C h a p t e r 8

STRONGLY NONLINEAR DYNAMIC REGIME: FREQUENCY BANDS OF STRONGLY NONLINEAR HOMOGENOUS GRANULAR SYSTSMS

Recent numerical studies on highly nonlinear, one-dimensional granular crystals composed of an infinite number of identical spherical beads in Hertzian contact showed the presence of frequency bands [Jayaprakash, et al., Nonlinear Dynamics, **63**: 359-385 (2011)]. These bands, denoted here as propagation and attenuation bands (PBs and ABs), are typically present in linear or weakly nonlinear periodic media; however their counterparts are not intuitive in essentially nonlinear periodic media where there is a complete lack of classical linear acoustics, i.e., in 'sonic vacua'. Here, we study the effects of PBs and ABs on the forced dynamics of ordered, uncompressed granular systems. Through numerical and experimental techniques, we find that the dynamics of these systems depend critically on the frequency and amplitude of the applied harmonic excitation. For fixed forcing amplitude, at lower frequencies, the oscillations are large in amplitude and governed by strongly nonlinear and non-smooth dynamics, indicating PB behavior. At higher frequencies the dynamics is weakly nonlinear and smooth, in the form of compressed low amplitude oscillations, indicating AB behavior. At the boundary between the PB and the AB large-amplitude oscillations due to resonance occur, giving rise to collisions between beads and chaotic dynamics; this renders the forced dynamics sensitive to initial and forcing conditions, and hence unpredictable. Finally, we study asymptotically the near field standing wave dynamics occurring for high frequencies, well inside the AB.

8.1 Introduction

This work numerically and experimentally examines the response of an uncompressed, harmonically driven two-bead system, similar to the one discussed in the theoretical model proposed by Jayaprakash et al.[132]. In contrast to the case of statically precompressed granular crystals, uncompressed granular crystals exhibit strongly nonlinear dynamic behavior. Their response is not linearizable and there is complete absence of classical linear acoustic response. Nesterenko characterized this essentially nonlinear medium as 'sonic vacuum'[35], since the linearized speed of sound (as defined in classical linear acoustics) is zero. Despite the fact that frequency bands are phenomena inherent to linear periodic systems, Jayaprakash et al.[132] demonstrated the existence of similar propagation and attenuation bands in essentially nonlinear uncompressed granular crystals. They predicted that a one-dimensional granular crystal of infinite extent exhibits either propagation or attenuation behavior dependent on both frequency (as is the case in coupled linear periodic oscillators) and amplitude (due to the nonlinearity of the system).

The propagation band (PB) of the system is realized at lower frequencies. It is characterized by strongly nonlinear and non-smooth dynamics, a result of bead separations and collisions. This gives rise to a time-periodic train of travelling pulses, similar to solitary waves analytically predicted and experimentally demonstrated by Nesterenko[35]. At higher frequencies, the attenuation band (AB) is characterized by a region where spatially periodic solutions cannot exist. In this regime, the system exhibits low-amplitude localized oscillations bounded by decaying spatial envelopes, similar to evanescent waves predicted in band gaps of linear media. In this high frequency regime the chain is dynamically compressed and weakly nonlinear dynamics govern the dynamical response. Jayaprakash et al.[132] predicted that these PB and AB exist as well in forced granular media of arbitrary length. We set out to demonstrate this behavior in a harmonically forced system of 2 beads.

8.2 Experimental results

We test the experimental setup shown in Fig. 3.2. We excite the first bead harmonically with amplitudes of approximately 0.4 μm (reproduced in the numerical simulations). Small deviations (±0.05 μm) from this excitation value occur due to inherent nonlinear behavior of the actuator. We measure the force exerted on the dynamic sensor and show the existence of a high-amplitude strongly nonlinear state at low frequencies (the PB), and a low-amplitude weakly nonlinear state at high frequencies (the AB).

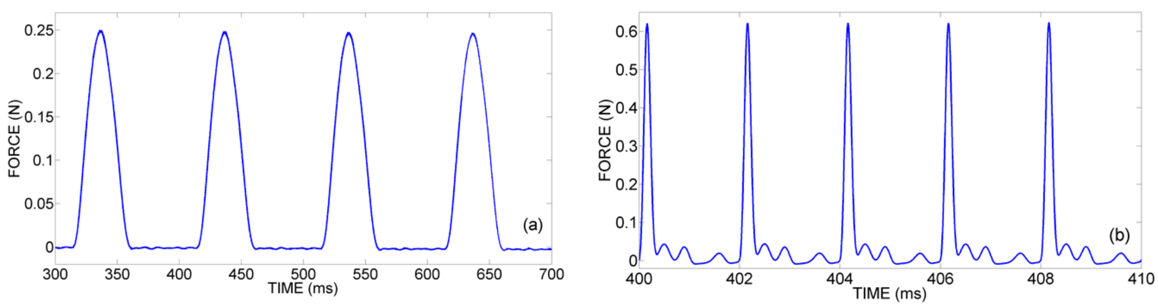

Figure 8.1: Experimental time series of the force at the dynamic sensor at (a) f = 10 Hz and A = 3.951x10^{-7} m and (b) f = 500 Hz and A = 3.775x10^{-7} m.

