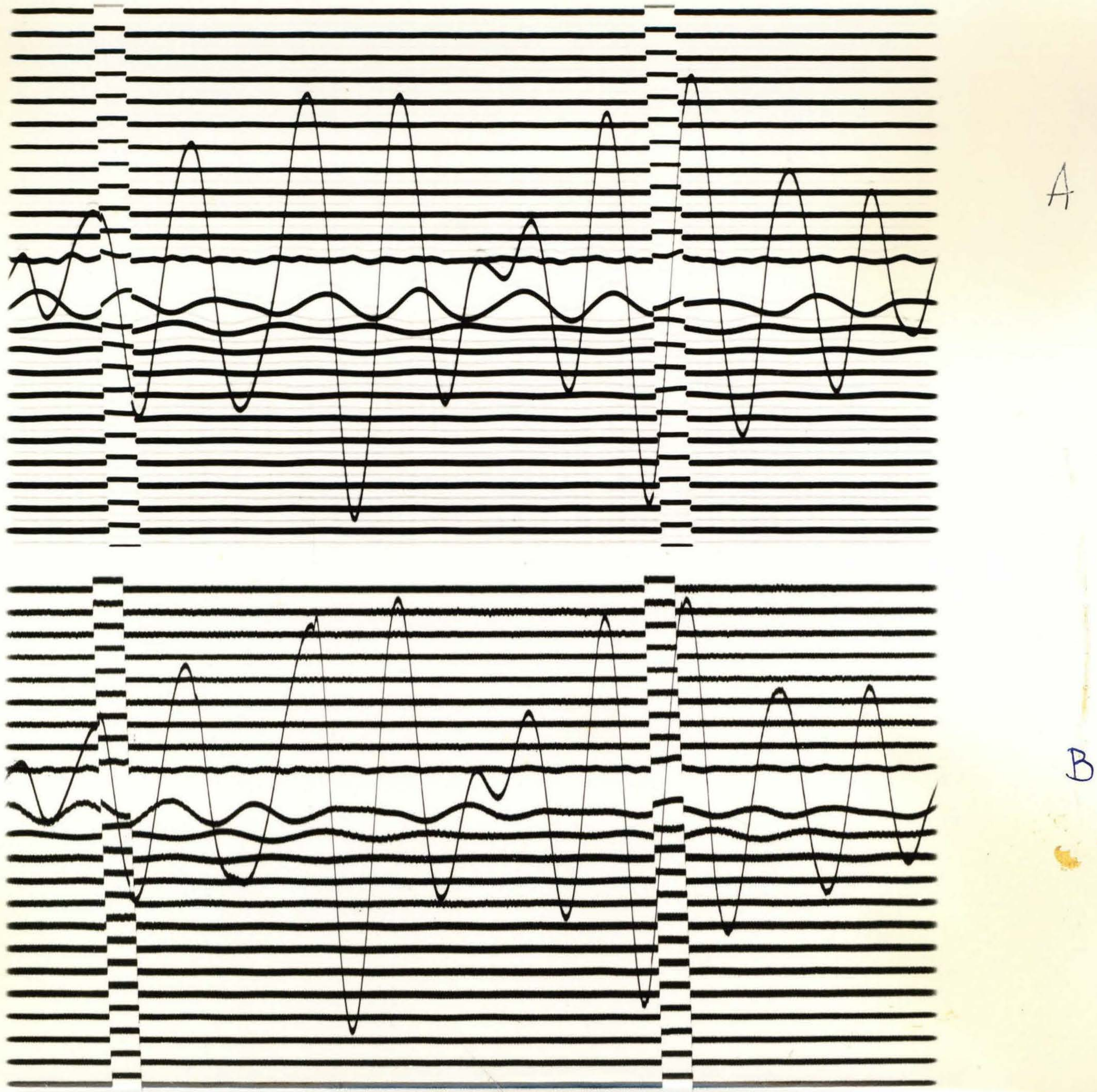


Fig. 13

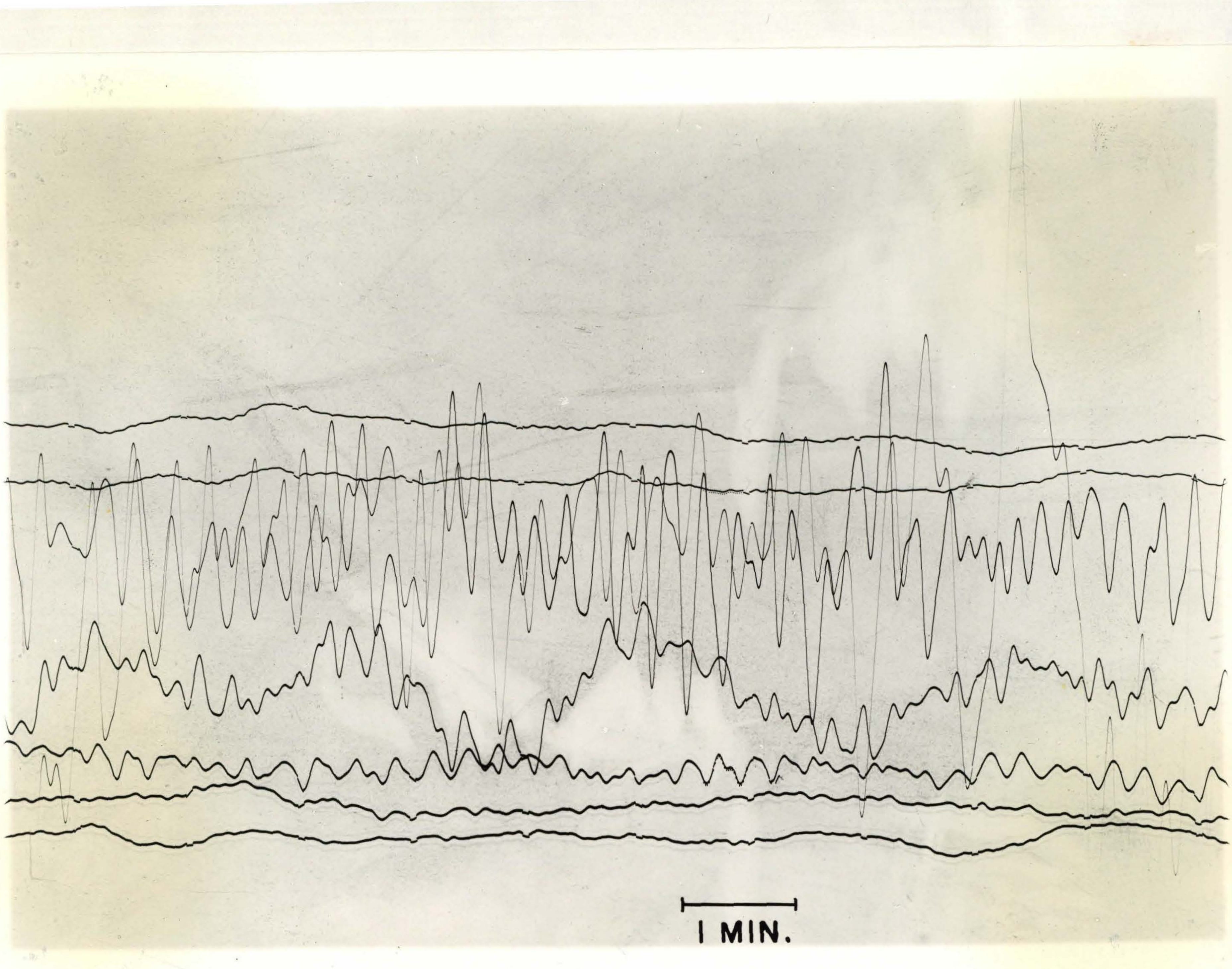


Fig. 14

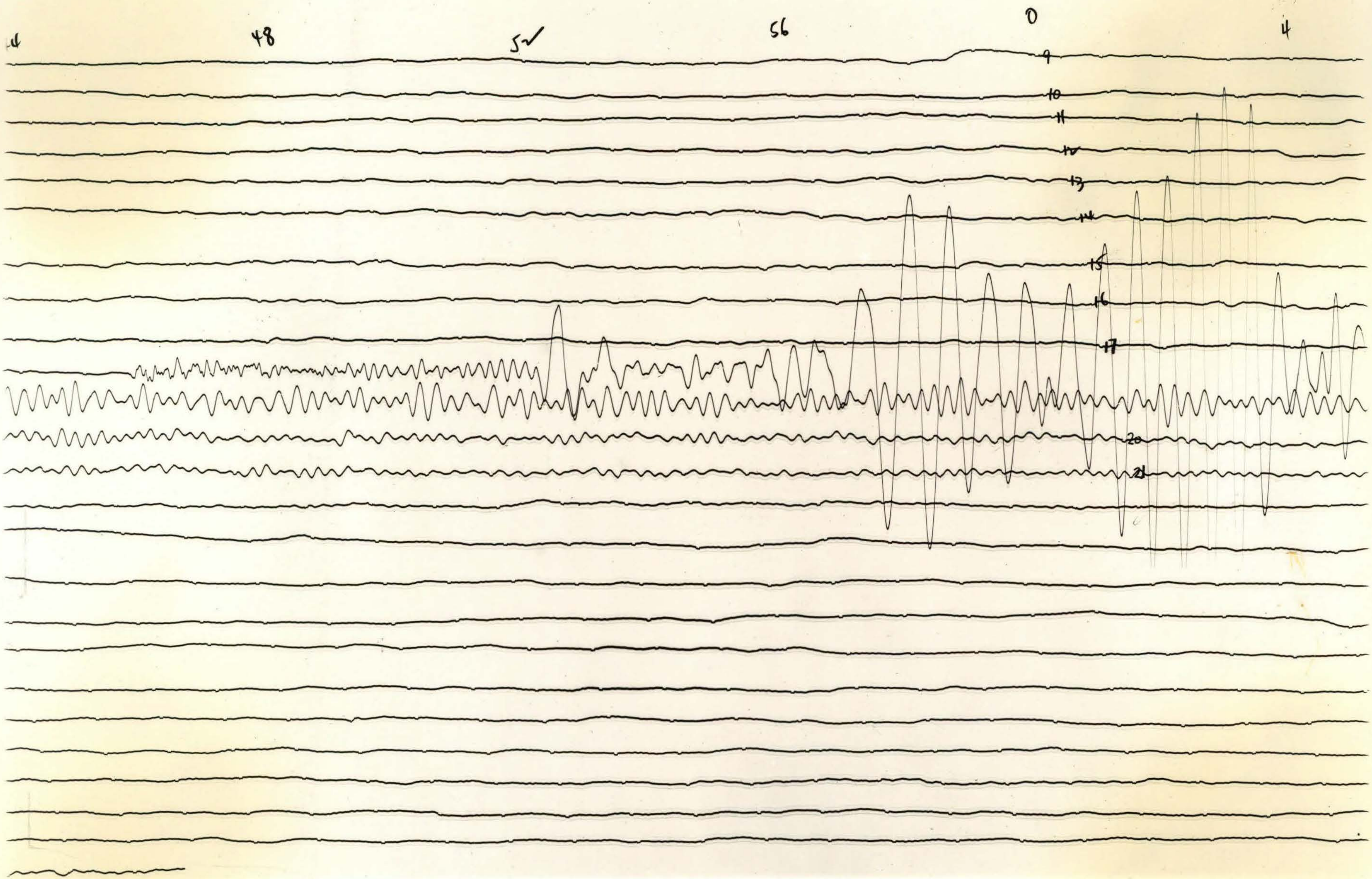


Fig 15

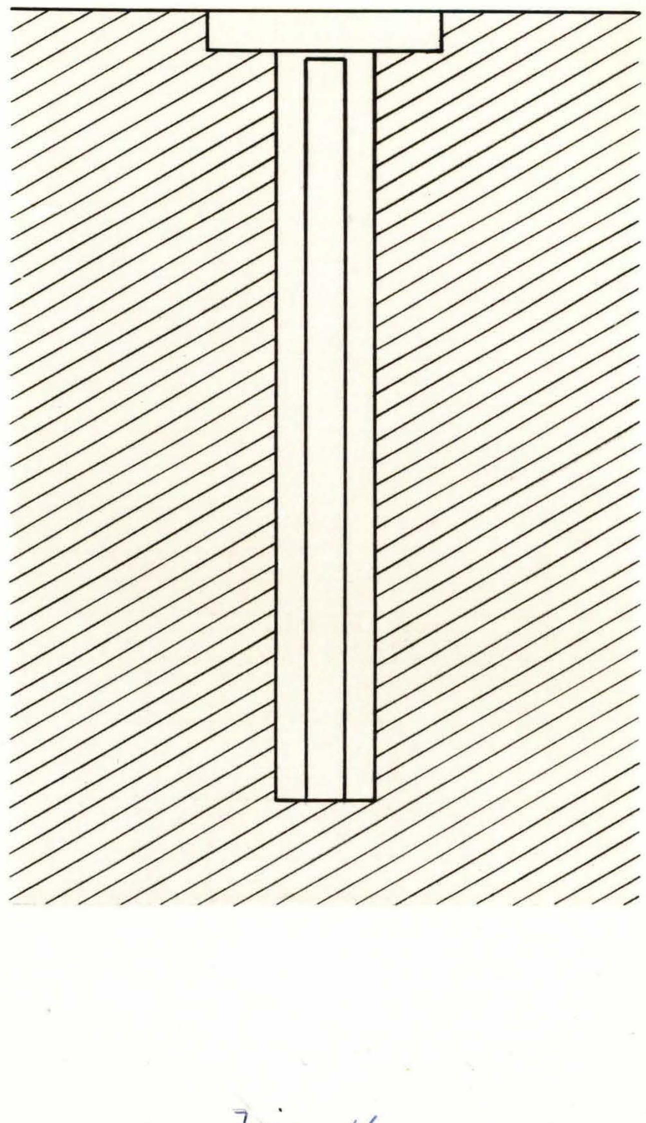

