A MEASUREMENT OF THE VALUE OF h/e BY THE DETERMINATION OF THE SHORT WAVELENGTH LIMIT OF THE CONTINUOUS X-RAY SPECTRUM AT 25 KILOVOLTS

Thesis by

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ABSTRACT

The apparatus used in the experiment is described in detail and the calibration of the instruments is discussed. The procedure followed in the experiment is described. Experimental results are presented in the form of curves and the analysis of the curves is given in detail. The final value of the ratio h/e obtained in this experiment is

$$\frac{h}{e} = 1.37912 \pm 0.00007 \times 10^{-17} \text{ erg-sec/esu}$$

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I. BACKGROUND OF THE EXPERIMENT

Measurement of the short wavelength limit of the continuous X-ray spectrum provides one of several methods for determining the natural constant h/e. The small group of h/e experiments in turn is a member of a somewhat larger group of experiments which together determine, and in fact overdetermine, the so-called fundamental atomic constants. These constants have been variously taken as 1) the charge on the electron e, the mass of the electron m, and Planck's constant of action h and 2) the Faraday F, the Avogadro number N, the mass of the electron m, and Planck's constant h.(1)

It is to be noted that most of the experiments on the atomic constants determine numerical values for functions of the fundamental constants rather than for the constants themselves. If one makes a least squares fitting of all of the experimentally determined quantities, one may solve for any constant or any function of these constants and obtain the same results without regard to the choice of explicit fundamental constants used in making the least squares adjustment. The complex of experiments can in fact be used to determine an ellipsoid of error. Clearly the characteristics of this ellipsoid can be specified with equal validity by alternative sets of axes.

The importance of the h/e (or hN/F) experiment lies in the significant reduction it can effect in the size of the above-mentioned ellipsoid of error. In consequence of this reduction our knowledge of those constants which are less accessible than h/e to physical measurement can be improved.

As mentioned above, the method employing the continuous X-ray spectrum is but one of several methods of measuring h/e. There are two basically different methods available. One consists of a measurement of the second radiation constant

$$c_2 = \frac{hc}{k} = \frac{Fc^2}{R} \cdot \frac{h}{e} \qquad (1)$$

where the auxiliary constants c and R are respectively the velocity of light and the gas constant. The radiation constant appears in the Planck radiation intensity equation and in the Wien displacement law. The other basic method is founded on the Einstein photoelectric equation

$$1/2mv^2 = h\nu - W \qquad (2)$$

where W is the work function of the photoelectric emitter, $1/2mv^2$ is the maximum kinetic energy of the emitted photoelectrons, ν is the frequency of the incident radiation and h is Planck's constant of action. The method of the continuous spectrum is, of course, a case of the inverse photoelectric type. Furthermore this X-ray method is to date the most accurate method for the direct measurement of h/e.

A partial list of previous experiments on h/e with their results is given below:

1. Duane, Palmer and Yeh(2) (1921)

   1.3749×10^{-17} (re-computed by DuMond(3))

2. Feder(4) (1929)

   1.3759×10^{-17} (re-computed by DuMond(3))

3. P. Kirkpatrick and Ross(5) (1934)

   1.3754×10^{-17} (re-computed by DuMond(3))

4. Schaitberger[6] (1935)

   1.3775×10^{-17} (re-computed by DuMond[3])

5. DuMond and Bollman[7] (1937)

   1.3765×10^{-17} (re-computed by DuMond[3])

6. Ohlin[8] (1939)

   1.3800×10^{-17} (corrected for cathode work function by Birge)

7. Bearden and Schwarz[9] (1941)

   1.3775×10^{-17} (mean value of copper and tungsten values)

8. Panofsky, Green and DuMond[10] (1942)

   1.3786×10^{-17}

9. Bearden and Schwarz[11] (1950)

   1.37938×10^{-17} (mean value for copper, nickel, molybdenum, tantalum, tungsten and gold)

10. Bearden, Johnson and Watts[12] (1950)

   1.37928×10^{-17} (computed by the authors with $c = 2.997898 \times 10^{10}$ cm/sec instead of $c = 2.99776 \times 10^{10}$ cm/sec)

To this list should be appended the value 1.37926×10^{-17} erg sec./e.s.u. calculated in 1947 by Dumond and Cohen[1] by least squares analysis of eleven of the most accurate experiments on functions of the fundamental atomic constants. In all cases but (10) the values for h/e have been computed with

$$\frac{\lambda g}{\lambda s} = 1.002030 \times 10^{-11} \text{ cm/x.u.}$$

and

$$c = 2.99776 \times 10^{10} \text{ cm/sec}$$

In the present experiment, new and more precise values for both auxiliary constants have been used.

II. APPARATUS

The main elements of the apparatus are the X-ray tube, the power supply, the voltage measuring equipment, the monochromater, and the detection and counting equipment. A block diagram of the complete system is shown in Fig. 1.

A. The X-ray Tube. The X-ray tube used in the experiment is commercially manufactured by the Eureka X-ray Tube Corporation of Chicago. It was intended primarily for industrial radiography and has a modest rating of 150 kvp on a half-wave rectified supply. The tube is of the "shockproof" type. That is, it is mounted in an oil-filled housing, and the electrodes are reached by means of co-axial cables which enter the housing through large, well-insulated bushings. The housing is equipped with a water jacket which permits continuous cooling of the oil in which the X-ray tube is immersed. The tungsten target is inlaid in a massive copper block; the filament is also of tungsten and is in the form of a coil one centimeter in length.

The X-ray tube is in fact rather ordinary compared with the tubes used in most of the previous X-ray measurements of h/e, and therein lie both its advantages and disadvantages. Among the advantages should be mentioned the compactness and general convenience of the "shockproof" type, freedom from the burden of a vacuum pumping system and, most important of all, cleanliness of the target surface. On the other hand, the tube has very definite power limitations - no rotating target -, the target material cannot be changed,

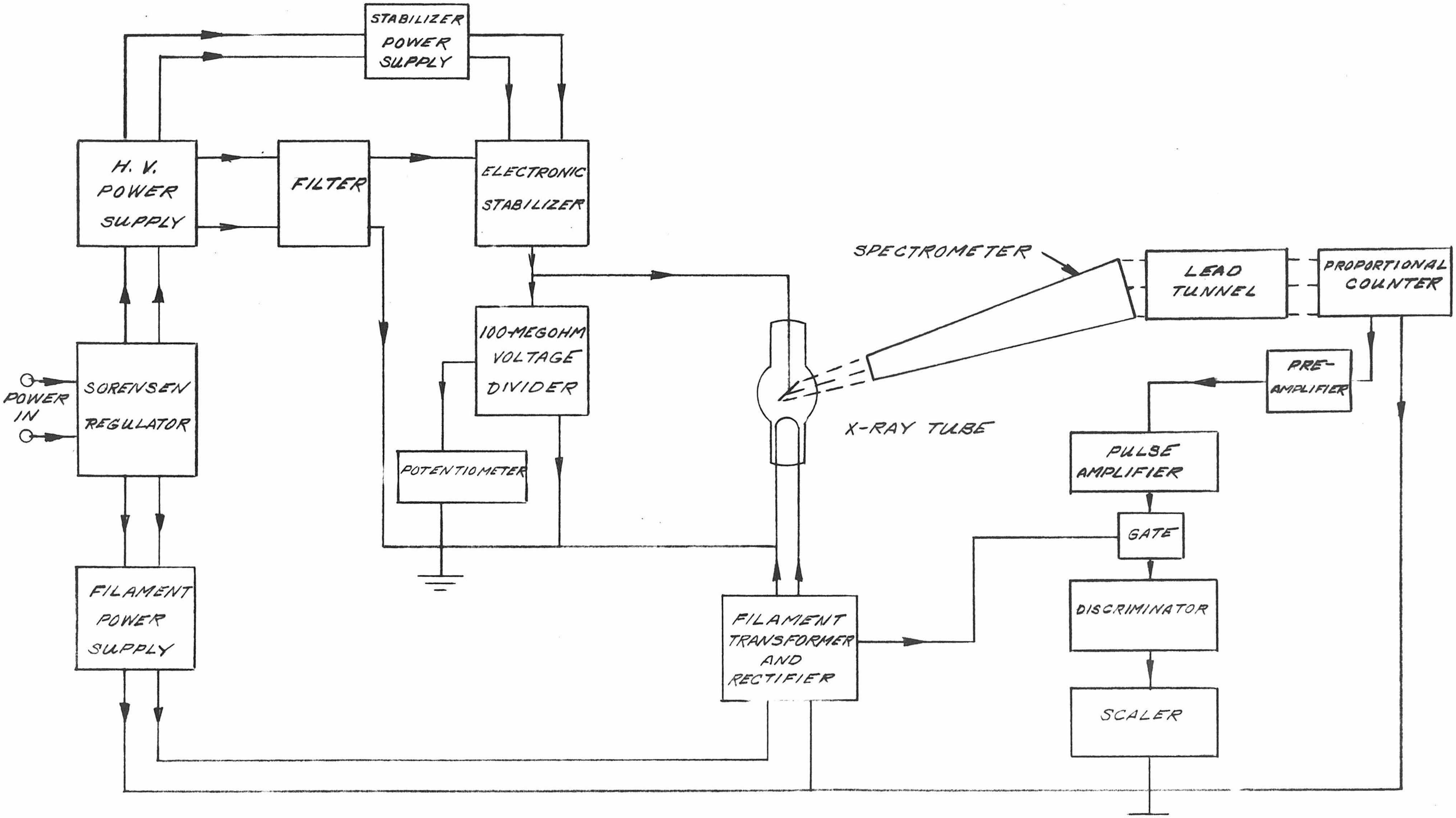


FIG. 1

APPARATUS BLOCK DIAGRAM

and the filament cannot be center-tapped. Though the tube is easy to work with, compact, portable, insulated against the high voltage, it is nonetheless not a versatile tube from the experimental point of view.

Of the disadvantages mentioned above, the power limitation is certainly the most important, since the h/e experiment is concerned with the X-rays emitted at the very edge of the continuous spectrum, at the point of theoretically zero intensity. However, the high luminosity of the gamma-ray spectrometer used in the measurement helped to offset this severe disadvantage. Furthermore, the use of any but the "shockproof" type tube with the spectrometer would be at best exceedingly difficult. The suitability of the tube for use with the spectrometer, the absence of the need for continuous pumping, and the certainty of cleanliness of the target in the hard vacuum of a sealed tube made the sacrifice of power and the sacrifice of interchangeability of targets seem both necessary and desirable.

B. The Power Supply. Since voltage is one of the two quantities measured in the experiment, it is necessary to have the greatest possible stability in the accelerating voltage applied to the X-ray tube. The elements composing the stabilized power supply are the primary voltage regulator, the main transformer and rectifiers, the filter, the electronic stabilizer, and the filament supply. These elements will be taken up in turn in the following sections.

(1). Voltage Regulator

The first stage in the voltage stabilization consisted of a 2 kva Sorensen electronic voltage regulator, Model 2000-2S, which regulates a nominal line voltage of 220 volts to 0.1% for large variations in both line voltage and load. The abilities of the Sorensen unit were not heavily taxed by our use. We imposed practically no change in load and our line voltage varied at most by about 5%. Nonetheless our success in achieving very high stability would have been considerably diminished without the help of this primary regulator. The beneficial effects of its use were particularly noticeable in the case of the X-ray tube current which, owing to the exponential dependence of emission on temperature, is extremely sensitive to very small changes in filament power. Of itself, variation in tube current, while undesirable because of the direct proportionality of X-ray emission to tube current, would not be crippling. Owing to the series resistance between the rectifiers and the X-ray tube however, variations in tube current appear also as variations in voltage applied to the tube. These variations in applied voltage can be partially bucked out by means of the electronic stabilizer, but the operation of the system is greatly enhanced by good regulation of the filament power. Both the high voltage and the filament power were taken through the Sorensen regulator.

The principle of operation of the Sorensen regulator can be mentioned briefly. The main elements consist of an autotransformer in series with a saturable core reactor across the regulator input. A part of the output voltage is fed through a transformer to the heater of a special diode located in one arm of a resistance bridge. The resistance of the diode is very sensitive to heater voltage. Any unbalance voltage in the bridge is applied to the grid of a pentode whose plate current flows through the secondary winding of the saturable reactor. A part of the input impedance is therefore controlled by the output voltage, and in this way the regulation is achieved.

(2). Main Transformer and Rectifiers

The main transformer and rectifiers are contained in a commercial unit manufactured by the North American Philips Company. The unit was intended to be operated, when used in an orthodox manner, from a control panel mounted in a handsome console. All of the necessary controls are mounted on this panel provided that the unit is used as intended. Our experiment required a few alterations.

The first alteration involved the rewiring of the rectifiers in order to provide a full-wave rectified output. For this modification it was necessary to pull the entire unit out of its tank of oil, cut the rectifier leads and insert banana plugs and banana couplings. It is now possible to convert the output in a few minutes from 180 kvp half-wave rectified (normal operation) to either positive or negative 75 kvp full-wave merely by reconnecting the banana plugs, which are accessible through ports on the top of the tank, and by placing a brass jumper bar between the corona cups atop the feed-through insulators. Fortunately the unit was designed for double-ended operation of an X-ray tube (both cathode and anode remote from ground potential), and both filament windings for the rectifier tubes are thoroughly insulated from the tank as well as from each other.

One side of the secondary winding of the X-ray tube filament transformer in the Philips set was permanently connected internally to one of the two leads through the cathode feed-through bushing. This connection was not disturbed. Our original intention was to disconnect the primary of this filament transformer, short out the secondary, and merely allow the secondary winding to ride at the high voltage. With our tube operating with grounded cathode, we could see no use for this built-in filament transformer. We later put it to a singular use which will be discussed in a later section.

A second alteration of the Philips set was not remarkable but proved to be a great convenience. The primary voltage for the high voltage transformer is taken from a large Variac. The high voltage can therefore be adjusted continuously over its entire range provided one does not regard a jump of several hundred volts as a discontinuity. In order to improve what we regarded as an awkward situation, we broke the lead from the Variac to the primary winding of the high voltage transformer and inserted the 6.3 volt winding of a filament transformer whose primary was supplied through another small Variac. We thereby achieved a very fine control of the voltage applied to the high voltage transformer. Through the use of this fine adjustment we could easily set the high voltage at the proper point for optimum operation of the electronic stabilizer.

(3). The High Voltage Filter

A filter was inserted between the high voltage output from the rectifiers and the electronic stabilizer. The filter consists essentially of three 0.25μf condensers separated by two 1000h chokes. Three resistors of large power capacity were later added to the filter in order to provide protection to the X-ray tube in the event of a breakdown. Two of these resistors were placed in series with the first two filter condensers respectively. The third was inserted between the output filter condenser and the input to the electronic stabilizer. The smoothing action of the filter for the 120-cycle ripple was thereby decreased approximately by a factor two (to 0.1 volt peak to peak at 25 ma), but sad experience with four successive X-ray tubes forced us to make this sacrifice.

The 0.25uf condensers were formed by combining four 0.25uf 50 kv DC Cornell Dubilier condensers in series-parallel to give 0.25uf at a rating of 100 kv DC. The two 1000h chokes are immersed in oil and are mounted on stands of well-seasoned wood. The condensers and chokes were equipment originally used in the Watters Memorial Generator[13] formerly in use in this laboratory.

The elements of the filter were connected by 3/8-inch flexible copper tubing equipped with special fittings to reduce corona losses to a minimum. Furthermore, all elements save the three

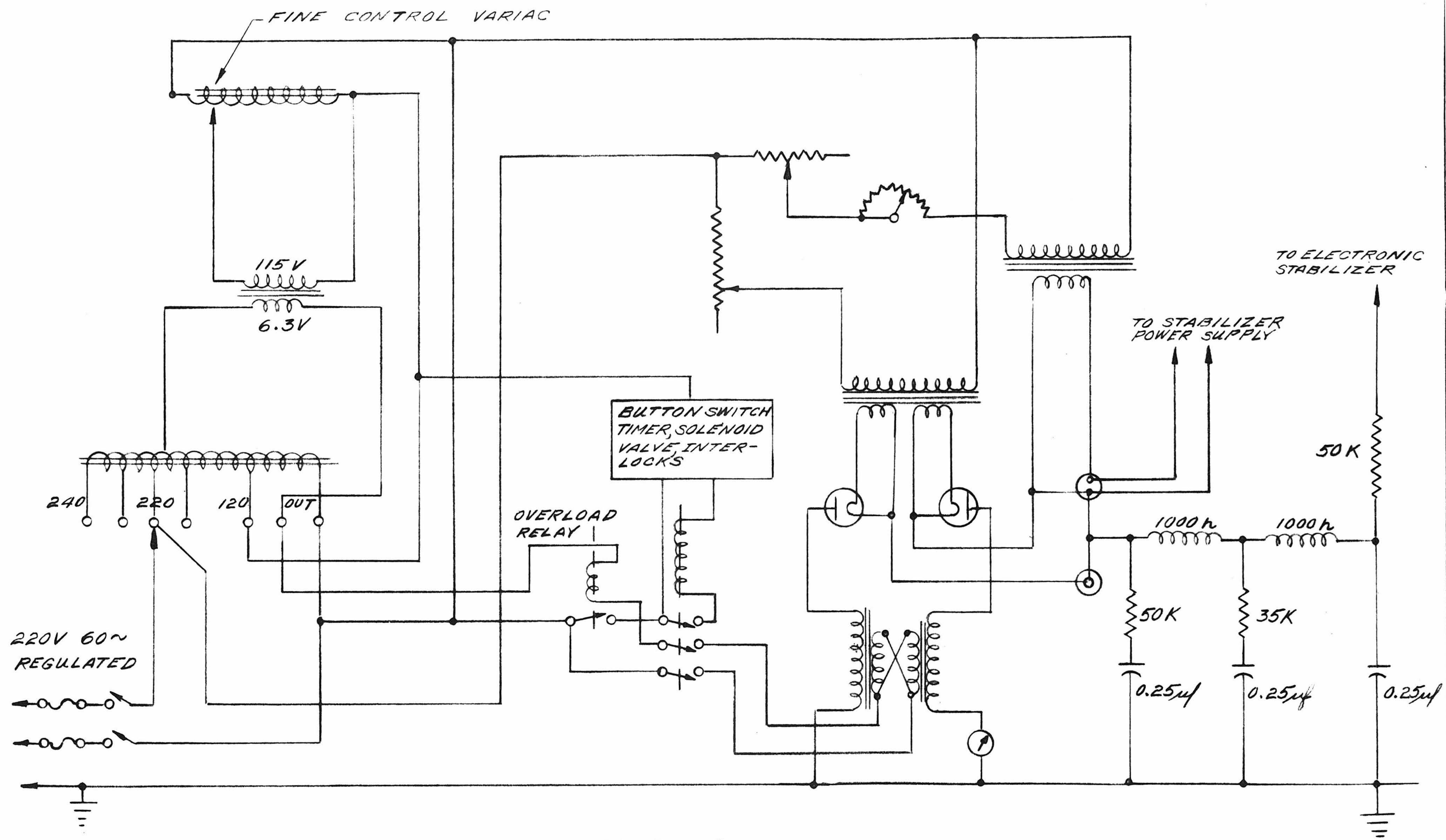


FIG. 2

H.V. POWER SUPPLY & FILTER

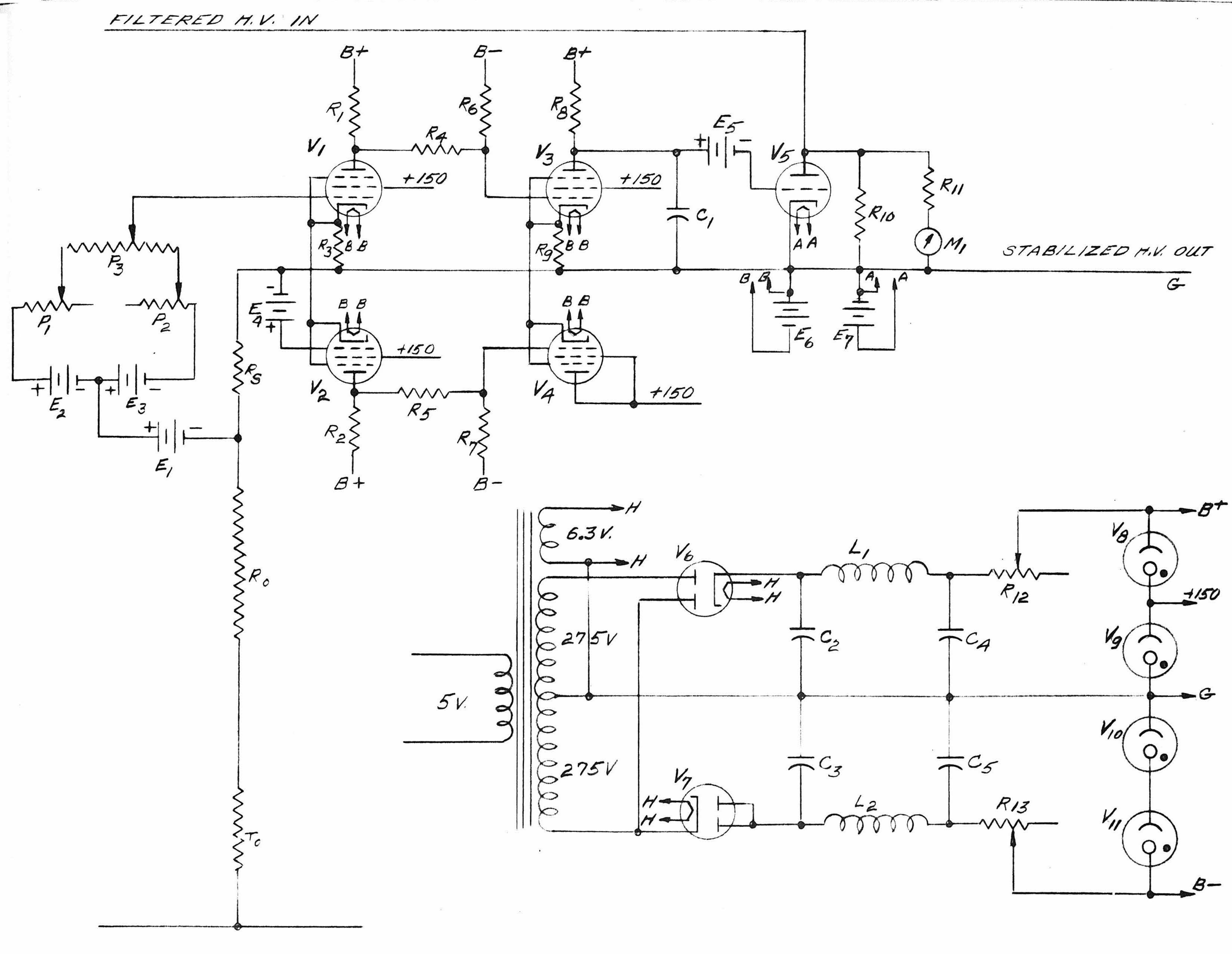
three resistors were protected by sphere gaps to prevent damage in the event of overload. The high voltage power supply and filter circuits are shown in Fig. 2.

