OBSERVATIONS ON THE THEORY AND CHARACTERISTICS OF ELECTRICAL FIGURES ON PLATES IN AIR

With Particular Reference to Their Speed of Growth

Thesis by

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CONTENTS

INDEX OF FIGURES

INDEX OF FIGURES

SUMMARY

The fact that electrical figures in air at atmospheric pressure grow with average velocities which lie in the range of from 10 to 10^3 km./sec., and which may extend beyond this range, depending on the magnitude of the voltage applied and the physical constants of the plate is verified. The velocity decreases with increasing size of the figure. The effect of a variation in the capacitance between surfaces per unit area of the plate has been found to be greater for negative than for positive figures, and has been found to decrease with increasing voltage, for figures at the same stage of development. The maximum current intake has been observed to be 22.5 amp. (an average for two figures in parallel) at 30 kv., rising to a value of 60 amp. at 42.5 kv.. The effective capacitance (of two figures in parallel) in one case rose from an initial value of practically zero to more than 8,000 cm., resulting in a reduction in maximum value of the voltage wave applied of 25% of the open line value. The reduction in rate of rise of voltage for the same case was 30%. The possibility of the use of such an arrangement for protection against over voltages is suggested. The polarity difference in the figures is a result of the difference in direction of the flow of charge, and is dependent on the constants of the plate, - - no predisposition to a fixed ratio being noted. The behavior of the figures is such as to verify the conclusions of other authors that figures of both polarities are formed by the movement of electrons alone.

My sincere thanks are due the "Verein deutscher Ingenieure", whose "Stipendiat" I was while engaged in the above work, and to Professor A. Matthias for the incentive to the problem. I am also indebted particularly to Dipl.-Ing. R. Andrieu and to the other gentlemen mentioned herein for their interest and advice.

INTRODUCTION AND OBJECT

The object of this paper is to give a review of some facts already known about electrical figures, and to offer some new data which may serve to throw light on the formation and characteristics of Lichtenberg figures and Toepler's "Gleitfunken" - - which are, as we might hereafter call them, "spreading discharges", so named after the man who has probably most extensively investigated them. The two sets of phenomena, which are closely related - - they arise in the same way, and the second is only a further developed form of the first - -, have been studied very exhaustively, and their mechanism has been explained on the basis of ionization by collision and the movement of the resulting space charges.[1] Certain points of importance in the observations obtained up to the present have, however, to my knowledge not been thoroughly cleared up, experimentally or theoretically. For example, the relation between the size of the positive and negative figures, the relation of the velocity of growth of the two, and the increase in size of the figure as a function of time. Additional observations therefore seemed desirable.

THE GENERAL BEHAVIOR OF ELECTRICAL FIGURES

In his very detailed work, Toepler has differentiated between two

1. See M. Toepler, Archiv f. Elektrotechnik - 1921, Vol. 10, p. 157;
   P. O. Pedersen, Annalen d. Physik - 1922, 4th Series, Vol. 69, p. 205;
   K. Przibram, Elektrotechnik u. Maschinenbau - 1923, Vol. 41, p. 97;
   C. E. Magnusson, Trans. A.I.E.E. - 1930, Vol. 49, p. 1384;
   H. Staack, Arch. f. Elek. - 1931, Vol. 25, p. 607.

stages of the figures, which he calls "Gleitkorona" and "Gleitfunken-korona", or "Polbüschel" and "Gleitbüschel", respectively. Let us take equally descriptive English names, and distinguish between "brush discharge" and "streamer discharge". Lichtenberg figures belong to the first class, where the voltage does not reach a high enough value (or the time of application of the voltage is too short) to allow the forming of glowing channels (streamers) of more or less good conductivity. The true "streamer discharges" belong in the second category, appearing only under the influence of more powerful voltage surges. These latter constitute the greatest danger of arc-over in the field.

The characteristics of these discharges which admit of investigation are:

1. Length attained.
2. Velocity of growth.
3. Current drawn.

These values can vary (assuming the customary and familiar arrangement of an electrode resting upon a glass plate coated underneath with metal forming the other electrode) with the voltage applied, the thickness of the glass, and the dielectric constant of the glass, - - assuming the upper electrode and the condition of the glass surface to remain unchanged, and the plate and coating to be large enough to avoid its limiting the growth of the figure. Toepler was chiefly concerned with 1. of the above list, and, to a lesser degree, with 2.

According to his results the size of the figures attained, <u>in their first stage</u> (Lichtenberg figures), depends <u>only</u> on the voltage wave, - -

i.e., on the polarity, the steepness of wave front, the maximum value, and - to a degree - on the steepness of the tail. No difference was noticeable "for plates of the most widely varied thicknesses". The voltage of transition from the first to the second stage is, however, dependent on the thickness of the plate. This critical voltage may be expressed as a constant times the square root of the thickness, in which the constant is somewhat different for the two polarities. As long as the discharge remains in the first stadium, that of Lichtenberg figures, the radius of the positive figure is about twice that of the negative figure for a given voltage wave, and the figures are symmetrical.

As the energy of the surge is increased, the discharges go over into the "streamer" state, in which glowing, branched channels appear to be going out from the electrode. The brush continues to go out from these channels, just as it did from the electrode in the first stage, and reaches a length, indeed, which is only slightly less than its length for the critical voltage at which the discharge changed state. The appearance of these branches causes the figure to lose its symmetry and to take on the appearance of a rather unbalanced, flower. Toepler's researches revealed that the maximum radial length reached by the figures in this stage depends not only on the form, size, and polarity of the voltage wave, but also on the capacity per unit area between the two surfaces of the plate, according to the relation, - -

$$\text{Maximum radial length} \propto (\text{unit capacity})^2.$$

In this stage, as the figures become larger and larger, the polarity difference in maximum radial length approaches zero.

THE MECHANISM OF SPREADING DISCHARGES

In order to explain this phenomenon of discharge along the surface of the glass, Toepler first proposed the familiar concept of a series of neighboring, elementary condensers formed by elementary areas of the upper surface and the coating of the lower surface. Under this assumption, the process would be somewhat as follows: - - A voltage wave, reaching the electrode, raises its potential high above that of the upper plate of the first elementary condenser (i.e., of the neighboring surface) so that an arcover occurs between electrode and first condenser. The latter is charged up through the spark, and, as soon as its voltage is high enough, arcs over to the second condenser. It in turn is charged and arcs to the third, which is then charged up, etc.. By the use of a formula derived by Binder[1] on the basis of the differential conception just outlined, Jezek[1] was able to confirm by calculation his measurements on the velocity and current intake of streamer discharges along glass tubes filled with mercury.

It remains to present a truer picture of the process, also suggested by Toepler in the paper referred to, in which consideration is given to

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1. See "Die Wanderwellenvorgänge auf experimentelle Grundlage", by L. Binder, published by Springer, Berlin, - p. 52.

the production and movement of the ions and electrons, the resulting space charge, and its influence on the original voltage distribution along the glass. Staack[1] also supports his theoretical discussion on this conception, which must, however, be modified in the light of the results of recent work done by Erwin Marx[2] and C. E. Magnusson[3]. These modifications will be dealt with later.

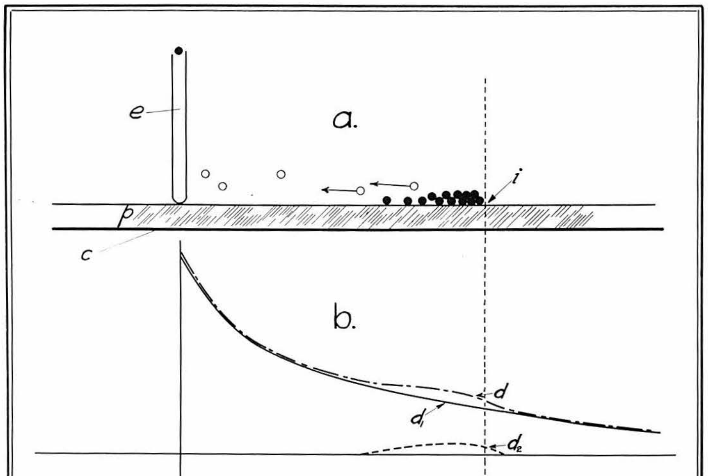

FIG. I

Fig. 1. Formation of Surface Charge

and Distortion of Original Voltage Distribution

With Growth of Electrical Figure

In Fig. 1., let e signify the electrode which is suddenly raised to a high voltage of either polarity against the metal coating, c. Let

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1. See Staack reference, p. 1.
2. E. Marx, Arch. f. Elek. - 1928, Vol. 20, p. 589,
   " " " - 1930, Vol. 24, p. 61.
3. C. E. Magnusson, as on page 1.

us indicate this polarity by the sign ●, and the opposite polarity by o. The field strength is greatest in the space immediately surrounding the point of the electrode, and ionization begins there. The ● particles are repelled, and at the same time pressed against the glass plate, p, by the vertical component of the field of force. They make their way, more or less retarded, along the surface, and, after a short time, form a ring-shaped space-charge around the electrode, as indicated in the figure. The o particles are, on the contrary, attracted directly to the electrode.

The voltage distribution due to the space charge, curve d_2 in Fig. 1, b, must now be superimposed on the original curve, d_1, so that the resulting distribution is given by the curve d. It is to be noted that the point, or circle, just at the outside edge of the space charge is subject to a very high gradient. This point is marked i, indicating that ionization now takes place here. It is very easy to picture the further expansion of the space-charge ring together with the concentric ionization area, - - the growth of the electrical figure. This growth continues until for some reason, such as the attenuation of the applied voltage or the dispersion of the space charge due to wide removal from the electrode, the gradient on the edge sinks below the value necessary to sustain ionization. The glowing, branched sparks which seem to the eye to shoot out from the electrode are in reality the paths followed by the o charges in flowing in to the electrode. The lighting, always a sign of recombination, is due to the union of some of these with fresh charges continually produced and repelled by the high gradient at the electrode.

With this general picture of the formation of these figures - - which, as I have said, must be qualified later, in view of proofs offered by Magnusson and Marx, that, practically, the movement of the electrons alone may be assumed - - let us leave the theory of formation of these figures, for the present, and proceed to an inspection of some of the experimental results attained by previous investigators.

RESULTS OF PREVIOUS EXPERIMENTS

Jezek[1] has determined simultaneously the maximum rate of voltage rise and the maximum current intake by spark gap methods. Using the current measurements in a formula, in which the velocity of growth (on the glass tubes mentioned before as having been used by these workers) appears as a function of current intake, voltage, and capacity per unit length between the two surfaces of the tube, he has succeeded in determining probable values of velocity. The velocity came out as directly proportional to the voltage, on tubes with values of capacity per unit length ranging between 4 and 9 cm. A voltage of 64 kv. gave a velocity of 5000 km./sec.. It must be remembered that this represents a value

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1. See reference to Binder, p. 4.

calculated by the insertion of observed data into a theoretical formula. It was not specified whether or not the velocity was maximum or average. The observed maximum current increased faster than directly proportional to the maximum voltage, going from 57 to 125 amperes as the maximum voltage was raised from 26.4 to 45 kv.

According to the measurements of Müller-Hillebrand[1] the velocity of growth of Lichtenberg figures, produced by voltages between 2.5 and 18 kv., can reach a value as high as 500 km./sec..

P. O. Pedersen[2] has even determined an expression for the radius of the figure as a function of time. By means of his observations on the line of division between figures going out from two neighboring electrodes of the same polarity, he was able to derive the following relation:

$r = R(1 - e^{-at})$, in which r = the radius at any instant, and R = the radius finally attained. From this it follows that velocity = V = $aRe^{-at} = a(R - r) = V_0 - ar$. For atmospheric pressure, V_0, or the initial velocity, was found to have a value of 700 km./sec. for positive and 300 km./sec. for negative figures. a was given as about 2.5×10^9 for positive and 1.9×10^9 for negative figures (for r in cm.).

Interesting, although of no great significance for this discussion of work done in air, are measurements of the velocity of growth of these

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1. See Siemenszeitschrift - 1927, No. 819, p. 547, 605.
2. See reference to Pedersen, p. 1.

figures on plates submerged in transformer oil. Staack[1] found velocities for one value of voltage, which were constant throughout the entire development of the figure, and amounted to 18 km./sec. for positive figures and 9 km./sec. for negative figures.

In his 1921 article, Toepler quoted measured or estimated values of velocity which varied from 3560 km./sec. for a positive figure reaching a length of 92 cm. under a voltage wave of 327 x 10^6 kv./sec. rise and 46.4 kv. maximum value, to 55 km./sec. for a negative figure reaching a length of 9.9 cm. under a voltage wave of 188.8 x 10^6 kv./sec. rise and 47.9 kv. maximum value. Whether or not the plate was the same in both cases is unknown to me.

DISCUSSION OF PREVIOUS RESULTS

Two features of marked interest are apparent upon scanning the material given above. Firstly, velocities of growth covering a very broad range seem to have been observed, - - the smallest differ by more than one order of magnitude from the largest. Secondly, how is the difference in the velocities of positive and negative figures to be explained, and what accounts for the fact of its practical disappearance when the figures are caused to become very large? The fact that in the results of Toepler, Pedersen, and Staack (in his work under oil) the ratio of approximately 2:1 between the positive and negative

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1. See reference to Staack, p. 1.

lengths and velocities in the first stage continually appears is very disturbing. If such a simple factor really belongs there, it should be exactly determinable experimentally, and justifiable theoretically. If not, it should be recognized as an accident.

At first it was necessary to consider the possibility that two different phenomena, with different characteristics, were to be dealt with, - - on the one hand, the case of the Lichtenberg figure, or brush discharge; and on the other, the case of the streamer discharge, with the glowing channels and greater extension. This was suggested by the fact that the previous observations can be divided into two groups:-

1. Jezek's results had to do with strongly developed streamer discharges. He made the assumption of a row of elementary condensers, and arrived at extremely high values of velocity, - - up to 5000 km./sec. Some of Toepler's observations on very long streamer discharges also yield velocities in this order of magnitude.

2. Müller-Hillebrand and Pedersen, on the other hand, working with Lichtenberg figures, published velocities one order of magnitude smaller, - - from 200 to 700 km./sec.

From these data it would be possible to suppose that upon going over from the first to the second stadium the velocity of growth of the figures increases to a much greater value, - - to a value, if you please, independent of polarity. Under this assumption it would be possible to explain the decrease of the difference in size and velocity of the positive and negative figures with increasing size.

I have rejected this hypothesis, however, - - first, because my own experimental results showed no such sudden increase in velocity, notwithstanding the fact that the figures investigated ranged from small Lichtenberg figures to marked streamer discharges, at high voltages and steep wave-fronts; and, secondly, because it is possible to explain the disappearance of the polarity difference by a different theory.

