GYROMAGNETIC RATIOS FOR IRON, COBALT AND CERTAIN BINARY ALLOYS OF IRON, COBALT AND NICKEL

Thesis by

George S. Kenny

In Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California

1952

ACKNOWLEDGMENT

The author takes great pleasure in expressing his appreciation to Professor Samuel J. Barnett for introducing him to the problem, and for the many helpful suggestions he offered.

The work was supported by the Office of Naval Research, the Carnegie Institution of Washington and the University of California

ABSTRACT

Gyromagnetic ratios for iron, cobalt and many binary alloys of iron, cobalt and nickel were determined by measurements on the Einstein-de Haas effect such as made in this laboratory previously. Improvements were made in the methods of eliminating large quadrature torques. For each series of binary alloys the gyromagnetic ratio was found to vary linearly or nearly linearly with concentration. For those materials studied here previously there is good agreement between the new results and the older.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Gyromagnetic effects are those effects which exist because the "magnetic element" has both angular momentum and a magnetic moment. Any change in the magnetization of a body will be accompanied by a corresponding change in its angular momentum. The first experiments designed to detect the concealed angular momentum were carried out by Maxwell about 1861.(1) He had an electromagnet constructed, mounted so it could turn about a horizontal axis within a ring. The ring was rotated about a vertical axis. When the circuit to the electromagnet was closed, any angular momentum associated with the mangetization of the magnet would tend to align the axis of the magnet with the vertical axis around which the ring was rotating. Since the disturbing effects of the earth's field were large, the results were rough, but no indication of any change in angle was found. Maxwell concluded that the angular momentum of any matter in rotation in a magnetized body must be very small compared with quantities he could measure. The following paragraphs describe the principal gyromagnetic effects in solids.

The first successful gyromagnetic experiments were presented in November, 1914, to the Ohio Academy of Science and in December, 1914, to the American Physical Society by S. J. Barnett.(2) Instead of attempting to detect a gross change in orientation of an entire magnet, he made an experiment in which each of the magnetic elements in a magnetic body replaced Maxwell's magnet and measured the change in orientation by measuring the resulting small change in magnetic moment. The experiments were very difficult but have been fully confirmed by much later work. The qualitative theory is simple. If the "magnetic elements" of a body have angular momentum, they will act like gyrostats and tend to align their angular momentum vectors parallel to any axis about which the body is rotated. This changes the magnetic moment along the axis of rotation. If the angular momentum is due to the motion of positive charges, the change in magnetic moment will be parallel to the angular velocity vector of the body; if due to the motion of negative charges it will be antiparallel. The latter is what happens.

The converse of the Barnett effect was discovered by Einstein and deHaas.(3) They obtained the rough magnitude of the effect in iron in 1915 and, working independently, established the sign in 1916. If the "magnetic elements" have angular momentum, then an alteration in the magnetization of a body must change its concealed angular momentum and to conserve total angular momentum the body must receive an equal and opposite angular momentum. The theory was worked out by O. W. Richardson (4) in 1907 but his experiments were not completed.

A fourth gyromagnetic effect looked for was gyroscopic magnetization by the rotation of a magnetic field (Fisher (5) and Barnett(6)). A bar of ferromagnetic material is placed in a transverse field which rotates rapidly about the axis of the bar. If the magnetic elements participate in the rotation, their gyroscopic properties should give rise to a longitudinal magnetization. Null effects, however, were obtained in all experiments. Several explanations(7) have been given, none of them at variance with the gyrostat picture of the magnetic element. Barnett has shown that Fisher's expectation of the magnetization must be reduced at least by the factor $3/2 \frac{I_t}{I_\infty}$, where I_t is the transverse magnetization of the bar and I_∞ the saturation value. This reduces the expected value to a few percent of Fisher's original estimate, even if the rotation of the magnetic element occurs. Einstein has suggested that the magnetic elements do not rotate with the intensity of magnetization vector, but that their moments are periodically reversed by the magnetic field. Block and Becker have also suggested, on the basis of domain theory, that no rotation would be expected in weak fields.

A fifth gyromagnetic effect was discovered by J. H. E. Griffiths in 1946(8). This is "ferromagnetic resonance absorption" at microwave frequencies. Many papers have been published on this effect. In a typical experimental arrangement, a thin foil of the ferromagnetic specimen is used as one wall of a rectangular cavity terminating a wave guide fed by a microwave generator. The cavity is made so the magnetic vector of the microwave field is parallel to the plane of this wall. A static magnetic field is applied, also in the plane of the wall but perpendicular to the microwave field. The energy loss in the cavity is found to go through a maximum as the intensity of the static field is increased. A classical theory has been developed by Charles Kittel.(9) The Quantum-Mechanical Theory, which gives results in agreement with the classical theory, was developed by J. H. Van Vleck and others.(10) The principal result of the theory is Kittel's resonance condition

$$\omega = \left(\frac{ge}{4\pi mc}\right) (BH)^{\frac{1}{2}}$$

(where "g" is the Lande g-factor and other symbols have their usual meaning) instead of the Larmor frequency $geH/4\pi mc$. The resonance frequency also depends on the shape of the specimen and involves the demagnetizing corrections. There are unexplained anomalies in the theory and results.

A sixth gyromagnetic effect was reported in 1949 by S. J. Barnett and L. A. Giamboni.(11)

A long cylinder of powdered ferromagnetic material is magnetized to saturation along its axis so the moments of all magnetic elements are essentially parallel to the axis. An oscillating magnetic field is applied normal to the axis of the specimen which causes the magnetic elements to oscillate through a small angle in the plane of the static field and the oscillating field. This oscillation of the magnetic elements, on account of the Barnett effect, gives rise to a gyromagnetic field whose intensity is normal to the intensity of the other two fields. Disturbing effects were large and the effect very small but its existence was established and its approximate magnitude determined.

In this report the following terminology will be used. The gyromagnetic ratio will be designated by ρ and will mean the ratio of the angular momentum M_0 of the elementary magnet to its magnetic moment μ_0 as

$$\rho = \frac{M_0}{\mu_0}$$

Thus the gyromagnetic ratio for orbital electronic motion is the angular momentum $l\hbar$ (l = orbital quantum number, $\hbar$ = Planck's constant $/2\pi$) divided by $l \cdot \frac{e\hbar}{2m}$ (where $\frac{e\hbar}{2m}$ = Bohr magneton, e = electronic charge, m = rest mass of electron), or

$$\rho = 2 \cdot \frac{m}{e}$$

Similarly for free electron "spin"

$$\rho = 1 \cdot \frac{m}{e}$$

These values will be spoken of loosely as gyromagnetic ratios of 2 and 1 respectively. The closely related Landé g-factors are defined so that g = 2 for electron spin and g = 1 for orbital motion.

Review of Experimental Methods and Results in the Einstein-de Haas Effect

In investigation on the Einstein-deHaas phenomenon, a cylindrical sample of the substance under investigation is suspended with its axis vertical by a wire or fiber coaxial with the cylinder. The cylinder is surrounded by a coaxial magnetizing coil or helix which may be wound on the cylinder or fixed to the surrounding structure. The rotational motion of the cylinder caused by changes in its axial magnetization is then studied.

There are three general experimental methods: First, the ballistic method in which the angular impulse associated with the reversal of the magnetization of the vertical cylinder is measured by the angular twist in the suspension of the cylinder and the constants of the system. Second, the simple resonance method in which the magnetization of the rod is reversed periodically at the resonance frequency of axial rotation of the suspended cylinder and the angular impulse determined from the resonance amplitude. Third, a modified resonance method in which the amplitude of vibration of the rotor due to the gyromagnetic torque is reduced to a minimum by the application of an opposing torque. A modified form of the third method was used in the present investigation.

A. Ballistic Methods

Ballistic methods were used by Richardson (14), J. Q. Stewart(12) and A. P. Chattock and L. F. Bates.(13) If the axial component of the suspended cylinder (called rotor hereafter) is "μ" the concealed angular momentum $H = \rho\mu$ will change at a rate $\frac{dH}{dt} = \rho\frac{d\mu}{dt}$ when μ changes. The angular momentum of the rotor J will change at a rate $\frac{dJ}{dt} = \frac{-dH}{dt} = -\rho\frac{d\mu}{dt}$ where $\frac{dJ}{dt}$ is the gyromagnetic torque on the rotor or

$$g = \frac{dJ}{dt} = -\rho\frac{d\mu}{dt} \qquad (1)$$

In ballistic experiments the magnetization of the rotor is changed very quickly and the total angular impulse is given by

$$\int g\,dt = \Delta J = -\rho\Delta\mu \qquad (2)$$

where ΔJ is the angular impulse imparted to the rotor because of the change in magnetic moment $\Delta\mu$. The angular impulse is found from the standard ballistic formula

$$\rho\Delta\mu = \theta\sqrt{IC} \qquad (3)$$

where θ is the angle of deflection for no damping, I is the moment of inertia of the rotor and C is the torsion constant of the suspension. In the standard treatment θ is found from the observed values θ_1 and θ_2 for the initial deflection to either side of zero from the expression

$$\theta = \theta_1\left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{\pi}\arctan\left(\frac{\pi}{\log\frac{\theta_1}{\theta_2}}\right)} \qquad (4)$$

Stewart annulled vertical and horizontal components of the earth's magnetic field; Chattock and Bates only the horizontal component. Stewart's work gave the values $\rho = 1.02\frac{m}{e}$ for iron and $\rho = 0.94\frac{m}{e}$ for nickel, with errors estimated at 15%. This was the first confirmation of the earlier values obtained by Barnett in the converse experiment.

Chattock and Bates found $\rho = 1.014\frac{m}{e}$ for soft iron, $1.002\frac{m}{e}$ for steel and $1.010\frac{m}{e}$ for nickel.

A very precise measurement by ballistic methods of the gyromagnetic ratio for pure iron has recently been reported by G. G. Scott.(14) The earth's magnetic field was compensated to about 1 part in 20,000 with continuous adjustments for variations in the horizontal components. The magnetizing coil was wound directly on the rotor which was statically and dynamically balanced. He obtained the value $1.0278 \pm 0.0014 \cdot \frac{m}{e}$.

B. Simple Resonance Methods

Resonance methods for measuring the gyromagnetic ratio in which the magnetizing impulses were timed to coincide with the resonance frequency of axial vibration of rotation were used in the original investigation of Einstein and deHaas and in most investigations since then. Let the fundamental (first harmonic) of the magnetic moment be given by $\mu = \mu_0 \cdot \cos \omega t$ when the magnetizing solenoid is supplied by an alternating current of frequency $\omega/2\pi$ The gyromagnetic torque on the system then is given by

$$-\rho\frac{d\mu}{dt} = \rho\mu\omega_0 \sin\omega t \qquad (5)$$

The equation of motion of the system then is

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + k\theta = \rho\mu\omega \sin\omega t \qquad (6)$$

where I is the moment of inertia of the system, b is the damping factor and k the torsion constant of the suspension. Higher harmonics will have negligible effects within present limits of error since the mechanical resonance must be very sharp in order to obtain sufficient sensitivity. The equation is of standard form and its solution is

$$\theta = \frac{\rho\mu}{\sqrt{b^2 + (\omega I - \frac{k}{\omega})^2}} \cos(\omega t - \phi) \qquad (7)$$

where $\tan\phi = \dfrac{\omega b}{k - \omega^2 I}$ (8)

If b is small and $\omega = \sqrt{\frac{k}{I}}$ then $\phi = \pi/2$

we have

$$\theta_M = \frac{\rho\mu}{b} \qquad (9)$$

Where θ_M = maximum value of θ*

The damping factor b is related to the logarithmic decrement λ by the approximate relation (for small damping)

$$b = \frac{I\omega_0\lambda}{\pi}$$

and

$$\rho = \frac{I\lambda\omega_0\theta_M}{\mu\pi} \qquad (10)$$

*Strictly speaking, if damping is present the maximum amplitude does not occur for $\omega = \sqrt{\frac{k}{I}}$ but at $\omega = \sqrt{\frac{k}{I} - \frac{b^2}{2I^2}}$.
The difference in amplitude becomes negligible, however for small damping.

Einstein and deHaas failed to obtain the correct value for ρ , but found $\rho = 2\frac{m}{e}$ with an error they estimated at about 15%. Later E. Beck(15) used this method successfully. He used a single or double German silver suspension for a ferromagnetic rod inside a vertical solenoid. He avoided direct measurement of λ by measuring other amplitudes and corresponding $\omega - \omega_0$ on the resonance curve in addition to the maximum amplitude. This permits elimination of λ and direct measurement of ω_0 is avoided. The value $\omega - \omega_0$ must be measured carefully, however, and Beck used a special frequency meter for this purpose. Beck found $\rho = 1.06 \times \frac{m}{e}$ for iron and $\rho = 1.14 \times \frac{m}{e}$ for nickel with errors estimated at 5%.

C. Modified Resonance Methods

In the first experiments of Einstein and deHaas, and in those of Beck, the only torque deliberately applied was the gyromagnetic torque. W. Sucksmith and L. F. Bates(16) developed a method suggested by A. P. Chattock which uses a torque impressed on the system 180° out of phase with the gyromagnetic torque to reduce the amplitude of vibration to zero (or a minimum if disturbing torques in quadrature with the gyromagnetic torque are present).

