

A REVIEW OF THE LITERATURE ON THE COMPTON EFFECT

Thesis by:

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I Introduction

As early as 1912 it had been found that scattered X-rays were softer than the incident rays. Sadler and Mesham⁽²⁾ tested a prediction of the Thomson theory in that year that the scattered rays should have the same absorption coefficient as the primary rays. They found quite the opposite to be true; namely, the scattered rays were softer than the primary rays. Florence⁽¹⁾ had found the same effect in the case of the gamma rays from radium as early as 1910. Any theoretical work which was done following these experiments was along classical lines and in no way approached the final explanation later put forward by Compton after more accurate measurements had been carried out in 1922.

Debye in 1915 attempted to extend the classical theory of scattering by taking into account the arrangement of the electron in the atom. He stated that there would be interference effects between the rays scattered from the various electrons in the same ring. But even this was not complete, for it assumed the electrons to be arranged in only one ring around the nucleus. A.H. Compton also attacked the problem of scattering from this point of view. But none of the methods seemed satisfactory.

In 1922 Compton first obtained experimental evidence for a change in wavelength upon scattering. Since it seemed impossible to explain this on any save the quantum theory, Compton was led to develop such an explanation.

Early in 1923 Compton published his extreme corpuscular theory in support of his previous quantitative experiments. Practically simultaneously, but independently, Debye applied the quantum theory to the scattering of X-rays and arrived at the same result as had Compton. The corpuscular picture and explanation are given later.

Immediately ~~other~~^{after} investigators started work to confirm Compton's data. Ross, in California, was the first to be successful. He improved on previous work by obtaining a photographic record, whereas Compton had been using an ionization chamber. Duane and his associates at Harvard, however, seemed to obtain negative results. But after repeated trials they too confirmed Compton and the last real opposition was removed. What Duane really observed at first was a so-called "Tertiary" effect, which will be briefly mentioned later.

At present the experiments and theory upon the amount of shift of the modified line, and its dependence upon the angle of scattering, are so well known that no further work needs to be done upon it. But the intensity distribution of the shifted line is still in need of much investigation.

Following the early period of experimental activity there came a large amount of theoretical work. This resulted in the explanation based upon the quantum theory. Then refinements of measurements and a few new evidences on intensity distribution were made. Hauncey's theory

of the scattered intensity and the wave mechanics' theory have been the latest important steps. Now it remains for experimentalists to produce more accurate intensity data and such as can be compared one with another for different angles of scattering. So far the variability of experimental conditions have made this at best a complicated and uncertain task. In the experimental chapter at the end of this paper we will briefly mention two methods which may accomplish this end.

C.T.R. Wilson⁽¹¹⁾ in 1923 obtained photographs of recoil electrons by means of his well-known cloud-expansion apparatus. Quantitative measurements on these plates verify the corpuscular explanation of Compton and give added weight to the evidence in favor of the quantum theory as opposed to the wave theory.

At first the experimental results differed from theory in that an unmodified line was found at the same time as the modified one. Also the modified line had a greater width than either the primary or the unmodified lines. Part of the width could be explained by slight variations in the value of the scattering angle, but not all, by any means. The explanations of these two points are taken up later on.

Inasmuch as the Compton effect can seemingly be explained upon the basis of the classical wave theory, the quantum theory, or the new wave mechanics, it is natural to try to obtain some correlation among the three theories.

No one of these three methods suits every problem in science. The wave theory can not explain the photo-electric effect, the quantum theory as yet fails to interpret optical interference, the wave mechanics is too new and difficult to have had complete success. But where these three basic theories overlap to explain one and the same phenomenon then it would seem possible to find some definite relation between them. Several attempts have been made in this direction and the correspondence principles so determined are mentioned, as they occur, in this paper. No one of them is generally valid. However, this phenomenon of Professor Compton's seems to give an excellent base from which to work. As more work is done on the subject it is hoped that more conclusive methods of bridging between them will be found. It is for this reason, partly, that so much importance is placed upon a correct solution of the intensity problem.

The wave theory of light has stood more precise tests than almost any other theory in the history of science. With the addition of Maxwell's equations and electromagnetic ideas there seemed to be nothing more to be desired in explaining the propagation of radiation. But Planck in studying black body radiation found discrepancies which seemed to indicate that the electrons could not be thought of as continuous radiators. This led him to postulate his corpuscular theory, or quantum theory of radiation. In the field of spectral analysis the quantum theory was especially attractive. The Compton effect derived much of its importance from the fact that it was the first.

direct experimental evidence favoring the then new quantum theory by disclosing encounters between quanta of radiation and "free" electrons.

Thus the Compton effect, with all the evidence which it presents, strongly confirms the idea that radiation travels in quanta of energy $h\nu$, and with momentum $h\nu/c$ "which never expand, or at all events always remain small enough to be swallowed up in one gulp by an atom, or to strike an electron with one single concentrated blow." (C)

However, it is not to be understood that the wave theory is by this entirely superceded. For, as is elsewhere pointed out, the phenomena of diffraction are best explained by the wave theory. The quantum idea may really be thought of as making accord between wave and corpuscular theories. Nor can it be forgotten that the quantum theory involves the wave theory in its most fundamental concept, for the "quantum of a given radiation is defined in terms of the frequency of the radiation, and the frequency is determined by applying the wave theory." (C)

The discussion of the Compton effect which follows is the basis upon which some of the present evidence for the quantum theory stands. As the yet unsolved questions are answered it can only be hoped that the discord of wave and corpuscular theories will be removed by some unifying generalization.

It has been impossible to find a definite test to

indicate the manner in which scattering takes place. The two proposed mechanisms are (a) delayed, (b) instantaneous. The delayed type supposes the atom to absorb the quantum, hold it for a finite small interval of time, and then emit it, after having moved an appreciable distance. This is a complicated process and one which is difficult to justify. At best the delayed theory could not explain unmodified scattering which has been shown to be "coherent"--i.e. to be definitely in phase relationship with the primary radiation. On the whole the instantaneous recoil of the quantum and emission of the electron are used throughout modern theory. It is on this latter assumption that Jauncey bases his intensity formula.

By placing absorbing screens in the path of the primary rays, and then in the path of the scattered rays it was originally shown that the scattered rays were softer than the incident rays. This same method has also been used to show that gamma rays from radium undergo a softening upon scattering. However, since the energy of the gamma rays is so great, none of the electrons are too tightly bound--or practically none--not to suffer ejection as recoil electrons from the parent atom. Hence the unmodified line is entirely absent or too faint to be observed. However, measurements on the change of absorption coefficient agree very closely with theory. This shows that as the limiting case of infinitely short wavelengths is approached the scattering of electrons obeys the same rules.

Duane's Tertiary radiation, so-called, was attributed to photo-electrons emitted when the primary X-rays struck the scatterer. It was supposed that some of these entered the scattering substance that some of these entered the scattering substance and struck other atoms making them emit other X-rays. Duane tried to show that these would have a maximum at about the same point as that at which the Compton maximum occurred. But this view had to be abandoned for it could not explain the dependence of the shift upon the angle of scattering.

Before going on the theory of the Compton effect it may be well to present a very schematic diagram, due to Richtmeyer, which readily shows the relations existing between the various phenomena accompanying the interaction of radiation and matter.

