A STUDY OF THE SELECTION OF FLIGHT PATHS IN AIR TRANSPORT OPERATIONS

Thesis by

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INTRODUCTION

1. GENERAL:

With the introduction of the modern high performance airplane into the air transportation systems of the country it has become increasingly necessary to make some study of the variables which influence the proper selection of flight paths for the efficient operation of the aircraft.

In the past it has been the practice in the air transport industry to make a very rough estimate of the effect of the winds and thus determine the best altitude at which to fly on the course. There has been a complete neglect of the possibilities which are presented by a consideration of the airplane performance.

It is the purpose of this thesis to analize the effects of all of these variables with a view to obtaining some practical knowledge which may be valuable to the air transportation industry.

2. STATEMENT OF THE PROBLEM:

The problem in general is the following: To provide a means of quickly determining before the start of a flight the path which the pilot should follow in order to fly between any two points in the minimum possible time. Thus for a given set of cruising specifications for the airplane, this path becomes the optimum, not only for time elapsed, but also for economy of operation. The method must take into account both aerodynamic and meteorological variations.

To be of practical use in air transport operation the method provided must have three definite qualifications.

1. It must be such as to be readily calculated for a given flight by a person completely unfamiliar with aerodynamics and airplane performance.
2. It must not require more than a few minutes to calculate for a given flight.
3. It must be such that the data can be presented to the pilot in a brief and useful form.

In addition to the above it is highly desirable to provide information which will be of use to the pilot in properly navigating the airplane along the course.

All of the above requirements have been carefully considered throughout the analysis and the attempt has been made to incorporate them into the resulting method to as great an extent as possible.

PART I

GENERAL ANALYSIS

1. DISCUSSION OF VARIABLES AND AVAILABLE DATA

Let us consider a flight to take place between two points, the first with horizontal position S_0 and with altitude H_0, and the second with the corresponding position (S_3, H_3). It will be assumed that the flight will be restricted to the vertical plane which includes the two points (S_0, H_0) and (S_3, H_3). The airplane will then have two dimensions in which to travel and an infinite number of paths from which to choose in these two dimensions.

Now assume that the variation of winds aloft and the performance for the given airplane for level flight would yield a maximum effective horizontal velocity, or velocity relative to the ground, at some altitude H_1. Then, if it were possible to start the flight from any point situated vertically over (S_0, H_0), and to terminate the flight at any point vertically over (S_3, H_3), the most efficient path along which to fly would be level flight at an altitude H_1.

Now finally, if the flight is of sufficient length that it becomes desirable to climb to the altitude H_1, during the flight, the problem is then reduced to one of finding the rate of climb from (S_0, H_0) to some point (S_1, H_1) which gives the minimum loss of time, and then of finding the rate of glide from some point (S_2, H_2)---(in this example H_1 is equal to H_2)---to the point (S_3, H_3) which will

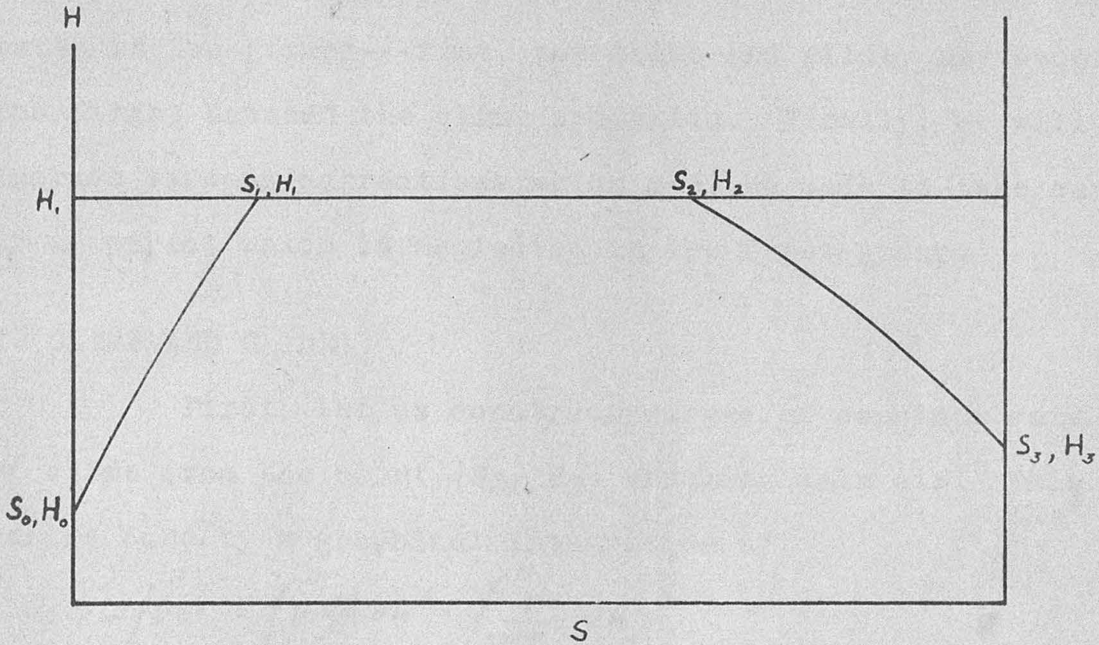


Fig. 1

give the maximum gain of time in the glide.

Thus the problem is resolved essentially into one of finding two control points, (S_1, H_1) and (S_2, H_2), which divide the flight up into three distinct parts, the first, a climb from the starting point at a given rate, the second, approximately level flight, and the third, a glide at a given rate into the terminal point. We shall consider these three parts in two groups---first, the climb and glide, and second, the flight between the climb and glide. Finally, we will include various corrections which must be made to take care of an effect which is neglected in the first group.

2. CLIMB AND GLIDE:

First, let us construct curves of constant rate of climb from the point (S_0, H_0) assuming calm air. This can be done by a graphical integration of

$$S = \int_{H_0}^{H} V\,dt = \int_{H_0}^{H} V \frac{dt}{dh}\,dh = \int_{H_0}^{H} \frac{V}{\frac{dh}{dt}}\,dh$$

where V, the horizontal ground velocity, is a function of both the altitude, H, and the rate of climb dH/dt. The values of this function must be computed by the aerodynamicist and, since it is a specialized and complex problem in itself, this calculation will not be discussed in this report with the exception of a few particular features which will be discussed in brief in Part II of this thesis.