A modification of this null method was used in the present investigation and the details are discussed later. Sucksmith and Bates reported the following results for $\rho\frac{e}{m}$ Iron 1.00, nickel 1.00 and Hensler alloy 1.00 with errors they estimated at 1%. Barnett[17] has given a thorough discussion of the sources of error in this and other experiments on the Einstein-deHaas effect.

Resonance methods were used by W. Sucksmith[18] in an investigation of certain paramagnetic rare earth oxides and salts of the iron group.

The amplitudes he obtained were very small (from 1 to $4\frac{1}{2}$ mm), in spite of a relatively low frequency of 1/3 cycle/sec, quartz fiber suspension and enclosing the system in an evacuated tube. He encountered difficulties from electric torques and particularly from traces of ferromagnetic impurities. The oxides were packed in thin glass-walled tubes, with great care to avoid contamination. The work was done at night to avoid mechanical and magnetic disturbances.

A special resonance technique employed by Sucksmith maintained the phase of the vibrating system at $\pi/2$ with the magnetizing current and thus $\pi/2$ with the first harmonic of the magnetization. Any quadrature torques which were in phase with the motion would feed no energy into the system and would not affect the amplitude.

Sucksmith found the value of ρ and thus Lande's g-factor $g = \frac{2m}{e} \cdot \frac{1}{\rho}$ for the salts of the iron group ($CrCl_3$, $MnSO_4$, $MnCO_3$, $FeSO_4$, $CoSO_4$, $CoCL_2$) and the rare earth group (Nd_2O_3, Eu_2O_3, $Rd_2O_3$and Dy_2O_3). He then compared these with values of g obtained from calculations of Van Vleck[19] and

Stoner.[20] In view of the great experimental difficulties, agreement was very satisfactory.

Another method for the partial elimination of disturbing quadrature torques was developed by F. Coeterier and P. Scherrer.[21] A narrow beam of light reflected from a mirror mounted on the rotor was allowed to fall on a photoelectric cell as the rotor passed through its equilibrium position. The photoelectric cell activated a series of relays which reversed the current supplied to the magnetizing solenoid. Torques in quadrature with the motion are thus eliminated, but lag in the magnetization behind the current prevents the elimination of all disturbing effects. They made a rough measurement on powdered iron and a careful measurement of the gyromagnetic ratio for pyrrhotite, obtaining $\rho\frac{e}{m} = 3.17$ or $g = 0.63$.

Measurements by this method have been made by F. Galavics[22] on iron, a ferromagnetic manganese-antimony alloy and a ferromagnetic iron-selenium alloy. He found $\rho\frac{e}{m} = 1.02$ for iron, $\rho\frac{e}{m}$ approximately 1 for the manganese - antimony alloy and $\rho\frac{e}{m} > 2$ for the iron-selenium alloy showing that it resembled pyrrhotite.

A recent series of measurements by this method have been reported by Andre Meyer.[23] He reports the following values for $\rho\frac{e}{m}$. Iron 0.996, nickel 1.002, FeNi 1.006, FeNi-Co 0.992, FeCo 1.001 and Fe_3Ni 0.966.

The work reported here is an extension with certain improvements in method of the very extensive investigations of S. J. Barnett[24] on the Einstein-deHaas effect. For some of the details of the experimental work reference will be made to these papers by numbers.

I. Researches on the rotation of permalloy and soft iron by magnetization and the nature of the elementary magnet. Proc. Amer. Acad. 66, 273-348, 1931

II. The rotation of Cobalt and Nickel by magnetization and the gyromagnetic ratios of their magnetic elements. Proc. Amer. Acad. 69, 119-135, 1934.

III. Gyromagnetic and Electron inertia effects, Reviews of Modern Physics 7; 129-166, 1935.

IV. Gyromagnetic ratios for ferromagnetic substances: New Determinations and a new discussion of earlier determinations. Proc. Amer. Acad. 73, 401-455, 1940.

In I Barnett modified the earlier resonance methods considerably. He obtained the values $\rho = 1.037 \pm 0.003 \frac{m}{e}$ for iron and $1.049 \pm 0.003 \frac{m}{e}$ for permalloy with errors estimated at perhaps $\frac{1}{2}\%$. In II, a quadrature coil first used by deHaas was introduced to reduce the effects of torques in quadratures with the gyromagnetic torque, and complex rotors were developed to eliminate the effects of magneto striction.

In IV, further improvements are made and gyromagnetic ratios reported.

$\rho \frac{e}{m}$

Armco iron	1.032	Hopkinson's alloy	1.024
Hondo iron	1.032	Permalloy	1.047
Yensen iron	1.034	Cobalt	1.090
Cold rolled steel	1.039	Cu-Cobalt	1.082
Nickel	1.052	Cobalt iron	1.026
Hipernik	1.052	Cobalt nickel	1.077

The methods used will be discussed in the next section with particular attention to changes and improvements.

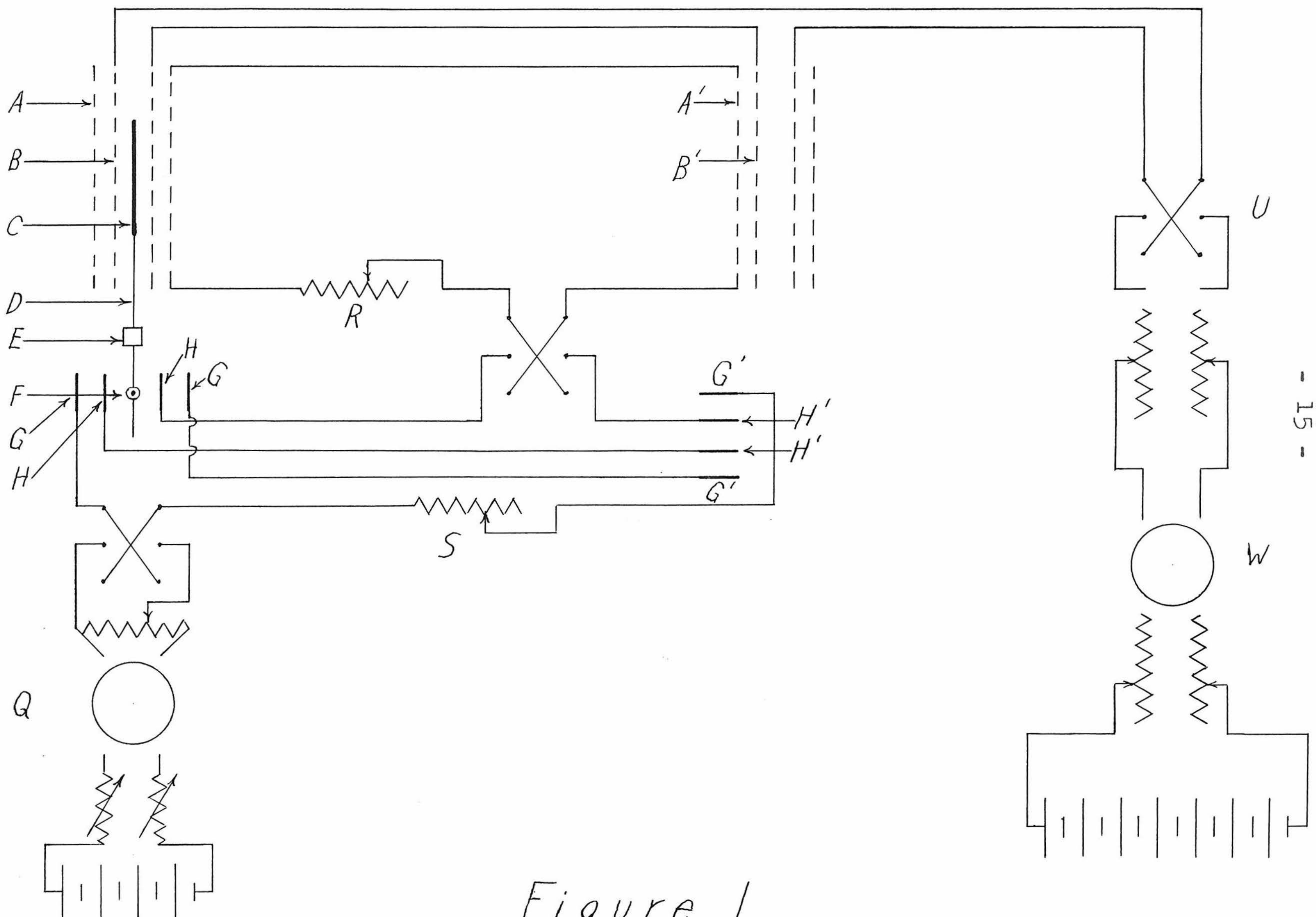

Figure 1

CHAPTER III

Experimental Methods and Sources of Error

A. Fundamental principles.

The methods of Einstein and deHaas, deHaas and Chattock as modified by Barnett were used in the present investigation, and an improved method of eliminating quadrature torques was developed. Figure 1 gives a diagram of the chief experimental circuits. All the work was done with unwound rotors, the magnetizing coil being fixed. "C" is the rotor proper, A, B the induction solenoid and magnetizing solenoid respectively, A'B' their mutual inductance compensators. D is the magnet mirror holder, E the mirrors, F the magnets of moment m_o, GG the Quadrature coil, HH the torque coil, G'G' and H'H' their mutual inductance compensators, R is the resistance in the induction circuit which is varied to find the null, T the induction circuit reversing switch, U the assymmetry reversing switch, Q and W the quadrature commutators and main square wave commutators respectively which are on the same shaft. The principle of the method is as follows:

From equation (1) the gyromagnetic torque on the rotor is

$$g = -\rho \frac{d\mu}{dt}$$

There will be an electromotive force induced in the secondary circuit by this changing magnetic moment given by

$$E = -\gamma \frac{d\mu}{dt} \tag{11}$$

where γ is the constant of the induction solenoid A. The current in the secondary circuit (Solenoids A, A', coils HH and H'H' etc.) will be

$$i = \frac{E}{R} = - \frac{\gamma}{R} \frac{d\mu}{dt} \qquad (12)$$

since the reactance is negligible compared to the resistance. The torque on the magnets L of moment m_o will be

$$L = i\Gamma m_o = - \gamma \frac{\Gamma m_o}{R} \frac{d\mu}{dt} \qquad (13)$$

where Γ is the field produced at the magnets for unit current in HH. By varying R and throwing switch T so the torques oppose each other we can make

$$g + L = o \qquad (14)$$

or, combining (1), (13) and (14)

$$\rho = -\frac{\gamma \Gamma m_o}{R_o} \qquad (15)$$

where R_o is the total resistance in the secondary circuit. The constants Γ, γ and m_o can be measured with precision; the chief difficulty is the determination of R_o. Since deflection methods were used to find the null point it will be convenient to enlarge the previous discussion.

If the first harmonic of the magnetic moment of the rotor is

$$\mu = \mu_o \sin \omega t \qquad (16)$$

the first harmonic of the gyromagnetic torque is (by equation 1)

$$g = -\rho\frac{d\mu}{dt} = -\rho\omega\mu_0\cos\omega t = G\cos\omega t \qquad (17)$$

From equation (13) the torque on the magnets is

$$L = \frac{-\omega\mu_0\gamma\Gamma m_0}{R}\cos\omega t = \frac{k}{R}\cos\omega t \qquad (18)$$

The total impressed torque on the system is

$$T = (G + \frac{k}{R})\cos\omega t \qquad (19)$$

In the steady state condition of vibration, the amplitude will be proportional to the amplitude of the torque or

$$A = \beta(G + \frac{k}{R}) \qquad (20)$$

In the null method, the torque on the magnets opposes the gyromagnetic torque and we may write

$$A = \beta(G - kX) \qquad (21)$$

where X, the conductance of the secondary circuit, is the reciprocal of R.

For $G = kX$, $A = 0$ and we have $\rho = \gamma\Gamma m_0 X_0$. The relationship between A and X is thus linear and the relative phases of the motion and torque change sign at the null point. If there is a torque of magnitude Δ in phase with G the null point would be given by $X_1 = X_0 + \delta X_0 = (G + \Delta)/k$. If the phase of Δ can be altered without changing its magnitude the null point $X_2 = X_0 - \delta X_0 = (G - \Delta)/k$ and the average of X_1 and X_2 gives the correct value X_0.

