ANALYTICAL PERFORMANCE STUDY OF TURBOJET CYCLE WITH NEARLY IDEAL COMPONENT EFFICIENCIES

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LIST OF SYMBOLS

A	turbine cross-sectional area (ft^2)
A_o	cross-sectional area of free air stream (ft^2)
c_p	specific heat of gas (BTU/lb ^oF)
C_F	thrust coefficient
F	thrust (lb)
f	fuel consumption (lb/sec)
H	heating value of fuel (BTU/lb)
M_o	flight Mach number
$\overset{o}{m}$	mass flow of gas (lb sec/ft)
P_o	ambient pressure (lb/ft^2)
P_{T_2}	total pressure diffuser outlet (lb/ft^2)
P_{T_3}	total pressure compresser outlet (lb/ft^2)
P_{T_4}	total pressure burner outlet (lb/ft^2)
P_{T_5}	total pressure turbine outlet (lb/ft^2)
π_c	compressor pressure ratio
q	dynamic pressure (incompressible) (lb/ft^2)
R	gas constant ($ft^2/{}^oF\ sec^2$)
S	specific fuel consumption (lb per lb/sec)
S^*	dimensionless parameter which varies as specific fuel consumption
T_o	ambient temperature (^oR)
T_{T_2}	total temperature diffuser outlet (^oR)
T_{T_3}	total temperature compressor outlet (^oR)

LIST OF SYMBOLS (Contd)

T_{T_4}	total temperature burner outlet (^oR)
T_{T_5}	total temperature turbine outlet (^oR)
T_6	static temperature nozzle exit (^oR)
τ_r	ratio of total temperature diffuser outlet to ambient temperature
τ_c	ratio of total temperature compressor outlet to total temperature diffuser outlet
τ_m	ratio of total temperature burner outlet to ambient temperature
U_o	flight velocity (ft/sec)
U_6	jet velocity (ft/sec)
γ^*	ratio of specific heats for combustion gas
γ	ratio of specific heats for air
η_b	combustion efficiency
η_c	compressor efficiency
η_d	diffuser efficiency
η_t	turbine efficiency
a_o	velocity of sound in free air (ft/sec)

ABSTRACT

The performance of the turbojet engine for high component efficiency is approximated by an ideal expression (efficiencies equal unity) plus the first terms of a Taylor Series expansion for the purpose of isolating the individual effects of the components.

The analytical solution to two optimizations is presented; namely:

(A) For fixed burner outlet temperature, flight Mach number, and component efficiency, what is the compressor pressure ratio corresponding to maximum jet velocity (maximum thrust per unit mass flow)?

(B) For fixed flight Mach number, compressor pressure ratio and component efficiency, what is the burner outlet temperature corresponding to minimum specific fuel consumption?

I. INTRODUCTION

Currently the turbojet engine, in the field of aircraft powerplants, holds a unique position in that it solely appears capable of satisfactory operation over the entire speed range defined by performance estimates of powerplant and airframe combinations to be available in the foreseeable future. Although its performance dominance over the conventional engine, turbine propeller and ramjet is analyzed to be restricted to a relatively narrow range in the vicinity of high subsonic and transonic speeds, the turbojet is singular in its indicated potentialities for spanning the extremes of static thrust operation and supersonic flight. This flexibility of operation is related to the integrated behavior of its principal components rather than the ability of individual elements to adapt their functioning to varied flight conditions. The principal components in the order of thermodynamic cycle process are the diffuser, compressor, burner, turbine and jet nozzle (Figure 1.) The compressor assumes prime importance for static thrust and low speed operation; at higher speeds the diffusion process contributes more effectively to performance and there is increasingly less return for energy expended in mechanical compression. Supersonic operation introduces shock phenomena which complicate the diffuser design in that the efficiency of pressure recovery, which is high and relatively independent of Mach number in the subsonic regime, becomes critically dependent upon the correct matching of diffuser configuration with Mach number. This effect is adverse to flexibility of operation bût it is an effect common to supersonic powerplants which are sustained by the atmosphere. The maximum thrust of the unit is determined principally by the heat resistivity of the materials that make up the turbine assembly. This limitation can be circumvented where the primary concern is thrust augmentation rather than economy of operation by the installation of an auxiliary burner downstream of the turbine (afterburner). The problem of determining the conditions corresponding to minimum specific fuel consumption is more subtle; it concerns the design for pressure and temperature levels at the turbine exhaust which will effect maximum jet velocity. Because the state of the exhaust gas is a function of the turbine work of compression, an optimum design is suggested.

The performance of the turbojet may be expressed as a function of flight Mach number, compressor pressure ratio, burner outlet temperature, and the efficiencies of the components:

$$C_F \text{ or } S = f(M_o, \pi_c, \tau_m, \eta_c, \eta_t, \eta_d) \qquad (1)$$

where these significant parameters are defined in the listing of the symbols. By the introduction of the thrust coefficient and the recognition of the functional independence of specific fuel consumption, the mass flow through the turbojet has been conveniently deleted from discussion. For the purposes of this investigation the parameters are considered functionally independent, an assumption which does not accurately represent the behavior of any single turbojet installation but which becomes significant when viewed as the description of a multiple number of units. For instance, if an arbitrary value is assigned to each of the parameters, presumably the resulting performance is representative of some particular unit. The performance comparisons presented are relative on the basis of submission to similar ambient conditions, thus the effect of altitude upon operation is not pertinent. The analysis is in the form of a set of dimensionless ratios for convenience of presentation.

The following assumptions will be common to all discussion:

a) The diffuser efficiency equals unity for the subsonic regime.

b) The velocity of the air at the burner inlet is low enough such that the drag of the burner component and the momentum pressure drop associated with the heat addition may be neglected.

(c) Energy expended in overcoming bearing friction in the transmission of turbine power to the mechanical compressor has been neglected.

(d) The static pressure at the nozzle exit is equal to ambient pressure, i.e. the nozzle is correctly expanded.

(e) The nozzle efficiency equals unity.

This investigation will be concerned with: (1) establishing the relative importance of the principal components for a given set of operating conditions; (2) determining analytical expressions for the relations between the significant parameters which correspond to optimum performance.

Because normal operation corresponds to high component efficiency, the performance of the turbojet can be approximated by an ideal expression (efficiencies equal unity) plus the first terms of a Taylor series expansion which correspond to the individual effects of the non-ideal behavior of the components. This representation has the advantage of directly indicating the contribution of design improvement in a particular component upon the overall performance of the unit, i.e. the objective of (1).

The condition of maximum performance imposes a functional relationship between flight speed, mechanical compression, and burner outlet temperature which is affected by component efficiency. Having isolated the effects of component efficiency by a limited Taylor series representation of performance, these expressions were first investigated for the optimum relation between parameters, i.e. the objective of (2). Difficulty was encountered because the approximations introduced prevented an accurate representation of turbojet operation where performance was critically sensitive to variation of the parameters. Also the complexity of the expressions obtained from the limited Taylor series representation of performance did not permit the derivative operation necessary to the determining of the optimum relations.

It was, however, possible to obtain an approximate analytical solution to two optimizations by the examination of the complex expressions for exact performance formulated from the assumptions previously set forth in this discussion; the optimization problems are:

A. For fixed burner outlet temperature, flight Mach number and component efficiency, what is the compressor pressure ratio corresponding to maximum jet velocity (maximum thrust per unit mass flow)?

B. For fixed flight Mach number, compressor pressure ratio and component efficiency, what is the burner outlet temperature corresponding to minimum specific fuel consumption?

II. THERMODYNAMICS OF TURBOJET

This section will be concerned with the derivation of exact expressions for turbojet performance within the assumptions prescribed in the INTRODUCTION. The symbols used are defined in a separate listing.

The performance of the turbojet may be described in terms of thrust output and specific fuel consumption defined by the following relations:

$$F = \overset{o}{m}(U_6 - U_o) \tag{2}$$

$$S = \frac{f}{F} = \frac{\overset{o}{m}\left\{T_{T_4} - T_{T_3}\right\}}{\overset{o}{m}\left\{U_6 - U_o\right\}} \frac{c_p}{H\eta_b} \tag{3}$$

To eliminate mass flow from further consideration we define the thrust coefficient as the ratio of the thrust to the product of free stream dynamic pressure (incompressible) and a characteristic area usually taken as the cross-sectional area of the turbine.

$$C_F = \frac{\rho_o A_o U_o\left[U_6 - U_o\right]}{\frac{1}{2}\rho_o A\, U_o^2} = 2\frac{A_o}{A}\left[\frac{U6}{U_o} - 1\right] \tag{4}$$

The specific fuel consumption can be conveniently nondimensionalized.

$$S = \frac{T_o(\tau_m - \tau_r\tau_c)}{U_o\left[\frac{U6}{U_o} - 1\right]} \frac{c_p}{H\eta_b} = \frac{1}{M_o} \frac{(\tau_m - \tau_r\tau_c)}{\left[\frac{U6}{U_o} - 1\right]} \frac{T_o c_p}{H\eta_b a_o} \tag{5}$$

$$S^* = S(\text{non-dimensionalized}) = \frac{1}{M_o} \frac{(\tau_m - \tau_r \tau_c)}{\left[U_6/U_o - 1 \right]} \tag{6}$$

It is evident for a given burner efficiency that S* varies as S. From an examination of relations (4) and (6) it is seen that it remains to express the velocity ratio (ratio of jet velocity to air speed) in terms of the significant parameters. This expression can be determined by an analysis of the various component processes.

<u>The Diffuser</u> (0 - 2)

The function of the diffuser is to convert free stream velocity to static pressure. The subsonic flight condition is a familiar design problem and can be effected efficiently. Efficient diffusion of supersonic flow depends upon the avoidance of an initial normal shock with accompanying large increment in entropy. Current design plans the pressure recovery by effecting a series of oblique shocks terminating in a normal shock followed by subsonic diffusion. The design for oblique shock critically involves the diffuser configuration. A variance in flight speed demands an alteration of geometry to maintain optimum diffusion.