Now, having established these basic curves, we can calculate the time required to fly from S_0, H_0) to any given position (S, H) on any one of the curves since

$$t = \frac{H - H_0}{\frac{dh}{dt}}$$

We can also calculate the time required to fly in level flight from the position (S_0, H) directly above the starting point to any position on the constant rate of climb curves. Thus, for every point we now have two values: one, the actual time which would be required to fly at a given rate of climb from the starting point to the point in question, and second, the time required to fly the same horizontal distance at the altitude H if the airplane were not required to climb. The difference between these two values is the time lost by reason of having to climb. Now let us assume that the value of this loss of time has been computed for each point along the curves. Also, a similar computation can be made for the glides into the point (S_3, H_3) except that in this case the time difference will be the time gained by reason of having to glide instead of the time lost in climb.

With the above we have from the practical standpoint, neglecting wind gradient, solved the problem of determining the optimum rate of climb up to the required altitude and the optimum rate of glide into the terminal. We have only to examine the values marked on the climb and glide curves at the given altitude and select the curve which has the minimum value in the case of the climb curves and the maximum value in the case of the glide curves. This gives us the rate of climb for the least loss of time and the rate of glide for the greatest gain in time. As mentioned above, the wind gradient, which is the only other factor which can effect the time lost or gained, has been neglected but will be taken care of later.

It should be mentioned here that all of the above data and curves can be entered on a chart and will be constant for a given airplane and a given course. Thus the chart can be printed and, for a given flight, the effects of the meteorological conditions which will vary from time to time can be superimposed as will be shown in the next section.

3. INTERMEDIATE FLIGHT BETWEEN CLIMB AND GLIDE:

Next comes the problem of determining the optimum flight path (approximately horizontal) in between the climb and glide. This problem resolves itself into combining the cruising velocity of the airplane in level flight with the wind velocity to give the effective ground speed. The only question is one of finding a means of solving the vector triangle, shown below, with sufficient rapidity to avoid a large amount of lost time in the calculation for a given flight.

The values of the cruising speeds of the airplane at the various altitudes must be calculated by the aerodynamicist and will be a function of altitude along since the airplane will be considered to be in level flight.

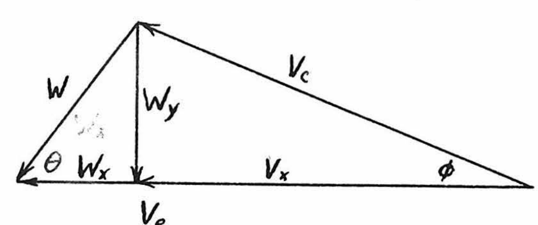


Fig. 2

In the above figure W represents the wind velocity vector, θ the angle between the wind vector and the course, V_c the vector representing the true air cruising speed and V_e the effective ground speed of the airplane. The other symbols are as indicated in the figure. The following felationships can be seen directly from the figure:

(1) $$W_x = W\cos\theta$$
$$W_y = W\sin\theta$$

Also $$V_x = \sqrt{V_c^2 - W_y^2}$$

Factoring out V_c from the right hand side.

$$V_x = V_c\sqrt{1 - \left(\frac{W_y}{V_c}\right)^2}$$

Expanding by the binomial theorem.

$$V_x = V_c\left[1 - \frac{1}{2}\left(\frac{W_y}{V_c}\right)^2 - \frac{1}{8}\left(\frac{W_y}{V_c}\right)^4 - \cdots\right]$$

Now let us consider an airplane which would rank near the lower limit of the present high performance airplane. This airplane should have a cruising speed in the neighborhood of 150 miles per hour. Let us also consider a direct cross wind of 50 miler per hour, θ will then be 90 degrees. The above values will give about maximum values to the terms involved in the expansion above. Considering the fourth order term $\frac{1}{8}\left(\frac{W_y}{V_c}\right)^4 = \frac{1}{8}\left(\frac{50}{150}\right)^4 = 0.0015$ or 0.15%

Thus this term is very small and can be neglected since it represents a variation in V_x of only 0.2 miles per hour.

Now consider the remaining equation.

$$V_x = V_c\left[1 - \frac{1}{2}\left(\frac{W_y}{V_c}\right)^2\right]$$
$$= V_c - \frac{1}{2}\left(\frac{W_y}{V_c}\right)^2 V_c$$

Let $$\Delta V = V_c - V_x$$

which is the amount by which the effective component of the cruising speed of the airplane is reduced by the crabbing effects of the wind.

Then $\Delta V = V_e - V_c + \frac{1}{2}\left(\frac{W_y}{V_c}\right)^2 V_c$

$$= \frac{1}{2}\left(\frac{W_y}{V_c}\right)^2 V_c$$

$$\Delta V = \frac{1}{2}\frac{W_y^2}{V_c}$$

Differentiating ΔV with respect to V_c

we get $d(\Delta V) = -\frac{1}{2}\frac{W_y^2}{V_c^2} dV_c$

Dividing by V_c $\frac{d(\Delta V)}{V_c} = -\frac{1}{2}\frac{W_y^2}{V_c^3} dV_c$

The above expression gives the proportionate variation in V_x which will be caused by an error in V_c. Now let us assume that the average cruising speed of the airplane proposed above is 150 miles per hour and that the maximum variation of V_c from this value, either plus or minus, is 20 miles per hour. Then substituting this value of the variation for dV_c

$$\frac{d(\Delta V)}{V_c} = -\frac{1}{2}\frac{(50)^2}{(150)^3} \times 20 = -0.0074 \quad \text{or} \quad 0.74\%$$

Now considering that this value represents just slightly more than one mile per hour in V_x and remembering that even this represents extremely pseeimistic conditions, it is ovvious that we are justified in using an average value of V_c in the expression for ΔV. In this way we get

(2) $\Delta V = \frac{1}{2}\frac{W_y^2}{V_{ave.}}$

Now combining (1) and (2) with V_c we see that

$$V_e = W_x + V_c - \Delta V$$

or rearranging

$$V_e = V_c + (W_x - \Delta V)$$

$$V_e = V_c + \Delta V_e$$

Where V_c is independent of winds and ΔV_e is independent of the airplane for a given airplane.

Thus $V_c = f(H)$

and $\Delta V_e = W\cos\theta - \frac{1}{2}\frac{(W\sin\theta)^2}{V_{ave.}}$

We have now broken up our calculation of V_e into two parts, one of which is dependent only upon the airplane performance and the other, only upon the meteorological conditions. We can build up with this material two tables, one a function of altitude and the other a function of wind direction and wind velocity for a given airplane.