If there is a torque $Q\sin\omega t$ in quadrature with G, equation (19) becomes

$$T = (G - kX)\cos\omega t - Q\sin\omega t$$
$$= \left[(G-kX)^2 - Q^2\right]^{\frac{1}{2}}\cos(\omega t - \alpha) \qquad (22)$$

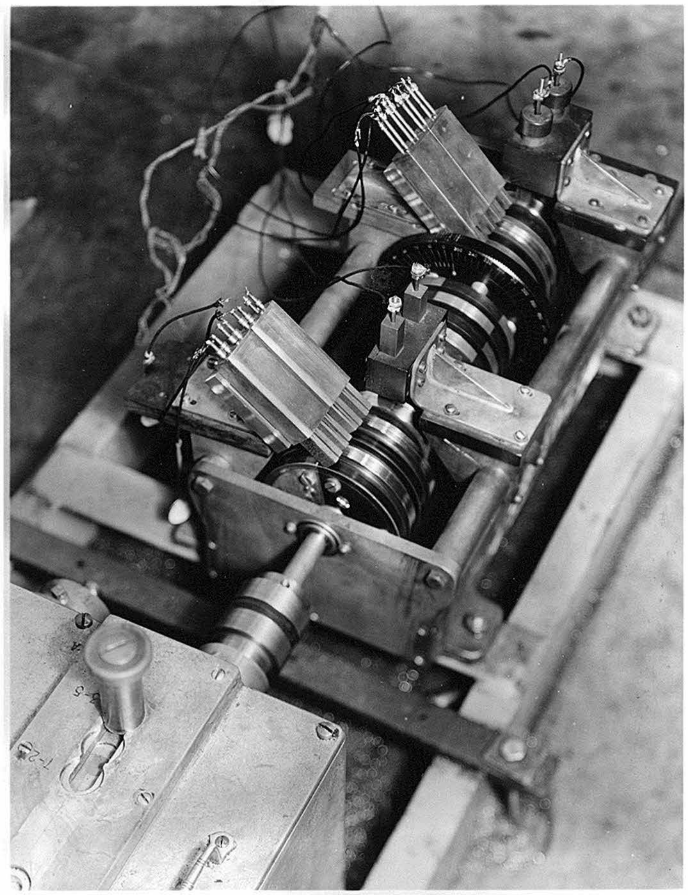

Figure 2

where $\tan\alpha = \frac{Q}{G - kx}$

The amplitude in steady state will then be

$$A = \beta\left[(G - kX)^2 - Q^2\right]^{\frac{1}{2}}. \qquad (23)$$

If we plot A against X the curve is one branch of a hyperbola with a minimum at $G - kX_0 = 0$ as before. Thus torques in quadrature with G will introduce no systematic error but large quadrature torques will make the curve so flat that X_0 cannot be accurately determined. The method developed for the elimination of large quadrature torques is discussed later.

Barnett used various experimental methods for determining the minimum. The most precise of these is the large deflection method discussed on pages 302 of I and 403-404 of II. In this method, $X_0 - \delta X_0$ and $X_0 + \delta X_0$ are determined by measuring the amplitudes A and A_2 respectively for X = 0 and $X = X_2$ where X_2 = approximately $2(X \pm \delta X_0)$; and measuring the amplitude A_0 for $X = (X_0 \pm \delta X_0)$. These values are substituted in the formula

$$X_0 \pm \delta X_0 = X_2 \cdot \frac{(A^2 - A_0{}^2)^{\frac{1}{2}}}{(A^2 - A_0{}^2)^{\frac{1}{2}} - (A_2{}^2 - A_0{}^2)^{\frac{1}{2}}} \qquad (24)$$

Since A_0 is usually small compared with A or A_2 it need not be determined with precision, and for A_0 negliginle (26) reduces to

$$X_0 \pm \delta X_0 = \frac{A}{A + A_2} \cdot X_2 \qquad (25)$$

Under good conditions, the use of the quadrature coil made A_0 negligible so equation (25) could be used.

B. Elimination of Quadrature torques.

The presence of disturbing torques in quadrature with the gyromagnetic torque flattens the vibration amplitude vs. conductance curve and makes the precise determination of the value of $X_0 \pm \delta X_0$ difficult. To find $X_0 \pm \delta X_0$ with precision it was necessary to find some means of eliminating these quadrature torques. A quadrature coil was first used by deHaas in his work at Leyden which was done at very low frequencies in order to observe directly the relative phases of the motion and the magnetization. Barnett used a quadrature coil in his earlier work on Nickel and Cobalt (see II) and in some of the later work. He obtained the current for his quadrature coil from the current supply for the magnetizing solenoid. If the magnetic moment of the rotor is exactly in phase with this current, then the gyromagnetic torque is in quadrature with the magnetizing current. Barnett used the EMF across a nearly non-inductive resistance in the magnetizing circuit to supply the quadrature coils. Since the resistance in the quadrature circuit was high and the inductance small, the current in this circuit was very nearly in phase with the magnetizing current. Time lags between field strengths and the corresponding magnetic induction, however, are well known and may be due to hysterisis, eddy currents within the material, or other causes.(25) If such a time lag is present, the torque of the quadrature coils on the magnets will not be strictly in quadrature with the gyromagnetic torque and will produce in-phase effects. Barnett had difficulty with in-phase torques from the quadrature coil but was sometimes able to eliminate the effect by determining ρ for several values of the quadrature torque. The correct value for ρ was then found by extrapolation to zero quadrature torque. This was not always possible, however, and it appeared desirable to supply the quadrature coils from a source with arbitrary phase angle with respect to the magnetizing current.

In preparation for the present investigation a double set of commutators was prepared (see I for details of design), mounted on the same shaft to give two square waves of exactly the same frequency. (Fig. 2). They were constructed so one set could be rotated to an arbitrary position with respect to the other, and the position was indicated on an accurately made scale and vernier. The two sets commutators were supplied from independent banks of lead storage cells. The commutators were driven by a $\frac{1}{2}$-horse power D.C. motor whose frequency was accurately controlled by a method described below. This apparatus gave two square waves having identical frequencies but arbitrary phase and independent amplitudes. These will be designated as the main square wave (which was supplied to the magnetizing helix) and the quadrature wave.

Let the fundamental of the main square wave be given by

$$I = I_0 \cos (\omega t + \alpha) \qquad (26)$$

and the fundamental of the magnetic moment of the rotor by

$$\mu = \mu_0 \cos \omega t \tag{27}$$

The gyromagnetic torque then is given by

$$G = -\rho \frac{d\mu}{dt} = \mu_0 \rho \omega \sin \omega t \tag{28}$$

The phase angle α was unknown and varied from rotor to rotor. To make effective use of the new quadrature supply it was necessary to determine the phase angle α or the phase of the first harmonic of quadrature wave with respect to the gyromagnetic torque. This was done by making use of the current in the secondary circuit.

From equation 12, the first harmonic of the current in the secondary circuit is given by

$$i = -\frac{\gamma}{R} \frac{d\mu}{dt} = -\frac{\gamma \omega \mu_0}{R} \sin \omega t \tag{29}$$

and is in phase with the gyromagnetic torque because of the negligible reactance of the secondary circuit.* The current in the secondary circuit was much different in form from the quadrature wave (a square wave), (See figures 9, 10, 11) and the problem was the determination of the phase angles between the fundamental of the wave in the secondary circuit and in the quadrature circuit. Consideration was given first to filtering the two waves to eliminate the higher harmonics and then comparing the phases of the fundamentals by an

*The frequency used was about 10 cycles per second, the inductance of the secondary circuit less than 0.05 henrys, resistance 10,000 ohms or more. The reactance $\frac{\omega L}{R}$ then is 3×10^{-4} radians.

oscilloscope. However, at frequencies of the order of 10 cycles, filter circuits become difficult to build, and a filter which would pass 10 cycles and cut out 20 cycles would also change the phase by an amount difficult to determine or control. It was finally decided to use a mechanical "filter", since these can be made easily and have very narrow resonance peaks. Two sets of concentric coils were constructed, wound on a single bakelite tube (Fig. 3). An alnico magnet with its axis horizontal and a mirror were suspended midway between these by a piano wire suspension of adjustable length. The piano wire suspension passed through a No. 0 Brown and Sharpe pin vise and out the top. The hollow handle was threaded with a 5-40 tap to a depth of some $1\frac{1}{2}$ inches. The piano wire passed through a hole in a special brass screw inserted in the pin vise. A collar around the wire above the brass screw was used to fix its length. With this arrangement it was possible to tune the system to the natural frequency of the rotor to within 1 part in 2,500. The self inductance of the inner set of coils was 15 millihenrys, the outer set 12 millihenrys as measured by a General Radio impedance bridge. Usually each set was connected in series with 4,000 to 6,000 ohms resistance. When in use, the flat topped waves from the quadrature commutators were supplied to the outer set of coils, the current from the secondary circuit was supplied to the inner set of coils.

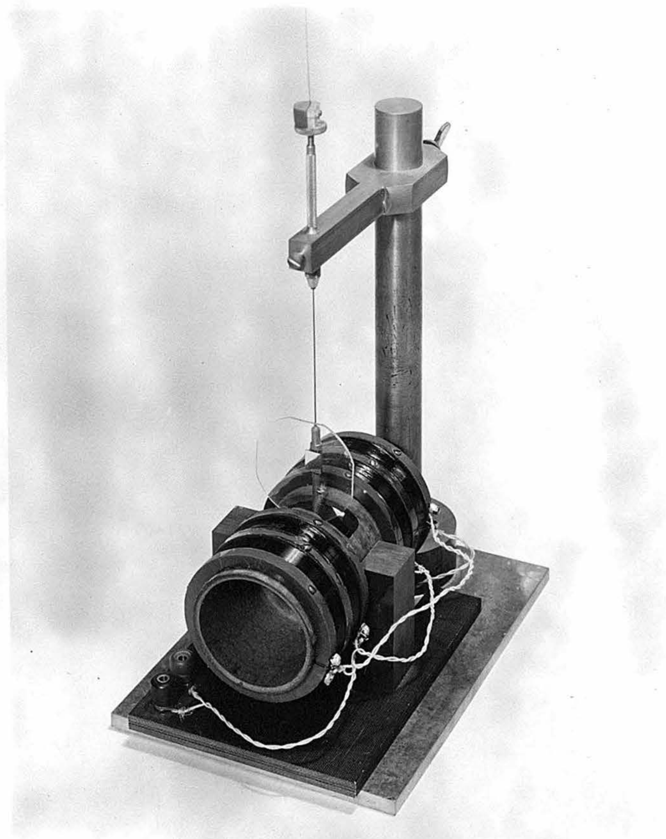

Figure 3

The main square wave was first tuned to the natural frequency of the rotor, then the magnet mirror system of the "Phaser" was tuned to the frequency of the main square wave by adjusting the length of the suspension. By adjusting the resistance and phase of the quadrature circuit it was possible to tune the system to a minimum amplitude of vibration. The quadrature wave was then 180° out of phase with fundamental of the current in the secondary circuit. Rotating the quadrature commutators 90° from this position, put the first harmonic of the quadrature wave in quadrature with the first harmonic of the current in the secondary circuit and thus with the gyromagnetic torque. With care, the null position could be determined to within $\pm \frac{1}{4}$ degree, but was usually determined to within $\pm \frac{1}{2}$ degree.

The settings for quadrature were constant (to within 1°) for one rotor for a period of several days, and appeared to change only as the brushes gradually wore down. The settings varied from rotor to rotor, however by angles up to 7°, depending on the effects of hysterisis, eddy currents and other factors. Examination of figures 10 and 11 will show that one would not expect the quadrature setting to be the same for different rotors because of the difference in the form of the waves they produce in the secondary circuit.

In summary, the "phaser" made it possible to set the fundamental of the quadrature wave at nearly exact quadrature with the gyromagnetic torque and the quadrature torques could be eliminated without introducing in-phase components.

C. Sources of Error

The sources of error in this experiment and the methods for eliminating or avoiding them have been discussed in detail in the papers by Barnett, references I, II and IV. Only a brief summary will be given here. The principle disturbing torques may be divided into three groups. (1) Torques due to the residual earths field. (2) Torques due to the action of the fixed magnetizing coil on the vibrating system. (3) Torques due to magnetostriction.

(1) The horizontal component of the residual earth's field acting on the horizontal component of the magnetic moment of the rotor (h) will produce a torque about a vertical axis. If the horizontal component of the magnetic moment (h) is in phase with the vertical component (v) which produces the gyromagnetic torque, this disturbing torque will be in quadrature with the gyromagnetic torque and cause no error. If h is not exactly in phase with v, this disturbing torque will have "in phase" components which would increase or decrease the observed value for the gyromagnetic ratio. If the suspended system is turned through 180^{o} about its axis, the torque will be unaltered in magnitude but will be reversed in sign and will be eliminated, provided the residual field remains constant.

(2) If the field of the magnetizing solenoid is not strictly vertical, it will produce a torque about a horizontal axis by acting on the horizontal component of the permanent

magnetism of the rotor. This torque also is reversed in sign by changing the azimuth 180°. There is a similar torque on the fixed magnets of the magnet mirror holder. In this work the magnets of the magnet mirror holder were much stronger than in the earlier work by Barnett; about 5.3 emu instead of some 0.7 emu. The stronger magnets gave certain advantages discussed later, but they increased the effects of the torques due to the magnetizing solenoid. Since "square waves" were supplied to the magnetizing solenoid, the magnetic moment of the rotor was nearly in phase with the fundamental of the square wave. The fundamental of the square wave hence, was nearly in quadrature with the gyromagnetic torque which depends on the rate of change of the magnetic moment. So any torque on the magnets due to the magnetizing solenoid would be largely a quadrature torque. The principal quadrature torques were due to magneto-striction (see below) and the residual earth's field. No certain effects due to the torque on the permanent magnets were observed. In any event, the torque is reversed in sign but not in magnitude when the suspended system is turned 180° in azimuth, and the effect is thus eliminated. It is necessary, of course, to be careful to have the suspended system very straight so the magnets do not change position when the system is rotated through 180°. Great care was taken to insure straightness of the suspended system.

(3) Longitudinal magnetostriction was large in most of

the alloys investigated. The vertical motions produced by magnetostriction may be partly converted into axial vibrations because of asymmetries present in rotor and wave form. Since the change in length due to magnetostriction is independent of the direction of magnetization, the frequency of the effect will be twice that of the magnetizing current and thus not in resonance. However, if the two half cycles of the magnetizing current are dissimilar, the difference between the two effects will have the frequency of the magnetizing current and may have any phase relation to the gyromagnetic torque. The effect may be eliminated by a reversing switch in the magnetizing circuit which interchanges the half-cycles of current producing a particular direction of magnetization.