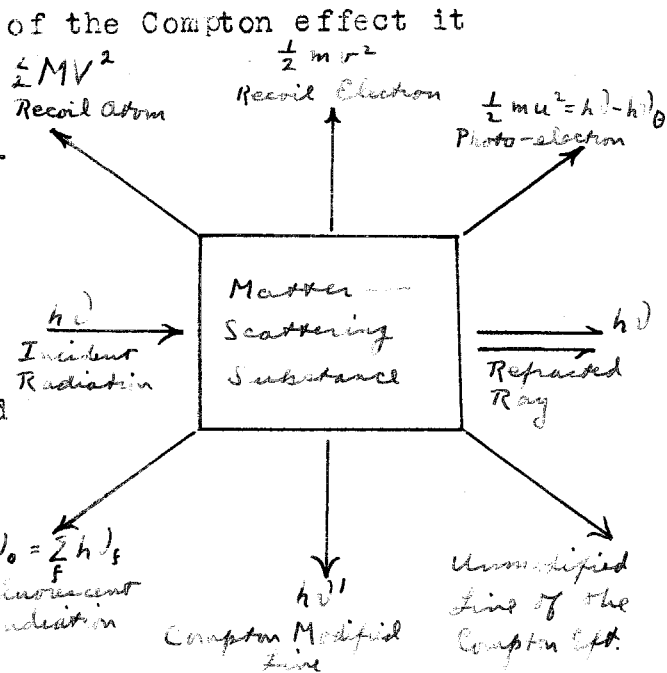


Fig I.

An imaginary horizontal line may be drawn through the middle of the square to separate the radiation (lower half) from the non-radiation (upper-half). It will be noticed that each radiation arrow is directly associated with a non-radiation arrow. This relation is indicated by diagonally opposite positions. Thus the shifted frequency which occurs in the Compton effect is directly opposite the kinetic energy of the recoil electron. (The directions of the arrows have nothing whatsoever to do

with the direction which these rays and corpuscles actually may assume in experiments.

Arrows at the top represent respectively: the recoil of the atom ($\frac{1}{2}MV^2$), the recoil of the electrons as in the Compton effect ($\frac{1}{2}mv^2$), and the energy of the photo-electron ($\frac{1}{2}mu^2$), where u is the velocity of the photo-electron.

In the bottom half we have the fluorescent radiation which is characteristic of the matter with the relation $\frac{1}{2}mu^2 = hv - hv_0$, where $hv_0 = \sum_f hv_f$, for the fluorescent radiation may be emitted at a number of different frequencies, which employs f to distinguish particular quantum transitions. Then follows hv' , the modified energy of the quantum thrown off in the Compton effect. Finally, there is the unmodified scattered quantum hv which causes the unmodified line.

II. Theoretical

Thomson Classical Theory:

The classical theory of the scattering, developed by J.J. Thomson and extended by Debye, accounted for many of the experimental properties. The successes of the theory led to the general belief that scattering could be placed side by side with interference and refraction as positive proof for the classical ideas of electrons and electromagnetic waves. However, Thomson's theory could in no way account for the Compton effect. And this new phenomenon, being quite opposed to the laws of electrodynamics, forced people to consider scattering to be direct evidence in favor of the quantum theory for radiant energy. It is true that the observed energy leaving a free electron upon scattering is in close agreement with Thomson's classical theory, but it does not agree when the matter of wavelength is concerned. The Thomson theory says that no change in frequency is to be expected upon the classical theory.

Even if one thinks of the Doppler principle as being active in causing the shifting phenomenon, the scattering particles would have to be moving in the direction of the primary beam at a speed comparable with, but considerably less than that of light. This is not possible on the classical (Thomson) theory, which supposes that all the electrons in the radiator are effective in scattering. Thus the classical electrodynamic theory appears irreconcilable with the view that the part of the secondary rays that are of greater wavelength than the primary are truly

scattered. This objection, though has lost its force since both the modified and the unmodified scattered radiation have been observed. It was advanced soon after the Compton effect was discovered. The reason for failure of the Thomson theory is that the magnetic field vector, and hence also the radiation pressure, is neglected.⁽⁹⁷⁾

Doppler Effect:

This method of explaining the Compton effect is based upon the fact that the electron after impact by the incident quantum is supposed to move off in a direction, dependent upon the initial conditions, and simultaneously to emit energy. This will be in the form of spherical waves. If the electron were stationary when it received the incident it were possible to assume that the radiation then there would be a change of wavelength for scattering straight forward in the direction of the primary radiation. But the electron must be considered to be in motion, and hence a double Doppler effect occurs. The electron is in motion both for the radiation it receives (primary) and for the radiation it emits (secondary) as well. Hence in the forward direction there is no change in wavelength. An observer on the moving electron receives the incident waves at a lower frequency than that at which a stationary observer would receive them--assuming that the electron is travelling in the direction away from the incident waves. And this decrease of frequency in the forward direction just compensates the shorter wavelength produced by the forward crowding effect of successively emitted wave fronts. Let it be

supposed that the straight lines in the accompanying fig. II. represent some particular phase in the nearly plane incident waves

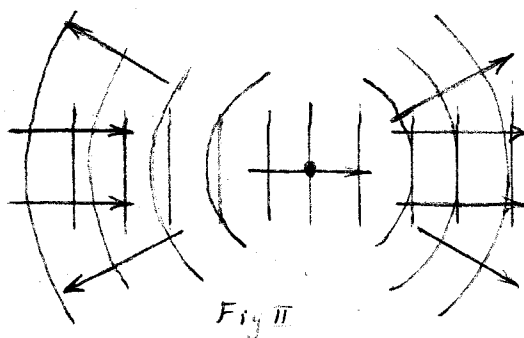


Fig II

and the circles represent that particular phase in the scattered wave which is always emitted from the electron when said particular phase of the incident wave arrives at the electron. By the definition of these phases then the corresponding sphere and plane are tangent at the instant of emission, and will remain tangent forevermore, in the forward direction, since in that direction the phase velocities are parallel and equal to the speed of light.

However, Compton has calculated that the oscillator must have a velocity equal to $\frac{c}{\frac{\lambda_0 mc}{h} + 1} \equiv \frac{c}{\frac{\lambda_0}{24 \times 10^8} + 1}$ in the direction of the primary ray to obtain the correct change of frequency. Where λ_0 is the incident wavelength, and the other terms have the usual meanings.

The mathematical derivation is unimportant for the purposes of this review, and if desired may readily be consulted in a number of early papers (cf. bibliography papers numbered 18, and 110). It is sufficient to state that the results obtained by the Doppler considerations --omitting relativistic corrections--are the same, to the first order, as those obtained by Compton's extreme corpuscular view. Using the relativity considerations the results are analagous up to second degree terms.

The end formula is (18): $\Delta \lambda = \frac{h}{m_0 c} (1 - \cos \theta)$ where ν is the changed frequency, and ν_0 is the incident frequency. From this it is readily seen that if the observation is made at an angle $\theta = 0$ the change of frequency is 0, as demanded by Compton's theory, and for $\theta = 180^\circ$: $\Delta \lambda = \frac{h}{m_0 c} [1 - (-1)] = 2 \frac{h}{m_0 c}$ also following Compton's derivation.

Perhaps the main values to be found in explaining the Compton effect by a wave theory are (1) to show that it is possible of explanation upon the basis of standard ideas, rather than having to think of an entirely new set of concepts, even though these new concepts are more probably correct, and (2) to give a method upon which to base a possible correspondence principle between wave and quantum theories.

With regard to this last point, P.S. Epstein has pointed out such a correspondence principle. He assumes a virtual oscillator radiating according to classical laws during the interval of impact. This has a velocity which is the vectorial mean of the initial and final velocities. This gives the correct shifted values from the Doppler effect.

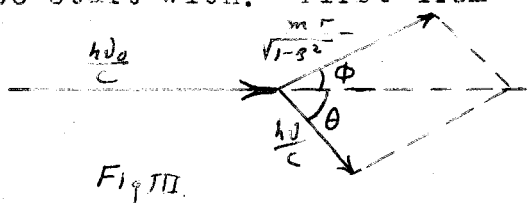
The main objection to the Doppler effect as an explanation of the Compton effect is that it requires a uniform velocity $\frac{hc}{mc\lambda_0 + h}$ for the electron. The radiation pressure, however, would give an acceleration, and while the electron was getting up to the required speed the Doppler effect would give the wrong frequency. But since the Compton line is diffuse and not sharp this last objection is not very strong.