We are now ready to superimpose the intermediate flight upon the climb and glide base charts. From the upper air balloon observations along the course we obtain the wind direction and velocity at the various altitudes. This we combine with our airplane performance for level flight by means of the tables compiled from the above equations. With the results of these calculations we are able to pick out immediately the correct altitude at which to carry out our intermediate flight. Then, examining the values of the time lost for the various rates of climb and the time gained for the various rates of glide at the optimum intermediate flight altitude, we are able to pick out our complete path in a very rapid and simple manner.

4. CORRECTIONS FOR WIND GRADIENT IN CLIMB AND GLIDE

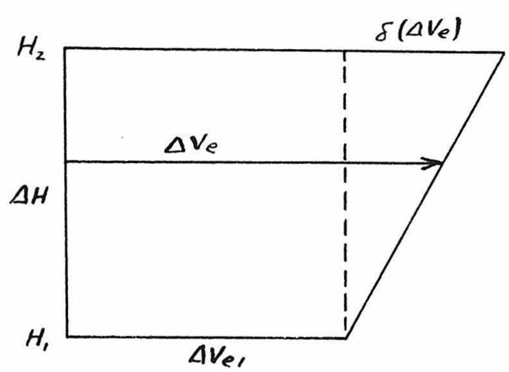
In all of the calculations in Section 2 the effect of the winds was completely neglected. The winds will have two distinct effects upon the results:

1. The position along the course at which the airplane will arrive after the climb to the optimum intermediate flight altitude, and the position along the course

at which the pilot must start his glide into the terminus will be shifted one way or another depending upon whether the airplane is flying in head or tail winds.

2. The actual time lost or gained ina glide or climb is affected by the vertical gradient of the wind aloft. This may alter the selection of the optimum rates of climb and glide.

First, we shall consider the shift in position. Considering the results in the last section we have an immediate method of finding the value of the actual effects of the wind upon the effective ground speed of the airplane. This is included in the expression for ΔV_e. Now let us assume a gradient of ΔV_e as shown in the diagram below. In this diagram we can divide the distribution



of ΔV_e up into the basic value of ΔV_e at the ground and the difference between the value of ΔV_e at an altitude H and that at the ground. We shall calculate the effects of each of these parts separately.

Let us first consider the shift in the horizontal position of the top of the climb due only to the basic wind effect ΔV_{e_1} at the ground. Here, the shift $\Delta_1 s$ is given as follows:

$$\Delta_1 s = \Delta V_{e_1} \times t$$

But $$t = \frac{\Delta H}{\frac{dH}{dt}}$$

Therefore $$\Delta_1 s = \Delta V_{e_1} \frac{\Delta H}{\frac{dH}{dt}}$$

or $$\frac{\Delta_1 s}{\Delta H} = \frac{\Delta V_{e_1}}{\frac{dH}{dt}}$$

Finally $$(3) \quad \frac{\Delta_1 s}{\Delta H} = \frac{\Delta V_{e_1}}{\frac{dH}{dt}} \frac{1000}{60}$$

where ΔV_{e1} is in miler per hour, dH/dt is in feet per minute, and $\Delta_1 S/\Delta H$ gives the horizontal shift per 1000 feet of climb.

Now consider the effect of the gradient of ΔV_e. Let $\delta(\Delta V_e)$ be the difference between ΔV_e at altitude and that at the ground. Then, since the rate of climb is constant, the shift $\Delta_2 S$ in S which will be caused by $\delta(\Delta V_e)$ will be as follows:

$$\Delta_2 s = \frac{\delta(\Delta V_e)}{2} \times t$$

$$= \frac{\delta(\Delta V_e)}{2} \frac{\Delta H}{\frac{dH}{dt}} \qquad \text{as before}$$

$$\frac{\Delta_2 s}{\Delta H} = \frac{\delta(\Delta V_e)}{2 \frac{dH}{dt}}$$

Finally $$(4) \quad \frac{\Delta_2 s}{\Delta H} = \frac{\delta(\Delta V_e)}{2 \frac{dH}{dt}} \frac{1000}{60}$$

where the units of $\delta(\Delta V_e)$, dH/dt, and $\Delta_2 S/\Delta h$ are the same as before. It must be pointed out that a tail wind will shift both the climb and glide curves away from the stations from which they emanate and a head wind will have the opposite effect. Thus a positive value of $\delta(\Delta V_e)$ will shift the curves farther from their respective stations while a negative value will have the same effect as that of a head wind.

With the use of the above formulae, (3) and (4), tables can be constructed which will give the shift in miles per 1000 feet of climb which will result from the wind aloft. This has been done in Part II together with the remainder of the optimum flight path data for the Douglas DC-1 transport plane.

Finally, in this section we must consider the effect which the wind will have upon the time lost in climb and that gained in glide. It is obvious that this will depend only upon the gradient of ΔV_e since the airplane will receive just as much aid, or hinderance, from the basic ΔV_e at the ground as at altitude. Thus it will be necessary to consider only the value of $\delta(\Delta V_e)$.

If the value of ΔV_e were a constant with respect to altitude and equal to ΔV_e at the ground plus $\delta(\Delta V_e)$, then the difference in the horizontal position of the airplane at the end of the climb for this condition and for the actual condition where a gradient existed would be equal to the $\Delta_2 S$ as calculated above. In other words, the airplane would be $\Delta_2 S$ farther along in its course if it were not for the existence of the gradient. Then the actual time lost because of the existence of the gradient must be this distance divided by the speed at which the airplane will fly along the course.

Thus $\Delta t = \dfrac{\Delta_2 S}{V_{ave}}$

$$= \frac{\delta(\Delta V_e)}{2} \frac{\Delta H}{\frac{dH}{dt}}$$

or $\dfrac{\Delta t}{\Delta H} = \dfrac{\delta(\Delta V_e)}{2\frac{dH}{dt} V_{ave}}$

Finally $\dfrac{\Delta t}{\Delta H} = \dfrac{\delta(\Delta V_e)}{2\frac{dH}{dt} V_{ave}}$

where $\delta(\Delta V_e)$ and dH/dt have the same units as before and $\Delta t/\Delta h$ is in minutes per 1000 feet of climb or glide. It must be noted that where $\delta(\Delta V_e)$ is positive (a tail wind) then $\Delta t/\Delta h$ will be time lost per 1000 feet for both climb and glide, while if $\delta(\Delta V_e)$ is negative (a head wind) the quantity represents a gain in time. As before the formulae derived is made up in the form of a table in Part II. In making up the table, V is taken as the average cruising speed of the airplane, this value giving results which are much closer than many of the other assumptions and measured quantities.