Fig. 16

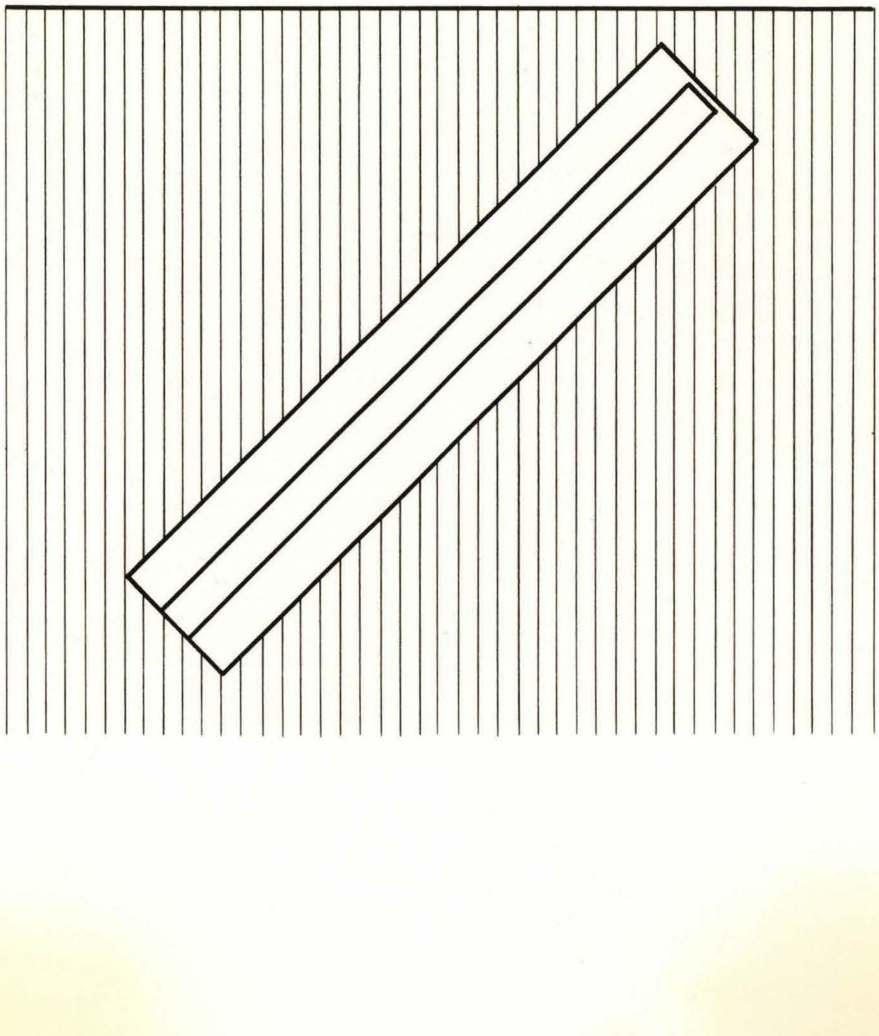

Fig. 17

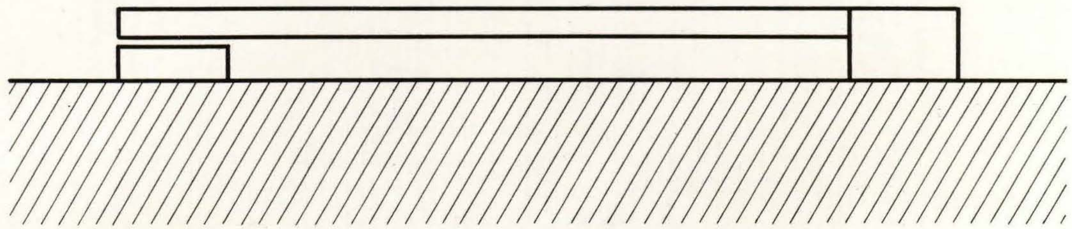

Fig. 18

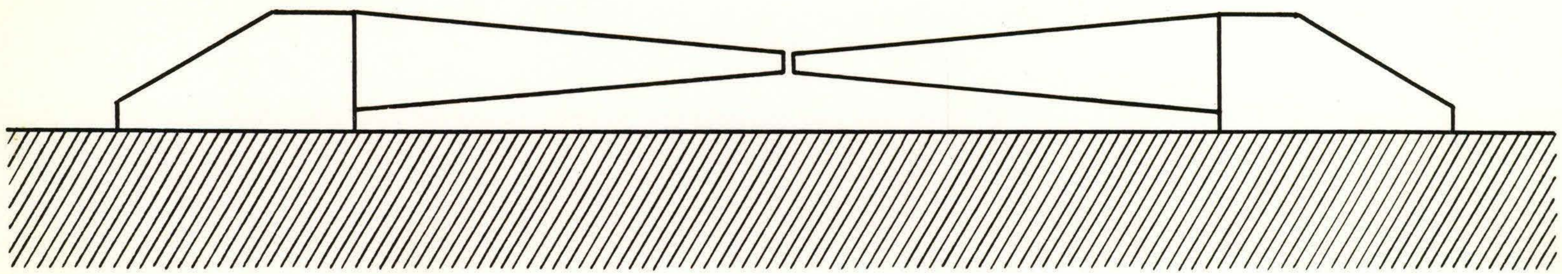

Fig. 19

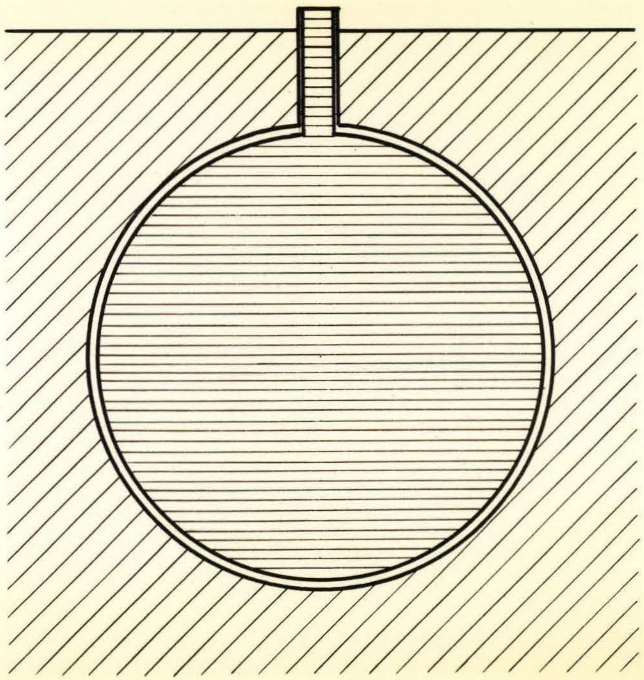

Fig. 20

THE PHYSICAL EVALUATION OF

SEISMIC DESTRUCTIVENESS

Thesis

by

Hugo Benioff

In partial fulfillment of

the requirements of the degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1935