Extreme Corpuscular View:

The extreme corpuscular theory in explanation of the Compton scattering was advanced almost simultaneously by Compton and Debye.⁽¹⁷⁾ To American writers it is known as the Compton theory and to Europeans as the Compton-Debye theory. We will enter into the explanation of this theory for the wavelength change of scattered X-rays more fully than for the other theories for several reasons. In the first place it is the most generally accepted explanation and gives a picture most easily understood. Second, the Compton effect is taken as being the first most important evidence for the light quantum theory, and this Compton-Debye explanation is based on that hypothesis. And finally the results of all the other theories may be shown to be analogous to those obtained by this method.

Briefly the picture may be given as follows. For simplicity we assume the scattering electron to be at rest at the instant of impact of the incident light quantum $h\nu_0$. Where h is, as usual, Planck's constant, and ν_0 the frequency of the incident beam--"nadelstrahlung"--in the notation of German writers. The simplest method of derivation is to omit relativity corrections for the electron after impact. We shall here consider such terms, as the difference is not very great, and it adds to the completeness of understanding.

There are three equations to start with. First from considerations of the law of conservation of energy we obtain: $h\nu = h\nu' + mc^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$ (13.1)



Then, considering the X and Y components respectively of the momenta, we have from the conservation of momentum

law:
$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \phi + \frac{m_0 c}{\sqrt{1-\beta^2}} \cos \theta \quad (14.1)$$

$$0 = \frac{h\nu'}{c} \sin \phi + \frac{m_0 c}{\sqrt{1-\beta^2}} \sin \theta. \quad (14.2)$$

From equations (14.1) and (14.2) we obtain, by elimination of the angle ϕ :
$$\left(\frac{h\nu'}{c}\right)^2 + \left(\frac{h\nu}{c}\right)^2 - 2\frac{h\nu'}{c} \cdot \frac{h\nu}{c} \cos \theta = \frac{m_0^2 c^2}{(1-\beta^2)} \quad (14.3)$$

which could have been obtained directly from the figure by the law of cosines.

The angle of scattering (ϕ) is determined by the point at which the observation is made, and is, consequently, a known quantity. The three unknowns to be determined by these three equations are ν' (changed frequency), $\beta = \frac{v}{c}$ and θ the angle of the recoil electron. Values of m (mass of the electron at rest), ν (frequency of the incident radiation), and c (the velocity of light) are all numerically known.

Partially following Compton's derivation ^(A) we write $\lambda' = \frac{c}{\nu'}$ for more convenient notation of the result. The solution, readily obtained is:

$$\lambda' = \lambda + \frac{h}{m_0 c} (1 - \cos \phi) \quad (14.4)$$

$$\delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \phi) \text{ or } \delta\lambda = 0.0484 \sin^2 \frac{\theta}{2} \quad (14.5)$$

Considerations of equation (14.4) show that the shifted wavelength must be larger than that of the incident beam. Also, from (14.5) it is apparent that for an angle of scattering $\phi = 0$ we have no change of wavelength, and for $\phi = 180^\circ$ the shift is a maximum, which is equal: $2 \frac{h}{m_0 c} = 0.0484 \text{ \AA}$

The relation between the angle of scattering and the

angle at which the recoil electron is ejected has been found approximately to be ⁽⁹⁷⁾: $\tan \phi = \cot \frac{\theta}{2}$. It holds where v' is nearly equal to v , because then the momentum triangle is nearly isosceles. This means that if the scattering angle ϕ increases from 0 to π , then θ decreases from $\pi/2$ to 0. This is directly evident, also, from fig. III.

Now if the initial motion of the scattering particle be taken into account--say with a velocity v_0 which makes an angle ϕ_0 with the direction of the incident ray, then the three initial equations become ⁽⁹⁷⁾:

$$mc^2 \left[\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} - 1 \right] + h\nu_0 = mc^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right] + h\nu \quad (15.1)$$

$$\frac{m v_0}{\sqrt{1-\frac{v_0^2}{c^2}}} = \cos \phi_0 + \frac{h\nu_0}{c} = \frac{m v}{\sqrt{1-\frac{v^2}{c^2}}} = \cos \phi + \frac{h\nu}{c} \cos \theta \quad (15.2)$$

$$-\frac{m v_0}{\sqrt{1-\frac{v_0^2}{c^2}}} \sin \phi_0 = -\frac{m v}{\sqrt{1-\frac{v^2}{c^2}}} \sin \phi + \frac{h\nu}{c} \sin \theta \quad (15.2')$$

$$0 = -\frac{m v_0}{\sqrt{1-\frac{v_0^2}{c^2}}} \sin \phi_0 + \frac{h\nu}{c} \sin \theta \quad (15.3)$$

Where the capital Greek letters denote a new set of angles, corresponding in position but differing in magnitude from those of the first set of equations. L. deBroglie has solved these equations and arrives at an answer of the same form as (14.5), but more complicated.

$$\Delta \lambda = \lambda - \lambda_0 = \lambda_0 \frac{v_0}{c} \frac{(\cos \phi_0 - \cos \phi)}{1 - \frac{v_0}{c} \cos \phi_0} + \frac{h\nu_0}{mc^2} \Lambda (1 - \cos \theta) \quad (15.4)$$

where: $\Lambda = \frac{1}{mc}$ and: $\cos \phi = \cos \phi_0 \cos \theta - \sin \phi_0 \sin \theta \cos \phi'$

If $v_0 = 0$ is substituted in (15.4) it then reverts to the form of (14.5). Further, if the initial velocity is sufficiently great ($\frac{v_0}{c} \gg \frac{h\nu}{\lambda_0}$) then the Compton effect entirely disappears and (15.4) gives the customary Doppler effect for scattering by a moving electron ⁽⁹⁷⁾.

If, in setting up the energy equation for this type of scattering, account were taken of all possible sources of loss of energy, we should have to add a term $\frac{1}{2}MV^2$ to equation (14.3), say. Where M is the mass of the atom and V its recoil velocity. But inasmuch as V is so small this term may readily be neglected in all cases. A further correction should, according to Compton in his book, be added-- $h\nu_0$ ---to denote the energy required to remove the scattering electron from the atom. But according to Jauncey's theory the process of scattering is conceived of as occurring in so short a space of time that the electron moves only a negligible amount through the atomic force field of the nucleus. During the process the light quant in its interaction with the electron does no work of ionization upon it. In this theory the electron obtains all its momentum and energy from the light quant in a sudden impulsive blow and subsequently, after the quant has travelled off with lost energy and changed frequency, the electron "coasts up-hill", so to speak, out of the atom upon the momentum gained from the collision.. But the equations of conservation governing the collision express only the equality of energy and momentum just before impact to the corresponding quantities just after impact. And since in this small time interval the electron had not moved appreciably with respect to the atom these equations should not contain a term expressing the work of ionization.

Half-Classical Theory:

This half-classical theory of the Compton effect was mainly advanced by Forsterling⁽⁶⁰⁾ and Halpern⁽⁵⁹⁾. It is built upon the fact that ~~that~~ the dependence of the wavelength change upon the angle of scattering is in agreement with that found by the Doppler principle for the scattering of a moving electron in the direction of the primary ray. In other words, it starts with ideas from the corpuscular theory and tries to explain them by the wave theory. Its chief work is an attempt to justify the velocity which it is necessary for an electron to have in order to yield the Doppler effect of the proper magnitude to explain the observed shift. Another way of expressing this is that it is desired to quantize the classical velocity equation $\beta c = c \frac{h\nu_0}{h\nu_0 + mc^2}$ and thus obtain the quantum theory for velocity.

By substituting one limit in the end equations (for which see the above noted articles) the Compton effect from a corpuscular view point is obtained, and using the other limit the classical Doppler effect is found. This yields a correspondence between the two explanations of the Compton phenomenon.

On the whole this theory is a complicated mixture of wave and quantum ideas which is now altogether superseded by more coherent, if not simpler treatments.