The quadrature torque usually reversed its phase while retaining nearly the same amplitude when the asymmetry reversing switch was thrown, indicating that most of the quadrature torque was due to magnetostriction.

D. Other Experimental Details

1. The torque and quadrature coils used were used in Barnett's earlier work and were designated torque and quadrature system E. (See page 410, ref. IV for a diagram). The constant of the torque coil was measured again and found to be unchanged from its original value of 1009.6 e.m.u. The mutual inductance between the torque and quadrature coils was again compensated by connecting a duplicate set of coils

so the mutual inductance of the second set was opposed to that of the main set. Tests showed the residual inductance to be entirely negligible.

2. The induction solenoid and fixed magnetizing coils had also been used previously by Barnett. Their mutual inductance was compensated as before by a duplicate set of coils. The balance was adjusted by moving the magnetizing coil of the compensator axially with respect to the induction coil. When the coils were adjusted properly, the residual mutual inductance was less than 0.02% of that of either set alone.

3. The magnet mirror holder used was the one designated as No. 7 in reference IV. The steel magnets previously used were replaced by magnets made from "Cunife" obtained from the General Electric Company. The moment of these magnets was about 5.3 e.m.u. instead of the 0.6 to 0.8 e.m.u. of the spring steel magnet previously used. This increased moment made the magnetic moment itself much easier to measure. No control magnet was necessary to increase the sensitivity of the magnetometer. The current in the torque coils was reduced in the same ratio as the increase in magnetic moment. Thus, any direct effect of the magnetic field of the torque coil on the rotor was reduced. The increased resistance in the secondary circuit decreased the already negligible time lag between the current in the secondary circuit and the gyromagnetic torque.

3. Rotor and suspension construction. All of the rotors were of the standard short type, unwound, and were constructed on the antimagnetostriction principle described in reference II and IV. The upper and lower suspensions were made of No. 30 German silver wire 6.9 cm long and 5 cm long respectively. Since the rotors were of nearly identical construction, and the same upper and lower suspensions were used for all of them, their resonance frequencies were all near 11 cycles per sec.

4. Electrical insulation. Since electrical leakage between primary and secondary circuits can cause errors, it is important to have the insulation of the secondary circuit as nearly perfect as possible. The insulation was high enough to retain an electrostatic charge on the circuit for a number of minutes.

5. The vibration-free mounting. The vibrations in the building due to traffic outside and people and equipment in other parts of Bridge Laboratory disturbed the moving system and made it necessary to protect it from the vibrations. As the vibration-free support used in Barnett's earlier investigations was in use it was necessary to construct a new support. Preliminary attempts were made with a modification of the Müller suspension(26) damped with two damping pendulums of the form developed by H. E. R. Becker.(27) The necessity of using non-magnetic material in the apparatus caused difficulties. The greatest freedom from vibrations is obtained

when the suspended system has a low natural frequency. Loading the suspension to increase the mass caused the rods to buckle before the period was sufficiently long to give good protection from vibration. In order to get rid of vibrations, the marble slab (15" x 30" x $2\frac{1}{2}$") on which the sensitive apparatus rested was suspended from the ceiling by 8 brass springs 44" long (before extension) about 1" O.D., made from 0.128" brass spring wire. In use, the springs were extended to 62 inches, making the period for vertical oscillations about 1.4 seconds. To provide for damping, the springs were wrapped with flat gum-rubber strips 2 inches wide. The high internal friction of the rubber and friction between the rubber and the springs damped the vertical oscillations very effectively.

Horizontal vibrations were a much more serious source of disturbance than vertical oscillations, and it was necessary to suppress their effects as much as possible. Internal damping is much superior to external damping in obtaining freedom from vibrations. If the apparatus is damped, for example, by a dashpot between the apparatus and vibrating surroundings, vibrations are transmitted through the dashpot to the apparatus as well as through the supporting springs. The more heavily the free vibrations of the system are damped by the dashpot, the more effectively the dashpot transmits vibrations to the apparatus. The same considerations apply to any kind of external damper. If the apparatus has internal

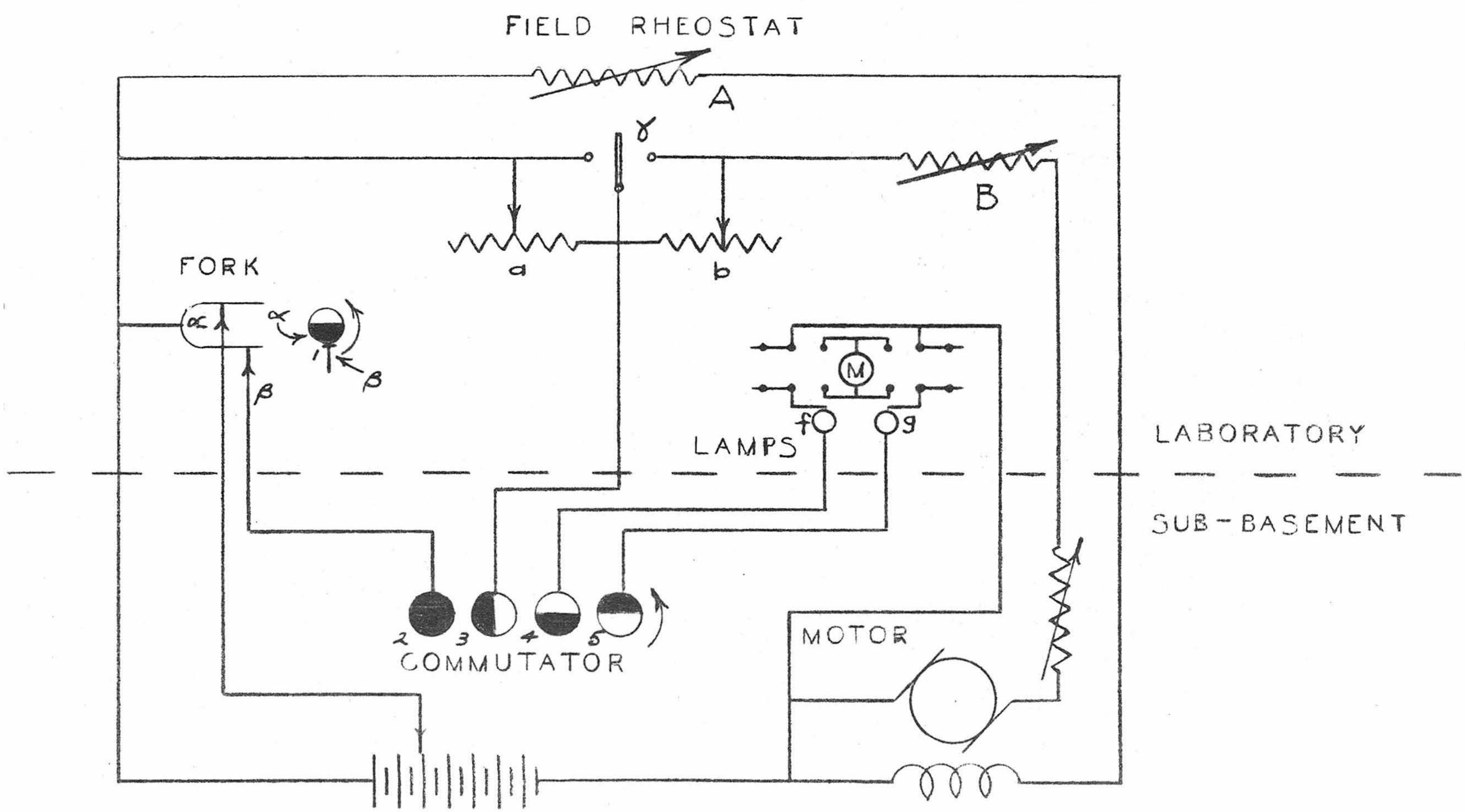
FIELD RHEOSTAT
A
γ
B
a
b
FORK
α
β
M
LAMPS
f
g
LABORATORY
SUB-BASEMENT
2
3
4
5
COMMUTATOR
MOTOR

FIGURE 4

damping, however, the heavier the damping the less it responds to external vibrations and the free vibrations, of course, die out more quickly.

The vibrations of the system in a horizontal direction, or oscillations about a vertical axis were damped by pendulums of the type developed by Becker mentioned previously. Fig. 5 shows the top of one of the pendulums. The two arms at right angles to each other carry pistons which dissipate the energy of oscillation of the pendulum by friction with heavy oil. The pendulum was supported by a wire instead of a pivot. At small amplitudes of vibration the friction at the pivot increased greatly so the pendulums did not swing but moved with the apparatus and provided no damping. The wire support, however, does not have increased friction for small amplitudes and the pendulums remained effective. It was necessary to tune the pendulums by adjusting the lengths until the pendulum period was approximately the same as the suspended system. The pendulum bobs weighed about 3.8 kg each, their length in proper adjustment was about 170 cm. The half time for horizontal vibrations was 14 seconds, for vertical motions about 30 seconds.

The freedom from mechanical disturbances was sufficient to permit work which did not depend upon precise anullment of the earth's magnetic field at any time of day except during the late afternoon when traffic by the laboratory was too heavy.

6. The Frequency Control. In order to maintain resonance, it was necessary to control the speed of the D. C. motor that drove the square wave commutators. The 3 h.p. motor used by Barnett in the previous Einstein-deHaas investigations used too much current for the present battery bank in the Norman Bridge Laboratory and was replaced by a 1/2 h.p. motor. To reduce the load, one gear box was used instead of two and the sleeve bearings of the commutators were replaced by sealed ball bearings. The frequency control system developed by F. Wenner (unpublished) used in the previous investigations was tried first. In this system a make and break contact on a tuning fork and a commutator on the shaft of the motor work together to short-circuit a resistance in series with the field coils of the motor. Together they act so as to increase the time of short circuit of the resistance (increasing the field) when the motor gains on the tuning fork and to decrease the time of short circuit when the motor lags behind the fork. Since an electric motor slows down if the field is increased and vice versa, this keeps the mean speed of the motor and the fork the same. If the control is strong enough the phase changes may be made very small.

This method proved unsatisfactory for the heavily loaded 1/2 h.p. motor. It had too little reserve power, so decreasing the field sometimes caused the motor to slow down because of insufficient torque. Power requirements were reduced further by changing the design of the brushes (see below) but the method remained unsatisfactory.

The armature current was only three amperes, so the control was transferred to the armature circuit in the belief the contacts would stand the current and the control would be more powerful. The attempt was immediately successful. A diagram of the circuit used is given in Fig. 4. "A" is the resistor automatically short circuited and (2)---(5) the commutator. The segments of the commutator used were 4 coaxial discs of brass or brass and bakelite, firmly screwed together, mounted on the motor shaft but electrically insulated from it. The brass parts of the commutator, all in electrical contact, are indicated by the black portions of the circles, the bakelite by the white. Contact was made by four graphite brushes in a line parallel to the top of the shaft. The phase relations of the segments of the commutator cannot vary since the segments are rigidly connected. The contact β on the tuning fork is adjusted until it is closed half the time. This contact is represented by the circle (1) and brush drawn beside it. In this position, the resistance "A" is short circuited 1/4 of the time by the contacts β , (2), and (3) when switch γ is open or thrown to the right. If the motor gains on the tuning fork, examination of the figure will show that the resistance "A" is short circuited less than 1/4 of the time so the armature current decreases and the motor slows down. Similarly, if the motor begins to lag behind the fork, the armature current will increase and the motor will speed up.

Control lamps f and g aid in starting the control. Examination of the diagram will show that when commutator (5) and the tuning fork are in phase lamp "g" will be brighter than lamp "f" because "g" is connected to the battery through the tuning fork (1/2 of the cycle) and through resistance (a) or switch γ (1/4 of a cycle); while lamp "f" is connected to the battery only 1/4 of a cycle through resistance "a" or switch γ. The details of the operation are the same as before and are described in detail in reference I, pages 318-322.

Although the motor ran at the same average frequency as the tuning fork, it tended to "hunt" and these changes in phase produced changes in the phase of the square wave which affected the amplitude of the vibrating rotor in the main experiment. To increase the inertia, a flywheel, 1 ft. in diameter and 65 pounds weight was constructed and connected rigidly to the motor shaft. It was mounted in ball bearings and carefully aligned so it added little to the load on the motor after the system reached its final speed. The voltage of the current supplied to the motor was stabilized by a set of 20 6-volt lead storage batteries across the line. After these changes, the frequency control was very satisfactory. It was necessary to use care in getting the system under control in order to avoid sparking at the brushes of the control commutator. These commutator surfaces had to be

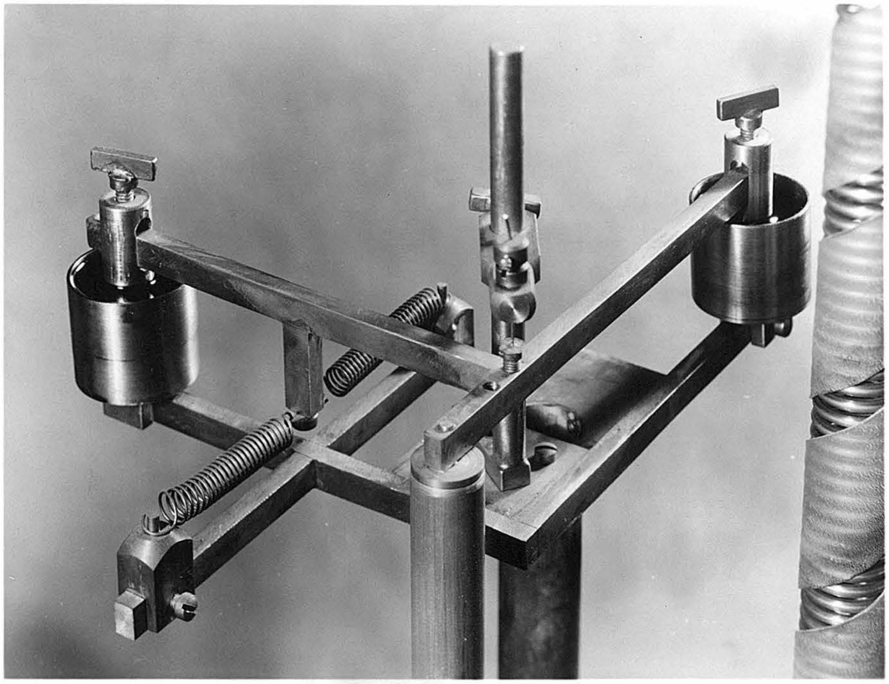

Figure 5

turned down on a lathe occasionally to keep them in good condition.