# THE PHYSICAL EVALUATION OF SEISMIC DESTRUCTIVENESS

By HUGO BENIOFF

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# THE PHYSICAL EVALUATION OF SEISMIC DESTRUCTIVENESS*

By Hugo Benioff

The problem of designing structures to withstand destructive earthquakes is not in a very satisfactory condition. On the one hand engineers do not know what characteristics of the ground motion are responsible for destruction, and on the other hand seismologists have no measurements of seismic motion which are sufficiently adequate to serve for design, even if the destructive characteristics were known. Consequently, engineers have been forced to proceed on an empirical basis. From past experience, chiefly in Japan, it has been found that buildings which are designed to withstand a constant horizontal acceleration of 0.1 gravity are, on the whole, fairly resistant to seismic damage. It is fortunate that such a simple formula works at all, in view of its inadequacy from the point of view of precise computation. We know that seismic motions do not exhibit constant accelerations; that instead they are made up of exceedingly variable oscillatory movements. A formula based upon constant acceleration may thus lead to large errors, especially when applied to new types of structures which have not been tested in actual earthquakes. In the following paragraphs a new formula for seismic destructiveness is proposed, in the belief that it is more accurate than previous ones. In addition to providing engineers with a more rational basis for design procedure, it determines a new type of seismographic instrument for recording and measuring the destructive characteristics of seismic motion.

The actual destructiveness of a given earthquake might be defined as the sum of all the material damage caused by the seismic movements. Such a definition does not lend itself to physical measurement and is therefore unsatisfactory from the physical point of view. The idea contained in this definition may be extended to permit physical measurements. Thus, if strain gauges were erected on all members of all the involved structures, the measure of seismic destructiveness could be taken as the sum of all the indicated strains. These definitions of destructive-

* [Received for publication May 26, 1934.]

ness, though ideal from the point of view of completeness, are impracticable to carry out. They do suggest, however, a formula having a high degree of practicability. Thus, suppose we substitute for the engineering structures a series of undamped pendulum seismometers having frequencies ranging from the lowest fundamental frequency of engineering structures to the highest significant overtones. During an earthquake each component seismometer would write a characteristic seismogram. Plotting the maximum recorded deflection of each pendulum against its frequency, we obtain a curve which may be termed the undamped pendular spectrum of the earthquake (see Fig. 1). We now define seismic

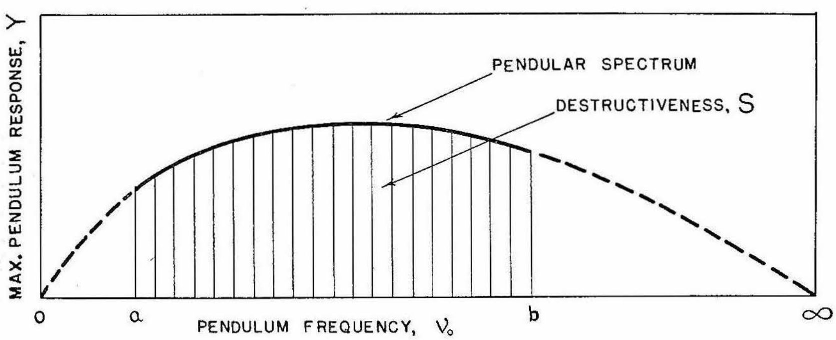


FIG. 1.—Pendular spectrum and seismic destructiveness

destructiveness as the area between this curve and the axis of abscissas. More precisely, seismic destructiveness is the integral with respect to pendulum frequency of the maximum displacement of an infinite series of undamped pendulums extending over the significant range of frequencies. Since the duration of violent movement is in general short, there will be no narrow humps in the pendular spectrum and consequently the integral will be closely approximated by the polygon given by the maximum deflections of a small finite number of pendulums. Probably twenty will be sufficient.

The application of this formula to design is straightforward. We shall first consider a pendular structure such as a water tower, in which the mass is effectively concentrated in the tank and the restoring force is concentrated in the supporting members. Such a structure responds to seismic movements substantially as a simple pendulum seismometer. Hence, if its frequency is known, the maximum displacement of its center of oscillation is given by the observed amplitude of the pendular spectrum at the same frequency. With this displacement given, all of the resulting structural strains may be readily calculated. Thus, with this formula the problem of design is reduced to the computation, or measurement, of the vibration frequencies of the structure.

Complex structures having more than one mode of vibration may be considered as made up of a group of pendulums with the frequencies of the individual modes. The resulting maximum displacements are given by the sum of the individual displacements, due regard being given, of course, to the existence of nodes and loops. In the case of structures which may be approximated by rods vibrating transversally, the natural frequencies and strains can be calculated by the formulas given by Le Conte and Younger.[1] The more complex structures are now being studied experimentally, or theoretically, by a number of investigators and it is to be expected that their results will be of great value in the application of the pendular spectrum method.

In the preceding discussion, structural damping has been neglected. In general, the errors introduced by this simplification are quite small. Thus, for example, we may consider the effects of damping on a single vibration mode of a structure. When subjected to a resonant vibratory force, it responds with an amplitude which increases asymptotically with the time to a maximum value determined by the damping constant. The time constant, defined as the time for the amplitude to rise to $1 - \frac{1}{e}$ of its final value, varies inversely with the damping constant. It is clear that if the resonant force acts for a time interval which is short compared to the time constant of the system, the resulting amplitude is independent of the damping. Now in the case of most engineering structures the damping is small. Furthermore, the time interval during which a finite resonant force may be effective in an earthquake, though not known precisely, must be quite short. Consequently we may expect the structural vibration amplitudes to be substantially independent of the damping. At any rate, calculations based upon the undamped pendular spectrum determine the upper limit for strains in any structure.

Whenever greater precision is required, it will be necessary to set up an additional series of critically damped pendulums for recording the critically damped pendular spectrum. This damped spectrum determines the lower limit of strains, since, in general, the damping constants of structures lie between zero and the critical value. With the two spectra

[1] Joseph N. Le Conte and John E. Younger, "Stresses in a Vertical Elastic Rod when Subjected to a Harmonic Motion of One End," *Bulletin of the Seismological Society of America,* **22**, **1**, March, 1932.

given, the strains can be computed with engineering accuracy for any value of structural damping. Observations with a set of seismographs designed by the writer for use by Professor Martel, of the California Institute of Technology, have shown that the tops of buildings exhibit vibration amplitudes of approximately three times those of the ground at the bases. On the basis of these experiments we may expect the undamped pendular spectrum to have amplitudes approximately three times those of the corresponding damped spectrum.

It should be emphasized that the formula of destructiveness defined in this paper refers solely to elastic deformations in which stress and strain are proportionate. In the event of actual destruction, elastic limits are exceeded and in this condition strains cannot be accurately calculated by this method. This limitation is of no serious consequence, since it is customary in good engineering practice to keep strains within the elastic limit.

It might be well to consider some of the characteristics of seismic pendular spectra which may be predicted on the basis of present knowledge. In the case of very small earthquakes, observations at a given point should indicate very nearly identical spectra for all shocks originating at a common focus. Larger shocks with a common focus may be expected to show greater spectral differences. In addition, the spectral energy of large earthquakes should be shifted toward the low frequencies, as compared with smaller shocks. The spectrum of a given earthquake changes from point to point, depending upon distance from the origin and upon ground characteristics. Increasing distance from the focus shifts the spectral energy toward the lower frequencies. A free vibration of the ground will be indicated by a sharp maximum in the undamped spectrum, which does not appear in the damped spectrum. This will be true regardless of whether the free vibration exists in the region of the focus or in the region of the observing station.