There is one other item which should be mentioned in this section. It will be noted that the calculation of the position at which the airplane should arrive at the end of the climb and the position at which to start the glide are not necessary in so far as finding the optimum flight path. However, the pilot must know these values, particularly the point to start the glide, in order to fly the course correctly. Thus this data is of navigational aid to the pilot. One other set of values which would be of extreme value to him would be the drift angle along the course. This is the angle ϕ in Figure 2 and is given by the expression

$$\phi = \arcsin\left(\frac{W \sin\theta}{V}\right)$$

Thus we can assume as before an average value for V and incorporate drift angles into one of the tables described in Part II. This will constitute a valuable aid to the pilot since without it he would be required to find the angles along the way purely by trial and error while flying.

5. OPERATIONS SCHEDULING

The results made available by the use of the above analysis present another interesting possibility. This is the problem of scheduling aircraft operations. In other word, given a certain airplane to be flown over a given route with certain existing meteorological conditions, how should the airplane be flown in order to make the flight in scheduled time?

By using the analysis described above we can very easily determine the average effect which the wind will have all along the course in addition to the effect of the climb and glide. Now let

V_e = the average effective ground speed along the intermediate path (as defined in Section 3)

V_c = the average cruising speed of the airplane

ΔV_e = the average effect of the wind along the intermediate path

s = the distance between the start and terminus of the flight

T = the scheduled time for the flight

Δt = time lost in climb and glide (due both to aerodynamic variations and winds) plus the time lost due to late starting

t = the time lost in take-off and landing. This should be taken as an average over several flights for the particular airplane.

P/P_o = the ratio of cruising power to full power

Now $$T = \frac{s}{V_e} + \Delta t + t$$

$$V_e = \frac{s}{T - \Delta t - t}$$

But $$V_e = V_c + \Delta V_e$$

Therefore

$$V_c = \frac{s}{T - \Delta t - t} - \Delta V_e$$

But, from the fundamentals of aerodynamics, power is proportional to the cube of the velocity.

Therefore

$$P/P_o = K V_c^3 \qquad K = \text{constant}$$

Substituting for V_c

$$P/P_o = K\left[\frac{s}{T - \Delta t - t} - \Delta V_e\right]^3$$

In which K, P_o, and t are constant for a given airplane.

Thus $$P/P_o = f(s, T - \Delta t, \Delta V_e)$$

For s in miles, T in minutes, Δt in minutes, t in minutes, ΔV_e in miles per hour, the equation becomes

$$P/P_o = K\left[\frac{s}{T - \Delta t - t} - \frac{\Delta V_e}{60}\right]^3$$

Using the above equation we can build up a chart for solving this equation for P/P_o for any given route and wind distribution. This has been done in Part II in connection with the Douglas Transport and will be discussed further later.

The above completes the general discussion of the optimum flight path analysis. In further chapters it will be discussed with reference to particular routes and airplanes. The analysis just completed, however, applies to any airplane.

PART II

APPLICATION OF METHOD TO DOUGLAS DC-1 TRANSPORT

1. GENERAL:

As an illustration of the application of the method presented in the previous chapter the actual numerical calculations developed were made for the Douglas DC-1 Transport airplane. In addition, through the courtesy of the Transcontinental and Western Air, the author was fortunate in having the opportunity of trying out these results to a limited extent in actual operations service. The results of these tests will be presented herein together with a brief discussion of the calculations themselves.

2. AIRPLANE PERFORMANCE CALCULATIONS:

A complete discussion of the method employed for the calculation of the airplane performance would be very long and involved and beyond the scope of this paper. Thus, only a brief mention of the salient points will be included.

The basis for all of the performance calculations was Technical Report 408 of the N.A.C.A. by Dr. W. B. Oswald. All of the work which is done in this paper is merely a variation of that discussed in the above report.

In the actual calculation of the cruising speed of the airplane at either climb or glide, the effect of the climb or glide was considered as a supercharging effect, positive in the case of a glide and negative in the case of a climb.

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~~Super charger~~

Performance calculation chart being printed.

The justification of this is apparent when we consider that the quantity W dh/dt represents the power which goes into raising the potential energy of the airplane due to change of altitude. This quantity, then, was added (or subtracted) to the brake horse power supplied by the motor and the total considered as the new brake horse power. The difficulties with this analysis are, of course, that there will be a change in the propeller efficiency and that in the case of a glide there is the danger of obtaining too high values for motor RPM. To correct for this it was necessary to consider the velocities obtained by the above method as first approximations and with these values finds the new efficiencies and correct for excessive motor RPM. Finally, going through the calculation a second time gives a much closer value to the correct cruising speed. This calculation had to be made for each rate of climb at each altitude. It should be noted here that a continuously variable pitch propeller would eliminate the difficulty of excessive motor RPM mentioned above.

In making the calculation described above the author devised a chart which shortened the calculation enormously and for this reason it is included in this paper. There is no need to explain the variables involved as anyone familiar with the Oswald analysis will recognise them immediately. The chart merely gives a rapid means of finding the correct value of the parameter Λ for an airplane with supercharging by assuming a value of any one of the variables and then making repeated circuits of the chart until the path converges upon an accurate value of Λ.

All of the calculations and charts for the Douglas DC-1 were made on the assumption of 75% full power for cruising.

3. MISCELLANEOUS TABLES AND CHARTS:

There are two sets of tables to consider, the first, the table giving "EFFECTIVE CRUISING SPEED" relative to the ground, and the second, the position and time correction tables for wind. In both sets the average cruising (air speed) speed where needed was taken as 190 miles per hour. However, the first portion of the first table will be reasonably correct in the case of other airplanes which have considerably different average cruising speeds than that of the DC-1. This is shown from calculations in PartI.

The position correction tables will, of course, be true for any airplane and the time correction table will not require change within the range for which the table discussed in the last paragraph holds true.

The "SCHEDULING CHART" is also based upon an average cruising speed of 190 miles per hour for 75% full power. This chart will give results which are accurate to about 1% in percent power which is greater accuracy than can be attained in flying the airplane. It was assumed that 10 minutes was lost in take off and landing.

