EFFECTIVE FIELD THEORIES IN THE STUDY OF $K_L \to \pi^+\pi^-e^+e^-$ AND HEAVY QUARK FRAGMENTATION

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Abstract

This thesis examines several situations in which effective field theories may be used to generate perturbative predictions for nonperturbative phenomena. The decay mode $K_L \to \pi^+\pi^-e^+e^-$ is analyzed in great detail using chiral perturbation theory, and the form factors for the decay are determined, along with the sizes of various CP violating observables. One of these variables turns out to be quite sizeable, approaching 20% for appropriate cuts on the lepton pair invariant mass. Fragmentation of a c quark to the excited charmed baryon doublet Λ_c^* is also studied within the framework of a chiral theory, and various decay distributions are expressed in terms of nonperturbative fragmentation parameters. A perturbative calculation of related fragmentation parameters is also briefly discussed.

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Chapter 1
Introduction

It has long been known[1] that our most promising candidate for a theory of the strong interactions, Quantum Chromodynamics (QCD), displays an energy dependent coupling of quark and gluon fields that decreases with increasing energy. Specifically, in the leading logarithm approximation, the dependence of the QCD coupling α_s on the momentum transfer q is given by[2]

$$\alpha_s(q^2) = \frac{12\pi}{(33 - 2N_f)ln(\frac{q^2}{\Lambda^2_{QCD}})}, \tag{1}$$

where N_f is the number of quark flavors having mass less than $\sqrt{q^2}$, and Λ_{QCD} is an empirically determined quantity, on the order of a few hundred MeV. Although (1) has become a very bad approximation by the time q^2 has fallen to Λ^2_{QCD}, it is sufficient to indicate that the QCD coupling will have become of order one at this point, rendering a perturbative treatment of such low energy processes impossible.

Unfortunately, this is precisely the energy range we find ourselves faced with if we hope to describe interactions of the low-lying mesons and hadrons as interactions of quark bound states. To avoid summing an infinite number of contributions to any physical process, one therefore employs an effective theory, which incorporates all of the symmetries of the full theory, but extends only to a given order in some small parameter. The challenge in constructing any such effective theory, of course, lies in finding that small parameter.

One child of such a prescription, Chiral Perturbation Theory, will be used to study CP violation in the decay $K_L \to \pi^+\pi^-e^+e^-$ in Chapter 2. Here, the relevant small parameters are the ratios of various pseudoscalar momenta to the chiral symmetry breaking scale, Λ_χ. An extension of Chiral Perturbation Theory to include heavy hadron fields, called Heavy Hadron Chiral Perturbation Theory, is then employed in

Chapter 3 to study the fragmentation of a heavy c quark into heavy baryons such as Λ_c, Σ_c, Σ_c^*, Λ_{c1}, and Λ_{c1}^*. This effective theory utilizes, in addition to the systematic expansion of ordinary Chiral Perturbation Theory, an expansion in the inverse mass of the heavy fields. The final section of this work will outline a purely perturbative QCD calculation that could possibly illuminate various parameters appearing in the fragmentation discussion of the previous chapter.

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2. See, e.g., J. F. Donoghue, E. Golowich, and B. R. Holstein, "*Dynamics of the Standard Model*" (Cambridge, MA: Cambridge University Press, 1992), p. 50.

Chapter 2

The Decay $K_L \to \pi^+\pi^-e^+e^-$

In this chapter, we apply the techniques of chiral perturbation theory to the study of the weak decay $K_L \to \pi^+\pi^-e^+e^-$. All of the form factors for the dominant one-photon contribution to the decay, in which the e^+e^- pair is produced by a single virtual photon, are calculated at leading order in Chiral Perturbation Theory. These factors depend on one unknown linear combination of coefficients in the chiral Lagrangian, and the unknown parameter may be fixed by a careful experimental study of the differential decay distributions. The magnitudes of two CP violating observables are also calculated, one of which receives significant contributions from direct CP violation, an effect heretofore unobserved experimentally.

1 Introduction

Although the experimental data set for the decay $K_L \to \pi^+\pi^-e^+e^-$ is fairly meager, the E731 fixed target experiment at Fermilab has already observed 10 such events[1], and its successor, E832, whose primary goal is to look for a nonzero value of ϵ'/ϵ, will reconstruct on the order of 1000 events in this rare mode[2]. The $K_L \to \pi^+\pi^-e^+e^-$ weak decay amplitude is dominated by the process, $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$, wherein a single virtual photon creates the e^+e^- pair. This one photon contribution to the decay amplitude has the form

$$M^{(1\gamma)} = i\,\frac{s_1 G_F \alpha}{4\pi f q^2}\left[iG\varepsilon^{\mu\lambda\rho\sigma}p_{+\lambda}\,p_{-\rho}\,q_\sigma \;+\; F_+ p_+^\mu \;+\; F_- p_-^\mu\right]\cdot \bar{u}(k_-)\gamma_\mu v(k_+) \qquad , \quad (1)$$

where G_F is Fermi's constant, α is the electromagnetic fine structure constant, $s_1 \simeq 0.22$ is the sine of the Cabibbo angle and $f \simeq 132$ MeV is the pion decay constant. The π^+ and π^- four-momenta are denoted by p_+ and p_-, while the e^+ and e^- four-momenta are denoted k_+ and k_-. The sum of the electron and positron four-momenta is defined to be $q = k_- + k_+$. The Lorentz scalar form factors G and $F_\pm$ depend on the scalar products of the four-momenta q, p_+ and p_-. Neglecting CP nonconservation, interchange of the pion four-momenta

$$p_+ \rightarrow p_- \quad \text{and} \quad p_- \rightarrow p_+, \tag{2}$$

transforms the form factors as follows:

$$G \rightarrow G \quad , \qquad F_+ \rightarrow F_- \quad , \qquad F_- \rightarrow F_+ \quad . \tag{3}$$

In the following sections, we compute the CP conserving contribution to the form factors $G, F_\pm$ using chiral perturbation theory at one-loop order. This is necessary, as the tree level amplitude vanishes. The coefficients of some of the local operators appearing at the same order in the chiral expansion are determined by the experimental value of the pion charge radius and the measured $K^+ \rightarrow \pi^+ e^+ e^-$ and $K_L \rightarrow \pi^+ \pi^- \gamma$ decay rates and spectra.

In addition, we compute an important tree level contribution to the form factors $F_\pm$ that arises from the small CP even component of the K_L state. This contribution to the $F_\pm$ form factors, arising from indirect CP nonconservation, has symmetry properties under interchange of the pion momenta opposite to those of the CP conserving contribution. If

$$p_+ \rightarrow p_- \quad \text{and} \quad p_- \rightarrow p_+, \tag{4}$$

then the CP violating one-photon form factors transform as

$$F_+ \rightarrow -F_- \quad , \qquad F_- \rightarrow -F_+ \quad . \tag{5}$$

The expression that results from squaring the invariant matrix element in eq. (1) and summing over e^+ and e^- spins is symmetric under interchange of e^+ and e^- momenta. Physical variables that are antisymmetric under such interchange arise from the interference of the short distance Z-penguin and W-box diagram contributions and the two photon piece with the one photon amplitude given in eq. (1).

In the minimal standard model, the coupling of the quarks to the W-bosons has the form

$$\mathcal{L}_{int} = -\frac{g_2}{\sqrt{2}}\, \bar{u}^j_L \gamma_\mu V^{jk} d^k_L \, W^\mu \; + \; H.c. \quad . \tag{6}$$

Repeated generation indices j, k are summed over 1,2,3 and g_2 is the weak SU(2) gauge coupling. V is the 3×3 unitary Cabibbo–Kobayashi–Maskawa matrix that arises from diagonalization of the quark mass matrices. By redefining phases of the quark fields, it is possible to write V in terms of the four angles $\theta_1, \theta_2, \theta_3$ and δ. The θ_j are analogous to the Euler angles and δ is a phase that, in the minimal standard model, is responsible for the observed CP violation. Explicitly,

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad , \tag{7}$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. It is possible to choose the θ_j to lie in the first quadrant. If this is done, the quadrant of δ has physical significance and cannot be chosen by a phase convention for the quark fields. A value of δ other than 0 or π gives rise to CP violation.

The short distance W-box and Z-penguin Feynman diagrams depend on the V_{ts} element of the Cabibbo–Kobayashi–Maskawa matrix, and it is very important to be able to determine this coupling experimentally. In this chapter we calculate the contribution to the $K_L \to \pi^+\pi^- e^+ e^-$ decay amplitude arising from the Z-penguin and W-box diagrams; this can be determined using chiral perturbation theory since the left-handed current $\bar{s}\gamma_\mu(1-\gamma_5)d$ is related to a generator of the chiral symmetry.

The $K_L \to \pi^+\pi^-e^+e^-$ mode is also fertile ground for constructing CP violating observables. At the present time, all observed CP nonconservation has its origin in $K^0 - \bar{K}^0$ mass mixing, also known as indirect CP violation. In the decay $K_L \to \pi^+\pi^-e^+e^-$, however, we may construct a CP violating variable that gets an important contribution from direct CP nonconservation in the Z-penguin and W-box diagrams. In the K_L rest frame, this variable,

$$A_{CP} = < \frac{(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)}{|(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)|} > \ , \tag{8}$$

is even under charge conjugation and odd under parity. It is also odd under interchange of $\vec{k}_+$ and $\vec{k}_-$. The real and imaginary parts of V_{ts} are comparable, and hence the CP conserving and CP violating parts of the Z-penguin and W-box diagrams are of roughly equal importance. A_{CP}, arising as it does from interference between the penguin and box diagrams and the one-photon pieces, therefore gets a significant contribution from this direct source of CP nonconservation. In the following sections, we calculate A_{CP} in the minimal standard model, but unfortunately find it to be small; $|A_{CP}| \approx 10^{-4}$.

The decay $K_L \to \pi^+\pi^-e^+e^-$ has been studied previously by Sehgal and Wanninger[3] and by Heiliger and Sehgal[4]. These authors adopted a phenomenological approach, relating the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude to the measured $K_L \to \pi^+\pi^-\gamma$ decay amplitude. In the systematic expansion of chiral perturbation theory, we find that there may be important additional contributions to the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude for $q^2 = (k_- + k_+)^2 \gg 4m_e^2$ that were not included in this previous work. It was pointed out in Refs. 3 and 4 that indirect CP nonconservation from $K^0 - \bar{K}^0$ mixing gives an important contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay rate, producing a CP violating observable, B_{CP}, that is quite large. We reexamine B_{CP} using the form factors determined in the following sections of this chapter.

2 Chiral Perturbation Theory

Chiral perturbation theory provides a systematic approach to understanding the one-photon part of the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude. It uses an effective field theory that incorporates the $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ chiral symmetry of QCD and an expansion in powers of momentum to reduce the number of operators that occur. The formal parameter of expansion is (p^2/Λ_χ^2), where Λ_χ is on the order of 1 GeV, and p is the momentum of any of the pseudo-Goldstone bosons. In the chiral Lagrangian, the π's, K's and η are incorporated into a 3×3 special unitary matrix Σ:

$$\Sigma = \exp\left(\frac{2iM}{f}\right) \quad , \tag{9}$$

where

$$M = \pi^a\lambda_a/\sqrt{2} = \begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \quad . \tag{10}$$

The λ_a are the Gell-Mann matrices for SU(3), and $\pi^1 \mp i\pi^2 = \sqrt{2}\pi^\pm, \pi^4 \mp i\pi^5 = \sqrt{2}K^\pm, \pi^6 - i\pi^7 = \sqrt{2}K_0, \pi^6 + i\pi^7 = \sqrt{2}\bar{K}^0, \pi^3 = \pi^0$, and $\pi^8 = \eta$. In order to construct our chiral Lagrangian, we must know how Σ transforms under an $\mathrm{SU}(3)_L\times$ $\mathrm{SU}(3)_R$ transformation. The general formalism for such transformations in the presence of spontaneously broken global symmetries has been worked out by Callan, Coleman, Wess, and Zumino[5], and we outline it briefly below.

In the limit of massless u, d, and s quarks, the QCD Lagrangian is symmetric under global transformations of the group $G = \mathrm{SU}(3)_L\times$ $\mathrm{SU}(3)_R$, under which the left and right-handed quark fields transform independently:

$$\psi_L(x) \to L\psi_L(x), \qquad \psi_R(x) \to R\psi_R(x). \tag{11}$$

This symmetry is spontaneously broken to the vector subgroup, H=$\mathrm{SU}(3)_V$, by the

$<\bar{\psi}\psi>$ condensate. The Goldstone bosons are a result of this spontaneous symmetry breaking, and are local parameters for transformations in the coset space $G/H \simeq$ SU(3). The CCWZ formalism instructs us that if we consider a quantity

$$\Xi(x) = e^{iX_a\pi^a}, \tag{12}$$

where X is a set of broken generators, then Ξ transforms under an element g of G in the fashion

$$\Xi(x) \to g\Xi(x)h^{-1}(g, \Xi(x)), \tag{13}$$

with $g \in G$ and $h \in H$. Note that h is a local transformation, since it depends on the space-time coordinate implicitly through its dependence on $\Xi(x)$. Working in the chiral representation, g takes the explicit form

$$g = \begin{bmatrix} L & 0 \\ 0 & R \end{bmatrix} \quad , \tag{14}$$

where L and R are the $\mathrm{SU}(3)_L$ and $\mathrm{SU}(3)_R$ transformations, respectively. The unbroken transformations, on the other hand, have the above form with L and R set equal:

$$h = \begin{bmatrix} U & 0 \\ 0 & U \end{bmatrix} \quad . \tag{15}$$

If we choose as the broken generators the $\mathrm{SU}(3)_L$–$\mathrm{SU}(3)_R$ generators,

$$X_a = \begin{bmatrix} \lambda_a/(\sqrt{2}f) & 0 \\ 0 & -\lambda_a/(\sqrt{2}f) \end{bmatrix} \quad , \tag{16}$$

then we find that

$$\Xi(x) = e^{iX_a\pi^a} = \begin{bmatrix} \xi(x) & 0 \\ 0 & \xi^\dagger(x) \end{bmatrix}, \tag{17}$$

with $\xi^2 = \Sigma$. Eq. (13) then immediately tells us that

$$\begin{bmatrix} \xi(x) & 0 \\ 0 & \xi^\dagger(x) \end{bmatrix} \to \begin{bmatrix} L & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \xi(x) & 0 \\ 0 & \xi^\dagger(x) \end{bmatrix} \begin{bmatrix} U^{-1} & 0 \\ 0 & U^{-1} \end{bmatrix} \quad , \tag{18}$$

which implies that, under SU(3)$_L\times$ SU(3)$_R$ transformations the Σ field transforms as

$$\Sigma(x) \to L\Sigma(x)R^\dagger \quad . \tag{19}$$

We now construct our chiral Lagrangian. Terms not proportional to the quark masses must be invariant under the transformation of eq. (19), while those proportional to the quark masses must transform in the same way as do the quark mass terms in the QCD Lagrangian, that is, like $(3_L,\, \bar{3}_R) + (\bar{3}_L, 3_R)$. At leading order, $\mathcal{O}(p^2)$, the strong and electromagnetic interactions of the pseudo–Goldstone bosons are described by the chiral Lagrange density

$$\mathcal{L}_S^{(1)} = \frac{f^2}{8} Tr(D_\mu \Sigma D^\mu \Sigma^\dagger) + vTr(m_q\Sigma + m_q\Sigma^\dagger) \quad , \tag{20}$$

where v is a parameter with dimensions of mass to the third power and m_q is the quark mass matrix:

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad . \tag{21}$$

The coefficient in front of the derivative term is chosen so as to give the conventional normalization for the kinetic energy. In this paper we neglect isospin violation in the quark mass matrix and set $m_u = m_d \equiv m_l$. The first term on the right-hand side of eq. (20) is clearly an $\mathcal{O}(p^2)$ term; it may at first seem strange, however, that we treat the second term in eq. (20) as an $\mathcal{O}(p^2)$ term as well, being proportional to m_q as it is. A quick calculation, however, shows that eq. (20) induces the pion, kaon, and eta masses:

$$m_\pi^2 = \frac{8vm_l}{f^2}$$

$$m_K^2 = \frac{8v}{f^2}(\frac{m_s + m_l}{2})$$

$$m_\eta^2 = \frac{8v}{f^2}(\frac{2m_s + m_l}{3}), \tag{22}$$

which lead immediately to the Gell-Mann–Okubo mass relation

$$3m_\eta^2 - 4m_K^2 + m_\pi^2 = 0 \quad . \tag{23}$$

Thus, we see that the two terms in eq. (20) are manifestly of the same order; one factor of quark mass counts as two factors of psuedoscalar boson mass, as far as power counting is concerned.