The above mentioned wide difference in the values observed by different experimenters is, probably, rather to be traced to variations in the individual experimental conditions such as, for instance, the thickness of the plate, the dielectric constant of the plate, the form of the voltage wave used, etc.. As already established by Toepler, the length attained by streamer discharges is strongly dependent on the capacity per unit area of the plate. The primary object of the present work was originally simply to investigate the dependence of the speed of growth of the figures on the thickness and dielectric constant of the glass. To quote immediately a few figures obtained from my results bearing on the influence of the thickness, I found that decreasing the plate thickness in the ratio 1.63:1 (dielectric constant the same) caused maximum ratios of increase of (average) velocity as follows:-

1.84:1 for positive figures, and

2.98:1 for negative figures.[1]

Upon decreasing the thickness in the ratio of 1.98:1, the increases in velocity were:-

2.51:1 for positive figures, and

3.97:1 for negative figures.[1]

These data serve to convince me that observed velocities might very easily differ by one order of magnitude, or more, when, as in the case of Jezek's tubes and my plates, the ratios of thicknesses varied between 6:1 and 11:1.

EXPERIMENTAL METHODS AND SET-UP USED IN PRESENT WORK

METHOD OF RECORDING FIGURES

As is commonly known, the customary method of recording electrical figures, Lichtenberg figures, is by means of photographic film or plates, used as in the klydonograph. The voltage above which this instrument becomes valueless, however, due to the occurrence of visible discharges (streamers) and the burning and blackening of the film, is just that voltage which forms the threshold to the interesting investigation of the electrical figures in their second stadium.

To circumvent this difficulty, recourse can be had to an old, familiar method, somewhat cumbersome for much present day research, but

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1. Of considerable significance for the theoretical explanation of the polarity difference is the fact that this effect is greater for negative than for positive figures.

just as effective now as when Toepler first used it successfully in his experiments to show the phenomena of electrical figures. I refer to the dust, or powder method. Lycopodium or any other fine powder may be strewn thinly on the surface of the glass, around the electrode. Upon the application of an electrical surge, to the electrode, the light powder around it is dashed aside by the rapidly moving charges, so that a very recognizable figure is left behind by the discharge of the surge. According to tests made by Toepler, the presence of the powder on the glass does not noticeably influence the formation of the figure.

Another method is to choose two powders which take opposite charges when shaken together, mix these in a shaker, and use the mixture to powder the plate immediately after the surface discharge. The powder with the proper charge settles on the boundary of the figure, where the space charge (or surface charge) is still resting on the glass, and clearly defines its position. Needless to say, if too much time is allowed to elapse between the voltage surge and the powdering of the glass, the figure will appear blurred, due to the leakage of the charge along the surface of the glass.

The mixture used in this work was red oxide of lead and sulphur, in which there were about two volumes of sulphur to one of lead oxide. The oxide takes a positive charge in the mixture, so that negative charges on the plate appear red-orange, the positive charges being yellow. This method, well known in the laboratory of the Berlin

Institute of Technology for its excellence on bakelite plate, was thought, at the beginning of my work, to be unsuitable for glass. As a matter of fact, it functions perfectly, provided the mixture is fresh, and the glass plate scrupulously clean.

In case of necessity, as, for instance, when due to experimental procedure it is impossible to give attention to the plates immediately after the surge, this same mixture as well as a neutral powder may be strewn uniformly on the plate beforehand, where it records the figure in the way first mentioned. Both methods were used here.

Figure 2 shows photographs of figures made in this way. The clearness of recording, and the unmistakable presence of the ring-shaped

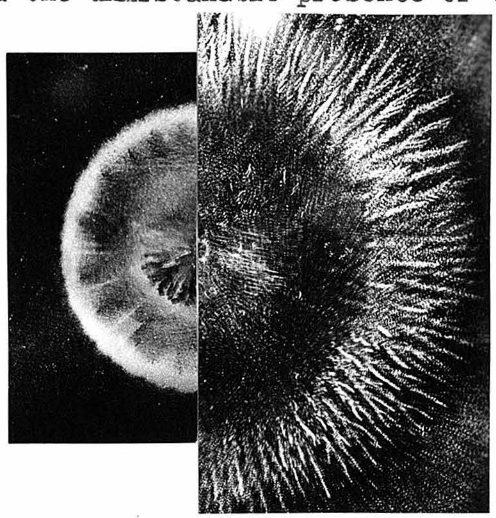


Figure 2. Right half of positive figure and left half of negative figure made under identical settings of firing gap and "cut-off" gap. Photograph natural size.

The plates were coated on one side with tinfoil. For the tests they were mounted, two at a time, side by side under a frame carrying two electrodes connected in parallel. The electrodes were brass rodes about 15 cm. long and 3 mm. in diameter, with the contact ends rounded off to a form somewhat flatter than hemispherical at the bottom.

In order to study the figures in different stages of development, it was necessary that provision be made whereby the surge voltage applied to the plates could be cut off after a small interval of time, variable at will. This was accomplished by means of a needle-gap connected in parallel with the plates. The time lag of such a gap, known to be appreciable, was such as to permit the figures to grow to sizes quite convenient for observation. The time lag, and the size of the figures, were varied by changing the spacing of the gap. The needles were ordinary darning needles of medium size, mounted in the ends of brass shafts 7 mm. in diameter.

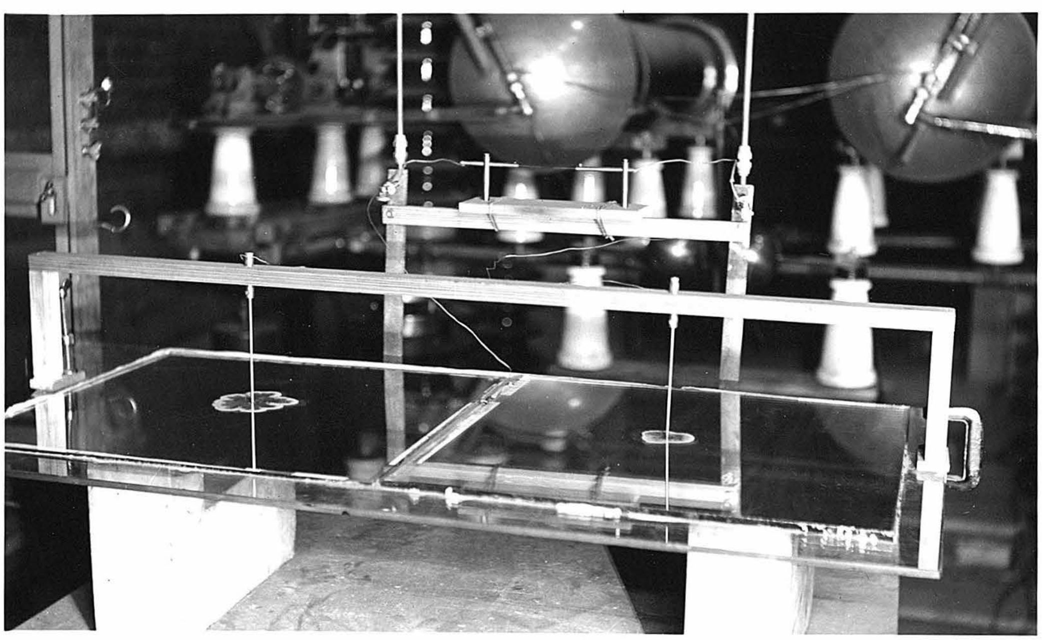


Figure 3. Test plate mounting, surge generator in background.

Figure 3 shows clearly the arrangement of plates, electrodes, and needle-gap. In addition, two negative figures which had been made just before taking this picture are to be seen. It is to be noted that the figures are in the second stage, - no longer round and symmetrical, but with somewhat scalloped rims, due to the appearance of the streamers.

SOURCE OF VOLTAGE SURGES

The ideal voltage wave for such an experiment is one which rises quickly to its maximum value, attenuates very slowly, and is entirely free from oscillations. A surge generator already at hand in the laboratory proved to be suitable, except for one objectionable feature which was overcome in the manner described later. The generator was connected symmetrically with respect to ground, in the customary manner. The diagram in Figure 4 shows the connections and the numerical constants.

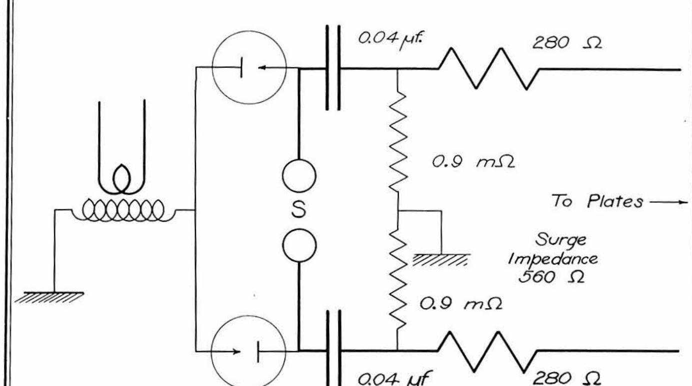


FIG. 4

Figure 4. Diagram of Connections of Surge Generator.

The surges were led to the recording apparatus over an indoor transmission line about 120 meters long, with a surge impedance of 560 ohms. As may be seen from the diagram, resistances of 280 ohms were placed in each side of the line near the surge generator. The purpose of these was to absorb any reflected voltage waves, and to prevent oscillations due to travelling waves.

In spite of this precaution, it was noticed in preliminary tests that a sphere gap located at some distance along the line indicated a higher voltage than that for which the firing gap, S, was set. This action was traced to oscillations arising from the self-induction of the surge generator circuit. After the inclusion of a carbon resistance of 1500 ohms (measured at steady current, low voltage) in series with the firing gap the test spark gap indicated the correct maximum value, and, as cathode-ray oscillograms later showed, no oscillations were present. Extreme steepness of wave-front was sacrificed, of course, by this procedure.

EXPERIMENTAL PROCEDURE AND RESULTS - FIRST PART

The test plates will henceforth be known by numbers, - from 1 to 4. To repeat, - - at my disposal were two thin plates and two thick ones. One thin plate and one thick plate each had a dielectric constant of a value usual for glass, and one thin one and one thick one had a particularly high dielectric constant. The thin plate with the lower constant was numbered 1 and the thin plate with the higher constant was No. 2.

The thick plates with low and high dielectric constants were numbered 3 and 4, respectively. According to the data desired, the plates were tested two at a time: - First 1 and 2, then 3 and 4, to show the effect of different dielectric constants in plates having the same thickness, for two different values of the thickness; and then 1 and 3, and 2 and 4, were used as pairs to study the dependence on the thickness.

All four arrangements were tested at four different surge voltages, - - 30 kv., 60 kv., 85 kv., and 112 kv., corresponding to spacings of 1, 2, 3, and 4 cm. of the firing gap (15 cm. spheres). At each of these voltages the "cut-off" needle gap was set successively at 0.5, 1.0, 1.5, 2.0 cm., etc., until either the needle gap failed to arc over, permitting the electric figures to grow to their maximum possible size for the voltage applied, or until (at higher voltages) one of the plates arced over. In recording the size of the electrical figures in the data, as soon as they had entered the second stage and become unsymmetrical the greatest radius of the figure was always measured.

Occasionally measurements could be repeated with consistent results, but more often the results were unsatisfactory. Obviously, to make possible consistent data it was necessary not only that the nature of development of the figures be such as to produce identical figures under identical conditions, but that the time lag of the "cut-off" needle gap remain always the same for the same voltage and spacing. In spite of the fact that the gap was pre-ionized by light from an open arc lamp brought quite near to it, a perfectly constant time lag could not be expected, and the variations in successive measurements may no doubt be traced in part to this fact.

Each measurement, for fixed conditions, was repeated three or four times, - until a good average was available, or, until the trend of the readings, made the most probable value evident. Evenso, the data and the curves which have been drawn from them can unfortunately be relied upon to give indications only, rather than exact information. The rule followed of always measuring the greatest radius of the figure did not always give trustworthy results in that successive figures, formed under the same voltage and needle gap settings, were often of markedly different shape. Sometimes one or two "principal" streamers would give rise to a very unsymmetrical figure, extended greatly in one or two directions. Again, the streamer formation would be more symmetrical, with more streamers, all developed to a lesser degree. The area of the figures would probably be a more reliable guide. The measurement of the area in every case would, however, have required too much time and was not attempted.

Since it is impossible to predict the curves mathematically, they were fitted as well as possible to the points found by test without regard to apparent inconsistencies. In some cases, such as the curves for the 60 kv. wave, curve Sheet 2, (see curves following page 21), negative figures, the points were so scattered as to render a justifiable choice of curve practically impossible. In other curves, such as the positive curves of the same sheet, and the negative curves on Sheet 3, the data seem reasonably consistent.

The data and curves obtained serve to throw some light on the effect of variations of the dielectric constant and thickness of the plate, for voltages of different magnitudes, and figures in different stages of development.

DATA

Properties of plates used.