7. Neutralization of the Earth's Magnetic Field. The vertical component of the earth's magnetic field was neutralized by the large square Helmholtz coil shown in figure 6. The north-south and east-west components were neutralized by Helmholtz coils stretched between bakelite supports. Brass spring wire was used for the coils because it could be stretched tightly between the supports, forming a regular coil without the need for a supporting framework. The currents for the three sets of coils were furnished by three independent banks of lead storage batteries. Precision ammeters were used to measure the currents in these circuits. The proper compensating currents were determined from the nullification of the electromotive force induced in a rotating coil. The coil was mounted inside a lucite cylinder, driven by an air stream.[28] The voltage induced in the coil was read by a Hewlett-Packard Model 400OC vacuum tube voltmeter. The sensitivity was sufficient to compensate the field intensity to 1 part in 5000, but the earth's field fluctuations were much larger than this. The compensating currents for the north-south and vertical components were determined to 1 part in 1000, that of the small east-west component to 1 part in 250.

In order to make this calibration, it was necessary to remove the magnetizing and induction solenoids. Replacing these solenoids and mounting the rotor properly within them was a troublesome, time-consuming operation, and it was necessary to have the rotor in place several hours before the measurements began (see below). Thus it was impossible to make a direct measurement of the compensating currents immediately before beginning a run.

In order to determine the correct compensating currents, an earth inductor was mounted near the apparatus. This was connected in series with a Hibbert standard of magnetic flux. The earth inductor was mounted with its axis vertical. It could be positioned so the net flux it cut was either the N-S component or the E-W component of the earth's magnetic field. At the time the primary calibrations were made, the deflections produced by the earth inductor D_{X_0} for the N-S component and the Hibbert standard D_{H_0} were observed. If X_0 was the correct compensating current at that time, the correct current at a later time is given by

$$X = \frac{D_X}{D_{X_0}} \cdot \frac{D_{H_0}}{D_H} \cdot X_0$$

where D_X and D_H are the deflections observed at the later time for the N-S component and the Hibbert standard respectively. The E-W neutralizing current was determined in a similar manner. The earth inductor could also have been used for the determination of the neutralizing current for the vertical component, but observations showed only very

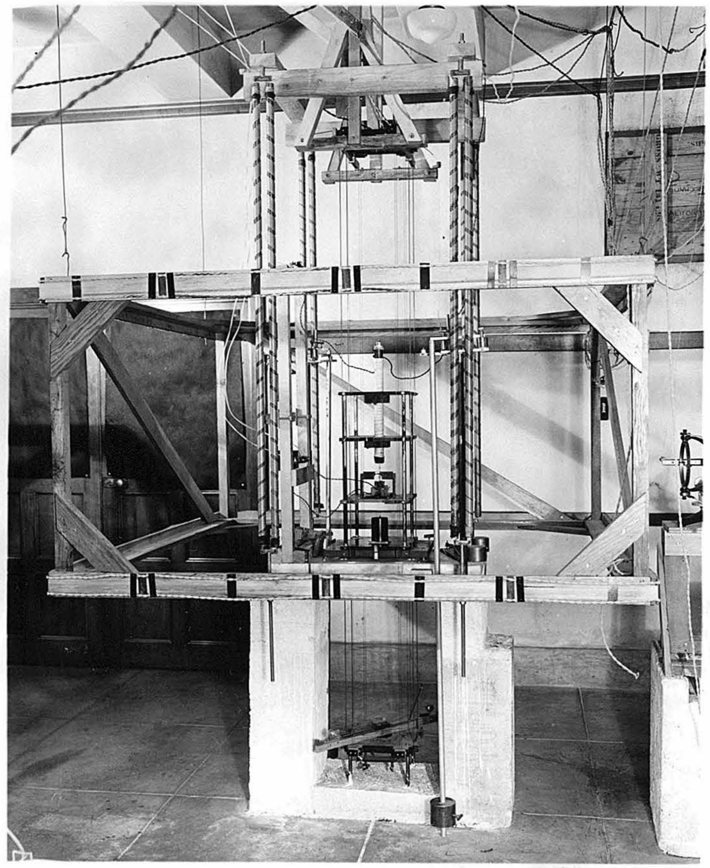

Figure 6

slight changes in this component over a period of months. Also, slight changes in the vertical field produce no effect on the observed value of gyromagnetic ratio.

The presence of iron in the building disturbed the uniformity of the earth's field near the apparatus, so the change in the field at the rotor was not the same as the change in the field at the earth inductor. If the changes were small, the earth inductor was satisfactory. Sometimes the compensating currents were determined by observing the amplitude of vibration of the rotor. Since the residual magnetic field interacts with the horizontal moment of the rotor, the smaller the residual earth's field, the smaller the amplitude of vibration (if no other torques are present). Thus the correct compensating currents, or nearly correct ones, could be determined just before beginning observations. The measurements using the earth inductor had to be made at least one hour before the observations began because it was necessary to have the compensation currents on for an hour before they became sufficiently steady. The earth inductor measurements could not be made when currents were flowing in the compensating coils. The currents were left the same for both azimuths of the rotor.

With the exception of two rotors which were unsymmetrical, and had very large horizontal moments, the residual earth's field introduced no difficulties.

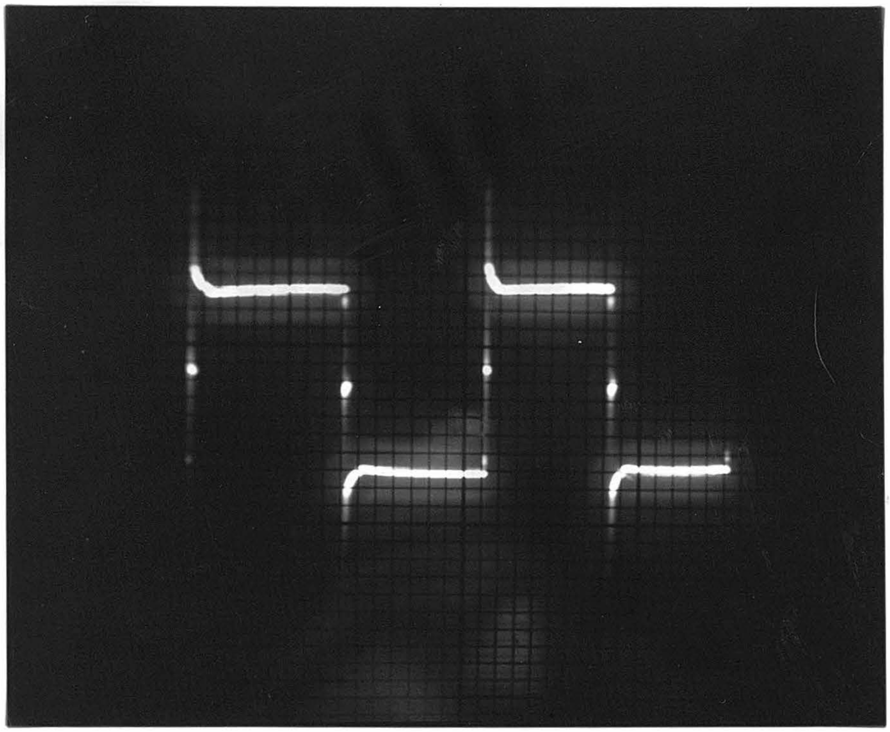

Figure 7

8. Commutators and Brushes. The commutator construction has been discussed above. It was necessary to turn the commutator surfaces on a lathe occasionally to keep the surfaces in good condition. This reduced sparking at the brushes, and kept the current more nearly uniform.

The blunt graphite brushes used in the previous work here had to be pressed against the commutator surfaces rather tightly to give good electrical contact. The frictional drag was quite heavy and loaded the one-half horsepower motor to capacity, making the speed control unsatisfactory. In order to reduce the load, new brushes and brushholders were constructed. Two independent brushes pressed on each commutator at an angle of 45_0 with the surface, so the frictional drag would tend to pull the brushes into closer contact. The pair of brushes insured more nearly uniform contact since it was unlikely that both would miss contact at the same time. Much less force was required to give satisfactory contact and the power input to the motor was reduced by one-third. The brushes gave very little trouble, but had to be reshaped at intervals. The reduced load on the motor made the speed control much easier.

9. Determination of Constants. The constants of the torque coil Γ_0 and the induction solenoid γ_0, the resistance R_0 of the secondary circuit and the moment of the magnets m_0 which enter into the fundamental equation

$$-\rho = \frac{\Gamma \gamma \, m_o}{R_o}$$

must be accurately known. The constants of the torque coil and induction solenoid were accurately measured for the earlier work by Barnett. The moment m_o of the magnets was measured bi-monthly by the magnetometer method described in I. The moment changed by only 0.1% per month, and the value for any particular night was found from the curve.

The resistance R was 8,5000 ohms or more, 124 ohms in the coils and leads, the rest in the resistance boxes. The standard resistance boxes were checked carefully and found to be accurate to 0.1% or better for the high resistances used. Below 40 ohms, the resistances were good to only 0.5% or 1%, but this error was a negligible contribution to the total resistance. So the total resistance of the induction circuit was known to an accuracy of at least 0.1%.

10. Oscillograms of Various emfs. Figure 7 shows the electromotive force in the primary circuit. The peak is due to the inductance of the circuit. Fig. 8 shows the current in the primary circuit. The rounded front is, of course, due to the induction, the commutator gaps also are visible as dots. Figures 9, 10 and 11 show the electromotive forces in the secondary circuit due to the rotor "F" (40% Ni 60% Fe), Rotor "A" (15% Ni 85% Co) and an iron rotor respectively. The primary current was the same for

each of these three rotors. The differences in shape show how the first harmonic of this wave form (which is essentially the same as the gyromagnetic torque on the rotor) may be shifted in phase by several degrees from exact quadrature with the fundamental of the magnetizing current. The differences in form are caused by the differences in the properties of the materials. Most of the lag in iron appears to be due to eddy currents since the decay is exponential. Ideally, one should want to have a very sharp peak and no lag to reduce the influence of the in-phase components of the disturbing torques. Figure 12 is the same material as figure 9. The peak is shown broadened to illustrate its double nature. With mechanical commutators, the current first falls to zero when the brush passes over the gap and then rises to a maximum in the opposite direction. The first part of the double peak is due to the change in magnetization when the current falls to zero, the second part is caused by the change in magnetization when the current rises to a maximum in the opposite direction. The change is so rapid that the rising part of the curve does not appear, but the decay is clearly visible.

11. Procedure. It is important to have stable conditions before beginning observations. In order to insure the necessary stability the motor driving the commutators, the tuning fork and the rotor current were all turned on for at least four hours

before observations were begun. The blinds were pulled down and were held against the window frame by long boards in order to reduce convection currents around the windows, and the air conditioning was turned off to eliminate the air currents it caused. After the motor was brought under the frequency control, the frequency was adjusted until the rotor amplitude was a maximum. Then the resistances of the secondary circuit and the quadrature circuit which gave minimum amplitude (see discussion of method above) were determined for both positions of the asymmetry reversing switch. The measurements then proceeded as described in detail in IV.

Most of the measurements were made after midnight, some were made in the evening before midnight and a few were attempted in daytime. On a few occasions two complete observations were made in one night. Work was made impossible on a few nights by magnetic storms, but the conditions between 1:00 A.M. and 4:30 A.M. were usually very good.