We may also determine the constant f at this point by considering the left-handed currents that arise from eq. (20) and from the QCD Lagraagian. Keeping the part of the current that involves π^-, for instance, one finds:

$$\overline{u}\gamma_\mu(1 - \gamma_5)d = -f\partial_\mu\pi^- + Terms\ with\ more\ powers\ of\ \pi. \tag{24}$$

Recalling the definition of the pion decay constant,

$$\langle 0|\overline{u}\gamma_\mu(1-\gamma_5)d|\pi^-(p)\rangle = if_\pi p_\mu, \tag{25}$$

we see immediately that $f = f_\pi \simeq 132$ MeV at leading order in chiral perturbation theory. Note also that, because of the symmetry of the first term on the right-hand side of eq. (20) with respect to π and K, the pion and kaon decay constants are identical at this order.

The effective Lagrangian for $\Delta S = 1$ weak nonleptonic decays transforms as $(8_L, 1_R)+(27_L, 1_R)$ under $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$. The $(8_L, 1_R)$ amplitudes are much larger than the $(27_L, 1_R)$ amplitudes and so we will neglect the $(27_L, 1_R)$ part of the effective Lagrangian. The effective Lagrangian for weak radiative kaon decay is obtained by gauging the effective Lagrangian for weak nonleptonic decays with respect to the $\mathrm{U}(1)_Q$ of electromagnetism. At leading order in chiral perturbation theory, the $\Delta S =$ 1 transitions are described by

$$\mathcal{L}_W^{(1)} = \frac{g_8 G_F s_1 f^4}{4\sqrt{2}} Tr\left[D_\mu\, \Sigma D^\mu\, \Sigma^\dagger\, T\right] + H.c. \quad . \tag{26}$$

The matrix T in (26) projects out the correct flavour structure of the octet:

$$T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad , \tag{27}$$

and g_8 is a constant determined by the measured $K_S \to \pi^+\pi^-$ decay rate, with magnitude $|g_8| \simeq 5.1$. In (20) and (26) D_μ represents a covariant derivative:

$$D_\mu\Sigma = \partial_\mu\Sigma + ieA_\mu[Q, \Sigma] \quad , \tag{28}$$

where

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \tag{29}$$

is the electromagnetic charge matrix for the three lightest quarks, u, d and s.

3 Power Counting in Chiral Perturbation Theory

We have already mentioned that chiral perturbation theory is a systematic expansion in powers of momentum. In this section we provide a brief derivation of the order of an arbitrary Feynman graph that arises in this theory. The complete chiral Lagrangian will contain terms with arbitrary even powers of derivatives, arbitrary integer powers of the quark mass matrix, m_q, and arbitrary combinations of the two. Let us denote by N_n the number of vertices in a given graph arising from a term in the chiral Lagrangian whose total derivative and quark mass matrix mass dimension is n, that is, from a term in the $\mathcal{O}(p^n)$ sector of the Lagrangian. Now, because the pseudo-Goldstone boson fields appear in the Lagrangian only within the dimensionless Σ, the coupling in front of an $\mathcal{O}(p^n)$ term must have mass dimension $4-n$. The contribution of couplings to the mass dimension of a given graph is therefore

$$[couplings] = \sum_n N_n(4-n). \tag{30}$$

Furthermore, in light of the expansion of Σ in terms of component fields, each boson field is accompanied by a factor $(1/f)$, so that if we have N_E external and N_I internal boson lines, we will pick up a factor $(1/f)^{2N_I+N_E}$. Using the relation

$$N_I = N_L + N_V - 1 = N_L + \sum_n N_n - 1 \tag{31}$$

where N_L is the number of loops in the graph and N_V the number of vertices, we find that the f factors contribute

$$[ffactors] = 2 - N_E - 2N_L - 2\sum_n N_n \tag{32}$$

to the mass dimension of a graph. Finally, because our boson states have mass dimension 1, we gain an additional factor of mass for each external boson line.

The only other quantities with mass dimension in the graph are the external particle momenta, which we shall denote generically by p. Let us say that a given graph depends on these momenta like p^D. Then, combining the results from eqs. (30), (31), and (32), we find that the total mass dimension of the interaction, $\mathcal{M}$, is

$$\mathcal{M} = \sum_n N_n(2-n) - 2N_L + 2 + D. \tag{33}$$

But this is just a new effective interaction in the Lagrangian, and so must have mass dimension 4. This restriction immediately determines D:

$$D = 2 + \sum_n N_n(n-2) + 2N_L \tag{34}$$

Eq. (34) shows us that $\mathcal{O}(p^2)$ terms in the Lagrangian used at tree level give us the leading $\mathcal{O}(p^2)$ contributions to a physical process, as we expect. More importantly, $\mathcal{O}(p^4)$ contributions arise both from $\mathcal{O}(p^4)$ terms in the Lagrangian used at tree level, and from $\mathcal{O}(p^2)$ terms used at one loop level. It is important to keep terms from both sources in order to have a consistent expansion.

The reason for taking $\Lambda_\chi \simeq 1$ GeV should now be apparent. In going from tree level to one loop level, we gain an f^2 in the denominator, a p^2 in the numerator, and a factor of $(1/16\pi^2)$ from the loop integration. Higher dimension local terms in the chiral Lagrangian must be suppressed by some relatively large mass scale, which is what we call Λ_χ. We require the infinite parts of the loop integrations, however, to be canceled by corresponding pieces of such higher dimension local terms and, for this reason, can argue that their coefficients be approximately equal. We therefore expect $\Lambda_\chi \simeq \sqrt{16\pi^2 f^2} = 4\pi f \simeq 1$ GeV.

4 The One Photon Amplitude

The K_L state

$$|K_L> \simeq |K_2> + \epsilon |K_1> \quad , \tag{35}$$

is mostly the CP odd state

$$|K_2> = \frac{1}{\sqrt{2}}(|K^0> + |\bar{K}^0>) \quad , \tag{36}$$

with small admixture of the CP even state

$$|K_1> = \frac{1}{\sqrt{2}}(|K^0> - |\bar{K}^0>) \quad . \tag{37}$$

The parameter ϵ characterizes CP nonconservation in $K^0 - \bar{K}^0$ mixing. At leading order in chiral perturbation theory, the $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$ decay amplitude arises via the CP even component of K_L. Writing the form factors contributing to $K_L \to \pi^+\pi^-\gamma^*$ as a power series in the chiral expansion,

$$F_\pm = F_\pm^{(1)} + F_\pm^{(2)} + ... \quad , \quad G = G^{(1)} + G^{(2)} + ... \quad , \tag{38}$$

we find that the Feynman diagrams in Fig. 1 give

$$
\begin{aligned}
G^{(1)} &= 0 \\
F_+^{(1)} &= -\frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{[q^2 + 2q \cdot p_+]} \\
F_-^{(1)} &= +\frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{[q^2 + 2q \cdot p_-]}
\end{aligned}
\quad . \tag{39}
$$

The subscripts denote the order of chiral perturbation theory at which each term arises, i.e., $F_\pm^{(m)}$ and $G^{(m)}$ give a contribution of order p^{2m-1} to the square brackets of eq. (1). $G^{(1)}$ vanishes simply because G is accompanied by three factors of momentum in eq. (1), and is therefore at most an $\mathcal{O}(p^3)$ effect.

Despite the fact that $\epsilon \simeq 0.0023\ e^{i44^\circ}$ (in a phase convention where the $K^0 \to \pi\pi(I=0)$ decay amplitude is real) is small, it is important to keep this part of the decay amplitude. Other contributions not proportional to ϵ occur only at higher order in chiral perturbation theory. We neglect direct sources of CP nonconservation in the one-photon part of the decay amplitude. Experimental information on ϵ' suggests that they are small.

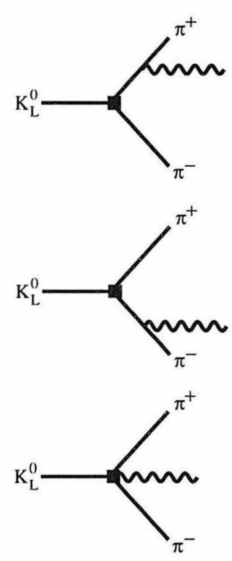

Fig. 1. Feynman diagrams contributing to $F_\pm^{(1)}$.

Proceeding to next order in the chiral expansion, the form factors $G^{(2)}$, $F_{\pm}^{(2)}$ arise both from $\mathcal{O}(p^4)$ local operators and from one-loop Feynman diagrams involving vertices from the leading Lagrange densities in (20) and (26). The form factor $G^{(2)}$, however, arises solely from local operators, as the one loop Feynman diagrams and tree graphs involving the Wess–Zumino term[6][7] do not contribute. The contribution of the $\mathcal{O}(p^4)$ local operators to $G^{(2)}$ is fixed by the measured $K_L \to \pi^+\pi^-\gamma$ decay rate[8][9] to be

$$|G^{(2)}| \simeq 40 \quad . \tag{40}$$

The experimentally observed $K_L \to \pi^+\pi^-\gamma$ Dalitz plot suggests that the form factor G has significant momentum dependence, indicating that $G^{(3)}$ is not negligible, and that our extraction of $G^{(2)}$ from the rate is not completely justified[10].

The form-factors $F_{\pm}^{(2)}$ get contributions both from local operators of $\mathcal{O}(p^4)$[11] and from one-loop diagrams involving vertices from the leading Lagrange densities in (20) and (26) . For $K_L \to \pi^+\pi^- e^+e^-$ the local operators that contribute are

$$\mathcal{L}_S^{(2)} = \frac{-ie\lambda_{cr}(\mu)}{16\pi^2} F^{\mu\nu} Tr\left[Q(D_\mu\Sigma D_\nu\Sigma^\dagger + D_\mu\Sigma^\dagger D_\nu\Sigma)\right] \tag{41}$$

and

$$\begin{aligned}\mathcal{L}_W^{(2)} = i\,\frac{G_F s_1 e f^2 g_8}{\sqrt{2}\;16\pi^2}\Big[& \; a_1(\mu)F^{\mu\nu}Tr[QT(\Sigma D_\mu\Sigma^\dagger)(\Sigma D_\nu\Sigma^\dagger)] \\ & + a_2(\mu)F^{\mu\nu}Tr[Q(\Sigma D_\mu\Sigma^\dagger)T(\Sigma D_\nu\Sigma^\dagger)] \\ & + a_3(\mu)F^{\mu\nu}Tr[T[Q,\Sigma]D_\mu\Sigma^\dagger\Sigma D_\nu\Sigma^\dagger - TD_\mu\Sigma D_\nu\Sigma^\dagger\Sigma[\Sigma^\dagger,Q]] \\ & + a_4(\mu)F^{\mu\nu}Tr[T\Sigma D_\mu\Sigma^\dagger[Q,\Sigma]D_\nu\Sigma^\dagger] \;\Big] + H.c.\end{aligned} \tag{42}$$

The coefficients $\lambda_{cr}, a_1, a_2, a_3$ and a_4 depend on the renormalization procedure used, and we employ dimensional regularization with $\overline{MS}$ subtraction. The dependence of the coefficients $\lambda_{cr}, a_{1,2,3,4}$ on the subtraction point μ cancels that coming from the one-loop diagrams. Note that the basis of operators in eq. (42) is slightly different than that used in Ref. 11. With this basis of operators, the combination of counterterms

$$w_L = a_3 - a_4 \tag{43}$$

is independent of the subtraction point μ at one loop.

The value of λ_{cr} is fixed by the measured π^+ charge radius; $< r_\pi^2 >= 0.44 \pm 0.02$ fm^2. The one-loop diagrams in Fig. 2 give, using $\overline{MS}$,

$$\lambda_{cr}(\mu) = -\left(\frac{2\pi^2}{3}\right) f^2 < r_\pi^2 > \ - \ \frac{1}{24}\left[2\ \ell n(m_\pi^2/\mu^2) + \ell n(m_K^2/\mu^2)\right] \quad , \tag{44}$$

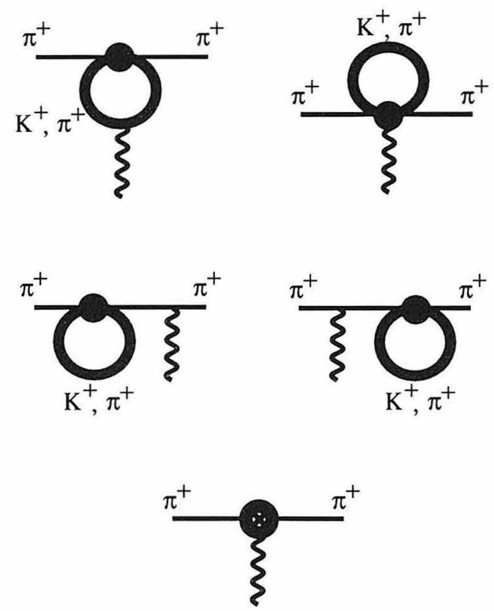

Fig. 2. Feynman diagrams contributing to the $\pi^\pm$ charge radius, $< r_\pi^2 >$, at leading order in chiral perturbation theory.

---

which implies that at the subtraction point $\mu = 1$GeV,

$$\lambda_{cr}(1GeV) = -0.91 \pm 0.06 \quad . \tag{45}$$

A linear combination of a_1 and a_2 is fixed by the measured $K^+ \rightarrow \pi^+ e^+ e^-$ decay amplitude. It is fortunately the same combination of a_1 and a_2 that enters into the $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ decay amplitude. The one-photon part of the $K^+ \rightarrow \pi^+ e^+ e^-$ decay amplitude can be written in terms of a single form factor $f(q^2)$:

$$M^{(1\gamma)}(K^+ \rightarrow \pi^+ e^+ e^-) = \frac{s_1 G_F}{\sqrt{2}} \frac{\alpha}{4\pi} f(q^2)\, p_\pi^\mu\, \bar{u}(k_-)\gamma_\mu v(k_+) \quad . \tag{46}$$

The one-loop diagrams in Fig. 3 and the operators in (41) and (42) combine to give

$$\begin{aligned} f(q^2) &= 2g_8 \left(\phi_K(q^2) + \phi_\pi(q^2) - \frac{1}{6}\ell n(m_K^2/\mu^2) - \frac{1}{6}\ell n(m_\pi^2/\mu^2) \right. \\ &\qquad \left. + \frac{2}{3}(a_1(\mu) + 2a_2(\mu)) - 4\lambda_{cr}(\mu) + \frac{1}{3} \right) \quad , \\ &= 2g_8 \left(\phi_K(q^2) + \phi_\pi(q^2) + w_+ \right) \end{aligned} \tag{47}$$

where

$$\phi_i(q^2) = \int_0^1 dx \left(\frac{m_i^2}{q^2} - x(1-x) \right) \ell n \left(1 - \frac{q^2}{m_i^2} x(1-x) \right) \quad . \tag{48}$$

