Appendix G Constraint on earthquake location implied by the observed time interval between the first two P wave arrivals

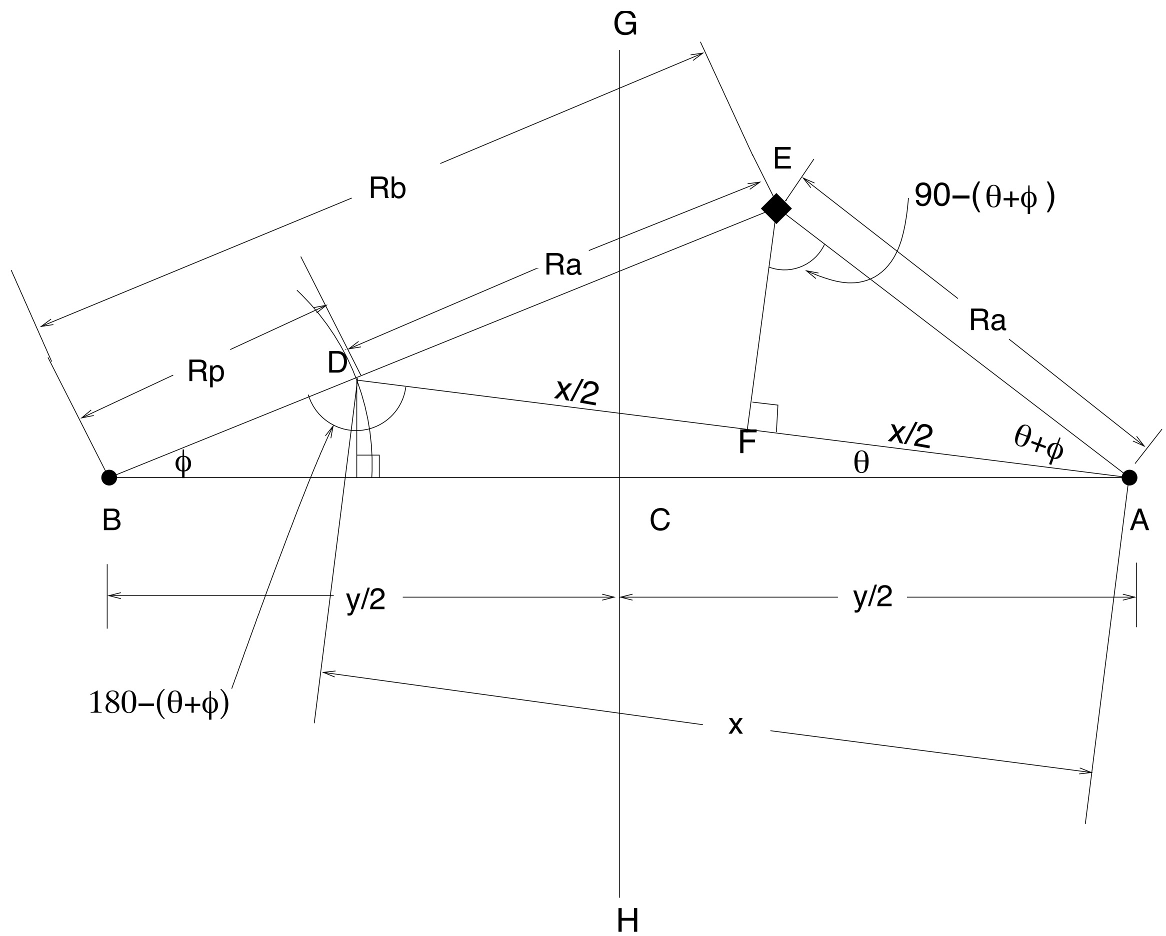

Figure G.1: Examining the constraint on earthquake location implied by the observed time interval between the first two P wave arrivals.

Let A and B be two adjacent stations. Let GH denote their shared Voronoi edge.

Let y be the distance between the two stations. Suppose the first P wave arrival from the event is detected at station A. Since the Voronoi cell of A consists of all locations closer to station A than any other station, the earthquake must have occurred in station A's Voronoi cell. Suppose that after Δt_p seconds, a subsequent P arrival is detected at the adjacent station B. We examine the constraint on possible locations for the event given this scenario (first arrival at station A, followed by a subsequent arrival at station B Δt_p seconds later).