(4). The Electronic Stabilizer

An electronic stabilizer was placed between the 50,000-ohm series resistor at the output end of the high voltage filter and the load. The load consisted of the 100-megohm voltage divider in parallel with either (1) the precision megohm divider or (2) the X-ray tube depending on whether the object was to check the divider ratio or to measure the continuous spectrum limit.

The stabilizer (Fig. 3) is a three-stage direct current negative feedback amplifier whose output stage is in series with the load. The basic design resembles that of Panofsky[10]. The control signal is developed across 100,000 ohms (R_s) at the high end of the 100-megohm divider (R_o and r_o). The signal is amplified primarily in the first two stages. This amplified signal is then applied to the grid of the series control tube (V_5).

Certain rather special features of the stabilizer will bear more extensive discussion. First, though battery supplies are generally considered more reliable for operation of a high gain DC amplifier, we found that for our purposes the battery drain was too great to be satisfactorily supplied by a sensible number of batteries. We therefore decided to provide the amplifier with a regulated power supply of the ordinary kind, at least for the plate and screen voltages. Rather than acquire an additional



T_o	4444.4Ω OIL IMMERSED	
R_o	100 MEG OIL IMMERSED	
R_s	100K WW RITEOHM	
R_1	150K WELWYN PRECISION	
R_2	"	" "
R_3	25K	" "
R_4	250K	" "
R_5	"	" "
R_6	1 MEG	" "
R_7	"	" "
R_8	250K	" "
R_9	25K	" "
R_{10}	125K 125W IRC	
R_{11}	10 MEG	
R_{12}	10 K	
R_{13}	25K	
P_1	25K MALLORY WW1	
P_2	" " "	
P_3	5K HELIPOT (10 TURN)	
C_1	0.1 μf	
C_2, C_3, C_4, C_5	4 μf	
L_1, L_2	10 h	
V_1, V_2	6SJ7	
V_3 V_4	6AC7	
V_5	EIMAC 3C24	
V_6	6X5	
V_7	6ZY5	
V_8, V_{11}	VR105	
V_9, V_{12}	VR150	
E_1	67.5V	
E_2 E_3	6V (DRY CELL)	
E_4	45V	
E_5	215V	
E_6, E_7	6V (STORAGE)	
M_1	300 μA	

FIG 3
ELECTRONIC STABILIZER

transformer rated to withstand 30 kv between two 110-volt windings, we decided to take our primary power from the unused filament transformer in the Philips set. This transformer, it will be recalled, had one end of its secondary winding already tied to the high voltage at the input end of the filter. Since the transformer was capable of supplying the necessary power (about 20 watts) at 5 volts, there remained only the problem of finding a power transformer whose windings could withstand the drop across the stabilizer and filter (up to 2500 volts). A standard power transformer proved adequate. Its 5-volt winding was used as a primary, its 6.3-volt and 300-volt windings as secondary. Two condenser input filtered power supplies were made, one half-wave with an output of -255 volts and the other full-wave with an output of +255 and +150 volts. The two rectifier tubes were heated through the 6.3-volt winding.

Since the amplifier power supply was regulated by VR tubes, whose performance is not notably precise, we felt that the amplifier should be balanced in such a way that spurious signals from the power supply should have the least possible effect. The desired result was accomplished by using two carefully matched tubes instead of a single tube for each of the first two stages and by tightly coupling the cathodes of the two tubes in each stage*. For a legitimate signal the amplifier behaves almost as if the balancing tubes (V_2 and V_4) were not present and there were no

*This modification was suggested by Mr. James E. Kohl of this laboratory.

cathode resistors. On the other hand, if a spurious signal arises from the power supply it will be applied symmetrically to the grids of the second stage, and, owing to the large cathode resistor, will be amplified very little.

The pentode heaters and the triode filament were supplied by two large storage batteries (E_6 and E_7) of approximately 200 ampere-hour capacity. Since the longest continuous demand on these batteries was only about twenty hours at about three amperes apiece, they did not need trickle charging during the runs on the experiment but could be charged between runs. The additional leakage due to the presence of a small motor-generator set could therefore be eliminated. No trouble from drifting heater voltage was encountered during the experiment.

Another comment about the stabilizer concerns its inherent stability. Stability in a high gain DC amplifier is not easy to come by, and our case was no exception. We found that so long as we attempted to retain a wide frequency band the stabilizer would oscillate spontaneously. We therefore sacrificed the wide band and eliminated the oscillations by placing a 0.luf condenser (C_1) from the plate of the second amplifier tube (V_3) to the stabilizer ground. The performance of the stabilizer was much improved by this addition; the high frequency response proved unnecessary after visual examination of the stabilizer output on an oscilloscope trace.

The complete stabilizer, including the regulated power supply and the storage batteries, was housed in an aluminum-paneled box which afforded both electric and thermal shielding. The box itself was well insulated from ground. The output end of the high voltage filter was connected to the plate of the bucking triode (V_5) through a porcelain feed-through insulator on the top of the stabilizer box. The 100,000-ohm signal resistor R_s was also mounted in the stabilizer box, next to the first pentode (V_1), and the connection between this resistor and the 100-megohm divider proper was made through a second feed-through insulator. The body of the box itself (stabilizer ground) was connected to a copper mercury cup mounted on the 100-megohm resistor rack for a reason which will be mentioned in a later section.

Proper operation of the stabilizer depends upon the setting of the grid bias of the input tube. The stabilizer was equipped with a range selector switch permitting operation between 20 and 30 kv and, though this second range was not used, between 62 and 72 kv. Since the normal cathode potential (above stabilizer ground) of the first stage was about 50 volts and the normal drop in the 100,000-ohm signal resistor, for the 25 kv range, was about 25 volts, provision was made to have the 25 kv position on the selector switch insert a 67.5-volt bucking battery (E_1). No current flowed in this battery under operating conditions. Its positive terminal was connected to the midpoint of a 12-volt battery (eight 1.5-volt flashlight batteries) across which there was a constant resistance of 30,000 ohms composed of two ganged 25,000-ohm Mallory controls

(P_1 and P_2) whose sliders were connected to the fixed ends of a 5,000-ohm ten-turn Helipot (P_3). The grid of the input tube was connected to the Helipot slider. Since the Helipot could be set to any 2-volt range in the twelve, the arrangement described above permitted a very delicate setting of the bias of the input grid over about a 12-volt range. All switches and controls, except for the filament power switches, could be operated by means of two removable lucite rods 42 inches long. The ends of these rods touched by the operator were permanently grounded.

Once the various elements of the experimental apparatus had reached thermal equilibrium, the electronic stabilizer could maintain a preset voltage, for several minutes at a time, within limits of 5 parts per million and even under rather poor conditions could hold a part in one hundred thousand. With a moderate amount of tending we were able to maintain each of the voltage settings in the experiment, stretching over a period of about twenty hours for the longest runs, to five parts in a million.

The amplifier gain without feedback for DC and for audio frequencies was measured at approximately 5×10^5. Since the input signal to the amplifier was attenuated in the ratio of 100,000 ohms to 100 megohms, the overall gain of the stabilizer was 500. Though the stabilizer would perform satisfactorily over rather wide limits (about $\pm$ 500 volts at the output of the high voltage filter), we found that its operation was optimum at a setting slightly above the midpoint of the range. In setting the high voltage we therefore took care that the drop across the stabilizer should have its proper value for the best operation. The small changes in voltage to one side or the other of the initial setting required by the exploration of the isochromat could be made by turning the Helipot control rod alone. The high voltage vernier Variac did not need to be reset during the runs except after bridge measurements to check the ratio of the 100-megohm divider.

(5). The Filament Supply

In view of the close connection between voltage stability and tube current fluctuations, the filament supply warrants discussion under this more general heading. The filament supply is shown in Fig. 4.

As mentioned above, the primary power source for the filament supply was the Sorensen regulator. The output of the Sorensen was first passed through a step-down transformer in order to give 110 volts RMS in place of 220. The primaries of three transformers and two Variacs were connected across the 110 volts. Two of the transformers had four 6.3-volt windings each. The third put out a secondary voltage of about 24 volts. The secondary voltages of the three transformers were connected in series, and each junction was tied to a contact on a selector switch. This switch had silver contacts of very low resistance. The Variac secondaries were connected to the primaries of a 12.5- and a 2.5-volt filament transformer respectively. The variable 12.5 and 2.5 volts were connected in series with the voltages on the selector switch. This arrangement provided nine coarse steps and

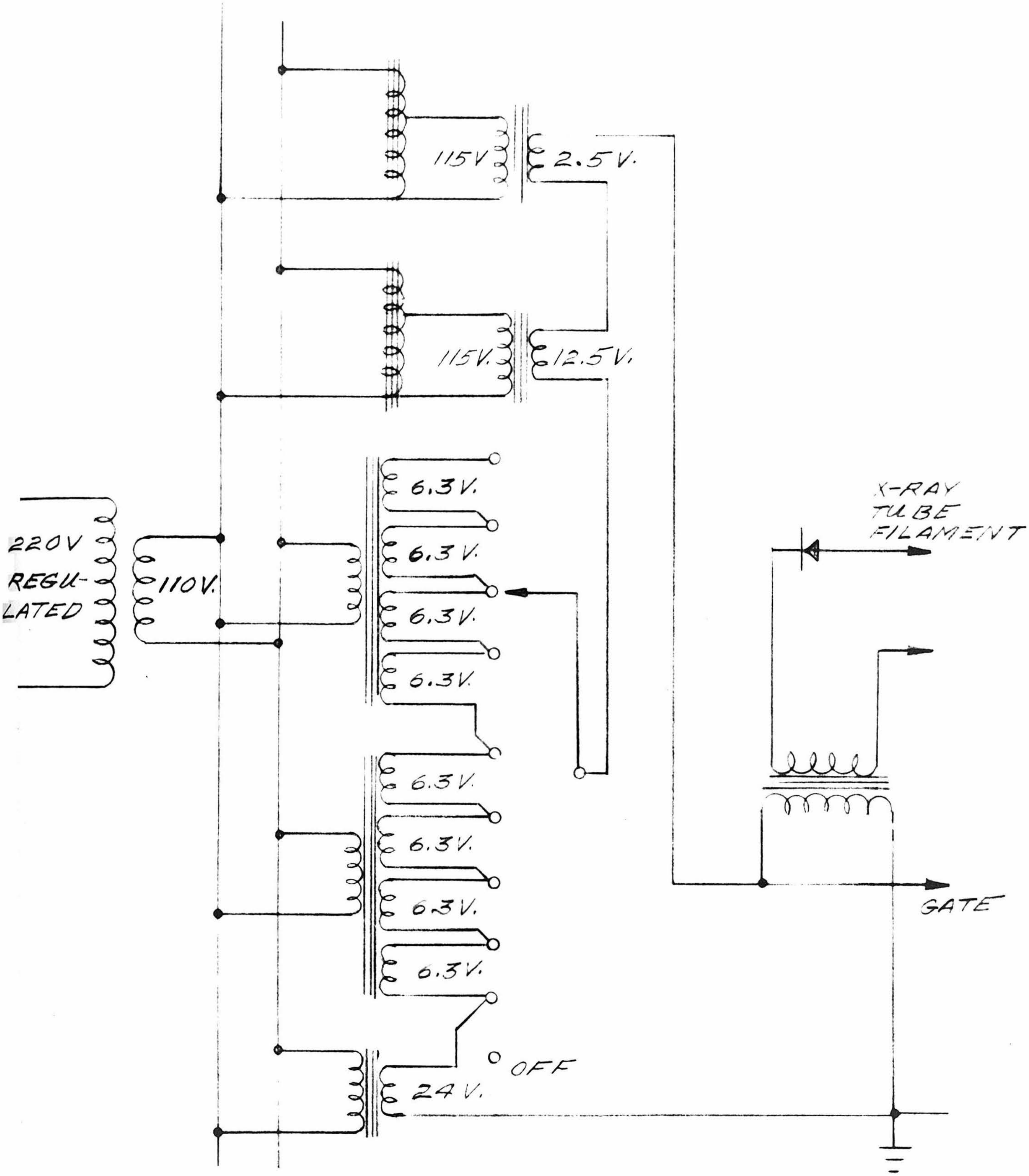


FIG. 4
FILAMENT SUPPLY

two continuously variable controls for the power supplied to the X-ray tube filament transformer. We found that a small isolation transformer between the filament transformer primary and the array of supply transformers was a desirable modification, since the voltage at the primary of the filament transformer was also used to activate the gating circuit on the differential discriminator. This gate will be described in section E below.

The filament transformer proper was of the standard type for X-ray tube operation. That is, its windings were insulated to withstand about 50kv, and the whole transformer was immersed in oil. This degree of insulation was, of course, not necessary for our work. One terminal of the primary was at ground potential, while one terminal of the secondary was grounded through the very low resistance of the tube current meter. The potential drop across this meter was about 0.03 volts for a tube current of 10 milliamperes; this voltage was in our case all that the filament transformer had to insulate.

Across the secondary of the filament transformer were connected the X-ray tube filament and, in series with the filament, a Selenium rectifier. The voltage applied to the filament was about 9 volts peak and was half-wave rectified. The filament was heated only during half a cycle. During the other half-cycle the filament, though still emitting electrons because of its slow thermal response, was essentially at ground potential over its entire length. The counting equipment was gated to record counts only during the intervals that the X-ray tube filament was not being supplied with power.

The power for the X-ray tube was piped from the supply in one room to the tube in an adjacent room. The high voltage and current return were Amphenol coaxial cables. The high voltage cable was RG 18/U armored cable with 0.188-inch diameter solid copper center conductor and 0.680-inch outside diameter polyethylene insulation. Though intended for radio frequency power transmission, this cable is reputed to withstand 150kvp without difficulty. The current return cable was RG 8/U. The filament power was transmitted through a shielded multi-conductor cable which carried in addition the two leads to the over-heating cut-out microswitch on the X-ray tube and the two leads to the solenoid valve in the tube water-cooling lines. All three cables were approximately fifty feet long and ran between the two rooms in a metal trough. Voltage drop in the H.V. and current return cables was negligible.

The high voltage transmission line was terminated in two 100kv stand-off insulators. Connections from these insulators were made by 3/8-inch flexible copper tubing. On the power supply end the copper tubing led to a copper mercury cup mounted on lucite on the 100-megohm resistor supporting rack. It will be recalled that the electronic stabilizer box was connected in similar fashion to another mercury cup on the resistor rack. These two cups, mounted ten inches apart and both well insulated from the resistor rack, were joined by a semi-circular link of 3/8-inch copper tubing. The X-ray tube could be disconnected simply by lifting this link out of the mercury cups.

The tube end of the cable was connected by copper tubing to a corona cup on one end of a polystyrene bar which supported the flexible "shockproof" tube cables. This polystyrene bar, 24 inches long and 1 1/2 inches in diameter, was itself supported by a lucite bracket fastened to a tall metal stand.

The power supply was satisfactory except in one respect. The electronic stabilizer was not able to buck out completely the fluctuations in tube voltage which appeared as a result of the half-wave rectified heating power supplied to the X-ray tube filament. Under working conditions a ripple of about 2 volts remained. Ripple does not, of course, affect the mean value of the high voltage, but it does reduce the sharpness of the quantum threshhold.

C. The Voltage Measuring Equipment. The elements of the voltage measuring equipment were the 100-megohm voltage divider, the precision megohm divider, the potentiometer and the galvanometer. The potentiometer and galvanometer were commercially manufactured instruments, whereas the two voltage dividers were built specifically for this experiment.

(1). The 100-Megohm Divider

The working divider, by means of which the voltage applied to the X ray tube could be continuously observed, consisted of

100 Shallcross 1-megohm Nichrome resistors connected in series with a 100,000-ohm Riteohm wire-wound resistor at the high voltage end (R_0 in Fig. 3), which provided the control signal for the electronic stabilizer, and with a specially wound Manganin low resistance (r_0 in Fig. 3) across which the voltage applied to the potentiometer was developed at the low end of the divider.

The 100 Shallcross resistors were mounted in six lucite tubes each 1 1/2 inches in inside diameter with 1/4-inch walls. These tubes were held in a horizontal position, one above the other, by bakelite brackets extending out from two wooden posts and gripping the lucite tubes 10 inches from their ends. The ends of the tubes were threaded and fitted with standard brass pipe caps (which proved to be porous; the condition was corrected by tinning the surfaces with solder). The backs of these caps were threaded to receive Imperial couplings for 3/8-inch flexible copper tubing which joined each tube with its neighbor above or below. This switchback mounting minimized the shunting effect of the oil which was pumped continuously through the entire system. The resistors were centered in the tubes by means of lucite spacers 1 1/2 inches in diameter and 1/2 inch thick. Five holes were drilled in each spacer. The center hole held a short piece of brass tubing threaded to engage the studs on the ends of the individual resistors. The remaining four holes permitted the oil to flow through the tubes. Three spaces at the low voltage end of the 100-megohm resistor were made of brass rather than lucite.

Electrical contact could be made to any of these three spacers in order to permit a reduction in the fixed value of the big resistor. These taps were not used in the experiment. Instead, all 100 resistors were used and a 100,000-ohm decade box (not shown in Fig. 3) was inserted, for bridge balancing, between the bottom Shallcross resistor and the low resistance member of the divider. Both the 100,000-ohm singnal resistor in the stabilizer box and the decade box were included in the nominal 100 megohms of the high resistance arm.

The oil was pumped through the lucite resistor tubes and through a heat exchanger by a small motor-driven gear pump backed by a reservoir with a 6-foot head. The oil flowed into the resistor tubes at the low voltage end and returned from the high end to the pump through a long lucite fall pipe.

The heat exchanger, originally built to be connected to a water faucet and drain, was converted into a closed circulation system. Water was circulated past the heat exchanger coils and past a thermoregulator and heater by means of a small bilge pump driven by the oil pump motor. The mercury regulator controlled the water temperature within 0.1° C. at 31° even with water flowing past it. Such a regulator will, under static conditions, hold a temperature within 0.01° C. Owing to the relatively large temperature coefficient of resistance of the Nichrome wire, the fluctuations introduced by the original tap water cooling system were intolerable. Though admittedly the thermo-regulator and heater should properly have been installed directly in the oil line, their location in a water bath was achieved with a good deal less disruption of equipment already completed at the time of the change than would otherwise have been incurred. It should be added that the system as used performed very satisfactorily once the fins on the Neoprene impeller in the bilge pump had been broken in.

As mentioned in the previous section, the 100,000-ohm stabilizer signal resistor at the top of the 100-megohm resistor was mounted in the electronic stabilizer box. The low resistance member of the divider was, on the other hand, immersed in oil (at room temperature) in a lucite box adjacent to the potentiometer and galvanometer. This resistance, of Manganin wire, was very insensitive to temperature changes compared with the top part of the divider. Any variations in the divider ratio could therefore be attributed with negligible error to the 100-megohm arm alone.

The electrical characteristics of this divider, as well as of the precision megohm divider, will be treated in the chapter on calibration.

(2). The 1-Megohm Precision Divider

The continuous measurement of the working voltage by means of the 100-megohm divider depends upon a very accurate determination of the divider ratio. The 1-Megohm precision divider was built in order to determine this ratio. Since the precision divider will be the subject of a separate paper, it will be discribed only briefly here.

The megohm divider consists of one hundred coils of 10,000 ohms each and twelve of low resistance, five 80-ohm, four 100-ohm, and three 133 1/3-ohm coils. The 10,000-ohm coils were wound with #36 Manganin wire and the low resistance coils with #30. All the coils were wound on enameled brass tubing 1 inch in diameter and 21 gauge. The 10,000-ohm coils occupy seven inches on each coil form.

The coils are mounted in the false bottom of a spacious lucite box. A well containing a thermoregulator, a heater and an impeller and shaft was placed in the center of the box with the impeller itself below the level of the false bottom on which the coils were mounted. The entire box is filled with oil. The rows of coils are separated by lucite baffles in such a way that oil forced up inside the coil forms is drawn down the outside of the coils before returning to the impeller through the central well.

The entire unit is maintained at its optimum working temperature in the oil bath. Since, in this divider, all of the wire has the same temperature characteristics, the resistance variation is very regular. In $\pm$ 2°C. from the operating temperature the divider changes by as little as 3 parts per million. The temperature control of this divider is therefore not very critical. Furthermore, because of the low heat loss through the lucite walls of the box, the heater power consumption is quite low.

Though the divider was intended to be operated at 10 kilovolts, the design was so generous that operation at 25 kilovolts for limited periods proved quite feasible. This happy circumstance permitted a determination of the 100-megohm divider ratio directly at the working voltage and thereby eliminated the necessity for making an extrapolation by means of the 100-megohm load coefficient.

(3). The Potentiometer

The potentiometer used in the experiment was a Rubicon High Precision Type B. This instrument has been carefully designed with compensating slide wire contacts which serve to reduce thermal emfs developed by the usual single slider to a very low value indeed. Since a part of the voltage developed across the low end of the 100-megohm divider was bucked out by a standard cell, the instrument could be used in its 0 to 160 millivolt range. At our settings the limit of error of the potentiometer was 10 microvolts.

(4). The Galvanometer

The galvanometer used for the voltage measurement (but not for calibration of the dividers) was a Leeds and Northrup 2430-D self-contained unit. This galvanometer has a sensitivity of 0.0004 microamperes per millimeter. In our work a deflection of one millimeter represented a deviation from the set voltage of 2 parts per million. With the potentiometer key locked, this galvanometer permitted continuous observation of the voltage on the X-ray tube as represented at the low end of the 100-megohm divider. The instrument was located next to the control rods of the electronic stabilizer for the convenience of the operator in setting the voltage increments and in maintaining the high voltage at the desired values.