This relation defines the μ independent constant w_+[12], which has been experimentally determined to be[13]

$$w_+ = 0.89^{+0.24}_{-0.14} \quad . \tag{49}$$

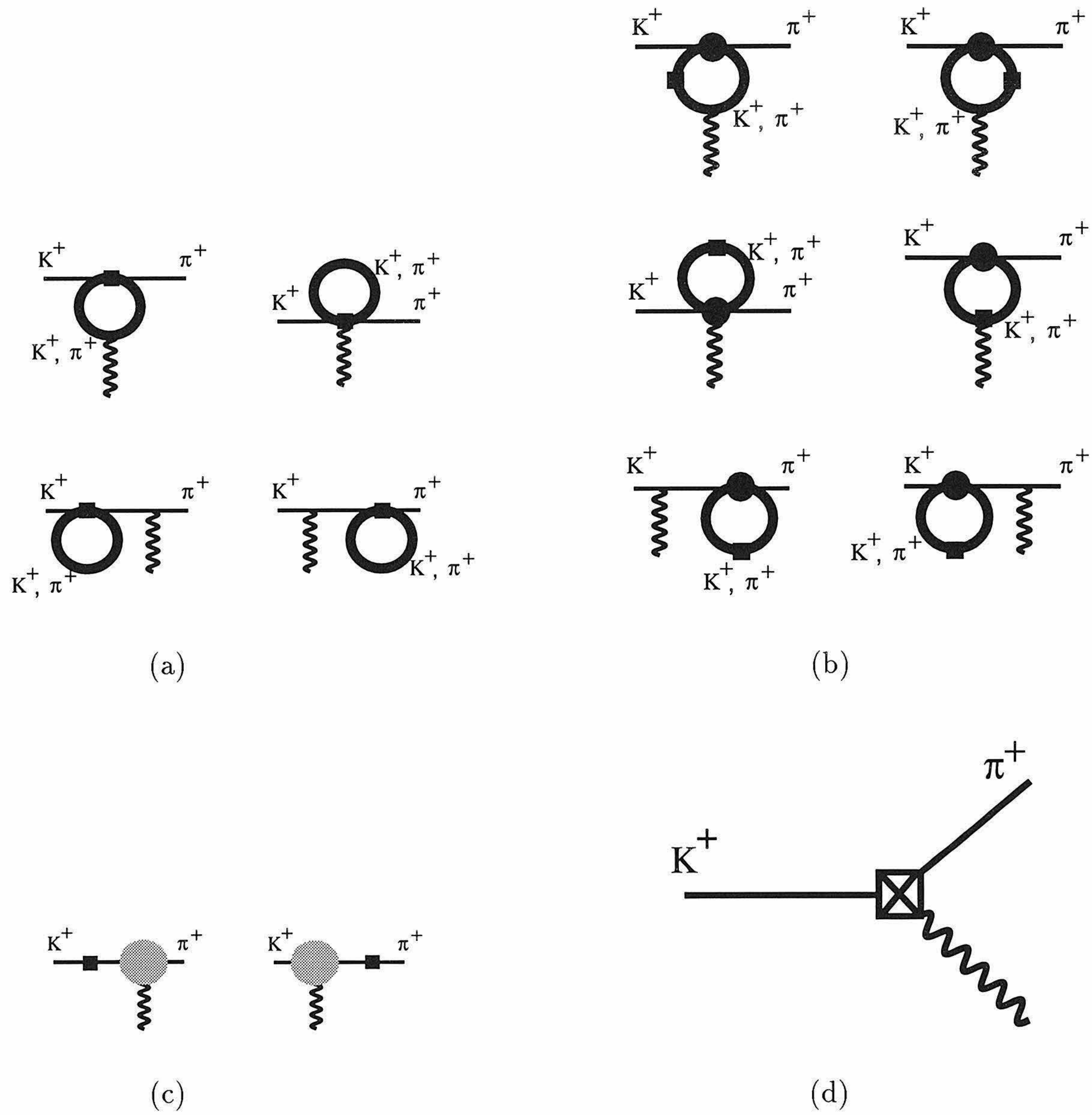

Fig 3. Feynman diagrams contributing to the amplitude for $K^+ \to \pi^+\gamma^*$ at leading order in chiral perturbation theory. The solid square denotes a vertex from the gauged weak Lagrangian (26), and the solid circle denotes a vertex from the gauged strong Lagrangian in (20). (a) involves only weak and electromagnetic vertices, while (b) also has a strong vertex. (c) is the contribution from the kaon and pion charge radii (including both loop graphs and the tree level counterterm). (d) is the contribution of the weak counterterm as given by (42). We have not shown the wavefunction renormalization of the tree graphs for the process as the sum of these graphs vanishes.

Using the central values of $\lambda_{cr}(1GeV)$ and w_+, we find that

$$a_1(1GeV) + 2a_2(1GeV) = -6.0 \quad , \tag{50}$$

with an associated error around 10%; this error is correlated with the uncertainty in $\lambda_{cr}(1GeV)$. Throughout the remainder of this work we will use the central values of $\lambda_{cr}(1GeV)$ and $a_1(1GeV) + 2a_2(1GeV)$, suppressing the associated uncertainties. Note that the contributions $\lambda_{cr}(1GeV)$ and $(a_1 + 2a_2)(1GeV)$ to $f(q^2)$ are separately quite large but nearly cancel against each other.

At $\mathcal{O}(p^4)$, the form factors $F_\pm^{(2)}$ for $K_L \to \pi^+\pi^-e^+e^-$ decay follow from the Feynman diagrams in Fig. 4 and tree level matrix elements of the operators in (41) and (42). We find, using $\overline{MS}$ subtraction, that

$$\begin{aligned} F_-^{(2)} = g_8 \Bigg(& -\frac{2}{3}q^2[a_1(\mu) + 2a_2(\mu) + 6a_3(\mu) - 6a_4(\mu)] - 4q^2\lambda_{cr}(\mu) + \frac{2}{3}q^2 + \phi_{K\eta} + \phi_{K\pi} \\ & -4\int_0^1 dx \left[q^2x(1-x)\ell n\left(\frac{m_\pi^2 - q^2x(1-x)}{\mu^2}\right) - m_\pi^2 \ell n\left(1 - \frac{q^2x(1-x)}{m_\pi^2}\right)\right] \\ & +2q^2\left(\frac{m_K^2 - m_\pi^2}{q^2 + 2q\cdot(p_+ + p_-)}\right)\left(\phi_K(q^2) - \phi_\pi(q^2) + \frac{1}{6}\ell n\left(\frac{m_\pi^2}{m_K^2}\right)\right) \Bigg) \end{aligned} \tag{51}$$

where

$$
\begin{aligned}
\phi_{K\eta} = \frac{2}{9}(m_K^2 - m_\pi^2)^2 & \left(\int\limits_0^1 dyy \int\limits_0^{1-y} dx \ \frac{1}{\mu_1^2} \right. \\
& \left. + \frac{1}{(q^2+2q\cdot p_-)} \int\limits_0^1 dx\ell n \left(1 - \frac{x(1-x)(q^2+2q\cdot p_-)}{m_K^2(1-x)+m_\eta^2 x - m_\pi^2 x(1-x)} \right) \right) \\
& + \frac{1}{3}(m_K^2 - m_\pi^2) \left(2\int\limits_0^1 dy \int\limits_0^{1-y} dx\ell n \left(1 - \frac{(q^2 x(1-x) + 2q\cdot p_- xy)}{m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y)} \right) \right. \\
& + 3\int\limits_0^1 dy \int\limits_0^{1-y} dx\ell n \left(1 - \frac{(q^2 x(1-x) + 2q\cdot p_+ xy)}{m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y)} \right) \\
& + \int\limits_0^1 dyy \int\limits_0^{1-y} dx \frac{1}{\mu_1^2} \Big[(q(1-x) + p_- y)\cdot(4q + 6p_+ + 4p_-) - 2m_K^2 - 2p_+\cdot(q+p_-) \Big] \\
& + \int\limits_0^1 dx \left(1 + x + \frac{(x-1)(3m_K^2 - 2m_\pi^2)}{q^2 + 2p_-\cdot q} \right) \cdot \ell n \left(1 - \frac{x(1-x)(q^2+2q\cdot p_-)}{m_K^2(1-x) + m_\eta^2 x - m_\pi^2 x(1-x)} \right)
\end{aligned}
\tag{52}
$$

with

$$
\mu_1^2 = m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y) - q^2 x(1-x) - 2q\cdot p_- xy \quad . \tag{53}
$$

The remaining function is

$$\phi_{K\pi} = (m_K^2 - m_\pi^2)\left(\int_0^1 dy \int_0^{1-y} dx\ell n\left(1 - \frac{(q^2x(1-x) + 2q\cdot p_- xy)}{m_K^2 y + m_\pi^2(1-y)^2}\right)\right.$$
$$+ \int_0^1 dy \int_0^{1-y} dx\ell n\left(1 - \frac{q^2x(1-x) + 2q\cdot p_+ xy}{m_K^2 y + m_\pi^2(1-y)^2}\right)$$
$$+2\int_0^1 dyy \int_0^{1-y} dx\,\frac{(p_+ + p_- + q)\cdot(q(1-x) + yp_- - p_+)}{\mu_2^2}$$
$$\left.+ \int_0^1 dx\left(1 + x + \frac{m_K^2(x-1)}{q^2 + 2q\cdot p_-}\right)\ell n\left(1 - \frac{x(1-x)(q^2 + 2q\cdot p_-)}{m_\pi^2(1-x)^2 + m_K^2 x}\right)\right) \tag{54}$$

with

$$\mu_2^2 = m_K^2 y + m_\pi^2(1-y)^2 - q^2x(1-x) - 2q\cdot p_- xy \quad . \tag{55}$$

The Gell-Mann–Okubo mass formula (23) has been used to simplify some of the dependence on the pseudoscalar masses in (52) and (54). $F_+^{(2)}$ is obtained from (51) by taking $p_+ \to p_-$ and $p_- \to p_+$. Notice that the terms $(a_1 + 2a_2)$ and λ_{cr} that appear in the expression for $F_\pm^{(2)}$ have a relative sign difference as compared to their appearance in the expression for $f(s)$ given in eq. (47), and will therefore reinforce each other, as opposed to the cancellation we observed previously. The uncertainty in $\lambda_{cr}(1GeV)$ and w_+ gives rise to an uncertainty of approximately 10% in the combination of counterterms that appears in (51). The one photon part of the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude is the largest and dominates the rate. In the next section, we use the form factors calculated here to obtain $d\Gamma(K_L \to \pi^+\pi^-e^+e^-)/dq^2$. One scale independent linear combination of counterterms, $w_L = a_3 - a_4$, remains

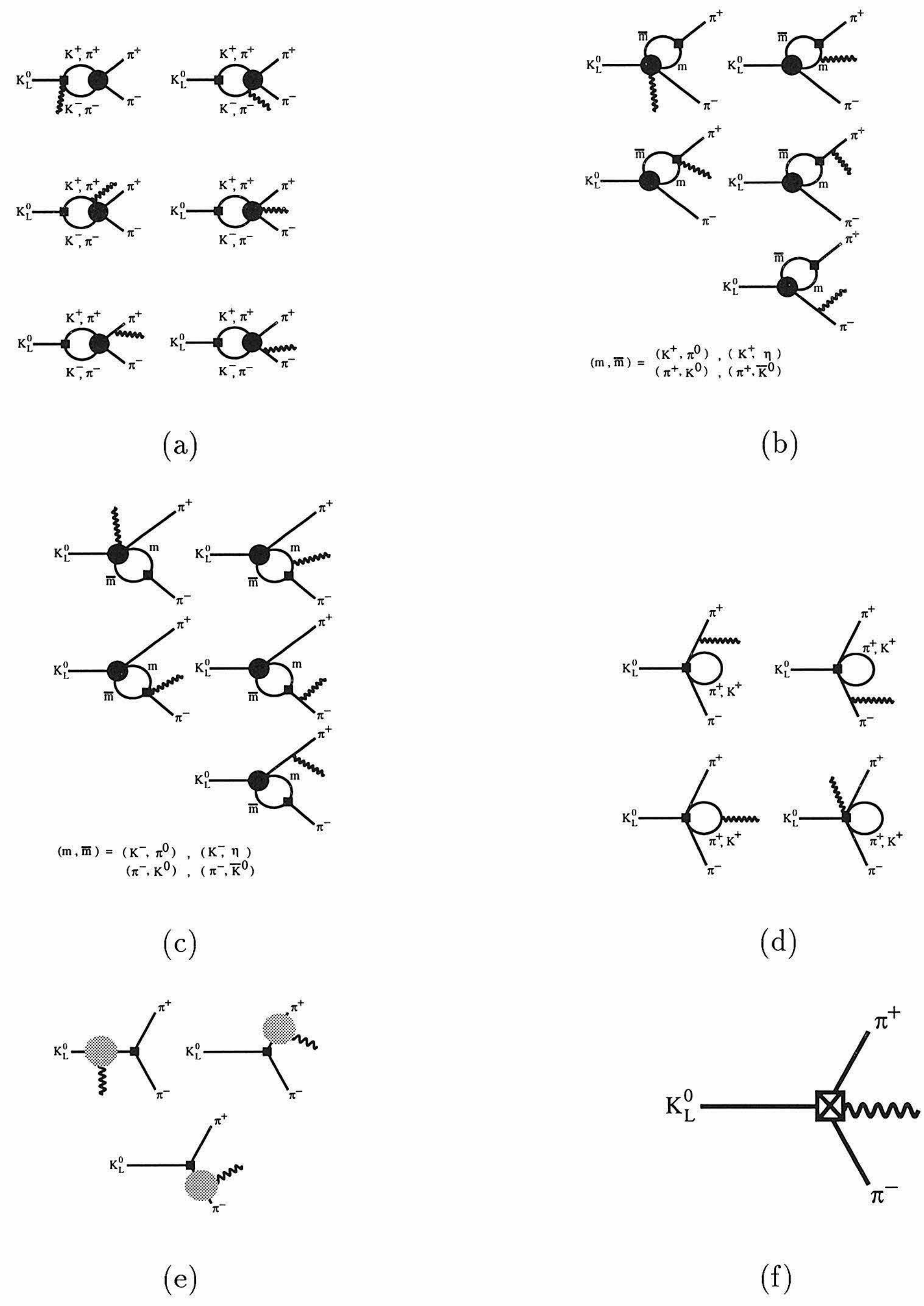

Fig 4. Feynman diagrams contributing to the CP-conserving amplitude for $K_L \to \pi^+\pi^-\gamma^*$ at leading order in chiral perturbation theory. The notation is the same as in Fig. 3, and we have not shown the wavefunction renormalization of the tree graphs for the process as the sum of these graphs vanishes.

undetermined by the present experimental data, and for this reason, we cannot predict the rate for $K_L \to \pi^+\pi^-e^+e^-$. This is the only undetermined constant, however, and the entire function $d\Gamma(K_L \to \pi^+\pi^-e^+e^-)/dq^2$ is experimentally accessible.

5 The Differential Decay Rate

The $K_L \to \pi^+\pi^-e^+e^-$ decay rate is obtained by squaring the invariant matrix element in eq. (1), summing over the e^+ and e^- spins, and integrating over the phase space. Since the e^+ and e^- four momenta occur only in the lepton trace, $Tr\left[\not{k}_-\gamma_\nu\not{k}_+\gamma_\mu\right]$, the phase space integrations over k_- and k_+ produce a factor

$$\begin{aligned}\int \frac{d^3k_-}{(2\pi)^3 2k_-^0}\int \frac{d^3k_+}{(2\pi)^3 2k_+^0}(2\pi)^4\delta^4(q-k_--k_+)Tr\left[\not{k}_-\gamma_\nu\not{k}_+\gamma_\mu\right] \\ = \frac{1}{6\pi}(q_\mu q_\nu - q^2 q_{\mu\nu})\end{aligned} \quad . \tag{56}$$

The remaining phase space integrations can be taken to be over q^2 and the sum and difference of the pion energies in the K_L rest frame, $E_S = p_+^0 + p_-^0$, $E_D = p_+^0 - p_-^0$. The contribution of the form factors $F_\pm$ and G to $d\Gamma/dq^2$ do not interfere. We may therefore write

$$\frac{d\Gamma}{dq^2}(K_L \to \pi^+\pi^-e^+e^-) = \frac{d\Gamma_G}{dq^2} + \frac{d\Gamma_F}{dq^2} \quad , \tag{57}$$

with

$$\frac{d\Gamma_G}{dq^2} = \frac{G_F^2\alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3q^2} \int dE_S \int dE_D |G|^2$$
$$\left[m_\pi^4 q^2 - m_\pi^2 (p_- \cdot q)^2 - m_\pi^2 (p_+ \cdot q)^2 + 2(p_+ \cdot p_-)(q \cdot p_+)(q \cdot p_-) - q^2 (p_+ \cdot p_-)^2\right]$$
$$\frac{d\Gamma_F}{dq^2} = \frac{G_F^2\alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3q^4} \int dE_S \int dE_D \Big[|F_+ \, q \cdot p_+ + F_- \, q \cdot p_-|^2$$
$$- q^2 \left(|F_+|^2 m_\pi^2 + |F_-|^2 m_\pi^2 + 2Re(F_+ F_-^*) p_+ \cdot p_-\right)\Big] \tag{58}$$

In eq. (58), the difference of pion energies is integrated over the region $-E_D^{(max)} < E_D < E_D^{(max)}$, where

$$E_D^{(max)} = \sqrt{\frac{2m_K E_S + q^2 - m_K^2 - 4m_\pi^2}{2m_K E_S + q^2 - m_K^2}} \sqrt{(m_K - E_S)^2 - q^2} \quad , \tag{59}$$

while the sum of pion energies is integrated over the region $E_S^{(min)} < E_S < E_S^{(max)}$ with boundaries

$$\begin{aligned} E_S^{(max)} &= m_K - \sqrt{q^2} \\ E_S^{(min)} &= \frac{m_K^2 - q^2 + 4m_\pi^2}{2m_K} \end{aligned} \quad . \tag{60}$$

The scalar products appearing in the expression for the rates are easily expressed in terms of E_S, E_D and q^2:

$$\begin{aligned} p_+ \cdot p_- &= \frac{1}{2}(q^2 - m_K^2 - 2m_\pi^2 + 2m_K E_S) \\ q \cdot p_+ &= \frac{1}{2}(-m_K E_S + m_K E_D - q^2 + m_K^2) \\ q \cdot p_- &= \frac{1}{2}(-m_K E_S - m_K E_D - q^2 + m_K^2) \end{aligned} \quad . \tag{61}$$

The form factors $F_\pm^{(1)}$ and $F_\pm^{(2)}$ have the opposite property under interchange of pion momenta, as indicated in eqs. (3) and (5), and consequently do not interfere in $d\Gamma/dq^2$. Neglecting terms in the chiral expansion of $\mathcal{O}(p^6)$ and higher, the differential decay rate given in (57) becomes

$$\frac{d\Gamma}{dq^2}(K_L \to \pi^+\pi^- e^+ e^-) = \frac{d\Gamma_{G^{(2)}}}{dq^2} + \frac{d\Gamma_{F^{(1)}}}{dq^2} + \frac{d\Gamma_{F^{(2)}}}{dq^2} \quad . \tag{62}$$

In Fig. 5 we have graphed

$$\frac{1}{\Gamma_{K_L}} \frac{d\Gamma}{dy} = 2y\ (m_K - 2m_\pi)^2 \frac{1}{\Gamma_{K_L}} \frac{d\Gamma}{dq^2} \tag{63}$$

for each of the three terms on the right-hand side of (62), where $y = \sqrt{q^2}/(m_K - 2m_\pi)$, Γ_{K_L} is the total width of the K_L, and we have set $w_L = 0$.