There is an additional feature which appears on the "SCHEDULING CHART" which was not explained in the previous chapter. There appear on the chart four broken lines with labels which indicate altitudes. These lines represent the limiting power which may be obtained from the engines at the

EFFECTIVE CRUISING SPEED

DOUGLAS DC-1 (75% POWER)

θ

W	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
2	2	2	2	2	2	1	1	1	0	0	0	-1	-1	-1	-2	-2	-2	-2	-2
4	4	4	4	4	3	3	2	1	1	0	-1	-1	-2	-3	-3	-4	-4	-4	-4
6	6	6	6	5	5	4	3	2	1	0	-1	-2	-3	-4	-5	-5	-6	-6	-6
8	8	8	8	7	6	5	4	3	1	0	-2	-3	-4	-5	-6	-7	-8	-8	-8
10	10	10	9	9	8	6	5	3	2	0	-2	-4	-5	-7	-8	-9	-9	-10	-10
12	12	12	11	10	9	8	6	4	2	0	-2	-4	-6	-8	-9	-11	-11	-12	-12
14	14	14	13	12	11	9	7	4	2	-1	-3	-5	-7	-9	-11	-12	-13	-14	-14
16	16	16	15	14	13	10	8	5	2	-1	-3	-6	-9	-11	-13	-14	-15	-16	-16
18	18	18	17	15	14	11	8	6	2	-1	-4	-7	-10	-12	-14	-16	-17	-18	-18
20	20	20	19	17	15	12	9	6	3	-1	-5	-8	-11	-14	-16	-18	-19	-20	-20
22	22	22	21	19	16	14	10	6	3	-1	-5	-9	-12	-15	-17	-19	-21	-22	-22
24	24	24	22	20	18	15	11	7	3	-1	-6	-10	-13	-16	-19	-21	-23	-24	-24
26	26	26	24	22	19	16	12	7	3	-2	-6	-10	-14	-18	-21	-23	-25	-26	-26
28	28	28	26	24	21	17	13	8	3	-2	-7	-11	-16	-19	-22	-25	-27	-28	-28
30	30	30	28	25	22	18	13	8	3	-2	-7	-12	-17	-21	-24	-27	-29	-30	-30
32	32	31	30	27	23	19	14	9	3	-3	-8	-13	-18	-22	-26	-28	-30	-31	-32
34	34	33	32	29	25	20	15	9	3	-3	-9	-14	-19	-24	-27	-30	-32	-33	-34
36	36	35	33	30	26	21	16	9	3	-3	-9	-15	-20	-25	-29	-32	-34	-35	-36
38	38	37	35	32	28	22	16	10	3	-4	-10	-16	-22	-27	-31	-34	-36	-37	-38
40	40	39	37	34	29	23	17	10	3	-4	-11	-17	-23	-28	-32	-36	-38	-39	-40
42	42	41	39	35	30	24	18	11	3	-4	-12	-18	-24	-30	-34	-38	-40	-41	-42
44	44	43	41	37	31	26	18	11	3	-5	-12	-19	-26	-31	-36	-39	-42	-43	-44
46	46	45	43	38	33	27	19	11	3	-5	-13	-20	-27	-33	-37	-41	-44	-45	-46
48	48	47	44	40	34	28	20	11	3	-6	-14	-22	-28	-34	-39	-43	-46	-47	-48
50	50	49	46	42	36	28	20	12	3	-6	-15	-23	-30	-36	-41	-45	-48	-49	-50
52	52	51	48	43	37	29	21	12	2	-7	-16	-24	-31	-37	-43	-47	-50	-51	-52
54	54	53	50	45	39	30	22	12	2	-7	-17	-25	-33	-39	-45	-49	-52	-53	-54
56	56	55	52	47	40	31	22	12	2	-8	-17	-26	-34	-41	-46	-51	-54	-55	-56
58	58	57	54	48	41	32	23	12	2	-8	-18	-27	-35	-42	-48	-52	-56	-57	-58
60	60	59	55	50	42	33	23	13	2	-9	-19	-28	-37	-44	-50	-54	-57	-59	-60

CRUISING SPEED AT ALTITUDE (75% POWER)

H	V_c	H	V_c
0	167	9000	192
1000	170	10000	194
2000	173	11000	196
3000	176	12000	198
4000	179	13000	199
5000	182	14000	201
6000	185	15000	199
7000	188	16000	196
8000	190	17000	193

CLIMB AND GLIDE POSITION CORRECTION (miles per 1000 ft.)

DUE TO BASIC WIND

	$\frac{dH}{dt}$			
ΔV_e	100	200	300	400
5	0.8	0.4	0.3	0.2
10	1.7	0.8	0.6	0.4
15	2.5	1.3	0.8	0.6
20	3.3	1.7	1.1	0.8
25	4.2	2.1	1.4	1.1
30	5.0	2.5	1.7	1.3
35	5.8	2.9	1.9	1.5
40	6.7	3.3	2.2	1.7
45	7.5	3.7	2.5	1.9
50	8.3	4.1	2.8	1.2