D. The Monochromater. An essential part of the experiment is the selection of a narrow wavelength band from the continuous spectrum emitted from the X-ray tube. The narrower one can make this band the more faithfully one reproduces the precise features of the spectrum. In the present experiment monochromatization was effected in two stages, firstly through the use of balanced filters, and secondly by the 2-meter curved crystal focusing spectrometer.

(1). Balanced Filters

The principle of balanced filters, an invention of P. A. Ross[14], may be aptly described as elegant. It is well known that two elements of adjacent position in the periodic table have their X-ray absorption edges slightly displaced in wavelength. Furthermore, the ratios of specific absorption on the two sides of these edges are very nearly the same. It is therefore possible, by choosing the proper thicknesses, to obtain two filters such that their absolute absorptions are almost identical on the long wavelength side of an absorption edge of the filter of lower atomic number and on the short wavelength side of the corresponding edge of the filter of higher atomic number. Between the two filter edges the absolute absorption of one can be made very different from the absolute absorption of the other. One should, of course, bear in mind that the selection of a wavelength band through the use of balanced filters does not correspond physically to band pass electrical filtering. The balanced filter technique is in a sense artificial inasmuch as the transmitted band is obtained by subtraction of the intensities observed through the two filters separately.

For the purposes of the present experiment, in which one expects zero intensity on the short wavelength side of the limit of the continuous X-ray spectrum, one adjusts the filter balance on the long wavelength side of the band and in addition chooses the thicknesses appropriate for the greatest difference in transmission within the band. The very simple calculation given in the following paragraphs shows how one obtains the proper thicknesses of the two filters.

Let I_o be the incident intensity in the narrow wavelength interval which includes the K absorption edge of element number Z. On the long wavelength side of this absorption edge, where the absolute absorptions are to be identical, the transmitted intensities are given by

$$I_Z = I_o e^{-\mu_Z \rho_Z X_Z} \qquad (1)$$

and

$$I_{Z+1} = I_o e^{-\mu_{Z+1} \rho_{Z+1} X_{Z+1}} \qquad (2)$$

where μ, ρ and X are the mass absorption coefficient, the density, and the thickness respectively. Clearly in order to have identical transmissions through the two absorbers one must have

$$\mu_{Z+1}\, \rho_{Z+1}\, X_{Z+1} = \mu_Z\, \rho_Z\, X_Z \qquad (3)$$

On the short wavelength side of the absorption edge of Z the absorption of Z will increase by the factor r_Z which is known as the absorption jump ratio. On the other hand the absorption of Z+1 will change practically not at all. On the short side of the edge the transmitted intensities will therefore be given by

$$I'_{Z+1} = I_o e^{-\mu_{Z+1}\, \rho_{Z+1}\, X_{Z+1}} \qquad (4)$$

for element Z+1 and

$$I'_Z = I_o e^{-\mu'_Z\, \rho_Z\, X_Z} \qquad (5)$$

or, since

$$\mu'_Z = r_Z\, \mu_Z \qquad (6)$$

$$I'_Z = I_o e^{-\mu_Z\, \rho_Z\, X_Z\, r_Z} \qquad (7)$$

for element Z. The primes indicate that the intensities are those transmitted on the short wavelength side of the K absorption edge of Z. To find the proper thickness of the two absorbers under the conditions that they shall have the same transmissions on the long wavelength side of the K absorption edge of Z and the greatest difference in transmission on the short wavelength side of this edge, one maximizes, after using (3), the expression

$$I'_{Z+1} - I'_{Z} = I_o\left[e^{-\mu_Z \rho_Z X_Z} - e^{-\mu_Z \rho_Z X_Z r_Z}\right] \quad (8)$$

with respect to the thickness X_Z. The result of this process is

$$X_Z = \frac{\ln r_Z}{(r_Z - 1)\mu_Z \rho_Z} \quad (9)$$

and, from (3) for the other filter

$$X_{Z+1} = \frac{\ln r_Z}{(r_Z - 1)\, \mu_{Z+1}\, \rho_{Z+1}} \quad (10)$$

In our experiment the filter elements were silver (Z=47) and palladium (Z=46). The thicknesses calculated from the above expressions were

$$X_{Pd} = 0.00109 \text{ inches}$$

$$X_{Ag} = 0.00113 \text{ inches}$$

Since the rolling of the foils even to an accuracy of 0.5 mils is not easy, the foils were rolled approximately to the right thicknesses and were then mounted in such a way that the effective thickness of either could be increased by tilting the foils away from the perpendicular—the X-ray beam was then transmitted obliquely. The silver foil turned out to be almost exactly the right thickness as predicted by the ratio I_{Ag}/I_o for the calculated value of X_{Ag}. The palladium foil was a little bit thin and had to be tilted about 15^o from perpendicular before good balance was achieved. Both filters were mounted on a turntable which by remote control inserted them alternately into the beam.

The use of filters does, it is true, cut down still further (to about 0.7 I_o for our silver filter) an intensity which is already very low, and for this reason may be undesirable. On the other hand, there is considerable advantage in being able to measure the background radiation in the presence of X-rays rather than the pure cosmic ray background which one measures by putting a lead block in the beam. The long wings of the spectrometer "window", which will be discussed in the following subsection, continue to admit radiation to the counter long after the short wavelength limit has receeded to wavelengths much longer than that corresponding to the center of the "window". The long tail on the observed isochromat can be virtually eliminated by the use of balanced filters, and the features of the true spectrum can in some cases [10] be sharpened.

(2). The Spectrometer

The spectrometer used in this experiment was the 2-meter curved crystal focusing spectrometer designed by DuMond. Since a complete discussion of this instrument and of work done with it is available elsewhere,[15, 16, 17, 18] a very brief description should be sufficient at this time. A simple diagram is shown in Fig. 5.

The instrument is of the transmission type. A quartz crystal 2 inches square and 2 millimeters thick is mounted in a stainless steel holder and bent to a cylindrical surface of radius of curvature

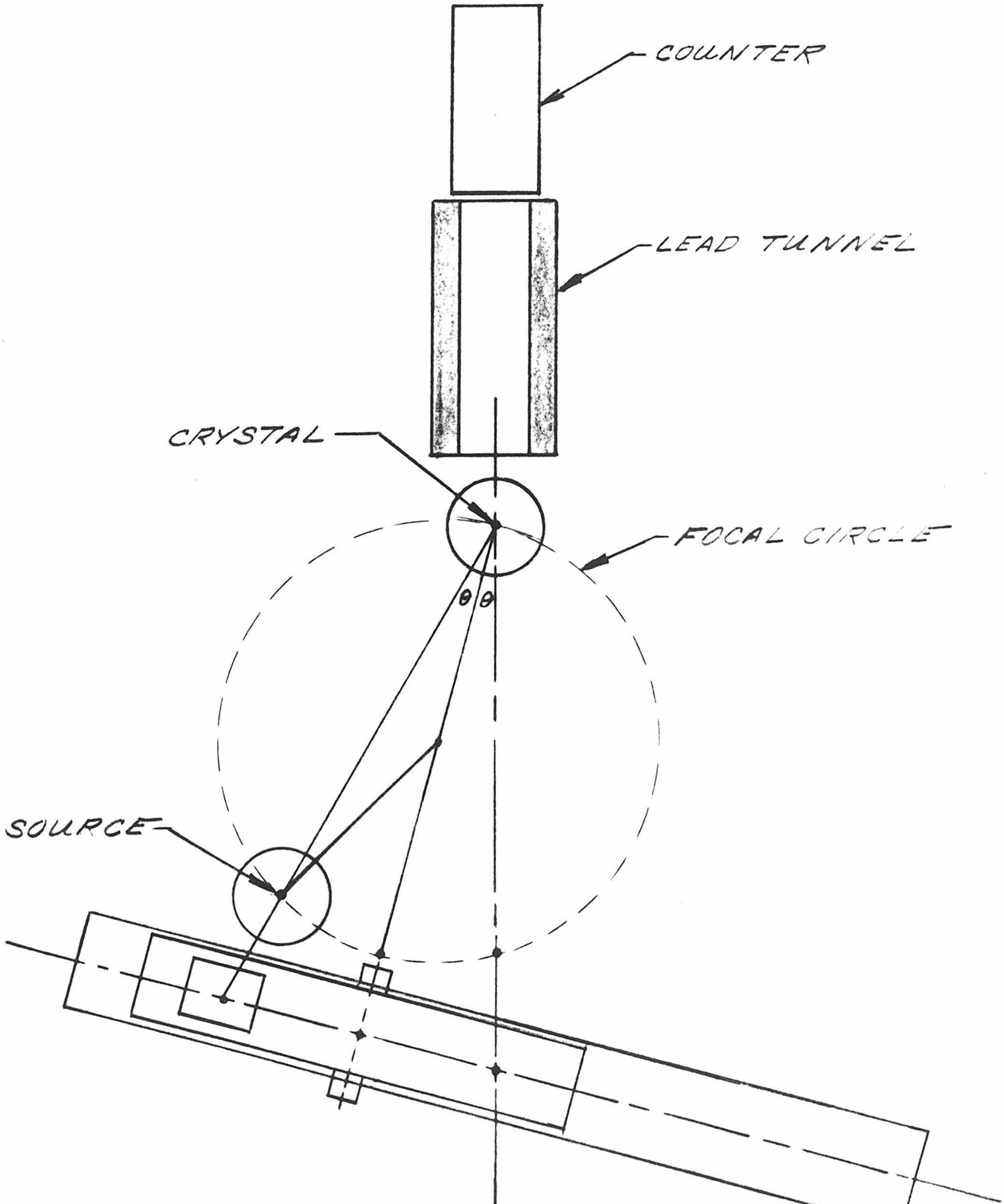


FIG. 5
SPECTROMETER SCHEMATIC

equal to 2 meters. The (310) atomic planes of the crystal lie in vertical planes which include the axis of the cylindrical surface. The focal circle of the crystal has a diameter of 2 meters. Apart from small aberrations, radiation from a source on the focal circle strikes all the crystal planes at the same angle. Conversely (whence the name focusing) radiation from an extended source on the far side of the crystal will appear on the focal circle as a line spectrum in focus. The center of the diffracting crystal also lies on the focal circle, but the ends of the neutral axis are, of course, slightly displaced therefrom.

The great advantage of the focusing spectrometer, particularly for X-ray spectroscopy, lies in the large solid angle from which monochromatic radiation is collected. It was also this feature of high luminosity which enabled us to do our X-ray experiment with the intrinsically low power delivered by our commercial X-ray tube.

The spectrometer is so designed that the diffracted radiation has a fixed direction independent of its wavelength. The crystal is rotated and the focal point is moved by two high precision drive screws of the same pitch (about 1 millimeter) which are geared together. For every setting of the spectrometer the crystal takes on the proper angle for first order Bragg reflection from the atomic planes. One of the drive screws is equipped with a carefully divided drum and vernier which enable one to set the screw to 0.001 revolutions. The dimensions of the instrument are such that one revolution of this drive screw corresponds very nearly to 1 Siegbahn X.U.* The calibration of the spectrometer will be discussed in the next section.

For γ-ray work a line source of small width is mounted on the spectrometer and is carried along the circumference of the focal circle. For most X-ray work the radioactive source is replaced by a narrow slit and the X-ray source, whether tube target or fluorescer, is mounted behind the slit on the same steel beam. The wavelength search is carried out in the same way for both radioactive and X-ray sources, namely by moving the focal line (the γ-ray source or the slit) along the circumference of the focal circle. For our work, in which interest centered on the intensity distribution in a very small wavelength region near the limit of the continuous X-ray spectrum, it was more convenient to change the voltage than to shift the spectrometer, though both methods of studying the limit are about equally feasible at the relatively long wavelength we used.

The "window" of the spectrometer is the fold of the selective diffraction curve (essentially a witch) into the slit (a rectangle) and the curve describing the aberrations of focus of various parts of the curved crystal (also approximately a rectangle). The aberrations result from the failure of the neutral axis of the curved crystal to coincide with the focal circle, as mentioned above, and also from the slight irregularities in curvature for various regions of the crystal. In its central portion the "window" curve is dominated

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*Hereafter in this paper the symbol X.U. will be used to differentiate between Siegbahn wavelength units and milliangstroms.

by the two rectangle components of the fold, since both the slit width and the aberration width are considerably greater than the intrinsic diffraction width of the crystal proper. The slit width can, at the expense of intensity, be narrowed. The aberration width is also reducible by stopping down the aperture, but again at the sacrifice of intensity. In using the full crystal and a fixed slit width, the "window" width remains sensibly constant over most of the range of the spectrometer. The resolution is therefore greatest at the longest wavelength the instrument can measure. Hence we chose to work at voltages such that the spectrum limit should be in this range (about 500 X.U.). Unfortunately the physical design of the present spectrometer did not permit going to longer wavelengths which would have offered still greater possibilities of higher resolution.

We used a slit width of 0.004 inches, which at 2 meters from the crystal corresponds approximately to 0.1 X.U. Many measurements, in other experiments with this instrument, of the "window" width for a slit of this size and the full crystal, indicate a "window" half-width at half maxium intensity of approximately 0.12 X.U. At our working voltage, the equivalent voltage width is about 6 volts. Since other considerations prevented us from placing the X-ray tube close enough to the slit for the finite area of the focal spot to furnish a sufficiently divergent beam to illuminate the entire crystal, this 6-volt half-width was reduced somewhat, and our resolution was improved.

It should be mentioned that for γ-ray and hard X-ray work a collimator is placed between the crystal and the counter. The collimator, formed of thin lead sheets which are slightly wedge-shaped with sides parallel to the diffracted (and slightly divergent) beam, has no effect whatsoever on the resolving power of the instrument but serves merely to cut out the radiation which is transmitted by the crystal but not diffracted. For the very small Bragg angles of hard radiation the collimator is mandatory, but for the large angles in our case (14^{o} or a deviation of 28^{o}) it is not at all necessary. Furthermore, in addition to selecting the diffracted beam, the collimator also absorbs about 65% of the diffracted radiation (because of the finite thickness of its partitions), and for this reason it was replaced in our experiment by a lead tunnel. Though the background from scattered radiation increased rather markedly, the signal counting rate went up almost by a factor three and the contrast was very definitely improved.

E. The Detection and Counting Equipment. The counting of the photons selected by the spectrometer under fixed conditions and over a given time interval was accomplished through the use of a xenon-filled proportional counter, a pre-amplifier and linear pulse amplifier, two discriminator channels and a double decade scaler. The electronic equipment was of standard design, but the counter was built specifically for this experiment.

(1). The Counter

Conventional argon-filled Geiger counters have a rather poor efficiency for X-ray counting owing to the combination of photon energy too low to eject photo-electrons from the counter walls and too high to be effectively absorbed by a reasonable pressure and volume of the gas. One solution is to use a heavier gas, either krypton or xenon, at as high a pressure as satisfactory operation of the counter will permit. Since we had originally intended to try a measurement of h/e at 70 kilovolts, we elected to use xenon as the counting gas.

Unfortunately xenon-filled counters are delicate in the extreme. We succeeded in making a small pilot model work in the Geiger region, but only after several failures and even then at a prohibitively high voltage on the collecting wire. Though the pilot counter worked beautifully when filled with argon up to atmospheric pressure, we found that a xenon pressure of 10 centimeters Hg was about the best we could do with the heavier gas. We were therefore resigned to trying a rather unsatisfactorily low xenon pressure in the large counter.

The counter case was similar to the cases of the γ-ray counters customarily used with the curved crystal spectrometer. It was 8 inches long and had an inside cross-section 3 inches square. The main part of the case was steel and was copper-plated. One of the ends was fitted with a plate in which the pumping line and four Kovar seals were set. The other end was covered by an aluminum window 0.006 inches thick. Because of the large area of the window necessary for admitting radiation from the entire quartz crystal, three thin spring-steel ribs were mounted inside the counter to support the window. Two 1/8-inch glass rods were mounted 1/2 inch back from the window, parallel to and in the shadow of the outer two ribs. Four tungsten collecting wires of 0.001-inch diameter were stretched between these glass rods and the four Kovar seals at the back of the counter. These four wires were parallel to each other, and each was 3/4 inch from its nearest side walls. Since the walls of the counter were rather rough, a 0.016-inch polished copper liner was inserted in the case.

Great care was taken at each stage in the construction and assembly of the counter to ensure that it should be as spotlessly clean as possible. The result was a very fine argon-filled counter and a fairly good xenon-filled counter, provided the xenon pressure was not too high. All attempts to make the counter work in the Geiger region at pressures greater than about 8 centimeters Hg gave unsatisfactory results, either very short plateaus or none at all, and plenty of multiple pulses.

At this point we decided to investigate the performance of our counter in the proportional region. For this purpose the counter was pumped down to a hard vacuum and refilled with a mixture of xenon and CO_2. Several mixtures were tried, but the final mixture consisted of 14 centimeters of xenon and 1 centimeter of CO_2 —not a vast improvement over our first attempts, yet nonetheless a stable and reliable counter. For 500-X.U. radiation the absorption in this counter is about 25%.

(2). The Electronic Circuits

The electronic parts of the counting apparatus were for the most part taken directly from the circuits used at Los Alamos and described by Elmore and Sands[19]. The pre-amplifier consisted of a single amplifing stage with a gain of ten and a cathode follower to drive the cable from the lead house, which enclosed the counter, to the pulse amplifier. The pre-amplifier was outside the counter house but was connected through a hole in the lead by a 5-inch length of coaxial cable.

The pulse amplifier was a modified version of the Los Alamos Model 100 and was built by the Synchrotron Laboratory of this Institute. It is a linear amplifier with a gain of 10^4. The chassis includes, in addition to the amplifier, an output discriminator (not used in this experiment) and a counter high voltage supply with a range of 500 to 2000 volts.

The discriminator circuit used in the experiment was the single channel differential discriminator described by Elmore and Sands. Both an integral and a differential discriminator are included on this chassis. The integral discriminator puts out a shaped pulse to a scaler for all input pulses larger than a certain pre-determined amplitude. The minimum pulse height can be set at the input of the discriminator. This integral discriminator, besides forming an output pulse, also supplies a gating pulse to the input which suppresses following pulses until after a pre-set time interval has passed. The purpose of this gate is to prevent the appearance of spurious pulses of large amplitude caused by the superposition two or more smaller pulses very close together. The action of this gate depends, however, on the level above which the integral discriminator will pass pulses. The lower this level the greater is the dead time of the circuit. At the low counting rates dealt with in this experiment, the piling up of pulses was, of course, unlikely, and the discriminator bias could be set at its optimum value for signal-to-noise contrast without appreciably affecting the pulse height distribution.

The differential discriminator puts out a pulse for each input pulse of amplitude lying between two adjustable limits. This discriminator can be used to find directly the true pulse height distribution from a counter. The width of the differential "window" can be set in steps from 0 to 7.5 volts by means of a selector switch and a battery or manually to any desired width up to 100 volts. In our experiment a second battery was added which enabled us to work with a fixed "window" width up to 15 volts.

An external gate (Fig. 6) was inserted between the pulse amplifier and the discriminator input. Since the two discriminators had the same input and internal gate, this external gate affected both of them. The purpose of the gate was to prevent counting during the time that the filament of the X-ray tube was being supplied with power. The output of the pulse amplifier was fed to the cathode of a diode (both halves of a 6AL5). The plate of the diode and the plate of the triode (both halves of a 6J6) were tied together to a dropping resistor from the 165-volt bus in the discriminator chassis and to the grid of a cathode follower (6AU6). The 90 volts RMS from the primary of the X-ray tube filament transformer was applied as a gating signal to the grid of the triode (6J6). So long as the triode was cut off, the plate of the diode followed the output of the pulse amplifier, but during the remaining half cycle, while

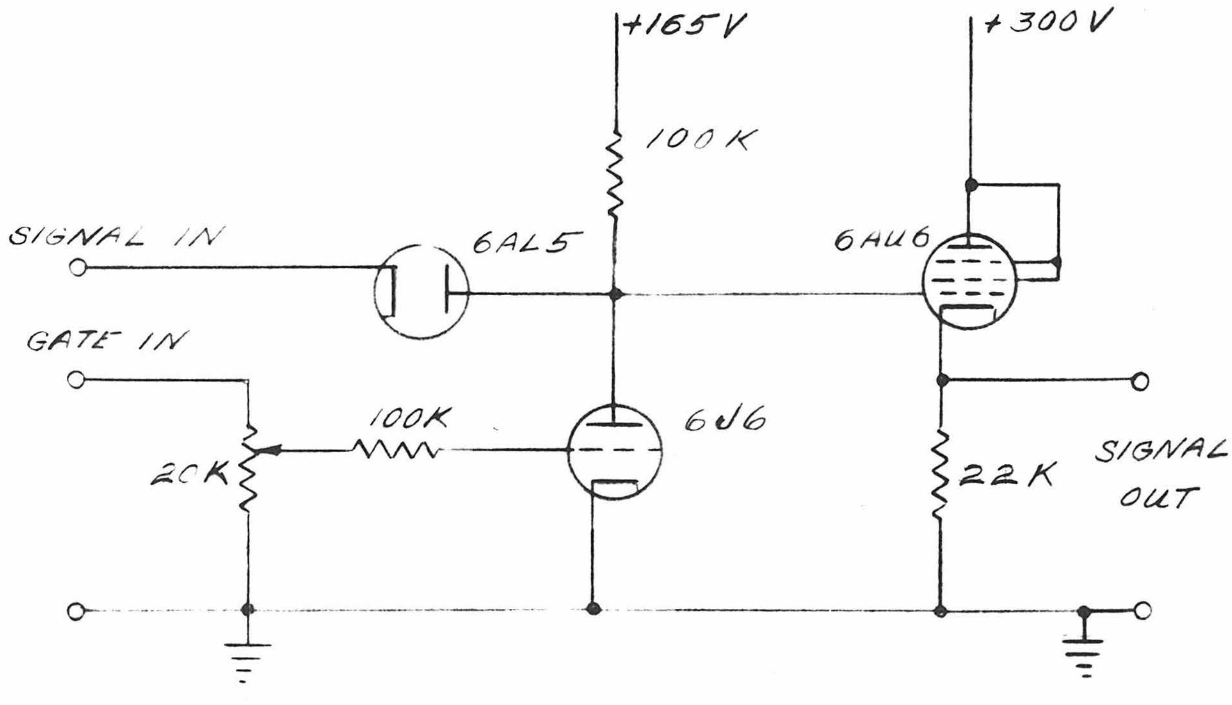

FIG. 6
EXTERNAL DISCRIMINATOR GATE

current flowed through the Selenium rectifier in the filament transformer secondary, pulses from the amplifier were very strongly attenuated and could not pass through the discriminators. We determined experimentally that the gate was closed for slightly more than half a cycle. The phasing of the gate with the filament heating current was accomplished very simply by means of an oscilloscope. By using this external gate we were able to observe radiation corresponding more closely to the conditions of an unipotential cathode.