The efficiency of subsonic diffusion has been taken to be unity. Writing the isentropic energy relation across the diffuser, the total pressure and temperature ratios are:

$$\frac{p_{t_2}}{p_o} = \left[1 + \frac{\gamma-1}{2} M_o^{\,2}\right]^{\frac{\gamma}{\gamma-1}} = \tau_r^{\;\frac{\gamma}{\gamma-1}} \quad \text{(subsonic)} \tag{7}$$

$$\frac{T_{t_2}}{T_o} = \tau_r = 1 + \frac{\gamma-1}{2} M_o^{\,2} \tag{8}$$

To differentiate supersonic from subsonic diffusion and to account for the unavoidable entropy increase of the former process, we introduce the diffuser efficiency. It is a measure of the departure from isentropic compression.

$$\eta_d = \frac{\left(\frac{p_{t_3}}{p_{t_2}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{t_3}}{T_{t_2}} - 1} \tag{9}$$

$$p_{t_2}/p_o = \left[1 + \eta_d(\tau_r - 1)\right]^{\frac{\gamma}{\gamma-1}} \quad \text{(supersonic)} \tag{10}$$

To establish a functional relation between the diffuser efficiency and the flight Mach number for purposes of this investigation, values have been assigned η_d which correspond to the diffusion of a single oblique shock followed by a normal shock as discussed in NACA T.M. No. 1140 (Figure 2). This diffusion is ideal in the sense that the efficiencies selected are those theoretically possible within the limitation of the described process.

The Mechanical Compressor (2 - 3)

The significant parameters are the compressor pressure ratio and the compressor efficiency. The definition of the latter is similar to the diffuser efficiency.

$$\frac{P_{t_3}}{P_{t_2}} = \pi_c \qquad \frac{T_{t_3}}{T_{t_2}} = \tau_c \tag{11}$$

$$\eta_c = \left[\left(\frac{P_{t_3}}{P_{t_2}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \Big/ \left[\frac{T_{t_3}}{T_{t_2}} - 1\right] \tag{12}$$

$$\frac{P_{t_3}}{P_{t_2}} = \left[1 + \eta_c\,(\tau_c - 1)\right]^{\frac{\gamma}{\gamma-1}} \tag{13}$$

The Burner (3 - 4)

The burner outlet temperature is significant because there is a limitation on its value set by the heat resistivity of the materials that make up the turbine assembly. The corresponding dimensionless parameter is:

$$\tau_m = \frac{T_{t_4}}{T_o} = \text{maximum temperature ratio of the cycle} \tag{14}$$

Loss in total pressure across the burner caused by the drag of the burner component and heat addition to the fluid has been neglected.

$$P_{t_3} = P_{t_4} \tag{15}$$

The Turbine (4 - 5)

The turbine extracts enthalpy from the fluid for the mechanical work of compression. It is necessary to determine the total temperature at turbine outlet in terms of the previously defined parameters to establish the quantity of available energy in the fluid for conversion to jet velocity. The principle that the enthalpy extracted from fluid at the turbine is equal to the enthalpy increase of the air at the mechanical compressor is utilized. The small difference between the mass flow through the turbine and compressor due to the addition of fuel and any difference in specific heat of the fluids has been neglected.

$$T_{t_4} - T_{t_5} = T_{t_3} - T_{t_2} \quad \text{or}$$

$$T_{t_5} = T_o\left[\tau_m - \tau_r(\tau_c - 1)\right] \tag{16}$$

The turbine efficiency relates total temperature ratio to total pressure ratio across turbine.

$$\eta_t = \frac{1 - \dfrac{T_{t5}}{T_{t4}}}{\left[1 - \left(\dfrac{p_{t_5}}{p_{t_4}}\right)\right]^{\frac{\gamma^* - 1}{\gamma^*}}} \tag{17}$$

$$\left(\frac{p_{t5}}{p_{t_4}}\right)^{\frac{\gamma^*-1}{\gamma^*}} = 1 + \frac{1}{\eta_t}\left[\frac{T_{t5}}{T_{t4}} - 1\right] = 1 + \frac{1}{\eta_t}\left[\frac{\tau_m - \tau_r(\tau_c - 1)}{\tau_m} - 1\right] = 1 - \frac{1}{\eta_t}\,\frac{\tau_r(\tau_c - 1)}{\tau_m} \tag{18}$$

The Nozzle (5 - 6)

The nozzle transforms the available enthalpy at the turbine to jet velocity. Equating energy between station 5 and 6 assuming the nozzle configuration is correct for the expansion process, the result is:

$$U_6{}^2 = \frac{2\gamma^* R}{\gamma^* - 1}\left[T_{t_5} - T_6\right] = \frac{2\gamma^* R}{\gamma^* - 1} T_{t_5}\left\{1 - \left(\frac{P_o}{P_{t_5}}\right)\right]^{\frac{\gamma^*-1}{\gamma^*}} \qquad (19)$$

but,

$$\left(\frac{p_o}{p_{t_5}}\right)^{\frac{\gamma^*-1}{\gamma^*}} = \left(\frac{p_o}{p_{t_2}}\,\frac{p_{t_2}}{p_{t_3}}\,\frac{p_{t_3}}{p_{t_4}}\,\frac{p_{t_4}}{p_{t_5}}\right)^{\frac{\gamma^*-1}{\gamma^*}}$$

and using expressions (10), (13), (16) and (18),

$$U_6{}^2 = \frac{2\gamma^* R}{\gamma^* - 1}\, T_o\left[\tau_m - \tau_r(\tau_c - 1)\right]\left\{1 - \frac{1}{\left[1+\eta_d(\tau_r-1)\right]\left[1+\eta_c(\tau_c-1)\right]\left[1-\frac{\tau_r(\tau_c-1)}{\eta_t \tau_m}\right]}\right\}$$

The velocity ratio is obtained finally after expression (8) is introduced. ($\gamma^* \doteq \gamma$)

$$\left(\frac{U_6}{U_o}\right)^2 = \left[\frac{\tau_m - \tau_r(\tau_c - 1)}{\tau_r - 1}\right]\left\{1 - \frac{1}{\left[1+\eta_d(\tau_r-1)\right]\left[1+\eta_c(\tau_c-1)\right]\left[1-\frac{\tau_r(\tau_c-1)}{\eta_t \tau_m}\right]}\right\} \qquad (20)$$

where $1 + \eta_d(\tau_r - 1)$ is reduced to τ_r for subsonic flight conditions. It is interesting to solve for the jet velocity corresponding to static thrust operation. Multiplying both sides of Equation (20) by the flight velocity squared and setting the ram total temperature ratio equal to unity, the result is:

$$U_6^{\,2} = \frac{2\gamma^* R}{\gamma^* - 1} T_o \left[\tau_m - (\tau_c - 1)\right] \left\{ 1 - \frac{1}{\left[1 + \eta_c(\tau_c - 1)\right]\left[1 - \frac{(\tau_c - 1)}{\eta_t \tau_m}\right]} \right\} \tag{21}$$

(static thrust operation)

It can be noted that in contrast to the ram-jet the turbojet is capable of producing static thrust. Finally the velocity ratio for the ideal behavior of the unit, i.e. component efficiencies equal unity, is the simple result:

$$\left(\frac{U_6}{U_o}\right)^2 \qquad \frac{\tau_m\left(1 - \frac{1}{\tau_r \tau_c}\right) - \tau_r(\tau_c - 1)}{\tau_r - 1} \tag{22}$$

(ideal operation)

III. EFFECT OF COMPONENT EFFICIENCY UPON PERFORMANCE

To gain insight into the effect of changes in the behavior of the various components upon the turbojet performance, the thrust coefficient and specific fuel consumption may be approximated by the first terms of a Taylor series expansion from ideal operation (efficiencies equal unity).

$$C_F \doteq C_{F_I} + \frac{\partial C_F}{\partial(1-\eta_c)}(1-\eta_c) + \frac{\partial C_F}{\partial(1-\eta_t)}(1-\eta_t) + \frac{\partial C_F}{\partial(1-\eta_d)}(1-\eta_d) \qquad (23)$$

and

$$S^* \doteq S^*_I + \frac{\partial S^*}{\partial(1-\eta_c)}(1-\eta_c) + \frac{\partial S^*}{\partial(1-\eta_t)}(1-\eta_t) + \frac{\partial S^*}{\partial(1-\eta_d)}(1-\eta_d) \qquad (24)$$

The advantage of this type of representation is that the effectiveness of individual components may be isolated and examined. Consider the thrust coefficient. The general expression for the partial derivatives is:

$$\frac{\partial C_F}{\partial \sigma} = \frac{dC_F}{d\left[{U_6}/{U_o}\right]} \frac{d\left[{U_6}/{U_o}\right]}{d\left[\left({U_6}/{U_o}\right)^2\right]} \frac{\partial \left({U_6}/{U_o}\right)^2}{\partial \sigma} = \cancel{2}\frac{A_o}{A}\frac{1}{\cancel{2}}\frac{U_o}{U_6}\frac{\partial \left({U_6}/{U_o}\right)^2}{\partial \sigma} \qquad (25)$$

where σ represents unity minus efficiency. The first two derivatives are common to all differentiations. The last is obtained by differentiating the exact expression for the velocity ratio (Equation (20)). It is specifically for the case of compressor efficiency:

$$\frac{\partial\left[\left(U_6/U_o\right)^2\right]}{\partial \eta_c} = -\frac{\partial\left[\left(U_6/U_o\right)^2\right]}{\partial(1-\eta_c)} = \frac{\tau_m-\tau_r(\tau_c-1)}{\tau_r - 1}\;\frac{\tau_c-1}{\left[1+\eta_c(\tau_c-1)\right]^2}\;\frac{1}{\tau_r\left[1-\frac{\tau_r(\tau_c-1)}{\eta_t\tau_m}\right]}$$