DUE TO WIND GRADIENT

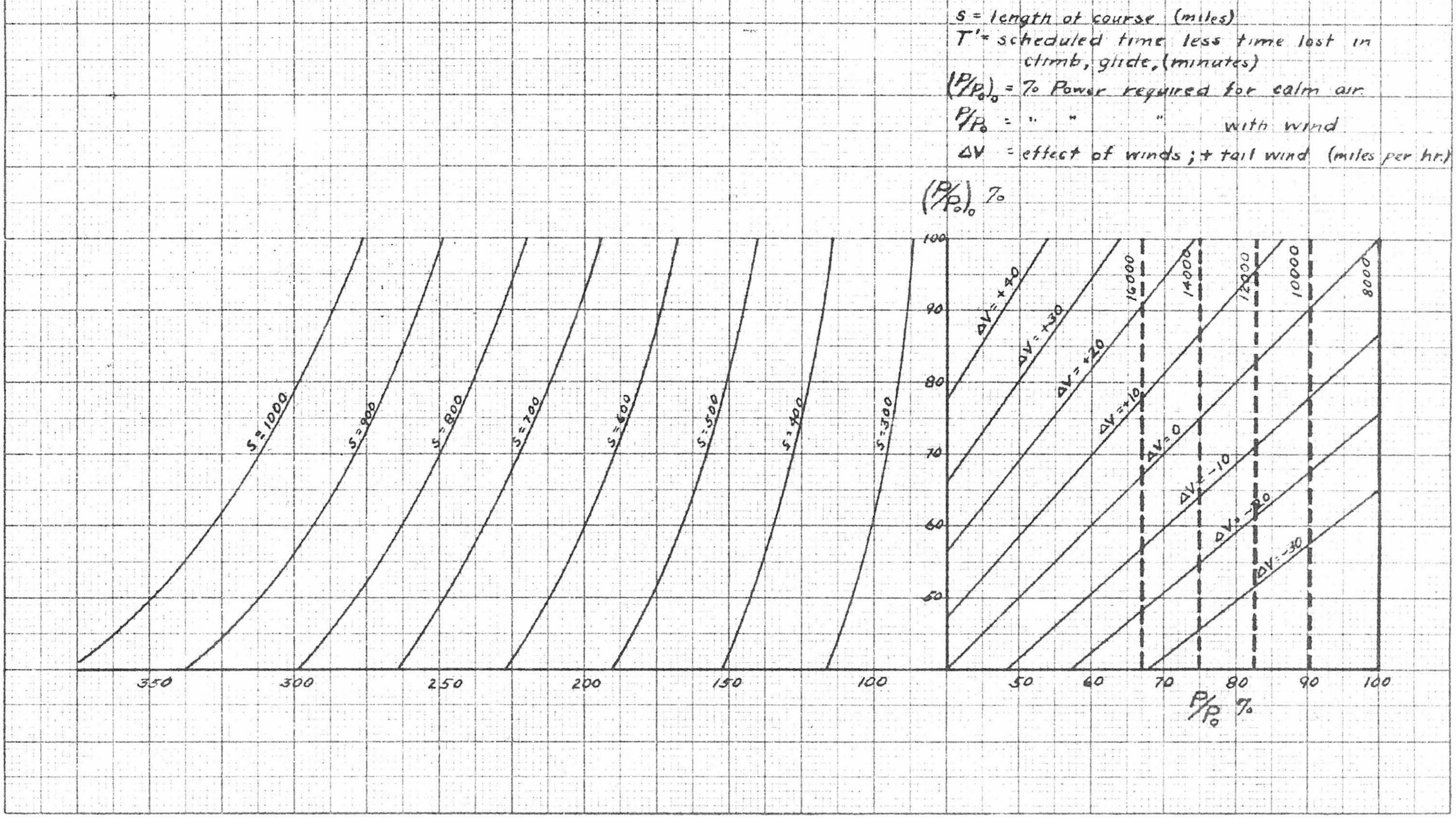
	$\frac{dH}{dt}$			
$\delta(\Delta V_e)$	100	200	300	400
5	0.4	0.2	0.1	0.1
10	0.8	0.4	0.3	0.2
15	1.3	0.7	0.4	0.3
20	1.7	0.9	0.6	0.4
25	2.1	1.1	0.7	0.5
30	2.5	1.3	0.9	0.6
35	2.9	1.5	1.0	0.7
40	3.3	1.7	1.1	0.8
45	3.8	1.9	1.3	0.9
50	4.2	2.1	1.4	1.0

TIME CORRECTION IN CLIMB AND GLIDE (minutes per 1000 ft.)

DUE TO WIND GRADIENT

	$\frac{dH}{dt}$			
$\delta(\Delta V_e)$	100	200	300	400
5	0.1	0.1	0.0	0.0
10	0.3	0.1	0.1	0.1
15	0.4	0.2	0.1	0.1
20	0.5	0.3	0.2	0.1
25	0.7	0.4	0.2	0.2
30	0.8	0.4	0.3	0.2
35	0.9	0.5	0.3	0.2
40	1.0	0.5	0.4	0.3
45	1.2	0.6	0.4	0.3
50	1.3	0.7	0.4	0.3