In Fig. 8.1a and 8.1b, we show the experimental time series of the force measured at the sensor at driving frequencies of 10 Hz and 500 Hz, respectively. These are qualitatively similar to the simulations at the same frequencies. We note a series of transmitted compressive force pulses, similar to the dynamics observed in simulation. At 10 Hz the width of this pulse is approximately half the period of the drive, however at 500 Hz this pulse width decreases below half the drive period. This is in agreement with the numerical results predicting decreasing pulse width for higher frequencies. The maximum transmitted force is higher at 500 Hz, also in qualitative agreement with the numerical simulations. However, in the numerical simulations we observed resonance phenomena where the maximum force recorded was much higher at 1000 Hz. The experimental observation of resonances is not included in this manuscript due to difficulty in experimental repeatability.

A number of experimental uncertainties, such as misalignment, surface roughness, and bead rotations, become important and difficult to avoid as the displacement amplitudes increases at resonance. In addition, we performed an extensive series of numerical simulations of harmonically forced ordered granular systems close to the boundary between their PB and AB where nonlinear resonances are excited and the granular media execute large-amplitude oscillations. In these regions, there occur strong collisions between beads, which are well known to give rise to chaotic dynamic[134]. Due to the existence of such chaotic (non-smooth) motions the forced dynamics of the forced granular systems exhibit sensitive dependence to initial and forcing conditions and become, in essence, unpredictable. This was verified in the experiments where in the resonance region (i.e., for frequencies close to the boundary between the PB and the AB) different experimental runs that were performed under identical forcing and initial conditions yielded completely different results. Hence, it appears that close to the boundary between the PB and AB the inherent chaotic dynamics of the harmonically forced system prevent the accurate measurement of the dynamic response and the resulting chaotic dynamics becomes unpredictable. We therefore omit these experimental results from further discussion herein until current research by the authors provides a better understanding and characterization of the dynamics in this regime.

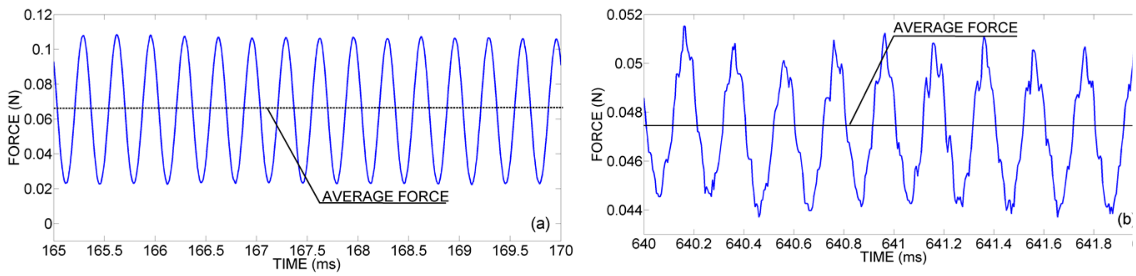

Figure 8.2: Experimental time series of the force at the dynamic sensor at (a) f = 3000 Hz and A = 3.408x10^{-7} m and (b) f = 5000 Hz and A = 3. 75x10^{-7} m.

By increasing the frequency of the excitation the dynamics become again regular and fully predictable (and reproducible). Indeed, a weakly nonlinear regime is found at higher frequencies (3000 Hz – Fig. 8.2a and 5000 Hz – Fig. 8.2b), with the nonzero mean force indicating a state of sustained compression. The small-amplitude oscillations of the measured force about this mean value indicates weakly nonlinear interactions in the dynamics. Moreover, increasing the frequency decreases the transmitted force amplitude for these oscillations. A comparison of the Fourier spectrum, calculated using a discrete Fast Fourier Transform, of the dynamics in the PB (Fig. 8.3a) with those in the AB (Fig. 8.3b) underlines that fewer harmonics are excited in the weakly nonlinear phase, as is in agreement with the simulations. It should be noted that, although the experimental results do not match the numerical results quantitatively, we have good qualitative agreement between the two responses. The mismatch can be attributed to the dry friction, material damping, and other uncertainties present in the experimental setup.

