Appendix A

Time-Domain Phase Analysis Using I/Q Demodulation

In this appendix we describe the in-phase and quadrature (I/Q) demodulation technique which is used for time-domain analysis of the locked-state OPLL phase error in chapter 6.

The goal of the technique is to separate the amplitude modulation $A(t)$ from the phase modulation $\theta(t)$ of a sinusoidal signal $y(t)$ with a known frequency $\omega_0$,

$$y(t) = A(t) \sin [\omega_0 t + \theta(t)]. \quad (A.1)$$

We form the in-phase signal $y_i(t)$ and the quadrature signal $y_q(t)$ by multiplying $y(t)$ with sine and cosine waveforms at a frequency of $\omega_0$, and low-pass filtering the results.

$$y_i(t) = h(t) \ast [y(t) \sin \omega_0 t] = h(t) \ast \left\{ \frac{A(t)}{2} \cos \theta(t) - \frac{A(t)}{2} \cos [2\omega_0 t + \theta(t)] \right\}, \quad (A.2)$$

$$y_q(t) = h(t) \ast [y(t) \cos \omega_0 t] = h(t) \ast \left\{ \frac{A(t)}{2} \sin \theta(t) + \frac{A(t)}{2} \sin [2\omega_0 t + \theta(t)] \right\}, \quad (A.3)$$

where $h(t)$ is the impulse response of the low-pass filter, and ‘$\ast$’ denotes the convolution operation. The filter is designed to average out the sum frequency terms at
frequency $2\omega_0$, while retaining the difference frequency terms at DC, yielding

$$y_i(t) = \frac{A(t)}{2} \cos \theta(t), \quad \text{and}$$  \hspace{1cm} (A.4)

$$y_q(t) = \frac{A(t)}{2} \sin \theta(t).$$  \hspace{1cm} (A.5)

The amplitude and phase modulations are recovered using

$$A(t) = 2 \sqrt{y_i^2(t) + y_q^2(t)}, \quad \text{and}$$  \hspace{1cm} (A.6)

$$\theta(t) = \text{atan2} [y_q(t), y_i(t)],$$  \hspace{1cm} (A.7)

where $\text{atan2}(y_q, y_i)$ is the four-quadrant inverse tangent function defined below.

$$\text{atan2}(y_q, y_i) \equiv \begin{cases} 
\tan^{-1} \left( \frac{y_q}{y_i} \right) & y_i > 0 \\
\tan^{-1} \left( \frac{y_q}{y_i} \right) + \pi & y_q \geq 0, \ y_i < 0 \\
\tan^{-1} \left( \frac{y_q}{y_i} \right) - \pi & y_q < 0, \ y_i < 0 \\
\frac{\pi}{2} & y_q > 0, \ y_i = 0 \\
-\frac{\pi}{2} & y_q < 0, \ y_i = 0 \\
\text{undefined} & y_q = 0, \ y_i = 0
\end{cases}$$  \hspace{1cm} (A.8)