SCHEDULING CHART
DC-1

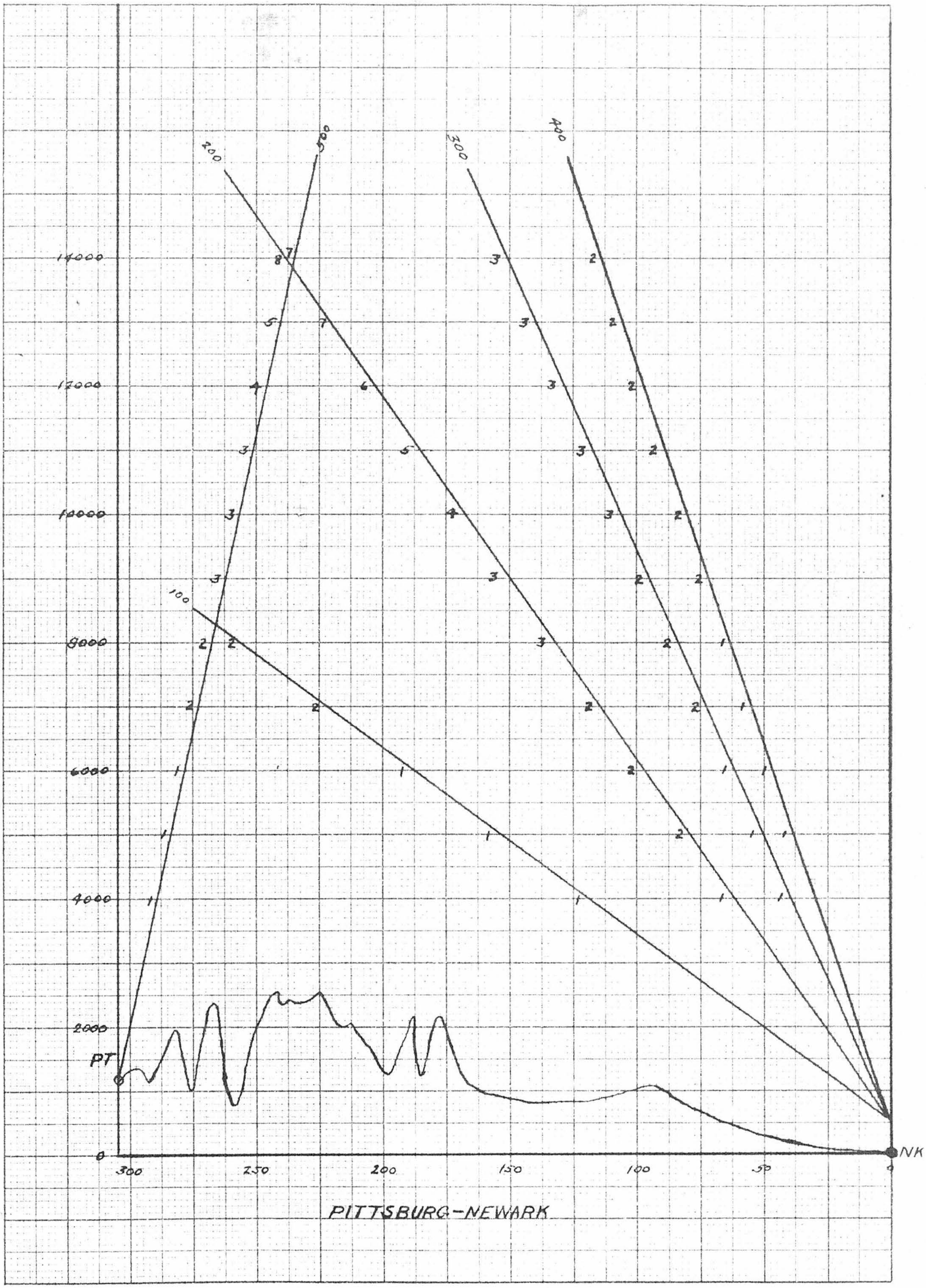


altitudes indicated. Thus, if it is found, for instance, that 80% power is required with an intermediate flight path at 14000 feet altitude, it is immediately apparent from the chart that this is impossible to attain. In this case, then, the plane would be unable to cover the course in scheduled time.

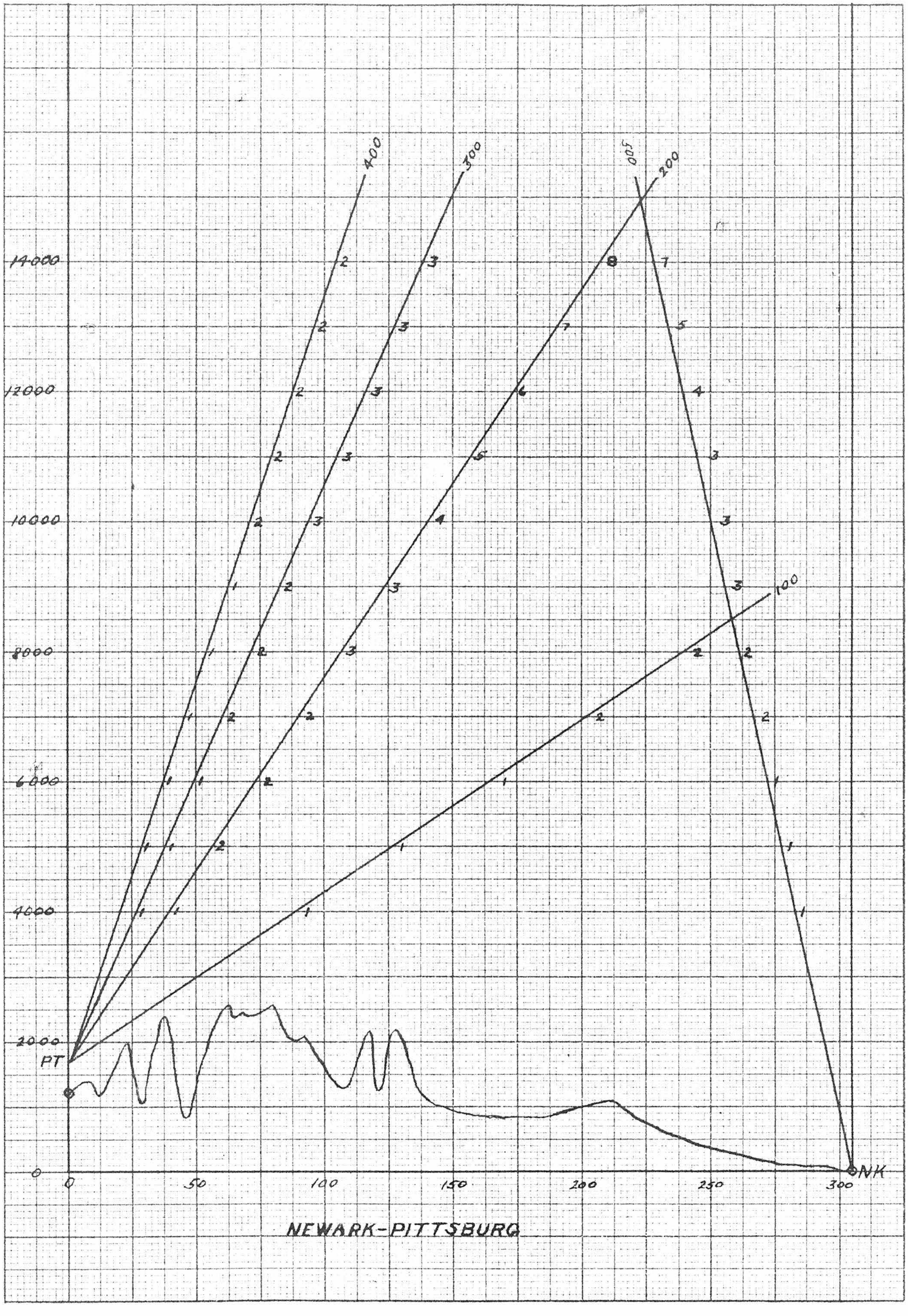
4. FLIGHT PATH CHARTS:

Finally, the author has drawn up two sets of basic flight path charts covering two different routes. These charts are of the type which would be used as the basis for computing the actual flight path for a given set of observed meteorological conditions.

These charts need no particular explanation in so far as the calculation is concerned. Each curve is the result of a graphical integration of the "cruising speed--altitude curve" mentioned in Part I. The figures alongside each curve give the time (in minutes) lost due to climb and the time gained due to glide. In this connection it should be mentioned that for the Douglas DC-1 there was no appreciable difference in time lost in climb up to reasonable altitudes between rates of climb ranging from 200 to 500 feet per minute. Therefore, since the wind gradient will in general increase up to the level of the intermediate flight altitude, the least loss in time will be realized with the maximum rate of climb so long as there is no appreciable loss due to an increase of rate of climb. For this reason the climbs were standardized at 500 feet per minute which rate appeared to be



14000
12000
10000
8000
6000
4000
2000
0
PT
NK
300
250
200
150
100
50
0
100
200
300
400
500
PITTSBURG-NEWARK



NEWARK-PITTSBURG

Pages 28 & 29

Two Flight Path Charts

Burbank to Albuquerque

being ~~[illegible]~~ printed

the best under almost all conditions.