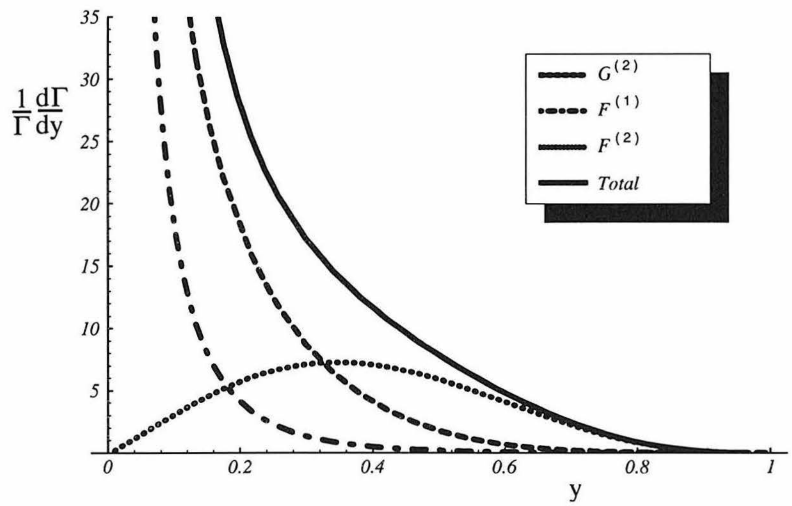

Fig. 5. The differential decay spectrum as a function of y, the invariant mass of the lepton pair normalized to $m_K - 2m_\pi$. The dot-dashed curve is the contribution from $F_\pm^{(1)}$, the dotted curve, the contribution from $F_\pm^{(2)}$ with w_L=0, and the dashed curve, the contribution from $G^{(2)}$. The total differential decay rate for w_L=0 is given by the solid curve.

Integrating the three terms on the right-hand side of (62) over the invariant mass interval $q^2 > (30MeV)^2$ (corresponding to $y > 0.13$), we find that for $w_L = 0$

$$10^8 \cdot Br(K_L \to \pi^+\pi^-e^+e^-; q^2 > (30MeV)^2) \;=\; 3.8 \;+\; 0.78 \;+\; 3.4 \;=\; 8.0 \quad . \tag{64}$$

The branching fraction over this range of e^+e^- invariant mass is dominated by the region of low q^2 and, for typical values of w_L, it receives comparable contributions from the form factors G and $F^{(2)}$. In the region of high q^2, however, the branching fraction is likely to be dominated by the $F_{\pm}^{(2)}$ form factor. For $q^2 > (80MeV)^2$, which corresponds to $y > 0.36$, and $w_L = 0$, the three terms on the right-hand side of eq. (62) contribute

$$10^8 \cdot Br(K_L \to \pi^+\pi^-e^+e^-; q^2 > (80MeV)^2) \;=\; 0.61 \;+\; 0.07 \;+\; 1.9 \;=\; 2.6 \quad . \tag{65}$$

A summary of our results for the rate can be found in Table 1. We have displayed the contribution to the branching ratio (in units of 10^{-8}) from the three form factors G, $F^{(1)}$ and $F^{(2)}$ for different values of the minimum lepton pair invariant mass $q^2_{\rm min}$. Since the loop contribution to the form factor $F_{\pm}^{(2)}$ is small, it will be difficult to extract a unique value for w_L from $d\Gamma/dq^2$ data alone; a two-fold ambiguity in the value of w_L will persist. The contributions to the rate from G and $F^{(1)}$ are numerically similar to those computed in Refs. 3 and 4, differing only because we have retained the q^2 dependence in $F^{(1)}$. For w_L=2, the contribution of $F_{\pm}^{(2)}$ to the rate is small, and our results are similar to those quoted in the above references.

Lower cut $q^2_{\rm min}$	$Br(10^{-8})_G$	$Br(10^{-8})_{F^{(1)}}$	$Br(10^{-8})_{F^{(2)}}$
$(10\mathrm{MeV})^2$	8.8	3.3	$3.6 - 3.4w_L + 0.8w_L^2$
$(20\mathrm{MeV})^2$	5.6	1.5	$3.5 - 3.3w_L + 0.8w_L^2$
$(30\mathrm{MeV})^2$	3.8	0.8	$3.4 - 3.2w_L + 0.8w_L^2$
$(40\mathrm{MeV})^2$	2.7	0.5	$3.1 - 3.0w_L + 0.7w_L^2$
$(60\mathrm{MeV})^2$	1.3	0.2	$2.6 - 2.4w_L + 0.6w_L^2$
$(80\mathrm{MeV})^2$	0.6	0.07	$1.9 - 1.8w_L + 0.4w_L^2$
$(100\mathrm{MeV})^2$	0.3	0.03	$1.3 - 1.2w_L + 0.3w_L^2$
$(120\mathrm{MeV})^2$	0.1	0.01	$0.74 - 0.68w_L + 0.16w_L^2$
$(180\mathrm{MeV})^2$	0.00072	0.0001	$0.027 - 0.025w_L + 0.006w_L^2$

Table 1: Contributions to the Branching Ratio (10^{-8}) **for a range of** $q^2_{\rm min}$

6 The Z-penguin and W-box Amplitude

The short distance W-box and Z-penguin diagrams give rise to the effective Lagrange density

$$\mathcal{L}_{SD} = \xi \, \frac{s_1 G_F \alpha}{\sqrt{2}} \, \bar{s}\gamma_\mu(1-\gamma_5)d \; \bar{e}\gamma^\mu\gamma_5 e \quad + \quad H.c. \quad . \tag{66}$$

Here we keep only the part that contains the lepton axial current; it is the axial current that gives rise to observables that are antisymmetric under interchange of e^+ and e^- four momenta, $k_+ \leftrightarrow k_-$.

In (66), the quantity ξ receives significant contributions from both top and charm quark loops and is given by

$$\xi = -\tilde{\xi}_c + \left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right) \tilde{\xi}_t \quad , \tag{67}$$

where

$$\tilde{\xi}_q = \tilde{\xi}_q^{(Z)} + \tilde{\xi}_q^{(W)} \tag{68}$$

is the sum of the contributions of the Z-penguin and W-box diagrams. It is convenient to express the combination of Cabibbo–Kobayashi–Maskawa matrix elements appearing in ξ in terms of $|V_{cb}|$ and the standard coordinates $\rho + i\eta$ of the unitarity triangle

$$V_{ts}^* V_{td} / V_{us}^* V_{ud} = (\rho - 1 + i\eta)|V_{cb}|^2 \quad . \tag{69}$$

A value of $|V_{cb}| \simeq 0.04$ is obtained from inclusive $B \to X_c e\bar{\nu}_e$ decay and from exclusive $B \to D^* e\bar{\nu}_e$ decay. Although the values of ρ and η are not determined by present data, they are expected to be of order unity.

The quantities $\tilde{\xi}_c$ and $\tilde{\xi}_t$ have been calculated including perturbative QCD corrections at the next to leading logarithmic level[14][15]. There is some sensitivity to the values of Λ_{QCD} , m_c, and m_t, but $\tilde{\xi}_c$ is of order 10^{-4} and $\tilde{\xi}_t$ is of order unity.

The quark level Lagrange density in eq. (66) can be converted into a Lagrange density involving the π, K and η hadrons by utilizing the Noether procedure. Equating the QCD chiral currents with those obtained from chiral variations of the effective lagrangian in eq. (20) leads to

$$\mathcal{L}_{SD} = -\xi \, \frac{iG_F \alpha s_1}{2\sqrt{2}} \, f^2 Tr(\partial^\mu \Sigma \Sigma^\dagger T) \;\; \bar{e}\gamma_\mu \gamma_5 e \; + \; H.c. \quad . \tag{70}$$

Expanding Σ in terms of the meson fields M, we find that the Lagrange density (70) produces a short distance contribution to the $K_L \to \pi^+\pi^- e^+ e^-$ decay amplitude from the W-box and Z-penguin diagrams given by

$$M^{(SD)} = i \, \frac{G_F s_1 \alpha}{f} \, (\xi \; p_-^\mu + \xi^* \; p_+^\mu) \;\; \bar{u}(k_-)\gamma_\mu \gamma_5 v(k_+) \quad . \tag{71}$$

7 The Asymmetry A_{CP}

It is the interference of $M^{(SD)}$ in eq. (71) with $M^{(1\gamma)}$ in eq. (1) that produces the asymmetry A_{CP} defined in (8). In calculating A_{CP}, it is convenient to use the phase space variables defined by Pais and Treiman[16] for $K_{\ell 4}$ decay, rather than those used for the total rate in Section 3. They are: $q^2 = (k_+ + k_-)^2$; $s = (p_+ + p_-)^2$; θ_π, the angle formed by the π^+ three momentum and the K_L three-momentum in the $\pi^+\pi^-$ rest frame; θ_ℓ, the angle between the e^- three momentum and the K_L three momentum in the e^+e^- rest frame; ϕ, the angle between the normals to the planes defined in the K_L rest frame by the $\pi^+\pi^-$ pair and the e^+e^- pair. In terms of these variables

$$\frac{(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)}{|(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)|} = \mathrm{sign}(\sin\phi) \quad , \tag{72}$$

and the asymmetry is

$$A_{CP} = \frac{1}{2^7 (2\pi)^6 m_K^3 \Gamma_{K_L}} \left(\int_0^{2\pi} d\phi \; \mathrm{sign}(\sin\phi) \right) \int dc_\pi \, dc_e \, ds \, dq^2 \beta \, X \, Re\left(M^{(SD)^*} M^{(1\gamma)} \right) \quad , \tag{73}$$

where $c_\pi = \cos\theta_\pi$, $c_e = \cos\theta_e$. The other kinematic functions appearing in this expression are

$$\begin{aligned} \beta &= [1 - 4m_\pi^2/s]^{1/2} \\ X &= \left[\left(\frac{m_K^2 - s - q^2}{2} \right)^2 - sq^2 \right]^{1/2} \quad . \end{aligned} \tag{74}$$

In order to evaluate the contributing form factors, the following scalar products of four vectors are required:

$$
\begin{aligned}
q \cdot p_+ &= \frac{1}{4}(m_K^2 - s - q^2) - \frac{1}{2}\beta X \cos\theta_\pi \\
q \cdot p_- &= \frac{1}{4}(m_K^2 - s - q^2) + \frac{1}{2}\beta X \cos\theta_\pi \\
p_+ \cdot p_- &= \frac{1}{2}(s - 2m_\pi^2) \\
\varepsilon_{\alpha\beta\sigma\rho} p_+^\alpha p_-^\beta k_+^\sigma k_-^\rho &= -\frac{1}{4}\beta X \sqrt{sq^2} \sin\theta_e \sin\theta_\pi \sin\phi
\end{aligned}
\quad . \tag{75}
$$

If the variables s and q^2 are not integrated over the complete phase space, then it is understood that the same is to be done for the K_L width Γ_{K_L} in the denominator of eq. (73).

The form factor G does not enter into $Re\left(M^{(SD)^*} M^{(1\gamma)}\right)$ (a sum over e^+ and e^- spins is understood). Integrating over $\cos\theta_e$ and ϕ, we find that

$$
A_{CP} = \frac{G_F^2 s_1^2 \alpha^2}{2^8 (2\pi)^6 f^2 m_K^3 \Gamma_{K_L}} \int dc_\pi \, ds \, dq^2 \; \sin\theta_\pi
$$
$$
\beta^2 \, X^2 \sqrt{\frac{s}{q^2}} \left[Im(\xi)\left(Re(F_+) + Re(F_-)\right) + Re(\xi)\left(Im(F_+) - Im(F_-)\right)\right] \tag{76}
$$

The integration over $\cos\theta_\pi$ implies that, at leading non-trivial order of chiral perturbation theory, $Im(F_+) - Im(F_-) \rightarrow Im(F_+^{(1)}) - Im(F_-^{(1)})$, reflecting indirect CP violation from ϵ, and $Re(F_+) + Re(F_-) \rightarrow Re(F_+^{(2)}) + Re(F_-^{(2)})$.

Using eqs. (67) and (69), we can write the CP violating asymmetry in terms of the real and imaginary parts of the CKM elements

$$
A_{CP} = A_1 \left((\rho - 1)|V_{cb}|^2 \tilde{\xi}_t - \tilde{\xi}_c \right) - A_2 \eta |V_{cb}|^2 \tilde{\xi}_t \quad , \tag{77}
$$

where A_1 arises from indirect CP nonconservation and A_2 arises from direct CP nonconservation. We predict only $|A_{CP}|$ since the sign of g_8 is not known. Our expressions for $F_\pm^{(1)}$ and $F_\pm^{(2)}$ with $w_L = 0$ give, up to an overall sign,

$$A_1 = 2.7 \times 10^{-2} \quad , \quad A_2 = 3.9 \times 10^{-2} \quad , \tag{78}$$

for $q^2 \geq (30MeV)^2$ and

$$A_1 = 2.4 \times 10^{-2} \quad , \quad A_2 = 8.4 \times 10^{-2} \quad , \tag{79}$$

for $q^2 \geq (80MeV)^2$. In Table 2 we present A_1 and A_2 for a range of values of the minimum lepton pair invariant mass, q^2_{min}, normalized to the branching ratios given in Table 1 assuming $w_L = 0$.

Lower cut q^2_{min}	A_1	A_2
$(10\text{MeV})^2$	2.0×10^{-2}	2.0×10^{-2}
$(20\text{MeV})^2$	2.5×10^{-2}	3.0×10^{-2}
$(30\text{MeV})^2$	2.7×10^{-2}	3.9×10^{-2}
$(40\text{MeV})^2$	2.8×10^{-2}	4.8×10^{-2}
$(60\text{MeV})^2$	2.7×10^{-2}	6.8×10^{-2}
$(80\text{MeV})^2$	2.4×10^{-2}	8.4×10^{-2}
$(100\text{MeV})^2$	2.1×10^{-2}	9.8×10^{-2}
$(120\text{MeV})^2$	1.8×10^{-2}	0.11
$(180\text{MeV})^2$	1.3×10^{-2}	0.13

Table 2: The CP violating quantities A_1, A_2 with $w_L = 0$ for different values of q^2_{min}

We find that direct and indirect sources of CP nonconservation give comparable contributions to A_{CP}. In our computation we have neglected final state $\pi^+\pi^-$ interactions which are formally higher order in chiral perturbation theory. With the values of A_1 and A_2 given in Table 2, $|A_{CP}|$ is only of order 10^{-4}, and further refinements of our calculation do not seem warranted.