Compton^(A) points out that the velocity βc which the

exponents of this idea desired to quantize "is just that which an electron would acquire by the absorption of the energy quantum $h\nu$ or by scattering such a quantum." They thought that the electron scattered its energy in all directions after it had acquired a velocity βc . But to do this all the recoil electrons should move forward with a velocity βc . However, Compton points out that his experiment with Simon found these \checkmark velocities to be actually twice as great as this theory predicts. "This βc represents merely an 'effective' velocity, not a real velocity, of the recoil electrons."

Statistical Theory of the Compton Effect:

Bohr suggested that inasmuch as the light quantum theory is meaningless without some of the notions of the wave theory (i.e. frequency) that there must be some dualistic theory to combine the best qualities of each. Bohr, Kramers, and Slater⁽⁴⁴⁾ sought to unite these two heterogeneous views by the startling assumption that the laws of conservation of momentum and energy were valid statistically, rather than actually for every individual process.

While experiments by Compton and by Bothe and Geiger later proved beyond a reasonable doubt that this new theory was untenable, nevertheless we shall briefly describe it as of historical interest, and also shall show the method by which it was disproved.

They assumed that "spherical electromagnetic waves

are scattered by virtual oscillators, one such oscillator corresponding to each electron in the scattering medium. These virtual oscillators scatter the radiation in spherical waves in a manner similar to that demanded by the classical theory; but to account for the change of wavelength they are supposed to scatter as if moving with such a velocity that the Doppler effect will give the same effect as that predicted by the quantum theory."^(A)

Further in their proof they find the radiation pressure to be due only to the momentum of a few recoil electrons. Also the difference between the energy spent on the virtual oscillators and that reappearing as scattered rays does not appear uniformly distributed among all the electrons, but rather as the kinetic energy of a small number of recoil electrons. Thus it is found that the conservation of momentum is only statistically valid.

Bothe and Geiger experimentally refuted this view in a striking manner.⁽⁵⁹⁾ The test of the theory is possible if one is able to detect individual recoil electrons and individual quanta of scattered X-rays. Following Bohr, Kramers, and Slater's statistical view the probability that a Beta-ray from the scattered X-ray should appear at the same time as the recoil electron is estimated to be less than 10^{-5} . Bothe and Geiger arranged two chambers with adjacent openings into which the scattered rays passed. One chamber contained an "hv-counter", the other an "e-counter". On the view of Bohr and his associates there should have been practically no e-count and a very

large hv-count. But the German experimenters found very many more coincidences--a ratio of one to eleven--and their apparatus could not detect every possible coincidence.

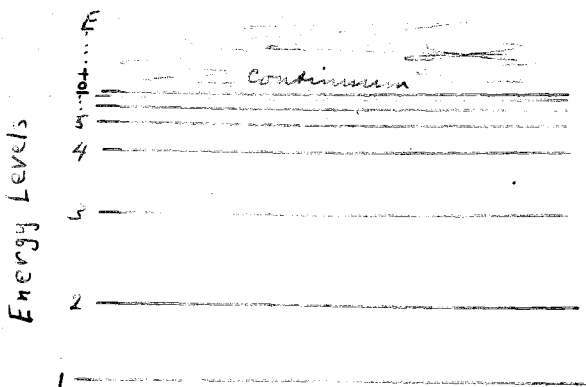
Compton and Simon⁽⁷²⁾ performed a second type of experiment which also refuted the statistical idea. "They passed very hard X-rays into a Wilson cloud chamber and examined the Beta ray tracks so produced. They measured the initial direction of the recoil electron track and so measured ψ . Occasionally the quantum scattered in the direction ϕ is either scattered again or ejects a photo-electron at a point outside the primary beam of X-ray. Joining the origin of the first Beta ray track to that of the second Beta-ray track, the angle between this line and the forward direction of the primary X-rays track is ϕ , while the angle at which the first Beta-ray track starts from its origin measures ψ ." (114)

These cloud expansion experiments show that the number of scattered quanta and recoil electrons are, on the average, identical. These results are quite opposite from the ideas of Bohr and his collaborators, and hence may be taken as nullifying their theory. Furthermore the experimental results speak favorably for the extreme light quantum theory.

The Wave Mechanics System:

Fundamentally the wave mechanics starts out with the consideration of this equation: $\nabla^2 \psi + \frac{8\pi^2 \mu}{h^2} (E - U) = 0$ where E is the total energy, U is the potential energy of the electron, μ is the mass of the electron. This equation

considers, for our purposes, the action of only one electron. More terms would have to be added to this equation for more complex forms. As it is we have reduced the problem to a hydrogen-like one, i.e. one electron and the rest of the system considered as more or less of a nuclear entity.



The equation has two particular cases, when E is positive and when it is negative. Certain restrictions are placed on ψ ---the "eigenfunktion"---namely: it must be finite when integrated over all space, it must be finite at all points in space, and finally it must vanish at infinity. In order for ψ to satisfy these conditions E must have discrete values when it is negative. On the other hand, if E is positive, then it (E) may assume any one of a continuum of values. The negative discrete values of E correspond to the old energy levels. The positive continuous values of E correspond to the energy of an electron in so-called hyperbolic orbits; i.e. an ionized electron.

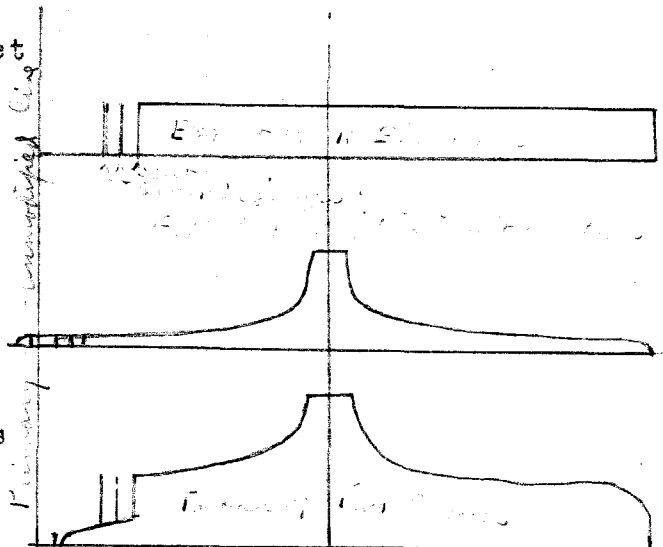
In Wentzel's theory of the Compton effect a quantum of radiant energy $h\nu$ (\gg the binding energy of an electron) is given to an electron which is initially in one of the negative energy levels. The electron, instead of absorbing this energy completely, and thus gaining kinetic energy (as a photo-electron), radiates part or all of the energy it has received, its final state being either the same as

its initial state or any one of the states discrete or continuous higher than the initial state. The absorption and radiation of energy together with the transition of radiation of the electron from the initial to the final state is here conceived as a single process. The radiation thus scattered will be unchanged in wavelength if the initial and final states of the electron are identical. These single processes are called Smeckal jumps. (Since first writing this a paper by Bergen Davis has appeared in the Physical Review for September, 1928. It presents good evidence for the Smeckal jumps where the initial and final electron levels are discrete energy levels.) The energy gained by the electron, if its final state is higher than its initial state, is abstracted from the radiation. The scattered radiation should thus contain frequencies less than the incident radiation by amounts corresponding to all such possible transitions of energy that the electron can make. Corresponding to jumps to discrete energy levels there should be shifted lines, differing from the incident radiation by amounts characteristic of the energy levels in the scattering atom. Since the scattering atom is light these lines are, in most cases, very close to the unshifted line. The intensity of such shifted lines is proportional to the probability of such an electron transition when the initial and final states of the electron coincide. Finally, there is some probability corresponding to a transition of the electron from its initial discrete level to any region dE of the continuum of positive energy levels. Wentzel's paper

purposes to show that these later probabilities have a maximum occurring at just the right continuous energy level to give the required shift: $h/mc(1-\cos\theta)$. In fact it appears from his paper that the analytical condition for the above mentioned maximum is formally identical with Compton's equation for the conservation of momentum. According to this the Compton shifted radiation should appear as a band whose maximum intensity occurs at the point observed and calculated by Compton for this shifted radiation.