Because of the presence of the gating circuit, the signal pulses were slightly attenuated even when the gate was open. The locations of the differential discriminator "window" and of the integral discriminator bias had to be adjusted from their optimum settings for un-gated counting. The general character of the pulse height distribution remained virtually unchanged.

The discriminators were followed by a double scaler which was also built in the Synchrotron Laboratory. The scaler consisted of two scale-of-one-hundred circuits each formed from two Berkeley decade scalers in tandem. The scalers were followed by two "Mercury" mechanical registers.

III. Calibration

In view of the extreme importance of the calibration of the measuring apparatus, a separate chapter has been set aside for this purpose. The two independently measured quantities are, of course, voltage and wavelength.

A. Voltage Calibration. The primary reference voltage consisted of a standard cell bank, but the comparison of the working voltage against the EMF of certified standard cells was complicated by the intervention of the two voltage dividers. The three steps in the voltage calibration will be discussed in the following subsections

(1). The Standard Cells

The standard cells used in the experiment fall into two groups. The first group consisted of the six saturated cells listed below:

Position No.	Cell No.	Electromotive Force (absolute volts at 30.00°C.)
1	358559	1.018104
2	358560	1.018105
3	358561	1.018104
4	358562	1.018104
5	358563	1.018104
6	358564	1.018105

These cells, calibrated and certified by the National Bureau of Standards in September 1947, have been guarded with care in the Calibration Laboratory of this Institute. Periodic intercomparisons of these cells have shown very little deviation of any one cell (a maximum of 3 microvolts for one of the six) from the mean of the group, and one may therefore safely assume that the mean of the group has not drifted appreciably from the original mean determined at the time of calibration three years ago*. These saturated cells were kept in a large well-insulated box whose temperature was controlled within 0.01° at $30.09^{\circ}C$. The temperature correction to the electromotive force was -5.1 microvolts for each cell.

Cell No. 358559, used more often in the last three years than any of the other five cells, has the lowest electromotive force of the group. This cell was therefore used as the intercomparison standard for the rest. The choice was made purely for convenience; the cell is no more holy than the others, and it has, of course, the greatest negative deviation from the mean of the group.

The second group of standard cells consisted of four cells of the unsaturated type. One of these was used as the potentiometer current standardizing cell. The remaining three were available for use as bucking cells in the voltage measurement.

The unsaturated cells were also kept at constant temperature in an insulated box. These cells, though calibrated and certified

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*This conclusion is based on a letter of Dec. 12, 1949, from G. W. Vinal of N.B.S. to J. N. Harris of this Institute in which the question of a possible drift in the mean of our cells was discussed. A more general treatment of the matter can be found in NBS Circular 475 (1949), pp. 9-11.

by the National Bureau of Standards, were recalibrated frequently during the course of the experiment using the saturated cell bank as a primary standard.

It should be noted that the electromotive force of the current standardizing cell need not be known to an accuracy greater than that of the potentiometer. The same applies to the bucking cell, but its electromotive force was measured frequently to an accuracy of 1 microvolt on the low range of the Rubicon potentiometer. The electromotive force of the bucking cell used in all final runs was

$$\text{emf S.C. "B"} = 1.019161 \text{ absolute volts}$$

with a conservative limit of error of 5 microvolts. This electromotive force was in opposition to the voltage developed across the low resistance arm of the 100-megohm divider.

(2). The Precision Divider

The determination of the dividing ratio of the 1-megohm precision divider involves a knowledge of the "cold" ratio—essentially no load—and of the load coefficient. The determination of the "cold" ratio can best be done by a method discussed by Wenner[20] and attributed by him to Lord Rayleigh. The method depends for its success on one's being able to connect the elements of the two arms of the divider in such a way that a ratio very close to unity is obtained.

The precision megohm divider was purposely designed such that Wenner's calibration technique could be applied. It will be recalled that each of the three groups of small resistors in the divider box could be connected in series to give 400 ohms. The one hundred 10,000-ohm coils can also be connected to give 400 ohms through the use of a large heavy copper grid which fits over the stud terminals of the individual coils and converts the resistor into fifty 20,000-ohm units in parallel. In our case the copper grid, with mercury amalgam contact surfaces, introduced negligible error. None of the coils, large or small, deviates from the mean of its group by as much as one part in 10^4 and most of them deviate by a good bit less.

If R_o is the mean value of the high-resistance elements (nominally 20,000 ohms) and r_o the mean value of the low (nominally 133 1/3 ohms for the group used in this experiment), then clearly for series connection

$$R_s = NR_o \tag{1}$$

and

$$r_s = nr_o \tag{2}$$

where N is the number of 20,000-ohm units and n is the number of 133 1/3-ohm units. In parallel connection one obtains

$$\frac{1}{R_p} = \frac{1}{R_o} \sum_{j=1}^{N} \frac{1}{1+\delta_j} \tag{3}$$

where δ_j is the deviation of the j^{th} resistor from the mean resistance R_o. Expanding (3) one obtains

$$\frac{1}{R_p} = \frac{1}{R_o} \sum_{j=1}^{N} (1 - \delta_j + \delta_j^2 \ldots\ldots)$$

$$\doteq \frac{1}{R_o} \sum_{j=1}^{N} (1 + \delta_j^2)$$

$$\doteq \frac{N}{R_o} (1 + \frac{1}{N} \sum_{j=1}^{N} \delta_j^2) \qquad (3')$$

Equation (3') follows since, from the definition of the mean, the linear term vanishes and terms higher than the squares in the deviations δ_j may be neglected. Similarly for the low resistances

$$\frac{1}{r_p} \doteq \frac{n}{r_o} (1 + \frac{1}{n} \sum_{K=1}^{n} \delta_K^2) \qquad (4)$$

Hence, using (1) and (3'), one obtains

$$\frac{R_s}{R_p} \doteq N^2 (1 + \frac{1}{N} \sum_{j=1}^{N} \delta_j^2) \qquad (5)$$

and similarly from (2) and (4) one finds

$$\frac{r_s}{r_p} \doteq n^2 (1 + \frac{1}{n} \sum_{K=1}^{n} \delta_K^2) \qquad (6)$$

The result of multiplying (5) and (6) and re-arranging the factors is

$$\frac{R_s}{r_p} \doteq \frac{R_p}{r_s} N^2 n^2 (1 + \frac{1}{N} \sum_{j=1}^{N} \delta_j^2)(1 + \frac{1}{n} \sum_{K=1}^{n} \delta_K^2)$$

$$\doteq \frac{R_p}{r_s} N^2 n^2 (1 + \frac{1}{N} \sum_{j=1}^{N} \delta_j^2 + \frac{1}{n} \sum_{K=1}^{n} \delta_K^2) \qquad (7)$$

Equation (7) is correct to terms in the squares of the deviations. In (7) the virtue of arranging matters to fulfill the condition

$$\frac{R_p}{r_s} \approx 1$$

becomes clear, since a ratio near unity can be measured very accurately by the bridge interchange method.

Consider the bridge formed from R_p, r_s, a variable resistance C and a fourth resistance D (Fig. 7). Neither C nor D need be accurate to better than 0.1% though in our experiment 0.01% resistors were used. Resistors C and D were specially wound of Manganin wire and placed in an oil-filled lucite box. The condition for balance of the bridge is evidently

$$\frac{R_p}{r_s} = \frac{C}{D} \tag{8}$$

If now the two arms R_p and r_s are interchanged and the new balancing value of the variable resistor is $C + \Delta C$, the condition for balance will be

$$\frac{r_s}{R_p} = \frac{C + \Delta C}{D} \tag{9}$$

Dividing (8) by (9), one obtains

$$\left(\frac{R_p}{r_s}\right)^2 = \frac{C}{C + \Delta C} \tag{10}$$

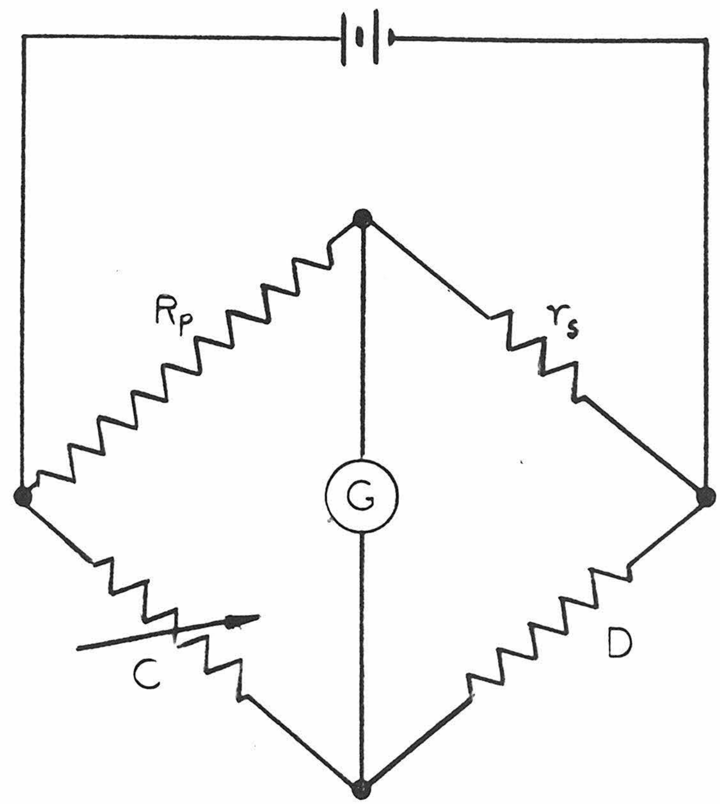


FIG. 7
RATIO BRIDGE

or, neglecting terms higher than the square in the binomial expansion,

$$\frac{R_p}{r_s} \doteq 1 - \frac{1}{2} \cdot \frac{\Delta C}{C} \qquad (11)$$

If the ratio $\Delta C/C$ differs from unity by less than 10^{-3}, higher terms may be neglected, and if in addition the deviations of the separate coils are 1 part in 10^{4} or less, then clearly one can determine the "cold" ratio of the divider by the above method to an accuracy of 1 part per million.

The "cold" ratio cannot be used alone to set the ratio of the 100-megohm divider for a working voltage of 25 kilovolts. In addition one must find the ratio of the precision divider under loaded conditions.

A convenient method of finding the load coefficient of the divider will be described below[21]. Since the low arm of the divider is operated far below its power capacity (about 1.1 volts across the three 133 1/3-ohm resistors in parallel), one need not consider the load coefficient for this arm but only for the 1-megohm arm, for which at 25 kilovolts the dissipation is 6.25 watts in each coil.

Omitting one of the 10,000-ohm coils, one may divide the remaining ninety-nine into three groups of thirty-three coils each. Let the "cold" resistance of one of these groups be taken as a standard. Call this resistance R_o. The "cold" resistances of the other two groups will deviate from R_o and may be given by

$$R_1 = R_o\,(1 + \delta_1) \qquad (12)$$

and

$$R_2 = R_o\ (1 + \delta_2) \qquad (13)$$

With a variable resistance C and another resistance D, approximately half the value of C, the bridge in Fig. 8a can be set up. A voltage E is applied as shown across this bridge. Now under the conditions that the voltage E is applied to the bridge and that the resistances C and D are both much smaller than the resistances in the other two arms, the new values of R_o, R_1 and R_2 will be given by

$$R_o' = R_o\ (1 + \alpha p) \qquad (14)$$

$$R_1' = R_o\ (1 + \frac{\alpha}{4} p)(1 + \delta_1) \qquad (15)$$

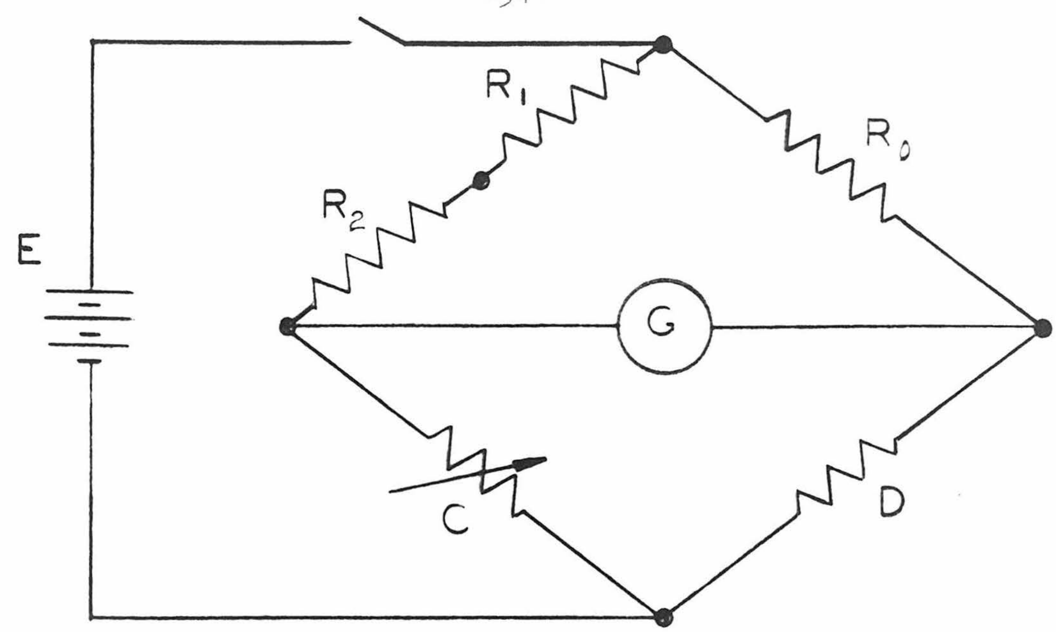
$$R_2' = R_o\ (1 + \frac{\alpha p}{4})(1 + \delta_2) \qquad (16)$$

where the primes indicate loaded conditions, α is a constant of proportionality, and p is approximately equal to the power dissipated in the resistor R. Since $R_o \approx R_1 \approx R_2$, the power dissipated in R_1 and in R_2 is approximately p/4. The balance condition for the bridge in Fig. 8a when the switch is closed is

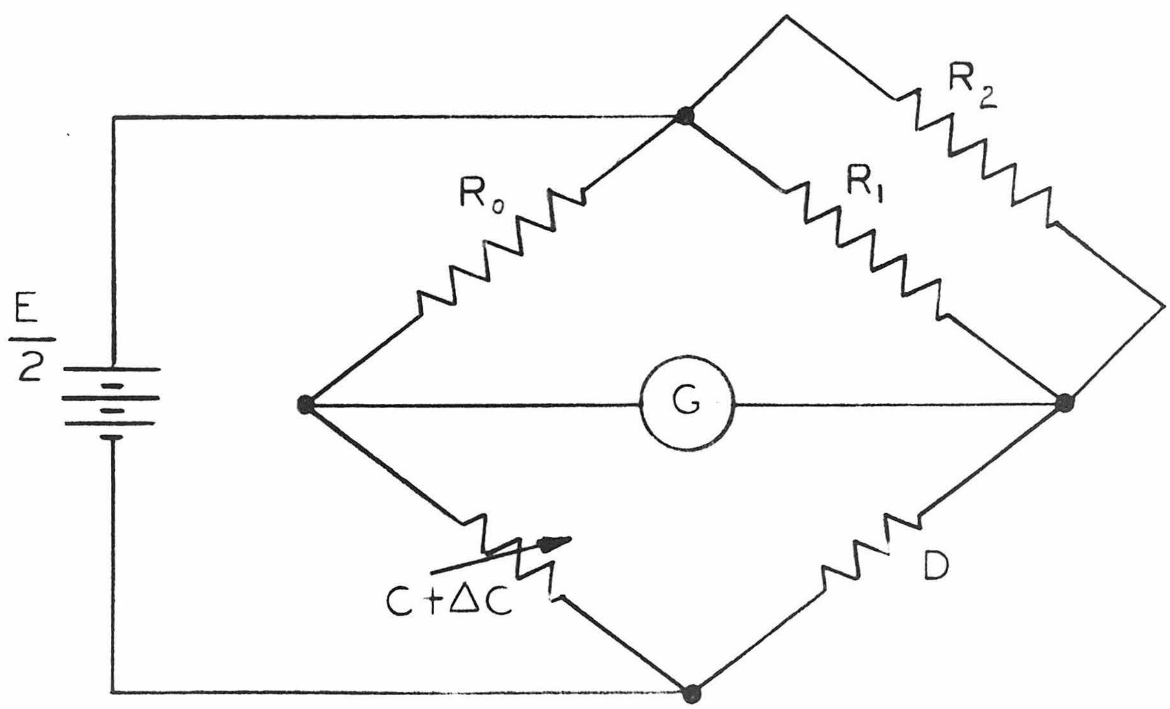
$$\frac{C}{D} = \frac{R_1' + R_2'}{R_o'} \qquad (17)$$

or, from (14), (15) and (16)

$$\frac{C}{D} = \frac{(1 + \frac{\alpha p}{4})(2 + \delta_1 + \delta_2)}{(1 + \alpha p)} \qquad (18)$$



(a)



(b)

FIG. 8
LOAD COEFFICIENT BRIDGES

Consider now the second bridge given in Fig. 8b. In this second bridge, resistors R_1 and R_2 are connected in parallel and placed in the arm formerly occupied by R_o. The voltage E/2 is applied to this bridge. Since the power dissipated in R_o under the new conditions is only 1/4 the power dissipated in the first case, whereas the power dissipated in R_1 and R_2 is the same, one has

$$R_o'' = R_o \left(1 + \frac{\alpha p}{4}\right) \tag{19}$$

$$R_1'' = R_1' = R_o \left(1 + \frac{\alpha p}{4}\right)(1 + \delta_1) \tag{20}$$

$$R_2'' = R_2' = R_o \left(1 + \frac{\alpha p}{4}\right)(1 + \delta_2) \tag{21}$$

and the balance equation is now

$$\frac{C + \Delta C}{D} = \frac{R_o''(R_1'' + R_2'')}{R_1'' R_2''} \tag{22}$$

or, from (19), (20) and (21)

$$\frac{C + \Delta C}{D} = \frac{(2 + \delta_1 + \delta_2)}{(1 + \delta_1)(1 + \delta_2)} \tag{23}$$

Dividing (23) by (18)

$$\frac{C + \Delta C}{C} = \frac{(1 + \alpha p)}{(1 + \delta_1)(1 + \delta_2)\left(1 + \frac{\alpha p}{4}\right)} \tag{24}$$

Since αp and the deviations are all very small compared with unity, one may expand the above result to obtain

$$1 + \frac{3}{4}\alpha p \doteq (1 + \frac{\Delta C}{C})(1 + \delta_1 + \delta_2) \qquad (25)$$

or, solving for αp,

$$\alpha p = \frac{4}{3}(\frac{\Delta C}{C} + \delta_1 + \delta_2) \qquad (26)$$

The quantity αp can be determined in this way for each of the groups of thirty-three 10,000-ohm resistors and the mean value applied to the megohm arm of the divider.

A further refinement should be mentioned. Though the voltage E produces the working voltage per coil in the load coefficient measurements, nonetheless because of the large power dissipation at this working voltage the temperature of the oil bath and of the resistor wire itself will rise, and the load coefficient will be time dependent. It is therefore desirable to find its time variation. Because the divider is operated at the temperature corresponding to the peak in the resistance-temperature curve for the Manganin wire, the divider ratio decreases almost linearly by about 10 parts per million in the period from 1 to 5 minutes after it has been subjected to the full working voltage. The correct value of the divider ratio can, of course, be found for any time after loading. We used the value at the end of 5 minutes. At the end of this interval the precision divider ratio was

$$\text{p.d.r.} = 22503.52 \pm 0.10 \text{ (L.E.)} \qquad (27)$$

A very conservative limit of error has been assigned to this ratio. Figures 9a and 9b show the slow variations of the "cold" ratio over some months and the time dependence of the load coefficient over 5 minutes.

(3). The Working Divider

The calibration of the 100-megohm divider can be discussed in little space since, in the strict sense, this divider is not really calibrated at all. Instead it is merely balanced from time to time against the precision divider treated in the preceding subsection.

It will be recalled that a 100,000-ohm decade box was placed in series with the 100 megohms at the low end of the high resistance arm. The bridge was balanced by means of this decade box to conform to the ratio of the precision divider at the end of five minutes' loading of the latter. The 100-megohm divider, which was always under full load except for the short time required to make the connections, returned to full-load equilibrium well within the five minutes allotted, as one may readily see by glancing at the precision divider load coefficient curve and the sample bridge measurement curves (Figs. 9b and 10).