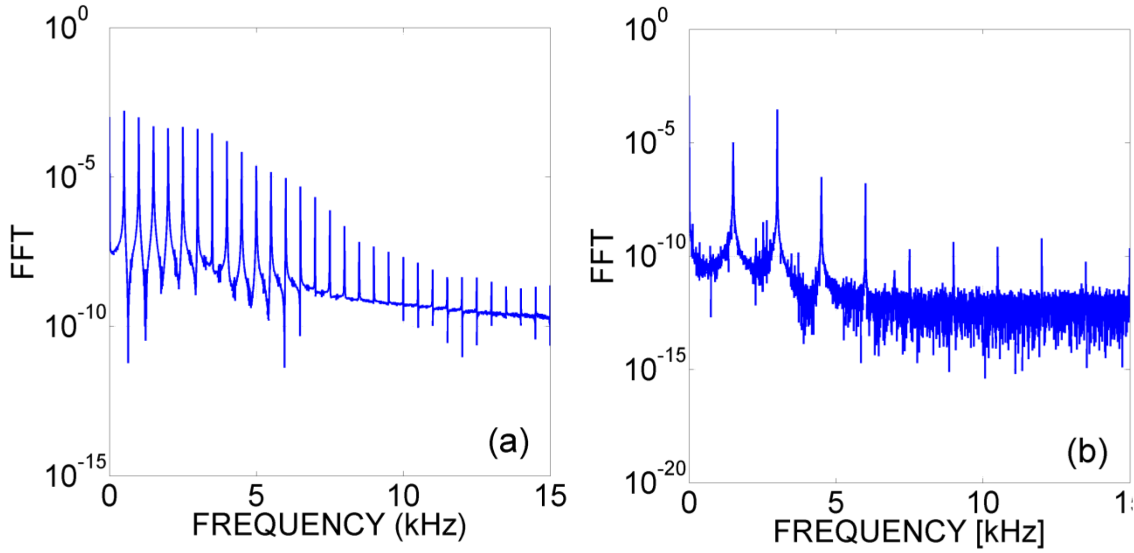

Figure 8.3: Experimental Power Spectral Densities of the force time series of (a) Fig. 8.1b: f = 500 Hz, and (b) Fig. 8.2a: f = 3000 Hz.

It should be emphasized that the reported experimental results are only for a system of two interacting beads. These experimental results show qualitative agreement with the numerical results for two beads. We believe this validates the modeling and suggests that the numerical results for longer chains hold true. At small initial compressions the experiment demonstrates similar dynamics to that predicted by our analysis for longer chains. We analytically examine the differences in dynamics for chains of arbitrary length below.

8.3 Conclusion

This paper explores in detail the presence of frequency bands in harmonically forced essentially nonlinear granular crystals. For fixed amplitude of excitation, the low-frequency dynamics is found to be strongly nonlinear, involving bead separations and collisions, and resulting in periodic trains of travelling solitary pulses. This represents the dynamics in a propagation band (PB) of the system. As we increase the drive frequency, the system enters into a state of permanent compression which results in weakly nonlinear and smooth dynamics. In this regime the response is localized close to the actuator's excitation and rapidly decays away from it. Hence, in contrast to the propagatory dynamics realized in the PB, the higher frequency dynamics is in the form of spatially decaying (and, hence, spatially localized) standing wave oscillations. This represents the attenuation band (AB) of the system. Between these two regimes, nonlinear resonance phenomena occur, where the dynamics become chaotic due to strong collisions between beads, and the dynamics exhibit sensitive dependence on initial and forcing conditions and, hence, become unpredictable. This regime was not considered in this work and is the focus of current research by the authors.

Finally, when the dynamics is realized in the AB, we employed an asymptotic technique based on static/dynamic partitions of the bead responses, and analytically deduced that the sustained state of compression realized in the granular crystal becomes independent of the excitation frequency. However, an increase in the size of the granular crystal does increase the permanent compression, which reduces the amount of energy transferred to the crystal. These results can contribute to designing of granular-based acoustic metamaterials as acoustic filters and attenuators of externally applied periodic or transient disturbances.

8.4 Author contributions

The results from this chapter are from "Frequency Bands of Strongly Nonlinear Homogeneous Granular Systems". Joseph Lydon performed the experiments. The analytics and numeric are done by K.R. Jayaprakash (see Appendix). Joseph Lydon, K.R. Jayaprakash, and Chiara Daraio all contributed to the writing of the manuscript.