It is well, at this point, to briefly point out that Jauncey's theory will give exactly the same result as to the position of the cut-offs and the Smeckal lines, and qualitatively the same general shape of continuous distribution as does the Wave Mechanics system. The principle difference between the two theories is in the assumption of the naïve Bohr atom with point electrons and elliptical orbits by Jauncey; while Wentzel's theory is more rigorous in taking the wave mechanics atom in which the electrons are not points but sets of waves. "The wave mechanics atom must be regarded as only a refinement on the Bohr atom, since in setting up the Hamilton-Jacobi equation for the wave mechanics atom the Hamilton function is identical with the old-fashioned mechanical function set up for the Bohr atom. Indeed, one sets up the Hamilton-Jacobi equation in identically the same way as though the Bohr atom model with point electrons in definite orbits were being

considered. But once having set it up one substitutes for the generalized momenta certain differential operators which then leaves the equation as a wave-equation instead of a mechanical equation, about mass points.



The accompanying rough diagrams show the sort of spectral distribution one is to expect from the types of jumps classified on the preceding page. The first part, showing a rectangle gives the effect of the K electrons of the scattering atom. The second part is the separate effect of an elliptic orbit electron. While the third part is a combination of the first two curves.

In the second paragraph above it is stated that the Compton radiations should appear as a band whose maximum intensity should occur at the point calculated by Compton for his shifted radiation. This predicted band for the Compton effect on the basis of the Quantum mechanics has been observed by Sharper and also by DuMond of this Institute.

Reason for the Unshifted Line:

It has previously been seen in the Compton corpuscular explanation of the shifted line that the absorption of some of the original $h\nu$ energy by the more loosely bound electrons accounted for the change in frequency and hence the scattered energy of the quantum. There obviously

follows from this that if the electron is more closely bound to the nucleus then there will be more energy absorbed in order to throw it loose as a recoil electron. The heavy atom, with closely bound electrons, will be harder to distort by quantum impacts, and hence the unmodified line will have the greater intensity. In order for modified radiation to be found the change in $h\nu$, due to the scattering process, must at least be equal to the critical ionization energy of the atom. One does not begin to get modified radiation until the frequency of the incident quantum not only exceeds, but even greatly exceeds, the critical ionization frequency. This is why there is no Compton effect for ordinary light. All of this will be seen to play a very important part in Jauncey's explanation of the intensity of the modified line. In the limiting case that the electron is so closely bound in the atom that the single quantum is unable to release it then it follows that the original amount of energy will be reradiated and no noticeable change of frequency will take place. The probability of multiple collisions at the same time is too small to be of value.

What happens ~~is~~ that the momentum of the entire atom is called into play for the case of $h\nu$ impact on a closely bound electron. That is, the electron may be thought of as absorbing the entire quantum, receiving on this account a severe shock which is not sufficient to knock it free. Then the entire atom must move--if at all--for the electron still remains in its original orbit. Since the mass of

the atom is several thousand times that of the electron then in the equation (6) of the Compton Corpuscular theory "m" must be replaced by "M" in the denominator, so that $\Delta\lambda$ becomes negligibly small.

In every case for which we find scattering according to the Compton theory we will also encounter bound electrons --either in the inner K orbit or other more closely associated sections of other orbits. Hence an unmodified line may nearly always be expected, in the X-ray range. The unmodified line can be absent in the X-ray range if the X-rays are hard and the atom of low atomic number and the angle of scattering large. It suffices that the recoil energy be sufficient to ionize the K electrons. A theoretical method has been worked out for obtaining a sufficient concentration of a true "electron gas" to result in a shifted line alone. If an experimental procedure to test this can be obtained then an unmodified line would not be expected. When gamma rays are used as an exciting energy source the modified line is very much more intense than the unmodified, showing that practically all of the electrons encountered have been changed.

Intensity Theories:

More formulae for the intensity and width of the modified line have been developed than theories of the explanation of the presence of the modified line. We can here but mention each briefly and give their relations to the other formulae. The two most satisfactory results

are obtained from Jauncey's formula and from the one developed on the basis of the wave mechanics. The physical picture behind Jauncey's reasoning we shall attempt to give. For the wave mechanics ideas the reader is referred to several papers listed in the bibliography: 146, 128, 120.

It is well to name specifically certain definite problems which exist in the intensity field. Briefly they are as follows. First, the question of total shifted intensity as a function of the angle of scattering. Second, that of the ratio of the total shifted intensity to the unshifted intensity as a function of the scattering angle. Third, the structure of the shifted line. That is, the intensity in any region $\Delta\lambda'$ as a function of the corresponding shift $\lambda' - \lambda$ for any given constant angle of scattering. Fourth, and finally, there is what might be called a problem, and for which no satisfactory solution has as yet been found, namely--what determines whether an electron will be ejected photo-electrically. The theories of Jauncey and Wentzel as outlined below are the only satisfactory solutions for the second and third intensity problems listed above. In answer to the first question Breit and Dirac have developed fairly satisfactory formulae.

The shifted intensity as worked out by Breit, and presented in Compton's book is: $I_{\phi} = I_0 \frac{1 + \cos^2 \phi}{2(1 + \alpha \cos \phi)}$

where I_0 is the incident intensity, and $\alpha = \frac{h\nu}{mc^2}$

"This represents the simplest possible average of the two limiting values of the intensity assigned by the correspondence principle." (A)

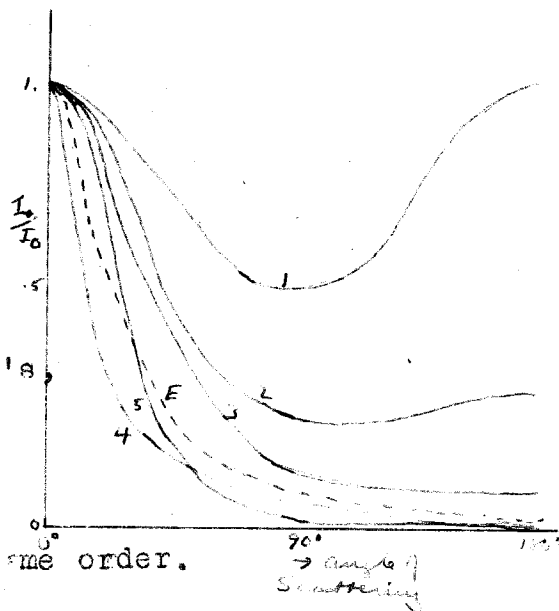
Wentzel gives a very compact and clear resume of the status of the classical and corpuscular theories upon the intensity problem. In the wave theory the frequency and intensity problems receive equal consideration, while in the quantum theory the intensity question is a special problem the solution of which is, in principle, very uncertain, and logically less satisfactory. In the experimental section we mention two methods which should in the near future go a long way towards clearing up this uncertainty. "The Bohr correspondence principle serves as one guide to the generally mentioned expression that qualitatively the classical calculation of intensity for the quantum process is necessary; in the boundary case, where the frequencies from classical and quantum considerations are exactly the same, then the computed intensities are exactly right. In our case, where the intensity distribution of the different directions of scattered rays are concerned, the statement of the correspondence principle is especially meager and has accordingly found frequent explanation. While in the frequency question the different theories lead to the same end formula."⁽⁹⁷⁾ But it can be shown that only two intensity formulae are in any way compatible with experiment--namely those by Jauncey and Compton and later the one based upon the wave mechanics.

Added importance is given to the need for an experimentally determined intensity law by the fact that it would make possible a knowledge of the velocity distribution of the recoil electrons.