8 The Asymmetry B_{CP}

Because the CP violating contribution to the $K_L \to \pi^+\pi^- e^+e^-$ decay amplitude occurs at a lower order of chiral perturbation theory than does the CP conserving contribution, one might anticipate the existence of a large CP violating observable with its origin in indirect CP violation. The observable B_{CP}, which, in terms of the kinematic variables of the previous section takes the form

$$B_{CP} =< \mathrm{sign}(\sin\phi\cos\phi) > \quad , \tag{80}$$

is just such an observable. Neglecting $M^{(SD)}$ we find, after integrating over ϕ and $cos\theta_e$, that

$$B_{CP} = \frac{G_F^2 s_1^2 \alpha^2}{3\ 2^7 (2\pi)^8 f^2 m_K^3 \Gamma_{K_L}} \int dc_\pi\ ds\ dq^2\ \sin^2\theta_\pi\ \beta^3\ X^2\ \frac{s}{q^2} Im\left[G\ (F_+^* - F_-^*)\right] \quad . \tag{81}$$

If the variables s and q^2 are not integrated over the entire phase space, then it is understood that the same is to be done for the K_L width Γ_{K_L} in the denominator of (81). The form factor G is real at leading order in chiral perturbation theory, and the imaginary part arises from the phase in $F_+ - F_-$ induced by $K^0 - \overline{K}^0$ mixing. The integration over $\cos\theta_\pi$ implies that $F_+ - F_- \to F_+^{(1)} - F_-^{(1)}$ in eq. (81). Using our expressions for $F_\pm^{(1)}$ and the value of $|G^{(2)}|$, we find that, with $w_L = 0$, $|B_{CP}| \simeq 6.3\%$ for $q^2 > (30\ MeV)^2$ and $|B_{CP}| \simeq 2.4\%$ for $q^2 > (80\ MeV)^2$. The asymmetries for a range of values of $q^2_{\min}$ are shown in Table 3.

Lower cut $q^2_{\min}$	$\vert B_{CP} \cdot Br(10^{-8})\vert$ (%)
$(10\text{MeV})^2$	134
$(20\text{MeV})^2$	78
$(30\text{MeV})^2$	50
$(40\text{MeV})^2$	33
$(60\text{MeV})^2$	14
$(80\text{MeV})^2$	6.3
$(100\text{MeV})^2$	2.5
$(120\text{MeV})^2$	0.92
$(180\text{MeV})^2$	0.0086

Table 3: The CP violating observable $|B_{CP} \cdot Br(10^{-8})|$ for a range of values of $q^2_{\min}$

Note that, in Table 3, $Br(10^{-8})$ denotes the $K_L \to \pi^+\pi^-e^+e^-$ branching ratio in units of 10^{-8} with the same cut on q^2 imposed.

We may refine our calculation somewhat by studying the effect of final state interactions on B_{CP}. To do so, we calculate the absorptive parts of G and $(F_+ - F_-)$ using chiral perturbation theory. The effect of final state interactions on B_{CP} was studied previously in Refs. 3 and 4. Our approach includes both $\pi\pi \to \pi^+\pi^-$ and $\pi\pi \to \pi^+\pi^-\gamma^*$ rescattering, whereas previous efforts used the measured pion phase shifts and neglected $\pi\pi \to \pi^+\pi^-\gamma^*$.

The influence of such final state interactions on B_{CP} is accounted for by setting

$$Im[G(F_+ - F_-)^*] \to Im[G^{(2)}(F_+^{(1)} - F_-^{(1)})^*]$$

$$+Re[AbsG^{(3)}(F_+^{(1)} - F_-^{(1)})^*] - Re[G^{(2)}(AbsF_+^{(2)} - AbsF_-^{(2)})^*], \qquad (82)$$

in (81), where $DispG^{(3)}$, $AbsG^{(3)}$, $DispF_\pm^{(2)}$, and $AbsF_\pm^{(2)}$ are real quantities, aside from their dependence on ϵ, defined by $G^{(3)} = DispG^{(3)} + iAbsG^{(3)}$ and $F_\pm^{(2)} =$

$DispF_{\pm}^{(2)} + iAbsF_{\pm}^{(2)}$. The first of the three terms on the right-hand side of eq. (82) was examined above and the last two represent the effects of final state interactions. The Feynman graph shown in Fig. 6 gives

$$AbsG^{(3)} = \frac{G^{(2)}}{48\pi}\left(\frac{s}{f^2}\right)\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2}. \tag{83}$$

Unfortunately, the dispersive part of $G^{(3)}$ is not calculable, as it receives a contribution not only from the loop graph in Fig. 6, but also from loop graphs involving the Wess–Zumino term and from new order p^6 local operators in the chiral Lagrangian for weak radiative kaon decay.

The absorptive parts of $F_{\pm}$ first arise at second order in chiral perturbation theory from the Feynman diagrams in Fig. 7. These give

$$AbsF_{+}^{(2)} = -g_8(m_K^2 - m_\pi^2)\pi\epsilon\left\{\frac{(4m_K^2 - 2m_\pi^2)}{q^2 + 2q\cdot p_+}\sqrt{1 - \frac{4m_\pi^2}{m_K^2}}\right.$$

$$-4\left[\int_0^{\xi_-} y_+ dy - \int_0^{\xi_+} y_- dx\right]$$

$$\left. - \frac{8q\cdot(p_+ - p_-)}{s}\left[\int_0^{\xi_+}\frac{xy_-}{(y_+ - y_-)}dx + \int_0^{\xi_-}\frac{xy_+}{(y_+ - y_-)}dx\right]\right\}. \tag{84}$$

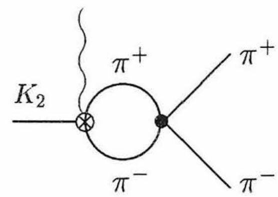

Fig. 6. Feynman diagram contributing to $AbsG^{(3)}$ at leading order. In this figure and those that follow, a solid circle denotes a vertex arising from the leading order strong and electromagnetic chiral Lagrangian. The other vertex in this figure arises from an $\mathcal{O}(p^4)$ counterterm in the chiral Lagrangian.

$AbsF_-^{(2)}$ is obtained from eq. (84) by interchanging p_+ with p_- using the symmetry property in eq. (5). The limits of integration in eq. (84) are given by

$$\xi_\pm = \frac{1 \pm \sqrt{1 - 4m_\pi^2/m_K^2}}{2}, \tag{85}$$

while the variables $y_\pm$ are defined by

$$y_\pm = \frac{(1-x)s + x(m_K^2 - q^2) \pm \sqrt{((1-x)s + x(m_K^2 - q^2))^2 - 4s(m_\pi^2 - q^2x(1-x))}}{2s}. \tag{86}$$

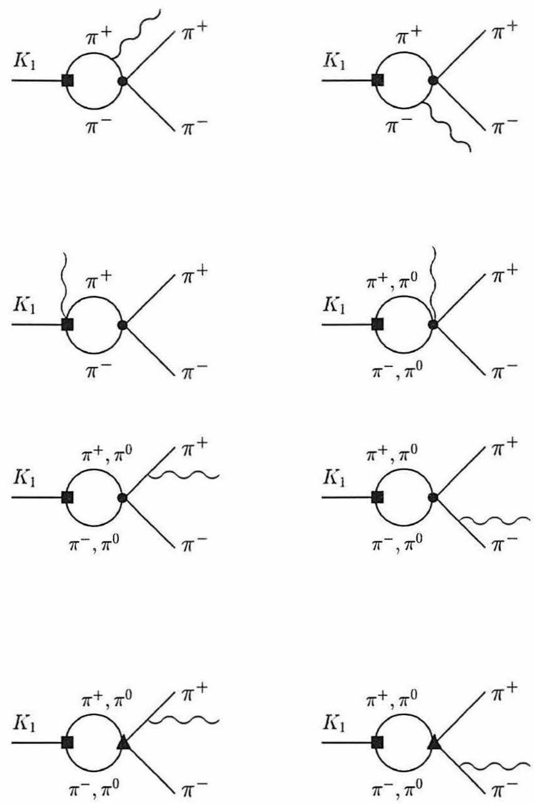

Fig. 7. Feynman diagrams contributing to $AbsF_{\pm}^{(2)}$ at leading order. A solid square denotes a vertex arising from the $\Delta S = 1$ part of the leading order gauged weak chiral Lagrangian. A solid triangle vertex arises from the piece of the leading order strong chiral Lagrangian proportional to the quark masses.

We find that final state interactions increase B_{CP} by about 45% over that presented above. The first term in eq. (84), and consequently the third term in eq. (82), is the dominant contribution from final state interactions, and it enhances B_{CP} by the factor

$$\frac{(4m_K^2 - 2m_\pi^2)}{32\pi f^2}\sqrt{1 - \frac{4m_\pi^2}{m_K^2}} \simeq 0.45 \tag{87}$$

over the leading order result obtained above. The trend that final state interactions increase B_{CP} is in agreement with Refs. 3 and 4. The rate Γ_{K_L} in the denominator of eq. (81) depends on the collection of counterterms defined as w_L above. Setting w_L to zero, we find that $|B_{CP}| \simeq 14\%$ with the cut $q^2 > (10MeV)^2$ imposed and $|B_{CP}| \simeq 4\%$ with the cut $q^2 > (80MeV)^2$ imposed. With $w_L = 2$, the asymmetry is even larger. We find in this case that $|B_{CP}| \simeq 18\%$ for each of the cuts listed above. Table 4 gives the predicted values for the magnitude of B_{CP} times the branching ratio for $K_L \to \pi^+\pi^-e^+e^-$(in units of 10^{-8}) for various cuts on the minimum lepton pair invariant mass squared, q^2_{min}. In this table, w_L has been set to zero.

Lower cut q^2_{min}	$\mid B_{CP}(\%) \mid \cdot Br(10^{-8})$
$(\ 10MeV)^2$	208
$(\ 20MeV)^2$	122
$(\ 30MeV)^2$	76
$(\ 40MeV)^2$	50
$(\ 60MeV)^2$	22
$(\ 80MeV)^2$	9.7
$(100MeV)^2$	3.9
$(120MeV)^2$	1.4
$(180MeV)^2$	0.013

Table 4: The CP violating observable $|B_{CP}| \cdot Br(10^{-8})$
for a range of values of q^2_{min}, including final state interactions

We have calculated the leading absorptive parts of the form factors G and $F_{\pm}$ using chiral perturbation theory and included, using eq. (82), their influence on B_{CP}. This is not a completely systematic approach, however, because $Im[DispG^{(3)}(F_{+}^{(1)} - F_{-}^{(1)})^{*}]$ and $Im[G^{(2)}(DispF_{+}^{(2)} - DispF_{-}^{(2)})^{*}]$ in eq. (82) were neglected, despite being the same order in the momentum expansion as the terms that were retained. This notwithstanding, including only the absorptive parts may be a good approximation as they are enhanced by a factor of π.

Finally we note that the absorptive parts of the form factors calculated here are also important for direct CP nonconservation in $K_L \to \pi^+\pi^-e^+e^-$. For example, the variable

$$D_{CP} =< sign(\cos\theta_e) > \tag{88}$$

is a CP violating observable that arises from interference of the one–photon amplitude in eq. (1) with the short distance contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude, eq. (71). In the kaon rest frame, the electron-positron energy difference is proportional to $\cos\theta_e$; D_{CP} is therefore a measure of this e^+e^- energy asymmetry. After integrating over ϕ and $\cos\theta_e$, we find that

$$D_{CP} = \frac{s_1^2 G_F^2 \alpha^2}{2^7(2\pi)^6 m_K^3 f^2 \Gamma_{K_L}} \int d\cos\theta_\pi \, ds \, dq^2 \, \beta^3 X^2 \sin^2\theta_\pi s Im G Im\xi. \tag{89}$$

At leading order in chiral perturbation theory $ImG = AbsG^{(3)}$. Unfortunately, we find that D_{CP} is around 10^{-7}, and is therefore too small to be measured in the next generation of kaon decay experiments. We do not provide more detailed data on D_{CP} since the CP violating variable A_{CP} discussed above is also a measure of direct CP violation, and has a much larger magnitude of $\sim 10^{-4}$.

It is important to note that, although we have estimated the final state interactions at leading order in the chiral expansion for strong interactions, higher order contributions which we have not computed may modify our results. Indeed, in the

I=J=1 channel, the ρ plays an important role[17], and one might expect such modifications.

9 Conclusions

In this chapter, we have calculated the one-photon contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay rate, and have used chiral perturbation theory to determine the form factors appearing in the decay amplitude. For e^+e^- pairs with high invariant mass ($q^2 >> 4m_e^2$), we have found important new contributions that were not included in previous work[3] [4]. The amplitude for $K_L \to \pi^+\pi^-e^+e^-$ depends on the undetermined (renormalization scale independent) combination of counterterms w_L. For $q^2 = (k_+ + k_-)^2 > (30\ MeV)^2$ the branching ratio for $K_L \to \pi^+\pi^-e^+e^-$ is approximately $(8.0 - 3.2w_L + 0.8w_L^2) \times 10^{-8}$ and for $q^2 > (80\ MeV)^2$ the branching ratio is approximately $(2.6 - 1.8w_L + 0.4w_L^2) \times 10^{-8}$.

One interesting aspect of this decay mode is that the CP even component of the K_L state contributes at a lower order in chiral perturbation theory than the CP odd component. This enhances CP violating effects in $K_L \to \pi^+\pi^-e^+e^-$ decay. For example, the CP violating observable B_{CP}, which is augmented by final state interactions by nearly 50%, is about 14% for $q^2 > (10MeV)^2$ if $w_L = 0$. The CP violating observables A_{CP} and D_{CP}, which arise from the interference of W-box and Z-penguin amplitudes with the one-photon part of the decay amplitude, have also been calculated. Unfortunately, we find that A_{CP} is of order 10^{-4}, and hence most likely unmeasurable in the near future. D_{CP}, at 10^{-7}, is even less accessible.

Chiral Perturbation theory has been extensively applied to nonleptonic, semileptonic and radiative kaon decays. The study of $K_L \to \pi^+\pi^-e^+e^-$ offers an opportunity to determine the linear combination of coefficients in the $\mathcal{O}(p^4)$ chiral lagrangian that we call w_L and to test the applicability of $\mathcal{O}(p^4)$ chiral perturbation theory for kaon decay.

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Chapter 3
Fragmentation to Excited Charmed Baryons

In this chapter, we employ heavy hadron chiral perturbation theory in studying the production of the excited charmed baryon doublet Λ_c^* via fragmentation. The flavor of this effective field theory is very similar to that in the previous chapter, with the exception that there is now an additional large mass scale present, the mass m_Q of a heavy quark. An expansion in inverse powers of this mass is irresistable. An analysis of the hadronic decays of the Λ_c^* doublet produces expressions for both the angular distribution of the decay products and the polarization of the final state heavy baryon in terms of various nonperturbative fragmentation parameters. Future experimental investigation of this system will determine these parameters. In addition, recent experimental results are shown to fix one of the parameters in the heavy hadron chiral Lagrangian.

1 Introduction

The production of a heavy quark at high energy via some hard process is a relatively well understood phenomenon, as we may bring the full apparatus of perturbative QCD to bear on the problem. Less well understood is the subsequent fragmentation of the heavy quark to form heavy mesons and baryons. It is the dynamics of this process that we propose to address in this chapter. We imagine that a heavy quark with mass $m_Q \gg \Lambda_{QCD}$ is produced on very short time scales in a hard reaction. It then travels out along the axis of fragmentation and hadronizes on a much longer time scale, at distances of order $1/\Lambda_{QCD}$. The fractional change in the heavy quark's velocity is therefore of order (Λ_{QCD}/m_Q), and vanishes at leading order in the heavy quark limit. Likewise, the heavy quark spin couples to the light degrees of freedom via the color magnetic moment operator

$$\frac{1}{m_Q}\,\bar{h}_v^{(Q)}\,\sigma_{\mu\nu}\,G^{a\mu\nu}\,T^a\,h_v^{(Q)} \quad , \tag{1}$$

which again vanishes in the heavy limit. We may therefore view the initial fragmentation process as leaving the heavy quark velocity and spin unchanged. Notice that, in this limit, the dynamics are also blind to the mass of the heavy quark, which therefore acts as a static color source in its interactions with the light degrees of freedom.

This simple result may not apply to the ultimate products of the strong fragmentation process, however, as was pointed out by Falk and Peskin[1]. Specifically, the polarization of the final state heavy baryons and mesons may not be determined solely by the heavy quark spin, but may depend in addition on the spin of the light degrees of freedom involved in the fragmentation process. This is the case when the initial fragmentation products decay to lower energy heavy baryons and mesons on a time scale long enough to allow interaction between the heavy quark spin and that of the light degrees of freedom. We will find that this is indeed the case in the Λ_c^* system.