	Plate 1	Plate 2	Plate 3	Plate 4
Surface	50 cm. squ.	- - - - - - - - - -	- - - - - - - - - -	- - - - - - - - - -
Thickness	0.42 cm.	0.54 cm.	0.83 cm.	0.88 cm.
Diel. constant	7.36	11.23	7.60	11.29

Surges on Plates 1 and 2 in Parallel

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure,(mm) on plate 1	on plate 2
30	0.5	21	19
"	"	19	17
"	"	20	17.5
"	"	20	17.5
"	1.0	21	17.5
"	"	22.5	19
"	"	22.5	19
"	"	22.5	18.5
"	1.5	23.5	35
"	"	36	35
- - - - New needles inserted in cut-off gap - - - - -			
"	"	47.5	72.5
"	"	40	40
"	"	45	92.5
"	2.0	32.5	100
"	"	50	100
"	"	45	105
60	0.5	25	22.5
"	"	21.5	22
"	"	25	22.5
"	"	25	25
"	1.0	33	30
"	"	31	32.5
"	"	33	30.5
"	"	32	32
"	1.5	50	55
"	"	52	61
"	"	50	52
"	"	43	46.5

Surges on Plates 1 and 2

Positive Figures

'eak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 2
60	2.0	160	220
"	"	130	175
"	"	150	198
"	"	115	170
85	0.5	25.5	27
"	"	27	25.5
"	"	28	28
"	"	25	27
"	1.0	38	37
"	"	36	36
"	"	38	38
"	1.5	49	58
"	"	52	56
"	"	47	51
"	2.0	153	192
"	"	110	133
"	"	90	115
"	"	115	138
- - - - - New needles inserted in cut-off gap - - -			
"	"	95	120
112	0.5	27.5	28
"	"	27	28
"	1.0	40	42.5
"	"	30	30
"	"	40.5	40
"	1.5	47.5	52
"	"	55	60
"	"	53	55
"	2.0	104	130
"	"	85	93
"	"	95	99

Negative Figures

30	0.5	9	7.5
"	"	7	7
"	"	7	7
"	1.0	10	10
"	"	9	10
"	"	10	10.5
"	1.5	12.5	31
"	"	12.5	12.5
"	"	17	32.5
"	"	12	13
"	2.0	13	27
"	"	25	30
"	"	15	15
"	"	20	15

Surges on Plates 1 and 2

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 2
30	2.5	15	25
"	"	14	25
"	"	15	25
60	0.5	12	14
"	"	13	14
"	1.0	15	15
"	"	18	20
"	"	15	20
"	1.5	23	36
"	"	31.5	26
"	"	30	36
"	"	28	35
"	2.0	50	99
"	"	68	100
"	"	70	90
"	2.5	98	120
"	"	89	120
"	"	59	95
"	"	120	166
85	0.5	15	16
"	"	15	17
"	1.0	22	25
"	"	22	23.5
"	1.5	27.5	50
"	"	35	47
"	"	35	50
"	2.0	71	87
"	"	83	120
"	"	100	150
"	"	89	120
"	2.5	172	232
"	"	112	179
"	"	113	157
112	0.5	- -	- -
"	1.0	27	32
"	"	27	27
"	"	25	32
"	1.5	44	47
"	"	42	49
"	"	45	55
"	2.0	86	120
"	"	73	115
"	"	73	115
"	2.5	140	185
"	"	133	175
"	"	130	208

Surges on Plates 3 and 4 in Parallel

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure(mm) on plate 3	on plate 4
30	0.5	20	16
"	"	19	16
"	"	20	17
"	1.0	24	25
"	"	22	20
"	"	22.5	20
"	"	23	20
"	1.5	43	50
"	"	41	50
"	"	40	50
"	"	40	45
"	2.0	40	34
"	"	37	70
"	"	40	40
"	"	40	70
"	2.5	40	50
"	"	40	40
60	0.5	27	23
"	"	26	23
"	"	26	23
"	1.0	35	30
"	"	34	30
"	"	35	31
"	1.5	45	45
"	"	50	53
"	"	47	48
"	2.0	68	96
"	"	71	76
"	"	68	85
"	2.5	90	132
"	"	85	145
"	"	90	135
85	0.5	30	28
"	"	30	27
"	1.0	40	36
"	"	40	36
"	1.5	50	50
"	"	50	50
"	2.0	73	90
"	"	65	80
"	"	70	80
"	2.5	90	146
"	"	100	135
"	"	105	130
"	3.0	125	192
"	"	110	160
"	"	125	195

Surges on Plates 3 and 4

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 3	on plate 4
112	0.5	27	22
"	"	31	31
"	"	31	30
"	"	29	30
"	1.0	42	42
"	"	43	40
"	"	44	41
"	1.5	56	55
"	"	57	55
"	"	60	57
"	2.0	80	89
"	"	79	84
"	"	70	88
"	2.5	95	125
"	"	104	118
"	"	92	120
"	3.0	112	158
"	"	143	196
"	"	125	207
"	3.5	180	245
"	"	170	260
"	"	155	245

- - - -New needles inserted in cut-off gap - - - - -

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 3	on plate 4
30	0.5	6	6
"	"	6.5	6
"	"	6.5	6.5
"	1.0	-	9
"	"	9	-
"	"	8.5	8.5
"	1.5	11.5	12
"	"	10	10
"	"	10.5	10.5
"	2.0	14	14
"	"	13	13
"	"	13	13
"	2.5	12	12.5
"	"	13	12.5
60	0.5	10	10.5
"	"	10	10
"	1.0	18	17
"	"	15	15
"	"	15	18.5
"	1.5	18	18
"	"	18	18
"	"	18	18

Surges on Plates 3 and 4

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 3	on plate 4
60	2.0	25	52
"	"	31.5	51
"	"	23	70
"	2.5	40	78
"	"	46	90
"	"	56	90
"	"	50	60
"	3.0	56	94
"	"	75	110
"	"	56	98
"	"	74	102
"	3.5	50	100
"	"	72	97
"	"	78	102
"	4.0	55	108
"	"	90	190
"	"	90	214
"	"	77	128
"	4.5	156	223
"	"	90	220
"	"	76	203
"	5.0	90	201
"	"	85	210
"	"	100	225
85	0.5	15	15
"	"	15	13
"	1.0	21	23
"	"	22.5	22.5
"	"	21.5	19
"	1.5	23	41
"	"	39	40
"	"	28	32.5
"	"	31	36
"	2.0	43	76
"	"	43	64
"	"	43	67
"	2.5	51	117
"	"	75	137
"	"	73	120
"	"	70	109
"	3.0	84	155
"	"	83	155
"	"	89	168
"	3.5	125	171
"	"	89	166
"	"	92	167
112	0.5	16	20
"	"	15	15
"	"	16	16

Surges on Plates 3 and 4

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 3	on plate 4
112	1.0	25	26
"	"	24	24
"	1.5	33	41
"	"	35	38
"	"	30	40
"	2.0	58	85
"	"	53	78
"	"	50	68
"	"	64	78
"	2.5	75	140
"	"	90	150
"	"	89	150
"	3.0	101	185
"	"	104	183
"	"	93	180
"	3.5	120	227
"	"	89	199
"	"	123	220

Surges on Plates 1 and 3 in Parallel

Positive Figures

		plate 1	plate 3
30	1.0	21.5	21
"	"	20	23
"	"	17	22
"	"	19	23
"	1.5	45	26
"	"	40	40
"	"	42	41
"	2.0	30	41
"	"	60	41
"	"	75	36
"	"	61	40
"	"	37	40
"	2.5	60	46
"	"	41	47
"	"	36	42
"	"	39	41
60	0.5	23	21
"	"	21	22
"	"	24	23
"	"	25	23
"	1.0	32	34
"	"	31	35
"	"	31	31
"	"	31	31

Surges on plates 1 and 3

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 3
60	1.5	45	49
"	"	53	40
"	"	50	50
"	"	50	45
"	2.0	104	60
"	"	95	62
"	"	155	66
"	"	107	65
"	2.5	130	65
"	"	146	77
"	"	270	120
"	"	253	99
- - - - - New needles inserted in cut-off gap - - - -			
"	"	148	70
"	"	155	77
85	0.5	26	24
"	"	25	24
"	"	23	21
"	"	25	23
"	1.0	35	37
"	"	34	36
"	"	36	36
"	1.5	51	50
"	"	50	50
"	"	51	51
"	2.0	84	62
"	"	80	60
"	"	105	66
"	"	94	65
"	2.5	166	82
"	"	125	75
"	"	185	78
"	"	187	95
"	3.0	202	85
"	"	222	84
112	0.5	24	30
"	"	21	22
"	"	25	23
"	1.0	36	36
"	"	39	38
"	"	35	36
"	1.5	50	50
"	"	52	50
"	2.0	80	65
"	"	77	67
"	"	79	63

Surges on Plates 1 and 3

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 3
112	2.5	159	92
"	"	200	81
"	"	175	80

Negative Figures

30	0.5	6	5.5
"	"	6	6
"	"	6.5	6
"	1.0	9.5	8.5
"	"	10	8
"	"	9.5	8
"	1.5	12.5	10
"	"	12	10.5
"	"	12	11
"	2.0	16	13
"	"	16	15
"	"	26	14
"	"	25	14
"	2.5	17.5	13
"	"	25	14
"	"	25	14
"	"	25	14
60	0.5	10.5	10
"	"	10	9.5
"	"	10	9
"	1.0	16	14
"	"	21	13.5
"	"	16	14
"	"	16	14
"	1.5	32	21
"	"	27	16.5
"	"	30	17
"	"	21	17.5
"	2.0	56	18.5
"	"	71	19
"	"	58	19
"	"	89	25
"	"	90	49
"	"	65	18.5
"	2.5	83	18.5
"	"	75	30
"	"	119	20
"	"	127	40
"	"	129	41
- - - - - New needles inserted in cut-off gap - - - -			
"	"	117	40

Surges on Plates 1 and 3

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 3
60	3.0	120	46.5
"	"	125	20
"	"	116	20
"	"	107	40
"	"	223	41
"	3.5	160	20
"	"	162	55
"	"	171	41
85	0.5	13	11
"	"	13	9
"	"	12	11
"	"	15	9
"	1.0	24	18
"	"	20	21
"	"	25	20
"	"	25	20
"	1.5	41	28
"	"	35	27
"	"	40	28.5
"	2.0	86	37
"	"	75	36
"	"	89	36
"	2.5	138	48
"	"	144	48
"	"	156	56
"	"	111	36
"	3.0	199	51
"	"	187	65
"	"	172	62
"	"	207	47
"	"	201	65
"	3.5	219	57
"	"	231	62
"	"	225	68
112	0.5	13	11
"	"	12	11
"	"	12	10
"	"	15	13
"	1.0	24	22
"	"	25	21
"	"	26	20
"	"	26	24
"	1.5	41.5	32
"	"	41	31
"	"	40	35
"	"	38	30

Surges on Plates 1 and 3

Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 1	on plate 3
112	2.0	87	45
"	"	89	54
"	"	99	57
"	"	92	54
"	2.5	170	60
"	"	225	66
"	"	202	84
"	"	170	70
"	3.0	240	77
"	"	215	75

Surges on Plates 2 and 4 in Parallel

Positive Figures

		plate 2	plate 4
30	0.5	17	15
"	"	17.5	18
"	"	17	20
"	"	17	19
"	1.0	32	20
"	"	20	20
"	"	35	20
"	"	20	20
"	1.5	83	40
"	"	52	34
"	"	75	50
"	"	62	38
"	2.0	105	51
"	"	101	46
"	"	96	68
"	2.5	115	45
60	0.5	24	24
"	"	24	24
"	1.0	30	31
"	"	31	30
"	1.5	55	50
"	"	62	50
"	"	55	45
"	2.0	114	70
"	"	115	78
"	"	160	85
"	"	100	63
"	2.5	180	85
85	0.5	27	26
"	"	27	27
"	1.0	36	36
"	"	36	36

Surges on Plates 2 and 4

Positive Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 2	on plate 4
85	1.5	51	45
"	"	58	49
"	"	56	50
"	"	52	45
"	2.0	125	85
"	"	110	77
"	"	115	75
"	"	115	79
112	0.5	29	28
"	"	28	27
"	1.0	41	41
"	"	40	40
"	"	40	39
"	1.5	52	49
"	"	57	50
"	"	55	50
"	2.0	95	77
"	"	100	70
"	"	109	75
"	2.5	270	165
"	"	295	165

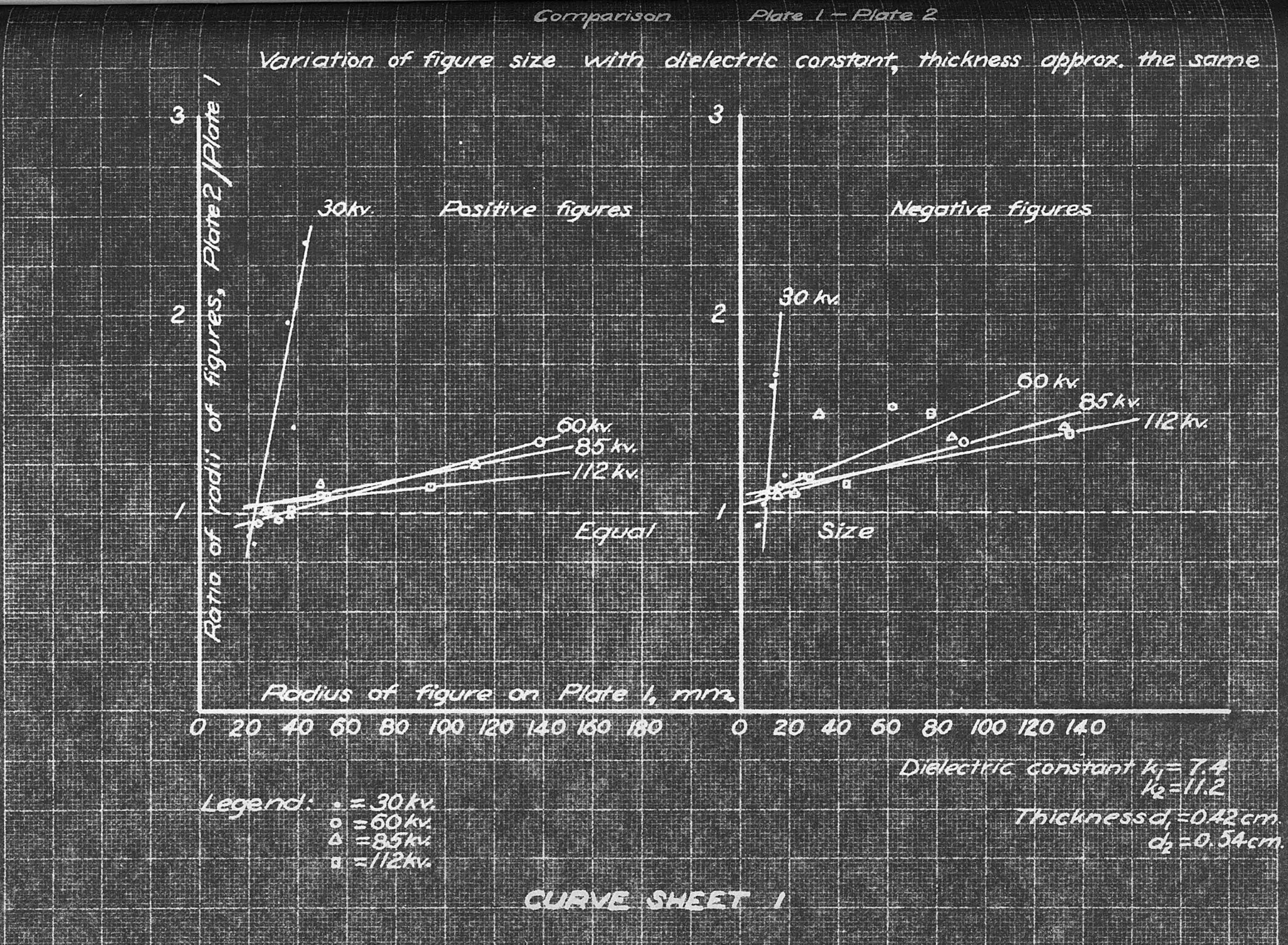
Negative Figures

30	0.5	7	6.5
"	"	7	6.5
"	1.0	10	8.5
"	"	9.5	8.5
"	1.5	25	15
"	"	12.5	10.5
"	"	26	11.5
"	"	17	10.5
"	2.0	27	11
"	"	37	14
"	"	36	14
"	"	30	24
"	"	38	26.5
60	0.5	10.5	10
"	"	10.5	9
"	"	10.5	10
"	1.0	16	13.5
"	"	18.5	13
"	"	15	16
"	"	20.5	12
"	1.5	26	21
"	"	40	22
"	"	30	15
"	"	25.5	15.5