The X-ray tube power supply was used in the bridge measurement. The high voltage was momentarily turned off and the X-ray tube link was removed from the two mercury cups on the resistor rack. A second link, from a cup at the high end of the precision divider, was then inserted between that divider and the cup connected permanently to the electronic stabilizer box. It was for ease in making this

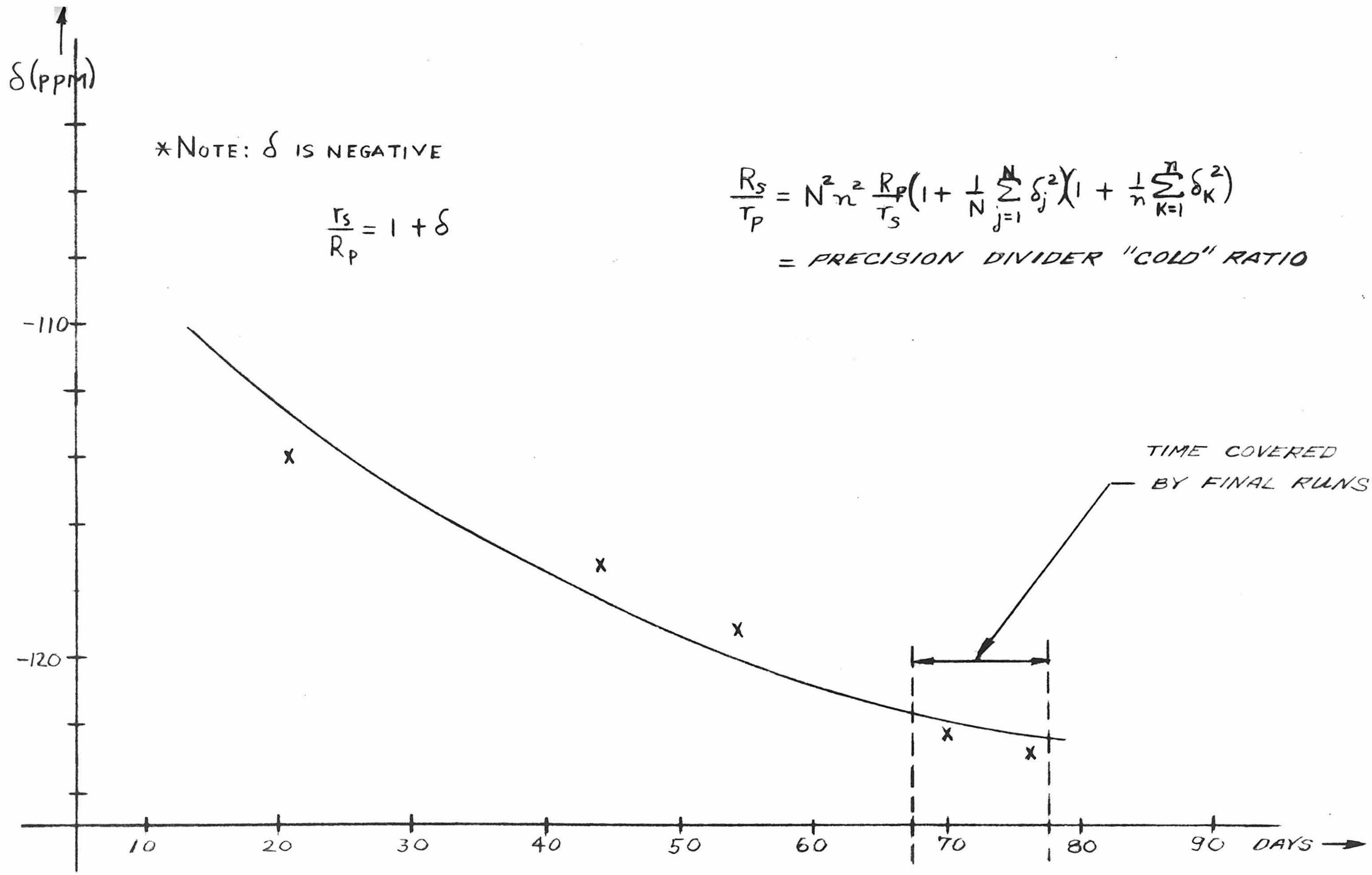


FIG. 9a
SLOW VARIATION OF PRECISION
DIVIDER "COLD" RATIO

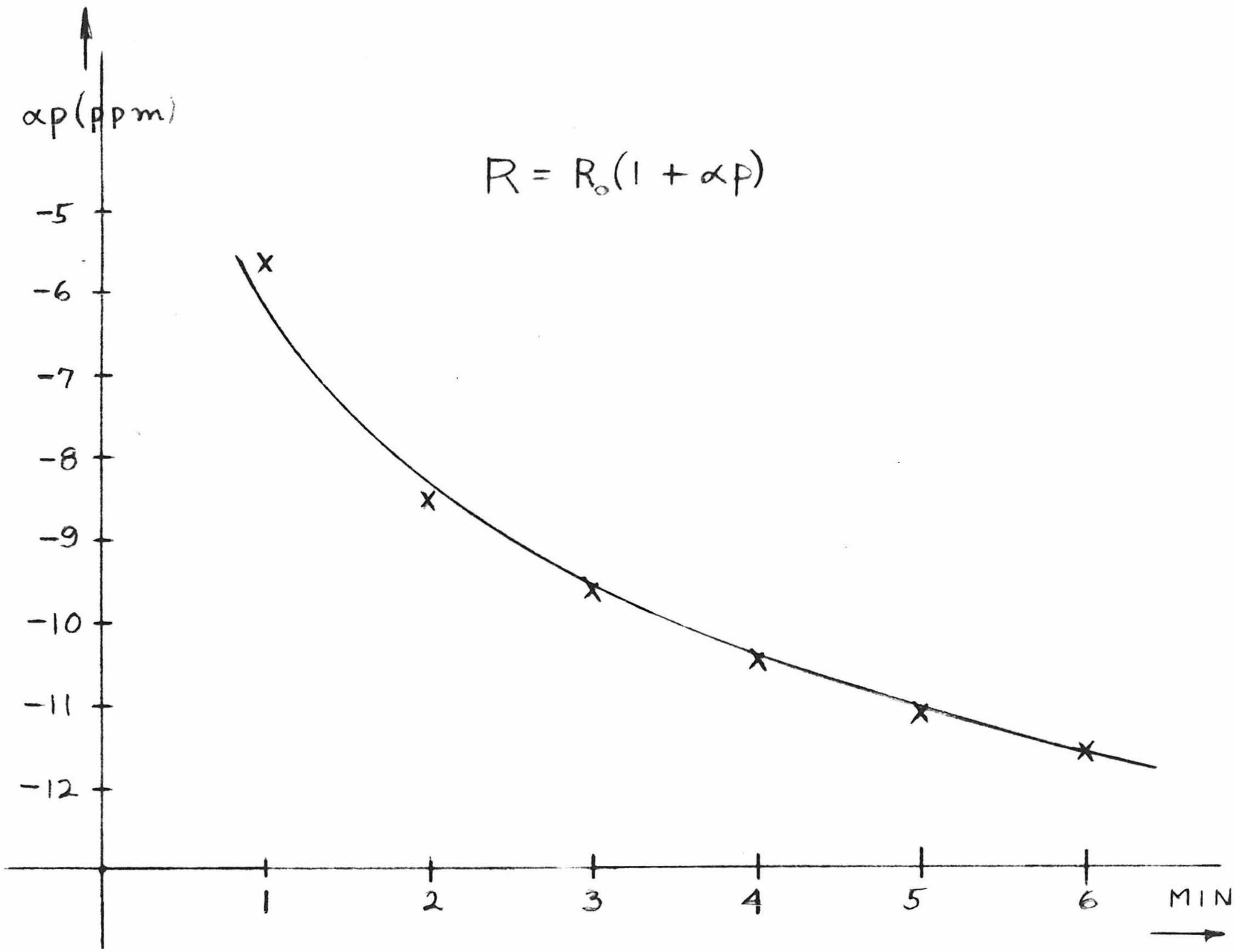


FIG. 9 b

TIME DEPENDENCE OF PRECISION DIVIDER LOAD COEFFICIENT

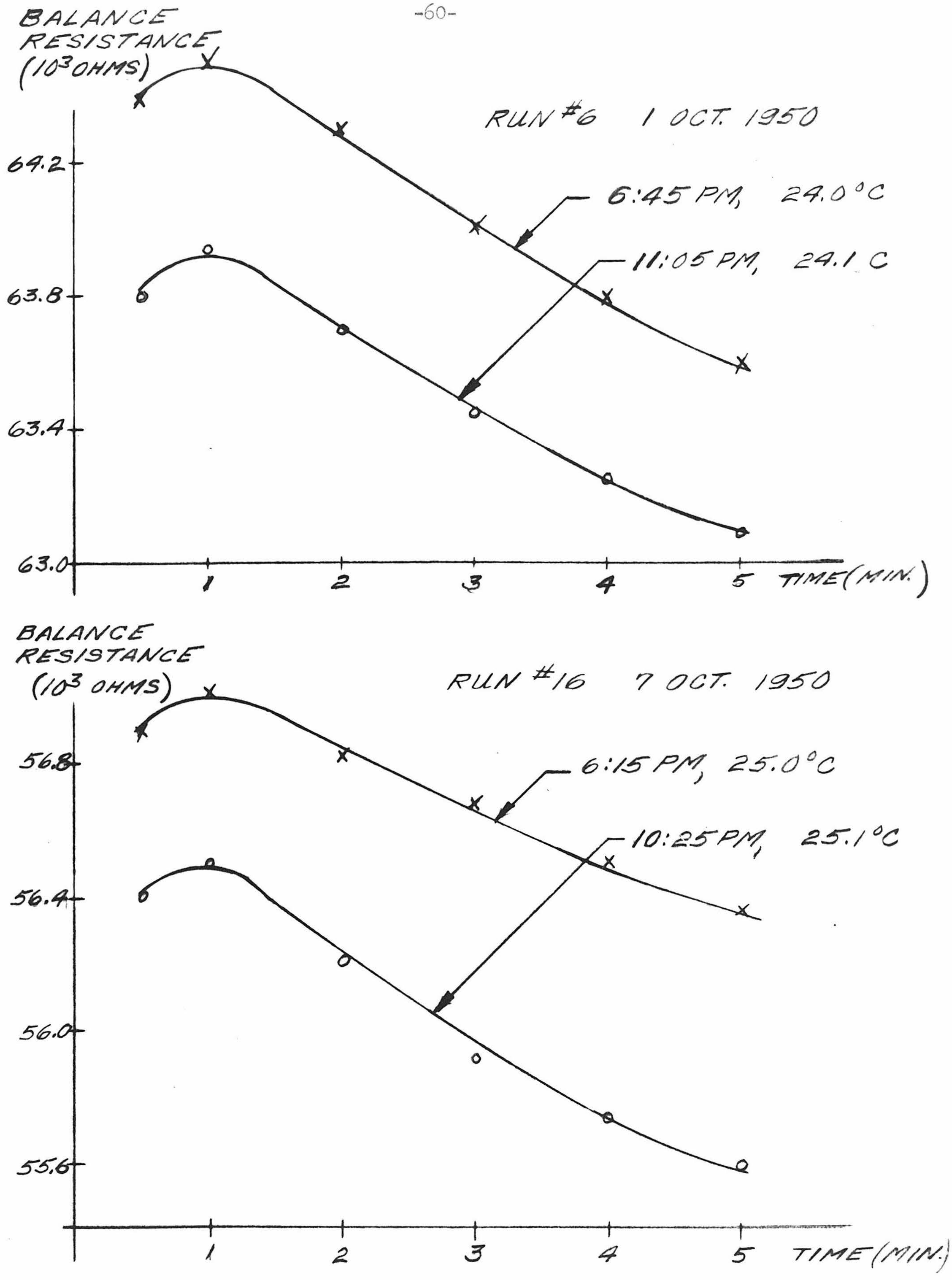


FIG. 10
SAMPLE BRIDGE RESPONSES

measurement that the system of mercury cups was used in place of more permanent connections. The switching from tube to precision divider could be made easily in less than 15 seconds.

Two aspects of the 100-megohm divider require further discussion. The first concerns its load coefficient. Since the divider was set while in equilibrium under its full working voltage, an accurate knowledge of its load coefficient was not necessary. Nonetheless its effective load coefficient was measured in order to determine how precisely the high voltage had to be set for good results in the bridge measurement. The result of this load coefficient measurement revealed that the effective resistance of the 100-megohm divider decreased approximately 2 ohms per volt over the entire range from 1 to 25 kilovolts. A voltage setting within 250 volts of the working value amounted therefore to a limit of error of 5 parts per million. Such a setting could be made quickly by means of an electrostatic voltmeter and was preferable to the longer process of setting by potentiometer. Further, the electronic stabilizer could be turned out of its stabilizing range and the hardship imposed on its power transformer by some 3000 volts' drop between its windings could be obviated.

The second aspect concerns departures of the divider from the ratio set at the start of a run. Though the divider was temperature controlled and under full load throughout a run, the divider ratio would nevertheless drift during a run. We found a pronounced dependence of the 100-megohm divider (but not the precision) on the ambient temperature in the power supply room (Fig. 11). In order to make a

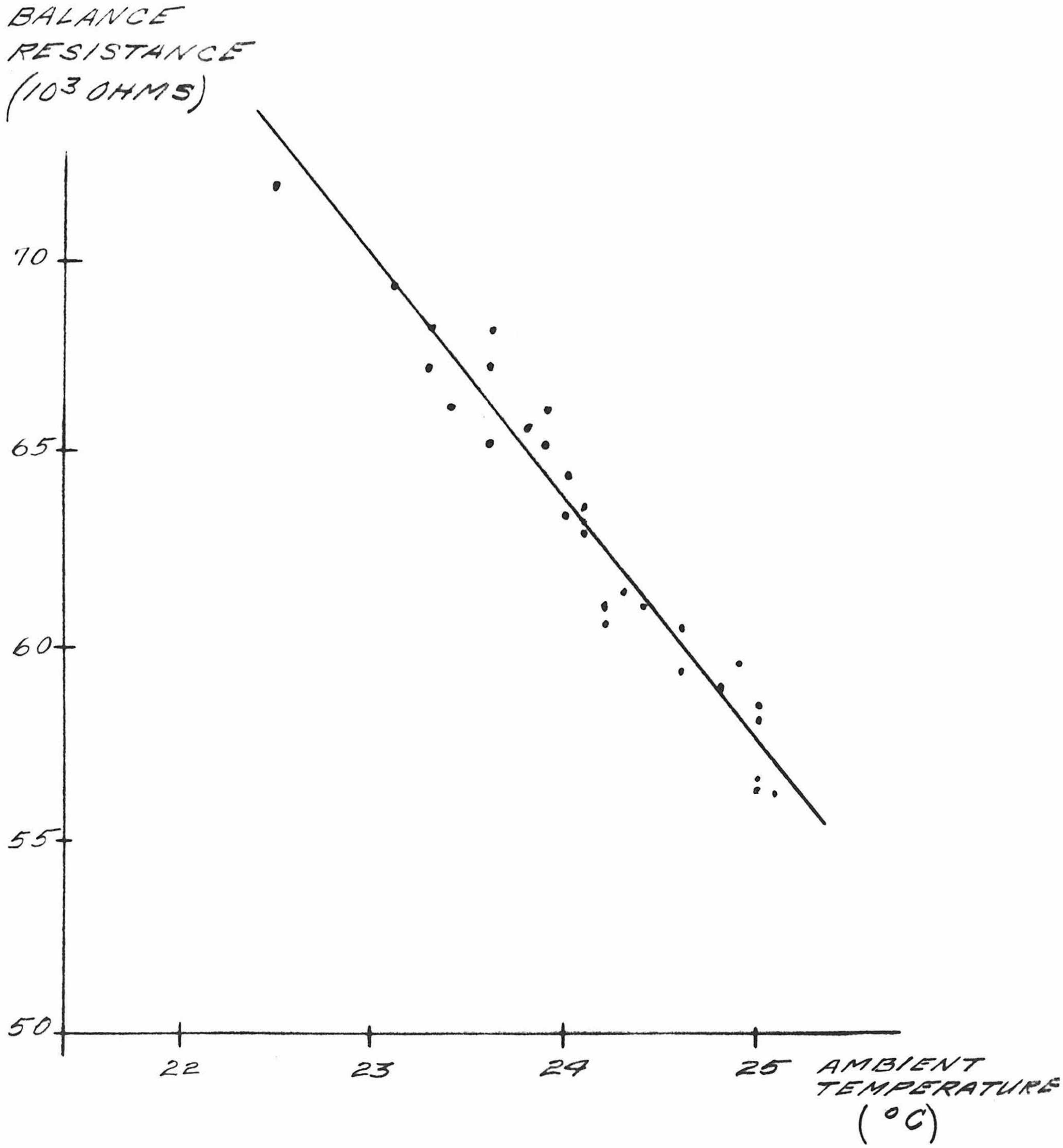


FIG. 11
DEPENDENCE OF 100 MEGOHM
DIVIDER ON AMBIENT TEMPERATURE

correction for this drift we recorded the ambient temperature at approximately half-hour intervals during runs as well as at the time of each bridge measurement. Because of overheating of the precision divider, it was inadvisable to make bridge measurements more frequently than every three hours, though by applying the proper temperature corrections it would have been possible to do so. We found in addition, however, that the X-ray tube emission tended to be unstable for some time after the high voltage had been turned on again even though it had been off for but five minutes. It was the tube rather than the precision divider which dictated our decision to measure the bridge at the start and finish of the runs only.

By using the curve of balancing resistance vs. ambient temperature we could obtain a correction to each setting of the potentiometer which was accurate within limits of 2 parts in 10^5 for runs in which temperature variations were sizable and a good bit better for most of the runs. The method of applying this correction for drift in the 100-megohm divider ratio will be discussed in a later section.

Before this subsection is concluded, mention should be made of the galvanometer used in the calibration measurements. This instrument was a Leeds and Northrup 2284-X with a sensitivity of 0.25 microvolts per millimeter at 1 meter and with circuit resistance of 1100 ohms. With our equipment this galvanometer gave the following deflections for errors of 1 part per million:

Standard cell intercomparison	4 mm.
Precision divider ratio	10 mm.
Bridge measurement	1.5 mm.

B. Wavelength Calibration. The determination of the wavelength setting of the spectrometer for the runs across the short wavelength limit of the continuous X-ray spectrum depends primarily upon the wavelength standards and secondarily upon the spectrometer calibration proper.

(1). Wavelength Standards

In order to avoid depending on the calibration of the spectrometer drive-screw alone for determining the wavelength setting to the desired precision, certain known X-ray fluorescence lines in the range near 500 X.U. were selected to provide local reference points. The four lines used are listed below:

Line[22]	Wavelength (Siegbahn X.U.)
Ag $K\beta_3$	496.65 ± 0.005
Ag $K\beta_1$	496.01 ± 0.005
Sn $K_{\alpha 2}$	494.02 ± 0.005
Sn $K_{\alpha 1}$	489.57 ± 0.005

These four lines, all known to 1 part in 10^5, were chosen both for their proximity to the desired spectrometer setting and for the ease with which they could be produced. The method of producing and locating the lines will be discussed in the following subsection.

(2). Spectrometer Calibration

There are two main aspects to the spectrometer calibration. They are the location of the fluorescence lines and the extrapolation to the setting for the runs across the short wavelength limit. These aspects will be discussed in turn.

The production of the silver and tin fluorescence lines listed above was accomplished by back illumination of foils placed between the tube target and the slit. One may well wonder at the decision to illuminate these foils from behind and suffer from high background when front illumination would alleviate this difficulty. The decision was based on the fact that the spectrometer, though massive, is nonetheless a very delicate instrument which can be put out of adjustment by the slight but unavoidable jarring incurred in mounting and dismounting a forty-pound X-ray tube. We felt that we could be much more certain of our settings if the continuous spectrum runs and the fluorescence measurements were made with the least possible disturbance of the spectrometer.

It was, of course, impossible to detect the fluorescence lines if radiation originating in the tube target and only slightly absorbed by the foils was allowed to pass through the slit. The tube was therefore mounted on a small carriage which consisted of two steel plates separated by ball bearings. One plate was fastened to the tube's counter-balancing sling—the counter-balance relieved the spectrometer of most of the weight of the tube—and the other to the source-mounting beam. Two simple drive screws permitted sufficient misalignment of the tube to reduce background to manageable size. In order that no error should result from the small change in weighting on the spectrometer source-mounting beam (caused by shifting the tube), the fluorescence lines were measured with both left and right misalignment of the X-ray tube. No shift in the lines was found to result from this process.

For best results in obtaining fluorescence radiation by back illumination, the fluorescer should have a certain calculable optimum thickness. The simple calculation given below ignores the fact that the foil is not illuminated by monochromatic radiation, but it nevertheless provides a remarkably satisfactory guide to the selection of the proper foil thickness*. The results indicated that a more elaborate calculation was unwarranted. The approximate calculation works well chiefly because only that incident radiation on the short wavelength side of the absorption edge of the fluorescer and lying in a range rather close to that edge is strongly absorbed, and because the absorption coefficient of the foil for its own characteristic radiation (particularly in the case of the silver $K\beta_{1,3}$ doublet) is not markedly different from the coefficient close to the long wavelength side of the absorption edge.

Consider incident radiation of mean intensity I_o, in the wavelength region just short of the K absorption edge of the fluorescer, falling normally on the back surface of a foil of thickness t (Fig. 12). If the mean linear absorption coefficient for this

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*This treatment was suggested by Professor J. W. M. DuMond.