Since the Taylor series expansion describes deviation from ideal behavior, the derivatives are evaluated for the condition of component efficiencies equal unity.

$$\underset{\text{(ideal)}}{\frac{\partial\left[\left(U_6/U_o\right)^2\right]}{\partial \eta_c}} = \frac{(\tau_c-1)\,\tau_m}{\tau_r\,(\tau_r-1)\,\tau_c^{\,2}} \tag{26}$$

The corresponding expressions for the diffuser and turbine efficiency are:

$$\underset{\text{(ideal)}}{\frac{\partial\left[\left(U_6/U_o\right)^2\right]}{\partial \eta_d}} = \frac{\tau_m}{\tau_c\,\tau_r^{\,2}} \quad ; \quad \underset{\text{(ideal)}}{\frac{\partial\left[\left(U_6/U_o\right)^2\right]}{\partial \eta_t}} = \frac{(\tau_c-1)\,\tau_m}{(\tau_r-1)\left[\tau_m-\tau_r(\tau_c-1)\right]\tau_c} \tag{27}$$

Equation (23) may now be written explicitly by the insertion of relations (25), (26) and (27).

$$C_F \doteq 2\,\frac{A_o}{A}\left[\frac{U_6}{U_o}-1\right]\left\{1-\frac{\frac{\tau_m}{\tau_c}\,\frac{\tau_c-1}{\tau_r-1}}{2\frac{U_6}{U_o}\left[\frac{U_6}{U_o}-1\right]}\left[\frac{1-\eta_c}{\tau_r\,\tau_c}+\frac{1-\eta_t}{\tau_m-\tau_r(\tau_c-1)}+\frac{\tau_r-1}{\tau_c-1}\;\frac{(1-\eta_d)}{\tau_r^{\,2}}\right]\right\} \tag{23a}$$

where the velocity ratio corresponds to ideal operation and is given by expression (22). The equivalent explicit expression for specific fuel consumption is obtained most directly by noting the effect upon S of a small change of C_F for fixed fuel consumption.

$$S = \frac{f}{F} = \frac{f}{C_F q_o A} \quad ; \quad S + \Delta S = \frac{f}{(C_F + \Delta C_F) q_o A} \doteq \frac{f}{C_F q_o A}\left(1 - \frac{\Delta C_F}{C_F}\right)$$

therefore,
$$\frac{\Delta S}{S} \doteq - \frac{\Delta C_F}{C_F} \tag{28}$$

Thus (24) may be rewritten as:

$$S^* \doteq \frac{\tau_m - \tau_r \tau_c}{M_o \left[\frac{U_6}{U_o} - 1\right]} \left\{ 1 + \frac{\frac{\tau_m}{\tau_c} \; \frac{\tau_c - 1}{\tau_r - 1}}{2 \frac{U_6}{U_o}\left[\frac{U_6}{U_o} - 1\right]} \left[\frac{1 - \eta_c}{\tau_r \tau_c} + \frac{1 - \eta_t}{\tau_m - \tau_r(\tau_c - 1)} + \frac{\tau_r - 1}{\tau_c - 1} \; \frac{1 - \eta_d}{\tau_r^{2}} \right] \right\} \tag{24a}$$

where the velocity ratio corresponds to ideal operation and is given by (22).

A comparison of (23a) and (24a) indicates within the approximation that the effect of variation of component efficiency upon thrust coefficient is equal to minus the effect upon specific fuel consumption. However, it is emphasized that (24a) represents a greater deviation from exact representation of performance because of the additional approximation introduced in the derivation of (28). The degree of approximation of the limited Taylor series representation of performance is shown by comparison with exact representation in Figures 3, 4 and 5. Figures 3 and 4 present the variation of thrust coefficient with compressor pressure ratio and burner outlet temperature for various flight speeds. Figure 5 presents the variation of specific fuel consumption with burner outlet temperature for various flight speeds. For turbine and compressor efficiency equal 0.90, (23a) is shown to accurately describe the thrust coefficient. The approximation to specific fuel consumption, (24a), is seen to be valid only for burner outlet temperatures above the condition of minimum specific fuel consumption. The accuracy of representation of both (23a) and (24a) increases with flight speed.

Comparisons of the relative effectiveness of the component efficiencies upon the thrust coefficient may be obtained directly from expression (23a). Effectiveness is defined as the partial derivative of the performance with respect to component efficiency. Relative effectiveness is the ratio of partial derivatives:

$$\partial C_F/\partial \eta_c \div \partial C_F/\partial \eta_d = \frac{\tau_r(\tau_c - 1)}{\tau_c(\tau_r - 1)} = \frac{T_{T_2}(T_{T_3} - T_{T_2})}{T_{T_3}(T_{T_2} - T_o)} \qquad (29)$$

$$\partial C_F/\partial \eta_c \div \partial C_F/\partial \eta_t = \frac{\tau_m - \tau_r(\tau_c - 1)}{\tau_r \tau_c} = \frac{T_{T_5}}{T_{T_4}} \qquad (30)$$

Expression (29) states that for equal diffuser and compressor efficiency, the effectiveness of compressor efficiency compared to diffuser efficiency varies directly with the ratio of compressor temperature rise to ram temperature rise and inversely as the ratio of compressor outlet absolute temperature to diffuser outlet absolute temperature. This relative effectiveness, for a flight Mach number of 0.7 and a compressor pressure ratio of five, has the value four. If the flight Mach number is increased to 1.7, the relative effectiveness is reduced to one. However, as previously noted, the character of diffusion of supersonic flow makes difficult the design of an efficient diffuser which enhances the effectiveness of improved diffuser performance compared with that of a more efficient component. It suffices to say that diffuser efficiency reaches a parity with mechanical compressor efficiency in contribution to thrust output near the transonic speed range.

The indication of relation (30) is that for equal efficiency the relative effectiveness of compressor efficiency compared to turbine efficiency varies inversely as the ratio of compressor outlet absolute temperature to turbine outlet absolute temperature. If the burner outlet temperature is limited to 2000°F and the flight Mach number and compressor ratio are 0.70 and 5 respectively, the relative effectiveness is 2.5. The compressor efficiency becomes relatively less dominant as flight Mach number or compressor pressure ratio is increased.

The effect of component efficiency upon specific fuel consumption may be inferred directly from the discussion of thrust coefficient.

IV. OPTIMIZATION A

The strength of the turbine assembly at elevated temperatures limits the magnitude of the jet velocity; however, there remains the following problem:

For fixed burner outlet temperature, flight Mach number, and component efficiencies, what is the compressor pressure ratio corresponding to maximum jet velocity (maximum thrust per unit mass flow)?

A direct approach to solution suggests differentiation of the expression for the velocity ratio (20) with respect to τ_c and solving for the critical value of τ_c. Unfortunately the complexity of the derivative of expression (20) does not permit an explicit solution. An alternate approach is to find a suitable approximation of the velocity ratio relation which will allow the derivative operation.

Expression (20) is rewritten with the following substitutions:

$$A^2 = \frac{1}{\tau_r - 1}\,;\; a = \tau_r(\tau_c - 1)\,;\; b = \frac{\tau_r(\tau_c - 1)}{\eta_t}\,;\; B = \frac{1}{\left[1+\eta_d(\tau_r - 1)\right]\left[1+\eta_c(\tau_c - 1)\right]}$$

$$\left(\frac{U_6}{U_o}\right)^2 = A^2\left[\tau_m - a\right]\left[1 - \frac{B}{1-\frac{b}{\tau_m}}\right] = A^2\left[\frac{\tau_m^{\,2}(1-B)+\tau_m(aB-b-a)+ab}{\tau_m - b}\right] \qquad (20)$$

$$= A^2\left[\tau_m(1-B)-b\right]\left[\frac{\tau_m - a}{\tau_m - b}\right] = A^2\left[\tau_m(1-B)-b\right]\left[1+\frac{b}{\tau_m - b}(1-\eta_t)\right]$$

If the last bracketed term has a value close to one for all operating conditions considered, a substantial simplification of (20) can be effected.

$$\frac{b}{\tau_m - b}(1-\eta_t) = \frac{\tau_r(\tau_c - 1)\dfrac{(1-\eta_t)}{\eta_t}}{\tau_m - \dfrac{\tau_r(\tau_c-1)}{\eta_t}} = a \qquad (30)$$

The conditions that a be small are that the turbine efficiency be large and the burner outlet temperature minus the mechanical compressor temperature rise $\left[\tau_m - \tau_r(\tau_c - 1)\right]$ shall not be small; both are consistent with maximum thrust operation. Restricting the turbine efficiency between 0.8 and 1.0 and checking the magnitude of a over operating conditions of interest, its value was found not to exceed 0.10. Neglecting the contribution of a to the magnitude of the velocity ratio, hence, underestimates by less than five per cent. The approximation of (20) therefore is:

$$\left(\frac{U_6}{U_o}\right)^2 = \frac{1}{\tau_r - 1}\left\{\tau_m\left(1 - \frac{1}{[1+\eta_d(\tau_r-1)][1+\eta_c(\tau_c-1)]}\right) - \frac{\tau_r(\tau_c-1)}{\eta_t}\right\} \qquad (20a)$$

Expression (20a) may now be maximized by equating

$\partial\left[\left(\frac{U_6}{U_o}\right)^2\right] / \partial\tau_c$ to zero.

$$\partial\left[\left(\frac{U_6}{U_o}\right)^2\right] / \partial\tau_c = \frac{1}{\tau_r - 1}\left\{\frac{\tau_m\eta_c}{[1+\eta_d(\tau_r-1)][1+\eta_c(\tau_c-1)]^2} - \frac{\tau_r}{\eta_t}\right\} = 0$$

$$\left[1+\eta_c(\tau_c-1)\right]^2 = \pi_c^{\frac{2(\gamma-1)}{\gamma}} = \frac{\tau_m\,\eta_c\,\eta_t}{\tau_r\left[1+\eta_d(\tau_r-1)\right]}$$

$$\pi_c = \left\{ \frac{\tau_m \eta_c \eta_t}{\tau_r \left[1 + \eta_d (\tau_r - 1) \right]} \right\}^{\frac{\gamma}{2(\gamma-1)}} \quad (31)$$

(maximum thrust per unit mass flow)

The optimum compressor ratio for maximum static thrust is:

$$\pi_c = \left[\tau_m \eta_c \eta_t \right]^{\frac{\gamma}{2(\gamma-1)}} \quad (32)$$

(maximum static thrust per unit mass flow)

For a burner outlet temperature to ambient temperature ratio of five and for component efficiencies of 0.80 and 0.90 optimum compressor pressure ratio for maximum static thrust per unit mass flow is 7.7 and 11.6, respectively.