Surges on Plates 2 and 4

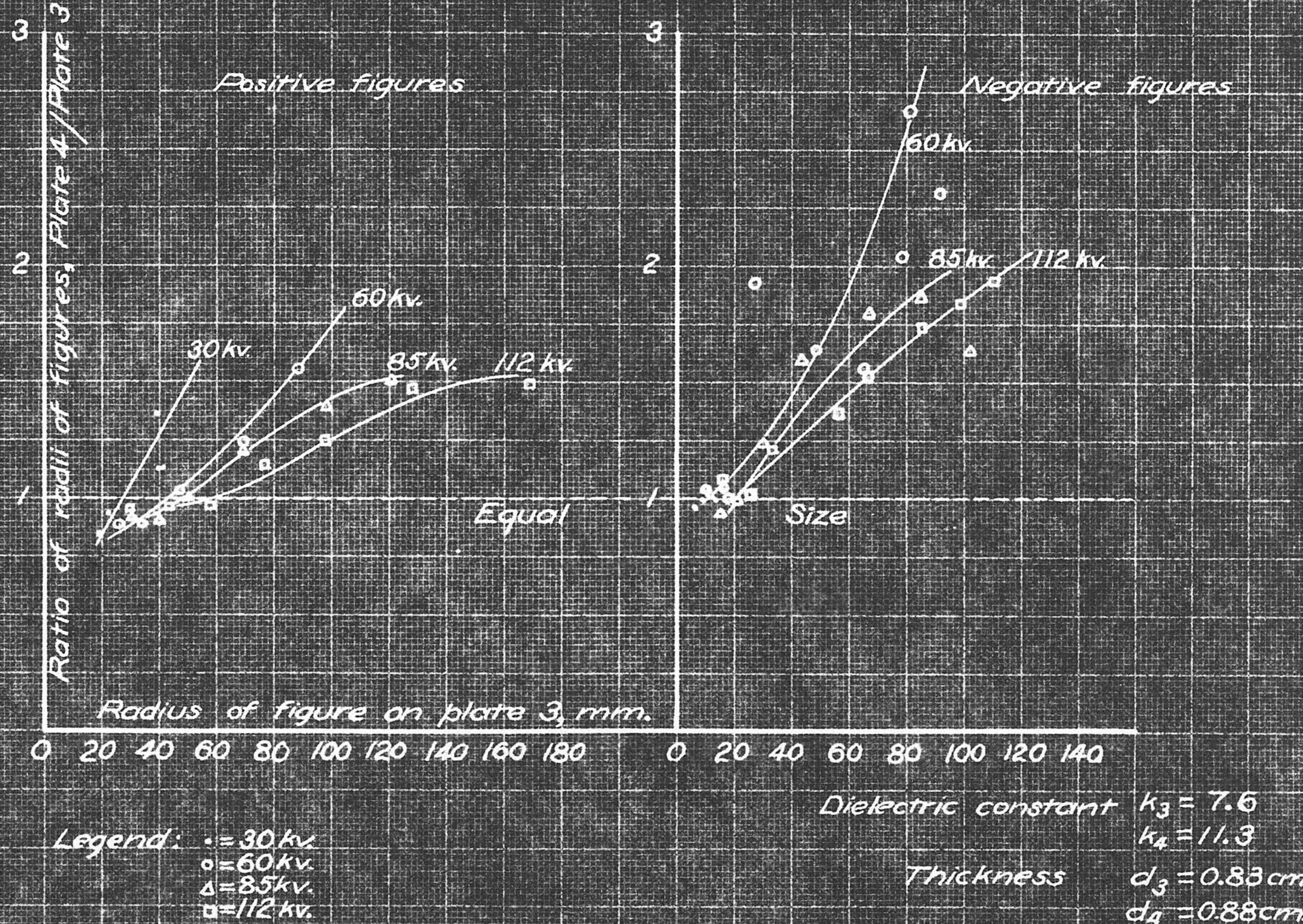
Negative Figures

Peak value of surge (kv)	Spacing of cut-off needle-gap (cm)	Greatest radius of figure (mm) on plate 2	on plate 4
60	2.0	69	31
"	"	78	32
"	"	69	42.5
"	"	86	42
85	0.5	17	15
"	"	16	15
"	"	15.5	14.5
"	1.0	26	21
"	"	27	21
"	"	25	19
"	"	26.5	21
"	1.5	49	30
"	"	49	35
"	"	56	34
"	"	49	33
"	2.0	105	46.5
"	"	125	59
"	"	114	47
"	"	111	39.5
"	2.5	170	61
"	"	191	67
"	"	201	65
"	"	180	55
112	0.5	16	16
"	"	16	16
"	1.0	30	26
"	"	30	24.5
"	"	30	26.5
"	1.5	55	35
"	"	55	35
"	"	48	35
"	"	51	34
"	2.0	105	60
"	"	146	64
"	"	100	53
"	"	103	52
"	2.5	230	71
"	"	196	75
"	"	149	63
"	"	192	75

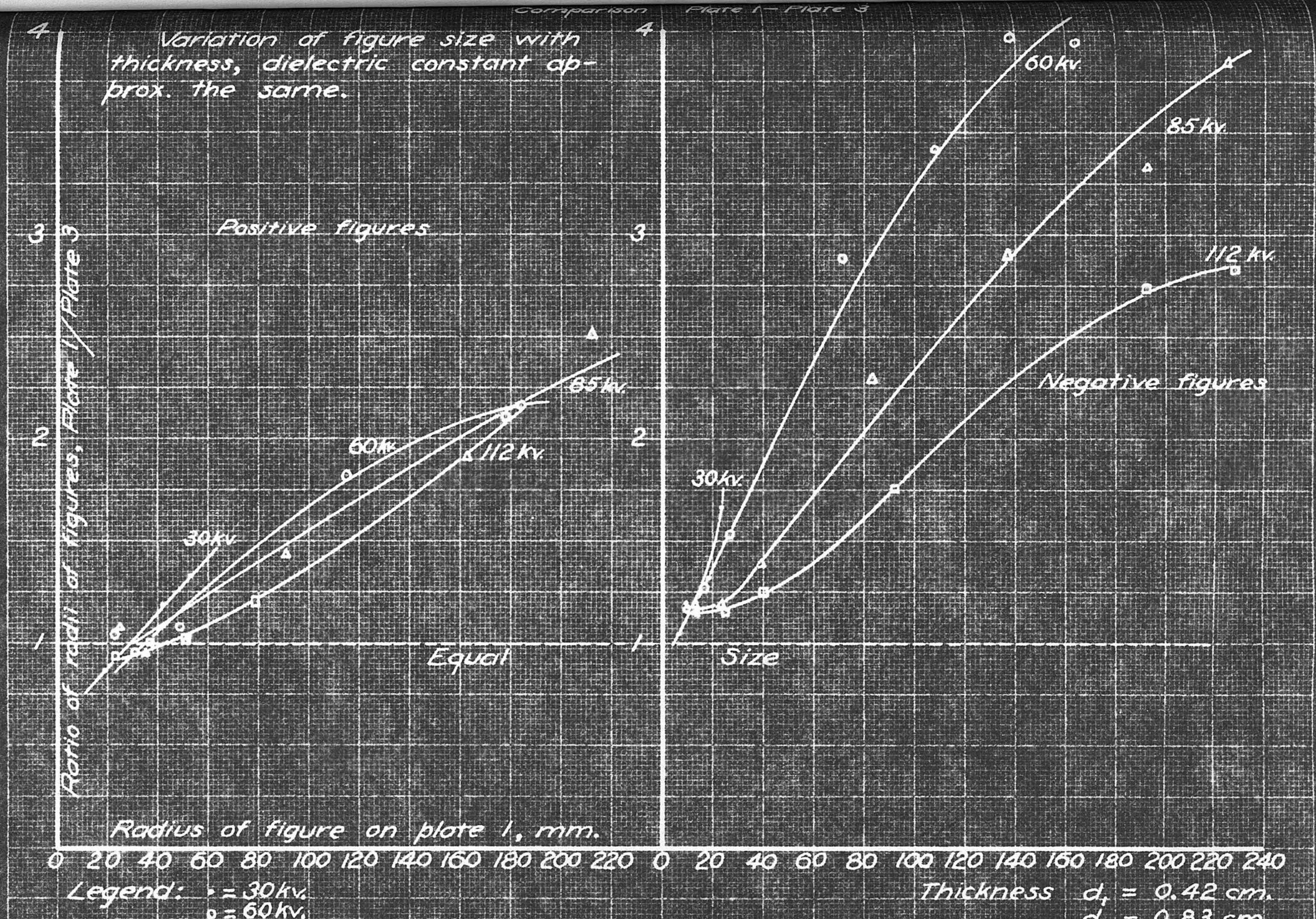

Comparison Plate 1 – Plate 2
Variation of figure size with dielectric constant, thickness approx. the same
Ratio of radii of figures, Plate 2 / Plate 1
3
2
1
30kv.
Positive figures
60 kv.
85 kv.
112 kv.
Equal
Negative figures
30 kv.
60 kv.
85 kv.
112 kv.
Size
Radius of figure on Plate 1, mm.
0 20 40 60 80 100 120 140 160 180
0 20 40 60 80 100 120 140
Dielectric constant k1 = 7.4
k2 = 11.2
Thickness d1 = 0.42 cm.
d2 = 0.54 cm.
Legend: • = 30 kv.
o = 60 kv.
Δ = 85 kv.
□ = 112 kv.
CURVE SHEET 1

Comparison Plate 3 - Plate 4

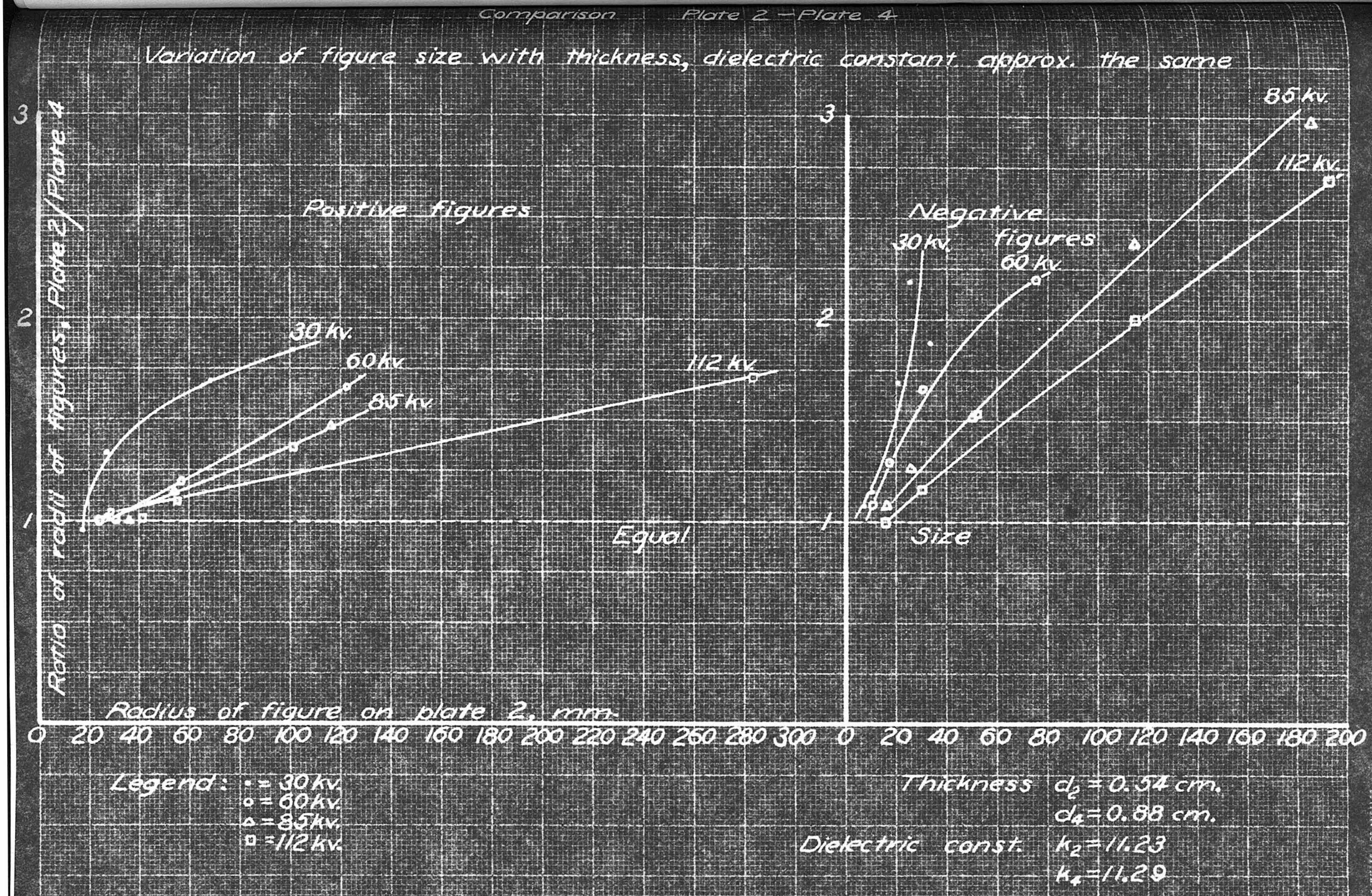
Variation of figure size with dielectric constant, thickness approx. equal



CURVE SHEET 2



CURVE SHEET 3



CURVE SHEET 4

DATA AND CURVES

Remembering that each point on the curve sheets represents the ratio of the radii of two figures caused by the identical voltage wave, and formed in the same length of time, it is immediately clear that the curves represent the ratio of the average velocities of growth of the figures on the two plates. Curve sheets 1 and 2 indicate differences in the velocity of the formation due to a difference in dielectric constant, sheets 3 and 4 show the effect of varying the thickness of the plate.

It is immediately apparent that the effect, in every case, increases with the size of the figures. An inspection of the curves, reveals that the velocity of formation on plates of a given thickness is actually greater for the plate which has the greater dielectric constant; or, if the dielectric constant is the same for both plates of a pair, for the plate with the smaller thickness, i.e., the plate with a greater unit capacity between surfaces will show the greater velocity of growth. This result was already known from the work of Toepler and others. By a more careful study of details, however, several other features of interest become apparent.

First, as the maximum voltage of the surge is increased, the ratio of greater average velocity to smaller average velocity, for a given size of figure, decreases. This is apparent from the decreasing steepness (with increasing voltage) of the four curves in each graph, and is to be noted on all of the curve sheets. The effect of varying the dielectric constant or thickness may, therefore, be said to decrease in importance, for any given stage of development of the figure, with increasing surge voltage.