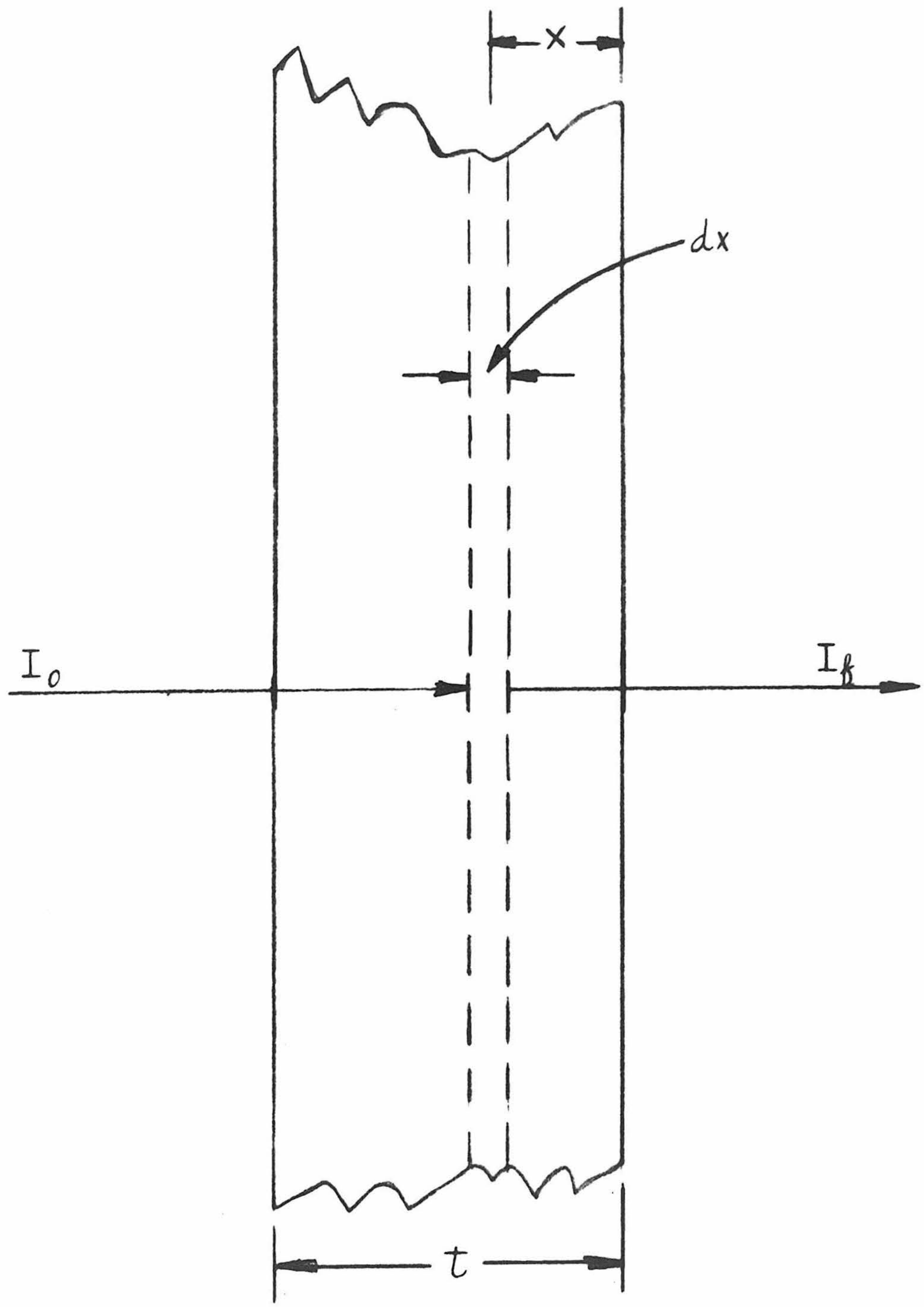

FIG. 12
FLUORESCENCE GENERATION

radiation is μ_1 then the intensity of incident radiation at a depth x from the front surface is given by

$$I_i(x) = I_o e^{-\mu_1(t - x)} \tag{1}$$

The fluorescent intensity originating in dx at x and emerging normally from the front surface is

$$dI_f = kI_o e^{-\mu_1(t - x)} e^{-\mu_2 x} \mu_1 dx \tag{2}$$

where μ_2 is the linear absorption coefficient of the fluorescer for its own characteristic radiation and k is a constant relating the fluorescence yield to the amount of incident radiation absorbed. The integral of (2), assuming no dependence on angle or on wavelength, is

$$I_f = \frac{k\mu_1 I_o}{\mu_1 - \mu_2} (e^{-\mu_2 t} - e^{-\mu_1 t}) \tag{3}$$

By maximizing (3) with respect to t one obtains, for the optimum foil thickness

$$t_{opt} = \frac{1}{\mu_1 - \mu_2} \ln \frac{\mu_1}{\mu_2} \tag{4}$$

Using (4) and the values of μ at the K absorption discontinuity we found for the two foils:

Foil	Thickness (inches)
silver	0.0015
tin	0.003

These foils were very satisfactory. Fluorescent radiation of sufficient intensity could be obtained from them with the X-ray tube operated at 35 kilovolts and 10 milliamperes. Each of the four fluorescence lines was measured several times in the course of the experiment. The lines were located by finding the intersection of straight lines tangent to their sides. The fluorescence lines are shown in Figs. 13 and 14. The mean values of the spectrometer settings are listed below:

Line	Spectrometer Setting (Readings in turns of drive-screw)
Ag $K\beta_3$	9503.20
Ag $K\beta_1$	9503.81
Sn $K_{\alpha 2}$	9505.830
Sn $K_{\alpha 1}$	9510.263

It was not possible to locate each of these lines with equal ease. The tin $K_{\alpha 1}$ line was much the sharpest of the lot, and in the extrapolation to the setting for the short wavelength limit runs this line was given more weight than the others.

For the runs across the limit the spectrometer was set at a screw reading of 9495.000; i.e., slightly more than 8 X.U. from the nearest reference line (Ag $K\beta_3$) and a little more than 15 X.U. from Sn $K_{\alpha 1}$. The spectrometer was set in this position in order to make the best use of the balanced filters. The setting was close to the palladium K absorption edge but still far enough from that

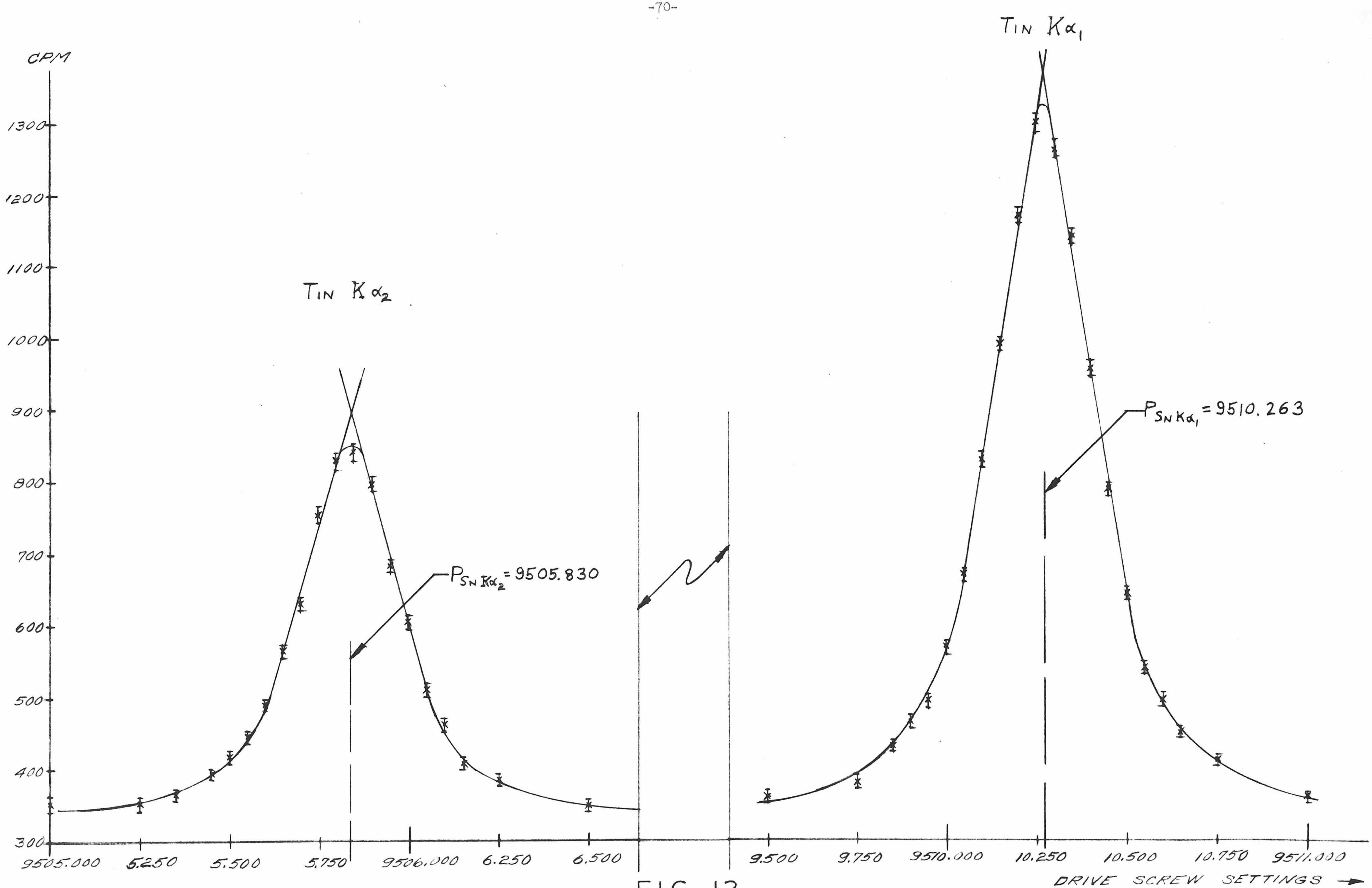

FIG. 13

TIN $K\alpha_{1,2}$ DOUBLET

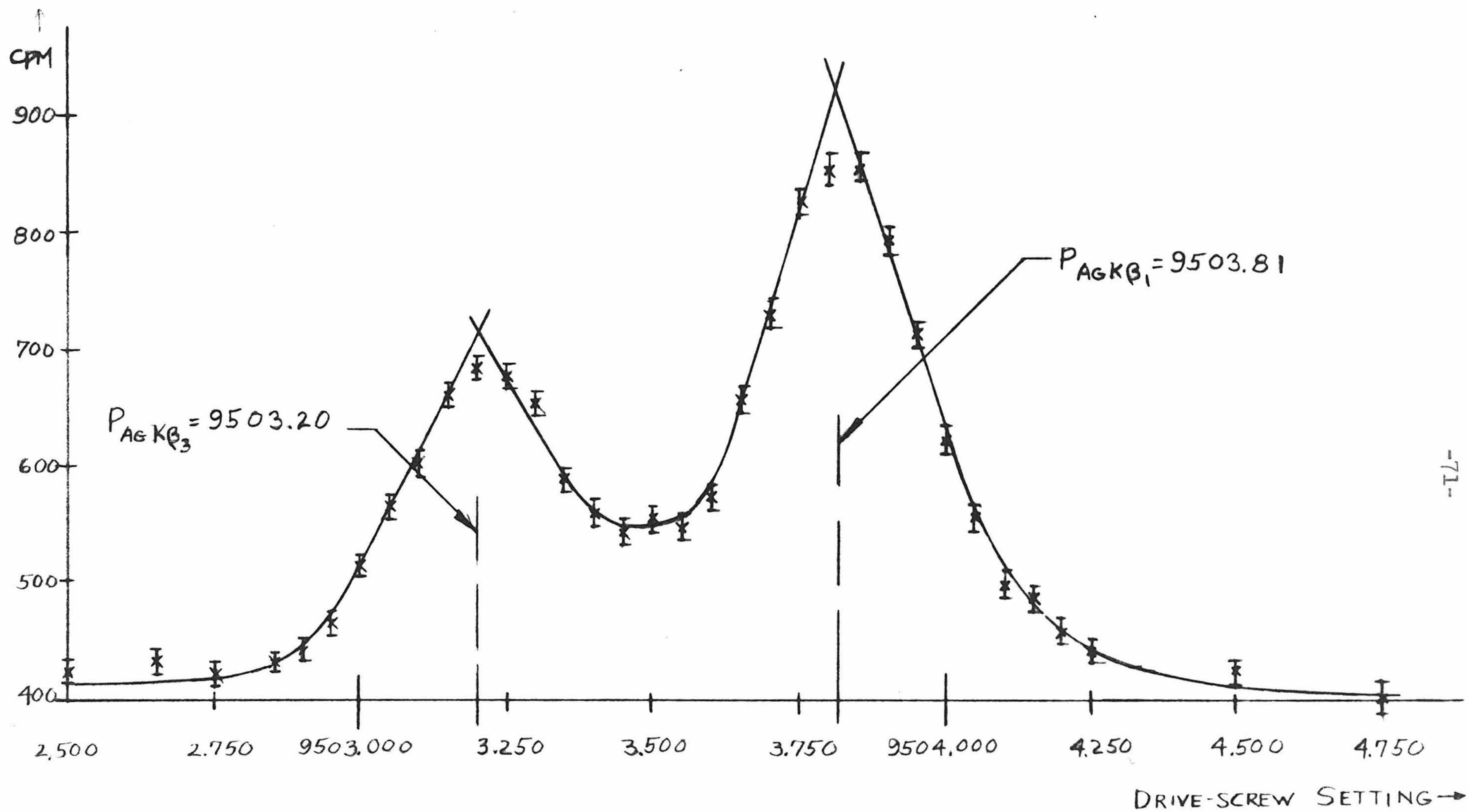

FIG. 14

SILVER $K\beta_{1,3}$ DOUBLET

edge that its structure did not interfere with the limit measurements.

Were it not for the fact that the reference lines are closely spaced, one could determine by means of them alone a local screw calibration which would suffice for extrapolation purposes. Such a local calibration can indeed be made, but because the lines are close together and each known only to 1 part in 10^5, the accuracy of such a local calibration, which depends on the difference of two lines, is necessarily low. Rather than rely too heavily on such a number we used the mean screw calibration number resulting from other recent work on the spectrometer and employed our local calibration as a rough check. The screw calibration factor we used was

$$SF = 1.00029 \text{ turns/X.U.}$$

In order to locate to 1 part in 10^5 an extrapolated position 15 X.U. from a reference point in the region of 500 X.U., one need know the screw calibration factor to but 1 part in 3000. Thus the entire significance of the decimal digits in the above screw correction factor is only barely necessary for our required precision. So far as they are able, the fluorescence lines confirm that the local screw calibration factor is close to unity.

One more topic concerning the wavelength calibration is the question of the effects of ambient temperature on the spectrometer. One may safely assume that the massive metal parts of the spectrometer are not subject to rapid temperature fluctuations. Furthermore these parts will tend to have very nearly the same relative expansions or contractions. Though lengths may change slightly, the angles between members will remain sensibly constant. The angles at which radiation strikes and leaves the atomic planes in the quartz crystal will therefore be unaffected by temperature changes. On the other hand the crystal grating constant is temperature dependent, and the wavelength diffracted by the crystal is proportional to the grating constant as well as to the sine of the grazing angle (referred, of course, to the crystal planes, not to its surface). At different temperatures a single setting of the spectrometer drive-screw will result in the diffraction of different wavelengths. The magnitude of this change in wavelength is not always negligible.

The relation holding between wavelength, grating constant, and grazing angle for radiation diffracted by the crystal is given by the Bragg law for first order

$$\lambda = 2d \sin\theta \tag{1}$$

Since the spectrometer is of the transmission type, no correction need be made for index of refraction[15]. For constant θ differentiation of (1) with respect to temperature and division of the result by (1) leads to

$$\frac{\delta\lambda}{\lambda\,\delta T} = \frac{1}{d}\,\frac{\delta d}{\delta T} = \alpha \tag{2}$$

For quartz the coefficient α is about 10^{-5} per degree C. Hence a deviation of 1 part in 10^5 per degree may be introduced into all wavelength measurements. For this reason the temperature of the quartz crystal was recorded during runs across the short wavelength limit and at the time of the fluorescence measurements, and all data were referred to 24°C. At this temperature the spectrometer setting 9495.000 corresponds to the wavelength

$$\lambda_{SWL}^{(S)} = 504.833 \pm 0.008 \text{ X.U.} \qquad (3)$$

Because of the drum and vernier the drive-screw can be set to 0.001 revolutions with negligible error. The complete calculation of (3) will be given in a later section.

IV. Procedures in the Experiment

It seems advisable before proceeding to a discussion of the results of the h/e experiment to insert a short chapter in which the various steps in making a run across the short wavelength limit are described in detail. The steps will be treated in the order in which they arise.

(1). Pre-heating

Almost all of the equipment required rather extensive preheating before its operation was reliable. For example, both voltage dividers were operated at a fixed point somewhat above room temperature, and about two hours were required to bring these instruments up to operating temperature. The heating process could be hastened considerably in the case of the precision divider by subjecting it to 25 kilovolts from the tube power supply and thereby adding 625 watts to the 250 watts supplied by the divider's Calrod heater. On the other hand, alongside the full-power output of the 300-watt Calrod in the 100-megohm divider the additional 6.25 watts available from the big power supply was a mere whisper. Incidentally, once operating temperature for the 100-megohm divider had been reached its heater was put on a 30% duty cycle at about 70 watts.

The electronic stabilizer, though protected from drafts by the aluminum panels of the stabilizer box, also required between two and three hours to reach very good stability. The stabilizer would operate after an hour's warm-up, but both vacuum tube emission and DC electrode potentials tended to be erratic until some time after the unit had been turned on.

Once we had become accustomed to the vagaries of these pieces of equipment and acquainted with the reasons for their troubles, we adopted the practice of leaving all three of them under power almost continuously. The two pump motors were oiled once a day but were allowed to run without stopping for many days at a time. The stabilizer, when not in operation at 25 kilovolts and deriving its power from storage batteries and, via a small power transformer, from the built-in filament transformer in the Philips set, could readily be switched to an auxiliary power supply. In this way both the vacuum tube heaters and the external circuit components could be kept warm and ready for immediate use.

We were not able to apply a similar technique to the X-ray tube or to the counting equipment. It would have been possible to keep the X-ray tube filament warm, and it was, of course, left on during the few minutes taken up by bridge measurements of the 100-megohm divider, but we could not maintain the entire tube at operating temperature through the nights without applying the high voltage. It seemed highly undesirable to leave the tube unattended while under voltage. It was therefore necessary to devote approximately an hour to warming the X-ray tube before a day's operations could be started.

The electronic counting equipment generally required little more than half an hour to reach satisfactory stability. The delay caused by the X-ray tube provided an opportunity to make the checks discussed in the next subsection.

(2). Equipment Checks

After the electronic counting equipment had warmed sufficiently and before the X-ray tube could be operated with the stability necessary for a run across the short wavelength limit, a check of the electronics could be made. The two scalers could be connected in parallel and their counting rates compared. Normally the two scalers behaved very well, but occasional difficulties were encountered with the scaler used on the differential channel of the discriminator. Further warming frequently, but not always, enabled this scaler to recover.

Another check concerned the differential discriminator itself. We found that, owing to small changes in voltage applied to the xenon counter from one day to the next, and even sometimes in the course of a day, the peak of the pulse distribution curve would shift with respect to the setting of the "window" in the differential discriminator channel, and spurious changes in counting rate would result. In the course of a run such a shift, which occured very rarely after sufficient warming, could be recognized and remedied.

While the X-ray tube was still warming up, counting rates were likely to decrease. Therefore the proper setting of the differential discriminator "window" was found by scanning the pulse distribution while using the integral channel as a monitor. The "window", 15 volts wide, was set to cover that range above low voltage background which gave the highest relative counting rate. Normally the differential channel passed pulses of amplitudes between

35 and 50 volts, about 1/3 of the pulses passed by the integral channel. The integral channel, which passed all pulses of amplitude greater than 20 volts, was virtually insensitive to the small changes which could introduce wild results in the differential channel.

The galvanometers were zeroed and the potentiometer current was standardized during this warm-up period. The galvanometers generally required very little adjustment, since neither of them was normally subjected to more than a very small deflection. The potentiometer current, supplied by a 6-volt storage battery, usually required a small adjustment at the beginning of each day. The electromotive force of the working standard cells was also frequently checked at this time.

(3). Initial Settings

The run proper began with the initial setting of the 100-megohm divider. Before the X-ray tube, now stable at the approximate operating voltage and current, was switched off and the precision divider connected in its place, the electrostatic voltmeter, permanently attached to the electronic stabilizer box, was checked against the potentiometer to make sure that it indeed read approximately the desired operating voltage. The calibration galvanometer was connected to the junction between the divider arms and the tracking galvanometer and potentiometer were removed. The high voltage was then momentarily switched off. The precision divider link was placed in the mercury cups upon removal of the tube link. The high voltage was rapidly returned to the operating point and held there for 5 minutes. During this time the bridge formed by the two dividers was balanced at intervals of 1 minute by means of the 100,000-ohm decade box in the high resistance arm of the 100-megohm divider. All elements of the bridge including the galvanometer, its reversing switch and the connecting wires were thoroughly protected from leakage by either lucite or amber insulation. The ground points of all the insulators were connected either by copper sheet or by copper wire, and the operator himself stood on a grounded copper sheet. The balancing resistance in the 100-megohm divider at the end of the 5-minute calibration period was left untouched throughout the remainder of the run.

When the bridge measurement had been completed the high voltage was again switched off and the tube and voltage measuring instruments were replaced. The voltage was returned approximately to the operating voltage by means of the main Variac and electrostatic voltmeter, and the tube current was adjusted to 10 milliamperes by means of the fine control Variacs in the filament power supply and the current milliammeter.

Usually the tube current would return immediately to the value obtaining before the bridge measurement. On the other hand, since the electronic stabilizer bias had been shifted to prevent a large stabilizer drop during the measurement, the high voltage always had to be reset. The resetting was a simple matter. The voltage was first set about 500 volts too high as indicated by the electrostatic meter. The stabilizer bias was then brought up by the coarse control rod until the stabilizer was at its optimum operating point, and the voltage was checked against the potentiometer setting for the first point of the run. Further adjustments were then made by working the fine control Variac in the primary of the high voltage supply against the stabilizer control rods until the voltage was correct and the stabilizer was operating in its best range. The whole process rarely took more than 3 minutes.

It will be noted that the X-ray tube was not operative during the bridge measurement. Though the filament power and water cooling remained on, the high voltage was disconnected. The 5-minute interval was sufficiently long to make the tube performance slightly irregular for the following half hour, and for this reason all runs were begun on the background some 35 volts away from the expected position of the limit. However, since the first half hour of a run covered less than 15 volts, small irregularities in this region could not appreciably affect the location of critical points on the isochromat.

As the spectrometer was not shifted during runs but only for wavelength calibration measurements, it did not require daily resetting. It was in fact not touched at all except for calibration. The same applies to the balanced filters which, once balanced, were not thereafter disturbed.

(4). Counting Procedure

Counting periods were divided into 5-minute intervals. At a minimum counting rate in the integral discriminator channel of about 200 to 225 counts per minute a fairly satisfactory sample could be obtained in 5 minutes and a neighboring point could be reached before any large change in conditions could take place.

The sequence of counting through the two filters and changing the voltage by increments followed one of two patterns. In all cases the potentiometer settings were the same but in some of the runs double readings through the silver filter were taken at points lying within about 15 volts on either side of the limit. In this way, though the background readings through the palladium filter were more widely separated in time, much more information could be obtained in this highly important region near the limit. Furthermore, since readings through the palladium filter were intended primarily as background readings and were not themselves to be used to locate the limit, these readings were taken at alternate voltage settings only in all but the first six runs.