For subsonic flight operation ($\eta_d = 1$) and fixed burner outlet temperature and component efficiency, expression (31) indicates that $\pi_c \tau_r^{\frac{\gamma}{\gamma-1}}$ should remain constant, i.e. that the product of ram pressure ratio and compressor pressure ratio should be invariant for maximum jet velocity. Increased flight speed demands a reduction in the enthalpy extracted from the fluid for the mechanical work of compression per unit mass flow.

Figure 6 presents the variation of optimum compressor pressure ratio with flight Mach number for various burner outlet to ambient temperature ratios and component efficiencies. It is seen that beyond Mach one the optimum compressor pressure ratio rapidly approaches unity. This trend is a direct result of the diminishing effect upon jet velocity of increased mechanical compression at the high pressure levels; the turbine exhaust temperature becomes dominant because of the excessive turbine work. Figure 3 indicates a 20 per cent increase in burner outlet temperature above 1500°F (τ_m increased from 4 to 5) at a flight Mach number of 1.5 effects approximately a 50% increment in the optimum compressor pressure ratio.

V. OPTIMIZATION B

In section II a dimensionless ratio denoted S* was defined which varied as specific fuel consumption.

$$S^* = \frac{1}{M_o} \frac{\tau_m - \tau_r \tau_c}{\left[\frac{U_6}{U_o} - 1\right]} \qquad (6)$$

where $$\frac{U_6}{U_o} = \left[\frac{\tau_m - \tau_r(\tau_c - 1)}{\tau_r - 1}\right]^{1/2} \left\{1 - \frac{1}{\left[1 + \eta_d(\tau_r - 1)\right]\left[1 + \eta_c(\tau_c - 1)\right]\left[1 - \frac{\tau_r(\tau_c - 1)}{\eta_t \tau_m}\right]}\right\}^{1/2}$$

Expression (6) suggests for fixed flight Mach number, compressor pressure ratio, and component efficiency, an optimum burner outlet temperature corresponding to minimum specific fuel consumption.

Figure 5 illustrates the functional dependence of the specific fuel consumption upon burner outlet temperature. It may be noted that the existence of an optimum burner outlet temperature is a property of the departure of the components from ideal operation; there exists no minimum specific fuel consumption condition for turbine and compressor efficiency equal unity. The optimum is more sharply defined for the lower flight Mach numbers. The trend for the level of S* to be increased with flight Mach number may be explained from an examination of the behavior of the jet velocity; the jet velocity increases at a much slower rate than the velocity of the air entering the unit as flight speed is increased because the effect upon the former of a buildup in ram pressure is relatively small for normal operation. For a compressor pressure ratio of five and a burner outlet temperature to ambient temperature ratio of four, the jet velocity increases less than 15 per cent as the free stream velocity is increased fourfold ($M_o = 0.25$ to 1.0). Another flight Mach number effect is that the specific fuel consumption is indicated to be less sensitive to burner outlet temperature at the higher flight speeds.

The solution of optimization B will yield an analytical expression for τ_m corresponding to minimum S* indicated in Figure 5. The analytical procedure will be to take the partial derivative of expression (6) with respect to τ_m and equate the result to zero.

$$\frac{\partial S^*}{\partial \tau_m} = \frac{\left[\frac{U_6}{U_o} - 1\right] - \left[\tau_m - \tau_r \tau_c\right]}{\left[\frac{U_6}{U_o} - 1\right]^2} \left[\frac{\partial\, U_6/U_o}{\partial \tau_m}\right] \tag{33}$$

The determination of the partial derivative of the velocity ratio with respect to τ_m which occurs in the above relation and the subsequent solution for the optimum burner outlet temperature is described in the Appendix. The result is a quadratic expression for τ_m which is approximate in that relation (20a) of section IV was introduced for the velocity ratio in the final analysis to permit an explicit solution. The solution is:

$$\tau_m \text{ (for min. S*)} = \left\{\frac{2c}{B} - \tau_r \tau_c\right\}\left\{1 + \sqrt{1 - \frac{\tau_r^2 \tau_c^2 - 4\frac{b}{B}\left(\tau_r \tau_c - \frac{c}{B}\right)}{\tau_r^2 \tau_c^2 - 4\frac{c}{B}\left(\tau_r \tau_c - \frac{c}{B}\right)}}\right\} \quad (34)$$

where $b = \dfrac{\tau_r(\tau_c - 1)}{\eta_t}$ $\qquad c = \dfrac{\tau_r(\tau_c - 1)}{\eta_t} + (\tau_r - 1)$

$$B = 1 - \frac{1}{\left[1 + \eta_d(\tau_r - 1)\right]\left[1 + \eta_c(\tau_c - 1)\right]}$$

The quotient under the radical represents the ratio of two terms nearly equal, however in extracting the square root, the contribution of the second bracketed term to the value of τ_m becomes significant and cannot be neglected.

Applying relation (34) to the operating conditions presented in Figure 5, the ratio of burner outlet temperature to ambient temperature corresponding to minimum specific fuel consumption for flight Mach numbers of 0.30, 0.80 and 1.5 is determined to be 2.45, 2.95 and 3.85 respectively, which check closely the values indicated by the graphical representation of Equation (6).

Figures 7 and 8 present the variation of optimum burner outlet temperature with flight Mach number for various compressor pressure ratios and component efficiencies. The influence of flight speed is similar for all turbojet operating conditions; that is, changes in compressor efficiencies affect the general level of τ_m values only. The burner outlet to burner inlet absolute temperature ratio for best economy operation is primarily a function of component efficiency, insensitive to moderate changes in compressor pressure ratio, and independent of flight speed. For component efficiencies equal 0.90, the optimum burner temperature ratio was approximately 1.5; for component efficiencies equal 0.80, the optimum burner temperature ratio was approximately 2.0.

The corresponding variation of minimum specific fuel consumption with operating conditions is shown in Figure 7. The curves have been discontinued for τ_m values larger than 6.5. An increase in component efficiency from 0.80 to 0.90 has a more pronounced effect upon economy of operation than an increase in compressor pressure ratio from 5 to 20. Small reduction in specific fuel consumption for increasing compressor pressure ratio above 10 is indicated.

CONCLUSIONS

1. For normal operating conditions, the expressions of turbojet performance may be closely approximated by an expression of ideal performance plus the first terms of a Taylor series expansion which represent the individual effects of non-ideal operation of the components.

2. The relative effectiveness of compressor efficiency compared with turbine efficiency in contributing to turbojet performance varies inversely as the ratio of compressor outlet absolute temperature to turbine outlet absolute temperature.

3. The relative effectiveness of compressor efficiency compared with diffuser efficiency in contributing to turbojet performance varies directly as the ratio of compressor temperature rise to diffuser temperature rise and inversely as the ratio of compressor outlet absolute temperature to diffuser outlet absolute temperature.

4. For maximum jet velocity the product of compressor pressure ratio and ram temperature ratio should remain constant with increasing subsonic flight Mach number.

5. The compressor pressure ratio corresponding to maximum jet velocity is given by:

$$\pi_c = \left\{ \frac{\tau_m \eta_t \eta_c}{\tau_r \left[1 + \eta_d (\tau_r - 1) \right]} \right\}^{1.75}$$

6. The ratio of burner outlet temperature to burner inlet temperature corresponding to minimum specific fuel consumption is primarily a function of component efficiency, being independent of flight speed and insensitive to moderate changes in compressor pressure ratio.

7. The optimum burner outlet temperature for minimum specific fuel consumption decreases with improved compressor or turbine efficiency and increases with compressor pressure ratio and flight Mach number.

8. For component efficiencies equal 0.90, the optimum ratio of burner outlet absolute temperature to burner inlet absolute temperature is approximately 1.5 for best economy operation; for component efficiencies equal 0.80, the optimum burner temperature ratio is approximately 2.0.