In this situation, one must know something about the spin of the light degrees of freedom in order to proceed further. The parity invariance of the strong interactions, coupled with heavy quark spin symmetry, demands that formation of light degrees of freedom with spin j depends only on the magnitude of the projection of j onto the axis of fragmentation, and not on its sign. That is, transverse may be preferred to longitudinal, but forward may not be preferred to back. Further, the light system may prefer to invest its angular momentum in orbital channels as opposed to spin channels. These preferences are catalogued by a set of fragmentation parameters: A and ω_1, defined in Ref. 1, and B and $\widetilde{\omega}_1$, defined in the following section.

Let us consider a fragmentation process in which light degrees of freedom of spin j are produced. They then associate with the heavy quark spin $s = \frac{1}{2}$ to form a doublet of total spin $J = j \pm \frac{1}{2}$. Two paths now lie open. The doublet (the two members of which have the same decay rate in the heavy quark limit) may decay rapidly enough that heavy quark spin flip processes have no time to occur. Then the doublet states decay coherently, the heavy quark retains its initial polarization in the final states, and the process begins anew with the decay products. On the other hand, heavy quark spin flip processes may have time to occur, in which case the doublet states decay incoherently, and the heavy quark polarization is altered. The two parameters responsible for determining which regime we are in are the total decay rate out of the doublet, Γ, and the mass splitting between the doublet states, Δ. The splitting Δ vanishes in the heavy quark limit, and is of the order of the rate for heavy quark spin flip processes within the doublet. We therefore expect that the situation $\Gamma \gg \Delta$ produces overlapping resonances which decay coherently out of the multiplet, and that the opposite extreme $\Gamma \ll \Delta$ allows for incoherent decays and the influence of the spin of the light degrees of freedom.

2 The Charmed Baryon System

In the charmed baryon system, the ground state is obtained by putting the light diquark in an antisymmetric $I = S = 0$ state with spin-parity $j^P = 0^+$. This yields the $J^P = \frac{1}{2}^+$ baryon Λ_c^+, with mass 2285 MeV. Alternatively, the light quarks may form a symmetric $I = S = 1$ state with spin-parity $j^P = 1^+$. The light spin then couples to that of the heavy quark to produce the symmetric $J^P = (\frac{3}{2}^+, \frac{1}{2}^+)$ doublet $(\Sigma_c^{*(0,+,++)}, \Sigma_c^{(0,+,++)})$ with mass (2530 MeV, 2453 MeV). Fragmentation through the $\Sigma_c^{(*)}$ system has already been considered in Ref. 1; we concern ourselves here with the $J^P = (\frac{3}{2}^-, \frac{1}{2}^-)$ doublet $(\Lambda_{c1}^*, \Lambda_{c1})$ that results when the light diquark is an $I = S = 0$ state with a single unit of orbital angular momentum. Allowing the light quarks to have both spin and orbital angular momentum produces a tremendous number of states, none of which have been observed to date. We ignore such states in the analysis that follows.

The fragmentation parameters A, B, ω_1, and $\tilde{\omega}_1$ may now be defined. A is taken to be the relative probability of producing any of the nine $I = S = 1$, $j^P = 1^+$ diquark states during fragmentation relative to that of producing the $I = S = 0$, $j^P = 0^+$ ground state. B is similarly the probability for producing any of the three $I = S = 0$, $j^P = 1^-$ diquark states relative to ground state production. The parameters ω_1 and $\tilde{\omega}_1$, on the other hand, encode the orientation of the light diquark angular momentum. The various helicity states of the spin-parity 1^+ and 1^- diquarks are populated with the probabilities

$$P[1] = P[-1] = \frac{\omega_1}{2}; \;\; P[0] = 1 - \omega_1 \;\; for \;\; j^P = 1^+ \quad , \tag{2}$$

and

$$P[1] = P[-1] = \frac{\tilde{\omega}_1}{2}; \;\; P[0] = 1 - \tilde{\omega}_1 \;\; for \;\; j^P = 1^- \quad . \tag{3}$$

The analysis of the excited D system in Ref. 1 has already indicated that $\omega_{3/2}$, the analog of ω_1 for the light degrees of freedom in the meson sector, is likely close to zero. One might also anticipate, therefore, that ω_1 would be close to zero. We will concentrate on $\tilde{\omega}_1$ most heavily in what follows.

The masses of the Λ_{c1}^* and Λ_{c1} are naively expected to be split by $\sim \frac{\Lambda_{QCD}^2}{m_c} \simeq 30MeV$, in fortuitously close agreement with the recently measured values $M_{\Lambda_{c1}^*} =$ 2625 MeV and $M_{\Lambda_{c1}} =$ 2593 MeV[2]. Decay of the Λ_{c1}^* to Λ_{c1} via pion emission is thus kinematically forbidden, and the corresponding electromagnetic transition is very slow compared with strong decays out of the doublet. Indeed, the dominant decay mode of both Λ_{c1}^* and Λ_{c1} is to Λ_c via pion emission. As both $(\Lambda_{c1}^*, \Lambda_{c1})$ and Λ_c are $I = 0$ states, single pion emission is forbidden by isospin conservation, and the dominant modes are $\Lambda_{c1}^* \to \Lambda_c\pi\pi$ and $\Lambda_{c1} \to \Lambda_c\pi\pi$. The mass differences $(M_{\Lambda_{c1}^*} - M_{\Lambda_c})$ =340 MeV and $(M_{\Lambda_{c1}} - M_{\Lambda_c})$ =308 MeV are very close to threshold, and the pions produced will be soft. We therefore expect the decays to be accurately described by heavy hadron chiral perturbation theory.

The CLEO collaboration recently measured the Λ_{c1} width to be $\Gamma_{\Lambda_{c1}} = 3.9^{+1.4+2.0}_{-1.2-1.0}$ MeV, and placed a new upper bound on the Λ_{c1}^* width: $\Gamma_{\Lambda_{c1}^*}$ <1.9 MeV[2]. It is an interesting breakdown of the naive heavy quark approximation that these rates are significantly different. The explanation is that, at leading order in the heavy hadron chiral Lagrangian, Λ_{c1}^* is connected to Λ_c only via an intermediate Σ_c^*, whereas Λ_{c1} is connected via an intermediate Σ_c. Kinematics allows the Σ_c, but not the Σ_c^*, to go on-shell. The Λ_{c1} thus enjoys a resonant amplification of its decay rate. We also note that the rates above place us securely in the regime $\Gamma \ll \Delta$, so that we anticipate interaction of the heavy quark spin with the light degrees of freedom in decays to the Λ_c. This will allow us to shed some light on the parameter $\tilde{\omega}_1$. In the following section, we provide a brief review of heavy hadron chiral perturbation theory before tackling the $(\Lambda_{c1}^*, \Lambda_{c1})$ decays.

3 Heavy Hadron Chiral Perturbation Theory

Heavy hadron chiral perturbation theory incorporates aspects of both ordinary chiral perturbation theory and the heavy quark effective theory, and describes the low energy interactions between hadrons containing a heavy quark and the light pseudo-Goldstone bosons. It has been discussed previously in a number of papers[3].

For definiteness we consider the charmed baryon system. Members of the ground state $J^P = \frac{1}{2}^+$ antitriplet are destroyed by the velocity dependent Dirac fields $\mathcal{T}_i(v)$, where

$$\mathcal{T}_1 = \Xi_c^0 \qquad \mathcal{T}_2 = -\Xi_c^+ \qquad \mathcal{T}_3 = \Lambda_c^+ \ . \tag{4}$$

The symmetric $J^P = \frac{1}{2}^+$ states are destroyed by the Dirac fields $S^{ij}(v)$ with components

$$\begin{aligned} S^{11} = \Sigma_c^{++} \qquad S^{12} = \sqrt{\frac{1}{2}}\Sigma_c^+ \qquad S^{22} = \Sigma_c^0 \\ S^{13} = \sqrt{\frac{1}{2}}\Xi_c^{+\prime} \qquad S^{23} = \sqrt{\frac{1}{2}}\Xi_c^{0\prime} \\ S^{33} = \Omega_c^0 \ , \end{aligned} \tag{5}$$

and their symmetric $J^P = \frac{3}{2}^+$ counterparts by the corresponding Rarita-Schwinger fields $S_\mu^{*ij}(v)$. Finally, we define Dirac and Rarita-Schwinger fields $R_i(v)$ and $R_{\mu i}^*(v)$ to annihilate the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ excited antitriplet states respectively. In our analysis the components of interest will be $R_3 = \Lambda_{c1}$ and $R_{\mu 3}^* = \Lambda_{c1,\mu}^*$.

As the heavy quark mass goes to infinity, the $J = \frac{3}{2}$ and $J = \frac{1}{2}$ members of the sextet and excited antitriplet multiplets become degenerate. It is then useful to combine them to form the superfields $\mathcal{R}_{\mu i}$ and $\mathcal{S}_\mu^{ij}$, defined by

$$\mathcal{R}_{\mu i} = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma^5 R_i + R^*_{\mu i} \quad , \tag{6}$$

$$\mathcal{S}_\mu^{ij} = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma^5 S^{ij} + S_\mu^{*ij} \quad . \tag{7}$$

If we are to discuss decay by π emission, we must also incorporate the pseudo-Goldstone boson octet into our Lagrangian. The Goldstone bosons are a product of the spontaneous breakdown of the chiral flavor symmetry $SU(3)_L \times SU(3)_R$ to $SU(3)_V$, its diagonal subgroup. They appear in the octet

$$M = \sum_a \pi^a T^a = \sqrt{\frac{1}{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \quad , \tag{8}$$

and are conveniently incorporated into the Lagrangian via the dimensionless fields $\Sigma \equiv e^{\frac{2iM}{f}}$ and $\xi \equiv e^{\frac{iM}{f}}$, where $f = f_\pi$ =93 MeV, the pion decay constant, at lowest order in chiral perturbation theory.

The goal is to combine these fields to produce a Lorentz invariant, parity even, heavy quark spin symmetric, and light chiral invariant Lagrangian. To this end, we now assemble various transformation properties of the fields. Under parity, P, the superfields transform as

$$P\mathcal{R}_\mu(\vec{r}, t)P^{-1} = \gamma_0 \mathcal{R}^\mu(-\vec{r}, t) \quad , \tag{9}$$

$$P\mathcal{S}_\mu(\vec{r}, t)P^{-1} = -\gamma_0 \mathcal{S}^\mu(-\vec{r}, t) \quad , \tag{10}$$

$$P\mathcal{T}(\vec{r},t)P^{-1} = \gamma_0 \mathcal{T}(-\vec{r},t) \quad . \tag{11}$$

They also obey the constraints

$$v^\mu \mathcal{R}_\mu = v^\mu \mathcal{S}_\mu = 0; \quad \not{v}\mathcal{R}_\mu = \mathcal{R}_\mu; \quad \not{v}\mathcal{S}_\mu = \mathcal{S}_\mu; \quad \not{v}\mathcal{T} = \mathcal{T} \quad . \tag{12}$$

The Rarita-Schwinger components obey the additional constraints

$$\gamma^\mu \mathcal{R}^*_{\mu i} = \gamma^\mu \mathcal{S}^{*ij}_\mu = 0 \quad . \tag{13}$$

We are also interested in how the various fields transform under chiral $SU(3)$. The Σ and ξ fields obey

$$\Sigma \to L\Sigma R^\dagger \quad , \tag{14}$$

$$\xi \to L\xi U^\dagger(x) = U(x)\xi R^\dagger \quad , \tag{15}$$

where L and R are global $SU(3)$ matrices, and $U(x)$ is a local member of $SU(3)_V$. If we further define the vector and axial vector fields

$$V^\mu = \frac{1}{2}[\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger] \quad , \tag{16}$$

$$A^\mu = \frac{i}{2}[\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger] \quad , \tag{17}$$

we find that, under chiral $SU(3)$,

$$V^\mu \rightarrow U V^\mu U^\dagger + U(\partial^\mu U^\dagger) \quad , \tag{18}$$

$$A^\mu \rightarrow U A^\mu U^\dagger \quad . \tag{19}$$

The only constraint imposed on the heavy fields is that they transform according to the appropriate sextet or antitriplet representation under transformations of the $SU(3)_V$ subgroup.

There remains one final symmetry to aid us in constructing our Lagrangian, and that is symmetry under reparameterization of the heavy field velocity. The momentum of a heavy hadron is written $p = Mv + k$, where k is termed the residual momentum of the hadron. If we make the following shifts in v and k

$$v \rightarrow v + \epsilon/M; \qquad k \rightarrow k - \epsilon \quad , \tag{20}$$

with $v \cdot \epsilon = 0$, then $p \rightarrow p$ and $v^2 \rightarrow v^2 + \mathcal{O}(1/M^2)$. Therefore, if we are working only to leading order in the $(1/M)$ expansion, we demand that our Lagrangian be invariant under such a transformation. The corresponding shifts induced in the fields are[4]

$$\delta \mathcal{R}_\mu = \frac{\not\epsilon}{2M} \mathcal{R}_\mu - \frac{\epsilon^\nu \mathcal{R}_\nu}{M} v_\mu \quad , \tag{21}$$

$$\delta \mathcal{S}_\mu = \frac{\not\epsilon}{2M} \mathcal{S}_\mu - \frac{\epsilon^\nu \mathcal{S}_\nu}{M} v_\mu \quad , \tag{22}$$

$$\delta \mathcal{T} = \frac{\not\epsilon}{2M} \mathcal{T} \quad . \tag{23}$$

Invariance of the Lagrangian under these shifts further restricts the terms that may appear, and leaves us with the following form for the most general Lorentz invariant, parity even, heavy quark spin symmetric, and light chiral invariant Lagrangian:

$$
\begin{aligned}
\mathcal{L}_v^{(0)} = \{ &\bar{\mathcal{R}}^i_\mu(-iv\cdot\mathcal{D}+\Delta M_{\mathcal{R}})\mathcal{R}^\mu_i + \bar{\mathcal{S}}^\mu_{ij}(-iv\cdot\mathcal{D}+\Delta M_{\mathcal{S}})\mathcal{S}^{ij}_\mu \\
&+\bar{\mathcal{T}}^i iv\cdot\mathcal{D}\mathcal{T}_i + ig_1\epsilon_{\mu\nu\sigma\lambda}\bar{\mathcal{S}}^\mu_{ik}v^\nu(A^\sigma)^i_j(\mathcal{S}^\lambda)^{jk} \\
&+ig_2\epsilon_{\mu\nu\sigma\lambda}\bar{\mathcal{R}}^{\mu i}v^\nu(A^\sigma)^i_j(\mathcal{R}^\lambda)_j \\
&+h_1[\epsilon_{ijk}\bar{\mathcal{T}}^i(A^\mu)^j_i\mathcal{S}^{kl}_\mu + \epsilon^{ijk}\bar{\mathcal{S}}^\mu_{kl}(A_\mu)^l_j\mathcal{T}_i] \\
&+h_2[\epsilon_{ijk}\bar{\mathcal{R}}^{\mu i}v\cdot A^j_l\mathcal{S}^{kl}_\mu + \epsilon^{ijk}\bar{\mathcal{S}}^\mu_{kl}v\cdot A^l_j\mathcal{R}_{\mu i}]\} \quad ,
\end{aligned}
\tag{24}
$$

where $\Delta M_{\mathcal{R}} = M_{\mathcal{R}} - M_{\mathcal{T}}$ is the mass splitting between the excited and ground state antitriplets, and $\Delta M_{\mathcal{S}} = M_{\mathcal{S}} - M_{\mathcal{T}}$ is the corresponding splitting between the sextet and the ground state antitriplet.

In defining the velocity dependent heavy fields which appear above, a common mass must be scaled out of all heavy fields

$$
H = e^{-iMv\cdot x}H_v \quad , \tag{25}
$$

despite the different masses of the various heavy baryons. In the above analysis we have chosen $M = M_{\Lambda_c}$.

It is also instructive at this point to examine the term proportional to h_2, which allows single π transitions between the excited antitriplet and sextet states. This term induces only S-wave transitions, although naive angular momentum and parity arguments would allow D-wave transitions as well. The D-wave transitions are induced by a higher dimension operator which is therefore suppressed by further powers of M and does not appear at leading order in the heavy hadron Lagrangian. This absence of D-wave transitions simplifies the way in which the π distributions depend on $\tilde{\omega}_1$ in the $\Lambda_{c1}^{(*)}$ decay process. Finally, we comment quickly on the errors induced by keeping only leading order terms. The relevant expansion parameter in our analyses is $(\frac{p_\pi}{M})$, so that we expect our results to be valid to $\sim (200/2285) \simeq 10\%$.