To be most useful, observations on pendular spectra should be maintained throughout populated regions for a long enough period to record strong earthquakes from all active foci. The resulting records of seismic spectra would then be sufficiently complete for all future engineering purposes. It may be, however, that a much less ambitious program would suffice. Thus, for example, a few observations on strong earthquakes may demonstrate a sufficient similarity in their spectra to warrant the practical use of a single generalized spectrum for all foci. Variations would be necessary, of course, to allow for differences in ground conditions at the building site. Furthermore, we may find that spectra of large and small earthquakes with a common focus are regularly related in such a way that the spectrum of a large earthquake may be easily computed from that of a corresponding small shock. If this be true, the time required for an adequate series of observations on the spectra of a region will be greatly reduced.

The spectral formula for seismic destructiveness has been developed in the preceding paragraphs on the basis of an intuitional argument. Originally, however, it was derived from a fundamental physical concept which will now be indicated briefly.

A wave disturbance $f(t)$, existing only in the finite interval $0 \leqq t \leqq T$, is represented analytically by the Fourier Integral[2]

$$f(t) = 2 \int_0^\infty F(\nu) \cos \left[2\pi\nu t + \vartheta(2\pi\nu t)\right] d\nu \qquad (1)$$

in which

$$[F(\nu)]^2 = \left[\int_0^T f(t) \cos 2\pi\nu t dt\right]^2 + \left[\int_0^T f(t) \sin 2\pi\nu t dt\right]^2$$

The significance of this Fourier Integral lies in the fact that it represents the sum of an infinite number of simple harmonic components of frequencies $\nu$ extending from zero to infinity, with definite phase angles $\vartheta$. Furthermore, the amplitudes of the individual components are infinitesimally small. Thus, we see that the Fourier Integral (1) represents a continuous spectrum. The response of an undamped pendulum to a simple harmonic displacement of the ground is

$$y = \frac{\nu^2}{\nu_0^2 - \nu^2} a \cos 2\pi\nu \, (t + \delta) \qquad (2)$$

in which

$a =$ the maximum amplitude.

$\nu_0 =$ the frequency of the pendulum.

$\nu =$ the frequency of the earth-wave.

$\delta =$ the phase angle of the earth-wave.

[2] J. R. Carson and O. J. Zobel, "Transient Oscillations in Electric Wave-Filters," *The Bell System Technical Journal,* **2**, 1, July, 1923; and J. R. Carson, "Selective Circuits and Static Interference," *ibid.,* **4**, 268, April, 1925. Also M. Biot, "Theory of Elastic Systems Vibrating under Transient Impulse, with an Application to Earthquake Proof Buildings," *Proceedings of the National Academy of Sciences,* **19**, 262–68, 1933; and M. Biot, "Acoustic Spectrum of an Elastic Body Submitted to a Shock," *Journal of the Acoustic Society of America,* **5**, 206, January, 1934.

From Equation 1 we find that a single component of the seismic disturbance is given by

$$2F(\nu)\cos\left[2\pi\nu t+\vartheta(2\pi\nu)\right]d\nu$$

Hence, the response of a pendulum to a single component is

$$\frac{2\nu^2}{\nu_0{}^2-\nu^2}F(\nu)\cos\left[2\pi\nu t+\vartheta(2\pi\nu)\right]d\nu$$

and the response to the whole spectrum is

$$y=2\int_0^{\infty}\frac{\nu^2}{\nu_0{}^2-\nu^2}F(\nu)\cos\left[2\pi\nu t+\vartheta(2\pi\nu t)\right]d\nu \tag{3}$$

At some time during the passage of the seismic wave-train the response of the pendulum will indicate a maximum value, $Y$. Taking an infinite number of pendulums, the pendular spectrum is given by the function

$$Y=\psi(\nu_0), \tag{4}$$

and seismic destructiveness is defined by the integral,

$$S=\int_a^b Y d\nu_0 \tag{5}$$

These are both shown in the diagram of Figure 1.

The limits $a$ and $b$ must be determined from observations on buildings. The integral $S$ defines a new intensity scale based upon physical measurements. For statistical study and comparison of earthquakes it should be much more accurate than the present scales based upon random observations. In effect it measures the potential destructiveness of a given earthquake to a standard city composed of simple standard structures.

From the engineering standpoint in the calculation and design of individual structures, the pendular spectrum, $Y$, is the function of primary interest, since it determines the structural response to earthquakes. The use of this function in the calculation of structural strains has the very great advantage that the true ground displacement (or its derivatives) does not have to be measured or calculated.

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# A METHOD FOR THE INSTRUMENTAL DETERMINATION OF THE EXTENT OF FAULTING

Thesis

By

Hugo Benioff

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A Method for the Instrumental Determination
of the Extent of Faulting

When a fault displacement is clearly visible at the surface of the ground, there is no difficulty in determining the extent of faulting. Thus for example, in the San Francisco earthquake of 1906, faulting was observed on land to the extent of some 180 miles. If the fault is deeply covered with sediments so that displacement can not be observed at the surface, it is necessary to employ indirect methods of measuring the extent of faulting.

The Long Beach earthquake produced no visible evidence to indicate the extent of faulting. The instrumental epicenter as determined by Gutenberg, Wood and Richter, is located a few kilometers off the coast at Newport. An instrumental epicenter determined, as this one was, by the arrival times of the first waves, gives no evidence as to the extent of faulting nor the point from which maximum energy radiates. It indicates solely, the point at which faulting originates.

The distribution of destruction in this earthquake was such as to suggest that faulting extended Northwest along the Inglewood fault in the direction of Long Beach. Long Beach and Compton, distant 18 and 25 miles from the epicenter respectively, exhibited severe damage. With the most liberal allowances for poor construction and bad ground, it is difficult to believe that an earthquake of such small magnitude could produce damage of this character at such large distances from the epicenter, unless faulting extended a substantial distance toward Long Beach. The total energy liberated by the Long Beach earthquake is estimated by

Dr. Richter, for example, at 1/1000 the energy of the San Francisco earthquake. The sharp falling off of destruction south of the epicenter indicates that there was no significant extension in that direction.

A rather crude but definite instrumental indication for extension of faulting is furnished by comparison of seismic wave periods in the principal shock with those in the aftershocks. At Pasadena the main shock was recorded on a strong motion seismograph having a period of 10 seconds, critical damping, and a magnification of 4. Aftershocks were recorded on a torsion seismograph having substantially the same constants with the exception of magnification which was approximately 600. It was clear from a casual inspection of the seismograms that the principal shock exhibited much longer periods than those of the aftershocks. This effect indicates that in the principal shock the fault displacement either took place more slowly than those of the aftershocks or extended an appreciably greater distance. Since the first alternative requires the principal shock to have less energy than the aftershocks it must be assumed that fault extension was responsible for the longer periods.