Assuming an average P wave velocity of α (typically 6 km/s), a time interval between arrivals of Δt_p means that the P-wave traveled $R_p = \alpha \cdot \Delta t_p$ km further to reach station B than the distance to station A. If R_a and R_b are the epicentral distances to stations A and B, then

$$R_b - R_a = R_p = \alpha \Delta t_P \tag{G.1}$$

Note that R_p (and hence the observed interval between arrivals, Δt_P) does not uniquely determine the epicentral distances R_a, R_b. In general, given an observed Δt_P, the epicentral distances R_a, R_bvary as a function of either of the angles θ or ϕ.

The simplest case, of course, is if $\phi = 0$, or when the earthquake occurs along the line CA. In this case, R_a, R_b are related by

$$\begin{aligned} R_a + R_b &= y && \text{(G.2)} \\ R_a + (R_a + R_p) &= y && \text{(G.3)} \\ \text{where } R_p &= \alpha \cdot \Delta t_P && \text{(G.4)} \\ \text{so } R_a &= \frac{y - \alpha \cdot \Delta t_P}{2} && \text{(G.5)} \\ \text{and } R_b &= y - R_a = \frac{y + \alpha \cdot \Delta t_P}{2} && \text{(G.6)} \end{aligned}$$

Figure G.1 depicts the more general case of $\phi \neq 0$. Let us consider $\triangle ABD$ for some non-zero ϕ. The quantities ϕ, Δt_P (and hence, R_p), and y (the distance between the two stations) are known. We wish to solve for R_a, R_b, the epicentral distances to stations A and B. (Thus far, x and θ are also unknowns.)

First, $\angle BCA = 180° - (\phi + \theta)$. Using the law of sines,

$$\frac{R_p}{\sin\theta} = \frac{x}{\sin\phi} = \frac{y}{\sin(180° - (\phi+\theta))} = -\frac{y}{\sin(\phi+\theta)} \tag{G.7}$$

Also,

$$\begin{aligned} R_p \cos\phi &+ x\cos\theta = y \\ \Rightarrow x\cos\theta &= y - R_p \sin\phi \\ \text{but from Eqn. G.7} & \\ x\sin\theta &= R_p \sin\phi \end{aligned}$$

Therefore,

$$\begin{aligned} x^2\sin^2\theta + x^2\cos^2\theta &= (y - R_p\cos\phi)^2 + R_p^2\sin^2\phi = x^2 && \text{(G.8)} \\ &= y^2 - 2yR_p\cos\phi + R_p^2\cos^2\phi + R_p^2\sin^2\phi && \text{(G.9)} \\ &= y^2 - 2yR_p\cos\phi + R_p^2 && \text{(G.10)} \\ \Rightarrow \quad x &= \sqrt{y^2 - 2yR_p\cos\phi + R_p^2} && \text{(G.11)} \end{aligned}$$

Since $\sin\theta = \frac{R_p\sin\phi}{x}$ (from Eqn. G.7), then

$$\Rightarrow \theta = \sin^{-1}\left(\frac{R_p\sin\phi}{x}\right) = \sin^{-1}\left(\frac{R_p\sin\phi}{\sqrt{y^2 - 2yR_p\cos\phi + R_p^2}}\right) \tag{G.12}$$

Let us now consider $\angle AED$ in Figure G.1. Let the line EF be the perpendicular bisector of the line AD. It is easy to see that

$$\begin{aligned} R_a &= \frac{x}{2}\cos(\theta+\phi) \\ &= \frac{1}{2}\left(\sqrt{y^2 - 2yR_p\cos\phi + R_p^2}\right)\cos(\theta+\phi) && \text{(G.13)} \\ \text{and} \quad & R_b = R_a + R_p \end{aligned}$$

Recall that $R_p = \alpha \cdot \Delta t_p$, where α is the average P wave velocity, and Δt_p is the observed interval between P wave arrivals at stations A and B. Eqn. G.13 gives the constraints on the epicentral distances R_a, R_b given the observed interval between P wave arrivals at stations A and B.