4 The Parameter h_2

The term proportional to h_2 in the leading order Lagrangian is responsible for the tree-level decay $\Lambda_{c1} \to \Sigma_c \pi$, the rate for which is easily calculated to be

$$\Gamma(\Lambda_{c1} \to \Sigma_c \pi) = \frac{|h_2|^2}{4\pi f^2} \frac{M_{\Sigma_c}}{M_{\Lambda_{c1}}} (M_{\Lambda_{c1}} - M_{\Sigma_c})^2 \sqrt{(M_{\Lambda_{c1}} - M_{\Sigma_c})^2 - m_\pi^2} \ , \tag{26}$$

as was done previously in 4. The Σ_c may then decay to $\Lambda_c \pi$ through the term proportional to h_1, producing a decay rate $\Gamma(\Lambda_{c1} \to \Lambda_c \pi\pi)$ that scales like the combination $|h_1|^2 |h_2|^2$. A quick calculation allows us to express $|h_1|^2$ in terms of the partial width $\Gamma(\Sigma_c \to \Lambda_c \pi)$,

$$\Gamma(\Sigma_c \to \Lambda_c \pi) = \frac{|h_1|^2}{12\pi f^2} \frac{M_{\Lambda_c}}{M_{\Sigma_c}} [(M_{\Sigma_c} - M_{\Lambda_c})^2 - m_\pi^2]^{3/2} \quad , \tag{27}$$

which is by far the dominant contribution to Γ_{Σ_c}. We may therefore view $\Gamma(\Lambda_{c1} \to \Lambda_c \pi\pi)$ as a function of h_2 and Γ_{Σ_c}. This decay is dominated by the pole region where Σ_c is close to being on-shell, and its rate coincides with that for $\Lambda_{c1} \to \Sigma_c \pi$ as $\Gamma_{\Sigma_c} \to 0$. In this narrow width approximation, we obtain

$$\Gamma(\Lambda_{c1} \to \Lambda_c \pi^+ \pi^-) = 4.6 |h_2|^2 \text{ MeV} \quad . \tag{28}$$

The result is modified slightly if we allow the Σ_c to have a finite width. The Σ_c is not expected to have a width greater than a few MeV. Setting $\Gamma_{\Sigma_c} = 2$ MeV, we find

$$\Gamma(\Lambda_{c1} \to \Lambda_c \pi^+ \pi^-) = 4.2 |h_2|^2 \text{ MeV} \quad . \tag{29}$$

Comparison with the CLEO measurement[2]

$$\Gamma(\Lambda_{c1} \to \Lambda_c \pi^+ \pi^-) = 3.9^{+1.4+2.0}_{-1.2-1.0} \text{ MeV} \tag{30}$$

then yields a central value of $|h_2| \simeq 0.9$ in the narrow width approximation, or $|h_2| \simeq 1.0$ with $\Gamma_{\Sigma_c} = 2$ MeV.

5 Production and Decay of Λ_{c1} and Λ_{c1}^*

The probabilities for fragmentation to the Λ_{c1} and Λ_{c1}^* states of various helicities may be expressed in terms of the parameters $\tilde{\omega}_1$ and B once the initial polarization of the heavy quark is given. For simplicity, we assume that the initial charm quark is completely left-hand polarized in the analysis that follows. With this assumption, the relative populations of the Λ_{c1}^* and Λ_{c1} states are

$$P[\Lambda_{c1}^*] = \frac{B}{1+A+B}[\frac{\tilde{\omega}_1}{2}, \frac{2}{3}(1-\tilde{\omega}_1), \frac{\tilde{\omega}_1}{6}, 0] \quad , \tag{31}$$

$$P[\Lambda_{c1}] = \frac{B}{1+A+B}[\frac{1}{3}(1-\tilde{\omega}_1), \frac{1}{3}\tilde{\omega}_1] \quad , \tag{32}$$

where the helicity states for Λ_{c1}^* read $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ from left to right, and those for Λ_{c1} read $-\frac{1}{2}, \frac{1}{2}$.

We now wish to calculate the double-pion distributions in the decays of these states to the ground state Λ_c. The differential decay rate may be written

$$\frac{d\Gamma}{d\Omega_1 d\Omega_2} = \frac{|M_{fi}|^2}{8M_{\Lambda_{c1}^{(*)}} M_{\Lambda_c}(2\pi)^5}\sqrt{(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)}\,\delta(M_{\Lambda_{c1}^{(*)}} - E_1 - E_2 - M_{\Lambda_c}) dE_1 dE_2 \ , \tag{33}$$

where Ω_1 and Ω_2 contain the angular variables for the two pions and E_1 and E_2 are their energies. A glance at the expression above indicates that we are conserving three momentum, but not energy. The explanation is simply that in the infinite mass limit, the charm baryon recoils to conserve momentum, but carries off a negligible amount of energy in the process.

Let us first address the case of Λ_{c1}^{*} and Λ_{c1} decay to $\Lambda_c\pi^0\pi^0$. The relevant Feynman diagrams which arise from the Lagrangian (24) are shown in Fig. 1.

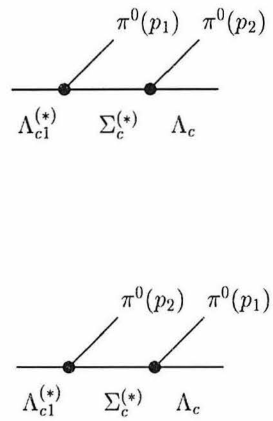

Fig. 1. Feynman diagrams contributing to $\Lambda_{c1}^{*} \to \Lambda_{c1}\pi\pi$ at leading order in the heavy hadron chiral Lagrangian.

In calculating the decays between Λ_{c1}^{*} and Λ_{c1} states of definite helicity, we find two distinct angular patterns, depending only on the change in the component of spin along the fragmentation axis, ΔS_z, between the initial and final state heavy hadrons:

$$F_1(\Omega_1,\Omega_2) = \frac{3}{32\pi^2}[\cos^2\theta_1 + \cos^2\theta_2 + \alpha\cos\theta_1\cos\theta_2] \quad , \qquad (34)$$

$$F_2(\Omega_1,\Omega_2) = \frac{3}{64\pi^2}[\sin^2\theta_1 + \sin^2\theta_2 + \alpha\sin\theta_1\sin\theta_2\cos(\phi_2-\phi_1)] \quad , \qquad (35)$$

where θ_1 and θ_2 are the angles between the two pion momenta and the fragmentation axis, and ϕ_1 and ϕ_2 are the azimuthal angles of the pion momenta about this axis.

These angles are defined in the rest frame of the decaying $\Lambda_{c1}^{(*)}$. The number α arises from interference between the two graphs depicted in Fig. 1, and is defined in eq. (36) below. Its dependence on the width $\Gamma_{\Sigma_c^*}$ is plotted in Fig. 2. To the order we are working, α=1.3 for any reasonable value of $\Gamma_{\Sigma_c^*}$.

$$
\begin{aligned}
\alpha &\equiv \alpha_1/\alpha_2 \ ; \\
\alpha_1 &= \int_{m_\pi}^{M_{\Lambda_{c1}^*}-M_{\Lambda_c}} dE_1 \int dE_2 \delta(M_{\Lambda_{c1}^*} - M_{\Lambda_c} - E_1 - E_2) \\
&\times \left(\frac{2E_1E_2(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)((M_{\Sigma_c^*}-M_{\Lambda_c}-E_1)(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)+(\Gamma_{\Sigma_c^*}/2)^2)}{((M_{\Sigma_c^*}-M_{\Lambda_c}-E_1)(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)+(\Gamma_{\Sigma_c^*}/2)^2)^2+(\Gamma_{\Sigma_c^*}/2)^2(E_1-E_2)^2}\right) \ ; \\
\alpha_2 &= \int_{m_\pi}^{M_{\Lambda_{c1}^*}-M_{\Lambda_c}} dE_1 \int dE_2 \delta(M_{\Lambda_{c1}^*} - M_{\Lambda_c} - E_1 - E_2)\left(\frac{E_1^2(E_2^2-m_\pi^2)^{3/2}(E_1^2-m_\pi^2)^{1/2}}{(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)^2+(\Gamma_{\Sigma_c^*}/2)^2}\right)
\end{aligned}
\tag{36}
$$

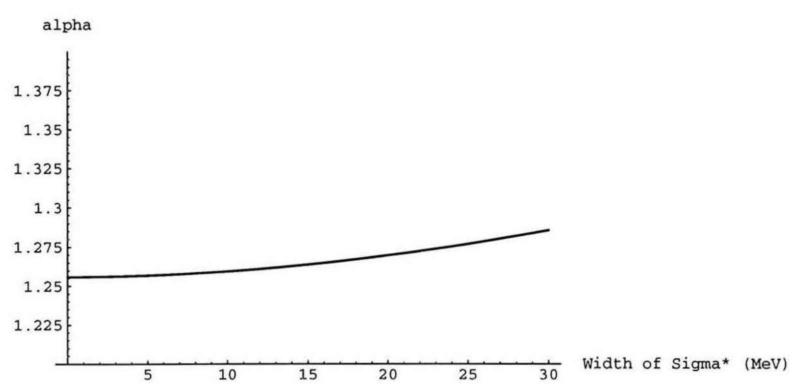

Fig. 2. The variation of the coefficient α as a function of the width of Σ_c^*.

The normalized differential rates $\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}$ for the various decays are then given in terms of F_1 and F_2 by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\{[\Lambda_{c1}^*(+\frac{1}{2}) \rightarrow \Lambda_c(+\frac{1}{2})], [\Lambda_{c1}^*(-\frac{1}{2}) \rightarrow \Lambda_c(-\frac{1}{2})]\} = F_1(\Omega_1, \Omega_2) \ , \tag{37}$$

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\{&[\Lambda_{c1}^*(+\frac{3}{2}) \rightarrow \Lambda_c(+\frac{1}{2})], [\Lambda_{c1}^*(+\frac{1}{2}) \rightarrow \Lambda_c(-\frac{1}{2})], \\ &[\Lambda_{c1}^*(-\frac{1}{2}) \rightarrow \Lambda_c(+\frac{1}{2})], [\Lambda_{c1}^*(-\frac{3}{2}) \rightarrow \Lambda_c(-\frac{1}{2})]\} = F_2(\Omega_1, \Omega_2) \ .\end{aligned} \tag{38}$$

The decays $\Lambda_{c1}^*(\pm\frac{3}{2}) \rightarrow \Lambda_c(\mp\frac{1}{2})$ are forbidden. A similar calculation for Λ_{c1} decays yields

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\{[\Lambda_{c1}(+\frac{1}{2}) \rightarrow \Lambda_c(+\frac{1}{2})], [\Lambda_{c1}(-\frac{1}{2}) \rightarrow \Lambda_c(-\frac{1}{2})]\} = G_1(\Omega_1, \Omega_2) \tag{39}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\{[\Lambda_{c1}(+\frac{1}{2}) \rightarrow \Lambda_c(-\frac{1}{2})], [\Lambda_{c1}(-\frac{1}{2}) \rightarrow \Lambda_c(+\frac{1}{2})]\} = G_2(\Omega_1, \Omega_2) \ , \tag{40}$$

where

$$G_1 = \frac{3}{32\pi^2}[\cos^2\theta_1 + \cos^2\theta_2 + \beta\cos\theta_1\cos\theta_2] \tag{41}$$

$$G_2 = \frac{3}{64\pi^2}[\sin^2\theta_1 + \sin^2\theta_2 + \beta\sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1)] \ . \tag{42}$$

The ratio β is defined analogously to α in (36), but with the substitutions $M_{\Lambda_{c1}^*} \rightarrow M_{\Lambda_{c1}}$, $M_{\Sigma_c^*} \rightarrow M_{\Sigma_c}$, and $\Gamma_{\Sigma_c^*} \rightarrow \Gamma_{\Sigma_c}$, that is, by removing all stars in (36). Its dependence on Γ_{Σ_c} is shown in Fig. 3. That β is much smaller than α is easily understood. Both α and β arise from the interference between Feynman graphs, but in the case of Λ_{c1} decay, the intermediate Σ_c may go on shell, and in fact, the rate is dominated by this region of phase space. The Λ_{c1} decay is thus essentially a two-step process, and interference effects are therefore relatively unimportant. The steep dependence of β on the intermediate state width does not significantly limit our predictions since it is numerically small.

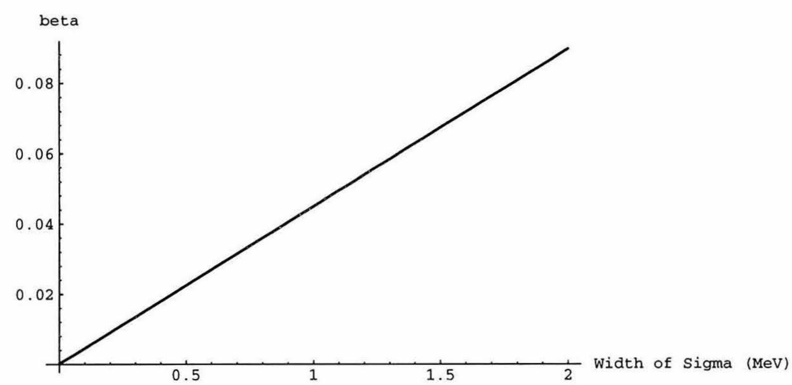

Fig. 3. The variation of the coefficient β as a function of the width of Σ_c.

We now take into account the initial populations of the various helicity states, as displayed in (31) and (32), and allow them to decay incoherently in light of the relation $\Gamma_{\Lambda_{c1}^{(*)}} \ll (M_{\Lambda_{c1}^*} - M_{\Lambda_{c1}})$. This produces, after summing final state helicities, the following double pion distributions for decay through Λ_{c1}^* and Λ_{c1} states separately:

$$
\begin{aligned}
\frac{1}{\Gamma}\frac{d\Gamma(\Lambda_{c1}^{*}\ only)}{d\Omega_1 d\Omega_2} =& \frac{3}{32\pi^2}\{[\frac{1}{3}+\frac{1}{2}(\cos^2\theta_1+\cos^2\theta_2)+\frac{2\alpha}{3}\cos\theta_1\cos\theta_2 \\
&+\frac{\alpha}{6}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)] \\
&+\tilde{\omega}_1[\frac{1}{2}-\frac{3}{4}(\cos^2\theta_1+\cos^2\theta_2)-\frac{\alpha}{2}\cos\theta_1\cos\theta_2 \\
&+\frac{\alpha}{4}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)]\} \quad ,
\end{aligned} \tag{43}
$$

$$
\frac{1}{\Gamma}\frac{d\Gamma(\Lambda_{c1}\ only)}{d\Omega_1 d\Omega_2} = \frac{1}{32\pi^2}\{2+\beta[\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)+\cos\theta_1\cos\theta_2]\} \quad . \tag{44}
$$

Combining both Λ_{c1}^{*} and Λ_{c1} decays incoherently yields

$$
\begin{aligned}
\frac{1}{\Gamma}\frac{d\Gamma(combined)}{d\Omega_1 d\Omega_2} =& \frac{1}{32\pi^2}\{[\frac{4}{3}+\cos^2\theta_1+\cos^2\theta_2+(\frac{4\alpha}{3}+\frac{\beta}{3})\cos\theta_1\cos\theta_2 \\
&+(\frac{\alpha}{3}+\frac{\beta}{3})\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)] \\
&+\tilde{\omega}_1[1-\frac{3}{2}(\cos^2\theta_1+\cos^2\theta_2)-\alpha\cos\theta_1\cos\theta_2 \\
&+\frac{\alpha}{2}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)]\} \quad .
\end{aligned} \tag{45}
$$

Note from Fig. 3 that β approaches zero as the width Γ_{Σ_c} vanishes. This means that the double pion distribution (44) resulting from Λ_{c1} decay becomes isotropic in this limit. This is easily understood as follows. As Γ_{Σ_c} approaches zero, Λ_{c1} decay is entirely dominated by production of a real intermediate Σ_c as discussed above, a process which may occur only via S-wave pion emission. The subsequent single pion decay of the Σ_c is also isotropic if Λ_c helicities are summed over, as previously observed in Ref. 1.

Integration of the combined distribution over azimuthal angles produces

$$\begin{aligned} \frac{1}{\Gamma}\frac{d\Gamma(combined)}{d\cos\theta_1 d\cos\theta_2} =& \frac{1}{8}\{[\frac{4}{3}+\cos^2\theta_1+\cos^2\theta_2+(\frac{4\alpha}{3}+\frac{\beta}{3})\cos\theta_1\cos\theta_2 \\ &+\tilde{\omega}_1[1-\frac{3}{2}(\cos^2\theta_1+\cos^2\theta_2)-\alpha\cos\theta_1\cos\theta_2]\}\ , \end{aligned} \tag{46}$$

which is plotted for a variety of $\tilde{\omega}_1$ values in Figs. 4 - 6.