A more precise method for determining the extent of faulting can be derived from the mechanism of faulting as given by the elastic rebound theory. Referring to Fig. 1,A we may suppose that in the neighborhood of a fault, indicated by a dotted line, a series of perpendicular lines is laid out on the ground at a time when the region is in an unstrained condition. With the passing of time, strains are set up and the consequent configuration of the region will be given by Fig. 1,B. As the strains increase further, there comes a time when the stress at some point exceeds the cohesive strength of the fault and in consequence the two fault surfaces slide by each other. See Fig. 2. This initial movement generates two seismic waves as shown in Fig 2, where the outer circle represents the front of a compressional wave and the inner circle represents the front of a slower shear wave. The immediate effect of the longitudinal wave is to increase the existing strains at the neighboring points along the fault. This effect is evident in Fig. 2 where the arrows indicate the direction of motion of the ground particles in the wave-front of the initial longitudinal wave. When the additional stress due to the wave is sufficient to raise the total stress to the breaking point a wave of faulting is propagated along the fault. The faulting velocity must necessarily be less than the longitudinal wave velocity, since the wave-front is not rectangular and in consequence a finite time is required for the incremental stress to build up to the slipping value. Since the faulting velocity does not differ greatly from the wave velocity, the elementary waves generated from successive slipping points are approximately in phase and consequently their effects are cumulative. Under such conditions, the incremental stress generated by the waves may be sufficient to cause the fault movement to overshoot the equilibrium position in the region of the end point, and thus to leave the structure in a state of strain opposite in direction to that of the original tectonic strain. Evidence will be given later to indicate that this was the condition following the Long Beach earthquake.

If the mechanism of faulting just described is correct the observed events at a given station may be predicted with the help of the diagram in Fig. 3, where E is the epicenter, O is the observing station, a is the length of the active segment of the fault, b is the distance from the station to the end point of the fault segment, and $\Delta$ is the distance of the epicenter to the station. We assume that the earthquake begins at E at the time zero. A wave of faulting is generated and proceeds along the fault with a velocity $V_F$. The faulting movement will therefore arrive at the end point P at a time $t_1 = a/V_F$. The shear wave which is generated at the end point arrives at O, the observing station, after an interval $t_2 = b/V_S$, from the time it started, where $V_S$ is the velocity of shear waves. The total apparent travel time of this shear wave is therefore

$t_1 \;\; t_2 = a/V_F \;\; b/V_S$. Under favorable conditions, this wave may arrive at the observing station earlier than the initial shear wave from the epicenter, for although its total path is longer than that of the direct wave, it has, in effect, travelled part of the way along the fault as a faulting impulse with a velocity which is higher than that of the normal shear wave. When the observed travel time is less than that of the direct wave, equation (1) can be used to calculate a, the extent of faulting. The simplest procedure is to plot the family of ellipses determined by equation (1) on a map of the region using faulting velocity as a parameter. The intersection of a given ellipse with the fault gives the extent of faulting for the assumed faulting velocity. The minimum extent of faulting is given by the ellipse which represents a faulting velocity equal to V , the velocity of longitudinal waves. As the assumed faulting velocity is reduced, the corresponding ellipses become smaller until finally a limiting value is reached at which the ellipse is just tangent to the fault. With further decrease in faulting velocity the curves no longer intersect the fault. The limiting tangent point represents therefore the maximum extent of faulting. Fig. 4 represents a map of the region of the Long Beach earthquake. The Inglewood fault is indicated by the heavy dotted line. The ellipse designated with the number 1 is derived using a faulting velocity equal to the velocity of longitudinal waves, 5.55 km./sec. The other curves were constructed using faulting velocities as indicated. The limiting value is approximately 0.75 V = 4.2 km/ sec. The corresponding maximum limit for the extent of faulting is indicated by the large dot near Signal Hill. Although it is known that the faulting velocity is less than the velocity of longitudinal waves, the precise value is not given by this method. Consequently the extent of faulting is determined only as to upper and lower limits. However, in the case of the Long Beach earthquake, there is evidence to indicate that the upper limit represents the actual limit of faulting.

The most reliable evidence comes from observations on aftershocks. Recent determinations of epicenters by Dr. Richter have shown that the aftershocks have been uniformly distributed throughout the length of the segment indicated by the large dots.

This distribution was maintained from the very first hours following the principal shock. No aftershocks occurred beyond the segment in either direction. The largest aftershock occurred on October 2. It was responsible for some minor damage in the region of North Long Beach. Its epicenter is shown on the map of Fig. 4. The direction of movement in this shock as indicated by the seismograms was opposite that of the principal earthquake. It is reasonable to believe therefore that during the main earthquake, the fault movement actually overshot the equilibrium position and that this large aftershock represented a return to equilibrium.

Further evidence for the extension of faulting to the maximum point is furnished by some observations of Dr. Thomas Clements which were published in Science. He measured the direction of fall of tombstones in a number of cemeteries located in the destructive region. In general, he found that for a given cemetery, the stones fell in two favored directions at right angles to each other. The number which fell in one direction was considerably larger than the number which fell in the other direction. Clement assumed that the epicenter of the earthquake is given by the intersection of the directions corresponding to the maximum number of falls. In other words he assumed that the larger number of stones was overthrown by longitudinal waves. The epicenter which he found in this manner was located near Compton. It is common experience, however, that the transverse waves of earthquakes exhibit larger amplitudes than the longitudinal waves. It is to be expected therefore that transverse waves are responsible for the larger number of falls. On this assumption Clement's data gives results in substantial agreement with those given in the preceding paragraphs. Clement's observations are shown in Fig. 5. The circles with the intersecting lines represent the five cemeteries which he studied. Each line represents the direction of fall of one or more stones. The thin dotted lines are drawn at right angles to the more common direction of fall and represent the azimuths of the source at the various cemeteries on the assumption that transverse waves are responsible for the larger number of falls. It is clear that all but one of the cemeteries indicate directions in excellent agreement with the results of this paper. The one discordant cemetery is located on the projection of the fault and would therefore receive much less energy in the transverse wave form. It can not be expected to indicate true directions.