The second point of interest is the polarity difference apparent in the curves. Each negative curve is steeper than the corresponding positive one, and higher values of the velocity ratio are observed for negative than for positive figures. The negative figures, therefore, are more sensitive to variations in the plate constants. This fact, already mentioned on page 12 , is of considerable significance for the theory of formation of electrical figures, and will be discussed at the end of the paper.

Lastly, a seeming inconsistency in the behavior of the figures, for very small radii, must be noted. In plotting the curves the ratio of the average velocities was always taken in such a way that it was greater than 1, and was increased with the figure size as already observed. Inspection of the curves reveals, however, that some points were obtained for which this ratio was less than 1, these points occurring always for the shortest radii observed. This means that - - as the observations were made, with the two plates on the testing stand - - for small settings of the "cut-off" needle gap, small "times of growth", and the resulting small figures, the larger figures occurred on the <u>other</u> plate from the one on which they appeared for larger spacings of the needle gap and the correspondingly larger figures. The effect is most marked on curve sheet 2, which gives in graphical form the data taken in observing figures on Plates 3 and 4, i.e., in studying the effect of an increase in the dielectric constant from 7.6 to 11.3, the thickness remaining fixed at about 0.85 cm. On curve sheet 1, representing about the same variation in dielectric constant for a thickness of approximately 0.5 cm., the effect is not so noticeable. On curve sheet 3 it is present to a very slight degree for the positive figures, and on curve sheet 4 it is entirely absent. The effect is stronger throughout for positive figures. This seeming inconsistency can be theoretically justified, as will be seen in the closing discussion.

EXPERIMENTAL PROCEDURE AND RESULTS - SECOND PART

We come now to a consideration of that part of the work for which a cathode-ray oscillograph was utilized in order to obtain data relative to the actual rate of growth of the figures. It was necessary, in short, to determine the lapse of time between the rise of voltage on the plate, and its removal caused by the arcing over of the needle gap. To serve this purpose, oscillograms showing the voltage on the plates were necessary.

An oscillograph, designed by Dipl.-Ing. Schaudinn and built under his supervision in the shop of the department of high voltage engineering at the Berlin Institute of Technology, and of the type developed there, was placed at my disposal. The instrument, which was several years old, did not possess all of the refinements which have been introduced in the later types since designed by that Institute, but it nevertheless proved equal to the task. It was a vertical type, distinguished from others by a sliding plate at the bottom, fitted to the main body by a ground joint rendered vacuum tight by the application of stop-cock grease. This plate used was movable, through a rack and pinion, by means of a hand wheel, and contained a recess near either end to receive photographic plates. By means of this arrangement it was possible to remove each plate for development directly after exposure, introducing a new plate at the same time, without losing the vacuum. Under ordinary conditions, exposures could be made at ten minute intervals. This much time was necessary to pump the adsorbed air and moisture out of the emulsion side of the plate. The other features of the instrument - - blocking plates, deflecting plates, focusing coil, etc. - - were of standard type and need no special mention in this paper. A schematic section of the instrument is shown in Figure 5.

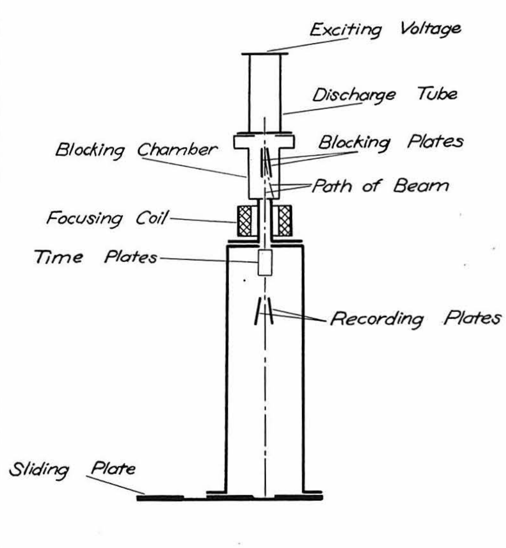


FIG. 5

Figure 5. Cathode-Ray Oscillograph Designed by Dipl.-Ing. Schaudinn, Berlin Institute of Technology.

A description of the oscillographic work will not be given here, as it is not considered an essential of this paper. Certain features of the work, however, particularly the relay circuit used, are of sufficient interest to warrant their inclusion in an appendix.

DATA OBTAINED

For the oscillograph tests two plates were again tested simultaneously. The needle-gap settings were not recorded, for any attempt at comparison with the results of the first part was of no particular value. Most of the tests were made with the firing gap of the surge generator set for 60 kv. A few were made at 30 kv., and a few at 85 kv., at which voltage the induced disturbances were so severe as to preclude any attempt to work at 112 kv..

For any voltage, the "cut-off" needle gap was adjusted so that both the rise and collapse of the voltage could be recorded on the oscillogram. Observing the wave on the viewing screen, the x-axis velocity of the beam was adjusted to the most desirable value. The test plates were carefully freed of any residual charges, after which they were sprinkled with powder, and on the next surge the picture was made. This was done because the necessity for immediately recording calibrating oscillations prevented giving any attention to the test plates for some seconds after the formation of the figures.

The maximum radial growth of the Lichtenberg figures was then recorded, and the exposed plate showing the form of the surge was removed from the oscillograph and developed. In the event of a successful exposure, the time of duration of the voltage was measured, and the average velocities of growth of the two figures calculated. The same procedure was then attempted for a different setting of the needle gap, in order that a value of the average velocity of growth of the figure for a different stage of development might be obtained.

Figure 6 shows a record of the surge voltage obtained in this way. Although not entirely free from disturbances, the length of time from voltage rise to voltage cut-off can be accurately determined. The wave length of the calibrating oscillation is 111 meters, the period 3.7×10^{-7} sec..

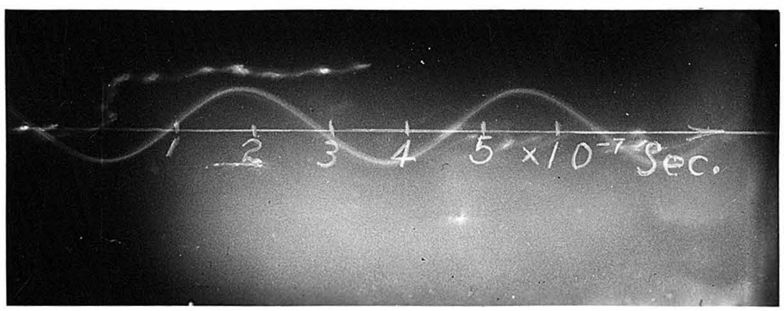


Figure 6. Oscillogram of Voltage on Test Plates

Most of the tests were made on Plates 1 and 2 in parallel, because this work was completed near the end of my stay in Berlin and only a little time remained for a few tests on Plates 3 and 4 in parallel. Needless to say the data obtained in this set of tests were much less numerous than those of the first part, - due to experimental difficulties and increased complication of procedure. The data obtained for Plates 1 and 2 are given in plotted form (average velocity of growth against maximum radius attained) in the accompanying diagrams. The entire data follow in tabular form.

DATA

Plate 1

Positive Figures

Peak value of surge (kv)	Radius attained (mm)	Average growth velocity (km/sec)
30	18	38.4
30	40	25.8
60	100	95.2
60	45	193.5
60	46	188
60	60	183
85	100	203
85	77	243

Negative Figures

30	35	20.3
60	35	217
60	50	167
85	69	168
85	160	152

Plate 2

Positive Figures

30	25	53.2
30	65	41.8
60	63	265
60	60	258
60	90	275
60	215	203
85	100	315
85	130	263

Negative Figures

30	60	34.8
60	25	311
60	85	283
60	165	200
85	122	297
85	230	219

Plate 3

Positive Figures

Peak value of surge (kv)	Radius attained (mm)	Average growth velocity (km/sec)
60	60	108
85	55	208

Negative Figures

60	21	45.5
85	60	89.5

Plate 4

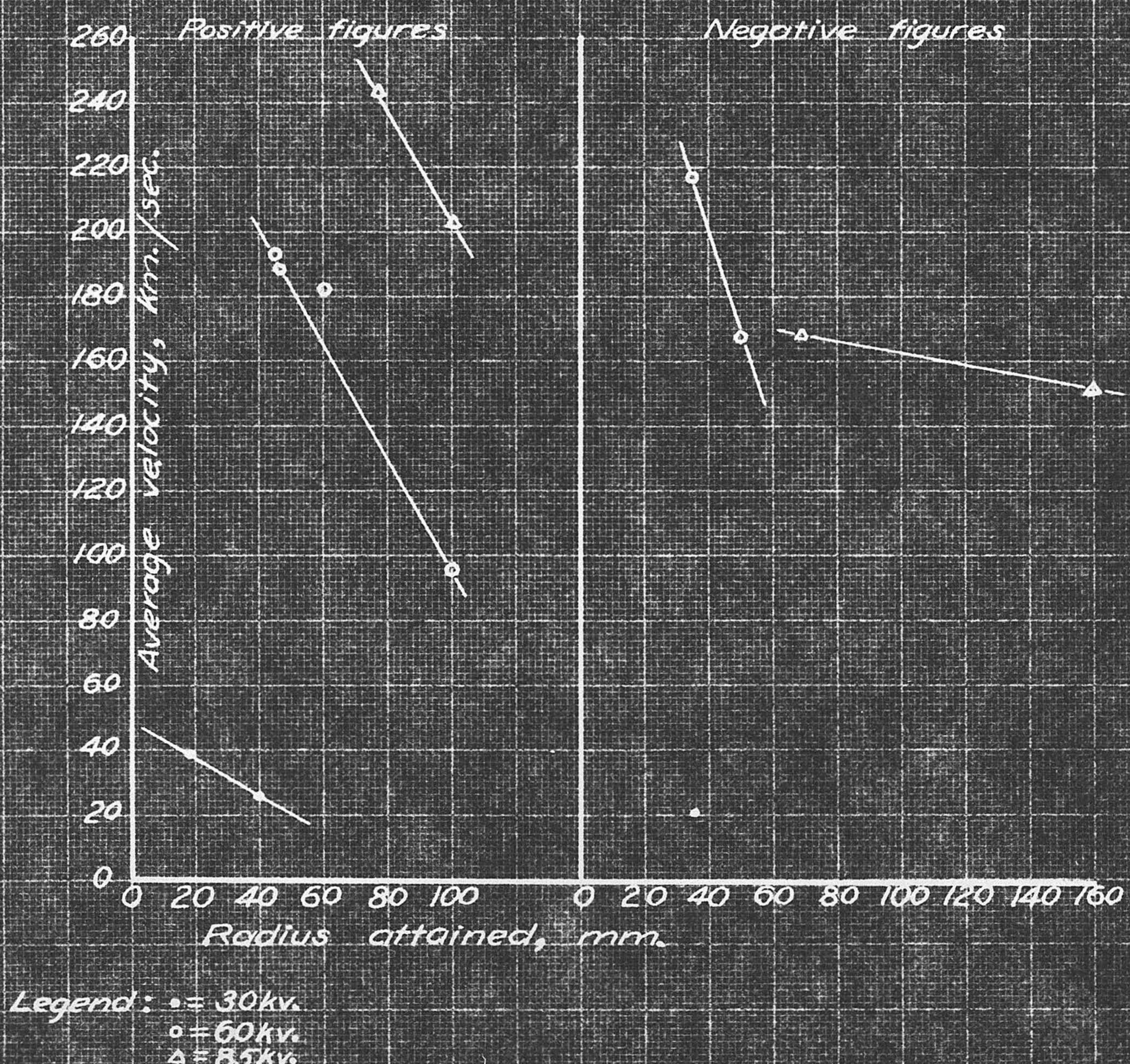
Positive Figures

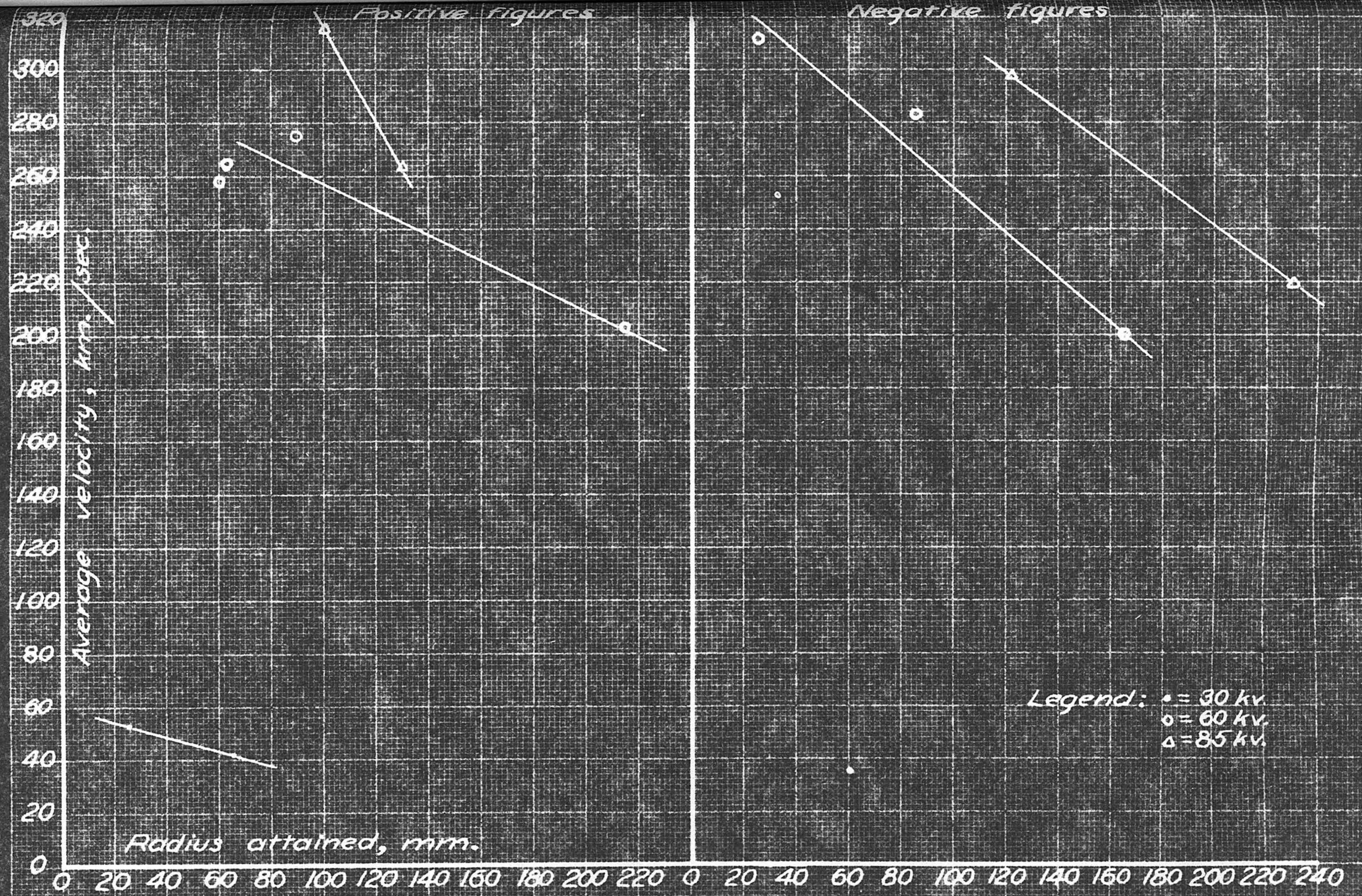
60	80	144
85	70	266

Negative Figures

60	80	173
85	160	239

Growth velocities of electrical figures on plate I dependent on radius attained ~ for different voltages





Growth velocities of electrical figures on plate 2 dependent on radius attained — for different voltages

The plots show that the average velocity of growth of the figures decreases for increasing radius, and is greater for the larger voltages. Further, the velocities found lie in the range from 20 to 350 km./sec., in agreement with the results of other experimenters.