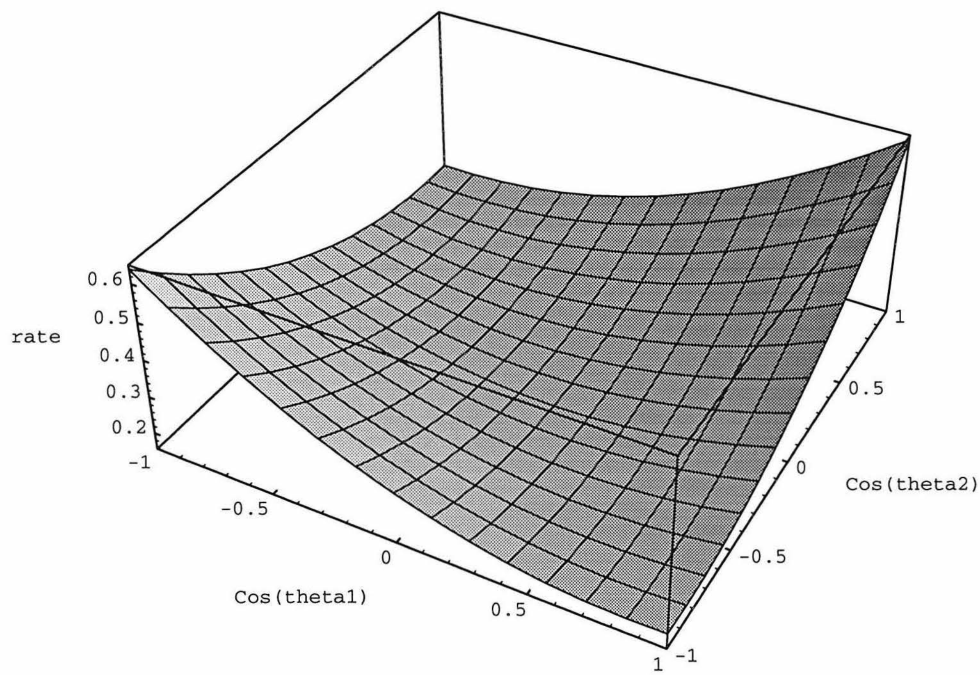

Fig. 4. Normalized differential decay rate for the case $\alpha = 1.3$, $\beta = 0.08$, and $\tilde{\omega}_1 = 0$.

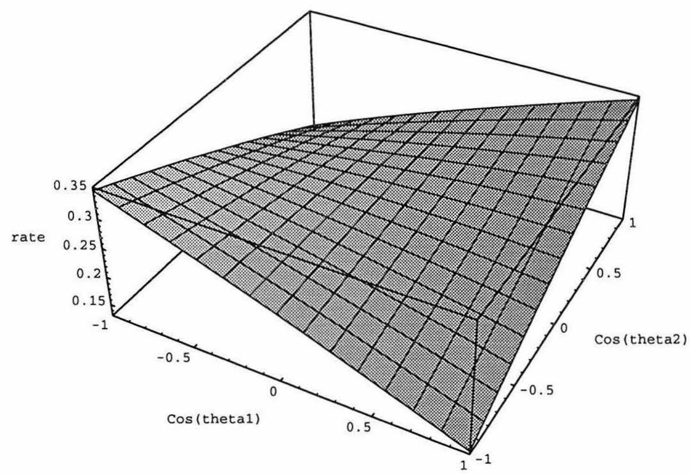

Fig. 5. Normalized differential decay rate for the case $\alpha = 1.3$, $\beta = 0.08$, and $\tilde{\omega}_1 = 0.7$.

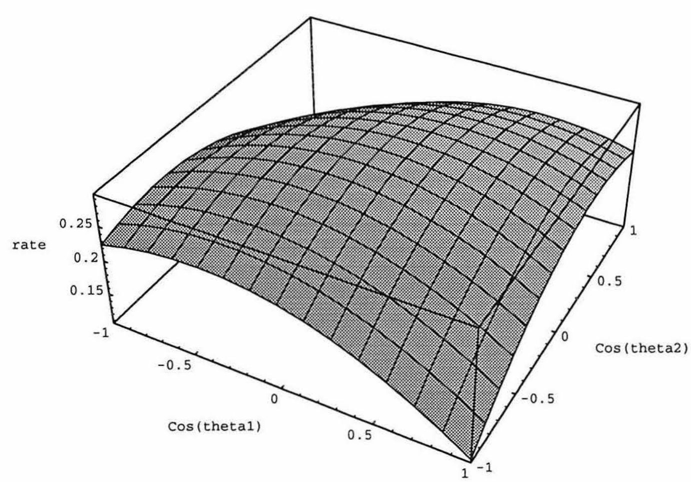

Fig. 6. Normalized differential decay rate for the case $\alpha = 1.3$, $\beta = 0.08$, and $\tilde{\omega}_1 = 1$.

Alternatively, we may prefer to integrate over pion angles and observe instead the polarization of the final Λ_c. We then find the population ratios

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})} = \frac{2 - \tilde{\omega}_1}{4 + \tilde{\omega}_1} \quad , \tag{47}$$

for fragmentation through Λ_{c1}^* alone,

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})} = \frac{2 - \tilde{\omega}_1}{1 + \tilde{\omega}_1} \quad , \tag{48}$$

for fragmentation through Λ_{c1} alone, and

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})} = \frac{4 - 2\tilde{\omega}_1}{5 + 2\tilde{\omega}_1} \quad , \tag{49}$$

for the incoherent combination of the two. To be consistent, however, we must include also the effects of initial fragmentation to (Σ_c^*, Σ_c) and Λ_c. This analysis was already carried out in Ref. 1, and including such effects leaves us with

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})} = \frac{2A(2 - \omega_1) + 2B(2 - \tilde{\omega}_1)}{A(5 + 2\omega_1) + B(5 + 2\tilde{\omega}_1) + 9} \quad . \tag{50}$$

We may define the polarization of the final state Λ_c in terms of the relative production probabilities for $\Lambda_c(+\frac{1}{2})$ and $\Lambda_c(-\frac{1}{2})$ as:

$$\mathcal{P} = \frac{Prob[\Lambda_c(-\frac{1}{2})] - Prob[\Lambda_c(+\frac{1}{2})]}{Prob[\Lambda_c(-\frac{1}{2})] + Prob[\Lambda_c(+\frac{1}{2})]} \quad . \tag{51}$$

For the case of a completely left-handed initial heavy quark, we find

$$\mathcal{P} = \frac{A(1+4\omega_1) + B(1+4\tilde{\omega}_1) + 9}{9(A+B+1)} \quad . \tag{52}$$

This function may never fall below $\frac{1}{9}$, so that the initial polarization information may never be entirely obliterated by the fragmentation process. As a first guess as to what polarization we may actually expect to measure, we may use the value $\omega_1 = 0$, suggested by experimental study of the charmed meson system 1, and $A = 0.45$, the default Lund value[5]. If we further assume that the light degrees of freedom fragment to $j^P = 1^+$ and $j^P = 1^-$ states indiscriminately so that $A=B$, we find that $\mathcal{P}$ ranges from 0.58 to 0.79 as $\tilde{\omega}_1$ ranges from 0 to 1. For a heavy quark with initial polarization $\mathbf{P}$, the above results for $\mathcal{P}$ are simply multiplied by $\mathbf{P}$. It is not unreasonable, therefore, to expect a significant fraction of the initial heavy quark's polarization to be observable in the final state Λ_c.

The parameters A and B are also of phenomenological interest. Accurate association of Λ_c with final state pions should measure the number of zero, one, and two pion events in the ratio:

$$\Lambda_c \ : \ \Lambda_c\pi \ : \ \Lambda_c\pi\pi \ = \ 1 \ : \ A \ : \ B \ . \tag{53}$$

Information on A and B may also be obtained by measuring the relative number of fragmentation events containing Σ_c as opposed to those containing Σ_c^*. Direct fragmentation to (Σ_c^*, Σ_c) produces them in the ratio $\Sigma_c^* : \Sigma_c = 2 : 1$. This ratio will be diminished, however, by Λ_{c1} that decay to real Σ_c on their way to Λ_c. The decays of Λ_{c1}^* are kinematically forbidden from producing such an enhancement in the Σ_c^* population. In the narrow width approximation for Σ_c, we find

$$\frac{events\ with\ \Sigma_c^*}{events\ with\ \Sigma_c} = \frac{2}{[1+\frac{B}{A}]} \quad . \tag{54}$$

An accurate measurement of such departure from naive spin counting could provide information on this interesting ratio, (B/A), and would be especially useful for checking the predictions of various fragmentation models.

A few remarks are in order concerning the decays to $\Lambda_c\pi^+\pi^-$. This case is slightly more complicated than the $\pi^0\pi^0$ case because the propagator connecting Λ_{c1}^* to Λ_c may be either $\Sigma_c^{(*)0}$ or $\Sigma_c^{(*)++}$. This fact, coupled with the different Σ_c masses

$$
\begin{aligned}
M[\Sigma_c^{++}] =& 2453.1 \pm 0.6 \ MeV \quad , \\
M[\Sigma_c^{+}] =& 2453.8 \pm 0.9 \ MeV \quad , \\
M[\Sigma_c^{0}] =& 2452.4 \pm 0.7 \ MeV \quad ,
\end{aligned}
\tag{55}
$$

produces distributions in Λ_{c1} decay that are not symmetric with respect to the π^+ and π^- momenta. Indeed, if we boldly accepted the central values of the sigma masses above, we would proceed to calculate an enhancement in the coefficient of $\cos^2\theta_{\pi^-}$ by approximately 10% with respect to that of $\cos^2\theta_{\pi^+}$ in (34) above, and a similar enhancement for the coefficient of $\sin^2\theta_{\pi^-}$ relative to that of $\sin^2\theta_{\pi^+}$ in (35). In light of the errors listed in (55) and the order to which we are working, however, such a conclusion would be inappropriate. The $\pi^+\pi^-$ distributions are, within the accuracy of this calculation, indistinguishable from those of the neutral pions.

6 Concluding Remarks

In this chapter, we have studied fragmentation through the $(\Lambda_{c1}^*, \Lambda_{c1})$ system, and have calculated the resultant double pion decay distributions in the well satisfied limit $\Gamma(\Lambda_{c1}^{(*)}) \ll (M_{\Lambda_{c1}^*} - M_{\Lambda_{c1}})$. In so doing, we have introduced the new fragmentation parameters $\tilde{\omega}_1$ and B, and have shown how $\tilde{\omega}_1$ may be extracted from pion angular data. We have also found that the final state Λ_c particles produced in the fragmentation process should retain a significant fraction of the initial heavy quark's polarization, allowing a test of the Standard Model's predictions for heavy quark polarization in such hard processes.

Experimental determinations of the ω parameters are extremely important in testing various ideas about fragmentation. Chen and Wise[6] have estimated $\omega_{3/2}$

using the $m_c/m_b \to 0$ limit of a *perturbative* QCD calculation of $b \to B_c^{**}$ done by Chen[7], and have found that $\omega_{3/2} = 29/114$. That this admittedly oversimplified approach gives reasonable agreement with the experimentally suggested $\omega_{3/2}$ <0.24[1] is of significant interest. Yuan[8] has augmented this analysis with a calculation of the dependence of $\omega_{3/2}$ on the longitudinal and transverse momentum fractions of the meson. Furthermore, fragmentation models such as the Lund model make predictions for parameters related to A[5][9]. Similar predictions will be possible for the remaining fragmentation parameters discussed in this paper, in either a limiting case of QCD, or in a model such as Lund, and the experimental extraction of these parameters will therefore provide non-trivial constraints on such methods. Determination of $\tilde{\omega}_1$ may in fact soon be possible at CLEO[10].

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Concluding Remarks and Outlook

As we have seen throughout this work, incorporating the symmetries of some complete theory into an effective theory that applies in a specific kinematic regime is a useful means for getting perturbative tools to do nonperturbative work. There are some quantities, however, notably ω_1 of the previous chapter, that simply cannot be calculated in a perturbative setting. In such situations, the best that we can do, in the absence of a full nonperturbative calculation, is to search for a similar quantity that *can* be determined perturbatively, and pray that the two are not completely unrelated. Such a quantity is available in the study of ω_1, and we spend the remainder of this work briefly sketching a procedure for its calculation.

Ideally, we would like to know how a heavy quark such as b fragments into a heavy baryon such as Λ_b or Σ_b. Consider instead the fragmentation of a b quark to a baryon composed of (bcc), where the two c quarks are placed in a relative $s = 1, l = 0$ state; there is also vanishing orbital angular momentum between the (cc) pair and the b quark. Fragmentation to this baryon, which we shall call Σ_{bcc}, is perturbatively calculable, since m_b and m_c are both large compared to Λ_{QCD}; a typical Feynman diagram contributing to the process is shown in Fig. 1. Moreover, the calculation is analogous to calculations already carried out for c quark fragmentation to quarkonia[1]. Note that graphs involving the three gluon vertex do not contribute to $b \to \Sigma_{bcc}\bar{c}\bar{c}$ fragmentation. The reason is that the three gluon vertex is proportional to the antisymmetric structure constants f^{abc}. Because both the final state c quark pair and the final state $\bar{c}$ quark pair have to be in antisymmetric color states, the three gluon vertex graphs cannot contribute. This reduction in graphs makes the calculation much more tractable.

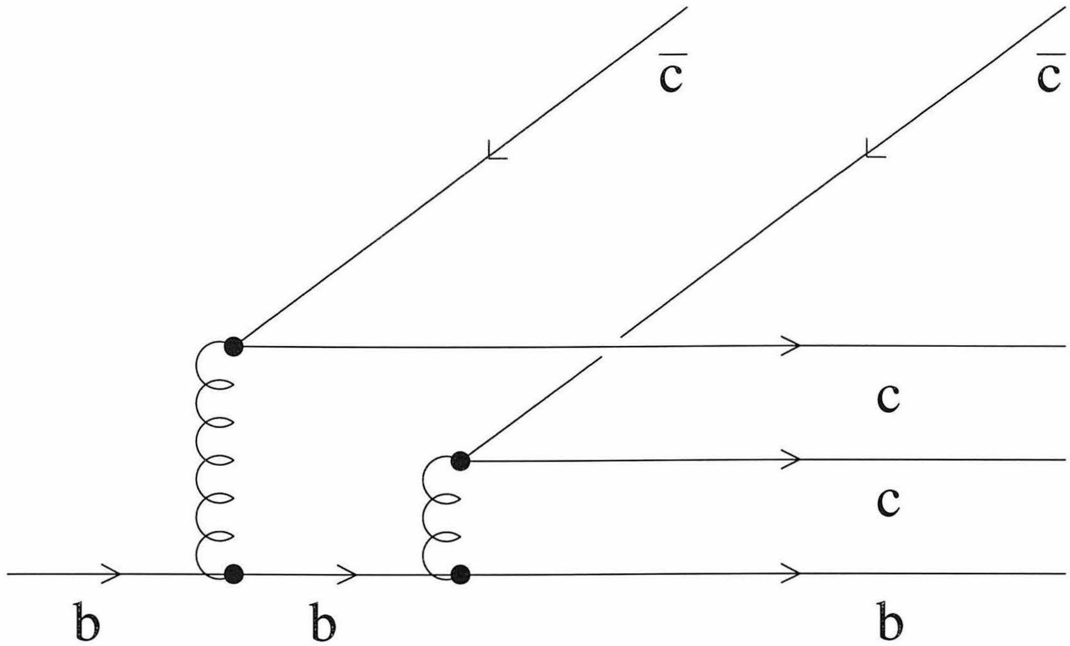

Fig. 1. One of twelve Feynman graphs contributing to $b \to \Sigma_{bcc}\bar{c}\bar{c}$.

All nonperturbative information relating to the formation of the bound baryon state may be assembled into the radial wavefunction at the origin, $R(0)$, for the non-relativistic bcc bound state. Spin information therefore remains in the perturbative part of the calulation, and we may calculate the analogue of ω_1 for fragmentation to this triply heavy baryon. The function thus obtained will depend on the ratio $(\frac{m_c}{m_b})$. One might expect that, making an expansion in this small parameter and keeping only the leading non-trivial term, we could arrive at a quantity that might naively approximate ω_1. We hope to carry out such a calculation in a future publication.

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