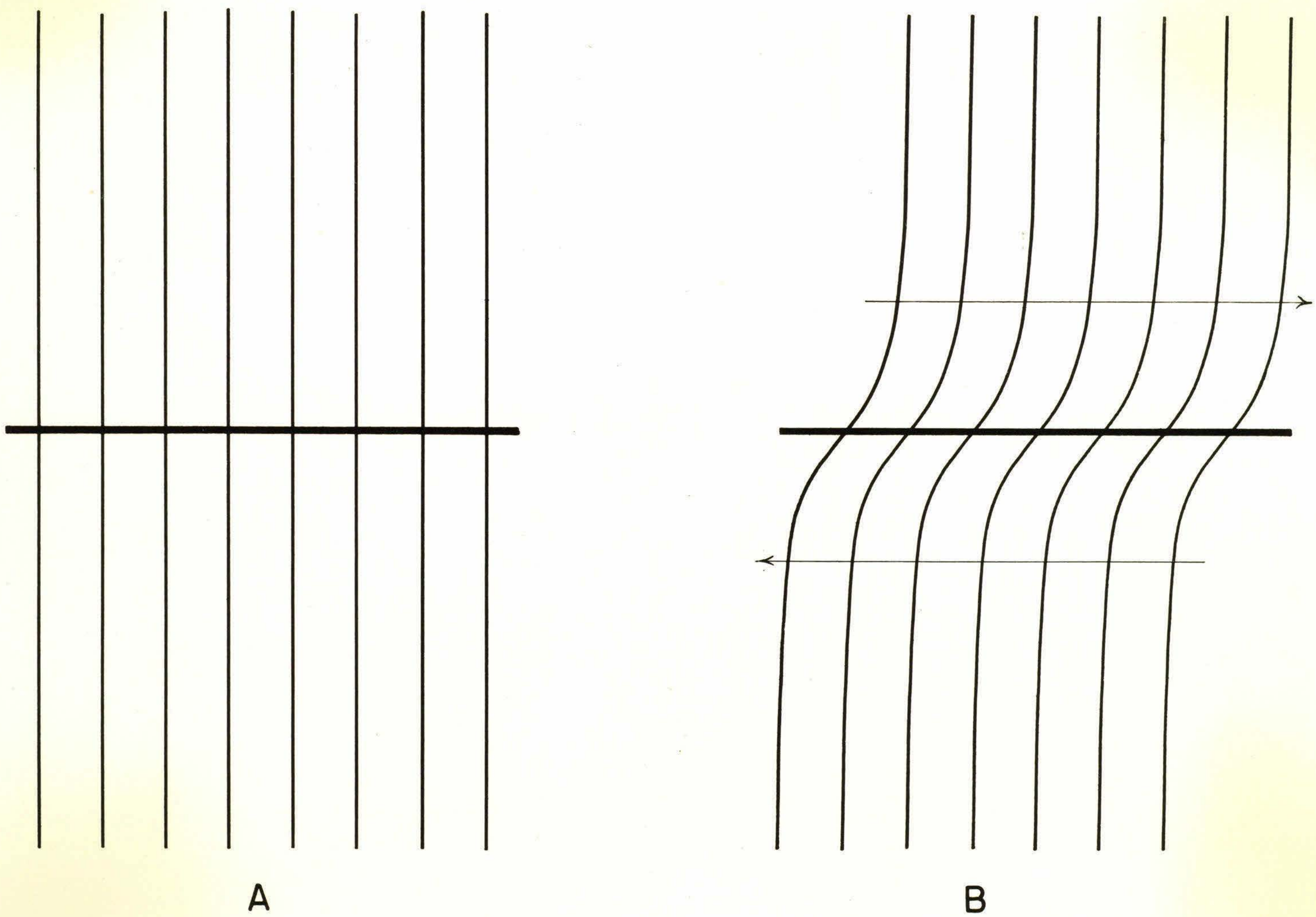


Fig. 1

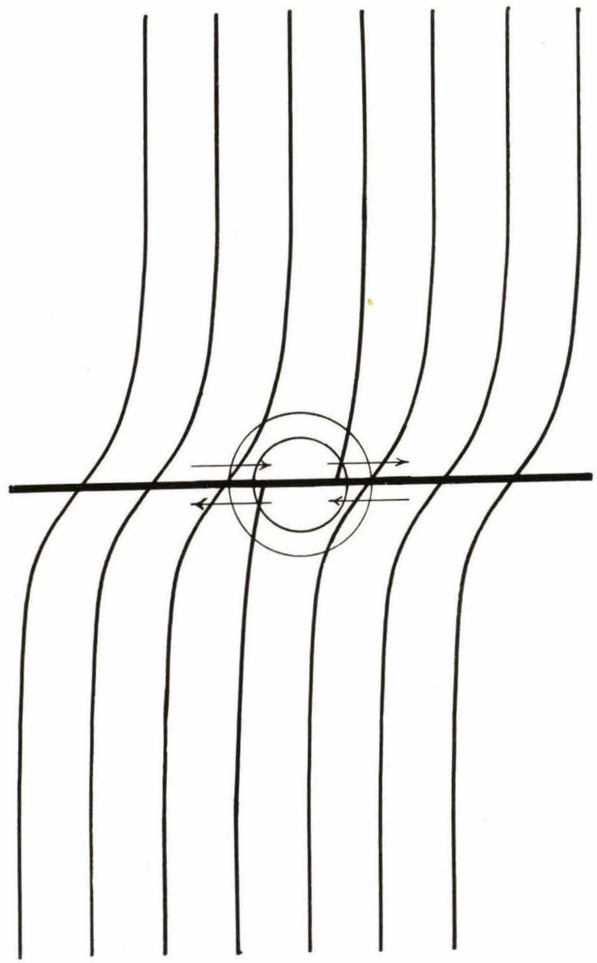

Fig. 2

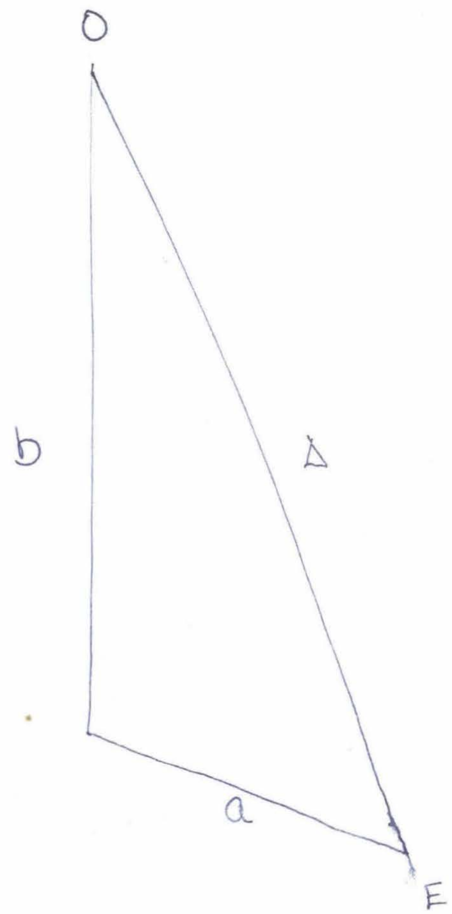


Fig. 3

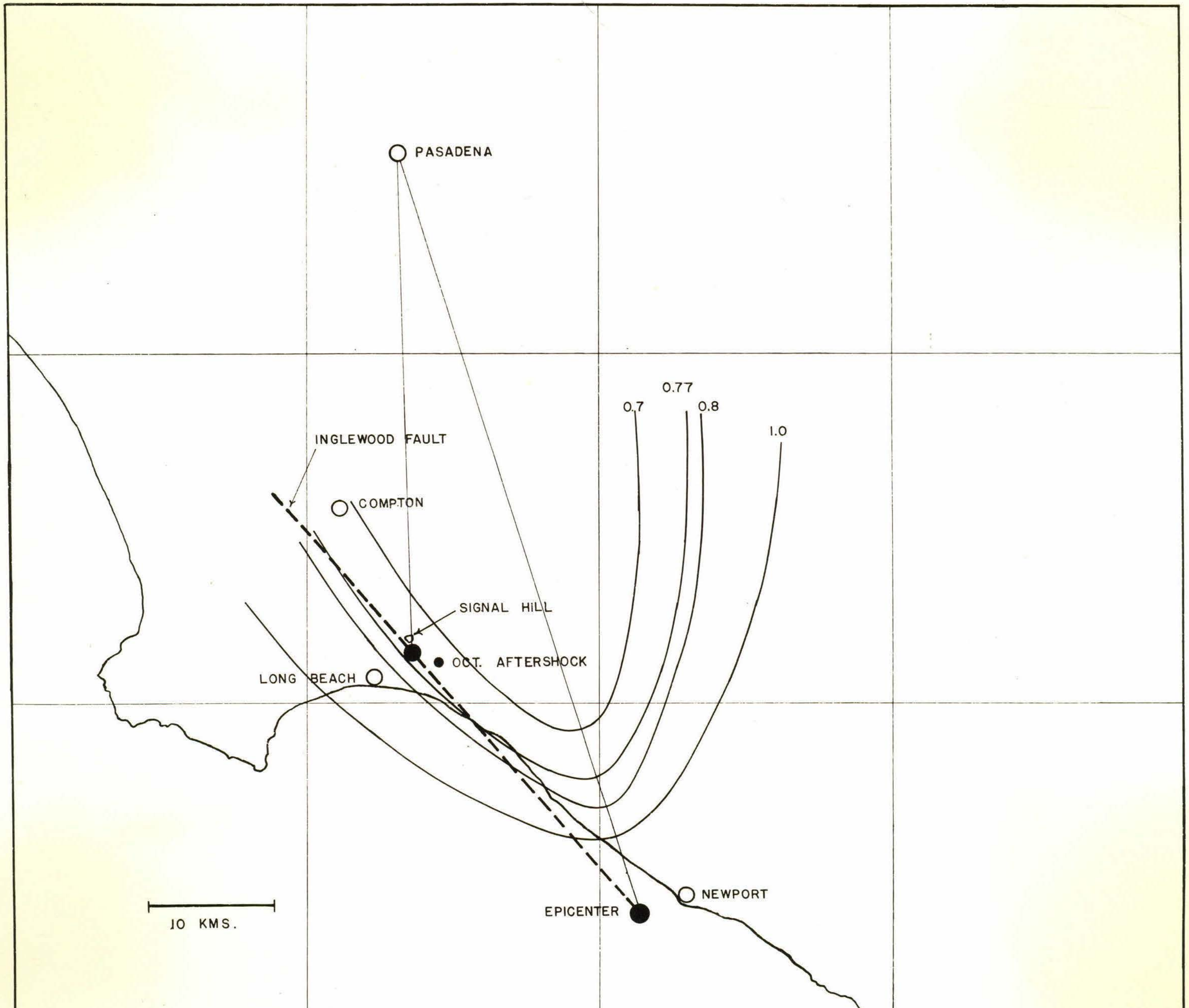