Unfortunately, the data here given cannot be used for a strict comparison of the relative velocities of positive and negative figures, for the following reason: The voltages hare given are the voltages for which the firing gap of the surge generator was set, - i.e., the maximum voltages which would have occurred on the <u>open</u> line. The presence of the test plates on the line, however, and of the rapidly increasing capacitance of the figures growing on them, served not only to decrease the steepness of the front of the travelling wave, but even to reduce its maximum value. This reducing effect was greater in the case of the positive figures, due to their (already known) greater initial speed of growth.

To verify this, an oscillogram was made on which three curves were written. The first, the 60 kv. wave on the open line; the second, the same wave with the test plates so attached as to develop negative figures; and the third, the same wave with positive figures formed on the test plates. Measurement of the curves thus obtained revealed that whereas with open line the 60 kv. maximum wave had a maximum rate of rise of approx. 850 x 10^6 kv./sec., with the negative figures the steepness was reduced to 520 x 10^6 kv./sec. and the maximum value attained to

44.2 kv., while with positive figures the two values were 236 x 10^6 kv./sec. and 31.6 kv. A similar oscillogram for 85 kv. showed a maximum value for positive figures of only 47.6 kv..

For a given setting of the firing gap the negative figures were formed under a certain advantage, - i.e., the actual voltage on the electrode remained higher throughout their formation than in the case of the positive figures. In spite of this predisposition to greater negative velocities, it will be observed from the plots that for the same firing gap voltages, and for the same radius attained, the positive velocities are larger. This is particularly noticeable in the case of the listed pair of observations on Plate 3.

CURRENT INTAKE AND EFFECTIVE CAPACITY OF THE FIGURES

The above-mentioned reduction of the travelling wave of voltage from its "open-line" value, caused by the current intake of the electrical figures, may be used to calculate this current intake. This calculation was made for the two voltage curves shown on the oscillogram in Figure 7. The upper trace gives the voltage recorded on open line, the lower the voltage appearing on Plates 1 and 2 in forming positive figures for the same setting of the surge generator firing gap.

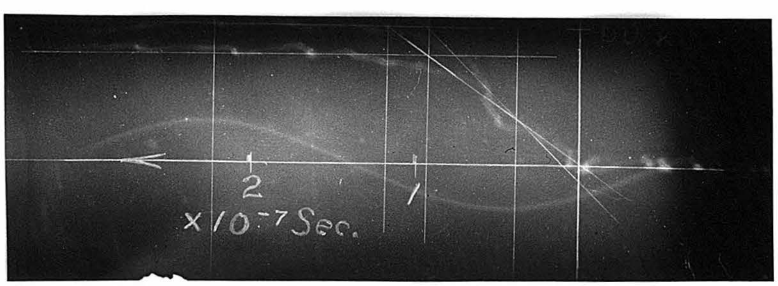


Figure 7. Oscillogram Showing Reduction of Surge by Formation of Electrical Figure.

The conditions in the two cases are made schematically clear in Figure 8. For open line, a voltage wave of instantaneous value e races along the line from the surge generator to the oscillograph, where it is registered. This is indicated in Figure 8, a. Upon the addition of the test plates to the circuit, the voltage wave enters their point of connection with the same instantaneous value, e, emerging, however, from this point with the reduced value e', due to the current, i, entering the plates. This reduced wave is registered by the oscillo-

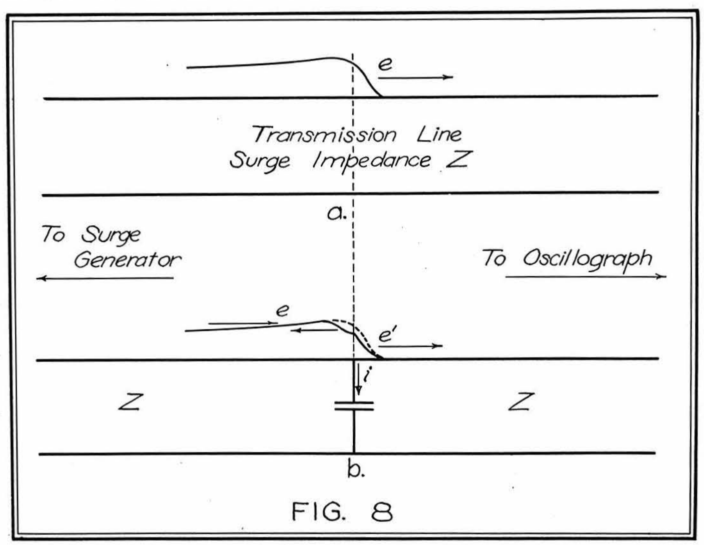


Figure 8. Effect of Capacitance on Travelling Wave

graph, as shown in Figure 8, b. For such a connection, where the voltage waves e and e' may be directly compared with each other as in the oscillogram of Figure 7, the instantaneous value of the current, i, taken by the plates can be calculated by the simple formula

$$i = \frac{2(e - e')}{Z} .$$

The voltage waves from Figure 7 are reproduced in the same relative scale in the drawing of Figure 9, with voltage and time calibration, together with the current curve calculated by the above formula. The curve in Figure 10 shows the instantaneous value of the effective capacitance of the figures on the two plates, as calculated from the current and e' curves of Figure 9.

The maximum current intake of Plates 1 and 2 in parallel, at this voltage, is seen to be 45 amps. Evaluation of a similar oscillogram for an open line maximum voltage of 85 kv. gave a maximum current intake of the same two plates of 120 amperes.

This arrangement of point-plate electrodes separated by an insulating plate of high breakdown strength and high dielectric constant is seen to possess, to a degree at least, characteristics rendering it suitable for the protection of other equipment against voltage surges, - i.e., very low capacitance under normal voltage, and rapidly rising capacitance, due to the growth of the streamers, under over-voltage surges. Such an apparatus, to be effective for the normal and surge

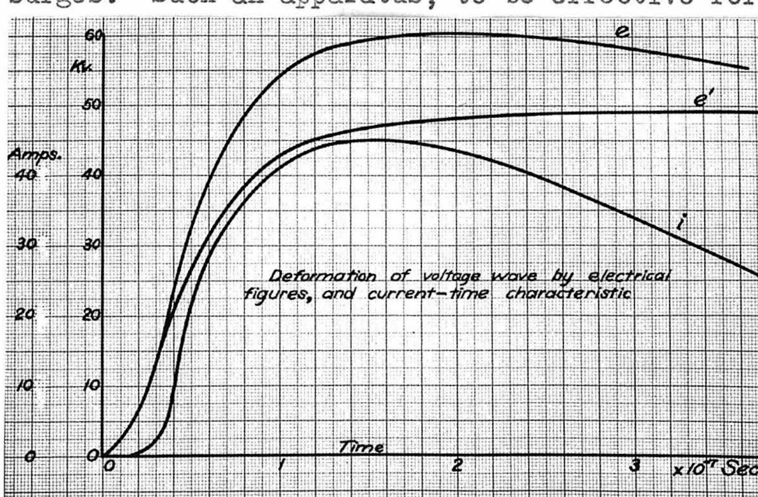


FIG. 9

Figure 9. Voltage-Time and Current-Time Characteristics of Electrical Figures.

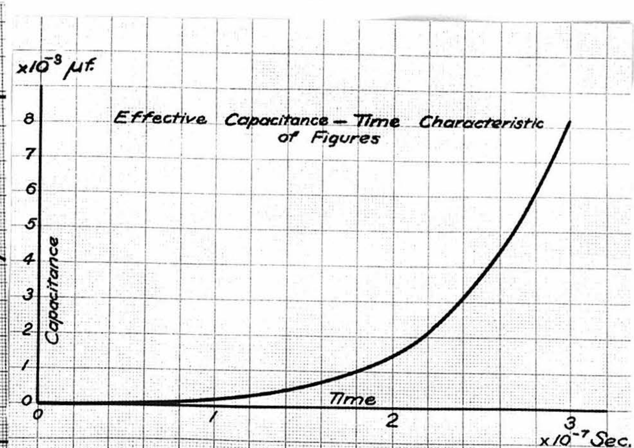


FIG. 10

Figure 10. Capacitance-Time Characteristics of Electrical Figures.

voltages encountered in practice, might prove too bulky, or it may be that a series arrangement of such units would prove feasible. There can be no doubt, in the light of the above results, that the formation of streamers on the insulators, accompanying the travel of a disturbance along a transmission line, contributes markedly to the attenuation of the travelling wave.

DISCUSSION OF THE THEORY OF FORMATION OF STREAMER DISCHARGES

The particular significance in connection with breakdown of the exceedingly great velocity of the electron in strong electric fields, as compared with that of the positive ion, has been quite generally recognized. In explaining the polarity difference in the arc-over voltage of the point-plate combination, Marx[1] based his space charge and field distortion analyses on the assumption that the positive charges remained fixed in space. By means of ingenious experiments, in which Lichtenberg figures were formed and recorded under the influence of a perpendicular magnetic field, Magnusson[2] has proven that these figures, both positive and negative, are formed by the movement of <u>electrons</u>. The velocity of growth observed for the electrical figures, of from 2×10^6 to 3×10^7 cm./sec. could hardly be approached by positive ions for the field strengths involved, at atmospheric

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1. Erwin Marx, Archiv für Elektrotechnik- 1930, Vol. 24, p. 61.
2. See reference to Magnusson, p. 1.

pressure.[1]

An interesting verification of this theory was afforded in the second part of the present work, in which the plates were powdered before the occurrence of the surge. Occasionally a figure would extend beyond the powdered area. If negative, the figure was still well defined, showing that the dust had been swept along ahead of the expanding ring of electrons. If positive, however, such was not the case, and the trace of the extended branch of the figure was lost.

The explanation of the mechanism of the growth of these figures, as given on page 6 , must thus be revised to the extent of being consistent with the theory that the movement of charges for figures of both polarities be limited to the negative charges only, the positive ions being regarded as fixed.

Under this assumption, the processes of formation for the two kinds of figures must be conceived of as radically different. At the risk of repeating much already well known about Lichtenberg figures, I shall review briefly the sequences of formation of a positive and a negative figure.

Immediately upon the ionization of the air directly around the positive electrode, the negative electrons are drawn rapidly into the

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1. Rogowski gives the velocity of charges moving in a field of 30 kv./cm. at room temperature and atmospheric pressure to be, for electrons 10^7 -10^8 cm./sec., for positive ions 10^5 cm./sec. (Arch. f. Elek., 1926, Vol. 16, p. 502.)

electrode, leaving the positive charges pressed against the plate, practically in their positions of origin, where we may consider them as remaining for the length of time involved in the entire process. The space charge produced by the isolation of the positive ions on the plate steepens the gradient at the outer rim of this charge, in the manner already discussed on page 6 , causing ionization at that point. The freshly produced electrons are attracted inward and slightly upward (the gradient at the surface of the plate has a vertical component) back over the positive charge on the plate. With the development of this process, the electrons flowing to the electrode will begin to ionize by collision there, due to the voltage drop existing around the electrode (see Figure 1, page 5), and at the points at which this ionization begins, channels especially rich in positive charge will develop. The presence of this positive charge assures the continued intensive supply of electrons to these particular channels, which continue to grow, expediting the ionization at their tips, on the very rim of the figure, and forming the streamers observed. Intensive recombination of

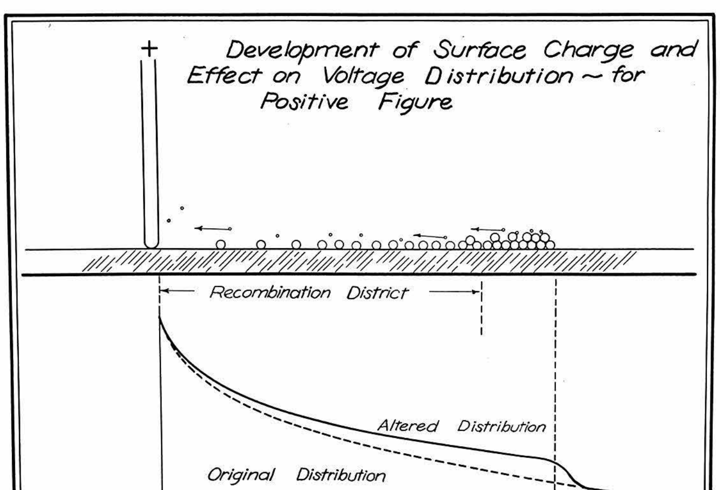

FIG. 11

the supplied electrons with the positive charge in these channels is evidenced, of course, by the light energy given off.

For the positive figure, at a stage of considerable development, the probable conditions are those shown in Figure 11.

The significant feature in the formation of the positive figure is the fact that the space charge is formed by the speedy withdrawal of the active particles, the electrons, from the freshly innized district. The electrons move freely, back and up from the surface, under the influence of the high potential drop at the edge of the figure.

Upon the ionization of the air about the electrode by the application of a high <u>negative</u> voltage, the positive ions experience a force toward the electrode, and the electrons are subject to a repelling force, directed somewhat against the plate. Due to their much greater movability, the electrons separate themselves rapidly from the positive ions and move out along the plate to form a negative, ring surface-charge. This ring impelled by the repulsion forces of the electrode and aided by ionization at the outside rim due to the high voltage drop there steadily expands. The electrons thus sweep outward, their movement impeded by friction against the plate, passing over, or through, the positive ions produced by ionization.

In this case, as indicated in Figure 12, there are two surface charges to be considered, - the negative charge on the outer rim of the figure, and the positive charge within, insofar as the new ions are not neutralized by the passing negative charges.