The counting intervals were timed by a stop-watch with a 30-second sweep.

(5). Voltage Increments

The voltage settings for a run were made by moving the large dial of the Rubicon potentiometer to the new position and then turning the stabilizer fine control rod to bring the tracking galvanometer back to zero deflection. Since the potentiometer dial was 5 inches in diameter and 30° represented 100 microvolts, the dial could be set with ease within 1 or 2 microvolts. Increments of 100 microvolts on the potentiometer represented approximately 2.25 volts on the X-ray tube.

Normally an entire run could be covered without readjustment of the stabilizer operating range. In the few cases in which a small adjustment was desirable it could be made simply by turning the fine control Variac in the high voltage primary. The effect of such a change was to relocate the stabilizer operating point without appreciably altering the voltage applied to the X-ray tube. The necessary adjustment could be made in 15 seconds between counting intervals.

During the counting intervals the voltage could be left pretty much to itself except for the first half hour or so of a run, while the X-ray tube was readjusting to working conditions. During this time frequent adjustment by means of the Helipot control rod was necessary, and the voltage was tracked constantly by the galvanometer. Thereafter it proved possible and often desirable to release the galvanometer key from the locked position and check the voltage about every half minute. This practice was, of course, not adopted until we were certain that the equipment was operating satisfactorily and was only then adopted in an effort to protect the galvanometer and the bucking cell from damage due to a breakdown of the high voltage. Such breakdowns occasionally happened, for reasons unknown, but we were fortunate in that no serious damage was done to any of the equipment after the set of final runs had been started.

(6). Final Bridge Measurement

At the completion of a run, which lasted either 3 1/2 or 4 1/2 hours, depending on whether double readings had been taken through the silver filter, the 100-megohm divider was again set according to the procedure described above in subsection (3). Except in one or two runs, the final bridge balance differed from the initial by less than 5000 ohms (5 parts in 10^5). Since the ambient temperature in the power supply room (as well as the crystal temperature) was recorded approximately every half hour throughout a run, and since the relation between the ambient temperature and the divider ratio was pretty well known, corrections to the divider ratio for intermediate potentiometer settings could readily be made. These corrections along with many others will be treated in the following chapter.

V. Results of the Experiment

A. The Data. The data were collected in the course of 16 "final" runs across the short wavelength limit. In this section we shall discuss the collection of raw data and their reduction.

(1). The Collection of Raw Data

The detailed account of the procedure followed in a run across the short wavelength limit has been given in the preceding chapter. In each of the sixteen runs an attempt was made to record all information which might conceivably be of use in the reduction of the final data.

The primary data consisted, of course, of the counts recorded in five-minute periods through the silver and palladium filters and of the corresponding potentiometer readings. In addition, we recorded the balance resistance in the 100-megohm divider, X-ray tube current, counter voltage, discriminator bias settings and the electromotive force of the bucking standard cell. We noted the ambient temperature in the power supply room and the temperature of the quartz crystal at half hour intervals.

These temperature readings were necessary for subsequent correction of the voltage applied to the X-ray tube. We were little troubled by severe temperature changes during the final runs, since preliminary work had apprised us of this danger. We found in our early work that the room temperature in our laboratory was held constant only within about 3° C. and that sudden changes would occur when the air-conditioning was turned on. We therefore arranged to have the vents blocked off and saw that the doors were not left open. As a result, though the room was not properly ventilated, its temperature was nevertheless not subject to rapid changes.

We could not so control the electronic counting equipment, and indeed we found that during the last half dozen runs one of our scalers, even after a long warm-up period, would frequently register counts when not connected to the discriminator output. This scaler was connected to the differential discriminator channel, and the unreliability of this scaler was one of the reasons for our abandoning the data taken through this channel.

We found also that our first runs made after the X-ray tube had been idle for some time were not quite so satisfactory as those made after the tube had been thoroughly warmed. However, since even in these runs the results were by no means extraordinarily wild, they were given equal weight in the final reduction of the data. This action is not so cavalier as one might think if one recalls that the irregularities arising from insufficient warming of the X-ray tube normally disappeared long before the critical regions of the isochromat were reached.

(2). Reduction of the Data

Though the taking of the data required long and tedious hours in attendance on the equipment, the process was essentially routine. The reduction of the data on the other hand required intensive effort over a relatively short period.

In bold outline, the reduction of the data consisted fundamentally of a revision of counting rates based on suitable equivalent voltage corrections. The data taken on a single run were subject to such large statistical fluctuations that much of the information used in making corrections was necessarily derived from mean values found by averaging.

As mentioned above, the primary data consisted of counts recorded in the 5-minute counting periods and the potentiometer readings for those periods. The problem which confronted us was that of adjusting recorded counts to correspond to the true voltage applied to the tube. Clearly the mere averaging of counts recorded at the same potentiometer setting was an inadequate solution, since in a few cases the actual voltage differed by 4 or even 5 parts in 10^5 from the value determined by means of the potentiometer. We must stress that such extreme cases, though they did occur, were infrequent. Nevertheless a satisfactory adjustment was made according to the procedure described below.

First the counts recorded for a given run were subjected to normalization. Because of the fact that the counter voltage and the discriminator biases were not exactly reproduced from one run to the next, it was clear that the number of counts might change by a small amount. The result, due to this reason alone, would be a slight increase or decrease in total counts recorded during a run and a general tendency for the number of counts at any given voltage setting to change in the same direction from the mean number of counts at these points. We found one or two runs which bore out this notion. In the end it developed that this normalization was not necessary. It was done to facilitate a statistical treatment which was not used.

Once the runs had been normalized, the counts recorded for each nominal value of applied high voltage—potentiometer settings gave nominal values for voltage abscissae—were averaged and the mean values were plotted. A smooth curve was drawn through these points. The slope of this rough curve was used to make adjustments in recorded counts taken at points whose voltage, because of drift in the ratio of the 100-megohm divider, did not in fact coincide with the value indicated by combining the initial setting of the divider ratio with the potentiometer reading. If C is the number of counts actually recorded and C' the equivalent number of counts corrected to the abscissa V, then the relation between C' and C is given approximately by

$$C' = C + \left(\frac{\partial C}{\partial V}\right)_V \Delta V \qquad (1)$$

The departure ΔV of the voltage from the desired value could be readily estimated even in the worst cases to an accuracy of 1 part in 10^5. Furthermore it developed that the rough curve used for this adjustment differed very little from the final curve and that the corrections to recorded counts were almost unnoticeable alongside the normal statistical fluctuations. Nonetheless the procedure permitted us to reduce our estimate of the probable error in the voltage abscissae. Since the correction applied was proportional to the slope of the isochromat, we were indeed fortunate that the steep slope just above the threshhold was pretty well defined and that the voltage deviations were always less than 3 parts in 10^5

in this critical region.

The voltage deviations ΔV due to drift in the divider ratio were computed on the assumption that the drift was very nearly linear throughout the runs in spite of the fact that the temperature readings did not always change so smoothly. We nevertheless assumed a linear variation in the divider ratio because of the known sluggishness of the response of the 100-megohm divider. It will be recalled that two hours were required to bring this resistor up to working temperature while supplying it with 300 watts from its Calrod heater. We were therefore quite confident that the divider ratio, though temperature sensitive, would not respond to any but slow temperature variations. Furthermore we never encountered a large discrepancy from proportionality between the 100-megohm divider ratio and the mean value of the ambient temperatures recorded (at the time of balance) for the same value of balancing resistance. The temperature readings made during a run, when combined with our knowledge of the general behavior of the divider and of the end-point measurements of its ratio, enabled us to make the above voltage corrections with great confidence. We must repeat that very few of these voltage deviations amounted to as much as 3 parts in 10^5. Most of them were less than 1 part in 10^5.

A second correction arising from temperature changes is perhaps less devious. This correction concerns the wavelength of the radiation diffracted by the quartz crystal. As mentioned in the chapter on calibration, the wavelength of the radiation selectively diffracted by the crystal will change by 1 part in 10^5 per degree

Centigrade. Because the crystal is held in a massive steel holder which is integral with the frame of the spectrometer and is futhermore protected from drafts (as well as from curious hands) by an aluminum house, it is not highly temperature sensitive and few runs were affected by changes in crystal temperature. Though the crystal temperature was generally quite constant throughout a given run, it was nevertheless not always the same from one run to the next. A voltage rather than a wavelength correction was applied to meet this situation.

Clearly, for the small changes involved, if the crystal selects radiation which is longer by 1 part in 10^5 than the radiation for which the spectrometer is presumed to be set, one may consider the voltage to have been lowered in the same proportion. Since the total variation in crystal temperature over all of the runs combined was about 2.5° C. and the variation within one run rarely was as great as 1° C. , one could, except for the wavelength calibration, have neglected variations in crystal temperature without appreciably affecting the results of the experiment.

Certain additional errors were not compensable as were those above, and correction was therefore not possible. The remaining errors will be discussed in section C below. The above corrections were applied chiefly in order that we might be able to reduce our estimated probable error, and in a few cases the correction was of some significance. For the most part they were unnecessary as all voltage corrections were swamped by the error in locating the limit on our isochromats.

Besides the reduction in error we had in mind the possibility that some excessively wild counts might be discarded on statistical grounds provided the counts recorded at a real voltage could be adjusted to equivalent counts at the same nominal setting in each run. In the end no data taken through the integral channel of the discriminator were discarded. The curves (Figs. 15 and 16) include all of the data, adjusted to the same abscissae for each run and averaged. Thus each ordinate plotted gives the mean number of counts observed in 5 minutes averaged for from 16 to 22 separate measurements. The indicated spread about the mean value for any ordinate is given by the square root of the total number of counts at that voltage divided by the number of separate 5-minute observations made at that point (16 or, for silver points in the middle of the curve, 22).

B. Isochromats. The reduced data were plotted on a large scale and smooth curves were drawn through them (Figs. 15 and 16). These curves are known as isochromats inasmuch as they represent the change in intensity at fixed wavelength as the electron bombarding voltage is increased and the spectrum is moved across the "window" of the monochromater. Plotted ordinates are counts in 5 minutes and abscissae are volts measured by the potentiometer. Because the monochromater is not infinitely sharp, the fine detail of the true spectrum is necessarily obscured. One can nevertheless determine the voltage abscissae corresponding to the wavelength of the center of the monochromater "window".

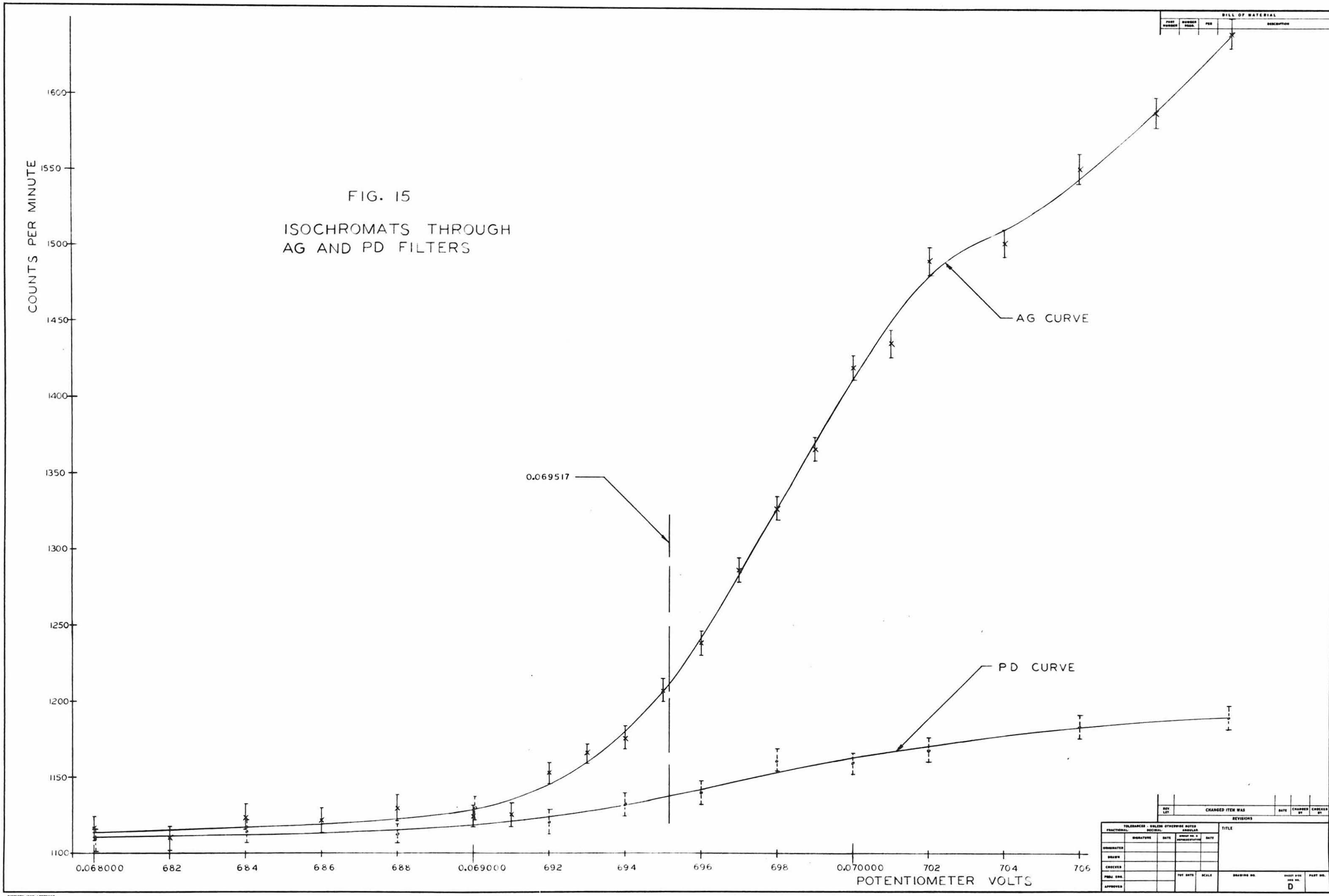

FIG. 15

ISOCHROMATS THROUGH AG AND PD FILTERS

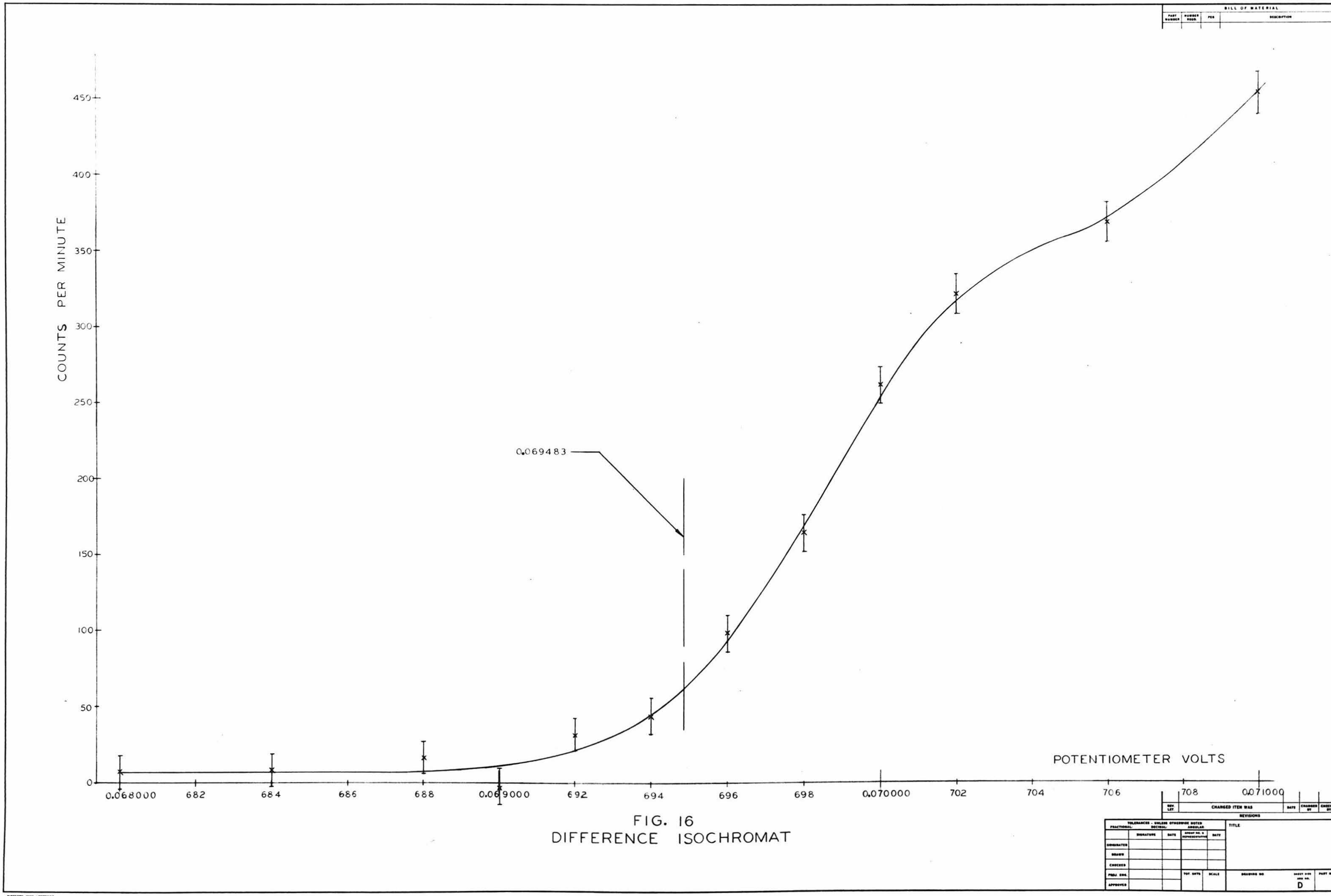

FIG. 16
DIFFERENCE ISOCHROMAT

(1). Theory of Maximum Bending

The theory of the interpretation of the isochromats has been given elsewhere [7, 10] and it is sufficient at this time to give merely a brief account of the salient features.

Let the function $y = f(z,x)$ represent the true spectrum of radiation whose structure is to be observed. In this expression y is proportional to the intensity at a position x, which represents voltage, wavelength, frequency or some other equivalent variable. The variable z represents one of the limiting values of x, e.g. the short wavelength limit. Observation of the spectrum is accomplished by means of an analyzer which in our case is fixed in position on the axis but is otherwise unspecified. It is the "window" of the spectrometer, the fold of the crystal, the slit, the aberrations, etc. In the experiment the spectrum is moved by increments of z. Furthermore within the small range of z relative to its absolute value (in volts, X.U., or other convenient units) the value of y for x close to its limit z will depend on the difference $(z-x)$ alone. This assumption implies that the shape of the spectrum, in the region of interest at least, is practically independent of z.

Let us represent the analyzer "window" by the function $g(x)$, and for convenience we shall consider x and z to be wavelength variables. Furthermore we shall reflect the wavelength scale such that the longest wavelength in the spectrum will be represented by $x = -\infty$ and the shortest by $x = z$. The observed ordinate corresponding to a fixed position of the short wavelength limit and of the analyzer will then be given by

$$Y = F(z) = \int_{-\infty}^{z} g(x)f(z - x)dx \tag{1}$$

In this equation Y is the observed number of counts when the short wavelength limit of the spectrum is at z. For each position of z the integral is taken over the entire spectrum from longest to shortest wavelength. In our case the function $f(z - x)$ is presumed to vanish for all values of $x \geq z$; that is, there is no radiation beyond the short wavelength limit. For this reason the upper limit of the integral (1) is set equal to z. In addition, the function $g(x)$ is presumed to vanish with sufficient rapidity that the function $f(z - x)$ need not be modified for different values of z. The problem before us is how best to locate the position of z with respect to the center of the "window" $g(x)$. It developes that one need not know this separation for every position of z. In fact this separation would be very difficult to determine. Now the wavelength z is related to the voltage through which the bombarding electrons in the X-ray tube fall by the Einstein photoelectric equation, and in our experiment the value of z is changed by changing this voltage. Though one cannot determine the separation of z from the center of the spectrometer "window" for all values of the bombarding voltage, one can, on the other hand, determine the voltage for which this separation is zero.

By differentiating (1) twice one obtains

$$F''(z(= g(z)f'(0) + \int_{-\infty}^{z} g(x)f''(z - x)dx \quad (2)$$

It has been shown elsewhere [10] that the second term on the right of (2), which represents the effect on the second derivative $F''(z)$ arising from changes in slope of the radiation spectrum, is negligibly small. The first term is the product of the slope of the true spectrum at the quantum limit and the "window" curve evaluated at that point (x = z). Clearly $F^{\pi}(z)$ will have its maximum value when z corresponds to the wavelength at the peak of the "window" curve g. For this reason the second derivative of the observed isochromat is examined. The quantum limit lies concealed under the point at which the rate of change of the slope of the isochromat is greatest.

(2). Practical Method of Locating the Short Wavelength Limit

In practice the variable z is not known in wavelength units until after the experiment has determined the ratio h/e. Instead one knows the voltage through which the bombarding electrons in the X-ray tube fall and assumes a proportionality between this voltage and the value of the wavelength at the spectrum limit. The observed isochromat is therefore a plot of counts against voltage. In our curves the voltage plotted is that measured by the Rubicon potentiometer reading corresponding to the point of maximum second derivative of the isochromat.

The point of maximum second derivative was located in the following way. First the measured points were plotted on large scale graph paper—50 millimeters corresponded approximately to 4.5 volts on the X-ray tube and exactly to 200 microvolts on the potentiometer, while 1 millimeter corresponded to 1 count in 5 minutes—and the "best" smooth curve was drawn through them. As always the "best" curve is largely a matter of opinion. The curve must be drawn with due consideration given to statistical fluctuations in the mean number of counts, to resolving power of the monochromater, etc. Also one's opinions may change, and for this reason several curves were drawn through the measured points before the final curve was selected. We found that the location of the limit varied by about 3 parts in 10^5 among the several "good" curves. In order to form a satisfactory estimate of probable error, we drew in addition several curves which were intentionally not good in that they ignored some of the data while remaining close to the statistically determined ranges of the rest. The process resulted in shifts of as much as 7 parts in 10^5.