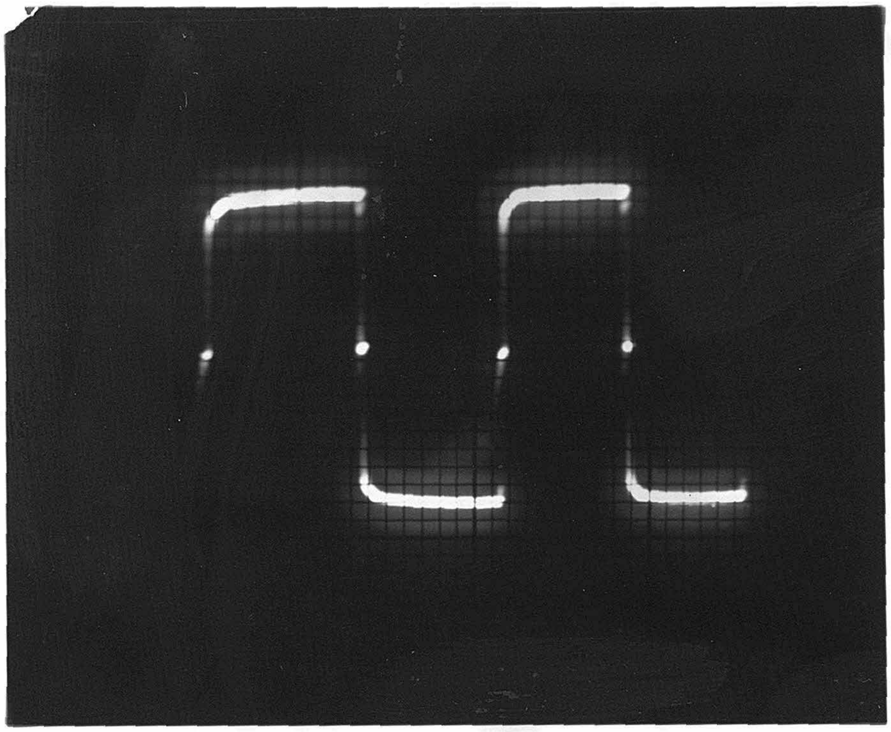

Figure 8

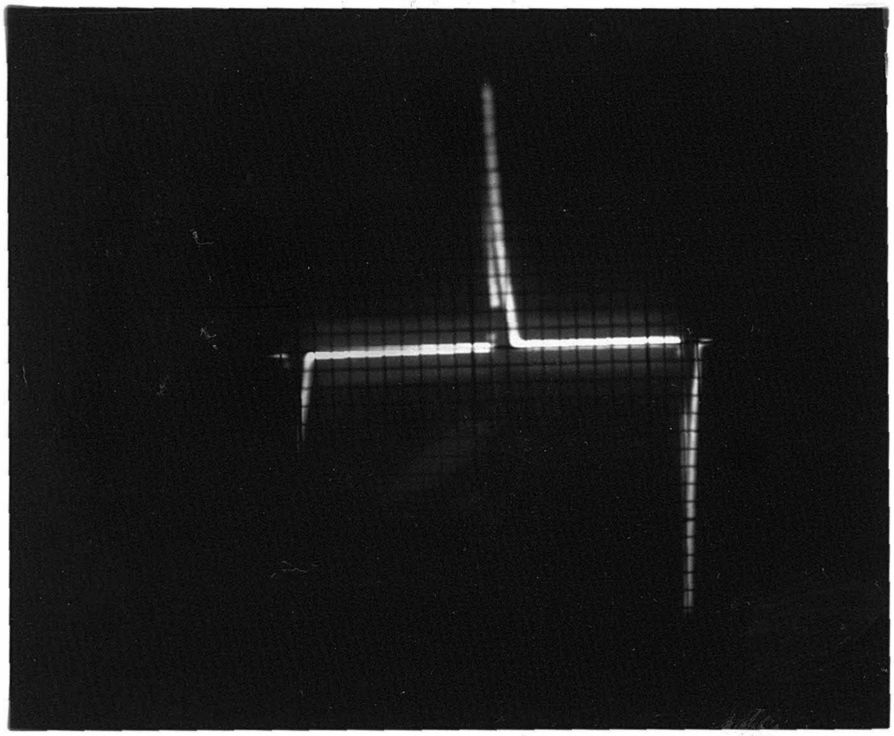

Figure 9

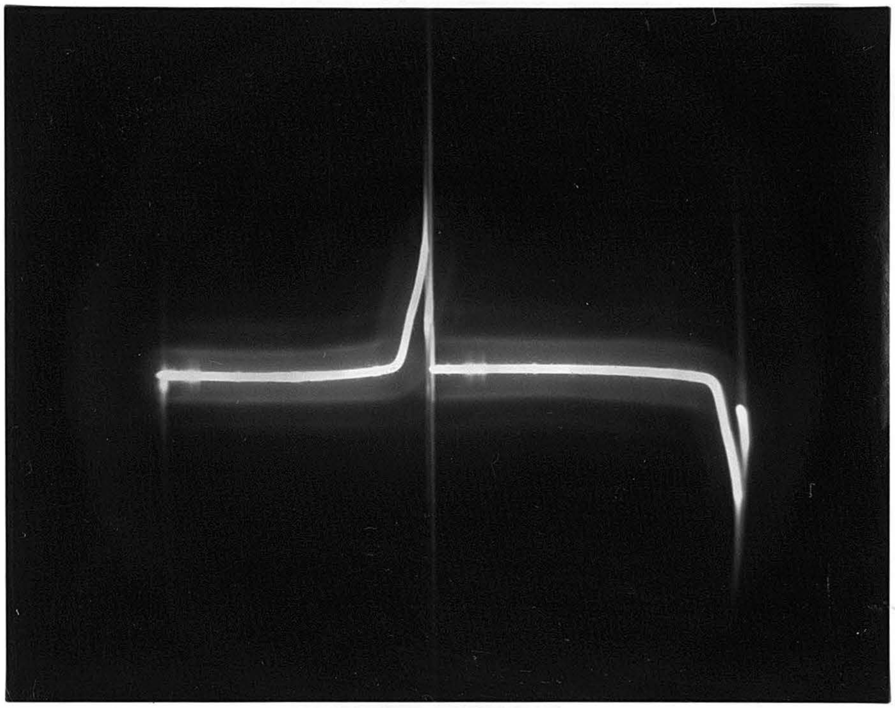

Figure 10

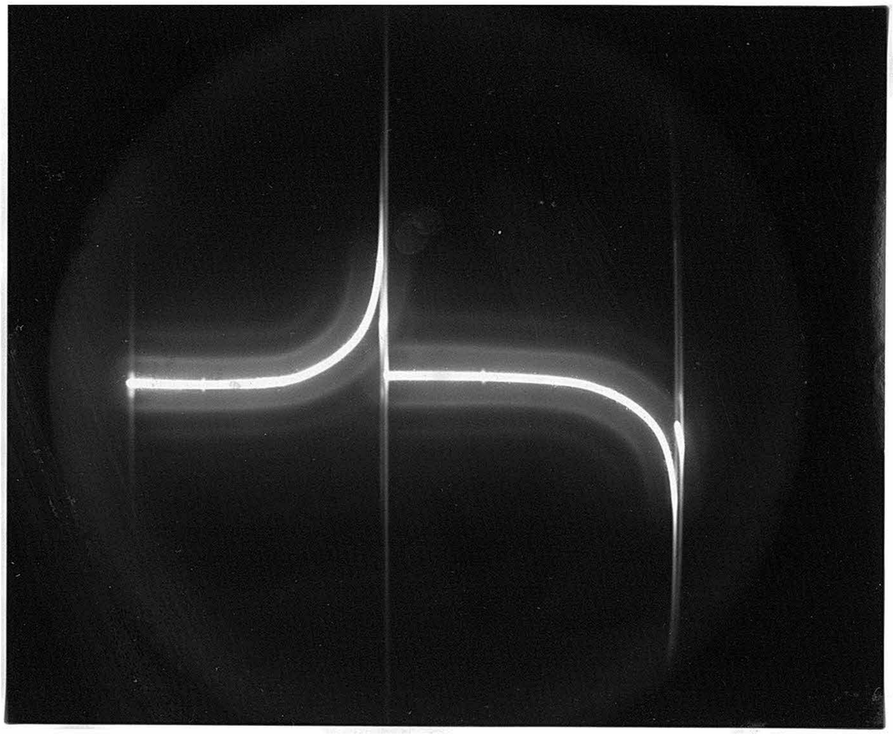

Figure 11

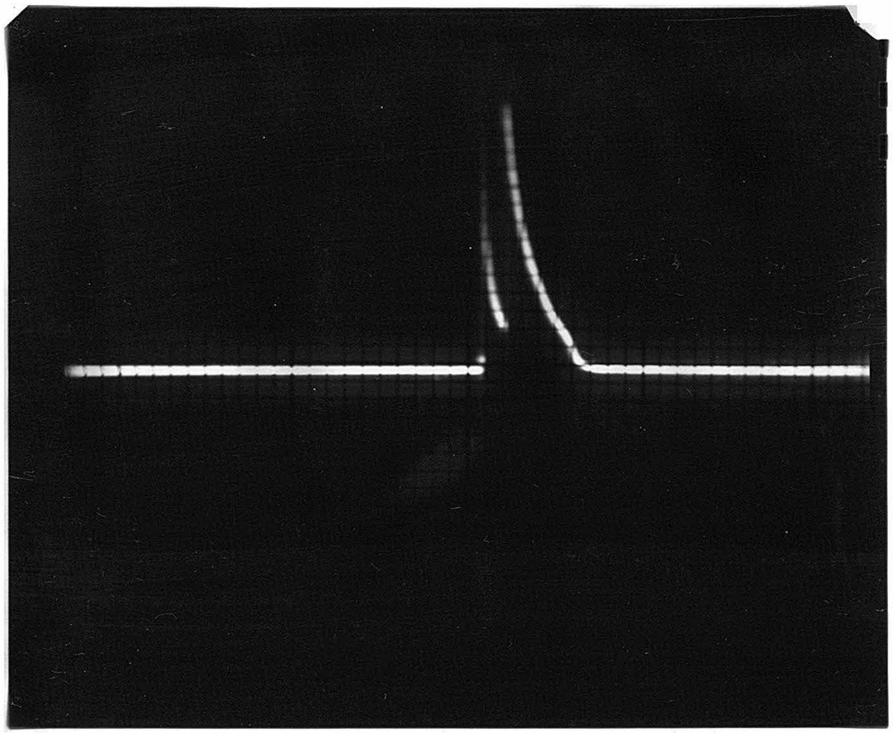

Figure 12

CHAPTER IV
Observations and Results

Gyromagnetic ratios were measured in 15 binary alloys of iron, cobalt and nickel, and 2 iron rotors. The results are given in the tables which follow. NPE and NPW refer to the two azimuths of the permanent magnet of the suspended system, I and II to the two positions of the asymmetry reversing switch. Thus, under the heading NPE II will be found the observed value of the gyromagnetic ratio for azimuth E of the suspended system when the asymmetry reversing switch is in position II. The average of the four values given is the observed gyromagnetic ratio found under the heading $\rho\frac{e}{m}$. The symbols day, eve, nt., under the heading T indicate that the observations were made in the daytime, in the evening before midnight, and after midnight, respectively. Sometimes special circumstances made it necessary to complete an observation in a shorter time than normal. This was done by reducing the number of measurements of the amplitude by 1/2 and is indicated under the heading OBS. by the fraction 1/2, full sets of observations are indicated by 1. The errors are expressed in terms of average departures from the mean. Full sets of observations were assigned the weight 2, the half-sets the weight 1.

TABLE I

Composition of Iron Cobalt Alloys

% Cobalt	% Manganese	% Iron	% Silicon	% Carbon
10.17	0.31	89.41	0.01	
20.04	0.47	79.22	Nil	
30.19	0.45	69.37	Nil	
40.09	0.44	59.44	Nil	
50.11	0.47	49.18	0.01	
59.82	0.58	39.45	0.01	
69.62	0.39	29.80	0.01	
79.36	0.48	20.18	0.01	
89.78	0.63	9.56	0.03	
98.90	0.63	0.18	0.09	0.20

A. Cobalt-Iron Alloys

Through the courtesy of the Bell Telephone Laboratories a series of iron-cobalt alloys was obtained. The analysis provided with the alloys is given in table I. The samples were all annealed for one-half hour at 900°C after swaging.

1. 10% Cobalt-Iron Rotor. This rotor had a very large horizontal magnetic moment which made measurements impossible. Repairs to the rotor were not completed in time for this work.

2. 20% Cobalt-Iron Rotor. The band of light on the scale was crossed by faint lines, indicating that the effects of magnetostriction were not completely eliminated by the complex rotor construction (See I, page 332). The pattern was steady. It was necessary to use the quadrature coil.

TABLE II

20% Cobalt-Iron Rotor
Double Amplitude 3.1-3.8 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
June 5, 1950	nt	1	1.020	1.029	0.993	1.036	1.020
6	nt	1	0.978	1.097	0.996	1.095	1.034
7	day	$\frac{1}{2}$	0.987	1.061	0.948	1.036	1.008
7	eve	$\frac{1}{2}$	0.968	1.097	0.963	1.091	1.030
10	nt	1	0.908	1.107	0.907	1.108	1.008
14	nt	1	1.025	1.093	0.958	1.111	1.046

The weighted mean is $\rho \frac{e}{m} = 1.025 \pm 0.013$

3. 30% Cobalt-Iron Rotor. At first, this rotor had a large horizontal magnetic moment and the effects of magnetostriction were evident. The rotor was remade, in an attempt to make it more nearly symmetrical. It then behaved quite well, although the I, II asymmetry was larger than usual. The quadrature coil was used. The rotor was inverted for the observation of July 27.

TABLE III

30% Cobalt-Iron Rotor
Double Amplitude 4.4-5.6 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
1950							
July 22	eve	$\frac{1}{2}$	1.145	0.925	1.117	0.906	1.023
24	nt	1	1.162	0.972	1.131	0.927	1.048
27	nt	1	0.649	1.447	0.673	1.412	1.045

The weighted mean is 1.042 ± 0.007

4. 40% Cobalt-Iron Rotor. This rotor broke during construction and could not be repaired or replaced.