APPENDIX

The following concerns the derivation of an expression for an optimum burner outlet temperature corresponding to minimum specific fuel consumption. The partial derivative of relation (6) is taken with respect to τ_m and equated to zero to find the best economy condition.

$$(6)\quad S^* = \frac{\tau_m - \tau_r\tau_c}{\left[\frac{U_6}{U_o} - 1\right] M_o}$$

$$\text{where}\quad \frac{U_6}{U_o} = \left[\frac{\tau_m - \tau_r(\tau_c - 1)}{\tau_r - 1}\right]^{\frac{1}{2}} \left\{ 1 - \frac{1}{\left[1 + \eta_d(\tau_r - 1)\right]\left[1 + \eta_c(\tau_c - 1)\right]\left[1 - \frac{\tau_r(\tau_c - 1)}{\eta_t\,\tau_m}\right]} \right\}^{\frac{1}{2}}$$

$$(i)\quad \frac{\partial S^*}{\partial \tau_m} = \frac{\left[\frac{U_6}{U_o} - 1\right] - \left[\tau_m - \tau_r\tau_c\right]\frac{\partial\, U_6/U_o}{\partial \tau_m}}{\left[\frac{U_6}{U_o} - 1\right]^2 M_o} = 0$$

$$\text{Let}\quad A = \frac{1}{\tau_r - 1};\quad B = \frac{1}{\left[1 + \eta_d(\tau_r - 1)\right]\left[1 + \eta_c(\tau_c - 1)\right]};\quad a = \tau_r(\tau_c - 1);\quad b = \frac{\tau_r(\tau_c - 1)}{\eta_t}$$

$$\text{Then}\quad \frac{U_6}{U_o} = A(\tau_m - a)^{\frac{1}{2}}\left[1 - \frac{B}{1 - \frac{b}{\tau_m}}\right]^{\frac{1}{2}} = A\left[\frac{\tau_m^2(1-B) + \tau_m(aB - b - a) + ab}{\tau_m - b}\right]^{\frac{1}{2}}$$

$$\frac{\partial\, U_6/U_o}{\partial \tau_m} = \frac{A^2}{2}\,\frac{U_o}{U_6}\left[\frac{\tau_m^2(1-B) - 2\tau_m b(1-B) - b(aB - b)}{(\tau_m - b)^2}\right]$$

Substituting into (i) and simplifying,

$$\left[\frac{U_6}{U_o} - 1\right] \frac{U_6}{U_o} (\tau_m - b)^2 - \frac{A^2}{2}\left[\tau_m - \tau_r \tau_c\right]\left\{\tau_m(\tau_m - 2b)(1-B) - b(aB-b)\right\} = 0$$

For turbine efficiencies nearly unity

$$-b(aB-b) = b^2(1 - \frac{a}{b}B) = b^2(1 - \eta_t B) \doteq b^2 (1 - B)$$

Therefore the condition for best economy operation becomes:

$$\text{(ii)} \qquad \left[\frac{U_6}{U_o} - 1\right] \frac{U_6}{U_o} \doteq \frac{A^2}{2}\left[\tau_m - \tau_r \tau_c\right]\left[1 - B\right]$$

An approximate expression which was derived in section IV is now introduced for the velocity ratio to permit an explicit solution for optimum τ_m; namely,

$$\frac{U_6}{U_o} \doteq A \left[\tau_m (1-B) - b\right]^{\frac{1}{2}}$$

The conditions which permit the approximation are that the turbine efficiency is nearly unity and the burner outlet temperature is large compared to the mechanical compressor temperature rise; both are consistent with best economy operation (See section IV).

With the substitution for velocity ratio expression (ii) becomes:

$$\frac{A}{2}\left[(\tau_m-\tau_r\tau_c)(1-B)-2b\right] = \left[\tau_m(1-B)-b\right]^{\frac{1}{2}}$$

A quadratic expression for τ_m is obtained:

$$\tau_m^{\,2}+\left\{2\tau_r\tau_c-4\left(\frac{b+\frac{1}{A^2}}{1-B}\right)\right\}\tau_m+\tau_r^{\,2}\tau_c^{\,2}-\frac{4b\tau_r\tau_c}{1-B}+\frac{4b\left(b+\frac{1}{A^2}\right)}{(1-B)^2}=0$$

$$\underset{\text{(best economy)}}{\tau_m} = \left\{2\frac{c}{B}-\tau_r\tau_c\right\}\left\{1+\sqrt{1-\frac{\tau_r^{\,2}\tau_c^{\,2}-4\frac{b}{B}\left(\tau_r\tau_c+\frac{c}{B}\right)}{\tau_r^{\,2}\tau_c^{\,2}-4\frac{c}{B}\left(\tau_r\tau_c+\frac{c}{B}\right)}}\right\}$$

where $b = \dfrac{\tau_r(\tau_c-1)}{\eta_t}$ $\qquad c = \dfrac{\tau_r(\tau_c-1)}{\eta_t} + (\tau_r-1)$

$$B = 1 - \frac{1}{\left[1+\eta_d(\tau_r-1)\right]\left[1+\eta_c(\tau_c-1)\right]}$$

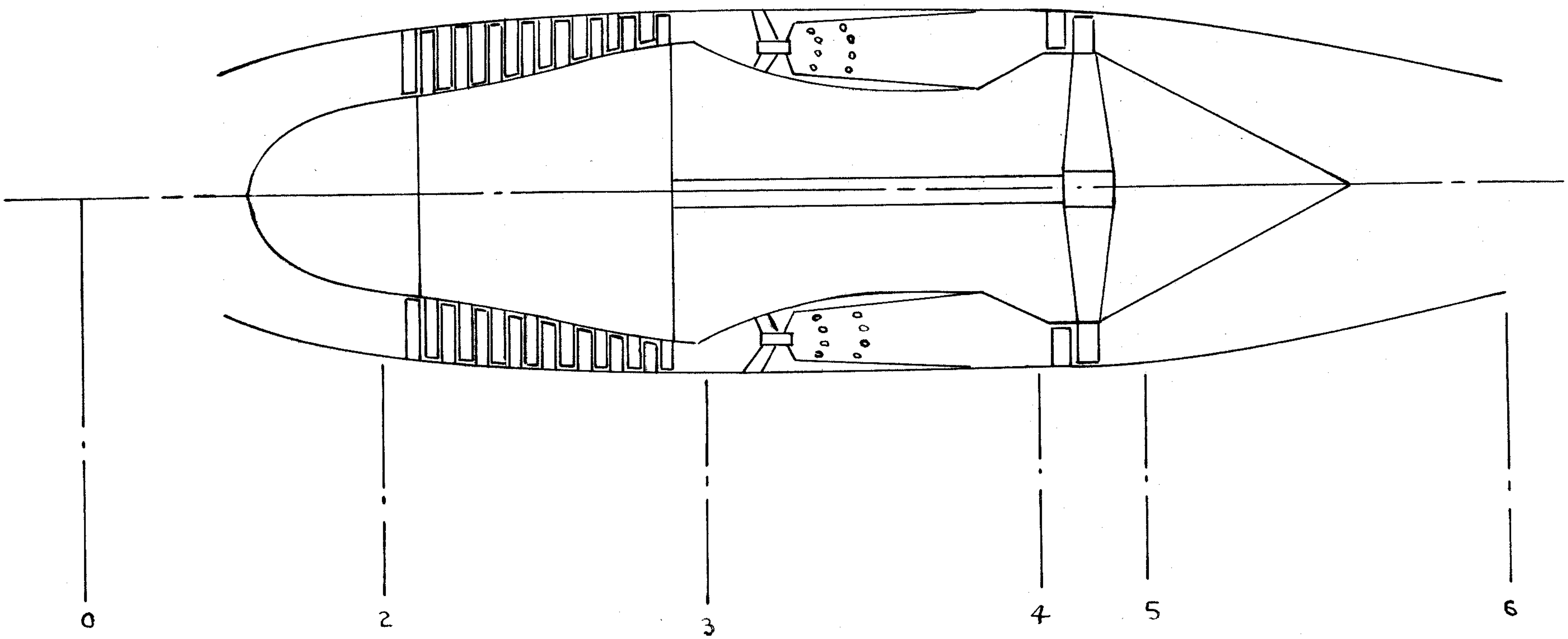

Figure 1. Schematic drawing of turbojet engine.