Each of the smooth curves whether "good" or "poor" was treated analytically by a method of numerical differentiation based on the calculus of finite differences given by Whittaker and Robinson[23]. The method consists essentially of the fitting of a high degree polynomial to an array of points read from the graph at equal intervals of the abscissae and a simultaneous differentiation of the resulting polynomial. Rather than calculate the second derivative we chose to calculate the third and locate the abscissa corresponding to the vanishing of the third derivative of the isochromat. The third derivatives of the curves in Figs. 15 and 16 are given in Fig. 17.

The virtue of this method is that by selecting abscissae rather widely spaced (30 or 40 microvolts on our graphs) one can include a large region of the curve in the calculation of the third derivative at the center of this region. Our calculations were based on regions 500 microvolts wide. On the other hand, this method of locating the limit is deceptive in that it is in reality no more precise than the smoothing of the curve is reliable. Since the restrictions imposed by the statistical spreads are more severe for the data taken through the silver filter ("silver curve") than for the difference curve obtained by point-for-point subtraction of measured palladium counts from corresponding silver counts and geometric combination of the spreads, one should presumably be able to draw better curves through the former. This conclusion is *a priori*, and in previous work [10] in which filters were used it was found to be strongly modified by experimental conditions. In our experiment, where the background in the presence of radiation leveled off very rapidly, the subtraction of the palladium curve from the silver did not noticeably facilitate interpretation of the data and in fact made the interpretation less precise by permitting a wider variety of possible curves. Nevertheless the results of both analyses are given in the following section.

The striking features of the true spectrum, the peaks found by Ohlin[8], by Bearden and Schwarz[11], and by Bearden, Johnson

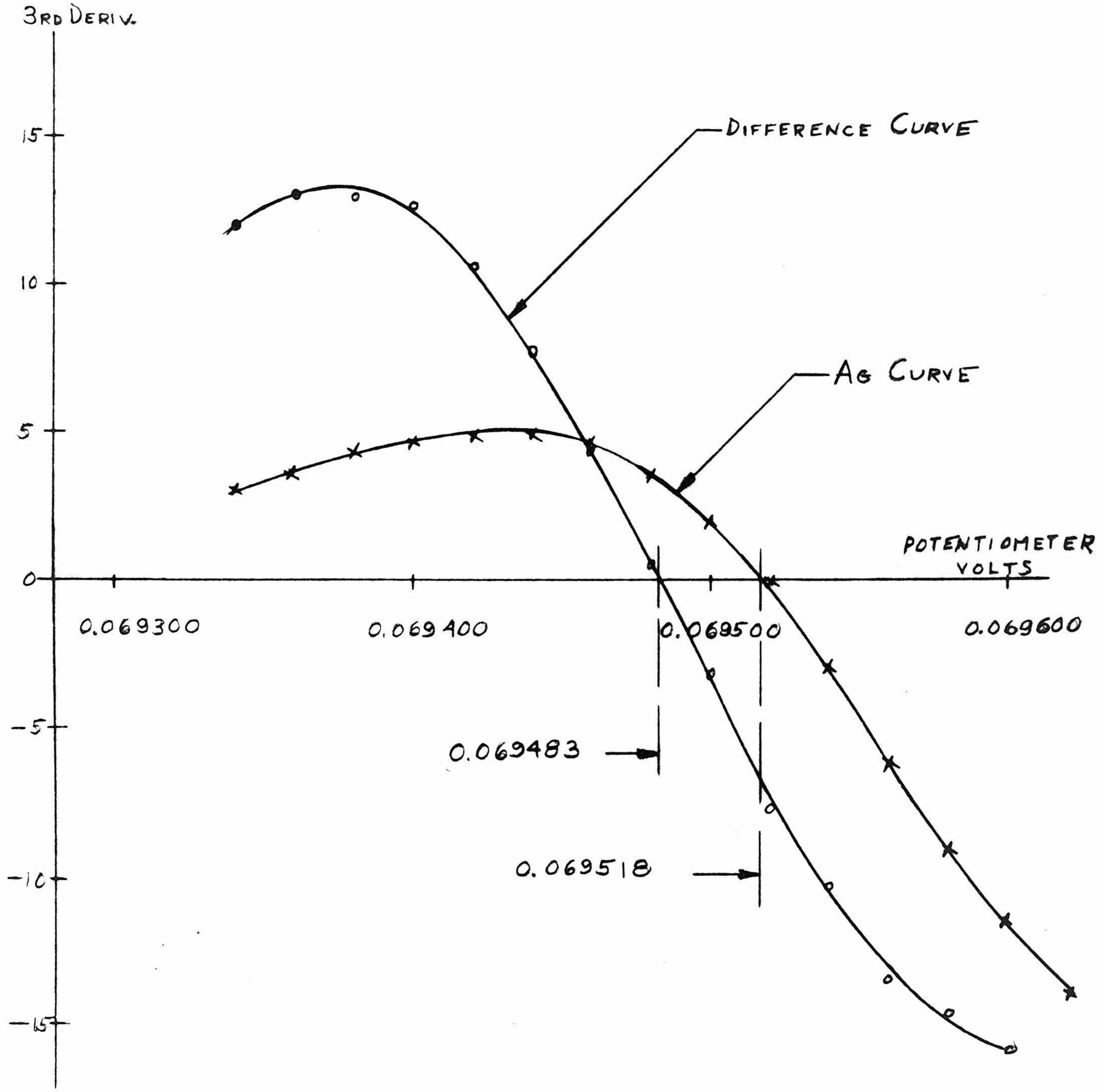

FIG. 17
3RD. DERIVATIVES OF ISOCHROMATS

and Watts[12] could not be completely resolved with our equipment, though our curves give a definite indication that the features are indeed present. Whereas these peaks are of considerable interest and may in the future play a significant role in the study of the solid state of metals, they are not of supreme importance for the present experiment. Because of the limitations of our equipment, we therefore concentrated our attention on the fillet at the end of the spectrum. The limit alone is sufficient to determine the ratio h/e. It is questionable, in view of other recent work[12], whether the peaks would serve this purpose, since their location relative to the spectrum limit appears to be voltage dependent. Our experiment, however, cannot very well be taken to give good confirmation of this hypothesis.

C. Final Results and Errors. The calculation of final results consists of two steps, the calculation of the center of the wavelength band selectively diffracted by the quartz crystal and the calculation of the electron energy just sufficient to produce X-rays of wavelength corresponding to the center of the band (and longer) but no shorter.

(1). Calculation of the Wavelength

Since known corrections, even those involving wavelength, have been made in terms of equivalent volts, the calculation of the wavelength at the center of the spectrometer "window" can be based entirely on the calibration measurements and on the constant $\lambda^{(g)}/\lambda^{(s)}$ which converts X.U. into centimeters. All calibration measurements were referred to a crystal temperature of 24° C. and the wavelength at the center of the band diffracted by the spectrometer has been given in Chapter III. This wavelength was

$$\bar{\lambda}^{(s)}_{SWL} = 504.833 \pm 0.008 \text{ X.U.} \qquad (1)$$

We shall now show how this value and its estimated probable error were obtained. The method of producing and locating the fluorescence lines was described in the chapter on calibration, and the spectrometer settings for their peaks were given. It will be convenient to subtract each of these numbers from 10,000, the reading of the spectrometer drive-screw register when the source-mounting beam makes essentially zero angle with the principal normal to the surface of the quartz crystal*. The spectrometer positions in turns from the setting 10,000 with the estimated probable errors in the location of the lines were

$$\begin{aligned} P_{SnK\alpha_1} &= 489.737 \pm 0.005 \text{ turns} \\ P_{SnK\alpha_2} &= 494.170 \pm 0.010 \text{ turns} \\ P_{AgK\beta_1} &= 496.19 \pm 0.015 \text{ turns} \\ P_{AgK\beta_3} &= 496.80 \pm 0.02 \text{ turns} \end{aligned} \qquad (2)$$

*The exact coincidence of this normal with the central axis of the source-mounting beam occurs at a drive-screw setting which is not precisely 10,000 but at a point, know as the β-point, which is very slightly displaced from 10,000. Since we are locating our settings by means of radiation of known wavelength rather than by absolute calibration of the drive-screw, the position of the β-point is in our case immaterial. Its small displacement results merely in an additive correction to all spectrometer readings which depend on drive-screw calibration alone. Our preliminary work, done before the location of the fluorescence lines, required a knowledge of the position of the β-point.

The estimated probable error in the location of the lines is based on examination of the measurements themselves and on the discrepancies arising from determination of the line location by means of intersecting tangents and by means of finding the position of the mid-point of the full width at half maximum intensity. Clearly the sharper lines are easier to locate than the others. The tin $K_{\alpha 1}$ line was exceedingly sharp (Fig. 13) and therefore could be located with good precision. On the other hand the silver doublet was only partially resolved (Fig. 14). The peaks of a close doublet tend to be drawn together and rounded off. It may be of interest that the estimated probable errors in line location are very nearly in inverse proportion to the ratios of peak to saddle-point counting rates.

The standard setting of the spectrometer for the runs across the short wavelength limit was

$$P_{SWL} = 505.000 \pm 0.001 \quad \text{turns} \tag{3}$$

As mentioned elsewhere in this paper, the spectrometer could be set with great precision by means of the divided drum and vernier. Furthermore, in order that no error should result from backlash of the drive-screw gears, the instrument was always set by rotating the drive-screw to the desired position from the long wavelength side.

Subtracting the set of position numbers (2) from the setting (3), one obtains the separation of each fluorescence line from the short wavelength setting. For convenience the complete subscripts in (2) may be abbreviated without loss in meaning. The separations were

$$\Delta P_{\alpha_1} = 15.263 \pm 0.005 \quad \text{turns}$$
$$\Delta P_{\alpha_2} = 10.830 \pm 0.010 \quad \text{turns} \qquad (4)$$
$$\Delta P_{\beta 1} = 8.81 \pm 0.015 \quad \text{turns}$$
$$\Delta P_{\beta 3} = 8.20 \pm 0.02 \quad \text{turns}$$

These separations are given in revolutions of the spectrometer drive-screw and must be converted into X.U. by means of the conversion factor

$$\text{S.F.} = 1.00029 \pm 0.0003 \quad \text{turns/X.U.} \qquad (5)$$

The estimated error in the screw conversion factor has been purposely increased above former estimates from other work because of the fact that our settings were located very near to one end of the spectrometer range. Furthermore we did not ourselves determine this factor. Upon division of the turn separations (4) by (5) one obtains the wavelength separations in X.U.

$$\Delta\lambda^{(s)}_{\alpha 1} = 15.259 \pm 0.0067 \quad \text{X.U.}$$
$$\Delta\lambda^{(s)}_{\alpha 2} = 10.827 \pm 0.0105 \quad \text{X.U.} \qquad (6)$$
$$\Delta\lambda^{(s)}_{\beta 1} = 8.807 \pm 0.015 \quad \text{X.U.}$$
$$\Delta\lambda^{(s)}_{\beta 3} = 8.198 \pm 0.02 \quad \text{X.U.}$$

The estimated probable errors above are compounded in the usual way of the error in initial location of the lines and of the error in the spectrometer screw factor. These errors rather than errors in line location alone were used to determine weights for calculation of the mean wavelength to which the spectrometer was set for the runs across the short wavelength limit. The weights were made proportional to the inverse squares of these errors.

The separations (6) were added to the wavelengths of the respective fluorescence lines given in Chapter III and listed again below

$$\begin{aligned}
\lambda^{(s)}_{\alpha 1} &= 489.57 \pm 0.005 \text{ X.U.} \\
\lambda^{(s)}_{\alpha 2} &= 494.02 \pm 0.005 \text{ X.U.} \\
\lambda^{(s)}_{\beta 1} &= 496.01 \pm 0.005 \text{ X.U.} \\
\lambda^{(s)}_{\beta 3} &= 496.65 \pm 0.005 \text{ X.U.}
\end{aligned} \tag{7}$$

The resulting weighted mean value of the wavelength at the center of the spectrometer "window" for the short wavelength limit runs was

$$\bar{\lambda}^{(s)}_{SWL} = 504.833 \pm 0.008 \text{ X.U.} \tag{8}$$

The estimated error is the error in the determination of the weighted mean arising from the four separate measurements with their corresponding errors. The determination of the error by internal consistency differs insignificantly from the error by external consistency given above.

One further step is required. The wavelength (8) is in X.U. and must be converted into centimeters. The conversion factor used was

$$\frac{\lambda^{(g)}}{\lambda^{(s)}} = 1.001995 \pm 0.000014 \times 10^{-11} \text{cm/X.U.} \qquad (9)$$

This value for the ratio of grating to Siegbahn wavelength units has been used in place of the more familiar value 1.002030×10^{-11} cm/X.U. as a result of an extensive re-examination by DuMond[24] of the complex of atomic constants in the light of recent (1949-50) very precise experiments. The conversion of (8) by multiplication into (9) gives the final value of the wavelength used in computing h/e

$$\bar{\lambda}^{(g)}_{SWL} = 505.840 \pm 0.011 \times 10^{-11} \text{cm} \qquad (10)$$

(2). Calculation of the Voltage

The calculation of the voltage applied to the bombarding electrons in the X-ray tube is straightforward. Since the voltage measurement was very precise the final error in the bombarding voltage was determined almost entirely by the error in locating the point of maximum bending of the isochromats.

The point of maximum bending was found for both the silver and the difference curves in spite of the fact that the difference curve was considered the less reliable of the two. The results were in very satisfactory agreement, as it happened, but as they arose from independent estimates based upon the same data rather than from estimates of independent data their values can be used to determine only that error inherent in the process of plotting, smoothing and differentiating of the curves. The finally assigned error was estimated by means of the "poor" curves. It will be recalled that the "poor" curves were drawn to exclude some data which did not seem regular and to pass through extreme points of the statistically expected spreads. The potentiometer readings corresponding to the points of maximum bending of the two "best" curves were

$$v_{Ag} = 0.069517 \text{ absolute volts} \tag{11}$$

and

$$v_{dif} = 0.069483 \text{ absolute volts} \tag{12}$$

where the subscript "Ag" refers to the curve based on data through the silver filter and the subject "dif" to the curved based on point-for-point subtraction of palladium readings. The final value adopted for the potentiometer reading corresponding to the wavelength $\bar{\lambda}^{(g)}_{SWL}$ was found by averaging (11) and (12) and allowing the respective weights 1.4 and 1.0. These weights were chosen in order to include the condition that the statistical spreads in counting rates near the limit were very nearly in the ratio of 1 to $\sqrt{2}$. The potentiometer reading adopted was therefore

$$v_p = 0.069503 \pm 0.000045 \text{ absolute volts} \tag{13}$$

The estimated probable error is based on our finding that the "poor" curves could shift the low voltage (standard cell voltage added to v_p above) by as much as 70 parts per million. We regard $\pm$ 0.000045 volts as a generous estimate for the probable error in determining the point of maximum bending on our "best" curves.

Alongside the above error the probable error in the potentiometer itself is very small indeed. The limit of error for the range used is 10 microvolts as against the 45-microvolt probable error above. Furthermore the limit of error of the bucking standard cell voltage is a scant 5 microvolts. This standard cell voltage was

$$v_{sc} = 1.019161 \pm 0.000005 \quad \text{(L.E.) absolute volts} \qquad (14)$$

Even if one regards the limits of error of the potentiometer and of the standard cell as probable errors, one obtains for the voltage across the low resistance arm of the divider

$$v_L = 1.088664 \pm 0.000046 \quad \text{absolute volts} \qquad (15)$$

The conversion from low to high voltage is accomplished by means of the precision divider ratio used to set the 100-megohm divider

$$\text{p.d.r.} = 22503.52 \pm 0.10 \text{ (L.E.)} \qquad (16)$$

where again a limit of error is given. The error in divider ratio is generous but still insignificant, and the nominal high voltage is, from (15) and (16),

$$V_N = 24{,}498.7 \pm 1.0 \quad \text{absolute volts} \qquad (17)$$

The error in high voltage does not yet include the effects of uncertainty in the 100-megohm divider ratio, which is no greater than

1 part in 10^5 at any given voltage abscissa, and of possible errors arising in the electronic stabilizer, which may also be estimated at 1 part in 10^5. The effects of ripple have been included implicitly in our original estimate of the location of the point of maximum bending of the isochromat inasmuch as ripple does not on the average dislocate the spectrum limit but merely tends to obscure its mean position. The inclusion of these errors lends to the correct high voltage applied to the tube (but not the full voltage through which the electrons fall),

$$V_H = 24{,}498.7 \pm 1.1 \text{ absolute volts} \qquad (18)$$

One further modification is the additive correction for cathode work function. The arguments leading to the inclusion of this correction have been so thoroughly treated elsewhere [7, 10, 11] that is seems unnecessary to repeat them here. The work function for the tungsten cathode was taken as

$$w = 4.52 \text{ absolute volts} \qquad (19)$$

Clearly any error in tungsten work function would necessarily have to be of the order of an unlikely 20% before the final error would be appreciably altered. Adding (18) and (19) one obtains for the final voltage

$$V_A = 24{,}503.2 \pm 1.1 \text{ absolute volts} \qquad (20)$$

(3). Computation of h/e

Having determined the wavelength at the short wavelength limit and the accelerating voltage corresponding to that limit, we may proceed at once to the calculation of the ratio h/e. The theoretical expression used is

$$\frac{h}{e} = \frac{\bar{\lambda}^{(g)}_{SWL} V_A}{c^2} \times 10^8 \quad \text{erg-sec./esu} \tag{21}$$

It is to be noted that the volocity of light enters twice in the above expression, once in the conversion from frequency to wavelength and again in the conversion from absolute to electrostatic units of voltage. For the velocity of light we have used the value

$$c = 299{,}790.0 \pm 0.9 \quad \text{km/sec} \tag{22}$$

This value for the velocity of light was accepted on the basis of a recent survey[24] of the experiments determining this highly important constant. Clearly in our experiment the estimated probable error in c will have negligible effect on the error in the ratio h/e. The value computed by the substitution of (10), (20), and (22) into (21) is

$$\frac{h}{e} = 1.37912 \pm 0.00007 \times 10^{-17} \text{erg-sec./esu} \tag{23}$$

APPENDIX

Suggestions for Improvement of the Equipment

Before further work can be done profitably with the basic equipment used in this measurement of h/e certain modifications should be made in various of its elements. The modifications discussed below will improve the precision of the experiment through increase in intensity, resolution and stability.

The most severe limitation in the experiment was the deficiency of X-ray intensity. This deficiency was directly responsible for the low counting rates and the long times required to accumulate sufficient data. The first improvement should certainly be the substitution of a more powerful X-ray tube, with the limitation that such a tube still be of the "shockproof" type and amenable to mounting on the spectrometer. New model hooded-anode tubes, which can be operated continuously at 25 kilovolts and 25 milliamperes, are available commercially.

These tubes can furthermore be obtained with focal spots large enough to illuminate the entire crystal without being mounted closer to the slit than was our tube. On the other hand an arrangement could be made without too much difficulty to place any X-ray tube at its best distance from the slit. The spectrometer work load at present precludes alterations to this extent.

Besides increasing the amount of incident radiation by the above two modifications one may in addition improve the detecting equipment in two ways. The counter sensitivity can certainly be improved. The xenon pressure can probably be increased by a factor two and possibly by a factor three without damaging the performance of the counter in the proportional region. It would probably be necessary, because of the higher pressures, to build a counter high voltage supply capable of 2500 to 3000 volts.

The counter geometry can be improved by surrounding each of the four counting wires with a separate copper shield. One would in effect have four small counters inside a single case, and the electric field would be somewhat stronger. The increase in counter sensitivity achieved by the change in geometry would not, however, be nearly so significant as the increase achieved by raising the xenon pressure.

Perhaps the simplest method of increasing the effective counting rate would not involve the counter sensitivity at all but would instead involve the discriminator gate and the X-ray tube filament. It appears quite possible to reduce the fraction of a cycle throughout which the filament is heated and the discriminator gate is closed. Such a change would nevertheless aggravate the stability problem and might for that reason be undesirable. On the other hand an improved stabilizer might be able to provide the necessary compensation.

Improvement of the resolution of the present spectrometer, which cannot be operated at wavelengths longer than about 500 X.U., can be accomplished by substituting a thinner quartz crystal for the 2-millimeter crystal used in this experiment and by narrowing the slit. Both of these modifications will reduce the X-ray intensity and should therefore not be made unless the intensity can be raised considerably by the methods mentioned in preceding paragraphs. One should furthermore recall that the wavelength equivalent of the slit width need not be reduced much below the wavelength equivalent of the crystal aberration; i.e., the focal characteristics of the thin crystal should be thoroughly investigated before a slit width is selected.

The modification of the present spectrometer to permit operation at wavelengths longer than 500 X.U. would be very difficult indeed and is not recommended. One should rather consider the construction of a simple, special instrument for the longer wavelength range. Such an instrument would not, of course, have the versatility of the 2-meter spectrometer; it would be very useful for the h/e experiment, but it would not be readily adaptable to other work unless it was made quite elaborate.

So far as stability is concerned, one should first improve the electronic stabilizer. It is questionable if the ordinary DC amplifier is the best approach to the stabilizer problem. An AC amplifier with a chopper and demodulator might well provide both higher gain and far greater inherent stability. On the other hand the increase in complexity of such a stabilizer might not be justified by the improved operation.

Also in connection with stability one should mention the voltage dividers. The 100-megohm divider has a temperature coefficient of resistance which is uncomfortably large. The complications of enclosing the entire unit in a thermal shield are severe, and one is therefore forced to conclude that the Nichrome resistors should be replaced by Manganin. The same ceramic coil forms could be used. Such a Manganin divider would still require periodic setting by means of the precision megohm divider but would be much less sensitive to ambient temperature. The alternative to such a modification would be the construction of another precision divider with enough resistance and power capacity to remain connected across the X-ray tube during the exploration of the isochromat. This alternative is the logical but costly approach to the problem of voltage measurement.

Some or all of the above-mentioned modifications should be made before the equipment built for the present experiment is used again. By these means it should be possible to achieve a reduction in errors markedly greater than that obtainable by the mere accumulation of more data.

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