PASADENA
0.77
0.7
0.8
1.0
INGLEWOOD FAULT
COMPTON
SIGNAL HILL
OCT. AFTERSHOCK
LONG BEACH
NEWPORT
EPICENTER
10 KMS.

Fig. 4

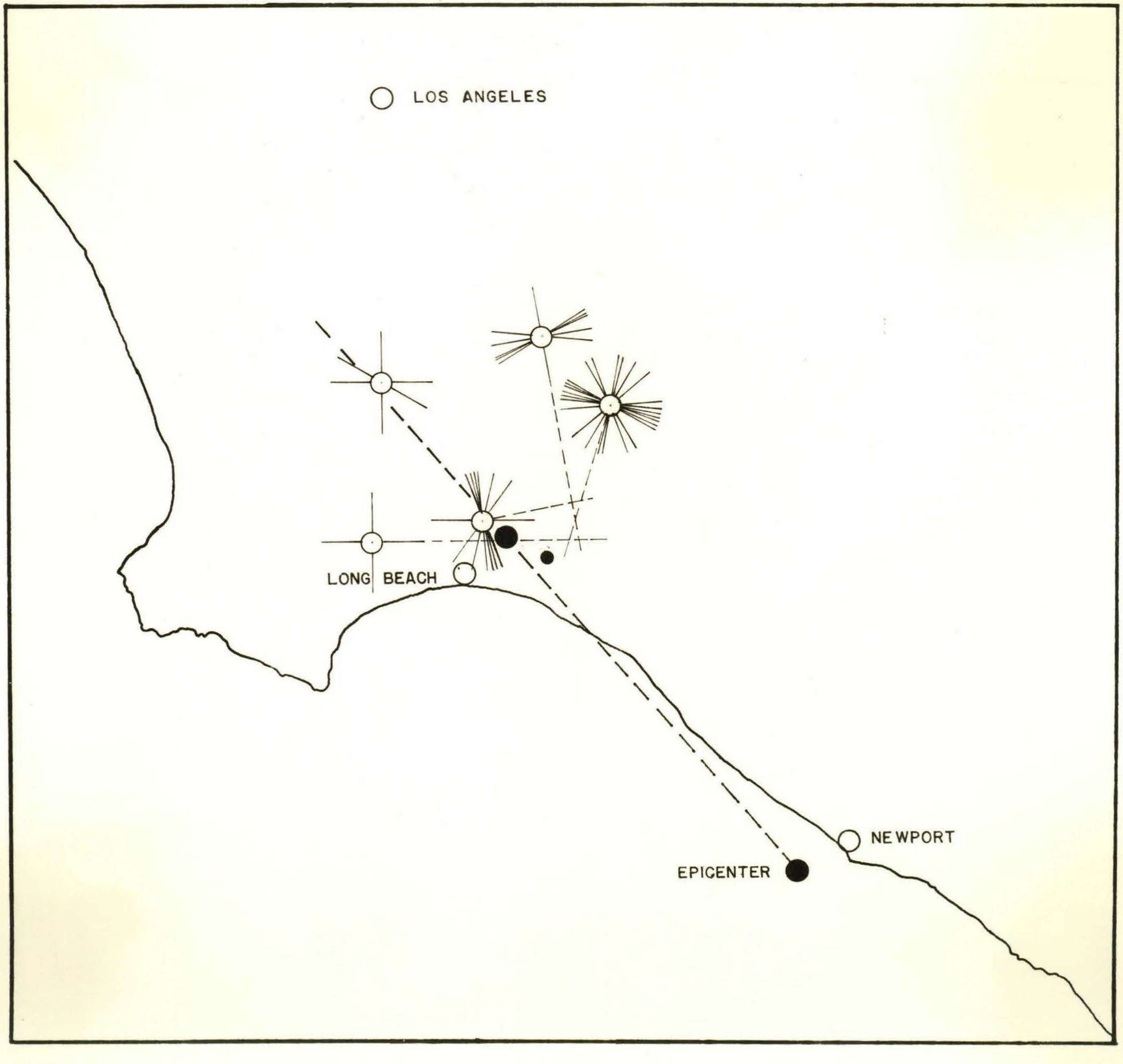


Fig. 5