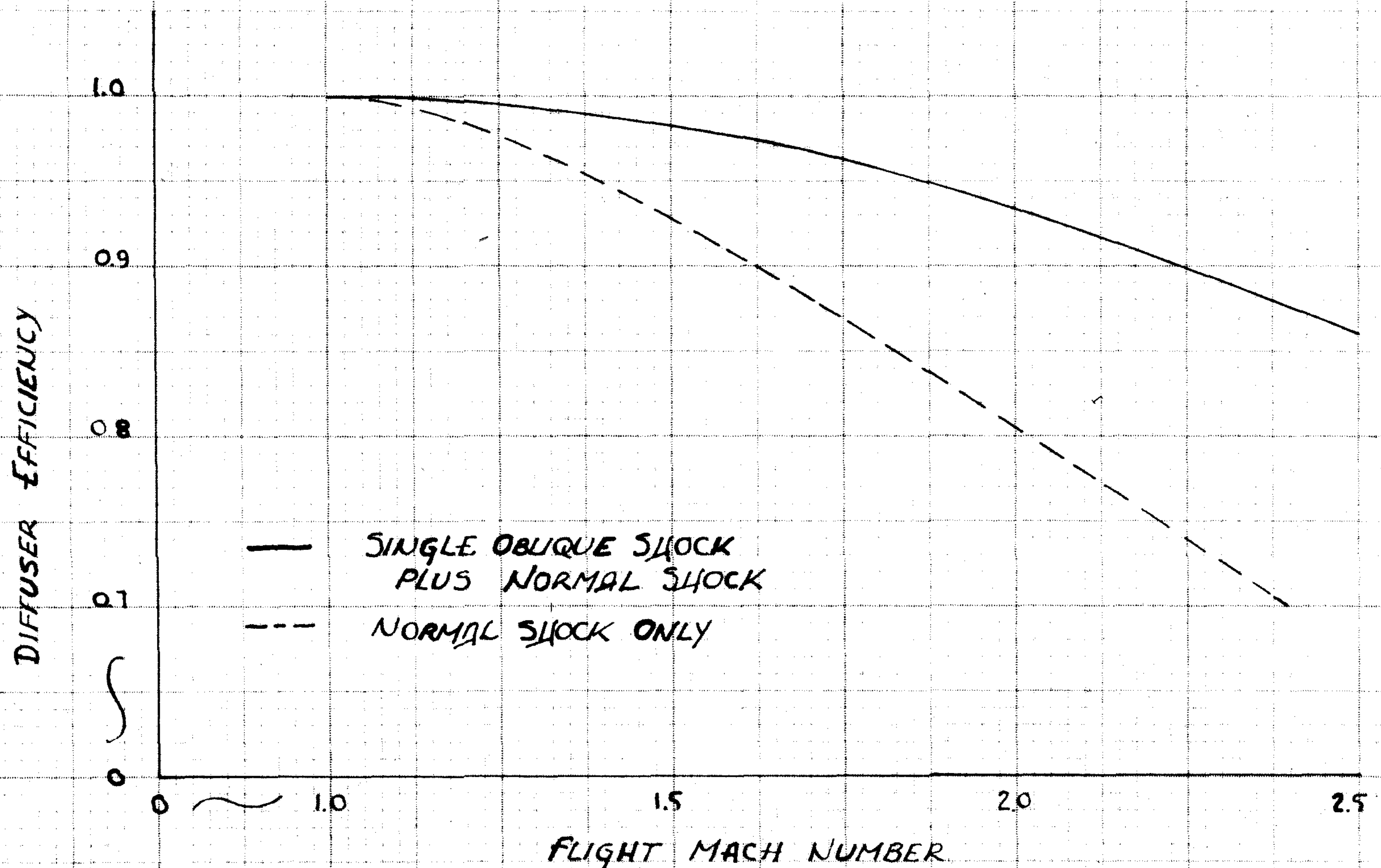

Figure 2. Variation of theoretical diffuser efficiency with supersonic flight Mach number corresponding to single oblique shock plus normal shock diffusion.

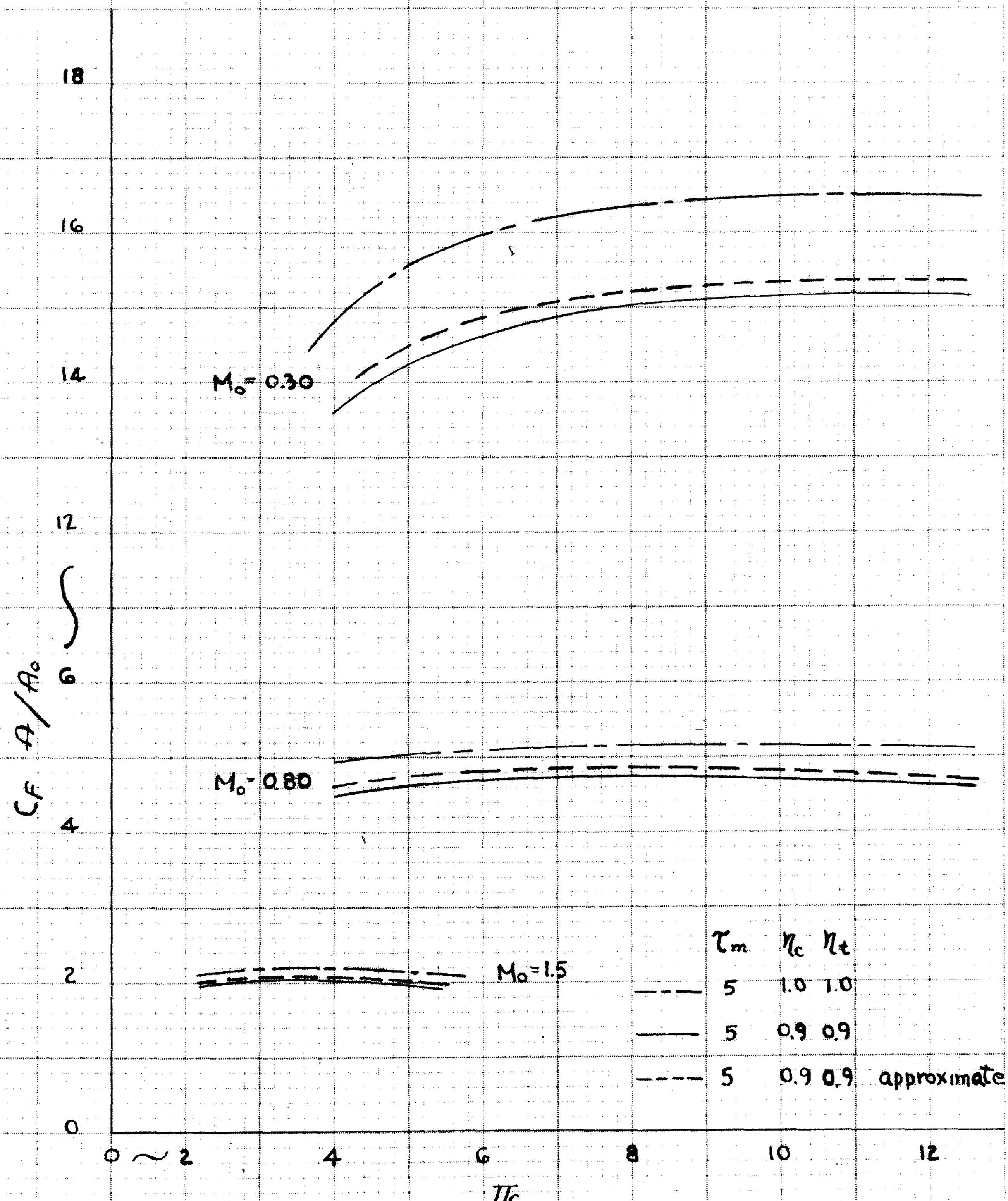

Figure 3. Variation of thrust coefficient with compressor pressure ratio for various flight speeds.

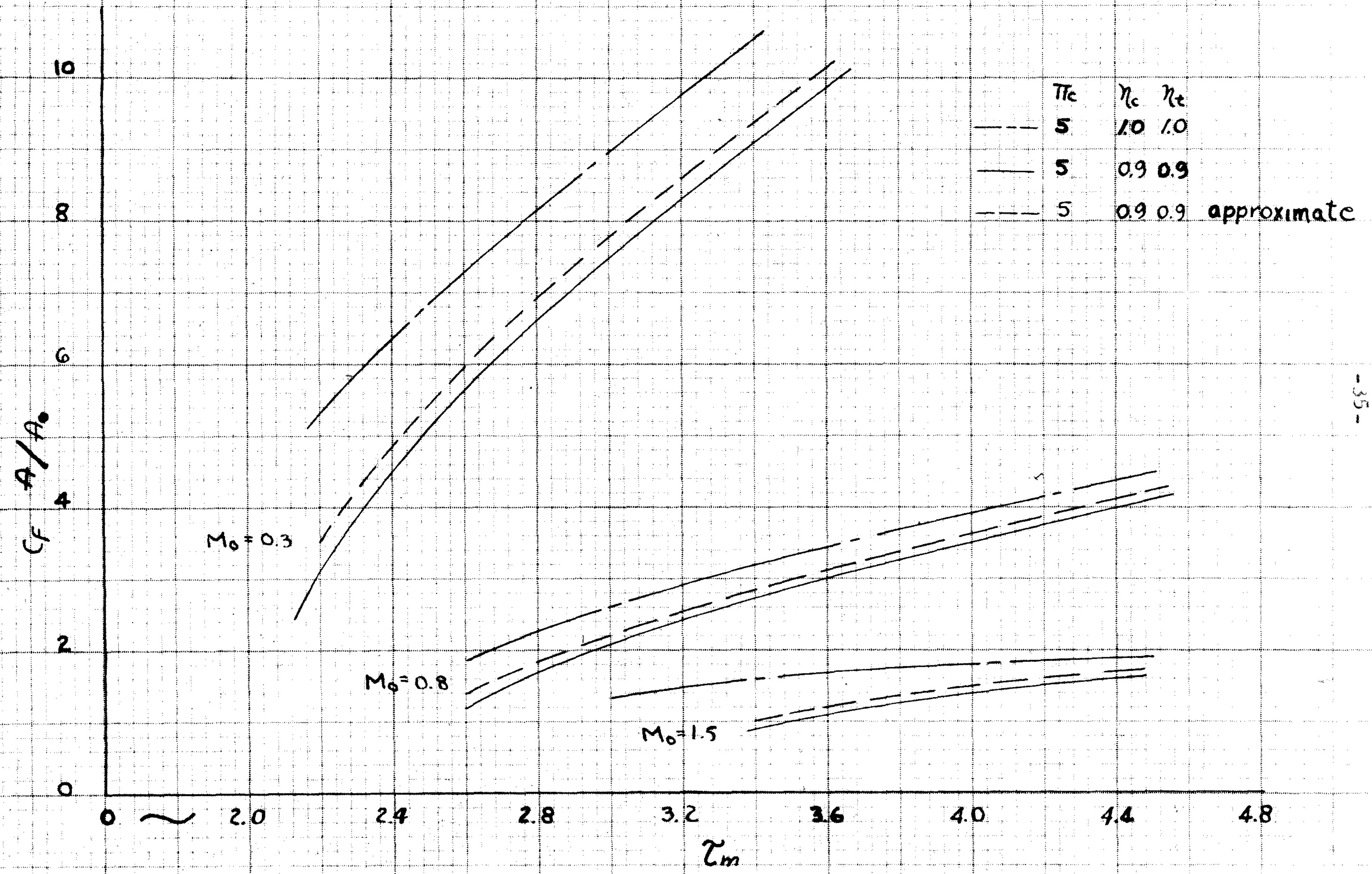

Figure 4. Variation of thrust coefficient with burner outlet temperature for various flight speeds.