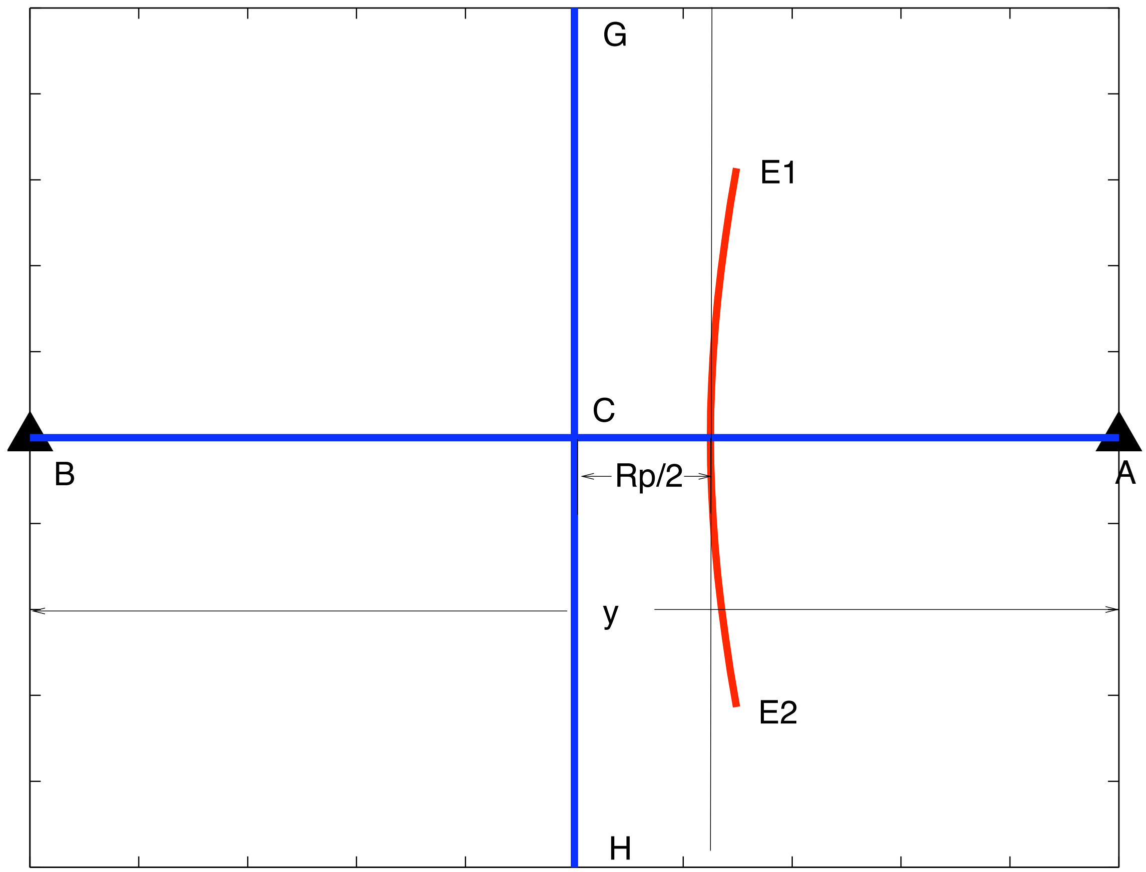

Figure G.2: Given the observed interval between P wave arrivals Δt_p at the two stations closest to an earthquake, the relationship between the possible epicentral distances, R_a, R_b is given by Eqn. G.13. The earthquake is constrained to be along the curve $E1 - E2$. The epicentral distance to station A, R_a is approximately $(y - R_p)/2$, where y is the distance between stations A and B. The error associated with this approximation increases with ϕ or θ.

Therefore, the time interval between the first and second P wave arrivals is an additional constraint on earthquake location. At the time of the first P wave arrival, the earthquake location is constrained to be within the Voronoi cell of the first triggered station. If t_1 is the time of the first arrival, some $t_1 + \delta t$ seconds later (before the P arrival at the station B), the region of possible locations is the area about station A obtained by translating all the Voronoi edges of station A a distance $(\alpha\delta t)/2$ towards station A. (In relation to Figure G.2, this would be the wedge-shaped region $E1 - E2 - A$.) Thus, the time interval δt after the initial P wave arrives in which nothing happens (no other arrivals, so $\delta t < \Delta t_p$) also provides information. In particular, for some $\delta t < \Delta t_p$, the constraint is

$$R_a < max((y_i - \alpha\delta t)/2) \tag{G.14}$$

where the index i corresponds to stations sharing a Voronoi edge with station A, and y_i is the distance between these stations and station A.

Once the second arrival is available, the location is constrained to the curve $E1 - E2$ in Figure G.2. Thus, if there is a reliable way to determine the angle from which the energy from the source arrived at station A (for example, from particle motion plots), a location can be uniquely determined once the P wave has arrived at the second station.

This is formulation is consistent with the method used by Rydelek and Pujol (2004) to constraint earthquake epicenters with a two-station subarray.