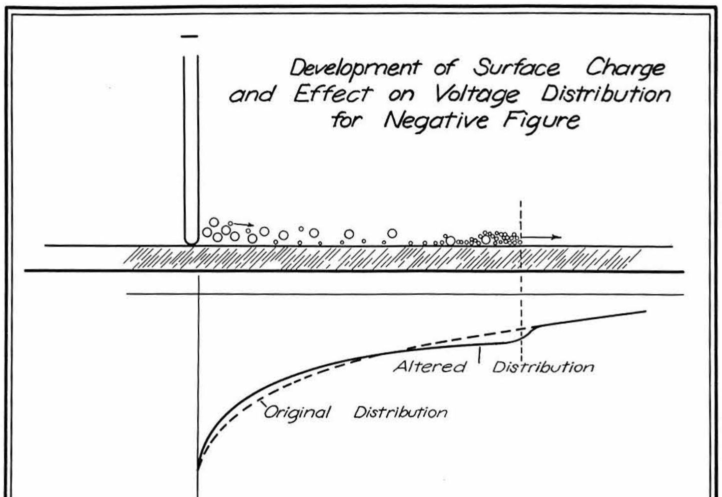

FIG. 12

This positive surface charge serves to increase the gradient at the electrode, and to support ionization there. As soon as ionization has started at any point, it is intensified by the positive space charge collecting there, - the new electrons being repelled along the surface and recombining with the ions generated at the rim of the figure.

The radical difference in the formation of the positive and negative figures is accordingly to be seen in the fact that in the former case the growth of the important, ring, surface-charge comes about through the free withdrawal of the electrons, inward and upward, with a high velocity corresponding to the high gradient at the rim of the figure; whereas in the latter case, the surface charge is formed by the outward movement of the electrons, in a field of approximately the same value, - a movement, however, which is impeded by friction on the plate.

Any exact analysis of the relation between the electron velocity and the speed of growth of the positive figure is difficult to make. It is my hypothesis, however, that the positive figure grows with a speed approximately equal to that of the electrons in the high gradient at the rim of the figure. Regarding the process as a differential one, let us suppose that ionization has just taken place in the elementary area of radial length δr. The theory that the prerequisite for ionization in the next greater elementary ring to take place be the removal of the newly formed electrons over the distance δr is a tenable one. Under this hypothesis, the difference in speed of growth of the positive and negative figures becomes the difference in speed of electrons moving freely in a high field, and electrons moving against a retarding, frictional force in the same field. The fact that this retarding force is of such a value as to have caused the ratio of size and velocity of the two types of figures to be 2 to 1, as found in the experiments of some authors, must be regarded as a coincidence. As to the frictional force itself, it can only be said that it varies with the magnitude of the vertical component of the field. The fact that for large figures the velocity of growth and size of the negative figure approach those of the positive figure may be ascribed to the decrease of the vertical component of the field with increase of distance from the electrode.

The speed of growth of the figures thus depends directly on the field strength at the rim. The original voltage distribution along the plate influendes the manner and rate of growth of the surface-charge, and thus, indirectly as well as directly, the field strength at the rim. Variations in the dielectric constant or thickness of the plate are significant because of their êffect on the original voltage distribution and on the superposed potential of the growing surface-charge.

Increasing the dielectric constant and reducing the thickness of the plate both cause a bunching of the lines of force toward the center, around the electrode. This increases the gradient, and its component along the plate, in the region directly around the electrode, with the result that it is diminished farther out. The curves of original voltage distribution along the plate for a (hypothetical) plate of dielectric constant 1 and for one of a higher constant may be plotted, as shown in Figure 13. The curve for a dielectric constant 1 gives the

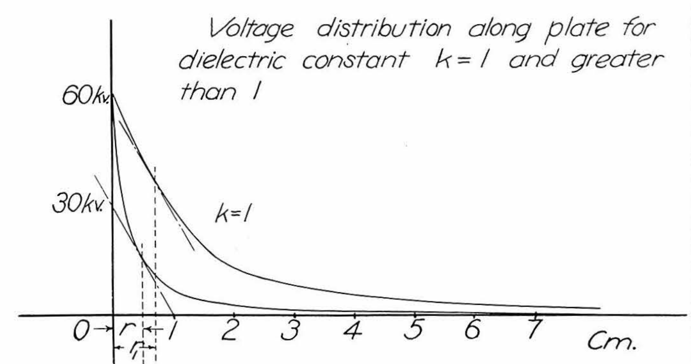

FIG. 13

Figure 13. Effect of an Increase in Dielectric Constant of Plate on Voltage Distribution.

actual voltage distribution for that case, having been drawn from a picture of the electrostatic field under that condition. Curves similar to those discussed can be drawn to show the influende of change in thickness.

The greater velocities and greater lengths of figures obtained on plates with higher values of capacitance per unit area between the surfaces can be attributed to the more intense ionization in the region around the electrode directly at the start of the formation of the figure. A surface charge of higher density, which is able to propagate itself along the plate with greater rapidity, is at once set up. Assuming an applied voltage of 60 kv. in Figure 13, and a gradient necessary to produce ionization of 30 kv./cm., the initial ionization in the two cases will extend to the radii marked r_1 and r, respectively.[1] Thus, the figure for the plate of lesser unit capacitance will initially be the greater, due to the greater extent, though with lesser intensity, of the ionization about the electrode. This explains the values of ratio of radii less than 1 observed in part 1 for very small figures.[2] The growth of the figure is further expedited, in the case of a plate with greater unit capacitance, by the bunching of the lines emanating from the surface-charge in its progress across the plate, and the resulting increase of the gradient at its rim.

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1. This is based on the assumption that the figure does not have any appreciable growth until the voltage has reached its maximum, - an assumption not strictly true, but satisfactory for the purposes of discussion. See curve for growth of capacity in Figure 10, p. 31).

2. See curve sheets 1-4.

The greater response of the speed of growth of the negative figures to an increase in the unit capacitance of the plate is probably traceable to two sources. First, the negative surface-charge, due to the nature of its advance along the plate, remains more dense than the positive surface charge.[1] The increase in the unit capacitance of the plate therefore probably heightens the rate of ionization at the rim of the negative figure more than for the positive. Second, the frictional retardation of the negative figure caused by the vertical component of the field must be taken into account. An examination of a picture of the electrostatic field in the two cases reveals that at some distance from the electrode the gradient at the surface of plate resulting from the initial voltage distribution is practically vertical. The reduction in this gradient, caused by the bunching of the lines at the center upon an increase in the unit capacitance of the plate, permits the negative surface charge to move across the plate with less frictional resistance.

The decrease in the ratio of growth velocities on plates with different dielectric constants or different thicknesses with increase of applied voltages (noted on the curves, sheets 1-4) is explainable through the decrease in significance of the space charge, - everything considered for a given radius of figure. The growth of the space charge on the plate, and its furtherance of the formation of the figure, is a cumula-

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1. See Figure 2, p. 14 .

tive process. A given difference in velocity and amount of space charge, at any stage of the simultaneous development of figures on two plates in parallel, is magnified with further growth. An extension of the region in which ionization can take place without help from the space charge, by producing a higher initial gradient along the plate, cancels some of this magnification, and reduces the difference in the figures.

APPENDIX

THE OSCILLOGRAPH CIRCUIT

The real problem was, of course, that of designing a system of connections for the oscillograph, - one which would place the beam in motion along the x-axis within the extremely short time necessary for the surge to travel from the generator to the test plates over the transmission line which was about 120 meters long. The method of discharging the blocking and time plates over a spark gap, held nearly at the sparking voltage, and tripped by an additional voltage induced from the operation of the surge generator, proved unsatisfactory, due to the irregular behavior of small spark gaps at the low voltages (less than 5000 volt) allowable on the oscillograph. The fastest operation obtainable would have been one using the firing gap of the surge generator itself to close the blocking and time circuits, - a scheme impossible due to the high voltages, 30 to 85 kv., applied to this gap. The use of a capacitive voltage of the firing gap suggested itself, and, chiefly through the ingenuity of Dipl.-Ing.s von Borries and Freundlich of the high voltage department of the Berlin Institute, a satisfactory connection was worked out.

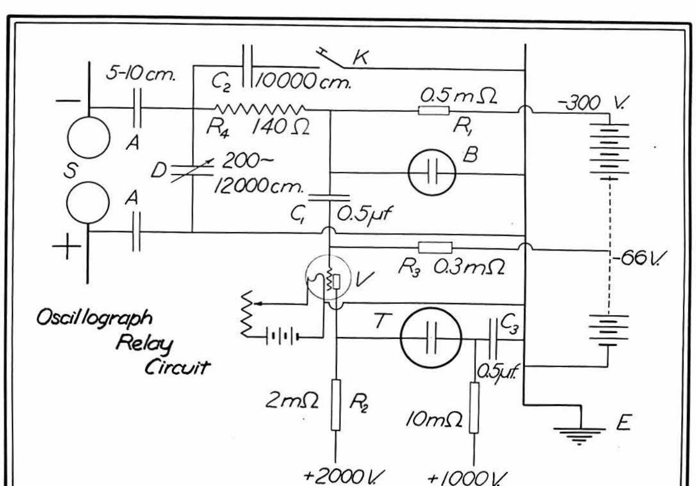


FIG. a

Figure a. shows the diagram of the tripping circuit, the explanation of the action of which follows.

The general principle followed was that of using a negative voltage of 300 volts on the blocking plates, B, and of reducing this suddenly to zero by the firing of the surge generator gap, S, the positive surge thus available through the collapse of the negative voltage being transmitted by means of condenser coupling (C_1) to the grid of a three element tube, V. This surge rendered the tube conducting, caused the voltage of one of the time plates, T, to drop swiftly from the previously maintained value of 2000 volts to practically ground potential. The beam of electrons, having been admitted to the main body of the oscillograph from the blocking chamber by the collapse of the -300 volt blocking voltage, became available for making the record, and was caused to sweep across the plate by the change of the 1000 volt difference in potential (between the +2000 volt and +1000 volt time plates) to a difference in potential of 1000 volt in the other direction (between the same +1000 volt plate and the +2000 volt plate which was now at zero potential).

The outstanding feature of the arrangement was the method used for causing the blocking voltage of -300 volts to collapse to zero. This took place simultaneously with and directly as a result of the collapse of the voltage across the firing gap, S, of the surge generator, through the use of the previously mentioned capacitive division of voltage.

The significant circuit is that formed by the gap, S, the two condensers, A - A, and the condenser D. The ratio of the capacitancies A and D (or, strictly, A and D+ the practically negligible capacitance of the blocking plates, B) was always so chosen that, with the sparking voltage on the gap S, the voltage on the condenser, D, and the parallel connected blocking plates, B, was 300 volts. Connection, through the resistance R_1 with the -300 volt terminal of a battery insured against the loss of blocking voltage through leakage of charge. It is clear that, with the proper ratio of capacitances, the distribution of charge between the condensers in the circuit was such that upon the release of their charges, through the sparking of the gap, they exactly neutralized each other, and the voltage of the blocking plates was brought to zero. For ease of adjustment, in providing for operation at various voltages, a variable, oil-filled condenser with a range of from 200 to 12,000 cm. was used for D. The blocking plates were connected in parallel with the condenser, D, through the low resistance R_4, which served to prevent any oscillations directly on the plates. By varying the value of the resistance R_1 in the -300 volt battery lead the short interval of time between the gap discharge and the reblocking of the beam by the charge coming from the battery could be regulated. The circuit including the condenser C_2 and the key K was added to permit hand operation of the blocking and timing circuits when desired, - as for instance when recording the calibrating wave.

The condenser C_1 which coupled the blocking and the timing circuits, as is to be seen from the diagram, was maintained at a positive voltage of 234 volts, reckoned from the blocking plates toward the grid of the vacuum tube, by virtue of having the blocking plate side connected to the -300 volt pole of the battery, and the grid side connected (over high resistance, R_3) to the -66 volt battery terminal. The sudden reduction to zero of the potential of the blocking plate side released a positive charge which at once brought the potential of the vacuum tube grid to +234 volts, rendering the tube conducting. After the desired lapse of time, regulated by the choice of the resistance R_3, the grid was brought again to a negative potential by the inflow of a negative charge from the battery. As soon as the high resistance of the tube was restored, the 2000 volt source, supplying charge to the "active" time plate over the resistance R_2, caused the return of the time plate to its original, "starting" voltage.

It was necessary to determine experimentally the most desirable absolute and relative values of the resistances R_1, R_2, and R_3. If, for instance, R_1 was chosen too large, the tube would regain its resistance and the beam would sweep back across the plate before being blocked, - thus writing an undesired, heavy, zero line. A too small value of R_1 caused the blocking of the electron beam to occur before the curve desired had been completely written. A too large value of R_2 caused such a slow return of the beam to its starting position that the stray electrons still entering the main body of the oscillograph (in spite of the blocking, which was in no case perfect) were able to cause an undesired darkening of the plate. The values of resistances which were found to give the best desired sequence of operation - i.e., admission of beam, writing of desired curve, blocking of beam, quick return of residual beam to starting position off the plate - , are given in the diagram.

FURTHER DEVELOPMENT OF OSCILLOGRAPHIC SET-UP

Before proceeding further, I will discuss at length the seriously unfavorable conditions under which this portion of the work was done, and the means of - at least partially - overcoming some of the difficulties encountered.

These difficulties arose, for the most part, in the form of external and undesired influences on the oscillograph circuit and on the electron beam itself, due to the necessary proximity of the instrument to the relatively high voltages and currents in the surge generator and on the test plates. The physical arrangement of the apparatus had, to a large degree, to be regarded as a fixed factor, and the experimental technique had to be modified to fit the conditions. I refer to the location of the surge generator, and the transmission line leading from it, and the thereby indicated placing of the test plates at the end of this line.

The laboratory was arranged with the surge generator in one corner, the transmission line running from this corner once around the room, and then along one side of the laboratory, returning some feet lower along the same side to the lead-down, which was located just in front of the surge generator. During the first part of the tests, the test plates were located at this lead-down, adjacent to the surge generator, - an arrangement which was not only natural and free from objection, but also necessary for one man operation of the set-up and recording of data. In applying the oscillograph to the second part of the work, it was placed immediately behind the table holding the test plates, between the latter and the surge generator, as shown in Figures b, g, and h.

This position of the oscillograph was the only one possible for several reasons. First, because of its inclusion of the surge generator, the test plate mounting, and the oscillograph together with its tables for auxiliary apparatus and the set-up to supply the exciting voltage for the oscillograph, the equipment involved required considerable floor space, and had to be compactly placed out of regard for the other equipment in the laboratory. Second, in order to maintain simplicity in the recording circuit of the oscillograph, and to avoid the risk of oscillations and induced disturbances arising from long leads from the test plates to the "voltage" plates of the oscillograph, it was desired to place the latter immediately adjacent to the end of the transmission line and the test plates (this adjacence to the test plates later proved undesirable and was eliminated, - not, however, by moving the oscillograph). Third, for easy application of the blocking and time circuits already described, and, above all, to keep the time required for bringing the time circuit into action, short, reasonable proximity to the surge generator firing gap was indicated. Lastly, the technique of operation of the set-up, already difficult enough, would have been impossible for one man with the oscillograph at some distance from the other apparatus.