At first it was thought that the predicted width of the modified line on theoretical grounds would give much wider lines than actually occur. But it is now found that the discrepancy is not as great as at first supposed. DuMond, in a paper soon to be published, has found the agreement to be as good as can be expected when the effect of all the electrons in the scattering atom is counted in. He points out that "those electrons which were thought to give too wide a line, as in the case of carbon K electrons, only contribute a part of the total modified intensity so that when the curve is added up for all electrons its general appearance is not far different from the experimental curve. Obviously those electrons which contribute breadth to the curve cannot contribute much intensity in any one spot according to Jauncey's theory--which assumes that all electrons have equal a-priori probability of collision (modified or unmodified scattering). For if the area under the component part of the curve, for each electron, is to be the same the component curves which are broad cannot be high." In the earlier Jauncey theory the discrepancy in the width of the theoretical and experimental curves was attributed to one or all of several causes. Namely, some electrons are moving in elliptic orbits and not in circular; there is an appreciable time of contact between electron and quantum; or because the electrons do not move with as great a velocity as the current form of the atomic theory supposes.

The adjacent figure V. shows the relation existing between

experiment and the several theories upon the question of total shifted intensity as a function of the angle of scattering. The dotted line is for experimental values. The other curves are: (1) Classical, (2) Debye's, (3) Compton's, (4) Jauncey's, (5) the Wave Mechanics'. They will now be mentioned and considered in the same order.



Classical Theory:

J.J.Thomson developed the classical theory of intensity which, as is seen from the accompanying figure, gives the same value for 180 degrees as for zero degrees. And except for its initial value is non-coincident with known experimental data. The intensity I of the scattered line is developed from classical considerations of electromagnetic waves, and becomes more complicated as additional factors such as motion of the secondary radiator are considered.

If I_0 is the primary intensity, then the scattered intensity for an angle θ is:
$$I_{\theta} = \frac{e^4 I_0}{R^2 m^2 c^4} \left[\frac{1 - \cos^2 \theta}{2} \right] \quad 30.1$$
 where R is the distance from the scatterer to the point of observation.

Quantum Theory:

Compton points out ^(A) that "all forms of the quantum theory of X-ray scattering are based on the assumption that for long wavelengths, where the motion imparted to the scattering electron is small, the intensity of the scattered

ray approaches that assigned by Thomson's classical theory."

The correspondence principle of Bohr requires a different limiting intensity for $\theta = 0$ than Thomson's theory. Breit shows this new limiting value to be: $I = I_0(1 + \alpha \cos \theta)^4$ 31.1
 From theoretical considerations and actual experimental data the limits should lie between those demanded by equation (30.1) and (31.1).

Debye in his original paper⁽¹⁷⁾, states that the intensity question (referred to as number 1, page 26) can be solved only by an assumption as to the probability of a single process of scattering at a certain angle. Or, stated in another way, the number of quanta scattered in any direction θ may be assigned by the classical theory, $\frac{I}{h\nu}$, but because of the change in wavelength the energy of each quantum would be reduced by $\frac{h\nu'}{h\nu}$. Consequently the scattered intensity according to Debye is: $I_\theta = I_0 \cdot \frac{1}{2} \cdot \frac{1 + \cos^2 \theta}{1 + \alpha \cos \theta}$
 The scattering absorption coefficient (fraction of primary energy removed by the scattering process) is the same as that of the classical theory: $\sigma = \sigma_0$.

However, it is known that the total absorption of hard X-rays is less than σ_0 ; hence Debye's formulae must be excluded from fulfilling the experimental curve completely.

Compton and Woo, from considerations of the Doppler formula and its relation to quantum theory, derived a form based on these two ideas. Woo used a Lorentz transformation and obtained: $I_\theta = I_0 \frac{e^4}{R^2 m^2 c^4} (1 + 2\alpha)$ (31.2)

where: $\alpha = \frac{h}{mc\lambda}$ or: $I_{\theta} = I_0 R^2 \frac{e^4}{m^2 c^4} \cdot \frac{1+2\alpha}{2(1+\alpha \cos \theta)^5}$

This approaches more nearly the experimental form, but is seen to be quite different even yet. It cannot be correct unless the form of correspondence principle used in its determination be inapplicable. Upon evaluation for sigma Woo found that he, too, had $\sigma = \sigma_0$, which is another argument against his formula.

Compton, by a different method, obtained:

$$I_{\theta} = I_0 R^2 \frac{e^4}{m^2 c^4} \cdot \frac{1}{2} \left[\frac{1 + \cos^2 \theta + 2\alpha(1+\alpha) \cos^2 \theta}{(1 + \alpha \cos \theta)^5} \right] \quad (32.1)$$

and $\sigma = \frac{\sigma_0}{(1+2\alpha)} \quad (32.2)$

This last equation indicates a reduction of the scattering absorption coefficient with frequency, which is found to be true experimentally, also.

Jauncey's formula gives results which are in good agreement with fact and the theory behind which seems to account for the physics for the scattering very completely. To obtain the theoretical curve for any practical case is somewhat of a task for it involves many computations. Briefly it is as follows.

For his intensity formula Jauncey makes the following six assumptions. He assumes.:

- (1) the Bohr-Sommerfeld atom---point electrons revolving in definite elliptical and circular orbits.
- (2) a priori probability of a collision between a light quantum and an electron equal for all electrons.
- (3) impulsive action between light quanta and electrons ---i.e. the time of interaction between light quanta and electron

compared to the orbital period of the electron is negligible.

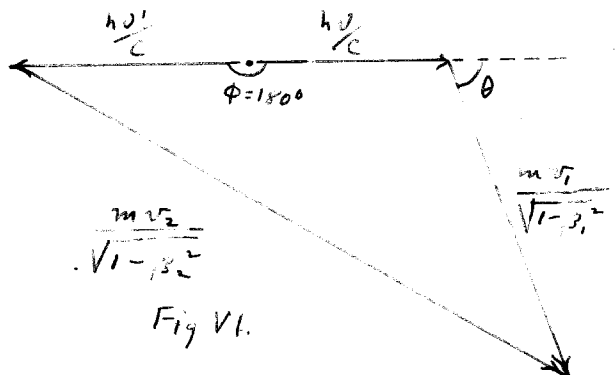
(4) a. conservation of energy and momentum between electron and light quantum when electron is ionized by light quant and (b) the same between atom and light quantum when electron is not ionized. The orbital momentum of the electron is taken into account.

(5) the electron is ionized when recoil energy, as calculated under (4a), exceeds the binding energy of the electron in question, and that it is not ionized in all other cases.

(6) unmodified scattered radiation is due to energy scattered under second part of (5).

The process involves separate calculations for circular and elliptical orbits of the electrons. We shall here proceed to outline the method followed for circular orbits and then the variations preliminary to solving the elliptical problem. The case considered is for 180 degree scattering, which is the simplest, and at the same time gives maximum displacements. Inasmuch as variation of displacement with scattering angle varies as (1-cosθ) it is readily seen that the curve will be approximately flat-topped at 180 degrees--so that this method will hold true for 176° scattering (which was the experimental angle used by DuMond).