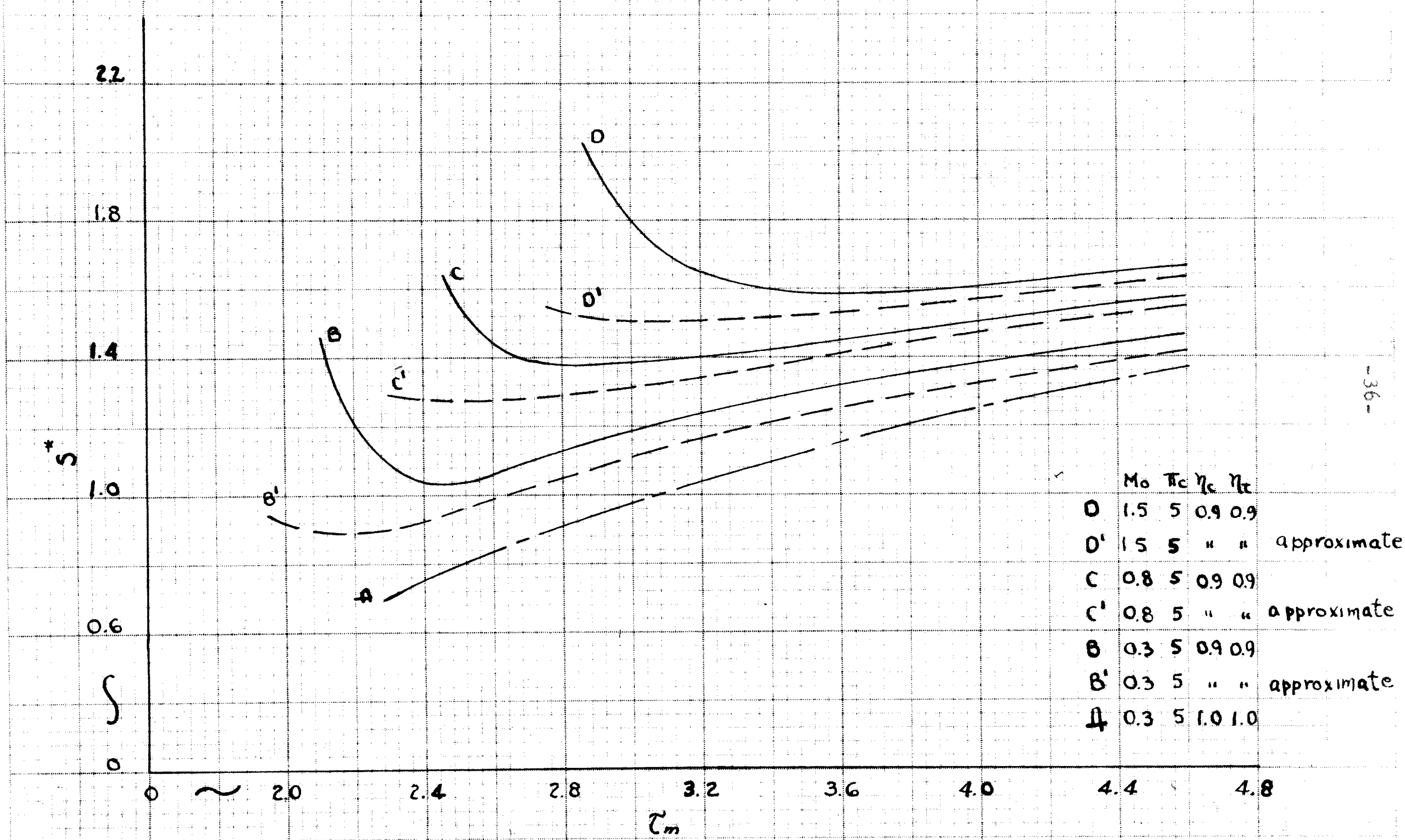

Figure 5. Variation of specific fuel consumption with burner outlet temperature for various flight speeds.

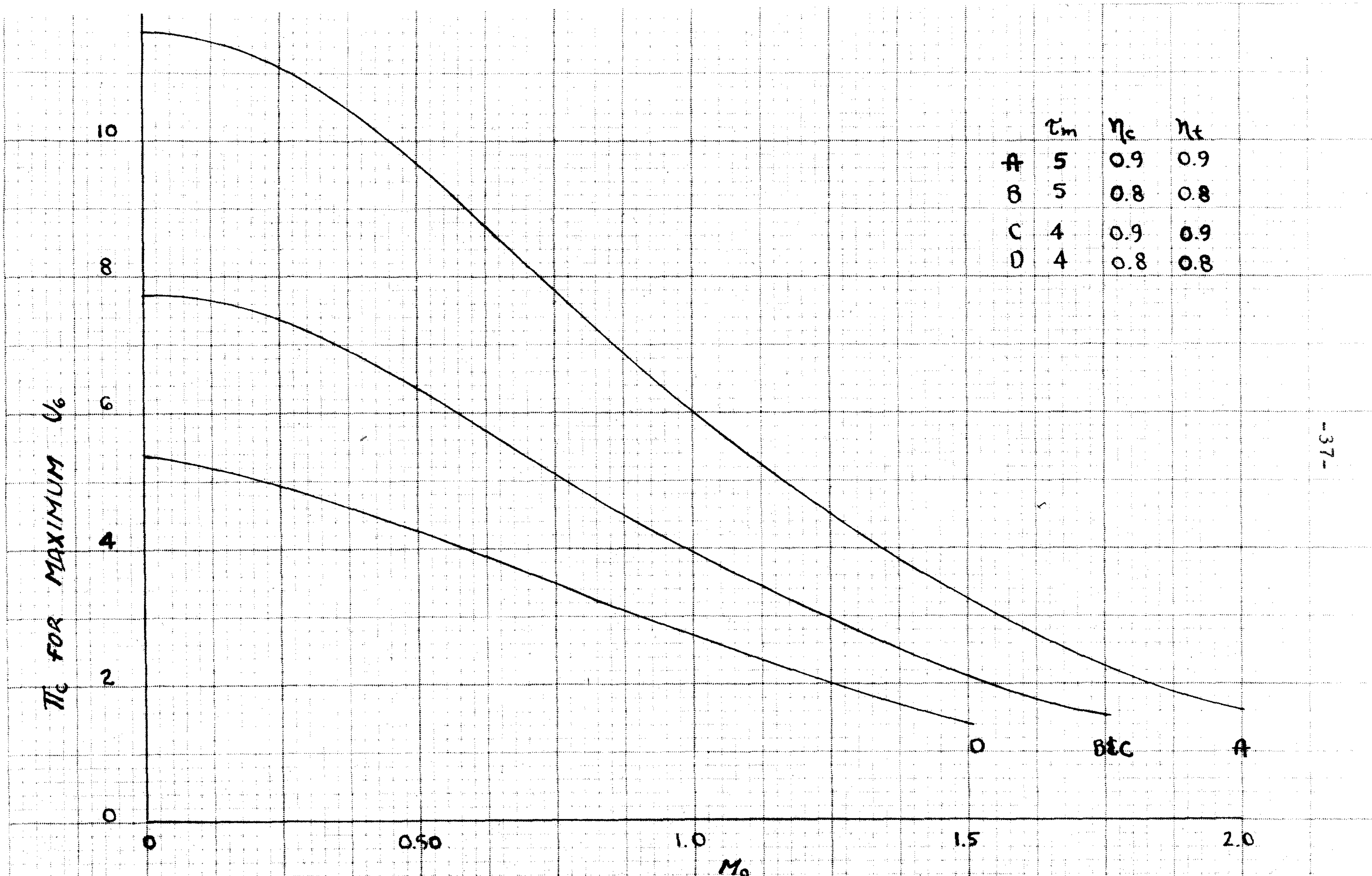

Figure 6. Variation of compressor pressure ratio corresponding to maximum jet velocity with flight Mach number.