The influence of the surge generator when too near the oscillograph in causing very serious disturbances to make their appearance on the records, is seen in Figure b, which shows a zero line (drawn with the voltage plates carefully connected together and grounded) and a "sine" wave from the oscillating circuit used for calibration. The irregularity of these disturbances shows they did not arise from any natural oscillations in the time circuit, but were all, rather, induced disturbances, caused simply by the proximity of the high voltages and currents in the surge generator. It will be noted that the disturbances are particularly severe in the time axis, being so great that the beam not only moved with varying velocity, but sometimes even reversed its motion (see particularly the <u>loops</u> in the calibrating wave). The vertical deviations in the oscillating circuit wave may be disregarded, or at once accounted for, in that this circuit, located near the surge generator and with no shielding whatsoever, served as an excellent antenna for disturbances. The small vertical deviations in the zero line are of much more interest, for they indicate an influence on the electron beam itself, even through the shielding body of the oscillograph (the "voltage", i.e., vertical axis, plates were bound to one another and directly grounded).

The steps taken to remedy this condition were two in number. Any further protection of the beam itself from magnetic influences was out of the question, as the construction of shielding of the kind needed would have involved too much time and expense. It remained to minimize the induced voltages in the circuit, and to annul their effect, so far as possible, once they were present. The 1000 volt lead to the "inactive" time plate, as originally attached, was of considerable length, and without intermediate high resistance. In order to block disturbances induced in this lead, the condenser C_3 (see Figure a) was connected directly at the plate, to act as a voltage reservoir, and the 1000 volt source was connected through a resistance of 10 megohms. This had a favorable influence, as may be seen by comparing the oscillogram shown in Figure c, taken with the changed connections, with the first oscillogram shown in Figure b. The disturbances were still unfortunately all too great for satisfactory records.

Finally, it was thought that oscillations imposed on the positive surge supplied by the condenser C_1 to the grid of the tube, arising from the induced oscillations in the blocking circuit, might be

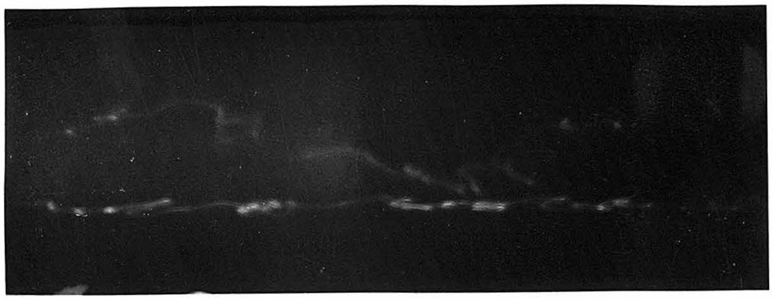


Figure b. A Timing Wave and Zero Line, Showing Induced Disturbances Resulting from Proximity of Oscillograph and Surge Generator.

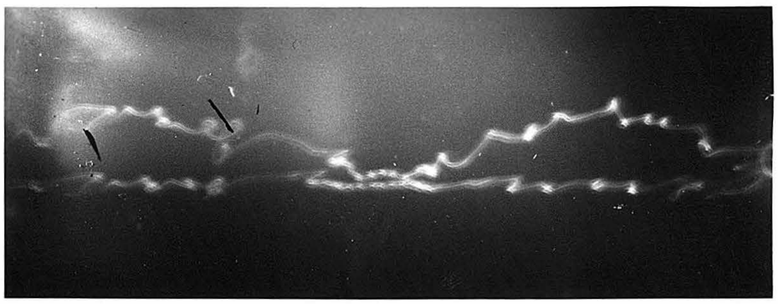


Figure c. Same as Figure b., Except Disturbances Slightly Reduced by Decreasing Length of Lead to One Time Plate.

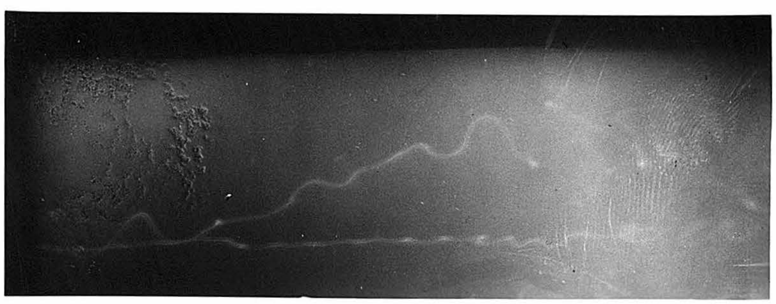


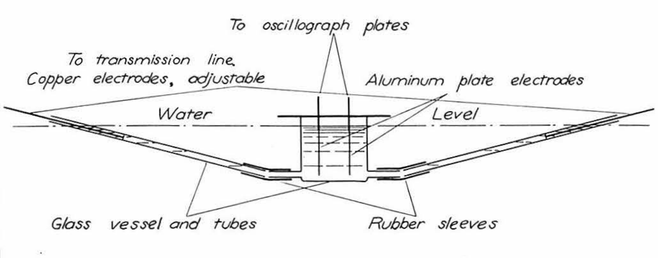
Figure d. Same as Figures b. and c., Showing Further Reduction in Disturbances Due to Increase in Magnitude of Positive Surge to Grid of Time-Circuit Vacuum Tube.

responsible for rapid variations in the conductivity of the vacuum tube, and resulting variations in the x-component of the beam velocity. To minimize the relative effect of such disturbances, the absolute magnitude of the positive surge was made as large as possible by reducing the original negative voltage on the grid to the lowest value at which the tube remained non-conducting. (This value was the -66 volts, given in the diagram of Figure a, - the voltage originally used was -150. An oscillogram taken with this new grid voltage and thought to show an improvement, is shown in Figure d. An exact comparison is difficult, because of the greater velocity with which this last oscillogram was written.

In spite of the unsatisfactory nature of the pictures obtainable, regarded from the point of view of an exercise in oscillography, they made possible determinations of the times of duration of the voltage surges with an accuracy greater than the probable per cent variation in the velocities of growth of successive figures (in view of the varying shape of successive figures, etc.). Respectable timing waves were obtained by the use of hand-tripping of the blocking and timing circuit.

In making the first exposures, it was noted that at the point of cut off of the voltage, the trace drawn by the beam, instead of returning at once to the zero line, entirely disappeared for an interval, to reappear in a wavy zero line farther on. It was suspected that this phenomenon was also due to magnetic influences on the beam caused by the sudden, large currents flowing in the needle-gap at its breakdown. To improve this condition, which, though undesirable, nonetheless permitted a determination of the time of duration of the voltage on the test plates, it was finally decided to make a new mounting of the test plates, farther away from the oscillograph, at some distance back along the transmission line. The "voltage" plates of the oscillograph, left connected (through a voltage divider) in exactly the same manner as before to the end of the line, received an undistorted record of the course of the voltage on the plates. This step proved helpful in obtaining a clearer record of the cutting off of the voltage wave.

A voltage of about 200 volts was desired on the recording plates of the oscillograph. A liquid potentiometer was used for this purpose. Its form is shown in Figure e. The electrolyte used was a copper sulfate solution.



Voltage divider for recording plates
Voltage ratio 150/1 to 450/1
Resistance between center plates ≅ 200Ω

FIG. e

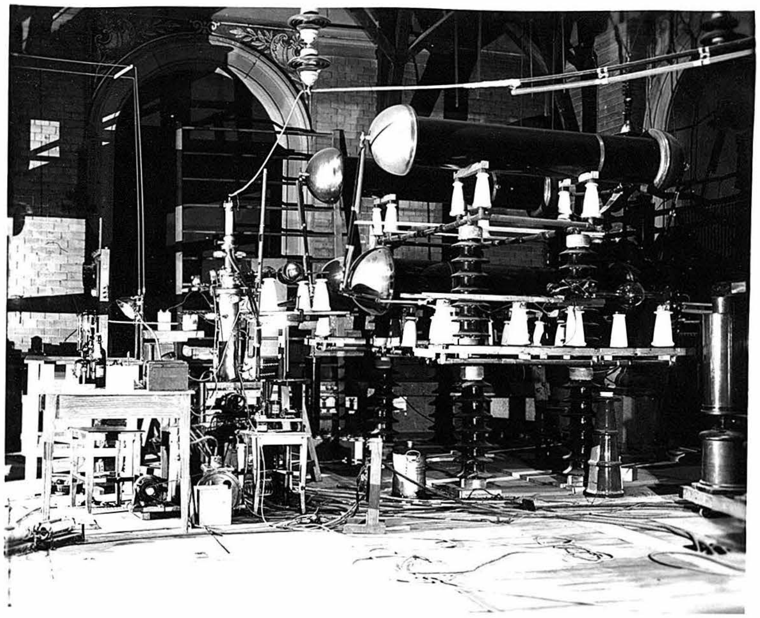


Figure f. View of Oscillograph and Surge Generator. Lead-down from Transmission Line at Left of Picture.

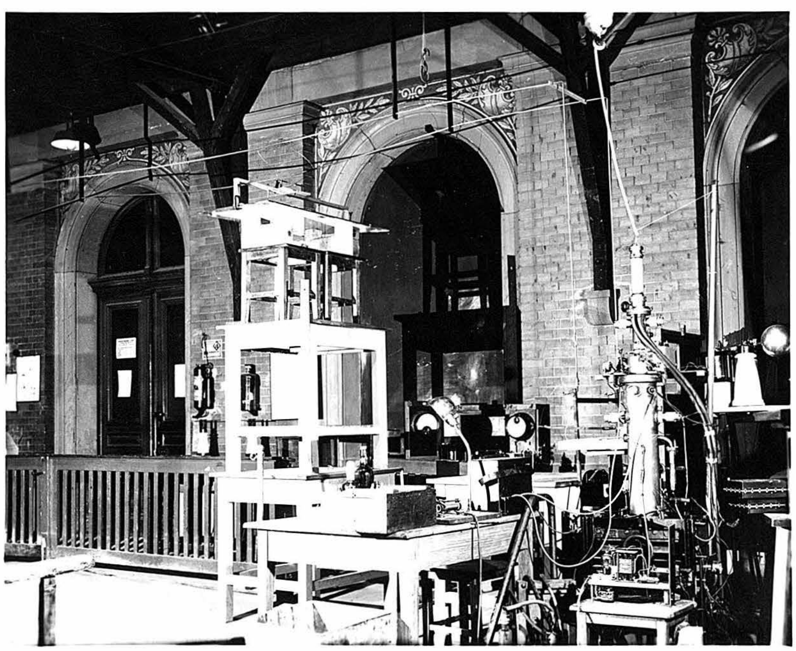


Figure g. View of Test Plate Mounting and Oscillograph.

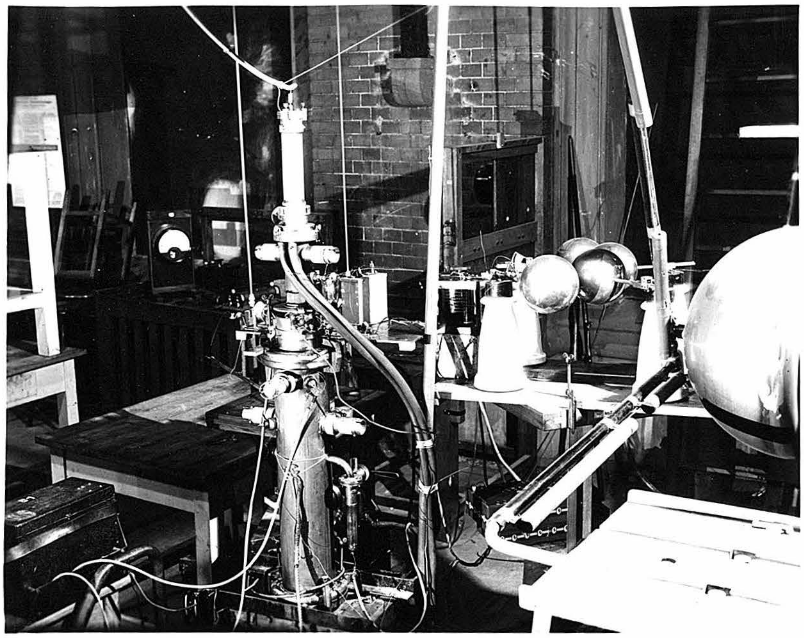


Figure h. Close-up View of Oscillograph, Showing Capacitive Potentiometer (Sphere in Picture) for Obtaining Blocking Plate Voltages.

Figures f, g, and h are views of the final arrangement of apparatus. In Figure f, the surge generator occupies the entire right-hand portion of the picture. Directly to its left the oscillograph standing adjacent to the lead-down from the indoor transmission line may be recognized. Figure g shows a view taken with the camera somewhat to the left. This figure shows the mounting of the test plates, directly under the transmission line. Figure h was taken from approximately the same angle as Figure g, but with the camera moved in very close to the oscillograph. The four spheres in the right center are to be particularly noted. Figure i shows their arrangement as seen from above.

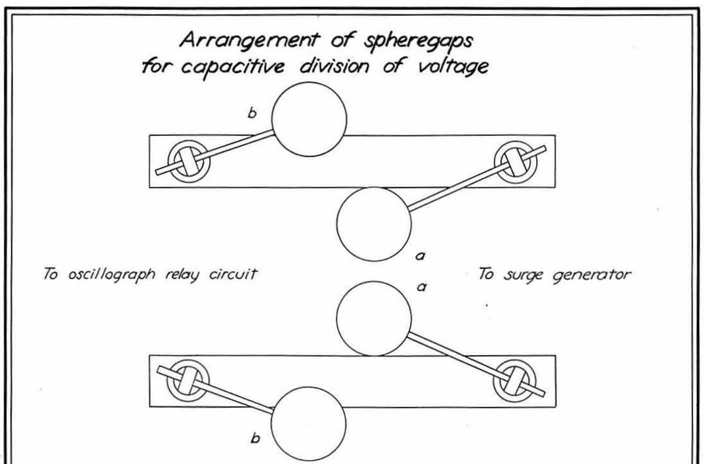


FIG. i

The two center spheres, a--a, served as the firing gap of the surge generator, while the two combinations a--b formed the capacitances A - A of the diagram in Figure a. The diameter of the spheres, 15 cm., may be used as a dimensional unit in Figure h, which indicates most clearly the close proximity of the oscillograph to the high voltages and currents of the surge generator.