5. 50% Cobalt-Iron Rotor. All observations on this rotor were made during the daytime or early evening. It had a rather small horizontal magnetic moment, however, and the observations were fairly good. The quadrature coil was used.

TABLE IV

50% Cobalt-Iron Rotor
Double Amplitude 2.7-3.3 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Mar. 14	day	$\frac{1}{2}$	1.130	1.014	1.058	1.033	1.059
16	day	$\frac{1}{2}$	1.164	1.057	1.089	1.058	1.092
16	eve	$\frac{1}{2}$	1.167	1.036	1.083	0.972	1.065

The mean of these observations is 1.072 ± 0.013

6. 60% Cobalt-Iron Rotor. The earth's magnetic field was unsteady through this series of observations, but the small horizontal moment of the rotor reduced the effects of the fluctuations. Quadrature torques were small. The quadrature coil was used.

TABLE V

60% Cobalt-Iron Rotor
Double Amplitude 3.3-4.2 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Aug. 6	nt	1	1.090	1.050	1.035	0.981	1.039
7	eve	1	1.079	1.087	1.037	1.013	1.054
7	nt	1	1.109	1.069	1.023	0.997	1.050
8	nt	1	1.080	1.099	0.998	1.017	1.049
8	nt	1	1.107	1.078	1.026	1.011	1.056

The mean of these observations is $\rho\frac{e}{m}$ = 1.050 ± 0.004

7. 70% Cobalt-Iron Rotor. The earth's magnetic field was steady during this series. The quadrature coil was used.

TABLE VI

70% Cobalt-Iron Rotor
Double Amplitude 3.0-3.8 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Aug. 1	eve	1	1.091	1.053	1.002	1.050	1.049
4	eve	1	1.020	1.079	1.089	1.065	1.063
5	nt	1	1.056	1.081	1.029	1.066	1.058
5	nt	1	1.016	1.078	1.080	1.109	1.070

The mean of these observations is $\rho\frac{e}{m} = 1.060 \pm 0.007$

8. 80% Cobalt-Iron Rotor. This rotor had a large magnetic moment and gave large amplitudes. The quadrature coil was used.

TABLE VII

80% Cobalt-Iron Rotor
Double Amplitude 4.7-7.0 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Mar. 20	nt	$\frac{1}{2}$	1.253	0.949	1.189	0.966	1.089
22	eve	1	1.163	1.016	1.064	1.083	1.081
23	day	1	1.199	1.032	1.095	1.040	1.092

The weighted mean of these observations is $\rho\frac{e}{m} = 1.087 \pm 0.005$

9. 90% Cobalt-Iron Rotor. This rotor was very good. The quadrature coil was used.

TABLE VIII

90% Cobalt-Iron Rotor
Double Amplitude 3.6-4.4 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
1950							
Apr. 6	day	$\frac{1}{2}$	1.121	1.141	1.048	1.071	1.095
	nt	1	1.130	1.114	1.066	1.06[illegible]	1.095
	nt	1	1.120	1.104	1.057	1.066	1.087
	nt	1	1.104	1.100	1.063	1.071	1.085

The weighted mean of these observations is $\rho \frac{e}{m}$ = 1.090_0.004

10. Cobalt Rotor. This rotor was exceptionally good. Quadrature torques were small and the amplitude very stable. The quadrature coil was used.

TABLE IX

Cobalt Rotor
Double Amplitude 2.0-2.2 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
1950							
June 27	nt	1	1.071	1.117	1.088	1.023	1.075
28	nt	1	1.063	1.106	1.089	1.044	1.076
30	eve	$\frac{1}{2}$	1.092	1.093	1.088	1.057	1.082
July 1	eve	1	1.105	1.079	1.053	1.101	1.085

The weighted mean of these observations is $\rho \frac{e}{m}$ = 1.079 - 0.004

B. Cobalt-Nickel Alloys.

Measurements of the gyromagnetic ratio were made on three cobalt-nickel alloys. The alloys were obtained from the General Electric Company. They were deoxidized with 0.2% aluminum, 0.2% silicon and 0.5% manganese. The percentages are "by addition"; the alloys were not analyzed.

1. 20% Cobalt-nickel Rotor. The band of light on the scale was crossed by lines showing that the effects of

magnetostriction were present. The pattern was steady. After 3 runs which were rejected because of large asymmetries and poor magnetic conditions, four good sets of measurements were obtained. The quadrature coil was used.

TABLE X

20% Cobalt-Nickel Rotor
Double Amplitude 1.7-2.4 cm

			NPE		NPW		
Date	T	Obs	I	II	I	II	$\rho\frac{e}{m}$
1950							
Aug. 16	1	eve	0.974	1.136	1.050	1.095	1.064
16	1	nt	0.957	1.137	1.050	1.100	1.061
17	1	nt	0.931	1.176	1.072	1.101	1.070
17	1	nt	0.971	1.139	1.029	1.083	1.056

The mean of these observations is $\rho\frac{e}{m} = 1.063 \pm 0.005$

2. 40% Cobalt-nickel Rotor. Quadrature torques were small. The quadrature coil was used.

TABLE XI

40% Cobalt-Nickel Rotor
Double Cmplitude 2.6-3.6 cm

			NPE		NPW		
Date	T	Obs	I	II	I	II	$\rho\frac{e}{m}$
1950							
Aug. 22	nt	1	0.979	1.193	0.968	1.163	1.076
	nt	$\frac{1}{2}$	0.963	1.251	0.913	1.176	1.076
	nt	$\frac{1}{2}$	1.089	1.122	1.038	1.101	1.087
	nt	1	1.125	1.094	1.076	1.077	1.093

The weighted mean of these observations is $\rho\frac{e}{m} = 1.084 \pm 0.008$

3. 70% Cobalt-nickel Rotor. There were enormous effects from magnetostriction in this rotor. Measurements were not attempted.

4. 85% Cobalt Rotor. This rotor gave rather small amplitudes, but very steady readings. Quadrature effects were small. The quadrature coil was used.

TABLE XII

85% Cobalt-Nickel Rotor
Double Amplitude 1.1-1.3 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Aug. 26	nt	1	1.040	1.166	1.100	1.035	1.085
26	nt	1	1.035	1.188	1.102	1.034	1.084
27	nt	1	1.055	1.157	1.106	1.029	1.087
27	nt	1	1.036	1.188	1.110	1.014	1.087

The mean of these observations gives $\rho\frac{e}{m} = 1.086 - 0.001$

C. Nickel-Iron Alloys

Measurements of gyromagnetic ratios were made on 4 nickel-iron alloys obtained from the General Electric Company. The alloys were deoxidized with 0.2% silicon and 0.2% aluminum. The percentages listed are "by addition."

1. 15% Nickel-Iron Rotor. Quadrature torques were so small with this rotor the quadrature coil was not used.

TABLE XIII

15% Nickel-Iron Rotor
Double Amplitude 3.1-3.4 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Aug. 30	nt	1	1.079	1.018	1.045	0.987	1.032
Sept. 1	nt	1	1.090	1.022	1.003	1.036	1.038
	nt	1	1.061	1.061	1.015	1.031	1.042

The mean of these observations is $\rho\frac{e}{m} = 1.037 \pm 0.004$

2. 40% Nickel-Iron Rotor. At small amplitudes, the band of light on the scale was crossed by faint lines, indicating small effects from magnetostriction were present. Quadrature torques were very small and the quadrature coil not used.

TABLE XIV

40% Nickel-Iron Rotor
Double Amplitude

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho\frac{e}{m}$
1950							
Sept. 8	nt	1	1.020	1.074	1.060	0.993	1.037
	nt	1	1.081	1.015	1.065	0.987	1.039
	nt	1	1.076	1.022	1.086	0.976	1.042

The mean of these observations gives $\rho\frac{e}{m} = 1.039 \pm 0.002$

3. 65% Nickel-Iron Rotor. One measurement was rejected because of very unstable conditions. Two good measurements were obtained. The rotor was very nearly symmetrical. The quadrature coil was used.

TABLE XV

65% Nickel-Iron Rotor
Double Amplitude 2.4-3.2 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
1950							
Sept. 14	nt	1	0.921	1.209	1.041	1.045	1.054
15	nt	1	1.004	1.116	1.003	1.078	1.050

The mean of these observations is $\rho \frac{e}{m} = 1.052 \pm 0.002$

4. 90% Nickel-Iron Rotor. The earth's magnetic field was not very steady during this series of measurements. The rotor was very true mechanically. The quadrature coil was used.

TABLE XVI

90% Nickel-Iron Rotor
Double Amplitude 2.3-3.1 cm

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho \frac{e}{m}$
1950							
Sept. 2	1	nt	1.019	1.066	0.901	1.143	1.032
4	1	nt	0.953	1.128	0.877	1.152	1.028
4	1	nt	0.991	1.160	0.893	1.158	1.051
5	1	nt	0.993	1.176	0.936	1.199	1.064
7	1	nt	0.974	1.139	0.943	1.186	1.060

The mean of these observations is $\rho \frac{e}{m} = 1.047 \pm 0.018$

D. Iron Rotors

Gyromagnetic ratios were measured for two electrolytic iron rotors. The material for one was obtained from Westinghouse, the other was obtained from the Bell Telephone Laboratories.

1. Westinghouse rotor. This rotor broke in the middle during construction and was welded together again. The quadrature coil was used.

TABLE XVI

Westinghouse Electrolytic Iron Rotor
Double Amplitude 3.0-3.9 cm

			NPE		NPW		
Date	T	Obs	I	II	I	II	$\rho \frac{e}{m}$
1950							
July 5	eve	1	1.031	1.029	1.028	1.047	1.034
6	eve	1	1.059	1.062	1.019	1.048	1.047
7	nt	1	1.072	1.077	1.009	1.034	1.048
11	nt	1	1.082	1.064	1.018	1.025	1.047

The mean of these observations is $\rho \frac{e}{m} = 1.044 \pm 0.005$

2. Bell Telephone Rotor. Quadrature effects were small. The quadrature coil was used.

TABLE XVII

Bell-Telephone Electrolytic Iron Rotor

			NPE		NPW		
Date	T	Obs	I	II	I	II	$\rho \frac{e}{m}$
1950							
July 12	nt	$\frac{1}{2}$	1.034	1.077	1.006	1.038	1.039
15	nt	1	1.021	1.074	1.004	1.045	1.036
20	nt	1	0.978	1.046	1.034	1.101	1.039

The mean of these observations is 1.038 ± 0.001

The mean value for electrolytic iron is 1.041 ± 0.003

Conclusion

Figures 13, 14 and 15 show the gyromagnetic ratio plotted against the concentration for each of these series of alloys. The solid circles represent the values obtained in the present investigation, the open circles, values obtained here in the course of earlier measurements by the Einstein-deHaas effect; those for pure iron, pure cobalt and permalloy (nearly identical with nickel) being considered particularly precise, the older values for iron, cobalt and nickel were taken as standards, and the dotted line is drawn connecting the end points. The most important characteristic of each of these curves is the general increase which it shows in the value of $\rho \frac{e}{m}$ with increasing concentration of the element which has the larger gyromagnetic ratio. Although the scatter is fairly large, the roughly linear relation between the gyromagnetic ratio and the concentration is unmistakable.

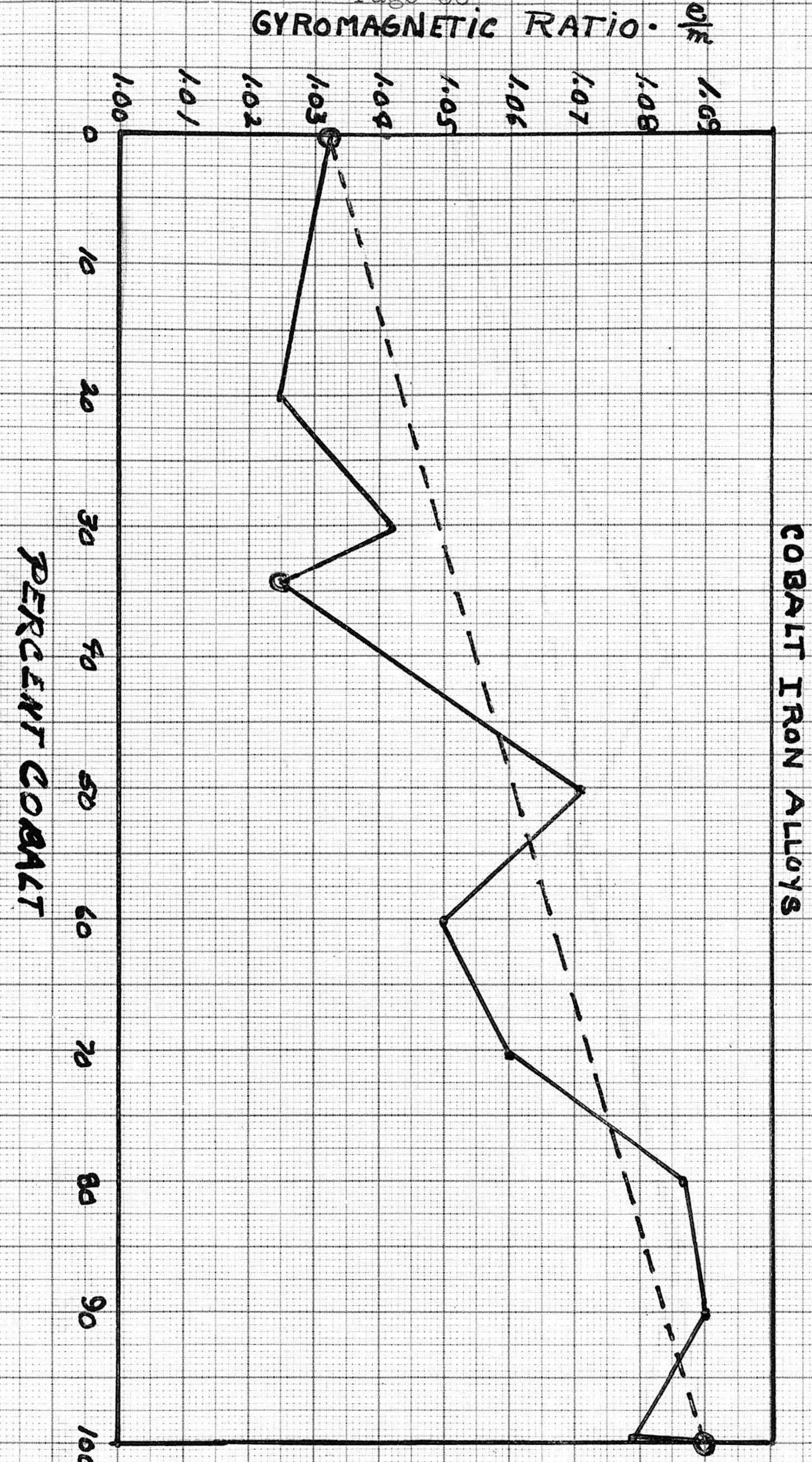
GYROMAGNETIC RATIO · e/m
1.00
1.01
1.02
1.03
1.04
1.05
1.06
1.07
1.08
1.09
0
10
20
30
40
50
60
70
80
90
100
PERCENT COBALT
COBALT IRON ALLOYS