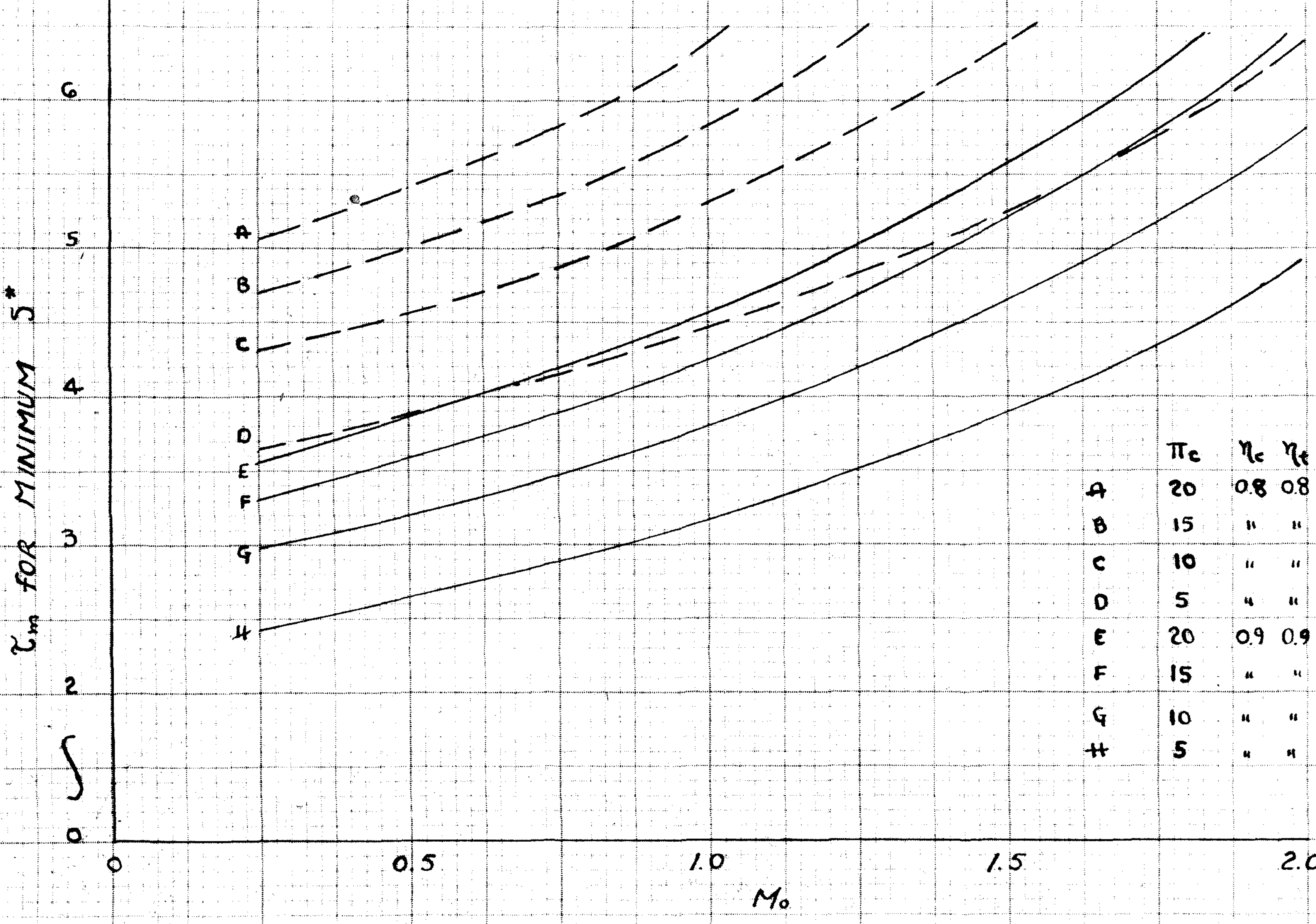

Figure 7. Variation of burner outlet temperature corresponding to minimum specific fuel consumption with flight Mach number.

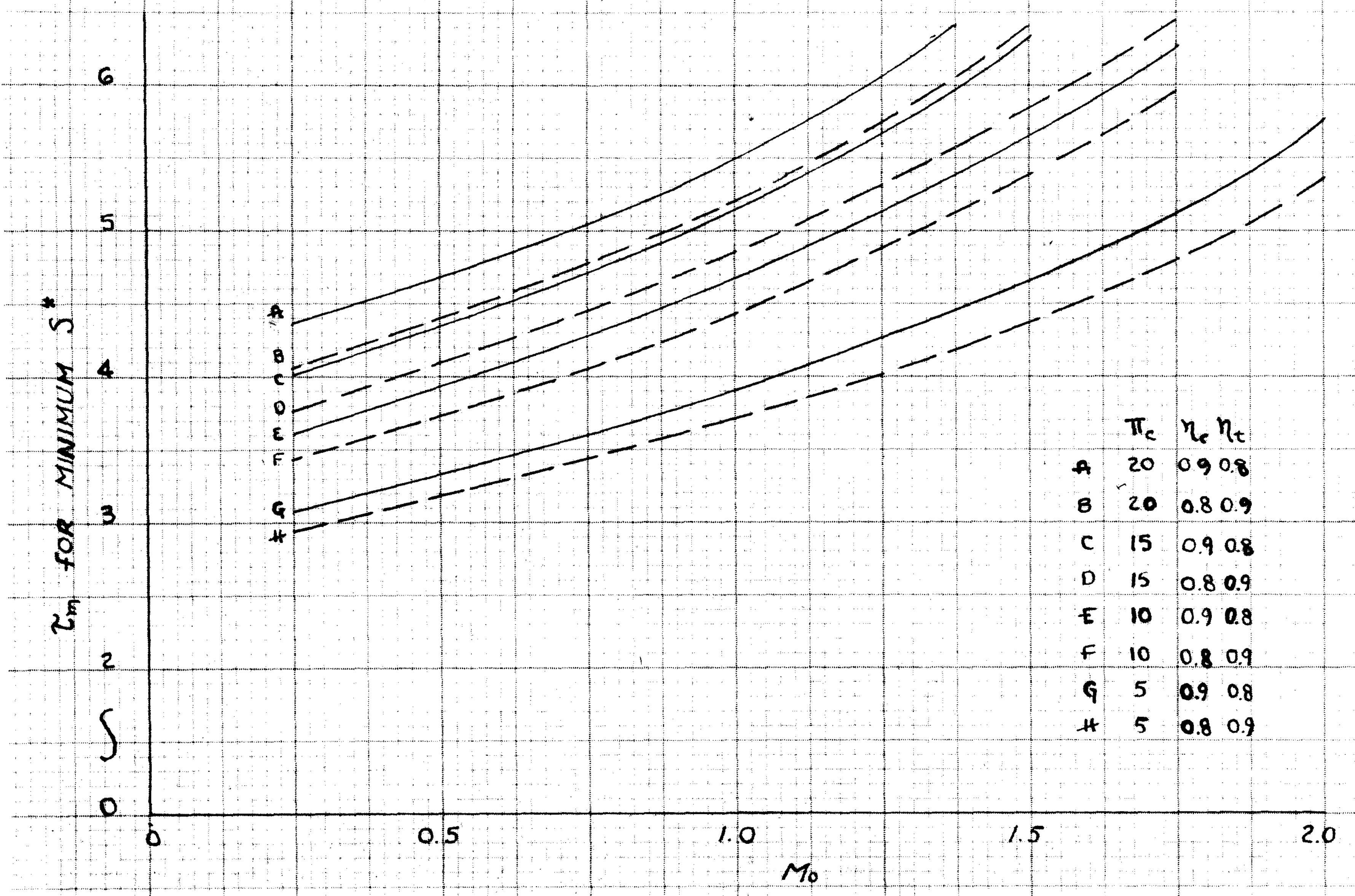

Figure 8. Variation of burner outlet temperature corresponding to minimum specific fuel consumption with flight Mach number.

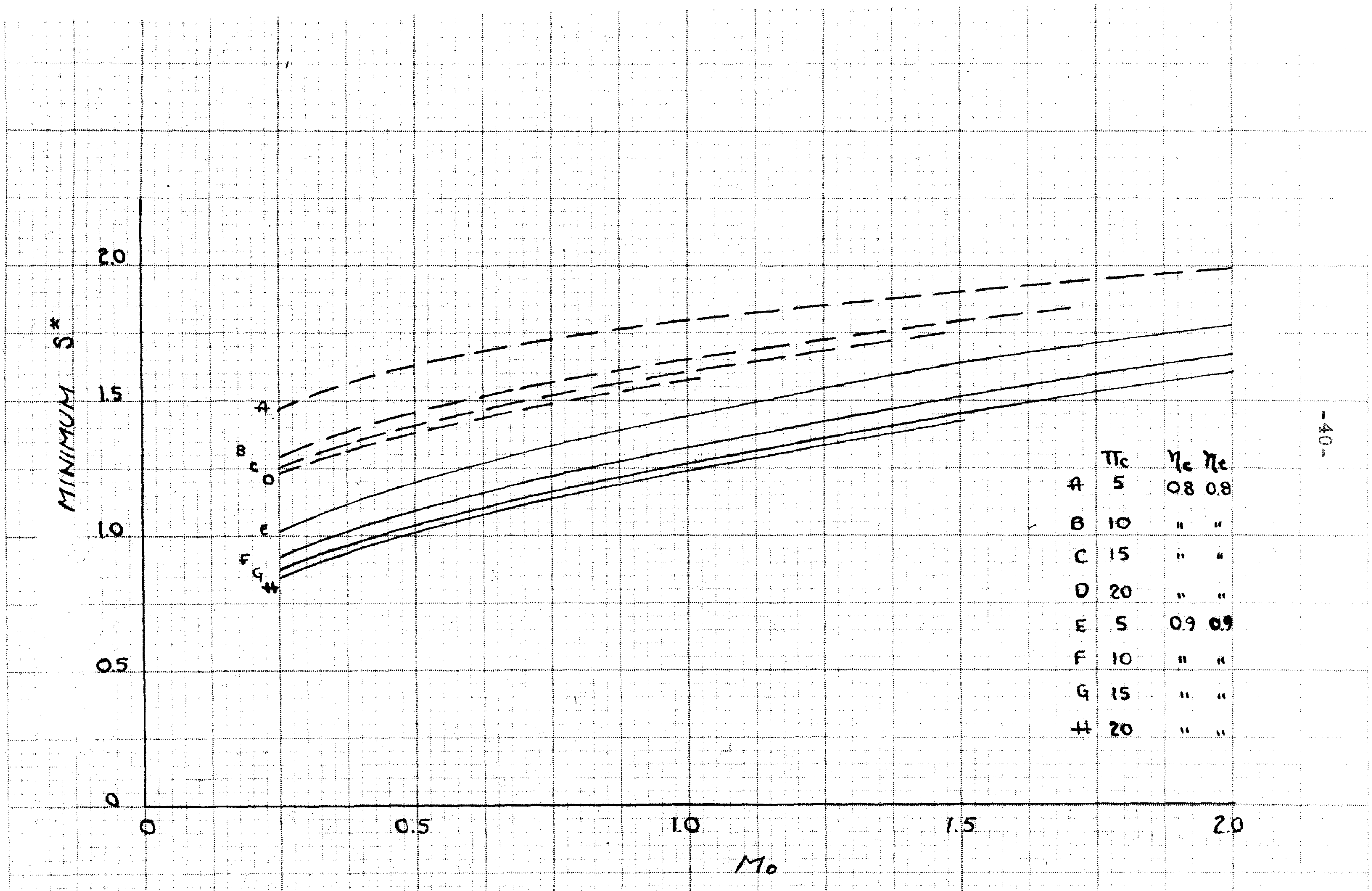

Figure 9. Variation of minimum specific fuel consumption with flight Mach number for various compressor pressure ratio.