Besides assuming electrons in circular orbits, we will further assume that there are an infinite number of them. The adjacent fig. VI is a vector picture of the



momenta for the case of electrons with initial velocity and scattering angle of Phi equals 180°. We will let θ be the angle between the incident quantum and the initial electron direction, ν the incident frequency, ν' the scattered frequency, $\beta_1 = \frac{v}{c}$, $\beta_2 = \frac{v'}{c}$, and the other quantities their usual meanings. The above figure is necessitated by the law of conservation of linear momentum. From this and the law of the conservation of energy we

are enabled to set up two simultaneous equations:

$$\left[\frac{h\nu}{c} + \frac{h\nu'}{c} \right]^2 + \left[\frac{m\nu v}{\sqrt{1-\beta_1^2}} \right]^2 + 2\cos\theta \left[\frac{h\nu}{c} + \frac{h\nu'}{c} \right] \left[\frac{m\nu v}{\sqrt{1-\beta_1^2}} \right] = \left[\frac{m\nu' v'}{\sqrt{1-\beta_2^2}} \right]^2 \quad (34.1)$$

$$h\nu + mc^2 \left[\frac{1}{\sqrt{1-\beta_1^2}} - 1 \right] = h\nu' + mc^2 \left[\frac{1}{\sqrt{1-\beta_2^2}} - 1 \right] \quad (34.2)$$

Upon solving for ν' we obtain the relation:

$$\nu' = \frac{1 - \beta_1 \cos\theta}{(1 + \beta_1 \cos\theta) \frac{h}{c} + 2 \frac{h}{mc} \sqrt{1-\beta_1^2}} \quad (34.3)$$

or:

$$\lambda' = \frac{(1 + \beta_1 \cos\theta) \lambda + 2 \frac{h}{mc} \sqrt{1-\beta_1^2}}{1 - \beta_1 \cos\theta} \quad (34.4)$$

hence the change in wavelength is:

$$\lambda' - \lambda = \frac{2\beta_1 \cos\theta}{1 - \beta_1 \cos\theta} \lambda + \frac{2h}{mc} \frac{\sqrt{1-\beta_1^2}}{(1 - \beta_1 \cos\theta)} \quad (34.5)$$

An examination of equation (5) shows that a change in wavelength is a nearly stationary quantity of amount $2(h/mc)$, with a slight correction factor, and also a variable term (the first term) whose importance diminishes with the incident wavelength, and which may be positive or negative depending upon θ . If all directions of electron velocity at the instant of collision are equally probable then this latter term should occur negatively as often as positively. This term accounts for the broadening of the line. The second term accounts for the wavelength shift. β_1 is always

a very small quantity, less than one tenth the speed of light. Hence β^2 is less than one percent and may, for our purposes, be neglected. If now we let Δ be the change in wavelength that is the shift of the radiation on either side of the shifted position for the case if initially stationary electrons (i.e. $2h/mc$) then the positive Δ s are for shifts greater than $2h/mc$, and negative ones for lesser shifts. Then, from equation (5) after simplifying and unifying we obtain:

$$\Delta = 2 \frac{\beta \lambda^* \cos \theta}{1 - \beta \cos \theta} \quad (35.1)$$

where: $\lambda^* = \lambda + h/mc$ and the subscript of β is dropped. And $d\Delta = 2\beta\lambda^* \sin \theta d\theta \frac{1 - 2\beta \cos \theta}{(1 - \beta \cos \theta)^2}$ (35.2)

It is known that the probability that an electron will be hit by a quantum with an angle θ between the two trajectories θ and $\theta + d\theta$ is $P_\theta = \frac{1}{2} \sin \theta d\theta$. Hence, neglecting β^2 again:

$$P_\Delta = -\frac{1}{4} \frac{d\Delta}{\beta \lambda^*} \quad (35.3)$$

In other words, for any Δ (shift from the position for resting electrons) the chance is independent of Δ and equal simply to the proportional part of the total spread -- $4\beta\lambda^*$ ---represented by $d\Delta$. The area of the rectangle (fig. VIII) is called unity if the probability of hitting the electron is taken as unity. Thus we obtain the width of the shifted line on either side of the $\lambda' - \lambda$ position. The height involving the intensity is com-

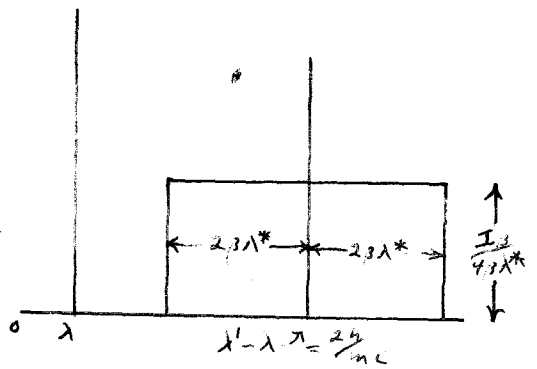


Fig VIII.

puted from more involved mathematical considerations. But it is readily seen that the area will be a total integrated intensity: $4\beta\lambda^* \frac{I_0}{4\beta\lambda^*} = I_0$

Now turning to the case of elliptical orbits. The curve

of Fig. VIII holds true in this case, except that it becomes more and more of a true curve as the ellipticity increases. This is because electron velocity varies in different portions of its orbit and account must be taken of this. So that it is necessary to plot, first, a graph of the relation of speed of each electron against its position in the orbit. The area of the rectangle is proportional to the number of electrons in the type of orbit under consideration, or, it is proportional to the length of time an electron is in a given speed interval of the orbit. The total intensity must, of necessity, be composed of components added together from the effects given by each of the orbits of the electrons of the scattering atom.

Above certain critical velocities the electron is so closely bound to the nucleus that a light quantum cannot knock it loose. This causes critical cut-off frequencies-- defined in a later paragraph under "Limits due to the Conservation of Energy"--. These cause irregularities in the intensity curve. That is, theoretically the width of the line will approach too closely to the unmodified position λ . But these cut-off frequencies govern where the cut-off for each particular electron orbit must occur. Hence part of some of the component curves are made inoperative. The area which is thus cut off is added to the intensity of the unshifted line.

The abscissae are plotted equally on either side of the $\Delta\lambda = \frac{h}{mc} \cos\theta$ position which is the shift for initially stationary electrons at the scattering angle of 180° . If there is more than one electron in the same type of orbit, then

the intensity for that type must be multiplied by the number of electrons in the orbit.

The agreement between Jauncey's theoretical curve, the theory of other investigators, and experiment cannot definitely be stated yet, for good experimental data is hard to obtain. But it has been shown that the maximum intensity demanded by Jauncey is far greater than that shown on the photographic plates. The maximum shift and the width of the line are, however, in good agreement, as is also the general shape.

Wave Mechanics:

As is seen from curve (5) of the graph on page (30) the intensity curve as calculated from the new wave mechanics is in good agreement both with Compton's theory and also with the experimental results. Dirac⁽¹²⁰⁾ evolves the intensity formula $I_{\theta} = \frac{a^2 c^4 v^2}{8\pi K_m c^4 v^2} (1 - \cos^2 \theta)$ where c is the velocity of light, v is 2π times the wave number of the incident radiation, and v' is the same for the scattered radiation. "a" is a small constant neglected in second order effects, and l_2 is a direction cosine. This he shows to be just $\frac{v'}{v}$ times as large as those formulae based on the quantum theory. In particular this formula agrees with the others at zero and 180 degrees. Dirac further points out that this is the first physical result obtained from the new mechanics which had not previously been known.

W. Gordan⁽¹²⁸⁾ showed that the values of the intensity

of the Compton effect on the basis of Schrodinger's wave mechanics is equal to the geometrical mean of the corresponding quantities for the beginning and end conditions of the process based on the corpuscular theory. This suggests a correspondence principle between quantum ideas and the wave mechanics which may, in the future, readily be extended.

Polarization of Shifted and Unshifted Lines:

Dirac⁽¹²⁰⁾ states that "the state of polarization of the scattered radiation is the same as on the classical theory". And further, "the radiation scattered through 90 degrees is thus plane polarized for unpolarized incident radiation." This has been experimentally verified by Kallmann and Mark in Germany⁽¹²²⁾. They investigated whether or not the primary intensity actually approaches zero, and also whether the secondary ray is linearly polarized. To accomplish this the secondary rays were analyzed spectroscopically at 90 degrees, and then the analyzing crystal was turned with respect to the secondary rays, when it was perpendicular to them. The amount of this turning is measured by an angle "A". The relation of the unmodified line is known, and also it is known that its intensity in the case of linear polarization detracts from the customary scattered radiation with the function $\cos^2 A$. It was thus observed that the ratio of the unmodified intensity to the modified intensity depended upon the angle A. If this ratio remains constant then it proves that the Compton radiation is linearly polarized--similarly to the normal scattering ray. The actual results which they obtained proved that the ratio of the shifted and unshifted intensities was

practically independent of the polarization angle, which showed that the intensity of the Compton radiation depends upon the normal scattered ray in the same manner that the polarization depends upon it.