Finally, it should be noted that the charts covering the Pittsburg-Newark course include a topographical profile of the region along the course. This feature would be incorporated into all of the charts when made up for practical use.

PART III

METHOD OF USING CHARTS

A. DETERMINATION OF INTERMEDIATE FLIGHT PATH

1. From balloon observations and by use of the "EFFECTIVE CRUISING SPEED" table calculate the effective speed for each level above each station. To do this look opposite the wind velocity in the left hand column and under column headed with the angle of the wind relative to the course. This gives ΔV_e which must be added (or subtracted according to sign) to the value of V_c corresponding to the altitude in question to obtain V_e. Write this value for V_e at the corresponding point on the base chart. Do this for all balloon observations. In addition to recording the above, write down on a separate sheet the values of V for each altitude for the end stations. These values will be used later.
2. Draw a line joining the maximum values of V_e which now appear on the base chart. This will give the intermediate flight path.

B. DETERMINATION OF BEST RATE OF GLIDE

1. Using the values of ΔV_e for the end stations assume a linear distribution of ΔV_e aloft. Extrapolating this linear distribution find the value of ΔV_e at the ground and the difference between the value at the intermediate flight level and that at the ground.

2. Using the "TIME CORRECTION" table find the value of time lost (or gained) in the glide per 1000 feet of glide for each rate of glide from the difference in value of ΔV_e at the flight level and at the ground. This will be time lost if the gradient increase upward and gained if downward.

3. Multiply this value by the total difference of altitude for the glide in thousands of feet and add (if it is time gained) this value to figure appearing on each glide curve at the intermediate flight altitude.

4. Choose the rate of glide which gives the greatest gain in time.

C. TO FIND POSITION TO START GLIDE:

1. Using the value of ΔV_e at the ground obtained in B.1. find the total position correction in miles due to basic winds in a manner similar to B.3 and B.4., multiplying by the difference in altitude as before. Do this only for the best rate of glide. This correction will add to the distances marked on the basic flight path chart if the wind is a tail wind and subtract if a head wind.

2. Do the same as in C.1. for the difference in ΔV_e aloft and at the ground. This will be added to the distance if the gradient increased aloft and subtractive if it decreases. The total of these two corrections gives the correct point at which to start the glide.

D. SCHEDULING THE FLIGHT:

1. Estimate the average effective cruising velocity which will be attained along the intermediate flight path. Subtract from this the value of V_c for the approximate altitude of the intermediate flight path. This gives the value of ΔV_e which will be used in the "SCHEDULING CHART".
2. Find from the calculation made in B. the total time lost in climb and glide (this may be negative). Subtract this value (algebraically) from the scheduled time for the flight in minutes. This gives T'.
3. Entering the chart with the value of T' found in D.2. go up to the value of S which gives the length of the course. Then move horizontally to the value of ΔV_e determined in D.1. (plus is tail wind; minus is head wind). Finally, move vertically downward to the axis which gives the percentage full power which must be used in order to make the flight on schedule. This value must not lie to the right of altitude line which corresponds with the altitude of the intermediate flight. If it does, then the flight cannot be made on time.

E. SCHEDULING A NEW ROUTE:

Obviously, the chart may be used backwards to determine proper schedule time to use on a new route. A statistical study can be made of the winds to determine the usual value of ΔV_e. Then, assuming a value of the power to be used, the correct scheduled time can be obtained. It must be remembered that the above chart takes the time lost in take off and landing into account automatically.

F. PRESENTATION OF DATA TO PILOT

The data which should be given to the pilot is as follows:

1. Rate of ascent from take-off.
2. Altitude to which to climb for beginning of intermediate flight path.
3. Altitudes at various points along the route if the intermediate flight path varies somewhat in altitude.
4. Point at which to begin glide in miles from destination or relative to some landmark.
5. Rate of glide into terminal.

PART IV

TESTS MADE OVER THE PITTSBURG-NEWARK ROUTE

Through the courtesy of Transcontinental & Western Air, the author was enabled to make a limited series of tests from May 11 to May 13, 1934 on the Douglas DC-1 transport plane in operation between Pittsburg and Newark. Although this was not a favorable route on which to make a test of the value of the method, several rather interesting and valuable results were brought out by these tests. Also, several minor changes were shown to be desirable and these have been incorporated in the analysis as it has been herein presented.

One of the most valuable results shown by the test was the ability to predict accurately by means of the method the point at which to start the glide in order to come in to the terminal at the proper altitude. This is a valuable navigation aid to the pilot since it avoids the great waste of time involved in spiraling down to the field after having overshot the proper point in the glide. Most of the glides extended for more than 125 miles, and in one case for about 150 miles.

Another interesting fact which was brought out during the tests and which is to be expected from the charts is that the plane will very seldom be given an opportunity to get up to its critical cruising altitude on so short a flight. Most of the flights for this course must necessarily consist of a climb up to a certain point followed directly by a glide without any intermediate flight between. Thus, such a short flight as the 305 miles between Pittsburg and Newark does not allow the maximum efficiency to be gained from the DC-1. Also, due to the same fact, it is harder to calculate the true optimum flight path for such a short path where the percentage saving in time cannot be very large because take-offs and landings take up an appreciable portion of the total time and none of this can be saved. For such a short path it is necessary to investigate every combination of climb and glide correcting each for position in order to determine the true optimum. In the case of a long flight, however, each part of the path is independent of the other and thus each can be investigated separately.

In regard to the time element involved in making the calculation it was found in practice that about 7 minutes was required to determine the proper path for the Pittsburg-Neward course along which there was only one pertinent balloon observation (which was sufficient for such a short flight). However, in calculation some trial situations on the Burbank-Albuquerque course, where there are four good balloon soundings, it was found that 15 minutes was sufficient time to complete the work. On a course such as this the saving in timeshould be considerable and the 15 minutes spent in calculation on the ground could well be paid for by the saving of time in the air.

No attempt was made to use the Scheduling Chart as this chart was not made up at that time.

CONCLUSION

The method appears to satisfy the conditions which have been set forth in the first part of the paper and, doing this, it should be of distinct practical advantage to the air transport operator.

In conclusion, the author wishes to thank Messrs. Frye, Fritz, Weaver, and Clover of the Operations and Meteorological Divisions of Transcontinental & Western Air who made available the possibility of the tests which were conducted on the use of this method on the Douglas DC-1. It was only through these tests that the author was able to make many essential additions and alterations and to determine how the method should be used under actual operating conditions.