Figure 13

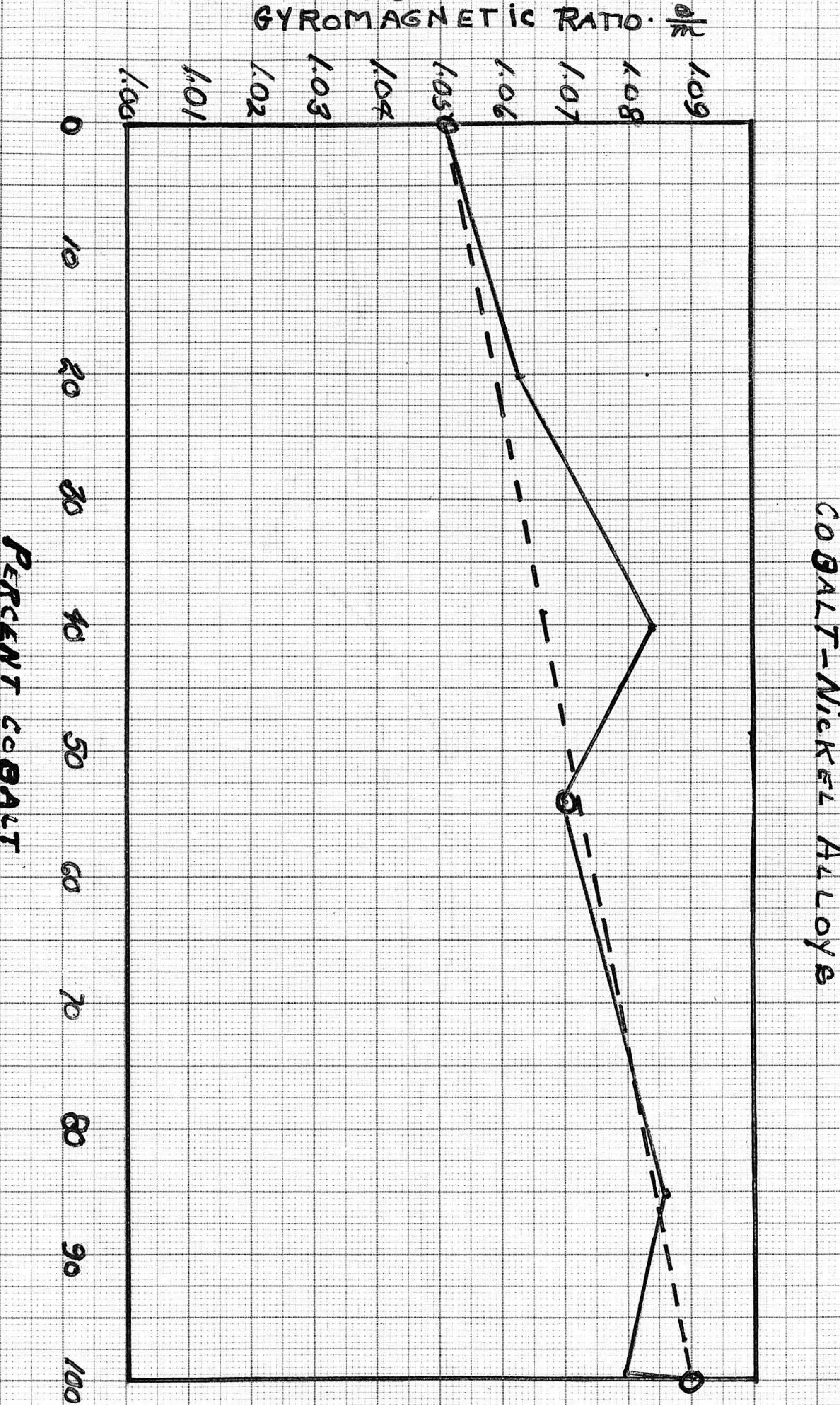
GYROMAGNETIC RATIO · e/m
1.00
1.01
1.02
1.03
1.04
1.05
1.06
1.07
1.08
1.09
0
10
20
30
40
50
60
70
80
90
100
PERCENT COBALT
COBALT-NICKEL ALLOYS

Figure 14

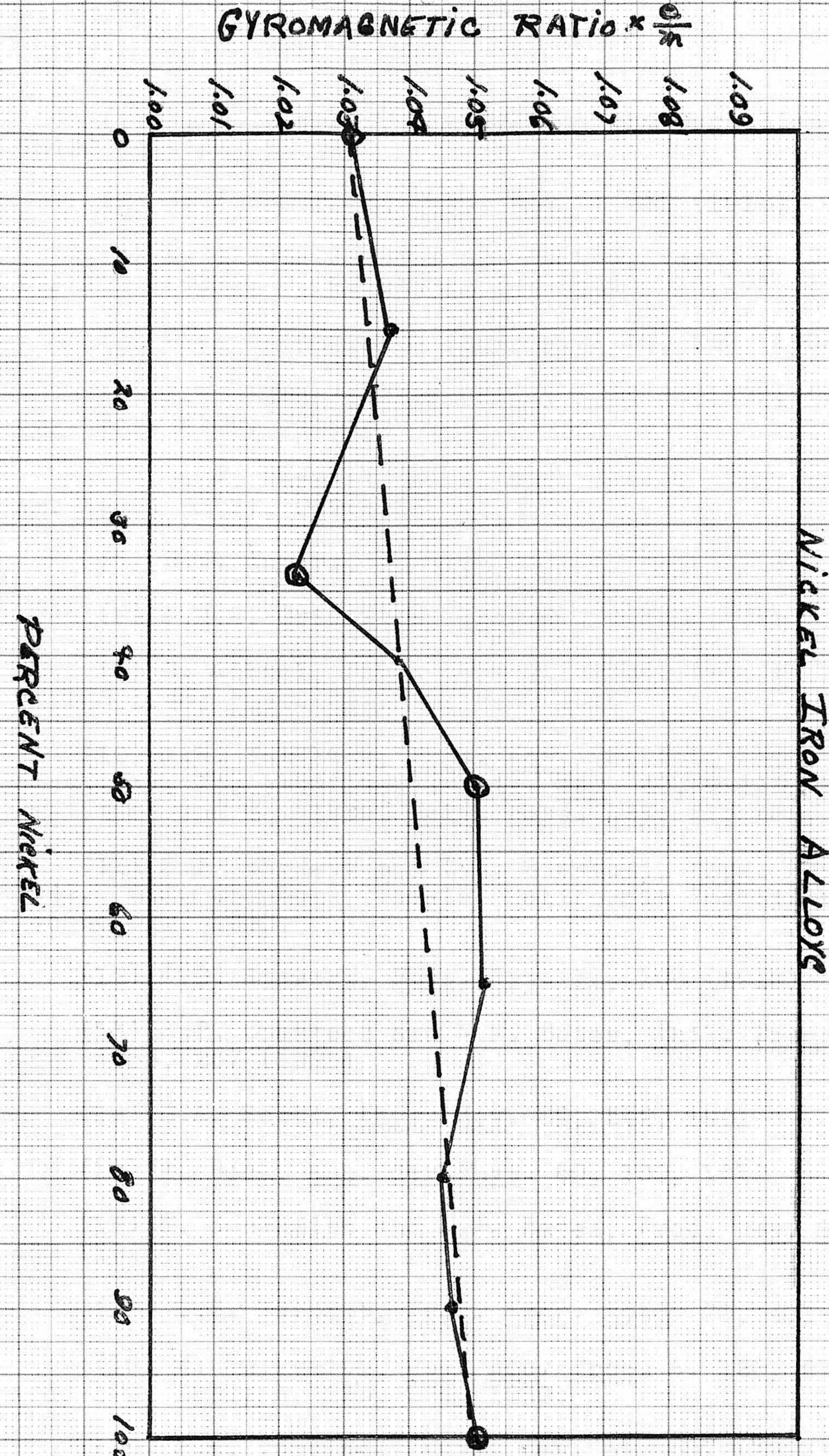
GYROMAGNETIC RATIO × e/m
1.00
1.01
1.02
1.03
1.04
1.05
1.06
1.07
1.08
1.09
NICKEL IRON ALLOYS
PERCENT NICKEL
0
10
20
30
40
50
60
70
80
90
100

Figure 15

References

(1) J. C. Maxwell, Electricity and Magnetism, Vol. II, Third Edition, p. 218

(2) S. J. Barnett, Phys. Rev. 6,239 (1915), 6,171,1915

(3) A. Einstein and W. J. deHaas, Verh. d. D. Phys. Ges., 17, 152 (1915); A. Einstein, loc. cit., 18,173 (1916) ---

(4) O.W. Richardson, Phys. Rev. 26, 248 (1908)

(5) J.W. Fisher, Proc. Phys. Soc. 34,177(1922); Proc. Roy. Soc. A109, 7(1925)

(6) S. J. Barnett, Phys. Rev. 27,115(1926) and Proc. Am. Acad. 68(7), 229(1933)

(7) O.V. Auwers, Naturwissen. 23,202(1935)

(8) J.H.E. Griffiths, Nature 158,670(1946)

(9) C. Kittel, Phys. Rev. 71, 270(1947), Phys. Rev. 73,155,1948; Phys. Rev. 76, 743, 1949

(10) J.H. VanVleck, Phys. Rev. 78, 266, 1950; D. Polder, Phys. Rev. 73, 1116(1948); Phil. Mag. 40,99-115, 1949; J.M. Luttinger and C. Kittel Helv. Phys. Acts 21, 480, 1948

(11) S.J. Barnett and L.A. Giamboni, Phys. Rev. 76,1542, 1949. L.A. Giamboni, Thesis, California Institute of Technology, 1950.

(12) J.G. Stewart, Phys. Rev. 11, 100 (1918)

(13) A.P. Chattock and L.F. Bates, Phil. Trans. Roy. Soc. A233, 257 (1923)

(14) G.A. Scott, Phys. Rev. 82, 542, 1951

(15) E. Beck, Ann. der Phys. 60, 109, 1919

(16) W. Sucksmith and L.F. Bates, Proc. Roy. Soc., 104A, 499 (1923)

(17) S.J. Barnett, Proc. Amer. Acad. 73, 401-455, 1940

(18) W. Sucksmith, Roy. Soc. Proc. A. 128, 276, 1930, A, 133, 179, 1931; A, 135, 276, 1932.

(19) J.H. VanVleck, Phys. Rev. 31, 587 (1928); Van Vleck and Frank, Phys. Rev. er, 1494, 1625 (1929). See also J.H. Van Vleck "Electric and Magnetic Susceptibilities", p.255,1932.

(20) E.C. Stoner, Phil. Mag. 8,250(1929

(21) F. Coeterier and P. Scherrer: Helv. Phys. Acta 5,217 (1932); F. Coeterier, Helv. Phys. Acta 6,483 (1933

(22) F. Galavics, Helv. Phys. Acta, 12, 581-608(1949)

(23) A. Meyer, Thesis, University of Strasbourg 1950

(24) S.J. Barnett, Proc. Amer. Acad. 66, 273-348, 1931; Proc. Amer. Acad. 69, 119-135, 1934; Rev. Mod. Phys. 7, 129-166, 1935; Proc. Amer. Acad., 73, 401-455, 1940.

(25) Bozorth, "Ferromagnetism", D. Van Nostrand Co., p. 769, 1951

(26) R. Muller, Ann. d. Physik, 1, 613 (1929

(27) H.E.R. Becker, Zeit. f. Techn. Physik, 21, 195, 1940

(28) Sheldon J. Brown, Thesis, University of California at Los Angeles, 1951.