Limits Due to the Conservation of Energy:

As was pointed out at the end of the section dealing with the extreme corpuscular view several minor terms must be added to the energy equation to make it theoretically complete. This gives, following Compton's book on page

286:

$$h\nu = h\nu' + h\nu_s + mc^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) + \frac{1}{2} MV^2 \quad (39.1)$$

To determine the limiting conditions we omit $\frac{1}{2}MV^2$ as being negligible, and $mc^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right)$ is 0 for ν' to be a maximum. Hence

we have:

$$h\nu'_{max} = h\nu - h\nu_s \quad (39.2)$$

or: $\lambda - \lambda'_{min} = \lambda - \lambda_s$. Since there is no finite lower limit to the frequency, the first equation being satisfied when ν' is 0, then the wavelength of the scattered ray may have any value for which: $\lambda' \geq \lambda + \frac{\lambda_s^2}{\lambda_s - \lambda}$. This equation gives the lower limit for the cut-off frequency. The only other limit easily obtainable is from the knowledge that ν must be greater than ν_s to throw an electron from the atom and form a scattered ray. Hence it follows that when the electron is firmly bound to the atom that no modified line may be expected.

The first of these limits tells why no Compton line is observed in the region of the visible spectrum.

Photo-electric Effect vs. Compton Effect:

For a short time there appeared to be a conflict between the ideas upon photoelectrons and the recoil electrons of the Compton effect. Millikan showed, in his experiments upon photo-electricity, that the electron which is ejected must be free. Then Compton showed that the momentum and hence the energy of the incident quantum becomes divided. This represented a definite disagreement between the two theories; and seemed to mean the complete abandonment of one or the other. But Klein and Rossland (Z.f.P. IV,46,1921) found an intermediate process, "Collisions of the second kind", which showed that the photo-electrons and recoil electrons were upon two very distinct bases. "Collisions of the second kind" admit of energy $h\nu$ being transferred indirectly, without loss, from the light wave to the free electron, thus obviating the necessity of a direct transfer. That is, energy can be transferred from the light wave to the free electron by being absorbed first by an atom, which is thus changed from the normal to an excited state. This excited atom can return to its normal condition without radiation by collisions of the second kind, which consist in transferring its whole absorbed $h\nu$ to a free electron.

In the Compton effect the light quantum collides directly with the free electron and can be changed in frequency if it collides with a bound electron only if the bound electron is raised to a higher energy level, discrete or continuous, by a collision. Hence the photo-electric effect cannot possibly represent the direct interaction between a light

quant and a free electron. Indirectly collisions of the second kind aid in clearing up the impasse between wave and quantum theories.

III Experimental

The experimental arrangement is rather fully set forth in Compton's book "X-rays and Electrons"^(A), so that there is no need to describe that here. The apparatus used by Wilson to obtain his "fishtracks" photographs also needs no description. However, it may be well to say a word as to the meaning of his photographs.

By means of the passage of the X-rays through the gas one obtains two types of figures: a) electrons of long range, and b) electrons of short range. The long range particles are photo-electrons and are ejected most copiously by the longer wave-length X-rays. The short range Beta particles (or recoil electrons) are found to be due to shorter wave X-rays. These are the "fish tracks" on the photographic plates.

The range of these particles and their direction of emission furnish strong evidence for the corpuscular theory of the Compton effect. The light quantum idea demands a strong concentration of recoil electrons with paths in the primary direction, which is in entire agreement with experiment. Pictures of these tracks have been taken which yield still more accurate measurements.

The time of exposure required to obtain a photograph of the Compton effect is about one hundred hours. It is obvious that conditions will change tremendously during that time so that intensity conditions will vary within

wide limits. Voltages change with shifting of the power line load unless much care is used to employ complicated ballast devices. Further the contamination of the walls of the tube with sputtered tungsten--or other target material ---which gradually cuts down the X-ray intensity is very harmful. Even the finite size of the grains of the emulsion of the photographic plate cause difficulties in obtaining a smooth photometric curve of intensities.

Each exposure of one hundred hours or more gives a result for just one angle of scattering. Now if a second angle of scattering is used for the next exposure it is obvious that no very accurate conclusion can be drawn from the two resulting intensities, for the variations are additive; they do not average out, i.e. in the progressive changes upon the tube.

Three experimental methods are now proposed for further study of the intensity problems and the structure of the modified line. The first is an X-ray tube, scatterer, and crystal spectrograph all contained in one evacuated tube and capable of 176° scattering. The second is a 50 crystal spectrometer, similar in theory to the well-known single Compton apparatus. The advantages hoped for this method of multiple crystals is (1) reduction of time of exposure, (2) increased ratio of intensity on film to primary intensity, thus reducing the relative importance of stray radiation, (3) constancy of scattering angle. The third method is a means of obtaining the displacement of the

modified line and its intensity simultaneously from a Seeman crystal spectrometer for practically a 270° range. Mr. J. W. M. DuMond is at present carrying on work to obtain experimental results with at least the first two of these devices.

Until these or some similar experimental methods present new data upon the intensity of the shifted line with regard to scattering angle and shift the theorist must halt or work most uncertainly upon this question of the Compton effect. When the problem is finally solved much new light will be furnished science upon the correspondence principles between the classical wave theory, the quantum theory, and the new wave mechanics. Further, a very definite addition will have been made to the knowledge of the structure of the atom and the speeds of the electrons in their orbits. Already, as we have previously pointed out, the Compton effect has done much to establish the quantum theory on a firm basis and has repealed the attacks upon the laws of conservation of energy and momentum.

In conclusion I take this opportunity of expressing my very great indebtedness and sincerest thanks to Mr. J. W. M. DuMond who has aided me with much valuable assistance and many suggestions, and who has most kindly read over the preliminary manuscript.

IV. BIBLIOGRAPHY.

The following bibliography of the Compton effect covers as fully as possible all articles dealing with the subject which have appeared since Professor Compton's original paper at the end of 1922. A few dealing indirectly with this type of scattering which appeared before then have been included as being of fundamental interest. The first arrangement is by years and sub-grouped by periodicals--in chronological order. The second list is an alphabetical author list with numbers corresponding to those appearing in the first index. The following abbreviations have been resorted to:

PR.--Physical Review.
PNAS.--Proceedings of the National Academy of Science.
JFS.--Journal of the Franklin Institute.
JOSA.--Journal of the Optical Society of America.
PM.--Philosophical Magazine of London.
PRS.--Proceedings of the Royal Society.
ZFP;--Zeitschrift fur Physik.
PZ.--Physikalische Zeitschrift.
NW.--Naturwissenschaft.
CR.--Comptes Rendues.
Cim.--Il Nuovo Cimento, Bologna.

A. Arranged Chronologically by Periodicals.

1910.

1. Primary and Secondary Gamma Rays. D.C.H. Flöhrance.
P.M. 20. 921-938. 1910

1912.

2. Rontgen Radiation from Substances of Low Atomic Weight.
P.M. 24. 138-149. 1912. Sadler and Mesham.

1922.

3. Secondary Radiation Produced by X-Rays. A.H. Compton.
Bul. Nat. Research Council. 4. 1-56. 1922.

1923.

4. Quantum Theory of Scattering of X-Rays by Light Elements.
PR. 21. 485-502. May 1923. A.H. Compton.

5. The Spectrum of Scattered X-Rays. A.H. Compton.
PR. 22. 409-413. Sept. 1923.

1923 (cont'd.)

6. Scattering of X-Rays by Light Elements. G.E.M. Jauncey
P.R. 22. 233-241. Sept. 1923.
 7. Change in Wavelength by Scattering. P.A. Ross.
PNAS. 9. 246-248. July 1923.
 8. Wavelength of Secondary X-Rays. Clark and Duane.
PNAS. 9. 413-418. and 419-424. Dec. 1923.
 9. X-Ray Scattering and J Radiations. Barkla and Sale
PM. 45. 737-750. April 1923.
 10. Absorption Measurements of the Change in Wavelength
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